

PROFESSIONAL STUDIES

Final Exam

Points possible: 100

Description: The final exam will cover topics from sessions 1-9.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, or basic graphing calculators (such as TI 83 or 84). Any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted.

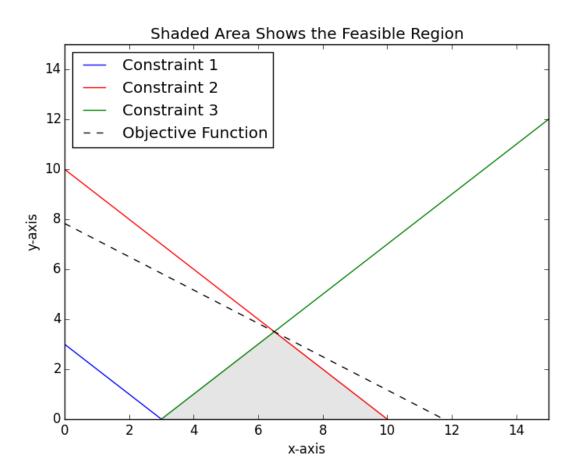
For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. For questions that specifically require Python, the code and output should be included with your answer. For questions that require graphs, only use Python.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

- 1. Answer all questions on the exam thoroughly. Create a Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.
- 2. Once you have completed your exam, return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. Maximize z = 4x + 6y subject to the constraints provided in the graph below. Using Python, recreate this graph and provide the optimal solution. You must also identify each linear inequality and the objective function in the legend of the graph that you create.



2. The manager of a concert hall estimates that 500 people attend each classical concert, 900 people attend each jazz concert, and 400 people attend each rock concert. In any given month, the total of the number of classical concerts and the number of jazz concerts may not exceed 11 and the number of rock concerts must be no more than 6. Furthermore, there should be twice as many rock concerts as classical concerts in any given month. How many of each type of concert should there be in a month in order to maximize attendance? What will that attendance be?

3. Researchers at Northwestern University have developed a piecewise function that can be used to estimate the body weight (in grams) of a female fox during the first 56 days of its life according to

$$W(t) = \begin{cases} 50 + 2.85t + 0.6519t^2 + 0.00804t^3 & if \ 0 \le t \le 28\\ -1097 + 68.9t & if \ 28 < t \le 56 \end{cases}$$

Is this function continuous? Why or why not? Using Python, graph this function.

4. \$2900.00 is deposited into an account with an interest rate of r% per year, compounded monthly. At the end of 8 years, the balance in the account is given by

$$A = 2900 \left(1 + \frac{r}{1200} \right)^{96}.$$

At what interest rate is the balance changing at a rate of \$372.62 per percent?

5. The natural resources of an island limit the growth of the population to a limiting value of 4,048. The population of the island over time is given by the logistic equation

$$P(t) = \frac{4048}{1 + 4.9e^{-0.37t}}$$

where *t* is the number of years after 1980. What is the rate of change of the population of the island in 1986?

6. The average monthly rent for a 1000 sq. ft. apartment in the Evanston area from 1998 through 2005 can be approximated by the function $f(t) = 1.603t^4 - 23.258t^3 + 62.12t^2 + 6.992t + 1010$, where t is the number of years since 1998. Using Python, find the year during which rents were increasing most rapidly. What was the highest rate of increase?

7. Phil is creating a flyer to contain 27 in² of printing with a 3-in margin at the top and bottom and a 1-in margin at each side. What overall dimensions will minimize the amount of paper used?

8. In 2015 the measles epidemic was growing according to the rate

$$N'(t) = \frac{106t}{t^2 + 2}$$

where N(t) is the number of people infected after t days. Find a formula for the number of people infected after t days, given that 47 people were initially infected. How many people were infected after 20 days?

9. The rate of change in a person's body temperature with respect to the dosage of *x* milligrams of a certain drug is given by

$$D'(x) = \frac{4}{x+5}.$$

One milligram raises the body temperature 2.5°C. Find a function giving the total temperature change. Using Python, graph both D(x) and D'(x).

10. The age of randomly selected, alcohol-impaired driver in a fatal car crash is a random variable with probability density function given by

$$f(x) = \frac{0.1972}{x^{0.5114}}$$

Find the following:

- (a) The expected age of a drunk driver in a fatal car crash.
- (b) The standard deviation of the distribution.
- (c) The probability that such a driver will be younger than 1 standard deviation below the mean.