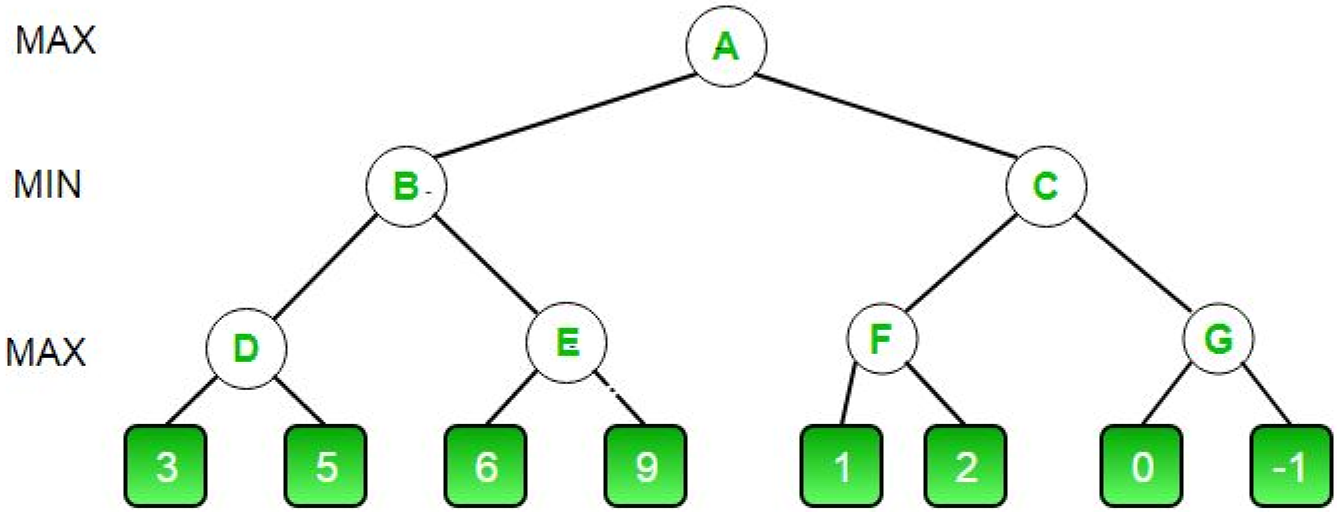
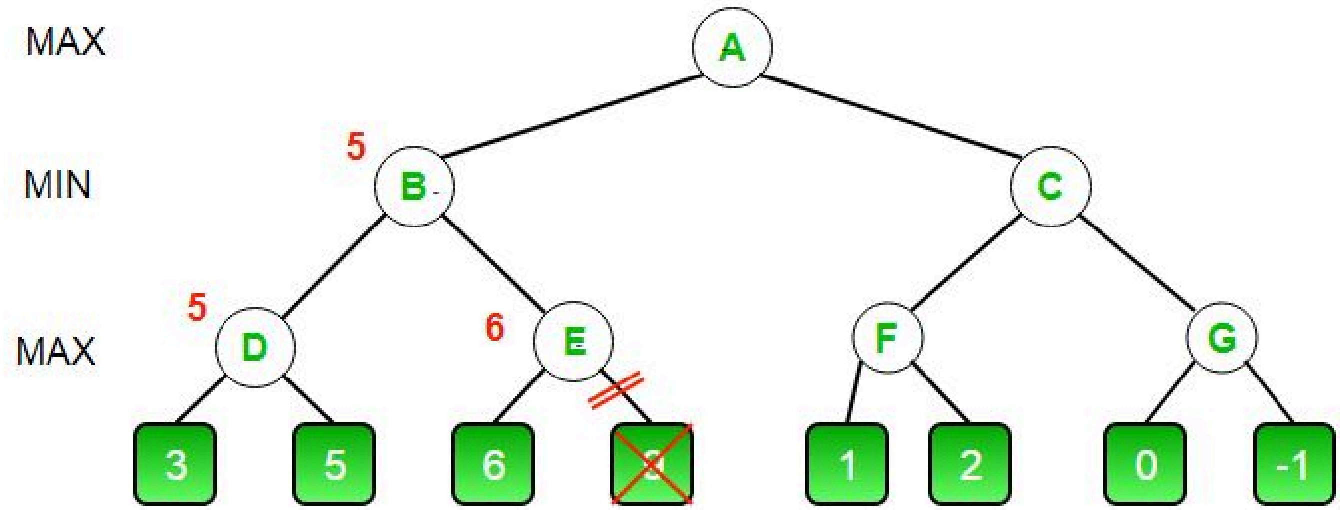
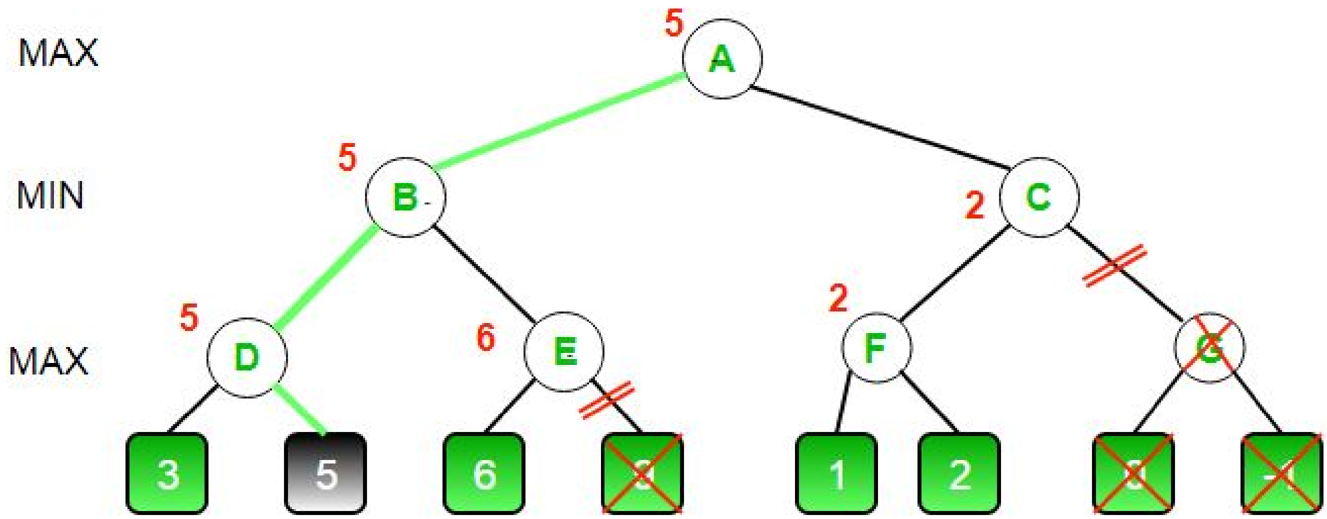


The initial call starts from A. The value of alpha here is -INFINITY and the value of beta is +INFINITY. These values are passed down to subsequent nodes in the tree. At A the maximizer must choose max of B and C, so A calls B first At B it the minimizer must choose min of D and E and hence calls D first. At D, it looks at its left child which is a leaf node. This node returns a value of 3. Now the value of alpha at D is max(-INF, 3) which is 3. To decide whether its worth looking at its right node or not, it checks the condition beta<=alpha. This is false since beta = +INF and alpha = 3. So it continues the search. D now looks at its right child which returns a value of 5.At D, alpha = max(3, 5) which is 5. Now the value of node D is 5 D returns a value of 5 to B. At B, beta = min(+INF, 5) which is 5. The minimizer is now guaranteed a value of 5 or lesser. B now calls E to see if he can get a lower value than 5. At E the values of alpha and beta is not -INF and +INF but instead -INF and 5 respectively, because the value of beta was changed at B and that is what B passed down to E Now E looks at its left child which is 6. At E, alpha = max(-INF, 6) which is 6. Here the condition becomes true. beta is 5 and alpha is 6. So beta<=alpha is true. Hence it breaks and E returns 6 to B Note how it did not matter what the value of E's right child is. It could have been +INF or -INF, it still wouldn't matter, We never even had to look at it because the minimizer was guaranteed a value of 5 or lesser. So as soon as the maximizer saw the 6 he knew the minimizer would never come this way because he can get a 5 on the left side of B. This way we didn't have to look at that 9 and hence saved computation time. E returns a value of 6 to B. At B, beta = min(5, 6) which is 5.The value of node B is also 5 So far this is how our game tree looks. The 9 is crossed out because it was never computed.



B returns 5 to A. At A, alpha = max(-INF, 5) which is 5. Now the maximizer is guaranteed a value of 5 or greater. A now calls C to see if it can get a higher value than 5. At C, alpha = 5 and beta = +INF. C calls F At F, alpha = 5 and beta = +INF. F looks at its left child which is a 1. alpha = max(5, 1) which is still 5. F looks at its right child which is a 2. Hence the best value of this node is 2. Alpha still remains 5 F returns a value of 2 to C. At C, beta = min(+INF, 2). The condition beta <= alpha becomes true as beta = 2 and alpha = 5. So it breaks and it does not even have to compute the entire sub-tree of G. The intuition behind this break-off is that, at C the minimizer was guaranteed a value of 2 or lesser. But the maximizer was already guaranteed a value of 5 if he choose B. So why would the maximizer ever choose C and get a value less than 2 ? Again you can see that it did not matter what those last 2 values were. We also saved a lot of computation by skipping a whole sub-tree. C now returns a value of 2 to A. Therefore the best value at A is max(5, 2) which is a 5. Hence the optimal value that the maximizer can get is 5 This is how our final game tree looks like. As you can see G has been crossed out as it was never computed.




```python

# Initial values of Alpha and Beta
MAX, MIN = 1000, -1000

# Returns optimal value for current player
#(Initially called for root and maximizer)
def minimax(depth, nodeIndex, maximizingPlayer,
            values, alpha, beta):

    # Terminating condition. i.e
    # leaf node is reached
    if depth == 3:
        return values[nodeIndex]

    if maximizingPlayer:

        best = MIN

        # Recur for left and right children
        for i in range(0, 2):

            val = minimax(depth + 1, nodeIndex * 2 + i,
                          False, values, alpha, beta)
            best = max(best, val)
            alpha = max(alpha, best)

            # Alpha Beta Pruning
            if beta <= alpha:
                break

        return best
```

```python

    else:
        best = MAX

        # Recur for left and
        # right children
        for i in range(0, 2):

            val = minimax(depth + 1, nodeIndex * 2 + i,
                            True, values, alpha, beta)
            best = min(best, val)
            beta = min(beta, best)

            # Alpha Beta Pruning
            if beta <= alpha:
                break

        return best

# Driver Code
if __name__ == "__main__":

    values = [3, 5, 6, 9, 1, 2, 0, -1]
    print("The optimal value is :", minimax(0, 0, True, values, MIN, MAX)

```

```
The optimal value is : 5
```