Probability Distributions for Discrete Random Variables

Probability Mass Function:

The **probability mass function** (p.m.f.) of a discrete random variable X is the function

$$p(x) = P(X = x).$$

The probability mass function is sometimes called the probability distribution.

Note: If the values of the probability mass function are added over all the possible values of X, the sum is equal to 1. That is,

$$\sum_x p(x) = \sum_x P(X = x) = 1, \quad \text{where the sum is over all the possible values of } X.$$

Cumulative Distribution Function:

The **cumulative distribution function** (c.d.f.) of a random variable X is the function

$$F(x) = P(X \leq x).$$

In general, for any discrete random variable X, the cumulative distribution function $F(x)$ can be computed by summing the probabilities of all the possible values of X that are less than or equal to x. That is,

$$F(x) = \sum_{t \leq x} p(t) = \sum_{t \leq x} P(X = t).$$

Note that $F(x)$ is defined for any number x, not just for the possible values of X. For a discrete random variable X, the graph of F will be a "step function."
