The Binomial Distribution

If a total of $\boldsymbol{n}$ **Bernoulli trials** are conducted, and

- **The trials are independent**
- **Each trial has the same success probability** $\boldsymbol{p}$
- $\boldsymbol{X}$ **is the number of successes in the** $\boldsymbol{n}$ **trials**

then X has the binomial distribution with parameters n and p, denoted $\boldsymbol{X \sim Bin(n, p)}$.

Note: Assume that a finite population contains items of two types, successes and failures, and that a simple random sample is drawn from the population. Then if the sample size is no more than 5% of the population, the binomial distribution may be used to model the number of successes. For example, a sampling "without replacement" experiment can be treated as binomial as long as the sample size (number of trials) n is at most 5% of the population size.

<u>Probability Mass Function of a Binomial Random Variable</u>:

If $X \sim Bin(n, p)$, the probability mass function of X is

$$p(x) = P(X = x) = \begin{cases} \dfrac{n!}{x!\,(n-x)!}\, p^x (1-p)^{n-x} & x = 0, 1, \ldots, n \\ 0 & otherwise \end{cases}$$

<u>Mean and Variance of a Binomial Random Variable</u>:

If $X \sim Bin(n, p)$, then the mean and variance of X are given by

$$\mu_X = np \qquad \text{and} \qquad \sigma_X^2 = np(1-p)$$