কাবীর খাঁন
AE-019-610

University of Dhaka
Department of Mathematics
Third Year B.S. (Hons.) 2020-21
Subject: Mathematics
Course No. MTH 350 Course Title: Math Lab III
Assignment No. 1
Deadline:

Write FORTRAN code for each of the following problems:

Root Finding Techniques

1. Consider Wallis' equation

$$f(x) = x^3 - 2x - 5 = 0 \qquad (1)$$

Use the methods (i) BISECTION METHOD, (ii) THE METHOD OF FALSE POSITION, (iii) THE SECANT METHOD to find the real root of the Wallis' equation.

2. We can rewrite the Eq. (1) in various forms:

$$(i)\ \phi_1(x) = (x^3 - 5)/2$$
$$(ii)\ \phi_1(x) = (2x + 5)^{1/3}$$
$$(iii)\ \phi_1(x) = (2x + 5)/x^2$$

Now use FIXED POINT ITERATIVE METHOD to find the real root of the Eq. (1) and find which one of the above forms converge.

3. Iteration formula for CHEBYSHEV METHOD is given below:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{1}{2}\frac{[f(x_n)]^2 f''(x_n)}{[f'(x_n)]^3}$$

Apply this formula to compute the real root of Eq. (1).

4. Using NEWTON'S RAPHSON METHOD, we can generate following iterative schemes:

(i) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for computing $\sqrt{a}$ (Hero's algorithm)

(ii) $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{a}{x_n^2}\right)$ for computing $\sqrt[3]{a}$.

(iii) $x_{n+1} = x_n(2 - ax_n)$ for computing a^{-1}.

(a) Please derive each of the iterative schemes using the formula of Newton's Raphson method.

(b) Let us assume that 'a' is a positive integer and hence find the values of $\sqrt{a}, \sqrt[3]{a}$, and a^{-1} using the above mentioned iterative schemes.

4 কাবীর
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Subject: Mathematics
Course No. MTH 350, Course Title: Math Lab III
Assignment No. 2
Deadline:

Write FORTRAN code for each of the following problems:

Solution of System of Equations

1. Solve the following system of equations using GAUSSIAN ELIMINATION METHOD without pivoting.

$$2x - 5y + 3z = -1$$
$$3x + y - 4z = -10$$
$$-2x - 3y + 6z = 10$$

2. Convert the following system of equations to diagonally dominant form and then solve the converted system using both JACOBI and GAUSS-SEIDEL METHODS with initial guess$(x_0, y_0, z_0) = (1.5, 1.5, -0.5)$.

$$6x + 5y + 3z = 14$$
$$8x - 3y + 2z = 11$$
$$10x - 7y - 8z = 21$$

3. For the following non-linear system with initial point $(x_0, y_0) = (2, 2)$, evaluate the Jacobian matrix at the given point and hence to estimate the root of the system to 3 significant figures using the NEWTON'S ITERATIVE METHOD starting with the given point.

$$\ln(x - y^2) - \sin(xy) - \sin\pi = 0$$
$$e^{xy} + \cos(x - y) - 2 = 0$$

4. Verify whether the iterative formula (ii) for the following non-linear system (i) will converge to the root near $(x_0, y_0) = (4.4, 1.0)$. If converges write a code to execute the iterative formula in (2) five times to estimate the root of the system (i).

$$\left.\begin{aligned} x^2 + y &= 7 \\ x - y^2 &= 4 \end{aligned}\right\} \qquad \text{(i)}$$

$$\left.\begin{aligned} x_{n+1} &= \sqrt{7 - y_n} \\ y_{n+1} &= -\sqrt{x_n - 4} \end{aligned}\right\} \qquad \text{(ii)}$$

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Assignment No. 3
Deadline:

Write FORTRAN code for each of the following problems:

Solution of Initial Value Problem (IVP)

1. Single step Method:

Consider the IVP:

$$\frac{dy}{dt} = f(t,y), 1 \le t \le 2, y(1) = 0.5$$

where, $f(t,y) = t^2 - ty + y^2$.

Estimate the values of (2.0) with step size h = 0.2 using

a. The Euler's method
b. The Modified Euler's method
c. The Taylor's method of order three/four
d. The Runge-Kutta method of order four

Multi-step Method:

Consider the IVP:

$$\frac{dy}{dt} = f(t,y), 0 \le t \le 2, y(0) = 0.5$$

where, $f(t,y) = y - t^2 + 1$.

Estimate the values of (2.0) with step size h = 0.25 using

a. Adams-Bashforth four/five-steps method
b. Adams-Moulton three/four-steps method
c. Adams fourth-order Predictor-corrector method

Runge-Kutta Method for Systems of Differential Equations:

A. The Kermack-McKendrick model:

The Kermack-McKendrick model is an SIR model for the number of people infected with a contagious illness in a closed population over time. It was proposed to explain the rapid rise and fall in the number of infected patients observed in epidemics such as the plague (London 1665-1666, Bombay 1906) and cholera (London 1865). It assumes that the population size is fixed (i.e., no births, deaths due to disease, or deaths by natural causes), incubation period of the infectious agent is instantaneous, and duration of infectivity is same as length of the disease. It also assumes a completely homogeneous population with no age, spatial, or social structure. The model consists of a system of three coupled nonlinear ordinary differential equations,

$$\frac{dS}{dt} = -\beta SI$$

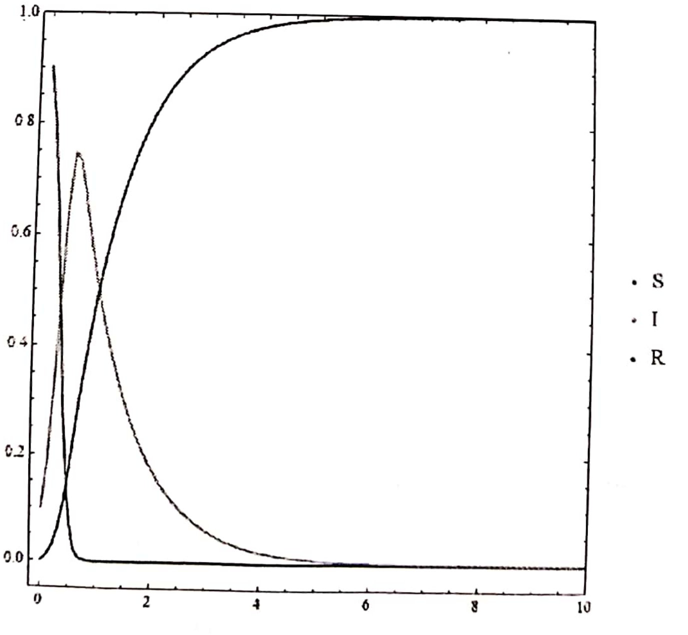
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

where t is time, $S(t)$ is the number of susceptible people, $I(t)$ is the number of people infected, $R(t)$ is the number of people who have recovered and developed immunity to the infection, beta is the infection rate, and gamma is the recovery rate.

Let us assume that $S(0) = S_0, I(0) = I_0, R(0) = R_0$.
Find the solution of $S(t), I(t)$ and $R(t)$ over the domain $0 \leq t \leq 10$.
Here, $S_0 = 0.9, I_0 = 0.1, R_0 = 0, \beta = 10, \gamma = 1, \Delta t = 0.1, n = 100$.

Sample Outcome:



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Assignment No. 4
Deadline:

Write FORTRAN code for each of the following problems:

Solution of Boundary Value Problem (BVP)

1. Consider the second order linear boundary value problem:

$$y'' = p(x)y' + q(x)y + r(x), a \le x \le b, y(a) = \beta, y(b) = \delta$$

where,

$$p(x) = -\frac{2}{x}, q(x) = \frac{2}{x^2}, r(x) = \frac{sin(lnx)}{x^2} \quad with \;\; y(1) = 1, y(2) = 2$$

Solve the above mentioned boundary value problem using the following methods:

a. Shooting Method b. Finite Difference Method.

Compare your results with the exact solution:

$$y(x) = C_1 x + C_2 \frac{1}{x^2} - \frac{3}{10} sin(lnx) - \frac{1}{10} cos(lnx)$$

where

$$C_1 \approx -0.03920701320, C_2 \approx 1.1392070132.$$

4/8/22

Dominant Eigenvalue

2. Use the Power method to approximate the dominant eigenvalue of the matrix, A:

$$A = \begin{bmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$