Chapter 8
Statistical Inference: Estimation for Single Populations

LEARNING OBJECTIVES

The overall learning objective of Chapter 8 is to help you understand estimating parameters of single populations, thereby enabling you to:

1. Estimate the population mean with a known population standard deviation with the *z* statistic, correcting for a finite population if necessary.
2. Estimate the population mean with an unknown population standard deviation using the *t* statistic and properties of the *t* distribution.
3. Estimate a population proportion using the *z* statistic.
4. Use the chi-square distribution to estimate the population variance given the sample variance.
5. Determine the sample size needed in order to estimate the population mean and population proportion.

CHAPTER OUTLINE

KEY WORDS

Bounds	Point Estimate
Chi-square Distribution	Robust
Degrees of Freedom (df)	Sample-Size Estimation
Interval Estimate	*t* Distribution
Lower bound of the confidence interval	*t* Value
Margin of error of the interval	Upper bound of the confidence interval

STUDY QUESTIONS

1. When a statistic taken from the sample is used to estimate a population parameter, it is called a(n) _______________ estimate.

2. When a range of values is used to estimate a population parameter, it is called a(n) _______________ estimate.

3. The z value associated with a two-sided 90% confidence interval is _______________.

4. The z value associated with a two-sided 95% confidence interval is _______________.

5. The z value associated with a two-sided 80% confidence interval is _______________.

6. Suppose a random sample of 40 is selected from a population with a standard deviation of 13. If the sample mean is 118, the 98% confidence interval to estimate the population mean is ____________________.

7. Suppose a random sample of size 75 is selected from a population with a standard deviation of 6.4. The sample yields a mean of 26. From this information, the 90% confidence interval to estimate the population mean can be computed as ____________________.

8. The following random sample of numbers are drawn from a population: 45, 61, 55, 43, 49, 60, 62, 53, 57, 44, 39, 48, 57, 40, 61, 62, 45, 39, 38, 56, 55, 59, 63, 50, 41, 39, 45, 47, 56, 51, 61, 39, 36, 57. Assume that the population standard deviation is 8.62. From these data, a 99% confidence interval to estimate the population mean can be computed as ____________________.

9. A random sample of 63 items is selected from a population of 400 items. The sample mean is 211. The population standard deviation is 48. From this information, a 95% confidence interval to estimate the population mean can be computed as ____________________.

10. Generally, when estimating a population mean and the population standard deviation is not known, you should use the _____ statistic.

11. The *t* test was developed by _______________.

12. In order to find values in the *t* distribution table, you must convert the sample size or sizes to _______________.

13. The table *t* value associated with 10 degrees of freedom and used to compute a 95% confidence interval is _______________.

14. The table *t* value associated with 18 degrees of freedom and used to compute a 99% confidence interval is _______________.

15. A researcher is interested in estimating the mean value for a population. She takes a random sample of 17 items and computes a sample mean of 224 and a sample standard deviation of 32. She decides to construct a 98% confidence interval to estimate the mean. The degrees of freedom associated with this problem are _______________. It can be assumed that these values are normally distributed in the population.

16. The table *t* value used to construct the confidence interval in question 15 is _______________.

17. The confidence interval resulting from the data in question 15 is _______________.

18. A researcher wants to estimate the proportion of the population which possesses a given characteristic. A random sample of size 800 is taken resulting in 380 items which possess the characteristic. The point estimate for this population proportion is ____________________.

19. A researcher wants to estimate the proportion of a population which possesses a given characteristic. A random sample of size 1250 is taken and .67 of the sample possess the characteristic. The 90% confidence interval to estimate the population proportion is ____________________.

20. A random sample of 255 items from a population results in 44% possessing a given characteristic. Using this information, the researcher constructs a 99% confidence interval to estimate the population proportion. The resulting confidence interval is ____________________.

21. What proportion of a population possesses a given characteristic? To estimate this, a random sample of 1700 people are interviewed from the population. Seven hundred and fourteen of the people sampled posses the characteristic. Using this information, the researcher computes an 88% confidence interval to estimate the proportion of the population who posses the given characteristic. The resulting confidence interval is ____________________.

22. A confidence interval to estimate the population variance can be constructed by using the sample variance and the __________________________ distribution.

23. Suppose we want to construct a confidence interval to estimate a population variance. A sample variance is computed from a sample of 14 items. To construct a 95% confidence interval, the chi-square table values are _______________ and _______________.

24. We want to estimate a population variance. A sample of 9 items produces a sample standard deviation of 4.29. The point estimate of the population variance is _______________________.

25. In an effort to estimate the population variance, a sample of 12 items is taken. The sample variance is 21.96. Using this information, it can be determined that the 90% confidence interval is ____.

26. In estimating the sample size necessary to estimate μ, the error of estimation, E, is equal to _______________.

27. In estimating sample size, if the population standard deviation is unknown, it can be estimated by using _______________.

28. Suppose a researcher wants to conduct a study to estimate the population mean. He/she plans to use a 95% level of confidence to estimate the mean and the population standard deviation is approximately 34. The researcher wants the error to be no more than 4. The sample size should be at least _______________.

29. A researcher wants to determine the sample size necessary to adequately conduct a study to estimate the population mean to within 5 points. The range of population values is 80 and the researcher plans to use a 90% level of confidence. The sample size should be at least _______________.

30. A study is going to be conducted in which a population mean will be estimated using a 92% confidence interval. The estimate needs to be within 12 of the actual population mean. The population variance is estimated to be around 2200. The necessary sample size should be at least _______________.

31. In estimating the sample size necessary to estimate p, if there is no good approximation for the value of p available, the value of _______________ should be used as an estimate of p in the formula.

32. A researcher wants to estimate the population proportion with a 95% level of confidence. He/she estimates from previous studies that the population proportion is no more than .30. The researcher wants the estimate to have an error of no more than .02. The necessary sample size is at least _______________.

33. A study will be conducted to estimate the population proportion. A level of confidence of 99% will be used and an error of no more than .05 is desired. There is no knowledge as to what the population proportion will be. The size of sample should be at least _______________.

34. A researcher conducts a study to determine what the population proportion is for a given characteristic. Is it believed from previous studies that the proportion of the population will be at least .65. The researcher wants to use a 98% level of confidence. He/she also wants the error to be no more than .03. The sample size should be at least _______________.

ANSWERS TO STUDY QUESTIONS

1. Point
2. Interval
3. 1.645
4. 1.96
5. 1.28
6. $113.2 \leq \mu \leq 122.8$
7. $24.8 \leq \mu \leq 27.2$
8. $46.6 \leq \mu \leq 54.2$
9. $200.1 \leq \mu \leq 221.9$
10. t
11. William S. Gosset
12. Degrees of Freedom
13. 2.228
14. 2.878
15. 16
16. 2.583
17. $203.95 \leq \mu \leq 244.05$
18. .475
19. $.648 \leq p \leq .692$
20. $.36 \leq p \leq .52$
21. $.401 \leq p \leq .439$
22. Chi-square
23. 5.00874, 24.7356
24. $s^2 = 18.4041$
25. $12.277 \leq \sigma^2 \leq 52.802$
26. $\bar{x} - \mu$
27. ¼ Range
28. 278
29. 44
30. 47
31. .50
32. 2,017
33. 664
34. 1,373

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 8

8.1 a) $\bar{x} = 25$ $\sigma = 3.5$ $n = 60$

95% Confidence $z_{.025} = 1.96$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 25 \pm 1.96\frac{3.5}{\sqrt{60}} = 25 \pm 0.89 = \mathbf{24.11 \le \mu \le 25.89}$$

b) $\bar{x} = 119.6$ $\sigma = 23.89$ $n = 75$

98% Confidence $z_{.01} = 2.33$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 119.6 \pm 2.33\frac{23.89}{\sqrt{75}} = 119.6 \pm 6.43 = \mathbf{113.17 \le \mu \le 126.03}$$

c) $\bar{x} = 3.419$ $\sigma = 0.974$ $n = 32$

90% C.I. $z_{.05} = 1.645$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 3.419 \pm 1.645\frac{0.974}{\sqrt{32}} = 3.419 \pm .283 = \mathbf{3.136 \le \mu \le 3.702}$$

d) $\bar{x} = 56.7$ $\sigma = 12.1$ $N = 500$ $n = 47$

80% C.I. $z_{.10} = 1.28$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = 56.7 \pm 1.28\frac{12.1}{\sqrt{47}}\sqrt{\frac{500-47}{500-1}} =$$

$$56.7 \pm 2.15 = \mathbf{54.55 \le \mu \le 58.85}$$

8.3 $n = 81$ $\bar{x} = 47$ $\sigma = 5.89$

90% C.I. $z_{.05}=1.645$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 47 \pm 1.645\frac{5.89}{\sqrt{81}} = 47 \pm 1.08 = \mathbf{45.92 \le \mu \le 48.08}$$

8.5 $n = 39$ $N = 200$ $\bar{x} = 66$ $\sigma = 11$
96% C.I. $z_{.02} = 2.05$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = 66 \pm 2.05\frac{11}{\sqrt{39}}\sqrt{\frac{200-39}{200-1}} =$$

66 ± 3.25 = **$62.75 \le \mu \le 69.25$**

$\bar{x}$ = 66 Point Estimate

8.7 $N = 1500$ $n = 187$ $\bar{x} = 5.3$ years $\sigma = 1.28$ years
95% C.I. $z_{.025} = 1.96$

$\bar{x}$ = 5.3 years Point Estimate

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = 5.3 \pm 1.96\frac{1.28}{\sqrt{187}}\sqrt{\frac{1500-187}{1500-1}} =$$

$5.3 \pm .17$ = **$5.13 \le \mu \le 5.47$**

8.9 $n = 36$ $\bar{x} = 3.306$ $\sigma = 1.17$
98% C.I. $z_{.01} = 2.33$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 3.306 \pm 2.33\frac{1.17}{\sqrt{36}} = 3.306 \pm .454 = \mathbf{2.852 \le \mu \le 3.760}$$

8.11 95% confidence interval $n = 45$

$\bar{x} = 24.533$ $\sigma = 5.124$ $z = \pm 1.96$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 24.533 \pm 1.96\frac{5.124}{\sqrt{45}} =$$

$$24.533 \pm 1.497 = \mathbf{23.036 \le \mu \le 26.030}$$

8.13 $n = 13$ $\bar{x} = 45.62$ $s = 5.694$ df = 13 – 1 = 12

95% Confidence Interval and $\alpha/2 = .025$

$t_{.025,12} = 2.179$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 45.62 \pm 2.179\frac{5.694}{\sqrt{13}} = 45.62 \pm 3.44 = \mathbf{42.18 \le \mu \le 49.06}$$

8.15 $n = 41$ $\bar{x} = 128.4$ $s = 20.6$ df = 41 – 1 = 40

98% Confidence Interval
$\alpha/2 = .01$

$t_{.01,40} = 2.423$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 128.4 \pm 2.423\frac{20.6}{\sqrt{41}} = 128.4 \pm 7.80 = \mathbf{120.6 \le \mu \le 136.2}$$

$\bar{x}$ **= 128.4 Point Estimate**

8.17 $n = 25$ $\bar{x} = 16.088$ $s = .817$ df = 25 – 1 = 24

99% Confidence Interval

$\alpha/2 = .005$

$t_{.005,24} = 2.797$

$\bar{x} \pm t\frac{s}{\sqrt{n}}$ = $16.088 \pm 2.797\frac{(.817)}{\sqrt{25}}$ = $16.088 \pm .457$ = $\mathbf{15.631 \le \mu \le 16.545}$

$\bar{x}$ **= 16.088 Point Estimate**

8.19 $n = 20$ df = 19 95% CI $t_{.025,19} = 2.093$

$\bar{x} = 2.36116$ $s = 0.19721$

$2.36116 \pm 2.093\frac{0.1972}{\sqrt{20}}$ = 2.36116 ± 0.0923 = $\mathbf{2.26886 \le \mu \le 2.45346}$

Point Estimate = **2.36116**

Error = **0.0923**

8.21 $n = 10$ $\bar{x} = 49.8$ $s = 18.22$ df = 10 – 1 = 9

95% Confidence $\alpha/2 = .025$ $t_{.025,9} = 2.262$

$\bar{x} \pm t\frac{s}{\sqrt{n}}$ = $49.8 \pm 2.262\frac{18.22}{\sqrt{10}}$ = 49.8 ± 13.03 = $\mathbf{36.77 \le \mu \le 62.83}$

8.23 $n = 41$ df = 41 – 1 = 40 99% confidence $\alpha/2 = .005$

$t_{.005,40} = 2.704$

from data: $\bar{x} = 11.098$ $s = 8.446$

confidence interval: $\bar{x} \pm t\frac{s}{\sqrt{n}} = 11.098 \pm 2.704 \frac{8.446}{\sqrt{41}} =$

$11.098 \pm 3.567 =$ $\mathbf{7.531 \le \mu \le 14.665}$

8.25 a) $n = 44$ $\hat{p} = .51$ 90% C.I. $z_{.05} = 1.645$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .51 \pm 1.645\sqrt{\frac{(.51)(.49)}{44}} = .51 \pm .124 = \mathbf{.386 \le p \le .634}$$

b) $n = 300$ $\hat{p} = .82$ 95% C.I. $z_{.025} = 1.96$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .82 \pm 1.96\sqrt{\frac{(.82)(.18)}{300}} = .82 \pm .043 = \mathbf{.777 \le p \le .863}$$

c) $n = 1150$ $\hat{p} = .48$ 90% C.I. $z_{.05} = 1.645$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .48 \pm 1.645\sqrt{\frac{(.48)(.52)}{1150}} = .48 \pm .024 = \mathbf{.456 \le p \le .504}$$

d) $n = 95$ $\hat{p} = .32$ 88% C.I. $z_{.06} = 1.555$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .32 \pm 1.555\sqrt{\frac{(.32)(.68)}{95}} = .32 \pm .074 = \mathbf{.246 \le p \le .394}$$

8.27 $n = 85$ $x = 40$ 90% C.I. $z_{.05} = 1.645$

$$\hat{p} = \frac{x}{n} = \frac{40}{85} = .47$$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .47 \pm 1.645\sqrt{\frac{(.47)(.53)}{85}} = .47 \pm .09 = \mathbf{.38 \le p \le .56}$$

95% C.I. $z_{.025} = 1.96$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .47 \pm 1.96\sqrt{\frac{(.47)(.53)}{85}} = .47 \pm .11 = \mathbf{.36 \le p \le .58}$$

99% C.I. $z_{.005} = 2.575$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .47 \pm 2.575\sqrt{\frac{(.47)(.53)}{85}} = .47 \pm .14 = \mathbf{.33 \le p \le .61}$$

All other things being constant, as the confidence increased, the width of the interval increased.

8.29 $n = 560$ $\hat{p} = .47$ 95% CI $z_{.025} = 1.96$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .47 \pm 1.96\sqrt{\frac{(.47)(.53)}{560}} = .47 \pm .0413 = \mathbf{.4287 \le p \le .5113}$$

$n = 560$ $\hat{p} = .28$ 90% CI $z_{.05} = 1.645$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .28 \pm 1.645\sqrt{\frac{(.28)(.72)}{560}} = .28 \pm .0312 = \mathbf{.2488 \le p \le .3112}$$

8.31 $n = 3481$ $x = 927$

$$\hat{p} = \frac{x}{n} = \frac{927}{3481} = .266$$

a) $\hat{p}$ = **.266 Point Estimate**

b) 99% C.I. $z_{.005} = 2.575$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .266 \pm 2.575\sqrt{\frac{(.266)(.734)}{3481}} = .266 \pm .019 =$$

$$\mathbf{.247 \le p \le .285}$$

8.33 $\hat{p} = .63$ $n = 672$ 95% Confidence $z_{.025} = \pm 1.96$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .63 \pm 1.96\sqrt{\frac{(.63)(.37)}{672}} = .63 \pm .0365 = \mathbf{.5935 \le p \le .6665}$$

8.35 a) $n = 12$ $\bar{x} = 28.4$ $s^2 = 44.9$ 99% C.I. df = 12 – 1 = 11

$\chi^2_{.995,11} = 2.60320$ $\chi^2_{.005,11} = 26.7569$

$$\frac{(12-1)(44.9)}{26.7569} \le \sigma^2 \le \frac{(12-1)(44.9)}{2.60320}$$

18.46 ≤ σ^2 ≤ 189.73

b) $n = 7$ $\bar{x} = 4.37$ $s = 1.24$ $s^2 = 1.5376$ 95% C.I. df = 7 – 1 = 6

$\chi^2_{.975,6} = 1.23734$ $\chi^2_{.025,6} = 14.4494$

$$\frac{(7-1)(1.5376)}{14.4494} \le \sigma^2 \le \frac{(7-1)(1.5376)}{1.23734}$$

0.64 ≤ σ^2 ≤ 7.46

c) $n = 20$ $\bar{x} = 105$ $s = 32$ $s^2 = 1024$ 90% C.I. df = 20 – 1 = 19

$\chi^2_{.95,19} = 10.11701$ $\chi^2_{.05,19} = 30.1435$

$$\frac{(20-1)(1024)}{30.1435} \le \sigma^2 \le \frac{(20-1)(1024)}{10.11701}$$

645.45 ≤ σ^2 ≤ 1923.10

d) $n = 17$ $s^2 = 18.56$ 80% C.I. df = 17 – 1 = 16

$\chi^2_{.90,16} = 9.31224$ $\chi^2_{.10,16} = 23.5418$

$$\frac{(17-1)(18.56)}{23.5418} \le \sigma^2 \le \frac{(17-1)(18.56)}{9.31224}$$

12.61 ≤ σ^2 ≤ 31.89

8.37 $n = 20$ $s = 4.3$ $s^2 = 18.49$ 98% C.I. df = 20 – 1 = 19

$\chi^2_{.99,19} = 7.63270$ $\chi^2_{.01,19} = 36.1908$

$$\frac{(20-1)(18.49)}{36.1908} \le \sigma^2 \le \frac{(20-1)(18.49)}{7.63270}$$

$\mathbf{9.71 \le \sigma^2 \le 46.03}$

Point Estimate = s^2 = 18.49

8.39 $n = 14$ $s^2 = 26{,}798{,}241.76$ 95% C.I. df = 14 – 1 = 13

Point Estimate = $s^2 = 26{,}798{,}241.76$

$\chi^2_{.975,13} = 5.00874$ $\chi^2_{.025,13} = 24.7356$

$$\frac{(14-1)(26{,}798{,}241.76)}{24.7356} \le \sigma^2 \le \frac{(14-1)(26{,}798{,}241.76)}{5.00874}$$

$\mathbf{14{,}084{,}038.51 \le \sigma^2 \le 69{,}553{,}848.45}$

8.41 a) $E = .02$ $p = .40$ 96% Confidence $z_{.02} = 2.05$

$$n = \frac{z^2 p \cdot q}{E^2} = \frac{(2.05)^2(.40)(.60)}{(.02)^2} = 2521.5$$

Sample 2522

b) $E = .04$ $p = .50$ 95% Confidence $z_{.025} = 1.96$

$$n = \frac{z^2 p \cdot q}{E^2} = \frac{(1.96)^2(.50)(.50)}{(.04)^2} = 600.25$$

Sample 601

c) $E = .05$ $p = .55$ 90% Confidence $z_{.05} = 1.645$

$$n = \frac{z^2 p \cdot q}{E^2} = \frac{(1.645)^2(.55)(.45)}{(.05)^2} = 267.9$$

Sample 268

d) $E = .01$ $p = .50$ 99% Confidence $z_{.005} = 2.575$

$$n = \frac{z^2 p \cdot q}{E^2} = \frac{(2.575)^2(.50)(.50)}{(.01)^2} = 16{,}576.6$$

Sample 16,577

8.43 $E = \$2$ $\sigma = \$12.50$ 90% Confidence $z_{.05} = 1.645$

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.645)^2(12.50)^2}{2^2} = 105.7$$

Sample 106

8.45 $p = .20$ $q = .80$ $E = .02$ 90% Confidence, $z_{.05} = 1.645$

$$n = \frac{z^2 p \cdot q}{E^2} = \frac{(1.645)^2(.20)(.80)}{(.02)^2} = 1082.41$$

Sample 1083

8.47 $E = .10$ $p = .50$ $q = .50$ 95% Confidence, $z_{.025} = 1.96$

$$n = \frac{z^2 p \cdot q}{E^2} = \frac{(1.96)^2(.50)(.50)}{(.10)^2} = 96.04$$

Sample 97

8.49 $\bar{x}$ = **12.03** (point estimate) $s = .4373$ $n = 10$ df = 9

For 90% confidence: $\alpha/2 = .05$ $t_{.05,9} = 1.833$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 12.03 \pm 1.833\frac{(.4373)}{\sqrt{10}} = 12.03 \pm .25$$

$11.78 \le \mu \le 12.28$

For 95% confidence: $\alpha/2 = .025$ $t_{.025,9} = 2.262$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 12.03 \pm 2.262\frac{(.4373)}{\sqrt{10}} = 12.03 \pm .31$$

$11.72 \le \mu \le 12.34$

For 99% confidence: $\alpha/2 = .005$ $t_{.005,9} = 3.25$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 12.03 \pm 3.25\frac{(.4373)}{\sqrt{10}} = 12.03 \pm .45$$

$11.58 \le \mu \le 12.48$

8.51 $n = 10$ $s = 7.40045$ $s^2 = 54.7667$ df = 10 – 1 = 9

90% confidence, $\alpha/2 = .05$ $1 - \alpha/2 = .95$

$\chi^2_{.95,9} = 3.32512$ $\chi^2_{.05,9} = 16.9190$

$$\frac{(10-1)(54.7667)}{16.9190} \le \sigma^2 \le \frac{(10-1)(54.7667)}{3.32512}$$

29.133 ≤ σ^2 ≤ 148.235

95% confidence, $\alpha/2 = .025$ $1 - \alpha/2 = .975$

$\chi^2_{.975,9} = 2.70039$ $\chi^2_{.025,9} = 19.0228$

$$\frac{(10-1)(54.7667)}{19.0228} \le \sigma^2 \le \frac{(10-1)(54.7667)}{2.70039}$$

25.911 ≤ σ^2 ≤ 182.529

8.53 $n = 17$ $\bar{x} = 10.765$ $s = 2.223$ df = 17 - 1 = 16

99% confidence $\alpha/2 = .005$ $t_{.005,16} = 2.921$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 10.765 \pm 2.921\frac{2.223}{\sqrt{17}} = 10.765 \pm 1.575$$

9.19 ≤ μ ≤ 12.34

8.55 $n = 17$ $s^2 = 4.941$ 99% C.I. df = 17 – 1 = 16

$\chi^2_{.995,16} = 5.14216$ $\chi^2_{.005,16} = 34.2671$

$$\frac{(17-1)(4.941)}{34.2671} \le \sigma^2 \le \frac{(17-1)(4.941)}{5.14216}$$

2.307 ≤ σ^2 ≤ 15.374

8.57 $n = 39$ $\bar{x} = 37.256$ $\sigma = 3.891$

90% confidence $z_{.05} = 1.645$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 37.256 \pm 1.645\frac{3.891}{\sqrt{39}} = 37.256 \pm 1.025$$

$36.231 \leq \mu \leq 38.281$

8.59 $n = 1,255$ $x = 714$ 95% Confidence $z_{.025} = 1.96$

$$\hat{p} = \frac{714}{1255} = .569$$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .569 \pm 1.96\sqrt{\frac{(.569)(.431)}{1,255}} = .569 \pm .027$$

$.542 \leq p \leq .596$

8.61 $n = 60$ $\bar{x} = 6.717$ $\sigma = 3.06$ $N = 300$

98% Confidence $z_{.01} = 2.33$

$$\bar{x} \pm z\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = 6.717 \pm 2.33\frac{3.06}{\sqrt{60}}\sqrt{\frac{300-60}{300-1}} =$$

6.717 ± 0.825

$5.892 \leq \mu \leq 7.542$

8.63 $n = 245$ $x = 189$ 90% Confidence, $z_{.05} = 1.645$

$$\hat{p} = \frac{x}{n} = \frac{189}{245} = .77$$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = .77 \pm 1.645\sqrt{\frac{(.77)(.23)}{245}} = .77 \pm .044$$

$$\mathbf{.726 \leq p \leq .814}$$

8.65 $n = 12$ $\bar{x} = 43.7$ $s^2 = 228$ df = 12 – 1 = 11 95% C.I.

$t_{.025,11} = 2.201$

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 43.7 \pm 2.201\frac{\sqrt{228}}{\sqrt{12}} = 43.7 \pm 9.59$$

$$\mathbf{34.11 \leq \mu \leq 53.29}$$

$\chi^2_{.99,11} = 3.05350$ $\chi^2_{.01,11} = 24.7250$

$$\frac{(12-1)(228)}{24.7250} \leq \sigma^2 \leq \frac{(12-1)(228)}{3.05350}$$

$$\mathbf{101.44 \leq \sigma^2 \leq 821.35}$$

8.67 $n = 77$ $\bar{x} = 2.48$ $\sigma = 12$

95% Confidence $z_{.025} = 1.96$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 2.48 \pm 1.96\frac{12}{\sqrt{77}} = 2.48 \pm 2.68$$

-0.20 $\le$ μ $\le$ 5.16

The point estimate is 2.48

The interval is inconclusive. It says that we are 95% confident that the average arrival time is somewhere between .20 of a minute (12 seconds) early and 5.16 minutes late. Since zero is in the interval, there is a possibility that, on average, the flights are on time.

8.69 $p = .50$ $E = .05$ 98% Confidence $z_{.01} = 2.33$

$$\frac{z^2 p\cdot q}{E^2} = \frac{(2.33)^2(.50)(.50)}{(.05)^2} = 542.89$$

Sample 543

8.71 $n = 23$ df = 23 – 1 = 22 $s = .0631455$ 90% C.I.

$\chi^2_{.95,22} = 12.33801$ $\chi^2_{.05,22} = 33.9245$

$$\frac{(23-1)(.0631455)^2}{33.9245} \le \sigma^2 \le \frac{(23-1)(.0631455)^2}{12.33801}$$

.0026 $\le$ σ^2 $\le$ .0071

8.73 $n = 1000$ $\hat{p} = .23$

80% Confidence $z_{.10} = 1.28$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\,\hat{q}}{n}} = .23 \pm 1.28\sqrt{\frac{(.23)(.77)}{1000}} = .23 \pm .017$$

$\mathbf{.213 \le p \le .247}$

8.75 The point estimate for the average length of burn of the new bulb is 2198.217 hours. Eighty-four bulbs were included in this study. A 90% confidence interval can be constructed from the information given. The error of the confidence interval is $\pm$ 27.76691. Combining this with the point estimate yields the 90% confidence interval of 2198.217 $\pm$ 27.76691 = 2170.450 $\le \mu \le$ 2225.984.

8.77 A poll of 781 American workers was taken. Of these, 506 drive their cars to work. Thus, the point estimate for the population proportion is 506/781 = .647887. A 95% confidence interval to estimate the population proportion shows that we are 95% confident that the actual value lies between .61324 and .681413. The error of this interval is $\pm$ .0340865.