Cheshire Cat Partial Beliefs

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In metaphysics, Nietzsche complains that Descartes' conclusions about the existence of a thinking self are based on grammar rather than sound logical inference. My current project is to pursue a Nietzschean-type criticism in formal epistemology. The culprit is again Cartesian, but this time it is the Cartesian coordinate system that implements a grammar misleading us to unwarranted epistemological conclusions.

Statisticians think of the set of normal distributions as manifolds—sets that behave locally like Euclidean space. Normal distributions, for example, are characterized by the mean and the standard deviation, which serve as coordinates that map normal distributions onto Euclidean space.

It is a current trend in formal epistemology to evaluate competing belief states using a scoring rule or some metric. Compromise is necessary between informative and accurate beliefs. Formal epistemologists want to use mathematical models to provide useful descriptions of this compromise. A scoring rule will ideally reveal the conditions of commensurability between informativeness and accuracy (given a number of assumptions). A *proper* scoring rule ensures that the belief state of all and only true beliefs fares well.

Coordinate systems are useful in creating these mathematical models. The Brier score or information entropy are interesting examples of these models. Deceptively, sometimes we begin to think of credal states as geometrically embodied in their parameters rather than represented by them.

My Nietzschean criticism encourages a move away from coordinates towards the manifolds of differential geometry. When first you learned what the constant angle sum of a plane triangle was you most likely absorbed the news in coordinate-free geometry. Later on, however, geometry became easier by using coordinates, usually Cartesian coordinates.

In modern physics, just as in epistemology, dependence on a particular representation in coordinates can become more of a liability than an asset. Modern physicists often do not want to think of space in terms of coordinates. Relativity theory especially has accelerated the transition from the vectors of the Cartesian grammar to the tensors and fiber bundles of differential geometry. The relevant relationships are now no longer between parametric representations (for example the mean and standard deviation of the normal distribution), but between derivations (generalized derivatives, thus the name differential geometry) and a metric based on an inner product defined on tangent spaces (such as the Fisher information matrix).

For the categorical distribution with a finite event space (for example, die rolls and coin tosses) the finite set of probabilities is usually considered to be the set of parameters or coordinates of the belief state—in order to characterize the probabilities 60% for heads and 40% for tails I would consider the point (0.6, 0.4) in a Cartesian coordinate system. But then highly counter-intuitive things happen!

When Foucault talks about sexuality, he uses the Cheshire Cat of *Alice in Wonderland* as an illustration of

"...smiles, happinesses, pleasures, and desires as qualities without an abiding substance to which they are said to adhere. As free-floating attributes, they suggest the possibility of a gendered experience that cannot be grasped through the substantializing and hierarchizing grammar of nouns and adjectives" (Judith Butler in *Gender Trouble*, page 32).

The current ambition in formal epistemology is to highlight parameter invariance as a discriminating feature between mathematical models. Following successes in physics and statistics, I am looking for ways in which the parameters can become an afterthought rather than a determining constituent of how we think about the relationships between different belief states. Let the doxastic landscape be, as Foucault expresses it in a very different context, "a world of pleasures in which grins hang about without the cat."