Image Thresolding & Region Based Segmentation

Computer Vision & Image Processing

Fundamentals

- Let R represent the entire spatial region occupied by an image. Image segmentation is a process that partitions R into *n* sub-regions, R_1, R_2, …, R_n, such that

(a) $\bigcup_{i=1}^{n} R_i = R.$

(b) R_i is a connected set. $i = 1,\ 2,\ \ldots,\ n.$

(c) $R_i \cap R_j = \Phi.$

(d) $\square\,(R_i) = \text{TRUE for } i = 1,\ 2,\ \ldots,\ n.$

(e) $\square\,(R_i \cup R_j) = \text{FALSE for any adjacent regions}$

R_i and R_j.

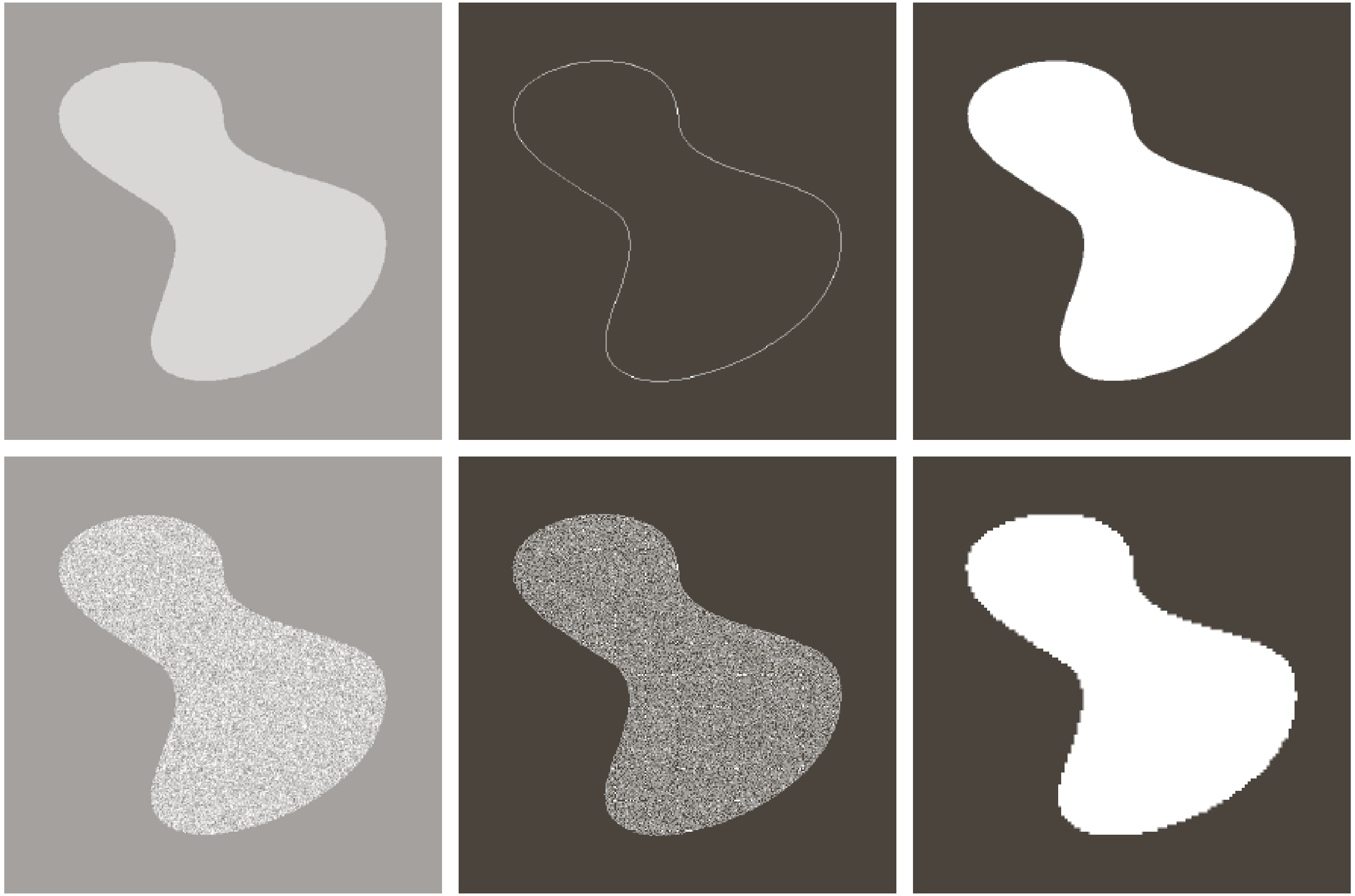
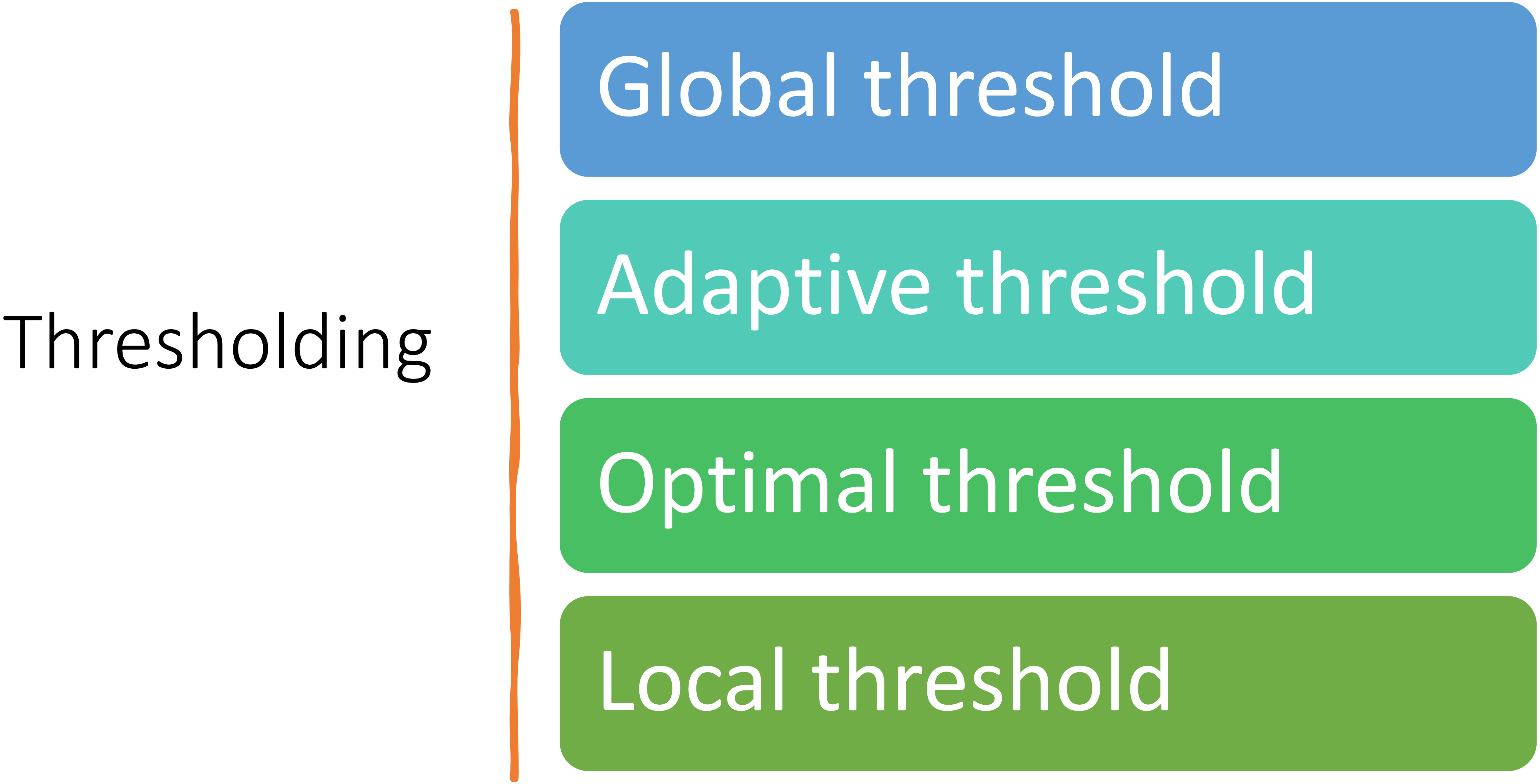


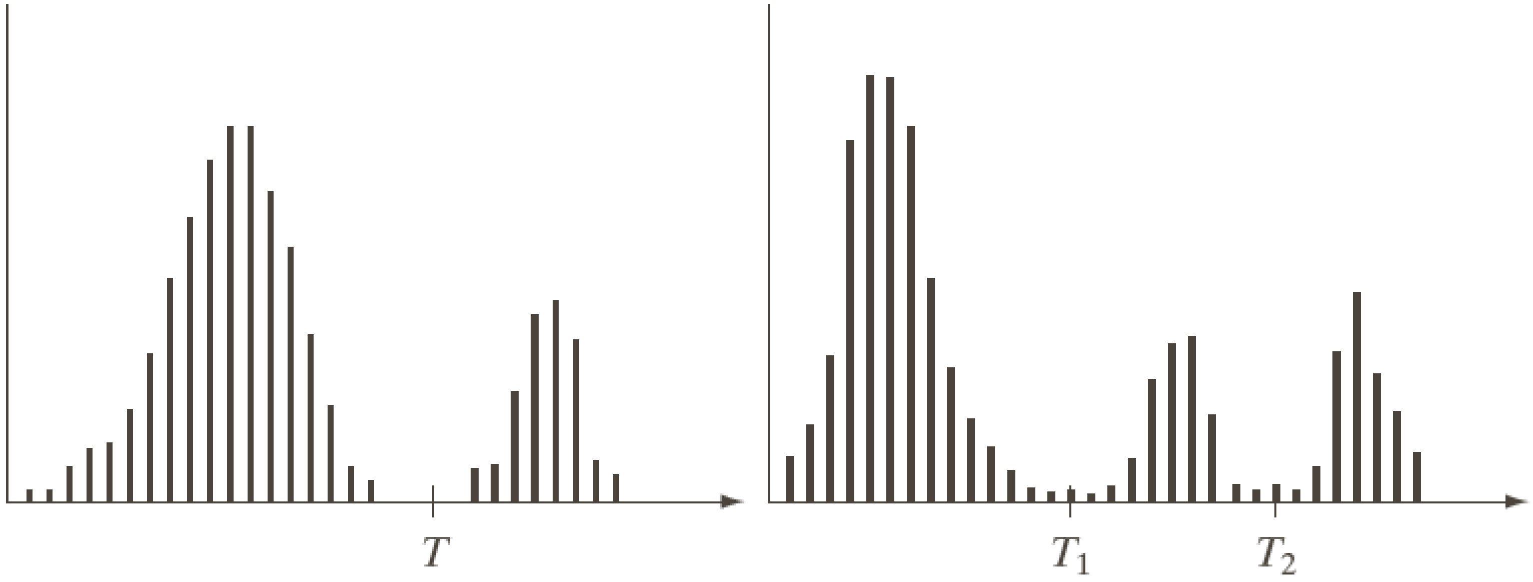
FIGURE 10.1 (a) Image containing a region of constant intensity. (b) Image showing the boundary of the inner region, obtained from intensity discontinuities. (c) Result of segmenting the image into two regions. (d) Image containing a textured region. (e) Result of edge computations. Note the large number of small edges that are connected to the original boundary, making it difficult to find a unique boundary using only edge information. (f) Result of segmentation based on region properties.



Thresholding

- Global threshold
- Adaptive threshold
- Optimal threshold
- Local threshold

Thresholding



a b

FIGURE 10.35 Intensity histograms that can be partitioned (a) by a single threshold, and (b) by dual thresholds.

Thresholding

$$g(x,y)=\begin{cases}1 & \text{if } f(x,y)>T \quad \text{(object point)}\\ 0 & \text{if } f(x,y)\le T \quad \text{(background point)}\end{cases}$$

$T:$ global thresholding

Multiple thresholding

$$g(x,y)=\begin{cases}a & \text{if } f(x,y)>T_2\\ b & \text{if } T_1< f(x,y)\le T_2\\ c & \text{if } f(x,y)\le T_1\end{cases}$$

Basic Global Thresholding

1. Select an initial estimate for the global threshold, T.
2. Segment the image using T. It will produce two groups of pixels: G1 consisting of all pixels with intensity values > T and G2 consisting of pixels with values $\leq$ T.
3. Compute the average intensity values m1 and m2 for the pixels in G1 and G2, respectively.
4. Compute a new threshold value.

$$T = \frac{1}{2}\left(m1 + m2\right)$$

5. Repeat Steps 2 through 4 until the difference between values of T in successive iterations is smaller than a predefined parameterΔT.

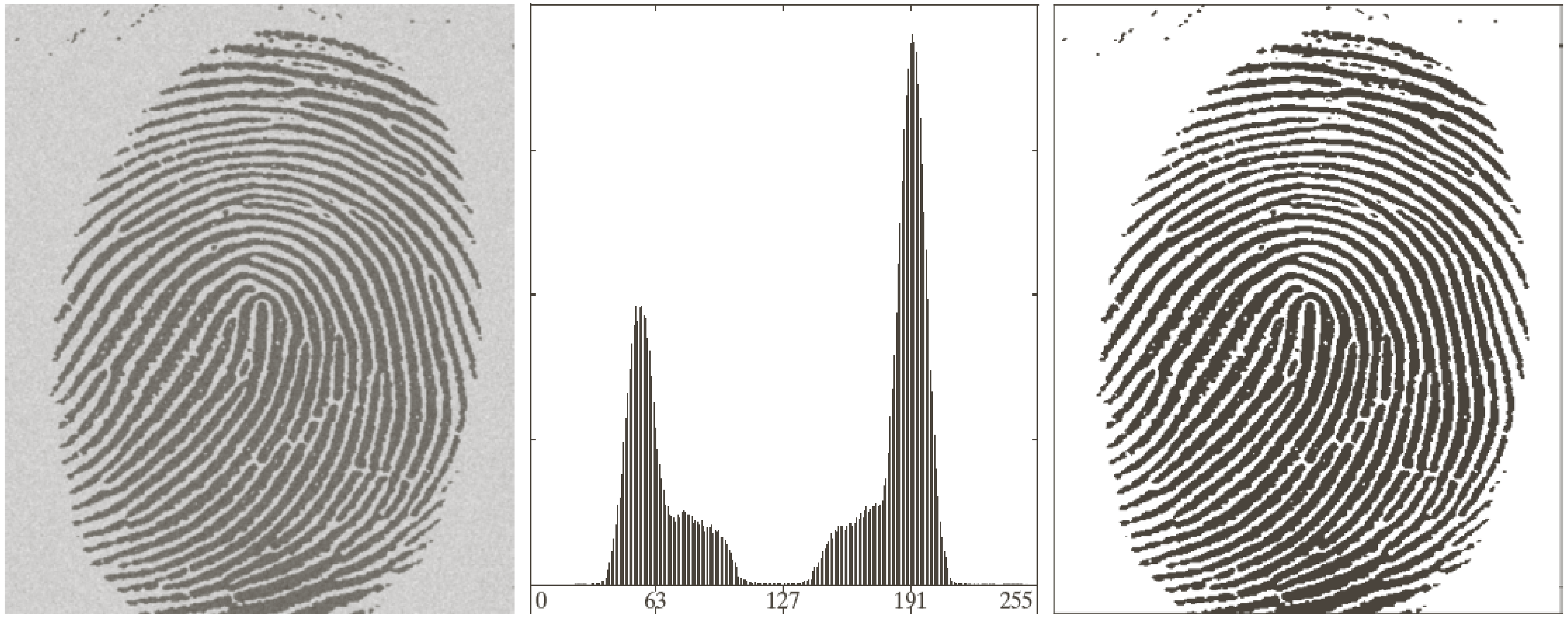


FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

The Role of Noise in Image Thresholding

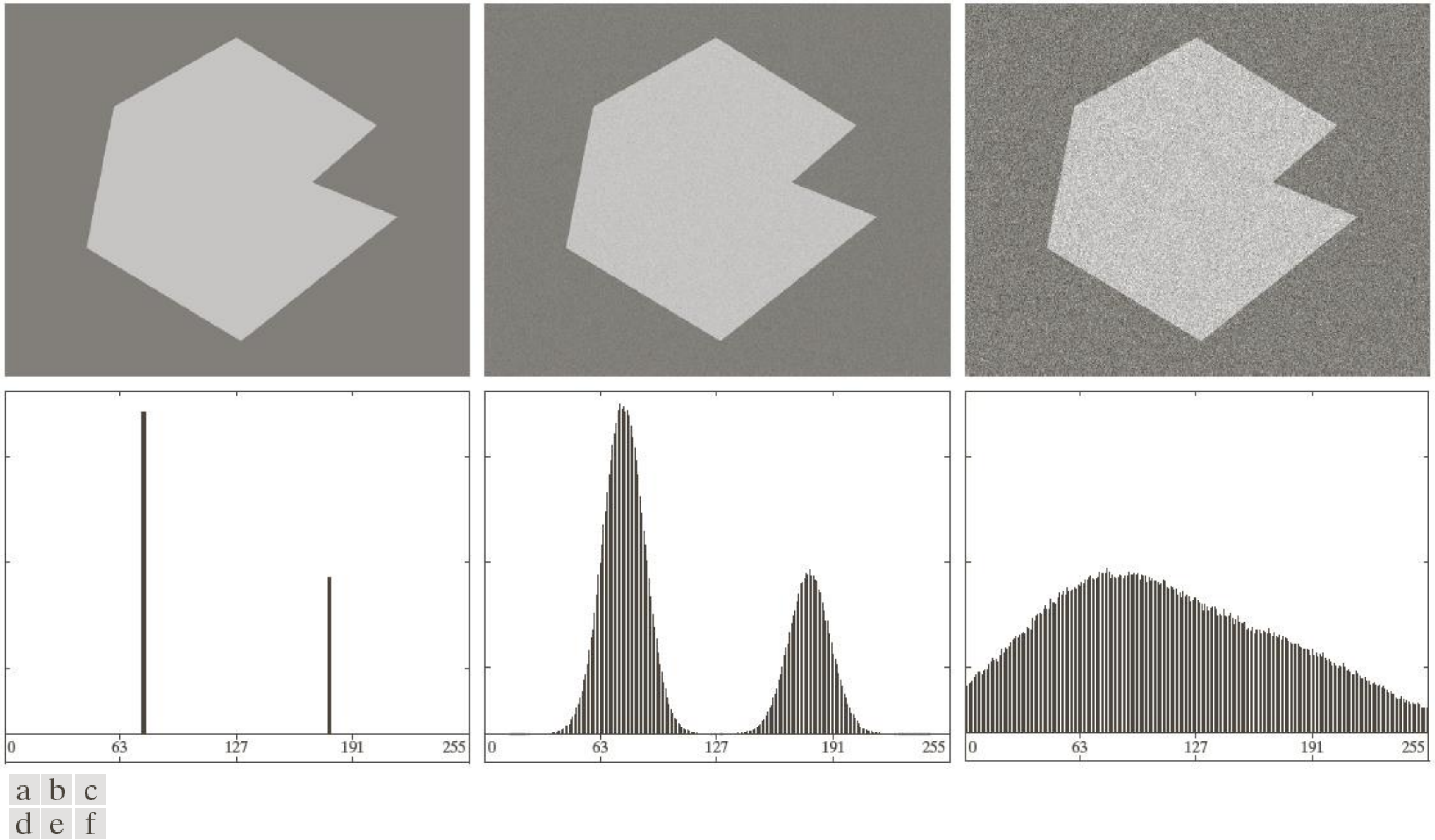


FIGURE 10.36 (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d)–(f) Corresponding histograms.

The Role of Illumination and Reflectance

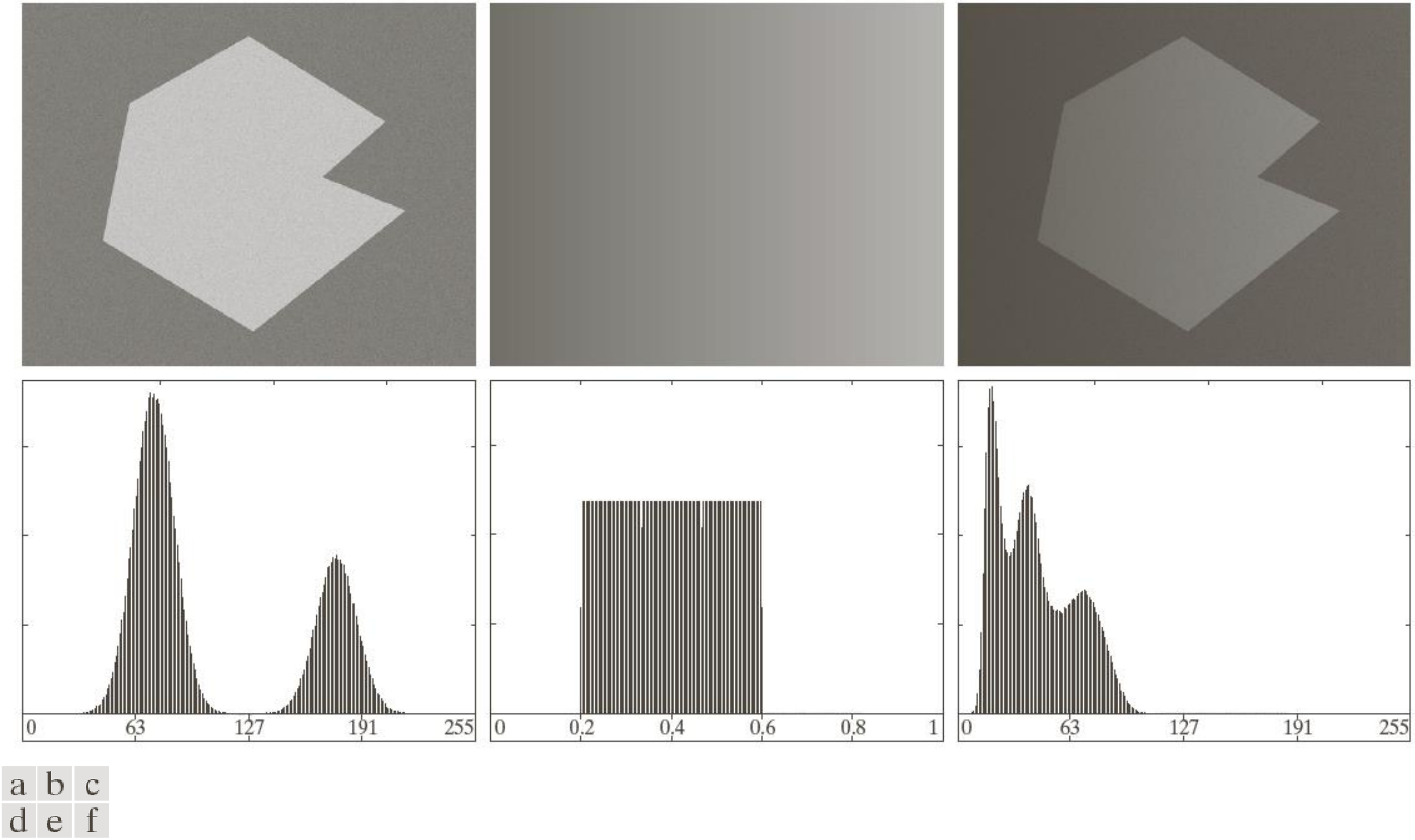


FIGURE 10.37 (a) Noisy image. (b) Intensity ramp in the range [0.2, 0.6]. (c) Product of (a) and (b). (d)–(f) Corresponding histograms.

Adaptive thresholding

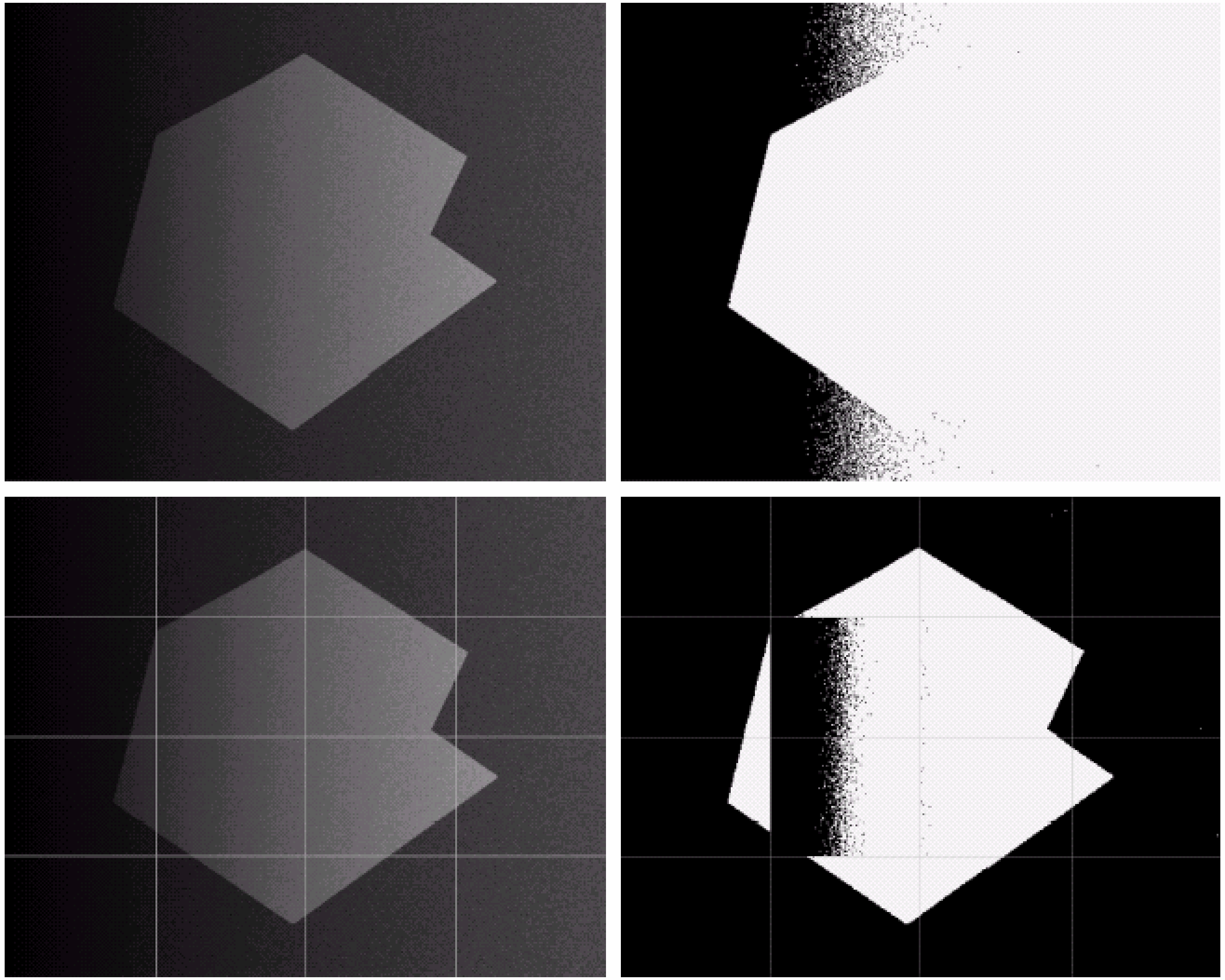
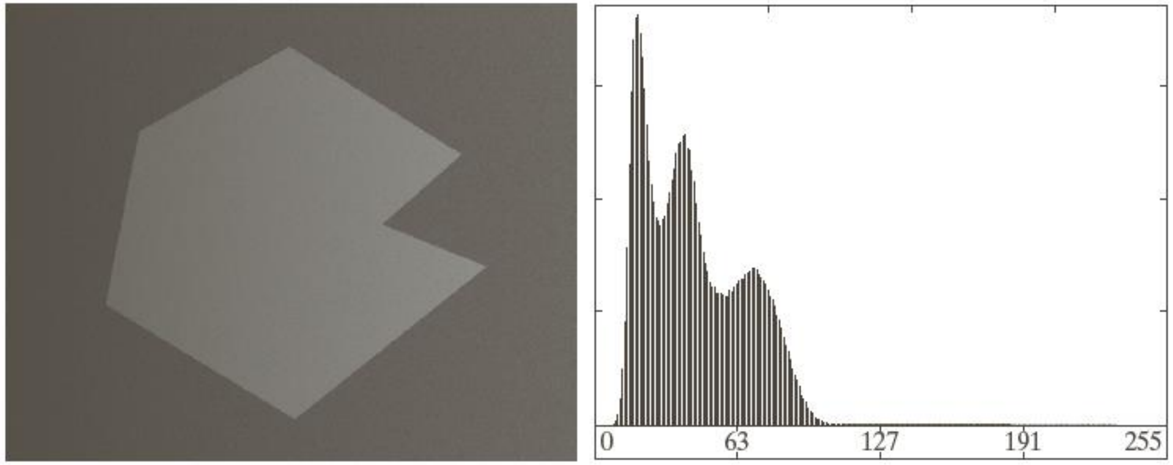
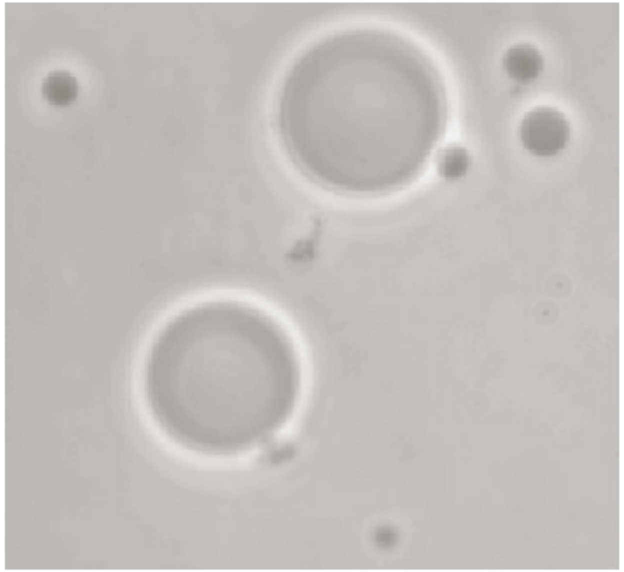


FIGURE 10.30 (a) Original image. (b) Result of global thresholding. (c) Image subdivided into individual subimages. (d) Result of adaptive thresholding.



a b c
d e f

FIGURE 10.46 (a) Noisy, shaded image and (b) its histogram. (c) Segmentation of (a) using the iterative global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (e) Image subdivided into six subimages. (f) Result of applying Otsu's method to each subimage individually.



a b
c d

FIGURE 10.39 (a) Original image. (b) Histogram (high peaks were clipped to highlight details in the lower values). (c) Segmentation result using the basic global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (Original image courtesy of Professor Daniel A. Hammer, the University of Pennsylvania.)

Optimum Global Thresholding Using Otsu’s Method

- Principle: maximizing the between-class variance

Let $\{0, 1, 2, ..., L\text{-}1\}$ denote the L distinct intensity levels in a digital image of size $M \times N$ pixels, and let n_i denote the number of pixels with intensity *i*.

$$p_i = n_i / MN \quad \text{and} \quad \sum_{i=0}^{L-1} p_i = 1$$

k is a threshold value, $C_1 \to [0, k]$, $C_2 \to [k+1, L\text{-}1]$

$$P_1(k) = \sum_{i=0}^{k} p_i \quad \text{and} \quad P_2(k) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1(k)$$

Optimum Global Thresholding Using Otsu’s Method

The mean intensity value of the pixels assigned to class C_1 is

$$m_1(k) = \sum_{i=0}^{k} iP(i / C_1) = \frac{1}{P_1(k)} \sum_{i=0}^{k} ip_i$$

The mean intensity value of the pixels assigned to class C_2 is

$$m_2(k) = \sum_{i=k+1}^{L-1} iP(i / C_2) = \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} ip_i$$

$$P_1 m_1 + P_2 m_2 = m_G \quad \text{(Global mean value)}$$

Optimum Global Thresholding Using Otsu’s Method

Between-class variance, σ_B^2 is defined as

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$$

Optimum Global Thresholding Using Otsu’s Method

The optimum threshold is the value, k*, that maximizes

$\sigma_B^2(k^*)$, $\quad \sigma_B^2(k^*) = \max\limits_{0 \le k \le L-1} \sigma_B^2(k)$

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > k^* \\ 0 & \text{if } f(x,y) \le k^* \end{cases}$$

Separability measure $\quad \eta = \dfrac{\sigma_B^2}{\sigma_G^2}$

Otsu’s Algorithm: Summary

1. Compute the normalized histogram of the input image. Denote the components of the histogram by p_i, i=0, 1, …, L-1.
2. Compute the cumulative sums, $P_1(k)$, for k = 0, 1, …, L-1.
3. Compute the cumulative means, m(k), for k = 0, 1, …, L-1.
4. Compute the global intensity mean, m_G.
5. Compute the between-class variance, for k = 0, 1, …, L-1.

Otsu’s Algorithm: Summary

6. Obtain the Otsu’s threshold, k*.

7. Obtain the separability measure.

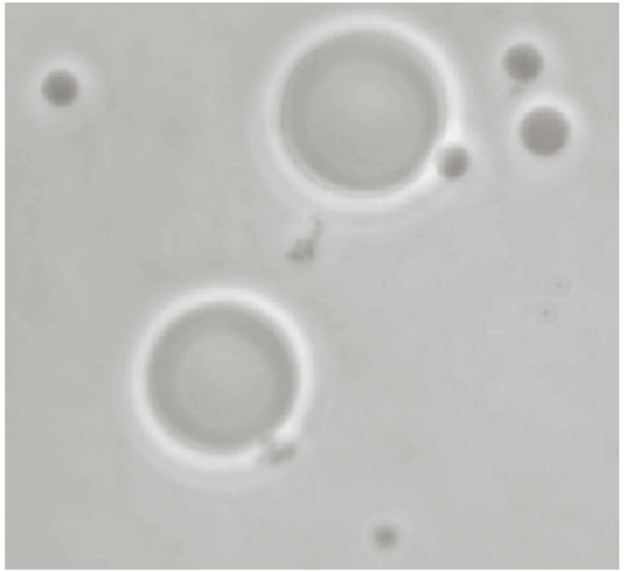
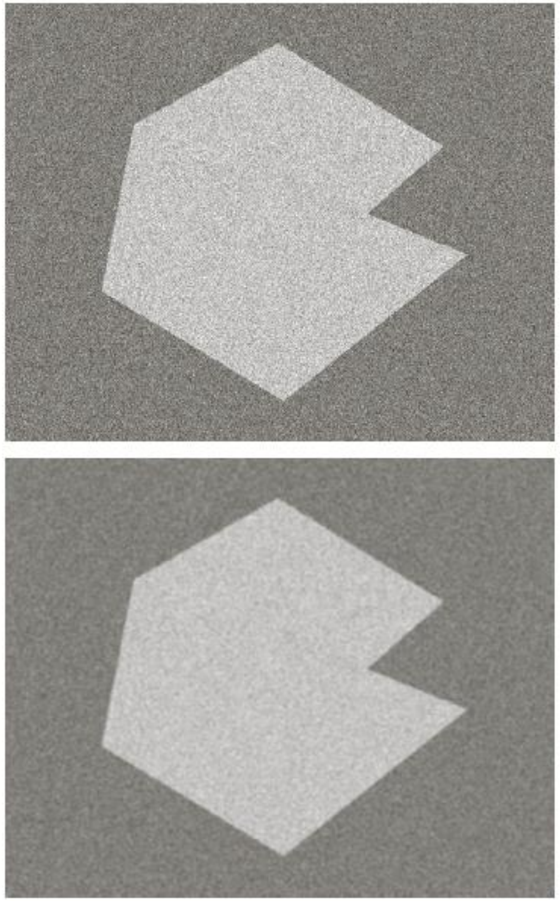


FIGURE 10.39 (a) Original image. (b) Histogram (high peaks were clipped to highlight details in the lower values). (c) Segmentation result using the basic global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (Original image courtesy of Professor Daniel A. Hammer, the University of Pennsylvania.)

Using Image Smoothing to Improve Global Thresholding



a b c
d e f

FIGURE 10.40 (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.

Using Edges to Improve Global Thresholding

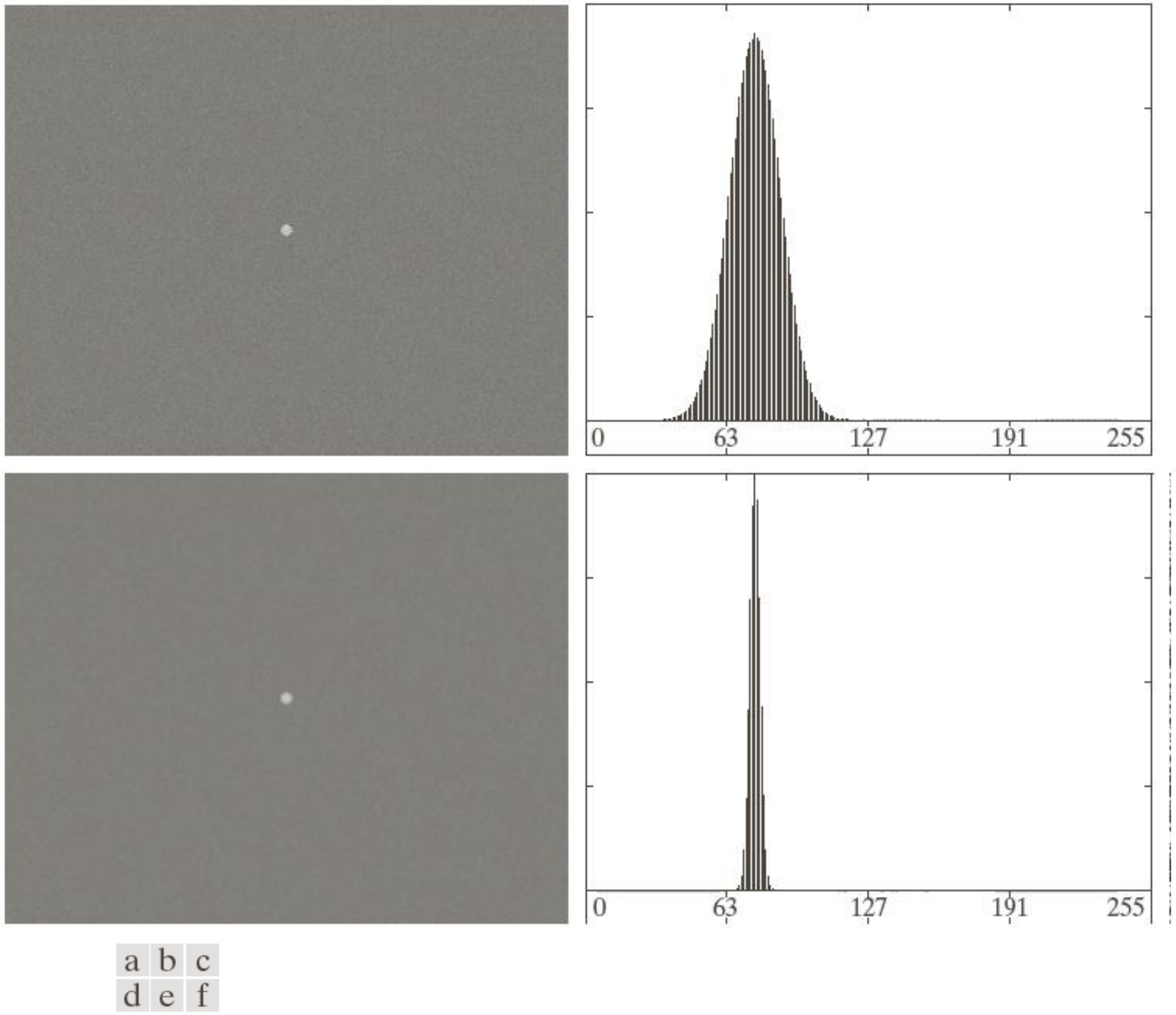
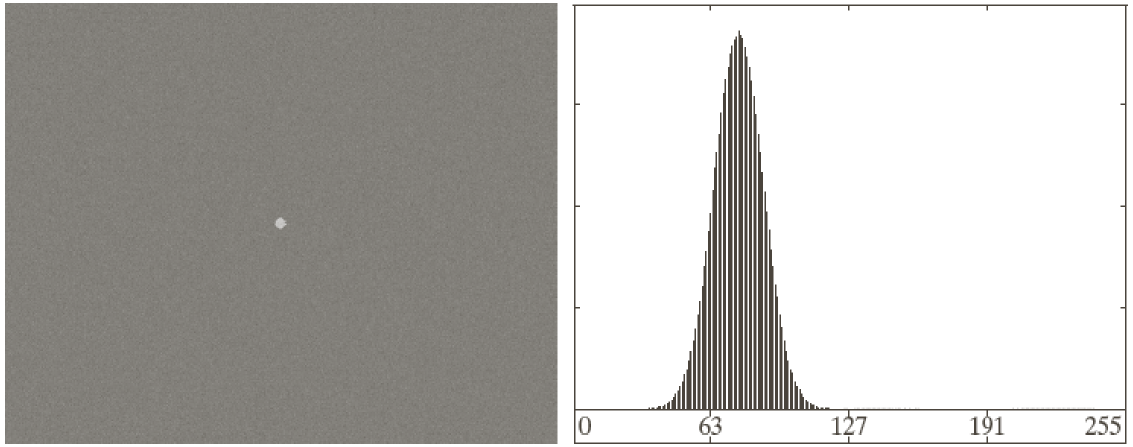


FIGURE 10.41 (a) Noisy image and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method. Thresholding failed in both cases.

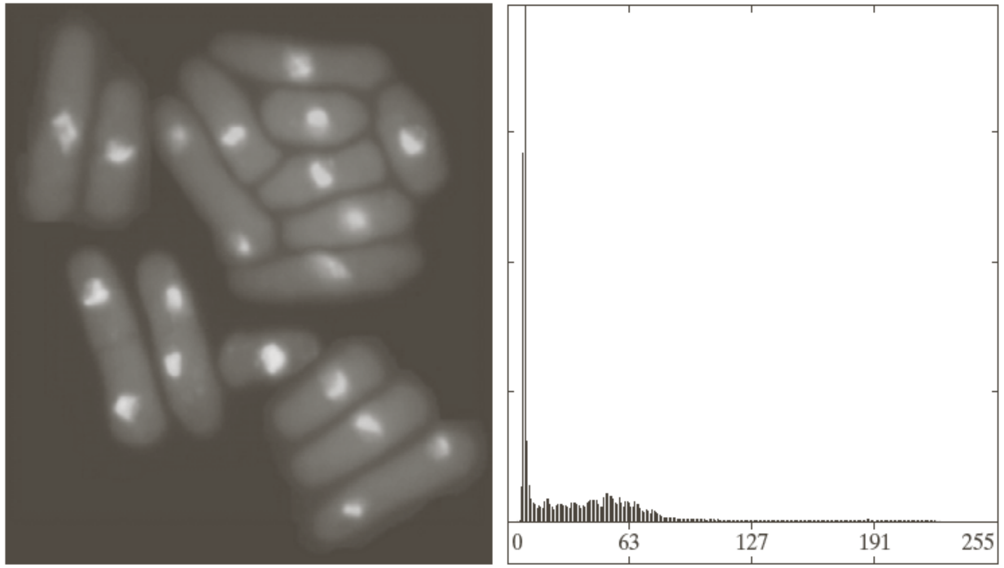
Using Edges to Improve Global Thresholding

1. Compute an edge image as either the magnitude of the gradient, or absolute value of the Laplacian of f(x,y)
2. Specify a threshold value T
3. Threshold the image and produce a binary image, which is used as a mask image; and select pixels from f(x,y) corresponding to “strong” edge pixels
4. Compute a histogram using only the chosen pixels in f(x,y)
5. Use the histogram from step 4 to segment f(x,y) globally



a b c
d e f

FIGURE 10.42 (a) Noisy image from Fig. 10.41(a) and (b) its histogram. (c) Gradient magnitude image thresholded at the 99.7 percentile. (d) Image formed as the product of (a) and (c). (e) Histogram of the nonzero pixels in the image in (d). (f) Result of segmenting image (a) with the Otsu threshold based on the histogram in (e). The threshold was 134, which is approximately midway between the peaks in this histogram.



a b c
d e f

FIGURE 10.43 (a) Image of yeast cells. (b) Histogram of (a). (c) Segmentation of (a) with Otsu's method using the histogram in (b). (d) Thresholded absolute Laplacian. (e) Histogram of the nonzero pixels in the product of (a) and (d). (f) Original image thresholded using Otsu's method based on the histogram in (e). (Original image courtesy of Professor Susan L. Forsburg, University of Southern California.)

Multiple Thresholds

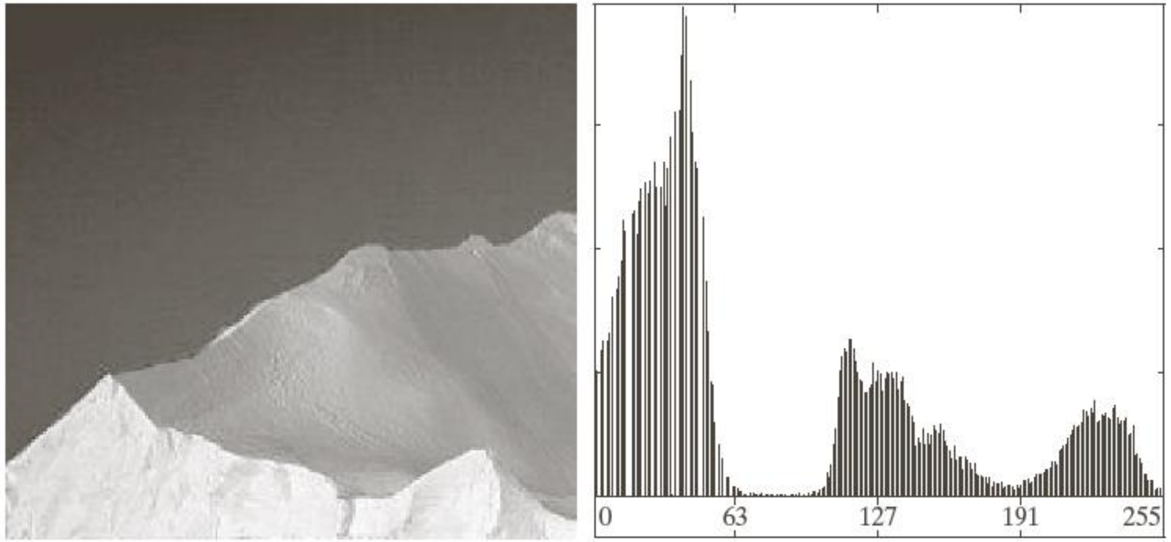
In the case of K classes, $C_1,\ C_2,\ ...,\ C_K$, the between-class variance is

$$\sigma_B^2 = \sum_{k=1}^{K} P_k \left(m_k - m_G\right)^2$$

where $\quad P_k = \sum_{i \in C_k} p_i \quad$ and $\quad m_k = \frac{1}{P_k} \sum_{i \in C_k} i p_i$

The optimum threshold values, $k_1*, k_2*,\ ..., k_{K-1}*$ that maximize

$$\sigma_B^2(k_1*, k_2*,\ ..., k_{K-1}*) = \max_{0 \le k \le L-1} \sigma_B^2(k_1, k_2,\ ..., k_{K-1})$$



a b c

FIGURE 10.45 (a) Image of iceberg. (b) Histogram. (c) Image segmented into three regions using dual Otsu thresholds. (Original image courtesy of NOAA.)

Variable Thresholding Based on Local Image Properties

Let σ_{xy} and m_{xy} denote the standard deviation and mean value of the set of pixels contained in a neighborhood S_{xy}, centered at coordinates $(x,\ y)$ in an image. The local thresholds,

$$T_{xy} \ = \ a\sigma_{xy} + bm_{xy}$$

If the background is nearly constant,

$$T_{xy} \ = \ a\sigma_{xy} + bm$$

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T_{xy} \\ 0 & \text{if } f(x,y) \le T_{xy} \end{cases}$$

Variable Thresholding Based on Local Image Properties

A modified thresholding

$$g(x,y)=\begin{cases}1 & \text{if } Q(\text{local parameters}) \text{ is true}\\ 0 & \text{otherwise}\end{cases}$$

e.g.,

$$Q(\sigma_{xy},m_{xy})=\begin{cases}\text{true} & \text{if } f(x,y)>a\sigma_{xy} \text{ AND } f(x,y)>bm_{xy}\\ \text{false} & \text{otherwise}\end{cases}$$

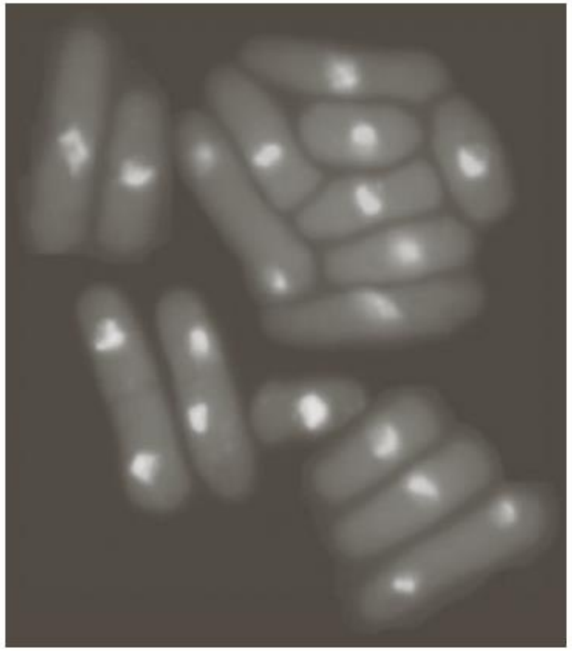


FIGURE 10.48 (a) Image from Fig. 10.43. (b) Image segmented using the dual thresholding approach discussed in Section 10.3.6. (c) Image of local standard deviations. (d) Result obtained using local thresholding.

a=30
b=1.5
$m_{xy} = m_G$

Variable Thresholding Using Moving Averages

- Thresholding based on moving averages works well when the objects are small with respect to the image size

- Quite useful in document processing

- The scanning (moving) typically is carried out line by line in zigzag pattern to reduce illumination bias

Variable Thresholding Using Moving Averages

Let z_{k+1} denote the intensity of the point encountered in the scanning sequence at step $k+1$. The moving average (mean intensity) at this new point is given by

$$m(k+1)=\frac{1}{n}\sum_{i=k+2-n}^{k+1} z_i = m(k)+\frac{1}{n}(z_{k+1}-z_k)$$

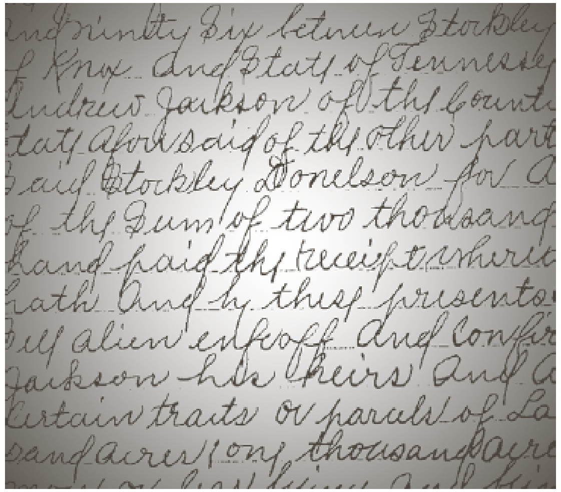
where n denotes the number of points used in computing the average and $m(1)=z_1/n$, the border of the image were padded with n-1 zeros.

Variable Thresholding Using Moving Averages

$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T_{xy} \\ 0 & \text{if } f(x,y) \le T_{xy} \end{cases}$$

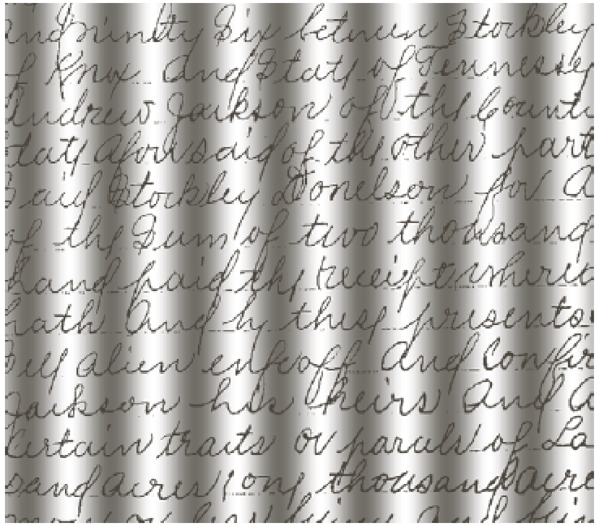
$$T_{xy} = bm_{xy}$$

N = 20
b=0.5



and ninety six between Stokley
of Knox and State of Tennessee
Andrew Jackson of the County
State aforesaid of the other part
said Stokley Donelson for a
of the Sum of two thousand
hand paid the receipt whereof
hath and by these presents
sell alien enfeoff and confirm
Jackson his heirs and a
certain traits or parcels of La
sand acres one thousand acre

FIGURE 10.49 (a) Text image corrupted by spot shading. (b) Result of global thresholding using Otsu's method. (c) Result of local thresholding using moving averages.



a b c

FIGURE 10.50 (a) Text image corrupted by sinusoidal shading. (b) Result of global thresholding using Otsu's method. (c) Result of local thresholding using moving averages.

Region-Based Segmentation

- Region Growing

1. Region growing is a procedure that groups pixels or subregions into larger regions.

2. The simplest of these approaches is ***pixel aggregation***, which starts with a set of “seed” points and from these grows regions by appending to each seed points those **neighboring pixels** that have **similar properties** (such as gray level, texture, color, shape).

3. Region growing based techniques are better than the edge-based techniques in noisy images where edges are difficult to detect.

Region-Based Segmentation

Example: Region Growing based on 8-connectivity

$f(x, y)$: input image array

$S(x, y)$: seed array containing 1s (seeds) and 0s

$Q(x, y)$: predicate

Region Growing based on 8-connectivity

1. Find all connected components in $S(x, y)$ and erode each connected components to one pixel; label all such pixels found as 1. All other pixels in S are labeled 0.

$$Q = \begin{cases} \text{TRUE} & \text{if the absolute difference of the intensities} \\ & \text{between the seed and the pixel at (x,y) is } \le \text{T} \\ \text{FALSE} & \text{otherwise} \end{cases}$$

Suppose that we have the image given below.

(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

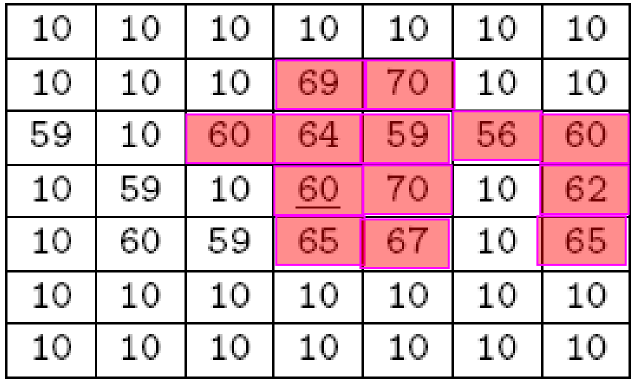
Table 1: Show the result of Part (a) on this figure.

10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	$\underline{60}$	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

Suppose that we have the image given below.

(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

Table 1: Show the result of Part (a) on this figure.



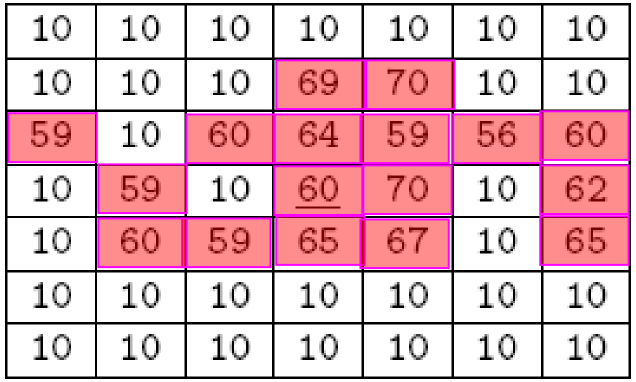
10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	60	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

4-connectivity

Suppose that we have the image given below.

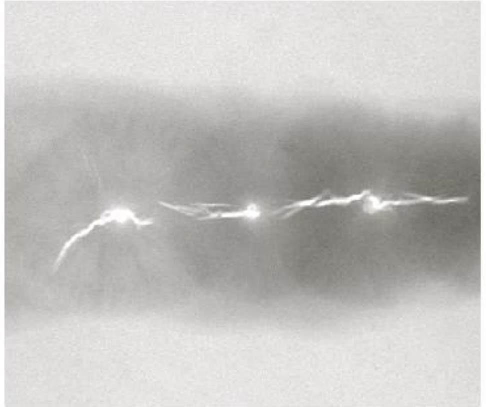
(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

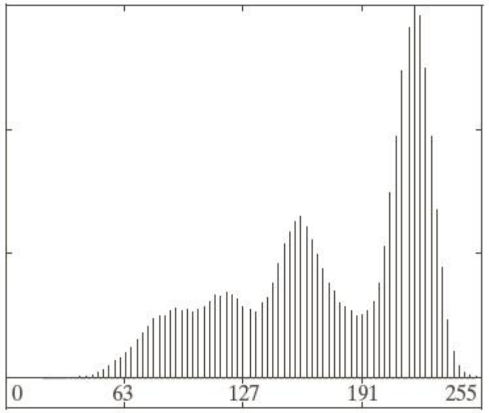
Table 1: Show the result of Part (a) on this figure.



10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	60	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

8-connectivity




0
63
127
191
255

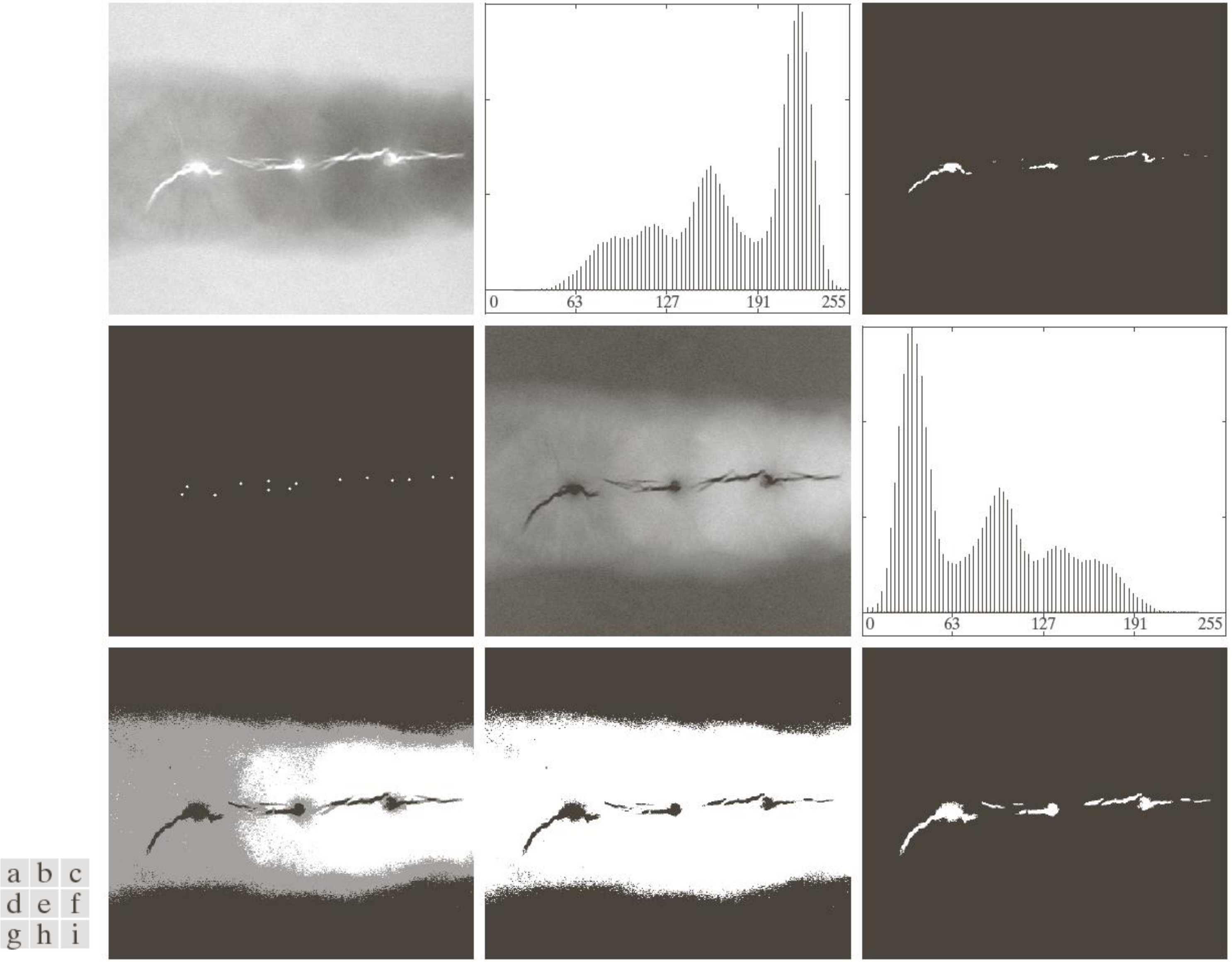
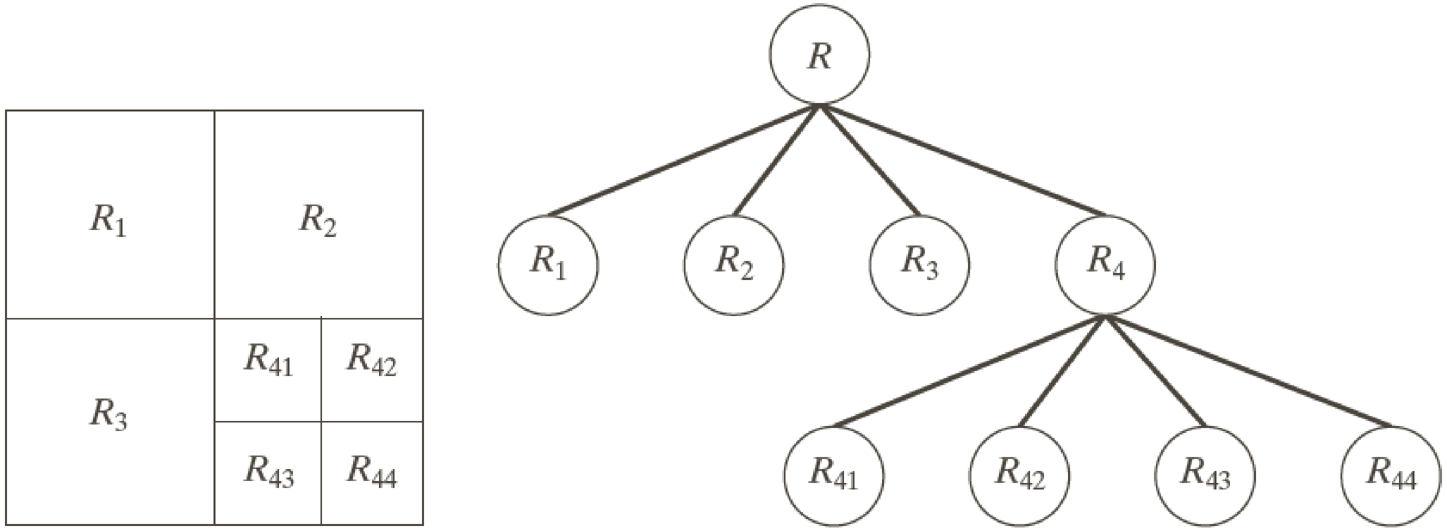


FIGURE 10.51 (a) X-ray image of a defective weld. (b) Histogram. (c) Initial seed image. (d) Final seed image (the points were enlarged for clarity). (e) Absolute value of the difference between (a) and (c). (f) Histogram of (e). (g) Difference image thresholded using dual thresholds. (h) Difference image thresholded with the smallest of the dual thresholds. (i) Segmentation result obtained by region growing. (Original image courtesy of X-TEK Systems, Ltd.)

Region Splitting and Merging

R : entire image $\quad R_i$:entire image $\quad Q$: predicate



a b

FIGURE 10.52 (a) Partitioned image. (b) Corresponding quadtree. *R* represents the entire image region.

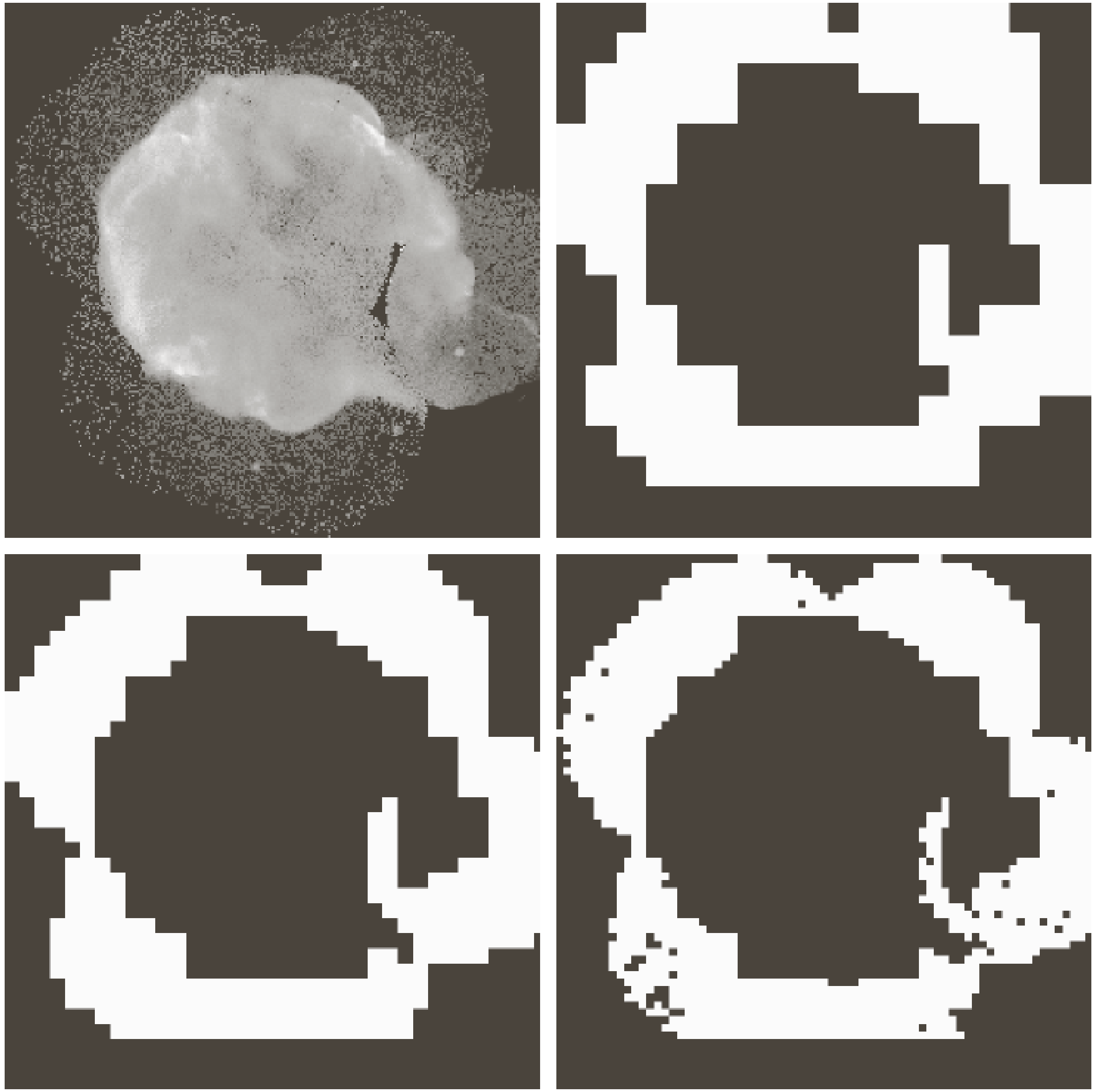


FIGURE 10.53 (a) Image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of limiting the smallest allowed quadregion to sizes of 32×32, 16×16, and 8×8 pixels, respectively. (Original image courtesy of NASA.)

$$Q = \begin{cases} \text{TRUE} & \text{if } \sigma > a \text{ and } 0 < m < b \\ \text{FALSE} & \text{otherwise} \end{cases}$$

K-means Clustering

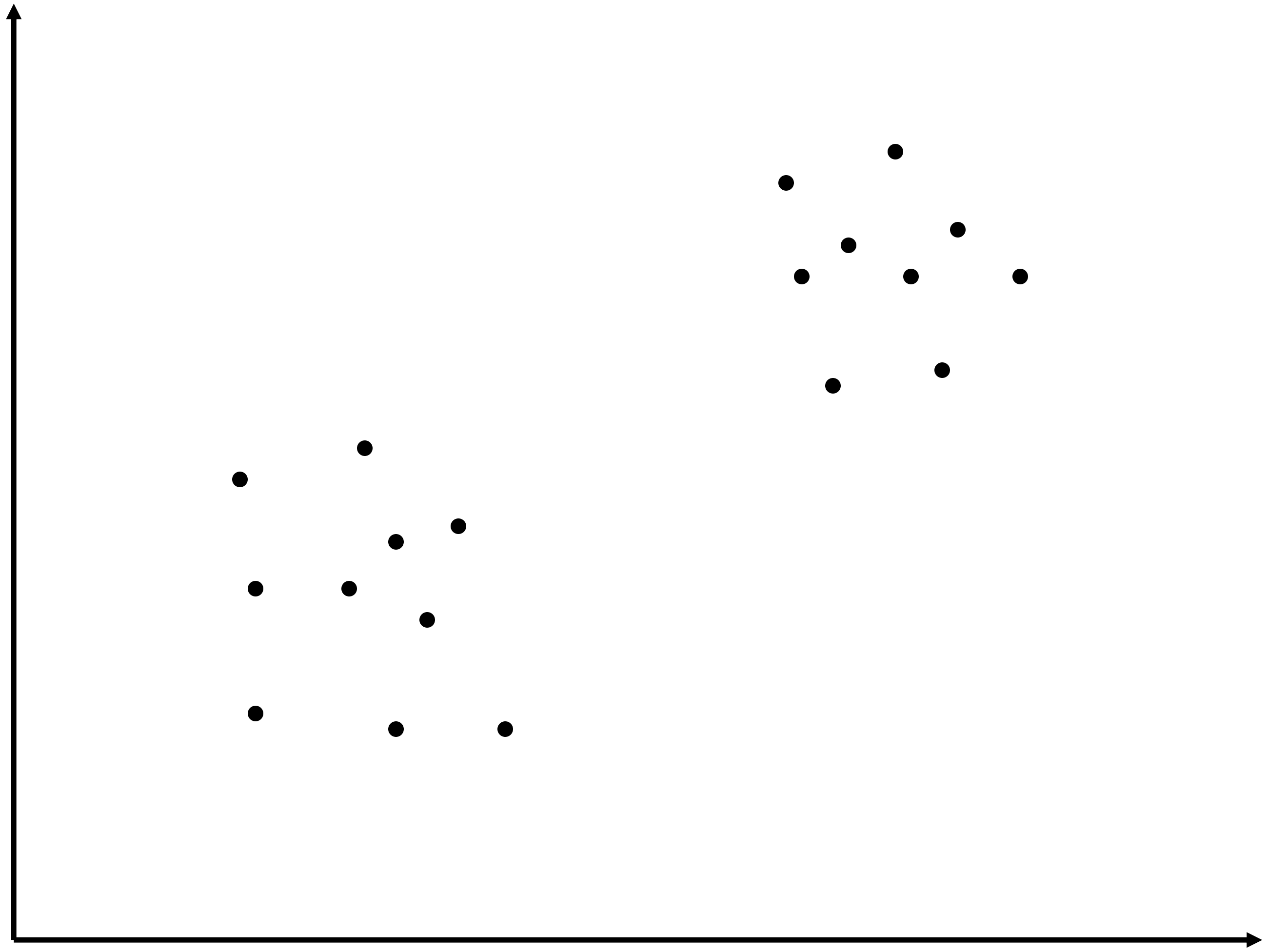
- Partition the data points into K clusters randomly. Find the centroids of each cluster.

- For each data point:
  - Calculate the distance from the data point to each cluster.
  - Assign the data point to the closest cluster.

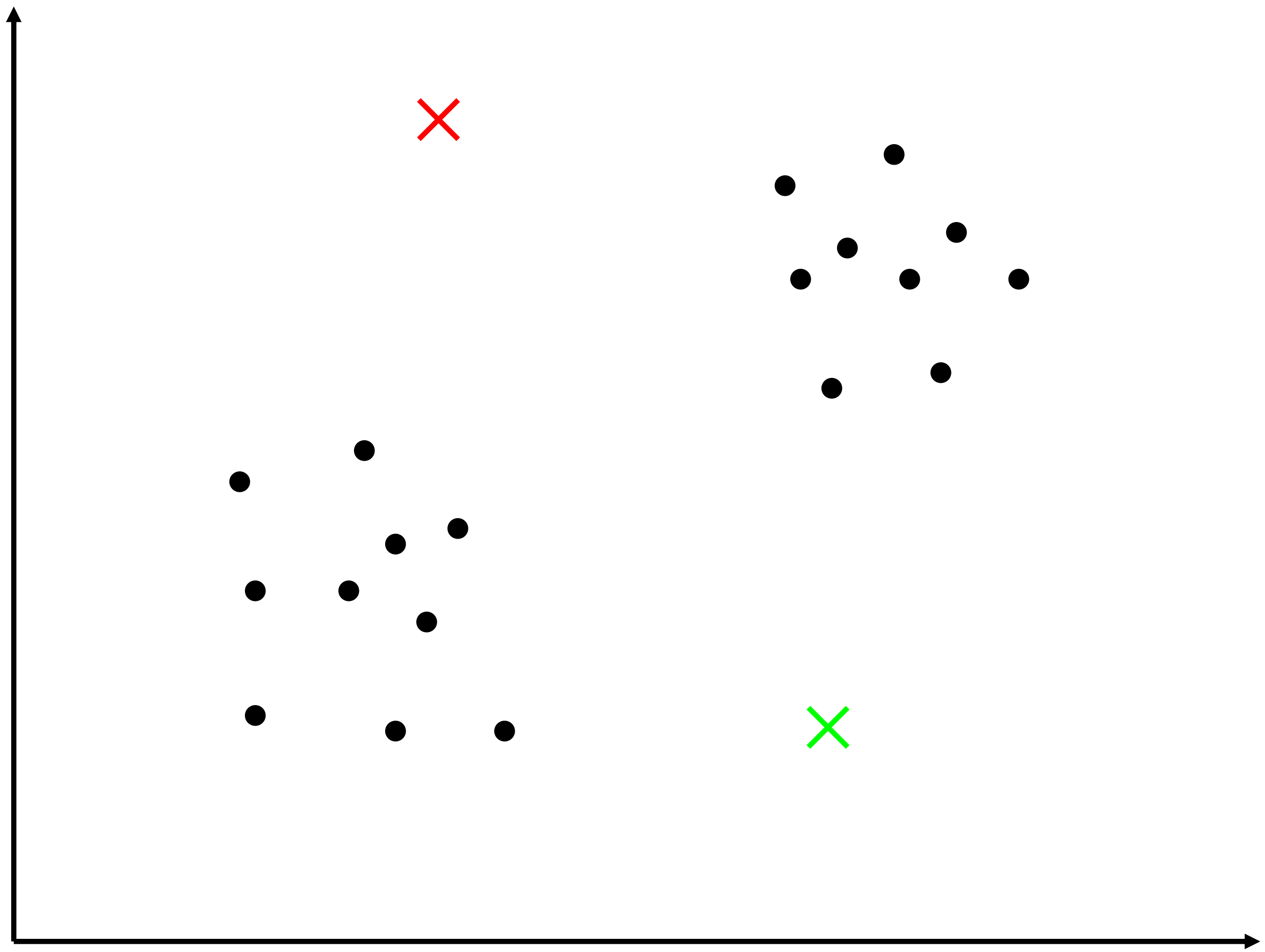
- Recompute the centroid of each cluster.

- Repeat steps 2 and 3 until there is no further change in the assignment of data points (or in the centroids).

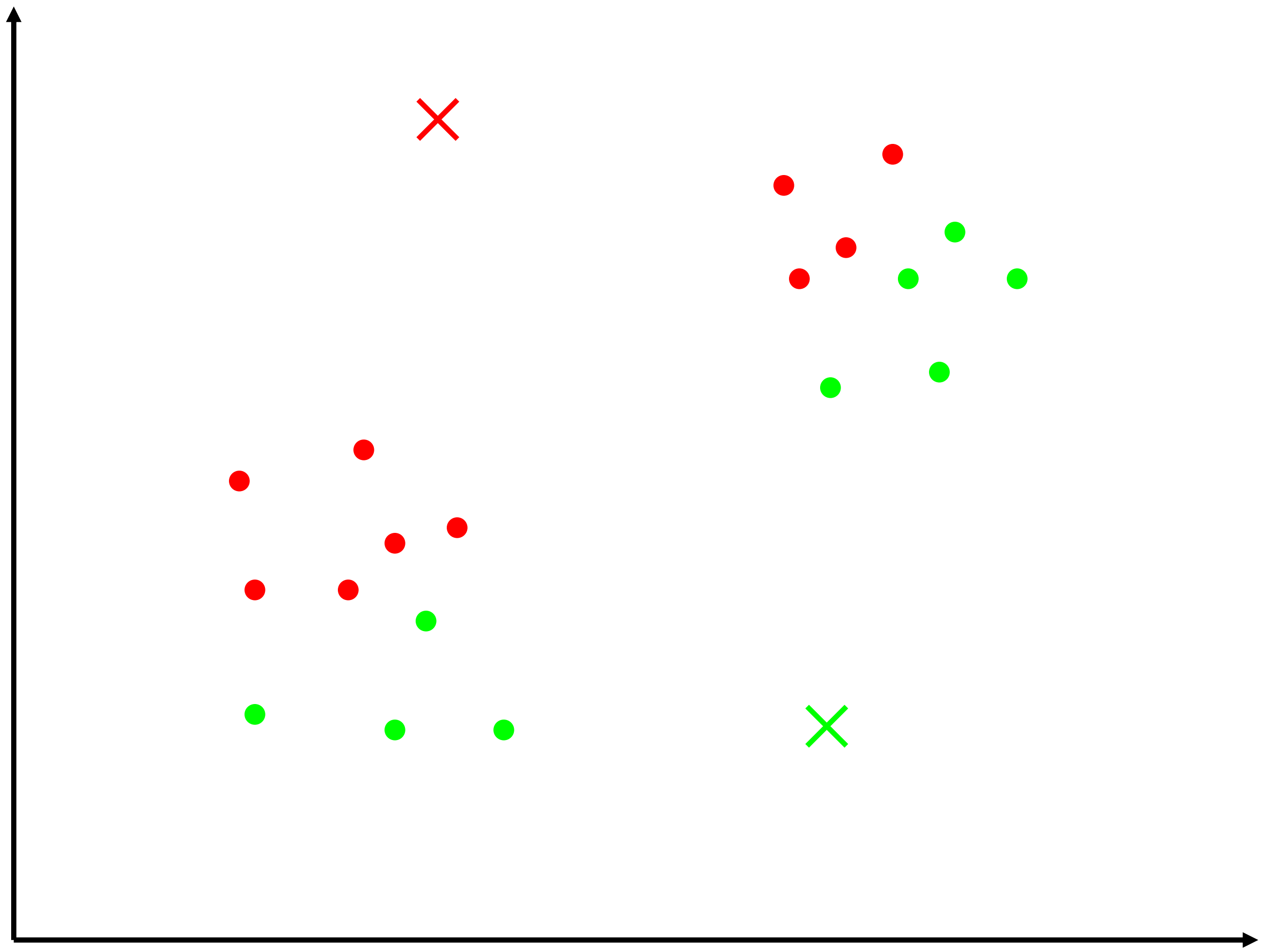
K-Means Clustering



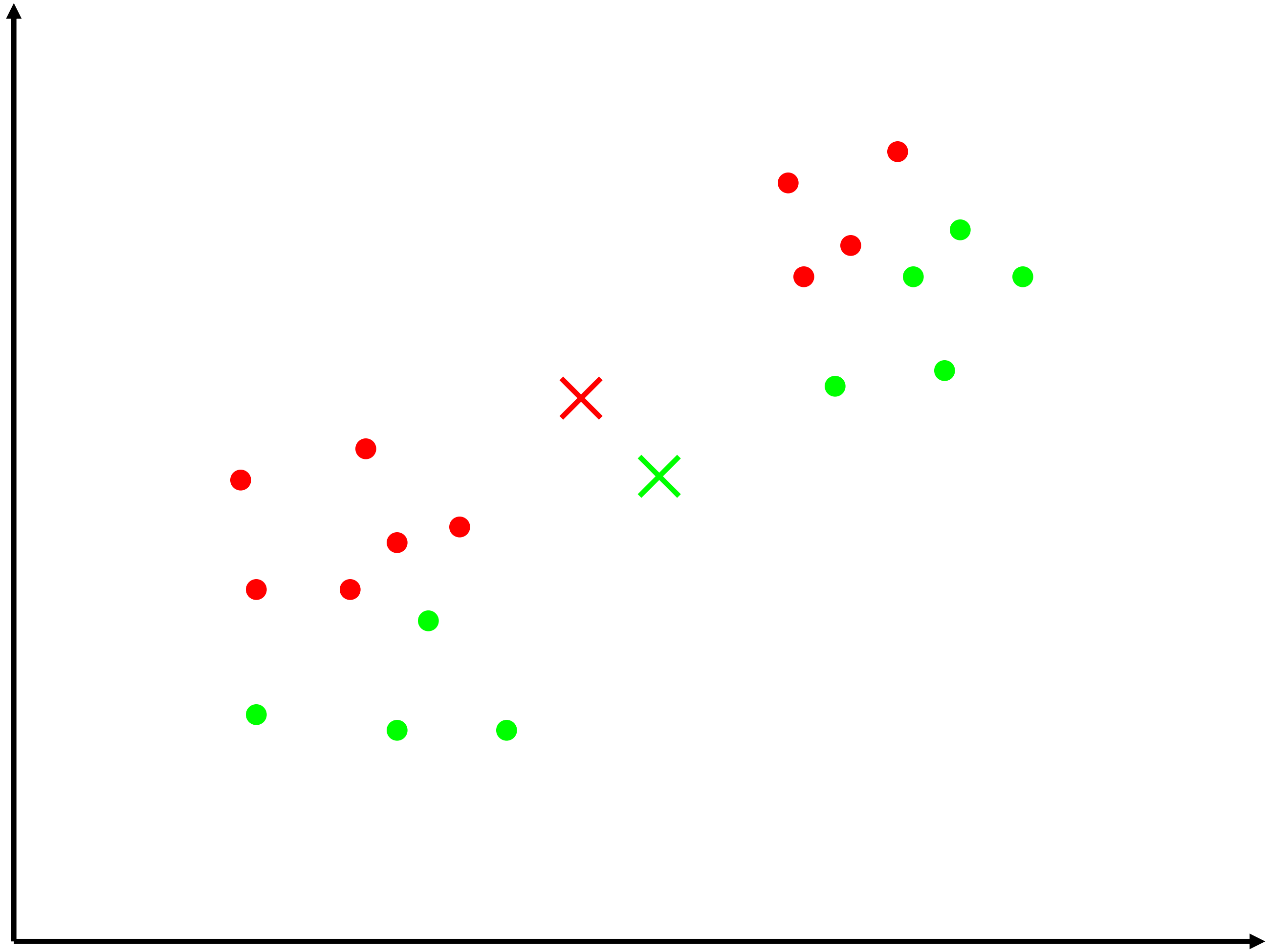
K-Means Clustering



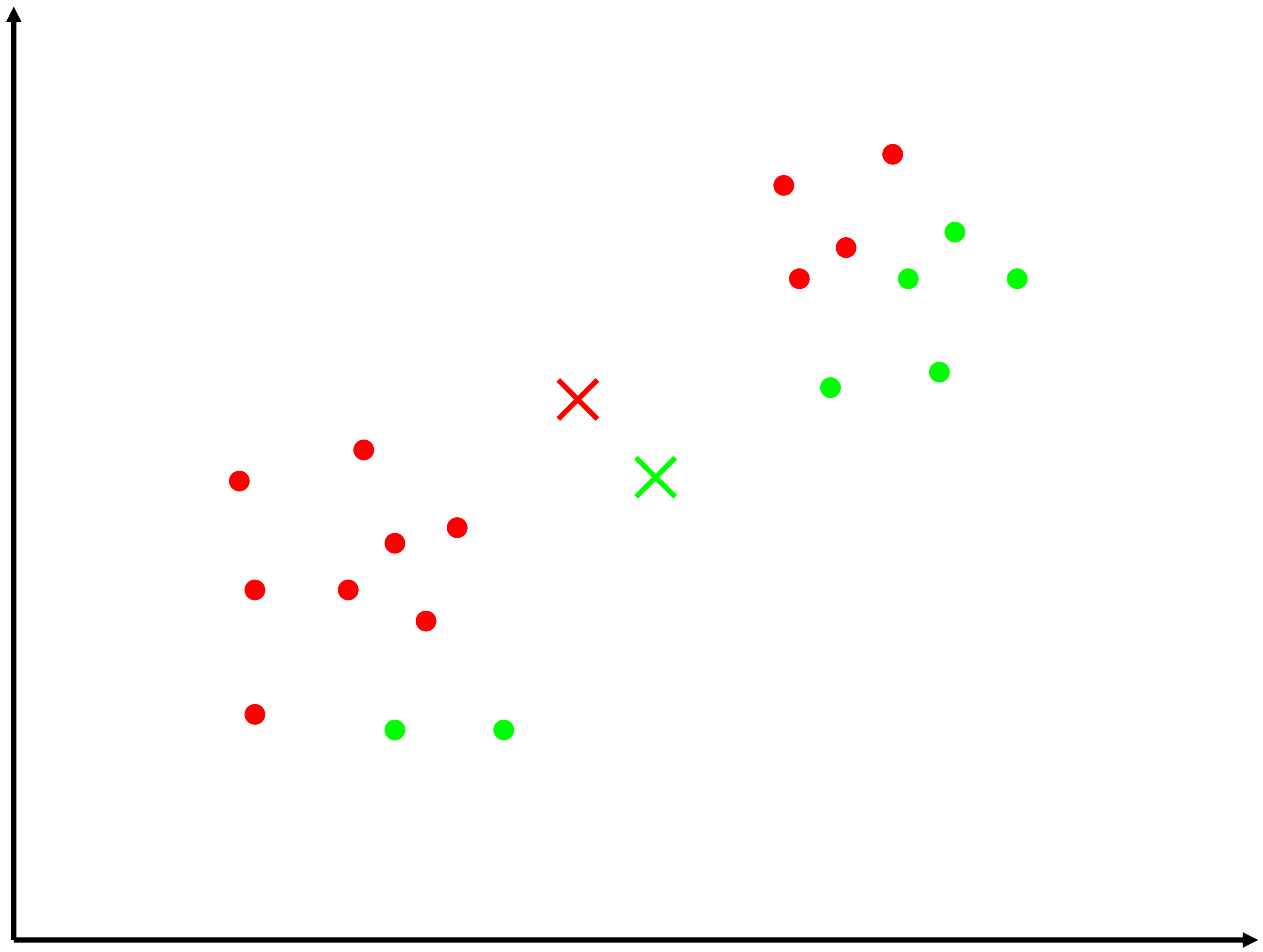
K-Means Clustering



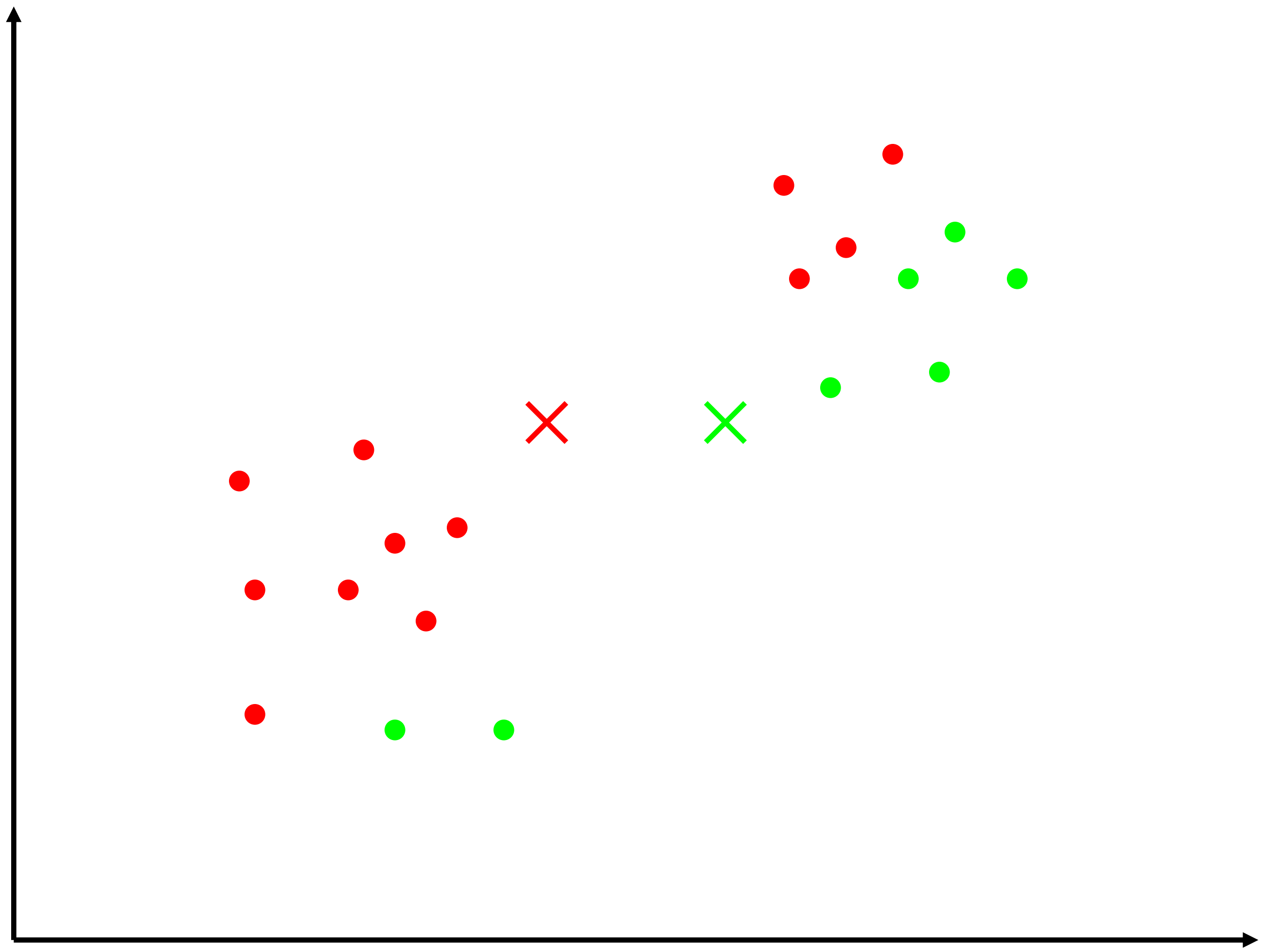
K-Means Clustering



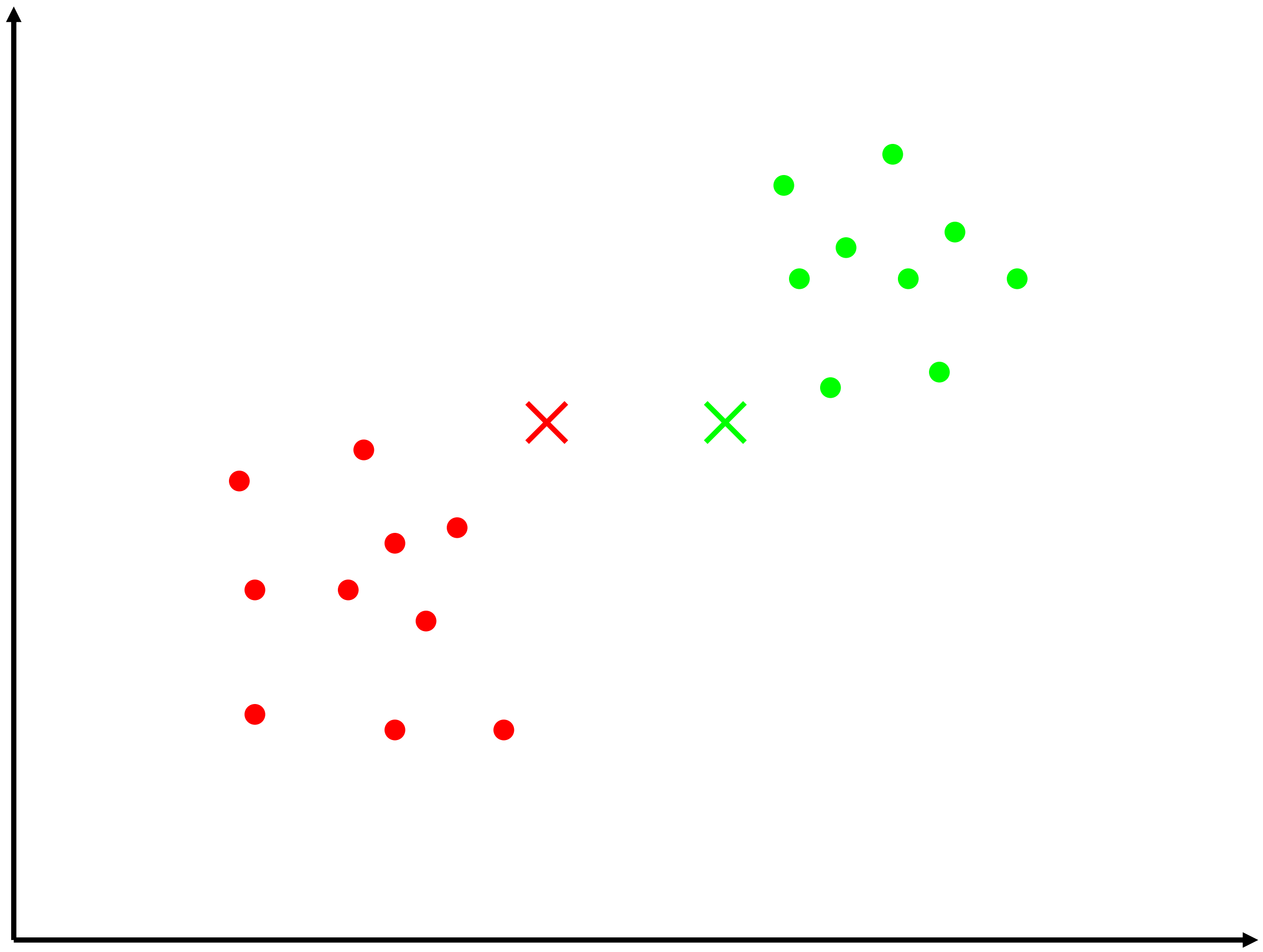
K-Means Clustering



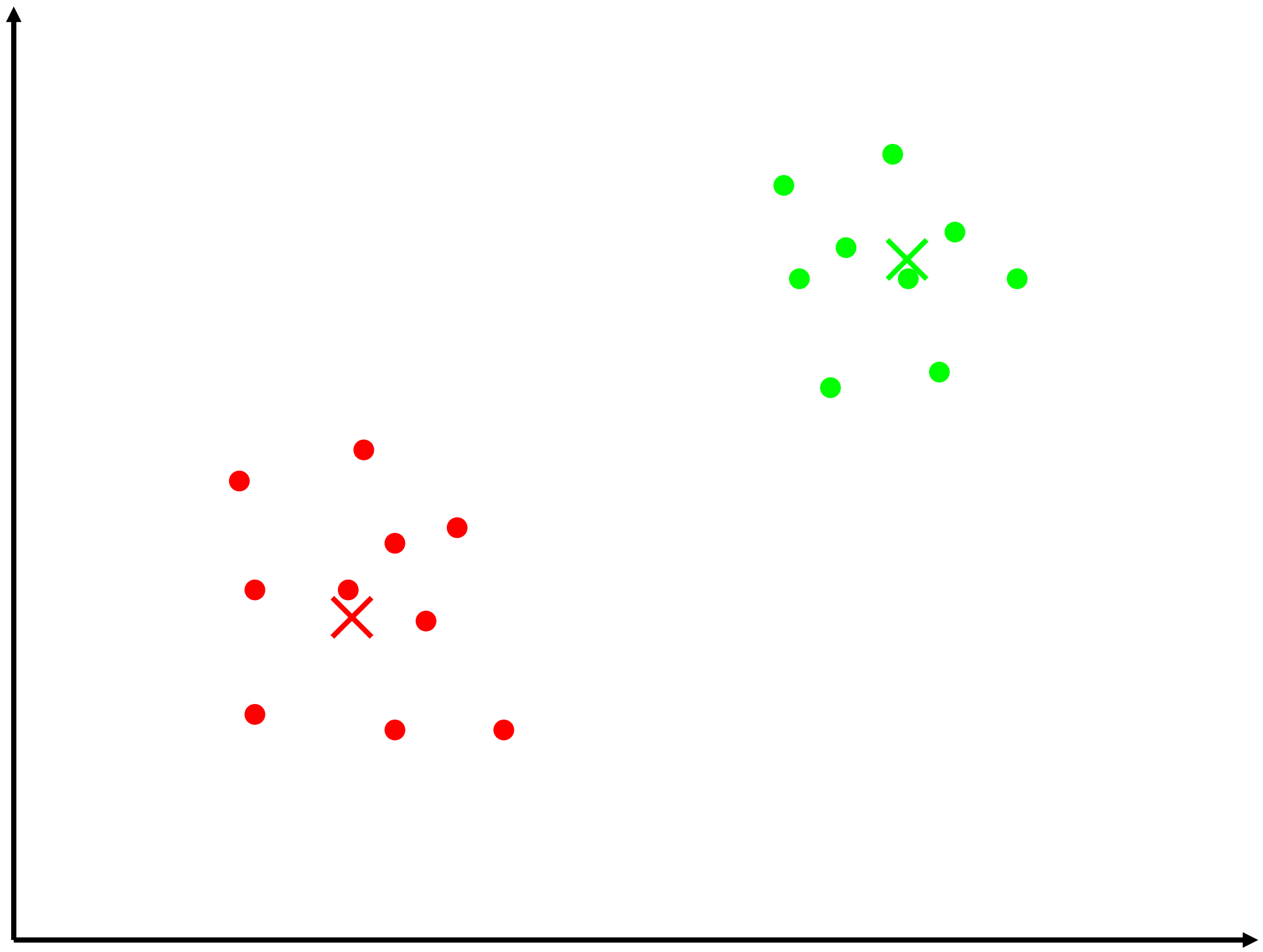
K-Means Clustering



K-Means Clustering

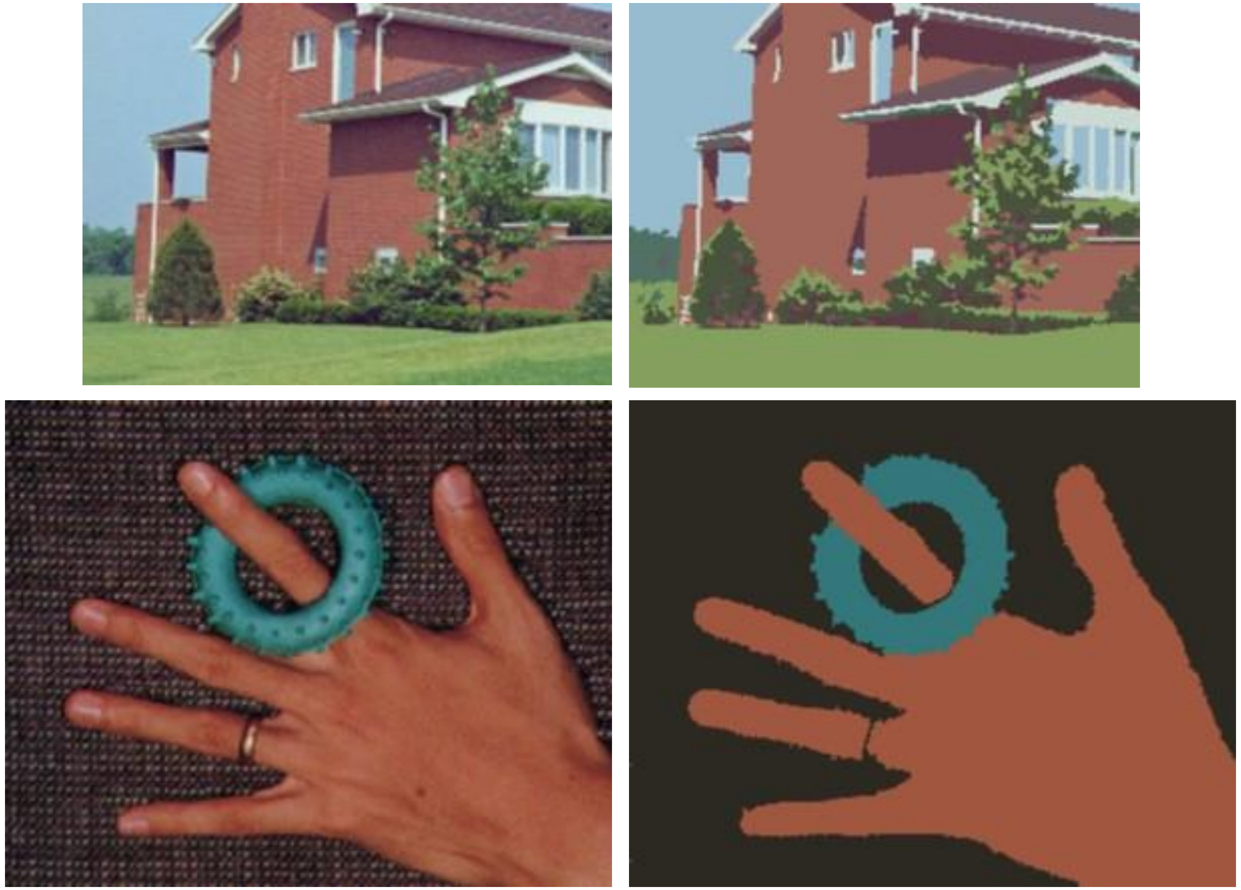


K-Means Clustering



Clustering

- Example



D. Comaniciu and P. Meer, *Robust Analysis of Feature Spaces: Color Image Segmentation*, 1997.