Chapter 10
Statistical Inferences about Two Populations

LEARNING OBJECTIVES

The general focus of Chapter 10 is on testing hypotheses and constructing confidence intervals about parameters from two populations, thereby enabling you to:

1. Test hypotheses and develop confidence intervals about the difference in two means with known population variances using the *z* statistic.
2. Test hypotheses and develop confidence intervals about the difference in two means of independent samples with unknown population variances using the *t* test.
3. Test hypotheses and develop confidence intervals about the difference in two dependent populations.
4. Test hypotheses and develop confidence intervals about the difference in two population proportions.
5. Test hypotheses about the difference in two population variances using the *F* distribution.

CHAPTER OUTLINE

KEY TERMS

Dependent Samples	Independent Samples
F Distribution	Matched-Pairs Test
F Value	Related Measures

STUDY QUESTIONS

1. A researcher wants to estimate the difference in the means of two populations. A random sample of 40 items from the first population results in a sample mean of 433 with a population standard deviation of 112. A random sample of 50 items from the second population results in a sample mean of 467 with a population standard deviation of 120. From this information, a point estimate of the difference of population means can be computed as _______________.

2. Using the information from question 1, the researcher can compute a 95% confidence interval to estimate the difference in population means. The resulting confidence interval is _________________________.

3. A random sample of 32 items is taken from a population which has a population variance of 93. The resulting sample mean is 45.6. A random sample of 37 items is taken from a population which has a population variance of 88. The resulting sample mean is 49.4. Using this information, a 98% confidence interval can be computed to estimate the difference in means of these two populations. The resulting interval is _________________________.

4. A researcher desires to estimate the difference in means of two populations. To accomplish this, he/she takes a random sample of 85 items from the first population. The sample yields a mean of 168 with a population variance of 783. A random sample of 70 items is taken from the second population yielding a mean of 161 with a population variance of 780. A 94% confidence interval is computed to estimate the difference in population means. The resulting confidence interval is _______________.

5. Is there a difference in the average number years of experience of assembly line employees between company A and company B? A researcher wants to conduct a statistical test to answer this question. He is likely to be conducting a _______________-tailed test.

6. The researcher who is conducting the test to determine if there is a difference in the average number of years of experience of assembly line workers between companies A and B is using an alpha of .10. The critical value of *z* for this problem is __________.

7. Suppose the researcher conducting an experiment to compare the ages of workers at two companies. The researcher randomly samples forty-five assembly-line workers from company A and discovers that the sample average is 7.1 years with a population standard deviation of 2.3. Fifty-two assembly-line workers from company B are randomly selected resulting in a sample average of 6.2 years and a population standard deviation of 2.7. The observed *z* value for this problem is _______________.

8. Using an alpha of .10 and the critical values determined in questions 6 and 7, the decision is to _______________ the null hypothesis.

9. A researcher has a theory that the mean for population A is less than the mean for population B. To test this, she randomly samples thirty-eight items from population A and determines that the sample average is 38.4 with a population variance of 50.5 She randomly samples thirty-two items from population B and determines that the sample average is 44.3 with a population variance of 48.6 Alpha is .05. She is going to conduct a _______________-tailed test.

10. Using the information from question 9, the critical *z* value is _______________.

11. Using the information from question 9, the observed value of *z* is _______________.

12. Using the results determined in question 10 and 11, the decision is to _______________ the null hypothesis.

13. A researcher is interested in testing to determine if the mean of population one is greater than the mean of population two. He uses the following hypotheses to test this theory:

$$H_o\text{: } \mu_1 - \mu_2 = 0$$
$$H_a\text{: } \mu_1 - \mu_2 > 0$$

He randomly selects a sample of 8 items from population one resulting in a mean of 14.7 and a standard deviation of 3.4. He randomly selects a sample of 12 items from population two resulting in a mean of 11.5 and a standard deviation 2.9. He is using an alpha value of .10 to conduct this test. The degrees of freedom for this problem are _____________. It is assumed that these values are normally distributed in both populations.

14. The critical table *t* value used to conduct the hypothesis test in question 13 is _______________.

15. The observed *t* value from the sample data is ______.

16. Based on the observed t value obtained in question 15 and the critical table t value in question 14, the researcher should _______________ the null hypothesis.

17. What is the difference in the means of two populations? A researcher wishes to determine this by taking random samples of size 14 from each population and computing a 90% confidence interval. The sample from the first population produces a mean of 780 with a standard deviation of 245. The sample from the second population produces a mean of 890 with a standard deviation of 256. The point estimate for the difference in the means of these two populations is _______________. Assume that the values are normally distributed in each population.

18. The table t value used to construct the confidence interval for the problem in question 17 is __________.

19. The confidence interval constructed for the problem in question 17 is __________.

20. The matched-pairs t test deals with _______________ samples.

21. A researcher wants to conduct a before/after study on 13 subjects to determine if a treatment results in higher scores. The hypotheses are:

$$H_o:\ D = 0$$
$$H_a:\ D < 0$$

Scores are obtained on the subjects both before and after the treatment. After subtracting the after scores from the before scores, the resulting value of $\bar{d}$ is -2.85 with a S_d of 1.01. The degrees of freedom for this test are _______________. Assume that the data are normally distributed in the population.

22. The critical table t value for the problem in question 21 is _______________ if $\alpha = .01$.

23. The observed t value for the problem in question 21 is _______________.

24. For the problem in question 21 based on the critical table t value obtained in question 22 and the observed t value obtained in question 23, the decision should be to _______________ the null hypothesis.

25. A researcher is conducting a matched-pairs study. She gathers data on each pair in the study resulting in:

Pair	Group 1	Group 2
1	10	12
2	13	14
3	11	15
4	14	14
5	12	11
6	12	15
7	10	16
8	8	10

Assuming that the data are normally distributed in the population, the computed value of $\overline{d}$ is ________________.

26. The value of s_d for the problem in question 25 is ________________.

27. The degrees of freedom for the problem in question 25 is ________________.

28. The observed value of t for the problem in question 25 is ________________.

29. A researcher desires to estimate the difference between two related populations. He gathers pairs of data from the populations. The data are below:

Pair	Group 1	Group 2
1	360	280
2	345	290
3	355	300
4	325	270
5	340	300
6	365	310

It is assumed that the data are normally distributed in the population. Using this data, the value of $\overline{d}$ is ________________.

30. For the problem in 29, the value of s_d is ___________.

31. The point estimate for the population difference for the problem in question 29 is ________________.

32. The researcher conducting the study for the problem in question 29 wants to use a 95% level of confidence. The table t value for this confidence interval is ________________.

33. The confidence interval computed for the problem in question 29 is ________________.

34. A researcher is interested in estimating the difference in two populations proportions. A sample of 1000 from each population results in sample proportions of .61 and .64. The point estimate of the difference in the population proportions is _______________.

35. Using the data from question 34, the researcher computes a 90% confidence interval to estimate the difference in population proportions. The resulting confidence interval is _________________________.

36. A random sample of 400 items from a population shows that 110 of the sample items possess a given characteristic. A random sample of 550 items from a second population resulted in 154 of the sample items possessing the characteristic. Using this data, a 99% confidence interval is constructed to estimate the difference in population proportions which possess the given characteristic. The resulting confidence interval is _________________________.

37. A researcher desires to estimate the difference in proportions of two populations. To accomplish this, he/she samples 338 and 332 items respectively from each population. The resulting sample proportions are .71 and .68 respectively. Using this data, a 90% confidence interval can be computed to estimate the difference in population proportions. The resulting confidence interval is _________________________.

38. A statistician is being asked to test a new theory that the proportion of population A possessing a given characteristic is greater than the proportion of population B possessing the characteristic. A random sample of 625 from population A has been taken and it is determined that 463 possess the characteristic. A random sample of 704 taken from population B results in 428 possessing the characteristic. The alternative hypothesis for this problem is _______________.

39. The observed value of z for question 38 is _______________.

40. Suppose alpha is .10. The critical value of z for question 38 is _______________.

41. Based on the results of question 39 and 40, the decision for the problem in question 38 is to _______________ the null hypothesis.

42. In testing hypotheses about two population variances, use the _______________________ distribution.

43. Suppose we want to test the following hypothesis:

H_0: $\sigma_1^2 = \sigma_2^2$ and H_a: $\sigma_1^2 > \sigma_2^2$

A sample of 9 items from population one yielded a sample standard deviation of 8.6.
A sample of 8 items from population two yielded a sample standard deviation of 6.9.
If alpha is .05, the critical F value is ___________________________.

44. The observed F value for question 45 is _______________________. The resulting decision is _______________________.

ANSWERS TO STUDY QUESTIONS

1. –34
2. $-82.07 \leq \mu_1 - \mu_2 \leq 14.07$
3. $-9.16 \leq \mu_1 - \mu_2 \leq 1.56$
4. $-1.48 \leq \mu_1 - \mu_2 \leq 15.48$
5. Two
6. $\pm$ 1.645
7. 1.77
8. Reject
9. One
10. –1.645
11. –3.50
12. Reject
13. 18
14. 1.33
15. 2.26
16. Reject
17. –110
18. 1.706
19. $-271.56 \leq \mu_1 - \mu_2 \leq 51.56$
20. Related
21. 12
22. –2.681
23. –10.17
24. Reject
25. –2.125
26. 2.232
27. 7
28. –2.69
29. 56.67
30. 12.91
31. 56.67
32. 2.571
33. $43.12 \leq D \leq 70.22$
34. -.03
35. $-.066 \leq p_1 - p_2 \leq .006$
36. $-.081 \leq p_1 - p_2 \leq .071$
37. $-.0285 \leq p_1 - p_2 \leq .0885$
38. $p_A - p_B > 0$
39. 5.14
40. 1.28
41. Reject
42. F
43. 3.73
44. 1.55, Fail to Reject the Null Hypothesis

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 10

10.1

Sample 1	Sample 2
$\bar{x}_1 = 51.3$	$\bar{x}_2 = 53.2$
$s_1^2 = 52$	$s_2^2 = 60$
$n_1 = 31$	$n_2 = 32$

a) H_o: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

For one-tail test, $\alpha = .10$ $z_{.10} = -1.28$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(51.3 - 53.2) - (0)}{\sqrt{\frac{52}{31} + \frac{60}{32}}} = \mathbf{-1.01}$$

Since the observed $z = -1.01 > z_c = -1.28$, the decision is to **fail to reject the null hypothesis**.

b) Critical value method:

$$z_c = \frac{(\bar{x}_1 - \bar{x}_2)_c - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$-1.28 = \frac{(\bar{x}_1 - \bar{x}_2)_c - (0)}{\sqrt{\frac{52}{31} + \frac{60}{32}}}$$

$(\bar{x}_1 - \bar{x}_2)_c = \mathbf{-2.41}$

c) The area for $z = -1.01$ using Table A.5 is .3438.
The p-value is .5000 - .3438 = **.1562**

10.3 a) Sample 1 Sample 2

$\bar{x}_1 = 88.23$ $\bar{x}_2 = 81.2$

$\sigma_1^2 = 22.74$ $\sigma_2^2 = 26.65$

$n_1 = 30$ $n_2 = 30$

H_o: $\mu_1 - \mu_2 = 0$

H_a: $\mu_1 - \mu_2 \neq 0$

For two-tail test, use $\alpha/2 = .01$ $z_{.01} = \pm 2.33$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(88.23 - 81.2) - (0)}{\sqrt{\frac{22.74}{30} + \frac{26.65}{30}}} = \mathbf{5.48}$$

Since the observed $z = 5.48 > z_{.01} = 2.33$, the decision is to **reject the null hypothesis**.

b) $$(\bar{x}_1 - \bar{x}_2) \pm z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(88.23 - 81.2) \pm 2.33\sqrt{\frac{22.74}{30} + \frac{26.65}{30}}$$

7.03 ± 2.99

$\mathbf{4.04 \leq \mu_1 - \mu_2 \leq 10.02}$

This supports the decision made in a) to reject the null hypothesis because zero is not in the interval.

10.5

A	B
$n_1 = 40$	$n_2 = 37$
$\bar{x}_1 = 5.3$	$\bar{x}_2 = 6.5$
$\sigma_1^2 = 1.99$	$\sigma_2^2 = 2.36$

For a 95% C.I., $z_{.025} = 1.96$

$$(\bar{x}_1 - \bar{x}_2) \pm z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(5.3 - 6.5) \pm 1.96\sqrt{\frac{1.99}{40} + \frac{2.36}{37}}$$

-1.2 ± .66 **$-1.86 \le \mu_1 - \mu_2 \le -.54$**

The results indicate that we are 95% confident that, on average, Plumber B does between 0.54 and 1.86 more jobs per day than Plumber A. Since zero does not lie in this interval, we are confident that there is a difference between Plumber A and Plumber B.

10.7

2001	2011	
$\bar{x}_1 = 190$	$\bar{x}_2 = 198$	
$\sigma_1 = 18.50$	$\sigma_2 = 15.60$	
$n_1 = 51$	$n_2 = 47$	$\alpha = .01$

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$
For a one-tailed test, $z_{.01} = -2.33$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(190 - 198) - (0)}{\sqrt{\frac{(18.50)^2}{51} + \frac{(15.60)^2}{47}}} = \mathbf{-2.32}$$

Since the observed $z = -2.32 > z_{.01} = -2.33$, the decision is to **fail to reject the null hypothesis**.

10.9 Canon Pioneer

Canon	Pioneer
$\bar{x}_1 = 1.3$	$\bar{x}_2 = 1.0$
$\sigma_1 = 0.7$	$\sigma_2 = 0.4$
$n_1 = 36$	$n_2 = 45$

H_o: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

For two-tail test, $\alpha/2 = .025$ $z_{.025} = \pm 1.96$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(1.3 - 1.0) - (0)}{\sqrt{\frac{(0.7)^2}{36} + \frac{(0.4)^2}{45}}} = \mathbf{2.29}$$

Since the observed $z = 2.29 > z_c = 1.96$, the decision is to **reject the null hypothesis**.

10.11 H_o: $\mu_1 - \mu_2 = 0$ $\alpha = .01$
H_a: $\mu_1 - \mu_2 < 0$ df = 8 + 11 - 2 = 17

Sample 1	Sample 2
$n_1 = 8$	$n_2 = 11$
$\bar{x}_1 = 24.56$	$\bar{x}_2 = 26.42$
$s_1^2 = 12.4$	$s_2^2 = 15.8$

For one-tail test, $\alpha = .01$ Critical $t_{.01,17} = -2.567$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(24.56 - 26.42) - (0)}{\sqrt{\frac{12.4(7) + 15.8(10)}{8 + 11 - 2}}\sqrt{\frac{1}{8} + \frac{1}{11}}} = \mathbf{-1.05}$$

Since the observed $t = -1.05 > t_{.01,19} = -2.567$, the decision is to **fail to reject the null hypothesis**.

10.13 H$_o$: $\mu_1 - \mu_2 = 0$ $\alpha = .05$

H$_a$: $\mu_1 - \mu_2 > 0$ df = $n_1 + n_2 - 2 = 10 + 10 - 2 = 18$

Sample 1	Sample 2
$n_1 = 10$	$n_2 = 10$
$\bar{x}_1 = 45.38$	$\bar{x}_2 = 40.49$
$s_1 = 2.357$	$s_2 = 2.355$

For one-tail test, $\alpha = .05$ Critical $t_{.05,18} = 1.734$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} =$$

$$t = \frac{(45.38 - 40.49) - (0)}{\sqrt{\frac{(2.357)^2(9) + (2.355)^2(9)}{10+10-2}}\sqrt{\frac{1}{10}+\frac{1}{10}}} = \mathbf{4.64}$$

Since the observed $t = 4.64 > t_{.05,18} = 1.734$, the decision is to **reject the null hypothesis**.

10.15

Peoria	Evansville	
$n_1 = 21$	$n_2 = 26$	
$\bar{x}_1 = 116{,}900$	$\bar{x}_2 = 114{,}000$	
$s_1 = 2{,}300$	$s_2 = 1{,}750$	df = 21 + 26 – 2

a) 90% level of confidence, $\alpha/2 = .05$ $t_{.05,45} = 1.684$ (used df = 40)

$$(\bar{x}_1 - \bar{x}_2) \pm t\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} =$$

$$(116{,}900 - 114{,}000) \pm 1.684\sqrt{\frac{(2300)^2(20) + (1750)^2(25)}{21+26-2}}\sqrt{\frac{1}{21}+\frac{1}{26}} =$$

$2{,}900 \pm 994.62$

1905.38 $\leq$ $\mu_1 - \mu_2$ $\leq$ 3894.62

b) H$_o$: $\mu_1 - \mu_2 = 0$ $\alpha = .10$
H$_a$: $\mu_1 - \mu_2 \neq 0$ df = 21 + 26 - 2

For two-tail test, $\alpha/2 = .05$ Critical $t_{.05,45} = 1.684$ (used df = 40)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(116{,}900 - 114{,}000) - (0)}{\sqrt{\frac{(2300)^2 20 + (1750)^2 25}{21+26-2}}\sqrt{\frac{1}{21}+\frac{1}{26}}} = \mathbf{4.91}$$

Since the observed $t = 4.91 < t_{.05,45} = 1.684$, **reject the null hypothesis.**

10.17 Let Boston be group 1

1) H$_o$: $\mu_1 - \mu_2 = 0$
H$_a$: $\mu_1 - \mu_2 > 0$

2) $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

3) $\alpha = .05$

4) For a one-tailed test and df = 8 + 9 - 2 = 15, $t_{.05,15} = 1.753$. If the observed value of t is greater than 1.753, the decision is to reject the null hypothesis.

5)

Boston	Dallas
$n_1 = 8$	$n_2 = 9$
$\bar{x}_1 = 47$	$\bar{x}_2 = 44$
$s_1 = 3$	$s_2 = 3$

6) $$\boldsymbol{t} = \frac{(47-44)-(0)}{\sqrt{\frac{7(3)^2 + 8(3)^2}{15}}\sqrt{\frac{1}{8}+\frac{1}{9}}} = \mathbf{2.06}$$

7) Since $t = 2.06 > t_{.05,15} = 1.753$, the decision is to **reject the null hypothesis.**

8) Boston rental rates are significantly higher. One might argue, however, that a $3 difference is not substantive.

10.19 H$_o$: $\mu_1 - \mu_2 = 0$
H$_a$: $\mu_1 - \mu_2 \neq 0$

df = $n_1 + n_2 - 2 = 11 + 11 - 2 = 20$

Toronto	Mexico City
$n_1 = 11$	$n_2 = 11$
$\bar{x}_1$ = \$67,381.82	$\bar{x}_2$ = \$63,481.82
s_1 = \$2,067.28	s_2 = \$1,594.25

For a two-tail test, $\alpha/2 = .005$ Critical $t_{.005,20} = \pm 2.845$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} =$$

$$t = \frac{(67{,}381.82 - 63{,}481.82) - (0)}{\sqrt{\frac{(2{,}067.28)^2(10) + (1{,}594.25)^2(10)}{11+11-2}}\sqrt{\frac{1}{11}+\frac{1}{11}}} = \mathbf{4.95}$$

Since the observed $t = 4.95 > t_{.005,20} = 2.845$, the decision is to **Reject the null hypothesis**.

95% Confidence Interval: $t_{.025,\,20} = 2.086$

$$(\bar{x}_1 - \bar{x}_2) \pm t\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} =$$

$$(67{,}381.82 - 63{,}481.82) \pm 2.086\sqrt{\frac{(2067.28)^2(10) + (1594.25)^2(10)}{11+11-2}}\sqrt{\frac{1}{11}+\frac{1}{11}}$$

$3900 \pm 2.086(1845.98)(.4264) = 3900 \pm 1641.94$

$$\mathbf{2258.06 \leq \mu_1 - \mu_2 \leq 5541.64}$$

10.21 H_o: $D = 0$
H_a: $D > 0$

Sample 1	Sample 2	d
38	22	16
27	28	-1
30	21	9
41	38	3
36	38	-2
38	26	12
33	19	14
35	31	4
44	35	9

$n = 9$ $\bar{d} = 7.11$ $s_d = 6.45$ $\alpha = .01$ df = n - 1 = 9 - 1 = 8

For one-tail test and $\alpha = .01$, the critical $t_{.01,8} = \pm 2.896$

$$t = \frac{\bar{d} - D}{\frac{s_d}{\sqrt{n}}} = \frac{7.11 - 0}{\frac{6.45}{\sqrt{9}}} = \mathbf{3.31}$$

Since the observed $t = 3.31 > t_{.01,8} = 2.896$, the decision is to **reject the null hypothesis**.

10.23 $n = 22$ $\bar{d} = 40.56$ $s_d = 26.58$

For a 98% Level of Confidence, $\alpha/2 = .01$, and df = n - 1 = 22 - 1 = 21

$t_{.01,21} = 2.518$

$$\bar{d} \pm t \frac{s_d}{\sqrt{n}}$$

$$40.56 \pm (2.518) \frac{26.58}{\sqrt{22}}$$

40.56 ± 14.27

$\mathbf{26.29 \leq \mathit{D} \leq 54.83}$

10.25

City	Cost	Resale	d
Atlanta	20427	25163	-4736
Boston	27255	24625	2630
Des Moines	22115	12600	9515
Kansas City	23256	24588	-1332
Louisville	21887	19267	2620
Portland	24255	20150	4105
Raleigh-Durham	19852	22500	-2648
Reno	23624	16667	6957
Ridgewood	25885	26875	- 990
San Francisco	28999	35333	-6334
Tulsa	20836	16292	4544

$\bar{d} = 1302.82$ $s_d = 4938.22$ $n = 11$, df = 10

$\alpha = .01$ $\alpha/2 = .005$ $t_{.005,10} = 3.169$

$$\bar{d} \pm t\frac{s_d}{\sqrt{n}} = 1302.82 \pm 3.169\frac{4938.22}{\sqrt{11}} = 1302.82 \pm 4718.42$$

$$\mathbf{-3415.6 \le D \le 6021.2}$$

10.27

Before	After	d
255	197	58
230	225	5
290	215	75
242	215	27
300	240	60
250	235	15
215	190	25
230	240	-10
225	200	25
219	203	16
236	223	13

$n = 11$ $\bar{d} = 28.09$ $s_d = 25.813$ df = n - 1 = 11 - 1 = 10

For a 98% level of confidence and $\alpha/2 = .01$, $t_{.01,10} = 2.764$

$$\bar{d} \pm t\frac{s_d}{\sqrt{n}} = 28.09 \pm (2.764)\frac{25.813}{\sqrt{11}} = 28.09 \pm 21.51$$

$$\mathbf{6.58 \le D \le 49.60}$$

10.29 $n = 21$ $\overline{d} = 75$ $s_d = 30$ df = 21 - 1 = 20

For a 90% confidence level, $\alpha/2 = .05$ and $t_{.05,20} = 1.725$

$$\overline{d} \pm t\frac{s_d}{\sqrt{n}} = 75 \pm 1.725\frac{30}{\sqrt{21}} = 75 \pm 11.29$$

$63.71 \le D \le 86.29$

10.31 a)

Sample 1	Sample 2
$n_1 = 368$	$n_2 = 405$
$x_1 = 175$	$x_2 = 182$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{175}{368} = .476 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{182}{405} = .449$$

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{175+182}{368+405} = \frac{357}{773} = .462$$

H_o: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 \neq 0$

For two-tail, $\alpha/2 = .025$ and $z_{.025} = \pm 1.96$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\overline{p}\cdot\overline{q}\left(\frac{1}{n_1} + \frac{1}{n}\right)}} = \frac{(.476 - .449) - (0)}{\sqrt{(.462)(.538)\left(\frac{1}{368} + \frac{1}{405}\right)}} = \mathbf{0.75}$$

Since the observed $z = 0.75 < z_c = 1.96$, the decision is to **fail to reject the null hypothesis**.

b) <u>Sample 1</u> <u>Sample 2</u>

$\hat{p}_1 = .38$ $\hat{p}_2 = .25$

$n_1 = 649$ $n_2 = 558$

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{649(.38) + 558(.25)}{649 + 558} = .32$$

H_o: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 > 0$

For a one-tail test and $\alpha = .10$, $z_{.10} = 1.28$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p} \cdot \bar{q}\left(\frac{1}{n_1} + \frac{1}{n}\right)}} = \frac{(.38 - .25) - (0)}{\sqrt{(.32)(.68)\left(\frac{1}{649} + \frac{1}{558}\right)}} = \mathbf{4.83}$$

Since the observed $z = 4.83 > z_c = 1.28$, the decision is to **reject the null hypothesis**.

10.33 H_0: $p_m - p_w = 0$
H_a: $p_m - p_w < 0$ $n_m = 374$ $n_w = 481$ $\hat{p}_m = .59$ $\hat{p}_w = .70$

For a one-tailed test and $\alpha = .05$, $z_{.05} = -1.645$

$$\bar{p} = \frac{n_m\hat{p}_m + n_w\hat{p}_w}{n_m + n_w} = \frac{374(.59) + 481(.70)}{374 + 481} = .652$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p} \cdot \bar{q}\left(\frac{1}{n_1} + \frac{1}{n}\right)}} = \frac{(.59 - .70) - (0)}{\sqrt{(.652)(.348)\left(\frac{1}{374} + \frac{1}{481}\right)}} = \mathbf{-3.35}$$

Since the observed $z = -3.35 < z_{.05} = -1.645$, the decision is to **reject the null hypothesis**.

10.35 Computer Firms Banks

$\hat{p}_1 = .48$ $\hat{p}_2 = .56$

$n_1 = 56$ $n_2 = 89$

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{56(.48) + 89(.56)}{56 + 89} = .529$$

H_o: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 \neq 0$

For two-tail test, $\alpha/2 = .10$ and $z_c = \pm 1.28$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p} \cdot \bar{q}\left(\frac{1}{n_1} + \frac{1}{n}\right)}} = \frac{(.48 - .56) - (0)}{\sqrt{(.529)(.471)\left(\frac{1}{56} + \frac{1}{89}\right)}} = \mathbf{-0.94}$$

Since the observed $z = -0.94 > z_c = -1.28$, the decision is to **fail to reject the null hypothesis**.

10.37 H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 \neq 0$

$\alpha = .10$ $\hat{p}_1 = .09$ $\hat{p}_2 = .06$ $n_1 = 780$ $n_2 = 915$

For a two-tailed test, $\alpha/2 = .05$ and $z_{.05} = \pm 1.645$

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{780(.09) + 915(.06)}{780 + 915} = .0738$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p} \cdot \bar{q}\left(\frac{1}{n_1} + \frac{1}{n}\right)}} = \frac{(.09 - .06) - (0)}{\sqrt{(.0738)(.9262)\left(\frac{1}{780} + \frac{1}{915}\right)}} = \mathbf{2.35}$$

Since the observed $z = 2.35 > z_{.05} = 1.645$, the decision is to **reject the null hypothesis**.

10.39 H$_0$: $\sigma_1^2 = \sigma_2^2$ $\alpha = .01$ $n_1 = 11$ $s_1^2 = 1013$
H$_a$: $\sigma_1^2 > \sigma_2^2$ $n_2 = 10$ $s_2^2 = 562$

$df_{num} = 11 - 1 = 10$ $df_{denom} = 10 - 1 = 9$

Table $F_{.01,10,9} = 5.26$

$$F = \frac{s_1^2}{s_2^2} = \mathbf{1.80}$$

Since the observed $F = 1.80 < F_{.01,10,9} = 5.26$, the decision is to **fail to reject the null hypothesis**.

10.41

City 1	City 2
3.43	3.33
3.40	3.42
3.39	3.39
3.32	3.30
3.39	3.46
3.38	3.39
3.34	3.36
3.38	3.44
3.38	3.37
3.28	3.38

$n_1 = 10$ $df_1 = 9$ $n_2 = 10$ $df_2 = 9$

$s_1^2 = .0018989$ $s_2^2 = .0023378$

H$_0$: $\sigma_1^2 = \sigma_2^2$ $\alpha = .01$ $\alpha/2 = .005$
H$_a$: $\sigma_1^2 \neq \sigma_2^2$

Upper tail critical F value = $F_{.005,9,9} = 6.54$
Lower tail critical F value = $F_{.995,9,9} = 0.153$

$$F = \frac{s_1^2}{s_2^2} = \frac{.0018989}{.0023378} = \mathbf{0.81}$$

Since the observed $F = 0.81$ is greater than the lower tail critical value of 0.153 and less than the upper tail critical value of 6.54, the decision is to **fail to reject the null hypothesis**.

10.43 H$_0$: $\sigma_1^2 = \sigma_2^2$ $\alpha = .05$ $n_1 = 11$ $s_1 = 7.52$
H$_a$: $\sigma_1^2 > \sigma_2^2$ $n_2 = 15$ $s_2 = 6.08$

$df_{num} = 11 - 1 = 10$ $df_{denom} = 15 - 1 = 14$

The critical table F value is $F_{.05,10,14} = 2.60$

$$F = \frac{s_1^2}{s_2^2} = \frac{(7.52)^2}{(6.08)^2} = \mathbf{1.53}$$

Since the observed $F = 1.53 < F_{.05,10,14} = 2.60$, the decision is to **fail to reject the null hypothesis**.

10.45 H$_o$: $\mu_1 - \mu_2 = 0$
H$_a$: $\mu_1 - \mu_2 \neq 0$

For $\alpha = .10$ and a two-tailed test, $\alpha/2 = .05$ and $z_{.05} = \pm 1.645$

Sample 1	Sample 2
$\bar{x}_1 = 138.4$	$\bar{x}_2 = 142.5$
$\sigma_1 = 6.71$	$\sigma_2 = 8.92$
$n_1 = 48$	$n_2 = 39$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(138.4 - 142.5) - (0)}{\sqrt{\frac{(6.71)^2}{48} + \frac{(8.92)}{39}}} = \mathbf{-2.38}$$

Since the observed value of $z = -2.38$ is less than the critical value of $z = -1.645$, the decision is to **reject the null hypothesis**. There is a significant difference in the means of the two populations.

10.47 H_o: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

Sample 1	Sample 2	
$\bar{x}_1 = 2.06$	$\bar{x}_2 = 1.93$	
$s_1^2 = .176$	$s_2^2 = .143$	
$n_1 = 12$	$n_2 = 15$	$\alpha = .05$

This is a one-tailed test with df = 12 + 15 - 2 = 25. The critical value is $t_{.05,25} = 1.708$. If the observed value is greater than 1.708, the decision will be to reject the null hypothesis.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(2.06 - 1.93) - (0)}{\sqrt{\frac{(.176)(11) + (.143)(14)}{25}}\sqrt{\frac{1}{12} + \frac{1}{15}}} = \mathbf{0.85}$$

Since the observed value of $t = 0.85$ is less than the critical value of $t = 1.708$, the decision is to **fail to reject the null hypothesis**. The mean for population one is not significantly greater than the mean for population two.

10.49 H_o: $D = 0$ $\alpha = .01$
H_a: $D < 0$

$n = 21$ df = 20 $\bar{d} = -1.16$ $s_d = 1.01$

The critical $t_{.01,20} = -2.528$. If the observed t is less than -2.528, then the decision will be to reject the null hypothesis.

$$t = \frac{\bar{d} - D}{\frac{s_d}{\sqrt{n}}} = \frac{-1.16 - 0}{\frac{1.01}{\sqrt{21}}} = \mathbf{-5.26}$$

Since the observed value of $t = -5.26$ is less than the critical t value of -2.528, the decision is to **reject the null hypothesis**. The population difference is less than zero.

10.51 H_o: $p_1 - p_2 = 0$ $\alpha = .05$ $\alpha/2 = .025$
H_a: $p_1 - p_2 \neq 0$ $z_{.025} = \pm\ 1.96$

If the observed value of z is greater than 1.96 or less than -1.96, then the decision will be to reject the null hypothesis.

Sample 1	Sample 2
$x_1 = 345$	$x_2 = 421$
$n_1 = 783$	$n_2 = 896$

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{345 + 421}{783 + 896} = .4562$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{345}{783} = .4406 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{421}{896} = .4699$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\overline{p} \cdot \overline{q}\left(\frac{1}{n_1} + \frac{1}{n}\right)}} = \frac{(.4406 - .4699) - (0)}{\sqrt{(.4562)(.5438)\left(\frac{1}{783} + \frac{1}{896}\right)}} = \mathbf{-1.20}$$

Since the observed value of $z = -1.20$ is greater than -1.96, the decision is to **fail to reject the null hypothesis**. There is no significant difference.

10.53 H_0: $\sigma_1^2 = \sigma_2^2$ $\alpha = .05$ $n_1 = 8$ $s_1^2 = 46$
H_a: $\sigma_1^2 \neq \sigma_2^2$ $n_2 = 10$ $s_2^2 = 37$

$df_{num} = 8 - 1 = 7$ $df_{denom} = 10 - 1 = 9$
The critical F values are: $F_{.025,7,9} = 4.20$ $F_{.975,9,7} = .238$

If the observed value of F is greater than 4.20 or less than .238, then the decision will be to reject the null hypothesis.

$$F = \frac{s_1^2}{s_2^2} = \frac{46}{37} = \mathbf{1.24}$$

Since the observed $F = 1.24$ is less than $F_{.025,7,9} = 4.20$ and greater than $F_{.975,9,7} = .238$, the decision is to **fail to reject the null hypothesis**. There is no significant difference in the variances of the two populations.

10.55

Morning	Afternoon	d
43	41	2
51	49	2
37	44	-7
24	32	-8
47	46	1
44	42	2
50	47	3
55	51	4
46	49	-3

$n = 9$ $\quad \bar{d} = -0.444$ $\quad s_d = 4.447$ $\quad$ df = 9 - 1 = 8

For a 90% Confidence Level: $\alpha/2 = .05$ and $t_{.05,8} = 1.86$

$$\bar{d} \pm t\frac{s_d}{\sqrt{n}}$$

$$-0.444 \pm (1.86)\frac{4.447}{\sqrt{9}} = -0.444 \pm 2.757$$

$$\mathbf{-3.201 \le D \le 2.313}$$

10.57

Accounting	Data Entry
$n_1 = 16$	$n_2 = 14$
$\bar{x}_1 = 26{,}400$	$\bar{x}_2 = 25{,}800$
$s_1 = 1{,}200$	$s_2 = 1{,}050$

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$ $\quad \alpha = .05$ and $\alpha/2 = .025$

$df_{num} = 16 - 1 = 15$ $\quad df_{denom} = 14 - 1 = 13$

The critical F values are: $F_{.025,15,13} = 3.05$ $\quad F_{.975,15,13} = 0.33$

$$F = \frac{s_1^2}{s_2^2} = \frac{1{,}440{,}000}{1{,}102{,}500} = \mathbf{1.31}$$

Since the observed $F = 1.31$ is less than $F_{.025,15,13} = 3.05$ and greater than $F_{.975,15,13} = 0.33$, the decision is to **fail to reject the null hypothesis**.

10.59 H_o: $\mu_1 - \mu_2 = 0$ $\alpha = .01$

H_a: $\mu_1 - \mu_2 \neq 0$ df = 20 + 24 - 2 = 42

Detroit	Charlotte
$n_1 = 20$	$n_2 = 24$
$\bar{x}_1 = 17.53$	$\bar{x}_2 = 14.89$
$s_1 = 3.2$	$s_2 = 2.7$

For two-tail test, $\alpha/2 = .005$ and the critical $t_{.005,40} = \pm 2.704$ (used df=40)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$t = \frac{(17.53 - 14.89) - (0)}{\sqrt{\frac{(3.2)^2(19) + (2.7)^2(23)}{42}}\sqrt{\frac{1}{20}+\frac{1}{24}}} = \mathbf{2.97}$$

Since the observed $t = 2.97 > t_{.005,40} = 2.704$, the decision is to **reject the null hypothesis**.

10.61

Specialty	Discount
$n_1 = 350$	$n_2 = 500$
$\hat{p}_1 = .75$	$\hat{p}_2 = .52$

Let $\alpha = .10$ $\alpha/2 = .05$ and $z_{.05} = \pm 1.645$

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1+n_2} = \frac{350(.75) + 500(.52)}{350+500} = .615$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\cdot\bar{q}\left(\frac{1}{n_1}+\frac{1}{n}\right)}} = \frac{(.75 - .52) - (0)}{\sqrt{(.615)(.385)\left(\frac{1}{350}+\frac{1}{500}\right)}} = 6.78$$

Since the observed $z = 6.78 > z_{.05} = 1.645$, the decision is to **reject the null hypothesis**. There is a significant difference between specialty stores and discount stores and a significantly higher proportion of specialty stores shoppers believe that quality of merchandise is a determining factor in their perception of the store's image.

10.63

Name Brand	Store Brand	d
54	49	5
55	50	5
59	52	7
53	51	2
54	50	4
61	56	5
51	47	4
53	49	4

$n = 8$ $\quad \bar{d} = 4.5$ $\quad s_d = 1.414$ $\quad$ df = 8 - 1 = 7

For a 90% Confidence Level, $\alpha/2 = .05$ and $t_{.05,7} = 1.895$

$$\bar{d} \pm t \frac{s_d}{\sqrt{n}}$$

$$4.5 \pm 1.895 \frac{1.414}{\sqrt{8}} = 4.5 \pm .947$$

$$\mathbf{3.553 \le D \le 5.447}$$

10.65

Wednesday	Friday	d
71	53	18
56	47	9
75	52	23
68	55	13
74	58	16

$n = 5$ $\quad \bar{d} = 15.8$ $\quad s_d = 5.263$ $\quad$ df = 5 - 1 = 4

H_o: $D = 0$ $\qquad \alpha = .05$
H_a: $D > 0$

For one-tail test, $\alpha = .05$ and the critical $t_{.05,4} = 2.132$

$$t = \frac{\bar{d} - D}{\frac{s_d}{\sqrt{n}}} = \frac{15.8 - 0}{\frac{5.263}{\sqrt{5}}} = \mathbf{6.71}$$

Since the observed $t = 6.71 > t_{.05,4} = 2.132$, the decision is to **reject the null hypothesis**.

10.67 Construction

$n_1 = 338$
$x_1 = 297$

Telephone Repair

$n_2 = 281$
$x_2 = 192$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{297}{338} = .879$ $\hat{p}_2 = \frac{x_2}{n_2} = \frac{192}{281} = .683$

For a 90% Confidence Level, $\alpha/2 = .05$ and $z_{.05} = 1.645$

$$(\hat{p}_1 - \hat{p}_2) \pm z\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$(.879 - .683) \pm 1.645\sqrt{\frac{(.879)(.121)}{338} + \frac{(.683)(.317)}{281}} = .196 \pm .054$$

$\mathbf{.142 \le p_1 - p_2 \le .250}$

10.69 Discount

$\bar{x}_1 = \$47.20$
$\sigma_1 = \$12.45$
$n_1 = 60$

Specialty

$\bar{x}_2 = \$27.40$
$\sigma_2 = \$9.82$
$n_2 = 40$

H_o: $\mu_1 - \mu_2 = 0$ $\alpha = .01$
H_a: $\mu_1 - \mu_2 \neq 0$

For two-tail test, $\alpha/2 = .005$ and $z_c = \pm 2.575$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(47.20 - 27.40) - (0)}{\sqrt{\frac{(12.45)^2}{60} + \frac{(9.82)^2}{40}}} = \mathbf{8.86}$$

Since the observed $z = 8.86 > z_c = 2.575$, the decision is to **reject the null hypothesis**.

10.71 H_o: $\mu_1 - \mu_2 = 0$ $\alpha = .01$
H_a: $\mu_1 - \mu_2 \neq 0$ df = 10 + 6 - 2 = 14

A	B
$n_1 = 10$	$n_2 = 6$
$\bar{x}_1 = 18.3$	$\bar{x}_2 = 9.667$
$s_1^2 = 17.122$	$s_2^2 = 7.467$

For two-tail test, $\alpha/2 = .005$ and the critical $t_{.005,14} = \pm 2.977$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(18.3 - 9.667) - (0)}{\sqrt{\frac{(17.122)(9) + (7.467)(5)}{14}}\sqrt{\frac{1}{10} + \frac{1}{6}}} = \mathbf{4.52}$$

Since the observed $t = 4.52 > t_{.005,14} = 2.977$, the decision is to **reject the null hypothesis**.

10.73 H_0: $D = 0$
H_a: $D \neq 0$

This is a related measures before and after study. Fourteen people were involved in the study. Before the treatment, the sample mean was 3.991 and after the treatment, the mean was 5.072. The higher number after the treatment indicates that subjects were more likely to "blow the whistle" after having been through the treatment. The observed t value was –2.47 which was more extreme than two-tailed table t value of $\pm$ 2.06 and as a result, the researcher rejects the null hypothesis. This is underscored by a p-value of .00204 which is less than $\alpha = .05$. The study concludes that there is a significantly higher likelihood of "blowing the whistle" after the treatment.

10.75 A test of differences of the variances of the populations of the two machines is being computed. The hypotheses are:

H$_0$: $\sigma_1^2 = \sigma_2^2$
H$_a$: $\sigma_1^2 > \sigma_2^2$

Twenty-six pipes were measured for sample one and twenty-eight pipes were measured for sample two. The observed F = 1.49 is not significant even at α = .10 for a one-tailed test since the associated p-value is .15766. The variance of pipe lengths for machine 1 is not significantly greater than the variance of pipe lengths for machine 2.