**Third part of the final exam of Algoritmos e Estruturas de Dados**

**February 4, 2022** **18h00m – 18h55m**

**Nome:** ____________________

**N. Mec.:** __________

**4.0** $\boxed{\textbf{1:}}$ The merge sort algorithm subdivides the array into two parts nearly equal parts, sorts recursively each of them, and then merges them. Its computational complexity is $\boldsymbol{\Theta(n\log n)}$. A student is convinced that instead of subdividing the array in two parts it is better to subdivide it into five nearly equal parts. By doing so, he claims that the computational complexity is even better,

1.0 **a)** What algoritmic strategy is used by merge sort?

2.0 **b)** The student is wright? Justify your answer.

1.0 **c)** The this merge sort variant, the merge phase is simpler or more complex than the merge phase of the original merge sort algorithm?

The *master theorem* states that if $\boldsymbol{T(n) = aT(n/b) + f(n)}$ then

- if $\boldsymbol{f(n) = O(n^{\log_b a-\epsilon})}$ for some $\boldsymbol{\epsilon > 0}$, then $\boldsymbol{T(n) = \Theta(n^{\log_b a})}$,
- if $\boldsymbol{f(n) = \Theta(n^{\log_b a})}$, then $\boldsymbol{T(n) = O(n^{\log_b a}\log n)}$,
- if $\boldsymbol{f(n) = \Omega(n^{\log_b a+\epsilon})}$ for some $\boldsymbol{\epsilon > 0}$ and if $\boldsymbol{af(\frac{n}{b}) \leq cf(n)}$ for $\boldsymbol{c < 1}$ nd $\boldsymbol{n}$ sufficiently large, then $\boldsymbol{T(n) = \Theta(f(n))}$.

**4.0** **2:** In a chess board we want to count the number of ways to go from the (0,0) corner to the (7,7) corner, using moves than can only increase either the x or the y coordinates. The following code presents one way of doing it.

```
1   long eval(int x,int y)
    {
2     if(x < 0 || x > 7 || y < 0 || y > 7)
2       return 0L;
3     else if(x == 7 && y == 7)
3       return 1L;
4     else
4       return eval(x + 1,y) + eval(x,y + 1);
    }
5   long count_paths(void)
    {
6     return eval(0,0);
    }
```

Answer to the following questions:

1.0 **a)** What algorithmic strategy is used in this code?

3.0 **b)** Explain in words that is the purpose of each of the numberes parts of the code.

**4.0** $\boxed{\textbf{3:}}$ Like in the previous problem, in a chess board we want to count the number of ways to go from the (0,0) corner to the (7,7) corner, using moves than can only increase either the x or the y coordinates. The following code presents one way of doing it.

```
1   long count_data[8][8];
2   long eval(int x,int y)
    {
3     if(x < 0 || x > 7 || y < 0 || y > 7)
3       return 0L;
4     if(count_data[x][y] < 0L)
4       count_data[x][y] = eval(x - 1,y) + eval(x,y - 1);
5     return count_data[x][y];
    }
6   long count_paths(void)
    {
7     for(int x = 0;x < 8;x++)
7       for(int y = 0;y < 8;y++)
7         count_data[x][y] = -1L;
8     count_data[0][0] = 1L;
9     return eval(7,7);
    }
```

Answer to the following questions:

1.0 **a)** What algorithmic strategy is used in this code?

3.0 **b)** Explain in words that is the purpose of each of the numberes parts of the code.

**4.0** $\boxed{\textbf{4:}}$ A five vertex graph, with vertices numbered from **1** to **5**, has an edge between vertex $\boldsymbol{i}$ and vertex $\boldsymbol{j}$ if and only if $\boldsymbol{i} < \boldsymbol{j}$. Answer to the following questions:

1.0 **a)** Drawthe graph.

1.0 **b)** Represent the graph using an adjacency matrix.

1.0 **c)** Represent the graph using adjacency lists.

1.0 **d)** Is it possible to represent this specific grph using a single 32-bit integer? If so, how? If not, why not?

**4.0** $\boxed{\textbf{5:}}$ You were captured by the Borg and taken to a Borg cube to be assimilated.[1] You managed to escape but are lost inside the Borg cube, which can be considered to be a tridimensional maze. Answer to the following questions:

2.0 **a)** To find an exit, which of the two algorithmic strategies would you use: depth search or breadth search?

2.0 **b)** What material would you take with you to help implement the strategy you have chosen?

[1]TYhe Borg are an alien race in the Star Trek universe. Their biggest ships are the Borg cube which have a volume of 27 cubic kilometers. Resistence is futile!