Optimization Assignment I

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Problem 1. (a)

- Cost of the portfolio is 2612.9048.
- Portfolio composition given by: $\boldsymbol{s} = (0,\ 0,\ 0,\ 0,\ 0,\ 0)^\top$,
  $\boldsymbol{x} = (0,\ 0,\ 10.1852,\ 0,\ 6.7151,\ 0,\ 2.5858,\ 0,\ 3.3202,\ 0,\ 0.2911,\ 0,\ 3.2911)^\top$

(b) The dual variables of constraints $\boldsymbol{F}\boldsymbol{x}+\boldsymbol{R}\boldsymbol{s} = \boldsymbol{l}$ are: $\boldsymbol{\mu} = (0.95,\ 0.8864,\ 0.8269,\ 0.7589,\ 0.7014,\ 0.6205)^\top$. The implied interest rates are calculated by $\widetilde{r}_t = 1/(\mu_t)^{1/t} - 1$. We have:

- $\widetilde{\boldsymbol{r}} = (0.0526,\ 0.0622,\ 0.0654,\ 0.0714,\ 0.0735,\ 0.0828)^\top$

Problem 2. Similar formulation as the in-class problem, except that we separate cash flow table and variables by bond rating:

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{s}}\ & \boldsymbol{p}_A^\top \boldsymbol{x}_A + \boldsymbol{p}_B^\top \boldsymbol{x}_B \\ s.t.\ & \boldsymbol{x}_A \succeq 0,\ \boldsymbol{x}_B \succeq 0,\ \boldsymbol{s} \succeq 0 \\ & \boldsymbol{F}_A \boldsymbol{x}_A + \boldsymbol{F}_B \boldsymbol{x}_B + \boldsymbol{R}\boldsymbol{s} = \boldsymbol{l} \\ & \boldsymbol{p}_A^\top \boldsymbol{x}_A \geq \boldsymbol{p}_B^\top \boldsymbol{x}_B \end{aligned} \tag{1}$$

- The cost of the portfolio is 2605.9465.
- The portfolio composition is $\boldsymbol{x}_B = (0,\ 0,\ 10.1852,\ 3.2954,\ 0,\ 0)^\top$; and
  $\boldsymbol{x}_A = (6.3614,\ 0,\ 3.3976,\ 0,\ 0.3614,\ 0,\ 3.3614)^\top$. The last constraint is binding.

Problem 3. Let $\boldsymbol{X} := (x_{ij})$, $\boldsymbol{y} := (y_k)$, $\boldsymbol{A}$ be the exchange rate. The following program solves a *sparse* $\boldsymbol{X}$ that admits an arbitrage:

$$\begin{aligned} \max_{\boldsymbol{X},\boldsymbol{y}}\ & y_1 - \lambda \left\|\boldsymbol{X}\right\|_1 \\ s.t.\ & y_1 \leq 1;\ y_k \geq 0,\ x_{ij} \geq 0\ \ \forall k,i,j = 1,...,5 \\ & y_k + \sum_{j=1,j\neq k}^{5} x_{kj} = \sum_{i=1,i\neq k}^{5} a_{ik} x_{ik}\ \ \forall k = 1,...,5 \end{aligned} \tag{2}$$

The solution contains a strictly positive $\boldsymbol{y}$, which implies that there is an arbitrage. The arbitrage is: 1000 USD $\to$ 894.6 EUR $\to$ 754.183584 GBP $\to$ 1302.248423 AUD $\to$ 1000.93527 USD $>$ 1000 USD, if the FX chain is transaction-cost-free.