

Diagram illustrating the decomposition of the Lie algebra $\mathfrak{so}(10)$ into irreducible representations of $\mathfrak{so}(8)$ and $\mathfrak{so}(2)$. The root system is shown in the complex plane, with roots labeled by their coordinates in the $(\alpha_1, \alpha_2, \alpha_3)$ basis. The diagram is organized into three rows, each representing a different level of the decomposition. The top row shows the full $\mathfrak{so}(10)$ root system. The middle row shows the decomposition into $\mathfrak{so}(8)$ and $\mathfrak{so}(2)$ components. The bottom row shows the further decomposition of the $\mathfrak{so}(8)$ components into irreducible representations of $\mathfrak{so}(6)$ and $\mathfrak{so}(2)$.