Asset Management HWI

Ze Yang (zey@andrew.cmu.edu)

October 31, 2018

1 Problem 1

(a) The Lagrangian of the problem, when x_t is *not observable*, is

$$\begin{aligned} & \mathcal{L}(w_t) = w_t\mu - \frac{\gamma}{2}w_t^2(\sigma_x^2 + \sigma_\epsilon^2) \\ \Rightarrow \ & \nabla\mathcal{L}(w_t) = \mu - \gamma w_t(\sigma_x^2 + \sigma_\epsilon^2) = 0 \quad \text{(first order condition)} \\ \Rightarrow \ & w_t^* = \frac{\mu}{\gamma(\sigma_x^2 + \sigma_\epsilon^2)} \end{aligned} \tag{1}$$

Therefore, the optimal weights is given by w_t^* above, and the portfolio return is $\mathbb{E}\left[w_t^* r_{t+1}\right] = w_t^*\mu = \mu^2/\gamma(\sigma_x^2 + \sigma_\epsilon^2) = S^2/\gamma$, due to the definition of S.

(b) The Lagrangian of the problem, when x_t is *observable*, is

$$\begin{aligned} & \mathcal{L}(w_t) = w_t(\mu + x_t) - \frac{\gamma}{2}w_t^2\sigma_\epsilon^2 \\ \Rightarrow \ & \nabla\mathcal{L}(w_t) = \mu + x_t - \gamma w_t\sigma_\epsilon^2 = 0 \quad \text{(first order condition)} \\ \Rightarrow \ & w_t^* = \frac{\mu + x_t}{\gamma\sigma_\epsilon^2} \end{aligned} \tag{2}$$

Therefore, the optimal weights is given by w_t^* above, and the portfolio return is

$$\begin{aligned} \mathbb{E}\left[w_t^* r_{t+1}\right] &= \mathbb{E}\left[\frac{\mu + x_t}{\gamma\sigma_\epsilon^2}(\mu + x_t + \epsilon_t)\right] \\ &= \frac{\mu^2}{\gamma\sigma_\epsilon^2} + \frac{2\mu\mathbb{E}\left[x_t\right]}{\gamma\sigma_\epsilon^2} + \frac{\mathbb{E}\left[x_t^2\right]}{\gamma\sigma_\epsilon^2} + \frac{\mu + x_t}{\gamma\sigma_\epsilon^2}\mathbb{E}\left[\epsilon_t\right] \\ &= \frac{\mu^2 + \sigma_x^2}{\gamma\sigma_\epsilon^2} = \frac{(S^2 + R^2)}{\gamma(1 - R^2)} \end{aligned} \tag{3}$$

By the definition of S and R.

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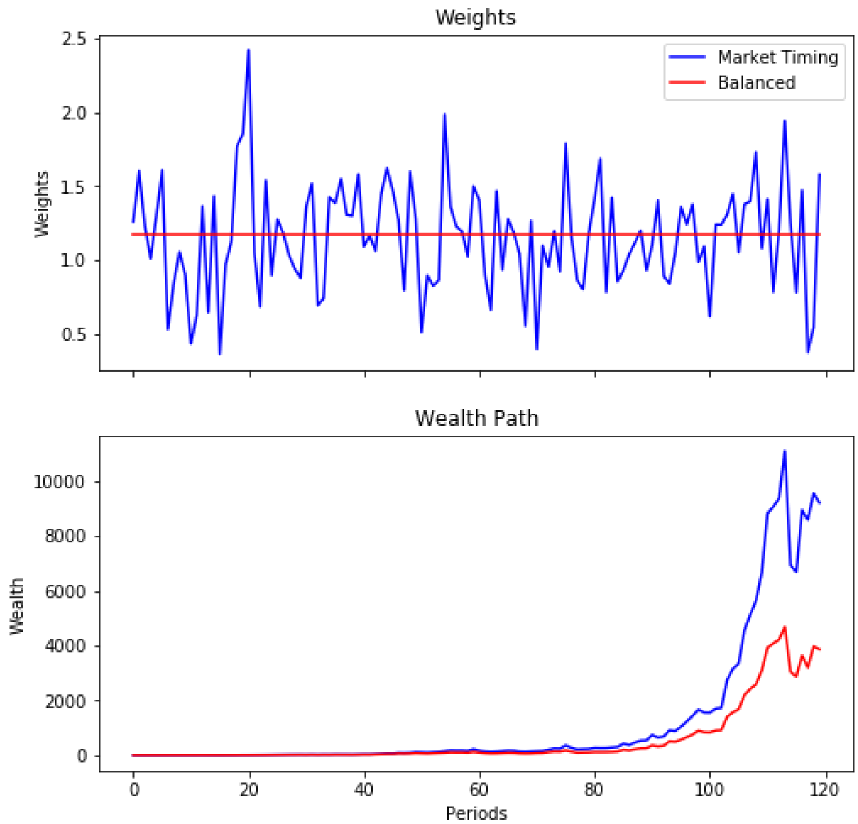
2 Problem 2



(Up): Weights plot for question (a), blue line stands for the market timing weights, and the red line is the constant "balanced" weight. **(Bottom):** Wealth path for the 2 portfolios.

In the code below, we run 30000 simulations and compare the average terminal wealth V.S. the expected terminal wealth implied by the expected portfolio return for the two cases. We can see that the simulated mean matches the theoretical value. On average (and in most cases), the "market timing" portfolio yields higher terminal wealth than the "balanced" one. In a few cases, we may also observe the opposite.

```
In [148]: sample = []
          from progressbar import ProgressBar
          bar = ProgressBar()
          for _ in bar(range(30000)):
              r1, r2 = sim_1path(plot=False)
              sample.append([r1, r2])
          sample = np.array(sample)
          sample.mean(axis=0)
```

```
100% (30000 of 30000) |###################| Elapsed Time: 0:00:05 Time: 0:00:05
```

```
Out[148]: array([ 31703.62661888,  12368.82899717])
```

```
In [137]: S = mu/np.sqrt(sigma_eps**2 + sigma_x**2)
          R = np.sqrt(sigma_x**2 / (sigma_eps**2 + sigma_x**2))
          r_expected_mkt_timing = (1/gamma) * (S**2 + R**2) / (1-R**2)
          r_expected_balanced = S**2/gamma
          r_expected_mkt_timing, r_expected_balanced

Out[137]: (0.090136054421768697, 0.081666666666666651)

In [138]: (1+r_expected_mkt_timing)**120, (1+r_expected_balanced)**120

Out[138]: (31454.617844235298, 12337.113778282106)
```

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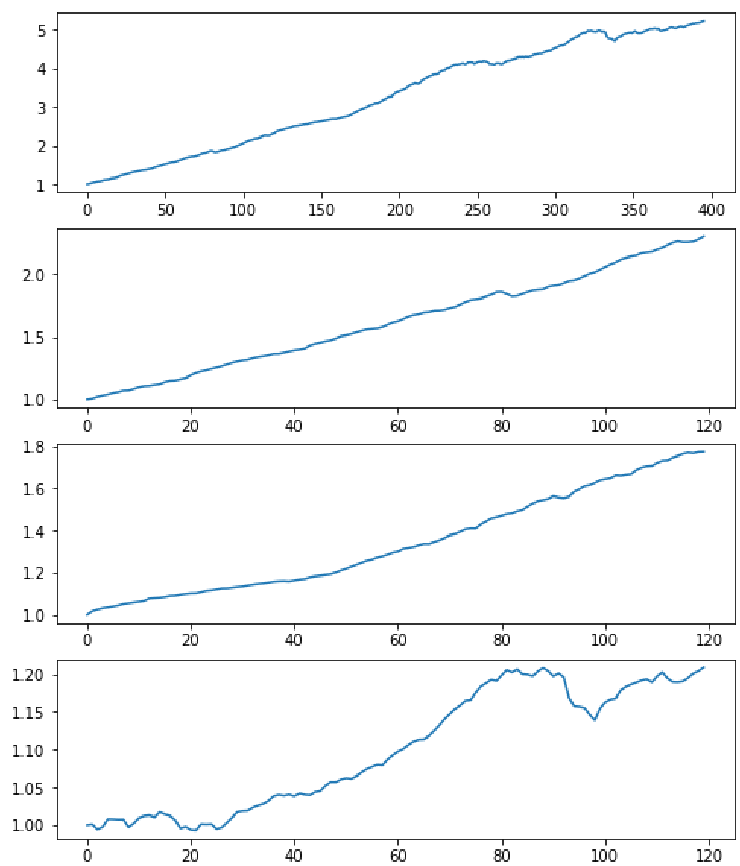
3 Problem 3

3.1 (a)

```
In [4]: // Code was hidden for succinctness

Out[5]: The optimal weight of risky asset is 0.11286468714593713

In [9]: // Code was hidden for succinctness
The plots below are the wealth curve in the given 4 time periods
using the optimal weight.
```



3.2 (b)

```
                          OLS Regression Results (Monthly)
==============================================================================
Dep. Variable:                      y   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.000
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0033      0.003      1.236      0.217      -0.002       0.009
x1            -0.0427      0.057     -0.745      0.456      -0.155       0.070
==============================================================================
```

```
                          OLS Regression Results (Quarterly)
==============================================================================
Dep. Variable:                      y   R-squared:                       0.002
Model:                            OLS   Adj. R-squared:                  0.000
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0042      0.010     -0.416      0.677      -0.024       0.016
x1             0.2254      0.214      1.053      0.293      -0.195       0.646
==============================================================================
```

```
                          OLS Regression Results (Yearly)
==============================================================================
Dep. Variable:                      y   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.007
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0209      0.041      0.506      0.614      -0.061       0.103
x1             0.0461      0.878      0.053      0.958      -1.689       1.781
==============================================================================
```

3.3 (c)

In [13]: // Code was hidden for succinctness

Out[14]: R^2 (OOS) = 4.2657276222435314e-05

3.4 (d)

```
In [15]: // Code was hidden for succinctness
```

```
Out[16]: R^2 (OOS, with restrictions) = -0.00062685117320127048
```

3.5 (e)

We tilt the mean-variance optimal constant weights towards a winsorized version (in order to remove extreme values of holdings) of the "market timing weights". Define $w_{\text{shrink},t} = 0.4\ \widetilde{w}^*_{\text{market timing},t} + 0.6\ w^*_{\text{mean-var}}$. See the detailed implementation in the code. The resulting portfolio outperforms the original mean-variance optimal portfolio in terms of Sharpe ratio in 3 (out of 4) testing time periods.

```
In [84]: // Code was hidden for succinctness
```

```
Sharpe (Constant Weight): 0.3271        Sharpe (Shrinkage): 0.3226
Sharpe (Constant Weight): 0.1217        Sharpe (Shrinkage): 0.1274
Sharpe (Constant Weight): 0.9831        Sharpe (Shrinkage): 1.0320
Sharpe (Constant Weight): -0.1143        Sharpe (Shrinkage): -0.0862
```