

The Multiplication Rule

(1) If A and B are two events with $P(B) \neq 0$, then $P(A \cap B) = P(B) \cdot P(A|B)$.

(2) If A and B are two events with $P(A) \neq 0$, then $P(A \cap B) = P(A) \cdot P(B|A)$.

If $P(A) \neq 0$ and $P(B) \neq 0$, then equations (1) and (2) above both hold.

When two events are independent, then $P(A|B) = P(A)$ and $P(B|A) = P(B)$ and we have the following:

If A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$.

This result can be extended to any number of events. If A_1, A_2, \dots, A_n are independent events, then for each collection A_{j_1}, \dots, A_{j_m} of events

$$P(A_{j_1} \cap A_{j_2} \cap \dots \cap A_{j_m}) = P(A_{j_1}) P(A_{j_2}) \dots P(A_{j_m})$$

In particular,

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) P(A_2) \dots P(A_n)$$