Independent Events

Definition:

Two events A and B are **independent** if the probability of each event remains the same whether or not the other occurs.

In symbols: If $P(A) \neq 0$ and $P(B) \neq 0$, then A and B are independent if

$$P(B|A) = P(B) \qquad \text{or, equivalently,} \qquad P(A|B) = P(A)$$

If either $P(A) = 0$ or $P(B) = 0$, then A and B are independent.

Note: If A and B are independent, then the following pairs of events are also independent: A and B^C, A^C and B, and A^C and B^C.

The concept of independence can be extended to more than two events:

Definition:

Events $A_1, A_2, \ldots, A_n$ are independent if the probability of each remains the same no matter which of the others occur.

In symbols: Events $A_1, A_2, \ldots, A_n$ are independent if for each A_i, and each $A_{j1}, \ldots, A_{jm}$ of events with $P\left(A_{j1} \cap \cdots \cap A_{jm}\right) \neq 0$,

$$P(A_i \mid A_{j1} \cap \cdots \cap A_{jm}) = P(A_i)$$