

Diagram illustrating the structure of the Lie algebra  $\mathfrak{so}(10)$  and its decomposition into irreducible representations of  $\mathfrak{so}(8)$  and  $\mathfrak{so}(6)$ . The diagram shows the root system of  $\mathfrak{so}(10)$  at the top, branching down into the root systems of  $\mathfrak{so}(8)$  and  $\mathfrak{so}(6)$ , and further into the root systems of the irreducible representations of  $\mathfrak{so}(8)$  and  $\mathfrak{so}(6)$ .

The root system of  $\mathfrak{so}(10)$  is shown at the top, with roots labeled by their coordinates in the  $\mathbb{R}^{10}$  space. The roots are:  $(1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ ,  $(-1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, 1, 0, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, -1, 0, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, -1, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, 1, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, -1, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, 1, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, -1, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, 1, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, -1, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, 1, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, -1, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, 1, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, -1, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, 0, 1, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, 0, -1, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$ ,  $(0, 0, 0, 0, 0, 0, 0, 0, 0, -1)$ .

The root system of  $\mathfrak{so}(8)$  is shown in the middle, with roots labeled by their coordinates in the  $\mathbb{R}^8$  space. The roots are:  $(1, 0, 0, 0, 0, 0, 0, 0)$ ,  $(-1, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, 1, 0, 0, 0, 0, 0, 0)$ ,  $(0, -1, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, 1, 0, 0, 0, 0, 0)$ ,  $(0, 0, -1, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, 1, 0, 0, 0, 0)$ ,  $(0, 0, 0, -1, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, 1, 0, 0, 0)$ ,  $(0, 0, 0, 0, -1, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, 1, 0, 0)$ ,  $(0, 0, 0, 0, 0, -1, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, 1, 0)$ ,  $(0, 0, 0, 0, 0, 0, -1, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, 1)$ ,  $(0, 0, 0, 0, 0, 0, 0, -1)$ .

The root system of  $\mathfrak{so}(6)$  is shown at the bottom, with roots labeled by their coordinates in the  $\mathbb{R}^6$  space. The roots are:  $(1, 0, 0, 0, 0, 0)$ ,  $(-1, 0, 0, 0, 0, 0)$ ,  $(0, 1, 0, 0, 0, 0)$ ,  $(0, -1, 0, 0, 0, 0)$ ,  $(0, 0, 1, 0, 0, 0)$ ,  $(0, 0, -1, 0, 0, 0)$ ,  $(0, 0, 0, 1, 0, 0)$ ,  $(0, 0, 0, -1, 0, 0)$ ,  $(0, 0, 0, 0, 1, 0)$ ,  $(0, 0, 0, 0, -1, 0)$ ,  $(0, 0, 0, 0, 0, 1)$ ,  $(0, 0, 0, 0, 0, -1)$ .

The diagram also shows the decomposition of the irreducible representations of  $\mathfrak{so}(8)$  and  $\mathfrak{so}(6)$  into irreducible representations of  $\mathfrak{so}(8)$  and  $\mathfrak{so}(6)$ . The representations are labeled by their highest weights in the  $\mathbb{R}^8$  and  $\mathbb{R}^6$  spaces, respectively.

The representations of  $\mathfrak{so}(8)$  are:  $(1, 0, 0, 0, 0, 0, 0, 0)$ ,  $(0, 1, 0, 0, 0, 0, 0, 0)$ ,  $(0, 0, 1, 0, 0, 0, 0, 0)$ ,  $(0, 0, 0, 1, 0, 0, 0, 0)$ ,  $(0, 0, 0, 0, 1, 0, 0, 0)$ ,  $(0, 0, 0, 0, 0, 1, 0, 0)$ ,  $(0, 0, 0, 0, 0, 0, 1, 0)$ ,  $(0, 0, 0, 0, 0, 0, 0, 1)$ .

The representations of  $\mathfrak{so}(6)$  are:  $(1, 0, 0, 0, 0, 0)$ ,  $(0, 1, 0, 0, 0, 0)$ ,  $(0, 0, 1, 0, 0, 0)$ ,  $(0, 0, 0, 1, 0, 0)$ ,  $(0, 0, 0, 0, 1, 0)$ ,  $(0, 0, 0, 0, 0, 1)$ .