Signals and Systems 1.1

---*Introduction*

School of Information & Communication Engineering，BUPT

Reference:

- *Textbook:* 1.1,1.2,1.4,1.5,
- 1.3 (optional)

Some Questions?

- Why to study Signals and Systems？
- What is a signal?
- What is a system?
- List some examples in your real life.
- In your opinion, What do you think about the signals and systems?

Outline of Today's Lecture

- Signals and systems introduction
- About our course
- Classification of signals
- Operation on Signals
- Summary

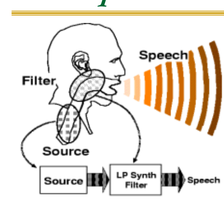
About the course

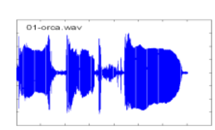
- This course is about **signals** and their **processing by systems**. It involves:
  - – Modelling of signals by mathematical functions.
  - – Modelling of systems by mathematical equations.
  - – Solution of the equations when excited by the functions.
  - – Stability of the systems

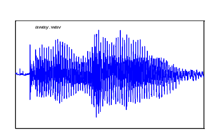
What is a Signal ?

- **Information:** meaning for exchange, purpose of communications.
- **A signal is formally defined as a function of one or more variables that *conveys information* on the nature of a physical phenomenon**
- Our world is full of signals, both natural and man-made.
  - Variation in air pressure when we speak.
  - Voltage waveform in a circuit.
  - The periodic electrical signals generated by the heart.
  - Stock prices

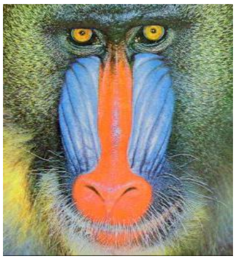
Examples of Signal / Voice

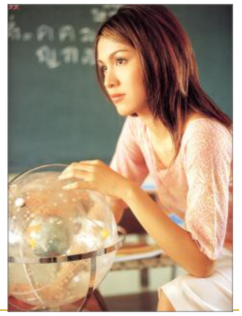






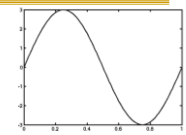
Examples of Signal / Image





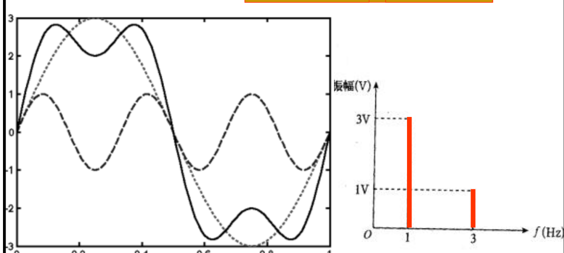
How to describe a Signal ?

- **In which shape**
  - Function: sin(2πt)
  - Waveform/image
- **In which domain**
  - In time domain: Signal is a function of time. (Time is an independent variable)
  - In frequency domain: signal is a function of frequency. (Frequency is an independent variable)



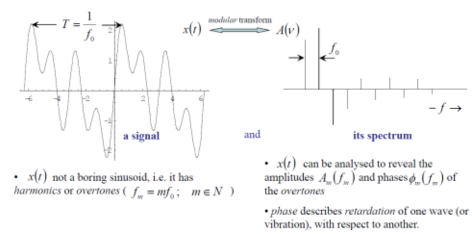
Sine Singals (sinusoid)

$3\sin(2\pi t)+\sin(6\pi t)$



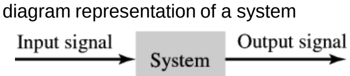
Examples of Signal ?

- A microphone (or generally a transducer/sensor) produces a voltage *V(t)*, proportional to the instantaneous air pressure *x(t)*
- *x(t)* can be displayed as a periodic signal on an oscilloscope

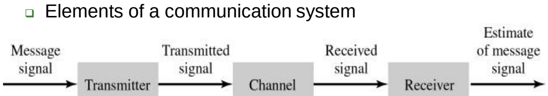


What is a System ?

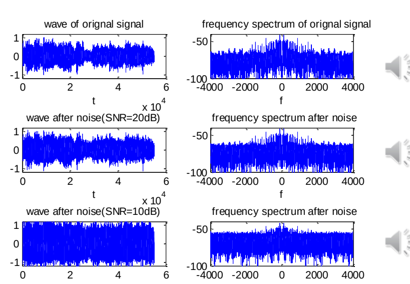
- A system is a generator of signals or it is a transformer of signals.
- **A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.**
- Block diagram representation of a system



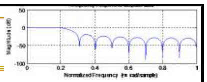
- Elements of a communication system

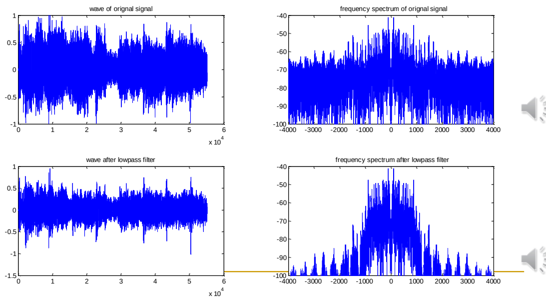


Add noise

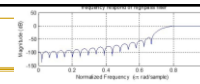


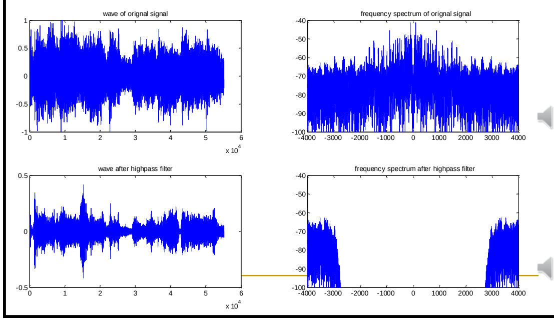
Low-pass Filter
(cutoff frequency = 1000HZ)



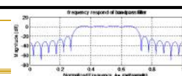


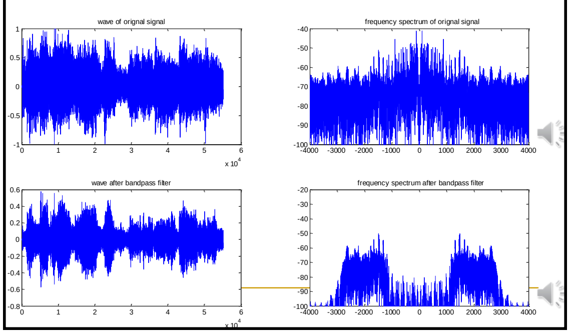
High-pass Filter
(f = 3000HZ)





Band-pass Filter
(f1 = 1000HZ, f2=3000HZ)

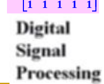




Examples of System / Signal Processing

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \qquad \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Digital
Signal
Processing



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Examples of System / Image Filtering 图像处理

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Overview of specific systems (1.3)

- Communication systems
- Control systems
- Microelectromechanical systems
- Remote sensing systems
- Biomedical signal processing
- Auditory system
- Analog versus digital signal processing

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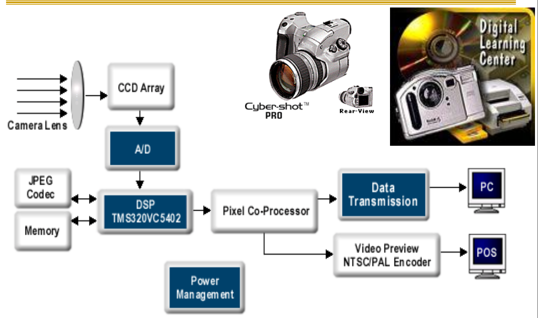
Digital Camera

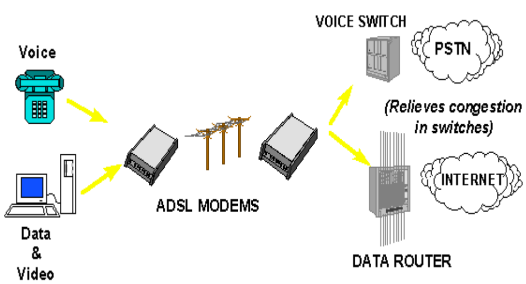


Digtal Signal Processing 数字信号处理

- HDTV

PDP (Plasma Display)TV

ADSL(Asymmetric Digital Subscriber Line)



Programmable switch 程控交换

Why study Signals and Systems?

- Signals and systems are fundamental to all of engineering!
- Steps involved in engineering are:
  - **Model system:** Involves writing a mathematical description of input and output signals.
  - **Analyze system:** Study of the various signals associated with the system.
  - **Design system:** Requires deciding on a suitable system architecture, as well as finding suitable system parameters.
  - **Implement system/test system:** Check system, and the input and output signals, to see that the performance is satisfactory.

Signals and systems introduction

- The course will serve as the prerequisites for additional coursework in the study of communications, signal processing and control.
- Although the signals and systems that arise across these diverse fields are naturally different in their physical make-up and application, the principles and tools of signals and systems are applicable to all of them.
- This course will expose to the students concepts like Fourier transform, Laplace transform, and z-Transform.

Textbook and Reference books

- **Textbook:**
  - **Signals and Systems, 2nd edition, by S. Haykin and B. Van Veen, John Wiley & sons, Inc 2003**
- **Reference books:**
  1. **Signals and Systems, 2nd edition, by By Alan V.Oppenheim, Alan S.Willsky and S.Hamid Nawab, 清华大学出版社，影印版 2002**
  2. **《信号与系统》，郑君里，应启珩，杨为理，高等教育出版社，2000**
  3. **Schaum's outline of signals and systems, Hwei P. Hsu, McGraw-Hill, 1995. Website: http://issuu.com/ek.korat/docs/schaum_s_outline_of_signals_and_systems**
- **Teaching hours：** **3hours, Mon. and Wed.**
- **Lab hours：** **12 hours，to be arranged, with Matlab software**

Overview of our course

- Chapter 1, Course Introduction
- Chapter 2, Time Domain Representations of LTI Systems
- Chapter 3, Fourier Representations of Signals and LTI Systems
- Chapter 4, Applications of Fourier Representations to Mixed Signal Classes
- Chapter 6, Representing Signals by Using Continuous-Time Complex Exponentials: the Laplace Transform
- Chapter 7, Representing Signals by Using Discrete-Time Complex Exponentials: the z-Transform

About Grading Scheme

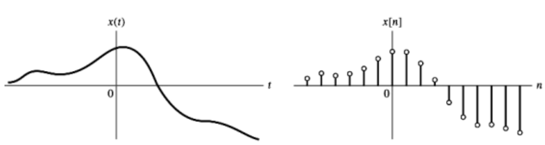
Homework, Attendance and Class Participation	**15%**
MATLAB Projects	**15%**
Final examination	**70%**
Total	**100%**

Classification of Signals

- ***Methods*** **used for** ***processing*** **a signal or** ***analyzing*** **the response of a system to a signal significantly depend on the characteristic attributes of the signal.**
- **Certain techniques apply to only specific types of signals – hence the need for classification**
  1. **continuous-time & discrete-time signals**
  2. **even and odd signals**
  3. **periodic and aperiodic signals**
  4. **deterministic signals and random signals**
  5. **energy and power signals**

Classification of Signals

- **1. Continuous-time & discrete-time**
  - **A *continuous-time signal* if it is defined for all time *t*, except at some discontinuous point.**
  - **A *discrete-time signal* is defined only at discrete instants of time.**



(a) (b)

Classification of Signals

- **1. Continuous-time & discrete-time**
  - A ***discrete-time signal*** is often derived from A ***continuous-time signal*** by sampling it at a uniform rate.

$$x[n]=x(t)|_{t=nT_s}=x(nT_s)$$

T_s：sampling period, n denote an integer

In this lecture, we use the symbol *t* to denote time for a continuous-time signal, and the symbol *n* to denote time for a discrete-time signal.

Continuous-time signals: ***x(t)*** — Parentheses (·)

Discrete-time signals: $x[n]=x(nT_s),\quad n=0,\ \pm1,\ \pm2,\ \ldots\ldots$ — Brackets [·] **where *t = nT*$_s$**

Classification of Signals

- **2. Even and odd signals**

Symmetric about vertical axis

Even signals: $x(-t)=x(t),\quad x[-n]=x[n]\ \ \forall\, t \text{ and } n$

Odd signals: $x(-t)=-x(t),\quad x[-n]=-x[n]\ \ \forall\, t \text{ and } n$

Antisymmetric about origin

Example 1.1

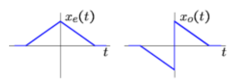
Consider the signal $x(t)=\begin{cases}\sin\left(\frac{\pi t}{T}\right), & -T\le t\le T\\ 0, & \text{otherwise}\end{cases}$

Is the signal $x(t)$ an even or an odd function of time?

<Sol.>

$$x(-t)=\begin{cases}\sin\left(-\frac{\pi t}{T}\right), & -T\le t\le T\\ 0, & \text{otherwise}\end{cases}=\begin{cases}-\sin\left(\frac{\pi t}{T}\right), & -T\le t\le T\\ 0, & \text{otherwise}\end{cases}=-x(t)\quad \text{for all } t$$

→ *odd function*



Classification of Signals

- **2. Even and odd signals,** Even-odd decomposition of $x(t)$:

$x(t)=x_e(t)+x_o(t)$

where $x_e(-t)=x_e(t)$

$x_o(-t)=-x_o(t)$

→ $x(-t)=x_e(-t)+x_o(-t)=x_e(t)-x_o(t)$

→ $x_e=\frac{1}{2}[x(t)+x(-t)]$ (1.4)

→ $x_o=\frac{1}{2}[x(t)-x(-t)]$ (1.5)

Example 1.2

Find the even and odd components of the signal

$x(t)=e^{-2t}\cos t$

<Sol.>

$x(-t)=e^{2t}\cos(-t)=e^{2t}\cos(t)$

Even component:

$x_e(t)=\frac{1}{2}(e^{-2t}\cos t+e^{2t}\cos t)=\cosh(2t)\cos t$

Odd component: $x_o(t)=\frac{1}{2}(e^{-2t}\cos t-e^{2t}\cos t)=-\sinh(2t)\cos t$

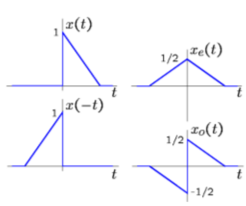
Example: Even and Odd Signals

Any signal is a sum of unique odd and even signals. Using

$$x(t)=x_e(t)+x_o(t) \text{ and } x(-t)=x_e(t)-x_o(t),$$

yeilds

$$x_e(t)=\frac{1}{2}(x(t)+x(-t)) \text{ and } x_o(t)=\frac{1}{2}(x(t)-x(-t)).$$

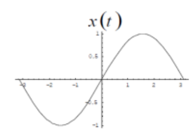


Classification of Signals

- **2. Even and odd signals,** PRODUCT rule

Note the PRODUCT rule:

ODD . ODD = EVEN
EVEN . EVEN = EVEN
EVEN . ODD = ODD
ODD . EVEN = ODD



Note also that:

$s=\int_{-T}^{T}x(t)dt=0$ ALWAYS if x(t) is ODD

$=0$ SOMETIMES if x(t) is EVEN

$s=\int_{-T}^{T}x(t)dt=2\int_{0}^{T}x(t)dt$ for x(t) EVEN

Problem: Even and Odd Signals

▶ **Problem 1.1** Find the even and odd components of each of the following signals:

(a) $x(t)=\cos(t)+\sin(t)+\sin(t)\cos(t)$
(b) $x(t)=1+t+3t^2+5t^3+9t^4$
(c) $x(t)=1+t\cos(t)+t^2\sin(t)+t^3\sin(t)\cos(t)$
(d) $x(t)=(1+t^3)\cos^3(10t)$

Answers:
(a) Even: $\cos(t)$
Odd: $\sin(t)(1+\cos(t))$
(b) Even: $1+3t^2+9t^4$
Odd: $t+5t^3$
(c) Even: $1+t^3\sin(t)\cos(t)$
Odd: $t\cos(t)+t^2\sin(t)$
(d) Even: $\cos^3(10t)$
Odd: $t^3\cos^3(10t)$

Classification of Signals

- **3. Periodic and Aperiodic Signals,** Continuous-Time Case

Periodic signals: $x(t)=x(t+T)\quad \text{for all } t$

T is a positive constant $T=T_0,\ 2T_0,\ 3T_0,\ \ldots\ldots$

$T=T_0\equiv\text{Fundamental period}$

Fundamental frequency: $f=1/T$ cycles per second

How frequency the periodic signal repeats itself.

Angular frequency: $\omega=2\pi f=2\pi/T$

Measured in radians per second.

Aperiodic signals:

Where T_0 otherwise does not exist, $x(t)$ is termed *aperiodic*.

Classification of Signals

- **3. Periodic and Aperiodic Signals,** Continuous-Time Case

◆ **Example of periodic and nonperiodic signals:** Fig. 1-14.

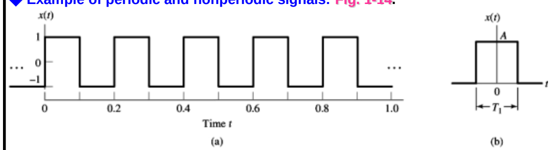


Figure 1.14 (p. 21)
(a) Square wave with amplitude $A = 1$ and period $T = 0.2$s.
(b) Rectangular pulse of amplitude A and duration T_1.

Classification of Signals

- **3. Periodic and Aperiodic Signals,** Discrete -Time Case

$x[n] = x[n+N] \quad \text{for} \quad \text{integer} \quad n$

Fundamental frequency of $x[n]$:

$\Omega = \frac{2\pi}{N}$

The smallest integer value of N for which the periodicity holds is the fundamental frequency N_0 where does not exist is termed *aperiodic*.

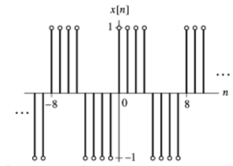


Figure 1.16 (p. 22)
Discrete-time square wave alternative between –1 and +1.

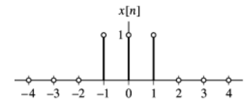


Figure 1.17 (p. 22)
Aperiodic discrete-time signal consisting of three nonzero samples.

Classification of Signals

- **3. Periodic and Aperiodic Signals---Notes**

■Note that a sequence obtained by uniform sampling of a periodic continuous-time signal may not be periodic.
■Note also that the sum of two continuous-time periodic signals may not be periodic but that the sum of two periodic sequences is always periodic.
■Note that the sum of two periodic sequences is always periodic.

Classification of Signals

- **4. Deterministic signals and random signals**

■ ***Deterministic signals*** **are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time.**
■ ***Random signals*** **are those signals that take random values at any given time and must be characterized statistically.**
■ **Random signals will not be discussed in this text.**

Classification of Signals

- **5. Energy and Power Signals**

$p(t) = \frac{v^2(t)}{R}$

$p(t) = Ri^2(t)$

If $R = 1\,\Omega$ and $x(t)$ represents a current or a voltage, then the instantaneous power is

$p(t) = x^2(t)$

The total energy of the continuous-time signal $x(t)$ is

$$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt$$

Time-averaged, or average, power is

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

For periodic signal, the time-averaged power is

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

◆ **Discrete-time case:**

Total energy of $x[n]$:

$$E = \sum_{n=-\infty}^{\infty} x^2[n]$$

Average power of $x[n]$:

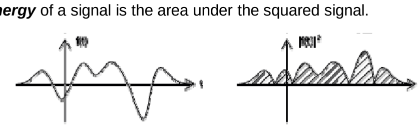
$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]$$

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

Classification of Signals

- **5. Energy and Power Signals**

■ Since we often think of signal as a function of varying amplitude through time, it seems to reason that a good measurement of the strength of a signal would be the area under the curve.
■ This suggests either squaring the signal or taking its absolute value, then finding the area under that curve. It turns out that what we call the ***energy*** of a signal is the area under the squared signal.

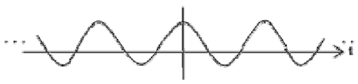


The energy of this signal is the shaded region.

Classification of Signals

- **5. Energy and Power Signals**

Our definition of energy seems reasonable. However, what if the signal does not decay? In this case we have infinite energy for any such signal. Does this mean that a sixty hertz sine wave feeding into your headphones is as strong as the sixty hertz sine wave coming out of your outlet? Obviously not. This is what leads us to the idea of signal power.
Power is a time average of energy (energy per unit time).
This is useful when the energy of the signal goes to infinity.



A simple, common signal with infinite energy.

Classification of Signals

- **5. Energy and Power Signals**

★ ***Energy signal***:

If and only if the total energy of the signal satisfies the condition

$0 < E < \infty$

★ ***Power signal***:

If and only if the average power of the signal satisfies the condition

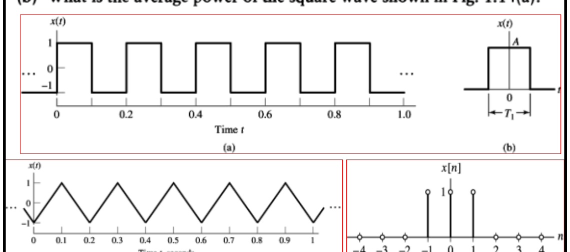
$0 < P < \infty$

- Energy signal has zero time-average power (why?)
- Power signal has infinite energy (why?)
- Energy signal and power signal are mutually exclusive
- Periodic signal and random signal are usually viewed as power signal
- Nonperiodic and deterministic are usually viewed as energy signal

Examples 1

(a) What is the total energy of the rectangular pulse shown in Fig. 1.14(b)?

(b) What is the average power of the square wave shown in Fig. 1.14(a)?



Answers: (a) A^2T_1. (b) 1 Power is 1/3 Energy is 3

Examples 2

▶ **Problem 1.9** Categorize each of the following signals as an energy signal or a power signal, and find the energy or time-averaged power of the signal:

$$x[n] = \begin{cases} n, & 0 \le n < 5 \\ 10 - n, & 5 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$$ $\boxed{\text{Energy signal, energy} = 85}$

$$x[n] = \begin{cases} \cos(\pi n), & -4 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$ $\boxed{\text{Energy signal, energy} = 9}$

$$x[n] = \begin{cases} \cos(\pi n), & n \ge 0 \\ 0, & \text{otherwise} \end{cases}$$ $\boxed{\text{Power signal, power} = \tfrac{1}{2}}$

Answers: (a) A^2T_1. (b) 1 Power is 1/3 Energy is 3

Classification of Signals

- **Orthogonality**

Orthogonality is fundamental to almost everything that is subsequent in signals and systems theory.
The definitions are:

Discrete signals:

•If the product of two signals averages to zero over the period T, then those two signals are ORTHOGONAL in that interval (T).

Continuous signals:

•If the product of two signals integrates to zero over the period T, then those two signals are ORTHOGONAL in that interval (T).

Operation on Signals

- **An issue of fundamental importance in the signals and systems is the use of systems to process or manipulate signals. This issue usually involves a combination of some basic operations in signals**
- Typical operations on signals:
  - Three transformation in amplitude
    - ***Amplitude scaling***
    - ***Addition***
    - ***Multiplication***
  - Three transformations in time domain
    - ***Time Scaling***
    - ***Time Reflection***
    - ***Time Shifting***

Operation on Signals --- in Amplitude

Amplitude scaling: $x(t)$ ➡ $y(t) = cx(t)$ — c = scaling factor

Discrete-time case: $x[n]$ ➡ $y[n] = cx[n]$ — Performed by amplifier

Addition:

$y(t) = x_1(t) + x_2(t)$

Discrete-time case: $y[n] = x_1[n] + x_2[n]$

Multiplication:

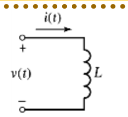
$y(t) = x_1(t)x_2(t)$ — AM modulation

$y[n] = x_1[n]x_2[n]$

Differentiation:

$y(t) = \frac{d}{dt}x(t)$

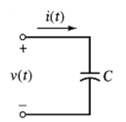
Inductor: $v(t) = L\frac{d}{dt}i(t)$ (1.25)



Integration:

$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

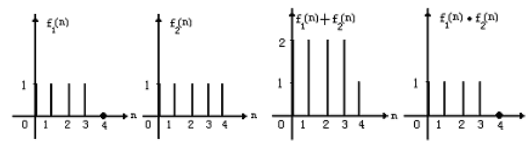
Capacitor: $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$



Addition and Multiplication Examples

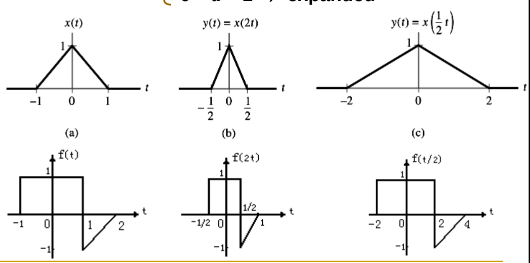
- Two methods
  - By waveform
  - By function

$f_1(n)$ $f_2(n)$ $f_1(n) + f_2(n)$ $f_1(n) \times f_2(n)$



Time Scaling in time domain

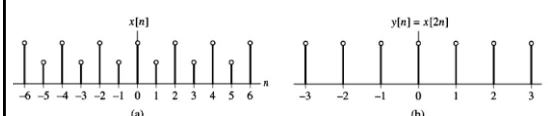
$y(t) = x(at)$ ➡ $\begin{cases} a > 1 \Rightarrow \text{compressed} \\ 0 < a < 1 \Rightarrow \text{expanded} \end{cases}$



Time Scaling in time domain

Discrete-time case: $y[n] = x[kn], \quad k > 0$ ***k* = integer**

➡ **Some values lost!**



Time Reflection in time domain

$y(t) = x(-t)$

➡ The signal *y*(*t*) represents a reflected version of *x*(*t*) about *t* = 0.

Ex. 1-3
Consider the triangular pulse x(t) shown in Fig. 1-22(a). Find the reflected version of x(t) about the amplitude axis (i.e., the origin).

<Sol.> Fig.1-2

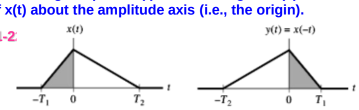


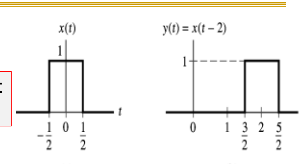
Figure 1.22 (p. 28)
Operation of reflection: (a) continuous-time signal *x*(*t*) and (b) reflected version of *x*(*t*) about the origin.

➡ $\begin{cases} x(t) = 0 \quad \text{for} \quad t < -T_1 \quad \text{and} \quad t > T_2 \\ y(t) = 0 \quad \text{for} \quad t > T_1 \quad \text{and} \quad t < -T_2 \end{cases}$

Time shifting in time domain

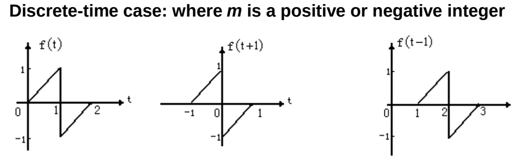
$y(t) = x(t - t_0)$

$t_0 > 0 \Rightarrow$ shift toward right
$t_0 < 0 \Rightarrow$ shift toward left



$y[n] = x[n - m]$

Discrete-time case: where *m* is a positive or negative integer



Time Shifting and Scaling in time domain

1. Combination of time shifting and time scaling:

$$y(t) = x(at - b)$$

$$y(0) = x(-b) \qquad y\left(\frac{b}{a}\right) = x(0)$$

2. Operation order:

1st step: time shifting $\quad v(t) = x(t - b)$

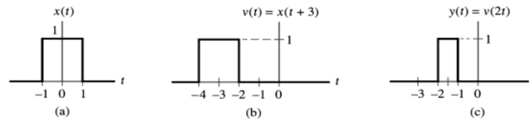
2nd step: time scaling $\quad y(t) = v(at) = x(at - b)$

To Obtain y(t) from x(t), the time-shifting and time-scaling operations must be performed in the correct order. The scaling operation always replaces *t* by *at*, while the time-shifting operation always replaces *t* by *t-b*.

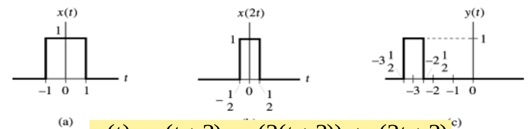
Time Shifting and Scaling in time domain

Ex. 1-5 **Find *y*(*t*)=*x*(2*t* + 3).**

<Sol.> **Case 1: Shifting first, then scaling**



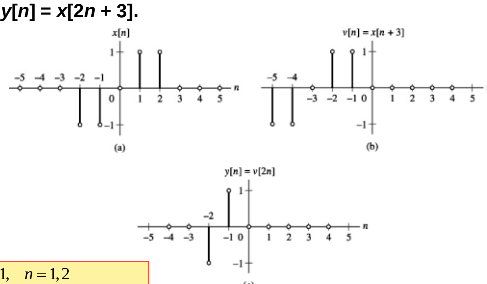
Case 2: Scaling first, then shifting



$$y(t) = v(t+3) = x(2(t+3)) \neq x(2t+3)$$

Example 1: Discrete-Time Signal

Find *y*[*n*] = *x*[2*n* + 3].

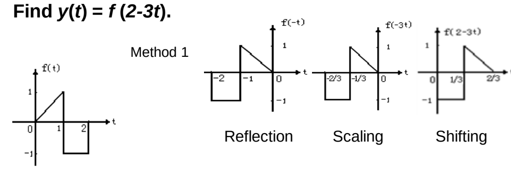


$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases}$$

Example 2: Continuous-time signal

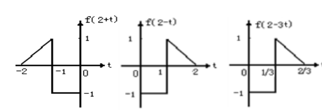
Find *y*(*t*) = *f* (2-3*t*).

Method 1



Reflection Scaling Shifting

Method 2



Shifting Reflection Scaling

Summary and Exercises

- **Summary**
  - Signals and systems introduction
  - Overview of our course
  - Classification of signals
  - Operation on Signals
- **Exercises**
  - P88-89: 1.42 (b, d, f, h), 1.44, 1.45, 1.46, 1.50, 1.51, 1.52 (a, b, f)