Linear Functions of Random Variables

Adding a Constant:

If X is a random variable and b is a constant, then

$$\mu_{X+b} = \mu_X + b \quad \text{and} \quad \sigma^2_{X+b} = \sigma^2_X$$

Using alternative notation,

$$E(X+b) = E(X) + b \quad \text{and} \quad V(X+b) = V(X)$$

(In general, when a constant is added to a random variable, the mean is shifted by that constant, and the variance is unchanged.)

Multipying by a Constant:

If X is a random variable and a is a constant, then

$$\mu_{aX} = a\mu_x \quad \text{and} \quad \sigma^2_{aX} = a^2\sigma^2_X$$

Using alternative notation,

$$E(aX) = a\,E(X) \quad \text{and} \quad V(aX) = a^2\,V(X)$$

(In general, when a random variable is multiplied by a constant, its mean is multiplied by the same constant but its variance is multiplied by the square of the constant.)

IN SUMMARY, combining the results above, we have the following:

If X is a random variable, and a and b are constants, then

$$\mu_{aX+b} = a\mu_X + b \quad \text{and} \quad \sigma^2_{aX+b} = a^2\sigma^2_X$$

Using alternative notation,

$$E(aX+b) = a\,E(X) + b \quad \text{and} \quad V(aX+b) = a^2\,V(X)$$

Ex. Assume that steel rods produced by a certain machine have a mean length of 5.0 in. and a variance of $\sigma^2 = 0.003$ in^2. Each rod is attached to a base that is exactly 1.0 in. long.

(a) What is the mean length of the assembly?

(b) Can you tell what the variance of the length of the assembly might be?

(c) Let X represent the length of a randomly chosen rod and let Y represent the length of the assembly. Write Y in terms of the variable X.

(d) Use statistical notation to show how to compute μ_Y.

(e) Use statistical notation to show how to compute σ_Y^2 .

(f) If we measure the lengths of the rods described above in centimeters rather than inches, what will be the mean length? (Use 2.54 cm = 1 in.)

(g) When the length X of a rod is measured in inches, the variance σ_X^2 must have units of in^2. If we measure the lengths of the rods in centimeters, what must be the units of the variance?

What must we multiply the variance σ_X^2 by in order to convert to the appropriate units?

(h) Let Z represent the length of a rod, in centimeters. Write Z in terms of the random variable X.

(i) Use statistical notation to show how to compute μ_Z .

(j) Use statistical notation to show how to compute σ_Z^2.