<u>Mecànica Quàntica: Formulari</u>

<u>H Pauli:</u> $\hbar=\frac{h}{2\pi}=1.0546\cdot10^{-34}\,J\cdot s$ $\mu_B=\frac{e^-}{m_e}\frac{\hbar}{2}=9{,}274\cdot10^{-24}\,J/T$ $\vec{\mu}=-g\frac{\mu_B}{\hbar}\vec{S}$ $g\approx2$ $\vec{S}=\frac{\hbar}{2}\vec{\sigma}$

$\boxed{H=-\vec{\mu}\,\vec{B}=g\frac{\mu_B}{2}\vec{\sigma}\,\vec{B}}$ $\boxed{\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}\quad\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\quad\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}}$ $[\sigma_i,\sigma_j]=2i\epsilon_{ijk}\sigma_k$ $\sigma_i\sigma_j+\sigma_j\sigma_i=2\delta_{ij}I$

$|S_x+\rangle=\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$ $|S_x-\rangle=\frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}$ $|S_y+\rangle=\frac{1}{\sqrt2}\begin{pmatrix}1\\i\end{pmatrix}$ $|S_y-\rangle=\frac{1}{\sqrt2}\begin{pmatrix}1\\-i\end{pmatrix}$ $|S_z+\rangle=\begin{pmatrix}1\\0\end{pmatrix}$ $|S_z-\rangle=\begin{pmatrix}0\\1\end{pmatrix}$

$$S_x=\frac{\hbar}{\sqrt2}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}\quad S_y=\frac{\hbar}{\sqrt2}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix}\quad S_z=\hbar\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$$

<u>Operadors:</u>

$[A,B]=AB-BA$ $[A,BC]=[A,B]C+B[A,C]$ si $[[A,B],B]=0\Rightarrow[A,f(B)]=[A,B]\cdot f'(B)$

$\langle\alpha v_1|\beta v_2\rangle=\alpha^*\beta\langle v_1|v_2\rangle$ $\langle e_i|A|e_j\rangle=\alpha_{ij}\Rightarrow\langle e_i|A^+|e_j\rangle=\alpha^*_{ji}$

VAP: $A\Rightarrow det(A-\lambda I)=0\Rightarrow\{\lambda_i\}\Rightarrow$ VEP: $A|v_i\rangle=\lambda_i|v_i\rangle$; si $A=A^+\Rightarrow\lambda_i\in\mathbb{R}$

Π projector $\Leftrightarrow\Pi=\Pi^+$ i $\Pi^2=\Pi,\lambda_i=\{0,1\}$ U unitari $\Leftrightarrow UU^+=I\Rightarrow\lambda_i=e^{i\varphi_i}$ $U=e^{iA}$(si $A=A^+$)

$A=\sum_{i=1}^{n=\#\{\lambda_i\}}\lambda_i|\lambda_i\rangle\langle\lambda_i|=\sum_{k=1}^{m=\#\{\lambda_k\,\text{diferents}\}}\lambda_k\Pi_k$ $[A,B]=0\Rightarrow e^{A+B}=e^Ae^B$ $[A,[A,B]]=0\Rightarrow e^{A+B}=e^Ae^Be^{-\frac12[A,B]}$

$\boxed{e^A=\sum e^{\lambda_i}|\lambda_i\rangle\langle\lambda_i|}$ $O'=e^{iA}Oe^{-iA}=e^{[A,-]}O$ $e^{-i\frac{\alpha}{2}\vec{n}\cdot\vec{\sigma}}=\cos\frac{\alpha}{2}I-i(\vec{n}\cdot\vec{\sigma})\sin\frac{\alpha}{2}$ $\vec{n}\cdot\vec{\sigma}=\begin{pmatrix}\cos\theta&\sin\theta\,e^{-i\varphi}\\\sin\theta\,e^{i\varphi}&-\cos\theta\end{pmatrix}$

$\boxed{P_{|\psi\rangle}(A:\lambda_i)=\|\Pi_i|\psi\rangle\|^2}$ $\langle A\rangle=\sum_{\forall\lambda_i}\lambda_iP_{|\psi\rangle}(A:\lambda_i)=\bar{\lambda}$

$\sigma_A^2=\sum_{\forall\lambda_i}(\lambda_i-\bar\lambda)^2P_{|\psi\rangle}(A:\lambda_i)=\langle\psi|A^2|\psi\rangle-\langle\psi|A|\psi\rangle^2=\|(A-\bar\lambda I)|\psi\rangle\|^2$ A ct moviment $\Leftrightarrow[A,H]+i\hbar\frac{\partial A}{\partial t}=0$

<u>Ev. temporal:</u>

$i\hbar\frac{\partial}{\partial t}|\psi[t]\rangle=H(t)|\psi[t]\rangle\Rightarrow$ $\boxed{|\psi[t]\rangle=e^{\frac{-i}{\hbar}Ht}|\psi[0]\rangle}$ $|\psi[t]\rangle=T\,e^{\frac{-i}{\hbar}\int_0^t dt'H(t')}|\psi[0]\rangle$ $U(t)=e^{\frac{-i}{\hbar}Ht}$

$X=\int dx\,x|x\rangle\langle x|$ $|\psi\rangle=\int dx\langle x|\psi\rangle|x\rangle$ $I=\int dx|x\rangle\langle x|$ $\langle p|X|\psi\rangle=i\hbar\frac{\partial\tilde\psi(p)}{\partial p}$ $\langle x|P|\psi\rangle=-i\hbar\frac{\partial\psi(x)}{\partial x}$

$\tilde\psi(p)=\int dx\frac{1}{\sqrt{2\pi\hbar}}e^{\frac{-i}{\hbar}px}\psi(x,t)$ $\psi(x)=\int dp\frac{1}{\sqrt{2\pi\hbar}}e^{\frac{i}{\hbar}px}\tilde\psi(p,t)$ $\boxed{\sigma_A\sigma_B\geq\frac12\|\langle\psi|[A,B]|\psi\rangle\|}$

$[X_i,X_j]=[P_i,P_j]=0$ $[X,P]=i\hbar I$ $[X^n,P]=i\hbar nX^{n-1}$ $[P^n,X]=-i\hbar nP^{n-1}$

$\frac{d}{dt}\langle x\rangle(t)=\frac1m\langle p\rangle(t)$ $\frac{d}{dt}\langle p\rangle(t)=-\langle\vec\nabla V\rangle(x)$ $\psi_n(x,t)=\psi_n(x)e^{\frac{-i}{\hbar}E_nt}$

<u>Osc. Harmònic:</u> $\boxed{H=\frac{P^2}{2\text{m}}+\frac12m\omega^2X^2}$ $E_n=\hbar\omega\left(n+\frac12\right)$ $b=\sqrt{\frac{\hbar}{m\omega}}$ $\tilde X=\frac{X}{b}$ $\tilde P=\frac{b}{\hbar}P$

$a=\frac{1}{\sqrt2}(\tilde X+i\tilde P)$ $a^+=\frac{1}{\sqrt2}(\tilde X-i\tilde P)$ $X=\frac{b}{\sqrt2}(a+a^+)$ $P=\frac{-i\hbar}{\sqrt2 b}(a-a^+)$ $\psi_1(x)=\frac{\sqrt2}{b}x\psi_0(x)$

$\boxed{\begin{matrix}a|n\rangle=\sqrt{n}|n-1\rangle\\a^+|n\rangle=\sqrt{n+1}|n+1\rangle\end{matrix}}$ $\begin{matrix}N=a^+a\\N|n\rangle=n|n\rangle\end{matrix}$ $[a,a^+]=I$ $\psi_0(x)=\frac{1}{(\pi b^2)^{1/4}}e^{-\frac12\left(\frac{x}{b}\right)^2}$ $\psi_n(x)=\frac{1}{\sqrt{2^nn!}}H_n\left(\frac{x}{b}\right)\psi_0(x)$

Imatges: $O(t)$ Operador unitari

$H_{est}'(t)=i\hbar\frac{\partial O(t)}{\partial t}O^+(t)+O(t)H(t)O^+(t)\to i\hbar\frac{\partial}{\partial t}|\psi'(t)\rangle=H_{est}|\psi'(t)\rangle$

$H_{op}(t)=-i\hbar\frac{dO(t)}{dt}O^+(t)\to i\hbar\frac{d}{dt}A'(t)=[A'(t),H_{op}(t)]$

Heisenberg: $H_{est}=0$ $H_{op}=U^+(t)H(t)U(t)$ Dirac: $H=H_0+H_1$ $\begin{matrix}H_{est}=U_0^+(t)H_1U_0(t)\\H_{op}=U_0^+(t)H_0U_0(t)\end{matrix}$

Schrödinger: $O(t)=I$

Simetries: $|\psi(t)\rangle$ solució Eq.Sch. $O_s(t)$ simetria $\Leftrightarrow |\psi'(t)\rangle = O_s(t)|\psi(t)\rangle$ solució Eq.Sch.

$O_s(t)$ invertible, conserva P.E $\quad O_s(t)=e^{isA(t)}$

Tr. Espacial: $O_a(t)=e^{\frac{-i}{\hbar}\vec{a}\cdot\vec{p}} \quad \rightarrow O_a(t)|\vec{x}\rangle = |\vec{x}+\vec{a}\rangle$

Rotació: $\vec{L}=\vec{X}\times\vec{P} \quad O(t)=e^{\frac{-i}{\hbar}\gamma\hat{n}\cdot\vec{L}} \quad \rightarrow O_\gamma(t)|\vec{x}\rangle = |R(\gamma,\hat{n})\vec{x}\rangle$ $\boxed{\begin{array}{l}[L_i,L_j]=i\hbar\epsilon_{ijk}L_k\\ [L_i,X_j]=i\hbar\epsilon_{ijk}X_k\\ [L_i,P_j]=i\hbar\epsilon_{ijk}P_k\end{array}}$ $[L_i,L^2]=0$

$L,S\rightarrow J$: $[J_i,J_j]=i\hbar\epsilon_{ijk}J_k \quad [J_i,J^2]=0 \quad J^2|\text{j, m}\rangle=\hbar^2 j(j+1)|\text{j, m}\rangle \quad J_z|\text{j,m}\rangle=\hbar m|\text{j,m}\rangle \quad m=\{-j..j\}$

$J_\pm=J_1\pm iJ_2 \quad J_+{}^+=J_- \quad J_-J_+=J^2-J_3^2-\hbar J_3 \quad J_+J_-=J^2-J_3^2+\hbar J_3$

$\boxed{J_\pm|\text{j,m}\rangle=\hbar\sqrt{j(j+1)-m(m\pm1)}|\text{j,m}\pm1\rangle}$

Rep. Posicions: $\vec{L}=\vec{X}\times\vec{P}=-i\hbar\vec{x}\times\vec{\nabla}$

$L_x=i\hbar\left(\sin\varphi\,\partial_\theta+\frac{\cos\varphi}{tg\,\theta}\partial_\varphi\right) \quad L_y=i\hbar\left(-\cos\varphi\,\partial_\theta+\frac{\sin\varphi}{tg\,\theta}\partial_\varphi\right) \quad L_z=-i\hbar\partial_\varphi$ $\boxed{\begin{array}{c}L_z\phi(\varphi)=\hbar m\phi(\varphi)\\ \phi(\varphi)=\frac{1}{\sqrt{2\pi}}e^{im\varphi}\end{array}}$

$L^2=-\hbar^2\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\,\partial_\theta)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$ $\boxed{L^2Y_{l,m}(\theta,\varphi)=\hbar^2 l(l+1)Y_{l,m}(\theta,\varphi)}$ P.164 Schaum

$\vec{J}=\vec{L}+\vec{S}$: $\langle J^2\rangle=\langle L^2\rangle+\langle S^2\rangle+2\cdot\langle\vec{L}\cdot\vec{S}\rangle \quad \vec{L}\vec{S}=L_zS_z+\frac{1}{2}(L_+S_-+L_-S_+)$

$|\vec{x}\rangle\otimes|j,m\rangle=|\vec{x},m\rangle \quad \psi_m(\vec{x})=\langle\vec{x},m|\psi\rangle \quad |\psi'\rangle=U(R)|\psi\rangle$ $\boxed{\psi_m'(x)=D_{m,m'}(R(\gamma,\hat{n}))\psi_{m'}'(R^{-1}\vec{x})}$

$D_{m,m'}(R(\gamma,\hat{n}))=e^{\frac{-i}{\hbar}\gamma\hat{n}S_{m,m'}} \quad s_{m,m'}=\langle j,m|\vec{S}|j,m'\rangle$

Euler: $R(\varphi,\hat{n})=R(\alpha,\hat{k})R(\beta,\hat{j})R(\gamma,\hat{k})\Rightarrow D_{m,m'}^{(j)}=\langle j,m|U(R)|j,m'\rangle=e^{-i(\alpha m+\gamma m')}d_{m,m'}^{(j)}(\beta)$

Clebsch-Gordan: $|j_1,m_1\rangle\otimes|j_2,m_2\rangle\rightarrow|j_1,j_2;j,m\rangle \quad j=\{|j_1-j_2|,\ldots,j_1+j_2\} \quad m=\{-j,\ldots,j\}$

$|j_1,j_2;j,m\rangle=\sum_{m_1,m_2}\left(\left(\langle j_1,m_1|\otimes\langle j_2,m_2|\right)|j_1,j_2;j,m\rangle\right)\left(|j_1,m_1\rangle\otimes|j_2,m_2\rangle\right)$

Pertorbacions: $H=H_0+\lambda H_1 \quad \langle\phi_n(\lambda)|\phi_n^{(i)}\rangle=0\,\forall i>0$

$H_0|\phi_n^{(0)}\rangle=E_n^{(0)}|\phi_n^{(0)}\rangle \quad |\phi_n\rangle=|\phi_n^{(0)}\rangle+\lambda|\phi_n^{(1)}\rangle+\ldots$

No degenerat: $\boxed{E_n^{(1)}=\langle\phi_n^{(0)}|H_1|\phi_n^{(0)}\rangle}$ $\quad E_n^{(2)}=\langle\phi_n^{(0)}|H_1|\phi_n^{(1)}\rangle=\sum_{m\neq n}\frac{\langle\phi_n^{(0)}|H_1|\phi_m^{(0)}\rangle\cdot\langle\phi_m^{(0)}|H_1|\phi_n^{(0)}\rangle}{E_n^{(0)}-E_m^{(0)}}$

Degenerat: $E_n^{(0)}$, $|\psi_{n,r}^{(0)}\rangle \quad |\psi_n(\lambda)\rangle=\alpha_r|\psi_n^{(0)}(\lambda)\rangle+\lambda|\psi_n^{(1)}(\lambda)\rangle+\ldots$

$H_{r,r'}=\langle\psi_{n,r'}^{(0)}|H_1|\psi_{n,r}^{(0)}\rangle\Rightarrow(H_{r,r'}-E_n^{(1)}\delta_{r,r'})\alpha_r=0\Rightarrow E_n^{(1)}\,VAP.$ Si $\#E_n^{(1)}=r$ Trenquem degeneració

Mètode variacional: $H|\varphi_n\rangle=E_n|\varphi_n\rangle \quad \langle H\rangle$ estacionari vora $|\phi(a)\rangle$ de prova

$\boxed{\langle H\rangle=\frac{\langle\phi|H|\phi\rangle}{\langle\phi|\phi\rangle}} \quad \frac{\partial\langle H\rangle}{\partial a}=0\rightarrow\begin{cases}\langle H\rangle\\ \phi(a)\end{cases}$

Cas: $H=H_0+\lambda V(t) \quad |\psi(0)\rangle=|\psi_i(0)\rangle \quad |\psi(t)\rangle=\sum_n\gamma_n(t)|\psi(t)\rangle \quad \Rightarrow\gamma_k^{(0)}=\delta_{ki}$

$i\hbar\dot{\vec{\gamma}}(t)=\lambda\tilde{V}(t)\vec{\gamma}(t)$

$\tilde{V}(t)=\tilde{V}_{m,n}(t)=\langle\psi_m(t)|V(t)|\psi_n(t)\rangle \quad \vec{\gamma}(t)=T\,e^{\frac{-i}{\hbar}\lambda\int_0^t\tilde{V}(t')dt'}\vec{\gamma}(0)$

$\gamma_k^{(1)}(t)=\frac{1}{i\hbar}\int_0^t dt'\tilde{V}_{ki}(t') \quad \tilde{V}_{ki}(t)=\langle\psi_k(t)|V(t)|\psi_i(t)\rangle=e^{\frac{it}{\hbar}(E_k-E_i)}\langle\psi_k(0)|V(t)|\psi_i(0)\rangle=e^{\frac{it}{\hbar}(E_k-E_i)}\bar{V}_{ki}(t)$

$P_{|\psi\rangle}(H:E_f)=\|\langle\psi_f(t)|\psi(t)\rangle\|^2=\lambda^2\left\|\frac{1}{i\hbar}\int_0^t\bar{V}_{fi}(t)e^{\frac{-it}{\hbar}(E_i-E_f)}\right\|^2$

Integrals útils: $\Gamma(1/2)=\sqrt{\pi};\ \Gamma(3/2)=\frac{\sqrt{\pi}}{2} \quad \int_0^\infty x^m e^{-ax^2}dx=\frac{\Gamma((m+1)/2)}{2a^{\frac{(m+1)}{2}}} \quad \int_0^\infty x^n e^{-ax}dx=\frac{\Gamma(n+1)}{a^{n+1}}$