1.Determine if the following signals are periodical, and determine the fundamental period.

(1) $x(n) = \cos(0.125n\pi) + \sin(0.05n\pi)$

(2) $x(n) = \cos(0.125n\pi) \cdot \sin(0.05n\pi)$

Remark:

For the discrete-time signal, its fundamental period is very different from the analog one. How to calculate the fundamental period? A simple equation is available with the relationship of phase between the discrete-time signal and the analog signal.

$N\omega = 2\pi k$ Where ω is the digital angle frequency, N is the number of the sample in one period, r is the corresponding number of the period.

So, the above equation means that N digital periods has the same phase shift as r analog periods.

Solution:

(1) For $\cos(0.125n\pi)$, its $\omega=0.125\pi$, then $N / r = 16$

For $\sin(0.05n\pi)$, its $\omega=0.05\pi$, then $N / r = 40$

So the fundamental period is lease common multiple [16,40] = 80.

(2) With $\cos A \sin B = 1/2[\sin(A+B) - \sin(A-B)]$

$x(n) = \cos(0.125n\pi) \cdot \sin(0.05n\pi) = 1/2[\sin(0.175n\pi)-\sin(0.075n\pi)]$

For $\sin(0.175n\pi)$, its $\omega=0.175\pi$, then $N / r = 80 / 7$

For $\sin(0.075n\pi)$, its $\omega=0.075\pi$, then $N / r = 80 / 3$

So the fundamental period is 80

Attention:

1，N / r 的物理含义，就是在数字域 N 个采样点对应着模拟域 r 个周期，所以只需要取数字域采样点的最小公倍数即可，无需一定具有相同的模拟周期的约束。

2 Determine the sequence x(n) from the following sampling process and calculate its fundament period:
$x_a(t) = 2\sin(20\pi t) - 4\cos(24\pi t) - 5\sin(120\pi t) - \cos(176\pi t)$, fs = 50Hz

Solution:

For $x_a(nTs) = x_a(t)|_{t=nTs}$
$= 2\sin(20\pi n/50) - 4\cos(24\pi n/50) - 5\sin(120\pi n/50) - \cos(176\pi n/50)$
$= 2\sin(0.4\pi n) - 4\cos(0.48\pi n) - 5\sin(2.4\pi n) - \cos(3.52\pi n)$
$= 2\sin(0.4\pi n) - 4\cos(0.48\pi n) - 5\sin(0.4\pi n) - \cos(0.48\pi n)$
$= -3\sin(0.4\pi n) - 5\cos(0.48\pi n)$

For $-3\sin(0.4\pi n)$, its $\omega = 0.4\pi$, then N / k = 5
For $-5\cos(0.48\pi n)$, its $\omega = 0.48\pi$, then N / k = 25 / 6
So the fundamental period is lease common multiple [5,25] = 25.

Attention:
(1) Many students don't know $\sin(0.4\pi n) = \sin(2.4\pi n)$.
(2) Some students write the unit for the fundamental period, such as s (second).

3. Consider the discrete-time signal $x(n) = \cos(n\pi/8)$, and try to find two different continuous-time signals which generate x(n) when sampled at 10Hz rate.

Solution:

For $\cos(nTs) = \cos(2\pi ft)|_{t=nTs}$

$\cos(n) = \cos(2\pi nf/f_s) = \cos(\pi nf/5)$ (2-1)
And $\cos(n\pi/8) = \cos(n\pi/8 + 2\pi kn)$, k is any integer. (2-2)

Let (2-1) is equal to (2-2), we get
$\pi nf/5 = n\pi/8 + 2\pi kn$
$f = 5/8 + 10k$, k is any integer

So the analog signal is $\cos[2\pi t(5/8 + 10k)]$, k is any integer
For k=0, it's the critical sample rate, the analog one is $\cos(5\pi t/4)$
And we can get the others for the different k.

Attention:
Many students don't know the relationship between the analog siganl and disrete signal, especially for the sin family functions.

4.Given discrete sequence $x(n)=2\cos(2\pi n/3)$, its

period $N=$ ______3______ ,assume the sample rate is 3000Hz， so the analogue

signal is_____ $x_a(t)=2\cos(2000\pi t)$ __________.

5.One analogue signal is $x(t)=5\sin 400\pi t+8\cos 1000\pi t$ .Its Nyquist sampling rate

is _1000_Hz.

6.For each of the following discrete-time systems, where y(n) and x(n) are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) time shift-invariant, (3) causal and (4) stable.

a) $y(n)=\sum_{m=-\infty}^{n} x(m)$.

Solution:

$$\sum_{m=-\infty}^{n}\left[Ax_1(m)+Bx_2(m)\right]=A\sum_{m=-\infty}^{n}x_1(m)+B\sum_{m=-\infty}^{n}x_2(m)$$

So the system is **linear**.

If $x(n)=x(n-n_0)$

Then $y(n)=\sum_{m=-\infty}^{n} x(m)=\sum_{m=-\infty}^{n-n_0} x(m)=y(n-n_0)$

So it is **time-invariant**.

Since $h(n)=\sum_{m=-\infty}^{n}\delta(m)=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}=u(n)$

Thus, if $n<0$, $h(n)=0$

So it is **causal**.

Since $\sum_{m=-\infty}^{\infty}|h(n)|=\sum_{m=-\infty}^{\infty}u(n)=\sum_{n=0}^{\infty}1=\infty$

So it is **not stable**.

b) $y(n)=2x^2(n)+3$.

Solution:

$$2[Ax_1(n)+Bx_2(n)]^2+3 \neq A[2x_1^2(n)+3]+B[2x_2^2(n)+3]$$

So it is **not linear**.

If $x(n)=x(n-n_0)$

Then $y(n)=2x^2(n)+3=2x^2(n-n_0)+3=y(n-n_0)$

So it is **time-invariant**.

Since there is no output before the input hence the system is **causal**.

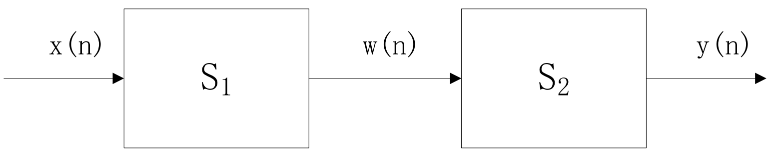
If $x(n)$ is a bounded input, $y(n)$ is a bounded output.

So it is **stable**.

7.Determine if the system $\boldsymbol{y(n)=Ax(n)}$ is a linear system. Linear(OR Yes)

8.Let the Nyquist frequency of $X_a(t)$ is Ω_S.The Nyquist frequency of $X_a(t)\cos(\Omega_0 t)$ is $\Omega_N=\Omega_S+2\Omega_0$. The Nyquist frequency of $X_a(2t)$ is $\Omega_N=2\Omega_S$.

9.Consider the two systems are connetected in cascade.



(a) If S_1 and S_2 are linear, time-invariant, stable and causal, then the system is linear, time-invariant or not?

(b) If S_1 is linear and S_2 is nonlinear, then the whole system is linear or nonlinear?

(a)For linear:

$$y_1(n) = T[x_1(n)] = S_2[S_1[x_1(n)]] = S_2[w_1(n)]$$

$$y_2(n) = T[x_2(n)] = S_2[S_1[x_2(n)]] = S_2[w_2(n)]$$

$$\begin{aligned} T[ax_1(n)+bx_2(n)] &= S_2[S_1[ax_1(n)+bx_2(n)]] \\ &= S_2[aS_1[x_1(n)]+bS_1[x_2(n)]] \\ &= S_2[aw_1(n)+bw_2(n)] \\ &= aS_2[w_1(n)]+bS_2[w_2(n)] \\ &= ay_1(n)+by_2(n) \end{aligned}$$

So linear

For time-invariant:

$$T[x(n-m)] = S_2[S_1[x(n-m)]] = S_2[w(n-m)] = y(n-m)$$

So time-invariant.

(b) for linear:

$$y_1(n) = T[x_1(n)] = S_2[S_1[x_1(n)]] = S_2[w_1(n)]$$

$$y_2(n) = T[x_2(n)] = S_2[S_1[x_2(n)]] = S_2[w_2(n)]$$

$$ay_1(n)+by_2(n) = aS_2[w_1(n)]+bS_2[w_2(n)]$$

$$\begin{aligned} T[ax_1(n)+bx_2(n)] &= S_2[S_1[ax_1(n)+bx_2(n)]] \\ &= S_2[aw_1(n)+bw_2(n)] \\ &\neq aS_2[w_1(n)]+bS_2[w_2(n)] \end{aligned}$$

So

non-linear .

10.Assuming using the same sampling frequency for analogue signals: $x_1(t) = 2\sin(200\pi t)$ and $x_2(t) = \sin(400\pi t)$. According the Shannon Sampling Theorem, the lowest sampling frequency is ___400Hz ___. After the sampling process, the discrete-time signals $x_1(n) =$ ___2sin(0.5πn)__, $x_2(n) =$ ___sin(πn)__.

The analogue angular frequencies for each signal are $\Omega_1 =$ _200πrad/s _,

$\Omega_2 =$ __400πrad/s _. And the digital angular frequencies for each signal are

$\omega_1 =$ __0.5π__, $\omega_2 =$ ___π____.

11.If the impulse response of a LTI system is $h(n)$, then the casual condition of this system is _$h[n]=0, n<0$_ and the stable condition is __$\sum_{n=-\infty}^{\infty}|h[n]|<\infty$_.

12.One analogy signal $x_a(t)=a\cdot\sin(100\pi t)$, a is nonzero constant, the sample frequency is $f_s=200\text{Hz}$, then the discrete sequence is $x(n)=x_a(nT_s)$, its period $N=$____4____ .

13.For each of the following discrete-time systems, where y(n) and x(n) are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) time shift-invariant, (3) causal and (4) stable.

i) $y(n)=nx(n)$

Solution:

nonlinear[2 marks], not time shift-invariant[2 marks], causal[2 marks], not stable[2 marks].

ii) $y(n)=\sum_{k=-\infty}^{\infty}x(k)x(n+k)$

Solution:

nonlinear[2 marks], not time shift-invariant[2 marks], not causal[2 marks], not stable[2 marks].

14.One analogue signal is

$x(t)=4\sin(20\pi t)-5\cos(24\pi t)+3\sin(120\pi t)+2\cos(176\pi t)$, If the sampling rate is 50Hz, then the digital signal could be expressed as:

$x[n]=4\sin\left(\frac{2\pi n}{5}\right)-5\cos\left(\frac{12\pi n}{25}\right)+3\sin\left(\frac{2\pi n}{5}\right)+2\cos\left(\frac{12\pi n}{25}\right)$. [2 marks]

15.onsidering the continuous-time signal ga(t) = sin(Ω_m t), it must be sampled at a rate __$\Omega_t \geq 2\Omega_m$___ to recover it fully from its samples.. [2 marks]

16.one signal $x(n) = \cos(0.125n\pi)$, its fundamental period N=___8___ . One signal $x(n) = \cos(0.125n\pi) + \sin(0.05n\pi)$, its fundamental period N= ___80___ . [4 marks]

17.For each of the following discrete-time systems, where y(n) and x(n) are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) time shift-invariant, (3) causal and (4) stable.

iii) $y(n) = 2x(n) + 3$

Solution:

$2(A x_1[n] + B x_2[n]) + 3 \neq A(2x_1(n)+3) + B(2x_2(n)+3)$

so it is not linear.

if $x[n]= x[n - n_0]$

then $y(n) = 2x(n)+3 = 2x[n - n_0]+3 = y[n - n_0]$

so it is time-invariant.

Since there is no output before the input hence the system is **causal**.

if x[n] is a bounded input

y[n] is a bounded output. So it is **stable**.

iv) $y(n) = x^2(n)$

Solution:

$(A x_1[n] + B x_2[n])^2 \neq A\, Ax_1^2(n) + Bx_2^2(n)$

So the system is **not linear**. [2 marks]

If $x_1[n]= x[n - n_0]$ then $y[n] = x^4[n] = x[n - n_0] = y[n - n_0]$.

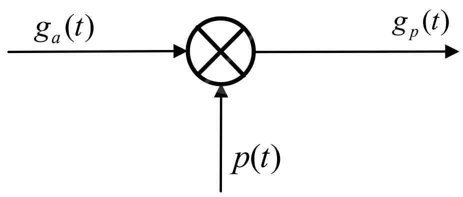
so it is time-invariant. [2 marks]

Since there is no output before the input hence the system is **causal**. [2 marks]

if x[n] is a bounded input

y[n] is a bounded output. so it is **stable**. [2 marks]

18.Consider the sampling process:



Given:

$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

$$g_p(t)=g_a(t)\cdot p(t)=\sum_{n=-\infty}^{+\infty}g_a(nT)\delta(t-nT)$$

Prove the sampling theorem: $G_p(j\Omega)=\dfrac{1}{T}\sum_{n=-\infty}^{+\infty}G_a(j(\Omega-n\Omega_T))$

Solution:

p(t) is a periodic signal with the period T, and satisfies the Dirichlet conditions, then it can be expanded as:

$$p(t)=\sum_{-\infty}^{+\infty}\delta(t-nT)=\sum_{-\infty}^{+\infty}a_n e^{jn\Omega_0 t}\ ,\ \ \Omega_0=\frac{2\pi}{T}$$

Here, $a_n=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}p(t)e^{-jn\Omega_0 t}dt=\frac{1}{T}$

Then p(t)=$\frac{1}{T}\sum_{-\infty}^{+\infty}e^{jn\Omega_0 t}$

By the Fourier transform, F[p(t)]=$\frac{2\pi}{T}\sum_{-\infty}^{+\infty}\delta(\Omega-n\Omega_0)$

Then, the Fourier transform of $g_p(t)=g_a(t)\cdot p(t)$ is:

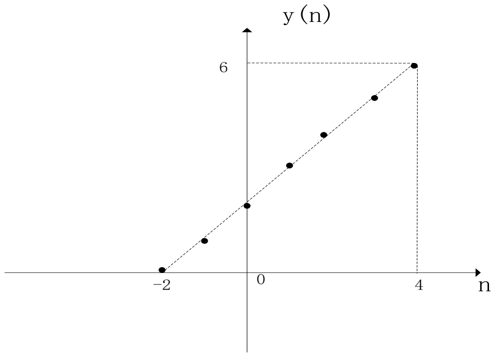
$$G_p(j\Omega)=\frac{1}{2\pi}[G_a(j\Omega)*p(j\Omega)]$$

$$=\frac{1}{2\pi}\left[G_a(j\Omega)*\frac{2\pi}{T}\sum_{-\infty}^{+\infty}\delta(\Omega-n\Omega_T)\right]\ ,\quad \Omega_T=\Omega_0 T$$

$$=\frac{1}{T}\sum_{-\infty}^{+\infty}G_a(\Omega-n\Omega_T)$$

Then $G_p(j\Omega) = \frac{1}{T}\sum_{-\infty}^{+\infty} G_a(j(\Omega - n\Omega_T))$

19. Let the signal $x(n) = (6-n)[u(n)-u(n-6)]$ .Sketch the signal $y(n) = x(4-n)$.



20. Let the Nyquist frequency of $X_a(t)$ is Ω_S .The Nyquist frequency of $X_a(t)\cos(\Omega_0 t)$ is $\Omega_N = \Omega_S + 2\Omega_0$. The Nyquist frequency of $X_a(2t)$ is $\Omega_N = 2\Omega_S$.

21. Determine if the system $\boldsymbol{y(n) = e^{x(n)}}$ is (a)Linear, (b)Time-Invariant, (c)Stable, (d) Causal. (a)Nonlinear, (b)Time-Invariant, (c)Stable, (d) Causal.

22.Given signal x(n) = cos(0.125nπ) + sin(0.05nπ), then its period N=___80______. [2 marks]

23.Given signal y(n) = cos(0.125nπ) · sin(0.05nπ), then its period N___80________ . [2 marks]

24.For each of the following discrete-time systems, where y(n) and x(n) are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) shift-invariant, (3) causal and (4) stable.

v) y(n)=ax(n)+b, a and b are nonzero constants.

Solution: nonlinear, shift-invariant, causal, stable

vi) $y(n) = a + \sum_{l=-3}^{3} x(n - l)$, a is a nonzero constant.

Solution: nonlinear, shift-invariant, noncausal, stable

25.One analogue signal is $x(t) = 3\sin 200\pi t + 7\sin 1200\pi t$ .Its Nyquist sampling rate is _**1200**_Hz.

[2 marks]

26.Given signal $x_a(t) = 3\sin(100\pi t)$, assume the sample frequency is $f_s = 300 Samples/s$, so the discrete sequence is $x(n) = x_a(nT_s)$, its period $N =$ ___**6**___

27.For each of the following discrete-time systems, where y(n) and x(n) are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) shift-invariant, (3) causal and (4) stable?

i) $y(n) = x^2(n)$

Solution:

$(A x_1[n] + B x_2[n])^2 \neq A Ax_1^2(n) + Bx_2^2(n)$

So the system is **not linear**.

If $x_1[n] = x[n - n_0]$ then $y[n] = x^4[n] = x[n - n_0] = y[n - n_0]$.

so it is **time-invariant**.

Since there is no output before the input hence the system is **causal**.

if x[n] is a bounded input

y[n] is a bounded output. so it is **stable**.

ii) $y(n) = 2x(n) + 3$

Solution:

2(A x_1[n] +B x_2[n]) +3≠ A($2x_1(n)+3$) +B($2x_2(n)+3$)

so it is **not linear.**

if x[n]= x[n − n_0]

then $y(n)=2x(n)+3$=2x[n − n_0]+3= y[n − n_0]

so it is time-invariant.

Since there is no output before the input hence the system is **causal**.

if x[n] is a bounded input
y[n] is a bounded output. So it is **stable**.

28.One analogue signal is

$x(t)=4\sin(20\pi t)-5\cos(24\pi t)+3\sin(120\pi t)+2\cos(176\pi t)$, If the sampling rate is 50Hz, then the digital signal could be expressed as:

___ $x[n]=4\sin\left(\frac{2\pi n}{5}\right)-5\cos\left(\frac{12\pi n}{25}\right)+3\sin\left(\frac{2\pi n}{5}\right)+2\cos\left(\frac{12\pi n}{25}\right)$ ___.

b) If the impulse response of a LTI system is $h(n)$, then the stable condition of this system is _ $\sum_{n=-\infty}^{\infty}|h[n]|<\infty$ _ and the casual condition is ____ $h[n]=0, n<0$ _______.

c) Consider the sequence defined by $\{g[n]\}=e^{j\pi n}$. Is it bounded or not? ___YES___ ('Yes' or 'No').

d) Considering the continuous-time signal $g_a(t)$ = sin(Ω_m t), it must be sampled at least at a rate ___ $\Omega_t > 2\Omega_m$ ____ to recover it fully from its samples..

29. For each of the following discrete-time systems, where y(n) and x(n) are, respectively, the output and the input sequences, determine whether or not the system is (1) linear, (2) shift-invariant, (3) causal and (4) stable.

i) $y(n) = x^3(n-1)$.

nonlinear, shift-invariant, causal, stable

ii) $y(n) = 2x(n+1) + 3$.

nonlinear, shift-invariant, noncausal, stable