AI, Headquarters and Guijie: A Principal-Agent Analysis

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1 Model

Consider a principal P (headquarters) and an agent A (local manager, "guijie"). The principal requires the agent to conduct a commercial campaign.

Campaign Success and Production Technology The campaign outcome is $y = 1$ (success) or $y = 0$ (failure). Success depends on two things:

$$y = q \cdot \theta$$

where q is the campaign quality and θ is the local condition.

The campaign quality q is exogenous with $\Pr\{q = 1\} = p$ and $\Pr\{q = 0\} = 1 - p$. The local condition θ is influenced by the agent's effort $a \in [0, 1]$ with $\Pr\{\theta = 1\} = a$ and $\Pr\{\theta = 0\} = 1 - a$. The local condition is exogenous.

Thus, the probability of campaign success is:

$$\Pr(y = 1) = \Pr(q = 1 \wedge \theta = 1) = p \cdot a$$

Agent's Cost of Effort The agent chooses effort level a at a cost $g(a)$, where the cost function satisfies standard properties: $g(0) = 0$, $g(1) = +\infty$, $g'(a) > 0$ and $g''(a) > 0$.

Contract and Payoffs The principal offers a contract specifying wage w for the campaign. The payoff of the principal is $v_P = ry - w$, where $r > 0$. The payoff of the agent is $v_A = w - g(a)$.

2 Analysis

2.1 First-Best Solution: Observable Efforts

In the first-best scenario, the principal can perfectly observe and contract directly on the agent's effort level a.

The principal maximizes expected payoff subject to the agent's participation constraint:

$$\begin{aligned} \max_{a,w} \quad & E[v_P] = r \cdot \Pr(y=1) - w = rpa - w \\ \text{s.t.} \quad & E[v_A] = w - g(a) \geq 0 \end{aligned}$$

Since the principal can directly specify a, she sets $w = g(a)$ to minimize compensation costs. The optimization simplifies to:

$$\max_a rpa - g(a)$$

Then we have the first order condition as

$$rp = g'(a)$$

The optimal effort level a^{FB} then satisfies:

$$g'(a^{FB}) = rp$$

And the principal's expected payoff is:

$$v_P^{FB} = rpa^{FB} - g(a^{FB})$$

2.2 Second-Best Solution: Unobservable Efforts

In the second-best scenario, the principal cannot observe the agent's effort a, but can observe the campaign outcome y. The principal must design an outcome-contingent contract $w(y)$ to provide incentives.

2.2.1 Agent's and Principal's Problem

The agent chooses effort a to maximize expected utility:

$$\max_a E[w - g(a)] = \Pr(y=1|a) \cdot w(1) + \Pr(y=0|a) \cdot w(0) - g(a)$$

Substituting the probabilities:

$$\max_a pa \cdot w_1 + (1 - pa) \cdot w_0 - g(a),$$

where $w_1 \equiv w(y = 1)$ and $w_0 \equiv w(y = 0)$.

Agent's first-order condition is given by

$$pw_1 - pw_0 = g'(a)$$

Then the Incentive compatibility constraint is

$$g'(a) = p(w_1 - w_0)$$

The principal chooses w_0, w_1, a to maximize expected payoff:

$$\max_{w_0,w_1,a} rpa - E[w] = rpa - [paw_1 + (1 - pa)w_0]$$

Subject to:

1. Incentive compatibility: $g'(a) = p(w_1 - w_0)$
2. Participation constraint: $E[w] - g(a) \geq 0$

2.2.2 Optimal Contract

Without loss of optimality, set $w_0 = 0$ (standard result in moral hazard models). Then:

$$w_1 = \frac{g'(a)}{p}$$

The expected wage becomes:

$$E[w] = paw_1 = pa \cdot \frac{g'(a)}{p} = ag'(a)$$

The Participation constraint, $ag'(a) - g(a) \geq 0$, is always satisfied by observing $ag'(a) - g(a) \mid_{a=0} = 0$ and this increases with a.

Principal's objective simplifies to:

$$\max_a rpa - ag'(a)$$

We have FOC as

$$rp = g'(a) + ag''(a)$$

The optimal effort a^{SB} then satisfies:

$$g'(a^{SB}) + a^{SB}g''(a^{SB}) = rp$$

Since $g''(a) > 0$ and $g'(a^{FB}) = rp$, we have:

$$a^{SB} < a^{FB}$$

Therefore, the second-best effort is lower than the first-best effort due to the moral hazard problem.