

## Maths for DSA

### • → Prime Numbers

$n \rightarrow (2 \text{ to } \sqrt{n})$

```
for(i=2; i ≤ n; i++) {  
    if(n % i == 0) {  
        → return "Non-Prime"  
    }  
}
```

$i^2$   
 $i \leq n$   
 $i \leq \sqrt{n}$

} → Inner Loop  
return "Prime";

If we want to find the prime numbers in the given range:

```
for(i=2; i ≤ N; i++) {  
    isprime = TRUE  
    Inner Loop
```

```
    isprime = false;  
    break;
```

Above approach would give TLE, So, the better approach is:

### Sieve of Eratosthenes

① we will be considering all the numbers as the Prime Numbers

② Then the number which we have considered as the Prime Number, multiple of these numbers will be discarded from the given range suppose 1 to  $N(50)$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

(15) is circled as a prime number.



```

vector<bool> isprime(n+1, TRUE) {
    for (i=2; i<=n; i++) {
        if (isprime[i]) {
            ans++;
            for (j=i*2; j<=n; j=j+i) {
                isprime[j] = false;
            }
        }
    }
}

```

→ Digits in a Number

We can do the following things by this

- Count of Digits
- Sum of Digits
- Print Digits

$n = 3586$   
 $3586 / 10 \rightarrow 6$   
 $358 / 10 \rightarrow 8$   
 $35 / 10 \rightarrow 5$   
 $3 / 10 \rightarrow 3$   
 $0$

Reverse digits

```

while (n != 0) {
    digit = n % 10;
    digit
    n = n / 10;
}

```

Count digits

```

while (n != 0) {
    digit = n % 10;
    count++;
    n = n / 10;
}
count

```

Sum of Digits

```

while (n != 0) {
    digit = n % 10;
    sum += digit;
    n = n / 10;
}
sum

```

OR

$\text{int}(\log_{10}(n)) + 1$

↓

will give the count as well



## → Armstrong Number

"Number that is equal to the sum of cubes of its digits."

$$153 \rightarrow 1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153$$

$$371 \rightarrow 3^3 + 7^3 + 1^3 = 27 + 343 + 1 = 371$$

```
bool IsArmstrong(int n)
```

```
{
    copy n = n
```

```
    Sum-of-cubes = 0
```

```
    while(n != 0)
```

it will be separating each digit and then adding their cubes as well.

```
        dig = n % 10
```

```
        Sum-of-cubes += (dig * dig * dig)
```

```
        n = n / 10
```

```
    }
    return Sum-of-cubes == copy n
```

→ GCD / HCF (Greatest Common Divisor / Highest Common Factor)

$$a = 20, b = 28$$

$$\begin{array}{r} 2 \overline{) 20} \\ 2 \overline{) 10} \\ 2 \overline{) 5} \\ 5 \overline{) 5} \\ 1 \overline{) 1} \\ 1 \overline{) 0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 28} \\ 2 \overline{) 14} \\ 7 \overline{) 7} \\ 1 \overline{) 1} \\ 1 \overline{) 0} \end{array}$$

$$20 = 2 \times 2 \times 5 \times 1 \quad (4)$$

$$28 = 2 \times 2 \times 7 \times 1 \quad (4)$$

Brute force

$O(\min(a, b))$

```
gcd = 1
for (i = 1; i <= min(a, b); i++) {
    if (a % i == 0 && b % i == 0) {
        gcd = i
    }
}
```

Corner cases

① -  $a = 0 \rightarrow b$

② -  $b = 0 \rightarrow a$

③ -  $a = b \rightarrow a \text{ or } b$



## Optimized Approach

### Euclid Algorithm

$$\text{gcd}(a, b) \leftarrow \begin{cases} \text{gcd}(a-b, b), & a > b \\ \text{gcd}(a, b-a), & b > a \end{cases}$$

$$\begin{aligned} \text{gcd}(20, 28) &\rightarrow \text{gcd}(20, 8) \rightarrow \text{gcd}(12, 8) \rightarrow \text{gcd}(4, 8) \rightarrow \text{gcd}(4, 4) \rightarrow \text{gcd}(0, 4) \\ &\quad \text{20-8, 12-8 = 4} \end{aligned}$$

$$\begin{aligned} &\quad \text{20} \div 8, 12 \div 8 = 4 \\ \text{gcd}(a, b) &\leftarrow \begin{cases} \text{gcd}(a \div b, b), & a > b \\ \text{gcd}(a, b \div a), & b > a \end{cases} \\ &\rightarrow \text{gcd}(20, 28) \rightarrow \text{gcd}(20, 8) \rightarrow \text{gcd}(4, 8) \\ &\quad \rightarrow \text{gcd}(4, 0) \end{aligned}$$

```
gcd(a, b) {  
    while (a > 0 && b > 0) {  
        if (a > b) {  
            a = a / b;  
        }  
        else {  
            b = b / a;  
        }  
    }  
    if (a == 0) return b;  
    return a;  
}
```

else

$b = b \div a$

if (a == 0)  
return b

return a;

OR Using Recursion

```
gcdrec(a, b)  
if (b == 0) return a;  
return gcdrec(b, a / b);
```



→ LCM (least Common factor)

LCM of two numbers would be like:

$$\text{LCM} = \frac{a * b}{\text{gcd}(a, b)} = \frac{20 * 28}{4} = \frac{560}{4}$$

$$= 140$$

$\text{LCM}(a, b) \{$

$\text{gcd} = \text{gcdRec}(a, b)$

$\text{return } a * b / \text{gcd};$

→ Reverse a Number

As we know the logic for extracting digits using the % operators in reverse order.

$\text{num} = 4537$

7  
3  
5  
4

$\text{revNum} = (\text{revNum} * 10) + \text{digit}$

$$70 + 3 = 73$$

$$730 + 5 = 735$$

$$7350 + 4 = 7354$$

→ Palindrome Number

$n = 353$

$\text{num} < 0 \rightarrow \text{false}$

- ① we will calculate the reverse.
- ② then compare with the original one.

$\text{isPalindrome}(n) \{$

$\text{if } (n < 0) \rightarrow \text{false}$

$\text{revNum} = \text{reverse}(n)$

$\text{return } n == \text{revNum}$



## → Modulo Arithmetic

Modulo Arithmetic are used to store the large or too much big values in the form of exponents. It is used to save us from the overflow condition.

$$x \% n \rightarrow [0, n-1]$$

$$100 \% 3 \rightarrow [0, 1, 2]$$

Properties

$$(x+y) \% n = x \% n + y \% n$$

$$(x-y) \% n = x \% n - y \% n$$

$$(x \cdot y) \% n = x \% n \cdot y \% n$$

$$((x \% n) \% n) \% n$$