Problem Chosen	2022	Team Control Number
E	MCM/ICM Summary Sheet	2200687

Multi-Factor Forest Management

Summary

Forests are an important ecological resource on the earth, which can be used as fuel and can also provide cellulose for paper making. An important role of forests is carbon sequestration, which can act as an important carbon sink to absorb a large amount of carbon dioxide. In short, forests can be said to have three aspects of economic, ecological and social performance. We model forest characteristics and socioeconomic impacts to derive better forest management practices.

We built two models: Model I: Carbon sequestration mode; Model II: Decision model. We optimized Model I by adding the relative growth rate α for more intuitive results.

For Model I: We need to consider carbon sequestration by plants, and plants use photosynthesis to convert carbon dioxide into glucose, etc. and store it in the body to achieve carbon sequestration. Then we established the carbon sequestration model of a single tree, and then calculated the maximum point of its carbon sequestration rate as our reference point. Then we apply it to forests in different temperature zones to get their carbon sequestration, and finally we get the most recent logging time period for each forest. We optimized Model I and added the coefficient of relative growth rate, which can more intuitively obtain the optimal time period for forest felling, so as to help forest managers to specify forest management plans more quickly.

For Model II: We first consult the paper to obtain the optimal harvest decision model for forests, then we use biomass per tree to represent the volume of wood per tree, and carbon sequestration to represent ecological benefits to simplify our decision model. We divide forest managers into four types: developed, developing, poor and private individuals. And we assign two coefficients τ and η to the model to determine the proportion of different forest manager types to economic, ecological and social factors. Then I used AHP to determine the weights of the two coefficients for different forest managers. Finally, we fed the data of different forest managers into our model to obtain their forest management schedule. Finally, we find the transition time point of the forest and analyze the choices in different situations.

We performed sensitivity analyses after both models were established to ensure that our models were accurate and stable. For Model I: Due to the competition between trees in the forest ecosystem, the optimal carbon sequestration effect will not be achieved in the end. For Model II: Because marginal harvesting costs and tree planting costs fluctuate due to economic and social impacts. We performed a sensitivity analysis on both models and determined that our model is robust.

Finally, we apply our model to the Amazon rainforest, and conclude that if we manage the forest according to our model, we can get better benefits than it is now. We conclude a strategies to satisfy forest managers and all those who use forests.

Keywords: Single tree model optimization; Carbon Sequestration; Relative growth rate; Sensitivity analysis; AHP

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1 Introduction

1.1 Problem Background

With the development of industrial technology, the concentration of greenhouse gases such as carbon dioxide in the atmosphere keeeps increasing, leading to aggravation of the greenhouse effect. This effect has threatened the development of social economy and the survival of human beings. In the ecosystem, the forest ecosystem is the main body of the carbon cycle and plays an important role in carbon fixation. The forest area accounts for 27.6% [5] of the global land area, and the carbon storage of the forest ecosystem accounts for about 46.6% of the carbon storage of the terrestrial ecosystem. Forest ecosystems have huge carbon storage capacity, and increasing forest carbon sinks is the most cost-effective and effective way to slow the rise of carbon dioxide concentration in the world.

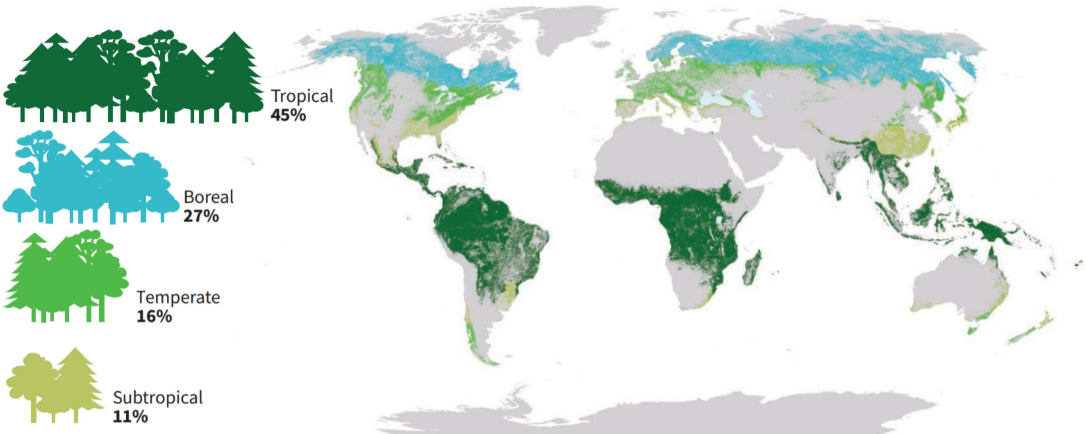

Figure 1: Distribution of the world's forest resources

1.2 Problem Restatement

Carbon Sequestration is a significant technology with many complexities. Through in-depth analysis and research on the background of the problem, combined with the specific constraints given, the restate of the problem can be expressed as follows:

- Build a mathematical model of carbon sequestration to determine carbon storage in forests at different times.
- Based on the model, find out most effective management plan at sequestering carbon dioxide in forest.
- Build a decision model to obtain the best way to use forests, considering other aspects of forest value.
- Apply the decision model on different forests and obtain detailed management plans and perform a sensitivity analysis.

- Write a one- to two-page non-technical newspaper article to explain why our plan includes harvesting.

1.3 Our Work

- Data were collected and two curves were fitted for individual trees based on a CAR model, biomass and carbon sequestration as a function of tree age.
- The relative growth rate α is defined to optimize the original model.
- Using AHP to calculate the influence of ecological, economic and social factors on determining forest management strategies, and optimize the $MaxS$ formula.
- Consider the density of trees in the forest, use $MaxS$ to predict and construct a decision model
- Test our model with the actual situation of the Amazon forest, proving the advantages and disadvantages of our model

In order to avoid complicated description, intuitively reflect our work process, the flow chart is shown in Figure 2:

2 Assumptions and Explanations

We make the following basic assumptions in order to simplify the problem. Each of our assumptions is justified and is consistent with the basic fact.

- Assumption 1: Each forest ecosystem is represented by one tree species.

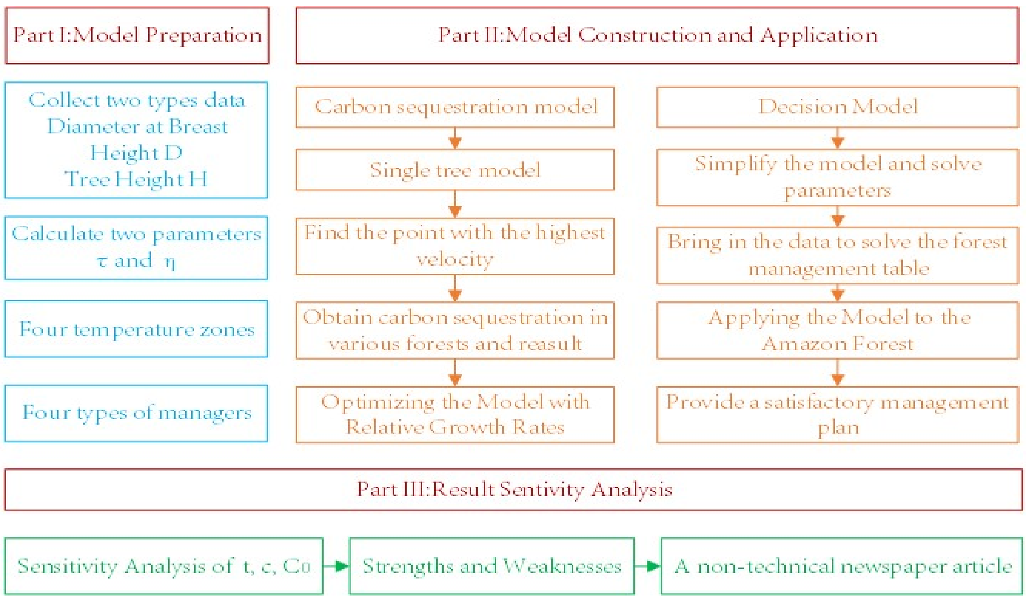

Figure 2: Flow Chart of Our Work

- Assumption 2: The biomass eventually reaches equilibrium
- Assumption 3: The discussion is about natural forests without human interference.
- Assumption 4: The stage of tree decay is not considered

3 Notations

We list symbols and notations used in this paper in Table1. There are some variables that are not listed here and will be discussed in detail in each section.

Table 1: Notations used in this paper

Symbol	Description
S	Biomass of tree
D_i	Diameter at Breast Height
H_i	Tree Height
a, b	Parameters of the regression equation
t_i	Age of tree
α_i	Growth rate of biomass
α_{max}	Maximum biomass growth rate
α	Growth factor
$MaxS$	Maximum total benefit net present value

4 Model I: Carbon sequestration model

4.1 Model Preparation

Plants absorb CO_2 mainly by absorbing H_2O and CO_2 to synthesize glucose through photosynthesis and store it in its bodies to sequester carbon. Considering the chemical formula of photosynthesis, we can exactly know how much carbon is in the body of a plant.

$$6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2 \tag{1}$$

We first find out the carbon sequestration model of a single tree, and then expand to an entire forest, and finally obtain the carbon sequestration model of the entire forest. By calculating the biomass of the forest, the carbon sequestration of the forest can be calculated.

4.2 Establishment of a single tree model

There are many mature models for the study of individual trees in forest ecosystems. As a result, we considered a lot of models and decided to use the model established by the average biomass method. [6]

$$S = \Sigma a(D_i^2 H_i)^b \tag{2}$$

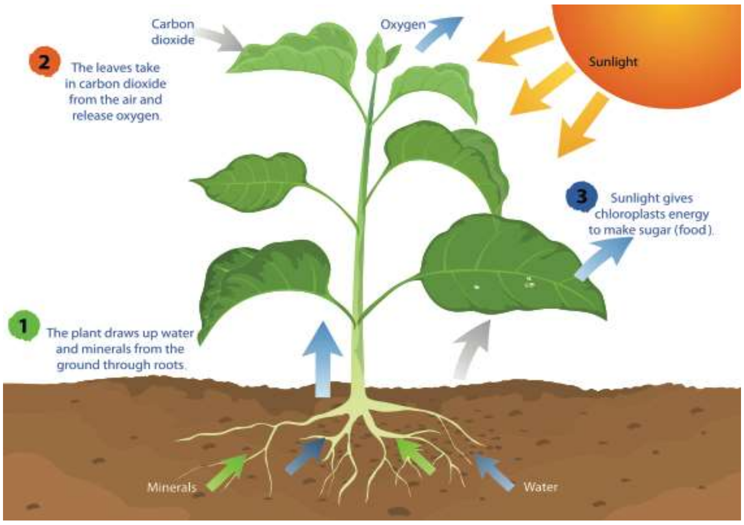

Figure 3: Process of photosynthesis
https://ssec.si.edu/stemvisions-blog/what-photosynthesis

For this model, we set both parameters a and b to 1 to simplify our model.

$$S = \Sigma D_i^2 H_i \tag{3}$$

The carbon element in green plants is accumulated in organic matter through photosynthesis, and the carbon content rate can be obtained by calculating the increase of organic matter in the plant. The main components of plants are cellulose, hemicellulose, and lignin, which account for more than 95% of the total wood, but the content of various substances varies with tree species. As long as the content of cellulose(μ), hemicellulose(γ) and lignin(ξ) in the tree species and the content of carbon elements in these three components are determined, the carbon content of the entire tree species can be determined. According to the molecular formula of cellulose $C_6H_{12}O_6$, carbon accounts for $4/9$ of its weight, carbon in the molecular formula of hemicellulose $(C_6H_8O_4)_n$ accounts for $5/11$ of its weight, and the proportion of carbon in lignin is 82.2%. From this, the carbon content of different trees(C) can be calculated by formula: [6]

$$C = \mu \times \frac{4}{9} + \gamma \times \frac{5}{11} + \xi \times 82.2\% \tag{4}$$

The amount of carbon sequestered by a single tree(W), can be expressed by the product of the biomass of the tree species and its carbon content.

$$W = S \times C \tag{5}$$

Since the molecular weight of carbon dioxide is 44 and the molecular weight of carbon element is 12, the carbon sequestration can be converted into carbon sink(M_{CO_2}).

$$M_{CO_2} = S \times C \times \frac{44}{12} \tag{6}$$

4.3 Application model

We first classify forest ecosystems by tropical, subtropical, temperate, subarctic, and frigid zones. Since each temperature zone contains multiple forest ecosystems, we select a representative forest ecosystem from each temperature zone for analysis. We first collected data [1] [7] [2] [4] [3] and then analyzed **temperate evergreen broad-leaved forests** as an example. Through the analysis of the problem, we need to collect the representative average tree height, DBH and other related data in each temperature zone. Due to the large amount of data, it is inconvenient to list them one by one.

We first bring the tree heights and diameters at breast height of trees of various ages into Equation (3) to obtain their corresponding relationships, and draw the Figure 3.

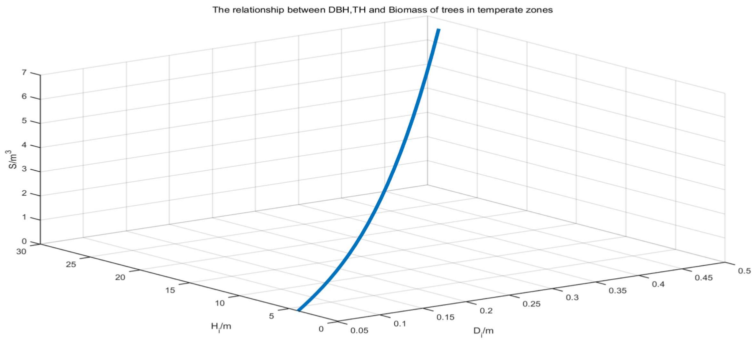

Figure 4: Growth of trees in temperate zones

From Figure 4 we can see that as the tree grows, the biomass S in the tree keeps increasing, but this way we have no way to known if we are going to rotate. According to the growth law of organisms, it is definitely impossible for trees to maintain a high rate of carbon dioxide absorption all the time, and there will definitely be a point where the carbon sequestration rate of trees is the largest.

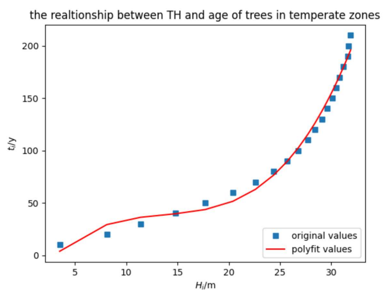

Figure 5: TH and age of trees

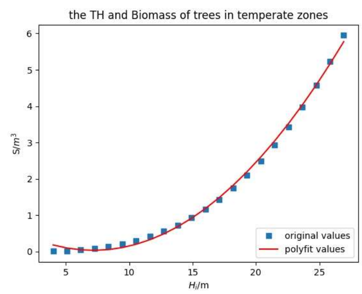

Figure 6: TH and Biomass of trees

We obtained the relationship between biomass and tree age by solving the relationship between tree height and biomass as shown in Figure 6, and the relationship between tree height and tree age as shown in Figure 5.Then we reduce the two equations into one equation. We obtained the relationship between biomass and tree age.

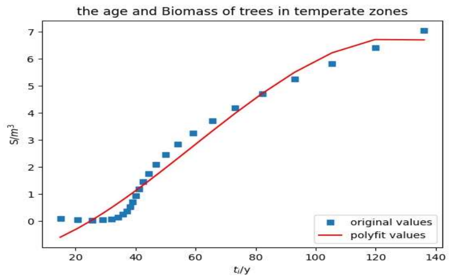

Figure 7: the age and Biomass of tress in the temperate zones

We can see that with the increase of tree age, the biomass of trees increases, but it tends to be flat in the end, indicating that the biomass is limited by a certain limit, that is, there is a maximum biomass.

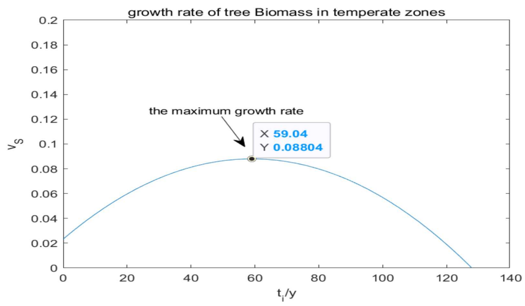

Figure 8: growrh rate of tree biomass in temperate zone

We differentiate the relationship equation between biomass and tree age to obtain the relationship between biomass growth rate and tree age, and find the time point when biomass growth rate is the largest.

When the growth rate of tree biomass reaches the maximum, the growth rate of tree biomass will slow down, and the corresponding carbon sequestration efficiency will begin to decrease. After this time point, it is suitable for felling. We intercept the biomass growth over a period of time after the maximum growth rate time point. When the biomass growth in the same time period is the same as in the case of replanting the tree, we consider is possible to fell, which we call optimal felling point in time. After the trees are felled, other wood products can be made.

We calculated that the optimal time interval is around **30 years** , at which time the biomass growth of trees in this time period after tree felling is the same as that from the beginning of tree growth to 30 years later. From here we can conclude that the optimal felling time is between **105-115 years**.

We bring the data of other temperature zones into Equation (3) and get the result as shown in Figure 9.

Finally, we add biomass into Equation (6) to obtain carbon sinks, and apply **the above method** to other temperature zones to obtain the corresponding results as shown in the figure below.We put

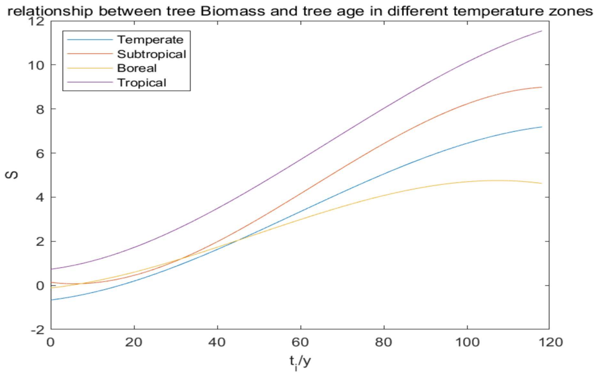

Figure 9: relationship between Biomass and tree age in different temperature zones

the results for the four temperature zones in Figure 10.

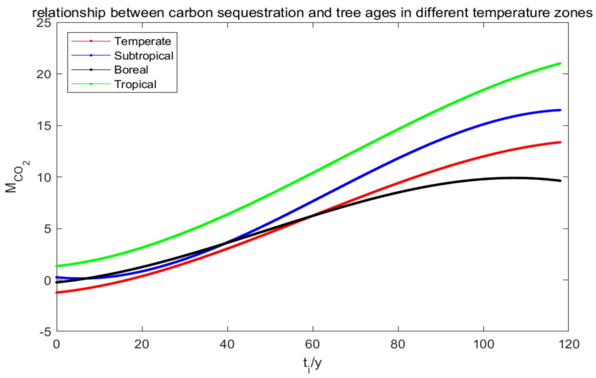

Figure 10: relationship between carbon sequestration and tree ages in different temperature zones

We put the results for the four temperature zones in Figure 11.

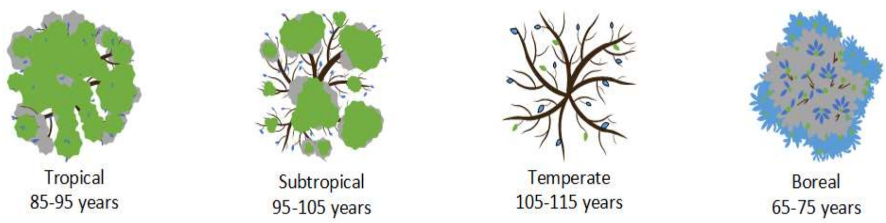

Figure 11: the reasult of different temperature zones

We present the pseudocode for Model I below.

4.4 Sensitivity analysis

Due to the competitive relationship between trees in the forest ecosystem, if the trees are cut down very late, the competition among trees will become more intense, and ultimately the optimal carbon sequestration effect will not be achieved. So an optimal felling period should be found after reaching

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Algorithm 1 CAR on single tree

Input: $(t_i, H_i), (D_i, H_i)(i = 1, 2...)$**,** T**(temperature zones' number),** $(\mu_m, \gamma_m, \xi_m)(m = 1, ..., T)$

Output: carbon sequestration function $M_{CO_2}(t)$ **of different temperature zones**

for a = 1 to T **do**
    fitting $t(H)$ and $D(H)$ with (t_i) and H_i
    calculating $S(t)$ with $S(D, H)$
    substitute (μ_a, γ_a, ξ_a) in
    determine the trend of $M_{CO_2}(t)$ and the maximum data point (t_{max}, x_{max})
    optimize the curve as follows:
    if $t >= t_{max}$ **then**
        $M_{CO_2}(t) = M_{max}$
    end if
    visualize $M_{CO_2}(t)$
end for

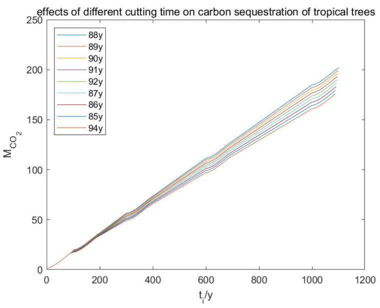

Figure 12: tropical trees

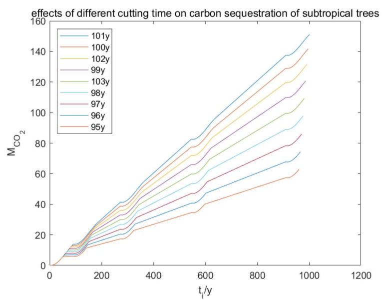

Figure 13: subtropical trees

the maximum growth rate. We choose to cut down at different time points, and then plotted their carbon sequestration after logging for comparison. The results are shown in the figure, in line with our obtained results.

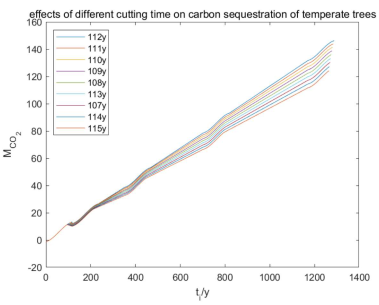

Figure 14: temperate trees

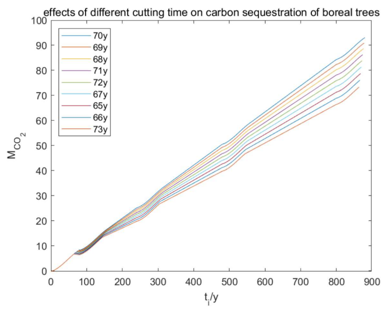

Figure 15: boreal trees

4.5 Optimization model based on RGR

We propose a new concept: relative growth rate – α. We define the relative growth rate (RGR) as a coefficient equal to the ratio of the current growth rate to the maximum growth rate.

$$\alpha = \frac{\alpha_i}{\alpha_{max}} \tag{7}$$

We multiply the relative growth rate by Equation 3 to obtain the biomass at the relative growth rate $(BRGR)$.We draw a diagram of $BRGR$.

$$S_\alpha = \alpha \Sigma D_i^2 H_i \tag{8}$$

Then we calculate the $BRGR$ by the same method for the forests in the four temperature zones,

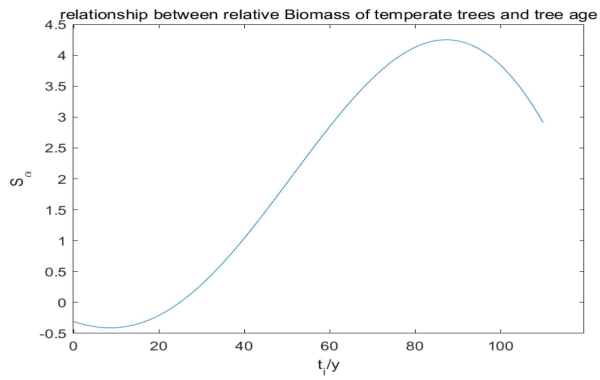

Figure 16: relationship between relative Biomass of temperate trees and tree age

and then bring it into Equation (6) to obtain the carbon sequestration as shown in the figure 18.

We can see that the results obtained by the optimized model are basically consistent with the results obtained by the previous model, indicating that our optimization is correct.

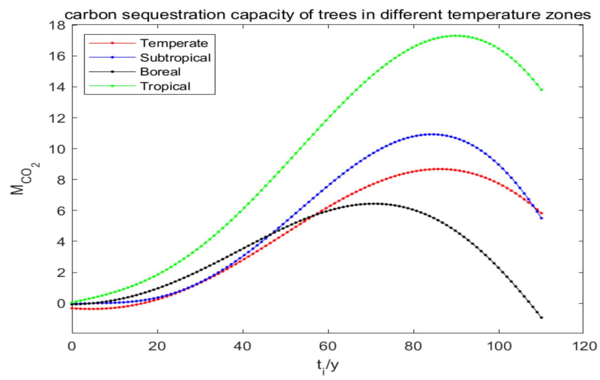

Figure 17: carbon sequestration capacity of trees in different temperature zones

We draw the flow of the two different methods as a flow chart as shown in Figure 17.

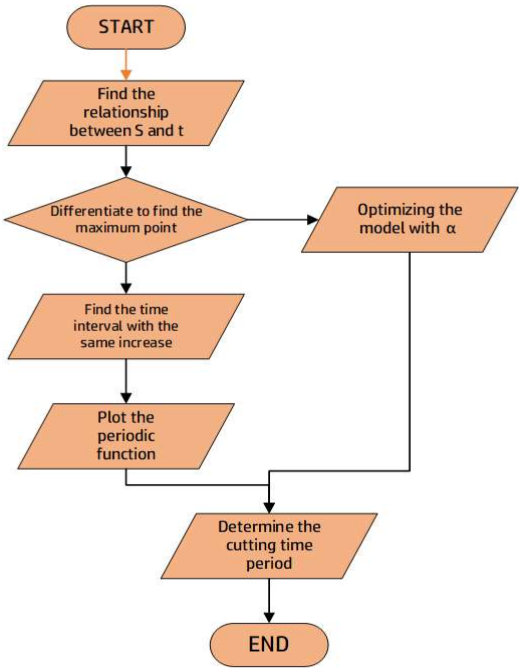

Figure 18: Flowchart of Model I

5 Model II: Decision model

5.1 Optimal Harvest Decision Model for the Forest

Because forests have more than one value for carbon sequestration, we also need to consider other aspects. After analysis, we believe that three aspects need to be considered: economy, society and ecology. [9] These three aspects are not independent of each other, so we need to build a model that considers three aspects.

$$MaxS = (p - c)V(t)e^{-rt} - C_0 \tag{9}$$

In Equation (9), p is the price of wood per unit volume, c is the marginal cost of harvesting, V is the volume of wood per plant, a function of tree age t, and C_0 is the cost of planting trees.

By solving the first-order condition (10), as long as $p \neq c$ is satisfied, harvesting when the tree grows to t^* year can maximize the net present value of the wood income

$$\frac{dS}{dt} = (p - c)e^{rt}\left[\frac{dV(t)}{dt} - V(t)r\right] = 0 \tag{10}$$

We can get r by solving Equation (11), where the numerator is the α_{max} we defined in the optimization model.

$$\frac{dV(t^*)/dt}{V(t^*)} = r = \frac{\alpha_{max}}{V(t^*)} \tag{11}$$

We let W be the ecological benefit of a tree in year t accumulation.Then we get Equation (12) considering the eco-efficiency.

$$MaxS = (p - c)V(t)e^{-rt} - C_0 + W(t) \tag{12}$$

To simplify our model, we replace the wood volume V per plant with the tree species biomass S, and replace the ecological benefit W with the carbon sink M to obtain Equation (13).

$$MaxS = (p - c)S(t)e^{-rt} - C_0 + M(t) \tag{13}$$

We know that c is the marginal felling costs and C_0 is tree planting costs in Equation (13), respectively, which are affected by social factors. We define $[(p - c)S(t)e^{-rt} - C_0]$ as the influence of economic factors, and finally M is the influence of ecological factors.

We know that c is and C_0 in Equation (13) are the marginal felling cost and tree planting cost, respectively, which are affected by social factors. We define $[(p - c)S(t)e^{-rt} - C_0]$ as the influence of economic factors, and finally M is the influence of ecological factors.

Because different countries and regions have different forest management plans, we divide the world into four types of forest management plans:**developed countries, developing countries, poor countries and private individuals.** To distinguish these four cases, we define two coefficients τ and η and obtain Equation (14).

$$MaxS = \tau \left[(p - c)S(t)e^{-rt} - C_0\right] + \eta M(t) \tag{14}$$

5.2 AHP determines parameters τ and η

In order to find τ and η in different regions, we surveyed more than 5,000 samples around the world through online questionnaires, and then used the analytic hierarchy process to find them.

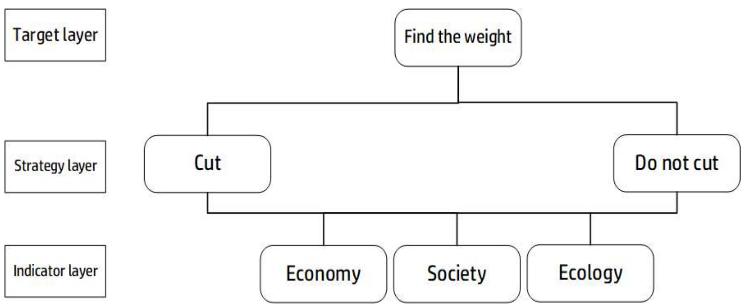

Figure 19: The framework of AHP

We obtain the individual private judgment matrix as follows. Its CR is $0.043 \leq 0.1$, and it passes the consistency check. Then we can get the weights of τ and η in different regions through the same method in Table 2.

$$\begin{bmatrix} 1 & 8 & 2 \\ \frac{1}{8} & 1 & \frac{1}{4} \\ \frac{1}{2} & 4 & 1 \end{bmatrix} \tag{15}$$

Table 2: τ and η in four regions

Region	**Economy / τ**	**Ecology / η**
Developed countries	0.257	0.743
Developing countries	0.689	0.302
Poor countries	0.794	0.206
Private individuals	0.850	0.150

5.3 Application model

We take a poor country as an example, bring τ and η into the Equation (14), calculate $MaxS$, and take the abscissa t when $MaxS$ is the largest as the optimal cutting time point.

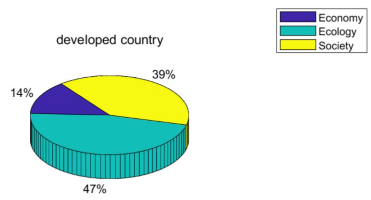

Figure 20: Developed countries

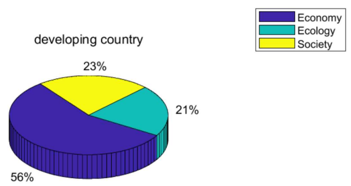

Figure 21: Developing countries

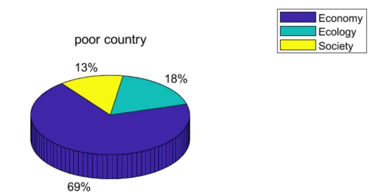

Figure 22: Poor countries

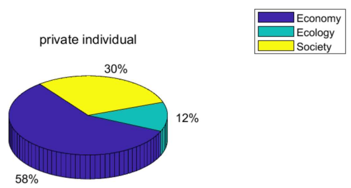

Figure 23: Private individuals

We make reasonable assumptions through social surveys and pass consistency tests, and find that social factors contribute to economic factors and ecological factors separately in each manager category.

Developed countries: $3:7$

Developing countries: $6:4$

Poor countries: $8:2$

Private individuals: $9:1$

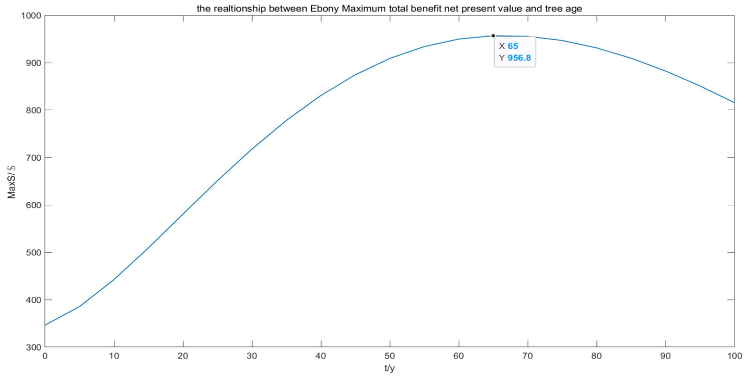

Figure 24: the relationship between Enony Maximum total benefit net present value and tree age

- **We cannot cut down trees when the $MaxS$ in the forest ecosystem is not at its maximum.**

The other three regions are brought into the access forest management plan in the same way.We mark the calculated optimal felling time for each type in Table 3.

We determine that the maximum area of each tree is $15m^2$ [8], and the area of a single tree cannot exceed this limit, otherwise it will affect the growth of other trees and reduce the benefits of the entire forest.

- **We believe there is a transition point.**
- **The definition of the transition point is the condition $S_0 = S$.Where S is the maximum biomass that a single tree can achieve without affecting the growth of other trees, and S_0 is the biomass under the optimal rotation period.**

When the forest manager changes, we have to change strategies for forest management. We take a pine tree as an example, and draw the change curve of maxs when the forest manager changes, as shown in Figure 26.

Let's take a pine tree as an example to find the transition point. First, we need to determine the time $t1$ when the tree grows to $15m^2$, and then find the time point $t2$ where MaxS is the largest in different forest managerment images time points for comparison.

Case 1: $t1 > t2$, we cut down trees at time $t1$, because trees will not affect the growth of other trees at this time.

Table 3: Forest management plan

Region	Species	50y	55y	60y	65y	70y	75y
Developed countries	Ebony	908.93	933.97	949.78	956.79	955.56	946.75
	Rosewood	948.43	974.52	990.99	998.29	996.99	987.80
	Locust wood	636.80	654.60	665.87	670.90	670.09	663.91
	Teak	617.79	635.08	646.03	650.92	650.14	644.14
Developing countries	Larch	54.11	57.65	59.61	60.14	59.42	57.63
	Hemlock	47.17	50.41	52.22	52.74	52.11	50.51
	White pine	66.83	70.91	73.15	73.72	72.83	70.68
	Poplar	99.78	105.29	108.25	108.91	107.56	104.49
Poor countries	Fir	7.12	8.07	8.75	9.21	9.48	9.60
	Pine	23.24	25.19	26.58	27.45	27.86	27.85
	Spruce wood	15.03	16.19	16.92	17.29	17.35	17.16
	Oak	113.18	118.99	122.35	123.47	122.64	120.11
Private individuals	Fir	16.23	17.57	17.96	17.55	16.51	14.95
	Larch	65.51	69.65	71.84	72.26	71.14	68.70
	Teak	660.77	679.21	690.86	696.02	695.11	688.62
	Pine	64.09	68.16	70.32	70.74	69.63	67.23

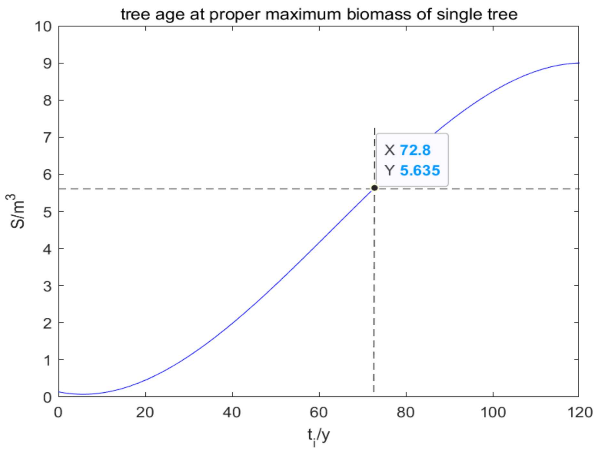

Figure 25: tree age at proper maximum biomass of single tree

Case 2**:** $t1 < t2$ **At this time, we cut down trees at time** $t2$**. At this time, trees have affected the growth of other trees and must be cut at time** $t2$**.**

5.4 Sensitivity analysis

Sensitivity analysis is necessary because marginal harvesting costs and tree-planting costs fluctuate due to economic and social impacts.

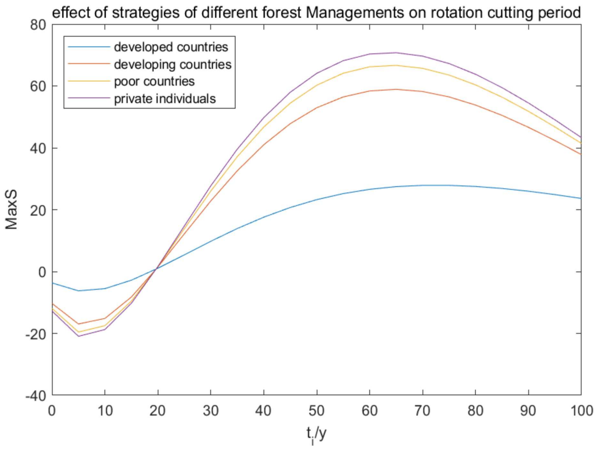

Figure 26: effect of strategies of different forest Managements on rotation cutting period

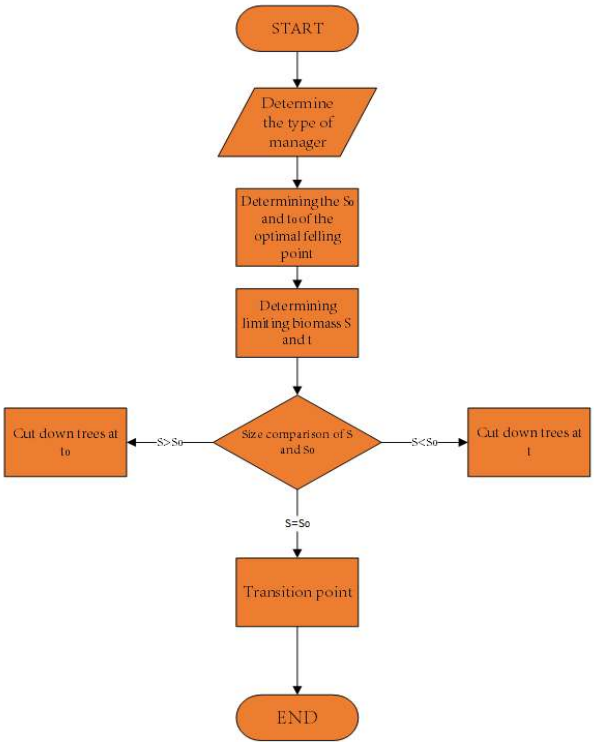

Figure 27: Mind map for finding transition points

We can see from Figure 28 and Figure 29 that our model has better robustness under different c and C_0.

5.5 Apply the model to the Amazon rainforest

We apply our model to the Amazon rainforest. We collect the data of 1990-2010, and then obtain the data of 2000-2010 by inputting the data of 1990-2000 into our model and compare it with the real data. It can be seen from Figure 30 that if the conclusion of our model is applied, we will

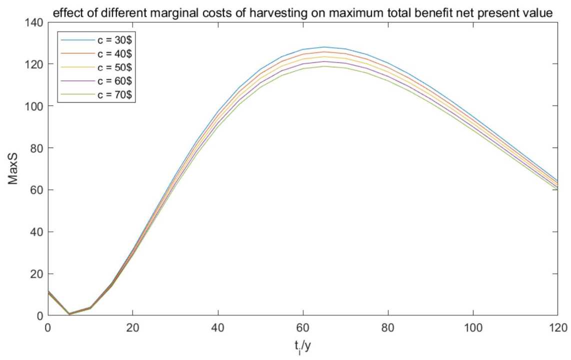

Figure 28: Marginal costs

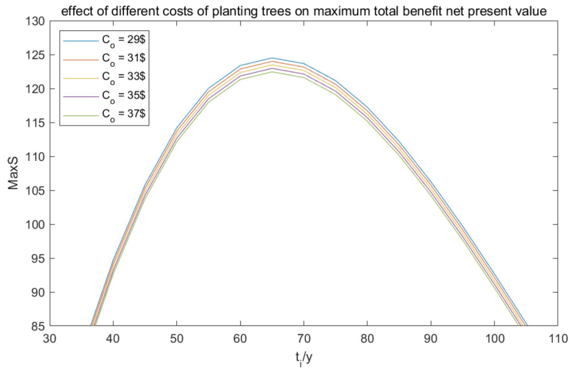

Figure 29: Costs of planting trees

obtain Better economic efficiency than now.

We assume that the size of the Amazon rainforest will remain the same for 100 years because the size of the rainforest fluctuates but the overall size will remain within a range. We divide the area by 15 to get the number of trees, calculate the amount of carbon sequestered at $t = 100$, and multiply the number of trees.

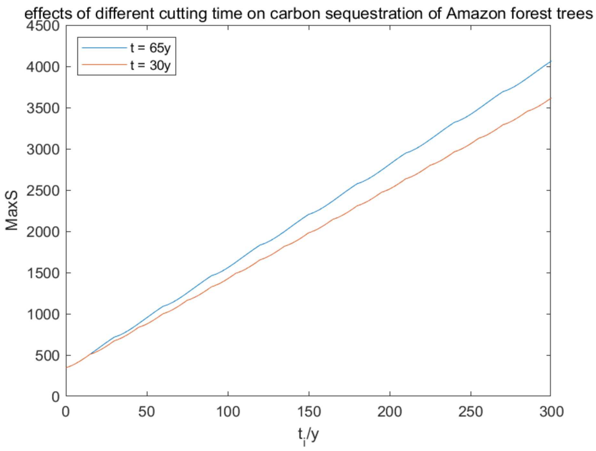

Figure 30: effects of different cutting time on carbon sequestration of Amazon forest trees

In the actual situation, the tree cutting time is delayed by one year, and the newly planted trees are also delayed by one year from the original planting time. For ten consecutive cycles, ten batches of trees are all cut longer than the original rotation time. After a one-year delay, after ten cycles, the management plan is ten years later than the current forest practice logging time. From the results shown in the image,The difference in $MaxS$ between this strategy and the optimal rotation strategy is so small as to be negligible that such a change can be made to the satisfaction of forest managers and all who use forests

6 Strengths, Weaknesses and Conclusion

6.1 Strengths

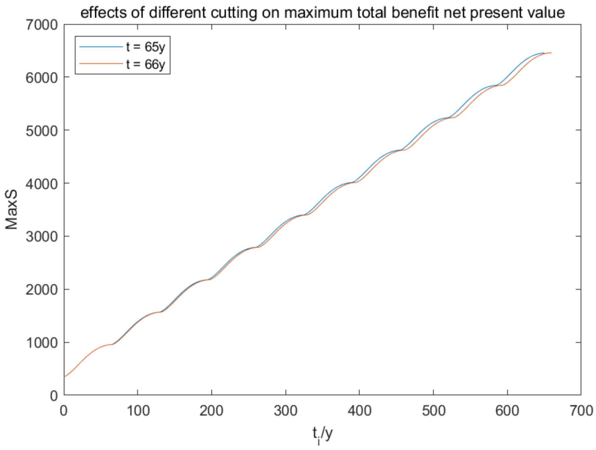

Figure 31: effects of different cutting on maximum total benefit net present value

- **Our model can be well applied to various places and has good stability.**

In the first model, we accurately analyzed the data by collecting a large number of growth cycle data of different tree species in different temperature zones, different countries and different forests, so the fitted curves can be well adapted to real life. The prediction of the biomass and carbon sequestration of the middle forest has high accuracy; for the second model, we not only reasonably simplify and optimize the previous model, but also fully explore the effect of density on tree growth conditions. Therefore, the optimal rotation period is further restricted, and the result makes the optimal rotation tree age we get more in line with the actual situation.

- **We have improved the carbon sequestration model for a single tree so that it can more intuitively reflect the optimal cutting time point.**

In our work, we have simplified the previous carbon sequestration model accordingly, and defined a new coefficient relative growth rate α based on the actual situation, which can well predict the amount of carbon sequestration while simplifying the calculation.

- **Our model can flexibly make calculation strategies according to changes in the actual situation.**

We fully consider the impact of ecological, economic and social factors in determining forest management strategies in real life. At the same time, we carefully consider the forest coverage rate, make reasonable assumptions about complex situations, and finally apply the resulting precise rotation strategy. Going to different countries and regions, the results can be well predicted, and we use the actual situation of the Amazon Forest to test our model. While 3 proves the advanced and correctness of our model, it also shows that our model is realistic.

6.2 Weaknesses

- **We have some randomness in the simplification of the biomass function.**

In the process of determining the biomass expression, we used the least squares method to solve the parameters a and b for different types of trees. In the previous calculations, we found that both a and b can be approximately 1, so they are roughly are assumed to be 1, so this randomness may lead to a certain bias in the results predicted by our model.

- **Our simplification of eco-efficiency is somewhat unreasonable.**

In our second model, we simply regard the ecological benefit in the MaxS function as carbon sequestration. When this is not the case, the forest also has the function of conserving water sources, etc., so the ecological benefit that we have obtained is relatively low. actually smaller.

- **We do not take into account the effects of oxygen, moisture and trace elements on trees.**

When we determine the impact of a single tree on other trees, we simply consider the footprint of a single tree, but we know that the competition between trees includes oxygen, water vapor, trace elements, etc. This is an extremely complex process, Because our assumptions may lead to certain incorrect results.

6.3 Conclusion

We first searched for relevant information, and based on the analysis of the collected data, we optimized the previous research model and developed a carbon sequestration model, which can well calculate the total carbon sequestration of different forests. For a more accurate prediction, the coefficient α proposed by us can simplify the original complex model without losing the accuracy, and then we apply the model to the actual situation, and the results are all very good approximations. We closely focus on the established carbon sequestration model and limit the optimal rotation period on the condition that it does not affect the growth of other trees, taking into account factors such as tree area, and come up with strategies with higher total benefits. Among them, we fully consider the impact of forests on economy, ecology and society, and use the AHP to determine their contribution rate, and determine the transition point according to the location and characteristics of the forest to re-determine the appropriate cutting time. Therefore, the obtained strategy is more in line with actual conditions.

According to different managers, we set up different models based on these evaluation factors, calculate the $MaxS$ obtained by different managers using different strategies, and select the strategy that maximizes $MaxS$, so that our model can respond to different managers decisions. Different strategies have good adaptability. Finally, we selected the Amazon forest to verify our carbon sequestration and strategy model, which proved the correctness and effectiveness of our model. At the same time, we also made the same predictions for other forests, which are in good agreement with the reality. Therefore our model is applicable worldwide, proving the power and robustness of this model.

However, there are still some areas for improvement in our model. For example, ecological benefits such as oxygen and water sources are not considered when calculating ecological benefits.

When calculating $MaxS$, the price of wood and tree planting costs will fluctuate to a certain extent according to time. The area of some trees does not reach when the value is determined in this paper, the growth of other trees has been affected, and the future direction of improvement will take these details into account.

7 A non-technical newspaper article

Scientific forest management plan should include cutting down trees

It is universally acknowledged that forest resources are a very important natural resource on the earth. The wood of trees in the forest can be used in human production and construction to bring us economic benefits. Growing trees can also bring ecological benefits such as evolutionary air, regulating water and soil, conserving water sources and preventing natural disasters. Therefore, it is very important to manage the forest scientifically, so as to obtain greater economic and ecological benefits while protecting the forest.

Figure 32: Amazon rainforest

With regard to scientific forest management, some people believe that we should never cut down trees. However, this idea of Never cut down trees is unscientific. The relationship between forest cutting and forest protection is not contradictory, but inseparable. A correct forest management plan should include both tree protection and cutting down. Scientific felling will not destroy the forest and can better protect the forest. Scientific felling can avoid over dense growth of trees, so as to reduce the competition between trees and be more conducive to the growth of trees. Cutting down some common tree species and planting new tree species can also increase the biodiversity of the forest, avoid too single tree species in the forest, prevent large-scale pests and bring greater ecological benefits. Moreover, cutting down some suitable trees can bring more economic benefits. After cutting down suitable trees, plant new trees in the area. The ecological benefits of old trees in a certain period of time in the future will not be greater than those newly planted after cutting, and the newly planted trees can also bring longer-term economic and ecological benefits, and then bring greater benefits.

For this kind of forest management plan that includes cutting down trees, there are some forests that have used this scientific plan and have been successful with great benefits. For example, Qiandaohu Forest: The manager of this forest has implemented a forest management plan including cutting down trees, carried out ecological surveys of the forest, investigated the tree species, soil, biological composition and other factors in the forest to scientifically determine the time of tree cutting down and Intensity, leading the surrounding residents to cut down trees, increasing the income of the residents in the area where the forest is located, not only reducing illegal logging by residents, but also benefiting the growth of trees in the forest. The ecological environment of the forest. Therefore, a reasonable forest management plan should include the cutting down trees to balance the economic and ecological benefits of forest resources, "to achieve a balanced development of forest resources and ecological benefits, so as to achieve optimal utilization of forest resources and maximum social welfare.

To improve the forest management plan, it is necessary to scientifically control the amount of deforestation, determine different deforestation intensity and time according to the main tree species, tree age and soil conditions in the forest, strengthen the protection and inspection of non deforestation, measure the progress of the forest management plan according to the forest income in a certain period, and implement corresponding adjustments when necessary. Only by implementing a scientific forest management plan can we promote forest ecological growth and effective economic benefits.

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Appendices

Here are periodic function of tropical trees we used in our model as follow.

Input matlab source:

```
% visualization of periodic function of tropic trees
coefficient = 44 / 12 *(0.45 * 4/9 + 0.2 * 5/11 + 0.25 * 0.822);
syms x;
MCO2 = (-7.01*10^-6*x.^3+0.0014*x.^2+0.024*x+0.735)*coefficient;
Sy = -7.01*10^-6*x.^3+0.0014*x.^2+0.024*x+0.735;
dy = diff(Sy,x);
dy_two = diff(dy, x);
max_x = double(solve(dy_two));
c = sym2poly(dy);
max_y = polyval(c,max_x);
c_two = sym2poly(Sy);
max_y_two = polyval(c_two,max_x);

r = -(max_y / max_y_two);
economy = 0.794;
ecology = 0.206;
p = 688;
c = 50;
C = 33;
optimal_x = 65;
change_x = 66;
independent_variable1 = 0:1:optimal_x;
temp_independent_variable1 = 0:1:optimal_x;
dependent_variable1 = [];
independent_variable2 = 0:1:change_x;
temp_independent_variable2 = 0:1:change_x;
dependent_variable2 = [];
zero = 0;

MaxS = economy *((p-c)*(-7.01*10^-6*independent_variable1.^3+0.0014*...
     independent_variable1.^2+0.024*independent_variable1+0.735).* ...
     exp(r * independent_variable1)-C)+ecology*(-7.01*10^-6*...
     independent_variable1.^3+0.0014*independent_variable1.^2+...
     0.024*independent_variable1+0.735)*coefficient;
optimal_MaxS = economy *((p-c)*(-7.01*10^-6*optimal_x.^3+0.0014*...
     optimal_x.^2+0.024*optimal_x+0.735).* exp(r * optimal_x)-C)+...
     ecology*(-7.01*10^-6*optimal_x.^3+0.0014*optimal_x.^2+...
     0.024*optimal_x+0.735)*coefficient;
MaxS_zero = economy *((p-c)*(0.735).* exp(r * zero)-C)+ecology*(0.735)*coefficient;
%clear all;
%disp(MaxS);
for i=1:10
    temp_y = MaxS + (i-1)*(optimal_MaxS-MaxS_zero);
    dependent_variable1=[dependent_variable1 temp_y];
end
for j=1:9
```

```
    temp_x = temp_independent_variable1 + optimal_x*(j);
    independent_variable1=[independent_variable1 temp_x];
end

MaxS_two = economy *((p-c)*(-7.01*10^-6*independent_variable2.^3+0.0014*...
     independent_variable2.^2+0.024*independent_variable2+0.735) .* exp(r * ...
     independent_variable2)-C)+ecology*(-7.01*10^-6*independent_variable2.^3+...
     0.0014*independent_variable2.^2+0.024*independent_variable2+0.735)*coefficient;
change_MaxS = economy *((p-c)*(-7.01*10^-6*change_x.^3+0.0014*...
     change_x.^2+0.024*change_x+0.735) .* exp(r * change_x)-C)+...
     ecology*(-7.01*10^-6*change_x.^3+0.0014*change_x.^2+0.024*...
     change_x+0.735)*coefficient;
for i=1:10
    temp_y = MaxS_two + (i-1)*(change_MaxS-MaxS_zero);
    dependent_variable2=[dependent_variable2 temp_y];
end
for j=1:9
    temp_x = temp_independent_variable2+ change_x*(j);
    independent_variable2=[independent_variable2 temp_x];
end

%disp(size(independent_variable1));
%disp(size(dependent_variable1));
%disp(size(independent_variable2));
%disp(size(dependent_variable2));
%disp(dependent_variable1);

plot(independent_variable1,dependent_variable1,independent_variable2,dependent_variable2);
legend('t = 65y','t = 66y','Location','NorthWest');
xlabel('t_i/y');
ylabel('MaxS');
title('effects of different cutting on maximum total benefit net present value');
```