

The diagram illustrates the decomposition of the tensor product of two irreducible representations of the Lie algebra  $E_6$ . The decomposition is organized into four rows, each representing a different level of the hierarchy of Lie algebras:  $E_6$ ,  $F_4$ ,  $C_3$ , and  $A_1$ .

**Row 1:  $E_6$  Irreps**

- Top node:  $E_6 - 4 - 2 - 1 - 1$  (Highest weight:  $(-4, -2, -1, -1)$ )
- Second node:  $E_6 - 4 - 1 - 2 - 1 - 1$  (Highest weight:  $(-4, -1, -2, -1, -1)$ )
- Third node:  $E_6 - 4 - 1 - 1 - 2 - 1$  (Highest weight:  $(-4, -1, -1, -2, -1)$ )
- Fourth node:  $E_6 - 4 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -2)$ )

**Row 2:  $F_4$  Irreps**

- Node 1:  $F_4 - 4 - 1 - 2 - 1$  (Highest weight:  $(-4, -1, -2, -1)$ )
- Node 2:  $F_4 - 4 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -2)$ )
- Node 3:  $F_4 - 4 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -2)$ )
- Node 4:  $F_4 - 4 - 1 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -1, -2)$ )

**Row 3:  $C_3$  Irreps**

- Node 1:  $C_3 - 4 - 1 - 2$  (Highest weight:  $(-4, -1, -2)$ )
- Node 2:  $C_3 - 4 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -2)$ )
- Node 3:  $C_3 - 4 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -2)$ )
- Node 4:  $C_3 - 4 - 1 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -1, -2)$ )

**Row 4:  $A_1$  Irreps**

- Node 1:  $A_1 - 4 - 1 - 2$  (Highest weight:  $(-4, -1, -2)$ )
- Node 2:  $A_1 - 4 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -2)$ )
- Node 3:  $A_1 - 4 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -2)$ )
- Node 4:  $A_1 - 4 - 1 - 1 - 1 - 1 - 2$  (Highest weight:  $(-4, -1, -1, -1, -1, -2)$ )

The connections between nodes are labeled with the multiplicity of the corresponding irreps in the decomposition. For example, the top node connects to the second and third nodes with multiplicity 1, and to the fourth node with multiplicity 2.