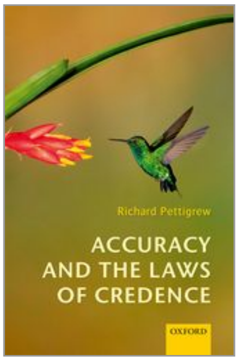


Accuracy and the Laws of Credence
Richard Pettigrew

Print publication date: 2016
Print ISBN-13: 9780198732716
Published to Oxford Scholarship Online: May 2016
DOI: 10.1093/acprof:oso/9780198732716.001.0001

Formulating the dominance principle

Richard Pettigrew

DOI:10.1093/acprof:oso/9780198732716.003.0003

Abstract and Keywords

This chapter strengthens the argument for Probabilism given in Chapter 1 by weakening the dominance principle to which it appeals. It shows why the naïve dominance principle from Chapter 1 is in fact too strong.

Keywords: Dominance principle, decision theory, rational choice

In our accuracy arguments for No Drop and then for Probabilism, we formulated the dominance principle of decision theory as follows:

Dominance If

(i) o is strongly $\mathfrak{U}$-dominated, then
(ii) o is irrational for any agent with utility function $\mathfrak{U}$.

Recall the framework in which we state these principles of decision theory: o is one from amongst a set of options $\mathcal{O}$ that are available to the agent; $\mathfrak{U}$ is a function that takes an option and a world and returns the utility of choosing that option at that world. We say that option o is strongly $\mathfrak{U}$-dominated if there is some other option in $\mathcal{O}$ that, by the lights of $\mathfrak{U}$, has greater utility than o at every world. And we say that o is weakly $\mathfrak{U}$-dominated if there is some other option in $\mathcal{O}$ that, by the lights of $\mathfrak{U}$, has at least as great utility as o at every world and greater utility at some worlds. In this chapter, we raise issues with this formulation of the dominance principle and we work gradually towards the correct formulation.[1]

2.1 From Dominance to Undominated Dominance

The formulation of the dominance principle just given is the formulation to which Joyce (1998) appeals in his own original formulation of the accuracy argument for Probabilism.[2] Yet there is a counterexample to this principle that is given by the following decision problem:

Name Your Fortune God tells you to pick a positive integer. If you pick k, God will give you k utiles.[3]

(p.21) Now, let's describe this decision problem using the framework in which our decision principles are stated. First, the set of options is $\mathcal{O} = \{o_1, o_2, \dots\}$, where o_1 is the act of picking integer 1, o_2 is the act of picking 2, and so on. Since there is no uncertainty about the outcomes of any of the options, there is just one possible world: so $\mathcal{W} = \{w\}$. And, by the description of the case, we know that the utility of o_k at that one possible world is k, so $\mathfrak{U}(o_k, w) = k$, for each $k =, 1, 2, \dots$. Thus, we can represent the decision as follows:

$\mathfrak{U}$	o_1	o_2	o_3	o_4	…	o_k	…
w	1	2	3	4	…	k	…

The problem is that each option is strongly $\mathfrak{U}$-dominated: picking integer k (that is, option o_k) is always strongly $\mathfrak{U}$-dominated by picking integer $k + 1$ (that is, option o_{k+1}). Thus, if Dominance is true, each option is irrational. Yet intuitively this seems wrong. Even if you think that there exist rational dilemmas—that is, decision problems in which no option is rationally permissible—this doesn't seem like one of them. In the light of this, a natural reformulation of the dominance principle runs as follows:

Dominance* If

(i) o is strongly $\mathfrak{U}$-dominated, and
(ii) there is o' that is not even weakly $\mathfrak{U}$-dominated, then
(iii) o is irrational for any agent with utility function $\mathfrak{U}$.

This says that a dominated option is irrational if there is some alternative option that isn't dominated. Thus, in Name Your Fortune, Dominance* would not rule out any option as irrational—each option is dominated, but since there is therefore no undominated option, none is ruled irrational on that basis.

However, there is an intuitive counterexample to this formulation as well that is given by the following decision problem:

Name Your Fortune* You have a choice: play a game with God or don't. If you don't, you receive 2 utiles for sure. If you do, you then pick an integer. If you pick k, God will then do one of two things: (i) give you k utiles, as before; or (ii) give you $2 - \frac{1}{2^{k-1}}$ utiles.

Again, let's describe this decision problem in our framework. This time, the options are $\mathcal{O} = (o, o_1, o_2, \ldots\}$, where o is the option of not playing the game with God; and o_k is the option of playing and picking k. And there are two worlds $\mathcal{W} = \{w_1, w_2\}$: if you play and pick k, then in w_1 God gives you k utiles, while in w_2 God gives you $2 - \frac{1}{2^{k-1}}$ utiles. Thus, the decision can be represented as follows: (p.22)

$\mathfrak{U}$	o	o_1	o_2	o_3	o_4	…	o_k	…
w_1	2	1	2	3	4	…	k…	…
w_2	2	1	$2-\frac{1}{2}$	$2-\frac{1}{4}$	$2-\frac{1}{8}$	…	$2-\frac{1}{2^{k-1}}$	…

This time, there is an option that is not even weakly $\mathfrak{U}$-dominated, namely, o. All other options, however, are strongly $\mathfrak{U}$-dominated: o_k is strongly $\mathfrak{U}$-dominatedby o_{k+1}. Thus, if Dominance* is correct, then each of o_1, o_2, … is irrational. But again this seems wrong. One way to appreciate the problem in this case is to notice that, for each probabilistic credence function on the two possibilities w_1 and w_2, there is an option o_k such that, relative to these credences, the expected utility of o_k exceeds that of o.[4] This suggests that we should not rule out each o_k as irrational.

This leads to our final formulation of the dominance principle:

Undominated Dominance If

(i) o^* strongly $\mathfrak{U}$-dominates o, and
(ii) there is no o' that weakly $\mathfrak{U}$-dominates o^* then
(iii) o is irrational for any agent with utility function $\mathfrak{U}$.

Thus, a dominated option is only ruled irrational if at least one of the dominating options is one that might be chosen without incurring a criticism similar to the one that would be levelled if one chose the dominated option, that is, if at least one of the dominating options is not itself dominated.

Now, Undominated Dominance is weaker than Dominance: that is, Dominance renders irrational all the options that Undominated Dominance renders irrational *and, in some cases, others besides*. Thus, we must ensure that Undominated Dominance is strong enough to provide a justification for Probabilism. However, a quick look at De Finetti's Dominance Theorem (stated above as Theorem 1.0.2) shows that it is. After all, De Finetti's Dominance Theorem shows that any non-probabilistic credence function is strongly Brier-dominated by a probabilistic credence function, and that no probabilistic credence function is even weakly Brier-dominated. Thus, each non-probabilistic credence function is strongly Brier-dominated by a credence function that is not even weakly Brier-dominated. And so Undominated Dominance rules out as irrational all of the non-probabilistic credence functions. Probabilism follows.

(p.23) 2.2 From Undominated Dominance to Immodest Dominance

We are not finished with the dominance principle yet. There are two outstanding issues, which we will treat in this section and the next. Both concern ways in which even dominating options that are not themselves dominated might turn out to be irrational; if this is the case, there is reason to think that the options they dominate should not be ruled out as irrational; and if that's the case, then Undominated Dominance is wrong.

Here is the first concern. As it stands, Undominated Dominance is, I think, the correct formulation of the dominance principle that governs practical situations, such as Name Your Fortune and Name Your Fortune*. But there is a problem when the situation is epistemic, and the options are credence functions. The insight that led us from Dominance* to Undominated Dominance was that, in order for a dominated option to be ruled out as irrational, it has to be that at least one of the dominating actions is not itself ruled out as irrational, either by appeal to the same principle or by appeal to some other. Now, as it stands, Undominated Dominance rules out a dominated option as irrational just in case at least one of the dominating actions is neither strongly nor weakly $\mathfrak{U}$-dominated. In the practical case, this is enough: dominance reasoning is only ever used when there is no relevant probability function available relative to which one might assess expected utility; and, in the absence of such an assessment, being strongly or weakly dominated seems to be the only way for an option to be ruled irrational.[5] However, in the epistemic case, the situation is different. While there is, initially, no relevant probability function available, at least one of the dominating options is a probabilistic credence function. Thus, there is a new way for that dominating option to be irrational: when it assesses the options in terms of their expected inaccuracies, it might not assess itself to be amongst the best; that is, it might expect itself to be *more* inaccurate than it expects some other credence function to be. If it does, we say that it is *extremely modest* relative to that particular way of measuring inaccuracy. (If it only expects some other credence function to be *at most as* inaccurate as it expects itself to be, we say that it is *moderately modest*, again, relative to the inaccuracy measure.)

Let's make all of this more precise. First, we have to define the expected inaccuracy of one credence function by the lights of another and relative to a particular inaccuracy measure. Crucially, the credence function by the lights of which we assess expected inaccuracy must be a probabilistic credence function; but, furthermore, it must be defined not just on the set of propositions $\mathcal{F}$—which may satisfy few closure conditions—but also on $\mathcal{F}^*$, which is the smallest algebra that extends $\mathcal{F}$.[6] The reason is that we need the credence function to assign a credence to each w in $\mathcal{W}_{\mathcal{F}}$, and that (p.24) isn't guaranteed if it is defined only on $\mathcal{F}$. Thus, suppose $\mathfrak{I}$ is a legitimate measure of inaccuracy; and suppose p is a probabilistic credence function on $\mathcal{F}^*$ and c is a credence function (not necessarily probabilistic) on $\mathcal{F}$. Then, we define the *expected inaccuracy of c by the lights ofp and relative to* $\mathfrak{I}$ as follows:

$$\mathrm{Exp}_{\mathfrak{I}}(c|p) := \sum_{w \in W_{\mathcal{F}}} p(w)\mathfrak{I}(c, w)$$

where we abuse notation and write *w* also for the proposition in $\mathcal{F}^*$ that specifies world *w* uniquely—thus, we write *w* not only for the possible world, but also for the proposition that is true at world *w* and only at world *w*. Thus, $\mathrm{Exp}_{\mathfrak{I}}(c|p)$ is the weighted sum of the inaccuracy of *c* at different worlds, where the weights are given by the credences that *p* assigns to these worlds. Now we can give the following definitions:

Definition 2.2.1 *Suppose p is a probabilistic credence function defined on* $\mathcal{F}$. *Then*

(i) *p* is extremely modest relative to $\mathfrak{I}$ (*or p* is extremely $\mathfrak{I}$-modest) *if there is another credence function* $c \neq p$ *such that* $\mathrm{Exp}_{\mathfrak{I}}(c|p^*) < \mathrm{Exp}_{\mathfrak{I}}(p|p^*)$ *for some probabilistic extension* p^* *of p to* $\mathcal{F}^*$.
(ii) *p* is moderately modest relative to $\mathfrak{I}$ (*or p* is moderately $\mathfrak{I}$-modest) *if there is a credence function* $c \neq p$ *such that* $\mathrm{Exp}_{\mathfrak{I}}(c|p^*) \leq \mathrm{Exp}_{\mathfrak{I}}(p|p^*)$ *for some probabilistic extension* p^* *of p to* $\mathcal{F}^*$.

Extremely modest credence functions relative to a legitimate measure of inaccuracy are irrational for one who endorses that measure ofinaccuracy—if a credence function expects another to be best, it is irrational to adopt that credence function; it is irrational to adopt a position from which some other position seems better. Moderately modest credence functions, on the other hand, need not be irrational; it is not irrational to adopt a position from which some other position seems just as good. Thus, if (i) a credence function is dominated and (ii) the only dominating options that are not themselves dominated are probabilistic credence functions and (iii) all of those are extremely modest, then it seems that we are not warranted in ruling out the original credence function as irrational.

In the light of this discussion, we might wish to move to the following version of the dominance principle, which only applies when the options are credence functions and which is weaker than Undominated Dominance—that is, anything ruled irrational by Immodest Dominance is ruled irrational by Undominated Dominance.

Immodest Dominance Suppose $\mathfrak{I}$ is a legitimate measure of inaccuracy. Then, if

(i) *c* is strongly $\mathfrak{I}$-dominated by probabilistic c^*, and

(ii) c^* is not extremely $\mathfrak{I}$-modest
then
(iii) c is irrational.

(p.25) There are two reasons one might resist this move. First, one might reason as follows. Suppose that *c* is dominated by a probabilistic credence function *p* that is not itself dominated. If Undominated Dominance is correct, this would be enough to rule out *c* as irrational. But if Immodest Dominance is the strongest principle in the vicinity, it is not enough, for we are worried about the possibility that *p* expects some other credence function c' to be more accurate than it expects itself to be. Suppose that's true. Then, providing we assume that our inaccuracy measure is real-valued and continuous— that is, if we assume the axioms Continuity and Finiteness introduced in Chapter 3 below—it is also true that there is a credence function c'' that has *minimal* expected inaccuracy by the lights of *p*.[7] That is, *p* expects c'' to be best from an epistemic point of view. Now, since *c* is dominated by *p*, we know that c'' must be a different credence function from *c*. Thus, one might think that *c* should be ruled out as irrational on these grounds: there is a doxastic state that is better than *c* at every world, and that doxastic state assesses some state other than *c* to be best, namely, c''. If one were giving advice to an agent with credence function *c*, one might naturally think that they should move to c''—they should move first to *p*, since it is guaranteed to be more accurate; but then, by the lights of *p*, c'' is best, so they should then move to c''. But I think this is mistaken. For one thing, there is no reason to think that c'' is not itself extremely modest. And, even if it weren't, c'' is the state recommended by a state that is irrational, namely, *p*. It would seem odd to take the advice given by that state. Thus, we do not wish to say that a credence function dominated by a probabilistic credence function is thereby irrational on the grounds that there is some credence function other than the dominated one that the dominating one expects to be best.

The second reason for concern about the tentative argument for moving from Undominated Dominance to Immodest Dominance is that it assumes, at least when a credence function is probabilistic, that the credence function assesses options by calculating their expected utility: that is, the assessment of an option is given by the weighted sum of its possible utilities, where the weights are given by the probabilities the credence function assigns to the option producing each of those possible utilities. But why should we think that probabilistic credence functions assess options in this way? Why not some other way? I think this is a serious question. It is often simply assumed in presentations of decision theory that credences and the possible utilities of an option ought to be combined in this way to give the evaluation of that option from the point of view of those credences. And one often finds the mistaken view that Savage-style representation theorems vindicate this assumption. But that is not the case. Rather, one must argue for that way of combining credences and utilities. (p.26) In fact, I believe that we can give something like a combined accuracy argument that establishes both that credences should be probabilities (i.e. Probabilism) and that evaluations should be expectations based on those probabilities. If we grant that an agent's evaluations of a set of quantities should be her best estimates of those quantities, and if we grant that these estimates are better or worse according to their proximity to the true quantities—where such proximity is measured using the sort of measures of distance that we will use in Chapter 4 as the basis of our measures of the inaccuracy of credence functions—then it is possible to adapt an argument due to de Finetti to create an accuracy-dominance argument for taking estimates, and thus evaluations, to be expectations based on the probabilities given by an agent's credences (de Finetti, 1974; Pedersen & Glymour, 2012; Pettigrew, to appear a).

In sum, I think we must move from Undominated Dominance to Immodest Dominance. How does our accuracy argument for Probabilism fare if we make this move? It goes through without a hitch due to the following theorem:

> **Theorem 2.2.2** *Every probabilistic credence function is not even moderately* $\mathfrak{B}$*-modest*.

If an inaccuracy measure $\mathfrak{I}$ has this property—that is, if all probabilistic credence functions are not extremely $\mathfrak{I}$-modest—we say that it is *proper*. If, furthermore, all probabilistic credence functions are not moderately $\mathfrak{I}$-modest either—that is, if every probabilistic credence function expects itself to be *least* inaccurate when inaccuracy is measured this way—we say that it is *strictly proper*. Thus, Theorem 2.2.2 says that the Brier score, $\mathfrak{B}$, is strictly proper. We will have a lot more to do with strictly proper inaccuracy measures in the coming section. Thus, replacing Dominance with Immodest Dominance in our original argument for Probabilism, and appealing to Theorem 2.2.2 in addition to Theorem 1.0.2, we have a new, stronger argument for Probabilism.

2.3 From Immodest Dominance to Deontological Immodest Dominance

How else might a dominating credence function that is itself undominated nonetheless end up being irrational? That is, what other conditions must we place on a dominating credence function so that its existence rules out the credence function it dominates as irrational? According to Easwaran & Fitelson (2012), a dominating credence function that violates the constraints imposed by the agent's evidence is thereby irrational, and thus its existence does not thereby rule out anything it dominates as irrational.

Let's consider an example that is close to the example described by Easwaran and Fitelson.[8] Taj has opinions only about two propositions, both of which concern the outcome of a coin toss: the first, *Heads*, says that the coin will land heads; the other, *Tails*, says that it will not. Taj has learned with certainty that the coin is not fair; it is biased. Indeed, she has learned with certainty that the chance of the coin landing heads (p.27) is 70%. That is her total evidence—no wizard has told her what he sees in his crystal ball concerning the outcome of the toss. Her credence function is as follows:

$$c(Heads) = 0.7 \quad c(Tails) = 0.6$$

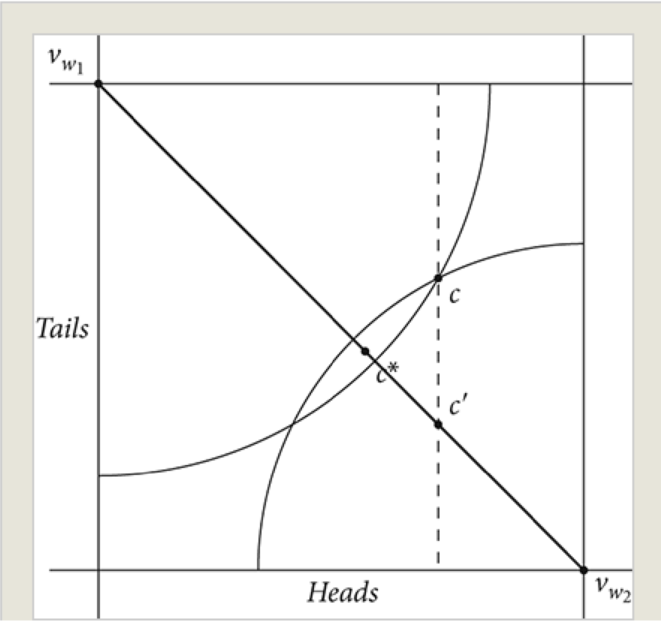
Thus, she violates Probabilism, since *Heads* and *Tails* are mutually exclusive and exhaustive propositions and yet the credences that Taj assigns to them do not sum to 1. By Theorem 1.0.2, her credence function is dominated by credence functions that are not themselves dominated; indeed, by Theorem 2.2.2, each credence function that dominates *c* and isn't itself dominated is a probabilistic credence function, and it is not even moderately modest—it expects itself to be best. Nonetheless, according to Easwaran and Fitelson, it turns out that all of the credence functions that dominate *c* violate the constraints that Taj's evidence seems to place on her credences—thus, they are irrational, and so their existence does not rule out Taj's credence function *c* as irrational. Let's see why they think this is the case. Taj has learned with certainty that the chance of heads is 70%. According to many versions of the Principal Principle, this imposes the following constraint on an agent's credence function: she must have credence 0.7 in *Heads* (Lewis, 1980).[9] But, as we can see from Figure 2.1, while Taj's credence function satisfies this constraint, all those that dominate her violate it, even though she has probabilistic dominators that are appropriately immodest. According to Easwaran and Fitelson, while Immodest Dominance rules out *c* as irrational, the correct dominance principle will not. The idea is that Immodest Dominance must be replaced with something like the following principle:

Deontological Immodest Dominance Suppose $\mathfrak{I}$ is a legitimate inaccuracy measure and suppose C are constraints on a credence function that might be imposed by an agent's evidence. Then, if

(i) c is strongly $\mathfrak{I}$-dominated by probabilistic c^*;
(ii) c^* is not extremely $\mathfrak{I}$-immodest;
(iii) if c satisfies constraints C, then c^* satisfies C. then
(iv) c is irrational for an agent whose evidence imposes constraints C on her credences.

According to Easwaran and Fitelson, Taj's credence function is not ruled irrational by Deontological Immodest Dominance, since it satisfies the constraints imposed by her evidence, but none of its dominators do —thus, clause (iii) is not satisfied.

There are two ways to respond to this charge: on the first, we accept Deontological Immodest Dominance and deny that Taj's credence function satisfies the true constraints imposed by her evidence; on the second, we deny Deontological Immodest (p.28)



Dominance (and possibly also deny that Taj's credence function satisfies the true constraints). In unpublished work, Jim Joyce takes the first approach (Joyce, ms). The idea, I think, is this: Joyce accepts that there are purely evidential constraints that may not be entailed by considerations of accuracy alone. However, he holds that such constraints are nonetheless subservient in some sense to accuracy considerations. Thus, he argues that a set of evidential constraints cannot be such that there are dominated credence functions that count as satisfying these constraints. That is, while the constraints do not all have their source entirely in considerations of accuracy, they are themselves constrained by such considerations. According to Joyce, the constraint that Taj's evidence places on her credences is not simply that her credence in *Heads* must be 0.7; it is also that her credence in *Tails* must be 0.3. In this way, Joyce argues that, contrary to Easwaran and Fitelson's claim, Taj's original credence function c does not satisfy the true constraints imposed by her evidence: thus, while its dominators (p.29) do not either, this does not prevent them from ruling it out as irrational, in line with Deontological Immodest Dominance.

Figure 2.1 Since Taj has credences in only two propositions, we can represent her credence function c—as well as the two omniscient credence functions v_{w1} and v_{w2} by points in the unit square. As in Figure 0.1, on this representation, the Brier score of c at v_{w1}, for instance, is the square of the Euclidean distance between the points that represent them. The thick diagonal line represents the credence functions that satisfy Probabilism; the dashed vertical line represents the credences that satisfy the Principal Principle; the two arcs represent the credence functions that are exactly as accurate as c at worlds w_1 and w_2, respectively. Thus, the points that lie in between them represent exactly the credence functions that are more accurate than c at both worlds; that is, the credence functions that accuracy dominate c. Joyce's argument turns on the fact that this area and the thick line intersect. That is, it relies on the fact that there are probabilistic credence functions that accuracy dominate c. Indeed, c^* is such a credence function—it assigns c^* (*Heads*) = 0.55 and c^* (*Tails*) = 0.45. Easwaran and Fitelson's objection turns on the fact that the dotted line does not intersect with the area between the two arcs. Thus, none of the credence functions that accuracy dominate c satisfy the Principal Principle. Indeed, the only credence function that satisfies Probabilism and the Principal Principle in this situation is $c'(X) = 0.7$, $c'(X) = 0.3$. And that doesn't dominate c—it is less accurate than c at world w_1.

Now, you might object that this can’t be correct. Suppose the agent knows that the chance of heads is 70%, but does not know that chances are always probabilities.[10] Surely then the chances still constrain her credence in *Heads*, but they cannot constrain her credence in *Tails*, because the agent does not know anything about the chance of that proposition. In fact, I think this is wrong. Until you know that the objective chances are probabilities, you do not have reason to set your credences to the known chances. After all, for all you know, that might lead you to set your credences in a nonprobabilistic way that is therefore accuracy dominated. That is, chances only place a constraint on your credence function once you know that those chances give rise to credences that are not accuracy dominated. Thus, the true constraint applies to credences in *Heads* and in *Tails*, and it only applies when you know that objective chances are always probabilities.

Joyce’s strategy here is in line with a broadly *accuracy-first epistemology*. For him, accuracy may not be the only source of value for credence functions—there is, perhaps, also the value that comes from respecting one’s evidence—but it is the primary source of value: it constrains how the other sources of value give rise to rational requirements and constraints. I favour a rather more radical accuracy-based epistemology. It might be better named an *accuracy-only epistemology*. It is embodied in Veritism. On this view, the only constraints that evidence can place on credence functions come from considerations of accuracy, together with decision-theoretic principles. Indeed, one of the central purposes of this book is to derive seemingly evidential principles from considerations of accuracy alone. As we will see in Part II, the Principal Principle is amongst them. Now, as we will see there, the constraints imposed by evidence like Taj’s are indeed those Joyce endorses: Taj’s evidence constrains her to have credence 0.7 in *Heads* and credence 0.3 in *Tails*. But the reason is rather different. The constraints don’t have their source both in the value of respecting evidence *and* in the value of accuracy, as Joyce would have it; they have their source only in the value of accuracy. Moreover, while this allows us to escape Easwaran and Fitelson’s argument by saying that Taj does not satisfy the evidential constraints, we do not need to, because we reject the amendment of Immodest Dominance that gives Deontological Immodest Dominance.

We need only move from Immodest Dominance to Deontological Immodest Dominance if we believe that there is some other source of value—such as respecting evidence—that might render an option irrational—or if we think there is another source of irrationality for credences beyond facts about their epistemic value—such as the sort of epistemic obligations posited by the epistemic deontologist. Without either of those, there is no need for the extra clause (iii). Now, it might seem that it makes very little difference whether we adopt Immodest Dominance or Deontological Immodest (p.30) Dominance. Immodest Dominance rules out Taj's credence function as irrational; Deontological Immodest Dominance does not. But, as we mentioned above, in Part II, we will present a decision-theoretic principle that rules out Taj's credence function as irrational solely on the grounds of accuracy. Indeed, that principle is strictly stronger than Immodest Dominance. Thus, it rules out *all* non-probabilistic credence functions as irrational; that is, it entails Probabilism. Thus, in the end, the same laws of credence result. So why quibble over Deontological Immodest Dominance?

There are, I think, two related reasons: Firstly, we are interested not only in the fact *that* a credence function is irrational; we are also interested in *why* it is so. It is over-determined that Taj's credence function is irrational: it is both strongly dominated by credence functions that are not even moderately modest; and it is what we will, in Part II of the book, call *chance dominated* by credence functions that are not even moderately modest. If we were to accept Deontological Immodest Dominance, its irrationality would therefore be a result only of the latter and not of the former. And that seems wrong.

The second reason not simply to acquiesce to Deontological Immodest Dominance is as follows. The principle, Chance Dominance, from which we will derive the Principal Principle below is less obvious than Immodest Dominance. While I believe it is true and will argue in its favour, I am not as certain of it as I am of Immodest Dominance. Indeed, since it is strictly stronger, this is hardly a surprise. In general, the decision-theoretic principles to which we will appeal in this book are not all equally plausible and equally certain. If we were to accept Deontological Immodest Dominance and rule out Taj's credence function as irrational on the basis of Chance Dominance alone, we would thereby render our belief in its irrationality less certain. Thus, I retain Immodest Dominance and do not accept Deontological Immodest Dominance.

Notes:

([1]) This chapter draws on some material in (Pettigrew, 2014a).

([2]) Hájek (2008) raises a related concern about Joyce's appeal to Dominance.

([3]) A *utile* is a unit of utility.

([4]) For instance, suppose we assign a credence of 0.2 to w_1 and 0.8 to w_2. Then we can see that option o_5 has greater expected utility than option o. After all, the expected utility of an option is the weighted sum of its utilities at different worlds, where the weights are provided by the credences assigned to the different worlds. Thus, the expected utility of o_5 by the lights of these credences is $(0.2 \times 5) + (0.8 \times (2 - \frac{1}{16})) = 2.55$. And the expected utility of o is $(0.2 \times 2) + (0.8 \times 2) = 2$. We will have more to say about expectations shortly.

([5]) Though, in the next section, we will discuss cases in which the dominating option is ruled out as irrational for reasons that appeal to more than just the utilities of its outcomes.

([6]) Of course, if $\mathscr{F}$ is already itself an algebra, then $\mathscr{F} = \mathscr{F}^*$.

([7]) If the inaccuracy of a credence function b at a world is a real-valued, continuous function of b, then the expected inaccuracy of b by the lights of a probabilistic credence function p is likewise a continuous, real-valued function of b. But the set of credence functions is a compact set–indeed, it is (isomorphic to) $[0,1]^n$, where n is the number of propositions in $\mathscr{F}$. And there is a well-known mathematical result that says that any continuous, real-valued function on a compact set is bounded and achieves its infimum and supremum values.

([8]) Here, I present some of the ideas that I began to explore in (Pettigrew, 2014a).

([9]) As we will see in Part II, there are many different versions of the Principal Principle. Here we appeal to perhaps the weakest and most intuitive: if an agent knows the objective chance of a proposition, then her credence in that proposition should be equal to that objective chance.

([10]) Cf. (Easwaran & Fitelson, 2012, Footnote 7).