Machine Learning HW6

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1 Equivalent Form of SVM Optimization Problem

(a) If we multiply $\boldsymbol{\beta}$ and β_0 by a fixed *positive* (I think the question forgot to mention that it must be non-negatively scaled) constant b, nothing changes about the meaning of

$$\begin{aligned} & y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) \geq M(1-\epsilon_i)\left\|\boldsymbol{\beta}\right\|_2 \\ \iff \quad & |b| y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) \geq |b| M(1-\epsilon_i)\left\|\boldsymbol{\beta}\right\|_2 \\ \iff \quad & y_i(\boldsymbol{x}_i^\top b\boldsymbol{\beta} + b\beta_0) \geq M(1-\epsilon_i)\left\|b\boldsymbol{\beta}\right\|_2 \end{aligned} \tag{1}$$

holds only when $b \geq 0$. We choose $\|\beta\| = \frac{1}{M}$, which is a non-negative scaling since $M \geq 0$. So the optimization can be rewritten as

$$\begin{aligned} (Form\ 2') \quad & \max_{\boldsymbol{\beta},\beta_0,\boldsymbol{\epsilon}} \frac{1}{\left\|\boldsymbol{\beta}\right\|_2} \\ s.t. \quad & y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) \geq 1-\epsilon_i,\ \forall i \\ & \sum_{i=1}^n \epsilon_i \leq C;\ \epsilon_i \geq 0,\ \forall i \end{aligned} \tag{2}$$

Maximizing $1/\left\|\boldsymbol{\beta}\right\|_2$ is equivalent to minimizing $\left\|\boldsymbol{\beta}\right\|_2$, which is equivalent to minimizing $\left\|\boldsymbol{\beta}\right\|_2^2$ since $(\cdot)^2$ is now a monotone transform due to the non-negativity of the norm.

$$\begin{aligned} (Form\ 2) \quad & \min_{\boldsymbol{\beta},\beta_0,\boldsymbol{\epsilon}} \frac{1}{2}\left\|\boldsymbol{\beta}\right\|_2^2 \\ s.t. \quad & y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) \geq 1-\epsilon_i,\ \forall i \\ & \sum_{i=1}^n \epsilon_i \leq C;\ \epsilon_i \geq 0,\ \forall i \end{aligned} \tag{3}$$

The information about the width of the margin is now encoded in the length of $\boldsymbol{\beta}$. Namely $2M = \frac{2}{\|\boldsymbol{\beta}\|}$. The $\boldsymbol{\beta}$ that solves this problem is but a positively scaled version of the $\boldsymbol{\beta}^{\text{prev}}$ that solves the previous problem. We have

$$\boldsymbol{\beta} = \frac{1}{M}\boldsymbol{\beta}^{\text{prev}};\ \beta_0 = \frac{1}{M}\beta_0^{\text{prev}}$$

(b)

$$\begin{aligned} (Form\ 3) \quad & \min_{\boldsymbol{\beta},\beta_0,\boldsymbol{\epsilon}} \frac{1}{2}\left\|\boldsymbol{\beta}\right\|_2^2 + D\sum_{i=1}^n \epsilon_i \\ s.t. \quad & y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) \geq 1-\epsilon_i;\ \epsilon_i \geq 0,\ \forall i \end{aligned} \tag{4}$$

Form 2 is equivalent to Form 3. I can't think of an one-sentence-or-two explanation, so I'll argue formally. The Lagrangian of form 2 is:

$$\mathcal{L}_2(\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}; u, \boldsymbol{v}, \boldsymbol{w}) = \frac{1}{2}\|\boldsymbol{\beta}\|_2^2 + u\left(\sum_{i=1}^n \epsilon_i - C\right) - \sum_{i=1}^n v_i \left(y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) - (1-\epsilon_i)\right) - \sum_{i=1}^n w_i \epsilon_i \quad (5)$$

The Lagrangian of form 3 is:

$$\mathcal{L}_3(\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}; \boldsymbol{v}, \boldsymbol{w}) = \frac{1}{2}\|\boldsymbol{\beta}\|_2^2 + D\sum_{i=1}^n \epsilon_i - \sum_{i=1}^n v_i \left(y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) - (1-\epsilon_i)\right) - \sum_{i=1}^n w_i \epsilon_i \quad (6)$$

Let $(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*)$ be the primal optimal, and $(u^*, \boldsymbol{v}^*, \boldsymbol{w}^*)$ the dual optimal of form 2. The necessity of KKT condition suggests:

$$\begin{aligned}
\textstyle\sum_{i=1}^n \epsilon_i^* - C &\le 0 \\
-\epsilon_i^* &\le 0, \ \forall i \\
(1-\epsilon_i^*) - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta}^* + \beta_0^*) &\le 0, \ \forall i \\
u^*, \boldsymbol{v}^*, \boldsymbol{w}^* &\succeq 0 \\
u^*(\textstyle\sum_{i=1}^n \epsilon_i^* - C) &= 0 \\
v_i^*[(1-\epsilon_i^*) - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta}^* + \beta_0^*)] &= 0, \ \forall i \\
w_i^* \epsilon_i^* &= 0, \ \forall i \\
\nabla_{\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}} \mathcal{L}_2(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*; u^*, \boldsymbol{v}^*, \boldsymbol{w}^*) &= 0
\end{aligned} \quad (7)$$

The strong duality holds for form 3, because:

1. The objective function of the primal problem is quadratic, thus convex. Each of the inequality constraints are affine, thus convex.

2. Slater's condition holds trivially for all the constraints which are affine.

Therefore, the sufficiency of KKT condition implies that $(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*)$ is the also the optimal solution of form 3, if and only if there exists dual optimal $(\widetilde{\boldsymbol{v}}, \widetilde{\boldsymbol{w}})$ such that

$$\begin{aligned}
-\epsilon_i^* &\le 0, \ \forall i \\
(1-\epsilon_i^*) - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta}^* + \beta_0^*) &\le 0, \ \forall i \\
\widetilde{\boldsymbol{v}}, \widetilde{\boldsymbol{w}} &\succeq 0 \\
\widetilde{v_i}[(1-\epsilon_i^*) - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta}^* + \beta_0^*)] &= 0, \ \forall i \\
\widetilde{w_i} \epsilon_i^* &= 0, \ \forall i \\
\nabla_{\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}} \mathcal{L}_3(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*) &= 0
\end{aligned} \quad (8)$$

Note that everything in (8) is automatically satisfied by choosing the same dual optimal as that of (7), i.e. choose $(\widetilde{\boldsymbol{v}}, \widetilde{\boldsymbol{w}}) = (\boldsymbol{v}^*, \boldsymbol{w}^*)$, except for $\nabla_{\boldsymbol{\epsilon}} \mathcal{L}_3(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*) = 0$. In form 2, we have

$$\nabla_{\boldsymbol{\epsilon}} \mathcal{L}_2 = u^* \mathbf{1} - \boldsymbol{v}^* - \boldsymbol{w}^* = \mathbf{0} \quad (9)$$

In form 3,

$$\nabla_{\boldsymbol{\epsilon}} \mathcal{L}_3 = D\mathbf{1} - \widetilde{\boldsymbol{v}} - \widetilde{\boldsymbol{w}} = \mathbf{0} \quad (10)$$

Therefore, for any given C, we choose $D = u^* = u^*(C)$ that solves (7), then it must make $(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*; \boldsymbol{v}^*, \boldsymbol{w}^*)$ the solution to (8), thus $(\boldsymbol{\beta}^*, \beta_0^*, \epsilon^*)$ is the primal optimal of form 3.

C represents our tolerance to the total proportional amount by which the predictions fall on the wrong side of their margin. Making C larger means we are more tolerant to this amount, so the cost of doing so, D, should be smaller. If $C = 0$, we return to the separable case, and falling on the wrong side is not allowed; consequently $D = \infty$ in this case.

(c) The inequality constraints can be viewed as:

$$\begin{aligned} \epsilon_i &\geq 0, \ \forall i \\ \epsilon_i &\geq 1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0), \ \forall i \end{aligned} \tag{11}$$

Following our analysis in (b), if $D = 0$, $C = \infty$, which implies that we don't care about whether or not our prediction falling on the wrong side at all. Clearly this is not what we want to do, so we will choose $D > 0$. Now (10) suggests

$$v_i^* + w_i^* = D > 0, \quad \forall i \tag{12}$$

That is, for any point i, $\widetilde{v}_i$ and $\widetilde{w}_i$ cannot both be zero. Therefore, the complementary slackness condition in (8) suggests that at least one of the constraints in (11) is binding, hence

$$\epsilon_i^* = \begin{cases} 0 & 0 \leq v_i^* < D, \quad w_i^* > 0, \quad 1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) \leq 0 \\ 1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0) & v_i^* = D, \quad w_i^* = 0, \quad \epsilon_i^* \geq 0 \end{cases} \tag{13}$$

So we conclude that

$$\epsilon_i^* = (1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0))_+ \tag{14}$$

Therefore we can rewrite the optimization problem

$$(Form\ 4) \quad \min_{\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}} \frac{1}{2} \|\boldsymbol{\beta}\|_2^2 + D \sum_{i=1}^n (1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0))_+ \tag{15}$$

(d)

$$(Form\ 5) \quad \min_{\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}} \frac{1/D}{2} \|\boldsymbol{\beta}\|_2^2 + \sum_{i=1}^n (1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0))_+ \tag{16}$$

Or define $\lambda := 1/D$,

$$(Form\ 5) \quad \min_{\boldsymbol{\beta}, \beta_0, \boldsymbol{\epsilon}} \frac{\lambda}{2} \|\boldsymbol{\beta}\|_2^2 + \sum_{i=1}^n (1 - y_i(\boldsymbol{x}_i^\top \boldsymbol{\beta} + \beta_0))_+ \tag{17}$$

Follow our discussion in (b): $C \nearrow \iff D \searrow \iff \lambda \nearrow \iff M \nearrow \iff \frac{1}{\|\boldsymbol{\beta}\|} \nearrow \iff \|\boldsymbol{\beta}\| \searrow$.

```
In [186]: import itertools
          from abc import ABCMeta, abstractmethod, abstractproperty
          import numpy as np
          import pandas as pd
          import sklearn as sk
          import matplotlib
          import seaborn as sns
          import matplotlib.pyplot as plt
          plt.style.use('ggplot')
          %matplotlib inline

          import scipy.stats
          from sklearn.base import ClassifierMixin, BaseEstimator, TransformerMixin
          from sklearn.model_selection import train_test_split
          from sklearn.svm import SVC, OneClassSVM
          from sklearn.calibration import calibration_curve
          from sklearn.metrics import accuracy_score, \
              roc_curve, confusion_matrix
          from copy import copy, deepcopy
          from progressbar import ProgressBar
          from tabulate import tabulate
```

2 Naive Bayes

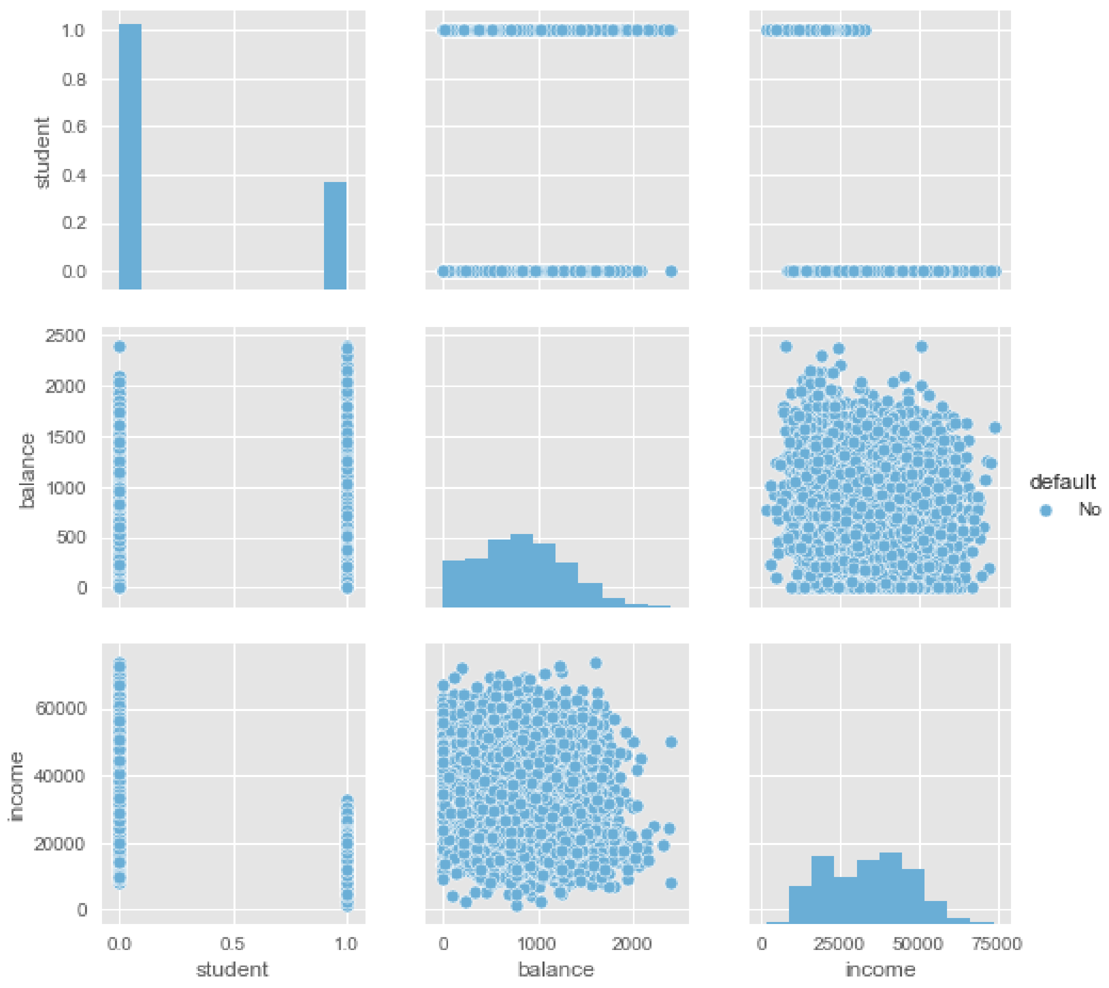
```
In [14]: df_train = pd.read_csv('default_train.csv')
         df_test = pd.read_csv('default_test.csv')
         df_train['student'] = np.where(
             df_train['student'] == 'Yes', 1, 0)
         df_test['student'] = np.where(
             df_test['student'] == 'Yes', 1, 0)
         X_train = df_train.iloc[:,1:].values
         y_train = np.where(df_train['default'] == 'Yes', 1, 0)
         X_test = df_test.iloc[:,1:].values
         y_test = np.where(df_test['default'] == 'Yes', 1, 0)
         X_names = list(df_train.columns)[1:]
         print(X_train.shape, X_test.shape)
         print(y_train.shape, y_test.shape)
```

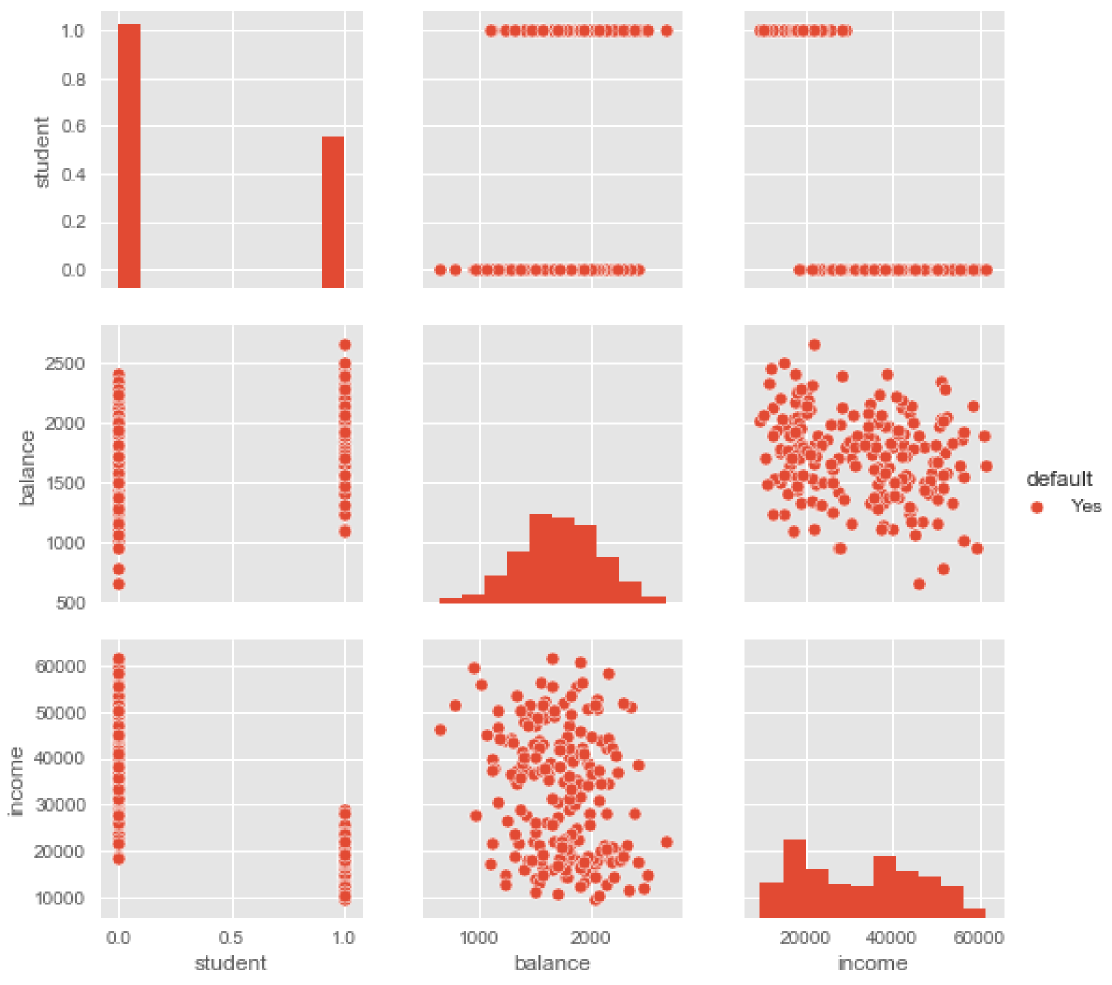
```
(6000, 3) (4000, 3)
(6000,) (4000,)
```

2.1 Exploratory Data Analysis

```
In [50]: sns.pairplot(
             df_train[df_train['default'] == 'No'],
             hue='default',palette="Blues")
         sns.pairplot(
             df_train[df_train['default'] == 'Yes'],
             hue='default')
```

```
Out[50]: <seaborn.axisgrid.PairGrid at 0x120d81080>
```


student
balance
income
default
No
1.0
0.8
0.6
0.4
0.2
0.0
2500
2000
1500
1000
500
0
60000
40000
20000
0
0.0
0.5
1.0
0
1000
2000
0
25000
50000
75000



For both classes, `balance` and `income` seem to be independent, but `student` and `income` are clearly correlated: students tend to have lower income. Moreover, for `default = 1` class, `balance` and `student` tend to be correlated as well. So we doubt the assumption of the Naive Bayes model.

2.2 Fit the Naive Bayes Model

```
In [154]: class Marginal:
              __metaclass__ = ABCMeta
              @abstractmethod
              def estimate(self, sample):
                  raise NotImplementedError

              @abstractmethod
              def log_likelihood(self, x):
                  raise NotImplementedError

          class BernoulliMarginal(Marginal):
              def __init__(self):
```

```
        self.p = None

    def __repr__(self):
        return "Bernoulli({})".format(self.p)

    def estimate(self, sample):
        self.p = np.sum(sample)/len(sample)

    def log_likelihood(self, x):
        return np.log(self.p)*x + np.log(
            1-self.p)*(1-x)

class GaussianMarginal(Marginal):
    def __init__(self):
        self.mu = None
        self.sigma_sq = None

    def __repr__(self):
        return "Gaussian({}, {})".format(
            self.mu, self.sigma_sq)

    def estimate(self, sample):
        self.mu = np.average(sample)
        self.sigma_sq = np.sum([
            (x-self.mu)**2 for x in sample])/len(sample)

    def log_likelihood(self, x):
        return -0.5*np.log(
            2*np.pi*self.sigma_sq) - (
            x-self.mu)**2/(2*self.sigma_sq)

class AwesomeNaiveBayes(ClassifierMixin, BaseEstimator, TransformerMixin):

    def __init__(self, marginals, n_cls, x_names=None):
        self.n_cls = n_cls
        self.marginals = {
            l: deepcopy(marginals) for l in range(n_cls)}
        self.d = len(marginals)
        self.priors = {l: None for l in range(n_cls)}
        self.x_names = x_names
        if not x_names:
            self.x_names = ['X'+str(i) for i in range(self.d)]
        self.fitted = False

    def __repr__(self):
        _repr, rows = 'NaiveBayesClassifier()\n', []
        if not self.fitted:
            return _repr
        for k in range(self.n_cls):
            rows.append(
                ['log P(Y={})'.format(k), self.priors[k]])
            for i, s in enumerate(self.x_names):
                rows.append([
                    s+'|Y={}'.format(k),
                    str(self.marginals[k][i])])
        _repr += tabulate(
            rows, headers=['', 'Estimate'])
        return _repr

    def fit(self, X_train, y_train):
        n, d = X_train.shape
        y_cls = np.unique(y_train)
        assert len(y_cls) == self.n_cls
        assert d == len(self.marginals[0])
        for k in range(self.n_cls):
            for j in range(d):
                self.marginals[k][j].estimate(
                    X_train[y_train == k, j])
            self.priors[k] = np.log(
                np.sum(y_train == k)/n)
        self.fitted = True
        return self

    def predict(self, X_test):
        log_proba = self.predict_log_proba(X_test)
        return np.argmax(log_proba, 1)

    def predict_log_proba(self, X_test):
        pred = np.zeros((len(X_test), self.n_cls))
        for i, row in enumerate(X_test):
            for k in range(self.n_cls):
                pred[i,k] = self.priors[k]
                for j, marginal in enumerate(self.marginals[k]):
                    pred[i,k] += marginal.log_likelihood(row[j])
        return pred
```

```
In [155]: clf = AwesomeNaiveBayes(
              [BernoulliMarginal(), GaussianMarginal(), GaussianMarginal()],
              2, ['student', 'balance', 'income'])
          clf.fit(X_train, y_train)
          # the parameter estimates are reported as the class representation
          # in the output box below.
```

```
Out[155]: NaiveBayesClassifier()
                       Estimate
          -----------  ------------------------------------------------
```

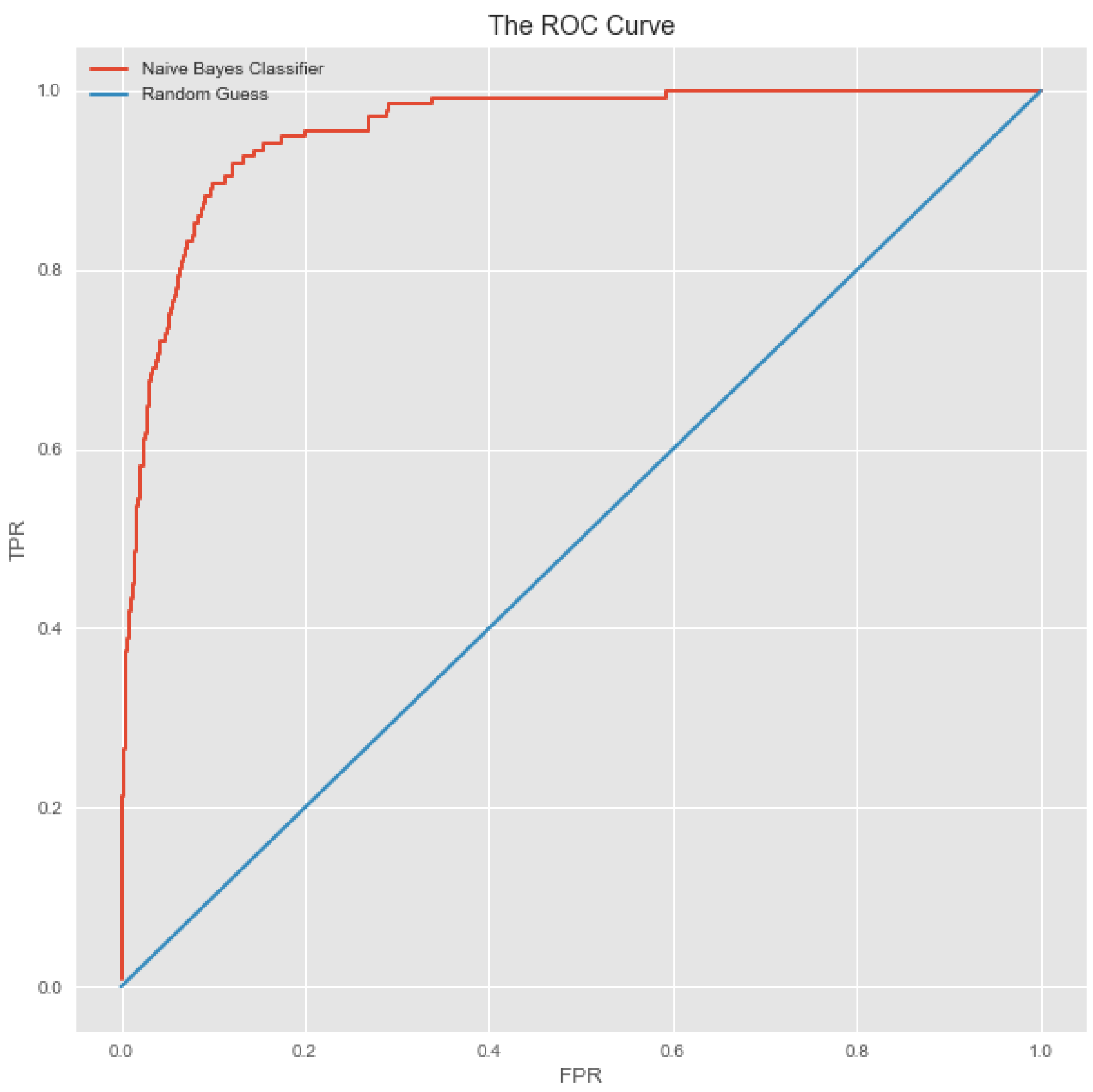
```
log P(Y=0)   -0.03338444401958382
student|Y=0  Bernoulli(0.28847148026882646)
balance|Y=0  Gaussian(806.4452220460979, 206827.67007859287)
income|Y=0   Gaussian(33451.15029361857, 177429727.3217092)
log P(Y=1)   -3.4163110194722037
student|Y=1  Bernoulli(0.36548223350253806)
balance|Y=1  Gaussian(1718.8534899175334, 127084.50907211454)
income|Y=1   Gaussian(32128.86080803881, 184822064.0125683)
```

2.3 Prediction and Model Evaluation

```
In [156]: y_pred = clf.predict(X_test)
          l = clf.predict_log_proba(X_test)
          score = l[:,1] - l[:,0]

          misclf_rate = 1-accuracy_score(
              y_test, y_pred, normalize=1)
          cm = confusion_matrix(y_test, y_pred)
          fpr, tpr, _ = roc_curve(y_test, score)
          print(
          """
          The misclassification rate is: {},
          The confusion matrix: \n {}
          """.format(misclf_rate, cm)
          )
```

```
The misclassification rate is: 0.028249999999999997,

The confusion matrix:
 [[3851   13]
 [ 100   36]]
```

```
In [157]: # ROC curve
          fig, ax = plt.subplots(1,1, figsize=(10,10))
          ax.plot(fpr, tpr, linewidth=2,
                  label='Naive Bayes Classifier')
          ax.plot([0,1], [0,1], linewidth=2,
                  label='Random Guess')
          ax.update({'xlabel':'FPR', 'ylabel':'TPR',
                     'title':'The ROC Curve'})
          _ = ax.legend()
```



---

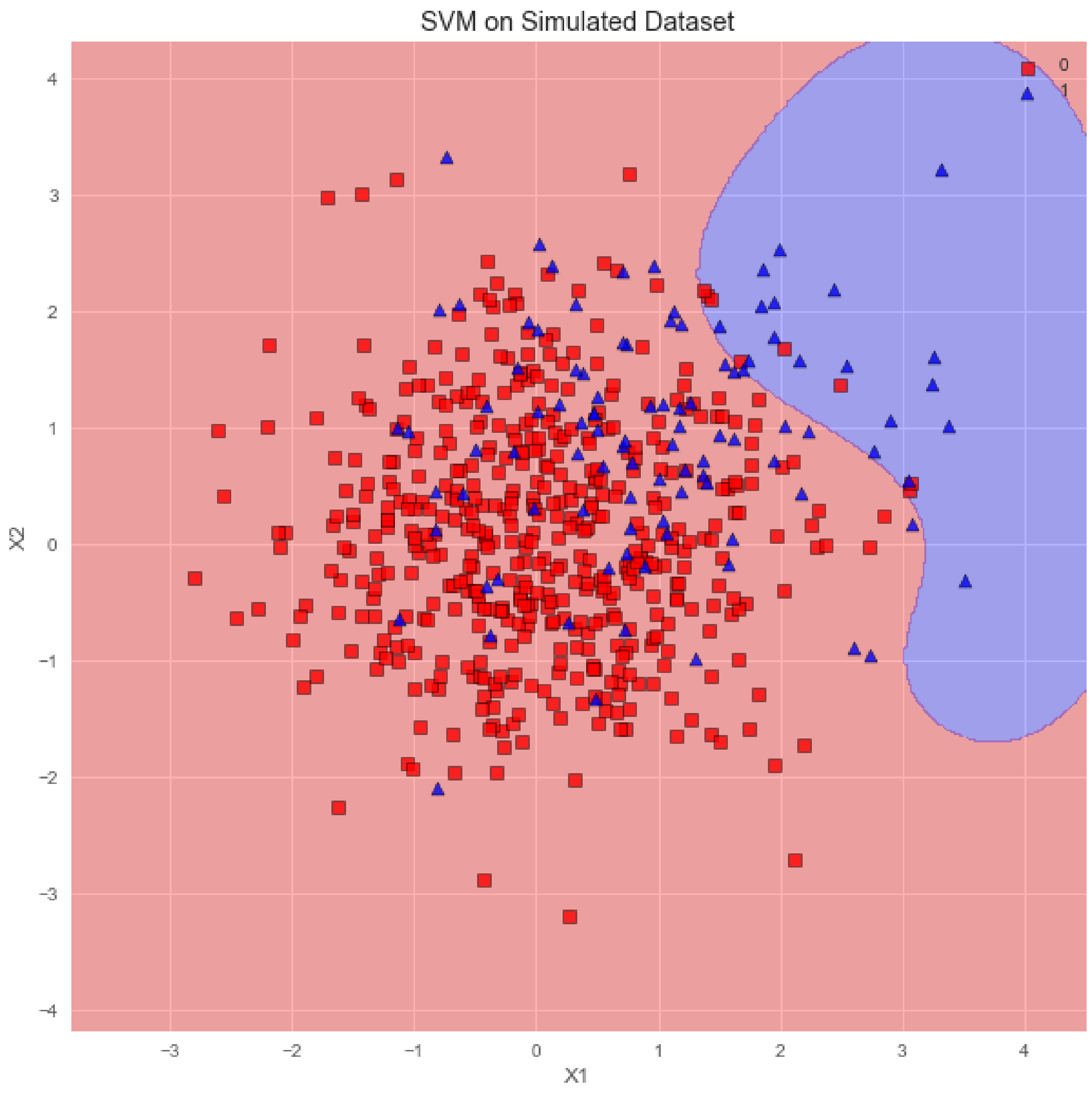
3 Class Weights

```
In [162]: from mlxtend.plotting import plot_decision_regions

In [161]: def get_data(n1,n2):
              dist1 = scipy.stats.multivariate_normal(
                  mean = np.zeros(2), cov = np.diag(np.ones(2)))
              dist2 = scipy.stats.multivariate_normal(
                  mean = np.ones(2), cov = np.diag(np.ones(2)))
              X = np.concatenate(
                  [dist1.rvs(size=n1), dist2.rvs(size=n2)], axis=0)
```

```
        y = np.array([0]*n1 + [1]*n2)
        return(X,y)
```

3.1 Linear SVM on Training Set

```
In [180]: X_train, y_train = get_data(500, 100)
          svc_clf = SVC(C=1)
          svc_clf.fit(X_train, y_train)

Out[180]: SVC(C=1, cache_size=200, class_weight=None, coef0=0.0,
            decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
            max_iter=-1, probability=False, random_state=None, shrinking=True,
            tol=0.001, verbose=False)

In [181]: fig, ax = plt.subplots(1,1, figsize=(10,10))
          plot_decision_regions(X_train, y_train, clf=svc_clf)
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'SVM on Simulated Dataset'})
```



3.2 Prediction on Testset

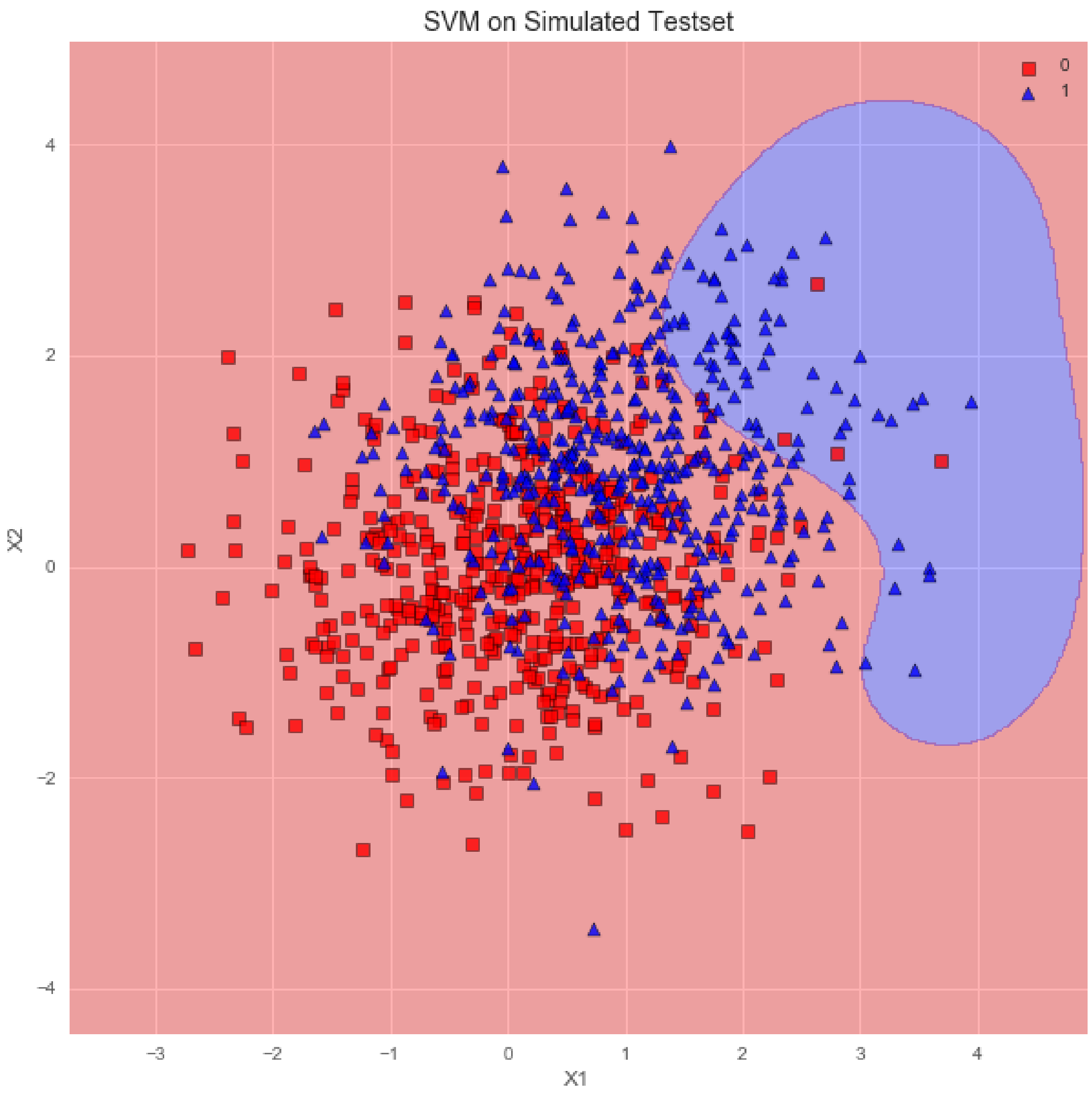
```
In [184]: X_test, y_test = get_data(500, 500)
          y_pred = svc_clf.predict(X_test)

          misclf_rate = 1-accuracy_score(
              y_test, y_pred, normalize=1)
          cm = confusion_matrix(y_test, y_pred)
          print(
          """
          The misclassification rate is: {},
          The confusion matrix: \n {}
          """.format(misclf_rate, cm)
          )
```

```
The misclassification rate is: 0.42100000000000004,
The confusion matrix:
 [[499   1]
 [420  80]]
```

The strange thing is that the False Negative is a lot more than the False positive, which implies that the classifier is too conservative on making positive prediction. And the misclassifaction rate is also very high.

The reason why we get such a weird result in a balanced problem is we trained with an imbalanced dataset, and assign equal weights to both classes. The classifier therefore thinks that the positive sample is much less than the negative samples in general, which is not the case in the test set. Consequently, it made too conservative prediction on the positive points.

```
In [183]: fig, ax = plt.subplots(1,1, figsize=(10,10))
          plot_decision_regions(X_test, y_test, clf=svc_clf)
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'SVM on Simulated Testset'})
```



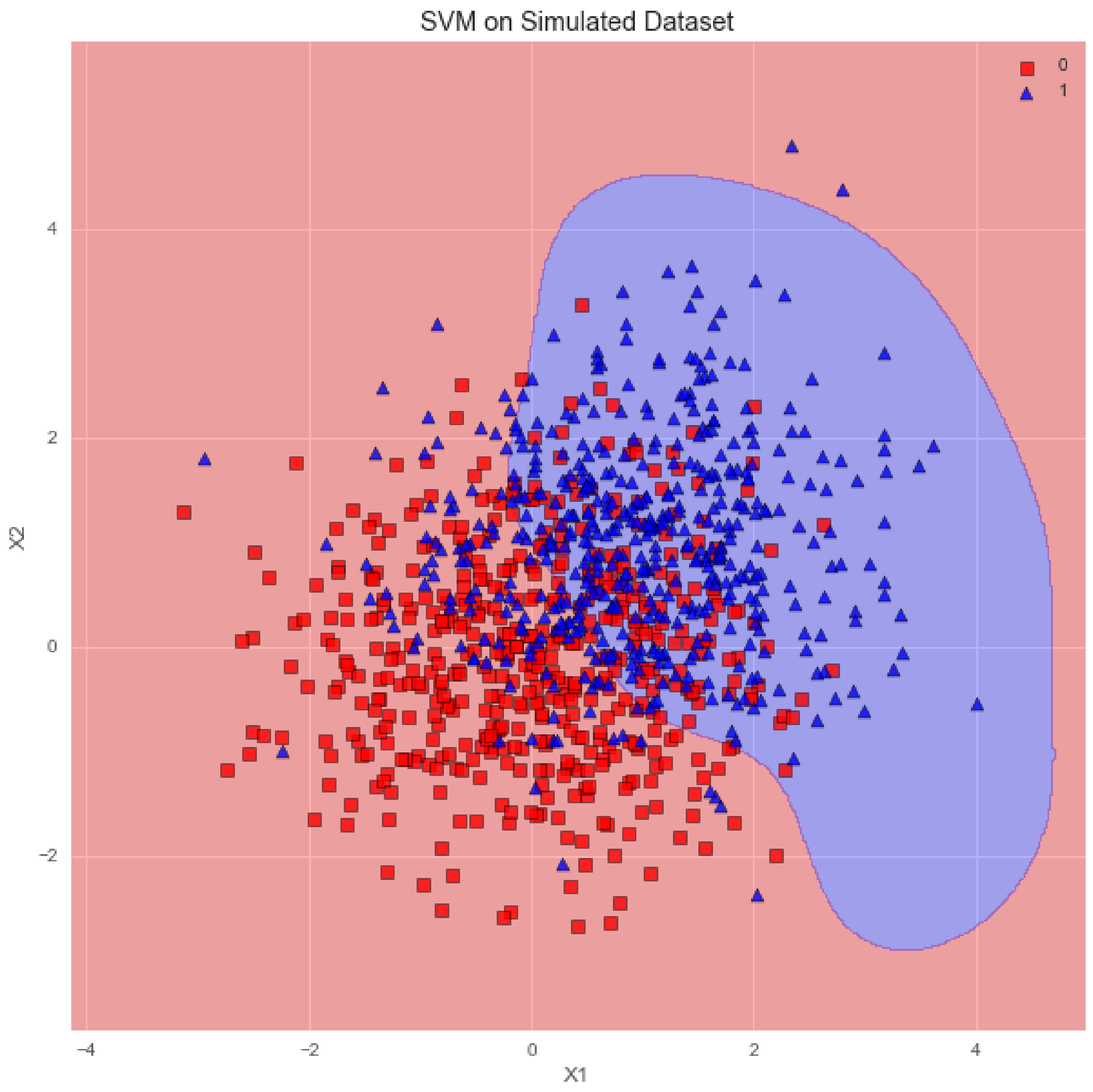
3.3 Refit SVC

```
In [185]: svc_clf_bal = SVC(C=1, class_weight='balanced')
          svc_clf_bal.fit(X_train, y_train)
          y_pred_2 = svc_clf_bal.predict(X_test)

          misclf_rate = 1-accuracy_score(
              y_test, y_pred_2, normalize=1)
          cm = confusion_matrix(y_test, y_pred_2)
          print(
          """
          The misclassification rate is: {},
          The confusion matrix: \n {}
          """.format(misclf_rate, cm)
          )
```

```
The misclassification rate is: 0.255,
The confusion matrix:
 [[382 118]
 [137 363]]
```

We can see that the classifier can make balanced predictions this time. The misclassification rate is lower, and we have a very balanced confusion matrix, with approximately the same number of FP and FNs.

```
In [179]: fig, ax = plt.subplots(1,1, figsize=(10,10))
          plot_decision_regions(X_test, y_test, clf=svc_clf_bal)
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'SVM on Simulated Dataset'})
```



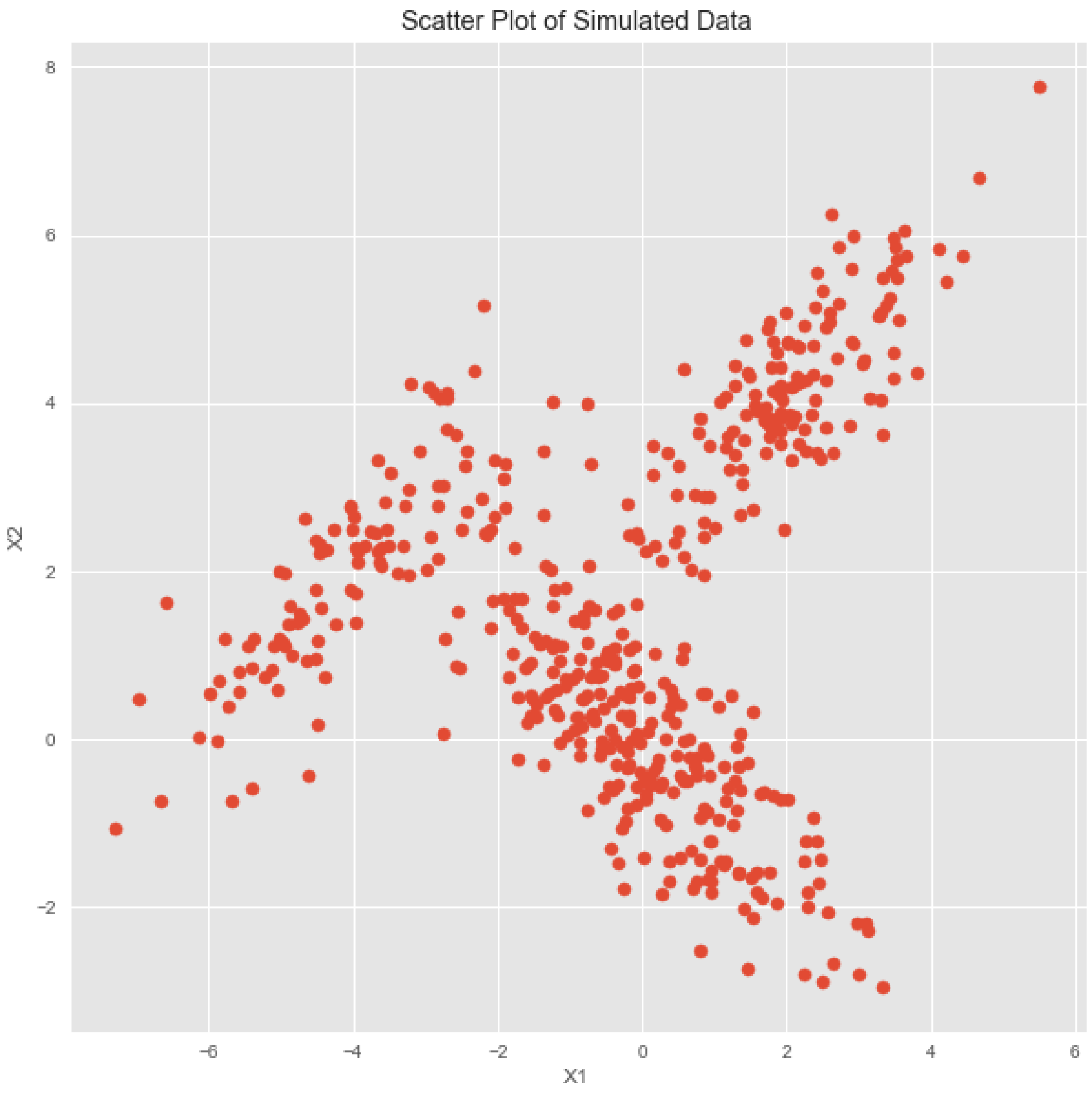
---

4 Anomaly Detection

```
In [187]: def get_data_anomaly(n):
              #prior probabilities
              pi0 = (.5,.3,.2)
              n_each = [int(n*pi) for pi in pi0]
              #means
              mus = ((0,0),(2,4),(-4,2))
              Sigmas = [
                  np.matrix(((1,-0.7),(-0.7,0.9)))*1.6,
                  np.matrix(((1,0.8),(0.8,0.9)))*1.6,
                  np.matrix(((1,0.8),(0.8,0.9)))*1.6
              ]
              gaussians = [scipy.stats.multivariate_normal(
                  mean = mus[i],cov = Sigmas[i]) for i in range(3)]
              X = np.concatenate([
                  gaussian.rvs(n_each[i])
                  for i, gaussian in enumerate(gaussians)], axis=0)
              return(X)
```

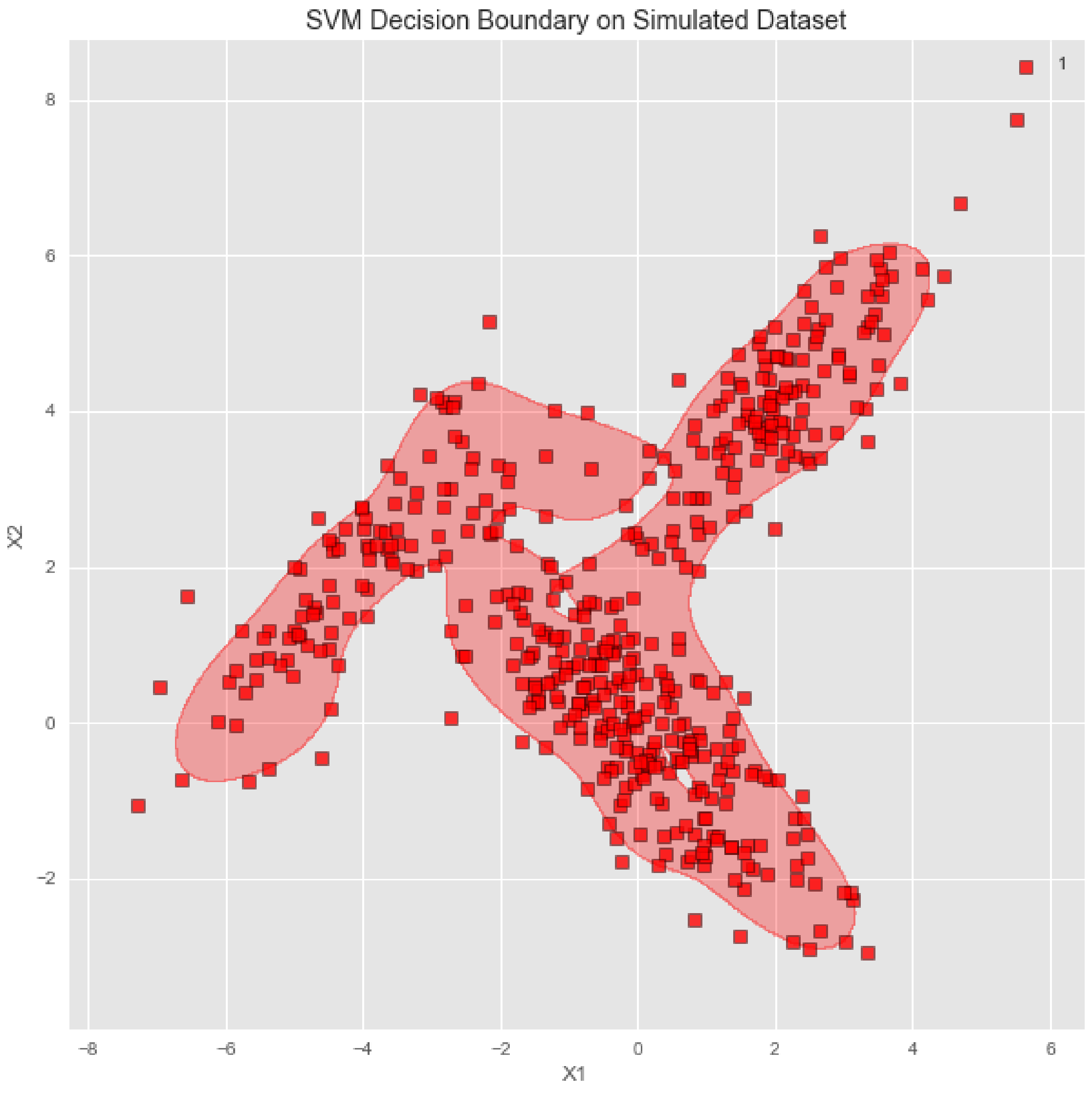
4.1 Exploratory Data Analysis

```
In [191]: X = get_data_anomaly(500)
          fig, ax = plt.subplots(1,1, figsize=(10,10))
          ax.scatter(X[:,0], X[:,1])
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'Scatter Plot of Simulated Data'})
```



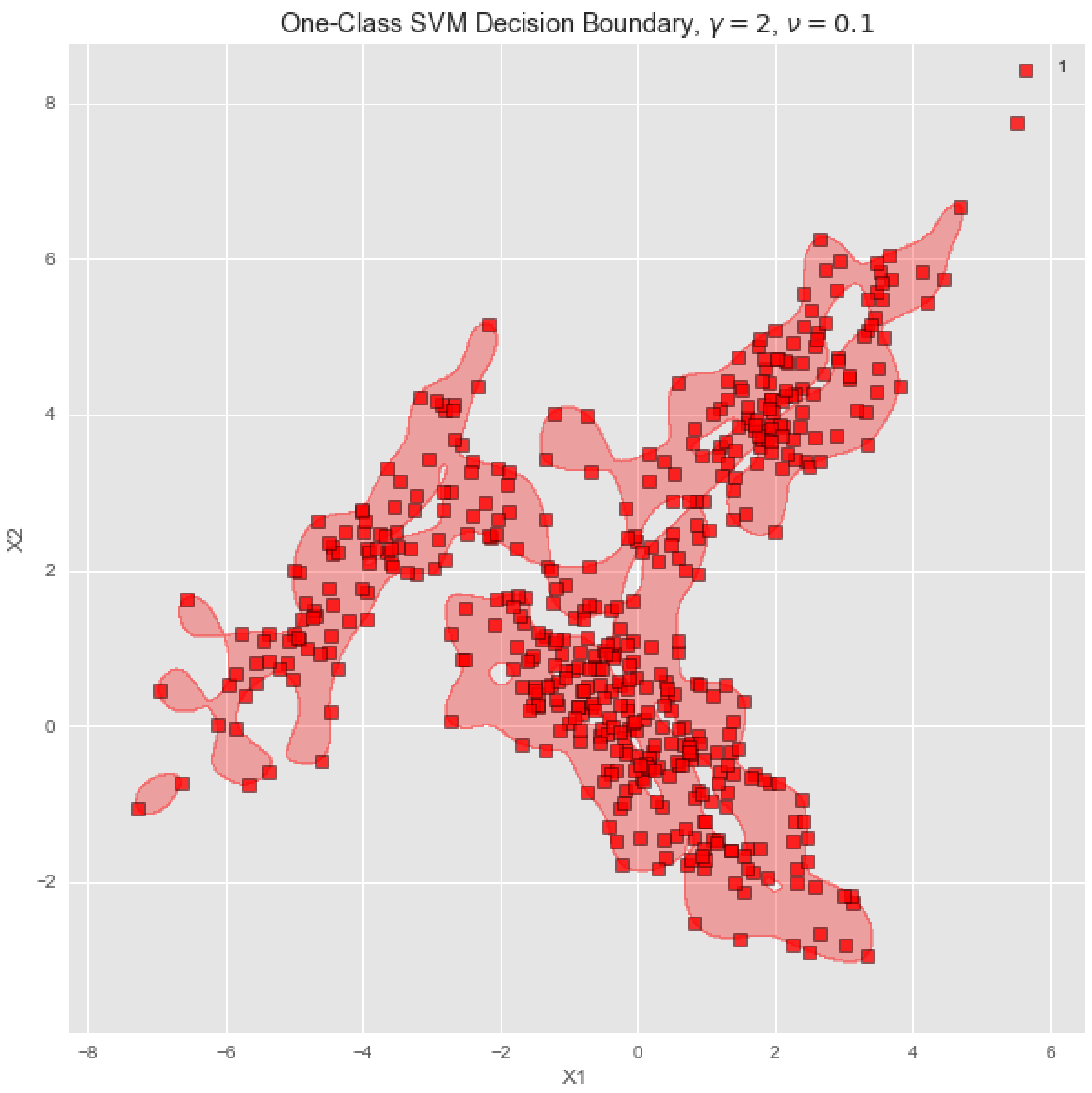
4.2 One-class SVM

```
In [199]: ocs_clf = OneClassSVM(nu=0.1, kernel="rbf", gamma=0.5)
          ocs_clf.fit(X)
          fig, ax = plt.subplots(1,1, figsize=(10,10))
          plot_decision_regions(X,np.ones(500,dtype=int), clf=ocs_clf)
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'SVM Decision Boundary on Simulated Dataset'})
```



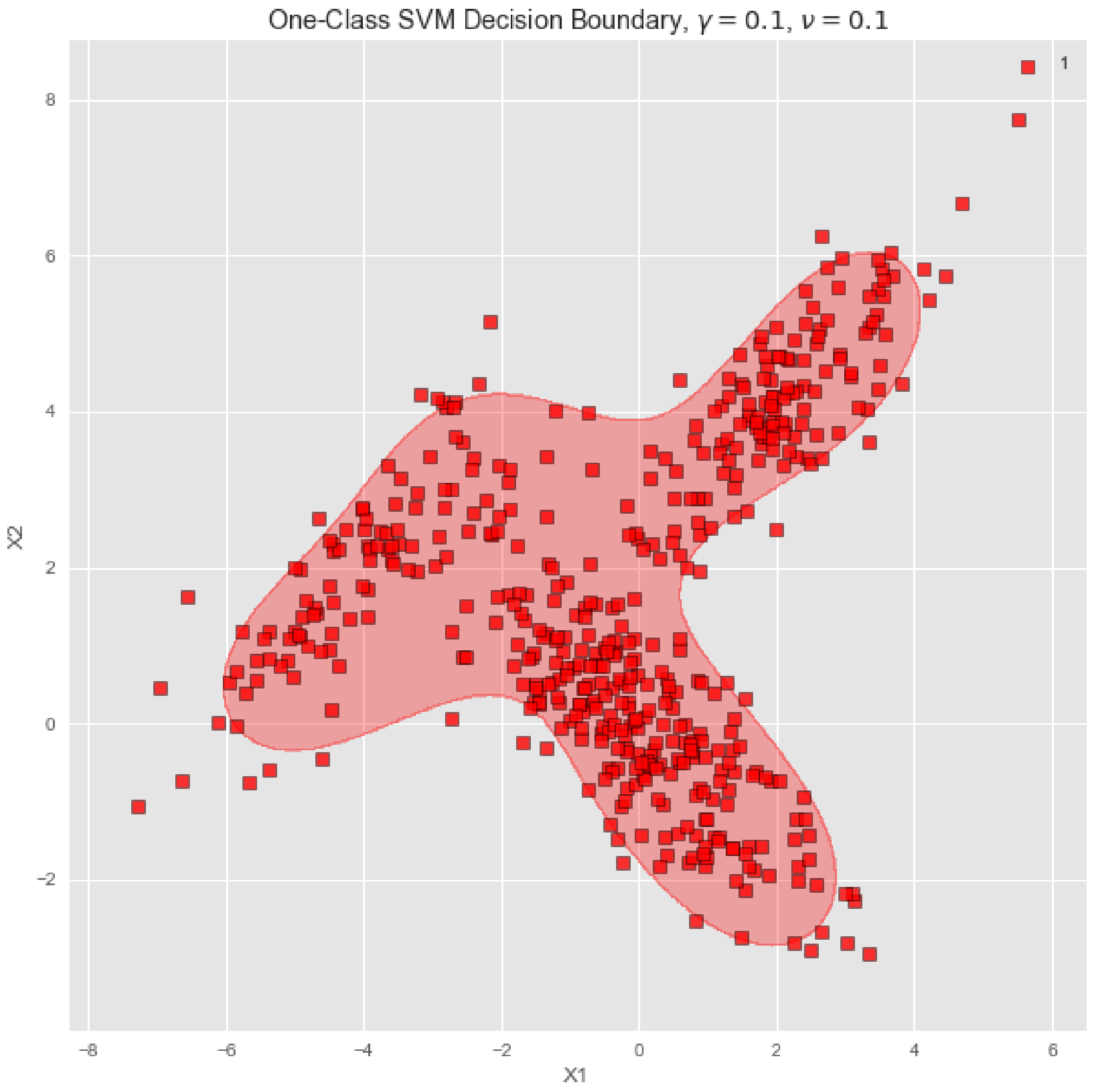
4.3 Meaning of γ

```
In [200]: ocs_clf = OneClassSVM(nu=0.1, kernel="rbf", gamma=2)
          ocs_clf.fit(X)
          fig, ax = plt.subplots(1,1, figsize=(10,10))
          plot_decision_regions(X,np.ones(500,dtype=int), clf=ocs_clf)
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'One-Class SVM Decision Boundary,'+
                         r' $\gamma={}$, $\nu={}$'.format(2, 0.1)})
```



```
In [201]: ocs_clf = OneClassSVM(nu=0.1, kernel="rbf", gamma=0.1)
          ocs_clf.fit(X)
          fig, ax = plt.subplots(1,1, figsize=(10,10))
```

```
plot_decision_regions(X,np.ones(500,dtype=int), clf=ocs_clf)
_ = ax.update({'xlabel':'X1', 'ylabel':'X2',
               'title':'One-Class SVM Decision Boundary,'+
               r' $\gamma={}$, $\nu={}$'.format(0.1, 0.1)})
```

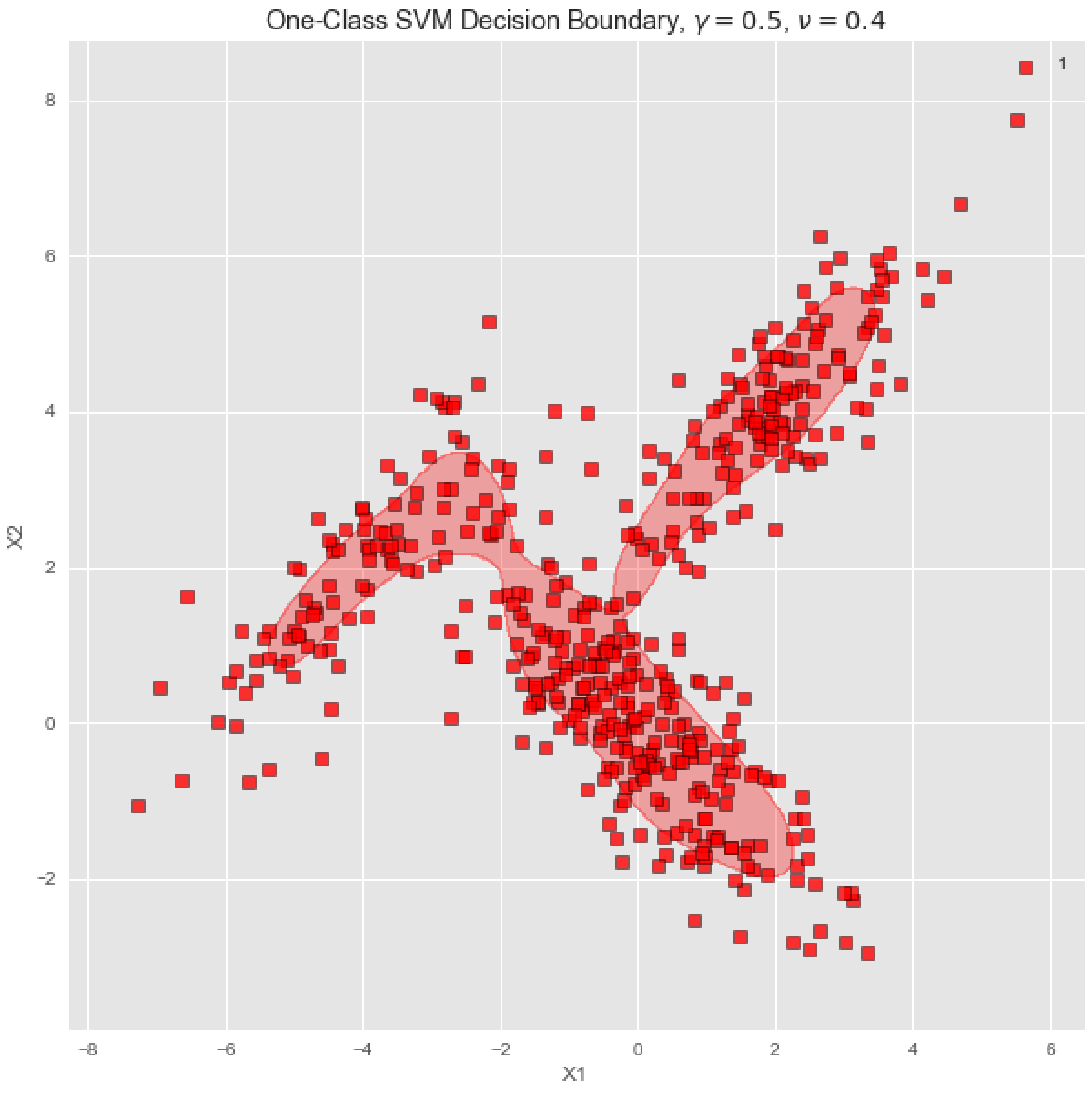


γ seems to affect the shape of the decision boundary. Larger γ results in a more complex shape of the boundary, and each sample point tends to affect a smaller local region. Smaller γ results in a smoother boundary, which appears to be the average of more sample points' influence, in other words, each sample point affects a bigger region.

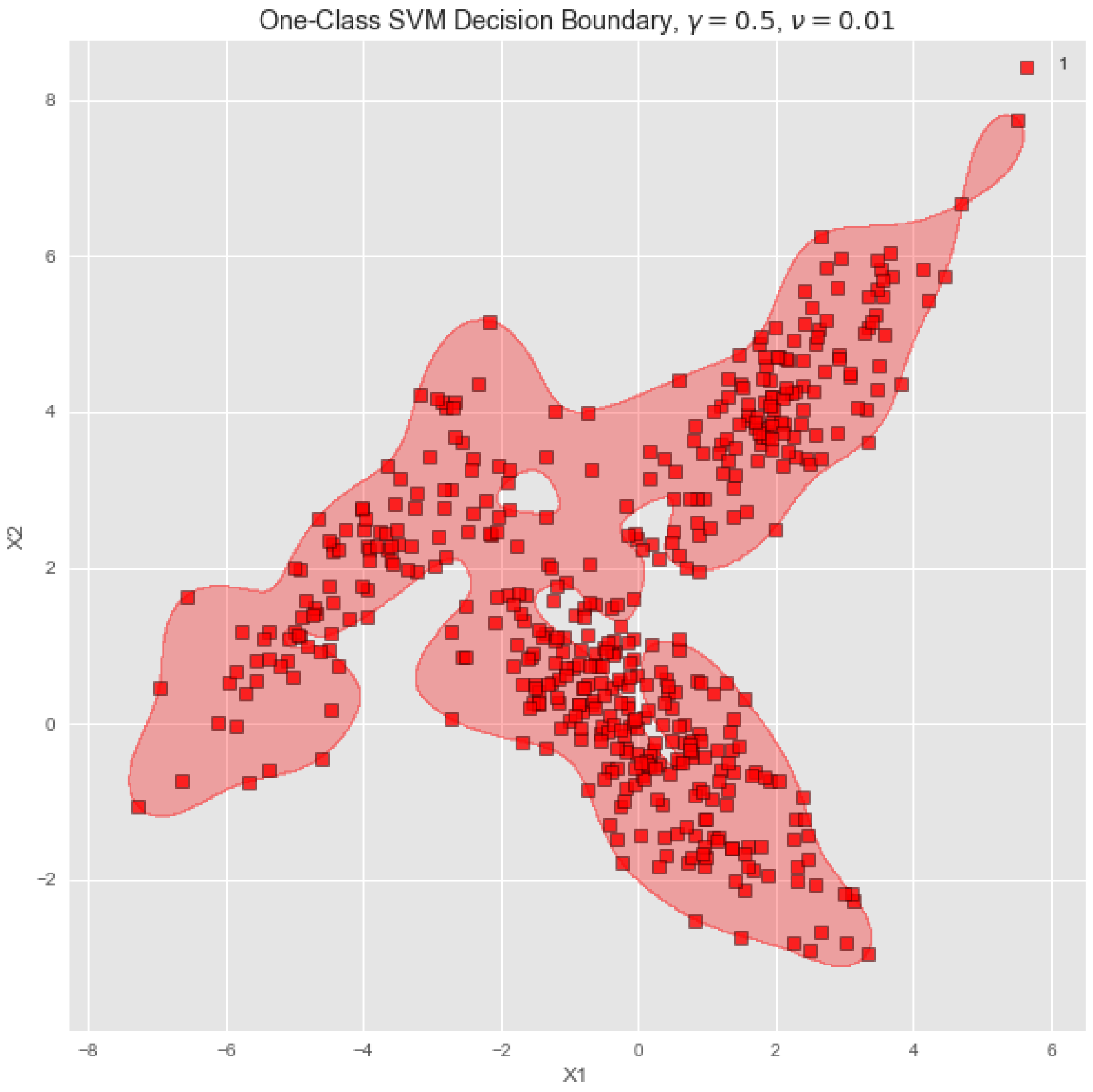
4.4 Meaning of ν

```
In [202]: ocs_clf = OneClassSVM(nu=0.4, kernel="rbf", gamma=0.5)
          ocs_clf.fit(X)
```

```
fig, ax = plt.subplots(1,1, figsize=(10,10))
plot_decision_regions(X,np.ones(500,dtype=int), clf=ocs_clf)
_ = ax.update({'xlabel':'X1', 'ylabel':'X2',
               'title':'One-Class SVM Decision Boundary,'+
               r' $\gamma={}$, $\nu={}$'.format(0.5, 0.4)})
```



```
In [203]: ocs_clf = OneClassSVM(nu=0.01, kernel="rbf", gamma=0.5)
          ocs_clf.fit(X)
          fig, ax = plt.subplots(1,1, figsize=(10,10))
          plot_decision_regions(X,np.ones(500,dtype=int), clf=ocs_clf)
          _ = ax.update({'xlabel':'X1', 'ylabel':'X2',
                         'title':'One-Class SVM Decision Boundary,'+
                         r' $\gamma={}$, $\nu={}$'.format(0.5, 0.01)})
```



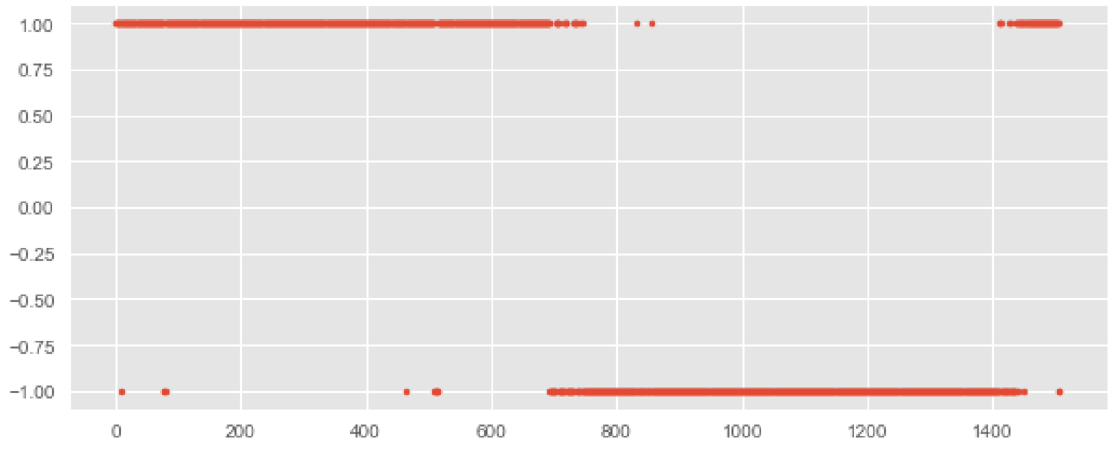
ν seems to control the tolerance to training error, which is similar to the role of C. If ν is small, the training error is low, more sample points are included in the decision region. Otherwise if ν is large, the training error is higher.

4.5 DJIA Anomalies

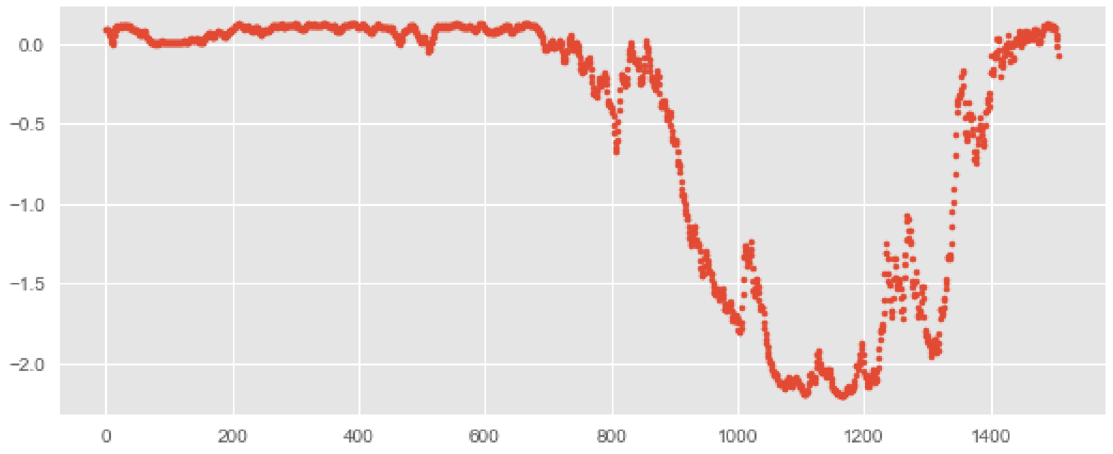
```
In [227]: djia = pd.read_csv('DJIA_transformed.csv', header=None)
          djia.columns = ['time', 'x1', 'x2', 'x3', 'x4']
          X_full = djia.iloc[:,1:]
          t_full = djia.iloc[:,0]
          X_train, X_test = djia.iloc[:467,1:], djia.iloc[467:,1:]
          t_train, t_test = djia.iloc[:467,0], djia.iloc[467:,0]

In [228]: ocs_clf_djia = OneClassSVM(nu=0.01, kernel="rbf", gamma=0.0000001)
```

```
        ocs_clf_djia.fit(X_train)
        y_pred = ocs_clf_djia.predict(X_full)
```

```
In [229]: ocs_clf.fit(X)
          fig, ax = plt.subplots(1,1, figsize=(10,4))
          ax.plot(y_pred, '.')

Out[229]: [<matplotlib.lines.Line2D at 0x12cfa0860>]
```



```
In [230]: dist_pred = ocs_clf_djia.decision_function(X_full)
          fig, ax = plt.subplots(1,1, figsize=(10,4))
          ax.plot(dist_pred, '.')

Out[230]: [<matplotlib.lines.Line2D at 0x12cd3d9b0>]
```



```
In [257]: # extract the points with scores < -2
          anomaly = djia['time'].iloc[(dist_pred < -2.0).reshape(1,len(djia))[0]]
          anomaly.iloc[0], anomaly.iloc[-1]

Out[257]: ('2007-05-02', '2008-01-03')
```

The resulting anomalous points with most negative scores are those between 2007-05-02 and 2008-01-03, during the financial crysis, as expected.