Probabilities

Given any experiment and any event A:

- The expression $P(A)$ denotes the probability that the event A occurs.
- $P(A)$ is the proportion of times that event A would occur in the long run, if the experiment were to be repeated over and over again.

The Axioms of Probability

1. Let **S** be a sample space. Then $P(\boldsymbol{S}) = 1$.

2. For any event A, $0 \leq P(A) \leq 1$.

3. If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

More generally, if $A_1, A_2, \ldots$ are mutually exclusive events, then
$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$.

Complementation Rule:

For any event A, $\quad P(A^C) = 1 - P(A)$

Let $\emptyset$ denote the empty set. Then $P(\emptyset) = 0$.

Ex.: A target on a test firing range consists of a bull’s eye with two concentric rings around it. A projectile is fired at the target. The probability that it hits the bull’s eye is 0.10, the probability that it hits the inner ring is 0.25, and the probability it hits the outer ring is 0.45.

(a) What is the probability that the projectile hits the target?

(b) What is the probability that it misses the target?

If A is an event containing outcomes $O_1, \ldots, O_n$, that is, if $A = \{O_1, \ldots, O_n\}$, then

$$P(A) = P(O_1) + P(O_2) + \cdots + P(O_n)$$

e.g.,

Sample Spaces with Equally Likely Outcomes

A population from which an item is sampled at random can be thought of as a sample space with equally likely outcomes.

If a sample space has N equally likely outcomes, the probability of each outcome is $1/N$.
The following theorem follows:

If $\boldsymbol{S}$ is a sample space containing N equally likely outcomes, and if A is an event containing k outcomes, then

$$P(A) = \frac{k}{N}$$

e.g.,

General Addition Rule

Let A and B be any events. Then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$