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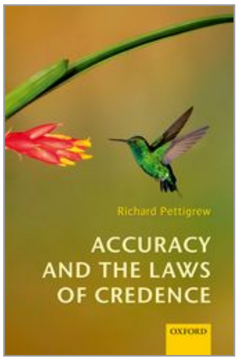

Accuracy and the Laws of Credence
Richard Pettigrew

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Dominance and chance

Richard Pettigrew

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Abstract and Keywords

This chapter introduces a second attempt at an accuracy-based argument for chance-credence principles. In this attempt, the veritist account of vindication, on which the accuracy-based argument for Probabilism depends, is retained. What is changed instead is the decision-theoretic principle: it is changed from a pure dominance principle to what is called a chance dominance principle. The chapter shows how the argument thus adapted can be used to establish various chance-credence principles and answers an objection that the justification it provides is circular.

Keywords: Chance dominance, Principal Principle, chance, veritism, accuracy

In this chapter, I'd like to present a different argument for ETP. It is one that isn't vulnerable to Caie's concern about a third sort of 'ought'; and it is not premised upon an account of vindication on which matching information-losing summaries of the truths is epistemically preferred to matching the truths themselves. To introduce it, I'd like first to present an argument for PP_0 that I gave in (Pettigrew, 2013). I no longer think it works. But we can adapt it to give a better argument, this time for ETP.

In the first accuracy-based argument for PP_0, we adapted the argument for Probabilism by altering the account of vindication. In Section 9.4, we saw that the alternative account of vindication to which we appealed is implausible. In this section, we give the second accuracy-based argument for PP_0. Taking heed of the arguments in the previous section, we abandon the move to Ur-Chance Initial Vindication—we retain Alethic Vindication. Instead of altering the account of vindication, we adapt Part I's argument for Probabilism by altering the decision-theoretic principle to which we appeal. That is, we replace Immodest Dominance. The new principle to which we'll appeal is called Ur-Chance Initial Immodest Dominance.

To understand this new principle, we begin with a general principle of decision theory—that is, one that applies to any decision problem concerning decisions between any sorts of options, whether the options are monetary bets, taking an umbrella or leaving it at home, or breaking an egg into a new bowl before transferring it to your main mixing bowl in case it is rotten. This is analogous to Undominated Dominance. After that, we state a principle that applies only to decision problems in which the options are credence functions and the utility function measures epistemic value. This is analogous to Immodest Dominance.

Throughout this section, we will assume that our agent has opinions only about eternal propositions. Thus, according to Alethic Vindication, the vindicated credence function for an agent is the omniscient credence function at her world. We need not relativize to times at this point (though we will later).

To state the analogue to Undominated Dominance and the analogue to Immodest Dominance, we require the following terminology. As above, suppose $\mathscr{O}$ is the set of options, $\mathscr{C}_0$ is the set of possible ur-chance functions, $\mathscr{W}$ is the set of possible worlds, and $\mathfrak{U}$ is a utility function. Suppose o and o^* are options in $\mathscr{O}$. Then: (p.124)

- We say that o^* *strongly ur-chance* $\mathfrak{U}$-*dominates* o If $\mathrm{Exp}_{\mathfrak{U}}(o|ch) < \mathrm{Exp}_{\mathfrak{U}}(o^*|ch)$ for all ch in $\mathscr{C}_0$.[1]
- We say that o^* *weakly ur-chance* $\mathfrak{U}$-*dominates* o if
  - (i) $\mathrm{Exp}_{\mathfrak{U}}(o|ch) \leq \mathrm{Exp}_{\mathfrak{U}}(o^*|ch)$ for all ch in C_0, and
  - (ii) $\mathrm{Exp}_{\mathfrak{U}}(o|ch) < \mathrm{Exp}_{\mathfrak{U}}(o^*|ch)$ for some ch in C_0.

We then state our first new decision-theoretic principle as follows:

Ur-Chance Initial Undominated Dominance Suppose $\mathcal{O}$ is the set of options, W is the set of possible worlds, and $\mathfrak{U}$ is a utility function. Suppose o, o' in $\mathcal{O}$. Then, if

(i) o^* strongly ur-chance $\mathfrak{U}$-dominates o, and
(ii) there is no o′ that weakly ur-chance $\mathfrak{U}$-dominates o^* then
(iii) o is irrational for any agent with utility function $\mathfrak{U}$ *at the beginning of her epistemic life*.

Thus, Ur-Chance Initial Undominated Dominance says the following: Suppose you are at the beginning of your epistemic life—that is, you have collected no evidence. You are considering which option to choose. The different possible ur-chance functions may well disagree on many things: for instance, they may disagree on which option has maximal objective expected utility relative to $\mathfrak{U}$. But suppose they all agree on at least one thing: they agree that the objective expected utility of o^* exceeds the objective expected utility of o; what's more, there's no option o' about which all the ur-chance functions agree that its objective expected utility is at least that of o* while some that think it higher. In that situation, o is irrational.

Let's consider an example: As so often in decision-theoretic examples, I am trying to decide whether or not to take an umbrella with me when I leave my house. My utilities are as follows:

	Rain	$\overline{Rain}$
Umbrella	4	5
No Umbrella	1	10

Let us suppose that my evidence tells me that the ur-chance of rain is at least 70%, but it tells me nothing more. Thus, all the epistemically possible ur-chance functions assign a probability of at least 70% to rain. Then it is straightforward to show that, relative to any possible ur-chance function, the objective expected utility of taking the umbrella exceeds the objective expected utility of not taking the umbrella. Thus, it is irrational for me to leave without the umbrella.

Next, we state our second new decision-theoretic principle. As mentioned above, this applies only in cases in which the options are credence functions and the utility function is a measure of epistemic value.

(p.125) **Ur-Chance Initial Immodest Dominance** Suppose $\mathfrak{I}$ is a legitimate measure of inaccuracy. Then, if

(i) c_0 is strongly ur-chance $\mathfrak{I}$-dominated by probabilistic c_0^*,

(ii) c_0^* is not itself even weakly ur-chance $\mathfrak{I}$-dominated by any credence function, and

(iii) c_0^* is not extremely $\mathfrak{I}$-modest, then

(iv) c_0 is irrational as an initial credence function for any agent with inaccuracy measure $\mathfrak{I}$.

Now, as a corollary of Theorems I.D.5,I.D.7, and 9.0.9 we have:

Theorem 10.0.1 *Suppose $\mathfrak{D}$ is an additive Bregman divergence and $\mathfrak{I}(c, w) = \mathfrak{D}(v_w, c)$. Then, if an initial credence function c_0 violates Probabilism or PP_0*, then there is an initial credence function c_0^* *such that*

(i) c_0^* *is probabilistic and satisfies PP_0;*

(ii) c_0^* strongly ur-chance $\mathfrak{I}$-dominates c_0;

(iii) c_0^* *is not even weakly ur-chance $\mathfrak{I}$-dominated by any initial credence function;*

(iv) c_0^* *is not even moderately $\mathfrak{I}$-modest.*

Thus, whichever inaccuracy measure we use—providing it is generated by an additive Bregman divergence in conjunction with Alethic Vindication—the following is true: any credence function that violates the Principal Principle is strongly ur-chance accuracy dominated by a credence function that is not itself even weakly ur-chance accuracy dominated and which is not even moderately immodest.

Of course, in order not to be vulnerable to the Bronfman objection, it is crucial that we endorse only one such inaccuracy measure. But this theorem shows that it doesn't matter which one. On the basis of the characterization given in Chapter 4, I endorse the Brier score. Thus, we have the following argument for PP_0:

($\mathrm{I}^{\star}_{PP_0}$) **Veritism** The sole fundamental source of epistemic value is accuracy.

($\mathrm{II}^{\star}_{PP_0}$) **Brier Alethic Accuracy** The inaccuracy of a credence function at a world is the squared Euclidean distance from the omniscient credence function at that world to the credence function.

($\mathrm{III}^{\star}_{PP_0}$) **Ur-Chance Initial Immodest Dominance**

($\mathrm{IV}^{\star}_{PP_0}$) **Theorems 10.0.1** and **I.B.2** Therefore,

($\mathrm{V}^{\star}_{PP_0}$) **Probabilism + PP**$_0$

This argument gives a different diagnosis of Cleo's irrationality from the diagnosis given in the previous chapter. Recall:

> *Cleo* knows nothing about the coin in front of her except that it is a trick coin that has either a 60% or a 70% chance of landing heads rather than tails. She is more confident that the coin will land tails on the next toss than she is that it will land heads.

(p.126) According to the first argument, Cleo's credence function is accuracy dominated: that is, there is an alternative credence function that is guaranteed to be closer to vindication than hers. Having reverted to the alethic account of vindication, this is no longer the case. Indeed, if the coin lands tails, then Cleo's credence function will be more accurate than any of the credence functions that satisfy PP_0. However, since each of the ur-chance functions finds this outcome less likely than the alternative, this greater accuracy counts less towards Cleo's objective expected accuracy than her inaccuracy if the coin lands heads—and that inaccuracy is greater than the inaccuracy of any credence function that satisfies PP_0. Indeed, there is a credence function that satisfies PP_0 that each ur-chance function expects to be more accurate than it expects Cleo's to be. As it turns out, in fact, the credence functions that accuracy dominate Cleo's relative to the ur-chance notion of vindication mooted in the previous chapter are precisely the credence functions that every ur-chance function expects to do better than they expect Cleo's to do: that is, a credence function strongly or weakly accuracy- dominates Cleo relative to the ur-chance notion of vindication iff it strongly or weakly ur-chance accuracy-dominates Cleo relative to the alethic notion of vindication. That is a consequence of Theorem I.D.7. Thus, adopting Cleo's credence function is like choosing to leave without an umbrella in the decision problem described above. They are both strongly ur-chance dominated relative to the relevant utility functions.

10.1 Adapting the argument

The foregoing, then, is (roughly) the argument for PP_0 that I gave in (Pettigrew, 2013). Now, just as Caie raised an objection to Ur-Chance Initial Vindication above, so there is an analogous objection to Ur-Chance Initial Undominated Dominance and its epistemological cousin Ur-Chance Initial Immodest Dominance. The objection is simple: Why should my *current* choice be rationally constrained by the possible *ur*-chances? Why should my choice now be constrained by facts about the chance functions that might have governed the earliest moment in the world I inhabit, if such exists, or in fact do not govern any particular moment at all, if the world has no earliest moment? Surely the correct decision-theoretic principle in the vicinity appeals to the unanimous verdicts of the possible chance functions that are contemporaneous with my decision, rather than those of the possible ur-chance functions.

This seems right to me. If it is, the correct decision-theoretic principle here is the following. By replacing the set of possible ur-chance functions in the definition of strong and weak ur-chance dominance with the set of possible current chance functions, we obtain the definition of *strong* and *weak current chance dominance*. Then we substitute those notions into Ur-Chance Initial Undominated Dominance and Ur-Chance Initial Immodest Dominance in place of the notions of strong and weak ur-chance dominance. We thereby obtain the general decision-theoretic principle of Current Chance Initial Undominated Dominance and its epistemological cousin (p.127) Current Chance Initial Immodest Dominance. The latter says that an initial credence function c_0 is irrational for an agent with inaccuracy measure $\mathfrak{I}$ if (i) there is an alternative probabilistic initial credence function c_0^* that the possible current chance functions unanimously expect to be less inaccurate than they expect c_0 to be relative to $\mathfrak{I}$, (ii) there is no alternative credence function that the possible current chance functions unanimously expect to be at most as inaccurate as c_0^* and some expect to be less inaccurate, and (iii) c_0^* expects itself to be least inaccurate relative to $\mathfrak{I}$.[2] This principle, together with Alethic Vindication, and the resulting Brier Alethic Accuracy, gives an argument for TPP_0. We will not stop to say why; the argument can be easily reconstructed from the much more general argument given at the end of this section.

Now, having moved to versions of these decision-theoretic principles that appeal to current chances rather than ur-chances, we can generalize the principles so that they govern any point in an agent's epistemic life, not only its earliest moment. The resulting principles we might call Current Chance Undominated Dominance and Current Chance Immodest Dominance. The latter says that a credence function c is irrational for an agent with inaccuracy measure $\mathfrak{I}$ if (i) there is an alternative credence function c^* that the possible current chance functions unanimously expect to be less inaccurate than they expect c to be relative to $\mathfrak{I}$, (ii) there is no alternative credence function that the possible current chance functions unanimously expect to be at most as inaccurate as c^* and some expect to be less inaccurate, and (iii) c^* expects itself to be least inaccurate relative to $\mathfrak{I}$. This principle, together with Brier Alethic Accuracy, gives an argument for TPP. Recall, however, that TPP is false: it gives the wrong verdict when we have evidence from the future.

Finally, then, we adjust Current Chance Undominated Dominance and Current Chance Immodest Dominance to give the correct decision-theoretic principle in this vicinity, together with its true epistemological cousin. As in the move from Current Chance Vindication to Current Chance Evidential Vindication, we must introduce some reference to our total evidence in order to avoid the sort of problems that TPP faces. Fortunately in this case, the move is well motivated and does not involve a third sense of 'ought' that sits uncomfortably between the familiar subjective and objective senses. First, some updated definitions. Let $\mathscr{C}$ be the set of possible current chance functions. And let E be a proposition. Then

- We say that o^* *strongly current chance* $\mathfrak{U}$*-dominates o conditional on E* if $\mathrm{Exp}_{\mathfrak{U}}(o|ch(-|E)) < \mathrm{Exp}_{\mathfrak{U}}(o^*|\mathrm{ch}(-|E))$ for all ch in $\mathscr{C}$.
- We say that o^* *weakly current chance* $\mathfrak{U}$*-dominates o conditional on E* if
  (i) $\mathrm{Exp}_{\mathfrak{U}}(o|ch(-|E)) \leq \mathrm{Exp}_{\mathfrak{U}}(o^*|ch(-|\mathrm{E}))$ for all ch in $\mathscr{C}$, and
  (ii) (p.128) $\mathrm{Exp}_{\mathfrak{U}}(o|ch(-|E)) < \mathrm{Exp}_{\mathfrak{U}}(o^*|ch(-|E))$ for some ch in $\mathscr{C}$.

And now the principles:

Current Chance Evidential Undominated Dominance
Suppose $\mathscr{O}$ is the set of options, $\mathscr{W}$ is the set of possible worlds, and $\mathfrak{U}$ is a utility function. Suppose o, o' in $\mathscr{O}$. Then, if

(i) o^* strongly current chance $\mathfrak{U}$-dominates o conditional on the agent's current total evidence E, and
(ii) there is no o' that weakly current chance $\mathfrak{U}$-dominates o^* conditional on E, then
(iii) o is irrational for any agent with utility function $\mathfrak{U}$ and evidence E.

This seems correct. Suppose I must choose whether or not to take my umbrella as I leave my flat. If every epistemically possible current chance function agrees that taking it is better than leaving it, I should not leave it, unless of course I have evidence from the future that it will not rain, even though I know the chance of rain is at least 70%. So Current Chance Evidential Undominated Dominance is the true requirement of rationality in this vicinity. And here is its epistemological cousin:

Current Chance Evidential Immodest Dominance Suppose $\mathfrak{I}$ is a legitimate measure of inaccuracy and E is a proposition. Then, if

(i) c is strongly current chance $\mathfrak{I}$-dominated by probabilistic c^* conditional on E,
(ii) there is no credence function that weakly current chance $\mathfrak{I}$-dominates c^* conditional on E, and
(iii) c^* is not extremely $\mathfrak{I}$-modest then
(iv) c is irrational as an credence function for any agent with inaccuracy measure $\mathfrak{I}$ and evidence E.

The latter can then be used to give an argument for ETP. Recall from the previous chapter:

Evidential Temporal Principle (ETP) If an agent has a credence function c *and total evidence* E, then rationality requires that

$$c(X|T_{ch}) = ch(X|E)$$

for all propositions X in $\mathscr{F}$, and all possible chance functions ch such that T_{ch} is in $\mathscr{F}$ and $c(T_{ch}) > 0$.

As above, the crucial lemma is Theorem 9.3.1 stated above. From Theorems I.D.5, I.D.7, and 9.3.1 we have: (p.129)

Theorem 10.1.1 *Suppose* $\mathfrak{D}$ *is an additive Bregman divergence and* $\mathfrak{I}(c, w) = \mathfrak{D}(v_w, c)$. *Then, if a credence function c violates Probabilism or ETP, then there is a credence function c* such that*

(i) c^* *is probabilistic and satisfies ETP*;
(ii) c^* *strongly current chance* $\mathfrak{I}$*-dominates c conditional on the agent's total evidence*;
(iii) c^* *is not even weakly current chance* $\mathfrak{I}$*-dominated by any credence function conditional on the agent's total evidence*;
(iv) c^* *is not even moderately* $\mathfrak{I}$*-modest*.

(I^*_{ETP}) **Veritism** The ultimate source of epistemic value is accuracy.

($\mathrm{II}^{\star}_{ETP}$) **Brier Alethic Accuracy** The inaccuracy of a credence function at a world is the squared Euclidean distance from the omniscient credence function at that world to the credence function.

($\mathrm{III}^{\star}_{ETP}$) **Current Chance Evidential Immodest Dominance**

($\mathrm{IV}^{\star}_{ETP}$) **Theorem 10.1.1** and **I.B.2** Therefore,

($\mathrm{V}^{\star}_{ETP}$) **Probabilism + ETP**

10.2 The circularity objection

This, then, is our final argument in favour of our final chance-credence principle. According to that principle, if you are rational, you will set your credence in a proposition X conditional on the current chance hypothesis T_{ch} to whatever probability ch assigns to X once it has been brought up to speed with your total evidence. And the reason it would be irrational not to do as the principle demands is this: if you violate the principle, there is an alternative credence function that is better than yours by the lights of every possible current chance function (once they've been brought up to speed with your total evidence); what's more, there is no further credence function that all the possible current chance functions (brought up to speed with your evidence) agree is better than this alternative credence function; and the alternative expects itself to be best.

However, you might worry that, while each premise of this argument is true and the argument itself is valid, nonetheless it does little to justify ETP. You might worry that one of the premises—in particular, the decision-theoretic one—is no more plausible than ETP itself. Worse than that, you might worry that that decision-theoretic principle—namely, Current Chance Evidential Immodest Dominance—is itself justified by appeal to ETP. If that's the case, then the argument simply begs the question. In this section, I'd like to address these concerns.

Before we consider these objections, it is worth noting that ETP is not the strongest chance-credence principle that our argument establishes. That principle is this:

> (p.130) **Evidential Temporal Principle^{+} (ETP^{+})** If an agent has a credence function c and total evidence E, then rationality requires that c is in $\mathrm{cl}(C_E^{+})$.

Note that $ETP^{+} \Rightarrow ETP$, providing all possible current chance functions are non-selfundermining relative to E—that is, $c(T_{ch}|E) = 1$. But $ETP \nRightarrow ETP+$. After all, ETP only governs the behaviour of c with respect to the possible current chance functions about which c has an opinion, that is, the possible chance functions ch such that T_{ch} is in $\mathscr{F}$. The restrictions it places on c are not strong enough to guarantee that c lies within the closure of the convex hull of *all* the possible current chance functions. In what follows, we will discuss ETP^{+} primarily, since it is the strongest principle that can be justified.

Let us return now to the concerns raised briefly above. First, let us address the concern that our reason for believing Current Chance Evidential Immodest Dominance is based ineliminably on ETP^{+}. We begin by noting that it is certainly true that we can use ETP$^+$, together with other plausible principles, to justify many instances of Current Chance Evidential Immodest Dominance. Here's how:

> Suppose an agent has a probabilistic credence function p that satisfies ETP^+. In particular, suppose that p is in $\mathscr{C}$+. Thus, there is a finite subset $\mathscr{C}_p \subseteq \mathscr{C}$ of the possible current chance functions and a positive weighting $\alpha_{ch} > 0$ assigned to each ch in $\mathscr{C}_p$ such that the weightings sum to 1—that is, $\sum_{ch \in \mathscr{C}p} \alpha_{ch} = 1$—and p is the weighted sum of the functions in $\mathscr{C}_p$ (brought up to speed with the agent's evidence) given by those weightings—that is, $p(–) = \sum_{ch \in \mathscr{C}p} \alpha_{ch} ch(–|E)$. Now, suppose that c and c^* are credence functions such that every probability function $ch(–|E)$ in $\mathscr{C}_E$ expects c^* to have lower inaccuracy than c. That is, for all ch in $\mathscr{C}$,
>
> $$\mathrm{Exp}_I(c^*|ch(-|E)) < \mathrm{Exp}_I(c|ch(-|E))$$
>
> Then it follows that the agent's subjective expectation of the inaccuracy of c exceeds her subjective expectation of the inaccuracy of c^*.[3] Thus, by a standard decision-theoretic principle, (p.131) which declares an option irrational if there is an alternative option with higher subjective expected utility, it follows that c is irrational, as required.

If there are only finitely many possible current chance functions and the current chance hypothesis corresponding to each belongs to $\mathscr{F}$, then we can run a similar argument using only ETP, since in that situation, $ETP \Leftrightarrow ETP^+$.

Thus, there is certainly an argument in favour of Current Chance Evidential Immodest Dominance that relies on Probabilism, ETP$^+$, and the decision-theoretic principle Maximize Subjective Expected Utility. But that in itself does not show that our accuracy argument for ETP$^+$ begs the question. After all, there is an argument for Dominance in its various forms that appeals to Probabilism and Maximize Subjective Expected Utility. It doesn't follow that Joyce's accuracy argument for Probabilism is circular. That would only be the case if the *only* compelling arguments for Dominance relied ineliminably on Probabilism. And that isn't the case. Dominance is in fact a more plausible and more basic principle than either Probabilism or Maximize Subjective Expected Utility. And that, I submit, is true also of Current Chance Evidential Immodest Dominance—it is more plausible and more basic than ETP$^+$.

Here are a couple of reasons to think that. First, like Dominance, Current Chance Evidential Immodest Dominance governs an agent whether or not she has a credence function and, if she does, whether or not that credence function is probabilistic. In contrast, ETP$^+$ and Maximize Subjective Expected Utility, apply only to agents with a probabilistic credence function. Thus, Current Chance Evidential Immodest Dominance is more general than either of the principles to which we appealed in the justification above—as I noted there, the justification only establishes some applications of Current Chance Evidential Immodest Dominance, namely, those in which the agent in question is equipped with a probabilistic credence function.

Second, you might worry that Current Chance Evidential Immodest Dominance and ETP$^+$ are too close for one to help justify the other because both take for granted that the chances should in some way guide us in our reasoning and action. It is true that they both do. The justification for ETP$^+$ given above will not satisfy someone who is not already convinced that we should defer to the chances in some way. Doing more is beyond the scope of this project. But note that ETP$^+$ specifies a very precise way in which we should defer to the chances in setting our credences. It demands that your credence in a proposition lies within the (closure of the) span of the possible current chances of that proposition conditional on your evidence. In contrast, Current Chance Evidential Immodest Dominance says nothing so precise about how we should (p.132) evaluate credence functions in the light of the verdict on the inaccuracy of those credence functions given by the possible chance functions. It merely says that, on the rare occasions on which they all agree in their ordering of two credence functions with respect to accuracy (once they have been brought up to speed with your evidence), then you should adopt that ordering yourself on pain of irrationality. So it seems to me that Current Chance Evidential Immodest Dominance is in fact the more basic principle. I suspect that it does not reside at normative bedrock: there is still work to be done justifying it. But it does serve to justify ETP$^+$ and thus ETP.

Notes:

([1]) Recall: $\mathrm{Exp}_{\mathrm{U}}(o|ch) := \sum_{w \in \mathrm{W}} ch(w) U(o, w)$.

([2]) Recall: $\mathrm{Exp}_{\mathrm{U}}(o|ch) = \sum_{w \in \mathscr{W}} ch(w) U(o, w)$. And recall that $\mathscr{W}$ is the set of worlds relative to $\mathscr{F}$: that is, it is the set of consistent truth-value assignments to the propositions in $\mathscr{F}$. Thus, since we are now working with non-eternal propositions as well as eternal propositions, $\mathscr{W}$ is no longer a coarse-graining of the set of possible worlds, as it was before.

It is now a coarse-graining of the set of *centred possible worlds*, that is, the set of pairs of possible worlds with times.

(3) After all, if p is in C^+, then it is a mixture of a finite number of possible chance functions. But it then follows that the expectation of a value relative to p is a mixture of the expectations of that value relative to the possible chance functions in that finite set. And if each possible chance function expects c^* to be less inaccurate than it expects c to be, then so does p. More precisely:

$$\begin{aligned}\mathrm{Exp}_I(c^*|p) &= \sum_{w\in W} p(w) I(c^*,\ w)\\ &= \sum_{w\in W}\left(\sum_{ch\in C_p} \alpha_{ch}\, ch(w|E)\right) I(c^*,\ w)\\ &= \sum_{w\in W}\sum_{ch\in C_p} \alpha_{ch}\, ch(w|E) I(c^*,\ w)\\ &= \sum_{ch\in C_p}\sum_{w\in W} \alpha_{ch}\, ch(w|E) I(c^*,\ w)\\ &= \sum_{ch\in C_p} \alpha_{ch} \sum_{w\in W} ch(w|E) I(c^*,\ w)\end{aligned}$$

And similarly for $\mathrm{Exp}_{\mathfrak{I}}(c|p)$. By hypothesis,

$$\sum_{w\in W} ch(w|E) I(c^*,\ w) < \sum_{w\in W} ch(w|E) I(c,\ w)$$

for all $ch(-|E)$ in $\mathscr{C}_E$. Thus,

$$\mathrm{Exp}_I(c^*|p) < \mathrm{Exp}_I(c|p)$$