# UNIVERSITY OF DHAKA

Department of Mathematics

Second Year B.S. (Honors) 2018-2019

Subject: Mathematics

Course No: **MTH 250** Course Title: **MATH Lab II**

## Assignment-3

| **Name:** | **Roll:** | **Group:** |
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***Write a Script file to solve each of the following problems.***

**Q1.** Create a function m-file that will take two vectors of same dimension and calculate their Euclidean inner product. Using that file solve the following problems.

**(a)** Let $\boldsymbol{u} = (3, -2), \boldsymbol{v} = (4, 5), \boldsymbol{w} = (-1, 6)$. Find

(i) Euclidean inner product $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \boldsymbol{u}.\boldsymbol{v}$

(ii) Verify $\langle \boldsymbol{u} + \boldsymbol{v}, \boldsymbol{w} \rangle = \langle \boldsymbol{u}, \boldsymbol{w} \rangle + \langle \boldsymbol{v}, \boldsymbol{w} \rangle$.

**(b)** Verify that the set of vectors $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is orthogonal with Euclidean inner product, i.e. by showing that $\langle \boldsymbol{u_1}, \boldsymbol{u_2} \rangle = \langle \boldsymbol{u_1}, \boldsymbol{u_3} \rangle = \langle \boldsymbol{u_2}, \boldsymbol{u_3} \rangle = \boldsymbol{0}$. Then convert it to an orthonormal set by normalizing the vectors, i.e. $\|\boldsymbol{v_i}\| = 1$, where

$$\boldsymbol{v_i} = \frac{\boldsymbol{u_i}}{\|\boldsymbol{u_i}\|}, i = 1, 2, 3$$

Given $\boldsymbol{u_1} = (1, 0, -1), \boldsymbol{u_2} = (2, 0, 2), \boldsymbol{u_3} = (0, 5, 0)$.

**(c)** Apply the Gram-Schmidt process to transform the basis vectors

$$v_1 = (1, 1, 1), v_2 = (0,1\ , 1), v_3 = (0, 0, 1)$$

into an orthogonal basis $\{u_1, u_2, u_3\}$; then normalize the vectors $\{u_1, u_2, u_3\}$ to obtain an orthonormal set $\{e_1, e_2, e_3\}$. The Gram-Schmidt orthogonalizing process is as follows

$$u_1 = v_1$$

$$u_k = v_k - \sum_{j=1}^{k-1} proj_{u_j}(v_k)\, , k = 2, 3, \ldots$$

**Q2.** Let $T$ be the transformation from the $uv$-plane to $xy$-plane defined by

$$T(u,v) = (x(u,v), y(u,v))$$
where, $x = \frac{1}{4}(u+v)$ and $y = \frac{1}{2}(u-v)$

(i) Find $T(1,3)$.

(ii) Find the coordinate point of the image under $T$ of the square region in the $uv$-plane bounded by the lines $u=-2, u=2, v=-2$ and $v=2$.

**Q3.** Verify that the quadratic function $f(x,y) = 2x^2 + 6xy - 5y^2$ can be written as $Q_A(\boldsymbol{x}) = \boldsymbol{x}^T A\boldsymbol{x}$, where $A$ is a symmetric matrix (compute it manually) and $\boldsymbol{x}$ is a column vector. Hence evaluate $Q_A$ for $x=2, y=1$.