

Chapter 11

Analysis of Variance and Design of Experiments

LEARNING OBJECTIVES

The focus of this chapter is the design of experiments and the analysis of variance, thereby enabling you to:

1. Describe an experimental design and its elements, including independent variables—both treatment and classification—and dependent variables.
2. Test a completely randomized design using a one-way analysis of variance.
3. Use multiple comparison techniques, including Tukey's honestly significant difference test and the Tukey-Kramer procedure, to test the difference in two treatment means when there is overall significant difference between treatments.
4. Test a randomized block design that includes a blocking variable to control for confounding variables.
5. Test a factorial design using a two-way analysis of variance, noting the advantages and applications of such a design and accounting for possible interaction between two treatment variables.

CHAPTER OUTLINE

- 11.1 Introduction to Design of Experiments
- 11.2 The Completely Randomized Design (One-Way ANOVA)
 - One-Way Analysis of Variance
 - Reading the F Distribution Table
 - Using the Computer for One-Way ANOVA
 - Comparison of F and t Values
- 11.3 Multiple Comparison Tests
 - Tukey's Honestly Significant Difference (HSD) Test: The Case of Equal Sample Sizes
 - Using the Computer to Do Multiple Comparisons
 - Tukey-Kramer Procedure: The Case of Unequal Sample Sizes
- 11.4 The Randomized Block Design
 - Using the Computer to Analyze Randomized Block Designs
- 11.5 A Factorial Design (Two-Way ANOVA)
 - Advantages of the Factorial Design
 - Factorial Designs with Two Treatments
 - Applications
 - Statistically Testing the Factorial Design
 - Interaction
 - Using a Computer to Do a Two-Way ANOVA

KEY TERMS

- | | |
|------------------------------|------------------------------|
| a posteriori | Factors |
| a priori | Independent Variable |
| Analysis of Variance (ANOVA) | Interaction |
| Blocking Variable | Levels |
| Classification Variable | Multiple Comparisons |
| Classifications | One-way Analysis of Variance |
| Completely Randomized Design | Post-hoc |
| Concomitant Variables | Randomized Block Design |
| Confounding Variables | Repeated Measures Design |
| Dependent Variable | Treatment Variable |
| Experimental Design | Tukey-Kramer Procedure |
| F Distribution | Tukey's HSD Test |
| F Value | Two-way Analysis of Variance |
| Factorial Design | |

STUDY QUESTIONS

1. A plan for testing hypotheses in which the researcher either controls or manipulates one or more variables is called a(n) _____.
2. A variable that is either controlled or manipulated is called a(n) _____ variable.
3. An independent variable is sometimes referred to as a _____ variable, a _____ variable, or a _____.
4. Each independent variable contains two or more _____ or _____.
5. The response to the different levels of the independent variables is called the _____ variable.
6. The experimental design that contains only one independent variable with two or more treatment levels is called a _____.
7. In chapter 11, the experimental designs are analyzed statistically using _____.
8. Suppose we want to analyze the data shown below using analysis of variance.

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
3	5	4	1
2	6	2	2
4	7	2	2
3	6	2	1
2	7	3	1
3			2

The degrees of freedom numerator for this analysis are _____.

The degrees of freedom denominator for this analysis are _____.

9. Assuming that $\alpha = .05$, for the problem presented in question 8, the critical F value is _____.
10. For the problem presented in question 8, the sum of squares between is _____ and the sum of squares error is _____. The mean square between is _____ and the mean square error is _____. The observed value of F for this problem is _____. The decision is to _____.
11. A set of techniques used to make comparisons between groups after an overall significant F value has been obtained is called _____.

12. The two types of multiple comparison techniques presented in chapter 11 are _____ and _____.
13. In conducting multiple comparisons with unequal sample sizes with techniques presented in chapter 11 of the text, you would use which procedure? _____
14. Suppose the following data are taken as samples from three populations and that an ANOVA results in an overall significant F value of 404.80. The mean square error for this ANOVA is 1.58.

<u>1</u>	<u>2</u>	<u>3</u>
11	24	27
9	25	30
10	25	29
12	26	28
11	24	31
8		29
10		

The Tukey-Kramer significant difference for groups 1 and 2 is _____. For groups 1 and 3, it is _____. For groups 2 and 3, it is _____. The following groups are significantly different _____ using $\alpha = .01$.

15. Suppose the following data represent four samples of size five which are taken from four populations. An ANOVA revealed a significant overall F value.

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
5	11	12	21
8	9	11	18
7	9	13	20
8	10	14	21
6	11	14	23

The mean square error for this problem is 1.92. The number of populations (C) for this problem is _____. The degrees of freedom error are _____. The value of q is _____. The value of HSD for this problem is _____. The following pairs of means are significantly different according to Tukey's HSD _____. Let $\alpha = .05$

16. A research design that is similar to the completely randomized design except that it includes a second variable referred to as a blocking variable is called a(n) _____.
17. In the randomized block design, the variable that the researcher desires to control but is not the treatment variable of interest is called the _____ variable.

18. Consider the following randomized block design.

Treatment Level

	<u>1</u>	<u>2</u>	<u>3</u>
<u>Block</u>			
1	2	4	8
2	3	4	9
3	2	5	7
4	4	6	6
5	3	5	9

The degrees of freedom treatment are _____. The degrees of freedom blocking are _____. The degrees of freedom error are _____.

19. For the problem in question 18, the sum of squares treatment is _____. The sum of squares blocking are _____. The sum of squares error are _____.
20. For the problem in question 18, the mean square treatment is _____. The mean square blocking is _____. The mean square error is _____. The observed F value for treatment is _____. The observed F value for blocking is _____. Using $\alpha = .01$, the following effects are significant based on these F values _____.
21. One advantage of a two-way design over the completely randomized design and the randomized block design is that the researcher can test for _____ if multiple measures are taken under every combination of treatment levels of the two treatments.
22. The ANOVA table shown below is compiled from the analysis of a two-way factorial design with three rows and four columns. There were a total of 48 values in this design.

<u>Effect</u>	<u>SS</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Row	29.3			
Column	17.1			
Interaction	14.7			
<u>Error</u>	<u>55.8</u>			
Total				

The sum of squares total is _____. The degrees of freedom for rows are _____. The degrees of freedom for columns are _____. The degrees of freedom for interaction are _____. The degrees of freedom for error are _____. The total degrees of freedom are _____. The mean square for rows is _____. The mean square for columns is _____. The mean squares for interaction is _____. The mean squares for error is _____. The observed F value for rows is _____. The observed F value for columns is _____. The observed F value for interaction is _____. The following effects are statistically significant using $\alpha = .05$ _____.

23. Perform a two-way ANOVA on the data given below.

Column Effects		1	2	3
Row Effects	1	2	5	5
		3	2	6
		2	4	5
	2	4	8	7
		6	4	6
		6	7	7

The sum of squares rows is _____. The sum of squares columns is _____. The sum of squares interaction is _____. The sum of squares error is _____. The degrees of freedom for rows are _____. The degrees of freedom for columns are _____. The degrees of freedom for interaction are _____. The degrees of freedom for error are _____. The mean square for rows is _____. The mean square for columns is _____. The mean squares for interaction is _____. The mean squares for error is _____. The observed F value for rows is _____. The observed F value for columns is _____. The observed F value for interaction is _____. The following effects are statistically significant using $\alpha = .05$ _____.

ANSWERS TO STUDY QUESTIONS

1. Experimental Design
2. Independent
3. Classification, Treatment, Factor
4. Levels, Classifications
5. Dependent
6. Completely Randomized Design
7. Analysis of Variance (ANOVA)
8. 3, 18
9. 3.16
10. 64.939, 10.333, 21.646, 0.574, 37.71, Reject the Null Hypothesis
11. Multiple Comparisons
12. Tukey's Honestly Significant Difference Test (HSD) and Tukey-Kramer Procedure
13. Tukey-Kramer Procedure
14. 2.514, 2.388, 2.60.
All are significantly different
15. 4, 16, 4.05, 2.51. All are significantly different
16. Randomized Block Design
17. Blocking
18. 2, 4, 8
19. 63.33, 2.40, 10.00
20. 31.67, 0.60, 1.25, 25.34, 0.48, Treatment
21. Interaction
22. 116.9, 2, 3, 6, 36, 47, 14.65, 5.70, 2.45, 1.55, 9.45, 3.68, 1.58, Rows and Columns
23. 24.50, 14.11, 2.33, 18.00, 1, 2, 2, 12, 24.50, 7.06, 1.17, 1.50, 16.33, 4.71, 0.78, Rows and Columns

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 11

- 11.1 a) Time Period, Market Condition, Day of the Week, Season of the Year
- b) Time Period - 4 P.M. to 5 P.M. and 5 P.M. to 6 P.M.
 Market Condition - Bull Market and Bear Market
 Day of the Week - Monday, Tuesday, Wednesday, Thursday, Friday
 Season of the Year - Summer, Winter, Fall, Spring
- c) Volume, Value of the Dow Jones Average, Earnings of Investment Houses
-
- 11.3 a) Type of Card, Age of User, Economic Class of Cardholder, Geographic Region
- b) Type of Card - Mastercard, Visa, Discover, American Express
 Age of User - 21-25 y, 26-32 y, 33-40 y, 41-50 y, over 50
 Economic Class - Lower, Middle, Upper
 Geographic Region - NE, South, MW, West
- c) Average number of card usages per person per month,
 Average balance due on the card, Average per expenditure per person,
 Number of cards possessed per person

11.5

Source	df	SS	MS	F
Treatment	2	22.20	11.10	11.07
Error	14	14.03	1.00	
Total	16	36.24		

$$\alpha = .05 \qquad \text{Critical } F_{.05, 2, 14} = 3.74$$

Since the observed $F = 11.07 > F_{.05, 2, 14} = 3.74$, the decision is to **reject the null hypothesis**.

11.7

Source	df	SS	MS	F
Treatment	3	544.3	181.4	13.00
Error	12	167.5	14.0	
Total	15	711.8		

$$\alpha = .01 \quad \text{Critical } F_{.01,3,12} = 5.95$$

Since the observed $F = 13.00 > F_{.01,3,12} = 5.95$, the decision is to **reject the null hypothesis**.

11.9

Source	SS	df	MS	F
Treatment	583.39	4	145.8475	7.50
Error	972.18	50	19.4436	
Total	1,555.57	54		

11.11

Source	df	SS	MS	F
Treatment	3	.007076	.002359	10.10
Error	15	.003503	.000234	
Total	18	.010579		

$$\alpha = .01 \quad \text{Critical } F_{.01,3,15} = 5.42$$

Since the observed $F = 10.10 > F_{.01,3,15} = 5.42$, the decision is to **reject the null hypothesis**.

11.13

Source	df	SS	MS	F
Treatment	2	29.61	14.80	11.76
Error	15	18.89	1.26	
Total	17	48.50		

$$\alpha = .05 \quad \text{Critical } F_{.05,2,15} = 3.68$$

Since the observed $F = 11.76 > F_{.05,2,15} = 3.68$, the decision is to **reject the null hypothesis**.

11.15 There are **4 treatment levels**. The sample sizes are **18, 15, 21, and 11**. The F value is **2.95** with a p -value of **.04**. There is an overall significant difference at alpha of .05. The means are **226.73, 238.79, 232.58, and 239.82**.

$$11.17 \quad C = 6 \quad \text{MSE} = .3352 \quad \alpha = .05 \quad N = 46$$

$$q_{.05,6,40} = 4.23 \quad n_3 = 8 \quad n_6 = 7 \quad \bar{x}_3 = 15.85 \quad \bar{x}_6 = 17.2$$

$$\text{HSD} = 4.23 \sqrt{\frac{.3352}{2} \left(\frac{1}{8} + \frac{1}{7} \right)} = \mathbf{0.896}$$

$$|\bar{x}_3 - \bar{x}_6| = |15.85 - 17.2| = 1.36$$

Since $1.36 > 0.896$, **there is a significant difference between the means of groups 3 and 6.**

$$11.19 \quad C = 3 \quad \text{MSE} = 1.002381 \quad \alpha = .05 \quad N = 17 \quad N - C = 14$$

$$q_{.05,3,14} = 3.70 \quad n_1 = 6 \quad n_2 = 5 \quad \bar{x}_1 = 2 \quad \bar{x}_2 = 4.6$$

$$\text{HSD} = 3.70 \sqrt{\frac{1.002381}{2} \left(\frac{1}{6} + \frac{1}{5} \right)} = \mathbf{1.586}$$

$$|\bar{x}_1 - \bar{x}_2| = |2 - 4.6| = 2.6$$

Since $2.6 > 1.586$, **there is a significant difference between the means of groups 1 and 2.**

$$11.21 \quad N = 16 \quad n = 4 \quad C = 4 \quad N - C = 12 \quad \text{MSE} = 13.95833 \quad q_{.01,4,12} = 5.50$$

$$\text{HSD} = q \sqrt{\frac{\text{MSE}}{n}} = 5.50 \sqrt{\frac{13.95833}{4}} = \mathbf{10.27}$$

$$\bar{x}_1 = 115.25 \quad \bar{x}_2 = 125.25 \quad \bar{x}_3 = 131.5 \quad \bar{x}_4 = 122.5$$

\bar{x}_1 and \bar{x}_3 are the only pair that are significantly different using the HSD test.

$$11.23 \quad C = 4 \quad \text{MSE} = .000234 \quad \alpha = .01 \quad N = 19 \quad N - C = 15$$

$$q_{.01,4,15} = 5.25 \quad n_1 = 4 \quad n_2 = 6 \quad n_3 = 5 \quad n_4 = 4$$

$$\bar{x}_1 = 4.03, \quad \bar{x}_2 = 4.001667, \quad \bar{x}_3 = 3.974, \quad \bar{x}_4 = 4.005$$

$$\text{HSD}_{1,2} = 5.25 \sqrt{\frac{.000234}{2} \left(\frac{1}{4} + \frac{1}{6} \right)} = .0367$$

$$\text{HSD}_{1,3} = 5.25 \sqrt{\frac{.000234}{2} \left(\frac{1}{4} + \frac{1}{5} \right)} = \mathbf{.0381}$$

$$\text{HSD}_{1,4} = 5.25 \sqrt{\frac{.000234}{2} \left(\frac{1}{4} + \frac{1}{4} \right)} = .0402$$

$$\text{HSD}_{2,3} = 5.25 \sqrt{\frac{.000234}{2} \left(\frac{1}{6} + \frac{1}{5} \right)} = .0344$$

$$\text{HSD}_{2,4} = 5.25 \sqrt{\frac{.000234}{2} \left(\frac{1}{6} + \frac{1}{4} \right)} = .0367$$

$$\text{HSD}_{3,4} = 5.25 \sqrt{\frac{.000234}{2} \left(\frac{1}{5} + \frac{1}{4} \right)} = .0381$$

$$|\bar{x}_1 - \bar{x}_3| = \mathbf{.056}$$

This is the only pair of means that are significantly different.

$$11.25 \quad \alpha = .05 \quad C = 3 \quad N = 18 \quad N - C = 15 \quad \text{MSE} = 1.259365$$

$$q_{.05,3,15} = 3.67 \quad n_1 = 5 \quad n_2 = 7 \quad n_3 = 6$$

$$\bar{x}_1 = 7.6 \quad \bar{x}_2 = 8.8571 \quad \bar{x}_3 = 5.8333$$

$$\text{HSD}_{1,2} = 3.67 \sqrt{\frac{1.259365}{2} \left(\frac{1}{5} + \frac{1}{7} \right)} = 1.705$$

$$\text{HSD}_{1,3} = 3.67 \sqrt{\frac{1.259365}{2} \left(\frac{1}{5} + \frac{1}{6} \right)} = \mathbf{1.764}$$

$$\text{HSD}_{2,3} = 3.67 \sqrt{\frac{1.259365}{2} \left(\frac{1}{7} + \frac{1}{6} \right)} = \mathbf{1.620}$$

$$|\bar{x}_1 - \bar{x}_3| = \mathbf{1.767 \text{ (is significant)}}$$

$$|\bar{x}_2 - \bar{x}_3| = \mathbf{3.024 \text{ (is significant)}}$$

- 11.27 $\alpha = .05$. There were five plants and ten pairwise comparisons. The Minitab output reveals that the only significant pairwise difference is between plant 2 and plant 3 where the reported confidence interval (0.180 to 22.460) contains the same sign throughout indicating that 0 is not in the interval. Since zero is not in the interval, then we are 95% confident that there is a pairwise difference significantly different from zero. The lower and upper values for all other confidence intervals have different signs indicating that zero is included in the interval. This indicates that the difference in the means for these pairs might be zero.

11.29 $H_0: \mu_1 = \mu_2 = \mu_3$ H_a : At least one treatment mean is different from the others

Source	df	SS	MS	F
Treatment	2	.001717	.000858	1.48
Blocks	3	.076867	.025622	44.13
Error	6	.003483	.000581	
Total	11	.082067		

 $\alpha = .01$ Critical $F_{.01,2,6} = 10.92$ for treatments

For treatments, the observed $F = 1.48 < F_{.01,2,6} = 10.92$ and the decision is to **fail to reject the null hypothesis**.

11.31

Source	df	SS	MS	F
Treatment	3	199.48	66.493	3.90
Blocks	6	265.24	44.207	2.60
Error	18	306.59	17.033	
Total	27	771.31		

 $\alpha = .01$ Critical $F_{.01,3,18} = 5.09$ for treatments

For treatments, the observed $F = 3.90 < F_{.01,3,18} = 5.09$ and the decision is to **fail to reject the null hypothesis**.

11.33

Source	df	SS	MS	F
Treatment	2	64.5333	32.2667	15.37
Blocks	4	137.6000	34.4000	16.38
Error	8	16.8000	2.1000	
Total	14	218.9300		

 $\alpha = .01$ Critical $F_{.01,2,8} = 8.65$ for treatments

For treatments, the observed $F = 15.37 > F_{.01,2,8} = 8.65$ and the decision is to **reject the null hypothesis**.

11.35 The p value for Phone Type, .00018, indicates that there is an overall significant difference in treatment means at alpha .001. The lengths of calls differ according to type of telephone used. The p -value for managers, .00028, indicates that there is an overall difference in block means at alpha .001. The lengths of calls differ according to Manager. The significant blocking effects have improved the power of the F test for treatments.

11.37 This is a two-way factorial design with two independent variables and one dependent variable. It is 4×3 in that there are four treatment levels and three column treatment levels. Since there are two measurements per cell, interaction can be analyzed.

$$df_{\text{row treatment}} = 3 \quad df_{\text{column treatment}} = 2 \quad df_{\text{interaction}} = 6 \quad df_{\text{error}} = 12 \quad df_{\text{total}} = 23$$

11.39

Source	df	SS	MS	F
Row	1	1.047	1.047	2.40
Column	3	3.844	1.281	2.94
Interaction	3	0.773	0.258	0.59
Error	16	6.968	0.436	
Total	23	12.632		

$$\alpha = .05$$

Critical $F_{.05,1,16} = 4.49$ for rows. For rows, the observed $F = 2.40 < F_{.05,1,16} = 4.49$ and decision is to **fail to reject the null hypothesis**.

Critical $F_{.05,3,16} = 3.24$ for columns. For columns, the observed $F = 2.94 < F_{.05,3,16} = 3.24$ and the decision is to **fail to reject the null hypothesis**.

Critical $F_{.05,3,16} = 3.24$ for interaction. For interaction, the observed $F = 0.59 < F_{.05,3,16} = 3.24$ and the decision is to **fail to reject the null hypothesis**.

11.41

Source	df	SS	MS	F
Treatment 1	1	1.24031	1.24031	63.67
Treatment 2	3	5.09844	1.69948	87.25
Interaction	3	0.12094	0.04031	2.07
Error	24	0.46750	0.01948	
Total	31	6.92719		

$$\alpha = .05$$

Critical $F_{.05,1,24} = 4.26$ for treatment 1. For treatment 1, the observed $F = 63.67 > F_{.05,1,24} = 4.26$ and the decision is to **reject the null hypothesis**.

Critical $F_{.05,3,24} = 3.01$ for treatment 2. For treatment 2, the observed $F = 87.25 > F_{.05,3,24} = 3.01$ and the decision is to **reject the null hypothesis**.

Critical $F_{.05,3,24} = 3.01$ for interaction. For interaction, the observed $F = 2.07 < F_{.05,3,24} = 3.01$ and the decision is to **fail to reject the null hypothesis**.

11.43

Source	df	SS	MS	F
Location	2	1736.22	868.11	34.31
Competitors	3	1078.33	359.44	14.20
Interaction	6	503.33	83.89	3.32
Error	24	607.33	25.31	
Total	35	3925.22		

$$\alpha = .05$$

Critical $F_{.05,2,24} = 3.40$ for rows. For rows, the observed $F = 34.31 > F_{.05,2,24} = 3.40$ and the decision is to **reject the null hypothesis**.

Critical $F_{.05,3,24} = 3.01$ for columns. For columns, the observed $F = 14.20 > F_{.05,3,24} = 3.01$ and decision is to **reject the null hypothesis**.

Critical $F_{.05,6,24} = 2.51$ for interaction. For interaction, the observed $F = 3.32 > F_{.05,6,24} = 2.51$ and the decision is to **reject the null hypothesis**.

Note: There is significant interaction in this study. This may confound the interpretation of the main effects, Location and Number of Competitors.

- 11.45 The null hypotheses are that there are no interaction effects, that there are no significant differences in the means of the valve openings by machine, and that there are no significant differences in the means of the valve openings by shift. Since the p -value for interaction effects is .8760, there are no significant interaction effects and that is good since significant interaction effects would confound that study. The p -value for columns (shifts) is .0078 indicating that column effects are significant at alpha of .01. There is a significant difference in the mean valve opening according to shift. No multiple comparisons are given in the output. However, an examination of the shift means indicates that the mean valve opening on shift one was the largest at 6.47 followed by shift three with 6.3 and shift two with 6.25. The p -value for rows (machines) is .9368 and that is not significant.

11.47

Source	df	SS	MS	F
Treatment	3	66.69	22.23	8.82
Error	12	30.25	2.52	
Total	15	96.94		

$$\alpha = .05 \quad \text{Critical } F_{.05,3,12} = 3.49$$

Since the treatment $F = 8.82 > F_{.05,3,12} = 3.49$, the decision is to **reject the null hypothesis**.

For Tukey's HSD:

$$MSE = 2.52 \quad n = 4 \quad N = 16 \quad C = 4 \quad N - C = 12$$

$$q_{.05,4,12} = 4.20$$

$$HSD = q \sqrt{\frac{MSE}{n}} = (4.20) \sqrt{\frac{2.52}{4}} = \mathbf{3.33}$$

$$\bar{x}_1 = 12 \quad \bar{x}_2 = 7.75 \quad \bar{x}_3 = 13.25 \quad \bar{x}_4 = 11.25$$

Using HSD of 3.33, there are significant pairwise differences between means 1 and 2, means 2 and 3, and means 2 and 4.

11.49

Source	df	SS	MS	<i>F</i>
Treatment	5	210	42.000	2.31
Error	36	655	18.194	
Total	41	865		

- 11.51 This design is a repeated-measures type random block design. There is one treatment variable with three levels. There is one blocking variable with six people in it (six levels). The degrees of freedom treatment are two. The degrees of freedom block are five. The error degrees of freedom are ten. The total degrees of freedom are seventeen. There is one dependent variable.

11.53

Source	df	SS	MS	F
Treatment	3	240.125	80.042	31.51
Blocks	5	548.708	109.742	43.20
Error	15	38.125	2.542	
Total	23			

$$\alpha = .05 \quad \text{Critical } F_{.05,3,15} = 3.29 \text{ for treatments}$$

Since for treatments the observed $F = 31.51 > F_{.05,3,15} = 3.29$, the decision is to **reject the null hypothesis**.

For Tukey's HSD:

Ignoring the blocking effects, the sum of squares blocking and sum of squares error are combined together for a new $SS_{\text{error}} = 548.708 + 38.125 = 586.833$. Combining the degrees of freedom error and blocking yields a new $df_{\text{error}} = 20$. Using these new figures, we compute a new mean square error, $MSE = (586.833/20) = 29.34165$.

$$n = 6 \quad C = 4 \quad N = 24 \quad N - C = 20 \quad q_{.05,4,20} = 3.96$$

$$HSD = q \sqrt{\frac{MSE}{n}} = (3.96) \sqrt{\frac{29.34165}{6}} = \mathbf{8.757}$$

$$\bar{x}_1 = 16.667 \quad \bar{x}_2 = 12.333 \quad \bar{x}_3 = 12.333 \quad \bar{x}_4 = 19.833$$

None of the pairs of means are significantly different using Tukey's $HSD = 8.757$. This may be due in part to the fact that we compared means by folding the blocking effects back into error and the blocking effects were highly significant.

11.55

Source	df	SS	MS	F
Treatment 2	3	257.889	85.963	38.21
Treatment 1	2	1.056	0.528	0.23
Interaction	6	17.611	2.935	1.30
Error	24	54.000	2.250	
Total	35	330.556		

$$\alpha = .01$$

Critical $F_{.01,3,24} = 4.72$ for treatment 2. For the treatment 2 effects, the observed $F = 38.21 > F_{.01,3,24} = 4.72$ and the decision is to **reject the null hypothesis**.

Critical $F_{.01,2,24} = 5.61$ for Treatment 1. For the treatment 1 effects, the observed $F = 0.23 < F_{.01,2,24} = 5.61$ and the decision is to **fail to reject the null hypothesis**.

Critical $F_{.01,6,24} = 3.67$ for interaction. For the interaction effects, the observed $F = 1.30 < F_{.01,6,24} = 3.67$ and the decision is to **fail to reject the null hypothesis**.

11.57

Source	df	SS	MS	F
Treatment	3	90477679	30159226	7.38
Error	20	81761905	4088095	
Total	23	172000000		

$$\alpha = .05 \quad \text{Critical } F_{.05,3,20} = 3.10$$

The treatment $F = 7.38 > F_{.05,3,20} = 3.10$ and the decision is to **reject the null hypothesis**.

11.59

Source	df	SS	MS	F
Treatment	2	9.555	4.777	0.46
Error	18	185.1337	10.285	
Total	20	194.6885		

$$\alpha = .05 \quad \text{Critical } F_{.05,2,18} = 3.55$$

Since the treatment $F = 0.46 < F_{.05,2,18} = 3.55$, the decision is to **fail to reject the null hypothesis**.

Since there are no significant treatment effects, it would make no sense to compute Tukey-Kramer values and do pairwise comparisons.

11.61

Source	df	SS	MS	F
Treatment	4	53.400	13.350	13.64
Blocks	7	17.100	2.443	2.50
Error	28	27.400	0.979	
Total	39	97.900		

$\alpha = .05$ Critical $F_{.05,4,28} = 2.71$ for treatments

For treatments, the observed $F = 13.64 > F_{.05,4,28} = 2.71$ and the decision is to **reject the null hypothesis**.

11.63 Excel reports that this is a two-factor design without replication indicating that this is a random block design. Neither the row nor the column p -values are less than .05 indicating that there are no significant treatment or blocking effects in this study. Also displayed in the output to underscore this conclusion are the observed and critical F values for both treatments and blocking. In both cases, the observed value is less than the critical value.

11.65 This is a two-way ANOVA with 4 rows and 3 columns. There are 3 observations per cell. $F_R = 4.30$ with a p -value of .014 is significant at $\alpha = .05$. The null hypothesis is rejected for rows. $F_C = 0.53$ with a p -value of .594 is not significant. We fail to reject the null hypothesis for columns. $F_I = 0.99$ with a p -value of .453 for interaction is not significant. We fail to reject the null hypothesis for interaction effects.

11.67 This one-way ANOVA has 4 treatment levels and 24 observations. The $F = 3.51$ yields a p -value of .034 indicating significance at $\alpha = .05$. Since the sample sizes are equal, Tukey's HSD is used to make multiple comparisons. The computer output shows that means 1 and 3 are the only pairs that are significantly different (same signs in confidence interval). Observe on the graph that the confidence intervals for means 1 and 3 barely overlap.