> "Shouldn't philosophers be permitted to rise above faith in grammar? All due respect for governesses—but hasn't the time come for philosophy to renounce the faith of governesses?" (Nietzsche in *Beyond Good and Evil*, Section 34.)

In metaphysics, there is a well-known Nietzschean criticism of Descartes' *cogito*. Nietzsche complains that Descartes' metaphysical conclusions about the existence of a thinking self are based on grammar rather than sound logical inference. My current project is to pursue a Nietzschean-type criticism in formal epistemology. The culprit is again Cartesian, but this time it is the Cartesian coordinate system rather than the Cartesian *cogito* that implements a grammar misleading us to unwarranted epistemological rather than metaphysical conclusions.

It is commonplace among statisticians to think of the set of normal distributions as a manifold. Manifolds are sets that behave locally like Euclidean space. Normal distributions, for example, are characterized by two parameters: the mean and the standard deviation. These two may serve as coordinates of sorts so that the set of normal distributions maps onto Euclidean space.

Once a manifold is defined, the formal epistemologist can begin her calculations on the mathematical structure of manifolds, especially with the aid of a metric or a "scoring rule." It is a current trend in formal epistemology to investigate how probability distributions score with respect to certain desiderata. Scoring rules are similar to metrics in the sense that they measure divergence in order to compare competing belief states.

Epistemic utilitarians consider the kinds of decisions we make whether or not to have certain beliefs as means to an epistemic end. You may want to have as many true beliefs as possible; or as few false beliefs as possible—but these are not useful ends, because believing everything (or, respectively, believing nothing) will optimally fulfill them. Epistemic agents usually want to have beliefs that are both informative and accurate. Compromise is necessary between these two desirable features of beliefs.

Formal epistemologists want to use mathematical models to provide useful descriptions of this compromise. A scoring rule will ideally reveal the conditions of commensurability between informativeness and accuracy. It measures in a model how belief states fare in relation to each other given a number

of assumptions. A proper scoring rule ensures that the belief state of all and only true beliefs fares well.

To create the mathematical models, formal epistemologists help themselves to features of beliefs that can be represented numerically. Coordinate systems are useful for this purpose. The parameters of a probability distribution or a probability density function such as the normal distribution often lend themselves to the definition of a metric or of scoring rules. The Brier score (a variant of mean squared error) or information entropy are interesting examples. The question is then whether the metric or the scoring rule is invariant with respect to the parameters. Sometimes it is not, but more deceptively, sometimes we begin to think of credal states as geometrically embodied in their parameters rather than represented by them.

The Nietzschean criticism that I am trying to articulate encourages a move away from coordinates towards the manifolds of differential geometry. When first you learned what the constant angle sum of a plane triangle is you most likely absorbed the news in coordinate-free geometry. Someone drew three congruent triangles for you and explained to you why the three angles form a straight line (a good kind of governess would have done this for you). Later on, however, geometry became easier by using coordinates, usually Cartesian coordinates. When two vectors in three-dimensional space form an angle, it is simple to calculate the angle in a Cartesian coordinate system; whereas calculating it using coordinate-free intuition is difficult.

Differential geometry became an important component of mathematical physics in the 20th century because in physics, just as in epistemology, dependence on a particular representation in coordinates can become more of a liability than an asset. We do not want our considerations in physics to depend on whether we think of space in terms of Cartesian coordinates or polar coordinates. Relativity theory especially accelerated the transition from the vectors of the Cartesian grammar to the tensors and fiber bundles of differential geometry. The relevant relationships are now no longer between parametric representations (for example the mean and standard deviation of the normal distribution), but between derivations (generalized derivatives, thus the name differential geometry) and a metric based on an inner product defined on tangent spaces (such as the Fisher information matrix).

For the categorical distribution with a finite event space (for example, die rolls and coin tosses) the finite set of probabilities is usually considered to

be the set of parameters or coordinates of the belief state. It indeed sounds intuitively plausible that in order to characterize the probabilities 60% for heads and 40% for tails I would consider the point $(0.6, 0.4)$ in a Cartesian coordinate system. But then highly counter-intuitive things happen! (I have written about this elsewhere.)

When Foucault talks about sexuality, he uses the Cheshire Cat of *Alice in Wonderland* as an illustration of

> ... smiles, happinesses, pleasures, and desires as qualities without an abiding substance to which they are said to adhere. As free-floating attributes, they suggest the possibility of a gendered experience that cannot be grasped through the substantializing and hierarchizing grammar of nouns and adjectives. (Judith Butler in *Gender Trouble*, page 32)

The current ambition in the literature of formal epistemology is to highlight parameter invariance as a discriminating feature between mathematical models. Following programs that have already succeeded in physics and statistics, I am looking for ways in which the parameters can become an afterthought rather than a determining constituent of how we think about the relationships between different belief states. Let the doxastic landscape be, as Foucault expresses it in a very different context, "a world of pleasures in which grins hang about without the cat."