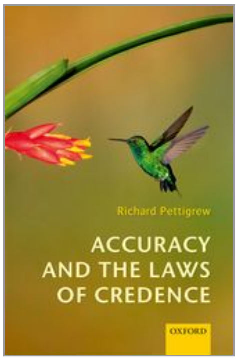

Accuracy and the Laws of Credence
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Where next for epistemic utility theory?

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Abstract and Keywords

This final chapter surveys various ways in which epistemic utility theory is being developed that go beyond the material covered in this book. The chapter also suggests a variety of research topics that are still to be fully explored.

Keywords: Infinitesimals, epistemic consequentialism, infinite algebras, self-locating propositions, risk aversion, incoherence

We have now considered accuracy-based arguments for Probabilism, the Principal Principle (and its variants), the Principle of Indifference (and its variants), and Conditionalization (at least as a synchronic principle governing updating plans). These are the central principles of credal rationality from which Bayesian epistemology is built. Without the Principle of Indifference, we have orthodox subjective Bayesianism; with that principle, we have its more restrictive objective cousin. In the case of each of these principles, our argument in its favour begins with Veritism, a principle that identifies accuracy—cashed out as proximity to the ideal credence functions—as the sole fundamental source of epistemic value. We then identify the ideal credence function—throughout, the omniscient credence function—and we characterize the legitimate ways of measuring the proximity of one credence function to another—throughout, squared Euclidean distance, though we have noted that all of our arguments are robust under taking any additive Bregman divergence to measure proximity. This characterizes the Brier score as the sole legitimate inaccuracy measure—though our arguments are robust under taking any additive and continuous strictly proper inaccuracy measure. We then combine this with a range of decision-theoretic principles— dominance principles, chance dominance principles, risk-sensitive principles, and expected utility maximization principles—and we derive the consequences. They are the familiar rational requirements of Bayesian epistemology listed above. These arguments can serve two purposes: assuming that Veritism is true, they can be used to support the Bayesian principles; assuming that the Bayesian principles are amongst the central principles of good reasoning, they can be used to support Veritism by eliminating the troubling objection that it is unable to explain the rational requirement of these principles, which can apparently be explained by the evidentialist.

This is where we will leave the project of this book, which has been to explore the consequences of an accuracy-first—indeed, an accuracy-only—epistemology. However, it isn't the end of the project. Indeed, it seems to be really just the end of the beginning—we have justified the basic principles in the most straightforward cases. Below, I describe some of the ways in which the project may be extended, or is already being extended.

(p.222) 16.1 Infinitely many credences

Throughout the book, we have assumed that the set $\mathscr{F}$ of propositions to which our agent assigns credences—that is, her opinion set—is finite. However, some will take this to be highly restrictive. If I know nothing about the bias of a coin, surely it is possible for me to have credences in each of the propositions *The bias of the coin is* θ, for all real numbers θ between 0 and 1. Or, if I am told that Hera has a favourite finite integer, surely it is possible for me to have credences in each of the propositions *Hera's favourite number is n*, for all finite integers *n*. How do the arguments presented in this book fare when we lift our self-imposed restriction and permit such opinion sets? Can we still justify Probabilism? Can we further justify the principle of Countable Additivity? As noted in Chapter 9, there are typically uncountably many possible current chance functions. Thus, if we allow $\mathscr{F}$ to be uncountably infinite, it would be possible for our agent to have credences in each possible current chance hypothesis. Would we still be able to establish the Temporal Principle in that case? What about the Principle of Indifference? Note that the Principle of Indifference, in the form we stated it above, and Countable Additivity are incompatible if $\mathscr{W}_{\mathscr{F}}$ is countably infinite, irrespective of accuracy considerations. This is the notorious problem of countably infinite fair lotteries that de Finetti raised for Countable Additivity—such a lottery is often known as a *de Finetti lottery* (de Finetti, 1974). For instance, if I know that Hera has a favourite finite integer, but no more, the Principle of Indifference and Finite Additivity require that I assign credence 0 to each proposition *Hera's favourite number is n*. But I must also assign credence 1 to the countably infinite disjunction of those propositions if I am to respect my evidence. Thus, I violate Countable Additivity. Finally, what if the agent deciding how she plans to update knows that her evidence will come from an infinite partition? Does Plan Conditionalization follow all the same?

In fact, in the last case, we know the answer already, due to work by Easwaran (2013) and Huttegger (2013). The justifications we gave for Plan Conditionalization and indeed for the Generalized Reflection Principle in Chapter 14 can be adapted to the infinite case. Huttegger, in particular, appeals to results by Banerjee et al. (2005) concerning Bregman divergences between credence functions on infinite opinion sets. These may well help us to adapt the accuracy-based justifications for the other principles of rational credence considered here. However, they assume already that the credence functions are probabilistic.

16.2 Infinitesimal credences

A number of philosophers have argued that, if we permit the set of propositions to which our agents assign credences to be infinite, we have to expand the possible values that those credences might take in order to save Regularity and possibly something akin to Countable Additivity (Lewis, 1980; Skyrms, 1980). The de Finetti lottery mentioned in the previous section is often used to motivate this claim (Wenmackers (p.223) & Horsten, 2013). How are we to measure inaccuracy when credences are allowed to take infinitesimal values—that is, values that, when multiplied by any natural number, do not exceed 1? What principles of rational infinitesimal credences can be justified using accuracy-based arguments? Benci et al. (2013) propose an axiomatization of infinitesimal probabilities—is there an accuracy-based argument in their favour along the lines of our accuracy dominance argument from Part I?

16.3 Self-locating propositions

In Part II, we were forced to consider agents with credences in temporally self-locating (or temporally centred) propositions in order to state the correct chance-credence principle, namely, the Evidential Temporal Principle. Self-locating propositions are those—such as *The current chance of heads is 60%* or *Cleo will be in France next week*—whose truth value can change from one time to another. Such propositions pose significant problems for the principles of rational credence. Adam Elga's Sleeping Beauty puzzle raises problems for our usual rules of update (Elga, 2000); his Dr Evil puzzle draws unsettling conclusions from the Principle of Indifference (Elga, 2004); see Titelbaum (2013) for an overview. Kierland & Monton (1999) apply an accuracy-based approach to the Sleeping Beauty case, and conclude that the two rival positions—Halfers and Thirders—can be derived from different accounts of the accuracy-related quantity that an epistemic agent wishes to maximize: expected average lifetime accuracy or expected total lifetime accuracy. How do these two approaches affect other puzzles in the literature on self-locating belief? Can they be used to adjudicate between the rival theories of update (Moss, 2012; Meacham, 2008; Titelbaum, to appear)?

16.4 Risk-sensitive decision principles

In Part IV, we considered the consequences of Veritism in the presence of certain principles of decision theory that encode attitudes to risk. In each case, the principle applied only to an agent at the beginning of her epistemic life, before she had set her credences. Indeed, we might think of such agents as using these principles to select their initial credences. However, we might consider an agent who remains risk-averse even after she has set her credences. Thus, we might consider the consequences of Veritism in the presence of a risk-sensitive decision-theoretic principle that governs an agent with credences. The most plausible and general such principle belongs to Lara Buchak's risk-weighted expected utility theory (Buchak, 2014b, a). According to Buchak, an agent has a probabilistic credence function c (defined over worlds in $\mathcal{W}$), a utility function $\mathfrak{U}$ (defined for options in $\mathcal{O}$ and worlds in $\mathcal{W}$), and a risk function $\mathfrak{r} : [0,1] \to [0,1]$, that is used to transform an agent's credences in order to give more weight than the credences alone give to the outcomes with lower utility (if the agent is (p.224) risk-averse) or to the outcomes with higher utility (if the agent is risk-seeking). Buchak requires r to be continuous, with $r(0) = 0$ and $r(1) = 1$.

Suppose we are evaluating an option o in $\mathcal{O}$. And suppose $\mathcal{W} = \{w_1, \ldots, w_n\}$. And suppose, without loss of generality, that $\mathfrak{U}(o, w_1) \leq \ldots \leq \mathfrak{U}(o, w_n)$. Then the expected utility of o by the lights of probabilistic credence function c is:

$$\mathrm{Exp}_{\mathrm{U}}(o|c) = \sum_{i=1}^{n} c(w_i)\mathrm{U}(o, w_i)$$

And this can be rewritten as follows:

$$\mathrm{Exp}_{\mathrm{U}}(o|c) = \mathrm{U}(o, w_1) + \sum_{i=2}^{n} c(w_i \vee \ldots \vee w_n)[\mathrm{U}(o, w_i) - \mathrm{U}(o, w_{i-1})]$$

Thus, the expected utility of an option o by the lights of c is obtained as follows: first, take the utility that o is guaranteed to give you, namely, the lowest utility; second, add to that the amount of extra utility you would get if it were to give the second lowest utility, weighted by the probability that you'll get at least that; and so on.

Now, given a risk function $\mathfrak{r}$, Buchak defines the risk-weighted expected utility as follows:

$$\mathrm{RExp}_{\mathrm{U},r}(o|c) = \mathrm{U}(o, w_1) + \sum_{i=2}^{n} \mathfrak{r}(c(w_i \vee \ldots \vee w_n))[\mathrm{U}(o, w_i) - \mathrm{U}(o, w_{i-1})]$$

Thus, the risk-weighted expected utility of an option o by the lights of c is obtained as follows: first, take the utility that o is guaranteed to give you, namely, the lowest utility; second, add to that the amount of extra utility you would get if it were to give the second lowest utility, weighted by the probability that you'll get at least that transformed by the risk function; and so on.

Thus, if $\mathfrak{r}(x) = x$ for all $0 \leq x \leq 1$, then risk-weighted expected utility coincides with expected utility. If $\mathfrak{r}(x) < x$ for all $0 \leq x \leq 1$, on the other hand, then the outcomes in which o has lower utility are given greater relative weight than the outcomes in which it has higher utility. Thus, an agent with such a risk function is risk-averse. If $\mathfrak{r}(x) > x$ for all $0 \leq x \leq 1$, then it is the outcomes in which o has higher utility that are given greater relative weight. Thus, an agent with such a risk function is risk-seeking.

Now, we might ask what happens when we replace all talk of expected utility above with risk-weighted expected utility. The short answer is that problems arise pretty quickly. Take, for instance, the definition of strict propriety: An inaccuracy measure $\mathfrak{I}$ is strictly proper if, for all probabilistic c and $c' \neq c$,

$$\mathrm{Exp}_I(c|c) < \mathrm{Exp}_I(c'|c)$$

Thus, given a risk function $\mathfrak{r}$, we say that $\mathfrak{I}$ *is strictly* $\mathfrak{r}$*-proper* if, for all probabilistic c and $c' \neq c$, (p.225)

$$\mathrm{RExp}_{I,\mathrm{r}}(c|c) < \mathrm{RExp}_{I,\mathrm{r}}(c'|c)$$

Now, suppose $\mathfrak{r}(x) = x$ for some $\frac{1}{2} < x \leq 1$. Then let us consider the set of probability functions p_z on $\mathrm{F} = \{X, \overline{X}\} : p_z(X) = z$ and $p_z(\overline{X}) = 1 - z$. Then, if $\frac{1}{2} < z$, then the worst-case for p_z is if $\overline{X}$ is true. Thus,

$$\begin{aligned}
\mathrm{RExp}_{I\mathrm{r}}(p_z|p_x) &= I(p_z, \overline{X}) + \mathrm{r}(x)[I(p_z, X) - I(p_z, \overline{X})] \\
&= (1 - \mathrm{r}(x))I(p_z, \overline{X}) + \mathrm{r}(x)I(p_z, X) \\
&= p_{\mathrm{r}(x)}(X)I(p_z, X) + p_{\mathrm{r}(x)}(\overline{X})I(p_z, \overline{X}) \\
&= \mathrm{Exp}_I(p_z|p_{\mathrm{r}(x)})
\end{aligned}$$

However, if $\mathfrak{I}$ is strictly proper, we know that $\mathrm{Exp}_{\mathfrak{I}}(p_z|p_{\mathfrak{r}(x)})$ will be minimized (as a function of z) for $z = \mathfrak{r}(x)$. And thus $\mathrm{RExp}_{\mathfrak{I},\mathfrak{r}}(p_z|p_x)$ will be minimized (as a function of z) for $z = \mathfrak{r}(x)$. But recall $\mathfrak{r}(x) \neq x$. Thus, $\mathrm{RExp}_{\mathfrak{I},\mathfrak{r}}(p_z|p_x)$ is not minimized (as a function of z) for $z = x$, as is required for $\mathfrak{I}$ to be strictly $\mathfrak{r}$-proper. So, if $\mathfrak{I}$ is strictly proper, then it is not strictly $\mathfrak{r}$-proper. And we can run an analogous argument if $\mathfrak{r}(x) \neq x$ for $0 < x \leq \frac{1}{2}$.

Thus, none of our familiar inaccuracy measures—namely, the strictly proper ones— will be strictly $\mathfrak{r}$-proper for any non-trivial $\mathfrak{r}$. So, if we are to combine Veritism with Buchak's risk-weighted expected utility theory, we will need a whole new set of inaccuracy measures.

For myself, while I find Buchak's theory the most plausible amongst the so-called non-expected utility theories, I nonetheless reject it. As I mentioned in Chapter 2 above, there is an accuracy-based argument in favour of standard expected utility theory as the correct theory of rational decision for agents with credences (de Finetti, 1974; Pedersen & Glymour, 2012; Pettigrew, to appear a). I side with the conclusion of that argument.

16.5 Measuring degrees of incoherence

In Part I of this book, we showed that any incoherent—that is, non-probabilistic— credence function is accuracy-dominated by an immodest probabilistic credence function. This, we claimed, renders it irrational. However, it seems that some nonprobabilistic credence functions are more incoherent and thus more irrational than others. If Rachel has credence 0.5 in *Rain* and credence 0.51 in $\overline{Rain}$, while Phil has 0.78 in *Rain* and 0.98 in $\overline{Rain}$, it seems that Phil's rational failure is the more egregious because his credences are the more incoherent. Julia Staffel proposes to measure the incoherence of a credence function by looking to certain aspects of their Dutch Book vulnerability (Staffel, 2015). And she notes a result by De Bona & Finger (2015) that relates this to a measure that involves the distance between credence functions. This latter measure takes the coherence of a credence function c to be the distance between c and π_c, where π_c is the probabilistic credence function that is closest (p.226) to c relative to the Manhattan or city block distance measures—recall: $\mathfrak{d}^1(c,c') = \| c - c'\|_1 = \sum_{X \in \mathcal{F}} |c(X) - c'(X)|$. A natural question arises: Why use the Manhattan distance measure? Why not use an additive Bregman divergence? Staffel defends the Manhattan distance, but gives no positive reasons to prefer it to any additive Bregman divergence. Thus, it is important to know what the consequences are of using different measures of distance—Manhattan, squared Euclidean Distance, other additive Bregman divergences—to measure incoherence. On what do they disagree? Do we have intuitions that favour one side or the other in such cases of disagreement?

16.6 Other doxastic states

Throughout, we have focussed on the credal states of an agent, though in Section 4.2 we touched briefly on the question of accuracy measures for full beliefs. It is natural to think that, for any doxastic state, Veritism holds and the sole fundamental source of epistemic value is their accuracy. If that's the case, the strategy of this book— in which we characterize the legitimate inaccuracy measures and apply decision-theoretic principles to establish principles of rationality—should be applicable to other doxastic states.

As mentioned above, Hempel (1962), Easwaran (to appear), and Fitelson (ms) have considered the equivalent of Part I of this book in the case of full beliefs, using the natural additive inaccuracy measures, which are proper in the relevant sense. And Fitelson (ms) also considers it in the case of comparative confidence, this time using the only strictly proper additive inaccuracy measure that is available in that context. Furthermore, a number of philosophers have considered the questions of Part I in the case of imprecise credences (Schoenfield, to appear; Konek, to appear). However, in this latter case, an impossibility result shows that there is no strictly proper inaccuracy measure (Seidenfeld et al., 2012).

A number of questions arise: What happens if we look to non-additive inaccuracy measures in the case of full belief and comparative confidence? Are these measures more appropriate in the non-credal cases? And what are the analogues of Parts II, III, and IV of this book for these other doxastic states?

16.7 Epistemic consequentialism

In Part I, we argued for Probabilism by appealing to Veritism and the dominance principle from decision theory, which says, roughly, that if one option is guaranteed to be better than another (and no option is guaranteed to be better than it), then the latter option is irrational. However, as has been well-known since the early days of decision theory, this principle is only correct in certain situations: it is only correct when the options between which we are choosing are independent of the possible states of the world over which we define our utilities for those options and our credences— that is, it is only correct under the assumption that is sometimes known as *act-state* (p.227) *independence*. Here's an example that illustrates why. Suppose that I will sit my driving test next week. I must choose whether to practise or not. I appeal to dominance reasoning as follows. I assign 100 utiles to passing the test, and 0 utiles to failing; I assign -10 utiles to practising, and 50 utiles to not. I take the options to be *Practise* and *Don't Practise* and the states of the world to be *Pass* and *Fail*. Then this table gives my utilities:

	Pass	*Fail*
Practise	$100 - 10 = 90$	$0 - 10 = -10$
Don't Practise	$100 + 50 = 150$	$0 + 50 = 50$

Clearly, *Don't Practise* dominates *Practise*. Thus, according to the sort of dominance reasoning we employed in Part I, I am irrational if I practise. But that is clearly the wrong result. The reason is that such dominance reasoning applies only in situations in which choosing an option does not affect which state of the world obtains. Although *Don't Practise* has greater utility if I pass, choosing not to practise makes it much less likely that I will pass.

How does this affect our argument for Probabilism? It means that it will go through only for agents for whom adopting a credence function does not affect the truth of the propositions on which that credence function is defined, and thus does not affect the inaccuracy of the credence function. Now, one response to this objection is to note that it is in fact very rare for our credences to affect the world in this way. For instance, in all good scientific investigation, we might hope, the credences of the investigator have no effect on whatever aspect of the world they are investigating. And similarly in everyday life: if I consider my credences that it will rain tomorrow, that there is food in fridge, that seven plus five is twelve, none of them will affect the truth of those propositions nor of any others about which I have an opinion. Thus, we might concede the objection, and note simply that our justification of Probabilism is restricted, but not in any very severe way.

However, we can extend the objection by asking what are the consequences of Veritism when combined with the correct decision principle for those cases in which our credal state is not independent of the world. If the recommendation is absurd, then this counts strongly against Veritism—a justification of a principle from a particular premise is undermined if that premise can, in some other situation, be used to justify an absurdity. Just such an objection has been raised by Jenkins (2007), Greaves (2013), Berker (2013), Caie (2013), and Carr (ms). Here is an example slightly adapted from Hilary Greaves that illustrates the point clearly.

> **Epistemic Imps** Maxine is standing in the Garden of Epistemic Imps. Graham is sitting in front of her. There are four children called Anna in the house.
>
> - If Maxine has credence greater than 0 that Graham is in front of her, then exactly two of the Annas will come out into the garden, but she doesn't know which two.
> - If Maxine has credence 0 that Graham is in front of her, then all four Annas will come out into the garden.

> (p.228) Maxine has credences in five propositions: *Graham* says that Graham is sitting in front of her; $Anna_1$ says that Anna_1 will come out into the garden;…; $Anna_4$ says that Anna_4 will come out into the garden. What credences in these five propositions is it rational for Maxine to have?

Intuitively, Maxine should be certain that Graham is in front of her. Thus, $c_{\text{Maxine}}(Graham) = 1$. But she then knows that two of the Annas will come out, but she doesn't know which. So she should assign credence 0.5 to each. Thus, $c_{\text{Maxine}}(Anna_i) = 0.5$, for $1 \leq i \leq 4$. Thus, at the world w at which she adopts c_{Maxine}, Maxine will be perfectly accurate in her credence concerning Graham's whereabouts, but quite inaccurate in each of her four credences concerning the Annas.

However, suppose Maxine can bring herself to have minimal credence that Graham is in front of her. Thus, $c'_{\text{Maxine}}(Graham) = 0$. Then she will know that all the Annas will come out, so she should be certain of all of those propositions. Thus, $c'_{\text{Maxine}}(Anna_i) = 1$, for $1 \leq i \leq 4$. Then, at the world w' at which she adopts c'_{Maxine}, she will be maximally inaccurate in her credence about Graham's location, but maximally accurate in her credences about the Annas. Indeed, it turns out that, at least on the Brier inaccuracy measure, c_{Maxine} is more inaccurate at w than c'_{Maxine} is at w'. The loss of accuracy that results from assigning minimal credence to a truth— namely, *Graham*—is outweighed by the gain in accuracy obtained by being certain of four truths.

Now, a natural decision principle in these situations, which is analogous to Dominance, is this: If one option is better in all worlds in which it is adopted than another option is in all worlds in which it is adopted, then the latter is irrational. This principle, together with Veritism and the Brier inaccuracy measure, says that Maxine should sacrifice the accuracy of her credence in *Graham* in order to gain greater accuracy in her credences in $Anna_i$. She ought to trade-off accuracy in one proposition for greater accuracy in many. She ought not to adopt c_{Maxine}. Yet intuitively this is wrong. Berker puts the point by saying that such a trade-off does not respect the separateness of propositions, much as certain crude forms of utilitarianism fail to respect the separateness of persons and allow us to trade-off the utility of one individual in order to obtain greater utility for a number of others.

How are we to respond to this version of the objection?[1] Konek & Levinstein (to appear) have argued that, in fact, the dominance principle to which we appealed in Part I is the correct dominance principle even when adopting a credal state that affects the truth values of the propositions to which the state assigns credences. According to that dominance principle, c_{Maxine} is not ruled out as irrational, so our intuitive verdict is saved. They appeal to the apparent fact that the sorts of actions between which we use practical decision theory to choose have a different direction of fit from the states between which we use epistemic decision theory to arbitrate: credal states have (p.229) mind-to-world direction of fit, whereas practical actions have world-to-mind direction of fit. I'm not convinced that the direction of fit distinction is robust enough to do the work required of it. Rather, I claim that we should simply bite the bullet and accept the consequences of Veritism when coupled with the principles of practical decision theory, whatever they are. To make that argument, I need an error theory, since this is clearly against the intuitions of many people. I leave that for another time.

These are only some of the future directions that research in this area—the area, broadly speaking, of epistemic utility theory—might take. There are many others. It has proved fertile philosophical ground already. (p.230)

Notes:

([1]) Cf. (Campbell-Moore, 2015) for a response particularly to Caie (2013).