

Mean and Variance for Continuous Random Variables

Mean of a Continuous Random Variable:

Let X be a continuous random variable with probability density function $f(x)$. Then the mean of X is given by

$$\mu_X = \int_{-\infty}^{\infty} x f(x) dx$$

The mean of X is sometimes called the expectation, or expected value, of X and may also be denoted by $E(X)$ or by μ .

Variance and Standard Deviation of a Continuous Random Variable:

Let X be a continuous random variable with probability density function $f(x)$. Then

- The variance of X is given by

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_X)^2 f(x) dx$$

- An alternate formula for the variance is given by

$$\sigma_X^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu_X^2$$

- The variance of X may also be denoted by $V(x)$ or by σ^2 .
- The standard deviation is the square root of the variance:

$$\sigma_X = \sqrt{\sigma_X^2}$$

Ex. A hole is drilled in a sheet-metal component, and then a shaft is inserted through the hole. The shaft clearance is equal to the difference between the radius of the hole and the radius of the shaft. Let the random variable X denote the clearance, in millimeters. The probability density function of X is

$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean clearance and the variance of the clearance.