Signals and Systems 3.4

--- *Frequency Representations of LTI systems*

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Reference:
1. Textbook: Chapter 3

Frequency Representations of LTI systems

- Frequency Response of LTI systems
- Filtering
- Representations and Solutions of LTI systems in frequency domain
- Conditions of Distortionless Transmission

Output of LTI System in frequency domain

$e^{j\omega t} \rightarrow$ **CT** $\rightarrow H(j\omega)e^{j\omega t}$

$$x(t)=\sum_{k=1}^{M} a_k e^{j\omega_k t} \qquad y(t)=\sum_{k=1}^{M} a_k H(j\omega) e^{j\omega_k t}$$

$e^{j\Omega n} \rightarrow$ **DT** $\rightarrow H(e^{j\Omega})e^{j\Omega n}$

$$x[n]=\sum_{k=1}^{M} a_k e^{j\Omega_k n} \qquad y[n]=\sum_{k=1}^{M} a_k H(e^{j\Omega}) e^{j\Omega_k n}$$

Convolution operation $\Rightarrow$ Multiplication

$x(t)$, $X(j\omega) \rightarrow$ **CT** $\rightarrow y(t)=x(t)*h(t)$, $Y(j\omega)=X(j\omega)H(j\omega)$

$x[n]$, $X(e^{j\Omega}) \rightarrow$ **DT** $\rightarrow y[n]=x[n]*h[n]$, $Y(e^{j\Omega})=X(e^{j\Omega})H(e^{j\Omega})$

$$H(j\omega)=\int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt \qquad H(e^{j\Omega})=\sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n}$$

Transfer function of LTI system

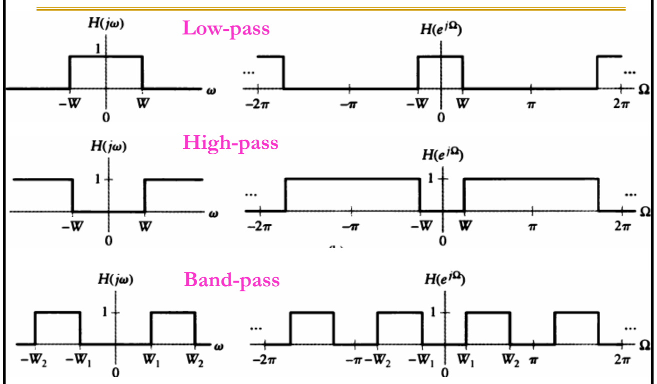
$x(t)$, $X(j\omega) \rightarrow$ **CT** $\rightarrow y(t)=x(t)*h(t)$, $Y(j\omega)=X(j\omega)H(j\omega)$

$$H(j\omega)=\int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt \qquad H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}$$

$x[n]$, $X(e^{j\Omega}) \rightarrow$ **DT** $\rightarrow y[n]=x[n]*h[n]$, $Y(e^{j\Omega})=X(e^{j\Omega})H(e^{j\Omega})$

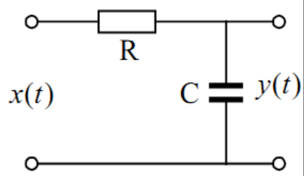
$$H(e^{j\Omega})=\sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n} \qquad H(e^{j\Omega})=\frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

Filtering: some frequency are eliminated while others are passed



Example: RC circuit (Filtering)

$$h(t)=\frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$



$$H(j\omega)=\frac{\frac{1}{RC}}{j\omega+\frac{1}{RC}}$$

Applications

- Find the output of the system, given its input and impulse response.

$$Y(j\omega)=X(j\omega)H(j\omega)$$

$$x(t)=3e^{-t}u(t)\Leftrightarrow X(j\omega)=\frac{3}{1+j\omega}$$

$$=\frac{2}{(2+j\omega)}\frac{3}{(1+j\omega)}$$

$$h(t)=2e^{-2t}u(t)\Leftrightarrow H(j\omega)=\frac{2}{2+j\omega}$$

$$y(t)=\tfrac{1}{6}e^{-t}u(t)-\tfrac{1}{6}e^{-2t}u(t)$$

- Identifying a system, given its input and output.

$$x(t)=e^{-2t}u(t)\Leftrightarrow X(j\omega)=\frac{1}{2+j\omega}$$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}$$

$$=1+\frac{1}{(1+j\omega)}$$

$$y(t)=e^{-t}u(t)\Leftrightarrow Y(j\omega)=\frac{1}{1+j\omega}$$

$$h(t)=\delta(t)+e^{-t}u(t)$$

Representations of LTI Systems in Frequency Domain

$$x(t)\longrightarrow\boxed{CT}\longrightarrow y(t)$$

$$\sum_{k=0}^{N}a_k\frac{d^k y(t)}{dt^k}=\sum_{k=0}^{M}b_k\frac{d^k x(t)}{dt^k}$$

$$x[n]\longrightarrow\boxed{DT}\longrightarrow y[n]$$

$$\sum_{k=0}^{N}a_i y[n-k]=\sum_{k=0}^{M}b_j x[n-k]$$

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Example1: solutions of the CT LTI system

$$y''(t)+3y'(t)+2y(t)=x'(t)+4x(t) \qquad x(t)=e^{-3t}u(t)$$

Find the impulse response h(t) and output y(t).

$$(j\omega)^2Y(j\omega)+3j\omega Y(j\omega)+2Y(j\omega)=j\omega X(j\omega)+4X(j\omega)$$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{4+j\omega}{(j\omega)^2+3(j\omega)+2}=\frac{-2}{j\omega+2}+\frac{3}{j\omega+1}$$

$$h(t)=-2e^{-2t}u(t)+3e^{-t}u(t)$$

$$Y(j\omega)=X(j\omega)H(j\omega)=\frac{1}{j\omega+3}\bullet\frac{j\omega+4}{(j\omega+2)(j\omega+1)}$$

$$=\frac{1/2}{3+j\omega}-\frac{2}{2+j\omega}+\frac{3/2}{1+j\omega}$$

$$y(t)=\left[\tfrac{1}{2}e^{-3t}-2e^{-2t}+\tfrac{3}{2}e^{-t}\right]u(t)$$

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Example2: solutions of the DT LTI system

$$y[n]=x[n]+x[n\text{-}1]+x[n\text{-}2]$$

Find the impulse response h(t) and H(e^jΩ).

$$(1+e^{j\Omega}+e^{j2\Omega})X(e^{j\Omega})=Y(e^{j\Omega})$$

$$H(e^{j\Omega})=\frac{Y(e^{j\Omega})}{X(e^{j\Omega})}=1+e^{j\Omega}+e^{j2\Omega}$$

$$h[n]=\delta[n]+\delta[n-1]+\delta[n-2]$$

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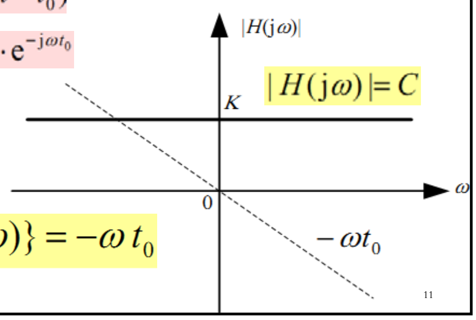
Conditions of Distorionless Transmission

$$y(t)=Cx(t-t_0)$$

$$h(t)=C\cdot\delta(t-t_0)$$

$$H(j\omega)=C\cdot e^{-j\omega t_0}$$



$$\arg\{H(j\omega)\}=-\omega t_0$$

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