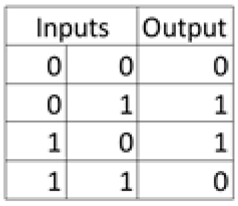
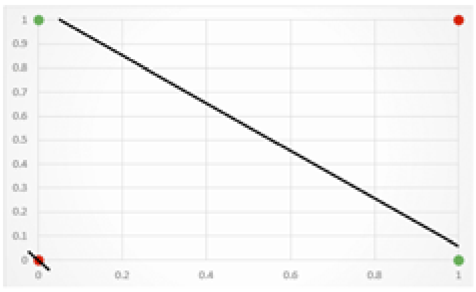
1. XOR problem in Tensorflow Playground

a) Describe why it is called the XOR problem, and why it provides a problem to other classifiers we've studied.

Answer: The Tensorflow Playground XOR problem is called the XOR problem because the data shown cannot be separated by a single straight line. As shown on website: https://playground.tensorflow.org/#dataset=xor (https://playground.tensorflow.org/#dataset=xor) ,there are two clusters of orange and blue dots , each representing a class, say orange and blue, sitting at a corner of a square. If we draw a line vertically or horizotally, we will end up having one cluster of each class in each partition. If we draw diagonal lines with different slopes and positions, we will always end up having some instances of both the classes in each parttion, thereby not allowing us to linearly classify the data. This is analogous to 2 bit XOR data with inputs and outputs as follows[3]:



Inputs		Output
0	0	0
0	1	1
1	0	1
1	1	0



It provides problem to other classifiers because the classifiers that we have studied like perceptron classifies data only if the data is linearly separable otherwise, it will never converge. Hence, those classifiers are not fit for classifying the XOR data in Tensor Playground XOR problem.

b) Notice how each of the neurons has some gradient shading. Describe what this shading represents.

Answer: The gradient shading showed by each of the neurons is the representation of the weights on features at each neuron. The weight decides the importance of the respective feature in predicting the corresponding class. In the given playground tensorflow model, the correct class or positive class is said to be blue and orange represents negative class. The blue shading shows positive weights and orange shading shows negative weights. The zero weights are represented by white color forming a white line at the separation of positive and negative regions. The intensity of the shades represents how big the negative or positive number is on weights inferring how confident the model is in predicting the respective class.

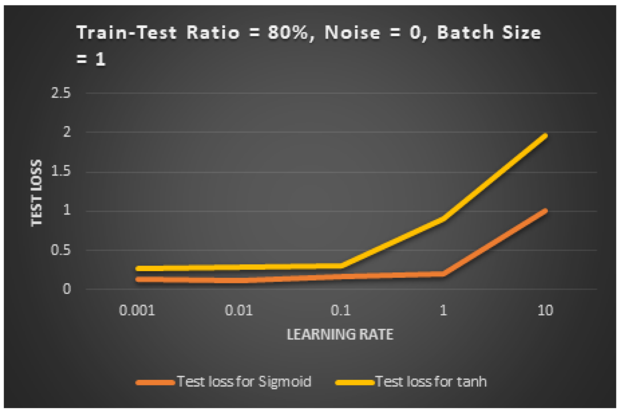
c) For 16 configurations of a model; train for 1,000 epochs and report on the final testing loss. Plot your results and discuss what you observed.

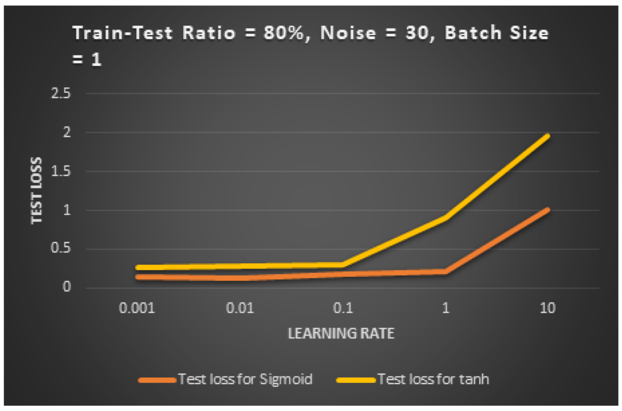
Answer: The 16 configurations were taken and the test loss were recorded for respective configurations as follows:

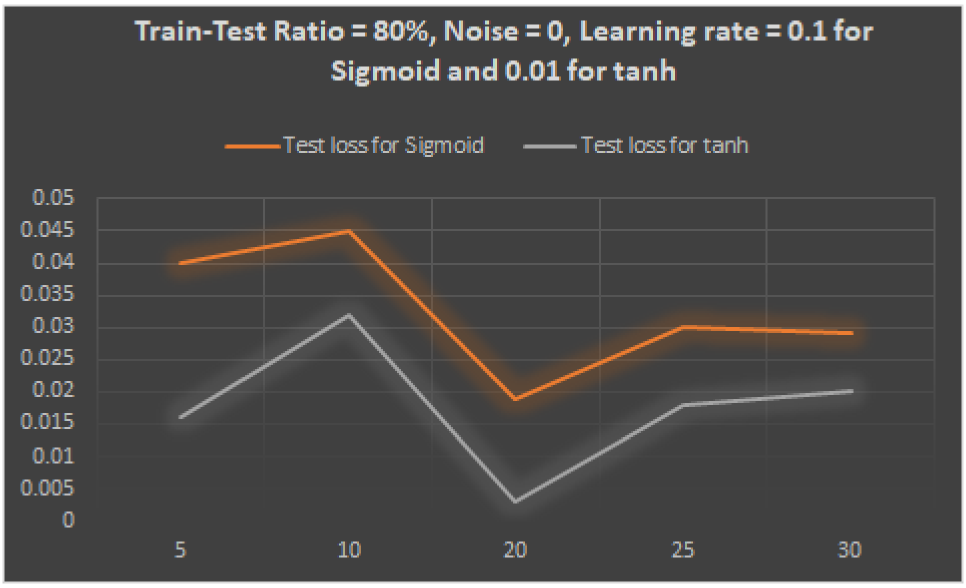
No . Of Hidden layers = 2 No. of neurons per hidden layer = 4			Regularization = None Regularization rate = 0			Epochs = 1000 30 configurations		
Configuration set #	**Configuration Details**	**Configuration #**	**Train to Test data Ratio**	**Noise**	**Batch Size**	**Learning Rate**	**Activation Function**	**Testing loss**
	Train-Test Ratio = 80%	1	0.8	0	1	10	Sigmoid	1.04
	Noise = 0	2	0.8	0	1	1	Sigmoid	0.345
	Batch Size = 1	3	0.8	0	1	0.1	Sigmoid	0
	Activation = Sigmoid	4	0.8	0	1	0.01	Sigmoid	0.001
1	Vary Learning rate	5	0.8	0	1	0.001	Sigmoid	0.25
	Train-Test Ratio = 80%	6	0.8	0	1	10	tanh	1.08
	Noise = 0	7	0.8	0	1	1	tanh	0.6
	Batch Size = 1	8	0.8	0	1	0.1	tanh	0.04
	Activation = tanh	9	0.8	0	1	0.01	tanh	0
2	Vary Learning rate	10	0.8	0	1	0.001	tanh	0.017
	Train-Test Ratio = 80%	11	0.8	0	5	0.1	Sigmoid	0.04
	Noise = 0	12	0.8	0	10	0.1	Sigmoid	0.045
	Learning rate = 0.1	13	0.8	0	20	0.1	Sigmoid	0.019
	Activation = Sigmoid	14	0.8	0	25	0.1	Sigmoid	0.03
3	Vary Batch Size	15	0.8	0	30	0.1	Sigmoid	0.029
	Train-Test Ratio = 80%	16	0.8	0	5	0.01	tanh	0.016
	Noise = 0	17	0.8	0	10	0.01	tanh	0.032
	Learning rate = 0.1	18	0.8	0	20	0.01	tanh	0.003
	Activation = tanh	19	0.8	0	25	0.01	tanh	0.018
4	Vary Batch Size	20	0.8	0	30	0.01	tanh	0.02
	Train-Test Ratio = 80%	21	0.8	30	1	10	Sigmoid	1.001
	Noise = 30	22	0.8	30	1	1	Sigmoid	0.212
	Batch Size = 1	23	0.8	30	1	0.1	Sigmoid	0.172
	Activation = Sigmoid	24	0.8	30	1	0.01	Sigmoid	0.12
5	Vary Learning rate	25	0.8	30	1	0.001	Sigmoid	0.138
	Train-Test Ratio = 80%	26	0.8	30	1	10	tanh	0.96
	Noise = 30	27	0.8	30	1	1	tanh	0.697
	Batch Size = 1	28	0.8	30	1	0.1	tanh	0.136
	Activation = tanh	29	0.8	30	1	0.01	tanh	0.163
6	Vary Learning rate	30	0.8	30	1	0.001	tanh	0.132

Explanation: Number of hidden layers taken were 2 and no. of neurons per hidden layer taken were 4 for all configurations. Also, regularization has been selected as None and regularization rate is set to 0 for all configurations. Six set of configurations have been considered, each with 5 variations resulting in 30 configurations in total. The model was trained on two activation functions, viz. `Sigmoid` and `tanh` , 3 sets of configurations for each activation. In the first set, the learning rate was varied keeping the remaining parameters fixed as shown in table. The second set was same as first except for the activation which was changed from Sigmoid to tanh. Similarly, for third and fourth set of configurations, the batch size was varied. Fifth and sixth set of configuration were same as first and second, except a noise of 30 was added. Each configuration was trained for approximately 1000 epochs.

Results:







Observations:

`Sigmoid function and Learning rate` :

- It was observed that for sigmoid function, when ratio of training data to test data was set to 80% with batch size = 1, and learning rate as 10, the test loss was as high as 1.00. When learning rate was reduced to 1 and then 0.1, the test loss reduced to approximately 30% and then to 0% inferring that the accuracy can be improved with lower learning rate as we are allowing our weights to update slowly with smaller steps to reach the minimum slope. And so there are less chances of skipping the minimum slope. However, when we further reduce the learning rate to 0.01 and then 0.001, we get test loss of 1% and then 25%. One explanation for this could be that at such low learning rate, the training model does not succeed fully in converging and hence does not reach the minimum slope.
- It was also observed that for initial epochs upto 100 the accuracy with learning rate = 1 for sigmoid function, the loss was still 50%. When gradually the data was trained further upto 1000 epochs, the test loss further reduced to 0.009 and so with more epochs since with more training, weights are further updated resulting in correct classification.

`Tanh function and Learning rate` :

- Similar behaviour as Sigmoid was observed for tanh, however, it needed further lower learning rate to yield 0% test loss with approx. 60% test loss at learning rate = 1 and 4% test loss at learning rate = 0.1. The model converges fully at learning rate = 0.01 and learning rate of 0.001 also proves to be much slower for tanh resulting in test loss of 0.017.

`Sigmoid function and Batch size` :

- Since sigmoid function enables the model to converge at learning rate of 0.1, this learning rate was kept fixed, and the batch size was varied.
- Based on the result data shown in table, it was observed that the batch size does not seem to have much effect on test loss. However, for all batch sizes, the minimum test loss of 0 was when batch size was smallest which was 1.
- One conclusion that can be drawn is for learning rate = 0.1, batch size of 1 is ideal. It may vary if the learning rate varies.

`Tanh function and Batch size` :

- Since tanh function enables the model to converge at learning rate of 0.01, this learning rate was kept fixed, and the batch size was varied.
- The increase in batch size intially increased the test loss, and then converges to as low as 0.003 at batch size = 20. When batch size is further increased, the test loss increases. In either of the cases, the test loss was still minimum when batch size was 1.
- It was also observed that when learning rate was changed to 0.1, and batch size was then set to 3 and 19, higher batch size gave better results. Hence, it can be concluded that for learning rate = 0.01, batch size of 1 is ideal. It may vary if the learning rate varies.

`Sigmoid function and Learning rate with noise = 30` :

- With one step lower learning rate than when tested without noise, the model was able to nearly converge the noisy data till test loss of 0.12 but could not achieve 0 loss.
- The test loss is still high with high learning rate of 10.

`Tanh function and Learning rate with noise = 30` :

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2. Derive the partial derivatives of the loss function for a feedforward multilayer perceptron

Given: a feedforward multilayer perceptron model with

- Input vector x_i with D dimensions, i.e., $x_i \epsilon R^D$
- Corresponding label vector y_i with K dimensions (one - hot label)
- L hidden layers, each hidden layer represented by l such that 1 ≤ l ≤ L + 1
- l' as cross entropy loss function to calculate error
- neuron index for each layer l is represented by j, and jth neuron is computed as

Eq-1:

$$h_{l,j} = f^{(l)}(< w_{l,j}, h_{l-1,*} >)$$

where $h_{l,j}$ is a scalar and $h_{l-1,*}$ is a vector of all of the neurons in layer $l-1$. $w_{l,j}$ is also a vector.

- Also $h_{0,*} = h_0 = x_i$ and
- $h_{L+1,*}$ is the output values we pass into our loss function.

a) What function must $f^{(L+1)}$ be to use a cross-entropy loss function?

Answer: Since the model contains L hidden layers, L+1 should be the output layer and so to use a cross entropy as the loss function for K dimensional label, we must use `softmax` as the activation function on output layer.

b) What are the parameters that must be learned?

Answer: The parameters that must be learned will be the weights between input layer and first hidden layer, between all hidden layers and between last hidden layer and output layer (at each neuron for each hidden layer), i.e. all $w_{l,j}$ or $w_{*,*}$ for all layer l and all neurons j at each layer l.

c) If D = 100, L = 2, K = 3, and each hidden layer (non-input or output layer) has 50 neurons, how many parameters are there in total?

Answer: Since there are 2 hidden layers (L = 2) and each hidden layer has 50 neurons, the model will look like as follows:
Input layer(D=100) ---▶ L1(50 neurons) ---▶ L2(50 neurons) ---▶ Output layer(K=3)
Therefore, 100∗50 parameters between Input layer & L1 + 50∗50 parameters between L1 & L2 + 50∗3 parameters between L2 and Output layer

$$= 100 * 50 + 50 * 50 + 50 * 3$$
$$= 5000 + 2500 + 150$$
$$= 7650$$

$==$ ▶ `7650 parameters`

d) Let D, L, K, and the number of neurons (call it E) be unknown. For each variable ♣, derive the partial derivate $\frac{\partial l'}{\partial ♣}$. Since you do not know D, L, K or E, you may provide a general partial derivative formula for some subset of the variables, a second formula for another subset, etc. The formulas you do provide must give a way of computing the partial derivative for each variable.

Answer: Given (if not, we assume):

- input vector x_k with D dimension
- target class label vector y_i^* with K dimension, where $i \epsilon K$
- predicted class label vector y_i
- $h_{l,j}$ represents hidden layer where l denotes hidden layers 1 to L, and j denotes the neurons from 1 to E in each each hidden layer l
- weights between the layers are represented by $\clubsuit$. Let weights between hidden layer l_j and output layer y_i be $\clubsuit_{ji}$. Similarly, let weights between input layer x_k and hidden layer l_j be $\clubsuit_{kj}^1$.
- Since each neuron j of each hidden layer l is computed by using activation as: $h_{l,j} = f^{(l)}(< \clubsuit_{l,j}, h_{l-1,*} >)$. For convenience to compute derivative [5] , I will use s_i to represent weighted sums of all neurons for a hidden layer l activation fed to output layer(l = L+1), and s_j^1 to represent weighted input sum between input layer x_k and hidden unit j of layer l. Therefore

  Eq-I:

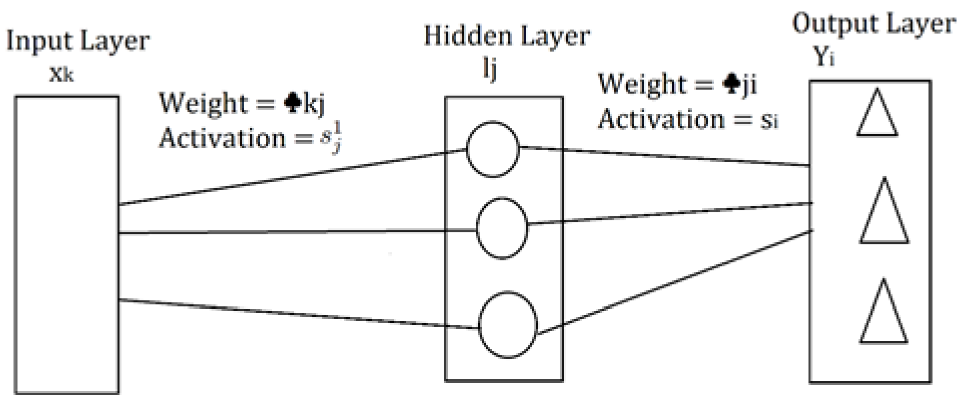
  $$s_i = \Sigma_{j=1} l_j . \clubsuit_{ji}$$

  and

  Eq-II:

  $$s_j^1 = \Sigma_{j=1} x_k . \clubsuit_{kj}^1$$

A rough sketch of what has been described above:



Since y_i is a vector of K dimenstion, and is not a binary class, I will use softmax activation at the output layer and so

Eq-III:

$$y_i = \frac{e^{s_i}}{\Sigma_c^{nclass} e^{s_c}}$$

where y_i is the softmax activation at ith output unit, s_c denotes activation for class c and the denominator denotes normalization among all classes nclass like $e^{s_1} + e^{s_2} + e^{s_3}..$ so on upto $e^{s_{nclass}}$.

Using cross entropy loss function l' for multiclass output:

Eq-IV:

$$l' = -\Sigma_i^{nclass} y_i^* \log(y_i)$$

Objective: We have to find $\frac{\partial l'}{\partial \clubsuit}$.

First we, compute error with respect to output by comparing predicted label with target label.

$$\frac{\partial l'}{\partial y_i} = -\frac{y_i^*}{y_i}$$

Computing the partial derivative for each variable will have two parts.

1. Error with respect to weights between hidden layer and output layer, $\clubsuit_{ji}$ in this case, $\frac{\partial l'}{\partial \clubsuit_{ji}}$
2. Error with respect to weights between input layer and hidden layer, $\clubsuit_{kj}^1$ in this case, $\frac{\partial l'}{\partial \clubsuit_{kj}^1}$

For part 1, computing the derivative using the chain rule:

Eq-V:

$$\frac{\partial l'}{\partial \clubsuit_{ji}} = \frac{\partial l'}{\partial y_i}\frac{y_i}{\partial s_i}\frac{s_i}{\partial \clubsuit_{ji}}$$

For computing the gradient w.r.t s_{i} - since it is a softmax function, and we normalize based on all classes, we consider two cases:

1. when we calculate the gradient of ith class label w.r.t ith activation (if we take s_i as s_k then i = k)
2. when we calculate the gradient of ith class label w.r.t all other activations except ith activation (if we take s_i as s_k then $i \neq k$)
   Therefore, by using Eq-III:
   $$\frac{\partial y_i}{\partial s_k} = \begin{cases} \frac{e^{s_i}}{\Sigma_c^{nclass} e^{s_c}} - (\frac{e^{s_i}}{\Sigma_c^{nclass} e^{s_c}})^2, i = k \\ -\frac{e^{s_i} e^{s_k}}{(\Sigma_c^{nclass} e^{s_c})^2}, i \neq k \end{cases}$$

   Eq-VI:
   $$\frac{\partial y_i}{\partial s_k} = \begin{cases} y_i(1 - y_i), i = k \\ -y_i y_k, i \neq k \end{cases}$$
   The above result has been derived for all activations when i=k and $i \neq k$, however we are interested only in case when i = k so we subtract the remaining term to yield $\frac{\partial l'}{s_i}$ as follows :
   Eq-VII:
   $$\frac{\partial l'}{s_i} = \Sigma_k^{nclass}\frac{\partial l'}{\partial y_k}\frac{\partial y_k}{\partial s_i} = \frac{\partial l'}{\partial y_i}\frac{\partial y_i}{\partial s_i} - \Sigma_{k \neq i}\frac{\partial l'}{\partial y_k}\frac{\partial y_k}{\partial s_i} = -y_i^*(1 - y_i) + \Sigma_{k \neq i} y_k^* y_i = -y_i^* + y_i \Sigma_k$$

Since the first two components of Eq-V has been solved in Eq-VI, now using Eq-I, we solve Eq-V as follows:
Eq-VIII:

$$\frac{\partial l'}{\partial \clubsuit_{ji}} = \Sigma_i \frac{\partial l'}{\partial s_i}\frac{\partial s_i}{\partial \clubsuit_{ji}} = (y_i - y_i^*) l_j$$

Eq-VII gives the gradients of the error with respect to the weights in the last layer of the network, now to compute the gradients with respect to the weights in lower layers we apply backpropagation in part 2.

For part 2, computing the derivative using the chain rule:

Eq-IX:

$$\frac{\partial l'}{\partial \clubsuit_{kj}^1} = \frac{\partial l'}{\partial s_j^1}\frac{\partial s_j^1}{\partial \clubsuit_{kj}^1}$$

The gradient for weights in units in the hidden layer l, indexed by j, is calculated as follows: For computing the gradient w.r.t s_j^1, and solving using Eq-VII, Eq-I and Eq-VI:
Eq-X:

$$\frac{\partial l'}{\partial s_j^1} = \Sigma_i^{nclass}\frac{\partial l'}{\partial s_i}\frac{\partial s_i}{\partial l_j}\frac{\partial l_j}{\partial s_j^1} = \Sigma_i^{nclass}(y_i - y_i^*)(\partial \clubsuit_{ji})(l_j(1 - l_j))$$

Note1:In the above equation consider l_j as new output layer and s_j^1 as activation fed into l_j similar to output layer y_i and activation s_i for part 1.
Also, gradient w.r.t hidden layer can be give as:

$$\frac{\partial l'}{\partial l_j} = \Sigma_{i=1}\frac{\partial l'}{\partial y_i}\frac{\partial y_i}{\partial s_i}\frac{\partial s_i}{\partial x_j} = \Sigma_i \frac{\partial l'}{\partial y_i}(y_i(1 - y_i))(\partial \clubsuit_{ji})$$

Finally, using Eq-II and Eq-X in Eq-IX gives:

$$\frac{\partial l'}{\partial \clubsuit^{1}_{kj}} = \Sigma_i^{nclass}(y_i - y_i^*)(\partial \clubsuit_{ji})(l_j(1-l_j))(x_k)$$

Hence, Part 1 and Part 2 lists down partial derivative for each variable for top layer and bottom layer of network. To calculate derivatives within multiple hidden layers, same computation as given in Eq-X can be done by updating the output layer and activation as mentioned in Note1.

e) [Eq-1] does not include an explicit bias; describe how you can include the bias without changing [Eq-1] (or the derived gradients).

Answer: To include the bias we simply add the bias element in the input vector (like we did for perceptron algorithm while calculating weight updates). This will change the dimension of input vector x_i,from (say) D to D+1, therefore $x_i \epsilon R^{D+1}$. Corresponding weight vectors will have similar dimensions as input vector for that layer. This means that for jth neuron, the weight is still represented as $w_{l,j}$; only the dimension of the vector will vary, leaving [Eq-1] or derived gradients unchanged.

3. Multiclass neural network classifier

a) Implementation and convergence criterion:

Answer: The multiclass neural network classifier with one hidden layer is implemented using python. The implementation has two parts –
(1) a class named MulticlassNN.py is implemented that contains the definition of training model and
(2) a train-test implementation named TrainTestNN.ipynb that uses training model MulticlassNN class to run the training and test data (dev data) with different configurations.

In MulticlassNN Neural network model,

- no. of layers (input, hidden and output), no. of neurons per hidden layer, activation function and cost function are initialized.
- An input layer to hidden layer and a hidden layer amongst themselves are connected using activation functions which implies, output of layer is input to the next layer after activation function is acted upon each set of inputs until it reaches final output layer. I have used three activation functions – sigmoid, tanh and softmax, implemented using NumPy package's inbuilt functions – tanh, exp, sum and divide.
- The training model has three steps, viz. forward pass, error computation and backward pass. Each instance will go through activation function while forward pass and error is computed at output. Based on error, it will be back propagated by taking the derivative deltas w.r.t activation and multiplying it with learning rate to decide on size of learning step, i.e., calculating the gradient descent.
- Error is computed based on two cost functions, viz. cross entropy or mean square error.
- Accuracy percentage is then computed based on total correctly classified predicted labels divided by total labels in the input data multiplied by 100.

In [60]:

```
import numpy as np
import dill as dill

class MulticlassNNClassifier:
    # Initialize no of layers( input, hidden and output), no. of neurons, activation an
d cost functions
    def __init__(self, noOfLayers, neurons, activation, costFunction):
        self.layers = []
        self.noOfLayers = noOfLayers
        self.neurons = neurons
        self.cost_function = costFunction

        # Declare vector size for each neuron in each layer, the elements in neuron lis
t should match no. of layers
        if not noOfLayers == len(neurons):
            raise ValueError("Layers and neuron count mismatch")

        # Include next layer neurons for input layer and all hidden layers (excluding o
utput layer)
        for x in range(noOfLayers):
            if x != noOfLayers-1:
                layer_x = layer(neurons[x], neurons[x+1], activation[x])
            else:
                layer_x = layer(neurons[x], 0, activation[x])
            self.layers.append(layer_x)

    # Each instance will go through activation function while forward pass, then error
 is computed at output;
    # based on error, it will be back propagated, weights are updated and again passed
 forward until the classifier
    # converges and a decent accuracy is attained.
    def trainNeuralNetwork(self, batch, trainingDataX, trainingLabelY, epochs, learning
Rate, filename):
        self.batch = batch
        self.learningRate = learningRate
        for j in range(epochs):
            i = 0
            while i+batch != len(trainingDataX):
                self.error = 0
                self.forwardPass(trainingDataX[i:i+batch])
                self.computeError(trainingLabelY[i:i+batch])
                self.backPropagate(trainingLabelY[i:i+batch])
                i += batch
            self.error /= batch
        dill.dump_session(filename)

    # Each forward pass will update weights based on an activation function
    def forwardPass(self, TrainingDataX):
        self.layers[0].activations = TrainingDataX
        for i in range(self.noOfLayers-1):
            tempMat = np.add(np.matmul(self.layers[i].activations, self.layers[i].curre
ntLayerWeights), self.layers[i].currentLayerBias)
            if self.layers[i+1].activation == "sigmoid":
                self.layers[i+1].activations = self.sigmoid(tempMat)
            elif self.layers[i+1].activation == "tanh":
                self.layers[i+1].activations = self.tanh(tempMat)
            elif self.layers[i+1].activation == "softmax":
                self.layers[i+1].activations = self.softmax(tempMat)
            else:
                self.layers[i+1].activations = tempMat

    # Activation function = sigmoid
    def sigmoid(self, layer):
        return np.divide(1, np.add(1, np.exp(np.negative(layer))))

    # Activation function = tanh
    def tanh(self, layer):
        return np.tanh(layer)

    # Activation function = softmax
    def softmax(self, layer):
        exp = np.exp(layer)
        if isinstance(layer[0], np.ndarray):
            return exp/np.sum(exp, axis=1, keepdims=True)
        else:
            return exp/np.sum(exp, keepdims=True)

    # Error can be calculate based on cost function as cross entropy or mean squared
    def computeError(self, trainingLabelY):
        if len(trainingLabelY[0]) != self.layers[self.noOfLayers-1].currentLayerNeurons
:
            print ("Error: Label Y and output layer matrix dimension mismatch.")
            return
        if self.cost_function == "meanSquared":
            self.error += np.mean(np.divide(np.square(np.subtract(trainingLabelY, self.
layers[self.noOfLayers-1].activations)), 2))
        elif self.cost_function == "crossEntropy":
            self.error += np.negative(np.sum(np.multiply(trainingLabelY, np.log(self.la
yers[self.noOfLayers-1].activations))))

    # Once we have error, we apply back propagation by differeniating w.r.t activation
 and update weights to improve accuracy
    def backPropagate(self, trainingLabelY):
        targets = trainingLabelY
        i = self.noOfLayers-1
        y = self.layers[i].activations
        deltab = np.multiply(y, np.multiply(1-y, targets-y))
        deltaw = np.matmul(np.asarray(self.layers[i-1].activations).T, deltab)
        new_weights = self.layers[i-1].currentLayerWeights - self.learningRate * deltaw
        new_bias = self.layers[i-1].currentLayerBias - self.learningRate * deltab
        for i in range(i-1, 0, -1):
            y = self.layers[i].activations
            deltab = np.multiply(y, np.multiply(1-y, np.sum(np.multiply(new_bias, self.
layers[i].currentLayerBias)).T))
            deltaw = np.matmul(np.asarray(self.layers[i-1].activations).T, np.multiply(
y, np.multiply(1-y, np.sum(np.multiply(new_weights, self.layers[i].currentLayerWeights
),axis=1).T)))
            self.layers[i].currentLayerWeights = new_weights
            self.layers[i].currentLayerBias = new_bias
            new_weights = self.layers[i-1].currentLayerWeights - self.learningRate * de
ltaw
            new_bias = self.layers[i-1].currentLayerBias - self.learningRate * deltab
        self.layers[0].currentLayerWeights = new_weights
        self.layers[0].currentLayerBias = new_bias


    def computeAccuracy(self, filename, inputDataX, labelY):
        dill.load_session(filename)
        self.batch = len(inputDataX)
        self.forwardPass(inputDataX)
        a = self.layers[self.noOfLayers-1].activations
        a[np.where(a==np.max(a))] = 1
        a[np.where(a!=np.max(a))] = 0
        total=0
        correct=0
        for i in range(len(a)):
            total += 1
            if np.equal(a[i], labelY[i]).all():
                correct += 1
        print("Accuracy percentage: ", correct*100/total)

class layer:
    def __init__(self, currentLayerNeurons, nextLayerNeurons, activation):
        self.currentLayerNeurons = currentLayerNeurons
        self.activation = activation
        self.activations = np.zeros([currentLayerNeurons,1])
        #Random distribution of weights for hidden layers and adding bias element
        if nextLayerNeurons != 0:
            self.currentLayerWeights = np.random.normal(0, 0.001, size=(currentLayerNeu
rons, nextLayerNeurons))
            self.currentLayerBias = np.random.normal(0, 0.001, size=(1, nextLayerNeuron
s))
        else:
            self.currentLayerWeights = None
            self.currentLayerBias = None
```

In train-test implementation,

- mnist data and MulticlassNN is imported. 60,000 samples are used as training data and 10,000 samples as test data or dev data.
- The data is then converted to numpy array format to perform numpy operations.
- A network is created using MulticlassNN neural network train model by plugging in appropriate configuration. The baseline model takes 3 layers, viz. input layer, one hidden layer and output layer. The input vector has 784 elements, hidden layer has 20 neurons and output layer has 10 elements corresponding to output class labels 0 to 9.
- The activation for input layer is set as none as no activation is needed at input layer. I have taken tanh as the activation function for hidden layer and softmax activation function for output layer.
- The error is calculated based on cost function as cross Entropy.
- Once the confirgurations are set, training data is passed to configured network to train the model with 5 iterations (epochs) and a learning rate of 0.001. The training is done collectively in a single batch. A filename 'Result.pkl is used for pickling, i.e, to store the intermediary result to perform numpy operations.
- Once the data is trained, the accuracy is calculated for the training data.
- The trained model is then applied to test data for its accuracy

In [65]:

```
#%pip install python-mnist
import numpy as np
from mnist import MNIST

#Import MNIST data and load training and Dev/Test data
mnist = MNIST('mnist-dataset')
TrainingDataX, TrainingLabelY = mnist.load_training() # 60000 training samples
TestDataX, TestLabelY = mnist.load_testing()          # 10000 test samples

#Converting data to numpy array format
TrainingData = np.asarray(TrainingDataX).astype(np.float32)
TrainingLabel = np.asarray(TrainingLabelY).astype(np.int32)
TestDataX = np.asarray(TestDataX).astype(np.float32)
TestLabelY = np.asarray(TestLabelY).astype(np.int32)

# Baseline: Train Neural Network - Single hidden layer using sigmoid activation and out
put layer using softmax activation
NumberOfLabels = 10
Trainingclass = np.eye(NumberOfLabels)[TrainingLabel] # converting to one -hot label
Network = MulticlassNNClassifier(3, [784, 100, 10], [None, "tanh", "softmax"], costFunc
tion="crossEntropy")
Network.trainNeuralNetwork(1, trainingDataX=TrainingData, trainingLabelY=Trainingclass,
epochs=5, learningRate=0.001, filename="Result.pkl")
print("Training Accuracy")
Network.computeAccuracy("Result.pkl", TrainingData, Trainingclass)

# Test Baseline Neural Network
TestClass = np.eye(NumberOfLabels)[TestLabelY]
print("Testing Accuracy")
Network.computeAccuracy("Result.pkl", TestDataX, TestClass)
```

```
Training Accuracy
Accuracy percentage:  4.043333333333333
Testing Accuracy
Accuracy percentage:  4.16
```

Convergence criteria:

- Weights and bias have been initialized based on random normal distribution.
- For each derivative w.r.t activation while back propagating, weights are updated by subtracting the gradients from current weight and again passed forward to check for error at each neuron.
- The process continues until the error stabilizes and stops reducing further based on cost function.
- Eventually the updated weights starts giving right classification of labels and converges to an accuracy of about 87%.

b) Validation using XOR input data:

Answer:

In [ ]:

```
import numpy as np
import MulticlassNN as nn
from mnist import MNIST

#XOR  data
datum_1 = [0, 0]
datum_2 = [0, 1]
datum_3 = [1, 0]
datum_4 = [1, 1]

training_dataX = [datum_1, datum_2, datum_3, datum_4]
training_labelY = [0, 1, 1, 0]

# #Converting data to numpy array format
X = np.array(training_dataX).astype(np.float32)
Y = np.asarray(training_labelY).astype(np.int32)

# Baseline: XOR data Train Neural Network - Single hidden layer using sigmoid activatio
n and output layer using softmax activation
NumberOfLabels = 2
Trainingclass = np.eye(NumberOfLabels)[Y] # all zeroes except ones on diagonals
print(Trainingclass)
Network = MulticlassNNClassifier(3, [3, 4, 2], [None, "tanh", "softmax"], costFunction=
"crossEntropy")
Network.trainNeuralNetwork(1, trainingDataX=X, trainingLabelY=Trainingclass, epochs=200
00, learningRate=0.2, filename="Result.pkl")
print("Training Accuracy")
Network.computeAccuracy("Result.pkl", TrainingDataX, TrainingLabelY)
```

c) Analysis of multiclass neural network classifier:

Answer: xbcx

4. Evaluate these models on the original 10,000 image test set

Answer: nxd

References

- [1] Playground Tensor XOR problem is taken from: https://playground.tensorflow.org/#dataset=xor (https://playground.tensorflow.org/#dataset=xor)
- [2] Lecture slides and homework problem has been taken from : https://www.csee.umbc.edu/courses/graduate/678/spring20 (https://www.csee.umbc.edu/courses/graduate/678/spring20)
- [3] XOR diagram for 1.(a) has been taken from: https://medium.com/@jayeshbahire/the-xor-problem-in-neural-networks-50006411840b (https://medium.com/@jayeshbahire/the-xor-problem-in-neural-networks-50006411840b)
- [4] MNIST Dataset has been loaded from: https://www.renom.jp/notebooks/tutorial/neuralnetwork/download_mnist/notebook.html (https://www.renom.jp/notebooks/tutorial/neuralnetwork/download_mnist/notebook.html)
- [5] Back propagation and generation of partial derivative is studied & referred from: https://www.ics.uci.edu/~pjsadows/notes.pdf (https://www.ics.uci.edu/~pjsadows/notes.pdf)
- [6] Report was prepared in Jupyter Notebook and imported to pdf.
- [7] Graphs and tables were prepared using MS Excel.

```
In [ ]:
```