First written examination of
Algoritmos e Estruturas de Dados

Outubro 3, 2016   **Duration: no more than 40 minutes**

Name:

Student number:

4.0 **1:** Consider that the function `f(x)` is defined in the following way

```
int f(int x) { return x * x; }
```

In the following code snippet, in which conditions and how many times is the function g(x) called? Justify your answer.

```
for(int i = 0;i <= 10;i++)
  if(((f(i) % 3) == 0) && (g(i) == 3))
    h(i);
```

Answer:

4.0 **2:** What are the final contents of the `a[]` array after execution of the following code

```
int a[100];
for(int i = 0;i < 100;i++) a[i] = i;
int *pa = &a[-10];
int *pb = pa + 23;
for(int i = 20;i <= 40;i++)
  a[i] = pb[i];
```

Answer:

4.0 $\boxed{\textbf{3:}}$ If $f(n) = O(g(n))$, what can we say about how $f(n)$ is upper and lower bounded?

Answer:

4.0 $\boxed{\textbf{4:}}$ Sort the following functions in increasing order of complexity (use the function number in your answer):

Function number	function
1	12
2	$\dfrac{n^2}{\log n}$
3	$12n \log n + n^{1.2}$
4	$7n^3$
5	$17n$
6	$11 + \dfrac{1}{n}$
7	$20n \log n + 100n$
8	1.01^n
9	$\dfrac{4}{n}$

Answer:

4.0 **5:** Give an expression (simplified if possible) for the value returned by the following function:

```
void f(int x)
{
  int i,j,r;

  r = 0;
  for(i = 0;i <= x;i++)
    for(j = 0;j <= i;j++)
      r += i * j;
  return r;
}
```

Answer:

Useful formulas:

- $\displaystyle\sum_{k=1}^{n} 1 = n$

- $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

- $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

- $\displaystyle\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

- $\displaystyle\sum_{k=1}^{n} \frac{1}{k} \approx \log n$

- $\displaystyle\sum_{k=n}^{m} f(k) = \sum_{k=1}^{m} f(k) - \sum_{k=1}^{n-1} f(k)$

- $n! \approx n^n e^{-n} \sqrt{2\pi n}$