UNIVERSITY OF DHAKA
Department of Mathematics
Fourth Year B.S. (Honors) 2021-2022
Subject: Mathematics

Course No: MTH 450
Assignment 1
Name:

Course Title: MATH Lab IV
Deadline: Three Lab Days
Roll:

Write a Script file to solve each of the following problems.

1. Consider the first order differential equation (DE) $\frac{dy}{dx} = x^2 - 2y$.
   i. Solve DE using 'dsolve' command and plot the solution trajectory passing through the point $(-3,2)$.
   ii. Construct a slope field for the differential equation and use it to sketch an approximate solution curve that passes through the points $(-3, y_0), y_0 = -2,0,2,4,6$.
   iii. Create a function `odefun` and use ode45 to solve the DE.
   iv. Use Euler's method to solve the DE.
2. Find the stability interval in Euler's method for the DE $y' = \lambda y$. Hence, perform the stability analysis for this method (graphically) for $\lambda = -2$.
3. Solve the following DE's by using Laplace Transform. Also Plot the solution trajectory. (Hint: See the commands "`laplace`" & "`ilaplace`").
   i. IVP $y'' + 4y' + 4y = 3e^{-3t},\ y(0) = 1,\ y'(0) = -1$.
   ii. BVP $y'' + y = -t,\ 0 \le t \le 1,\ y(0) = 0,\ y(1) = 0$.
   iii. IVP $y'' + 3y' + 2y = f(t),\ y(0) = 0,\ y'(0) = -2,\ f(t) = \begin{cases} 2 & t < 6 \\ t & 6 \le t < 10 \\ 4 & t \ge 10 \end{cases}$

      (Hint: "heaviside")
   iv. IVP $y'' + 2y' + 10y = 1 + 5\,\delta(t-5),\ y(0) = 1,\ y'(0) = 2$.

      (Hint: "`dirac`")
   v. System of DE's $\begin{array}{ll} x_1' = 3x_1 - 3x_2 + 2 & x_1(0) = 1 \\ x_2' = -6x_1 - t & x_2(0) = -1 \end{array}$
4. Solve the one-dimensional heat equation (PDE): $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(0,t) = u(\pi,t) = 0, u(x,0) = 100$. Interpret your result. (Hint: "`pdepe`")
5. Let $u(x,t)$ be the vertical displacement of a string of length $1m$ from a fixed horizontal axis (X-axis) at position x and time t. The ends of the string are fixed at height zero and its initial configuration and speed are given by $u(x,0) = x(1-x)$and $u_t(x,0) = 0$ respectively. If the vibration of the string is governed by $u_{tt} = u_{xx}$, find the configuration $u(x,t)$ for all $x,\ t$. Interpret your result geometrically.
6. Solve system of PDEs:

$$\begin{gathered} u_t = u_{xx} + u(1 - u - v) \\ v_t = v_{xx} + v(1 - u - v) \\ u_x(t,0) = 0;\ u(t,1) = 1;\ v(t,0) = 0;\ v_x(t,1) = 0; \\ u(0,x) = x^2;\ v(0,x) = x(x-2) \end{gathered}$$

   Plot the solutions.

UNIVERSITY OF DHAKA

Department of Mathematics
Fourth Year B.S. (Honors) 2021-2022
Subject: Mathematics
Course No: **MTH 450** Course Title: **MATH Lab IV**

Assignment-2

1. **Continuous-time Logistic model:** Suppose that the spruce budworm, in the absence of predation by birds, will grow according to a simple logistic equation of the form
$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right)$$
Budworms feed on the foliage of trees. The size of the carrying capacity, K, will therefore depend on the amount of foliage on the trees; we take it to be constant for this model.
   (a) Draw graphs for how the population might grow if r were 0.48 and K were 15. Use several initial values.

2. **Discrete-time Logistic model:** Consider the model equation
$P_{n+1} = P_n + rP_n\left(1 - \frac{P_n}{K}\right)$.
   (a) Find the fixed point and analysis its stability.
   (b) Plot the solution trajectory (orbit) for fixed values of r, K & P_0. Set $r = 3.5, K = 5, P_0 = 0.5$.
   (c) Comments on the dynamics of its orbit by varying r. You may use $2.5 \leq r \leq 3.9$.

3. The original Lotka–Volterra equations are
$$\begin{aligned} \frac{dx}{dt} &= rx - axy \\ \frac{dy}{dt} &= -my + bxy \end{aligned}$$
where the positive constants r, m, a, and b are parameters. The model was meant to treat predator–prey interactions. In this, x denotes the population size of the prey, and y the same for the predators.

   (a) Plot the solution trajectory for the fixed values of model parameters and initial values, and interpret the result.
   (b) Find the nonnegative critical points and determine the nature (stability)
   (c) Draw the phase plane diagram for different set of initial conditions.

4. Imagine a three-species predator–prey problem that we identify with grass, sheep, and wolves. The grass grows according to a logistic equation in the absence of sheep. The sheep eat the grass and the wolves eat the sheep. We model this with the equations that follow. Here x represents the wolf population, y represents the sheep population, and z represents the area in grass:

$$\frac{dx}{dt} = -x + xy$$
$$\frac{dy}{dt} = -y + 2yz - xy$$
$$\frac{dz}{dt} = 2z - z^2 - yz$$

(a) What would be the steady state of grass with no sheep or wolves present?

(b) What would be the steady state of sheep and grass with no wolves present?

(c) What is the revised steady state with wolves present?

(d) Does the introduction of wolves benefit the grass?

5. Consider SIR model

$$\dot{S} = -aSI \qquad S = susceptible \text{ individuals}$$
$$\dot{I} = aSI - bI, \qquad I = infected \text{ individuals}$$
$$\dot{R} = bI \qquad R = recovered \text{ individuals}$$

(a) Plot the solution trajectory and interpret

(b) What happens if the infectious coefficient increases?

6. Describe the dynamics of Lorenz system

$$\dot{x} = \sigma(-x + y)$$
$$\dot{y} = \rho x - y - xz,$$
$$\dot{z} = xy - \beta z$$

(a) Plot the solution trajectory for the fixed values of model parameters and initial values, and interpret the result

(b) Find the nonnegative critical points and determine the nature (stability)

(c) Draw the phase plane diagram.