Chapter 9
Statistical Inference:
Hypothesis Testing for Single Populations

LEARNING OBJECTIVES

The main objective of Chapter 9 is to help you to learn how to test Hypotheses on single populations, thereby enabling you to:

1. Develop both one- and two-tailed null and alternative hypotheses that can be tested in a business setting by examining the rejection and non-rejection regions in light of Type I and Type II errors.
2. Reach a statistical conclusion in hypothesis testing problems about a population mean with a known population standard deviation using the z statistic.
3. Reach a statistical conclusion in hypothesis testing problems about a population mean with an unknown population standard deviation using the t statistic.
4. Reach a statistical conclusion in hypothesis testing problems about a population proportion using the z statistic.
5. Reach a statistical conclusion in hypothesis testing problems about a population variance using the chi-square statistic.
6. Solve for possible Type II errors when failing to reject the null hypothesis.

CHAPTER OUTLINE

9.1 Introduction to Hypothesis Testing
- Types of Hypotheses
- Research Hypotheses
- Statistical Hypotheses
- Substantive Hypotheses
- Eight-Step Process for Testing Hypotheses
- Rejection and Non-rejection Regions
- Type I and Type II errors
  - Comparing Type I and Type II Errors

KEY TERMS

Alpha(α)
Alternative Hypothesis
Beta(β)
Critical Value
Critical Value Method
Hypothesis
Hypothesis Testing
Level of Significance
Nonrejection Region
Null Hypothesis
Observed Significance Level
Observed Value
One-tailed Test
Operating-Characteristic Curve (OC)
p-Value
Power
Power Curve
Rejection Region
Research Hypothesis
Statistical Hypothesis
Substantive Result
Two-Tailed Test
Type I Error
Type II Error

STUDY QUESTIONS

1. The first step in testing a hypothesis is to establish a(n) _______________ hypothesis and a(n) _______________ hypothesis.

2. In testing hypotheses, the researcher initially assumes that the _______________ hypothesis is true.

3. The region of the distribution in hypothesis testing in which the null hypothesis is rejected is called the _______________ region.

4. The rejection and acceptance regions are divided by a point called the _______________ value.

5. The portion of the distribution which is not in the rejection region is called the _______________ region.

6. The probability of committing a Type I error is called _______________.

7. Another name for alpha is _______________ _______________ _______________.

8. When a true null hypothesis is rejected, the researcher has committed a _______________ error.

9. When a researcher fails to reject a false null hypothesis, a _______________ error has been committed.

10. The probability of committing a Type II error is represented by _______________.

11. Power is equal to _______________.

12. Whenever hypotheses are established such that the alternative hypothesis is directional, then the researcher is conducting a _______________-tailed test.

13. A _______________-tailed test is nondirectional.

14. If in testing hypotheses, the researcher uses a method in which the probability of the observed statistic is compared to alpha to reach a decision, the researcher is using the _______________ method.

15. Suppose H_o: $\mu = 95$ and H_a: $\mu \neq 95$. If the sample size is 50, the population standard deviation is known, and $\alpha = .05$, the critical value of z is ________.

16. Suppose H_o: $\mu = 2.36$ and H_a: $\mu < 2.36$. If the sample size is 64, the population standard deviation is known, and $\alpha = .01$, the critical value of z is ________.

17. Suppose H_o: $\mu = 24.8$ and H_a: $\mu \neq 24.8$. If the sample size is 49, the population standard deviation is known, and $\alpha = .10$, the critical value of z is ________.

18. Suppose a researcher is testing a null hypothesis that $\mu = 61$. A random sample of $n = 38$ is taken resulting in $\bar{x} = 63$ and $\sigma = 8.76$. The observed z value is ________________.

19. Suppose a researcher is testing a null hypothesis that $\mu = 413$. A random sample of $n = 70$ is taken resulting in $\bar{x} = 405$. The population standard deviation is 34. The observed z value is ________________.

20. A researcher is testing a hypothesis of a single mean. The critical z value for $\alpha = .05$ and a one-tailed test is 1.645. The observed z value from sample data is 1.13. The decision made by the researcher based on this information is to ________________ the null hypothesis.

21. A researcher is testing a hypothesis of a single mean. The critical z value for $\alpha = .05$ and a two-tailed test is $\pm$ 1.96. The observed z value from sample data is -1.91. The decision made by the researcher based on this information is to ________________ the null hypothesis.

22. A researcher is testing a hypothesis of a single mean. The critical z value for $\alpha = .01$ and a one-tailed test is -2.33. The observed z value from sample data is -2.45. The decision made by the researcher based on this information is to ________________ the null hypothesis.

23. A researcher has a theory that the average age of managers in a particular industry is over 35-years-old. The null hypothesis to conduct a statistical test on this theory would be ________________.

24. A company produces, among other things, a metal plate that is supposed to have a six inch hole punched in the center. A quality control inspector is concerned that the machine which punches the hole is "out-of-control". In an effort to test this, the inspector is going to gather a sample of metal plates punched by the machine and measure the diameter of the hole. The alternative hypothesis used to statistical test to determine if the machine is out-of-control is ________________.

25. The following hypotheses are being tested:

H_o: $\mu = 4.6$
H_a: $\mu \neq 4.6$

The value of alpha is .05. To test these hypotheses, a random sample of 22 items is selected resulting in a sample mean of 4.1 with a sample standard deviation of 1.8. It can be assumed that this measurement is normally distributed in the population. The degrees of freedom associated with the t test used in this problem are ________________.

26. The *critical t* value for the problem presented in question 25 is ________________.

27. The problem presented in question 25 contains hypotheses which lead to a ___________-tailed test.

28. The observed value of t for the problem presented in question 25 is ________________.

29. Based on the results of the observed *t* value and the critical table *t* value, the researcher should _______________ the null hypothesis in the problem presented in question 25.

30. It is believed that the average time to assemble a given product is less than 2 hours. To test this, a sample of 18 assemblies is taken resulting in a sample mean of 1.91 hours with a sample standard deviation of 0.73 hours. Suppose α = .01. If a hypothesis test is done on this problem, the table value is _______________. The observed value is __________________. The decision is _________________.

31. A political scientist want to statistically test the null hypothesis that her candidate for governor is currently carrying at least 57% of the vote in the state. She has her assistants randomly sample 550 eligible voters in the state by telephone and only 300 declare that they support her candidate. The observed *z* value for this problem is _______________.

32. Problem 31 is a _______________-tailed test.

33. Suppose that the value of alpha for problem 31 is .05. After comparing the observed value to the critical value, the political scientist decided to _______________ the null hypothesis.

34. A company believes that it controls .27 of the total market share in the South for one of its products. To test this belief, a random sample of 1150 purchases of this product in the South are contracted. 385 of the 1150 purchased this company's brand of the product. If a researcher wants to conduct a statistical test for this problem, the alternative hypothesis would be _______________.

35. The observed value of *z* for problem 34 is _______________.

36. Problem 34 would result in a _______________-tailed test.

37. Suppose that a .01 value of alpha were used in problem 34. The critical value of *z* for the problem is _______________.

38. Upon comparing the observed value of *z* to the critical value of *z*, it is determined to _______________ the null hypothesis in problem 34.

39. A production process produces parts with a normal variance of 27.3. Engineers are concerned that the process may now be producing parts with greater variance than that. To test this concern, a sample of 9 newly produced parts is taken. The sample standard deviation is 5.93. Let α = .01. The null hypothesis for this problem is _______________________.

40. The critical table value of σ^2 for problem 39 is ______________________.

41. The observed value of chi-square in problem 39 is ______________________.

42. The decision reached for problem 39 is _______________.

43. The null hypothesis for a test is H_0: $\mu = 30$. The population standard deviation is known to be 0.63. A one-tailed test is being conducted in the lower tail of the distribution. After taking a sample of 49 items and computing a mean, it is decided to fail to reject the null hypothesis. Let $\alpha = .05$. If the null hypothesis is not true and if the true alternative hypothesis is 29.6, the value of beta is ________________________.

44. Suppose the alternative mean in problem 43 is really 29.9, the value of beta is ________________________.

45. Plotting the power values against the various values of the alternative hypotheses produces a ________________________ curve.

46. Plotting the values of β against various values of the alternative hypothesis produces a ______________________ curve.

47. The *p*-value for an observed *z* of 2.73 is __________.

48. The *p*-value for an observed *z* of 0.85 is __________.

49. In a hypothesis-testing problem, a *p*-value of .0046 is obtained for the observed statistic. If a one-tailed test is being conducted and α is .01, then the decision is to _________________ the null hypothesis.

50. A researcher is conducting a two-tailed hypothesis test using a 5% level of significance. As a result of the test, a *p*-value of .032 is obtained for one of the tails. The decision should be to __________________ the null hypothesis.

ANSWERS TO STUDY QUESTIONS

1. Null, Alternative
2. Null
3. Rejection
4. Critical
5. Nonrejection Region
6. Alpha
7. Level of Significance

26. $\pm$ 2.08
27. Two
28. – 1.30
29. Fail to Reject
30. – 2.567, - 0.52, Fail to Reject
31. – 1.16
32. One

8. Type I
9. Type II
10. Beta
11. 1 - β
12. One
13. Two
14. *p*-value
15. $\pm$1.96
16. – 2.33
17. $\pm$ 1.645
18. 1.41
19. – 1.97
20. Fail to Reject
21. Fail to Reject
22. Reject
23. $\mu = 35$
24. $\mu \neq$ 6"
25. 21
33. Fail to Reject
34. $p \neq .27$
35. 4.95
36. Two
37. $\pm$ 2.575
38. Reject
39. H_0: $\sigma^2 = 27.3$
40. 20.0902
41. 10.3047
42. Fail to Reject
43. .0026
44. .7019
45. Power
46. Operating Characteristic
47. .0032
48. .1977
49. reject
50. fail to reject

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 9

9.1 a) Two-Tailed
b) One-Tailed
c) One-Tailed
d) Two-Tailed

9.3 a) H_o: $\mu = 25$
H_a: $\mu \neq 25$ $\bar{x} = 28.1$ $n = 57$ $\sigma = 8.46$ $\alpha = .01$

For two-tail, $\alpha/2 = .005$ $z_c = 2.575$

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{28.1-25}{\frac{8.46}{\sqrt{57}}} = \mathbf{2.77}$$

observed $z = 2.77 > z_c = 2.575$

Reject the null hypothesis

b) from Table A.5, inside area between $z = 0$ and $z = 2.77$ is .4972

p-value = .5000 - .4972 = **.0028**

Since the p-value of .0028 is less than $\alpha/2 = .005$, the decision is to:
Reject the null hypothesis

c) critical mean values:

$$z_c = \frac{\bar{x}_c-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\pm 2.575 = \frac{\bar{x}_c-25}{\frac{8.46}{\sqrt{57}}}$$

$$\bar{x}_c = 25 \pm 2.885$$

$\bar{x}_c$ = **27.885 (upper value)**
$\bar{x}_c$ = **22.115 (lower value)**

9.5 a) H_o: $\mu = 1{,}200$
H_a: $\mu > 1{,}200$

$\bar{x} = 1{,}215$ $n = 113$ $\sigma = 100$ $\alpha = .10$

For one-tail, $\alpha = .10$ $z_c = 1.28$

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1{,}215-1{,}200}{\frac{100}{\sqrt{113}}} = \mathbf{1.59}$$

observed $z = 1.59 > z_c = 1.28$

Reject the null hypothesis

b) Probability > observed $z = 1.59$ is .5000 - .4441 = **.0559** (the *p*-value) which is less than $\alpha = .10$.

Reject the null hypothesis.

c) Critical mean value:

$$z_c = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$1.28 = \frac{\bar{x}_c - 1{,}200}{\frac{100}{\sqrt{113}}}$$

$\bar{x}_c = 1{,}200 + 12.04$

Since the observed $\bar{x} = 1{,}215$ is greater than the critical $\bar{x} = 1212.04$, the decision is to reject the null hypothesis.

9.7 H$_0$: μ = \$657.49
H$_a$: $\mu \neq$ \$657.49

$\bar{x}$ = \$673.58 n = 54 σ = \$63.90 α = .05

2-tailed test, $\alpha/2$ = .025 $z_{.025} = \pm$ 1.96

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{673.58 - 657.49}{\frac{63.90}{\sqrt{54}}} = \mathbf{1.85}$$

Since the observed z = 1.85 < $z_{.025}$ = 1.96, the decision is to **fail** to reject the null hypothesis.

9.9 H$_0$: μ = 5
H$_a$: $\mu \neq$ 5

$\bar{x}$ = 5.0611 n = 42 N = 650 σ = 0.2803 α = .10

2-tailed test, $\alpha/2$ = .05 $z_{.05} = \pm$ 1.645

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}} = \frac{5.0611 - 5}{\frac{0.2803}{\sqrt{42}}\sqrt{\frac{650-42}{650-1}}} = \mathbf{1.46}$$

Since the observed z = 1.46 < $z_{.05}$ = 1.645, the decision is to **fail to reject** the null hypothesis.

9.11 H_o: μ = \$50
H_a: μ > \$50

$\bar{x}$ = \$52.17 n = 23 σ = \$3.49 α = .10

For one-tailed test, α = .10, $z_{.10}$ = 1.28

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\$52.17-\$50}{\frac{\$3.49}{\sqrt{23}}} = 2.98$$

Since the observed z = 2.98 > $z_{.10}$ = 1.28, the decision is to **Reject the null Hypothesis.**

The table value for z = 2.98 is .4986. The p-value is .5000 - .4986 = **.0014**. Since this is less than α = .10, the decision using the p-value is to reject the null hypothesis.

In our decision to reject the hypothesized mean of \$50, we are saying that we have concluded that the mean is greater than \$50. However, our sample mean is only \$2.17 more than \$50. While this may be an indication of carpet cleaning inflation, for many customers, an additional \$2.17 may not be substantial nor cause them to forgo the cleaning.

9.13 n = 20 $\bar{x}$ = 16.45 s = 3.59 df = 20 - 1 = 19 α = .05

H_o: μ = 16
H_a: $\mu \neq$ 16

For two-tail test, $\alpha/2$ = .025, critical $t_{.025,19}$ = ±2.093

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{16.45-16}{\frac{3.59}{\sqrt{20}}} = \mathbf{0.56}$$

Observed t = 0.56 < $t_{.025,19}$ = 2.093

The decision is to **Fail to reject the null hypothesis**

9.15 $n = 11$ $\bar{x} = 1{,}236.36$ $s = 103.81$ df = 11 - 1 = 10 $\alpha = .05$

H_o: $\mu = 1{,}160$
H_a: $\mu > 1{,}160$
or one-tail test, $\alpha = .05$ critical $t_{.05,10} = 1.812$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{1{,}236.36 - 1{,}160}{\frac{103.81}{\sqrt{11}}} = \mathbf{2.44}$$

Observed $t = 2.44 > t_{.05,10} = 1.812$
The decision is to **Reject the null hypothesis**

9.17 $n = 12$ $\bar{x} = 1.85083$ $s = .02353$ df = 12 - 1 = 11 $\alpha = .10$

H_0: $\mu = 1.84$
H_a: $\mu \neq 1.84$
For a two-tailed test, $\alpha/2 = .05$ critical $t_{.05,11} = 1.796$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{1.85083 - 1.84}{\frac{.02353}{\sqrt{12}}} = \mathbf{1.59}$$

Since $t = 1.59 < t_{11,.05} = 1.796$,
The decision is to **fail to reject the null hypothesis**.

9.19 $n = 49$ $\bar{x} = \$31.67$ $s = \$1.29$ df = 49 – 1 = 48 $\alpha = .05$

H_0: $\mu = \$32.28$
H_a: $\mu \neq \$32.28$
Two-tailed test, $\alpha/2 = .025$ for 40 degrees of freedom, $t_{.025,40} = \pm 2.021$;
for 50 degrees of freedom $t_{.025,50} = \pm 2.009$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{31.67 - 32.28}{\frac{1.29}{\sqrt{49}}} = -3.31$$

The observed $t = -3.31 < t_{.025,40} = \pm 2.021$ or , $t_{.025,50} = \pm 2.009$
The decision is to **reject the null hypothesis.**

9.21 $n = 22$ $\bar{x} = 1031.32$ $s = 240.37$ df = 22 – 1 = 21 $\alpha = .05$

H$_0$: $\mu = 1135$
H$_a$: $\mu \neq 1135$

Two-tailed test, $\alpha/2 = .025$ $t_{.025,21} = \pm 2.080$

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{1031.32-1135}{\frac{240.37}{\sqrt{22}}} = \mathbf{-2.02}$$

The observed $t = -2.02 > t_{.025,21} = -2.080$,
The decision is to **fail to reject the null hypothesis**

9.23 $n = 26$ $\bar{x} = 19.534$ minutes $s = 4.100$ minutes $\alpha = .05$

H$_0$: $\mu = 19$
H$_a$: $\mu \neq 19$

Two-tailed test, $\alpha/2 = .025$, critical t value = $\pm$ 2.06

Observed t value = 0.66. Since the observed $t = 0.66 <$ critical t value = 2.06,

The decision is to **fail to reject the null hypothesis**.
Since the Excel p-value = .256 > $\alpha/2$ = .025 and MINITAB p-value =.513 > .05, the decision is to **fail to reject the null hypothesis.**
She would <u>not</u> conclude that her city is any different from the ones in the national survey.

9.25 H_o: $p = 0.63$
H_a: $p < 0.63$

$n = 100$ $x = 55$ $\hat{p} = \frac{x}{n} = \frac{55}{100} = .55$

For one-tail, $\alpha = .01$ $z_{.01} = -2.33$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.55 - .63}{\sqrt{\frac{(.63)(.37)}{100}}} = \mathbf{-1.66}$$

observed $z = -1.66 > z_c = -2.33$

The decision is to **Fail to reject the null hypothesis**

9.27 H_o: $p = .48$
H_a: $p \neq .48$

$n = 380$ $x = 164$ $\alpha = .01$ $\alpha/2 = .005$ $z_{.005} = \underline{+}2.575$

$$\hat{p} = \frac{x}{n} = \frac{164}{380} = .4316$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.4316 - .48}{\sqrt{\frac{(.48)(.52)}{380}}} = \mathbf{-1.89}$$

Since the observed $z = -1.89$ is greater than $z_{.005} = -2.575$, The decision is to **fail to reject the null hypothesis**. There is not enough evidence to declare that the proportion is any different than .48.

9.29 H_o: p = .31
H_a: $p \neq .31$

n = 600 x = 200 α = .10 $\alpha/2$ = .05 $z_{.005} = \underline{+}1.645$

$$\hat{p} = \frac{x}{n} = \frac{200}{600} = .3333$$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{p\cdot q}{n}}} = \frac{.3333-.31}{\sqrt{\frac{(.31)(.69)}{600}}} = \mathbf{1.23}$$

Since the observed z = 1.23 is less than $z_{.005}$= 1.645, The decision is to **fail to reject the null hypothesis**. There is not enough evidence to declare that the proportion is any different than .31.

H_o: p = .24
H_a: $p > .24$

n = 600 x = 158 α = .05 $z_{.05}$ = 1.645

$$\hat{p} = \frac{x}{n} = \frac{158}{600} = .2633$$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{p\cdot q}{n}}} = \frac{.2633-.24}{\sqrt{\frac{(.24)(.76)}{600}}} = 1.34$$

Since the observed z = 1.34 is less than $z_{.05}$= 1.645, The decision is to **fail to reject the null hypothesis**. There is not enough evidence to declare that the proportion is less than .24.

9.31 H_o: $p = .32$
H_a: $p < .32$

$n = 118$ $x = 22$ $\hat{p} = \frac{x}{n} = \frac{22}{118} = .1864$ $\alpha = .05$

For one-tailed test, $z_{.05} = -1.645$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.1864 - .32}{\sqrt{\frac{(.32)(.68)}{118}}} = \mathbf{-3.11}$$

Observed $z = -3.11 < z_{.05}$ –1.645

Since the observed $z = -3.11$ is less than $z_{.05} = -1.645$, the decision is to **reject the null hypothesis**.

9.33 a) H_0: $\sigma^2 = 20$ $\alpha = .05$ $n = 15$ df = 15 – 1 = 14 $s^2 = 32$
H_a: $\sigma^2 > 20$

$\chi^2_{.05,14} = 23.6848$

$$\chi^2 = \frac{(15-1)(32)}{20} = \mathbf{22.4}$$

Since $\chi^2 = 22.4 < \chi^2_{.05,14} = 23.6848$, the decision is to **fail to reject the null hypothesis**.

b) H_0: $\sigma^2 = 8.5$ $\alpha = .10$ $\alpha/2 = .05$ $n = 22$ df = n-1 = 21 $s^2 = 17$
H_a: $\sigma^2 \neq 8.5$

$\chi^2_{.05,21} = 32.6706$

$$\chi^2 = \frac{(22-1)(17)}{8.5} = \mathbf{42}$$

Since $\chi^2 = 42 > \chi^2_{.05,21} = 32.6706$, the decision is to **reject the null hypothesis**.

c) H_0: $\sigma^2 = 45$ $\alpha = .01$ $n = 8$ df = $n - 1 = 7$ $s = 4.12$
H_a: $\sigma^2 < 45$

$\chi^2_{.01,7} = 18.4753$

$$\chi^2 = \frac{(8-1)(4.12)^2}{45} = \mathbf{2.64}$$

Since $\chi^2 = 2.64 < \chi^2_{.01,7} = 18.4753$, the decision is to **fail to reject the null hypothesis**.

d) H_0: $\sigma^2 = 5$ $\alpha = .05$ $\alpha/2 = .025$ $n = 11$ df = $11 - 1 = 10$ $s^2 = 1.2$
H_a: $\sigma^2 \neq 5$

$\chi^2_{.025,10} = 20.4832$ $\chi^2_{.975,10} = 3.24696$

$$\chi^2 = \frac{(11-1)(1.2)}{5} = \mathbf{2.4}$$

Since $\chi^2 = 2.4 < \chi^2_{.975,10} = 3.24696$, the decision is to **reject the null hypothesis**.

9.35 H_0: $\sigma^2 = .001$ $\alpha = .01$ $n = 16$ df = $16 - 1 = 15$ $s^2 = .00144667$
H_a: $\sigma^2 > .001$

$\chi^2_{.01,15} = 30.5780$

$$\chi^2 = \frac{(16-1)(.00144667)}{.001} = \mathbf{21.7}$$

Since $\chi^2 = 21.7 < \chi^2_{.01,15} = 30.5780$, the decision is to **fail to reject the null hypothesis**.

9.37 H$_0$: $\sigma^2 = .04$ $\alpha = .01$ $n = 7$ df = 7 – 1 = 6 $s = .34$ $s^2 = .1156$
H$_a$: $\sigma^2 > .04$

$\chi^2_{.01,6} = 16.8119$

$$\chi^2 = \frac{(7-1)(.1156)}{.04} = \mathbf{17.34}$$

Since $\chi^2 = 17.34 > \chi^2_{.01,6} = 16.8119$, the decision is to **reject the null hypothesis**

9.39 $\alpha = .05$ $\mu = 100$ $n = 48$ $\sigma = 14$

a) $\mu_a = 98.5$ $z_c = -1.645$

$$z_c = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$-1.645 = \frac{\bar{x}_c - 100}{\frac{14}{\sqrt{48}}}$$

$\bar{x}_c = 96.68$

$$z = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{96.68 - 98.5}{\frac{14}{\sqrt{48}}} = -0.90$$

from Table A.5, area for $z = -0.90$ is .3159

β = .3159 + .5000 = **.8159**

b) $\mu_a = 98$ $z_c = -1.645$

$\bar{x}_c = 96.68$

$$z_c = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{96.68 - 98}{\frac{14}{\sqrt{48}}} = -0.65$$

from Table A.5, area for $z = -0.65$ is .2422

β = .2422 + .5000 = **.7422**

c) $\mu_a = 97$ $z_{.05} = -1.645$

$\bar{x}_c = 96.68$

$$z = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{96.68 - 97}{\frac{14}{\sqrt{48}}} = -0.16$$

from Table A.5, area for $z = -0.16$ is .0636

β = .0636 + .5000 = **.5636**

d) $\mu_a = 96$ $z_{.05} = -1.645$

$\bar{x}_c = 96.68$

$$z = \frac{\bar{x}_c - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{96.68 - 96}{\frac{14}{\sqrt{48}}} = 0.34$$

from Table A.5, area for $z = 0.34$ is .1331

β = .5000 - .1331 = **.3669**

e) As the alternative value gets farther from the null hypothesized value, the probability of committing a Type II error reduces (all other variables being held constant).

9.41 a) H_o: $p = .65$
H_a: $p < .65$

$n = 360$ $\alpha = .05$ $p_a = .60$ $z_{.05} = -1.645$

$$z_c = \frac{\hat{p}_c - p}{\sqrt{\dfrac{p \cdot q}{n}}}$$

$$-1.645 = \frac{\hat{p}_c - .65}{\sqrt{\dfrac{(.65)(.35)}{360}}}$$

$\hat{p}_c = .65 - .041 = .609$

$$z = \frac{\hat{p}_c - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{.609 - .60}{\sqrt{\dfrac{(.60)(.40)}{360}}} = 0.35$$

from Table A.5, area for $z = -0.35$ is .1368

$\beta = .5000 - .1368 =$ **.3632**

b) $p_a = .55$ $z_{.05} = -1.645$ $\hat{p}_c = .609$

$$z = \frac{\hat{p}_c - P}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{.609 - .55}{\sqrt{\dfrac{(.55)(.45)}{360}}} = 2.25$$

from Table A.5, area for $z = -2.25$ is .4878

$\beta = .5000 - .4878 =$ **.0122**

c) $p_a = .50$ $z_{.05} = -1.645$ $\hat{p}_c = .609$

$$z = \frac{\hat{p}_c - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{.609 - .50}{\sqrt{\dfrac{(.50)(.50)}{360}}} = -4.14$$

from Table A.5, the area for z = -4.14 is .5000

$\beta = .5000 - .5000 =$ **.0000**

9.43 H_0: $p = .71$
H_a: $p < .71$

$n = 463$ $x = 324$ $\hat{p} = \dfrac{324}{463} = .6998$ $\alpha = .10$

$z_{.10} = -1.28$

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{.6998 - .71}{\sqrt{\dfrac{(.71)(.29)}{463}}} = \mathbf{-0.48}$$

Since the observed $z = -0.48 > z_{.10} = -1.28$, the decision is to **fail to reject the null hypothesis**.

Type II error:

Solving for the critical proportion, $\hat{p}_c$:

$$z_c = \frac{\hat{p}_c - p}{\sqrt{\dfrac{p \cdot q}{n}}}$$

$$-1.28 = \frac{\hat{p}_c - .71}{\sqrt{\dfrac{(.71)(.29)}{463}}}$$

$$\hat{p} = .683$$

For $p_a = .69$

$$z = \frac{.683 - .69}{\sqrt{\dfrac{(.69)(.31)}{463}}} = -0.33$$

From Table A.5, the area for $z = -0.33$ is .1293

The probability of committing a Type II error = .1293 + .5000 = **.6293**

For $p_a = .66$

$$z = \frac{.683 - .66}{\sqrt{\frac{(.66)(.34)}{463}}} = 1.04$$

From Table A.5, the area for $z = 1.04$ is .3508

The probability of committing a Type II error = .5000 - .3508 = **.1492**

For $p_a = .60$

$$z = \frac{.683 - .60}{\sqrt{\frac{(.60)(.40)}{463}}} = 3.65$$

From Table A.5, the area for $z = 3.65$ is essentially, .5000

The probability of committing a Type II error = .5000 - .5000 = **.0000**

9.45 8 steps:

1) H_o: $\mu = 7.82$
   H_a: $\mu < 7.82$

2) The test statistic is

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

3) $\alpha = .05$

4) df = n - 1 = 16, $t_{.05,16} = -1.746$. If the observed value of t is less than -1.746, then the decision will be to reject the null hypothesis.

5) $n = 17$ $\bar{x} = 7.01$ $s = 1.69$

6) $$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{7.01 - 7.82}{\frac{1.69}{\sqrt{17}}} = \mathbf{-1.98}$$

7) Since the observed t = -1.98 is less than the table value of t = -1.746, the decision is to **reject the null hypothesis**.

8) The population mean is significantly less than 7.82.

9.47 8 steps:

1) H_0: $\sigma^2 = 15.4$
H_a: $\sigma^2 > 15.4$

2) $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

3) $\alpha = .01$

4) $n = 18$, df = 17, one-tailed test

$\chi^2_{.01,17} = 33.4087$

5) $s^2 = 29.6$

6) $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(17)(29.6)}{15.4} = \mathbf{32.675}$

7) Since the observed $\chi^2 = 32.675$ is less than 33.4087, the decision is to **fail to reject the null hypothesis**.

8) The population variance is not significantly more than 15.4.

9.49 H_0: $p = .32$
H_a: $p \neq .32$

$n = 80$ $\alpha = .01$ $\hat{p} = .25$ $z_{.005} = \pm 2.575$

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{.25 - .32}{\sqrt{\dfrac{(.32)(.68)}{80}}} = -1.34$$

Since the observed $z = -1.34 > z_{.005} = -2.575$, the decision is to **fail to reject the null hypothesis**.

9.51 $n = 210$ $x = 93$ $\alpha = .10$ $\hat{p} = \dfrac{x}{n} = \dfrac{93}{210} = .443$

H_o: $p = .57$
H_a: $p < .57$

For one-tail, $\alpha = .10$ $z_c = -1.28$

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{.443 - .57}{\sqrt{\dfrac{(.57)(.43)}{210}}} = \mathbf{-3.72}$$

Since the observed $z = -3.72 < z_c = -1.28$, the decision is to **reject the null hypothesis**.

9.53 H_0: $\mu = 8.4$ $\alpha = .01$ $\alpha/2 = .005$ $n = 7$ df = 7 – 1 = 6 $s = 1.3$
H_a: $\mu \neq 8.4$

$\bar{x} = 5.6$ $t_{.005,6} = \pm 3.707$

$$t = \frac{5.6 - 8.4}{\dfrac{1.3}{\sqrt{7}}} = \mathbf{-5.70}$$

Since the observed $t = -5.70 < t_{.005,6} = -3.707$, the decision is to **reject the null hypothesis**.

9.55 H_0: $\sigma^2 = 4$ $n = 8$ $s = 7.80$ $\alpha = .10$ df = 8 – 1 = 7
H_a: $\sigma^2 > 4$

$\chi^2_{.10,7} = 12.0170$

$$\chi^2 = \frac{(8-1)(7.80)^2}{4} = \mathbf{106.47}$$

Since observed $\chi^2 = 106.47 > \chi^2_{.10,7} = 12.017$, the decision is to **reject the null hypothesis**.

9.57 $n = 16$ $\bar{x} = 175$ $s = 14.28286$ df = 16 - 1 = 15 $\alpha = .05$

H$_0$: $\mu = 185$
H$_a$: $\mu < 185$

$t_{.05,15} = -1.753$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{175 - 185}{\frac{14.28286}{\sqrt{16}}} = \mathbf{-2.80}$$

Since observed $t = -2.80 < t_{.05,15} = -1.753$, the decision is to **reject the null hypothesis**.

9.59 H$_o$: $\mu = \$15$
H$_a$: $\mu > \$15$

$\bar{x} = \$19.34$ $n = 17$ $\sigma = \$4.52$ $\alpha = .10$

For one-tail and $\alpha = .10$ $z_c = 1.28$

$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{19.34 - 15}{\frac{4.52}{\sqrt{17}}} = 3.96$$

Since the observed $z = 3.96 > z_c = 1.28$, the decision is to **reject the null hypothesis**.

9.61 H$_0$: $\mu = 2.5$ $\bar{x} = 3.4$ $s = 0.6$ $\alpha = .01$ $n = 9$ df = 9 – 1 = 8
H$_a$: $\mu > 2.5$

$t_{.01,8} = 2.896$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.4 - 2.5}{\frac{0.6}{\sqrt{9}}} = \mathbf{4.50}$$

Since the observed $t = 4.50 > t_{.01,8} = 2.896$, the decision is to **reject the null hypothesis**.

9.63 $n = 12$ $\bar{x} = 12.333$ $s^2 = 10.424$

H_0: $\sigma^2 = 2.5$
H_a: $\sigma^2 \neq 2.5$

$\alpha = .05$ df = 11 two-tailed test, $\alpha/2 = .025$

$\chi^2_{.025,11} = 21.9200$

$\chi^2_{..975,11} = 3.81574$

If the observed χ^2 is greater than 21.9200 or less than 3.81574, the decision is to reject the null hypothesis.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{11(10.424)}{2.5} = \mathbf{45.866}$$

Since the observed $\chi^2 = 45.866$ is greater than $\chi^2_{.025,11} = 21.92$, the decision is to **reject the null hypothesis**. The population variance is significantly more than 2.5.

9.65 The sample size is 22. $\bar{x}$ is 3.969 $s = 0.866$ df = 21

The test statistic is:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

The observed $t = -2.33$. The p-value is .015.

The results are statistical significant at $\alpha = .05$.

The decision is to reject the null hypothesis.

9.67 H_0: $\mu = 2.51$
H_a: $\mu > 2.51$

This is a one-tailed test. The sample mean is 2.55 which is more than the hypothesized value. The observed *t* value is 1.51 with an associated *p*-value of .072 for a one-tailed test. Because the *p*-value is greater than $\alpha = .05$, the decision is to fail to reject the null hypothesis.
There is not enough evidence to conclude that beef prices are higher.