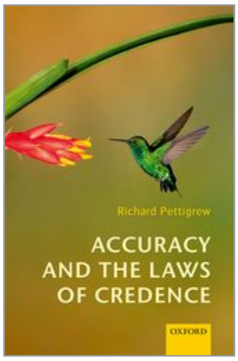

Accuracy and the Laws of Credence
Richard Pettigrew

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Measuring accuracy: existing accounts

Richard Pettigrew

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Abstract and Keywords

This chapter surveys the existing attempts to characterize the legitimate inaccuracy measures. It considers characterizations due to James M. Joyce as well as the characterization offered by Hannes Leitgeb and Richard Pettigrew. In each case, some of the assumptions required for the characterization are seen to be unjustified.

Keywords: Scoring rule, inaccuracy measure, James M. Joyce, epistemic dilemmas

In Chapter 1, we showed how to extend the accuracy argument for No Drop to give an argument for Probabilism. In Chapter 2, we strengthened the argument by replacing the implausibly strong version of the decision-theoretic dominance principle (Dominance) with a weaker and more plausible version (Undominated Dominance) and then by a version that applies only when the options in question are credence functions (Immodest Dominance). In this chapter and the next, we consider ways in which we might strengthen the argument further by weakening premise (II*p*), namely, Brier Alethic Accuracy. In that premise, we assume not only that the Brier score is a legitimate measure of the accuracy of a credence function but moreover that it is the only legitimate measure. If we hadn't assumed the latter also, the argument would be invalid, for the premises would have left open the possibility that there might be another legitimate measure of inaccuracy relative to which some non-probabilistic credence functions aren't dominated, or are dominated only by credence functions that are themselves dominated or that are extremely modest.

In this chapter and the next, we explore various ways in which we might characterize the legitimate ways of measuring inaccuracy. All of them permit the Brier score; some of them (including my favoured characterization) also mandate it, thereby endorsing Brier Alethic Accuracy and premise (II*p*); and others allow a wider range of alternatives.

While we will learn some lessons from considering the characterizations we do, and our favoured characterization will share some features in common with some of them, the reader can skip to that favoured characterization—given in Chapter 4—without great loss.

3.1 Joyce on convexity

Let's begin with Jim Joyce's characterization of the legitimate accuracy measures in his original 1998 paper. Recall our initial statement of Brier Alethic Accuracy in the Introduction: we split the claim into three more basic claims, namely, Perfectionism, Vindication, and Squared Euclidean Distance. Some of the characterizations we'll (p.32) consider take a similar route, assuming Perfectionism and Vindication and then arguing that the distance between two credence functions ought to be measured in a particular way. Joyce does not take this route. Rather, he characterizes the inaccuracy measures directly by stating properties that they must have, rather than by characterizing the distance measures that give rise to them.[1]

Some of these properties are reasonably innocuous. One consequence of Joyce's Structure axiom (Joyce, 1998, 591), which will recur in other characterizations, is the following:

> **Continuity** If $\mathfrak{I}$ is a legitimate inaccuracy measure, then $\mathfrak{I}(c, w)$ is a continuous function of c, for all worlds w.

Very roughly: there are no 'jumps' in inaccuracy; a small change in one's credences should never give rise to a large change in one's inaccuracy. I will assume this axiom in my own favoured characterization, so I will leave it until then to argue for it—see Section 4.2.

Other properties are more controversial. We will focus our attention on Joyce's Weak Convexity axiom (Joyce, 1998, 596), which says that, for each world, the inaccuracy of a credence function at that world is a strictly convex function of it. That is:

> **Weak Convexity** If $\mathfrak{I}$ is a legitimate inaccuracy measure and c and c' are distinct credence functions that are equally inaccurate at w, then the equal mixture of c and c' is less inaccurate than either c or c'.
>
> That is: If $\mathfrak{I}(c, w) = \mathfrak{I}(c', w)$, then $I(\frac{1}{2}c + \frac{1}{2}c', w) < I(c, w) = I(c', w)$.

Now, the Brier score certainlyhas this property, as we can see from Figure 3.1. But why assume it is true of every legitimate measure of inaccuracy?

Joyce gives the following philosophical argument for Weak Convexity:

> Weak Convexity is motivated by the intuition that extremism in the pursuit of accuracy is no virtue. It says that if a certain change in a person's degrees of belief does not improve accuracy then a more radical change in the same direction and of the same magnitude should not improve accuracy either. Indeed, this is just what the principle says. (Joyce, 1998, 596)

However, there are two problems with Joyce's argument: first, an inaccuracy measure that violates Weak Convexity in fact no more encourages extremism than one that satisfies it; second, while Weak Convexity does rule out the sort of situation that Joyce envisages in the passage just quoted, it also rules out others and Joyce's argument says nothing about why we should rule out those—that is, it does not say only what Joyce claims it says; it says something stronger. The latter point is due to Maher (2002). (p.33)

First point. Joyce writes:

> [T]he change in belief involved in going from c to c' has the *same direction* but a *doubly greater magnitude* than the change involved in going from c to m $[= \frac{1}{2}c + \frac{1}{2}c']$. This means that the former change is more *extreme* than the latter in the sense that, for every proposition X, both changes alter the agent's degree of belief for X in the same direction, either by moving it closer to one or closer to zero, but the c to c' change will always move $c(X)$ twice as far as the c to m change moves it. (Joyce, 1998, 596)

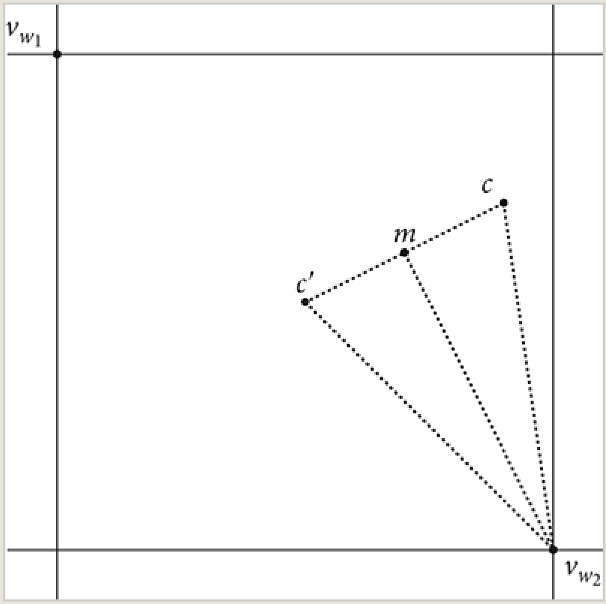

Figure 3.1 The Brier score satisfies Weak Convexity. This figures illustrates a particular instance of this. In the figure, c, c', and m are three credence functions defined on a two-element partition. They are represented by the following points in the unit square: $c = (0.9, 0.7)$, $c' = (0.5, 0.5)$, and $m = \frac{1}{2}c + \frac{1}{2}c' = (0.7,\ 0.6)$. m is the midpoint between c and c'. As we can see, $\mathfrak{B}(c, W_2) = \mathfrak{B}(c', W_2)$. Thus, by Weak Convexity, $\mathfrak{B}(c, W_2) = \mathfrak{B}(c', W_2) < \mathfrak{B}(m, W_2)$.

However, as we can see from Figure 3.1, moving closer to 0 or to 1 by double a given amount does not necessarily entail a greater move towards extremity than moving closer to 0 or to 1 by that amount. Indeed, such a move can just as often constitute a move *away* from extremity. In Figure 3.1, the move from c to c' in fact constitutes a greater move towards *moderation* and away from *extremity* than the move from c to m.

Moreover, just because m is more accurate than c or c' at w_2, this does not necessarily encourage a move from c to m and not to c' unless one knows that one is at world w_2. After all, as we can see from Figure 3.1, m is less accurate than c' at world w_1. And, if you know you are at world w_2, you ought to adopt v_{w2}, which is maximally accurate at w_2, not c, m, or c', which are not!

Second point. Joyce would like Weak Convexity to rule out the following sort of situation: An agent begins with credence function c, which has a certain level of inaccuracy; she then shifts her credences halfway towards c'—that is, to m, the equal mixture of c and c'—and her accuracy decreases; she then moves by the same amount and in the same direction—taking her from m to c'—and her accuracy increases again to its original level. And indeed Weak Convexity does rule out that sort of (p.34) situation. The problem is that it rules out the following situation as well: An agent begins with c, which has a certain level of inaccuracy; she then shifts her credences halfway towards c'—that is, to m, the equal mixture of c and c'—and her accuracy stays the same; she then moves by the same amount and in the same direction—taking her from m to c'—and her accuracy still remains the same. However, there seems to be nothing wrong with this situation and therefore no reason for us to rule it out. Or, if there is such a reason, Joyce's argument has not identified it. Thus, if Joyce's argument establishes anything, it establishes something weaker than Weak Convexity; it establishes that, for each world, the inaccuracy of a credence function at that world is a *convex* (but not necessarily *strictly convex*) function of it. But this is too weak to establish the central theorem of Joyce's paper, which underlies his accuracy argument for Probabilism.

This second point is due originally to Maher (2002). In much the same way as I have just done, Maher points out that Joyce's argument for Weak Convexity doesn't establish that principle. He then goes on to argue moreover that Weak Convexity is false. To do this, he appeals to an inaccuracy measure that he takes to be intuitively legitimate. It is known as the *absolute value measure*. It is generated by Perfectionism and Vindication along with the following distance measure, just as the Brier score is generated by Perfectionism and Vindication along with Squared Euclidean Distance:

$$\mathrm{d}^1(c, c') := \sum_{X \in \mathcal{F}} |c(X) - c'(X)|$$

(The distance measure $\partial^1(c, c')$ is sometimes written $\|c - c'\|_1$.) This is very similar to squared Euclidean distance, except that the difference between the credences in each proposition is taken, but not squared. It generates the following inaccuracy measure:

$$\mathfrak{A}(c, w) := d^1(v_w, c) = \| v_w - c \|_1 = \sum_{X \in \mathcal{F}} |v_w(X) - c(X)|$$

Maher points out that $\mathfrak{A}$ violates Weak Convexity, though it does satisfy the weaker condition that Joyce's argument for Weak Convexity might be taken to establish—for each world, the inaccuracy of a credence function at that world given by the absolute value measure is a convex (though not necessarily strictly convex) function of the credence function. He then points out that Joyce's Main Theorem—which generalizes the first part of De Finetti's Dominance Theorem (Theorem 1.0.2(i) above) to a broader class of inaccuracy measures—will not go through if he permits $\mathfrak{A}$ to be a legitimate measure of accuracy.

Indeed, it turns out that, relative to the absolute value measure, straightforward and natural probabilistic credence functions are in fact themselves dominated by nonprobabilistic credence functions that are themselves not dominated. Suppose the agent has credences only in three mutually exclusive and exhaustive propositions X_1, X_2, X_3—that is, her opinion set is $\{X_1, X_2, X_3\}$. Then it turns out that, when we measure inaccuracy using the absolute value measure, the probabilistic credence function that (p.35) assigns $\frac{1}{3}$ to each proposition is dominated by the non-probabilistic credence function that assigns 0 to each proposition, while the latter is undominated by anything.[2]

As we will see, ruling out the absolute value measure as a legitimate measure of inaccuracy in a way that does not beg any questions is one of the most difficult challenges facing any characterization of inaccuracy measures that will serve to underpin an accuracy argument for Probabilism. Indeed, in the next section, I consider a characterization of inaccuracy measures that is due to joint work I carried out with Hannes Leitgeb (Leitgeb & Pettigrew, 2010a), and my main objection to that characterization is that it rules out the absolute value measure essentially by fiat.

Before we move on to consider that characterization, let me raise one further worry about Joyce's characterization—it is a worry that we met already in Chapter 2. It is not so much a worry about the characterization itself; rather, it is a concern that the characterization cannot support a plausible argument for Probabilism. After all, the Main Theorem of Joyce's paper gives us only the resources to infer Probabilism if we assume Dominance from above. Recall, Joyce's Main Theorem generalizes only the first part of De Finetti's Dominance Theorem; that is, it generalizes Theorem 1.0.2(i). It does not include a generalization of the second part, Theorem 1.0.2(ii).[3] That is, Joyce shows only that, if $\mathfrak{I}$ is an inaccuracy measure that satisfies his conditions, any nonprobabilistic credence function is strongly $\mathfrak{I}$-dominated by a probabilistic credence function. If we assume only the weaker Undominated Dominance or Immodest Dominance, we cannot infer Probabilism (better: Joyce's theorem gives us no reason to think that we can infer Probabilism). Now, as I argued in Section 2.1 above, Dominance is too strong. Thus, whatever the merits of Joyce's characterization, it seems that it cannot be used in the service of an accuracy argument for Probabilism.

3.2 Leitgeb and Pettigrew on agreement and epistemic dilemmas

The next characterization I will consider is due to joint work I carried out with Hannes Leitgeb (Leitgeb & Pettigrew, 2010a). In fact, we gave three separate characterizations, but the problem with each is the same, so I will consider only one of them here.[4] (p.36) Leitgeb and I were interested in jointly characterizing two sorts of inaccuracy measure: a local one $\mathfrak{s}$ and a global one $\mathfrak{I}$.

- First, $\mathfrak{s}$. Given a proposition X, a credence x, and a world w, $\mathfrak{s}$ takes the omniscient credence in X at w and the credence x and gives a measure $\mathfrak{s}(v_w(X), x)$ of the inaccuracy of having credence x in X at w. Such a function is sometimes called a *scoring rule*. We will talk a lot more about scoring rules in Chapter 4. They are essentially measures of the inaccuracy of individual credences rather than entire credence functions.
- Second, $\mathfrak{I}$. As above, given a credence function c and a world w, $\mathfrak{I}(c, w)$ is a measure of the inaccuracy of having c at w.

Each of the three characterizations of $\mathfrak{s}$ and $\mathfrak{I}$ consists of three axioms: the first is a claim about what local and global inaccuracy supervene upon; the second and third make claims about how the local and global measures of inaccuracy should interact. Each narrows the field of legitimate inaccuracy measures to a small family of candidates, which consists of those global inaccuracy measures $\mathfrak{I}$ such that $\mathfrak{I}$ is a positive linear transformation of the Brier score, and those local inaccuracy measures (or scoring rules) $\mathfrak{s}$ such that $\mathfrak{s}$ is a positive linear transformation of the so-called *quadratic scoring rule* q.[5]

The first axiom consists of two claims about the way in which local and global inaccuracy measures are generated by an underlying distance measure:

Local Normality and Dominance The inaccuracy of a credence x in a proposition X at a world w is a strictly increasing function of the difference between x and the omniscient credence in X at w.

That is, if $\mathfrak{s}$ is a legitimate local inaccuracy measure (that is, a scoring rule), then there is strictly increasing $f : [0, \infty) \to [0, \infty)$ such that $f(0) = 0$ and

$$\mathfrak{s}(v_w(X),\ x) = f(|v_w(X) - x|).$$

Global Normality and Dominance[6] The inaccuracy of a credence function c at a world w is a strictly increasing function of the Euclidean distance between v_w and c when we consider those credence functions as vectors.

(p.37) That is, if $\mathfrak{I}$ is a legitimate global inaccuracy measure, there is a strictly increasing f: $[0, \infty) \to [0, \infty)$ such that

$$\mathfrak{I}(c, w) = f(\| v_w - c \|_2)$$

where

$$\| v_w - c \|_2 := \sqrt{\sum_{X \in \mathcal{F}} |v_w(X) - c(X)|^2}$$

The second half of this axiom is the claim with which I now take exception. I will explain why below. First, note that the Global Normality and Dominance entails the following:

Finiteness For all c and w, $\mathfrak{I}(c, w) < \infty$.

Elsewhere—and, in particular, in my own favoured characterization of the legitimate inaccuracy measures—we will allow credence functions to have infinite inaccuracy. This is not something about which our concept of accuracy contains much information. It is hard to assess this axiom for plausibility, since it is very much an assumption about the mathematical nature of the inaccuracy measure. It is hard to believe we have any intuitions one way or the other about whether a credence function can ever be infinitely inaccurate. However, one consideration that speaks against Finiteness is that the following popular measures of inaccuracy violate it:

- Logarithmic inaccuracy measure: $\mathfrak{L}(c, w) := -ln\ c(w)$.[7]
- Additive logarithmic inaccuracy measure: $$\mathfrak{LA}(c, w) := \sum_{X \in \mathcal{F}} -ln(|1 - v_w(X) - c(X)|)$$

Notice that $\mathfrak{L}(c, w) = \infty$ if c assigns minimal credence to w; and $\mathfrak{LA}(c, w) = \infty$ if c assigns minimal credence to a truth or maximal credence to a falsehood. My favoured characterization below does not assume

Finiteness, but it does entail it. (Note that $\mathfrak{L}$ cannot be used in an argument for Probabilism since it does not take into account all of the agent's credences when assigning inaccuracy; $\mathfrak{LA}$ rectifies this problem.)

The second axiom concerns the ways in which one can generate a global inaccuracy measure from a local one and vice versa:

> **Local and Global Comparability** For any function $f : [0,1] \to [0, \infty)$, the following holds: $\mathfrak{s}(v_w(X), x) = f(|v_w(X) - x|)$ is a legitimate local inaccuracy measure $\Leftrightarrow \mathfrak{I}(c, w) = f(\|v_w - c\|_2)$ is a legitimate global inaccuracy measure.

(p.38) When $\mathfrak{s}$ and $\mathfrak{I}$ are related in this way, we say that they are both *generated by f*.

The third axiom of the characterizations that Leitgeb and I gave is one of two claims about the ways in which $\mathfrak{s}$ and $\mathfrak{I}$ ought to agree. Here is the first:

> **Agreement on Inaccuracy** If a local and a global inaccuracy measure are both generated by the same function, the global inaccuracy of c at w ought to be the sum of the local inaccuracies at w of the credences that c assigns to the various propositions in $\mathcal{F}$.

That is, if $\mathfrak{s}(V_w(X), x) = f(|V_w(X) - x|)$ and $\mathfrak{I}(c, w) = f(\|v_w - c\|_2)$, then

$$I(c, w) = \sum_{X \in F} s(v_w(X), c(X))$$

for all c and w.

The idea is this: In the presence of a local and a global inaccuracy measure, an agent must decide which to use to evaluate her credence function. She might simply apply the global inaccuracy measure to her whole credence function, or she might work through each of her credences one by one, evaluating each using the local inaccuracy measure, and then summing them together. Leitgeb and I argued that, if these two ways of evaluating oneself failed to agree, the agent would face an unacceptable epistemic dilemma. Thus, we claimed, they must agree.

Notice also that Agreement on Inaccuracy is strictly stronger than an assumption that is often made about measures of inaccuracy, namely, Additivity, which says that a measure of the inaccuracy of a credence function is given by the sum of the inaccuracies of the individual credences.

Additivity If $\mathfrak{I}$ is a legitimate (global) measure of inaccuracy, then there is a local measure of inaccuracy $\mathfrak{s}$ such that

$$I(c, w) = \sum_{X \in F} S(v_w(X), c(X))$$

Agreement on Inaccuracy adds to Additivity the assumption that the global and the local measures of inaccuracy are generated by the same function *f*. I will assume Additivity in my own favoured characterization in Chapter 4, where I will explain why inaccuracy measures ought to obey it.

Leitgeb and I proved the following theorem (Leitgeb & Pettigrew, 2010a, Theorems 3 and 4):

Theorem 3.2.1 (**Leitgeb and Pettigrew**) *The following axioms*:

(i) *Local and Global Normality and Dominance*
(ii) *Local and Global Comparability*
(iii) *Agreement on Inaccuracy*

entail that, if $\mathfrak{s}$ *is a local inaccuracy measure and* $\mathfrak{I}$ *is a global inaccuracy measure and* $\mathfrak{s}$ *and* $\mathfrak{I}$ *are generated by the same function, then there is* $\lambda > 0$ *such that* (p.39)

(a) $\mathfrak{s}(v_w(X), x) = \lambda \mathrm{q}(v_w(X), x)$.
(b) $\mathfrak{I}(c,w) = \lambda \mathfrak{B}(c,w)$.

Thus, if this argument works, we have exactly the premise we used to derive Probabilism in the argument at the end of Chapter 1, namely, Brier Alethic Accuracy. Unfortunately, the argument doesn't work. As I mentioned above, the main problem lies in Global Normality and Dominance. In that premise, we assume that the global inaccuracy of a credence function c at a world w supervenes in a particular way on the Euclidean distance between the vectors that represent v_w and c. However, this is too close to what we would like to prove. To see this, observe what happens if we replace this axiom with one that says that the global inaccuracy of c at w supervenes instead on the so-called *taxicab distance* between the vectors that represent v_w and c.[8] That is,

Global Normality and Dominance* If $\mathfrak{I}$ is a legitimate global inaccuracy measure, there is a strictly increasing $f : [0, \infty) \to [0, \infty)$ such that

$$I(c,\ w) = f(\| v_w - c \|_1)$$

where $\| c - c' \|_1 = \sum_{X \in F} |c(X) - c'(X)|$.

Of course, we may not be so familiar with the taxicab distance measure, but it is certainly very natural. It is the more natural choice of distance measure in some applications and less natural in others. But in any case, it shares with Euclidean distance the three defining characteristics of a metric, which is the mathematical axiomatization of a distance measure. Thus, without further argument, we seem to have no reason to prefer Euclidean distance to the taxicab distance for the particular application with which we are concerned here. Thus, without further argument Global Normality and Dominance* seems every bit as justified as Global Normality and Dominance. The problem is that we can now prove the following theorem:

Theorem 3.2.2 *The following axioms*:

(i) *Local and Global Normality and Dominance**
(ii) *Local and Global Comparability*
(iii) *Agreement on Inaccuracy*

entail

(a) $\mathfrak{s}(v_w(X), x) = \lambda |v_w(X) - x|$.
(b) $\mathfrak{I}(c, w) = \lambda \mathfrak{A}(c, w)$.

Thus, if we demand that global inaccuracy supervene on taxicab distance in the way that we previously demanded that it supervene on Euclidean distance, we thereby characterize not the Brier score $\mathfrak{B}$, but the absolute value measure $\mathfrak{A}$, which we met above. And we know from the previous section that this cannot be used in (p.40) an accuracy argument for Probabilism: indeed, on this measure, many probabilistic credence functions are strongly dominated. Thus, I conclude, the characterization of inaccuracy measures that Leitgeb and I offered fails, for it offers no principled reason to assume that global inaccuracy supervenes on Euclidean distance rather than taxicab distance (or some other, more exotic metric, for that matter).

3.3 Joyce on coherent admissibility

A little over ten years after he published the paper that launched this particular project in epistemology—the project to which this book hopes to make a contribution— Joyce wrote a follow-up paper in which he accepted at least Maher's criticism of the characterization he had originally offered (Joyce, 2009). In that paper, he formulated an alternative characterization of inaccuracy measures that avoids Maher's criticisms.[9]

Joyce begins with a handful of innocuous principles. He assumes Finiteness, as Leitgeb and I did in our characterization; and he assumes Continuity as he did in his original characterization. Another innocuous principle he assumes is the following (Joyce, 2009, 269):

> **Truth-Directedness** If c is uniformly at least as close to the omniscient credence function at w as c' is, and sometimes closer, then $\mathfrak{I}(c, w) < \mathfrak{I}(c', w)$.
>
> That is, if
>
> (i) $c'(X) \leq c(X) \leq 1$ for all X true at w; and
> (ii) $0 \leq c(X) \leq c'(X)$ for all X false at w; and
> (iii) (a) $c'(X) < c(X) \leq 1$ for some X true at w; or (b) $0 \leq c(X) < c'(X)$ for some X false at w;
>
> then $\mathfrak{I}(c, w) < \mathfrak{I}(c', w)$.

This seems almost constitutive of the notion of accuracy. We will consider it again in Section 4.3.

However, Joyce's characterization gains nearly all of its power from the following axiom (Joyce, 2009, 280):[10]

> **Coherent Admissibility** Suppose p is a probabilistic credence function. Then there is no credence function $c \neq p$ such that, for all w,
>
> $$I(c, w) \leq I(p, w)$$

(p.41) This is a strong claim. What's more, at first sight it seems to beg the question: Joyce hopes to use his characterization of inaccuracy measures in an accuracy argument for Probabilism of the sort we have been developing in this part of the book. Yet Coherent Admissibility seems to assume something close to Probabilism. At least, it assumes that there is something special about the probability functions, something that should render them not only undominated but also not equalled by an alternative at all worlds. However, Joyce goes on to argue for this special treatment.

Here is that argument: Fix an arbitrary probabilistic credence function p. We want to show that there is an evidential situation in which an agent might find herself such that p is the unique rational response to her evidence in that situation. If this is the case, and if one subscribes to veritism, then it seems that there should be no alternative credence function that is at least as accurate as *p* at all worlds. If there were, it would be permissible to move to that credence function from *p*, since it is guaranteed to be at least as accurate, and thus by veritism, at least as epistemically good. But, by hypothesis, the agent is in a situation in which *p* and only *p* is rationally permitted.

Thus, the burden in Joyce's argument is to show that, for each probabilistic credence function *p*, there is an evidential situation to which *p* is the unique rational response. Here is Joyce's argument for that intermediate conclusion (Joyce, 2009, 279). First, he claims that there must be some possible world at which the objective chances are given by *p*. Second, he appeals to a weak version of David Lewis' Principal Principle that says that, if an agent were to learn with certainty that the objective chances are given by *p*, the *unique* rational response to that evidence would be to adopt *p* (Lewis, 1980).[11] Thus, the evidential situation we require is simply that in which the agent learns with certainty that *p* gives the objective chances.

This, then, is the justification for giving the probabilistic credence functions the sort of special treatment required by Coherent Admissibility: they are precisely the credence functions that might end up giving credences that match the true chances at some world; and therefore they are the ones that the Principal Principle might end up mandating.

Let me consider four objections to this line of argument: the first, due to Alan Hájek, can be circumvented provided we make an assumption that Joyce might be happy to make; the second shows that the argument cannot be used by someone pursuing the sort of project pursued in this book; the third shows that the argument cannot be used by someone pursuing an accuracy argument for Probabilism; and the fourth questions not the characterization itself but the version of the accuracy argument for Probabilism to which it gives rise.

We begin with Hájek's objection (Hájek, 2008). He objects to the initial claim in the argument for the intermediate conclusion: that is, he objects to the claim that, for any probabilistic credence function *p*, there is a world at which the objective chances are (p.42) given by *p*. The problem, he claims, lies in the content of the propositions on which *p* is defined. If they are propositions concerning one-off physical events —for instance, *It will rain in Bristol on New Year's Day 2016*—there is no problem, since chances can certainly attach to such propositions and can plausibly take any value from 0 to 1. But if they include moral propositions or aesthetic propositions, self-locating propositions or propositions about the objective chances themselves, mathematical propositions or propositions about the fundamental constants of the universe or its fundamental laws, then it is not obvious that chances can attach to them; or, if chances can attach to them, it is not obvious they can take any value from 0 to 1. For instance, suppose p is defined on the proposition *Torture is always wrong* and takes value 0.98 at that proposition. Then it seems that p does not give the objective chances at any world, since the objective chances simply aren't defined on moral propositions at any world. Of course, we might try to fix this by stipulating that moral propositions when true are necessarily true and when false are necessarily false; thus, chances do attach to them, but they always receive a chance of 1 or 0. But then there is still no possible world in which p is the objective chance function since p assigns 0.98 to a moral proposition. Therefore it is not obvious that there is any world in which p matches the objective chance function.

However, I think we can respond to Hájek on Joyce's behalf, provided we can appeal to a strong axiom of extensionality.[12] Axioms of extensionality are concerned with what factors get to determine the inaccuracy of a credence function. The strong axiom of extensionality that I propose here says that the inaccuracy of a credence function at a world ought to be determined only by the following factors: (i) the truth values at that world of the propositions on which the credence function is defined; and (ii) the credences assigned to those propositions by the credence function. To state this precisely, we define the *accuracy profile* of a credence function $c : \mathscr{F} \to [0,1]$ at a world w to be the following multiset:[13]

$$\{\{(v_w(X), c(X)) : X \in \mathrm{F}\}\}$$

So, the accuracy profile of c at w collects together, for each proposition in $\mathscr{F}$, the pair consisting of the truth value of that proposition at w, and the credence that c assigns to that proposition.

Our strong axiom of extensionality says that the inaccuracy of c at w is determined entirely by the accuracy profile of c at w. That is:

> (p.43) **Strong Extensionality**[14] Suppose $c : \mathscr{F} \to [0,1]$ and $c' : \mathscr{F}' \to [0,1]$. If the inaccuracy profile of c at w in $W_{\mathscr{F}}$ is the same as the inaccuracy profile of c' at w' in $W_{\mathscr{F}'}$, then $\mathfrak{I}(c, w) = \mathfrak{I}(c', w')$.

If we assume Strong Extensionality, we can respond to Hájek's objection as follows: Suppose p is a probability function defined on a finite set $\mathscr{F}$ of propositions. And suppose that some of the propositions in $\mathscr{F}$ are of the sort to which chances cannot attach. Then proceed as follows: First, we create a set $\mathscr{F}^\dagger$ of propositions and a probability function $p\dagger$ defined on $\mathscr{F}\dagger$ such that $p\dagger$ genuinely could be the chance function at some world and such that $p\dagger$ has the same inaccuracy profile at each of the worlds relative to $\mathscr{F}\dagger$ that p has at each of the worlds relative to $\mathscr{F}$. An example will help to illustrate the strategy: Suppose Anna has credence function p defined only on the proposition that torture is always wrong and its negation: that is, her opinion set is $\mathrm{F} = \{Torture, \overline{Torture}\}$. And suppose

$$p(Torture) = 0.98, \quad p(\overline{Torture}) = 0.02$$

Hájek's concern is that p is not the objective chance function at any world. But now we define a corresponding credence function $p\dagger$ on the proposition that it will rain in Bristol on New Year's Day 2016 (call this *Rain*) and its negation $(\overline{Rain})$. That is, $\mathrm{F}^{\dagger} = \{Rain, \overline{Rain}\}$. And define the following mapping from $\mathscr{F}$ to $\mathscr{F}\dagger$: $Torture\dagger = Rain$ and $\overline{Torture}^{\dagger} = \overline{Rain}$. Then, given a credence function c on $\mathscr{F}$, define $c\dagger$ on $\mathscr{F}\dagger$ in the natural way: $c\dagger(X\dagger) = c(X)$. Thus, for instance,

- $p\dagger(Rain) = p(Torture) = 0.98$,
- $p^{\dagger}(\overline{Rain}) = p(\overline{Torture}) = 0.02$

(p.44) Now $p\dagger$ can certainly be the objective chance function of a possible world.[15] Moreover, if $\mathfrak{I}$ satisfies Strong Extensionality, c is defined on $\mathscr{F}$, and w is in $W_{\mathscr{F}}$, then $\mathfrak{I}(c, w) = \mathfrak{I}(c\dagger, w\dagger)$, where $w\dagger$ is the assignment of truth-values to the propositions in $\mathscr{F}\dagger$ that makes a proposition $X\dagger$ true exactly when w makes X true. Thus, there is a credence function c defined on $\mathscr{F}$ such that $\mathfrak{I}(c, w) \leq \mathfrak{I}(p, w)$ for all w in $W_{\mathscr{F}}$ iff there is $c\dagger$ on $\mathscr{F}\dagger$ such that $\mathfrak{I}(c\dagger, w\dagger) \leq \mathfrak{I}(p\dagger, w\dagger)$ for all $w\dagger$ in $W_{\mathscr{F}\dagger}$. But, since $p\dagger$ could be the objective chance function of a possible world, Joyce's argument shows that the latter should never happen. So we can infer that the former should never happen either. Thus, there is no c such that $\mathfrak{I}(c, w) \leq \mathfrak{I}(p, w)$ for all w in $W_{\mathscr{F}}$. Now, it should be clear how to generalize this procedure so that we can produce, for any probabilistic credence function on any set of propositions, a corresponding probability function on a corresponding set of propositions, where the corresponding probability function could be the objective chance function at some possible world and where the inaccuracy profiles of the probability function and the corresponding probability function pair up. From this, we can appeal to Joyce's argument to show that $\mathfrak{I}$ ought to satisfy Coherent Admissibility. But of course Joyce must accept Strong Extensionality if he wishes to make this defence.

The second issue I wish to raise for Joyce's argument concerns the role it might play in an epistemology motivated by veritism. Admittedly, as noted in footnote 9 above, Joyce no longer seems to accept veritism. But since this book is devoted in part to a defence of that thesis, it is worth saying why Joyce's argument could not be used in such a defence, whatever its other merits. The problem lies in the appeal to the Principal Principle. The Principal Principle, like the Principle of Indifference, is a law of credence that seems most naturally to follow from evidential considerations. Thus, it is one of the laws of credence that we will be most concerned to establish on the basis of veritism alone, if we are to answer the complaint that veritism fails to account for all the laws of credence. If we were to appeal to the Principal Principle in our characterization of the legitimate inaccuracy measures, this would preclude a non-circular justification of that law on the basis of considerations of accuracy. (Cf. Part II for our accuracy argument for the Principal Principle.)

My third concern about Joyce's style of argument is this: In the absence of Probabilism, it proves too much; thus, it cannot be used to justify Probabilism since it needs that law of credence as a premise if it is to avoid over-generating. To see why this is the case, consider the following argument, which is analogous to Joyce's. Fix an arbitrary *non*-probabilistic credence function c. Then there is an evidential situation in which the unique rational response is to adopt c: it is the situation in which you learn that God, or some other agent to whom you should defer completely as an epistemic expert, has credence function c. Therefore, there can be no c' such that $\mathfrak{I}(c', w) \leq \mathfrak{I}(c, w)$ for all worlds w. If there were, it would be rationally permissible to move from c to c' even (p.45) when you have heard that God knows c. But again, that contradicts the claim that c is the unique rational response to the evidence. Thus, we can conclude:

Incoherent Admissibility Suppose c is a non-probabilistic credence function. Then there is no credence function $c' \neq c$ such that, for all w,

$$I(c', w) \leq I(c, w)$$

However, we know that there is no inaccuracy measure $\mathfrak{I}$ that satisfies Coherent Admissibility and Incoherent Admissibility. If $\mathfrak{I}$ satisfies Coherent Admissibility then, at least for credence functions defined over a particular sort of opinion set, every non-probabilistic credence function is dominated—this is the content of the main theorem (Theorem 2) of (Joyce, 2009). Thus, $\mathfrak{I}$ does not satisfy Incoherent Admissibility. But, if Joyce's argument for Coherent Admissibility works, then so does the argument for Incoherent Admissibility. And together they characterize the empty set of inaccuracy measures. And that is not going to help to support an accuracy argument for Probabilism.

Of course, one might respond that the expert principle to which we appealed in the argument for Incoherent Admissibility is false. There are various ways to make this point: We might say that an expert principle is defeasible—one must adopt the recommendation of the expert *provided that recommendation satisfies the laws of credence*. If that's the case, c need not be the unique rational response to learning that it is the expert's credence function—indeed, it may not be a rational response at all. Or one might say that an expert who recommends a credence function that violates the laws of credence thereby forfeits their status as an expert. Either way, the upshot is the same: the expert principle in the argument for Incoherent Admissibility doesn't do what the argument requires of it.

In fact, I agree with this. I agree that this is not a case in which misleading higher-order evidence about rationality creates an epistemic dilemma; it simply fails to create a rational requirement at all. The expert principle does need to be amended in one of the ways just described. But that will not help someone who wishes to use Joyce's argument to establish Coherent Admissibility and then hopes to use Coherent Admissibility to establish Probabilism. For the argument in favour of restricting the expert principle in the way required to block the argument for Incoherent Admissibility depends essentially on Probabilism. Thus, to use it in the service of characterizing inaccuracy measures in order to establish Probabilism is circular. Put another way, there is a lacuna in Joyce's argument: the Principal Principle is not enough on its own to show that the agent who learns that p gives the objective chance is rationally required to adopt p. We must know further that p is not ruled out by a rational requirement. There are many ways in which p might violate rational requirements. The agent might have what Lewis calls inadmissible evidence that defeats the application of the Principal Principle. Or there may be some general coherence requirements that p violates—requirements that hold of any agent at any time regardless of their evidence or the content of the (p.46) propositions to which they assign credence. Now, we can set up the evidential situation in order to rule out inadmissible evidence. But, unless we know in advance what they are, we cannot assume that p satisfies the coherence requirements. Thus, we need to know Probabilism before we can accept Joyce's argument for Coherent Admissibility.

My final concern with Joyce's characterization is a slightly weaker version of a concern raised above about his earlier characterization. The central theorem (Main Theorem) of (Joyce, 1998)—which is a generalization of Theorem 1.0.2(i)—supports an argument for Probabilism only in the presence of Dominance, which is too strong. The central theorem of (Joyce, 2009) (Theorem 2)—which is a generalization of Theorem 1.0.2(i), (ii)—supports an argument for Probabilism only in the presence of Undominated Dominance, which is also too strong. Thus, in this later paper, Joyce leaves open the following possibility: there is an inaccuracy measure $\mathfrak{I}$ that satisfies his characterization and there is a non-probabilistic credence function c that is $\mathfrak{I}$-dominated only by probabilistic functions that are extremely $\mathfrak{I}$-modest. As I argued in Chapter 2, if that were the case, it seems at least questionable whether c would be ruled out as irrational.

Notes:

([1]) Having said that, in his proof of his Main Theorem, Joyce does extract a distance measure from an inaccuracy measure (Joyce, 1998, 598). He defines $D(c, c') := \mathfrak{I}(v_w + c - c', w)$, where w is some possible world. It is well-defined because his axioms guarantee that any choice of w gives the same function.

([2]) The absolute value measure $\mathfrak{A}$ assigns an inaccuracy of

$$\left|\frac{1}{3} - 1\right| + \left|\frac{1}{3} - 0\right| + \left|\frac{1}{3} - 0\right| = \frac{4}{3}$$

to the former credence function at all worlds. It assigns

$$|0 - 1| + |0 - 0| + |0 - 0| = 1$$

to the latter credence function at all worlds.

([3]) See (Pettigrew, 2012) for a generalization of the second part based on a slight strengthening of Joyce's characterization.

([4]) I will consider the characterization that appeals to Agreement on Inaccuracy. The same problem arises for the characterizations that appeal to Separability of Global Inaccuracy and Agreement on Directed Urgency, since the problem lies in an assumption shared by all three characterizations, namely, Global Normality and Dominance. I state this assumption below.

([5]) We define the quadratic scoring rule as follows: $q(1, x) = (1 - x)^2$ and $q(0, x) = x^2$. It is easy to see that the Brier score of a credence function at a world is the sum of the quadratic scores of the individual credences it assigns at that world. That is, $\mathfrak{B}(c, w) = \sum_{X \in \mathcal{F}} |v_w(X) - c(X)|^2 = \sum_{X \in \mathcal{F}} \mathfrak{q}(v_w(X), \ c(X))$.

([6]) The version of Global Normality and Dominance that I state here is slightly different from the version we gave in our paper. This version allows us to characterize the Brier score itself, rather than the close cousin of it that Leitgeb and I characterized. I make this change because that close cousin cannot be used in an accuracy dominance argument of the sort we have been developing. That is not an objection to our earlier characterization, since we were pursuing a different accuracy argument for Probabilism—we were pursuing a justification of Probabilism based on minimizing expected inaccuracy. But my concern here is with characterizations of inaccuracy measures that can underpin an accuracy dominance argument. The objection I raise below applies to this version and to the version in our original paper.

([7]) Recall from above that we abuse notation and let w also be the proposition in $\mathscr{F}^*$ that specifies w uniquely—that is, the proposition that is true at w and only at w. And *In* x is the natural logarithm of x, viz., loge(x). Thus, $\mathfrak{L}(c, w)$ is well-defined only if c is probabilistic and every probabilistic extension of c to $\mathscr{F}^*$ —the smallest algebra that includes the set $\mathscr{F}$ on which c is defined—assigns the same credence to w.

([8]) This is sometimes called the *Manhattan* or *city block* distance measure.

([9]) In fact, Joyce takes himself to be characterizing epistemic disutility functions rather than inaccuracy measures. Of course, in the presence of veritism, these amount to the same thing. But, by the time he wrote the later paper, Joyce was no longer committed to veritism, if, indeed, he ever was. On the other hand, I am. Thus, I will assume veritism and consider Joyce's characterization of epistemic disutility measures as a characterization of inaccuracy measures.

([10]) As Joyce states this axiom in the main body of the paper, it is weaker than this formulation. But, for the proof of his central theorem, he requires the stronger version stated here.

([11]) Cf. Part II of this book for more on the Principal Principle and other chance-credence principles.

([12]) This is an extension of a response I first proposed in Pettigrew (2014b).

([13]) Like a set, a multiset is unordered; unlike a set, it may contain the same object many times over. Thus, {{1,2,3}} = {{1,3,2}}, because both multisets contain the same elements; but {{1,1,2}} ≠ {{1,2}}, since the former contains the number 1 twice, whereas the latter contains it only once.

([14]) In fact, this axiom is stronger than one might expect. It rules out all but one of the following family of inaccuracy measures, which we call the *weighted Brier measures*. Suppose that, for each set of propositions $\mathscr{F}$, we have aset $\Lambda_{\mathscr{F}}$ of non-negative real numbers, one for each proposition in $\mathscr{F}$; and suppose that, for each $\mathscr{F}$, these numbers sum to 1 —that is, $\Lambda_{\mathcal{F}} = \{\lambda^{\mathcal{F}}_X : X \in \mathcal{F}\}$ and $\lambda^{\mathcal{F}}_X \geq 0$, for all $X \in \mathscr{F}$, and $\sum_{X \in \mathcal{F}} \lambda^{\mathcal{F}}_X = 1$. Then, we can define the *weighted Brier score relative to* Λ as follows: if $c: \mathscr{F} \to [0,1]$ and $w \in W_{\mathscr{F}}$,

$$\mathrm{B}_{\Lambda}(c, w) := \sum_{X \in \mathcal{F}} \lambda^{\mathcal{F}}_X |v_w(X) - c(X)|^2$$

Thus, while the Brier score of c at w is the sum of the squares of the differences between the credences assigned by c and by v_w, the weighted Brier score relative to Λ is the *weighted* sum of those differences, where the weights are given by the real numbers in $\Lambda_{\mathscr{F}}$. Then the only member of that family of inaccuracy measures that satisfies Strong Extensionality is $\mathfrak{B}_{\Gamma}$, where $\Gamma_{\mathcal{F}} = \{\gamma^{\mathcal{F}}_X = \frac{1}{|\mathcal{F}|} : X \in \mathcal{F}\}$ —that is,

$$\mathrm{B}_{\Gamma}(c, w) := \frac{1}{|\mathcal{F}|} \sum_{X \in \mathcal{F}} |v_w(X) - c(X)|^2$$

In other words, the only weighted Brier score that satisfies Strong Extensionality is the one that takes the straight average of the squared differences between the credences assigned by c and by v_w.

([15]) Indeed, based on my experience of the city, it is likely close to the objective chance function of the actual world on New Year’s Eve 2016.