

Diagram illustrating the structure of the Lie algebra $\mathfrak{so}(10)$ and its decomposition into irreducible representations of $\mathfrak{so}(8)$ and $\mathfrak{so}(6)$. The diagram shows the branching rules for the adjoint representation of $\mathfrak{so}(10)$ and the decomposition of the adjoint representation of $\mathfrak{so}(8)$ into irreducible representations of $\mathfrak{so}(6)$.

The diagram is organized into three main sections, each showing a branching rule for a specific representation of $\mathfrak{so}(10)$ into irreducible representations of $\mathfrak{so}(8)$ and $\mathfrak{so}(6)$.

Section 1: $\mathfrak{so}(10)$ Adjoint Representation

The adjoint representation of $\mathfrak{so}(10)$ (dimension 45) branches into the adjoint representation of $\mathfrak{so}(8)$ (dimension 28) and the adjoint representation of $\mathfrak{so}(6)$ (dimension 15).

Section 2: $\mathfrak{so}(8)$ Adjoint Representation

The adjoint representation of $\mathfrak{so}(8)$ (dimension 28) branches into the adjoint representation of $\mathfrak{so}(6)$ (dimension 15) and the adjoint representation of $\mathfrak{so}(4)$ (dimension 6).

Section 3: $\mathfrak{so}(6)$ Adjoint Representation

The adjoint representation of $\mathfrak{so}(6)$ (dimension 15) branches into the adjoint representation of $\mathfrak{so}(4)$ (dimension 6) and the adjoint representation of $\mathfrak{so}(2)$ (dimension 1).

The diagram uses the following notation for representations:

- $\mathfrak{so}(10)$ Adjoint: 45
- $\mathfrak{so}(8)$ Adjoint: 28
- $\mathfrak{so}(6)$ Adjoint: 15
- $\mathfrak{so}(4)$ Adjoint: 6
- $\mathfrak{so}(2)$ Adjoint: 1

The diagram also includes the following branching rules for the adjoint representation of $\mathfrak{so}(10)$ into irreducible representations of $\mathfrak{so}(8)$ and $\mathfrak{so}(6)$:

- $45 \rightarrow 28 + 15$
- $28 \rightarrow 15 + 6$
- $15 \rightarrow 6 + 1$

The diagram also includes the following branching rules for the adjoint representation of $\mathfrak{so}(8)$ into irreducible representations of $\mathfrak{so}(6)$ and $\mathfrak{so}(4)$:

- $28 \rightarrow 15 + 6$
- $15 \rightarrow 6 + 1$

The diagram also includes the following branching rules for the adjoint representation of $\mathfrak{so}(6)$ into irreducible representations of $\mathfrak{so}(4)$ and $\mathfrak{so}(2)$:

- $15 \rightarrow 6 + 1$

The diagram also includes the following branching rules for the adjoint representation of $\mathfrak{so}(4)$ into irreducible representations of $\mathfrak{so}(2)$:

- $6 \rightarrow 1 + 1$