Options Assignment I

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Problem 1.

Solution. (Part-a)

Table 1: State Prices and RN Probabilities

Macro State	Objective Prob	State Price	RN Prob	$\mathbb{E}[\text{Ret}] - R_f$	Pricing Kernel
Strong	0.1000	\$ 0.0950	0.0964	0.0374	0.9500
Weak	0.3000	\$ 0.2800	0.2843	0.0562	0.9333
Flat	0.2500	\$ 0.2500	0.2538	-0.0152	1.0000
Continued Recession	0.3500	\$ 0.3600	0.3655	-0.0430	1.0286
Sums	1.0000	0.9850	1.0000		
$T - t$	0.5				
$r_{t,T}(a_t)$	0.03023				

Let $a_T \in A_T = \{strong, weak, flat, continued\ recession\}$ be the four future states. Our inputs are the objective probabilities $p(a_T|a_t)$ in the second column of table 1, and the state prices $\pi(a_T|a_t)$ in the third column of table 1.
The risk neutral probabilities are listed in the fourth column:

$$q^{RN}(a_T|a_t) = \frac{\pi(a_T|a_t)}{\sum_{a_T \in A_T} \pi(a_T|a_t)} = \pi(a_T|a_t)e^{-r_{t,T}(a_t)(T-t)} \tag{1}$$

Therefore we can calculate the continuously compounded annualized risk-free interest rate:uo

$$\begin{aligned} r_{t,T}(a_t) &= -\frac{1}{T-t}\log\left(\sum_{a_T \in A_T} \pi(a_T|a_t)\right) \\ &= -\frac{1}{0.5}\log(0.985) \approx 0.03023 \end{aligned} \tag{2}$$

The pricing kernels are listed in the sixth column:

$$m(a_T|a_t) = \frac{\pi(a_T|a_t)}{p(a_T|a_t)} \tag{3}$$

The expected risk premium are in the fifth column:

$$\mathbb{E}_t[R_i] - R_f = \left(\frac{p(a_T|a_t)}{q^{RN}(a_T|a_t)} - 1\right)e^{r_{t,T}(a_t)(T-t)} \tag{4}$$

Now we calculate the option price via risk neutral valuation, see the third column of table-2. We have:

$$c_{jt} = \sum_{a_T \in A_T} \pi(a_T|a_t)c_j^*(a_T) \tag{5}$$

- Price of reverse macro straddle that pays \$1 million if $a_T \in \{weak, flat\}$: $c_{1t} = 0.53$ million \$.
- Price of reverse macro straddle that pays \$25 million if $a_T \in \{weak, flat\}$: $c_{2t} = 13.25$ million \$.
- Price of long macro straddle that pays \$1 million if $a_T \in \{strong, recession\}$: $c_{3t} = 0.46$ million \$.

By the DCF valuation:

$$c_{jt} = \frac{\sum_{a_T \in A_T} p(a_T|a_t) c_j^*(a_T)}{1 + RAD_{jt}} \tag{6}$$

- RAD of reverse macro straddle that pays \$1 million if $a_T \in \{weak, flat\}$: $RAD_{1t} = 0.0377$.
- RAD of reverse macro straddle that pays \$25 million if $a_T \in \{weak, flat\}$: $RAD_{2t} = 0.0377$.
- RAD of long macro straddle that pays \$1 million if $a_T \in \{strong, recession\}$: $RAD_{3t} = -0.0110$.

Explaination:

- As we have seen in the table, the reverse macro straddle is a portfolio of state claim for $flat$ and $weak$. Hence $c_{1t} = \pi(weak|a_t) + \pi(flat|a_t)$. By our calculation, the risk premium of state claim $weak$ is positive (0.0562), and the risk premium of state claim $flat$ is negative (-0.0152), but a lot smaller than that of $weak$ in absolute value. The positive risk premium dominates. As a result, the overall impact on risk-adjusted discount rate is a positive risk premium. Hence RAD_{1j} is greater than the risk-free rate.
- The Second case is just 25 shares of the reverse macro straddle in the First case. $c_{2t} = 25c_{1t}$. Holding more shares of options does **not** change the payoff probabilites, preferences or timing. So the RAD does not change for the second one.
- For the third case, the long macro straddle is a portfolio of state claim for *strong* and *recession*. Hence $c_{3t} = \pi(strong|a_t) + \pi(recession|a_t)$. By our calculation, the risk premium of state claim *strong* is positive (0.0374), and the risk premium of state claim *recession* is negative (-0.0430), but greater than that of *strong* in absolute value. The negative risk premium dominates. As a result, the overall impact on risk-adjusted discount rate is a negative risk premium. The negative risk-premium is so deep that it wiped out risk-free rate. As a result, the RAD ends up to be negative.

Table 2: Option Valuations

Reverse Macro Straddle Macro State	Payoffs $c^*(a_T)$	RN Valuations	$p(a_T\|a_t)c^*(a_T)$	RAD
Strong	0	0.00	0.00	
Weak	1	0.28	0.30	
Flat	1	0.25	0.25	
Continued Recession	0	0.00	0.00	
value		\$0.53	\$0.55	0.0377358

Reverse Macro Straddle (25) Macro State	Payoffs $c^*(a_T)$	RN Valuations	$p(a_T\|a_t)c^*(a_T)$	RAD
Strong	0	0.00	0.00	
Weak	25	7.00	7.50	
Flat	25	6.25	6.25	
Continued Recession	0	0.00	0.00	
option value		\$13.25	\$13.75	0.0377358

Long straddle Macro State	Payoffs $c^*(a_T)$	RN Valuations	$p(a_T\|a_t)c^*(a_T)$	RAD
Strong	1	0.10	0.10	
Weak	0	0.00	0.00	
Flat	0	0.00	0.00	
Continued Recession	1	0.36	0.35	
option value		\$0.46	\$0.45	-0.010989

(Part-b)

$$F_{t,T} = \mathbb{E}_t^{RN}\left[S_T^*\right] = 1074.7462 \tag{7}$$

Table 3: Future Price

Macro State	Payoffs	RN Probs	$S_T^* q^{RN}(a_T\|a_t)$
Strong	1195	0.0964	115.25381
Weak	1150	0.2843	326.90355
Flat	1110	0.2538	281.72589
Continued Recession	960	0.3655	350.86294
Future Price			1074.7462

(Part-c) State price $\pi(a_T|a_t)$ summarizes combined valuation impact of timing, probabilities and preferences.
Since the objective probabilities and the timing of the states are unchanged, the **preferences** would need to have changed to account for the changes in state prices.