Chapter 4
Probability

LEARNING OBJECTIVES

The main objective of Chapter 4 is to help you understand the basic principles of probability, thereby enabling you to:

1. Describe what probability is and when one would use it
2. Differentiate among three methods of assigning probabilities: the classical method, relative frequency of occurrence, and subjective probability
3. Deconstruct the elements of probability by defining experiments, sample spaces, and events, classifying events as mutually exclusive, collectively exhaustive, complementary, or independent, and counting possibilities
4. Compare marginal, union, joint, and conditional probabilities by defining each one.
5. Calculate probabilities using the general law of addition, along with a joint probability table, the complement of a union, or the special law of addition if necessary
6. Calculate joint probabilities of both independent and dependent events using the general and special laws of multiplication
7. Calculate conditional probabilities with various forms of the law of conditional probability, and use them to determine if two events are independent.
8. Calculate conditional probabilities using Bayes' rule

CHAPTER OUTLINE

4.1 Introduction to Probability

4.2 Methods of Assigning Probabilities
- Classical Method of Assigning Probabilities
- Relative Frequency of Occurrence
- Subjective Probability

4.3 Structure of Probability
- Experiment
- Event
- Elementary Events
- Sample Space
- Unions and Intersections
- Mutually Exclusive Events
- Independent Events
- Collectively Exhaustive Events
- Complimentary Events
- Counting the Possibilities
  - The *mn* Counting Rule
  - Sampling from a Population with Replacement
  - Combinations: Sampling from a Population Without Replacement

4.4 Marginal, Union, Joint, and Conditional Probabilities

4.5 Addition Laws
- Joint Probability Tables
  - Complement of a Union
- Special Law of Addition

4.6 Multiplication Laws
- General Law of Multiplication
- Special Law of Multiplication

4.7 Conditional Probability
- Independent Events

4.8 Revision of Probabilities: Bayes' Rule

KEY TERMS

A Priori
Bayes' Rule
Classical Method of Assigning Probabilities
Collectively Exhaustive Events
Combinations
Complement of a Union
Complement
Conditional Probability
Cross-Tabulation Table
Elementary Events
Event
Experiment
Independent Events
Intersection
Joint Probability
Joint Probability Table
Marginal Probability
mn Counting Rule
Mutually Exclusive Events
Relative Frequency of Occurrence
Sample Space
Set Notation
Subjective Probability
Union
Union Probability

STUDY QUESTIONS

1. The ________________ method of assigning probabilities relies on the insight or feelings of the person determining the probabilities.

2. If probabilities are determined "a priori" to an experiment using rules and laws, then the ________________ method of assigning probabilities is being used.

3. The range of possibilities for probability values is from ________________ to ________________.

4. Suppose a technician keeps track of all defects in raw materials for a single day and uses this information to determine the probability of finding a defect in raw materials the next day. She is using the ________________ method of assigning probabilities.

5. The outcome of an experiment is called a(n) ________________. If these outcomes cannot be decomposed further, then they are referred to as ________________ ________________.

6. A computer hardware retailer allows you to order your own computer monitor. The store carries five different brands of monitors. Each brand comes in 14", 15" or 17" models. In addition, you can purchase either the deluxe model or the regular model in each brand and in each size. How many different types of monitors are available considering all the factors? ______________ You probably used the _______ rule to solve this.

7. Suppose you are playing the Lotto game and you are trying to “pick” three numbers. For each of the three numbers, any of the digits 0 through 9 are possible (with replacement). How many different sets of numbers are available? ______________

8. A population consists of the odd numbers between 1 and 9 inclusive. If a researcher randomly samples numbers from the population three at a time, the sample space is ________________. Using combinations, how could we have determined ahead of time how many elementary events would be in the sample space? ________

9. Let A = {2,3,5,6,7,9} and B = {1,3,4,6,7,9}
   $A \perp B$ = ________________ and A _ B = ________________.

10. If the occurrence of one event does not affect the occurrence of the other event, then the events are said to be ________________.

11. The outcome of the roll of one die is said to be ________________ of the outcome of the roll of another die.

12. The event of rolling a three on a die and the event of rolling an even number on the same roll with the same die are ________________.

13. If the probability of the intersection of two events is zero, then the events are said to be _______________.

14. If three objects are selected from a bin, one at a time with replacement, the outcomes of each selection are _______________.

15. Suppose a population consists of a manufacturing facility's 1600 workers. Suppose an experiment is conducted in which a worker is randomly selected. If an event is the selection of a worker over 40 years old, then the event of selecting a worker 40 years or younger is called the _______________ of the first event.

16. The probability of selecting X given that Y has occurred is called a _______________ probability.

17. The probability of X is called a _______________ probability.

18. The probability of X or Y occurring is called a _______________ probability.

19. The probability of X and Y occurring is called a _______________ probability.

20. Only one of the four types of probability does not use the total possible outcomes in the denominator when calculating the probability. This type of probability is called _______________ probability.

21. If the $P(A|B) = P(A)$, then the events A, B are _______________ events.

22. If the $P(X) = .53$, the $P(Y) = .12$, and the $P(X \_ Y) = .07$, then $P(X \perp Y)$ = _______________.

23. If the $P(X) = .26$, the $P(Y) = .31$, and X, Y are mutually exclusive, then $P(X \perp Y)$ = _______________.

24. In a company, 47% of the employees wear glasses, 60% of the employees are women, and 28% of the employees are women and wear glasses. Complete the probability matrix below for this problem.

		Wear Glasses? Yes	No
Gender	Men		
	Women		

25. Suppose that in another company, 40% of the workers are part time and 80% of the part time workers are men. The probability of randomly selecting a company worker who is both part time and a man is _______________.

26. The probability of tossing three coins in a row and getting all tails is _______________. This is an application of the _______________ law of multiplication because each toss is _______________.

27. Suppose 70% of all cars purchased in America are U.S.A. made and that 18% of all cars purchased in America are both U.S.A. made and are red. The probability that a randomly selected car purchased in America is red given that it is U.S.A. made is _______________.

Use the matrix below to answer questions 28-37:

	C	D	
A	.35	.31	.66
B	.14	.20	.34
	.49	.51	1.00

28. The probability of A and C occurring is __________.

29. The probability of A or D occurring is __________.

30. The probability of D occurring is __________.

31. The probability of B occurring given C is __________.

32. The probability of B and D occurring is __________.

33. The probability of C and D occurring is __________.

34. The probability of C or D occurring is __________.

35. The probability of C occurring given D is __________.

36. The probability of C occurring given A is __________.

37. The probability of C or B occurring is __________.

38. Suppose 42% of all people in a county have characteristic X. Suppose 17% of all people in this county have characteristic X and characteristic Y. If a person is randomly selected from the county who is known to have characteristic X, then the probability that they have characteristic Y is _________________.

39. Suppose 22% of all parts produced at a plant have flaw X and 37% have flaw Y. In addition, suppose 53% of the parts with flaw X have flaw Y. If a part is randomly selected, the probability that it has flaw X or flaw Y is ________________.

40. Another name for revision of probabilities is _______________.

41. Suppose the prior probabilities of A and B are .57 and .43 respectively. Suppose that $P(E|A) = .24$ and $P(E|B) = .56$. If E is known to have occurred, then the revised probability of A occurring is _______________ and of B occurring is _______________.

ANSWERS TO STUDY QUESTIONS

1. Subjective
2. Classical
3. 0, 1
4. Relative Frequency
5. Event, Elementary Events
6. 30, *mn* counting rule
7. 10^3 = 1000 numbers
8. {(1,3,5), (1,3,7), (1,3,9), (1,5,7), (1,5,9), (1,7,9), (3,5,7), (3,5,9), (3,7,9), (5,7,9)}, ${}_5C_3$ = 10
9. {1,2,3,4,5,6,7,9}, {3,6,7,9}
10. Independent
11. Independent
12. Mutually Exclusive
13. Mutually Exclusive
14. Independent
15. Complement
16. Conditional
17. Marginal
18. Union
19. Joint or Intersection
20. Conditional
21. Independent
22. .58
23. .57
24. Wear Glasses

	Yes	No	
Men	.19	.21	.40
Women	.28	.32	.60
	.47	.53	1.00

25. .32
26. 1/8 = .125, Special, Independent
27. .2571
28. .35
29. .86
30. .51
31. .2857
32. .20
33. .0000
34. 1.00
35. .0000
36. .5303
37. .69
38. .4048
39. .4734
40. Bayes' Rule
41. .3623, .6377

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 4

4.1 Enumeration of the six parts: D_1, D_2, D_3, A_4, A_5, A_6

D = Defective part
A = Acceptable part

Sample Space:

$D_1 D_2$, $D_2 D_3$, $D_3 A_5$
$D_1 D_3$, $D_2 A_4$, $D_3 A_6$
$D_1 A_4$, $D_2 A_5$, $A_4 A_5$
$D_1 A_5$, $D_2 A_6$, $A_4 A_6$
$D_1 A_6$, $D_3 A_4$, $A_5 A_6$

There are **15** members of the sample space

The probability of selecting exactly one defect out of two is:

9/15 = **.60**

4.3 If A = {2, 6, 12, 24} and the population is the positive even numbers through 30,

A' = **{4, 8, 10, 14, 16, 18, 20, 22, 26, 28, 30}**

4.5 Enumeration of the six parts: D_1, D_2, A_1, A_2, A_3, A_4

D = Defective part
A = Acceptable part

Sample Space:

$D_1\ D_2\ A_1$, $D_1\ D_2\ A_2$, $D_1\ D_2\ A_3$,
$D_1\ D_2\ A_4$, $D_1\ A_1\ A_2$, $D_1\ A_1\ A_3$,
$D_1\ A_1\ A_4$, $D_1\ A_2\ A_3$, $D_1\ A_2\ A_4$,
$D_1\ A_3\ A_4$, $D_2\ A_1\ A_2$, $D_2\ A_1\ A_3$,
$D_2\ A_1\ A_4$, $D_2\ A_2\ A_3$, $D_2\ A_2\ A_4$,
$D_2\ A_3\ A_4$, $A_1\ A_2\ A_3$, $A_1\ A_2\ A_4$,
$A_1\ A_3\ A_4$, $A_2\ A_3\ A_4$

Combinations are used to counting the sample space because sampling is done without replacement.

$${}_6C_3 = \frac{6!}{3!3!} = \mathbf{20}$$

Probability that one of three is defective is:

12/20 = 3/5 **.60**

There are 20 members of the sample space and 12 of them have exactly 1 defective part.

4.7 $${}_{20}C_6 = \frac{20!}{6!14!} = \mathbf{38{,}760}$$

It is assumed here that 6 different (without replacement) employees are to be selected.

4.9

	D	E	F	
A	5	8	12	25
B	10	6	4	20
C	8	2	5	15
	23	16	21	60

a) $P(A \cup D) = P(A) + P(D) - P(A \cap D) =$ 25/60 + 23/60 - 5/60 = 43/60 = **.7167**

b) $P(E \cup B) = P(E) + P(B) - P(E \cap B) =$ 16/60 + 20/60 - 6/60 = 30/60 = **.5000**

c) $P(D \cup E) = P(D) + P(E) =$ 23/60 + 16/60 = 39/60 = **.6500**

d) $P(C \cup F) = P(C) + P(F) - P(C \cap F) =$ 15/60 + 21/60 - 5/60 = 31/60 = **.5167**

4.11 O = buyer uses on-line search engine to buy car
S = buyer skips test drive

$P(O) = .65$ $P(S) = .11$ $P(O \cap S) = .07$

a.) $P(O \cup S) = P(O) + P(S) - P(O \cap S) = .65 + .11 - .07 =$ **.69**

b.) $P(\text{not } O \cup S) = P(\text{not } O) + P(S) - P(\text{not } O \cap S)$
.
but $P(\text{not } O) = 1 - P(O) = 1 - .65 = .35$
and $P(\text{not } O \cap S) = P(S) - P(O \cap S) = .11 - .07 = .04$
Thus, $P(\text{not } O \cup S) = P(\text{not } O) + P(S) - P(\text{not } O \cap S) = .35 + .11 - .04 =$ **.42**

c.) $P(O \cup \text{not } S) = P(O) + P(\text{not } S) - P(O \cap \text{not } S)$
but $P(\text{not } S) = 1 - P(S) = 1 - .11 = .89$
and $P(O \cap \text{not } S) = P(O) - P(O \cap S) = .65 - .07 = .58$
Thus, $P(O \cup \text{not } S) = P(O) + P(\text{not } S) - P(O \cap \text{not } S) = .65 + .89 - .58 =$ **.96**

The joint probability table:

		S		
		Yes	No	
O	Yes	.07	.58	.65
	No	.04	.31	.35
		.11	.89	1.00

4.13 Let D = have digital cable TV
Let M = have multiple TV sets
$P(\text{D}) = .52$, $P(\text{M}) = .84$, $P(\text{D} \cap \text{M}) = .45$

a) $P(\text{D} \cup \text{M}) = P(\text{D}) + P(\text{M}) - P(\text{D} \cap \text{M}) = .52 + .84 - .45 = \mathbf{.91}$

b) $P(\text{ND} \cup \text{M}) = P(\text{ND}) + P(\text{M}) - P(\text{ND} \cap \text{M})$
but $P(\text{ND}) = 1 - P(\text{D}) = 1 - .52 = .48$
$P(\text{ND} \cap \text{M}) = P(\text{M}) - P(\text{D} \cap \text{M}) = .84 - .45 = .39$
$P(\text{ND} \cup \text{M}) = P(\text{ND}) + P(\text{M}) - P(\text{ND} \cap \text{M}) = .48 + .84 - .39 = \mathbf{.93}$

c) $P(\text{D} \cup \text{NM}) = P(\text{D}) + P(\text{NM}) - P(\text{D} \cap \text{NM})$
but $P(\text{NM}) = 1 - P(\text{M}) = 1 - .84 = .16$
$P(\text{D} \cap \text{NM}) = P(\text{D}) - P(\text{D} \cap \text{M}) = .52 - .45 = .07$
$P(\text{D} \cup \text{NM}) = P(\text{D}) + P(\text{NM}) - P(\text{D} \cap \text{NM}) = .52 + .16 - .07 = \mathbf{.61}$

d) $P(\text{ND} \cup \text{NM}) = P(\text{ND}) + P(\text{NM}) - P(\text{ND} \cap \text{NM})$
but $P(\text{ND} \cap \text{NM}) = P(\text{NM}) - P(\text{D} \cup \text{NM}) = .16 - .07 = .09$
$P(\text{ND} \cup \text{NM}) = P(\text{ND}) + P(\text{NM}) - P(\text{ND} \cap \text{NM}) = .48 + .16 - .09 = \mathbf{.55}$

The joint probability table:

		M: Yes	M: No	
D	Yes	.45	.07	.52
	No	.39	.09	.48
		.84	.16	1.00

4.15

	C	D	E	F	
A	5	11	16	8	40
B	2	3	5	7	17
	7	14	21	15	57

a) $P(\text{A} \cap \text{E})$ = 16/57 = **.2807**

b) $P(\text{D} \cap \text{B})$ = 3/57 = **.0526**

c) $P(\text{D} \cap \text{E})$ = **.0000**

d) $P(\text{A} \cap \text{B})$ = **.0000**

4.17 Let D = Defective part

a) (without replacement)

$$P(\text{D}_1 \cap \text{D}_2) = P(\text{D}_1) \cdot P(\text{D}_2 \mid \text{D}_1) = \frac{6}{50} \cdot \frac{5}{49} = \frac{30}{2450} = \mathbf{.0122}$$

b) (with replacement)

$$P(\text{D}_1 \cap \text{D}_2) = P(\text{D}_1) \cdot P(\text{D}_2) = \frac{6}{50} \cdot \frac{6}{50} = \frac{36}{2500} = \mathbf{.0144}$$

4.19 Let S = stockholder   Let C = college

$P(\text{S}) = .43$ $P(\text{C}) = .37$ $P(\text{C} \mid \text{S}) = .75$

a) $P(\text{S} \cap \text{C}) = P(\text{S})\cdot P(\text{C} \mid \text{S}) = (.43)(.75) = \mathbf{.3225}$

b) $P(\text{NS} \cap \text{C}) = P(\text{C}) - P(\text{S} \cap \text{C}) = .37 - .3225 = \mathbf{.0475}$

c) $P(\text{S} \cap \text{NC}) = P(\text{S}) - P(\text{S} \cap \text{C}) = .43 - .3225 = \mathbf{.1075}$

d) $P(\text{NS} \cap \text{NC}) = P(\text{NS}) - P(\text{NS} \cap \text{C})$
but $P(\text{NS}) = 1 - P(\text{S}) = 1 - .43 = .57$
$P(\text{NS} \cap \text{NC}) = P(\text{NS}) - P(\text{NS} \cap \text{C}) = .57 - .0475 = \mathbf{.5225}$

The joint probability table:

		C Yes	C No	
S	Yes	.3225	.1075	.43
	No	.0475	.5225	.57
		.37	.63	1.00

4.21 Let S = safety   Let A = age

$P(\text{S}) = .30$ $P(\text{A}) = .39$ $P(\text{A} \mid \text{S}) = .87$

a) $P(\text{S} \cap \text{NA}) = P(\text{S})\cdot P(\text{NA} \mid \text{S})$

but $P(\text{NA} \mid \text{S}) = 1 - P(\text{A} \mid \text{S}) = 1 - .87 = .13$
$P(\text{S} \cap \text{NA}) = (.30)(.13) = \mathbf{.039}$

b) $P(\text{NS} \cap \text{NA}) = 1 - P(\text{S} \cup \text{A}) = 1 - [P(\text{S}) + P(\text{A}) - P(\text{S} \cap \text{A})]$
but $P(\text{S} \cap \text{A}) = P(\text{S}) \cdot P(\text{A} \mid \text{S}) = (.30)(.87) = .261$
$P(\text{NS} \cap \text{NA}) = 1 - (.30 + .39 - .261) = \mathbf{.571}$

c) $P(\text{NS} \cap \text{A}) = P(\text{NS}) - P(\text{NS} \cap \text{NA})$
but $P(\text{NS}) = 1 - \text{P(S)} = 1 - .30 = .70$
$P(\text{NS} \cap \text{A}) = .70 - 571 = \mathbf{.129}$

The joint probability table:

		A Yes	A No	
S	Yes	.261	.039	.30
	No	.129	.571	.70
		.39	.61	1.00

4.23

	E	F	G	
A	15	12	8	35
B	11	17	19	47
C	21	32	27	80
D	18	13	12	43
	65	74	66	205

a) $P(\text{G} \mid \text{A}) = 8/35 =$ **.2286**

b) $P(\text{B} \mid \text{F}) = 17/74 =$ **.2297**

c) $P(\text{C} \mid \text{E}) = 21/65 =$ **.3231**

d) $P(\text{E} \mid \text{G}) =$ **.0000**

4.25

		Calculator Yes	Calculator No	
Computer	Yes	46	3	49
	No	11	15	26
		57	18	75

Select a category from each variable and test

$$P(\text{V}_1 \mid \text{V}_2) = P(\text{V}_1).$$

For example, $P(\text{Yes Computer} \mid \text{Yes Calculator}) = \text{P(Yes Computer)}$?

$$\frac{46}{57} = \frac{49}{75}?$$

$$.8070 \neq .6533$$

Since this is one example that the conditional does not equal the marginal in is matrix, the variable, computer, is not independent of the variable, calculator.

4.27 Let E = Economy Let Q = Qualified

$P(E) = .46$ $P(Q) = .37$ $P(E \cap Q) = .15$

a) $P(E|Q) = P(E \cap Q)/P(Q) = .15/.37 =$ **.4054**

b) $P(Q|E) = P(E \cap Q)/P(E) = .15/.46 =$ **.3261**

c) $P(Q|NE) = P(Q \cap NE)/P(NE)$

but $P(Q \cap NE) = P(Q) - P(Q \cap E) = .37 - .15 = .22$

$P(NE) = 1 - P(E) = 1 - .46 = .54$

$P(Q|NE) = .22/.54 =$ **.4074**

d) $P(NE \cap NQ) = 1 - P(E \cup Q) = 1 - [P(E) + P(Q) - P(E \cap Q)]$

$= 1 - [.46 + .37 - .15] = 1 - (.68) =$ **.32**

The joint probability table:

		Q: Yes	Q: No	
E	Yes	.15	.31	.46
	No	.22	.32	.54
		.37	.63	1.00

4.29 Let S = believe they need to use social media
Let E = started career at entry-level position
$P(S) = .77 \quad P(E) = .80 \quad P(S|E) = .83$

a) $P(S \cap E) = P(E) \cdot P(S|E) = (.80)(.83) =$ **.664**

b) $P(S \cup E) = P(S) + P(E) - P(S \cap E) = .77 + .80 - .664 =$ **.906**

c) $P(NS|E) = P(NS \cap E)/P(E)$
but $P(NS \cap E) = P(E) - P(S \cap E) = .80 - .664 = .136$
$P(NS|E) = .136/.80 =$ **.17**

d) $P(S|NE) = P(S \cap NE)/P(NE)$
but $P(NE) = 1 - P(E) = 1 - .80 = .20$
$P(S \cap NE) = P(S) - P(S \cap E) = .77 - .664 = .106$
$P(S|NE) = .106/.20 =$ **.53**

e) $P(NE|NS) = P(NE \cap NS)/P(NS)$
but $P(NE \cap NS) = P(NS) - P(E \cap NS)$
and $P(NS) = 1 - P(S) = 1 - .77 = .23$
so $P(NE \cap NS) = .23 - .136 = .094$
$P(NE|NS) = .094/.23 =$ **.4087**

The joint probability table:

		E: Yes	E: No	
S	Yes	.664	.106	.77
	No	.136	.094	.23
		.80	.20	1.00

4.31 Let A = product produced on Machine A
B = product produces on Machine B
C = product produced on Machine C
D = defective product

P(A) = .10 P(B) = .40 P(C) = .50
P(D|A) = .05 P(D|B) = .12 P(D|C) = .08

Event	Prior	Conditional	Joint	Revised
	$P(E_i)$	$P(D\|E_i)$	$P(D \cap E_i)$	
A	.10	.05	.005	.005/.093=**.0538**
B	.40	.12	.048	.048/.093=**.5161**
C	.50	.08	.040	.040/.093=**.4301**
			P(D)=.093	

Revise: P(A|D) = .005/.093 **= .0538**

P(B|D) = .048/.093 = **.5161**

P(C|D) = .040/.093 = **.4301**

4.33 Let T = lawn treated by Tri-state
G = lawn treated by Green Chem
V = very healthy lawn
N = not very healthy lawn

P(T) = .72 P(G) = .28 P(V|T) = .30 P(V|G) = .20

Event	Prior	Conditional	Joint	Revised
	$P(E_i)$	$P(V\|E_i)$	$P(V \cap E_i)$	$P(E_i\|V)$
A	.72	.30	.216	.216/.272=**.7941**
B	.28	.20	.056	.056/.272=**.2059**
			P(V)=.272	

Revised: P(T|V) = .216/.272 = **.7941**

P(G|V) = .056/.272 = **.2059**

4.35

		Variable 1		
		D	E	
	A	10	20	30
Variable 2	B	15	5	20
	C	30	15	45
		55	40	95

a) P(E) = 40/95 = **.42105**

b) P(B ∪ D) = P(B) + P(D) - P(B ∩ D)

= 20/95 + 55/95 - 15/95 = 60/95 = **.63158**

c) P(A ∩ E) = 20/95 = **.21053**

d) P(B|E) = 5/40 = **.1250**

e) P(A ∪ B) = P(A) + P(B) = 30/95 + 20/95 =

50/95 = **.52632**

f) P(B ∩ C) = .0000 (mutually exclusive)

g) P(D|C) = 30/45 = **.66667**

h) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.0000}{20/95} =$ **.0000 (A and B are mutually exclusive)**

i) P(A) = P(A|D)??

Does 30/95 = 10/95 ??

Since, .31579 ≠ .18182, Variables 1 and 2 are <u>not</u> independent.

4.37

	D	E	F	G	
A	3	9	7	12	31
B	8	4	6	4	22
C	10	5	3	7	25

		Age(years)					
		<35	35-44	45-54	55-64	>65	
Gender	Male	.11	.20	.19	.12	.16	.78
	Female	.07	.08	.04	.02	.01	.22
		.18	.28	.23	.14	.17	1.00

a) P(35-44) = **.28**

b) P(Woman ∩ 45-54) = **.04**

c) P(Man ∪ 35-44) = P(Man) + P(35-44) - P(Man ∩ 35-44) = .78 + .28 - .20 = **.86**

d) P(<35 ∪ 55-64) = P(<35) + P(55-64) = .18 + .14 = **.32**

e) P(Woman | 45-54) = P(Woman ∩ 45-54)/P(45-54) = .04/.23= **.1739**

f) P(not W ∩ not 55-64) = .11 + .20 + .19 + .16 = **.66**

4.39 Let R = retirement Let L = life insurance

$P(\text{R}) = .42$ $P(\text{L}) = .61$ $P(\text{R} \cap \text{L}) = .33$

a) $P(\text{R}|\text{L}) = P(\text{R} \cap \text{L})/P(\text{L}) = .33/.61 = \mathbf{.5410}$

b) $P(\text{L}|\text{R}) = P(\text{R} \cap \text{L})/P(\text{R}) = .33/.42 = \mathbf{.7857}$

c) $P(\text{L} \cup \text{R}) = P(\text{L}) + P(\text{R}) - P(\text{L} \cap \text{R}) = .61 + .42 - .33 = \mathbf{.70}$

d) $P(\text{R} \cap \text{NL}) = P(\text{R}) - P(\text{R} \cap \text{L}) = .42 - .33 = \mathbf{.09}$

e) $P(\text{NL}|\text{R}) = P(\text{NL} \cap \text{R})/\text{P}(\text{R}) = .09/.42 = \mathbf{.2143}$

The joint probability table:

		L: Yes	L: No	
R	Yes	.33	.09	.42
	No	.28	.30	.58
		.61	.39	1.00

4.41 Let M = MasterCard A = American Express V = Visa

$P(\text{M}) = .30$ $P(\text{A}) = .20$ $P(\text{V}) = .25$
$P(\text{M} \cap \text{A}) = .08$ $P(\text{V} \cap \text{M}) = .12$ $P(\text{A} \cap \text{V}) = .06$

a) $P(\text{V} \cup \text{A}) = P(\text{V}) + P(\text{A}) - P(\text{V} \cap \text{A}) = .25 + .20 - .06 = \mathbf{.39}$

b) $P(\text{V}|\text{M}) = P(\text{V} \cap \text{M})/P(\text{M}) = .12/.30 = \mathbf{.40}$

c) $P(\text{M}|\text{V}) = P(\text{V} \cap \text{M})/P(\text{V}) = .12/.25 = \mathbf{.48}$

d) $P(\text{V}) = P(\text{V}|\text{M})$??

$.25 \neq .40$

Possession of Visa is not independent of possession of MasterCard

e) American Express is not mutually exclusive of Visa
because $\text{P}(\text{A} \cap \text{V}) \neq .0000$

4.43 Let M = expect to save more
R = expect to reduce debt
NM = don't expect to save more
NR = don't expect to reduce debt

$P(\text{M}) = .43$ $P(\text{R}) = .45$ $P(\text{R}|\text{M}) = .81$
$P(\text{NR}|\text{M}) = 1 - P(\text{R}|\text{M}) = 1 - .81 = .19$
$P(\text{NM}) = 1 - P(\text{M}) = 1 - .43 = .57$
$P(\text{NR}) = 1 - P(\text{R}) = 1 - .45 = .55$

a) $P(\text{M} \cap \text{R}) = P(\text{M})\cdot P(\text{R}|\text{M}) = (.43)(.81) = \mathbf{.3483}$

b) $P(\text{M} \cup \text{R}) = P(\text{M}) + P(\text{R}) - P(\text{M} \cap \text{R}) = .43 + .45 - .3483 = \mathbf{.5317}$

c) $P(\text{M} \cap \text{NR}) = P(\text{M})\cdot P(\text{NR}|\text{M}) = (.43)(.19) = \mathbf{.0817}$

d) $P(\text{NM}|\text{R}) = P(\text{NM} \cap \text{R})/P(\text{R})$
but $P(\text{NM} \cap \text{R}) = P(\text{R}) - P(\text{M} \cap \text{R}) = .45 - .3483 = .1017$
$P(\text{NM}|\text{R}) = .1017/.45 = \mathbf{.226}$

Joint probability table for problem 4.43:

		Reduce		
		Yes	No	
Save	Yes	.3483	.0817	.43
	No	.1017	.4683	.57
		.45	.55	1.00

4.45 Let Q = keep quiet when they see co-worker misconduct
Let C = call in sick when they are well

P(Q) = .35 P(NQ) = 1 - .35 = .65 $P(C|Q)$ = .75 $P(Q|C)$ = .40

a) $P(C \cap Q) = P(Q) \cdot P(C|Q)$ = (.35)(.75) = **.2625**

b) $P(Q \cup C) = P(Q) + P(C) - P(C \cap Q)$

but P(C) must be solved for:
$P(C \cap Q) = P(C) \cdot P(Q|C)$

.2625 = P(C) (.40)

Therefore, P(C) = .2625/.40 = .65625
and $P(Q \cup C)$ = .35 + .65626 - .2625 = **.74375**

c) $P(NQ|C) = P(NQ \cap C)/P(C)$
but $P(NQ \cap C) = P(C) - P(C \cap Q)$ = .65625 - .2625 = ..39375
Therefore, $P(NQ|C) = P(NQ \cap C)/P(C)$ = .39375/.65626 = **.60**

d) $P(NQ \cap NC) = 1 - P(Q \cup C)$ = 1 - .74375 = **.25625**

e) $P(Q \cap NC) = P(Q) - P(Q \cap C)$ = .35 - .2625 = **.0875**

Joint probability table for problem 4.45:

		C: Y	C: N	
Q	Y	.2625	.0875	.35
	N	.39375	.25625	.65
		.65625	.34375	1.00

4.47 Let R = retention Let P = process improvement

$P(\text{R}) = .56 \quad P(\text{P} \cap \text{R}) = .36 \quad P(\text{R}|\text{P}) = .90$

a) $P(\text{R} \cap \text{NP}) = P(\text{R}) - P(\text{P} \cap \text{R}) = .56 - .36 =$ **.20**

b) $P(\text{P}|\text{R}) = P(\text{P} \cap \text{R})/P(\text{R}) = .36/.56 =$ **.6429**

c) $P(\text{P})$ = ??
Solve $P(\text{R}|\text{P}) = P(\text{R} \cap \text{P})/P(\text{P})$ for $P(\text{P})$:
$P(\text{P}) = P(\text{R} \cap \text{P})/P(\text{R}|\text{P}) = .36/.90 =$ **.40**

d) $P(\text{R} \cup \text{P}) = P(\text{R}) + P(\text{P}) - P(\text{R} \cap \text{P}) =$

$.56 + .40 - .36 =$ **.60**

e) $P(\text{NR} \cap \text{NP}) = 1 - P(\text{R} \cup \text{P}) = 1 - .60 =$ **.40**

f) $P(\text{R}|\text{NP}) = P(\text{R} \cap \text{NP})/P(\text{NP})$

but $P(\text{NP}) = 1 - P(\text{P}) = 1 - .40 = .60$

$P(\text{R}|\text{NP}) = .20/.60 =$ **.3333**

		P: Y	P: N	
R	Y	.36	.20	.56
	N	.04	.40	.44
		.40	.60	1.00

Note: In constructing the matrix, we are given $P(\text{R}) = .56$, $P(\text{P} \cap \text{R}) = .36$, and $P(\text{R}|\text{P}) = .90$. That is, only one marginal probability is given.
From $P(\text{R})$, we can get $P(\text{NR})$ by taking 1 - .56 = .44.
However, only these two marginal values can be computed directly.
To solve for $P(\text{P})$, using what is given, since we know that 90% of P lies in the intersection and that the intersection is .36, we can set up an equation to solve for P:
.90P = .36
Solving for P = .40.

4.49 Let F = Flexible Work Let V = Gives time off for Volunteerism

$P(F) = .41$ $P(V|NF) = .10$ $P(V|F) = .60$
from this, $P(NF) = 1 - .41 = .59$

a) $P(F \cup V) = P(F) + P(V) - P(F \cap V)$
$P(F) = .41$ and $P(F \cap V) = P(F) \cdot P(V|F) = (.41)(.60) = .246$
Find $P(V)$ by using $P(V) = P(F \cap V) + P(NF \cap V)$
but $P(NF \cap V) = P(NF) \cdot P(V|NF) = (.59)(.10) = .059$
so, $P(V) = P(F \cap V) + P(NF \cap V) = .246 + .059 = .305$
and $P(F \cup V) = P(F) + P(V) - P(F \cap V) = .41 + .305 - .246 = \mathbf{.469}$

b) $P(F \cap NV) = P(F) - P(F \cap V) = .41 - .246 = \mathbf{.164}$

c) $P(F|NV) = P(F \cap NV)/P(NV)$
$P(F \cap NV) = .164$

$P(NV) = 1 – P(V) = 1 - .305 = .695.$
$P(F|NV) = P(F \cap NV)/P(NV) = .164/.695 = \mathbf{.2360}$

d) $P(NF|V) = P(NF \cap V)/P(V) = .059/.305 = \mathbf{.1934}$

e) $P(NF \cup NV) = P(NF) + P(NV) - P(NF \cap NV)$
$P(NF) = .59$ $P(NV) = .695$
Solve for $P(NF \cap NV) = P(NV) – P(F \cap NV) = .695 - .164 = .531$
$P(NF \cup NV) = P(NF) + P(NV) - P(NF \cap NV) = .59 + .695 - .531 = \mathbf{.754}$

Joint probability table for problem 4.49:

		V: Y	V: N	
F	Y	.246	.164	.41
	N	.059	.531	.59
		.305	.695	1.00

4.51 Let R = regulations T = tax burden

$P(\text{R}) = .30$ $P(\text{T}) = .35$ $P(\text{T}|\text{R}) = .71$

a) $P(\text{R} \cap \text{T}) = P(\text{R})\cdot P(\text{T}|\text{R}) = (.30)(.71) =$ **.2130**

b) $P(\text{R} \cup \text{T}) = P(\text{R}) + P(\text{T}) - P(\text{R} \cap \text{T}) =$

$.30 + .35 - .2130 =$ **.4370**

c) $P(\text{R} \cup \text{T}) - P(\text{R} \cap \text{T}) = .4370 - .2130 =$ **.2240**

d) $P(\text{R}|\text{T}) = P(\text{R} \cap \text{T})/P(\text{T}) = .2130/.35 =$ **.6086**

e) $\text{P}(\text{NR}|\text{T}) = 1 - P(\text{R}|\text{T}) = 1 - .6086 =$ **.3914**

f) $P(\text{NR}|\text{NT}) = P(\text{NR} \cap \text{NT})/P(\text{NT}) = [1 - P(\text{R} \cup \text{T})]/P(\text{NT}) =$

$(1 - .4370)/.65 =$ **.8662**

Joint probability table for problem 4.51:

		T: Y	T: N	
R	Y	.213	.087	.30
	N	.137	.563	.70
		.35	.65	1.00

4.53 Let B = Believe plastic shopping bags should be Banned
Let R = Recycle aluminum cans

$P(\text{B}) = .54$ $P(\text{R} \cap \text{B}) = .41$ $P(\text{R} \mid \text{NB}) = .60$

a) $P(\text{R} \cap \text{NB}) = P(\text{NB} \cap \text{R}) = P(\text{NB}) \cdot P(\text{R} \mid \text{NB})$
but $P(\text{NB}) = 1 - P(\text{B}) = 1 - .54 = .46$ and $P(\text{R} \mid \text{NB}) = .60$.
$P(\text{R} \cap \text{NB}) = P(\text{NB} \cap \text{R}) = P(\text{NB}) \cdot P(\text{R} \mid \text{NB}) = (.46)(.60) =$ **.2760**.

b) $P(\text{R}) = P(\text{R} \cap \text{B}) + P(\text{R} \cap \text{NB}) = .41 + .2760 =$ **.6860**.

c) $P(\text{R} \cup \text{B}) = P(\text{R}) + P(\text{B}) - P(\text{R} \cap \text{B}) = .6860 + .54 - .41 =$ **.8160**

d) $P(\text{NR} \cup \text{NB}) = P(\text{NR}) + P(\text{NB}) - P(\text{NR} \cap \text{NB})$
But $P(\text{NB}) = .46$, $P(\text{NR}) = 1 - P(\text{R}) = 1 - .6860 = .3140$, and
$P(\text{NR} \cap \text{NB}) = 1 - P(\text{R} \cup \text{B}) = 1 - .8160 = .1840$, therefore
$P(\text{NR} \cup \text{NB}) = .3140 + .46 - .1840 =$ **.5900**

e) $P(\text{NB} \mid \text{R}) = P(\text{NB} \cap \text{R}) / P(\text{R}) =$
$P(\text{NB} \cap \text{R}) = P(\text{R} \cap \text{NB}) = .2760$ and $P(\text{R}) = .6860$, therefore
$P(\text{NB} \mid \text{R}) = P(\text{NB} \cap \text{R}) / P(\text{R}) = .2760/.6860 =$ **.4023**.

Initial joint probability table for problem 4.53:

		B		
		Y	N	
R	Y	.41		
	N			
		.54	.46	1.00

Multiply .60 by the .46 to get the intersection of R and not B = .2760.
Fill in the rest of the cells.

Final joint probability table for problem 4.53:

		B		
		Y	N	
R	Y	.41	.276	.686
	N	.13	.184	.314
		.54	.46	1.000