

Chapter 7

Sampling and Sampling Distributions

LEARNING OBJECTIVES

The two main objectives for Chapter 7 are to give you an appreciation for the proper application of sampling techniques and an understanding of the sampling distributions of two statistics, thereby enabling you to:

1. Contrast sampling to census and differentiate among different methods of sampling, which include simple, stratified, systematic, and cluster random sampling; and convenience, judgment, quota, and snowball nonrandom sampling, by assessing the advantages associated with each
2. Describe the distribution of a sample's mean using the central limit theorem, correcting for a finite population if necessary
3. Describe the distribution of a sample's proportion using the z formula for sample proportions

CHAPTER OUTLINE

7.1 Sampling

- Reasons for Sampling
- Reasons for Taking a Census
- Frame
- Random Versus Nonrandom Sampling
- Random Sampling Techniques
 - Simple Random Sampling
 - Stratified Random Sampling
 - Systematic Sampling
 - Cluster (or Area) Sampling
- Nonrandom Sampling
 - Convenience Sampling
 - Judgment Sampling
 - Quota Sampling
 - Snowball Sampling
- Sampling Error
- Nonsampling Errors

7.2 Sampling Distribution of \bar{x}

- Sampling from a Finite Population

7.3 Sampling Distribution of \hat{p}

KEY TERMS

Central Limit Theorem	Quota Sampling
Cluster (or Area) Sampling	Random Sampling
Convenience Sampling	Sample Proportion
Disproportionate Stratified Random Sampling	Sampling Error
Finite Correction Factor	Simple Random Sampling
Frame	Snowball Sampling
Judgment Sampling	Standard Error of the Mean
Nonrandom Sampling	Standard Error of the Proportion
Nonrandom Sampling Techniques	Stratified Random Sampling
Nonsampling Errors	Systematic Sampling
Overregistration	Two-Stage Sampling
Proportionate Stratified Random Sampling	Underregistration

STUDY QUESTIONS

1. Saving time and money are reasons to take a _____ rather than a census.
2. If the research process is destructive, taking a _____ may be the only option in gathering data.
3. A researcher may opt to take a _____ to eliminate the possibility that by chance randomly selected items are not representative of the population.
4. The directory or map from which a sample is taken is called the _____.
5. If the population list from which the researcher takes the sample contains fewer units than the target population, then the list has _____.
6. There are two main types of sampling, _____ sampling and _____ sampling.
7. If every unit of the population does not have the same probability of being selected to the sample, then the researcher is probably conducting _____ sampling.
8. Nonrandom sampling is sometimes referred to as _____ sampling.
9. The most elementary type of random sampling is _____ random sampling.
10. In _____ random sampling, the population is divided into nonoverlapping subpopulations called strata.
11. Whenever the proportions of the strata in the sample are different than the proportions of the strata in the population, _____ random sampling occurs.
12. With _____ random sampling, there is homogeneity within a subgroup or stratum.
13. If a researcher selects every k th item from a population of N items, then he/she is likely conducting _____ random sampling.
14. When the population is divided into nonoverlapping areas and then random samples are drawn from the areas, the researcher is likely conducting _____ or _____ sampling.

15. A nonrandom sampling technique in which elements are selected for the sample based on the convenience of the researcher is called _____ sampling.
16. A nonrandom sampling technique in which elements are selected for the sample based on the judgment of the researcher is called _____ sampling.
17. A nonrandom sampling technique which is similar to stratified random sampling is called _____ sampling.
18. A nonrandom sampling technique in which survey subjects are selected based on referral from other survey respondents is called _____ sampling.
19. _____ error occurs when, by chance, the sample is not representative of the population.
20. Missing data and recording errors are examples of _____ errors.
21. The central limit theorem states that if n is large enough, the sample means are _____ distributed regardless of the shape of the population.
22. According to the central limit theorem, the mean of the sample means for a given size of sample is equal to the _____.
23. According to the central limit theorem, the standard deviation of sample means for a given size of sample equals _____.
24. If samples are being drawn from a known population size, the z formula for sample means includes a _____ factor.
25. Suppose a population has a mean of 90 and a standard deviation of 27. If a random sample of size 49 is drawn from the population, the probability of drawing a sample with a mean of more than 95 is _____.
26. Suppose a population has a mean of 455 and a variance of 900. If a random sample of size 65 is drawn from the population, the probability that the sample mean is between 448 and 453 is _____.
27. Suppose .60 of the population possesses a given characteristic. If a random sample of size 300 is drawn from the population, then the probability that .53 or fewer of the sample possess the characteristic is _____.
28. Suppose .36 of a population possesses a given characteristic. If a random sample of size 1200 is drawn from the population, then the probability that less than 480 possess that characteristic in the sample is _____.

ANSWERS TO STUDY QUESTIONS

- | | |
|---------------------------------|-------------------------------|
| 1. Sample | 15. Convenience |
| 2. Sample | 16. Judgment |
| 3. Census | 17. Quota |
| 4. Frame | 18. Snowball |
| 5. Underregistration | 19. Sampling |
| 6. Random, Nonrandom | 20. Nonsampling |
| 7. Nonrandom | 21. Normally |
| 8. Nonprobability | 22. Population Mean |
| 9. Simple | 23. $\frac{\sigma}{\sqrt{n}}$ |
| 10. Stratified | 24. Finite Correction |
| 11. Disproportionate Stratified | 25. .0968 |
| 12. Stratified | 26. .2645 |
| 13. Systematic | 27. .0068 |
| 14. Area, Cluster | 28. .9981 |

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 7

- 7.1
- a)
 - i. A union membership list for the company.
 - ii. A list of all employees of the company.
 - b)
 - i. White pages of the telephone directory for Utica, New York.
 - ii. Utility company list of all customers.
 - c)
 - i. Airline company list of phone and mail purchasers of tickets from the airline during the past six months.
 - ii. A list of frequent flyer club members for the airline.
 - d)
 - i. List of boat manufacturer's employees.
 - ii. List of members of a boat owners association.
 - e)
 - i. Cable company telephone directory.
 - ii. Membership list of cable management association.
- 7.5
- a) Under 21 years of age, 21 to 39 years of age, 40 to 55 years of age, over 55 years of age.
 - b) Under \$1,000,000 sales per year, \$1,000,000 to \$4,999,999 sales per year, \$5,000,000 to \$19,999,999 sales per year, \$20,000,000 to \$49,000,000 per year, \$50,000,000 to \$99,999,999 per year, over \$100,000,000 per year.
 - c) Less than 2,000 sq. ft., 2,000 to 4,999 sq. ft., 5,000 to 9,999 sq. ft., over 10,000 sq. ft.
 - d) East, southeast, midwest, south, southwest, west, northwest.
 - e) Government worker, teacher, lawyer, physician, engineer, business person, police officer, fire fighter, computer worker.
 - f) Manufacturing, finance, communications, health care, retailing, chemical, transportation.
- 7.7 $N = n \cdot k = 75(11) = \mathbf{825}$

- 7.9 a) i. Counties
 ii. Metropolitan areas
- b) i. States (beside which the oil wells lie)
 ii. Companies that own the wells
- c) i. States
 ii. Counties

- 7.11 Go to a conference where some of the Fortune 500 executives attend.
 Approach those executives who appear to be friendly and approachable.

7.13 $\mu = 50, \quad \sigma = 10, \quad n = 64$

a) $P(\bar{x} > 52)$:

$$z = \frac{\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}{\frac{10}{\sqrt{64}}} = \frac{52 - 50}{10} = 1.6$$

from Table A.5, Prob. = .4452

$$P(\bar{x} > 52) = .5000 - .4452 = \mathbf{.0548}$$

b) $P(\bar{x} < 51)$:

$$z = \frac{\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}{\frac{10}{\sqrt{64}}} = \frac{51 - 50}{10} = 0.80$$

from Table A.5 prob. = .2881

$$P(\bar{x} < 51) = .5000 + .2881 = \mathbf{.7881}$$

c) $P(\bar{x} < 47)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{\sigma}{\sqrt{n}}} = \frac{47 - 50}{\frac{10}{\sqrt{64}}} = -2.40$$

from Table A.5 prob. = .4918

$$P(\bar{x} < 47) = .5000 - .4918 = \mathbf{.0082}$$

d) $P(48.5 \leq \bar{x} \leq 52.4)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{\sigma}{\sqrt{n}}} = \frac{48.5 - 50}{\frac{10}{\sqrt{64}}} = -1.20$$

from Table A.5 prob. = .3849

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{\sigma}{\sqrt{n}}} = \frac{52.4 - 50}{\frac{10}{\sqrt{64}}} = 1.92$$

from Table A.5 prob. = .4726

$$P(48.5 \leq \bar{x} \leq 52.4) = .3849 + .4726 = \mathbf{.8575}$$

e) $P(50.6 \leq \bar{x} \leq 51.3)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{\sigma}{\sqrt{n}}} = \frac{50.6 - 50}{\frac{10}{\sqrt{64}}} = 0.48$$

from Table A.5, prob. = .1844

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{\sigma}{\sqrt{n}}} = \frac{51.3 - 50}{\frac{10}{\sqrt{64}}} = 1.04$$

from Table A.5, prob. = .3508

$$P(50.6 \leq \bar{x} \leq 51.3) = .3508 - .1844 = \mathbf{.1644}$$

$$7.15 \quad n = 36 \quad \mu = 278 \quad P(\bar{x} < 280) = .86$$

.3600 of the area lies between $\bar{x} = 280$ and $\mu = 278$. This probability is associated with $z = 1.08$ from Table A.5. Solving for σ :

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$1.08 = \frac{280 - 278}{\frac{\sigma}{\sqrt{36}}}$$

$$1.08 \frac{\sigma}{6} = 2$$

$$\sigma = \frac{12}{1.08} = \mathbf{11.11}$$

$$7.17 \quad \text{a) } N = 1,000 \quad n = 60 \quad \mu = 75 \quad \sigma = 6$$

$$P(\bar{x} < 76.5):$$

$$z = \frac{\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}}}{\frac{6}{\sqrt{60}} \sqrt{\frac{1000-60}{1000-1}}} = \frac{76.5 - 75}{\frac{6}{\sqrt{60}} \sqrt{\frac{1000-60}{1000-1}}} = 2.00$$

from Table A.5, prob. = .4772

$$P(\bar{x} < 76.5) = .4772 + .5000 = \mathbf{.9772}$$

$$\text{b) } N = 90 \quad n = 36 \quad \mu = 108 \quad \sigma = 3.46$$

$$P(107 < \bar{x} < 107.7):$$

$$z = \frac{\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}}}{\frac{3.46}{\sqrt{36}} \sqrt{\frac{90-36}{90-1}}} = \frac{107 - 108}{\frac{3.46}{\sqrt{36}} \sqrt{\frac{90-36}{90-1}}} = -2.23$$

from Table A.5, prob. = .4871

$$z = \frac{\frac{\bar{x} - \mu}{\sigma} \sqrt{\frac{N-n}{N-1}}}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}} = \frac{107.7 - 108}{3.46 \sqrt{\frac{90-36}{90-1}}} = -0.67$$

from Table A.5, prob. = .2486

$$P(107 < \bar{x} < 107.7) = .4871 - .2486 = \mathbf{.2385}$$

$$\text{c) } N = 250 \quad n = 100 \quad \mu = 35.6 \quad \sigma = 4.89$$

$P(\bar{x} \geq 36)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma} \sqrt{\frac{N-n}{N-1}}}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}} = \frac{36 - 35.6}{4.89 \sqrt{\frac{250-100}{250-1}}} = 1.05$$

from Table A.5, prob. = .3531

$$P(\bar{x} \geq 36) = .5000 - .3531 = \mathbf{.1469}$$

$$\text{d) } N = 5000 \quad n = 60 \quad \mu = 125 \quad \sigma = 13.4$$

$P(\bar{x} \leq 123)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma} \sqrt{\frac{N-n}{N-1}}}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}} = \frac{123 - 125}{13.4 \sqrt{\frac{5000-60}{5000-1}}} = -1.16$$

from Table A.5, prob. = .3770

$$P(\bar{x} \leq 123) = .5000 - .3770 = \mathbf{.1230}$$

$$7.19 \quad N = 1500 \quad n = 100 \quad \mu = 227,000 \quad \sigma = 8,500$$

$$P(\bar{X} > \$229,000):$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}} = \frac{229,000 - 227,000}{\frac{8,500}{\sqrt{100}} \sqrt{\frac{1500 - 100}{1500 - 1}}} = 2.43$$

From Table A.5, prob. = .5000

$$P(\bar{X} > \$229,000) = .5000 - .4925 = \mathbf{.0075}$$

$$7.21 \quad \mu = 50.4 \quad \sigma = 11.8 \quad n = 42$$

$$\text{a) } P(\bar{x} > 52):$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{52 - 50.4}{\frac{11.8}{\sqrt{42}}} = 0.88$$

from Table A.5, the area for $z = 0.88$ is .3106

$$P(\bar{x} > 52) = .5000 - .3106 = \mathbf{.1894}$$

$$\text{b) } P(\bar{x} < 47.5):$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{47.5 - 50.4}{\frac{11.8}{\sqrt{42}}} = -1.59$$

from Table A.5, the area for $z = -1.59$ is .4441

$$P(\bar{x} < 47.5) = .5000 - .4441 = \mathbf{.0559}$$

c) $P(\bar{x} < 40)$:

$$z = \frac{\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}{\frac{\sigma}{\sqrt{n}}} = \frac{40 - 50.4}{\frac{11.8}{\sqrt{42}}} = -5.71$$

from Table A.5, the area for $z = -5.71$ is .5000

$$P(\bar{x} < 40) = .5000 - .5000 = \mathbf{.0000}$$

d) 71% of the values are greater than 49. Therefore, 21% are between the sample mean of 49 and the population mean, $\mu = 50.4$.

The z value associated with the 21% of the area is -0.55

$$z_{.21} = -0.55$$

$$z = \frac{\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}{\frac{\sigma}{\sqrt{n}}}$$

$$-0.55 = \frac{49 - 50.4}{\frac{\sigma}{\sqrt{42}}}$$

$$\sigma = \mathbf{16.4964}$$

$$7.23 \quad p = .58 \quad n = 660$$

a) $P(\hat{p} > .60)$:

$$z = \frac{\frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.60 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = 1.04$$

from table A.5, area = .3508

$$P(\hat{p} > .60) = .5000 - .3508 = \mathbf{.1492}$$

b) $P(.55 < \hat{p} < .65)$:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.65 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = 3.64$$

from table A.5, area = .4998

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.55 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = -1.56$$

from table A.5, area = .4406

$$P(.55 < \hat{p} < .65) = .4998 + .4406 = \mathbf{.9404}$$

c) $P(\hat{p} > .57)$:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.57 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = -0.52$$

from table A.5, area = .1985

$$P(\hat{p} > .57) = .1985 + .5000 = \mathbf{.6985}$$

d) $P(.53 \leq \hat{p} \leq .56)$:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.56 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = -1.04 \qquad z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.53 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = -2.60$$

from table A.5, area for $z = -1.04$ is .3508

from table A.5, area for $z = -2.60$ is .4953

$$P(.53 \leq \hat{p} \leq .56) = .4953 - .3508 = \mathbf{.1445}$$

e) $P(\hat{p} < .48)$:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.48 - .58}{\sqrt{\frac{(.58)(.42)}{660}}} = -5.21$$

from table A.5, area = .5000

$$P(\hat{p} < .48) = .5000 - .5000 = \mathbf{.0000}$$

$$7.25 \quad p = .28 \quad n = 140 \quad P(\hat{p} < \hat{p}_0) = .3000$$

$$P(\hat{p} \leq \hat{p}_0 \leq .28) = .5000 - .3000 = .2000$$

from Table A.5, $z_{.2000} = -0.52$

Solving for \hat{p}_0 :

$$z = \frac{\hat{p}_0 - p}{\sqrt{\frac{p \cdot q}{n}}}$$

$$-0.52 = \frac{\hat{p}_0 - .28}{\sqrt{\frac{(.28)(.72)}{140}}}$$

$$-.02 = \hat{p}_0 - .28$$

$$\hat{p}_0 = .28 - .02 = \mathbf{.26}$$

$$7.27 \quad p = .48 \quad n = 200$$

a) $P(x < 90)$:

$$\hat{p} = \frac{90}{200} = .45$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.45 - .48}{\sqrt{\frac{(.48)(.52)}{200}}} = -0.85$$

from Table A.5, the area for $z = -0.85$ is .3023

$$P(x < 90) = .5000 - .3023 = \mathbf{.1977}$$

b) $P(x > 100)$:

$$\hat{p} = \frac{100}{200} = .50$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.50 - .48}{\sqrt{\frac{(.48)(.52)}{200}}} = 0.57$$

from Table A.5, the area for $z = 0.57$ is .2157

$$P(x > 100) = .5000 - .2157 = \mathbf{.2843}$$

c) $P(x > 80)$:

$$\hat{p} = \frac{80}{200} = .40$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.40 - .48}{\sqrt{\frac{(.48)(.52)}{200}}} = -2.26$$

from Table A.5, the area for $z = -2.26$ is .4881

$$P(x > 80) = .5000 + .4881 = \mathbf{.9881}$$

$$7.29 \quad \mu = 76, \quad \sigma = 14$$

$$\text{a) } n = 35, \quad P(\bar{x} \geq 79):$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{79 - 76}{\frac{14}{\sqrt{35}}} = 1.27$$

from table A.5, area = .3980

$$P(\bar{x} \geq 79) = .5000 - .3980 = \mathbf{.1020}$$

$$\text{b) } n = 140, \quad P(74 \leq \bar{x} \leq 77):$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{74 - 76}{\frac{14}{\sqrt{140}}} = -1.69$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{77 - 76}{\frac{14}{\sqrt{140}}} = 0.85$$

from table A.5, area for $z = -1.69$ is .4545

from table A.5, area for 0.85 is .3023

$$P(74 \leq \bar{x} \leq 77) = .4545 + .3023 = \mathbf{.7568}$$

$$\text{c) } n = 219, \quad P(\bar{x} < 76.5):$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{76.5 - 76}{\frac{14}{\sqrt{219}}} = 0.53$$

from table A.5, area = .2019

$$P(\bar{x} < 76.5) = .5000 + .2019 = \mathbf{.7019}$$

7.31	Under 18	$250(.22) =$	55
	18 – 25	$250(.18) =$	45
	26 – 50	$250(.36) =$	90
	51 – 65	$250(.10) =$	25
	over 65	$250(.14) =$	35
		$n =$	250

7.33 a) Roster of production employees secured from the human resources department of the company.

b) Ralphs store records kept at the headquarters of their California division or merged files of store records from regional offices across the state.

c) Membership list of Maine lobster catchers association.

7.35 Number the employees from 0001 to 1250. Randomly sample from the random number table until 60 different usable numbers are obtained. You cannot use numbers from 1251 to 9999.

7.37 $n = 1100$

a) $x > 810, \quad p = .73$

$$\hat{p} = \frac{x}{n} = \frac{810}{1100}$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.7364 - .73}{\sqrt{\frac{(.73)(.27)}{1100}}} = 0.48$$

from table A.5, area = .1844

$$P(x > 810) = .5000 - .1844 = \mathbf{.3156}$$

b) $x < 1030, \quad p = .96,$

$$\hat{p} = \frac{x}{n} = \frac{1030}{1100} = .9364$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.9364 - .96}{\sqrt{\frac{(.96)(.04)}{1100}}} = -3.99$$

from table A.5, area = .49997

$$P(x < 1030) = .5000 - .49997 = \mathbf{.00003}$$

c) $p = .85$

$$P(.82 \leq \hat{p} \leq .84):$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.82 - .85}{\sqrt{\frac{(.85)(.15)}{1100}}} = -2.79$$

from table A.5, area = .4974

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.84 - .85}{\sqrt{\frac{(.85)(.15)}{1100}}} = -0.93$$

from table A.5, area = .3238

$$P(.82 \leq \hat{p} \leq .84) = .4974 - .3238 = \mathbf{.1736}$$

- 7.39 Divide the factories into geographic regions and select a few factories to represent those regional areas of the country. Take a random sample of employees from each selected factory. Do the same for distribution centers and retail outlets. Divide the United States into regions of areas. Select a few areas. Take a random sample from each of the selected area distribution centers and retail outlets.

7.41 $p = .54$ $n = 565$

a) $P(x \geq 339)$:

$$\hat{p} = \frac{x}{n} = \frac{339}{565} = .60$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.60 - .54}{\sqrt{\frac{(.54)(.46)}{565}}} = 2.86$$

from Table A.5, the area for $z = 2.86$ is .4979

$$P(x \geq 339) = .5000 - .4979 = \mathbf{.0021}$$

b) $P(x \geq 288)$:

$$\hat{p} = \frac{x}{n} = \frac{288}{565} = .5097$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.5097 - .54}{\sqrt{\frac{(.54)(.46)}{565}}} = -1.45$$

from Table A.5, the area for $z = -1.45$ is .4265

$$P(x \geq 288) = .5000 + .4265 = \mathbf{.9265}$$

c) $P(\hat{p} \leq .50)$:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.50 - .54}{\sqrt{\frac{(.54)(.46)}{565}}} = -1.91$$

from Table A.5, the area for $z = -1.91$ is .4719

$$P(\hat{p} \leq .50) = .5000 - .4719 = \mathbf{.0281}$$

$$7.43 \quad \mu = 56.8 \quad n = 51 \quad \sigma = 12.3$$

a) $P(\bar{x} > 60)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{60 - 56.8}{\frac{12.3}{\sqrt{51}}} = 1.86$$

from Table A.5, Prob. = .4686

$$P(\bar{x} > 60) = .5000 - .4686 = \mathbf{.0314}$$

b) $P(\bar{x} > 58)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{58 - 56.8}{\frac{12.3}{\sqrt{51}}} = 0.70$$

from Table A.5, Prob. = .2580

$$P(\bar{x} > 58) = .5000 - .2580 = \mathbf{.2420}$$

c) $P(56 < \bar{x} < 57)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{56 - 56.8}{\frac{12.3}{\sqrt{51}}} = -0.46$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{57 - 56.8}{\frac{12.3}{\sqrt{51}}} = 0.12$$

from Table A.5, Prob. for $z = -0.46$ is .1772

from Table A.5, Prob. for $z = 0.12$ is .0478

$$P(56 < \bar{x} < 57) = .1772 + .0478 = \mathbf{.2250}$$

d) $P(\bar{x} < 55)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{55 - 56.8}{\frac{12.3}{\sqrt{51}}} = -1.05$$

from Table A.5, Prob. = .3531

$$P(\bar{x} < 55) = .5000 - .3531 = \mathbf{.1469}$$

e) $P(\bar{x} < 50)$:

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{\sigma}{\sqrt{n}}} = \frac{50 - 56.8}{\frac{12.3}{\sqrt{51}}} = -3.95$$

from Table A.5, Prob.= .5000

$$P(\bar{x} < 50) = .5000 - .5000 = \mathbf{.0000}$$

7.45 $p = .73 \quad n = 300$

a) $P(210 \leq x \leq 234)$:

$$\hat{p}_1 = \frac{x}{n} = \frac{210}{300} = .70 \qquad \hat{p}_2 = \frac{x}{n} = \frac{234}{300} = .78$$

$$z = \frac{\frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}}{\sqrt{\frac{p \cdot q}{n}}} = \frac{\frac{.70 - .73}{\sqrt{\frac{(.73)(.27)}{300}}}}{\sqrt{\frac{(.73)(.27)}{300}}} = -1.17$$

$$z = \frac{\frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}}{\sqrt{\frac{p \cdot q}{n}}} = \frac{\frac{.78 - .73}{\sqrt{\frac{(.73)(.27)}{300}}}}{\sqrt{\frac{(.73)(.27)}{300}}} = 1.95$$

from Table A.5, the area for $z = -1.17$ is .3790
the area for $z = 1.95$ is .4744

$$P(210 \leq x \leq 234) = .3790 + .4744 = \mathbf{.8534}$$

b) $P(\hat{p} \geq .78)$:

$$z = \frac{\frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}}{\sqrt{\frac{p \cdot q}{n}}} = \frac{\frac{.78 - .73}{\sqrt{\frac{(.73)(.27)}{300}}}}{\sqrt{\frac{(.73)(.27)}{300}}} = 1.95$$

from Table A.5, the area for $z = 1.95$ is .4744

$$P(\hat{p} \geq .78) = .5000 - .4744 = \mathbf{.0256}$$

$$\text{c) } p = .73 \quad n = 800 \quad P(\hat{p} \geq .78):$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.78 - .73}{\sqrt{\frac{(.73)(.27)}{800}}} = 3.19$$

from Table A.5, the area for $z = 3.19$ is .4993

$$P(\hat{p} \geq .78) = .5000 - .4993 = \mathbf{.0007}$$

- 7.47 By taking a sample, there is potential for obtaining more detailed information. More time can be spent with each employee. Probing questions can be asked. There is more time for trust to be built between employee and interviewer resulting in the potential for more honest, open answers.

With a census, data is usually more general and easier to analyze because it is in a more standard format. Decision-makers are sometimes more comfortable with a census because everyone is included and there is no sampling error. A census appears to be a better political device because the CEO can claim that everyone in the company has had input.

$$7.49 \text{ a) Switzerland: } n = 40 \quad \mu = \$ 30.67 \quad \sigma = \$ 3$$

$$P(30 \leq \bar{x} \leq 31):$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{31 - 30.67}{\frac{3}{\sqrt{40}}} = 0.70$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{30 - 30.67}{\frac{3}{\sqrt{40}}} = -1.41$$

from Table A.5, the area for $z = 0.70$ is .2580
the area for $z = -1.41$ is .4207

$$P(30 \leq \bar{x} \leq 31) = .2580 + .4207 = \mathbf{.6787}$$

b) Japan: $n = 35$ $\mu = \$ 20.20$ $\sigma = \$3$

$$P(\bar{x} > 21):$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{21 - 20.20}{\frac{3}{\sqrt{35}}} = 1.58$$

from Table A.5, the area for $z = 1.58$ is .4429

$$P(\bar{x} > 21) = .5000 - .4429 = \mathbf{.0571}$$

c) U.S.: $n = 50$ $\mu = \$ 23.82$ $\sigma = \$ 3$

$$P(\bar{x} < 22.75):$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{22.75 - 23.82}{\frac{3}{\sqrt{50}}} = -2.52$$

from Table A.5, the area for $z = -2.52$ is .4941

$$P(\bar{x} < 22.75) = .5000 - .4941 = \mathbf{.0059}$$

7.51 $\mu = \$281$ $n = 65$ $\sigma = \$47$

$$P(\bar{x} > \$273):$$

$$z = \frac{\frac{\bar{x} - \mu}{\sigma}}{\frac{1}{\sqrt{n}}} = \frac{273 - 281}{\frac{47}{\sqrt{65}}} = -1.37$$

from Table A.5 the area for $z = -1.37$ is .4147

$$P(\bar{x} > \$273) = .5000 + .4147 = \mathbf{.9147}$$