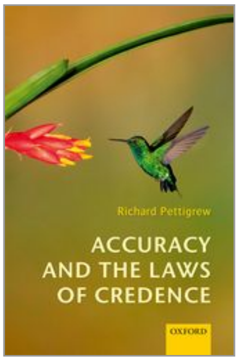

Accuracy and the Laws of Credence
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Plan Conditionalization

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Abstract and Keywords

This chapter begins Part IV of the book, which treats the accuracy arguments for principles that govern how agents should change their credences in the light of new evidence. The chapter considers the synchronic versions of these principles, which govern how a rational agent will plan to update, even though they say nothing about how such an agent will in fact update. The chapter treats Conditionalization in particular.

Keywords: Conditionalization, synchronic rationality, evidence, update rules

In this chapter, we consider three arguments for the synchronic version of Bayesian conditionalization, which renders it a law of credence that governs how agents plan to update in the light of new evidence, not how they in fact do update. To give a precise statement of this version of conditionalization, we must give a couple of definitions.

First, given a partition $\mathcal{E}$, an *updating plan on* $\mathcal{E}$ is a function $R_{\mathcal{E}}$ that associates with each element E of $\mathcal{E}$ a credence function R_E. Thus, given an updating plan $R_{\mathcal{E}}$ and an element E of $\mathcal{E}$, R_E is the credence function that this updating rule recommends to the agent who learns E and who knew in advance that they would learn some element of $\mathcal{E}$.

Of course, there are many updating rules on $\mathcal{E}$. For instance, for any credence function c, there is the *constant rule on c*. This is the rule that simply recommends that the agent adopt c regardless of what they learn. Also, for any probabilistic credence function c that assigns positive probability to each E in $\mathcal{E}$, there is the unique conditionalization rule on $\mathcal{E}$ for c: given E in $\mathcal{E}$, this rule recommends the credence function $c(-|E) = \frac{c(- \& E)}{c(E)}$. If, on the other hand, $c(E) = 0$ for some E in $\mathcal{E}$, there are multiple conditionalization rules for c on E, since conditionalization makes no particular demands on how an agent should plan to update in the event that she learns as evidence a proposition that she was initially certain isn't true. The following definition covers both cases:

Definition 14.0.1 (**Conditionalization rule**) *Given a rule $R_{\mathcal{E}}$, we say that $R_{\mathcal{E}}$* is a conditionalization rule on $\mathcal{E}$ for *c if the following holds for each E in* $\mathcal{E}$:

$$c(- \ \& \ E) = c(E)R_E(-)$$

If $c(E) > 0$, *this entails that* $R_E(-) = c(-|E) := \frac{c(-E)}{c(E)}$. *If* $c(E) = 0$, *this imposes no constraints whatsoever on* R_E.

With this terminology in hand, we can now state the synchronic version of Bayesian conditionalization. It runs as follows. (I take the name from Easwaran (2013, 132).)

Plan Conditionalization Suppose an agent has probabilistic credence function c at t. Suppose she knows that she will receive evidence from $\mathcal{E}$ between t and t'. Then, if she adopts an updating rule at t, it is a requirement of rationality that she adopts a conditionalization rule on $\mathcal{E}$ for c.

(p.188) Note: we don't assume that an agent who fails to adopt an updating rule is irrational. Of course, there seems to be something defective about someone who doesn't; but I can't put my finger on what it is, and I'm inclined to think that the defect is not a failure of rationality.

You might worry that Plan Conditionalization is a rather weak principle. It only applies to an agent who makes a plan, and an agent can only make a plan when she knows the partition from which her evidence is guaranteed to come. But, you might worry, we rather rarely know such a partition and so we're rather rarely in a position to make such a plan. For instance, I may know that when I check the news a little while after polls close on election night, I'll either learn that Party A has won, or that party B has won, or that party C has won, or—because it's still too close to call between them so soon after polls close—I'll learn that Party

A or Party B has won and I'll have to wait longer to learn which. But these four possible learned propositions do not form a partition. And this, it seems, is the norm. As a result, Plan Conditionalization has a very narrow range of application.

It's worth saying to begin with that Plan Conditionalization is also the strongest principle that the so-called diachronic Dutch book argument can establish (Lewis, 1999). And it is also the strongest principle that the less well-known expected utility argument can establish (Brown, 1976). As Lewis recognized in his own presentation of the former argument, it is only if the agent makes a plan on a partition and only if that plan violates Plan Conditionalization that we can construct a series of bets that she will accept that is guaranteed to lose her money. And as Peter M. Brown is well aware, the expected utilities required to formulate his alternative pragmatic argument are only well-defined if the plan is formed over a partition. But just because other arguments only establish an equally narrow principle, that doesn't mean that we should be satisfied with it.

In fact, I think that the principle isn't as narrow as we initially imagine. Consider again my anticipated learning experience when I check the news on election night. When I stated the example above, I assumed that the possible propositions I might learn were the following: *Party A wins, Party B wins, Party C wins, Party A or Party B wins*. And of course these are not mutually exclusive, even if they are exhaustive. But, when I am planning how to update, my plans don't have the form *If X, I will adopt such-and-such a credence function*, where *X* is a proposition I might learn. They have the form *If X and I learn X, I will adopt such-and-such a credence function*. But of course, once we conjoin to each proposition the proposition that I learn it, we now do have a partition. It is this: *Party A wins & I learn that Party A wins, Party B wins & I learn that Party B wins, Party C wins & I learn that Party C wins, Party A or Party B wins & I learn that Party A or Party B wins*. It is this partition on which I define my updating plans (cf. (Schoenfield, ms)). Thus, Plan Conditionalization has a rather broader scope than at first we imagine.

(p.189) 14.1 Forward-facing argument

The first accuracy argument for Plan Conditionalization was given by Hilary Greaves and David Wallace (though a prototype argument was given earlier by Graham Oddie) (Oddie, 1997; Greaves & Wallace, 2006). It is based on the same mathematical facts to which Peter M. Brown appeals in his expected utility argument for Plan Conditionalization (Brown, 1976; Okasha, 2014).

Just as we have been doing throughout this book, Greaves and Wallace appeal to a decision-theoretic principle in order to establish the credal principle. What sort of decision-theoretic principle governs this case? By contrast with the other cases considered in this book, Plan Conditionalization governs an agent's adoption of a *plan to have a credence function conditional on some learning event rather than her adoption of a credence function*. Thus, we need a decision-theoretic principle that governs the adoption of a plan to choose an option conditional on some event rather than a principle that governs simply the choosing of an option. How do we deal with this in decision theory? In fact, it is straightforward. Suppose I have a plan $O_{\mathcal{E}}$ defined on a partition $\mathcal{E}$. It consists of a set of conditionals, *If E, then choose option* o_E for each E in $\mathcal{E}$. Then I take the utility of this *plan* at a given possible world w to be given by the utility of the *option* that it would recommend I choose at w; that is, the utility of the option o_{E_w} that it recommends if E_w comes to pass (where E_w is the element of $\mathcal{E}$ that is true at w). Thus, we extend the utility function $\mathfrak{U}$, which is usually defined only for options, so that it is also defined for plans:

$$\mathrm{U}(O_{\mathrm{E}}, w) := \mathrm{U}(o_{E_w}, w)$$

We can then apply the usual principles of decision theory to these choices.

Thus, by analogy, we define the inaccuracy of an updating rule $R_{\mathcal{E}}$ at w to be the inaccuracy of the credence function it recommends at w; that is, the credence function R_{E_w} that it recommends to an agent who learns E_w. So we can extend the definition of an inaccuracy measure for credence functions so that it applies to updating rules as follows:

$$I(R_{\mathrm{E}}, w) := I(R_{E_w}, w)$$

This definition makes it clear why it is so important that $\mathcal{E}$ is a partition. If there are E, E' in $\mathcal{E}$ with E and E' both true at w (that is, if the elements of $\mathcal{E}$ are not disjoint), then E_w is not well defined and $\mathfrak{I}(R_{\mathcal{E}}, w)$ is ambiguous between $\mathfrak{I}(R_E, w)$ and $I(R_{E'}, w)$. And if there is w at which all E in $\mathcal{E}$ are false (that is, if the elements of $\mathcal{E}$ are not exhaustive), then there is no value to assign to $\mathfrak{I}(R_{\mathcal{E}}, w)$ at all.

Having given an inaccuracy measure for updating rules, we can state the very well known principle of decision theory to which Greaves and Wallace appeal.

> **Maximize Subjective Expected Utility** Let $\mathcal{O}$ be a set of options. Let $\mathcal{W}$ be the set of possible worlds. Let $\mathfrak{U}$ be a utility function and

let c be a probabilistic credence function defined on the algebra over $\mathscr{F}$. Then, if

(p.190) (i) $\mathrm{Exp}_{\mathfrak{U}}(o|c) < \mathrm{Exp}_{\mathfrak{U}}(o'|c)$,and
(ii) there is no o'' in $\mathscr{O}$ such that $\mathrm{Exp}_{\mathfrak{U}}(o'|c) < \mathrm{Exp}_{\mathfrak{U}}(o''|c)$ then
(iii) o is irrational for any agent with utility function $\mathfrak{U}$.

Having defined the inaccuracy of $R_{\mathscr{E}}$ at a world in terms of the inaccuracy of the credence function it recommends at w, the expected inaccuracy is then defined as follows:

$$\mathrm{Exp}_I(R_{\mathbb{E}}|c) := \sum_w c(w) I(R_{\mathbb{E}}, w) = \sum_w c(w) I(R_{E_w}, w)$$

Greaves and Wallace then prove the following theorem:

Theorem 14.1.1 (**Greaves and Wallace**) *Suppose $\mathfrak{D}$ is an additive Bregman divergence and $\mathfrak{I}(c, w) = \mathfrak{D}(\nu_w, c)$. So $\mathfrak{I}$ is an additive and continuous strictly proper inaccuracy measure. Suppose c is a probabilistic credence function. And suppose $\mathscr{E}$ is a partition. Then an updating rule $R_{\mathscr{E}}$ minimizes expected inaccuracy by the lights of c iff $R_{\mathscr{E}}$ is a conditionalization rule on $\mathscr{E}$ for c.*

That is,

(i) *If $R_{\mathscr{E}}$ and $R'_{\mathbb{E}}$ are both conditionalization rules on $\mathscr{E}$ for c, then*

$$\mathrm{Exp}_I(R_{\mathbb{E}}|c) = \mathrm{Exp}_I(R'_{\mathbb{E}}|c)$$

(ii) *If $R_{\mathscr{E}}$ is a conditionalization rule on $\mathscr{E}$ for c and $R'_{\mathbb{E}}$ is not, then*

$$\mathrm{Exp}_I(R_{\mathbb{E}}|c) < \mathrm{Exp}_I(R'_{\mathbb{E}}|c)$$

It might seem at first that this cannot be correct. After all, when inaccuracy is measured by a strictly proper inaccuracy measure, surely a probabilistic credence function c expects itself to be least inaccurate. Thus, surely it is the constant updating rule on c—that is, the rule that recommends that the agent adopt c come what may—that minimizes expected inaccuracy by the lights of c; and this rule is certainly not a conditionalization rule on $\mathscr{E}$ for c. The reason this doesn't happen is that strict propriety entails only this: for each probabilistic credence function c, if we look only at the options that amount to adopting the same credence function in every world, then the option that amounts to adopting c at every world is the option that c expects to be least inaccurate. But, if we consider instead options that amount to adopting different credence functions at different worlds, then c will not necessarily expect the constant rule on c to be the least inaccurate. Indeed, every probabilistic credence function will expect the option that amounts to adopting the omniscient credence function ν_w at w for all worlds w to be least inaccurate—each will give an expected inaccuracy of 0 to that option! But of course that is not an updating rule on the partition $\mathcal{E}$: updating rules on $\mathcal{E}$ have to give the same recommendation for any two worlds at which the same element of the partition is true. The reason for this restriction is that updating (p.191) rules have to be something the agent might follow; and she can only follow a rule if, whenever she can't distinguish two possibilities, the rule doesn't distinguish those possibilities either, and gives the same recommendation for both.

With this theorem in hand, we can formulate the first accuracy argument for conditionalization:

(I_{PC}) **Veritism**
(II_{PC}) **Brier Alethic Accuracy**
(III_{PC}) **Maximize Subjective Expected Utility**
(IV_{PC}) **Theorems 14.1.1** and **I.B.2**
Therefore,
(V_{PC}) **Plan Conditionalization**

Recall from the previous part of the book that our arguments for the Principle of Indifference all go through even if we do not narrow down the class of legitimate inaccuracy measures to a single measure—such as the Brier score—and instead take the supervaluationist or epistemicist approach mooted in Chapter 5. The Bronfman objection arises for the accuracy argument for Probabilism because, given a nonprobabilistic credence function, different sets of credence functions will dominate it relative to different additive and continuous strictly proper inaccuracy measures; and there may be no credence function that lies in all of those dominating sets. The objection doesn't arise for the accuracy argument for the Principle of Indifference, because, relative to the decision-theoretic principle that we use in that argument, all those inaccuracy measures agree that the uniform distribution is the only credence function that minimizes maximal inaccuracy. The same is true of the argument just given for Plan Conditionalization. Given a credence function c and a partition $\mathcal{E}$, every additive and continuous strictly proper inaccuracy measure agrees that the update plans on $\mathcal{E}$ that have minimal expected inaccuracy by the lights of c are the conditionalization plans on c. So, as in the argument for the Principle of Indifference, the above argument for Plan Conditionalization will still go through if we replace Brier Alethic Accuracy with Supervaluationism about Inaccuracy Measures or with Epistemicism about Inaccuracy Measures. Of course, I favour Brier

Alethic Accuracy, because I am convinced by the principle Symmetry from Chapter 4. But for those who aren’t, this argument will still go through.

14.2 Backwards-facing argument

Bas van Fraassen's Reflection Principle is an expert principle, just as the Principal Principle is (van Fraassen, 1984, 1995). Where the Principal Principle says that rationality requires an agent to defer to the objective chances when setting her credences, the Reflection Principle says that it requires her (also) to defer to her future credences. Thus, in its original form, the Reflection Principle runs as follows:

> (p.192) **Reflection Principle** Suppose an agent has credence function c at t. If p is a probability function on the same set of propositions, then let F_p be the following proposition: *My credence function at t' is p*. Then the following is a requirement of rationality: for all $X \in \mathscr{F}$,
>
> $$c(X|F_p) = p(X)$$

Famously, van Fraassen showed that there is a Dutch book argument for the Reflection Principle that seems to be valid just in case the Dutch book argument for Conditionalization is valid.[1]

Later, van Fraassen formulated another version of the principle(van Fraassen, 1999):

> **Generalized Reflection Principle** Suppose an agent has credence function c at t. And suppose she knows her evidence between t and t' will come from the finite partition $\mathscr{E}$. Then it is a requirement of rationality that, if she plans to update in accordance with update rule $R_{\mathscr{E}}$, then
>
> (i) $R_E(E) = 1$ for all E in $\mathscr{E}$;
> (ii) c is in the convex hull of $R := \{R_E : E \in \mathscr{E}\}$.
> That is, $c \in R^+$.
> That is, there are $\{\lambda_E : E \in \mathscr{E}\}$ such that $\lambda_E \geq 0$ for all $E \in \mathscr{E}$ and $\sum_{E \in \mathrm{E}} \lambda_E = 1$ such that, for all X in $\mathscr{F}$,
>
> $$c(X) = \sum_{E \in \mathrm{E}} \lambda_E R_E(X)$$

Clause (i) makes the straightforward claim that an agent who plans to update upon evidence E in a way that makes them less than certain of E is irrational. Clause (ii) looks technical, but it is motivated by the following thought: Suppose I plan to update in such a way that, come what may, my future credence will lie between 0.7 and 0.8. And suppose that my current credence is 0.5. Then this seems irrational: if I really endorse the credences that my updating rule recommends, I should raise my current credence at least to 0.7.

This time, van Fraassen went further than showing that his principle is justified by an argument that is analogous to the justification of conditionalization. He showed that the two principles are equivalent: that is,

Theorem 14.2.1 (**van Fraassen**) *Plan Conditionalization* ⇔ *Generalized Reflection Principle*.

Thus, one way to argue for Plan Conditionalization is to argue for the Generalized Reflection Principle. That is what we will do in this section.

First, though, let us note an important way in which the Reflection Principle and the Generalized Reflection Principle differ. After all, there are many apparent (p.193) counterexamples to the Reflection Principle. Suppose I know that, between the earlier time t and the later time t', I will not update in the way I would like to: perhaps I will forget things I currently know; perhaps I will form new beliefs in a random and irrational way because of a drug I will take (Talbott, 1991; Christensen, 1991). Then this defeats my standing reason to defer to my future credences. Thus, the Reflection Principle is false: there are many situations in which violating it is perfectly rational; indeed, there are situations in which obeying it is irrational. The feature of the Generalized Reflection Principle that allows it to escape the counterexamples that assail the Reflection Principle is that it concerns how an agent plans to update, not how she responds to how she believes she will update. All of the counterexamples to the Reflection Principle concern cases in which the agent knows that she will update in some way other than the way she would ideally like to update—that is, she will not update in the way she plans to update. In these cases, an agent ought not to defer to her future credences, since she does not currently endorse them; they are the result of some external influence that causes her plans to be disrupted. The Generalized Reflection Principle, on the other hand, concerns only an agent’s plans. It says that an agent ought to defer in a particular way to the credences she plans to adopt. As a result, the counterexamples fail to affect the Generalized Reflection Principle. Of course, van Fraassen’s voluntarism amounts to the claim that an agent who doubts that she will update in the way she plans to update is irrational. Thus, the Reflection Principle and the Generalized Reflection Principle come closer together for van Fraassen. But we needn’t accept this claim in order to accept the Generalized Reflection Principle.

It is now time to give our accuracy argument for the Generalized Reflection Principle. Recall our argument for the Principal Principle and its variants in Part II of this book. It was based on the following idea: we don't know what the chance function is; but we do know what the possibilities are; and if those possibilities are unanimous in their judgement about a particular issue, then it is irrational not to follow the recommendations that follow from those judgements. This gave us the following principle, where $\mathcal{C}$ is the set of possible chance functions.

> **Current Chance Evidential Immodest Dominance** Suppose $\mathfrak{I}$ is a legitimate measure of inaccuracy. Then, if
>
> (i) c is strongly current chance $\mathfrak{I}$-dominated by probabilistic c^* relative to E,
> (ii) there is no credence function that weakly current chance $\mathfrak{I}$-dominates c^* relative to E, and
> (iii) c^* is not extremely $\mathfrak{I}$-modest then
> (iv) c is irrational for an agent with total evidence E.

Our accuracy argument for the Reflection Principle is almost exactly the same. It is based on the following idea: we don't know what evidence we will obtain; and we don't know how we will respond to it; but we can plan how we would like to respond to it; (p.194) so we know what we would like our possible future credence functions to be; and if those possibilities are unanimous in their judgement about a particular issue, then it is irrational to ignore them. To state the principle precisely, we need the following definitions, which are just exact analogues of the corresponding notions for chances. Suppose an agent adopts updating plan $R_{\mathcal{E}}$. And suppose o, o' are options. Then:

- We say that *o' strongly future credence $\mathfrak{U}$-dominates o relative to $R_{\mathcal{E}}$* if, for each E in $\mathcal{E}$,
$$\mathrm{Exp}_{\mathrm{U}}(o|R_E) < \mathrm{Exp}_{\mathrm{U}}(o|R_E)$$
- We say that *o' weakly future credence $\mathfrak{U}$-dominates o relative to $R_{\mathcal{E}}$* if
    (i) for each E in $\mathcal{E}$,
$$\mathrm{Exp}_{\mathrm{U}}(o|R_E) \leq \mathrm{Exp}_{\mathrm{U}}(o|R_E)$$
    (ii) for some E in $\mathcal{E}$,
$$\mathrm{Exp}_{\mathrm{U}}(o|R_E) < \mathrm{Exp}_{\mathrm{U}}(o|R_E)$$

And now the principle:

> **Future Credence Immodest Dominance** Suppose $R_{\mathcal{E}}$ is an updating rule on $\mathcal{E}$. Suppose $\mathfrak{I}$ is a legitimate measure of inaccuracy. Then, if

(i) c is strongly future credence $\mathfrak{I}$-dominated by probabilistic c^* relative to $R_{\mathscr{E}}$,
(ii) there is no credence function that weakly future credence $\mathfrak{I}$-dominates c^* relative to $R_{\mathscr{E}}$, and
(iii) c^* is not extremely $\mathfrak{I}$-modest, then
(iv) c is irrational for an agent who adopts updating plan $R_{\mathscr{E}}$.

Together with the following theorem, this gives van Fraassen's Generalized Reflection Principle, providing we assume that each credence function R_E that $R_{\mathscr{E}}$ might endorse is probabilistic, and providing that each R_E is certain of E—that is, $R_E(E) = 1$.

Theorem 14.2.2 *Suppose c is a probabilistic credence function.*

(I) *If $c \notin R^+$, then there is c^* such that c^* strongly future credence $\mathfrak{I}$-dominates c relative to $R_{\mathscr{E}}$. That is, for all $\mathscr{E}$ in $\mathscr{E}$,*

$$\mathrm{Exp}_I(c^*|R_E) < \mathrm{Exp}_I(c|R_E)$$

That is, c is strongly future credence $\mathfrak{I}$-dominated by c^* relative to $R_{\mathscr{E}}$.
(II) *If $c \in R^+$, then there is no c^* that even weakly future credence $\mathfrak{I}$-dominates c relative to $R_{\mathscr{E}}$. In fact, if $c^* = c$, there is E in $\mathscr{E}$ such that*

$$\mathrm{Exp}_I(c|R_E) < \mathrm{Exp}_I(c^*|R_E)$$

(p.195) *That is, c is not weakly future credence $\mathfrak{I}$-dominated relative to $R_{\mathscr{E}}$.*
(III) *If $c \in R^+$, then*

$$\mathrm{Exp}_I(c|c) < \mathrm{Exp}_I(c^*|c)$$

That is, c is $\mathfrak{I}$-immodest.

Thus, we have the following argument for Generalized Reflection Principle, and therefore for Plan Conditionalization:

($\mathrm{I}^{\star}_{PC}$) **Veritism**
($\mathrm{II}^{\star}_{PC}$) **Brier Alethic Accuracy**
($\mathrm{III}^{\star}_{PC}$) **Future Credence Immodest Dominance**
($\mathrm{IV}^{\star}_{PC}$) **Theorem 14.2.2**
Therefore,
($\mathrm{V}^{\star}_{PC}$) **Generalized Reflection Principle**
($\mathrm{VI}^{\star}_{PC}$) **Theorems 14.2.1** and **I.B.2**
Therefore,
($\mathrm{VII}^{\star}_{PC}$) **Plan Conditionalization**

Note that this argument is a sort of dual to the first accuracy argument, due to Greaves and Wallace, presented in the previous section. The Greaves and Wallace argument shows that, if you have a probabilistic credence function c and you know that your future evidence will come from $\mathcal{E}$, then rationality requires that, if you adopt an updating rule at all, it should be a conditionalization rule on $\mathcal{E}$ for c.Thus, the argument determines rational constraints on the adoption of an updating rule by appealing to a fixed current credence function. The argument of this section, on the other hand, appeals to a fixed updating rule and places rational constraints on the current credence function you should adopt. Note that any updating rule $R_{\mathcal{E}}$, where R_E is probabilistic and $R_E(E) = 1$, is a conditionalization rule for some credence function. Thus, the argument of this section shows that, having picked your updating rule, you should pick as your current credence function one of those for which your updating rule is a conditionalization rule.

In fact, we need not see these two arguments as rivals. It could be that each heads off a worry about the other. Greaves and Wallace show that, if you adopt an updating plan—a way to update your current credences in response to evidence—rationality requires that it is a conditionalizing plan. For each potential piece of evidence, this plan endorses a response. But what if there is some other credence function such that each of the responses that this updating rule endorses judges this other credence function to be better than it judges your current credence function to be? That is, you use your current credence function to pick your updating rule; but then that very updating rule judges your prior to be suboptimal—it unequivocally judges some other credence to be better than yours. Is it still rationally required to adopt the updating rule? I think not. Your way of picking an updating rule would have been shown to (p.196) be self-undermining. Thus, the backward-looking argument presented in this section fills a lacuna in Greaves and Wallace's original argument. For the original argument to go through, we have to know that the way of choosing the updating rule is not selfundermining in the way just described. The argument above—via Theorem 14.2.2 and Future Credence Immodest Dominance—does that.

After we stated Greaves and Wallace's forward-facing argument in the previous section, I noted that it is not vulnerable to the Bronfman objection and would go through even if we did not narrow down the class of legitimate inaccuracy measures to a single one and instead took a supervaluationist or epistemicist view of a plurality of those measures. However, the backwards-looking argument of this section, just like the analogous argument for the Temporal Principle given in Chapter 10, is vulnerable to the Bronfman objection. Suppose c is not in

R^+ (where, recall, $R := \{R_E : E \in \mathcal{E}\}$). Then Theorem 14.2.2 says that, for any inaccuracy measure, there are credence functions that future credence dominate c relative to $R_{\mathcal{E}}$. But of course, different sets of credence functions will future credence dominate c for different measures of inaccuracy. And Theorem 14.2.2 gives us no reason to think that there is any credence function that lies in all of these sets. So, in order to give the backward-facing argument, we must accept some characterization of the legitimate inaccuracy measures—such as the characterization proposed in Chapter 4—that narrows down to just one such measure.

14.3 Neither-facing argument

The final accuracy argument for Plan Conditionalization that I will consider is due to Rachael Briggs (Briggs & Pettigrew, ms): it was based initially on a theorem by de Finetti, which I later generalized.[2] The previous arguments sought to establish what we might think of as narrow-scope requirements of rationality. That is, they had the following form: If an agent has doxastic attitude X, then rationality requires that she should have doxastic attitude Y. Greaves and Wallace's argument—the forward-facing argument—says: If an agent has credence function c, then rationality requires that she should plan to update by conditionalizing on c. Thus, to establish this, we fixed the credence function and used it to evaluate the updating rules. The second argument I presented—the backwards-facing argument—says: If an agent adopts updating rule $R_{\mathcal{E}}$, then rationality requires that she should have a credence function c such that $R_{\mathcal{E}}$ is a conditionalization rule on $\mathcal{E}$ for c. Thus, to establish this, we fixed the updating rule and used it to evaluate the credence functions. Briggs' argument, on the other hand, attempts to establish a wide-scope requirement of rationality. That is, Briggs argues that the following is a requirement of rationality: If an agent has credence function c, she adopts a conditionalization rule on $\mathcal{E}$ for c. Equivalently: it is rationally required (p.197) that she not simultaneously have credence function c and have an updating rule that is not a conditionalization rule on $\mathcal{E}$ for c. Thus, Briggs' argument does not begin by assuming that an agent has a particular doxastic attitude—a credence function or an updating rule—and it does not proceed by evaluating another doxastic attitude from that perspective. Rather, it treats both credence function and updating rule together; it gives a measure of the inaccuracy of them taken as a pair; and then it shows that only pairs $\langle c, R_{\mathcal{E}}\rangle$—where $R_{\mathcal{E}}$ is a conditionalization rule on $\mathcal{E}$ for c—are not dominated by other pairs. When $R_{\mathcal{E}}$ is a conditionalization rule on $\mathcal{E}$ for c, we say that $\langle c, R_{\mathcal{E}}\rangle$ is a *conditionalization pair*.

Now, in the Greaves and Wallace argument, we moved from using inaccuracy measures to evaluate credence functions only to using them also to evaluate updating rules. In Briggs' argument we move to using them also to evaluate pairs consisting of credence functions and updating rules. Given such a pair $\langle c, R_{\mathscr{E}}\rangle$,wedefine itsaccuracy at a given world w to be the sum of the inaccuracy of c at w and the inaccuracy of $R_{\mathscr{E}}$ at w:

$$I(\langle c, R_{\mathrm{E}}\rangle, w) := I(c, w) + I(R_{\mathrm{E}}, w)\ (= I(c, w) + I(R_{E_w}, w))$$

Now we prove the following theorem:

Theorem 14.3.1 *Suppose $\mathfrak{D}$ is an additive Bregman divergence and $\mathscr{I}(c, w) = \mathfrak{D}(\nu_w, c)$. So $\mathscr{I}$ is an additive and continuous strictly proper inaccuracy measure.*

(I) *If $\langle c, R_{\mathscr{E}}\rangle$ is not a conditionalizing pair, then there is a conditionalizing pair $\langle c^*, R^*_{\mathrm{E}}\rangle$ such that, for all w in $\mathsf{W}_{\mathscr{F}}$,*

$$I(\langle c^*, R^*_{\mathrm{E}}\rangle, w) < I(\langle c, R_{\mathrm{E}}\rangle, w)$$

(II) *If $\langle c, R_{\mathscr{E}}\rangle$ is a conditionalizing pair, then for any pair $\langle c^*, R^*_{\mathrm{E}}\rangle \neq \langle c, R_{\mathrm{E}}\rangle$,*

$$\mathrm{Exp}_I(\langle c, R_{\mathrm{E}}\rangle | c) \leq \mathrm{Exp}_I(\langle c^*, R^*_{\mathrm{E}}\rangle | c)$$

with equality iff $c = c^*$ and R^*_{E} is a conditionalization rule for $c = c^*$.

Thus, we have the following argument for Plan Conditionalization:

(I^{**}_{PC}) **Veritism**
(II^{**}_{PC}) **Brier Alethic Accuracy**
(III^{**}_{PC}) **Immodest Dominance**
(IV^{**}_{PC}) **Theorems 14.3.1 and I.B.2**
Therefore,
(V^{**}_{PC}) **Plan Conditionalization**

Again, as with the backwards-facing argument of the previous section, this argument requires as its second premise the claim that some specified additive and continuous strictly proper inaccuracy measure is the only legitimate measure of inaccuracy. (p.198) Without that, it is vulnerable to the Bronfman objection. I have included Brier Alethic Accuracy as the second premise, which says of the Brier score that it is the only legitimate measure, because I endorse the characterization given in Chapter 4. But any other such premise would work just as well. What will *not* work is Epistemicism about Inaccuracy Measures or Supervaluationism about Inaccuracy Measures.

Notes:

([1]) Cf. Briggs (2009) for an interesting dissenting view; and Mahtani (2012, 2014) for a reply.

([2]) While I haven't been able to find this result in the literature, I find it very hard to believe that it is not known.