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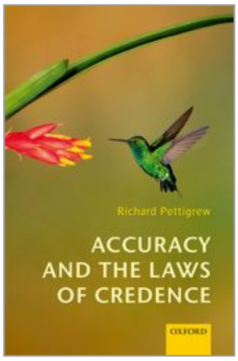

Accuracy and the Laws of Credence
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The Bronfman objection

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Abstract and Keywords

This chapter considers an objection that has been raised against applications of dominance reasoning in those situations in which there is not a single objective measure of utility. In the context of the book, the objection is due to Aaron Bronfman. The chapter concludes that the accuracy argument for Probabilism can be saved only by narrowing down the class of legitimate inaccuracy measures to a single measure.

Keywords: Aaron Bronfman, dominance principle, imprecise utility, epistemicism, supervaluationism, subjectivism

Suppose that, unlike me, you are not convinced by the argument for Symmetrymooted at the end of the last chapter. You accept only that the legitimate inaccuracy measures must be additive, continuous, and strictly proper; you do not accept further that the divergences that generate them must be symmetric. Then you are left with a problem: how are we to formulate the first premise of the accuracy argument? This problem was first raised by Aaron Bronfman in unpublished work and it has come to be known as the *Bronfman objection* against Joyce's original argument for Probabilism (Bronfman, ms).[1]

As I will present it, the Bronfman objection begins with an example. Suppose Phil has credences in only two propositions, X and its negation $\overline{X}$. And suppose his credence function is $c(X) = 0.9$ and $c(\overline{X}) = 0.2$. So, c is non-probabilistic. Then Theorem 4.3.4 establishes the following: For each legitimate inaccuracy measure $\mathfrak{I}$, there is a credence function c^* such that $\mathfrak{I}(c^*, w) < \mathfrak{I}(c, w)$ for all worlds w. Indeed, there are many credence functions that $\mathfrak{I}$-dominate c in this way. Moreover, amongst those credence functions that $\mathfrak{I}$-dominate c, there are some that are not themselves $\mathfrak{I}$-dominated; and those are all probabilistic and $\mathfrak{I}$-immodest.[2] It is this fact that is supposed to render c irrational. But notice the following: for all we have said, given two legitimate inaccuracy measures $\mathfrak{I}$ and $\mathfrak{I}'$, the set of credence functions each of which $\mathfrak{I}$-dominates c might be different from the set of credence functions each of which $\mathfrak{I}'$-dominates c. Indeed, the two sets might be disjoint—there might be no credence function that both $\mathfrak{I}$-dominates and $\mathfrak{I}'$-dominates c. In fact, this is true of Phil's credence function c and the following two inaccuracy measures:

- *The additive logarithmic inaccuracy measure* $\mathfrak{LA}$. We met this in Section 3.2 above. It is defined as follows. First, we define the logarithmic scoring rule:

$$\mathfrak{l}(1, x) := -\ln x \quad \mathfrak{l}(0, x) := -\ln(1-x)$$

(p.70) Next, we define the additive logarithmic inaccuracy measure:

$$\mathfrak{LA}(c, w) = \sum_{X \in \mathcal{F}} \mathfrak{l}(v_w(X), c(X))$$

$\mathfrak{l}$ is a continuous strictly proper scoring rule, so $\mathfrak{LA}$ is a continuous and additive strictly proper inaccuracy measure. Thus, $\mathfrak{LA}$ satisfies our necessary conditions on a legitimate inaccuracy measure from the previous chapter.

- *The additive spherical inaccuracy measure* $\mathfrak{SA}$. This is defined as follows. First, we define the spherical scoring rule:

$$\mathfrak{s}(1, x) = -\frac{x}{\sqrt{x^2+(1-x)^2}} \qquad \mathfrak{s}(0, x) = -\frac{1-x}{\sqrt{x^2+(1-x)^2}}$$

Next, we define the additive spherical inaccuracy measure:

$$\mathfrak{SA}(c, w) = \sum_{X \in \mathcal{F}} \mathfrak{s}(v_w(X), c(X))$$

$\mathfrak{s}$ is a continuous strictly proper scoring rule, so $\mathfrak{SA}$ is a continuous and additive strictly proper inaccuracy measure. Thus, $\mathfrak{SA}$ satisfies our necessary conditions on a legitimate inaccuracy measure from the previous chapter.

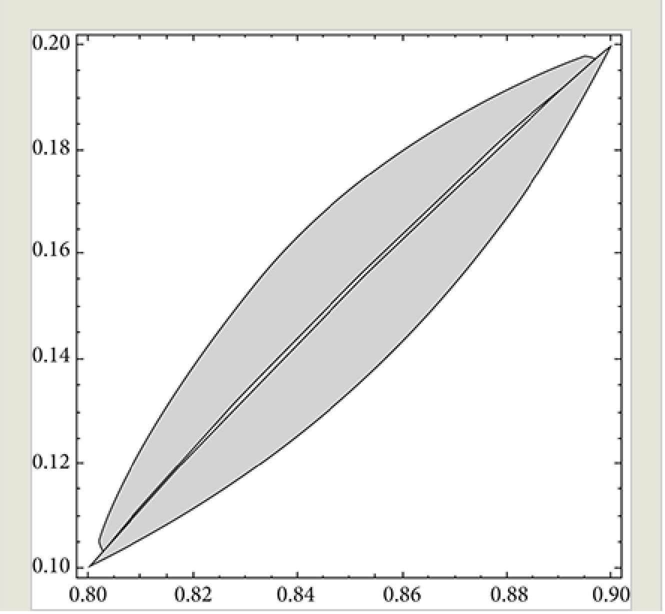
Now, as we can see from Figure 5.1, although there are credence functions that $\mathfrak{LA}$A-dominate c and (other) credence functions that $\mathfrak{SA}$-

dominate Phil's credence function c, there is no credence function that both $\mathfrak{LA}$-dominates and $\mathfrak{SA}$-dominates c.

How we incorporate the moral of this example depends on what we take to be the status of the characterization of legitimate inaccuracy measures given in the previous chapter. There are three main options:

- *Supervaluationism* There is an objective notion of inaccuracy, but it is too indeterminate for there to be a single correct numerical measure of it. Each continuous and additive strictly proper inaccuracy measure is an acceptable precisification of it.
- *Epistemicism* There is an objective notion of inaccuracy, and it is determinate enough for there to be a single correct measure of it. But we do not have perfect epistemic access to it at this stage; all we know is that, whichever measure is the single correct one, it is amongst the continuous and additive strictly proper inaccuracy measures.
- *Subjectivism* There is an objective notion of inaccuracy, but it is too indeterminate for there to be a single correct measure of it. Each continuous and additive strictly proper inaccuracy measure is an acceptable measure for an agent to adopt as her own subjective inaccuracy measure.

Let us consider each in turn to see how they might affect our accuracy argument for Probabilism. (p.71)



5.1 Epistemicism

Figure 5.1 The upper region in this diagram shows the set of credence functions that $\mathfrak{SA}$- dominates Phil's credence function c; the lower region shows the set that $\mathfrak{LA}$-dominates it. As is clear from the diagram, these two regions do not intersect.

First, Epistemicism. Suppose we replace Brier Alethic Accuracy with Epistemicism. That is, we take as a premise that there is a single legitimate measure of inaccuracy and that it is a continuous and additive strictly proper inaccuracy measure. But we do not assume that we can narrow down the field any further than this. Then it might seem that we will be able to establish Probabilism. After all, while we don't know which scoring rule is the one true measure of inaccuracy, we do know that, whichever it is, every non-probabilistic credence function is dominated relative to it; and, moreover, we know that amongst the dominating credence functions are some that are immodest. It is surely irrational to adopt an option that you know to be dominated, even if you do not know the identity of its dominators.

In fact, there is reason to think this isn't so. Consider the following case, which shares something in common with Parfit's so-called Miners Paradox (Parfit, ms), but takes its main inspiration from L. A. Paul's work on transformative experience (Paul, 2014). It is a case in which an agent doesn't know her own utility function because she has not had certain experiences that are required to assess the value she attaches to some of the outcomes of the available acts.

> **Sandwich** I must choose between three sandwich options: Marmite, cheese, and Pater Peperium (or Gentleman's Relish). I have tried a cheese sandwich before, but not a Marmite one and not a Pater Peperium one. Thus, I know the utility I assign to (p.72) eating a cheese sandwich—I'm pretty indifferent to them. But I don't know the utility I assign to eating a Marmite sandwich and I don't know the utility I assign to eating a Pater Peperium sandwich. What I do know is that everyone either hates Marmite and loves Pater Peperium or hates Pater Peperium and loves Marmite. Thus, since I know that I am indifferent to a cheese sandwich—I neither love it nor hate it—I know that I order the options in one of the following two ways:
>
> - Marmite < Cheese < Pater Peperium
> - Pater Peperium < Cheese < Marmite

Now, for our purposes, the important feature of this example is this: Relative to whichever utility function I in fact have, the cheese sandwich option is dominated; but relative to one possible utility function it is dominated by the Marmite option while according to the other it is dominated by the Pater Peperium option. Does this rule out the cheese sandwich as irrational? It seems not. After all, while I might increase my utility by opting for the Marmite sandwich instead—if I love Marmite—I might also decrease my utility significantly—if I hate it. Thus, there seems to be nothing irrational about sticking with the cheese sandwich.

Notice that this is exactly the position in which Phil finds himself in our example above. His credence function is non-probabilistic. If he accepts Epistemicism, he knows that it is dominated. However, since he doesn't know which credence functions dominate it—after all, he doesn't know which epistemic utility function is his because he doesn't know which is the one true measure of accuracy—no irrationality attaches to him for adopting it.

Thus, the upshot of this example is that the following initially plausible decision- theoretic principle is false:

> **Epistemic Dominance** Suppose $\mathcal{O}$ is a set of options, $\mathcal{W}$ is a set of possible worlds, and $\mathcal{U}$ is a set of utility functions on $\mathcal{O}$ and $\mathcal{W}$. Suppose o is in $\mathcal{O}$. Then, if, for all $\mathfrak{U}$ in $\mathcal{U}$, there is o^*_{U} in $\mathcal{O}$ such that
>
> (i) $\mathrm{U}(o^*_{\mathrm{U}}, w) < \mathrm{U}(o, w)$, for all w in $\mathcal{W}$, and
>
> (ii) there is no o'_{U} such that $\mathrm{U}(o'_{\mathrm{U}}, w) < U(o^*_{\mathrm{U}}, w)$, for all w in $\mathcal{W}$
>
> then
>
> (iii) o is irrational for an agent who knows that her utility function lies in $\mathcal{U}$.

It is this decision-theoretic principle that we require if we are to establish Probabilism from Epistemicism.

5.2 Supervaluationism

Second, Supervaluationism. Suppose we replace Brier Alethic Accuracy with Super- valuationism. Again, we must alter Dominance if we are to establish Probabilism from this. Dominance presupposes that there is a single determinate utility function (p.73) that the agent knows. In the previous section, we saw what happens if we relax the presupposition that the agent knows the unique determinate utility function. In this section, we ask what happens when we relax the presupposition that there is a unique determinate function to know. We consider the case in which the utility function is indeterminate, but we know the limits of the indeterminacy: that is, we know the set of acceptable precisifications of the utility function. Indeed, in the case in which the options are credence functions, we know that the acceptable precisifications are the continuous and additive strictly proper inaccuracy measures. What is the dominance principle that governs this situation? We might be tempted by the following decision- theoretic principle, which is analogous to Epistemic Dominance from the previous section. It is also the principle that we would need to derive Probabilism from Supervaluationism:

Supervaluationist Dominance Suppose $\mathcal{O}$ is a set of options, $\mathcal{W}$ is a set of possible worlds, and $\mathcal{U}$ is a set of utility functions on $\mathcal{O}$ and $\mathcal{W}$. Suppose *o* is in $\mathcal{O}$. Then, if, for all $\mathfrak{U}$ in $\mathcal{U}$, there is o^*_{U} in $\mathcal{O}$ such that

(i) $\mathrm{U}(o^*_{\mathrm{U}}, w) < \mathrm{U}(o, w)$, for all *w* in ??, and

(ii) there is no o_{U} such that $\mathrm{U}(o'_{\mathrm{U}}, w) < U(o^*_{\mathrm{U}}, w)$, for all *w* in $\mathcal{W}$,

then

(iii) *o* is irrational for an agent who knows that the acceptable precisifications of the indeterminate utility function are in $\mathcal{U}$.

According to this principle, all that is required for an option to be irrational is that it be super-true—that is, true on all acceptable precisifications—that the option is dominated. This is precisely the situation of each non-probabilistic credence function with respect to the additive and continuous strictly proper inaccuracy measures—according to all of those inaccuracy measures, it is dominated. Thus, if we accept Supervaluationist Dominance, we get Probabilism via Theorem 4.3.4.

How plausible is Supervaluationist Dominance? Only as plausible, it seems to me, as the following natural argument in its favour—and we will see that this is invalid. According to supervaluationist semantics, a sentence is true *tout court* iff it is true relative to each of the acceptable precisifications—we might call this the *supervaluationist biconditional*. Thus, ‘This pen is red’ is true *tout court* iff ‘This pen is red’ is true relative to all acceptable precisifications of the concept Red. Now, suppose that, relative to each acceptable precisification of the utility function, option *o* is dominated by an undominated option. Then, relative to this precisification, option *o* is irrational for our agent. Thus, relative to all acceptable precisifications, *o* is irrational. Thus, it is true *tout court* that *o* is irrational.

Unfortunately, that argument is invalid. Although many informal presentations of supervaluationist semantics in various areas assert the supervaluationist biconditional as true of all sentences, it is not: the inference from truth on all acceptable precisifications to truth *tout court* is not valid. Consider, for instance, the following inference: (p.74) Relative to every acceptable precisification of the concept Red, the concept Red is not vague (where to be vague is to have more than one acceptable precisification); thus, it is true *tout court* that the concept Red is not vague. In this inference, the premise is true, while the conclusion is false. The point is that, when we apply supervaluationist semantics to a particular area of language, we must specify in advance exactly the sentences to which the supervaluationist biconditional applies. When we apply that semantics to our talk of accuracy in epistemology, we need not say that it applies to that part of our talk that deals with the notion of rationality.

We need not, but we could. Should we? I think not. Consider the following case, which is a version of Sandwich adapted to consider the decisions of a group of agents. This might seem irrelevant to our purposes at first, since we are concerned with the decisions of a single agent, but we'll see its relevance shortly.

> **Sandwich*** Two friends, Rachel and Bert, must choose between three sandwich options: they both get Marmite, they both get cheese, or they both get Pater Peperium—those are their only options. Both friends are pretty indifferent to cheese. Rachel loves Marmite and hates Pater Peperium, whilst Bert loves Pater Peperium and hates Marmite. Thus, their preference orderings are as follows (even once we factor in that they care to some extent about the other being happy!):
>
> - Rachel: Marmite < Cheese < Pater Peperium
> - Bert: Pater Peperium < Cheese < Marmite

What should they collectively choose? Their collective utility function is indeterminate: the value that it assigns to the cheese option is fixed, but the values it assigns to the Marmite option and the Pater Peperium option are indeterminate. Nonetheless, on all acceptable precisifications—which, in this case, are Rachel's and Bert's individual utility functions—the cheese option is dominated. Nonetheless, as in the epistemic example Sandwich considered above, it seems wrong to say that that option is ruled out as irrational in their collective choice.

Of course, collective decision-making amongst a group of agents each of whom has a single determinate utility function is different from individual decision-making for an agent with indeterminate utilities. We shouldn’t expect everything we can say about one to be true of the other. Nonetheless, an agent with indeterminate utilities is often best understood as a collective of agents each of whom takes a different acceptable precisification of the actual agent’s indeterminate utility function as their utility function.[3] So if Supervaluationist Dominance fails for the groups of agents making collective decisions—whose utility functions are indeterminate if the utility functions of the group members differ—this should make us less confident in it as a principle that governs individuals who have indeterminate utilities.

(p.75) 5.3 Subjectivism

Finally, Subjectivism. Subjectivism agrees with Supervaluationism that there is no unique determinate objective measure of inaccuracy, but instead of reading the necessary conditions on legitimate determinate inaccuracy measures as circumscribing the acceptable precisifications of this indeterminate objective notion, as Supervaluationism does, Subjectivism reads them as circumscribing the permissible subjective measures of inaccuracy that an individual agent might adopt. If, in the argument for Probabilism, we replace Brier Alethic Accuracy with Subjectivism, a limited version of the argument goes through. This limited version establishes that Probabilism holds for agents whose attitudes towards inaccuracy are so rich and detailed that they specify a unique legitimate measure of inaccuracy, such as the Brier score or the additive logarithmic inaccuracy measure or the additive spherical inaccuracy measure. For such an agent, if she violates Probabilism, there is a probabilistic credence function that dominates her credence function relative to her single determinate measure of inaccuracy and that credence function is immodest relative to that same measure. On the other hand, if an agent's attitudes towards inaccuracy are not so rich that they specify a unique measure of inaccuracy, then the Subjectivist is in the same position as the Supervaluationist—for any non-probabilistic credence function, it is super-true that it is dominated, but there is no alternative credence function such that it is supertrue that this alternative dominates the original credence function.

Now, there are clearly agents whose attitudes concerning inaccuracy are not rich or detailed enough to specify a unique determinate measure of inaccuracy. But that would not be a problem if we were able to argue that such an agent is irrational. After all, Probabilism claims only that a *rational* agent will have a probabilistic credence function. If it is a requirement of rationality that your attitudes towards inaccuracy specify a unique measure, and if the arguments given so far show that Probabilism holds for all agents with such specific attitudes, then Probabilism holds quite generally. Anyone who violates it either (i) has a single inaccuracy measure and is immodestly dominated relative to it and is thereby irrational, or (ii) has no single inaccuracy measure and is thereby irrational.

Unfortunately, however, I don't think there could be any rational requirement to have attitudes concerning inaccuracy that are so detailed and rich that they specify a unique determinate measure of it. After all, what could possibly ground such a requirement? Subjectivism admits that there are simply not enough objective constraints on the notion of inaccuracy to specify a unique measure. How could we fault an agent whose only attitudes to inaccuracy are those that we have seen to be objectively mandated? I don't think that we can. So, like Supervaluationism and Epistemicism, I think Subjectivism fails to give a wholly satisfactory answer to the Bronfman objection.

I conclude, then, that the Bronfman objection is decisive against an accuracy argument that counts as legitimate, for instance, both the additive logarithmic and (p.76) additive spherical inaccuracy measures (which we met at the beginning of this chapter) whether that means that they are acceptable precisifications of an indeterminate objective notion of inaccuracy (Supervaluationism), or epistemically possible determinate objective measures (Epistemicism), or permissible determinate subjective measures that individual agents might have (Subjectivism). In response, I will assume Symmetry in much of what follows. In combination with the other conditions on legitimate inaccuracy measures considered above, Symmetry gives us Brier Alethic Accuracy. And that, in turn, ensures that, for each non-probabilistic credence function, there is a probabilistic credence function that dominates it and is immodest relative to *all* legitimate measures of inaccuracy. It ensures that because, by Brier Alethic Accuracy, there is essentially only one legitimate measure of inaccuracy, namely, the Brier score. Thus, the argument for Probabilism goes through.

Notes:

([1]) As we saw in Chapter 3 above, Joyce provided different necessary conditions for being a legitimate inaccuracy measure—different, that is, from the conditions we provided in the previous chapter. Nonetheless, the Bronfman objection still applies when we assume Joyce's conditions.

([2]) A probabilistic credence function is $\mathfrak{I}$-*immodest* if it is not even moderately $\mathfrak{I}$-modest. That is, if it expects every other credence function to be more inaccurate than it expects itself to be relative to the inaccuracy measure $\mathfrak{I}$.

([3]) This is akin to Joyce's analogy between a single agent with imprecise credences and a committee of agents each with precise credences (Joyce, 2010a).