Probability Distributions for Continuous Random Variables

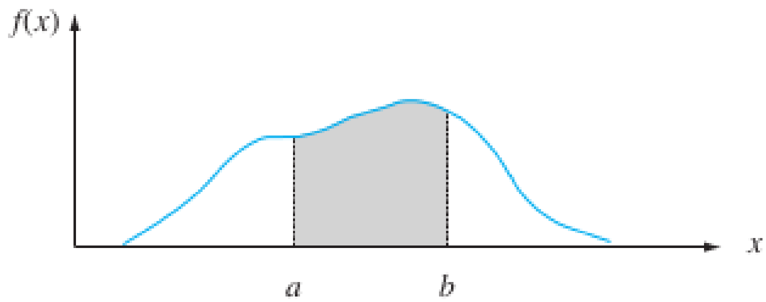
Probability Density Function:

A random variable is **continuous** if its probabilities are given by areas under a curve. This curve is called a **probability density function** (p.d.f.) for the random variable. The probability density function is sometimes called the probability distribution.

Let X be a continuous random variable with probability density function $f(x)$. Let a and b be any two numbers, with $a < b$.

The proportion of the population whose values of X lie between a and b is given by $\int_a^b f(x)dx$, the area under the probability density function between a and b. This is the probability that the random variable X takes on a value between a and b. Note that the area under the curve does not depend on whether the endpoints a and b are included in the interval. So, probabilities involving X do not depend on whether endpoints are included. That is,

$$P(a \le X \le b) = P(a \le X < b) = P(a < X \le b) = P(a < X < b) = \int_a^b f(x)dx$$



In addition,

$$P(X \le b) = P(X < b) = \int_{-\infty}^{b} f(x)dx$$

$$P(X \ge a) = P(X > a) = \int_{a}^{\infty} f(x)dx$$

Note also, for $f(x)$ to be a legitimate p.d.f., it must satisfy the following conditions:

1. $f(x) \ge 0$ for all x
2. $\int_{-\infty}^{\infty} f(x)dx = 1$ (The area under the entire curve is equal to 1.)

Ex. A hole is drilled in a sheet-metal component, and then a shaft is inserted through the hole. The shaft clearance is equal to the difference between the radius of the hole and the radius of the shaft. Let the random variable X denote the clearance, in millimeters. The probability density function of X is

$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Components with clearances larger than 0.8 mm must be scrapped. What proportion of components are scrapped?