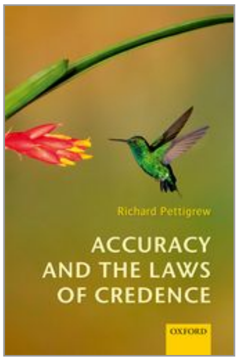

Accuracy and the Laws of Credence
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From No Drop to Probabilism

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Abstract and Keywords

This chapter begins Part I of the book, which treats the accuracy argument for Probabilism. This chapter generalizes the argument given in the Introduction in favour of the principle No Drop. It shows that a similar argument can be given for Probabilism, the principle that says that a rational agent’s credences will satisfy the laws of probability.

Keywords: Probability, Probabilism, accuracy, veritism, Bruno de Finetti

Yasho, then, is irrational because he violates No Drop. And an agent who violates No Drop is irrational because there is a credence function defined on the same opinion set as hers—that is, a credence function that assigns credences to exactly the same propositions as hers does—that is guaranteed to be more accurate than hers—and thus guaranteed to have more epistemic value—regardless of how the world turns out to be. That was the argument of the Introduction. It consisted of an account of epistemic value (Brier Alethic Accuracy), a principle of decision theory (Dominance), and a theorem (Theorem 0.0.1) showing that the principle of decision theory applied to the account of epistemic value entails No Drop.

However, the argument given there applies only to a very limited group of agents, namely, those who—like Yasho—assign credences only to two propositions, one of which is stronger than the other. But of course most agents assign credences to a much richer set of propositions than this. I assign credences to propositions about the chemical composition of water, where I left my house keys, my mother's first name, my niece's favourite ride at Disney World, and so on. You might assign credences to propositions about your immediate physical environment, the colour of a rainbow lorikeet's beak, the theory of natural selection, the chance that humans will colonize Mars, and so on. In this chapter, we seek the requirements that rationality places on such agents.

No Drop is a consequence of Probabilism, the law that requires an agent to have probabilistic credences.[1] Indeed, No Drop simply is the requirement that Probabilism makes of an agent, like Yasho, whose opinion set is $\{A, B\}$. That is, if A is stronger than B, then an agent with credences only in those two propositions satisfies Probabilism iff she satisfies No Drop. But No Drop is also the requirement made of Yasho by a host of other principles of rationality. For instance, if one holds that a rational credence function must be a *plausibility function*—another type of credence function—then No Drop is the requirement that rationality makes of Yasho (Halpern, 2003, 51). Similarly, if one holds that a rational credence function must be a *Dempster-Shafer function* (Dempster, 1968; Shafer, 1976)—another type of credence function still. Thus, establishing No Drop does not allow us to tell between a host of more general rival accounts of the laws of rational credence. It is entailed by all of them. In this chapter, (p.16) we ask which of these more general rival accounts, if any, are supported by the sort of argument from accuracy proposed in the Introduction.

In the argument of the Introduction, we restricted attention to agents with a particular sort of opinion set. In this chapter, we lift that restriction. Indeed, the only restriction we impose is that an agent's opinion set is finite. We do not impose any closure conditions: we do not demand that our agent's opinion set is closed under negation or conjunction, or that it includes a tautology or a contradiction. The question, then, is this: What follows from the argument from accuracy for an agent with a finite opinion set? What requirements of rationality follow for such an agent from the veritist account of epistemic value embodied in the first two premises (Veritism and Brier Alethic Accuracy) together with the principle of decision theory to which we appeal in the second premise (Dominance)? The answer, as we will see, is Probabilism.

To state this rational requirement for credences precisely, let us begin with a definition:

Definition 1.0.1 (Probability function) *Suppose $\mathscr{F}$ is a finite set of propositions. Suppose* $c : \mathscr{F} \to [0,1]$ is a credence function on $\mathscr{F}$. *Then we say that c is a probability function on $\mathscr{F}$ if*

(i) *$\mathscr{F}$ is an algebra and*

(a) *c is normalized. That is,* $c(\bot) = 0$ *and* $c(\top) = 1$.[2]

(b) *c is additive. That is,* $c(A \vee B) = c(A) + c(B) - c(A \,\&\, B)$. or

(ii) *$\mathscr{F}$ is not an algebra, $\mathscr{F}^*$ is an algebra with $\mathscr{F} \subseteq \mathscr{F}^*$, and there is a credence function* $c^* : \mathscr{F}^* \to [0,1]$ *such that c^* is a probability function and $c(X) = c^*(X)$ for all X in $\mathscr{F}$. (That is, $c = c^* |_{\mathscr{F}}$.)*

That is, if an opinion set is an algebra, a credence function on that set is a probability function if it is normalized and additive; and if an opinion set is not an algebra, a credence function on that set is a probability function if it can be extended to a normalized and additive credence function on an algebra.

Thus, if I have credences in the four-element algebra (⊥, Rain, $\{\bot, \mathit{Rain}, \overline{\mathit{Rain}}, \top\}$, where *Rain* is the proposition that it rained in Bristol on 31 May 2015, then my credence function is a probability function if, and only if: (1) it assigns maximal credence, i.e. 1, to the tautology ⊤; (2) it assigns minimal credence, i.e. 0, to the contradiction ⊥; (3) the credences in *Rain* and $\overline{\mathit{Rain}}$ sum to 1.[3]

(p.17) Probabilism is the credal principle that says that rational credence functions are probability functions:

Probabilism If an agent has a credence function over $\mathscr{F}$, then it is a requirement of rationality that c is a probability function on $\mathscr{F}$.

Some notation: Suppose $\mathscr{F}$ is a finite opinion set. Then

- Let $\mathscr{B}_{\mathscr{F}}$ be the set of credence functions $c : \mathscr{F} \to [0,1]$ on $\mathscr{F}$.
- Let $\mathscr{P}_{\mathscr{F}}$ be the set of credence functions that are probability functions on $\mathscr{F}$.

Thus, $\mathscr{P}_{\mathscr{F}} \subseteq \mathscr{B}_{\mathscr{F}}$.

The following theorem is due to Bruno de Finetti and it provides the mathematical result that we need to generalize our argument for No Drop to give an argument for Probabilism (de Finetti, 1974, 87–90):

Theorem 1.0.2 (De Finetti's Dominance Theorem) *Suppose* $\mathscr{F}$ *is a finite opinion set.*

(i) *Each non-probabilistic credence function is strongly Brier-dominated by a probabilistic credence function.*
That is: If c is not in $\mathscr{P}_{\mathscr{F}}$, *then there is c* in* $\mathscr{P}_{\mathscr{F}}$ *such that, for all worlds w in* $\mathscr{W}_{\mathscr{F}}$, $\mathscr{B}(c^*, w) < \mathscr{B}(c, w)$.
That is,

$$(\forall\, c \notin \mathrm{P}_{\mathrm{F}})(\exists\, c^* \in \mathrm{P}_{\mathrm{F}})(\forall\, w \in \mathrm{W}_{\mathrm{F}})[\mathrm{B}(c^*, w) < \mathrm{B}(c, w)]$$

(ii) *No probabilistic credence function is weakly Brier-dominated by any credence function.*
That is: If c is in $\mathscr{P}_{\mathscr{F}}$, *then there is no c* in* $\mathscr{B}_{\mathscr{F}}$ *such that* (i) *for all worlds w in* $\mathscr{W}_{\mathscr{F}}$, $\mathscr{B}(c^*, w) \leq \mathscr{B}(c, w)$ *and* (ii) *for some worlds w in* $\mathscr{W}_{\mathscr{F}}$, $\mathscr{B}(c^*, w) < \mathscr{B}(c, w)$.
That is,

$$(\forall\, c \in \mathrm{P}_{\mathrm{F}}) \neg (\exists\, c^* \in B_{\mathrm{F}})$$
$$((\forall\, w \in \mathrm{W}_{\mathrm{F}})[\mathrm{B}(c^*, w) \leq \mathrm{B}(c, w)] \,\&\, (\exists\, w \in \mathrm{W}_{\mathrm{F}})[\mathrm{B}(c^*, w) < \mathrm{B}(c, w)])$$

A consequence: The credence functions that are either weakly or strongly Brier-dominated are precisely the non-probabilistic ones. This theorem is a corollary of Theorem 4.3.4, the main theorem of this part of the book, which we state in Chapter 4 and prove in Appendix I.

Figures 1.1 and 1.2 provide diagrams analogous to Figure 0.1 that show this result in action.

We can now replace Theorem 0.0.1 with Theorem 1.0.2 in the accuracy argument from the Introduction. This allows us to draw a stronger conclusion from the dominance principle to which we appealed there. It allows us to conclude Probabilism. Here is the new argument: (p.18)

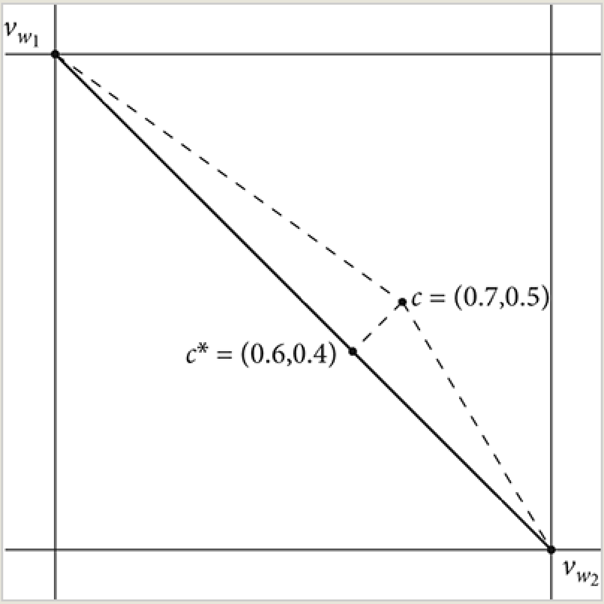

Figure 1.1 As in the Introduction, we plot a credence function *c* as a point in the unit square. In this example, the agent has credences only in *A* and Ā. At world w_1, *A* is true; and world w_2, *A* is false. The thick, solid diagonal line represents the set of credence functions that satisfy Probabilism—they are those for which $c(A) + c(\bar{A}) = 1$. In this figure, *c* is a credence function that violates Probabilism. c^* is the nearest point to *c* that lies on the thick, solid line. Thus, it satisfies Probabilism. Moreover, as the thin, dashed lines show, c^* is closer to each v_{wi} than *c* is.

(p.19)

(I_p) **Veritism** The ultimate source of epistemic value is accuracy.

(II_p) **Brier Alethic Accuracy** Inaccuracy is measured by the Brier score. So the inaccuracy of credence function *c* at world *w* is

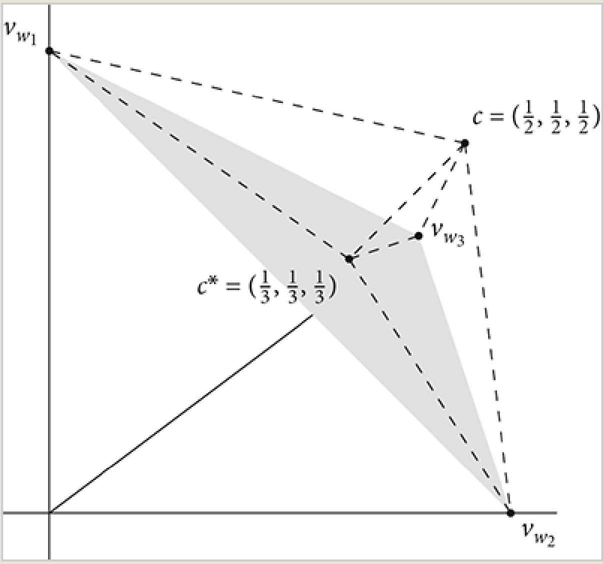

Figure 1.2 In this example, our agent has credences in the three elements of a partition $\{X_1, X_2, X_3\}$. We represent a credence function c on this opinion set as the point $(c(X_1), c(X_2), c(X_3))$ in the unit cube. Let w_1, w_2, and w_3 be the possible worlds relative to this set of propositions, where X_{i} is true at world w_{i}. Probabilism imposes the constraint that credences in the three propositions should sum to 1. The shaded triangle represents the set of credence functions that satisfy that constraint. c violates the constraint; c^* satisfies it. As the dashed lines show, c^* is closer to each omniscient credence function v_{wi} than c is.

$$\mathrm{B}(c, w) := \mathrm{d}^2(v_w, c) = \| v_w - c \|_2^2.$$

(III_p) **Dominance** If

(i) *o* is strongly $\mathfrak{U}$-dominated

then

(ii) *o* is irrational for any agent with utility function $\mathfrak{U}$.

(IV_p) **Theorem 1.0.2.**

Therefore,

(V_p) **Probabilism.**

We have now generalized our accuracy argument for No Drop to give an accuracy argument for Probabilism. Our next job is to strengthen that argument in two ways:

- There is a problem with our current formulation of the dominance principle: it is too strong. We will weaken it in Chapter 2.
- We have yet to motivate our assumption that inaccuracy must be measured by the Brier score. In Chapter 3, we will consider existing attempts to motivate this or a weaker assumption. In Chapter 4, we will give our own motivation.

Notes:

([1]) We will formulate Probabilism precisely below.

([2]) Recall: ⊥ is the contradictory proposition; ⊤ is its negation, the tautologous proposition.

([3]) To see that the latter constraint holds:

$$\begin{aligned}1 &= c(\top) \text{ since } c \text{ is normalized}\\ &= c(Rain \vee \overline{Rain})\\ &= c(Rain) + c(\overline{Rain}) - c(Rain \,\&\, \overline{Rain}) \text{ since } c \text{ is additive}\\ &= c(Rain) + c(\overline{Rain}) \text{ since } Rain \,\&\, \overline{Rain} \equiv \bot \text{ and } c \text{ is normalized}\end{aligned}$$