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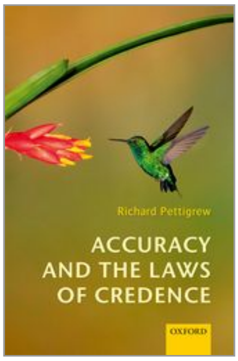

Accuracy and the Laws of Credence
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The Principal Principle

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Abstract and Keywords

This chapter begins Part II of the book, which treats the accuracy argument for various chance-credence principles. This chapter considers David Lewis' Principal Principle. It introduces the framework in which that principle is stated and discusses some of its features.

Keywords: David Lewis, Principal Principle, chance-credence principles, chance

All of the chance-credence principles we will consider in this book are attempts to make precise the following law: It is rationally required to defer to chance as an expert when setting credences. We begin in this chapter by formulating the weakest precise version of the requirement. The formulation is due to David Lewis and it is called the Principal Principle (Lewis, 1980).

To formulate the Principal Principle, we introduce the notion of an *ur-chance function* (Hall, 2004). Consider a possible world w. If w contains an earliest moment, then the ur-chance function of w—denoted ch_w—is just the function that takes a proposition and returns its chance at the initial moment of world w—that is, it is the chance function at that world at its initial moment. The chance function in w at any later moment—that is, the function that takes a proposition and returns the chance of that proposition at that later moment—is then obtained from that ur-chance function by conditionalizing on the history of w up to, but not including, that moment. Thus, if H_{tw} is the history of w up to moment t, then the chances in w at t are given by $ch_w(- \mid H_{tw})$. Indeed, this is the defining feature of an ur-chance function, and we can use it to define the ur-chance function of a world that does not have an initial moment. Suppose world w does not have an initial moment. Then we simply define the ur- chance function of w to be the probability function ch_w with the following property: for every moment t that w contains, the chances in w at t are given by the function $ch_w(-|H_{tw})$, where H_{tw} is the history of w leading up to t.[1]

Note that, throughout, we will assume that chances are defined for all propositions. At first sight, this seems implausible. But we make it true essentially by fiat. If a proposition is genuinely chancy, the ur-chance function assigns to it the ur-chance of that proposition; if it is not chancy, and it is true, it assigns 1; if it is not chancy, and it is false, it assigns 0.

Fix the set of propositions $\mathscr{F}$ over which our agent's credence function is defined. Given a particular probability function ch defined on $\mathscr{F}$, we let C_{ch} denote the proposition that says that ch is the ur-chance function. Thus, C_{ch} is the proposition (p.102) that is true at world w iff $ch = ch_w$. We call a proposition of this form an *ur-chance hypothesis*.

With this in hand, we are ready to state the Principal Principle.

> **Principal Principle ($\mathbf{PP_o}$)** If an agent has an initial credence function c_0 defined on $\mathscr{F}$, then rationality requires that
>
> $$c_0(X|C_{ch}) = ch(X)$$
>
> for all propositions X in $\mathscr{F}$ and all possible ur-chance functions ch such that C_{ch} is in $\mathscr{F}$ and $c_o(C_{ch}) > 0$.

That is, rationality requires that an agent's credence in a proposition conditional on the ur-chance hypothesis that says that ch is the ur-chance function must match the probability that ch assigns to that proposition.

As at all points in this book, we assume that $\mathscr{F}$ is finite. Thus, while there might well be an uncountable infinity of epistemically possible ur-chance functions, we only consider agents who have explicit opinions about finitely many of them.

Throughout this part of the book, we will encounter many different chance- credence principles like PP_0. Many of them differ only slightly from others. As a result, there is a risk of confusion. In Appendix II, I've included a summary of all the principles, together with the problems they face that lead us to seek improvements. Hopefully, this will mitigate the risk of confusion.

It is straightforward to show that Cleo violates PP_0. Suppose ch_1 is the ur-chance function on which the chance of heads is 60% and ch_2 is the one on which it is 70%. Then $c_{\text{Cleo}}(C_{ch_1} \vee C_{ch_2}) = 1$. Thus, if Cleo satisfies PP_0, then by the Theorem of Total Probability, we have:[2]

$$\begin{aligned} c_{\text{Cleo}}(X) &= c_{\text{Cleo}}(C_{ch_1})c_{\text{Cleo}}(X|C_{ch_1}) + c_{\text{Cleo}}(C_{ch_2})c_{\text{Cleo}}(X|C_{ch_2}) \\ &= c_{\text{Cleo}}(C_{ch_1})ch_1(X) + c_{\text{Cleo}}(C_{ch_2})ch_2(X) \end{aligned}$$

That is, if Cleo satisfies PP_0, then her credence in a proposition is a convex combination of the chances assigned to that proposition by the two possible ur-chance functions ch_1 and ch_2—and thus, it lies somewhere between these two values. However, it is clear that this isn't the case for the proposition *Heads*, since $ch_1(Heads) = 0.6$, $ch_2(Heads) = 0.7$ and $c_{\text{Cleo}}(Heads) < 0.5$.

Indeed, the above fact is generally true. If c satisfies PP_0 and there are $C_{ch_1}, \ldots, C_{ch_n}$ in $\mathscr{F}$ such that $c(C_{ch_1} \vee \ldots \vee C_{ch_n}) = 1$, then c is a weighted sum of the possible (p.103) ur-chance functions $ch_1, \ldots, ch_n$, where the weights are given by the credences $c(C_{ch_1}), \ldots, c(C_{ch_n})$ that c assigns to the respective ur-chance hypotheses.

Note that PP_0 is equivalent to Lewis' second formulation of the Principal Principle in (Lewis, 1980, 277). Written in our notation—that is, using ur-chance hypotheses instead of conjunctions of history-to-chance conditionals—Lewis' second formulation runs as follows:

> **PP**$_{\text{Lewis2}}$ If an agent has an initial credence function c_0 defined on $\mathscr{F}$, then rationality requires that
>
> $$c_0(X|C_{ch}\&H_t) = ch_{H_t}(X)$$
>
> for all propositions X in $\mathscr{F}$ and all epistemically possible ur-chance functions ch such that C_{ch} is in $\mathscr{F}$ and $c_0(C_{ch}) > 0$ (where H_t is a proposition that details a possible history up to a moment t and ch_{H_t} is the chance function at t at a world whose ur- chance function is ch and whose history up to t is H_t—that is, $ch_{H_t}(-) = ch(-|H_t)$).

That PP_0 is equivalent to $\text{PP}_{\text{Lewis2}}$ is a consequence of the following more general fact: if c_0 satisfies PP_0, then

$$c_0(X \mid C_{ch} \,\&\, E) = ch(X \mid E)$$

for all $X, E \in \mathscr{F}$ such that $c_0(C_{ch} \,\&\, E)$, $ch(E) > 0$. And it is this fact that allows us to state the Principal Principle without referring to Lewis' notorious admissibility condition. In Lewis' first formulation of his principle, he states it as follows (Lewis, 1980, 266):

> **PP**$_{\text{Lewis1}}$ If an agent has an initial credence function c_0 defined on $\mathscr{F}$, then rationality requires that
>
> $$c_0(X \mid C_{ch} \;\&\; H_t \;\&\; E) = ch_{H_t}(X)$$
>
> for all propositions X in $\mathscr{F}$, all epistemically possible ur-chance functions ch such that C_{ch} is in $\mathscr{F}$ and $c_0\,(C_{ch}) > 0$, *and all propositions E in $\mathscr{F}$ such that E is admissible evidence for X relative to the chances at time t.*

Lewis introduced the admissibility criterion in order to account for the following sort of case: Suppose I learn the true chance hypothesis—that is, I learn C_{ch}—and I learn the history of the world up to t—that is, I learn H_t. So I know the chance of X at t—it is $ch_{H_t}(X)$. And let's suppose that $ch_{H_t}(X) < 0$. But suppose I also learn, perhaps from a clairvoyant friend with a crystal ball, that X will in fact turn out to be true. Then, in that situation, it would seem wrong for me to set my credence in X equal to $ch_{H_t}(X) < 0$. After all, I know that X is true! Lewis avoids this consequence of the Principal Principle by insisting that an agent is only obliged to set her credence in proposition X conditional on chance hypothesis C_{ch} and history H_t and evidence E to $ch_{H_t}(X)$ if evidence E is admissible. He then laments that he can give no (p.104) precise account of admissibility, though he does give examples of obviously admissible evidence and obviously inadmissible evidence—information about the truth of X received from the future is obviously inadmissible, whereas information purely about the past is obviously admissible. But, by the point just noted, if we accept PP$_0$, we obtain an account of admissibility for free. After all, PP$_0$ entails

$$c_0(X \mid C_{ch} \;\&\; H_t \;\&\; E) = ch_{H_t}(X \mid E)$$

Thus, E is admissible for X relative to the chances ch_{H_t} at time t iff $ch_{H_t}(X|E) = ch_{H_t}(X)$. That is, iff E andX are stochastically independent by the lights of ch_{H_t}. Thus, there is no such thing as evidence that is admissible *tout court*. Evidence is admissible relative to a particular ur-chance function and history.

Now, as is well known—indeed, as Lewis (1980) himself observed—PP$_0$ has unwelcome consequences for certain accounts of chance. In particular, it has unwelcome consequences for accounts of chance on which there are possible ur-chance functions that assign a probability less than 1 to the proposition that says that that very possible ur-chance function gives the ur-chances; that is, accounts on which there are possible ur-chance functions ch such that $ch(C_{ch}) < 1$. Lewis said that such chance functions admit ‘undermining futures’ (Lewis, 1994, 482ff.). I will call them *self-undermining*. If an ur-chance function is not self-undermining, I will call it *non-self-undermining*. Later, we will also have cause to talk of ur-chance functions that are self-undermining or non-self-undermining relative to some evidence. *ch is self-undermining relative to E* iff $ch(C_{\mathrm{ch}}|E) < 1$; otherwise, it is *non-self-undermining relative to E*.

Let us see what trouble these self-undermining ur-chance functions cause. Suppose ch is a self-undermining possible ur-chance function. And suppose that C_{ch} is in $\mathscr{F}$— recall that C_{ch} is the proposition that says that ch gives the ur-chances. Then PP_0 demands the following:

$$c_0(X \mid C_{ch}) = ch(X)$$

if X is in $\mathscr{F}$ and $c_0\,(C_{ch}) > 0$. Thus, in particular, it demands that, if $c_0\,(C_{ch}) > 0$, then

$$c_0(C_{ch} \mid C_{ch}) = ch(C_{ch}) < 1$$

Now, by the ratio definition of conditional probability—which says that $p(X|Y) := \frac{p(XY)}{p(Y)}$ (providing $p(Y) > 0$)—we have that, if $c_0(C_{ch}) > 0$, then

$$c_0(C_{ch}|C_{ch}) = \frac{c_0(C_{ch} \,\&\, C_{ch})}{c_0(C_{ch})} = 1$$

Thus, if $c_0\,(C_{ch}) > 0$, then

(i) $c_0(C_{ch}|C_{ch}) < 1$ (by PP_0)
(ii) $c_0(C_{ch}|C_{ch}) = 1$ (by the ratio definition of conditional probability)

(p.105) But this cannot be. So, a rational agent must assign $c_0(C_{ch}) = 0$ in order to avoid the incompatible demands of the ratio definition of conditional probability and PP_0 That is, she must be certain that the ur-chances are not self-undermining. But that does not seem to be a requirement of rationality. Indeed, Lewis at least held that it is rationally required of an agent that she assign a positive credence to each possibility that is compatible with her evidence—this is the Principle of Regularity. And, on certain theories of chance, it is very rarely incompatible with one’s evidence that the chances are self-undermining. Thus, in many cases, the Principal Principle and the Principle of Regularity conflict.

In Chapter 11, we will consider in more detail why you might think that the ur- chances might be self-undermining and what alternative principles you might adopt if you do. However, in the meantime, we will simply assume that $ch(C_{ch}) = 1$ for all possible ur-chance functions ch such that C_{ch} is in $\mathscr{F}$. There are plenty of accounts of chance that make this so. For instance, suppose you take chances to be primitive modal features of the world. Then you likely think that, though the chances change over time, later chances evolve from earlier ones by conditionalizing on the intervening history in such a way that the ur-chance function of a world is a fixed and eternal feature of it, determined at all moments that the world contains. Thus, the true ur-chance hypothesis at a world is a truth that is determinedly true at all moments and thus should be assigned a probability of 1 by the ur-chance function. If that's the case, the ur-chance function is not self-undermining.

Notes:

([1]) For those familiar with Lewis' second formulation of the Principal Principle in (Lewis, 1980), it might be useful to note that an ur-chance function encodes the same information as Lewis' conjunctions of history- to-chance conditionals: in both cases, they take in the history of a world leading up to a moment and return the chance at that moment.

([2]) Recall:

> **Theorem of Total Probability** If c is a probability function defined on propositions $E_1, \ldots, E_n$, and $c(E_1 \vee \ldots \vee E_n) = 1$ and $c(E_i \,\&\, E_j) = 0$ for all $i \neq j$, and $c(E_i) > 0$ for all i, then

$$c(X) = \sum_{i=1}^{n} c(E_i)c(X|E_i)$$