Thinking and Execution

October 9, 2025

1 Remarks and Intuitions

Remarks

- This version is the step-3 model we've discussed. Individuals differ both in thinking and execution. The impact of AI is only discussed in the homogeneous execution case. No other complications.
- I made a departure (not sure if it is a major or a mild one) from what we've discussed previously. Individuals now cannot think and execute at the same time. The benefit of doing so is that, all employees now become homogeneous executors when they share the same execution level. This simplifies the analysis of equilibrium wage a lot, though making it slightly harder to find the output of a solo worker.

Intuitions

- Much like the Lucas model on firm size and management skill, the equilibrium scenario can be summarized by two necessary conditions:
  - Condition 1 (indifference): the cutoff separating employees and employers is the one where the marginal individual is indifferent between hiring, being hired and working solo;
  - Condition 2 (market clearing): the measure of execution from employees should be equal to the measure of thinking from employers in the economy.

- All employees are paid a wage proportional to their execution level, because employment is competitive and it doesn't matter who the employees are. Employers with a higher thinking level has a larger profit from scaling up.
- In equilibrium, an employer is indifferent between hiring an employee with 2 execution level or two employees with 1 execution level. I cannot determine the matching between employers and employees.

2 Model

Individuals and skills. There is a unit mass of individuals in the economy. Each individual i has two types of skills, thinking s_i and execution e_i. We denote this individual as $z_i = (s_i, e_i)$.

For the time being, we assume $e_i = 1$ for all i, so types are ordered by thinking skill and write $z_j < z_i$ iff $s_j < s_i$. Each individual's type is perfectly observable to himself and others. The skill distribution z_i is drawn from a distribution F. We assume that F is absolutely continuous with a density function f fully supported on $[0, \infty)$. In addition, $\mathbb{E}[s] < \infty$.

Production. An individual z_i may employ a number of other individuals, who then altogether form a group of individuals which we call a *firm*. We have only one employer in a firm. An individual is a solo worker if he neither employs anyone nor is employed by anyone.

Each day, a solo worker $z_i = (s_i, e_i)$ faces a unit time budget. Thinking and execution cannot be performed simultaneously. For an individual, he can either use one day to think and produce s_i intermediate good (waiting to be executed), or use one day to execute and produce e_i final output. A solo worker then optimally assign his time to think, execute, and repeat this over time, whose optimal per day output equals $\frac{s_i e_i}{s_i + e_i}$.[1]

Under employment, we assume that an employer only thinks, and an employee only executes. [2] A firm with an employer of thinking level s and a total amount of execution e produces an output of $\min(s, e)$.

[1] The average output of a solo worker can be found in this way: optimal production needs thinking and execution to be balanced, so for a (s_i, e_i) worker, he should assign his time to think and execute in the ratio of e_i to s_i.

[2] If employees were also allowed to think within a firm, the configuration is essentially a "firm within a firm." Any arrangement with more than one thinker inside a firm can be decomposed into two firms—each with a single thinker and some executors—yielding the same results. Put differently, the second thinker can be treated as a separate manager/firm selling thinking to the first at the competitive wage. Hence, without loss of generality, we may restrict attention to configurations with exactly one thinker per firm while all employees execute.

Wages and payoffs. Individuals with relatively high thinking skills competitively offer wages to those with relatively low thinking skills to attract them to join their firm. That is, for an individual with a relatively low type, it receives wage offers from each employer, and then chooses the one that offers the highest wage.

Thinking and execution has no costs. If a task is finished, the individual receives a payoff of v, normalized to 1. An employer's payoff is the total value of the outputs net the wages paid to the employees. An employee receives the wage paid to him. Employment also incurs no costs.

3 Equilibrium

Competitive equilibrium. In a competitive equilibrium, each employer weakly profits from hiring people compared with working solo or being employed. A wage w is a competitive equilibrium wage if: (a) each employer maximizes his profit, which is his output minus wages paid to employees; (b) each employee accepting w cannot strictly do better by becoming an employer or a solo worker.

Let S_1 denote the set of thinking levels of employees, S_{solo} the set of solo workers, and S_2 the set of employers. With a slight abuse of notation, we also use these sets to denote the sets of individuals with their different roles. In addition, write $\sup S_i \equiv \overline{s}_i$ and $\inf S_i \equiv \underline{s}_i$, $i \in \{1, 2\}$.

In equilibrium, the integer constrained is ignored and each employer with a thinking level s produces fully without excessively. This can be explained by considering that one employee works for different employers at different times, though this is not explicitly modeled.

Proposition 1. *The competitive equilibrium is characterized by what follows. There always exists a unique cutoff $s^* \in [0, \infty)$ such that:*

- $S_1 = [0, s^*]$, $S_2 = [s^*, \infty)$. *Either* $S_{solo} = \emptyset$ *or* $S_{solo} = \{s^*\}$.
- *all employees are paid a common wage* $w = \frac{s^*}{s^*+1}$;
- *the profit of an employer with thinking level* $s \geq s^*$ *equals* $s - ws = \frac{s}{s^*+1}$, *which is strictly increasing in* s.

The cutoff s^* is pinned down by market clearing (the measure of execution from employees should be equal to the measure of thinking from employers),

$$\int_0^{s^*} dF(s) = \int_{s^*}^{\infty} s\, dF(s),$$

equivalently,

$$\int_{s^*}^{\infty} (s+1)\, dF(s) = 1.$$

Proof. Please see the appendix. □

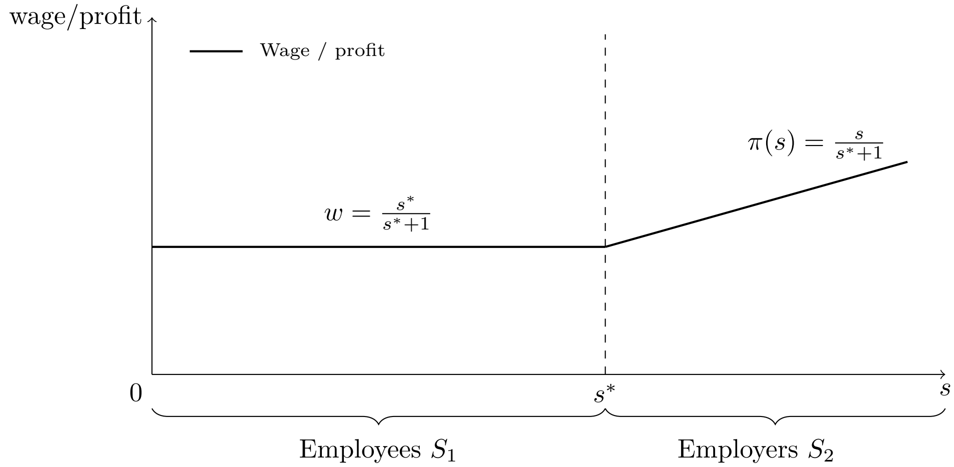

Figure 1: Equilibrium characterization with $e = 1$

4 The Impact of AI Technology

AI may have an impact on the execution skills in the economy. I separately consider two types of AI: (1) AI evenly enhances the execution level of all individuals; (2) AI serves as a pool of execution resources and employers bid for using it. The two cases are called augmentation AI and substitution AI, respectively.

4.1 Augmentation AI

Suppose there is an execution-enhancing AI which increases the execution level of each individual by a factor of $\gamma > 1$. For an individual with a skill of $(s, 1)$, his output as a solo worker is now $\frac{s\gamma}{s+\gamma} > \frac{s}{s+1}$. Our analysis in the previous section applies here. In this post-AI economy some previous employees will become employers, as the execution skill now becomes relatively abundant. Correspondingly, the cutoff skill level in the market clearing equation shifts to the left. Whether the equilibrium wage increases or decreases depends on the shape of F and the magnitude of γ. On one hand, as each individual's execution increase, the outside option value of working solo also goes higher, thus making it more costly for an employer to recruit people. On the other hand, the individual at the threshold has a lower execution skill. These results are summarized in the following proposition.

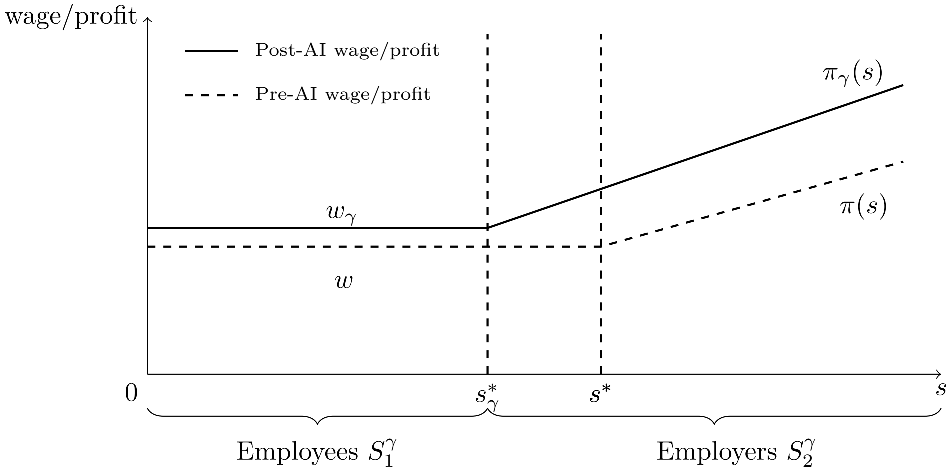

Figure 2: Equilibrium with augmentation AI, $F(s) = 1 - \frac{1}{(1+s)^2}$

Proposition 2. *With augmentation AI, the cutoff skill level separating employees and employers decreases to $s^*_\gamma < s^*$. Market clearing determines s^*_γ as*

$$\gamma \int_0^{s^*_\gamma} dF(s) = \int_{s^*_\gamma}^{\infty} s\, dF(s),$$

or equivalently,

$$\int_{s^*_\gamma}^{\infty} (s+\gamma) dF(s) = \gamma.$$

The equilibrium wage is given by $w_\gamma \equiv \frac{\gamma s^*_\gamma}{s^*_\gamma + \gamma}$. Compared with the pre-AI equilibrium, it depends on F whether the equilibrium wage increases or decreases. The profit of an employer with a thinking skill s is given by $\pi_\gamma(s) = \frac{s}{\frac{s^*_\gamma}{\gamma}+1}$.

Proof. Please see the appendix. $\square$

Observation 1. *If we assume that* $F(s) = 1 - \frac{1}{(1+s)^2}$*, which has a strictly decreasing density, then the equilibrium wage increases with* $\gamma > 1$*. The profit of an employer with a thinking skill* s *has a larger slope than the pre-AI equilibrium.*

Proof. Please see the appendix. □

4.2 Substitution AI

Now we consider the substitution AI. Suppose that AI now provides a measure of μ execution resources, which are perfectly divisible. Employers can bid for using these resources. Alternatively speaking, the price of computation is determined endogenously, and in equilibrium employer should be indifferent between hiring employees and using the AI.

Intuitively, the employers now are able to the AI to substitute for hiring employees. This gives us the new market clearing condition as

$$\int_0^{s_\mu^*} dF(s) + \mu = \int_{s_\mu^*}^{\infty} s\, dF(s),$$

This in turn has an impact and the cutoff level and wage in equilibrium, which is summarized in the following proposition.

Proposition 3. *With substitution AI, the cutoff skill level separating employees and employers decreases to* $s_\mu^* < s^*$*. Market clearing determines* s_μ^* *as*

$$\int_0^{s_\mu^*} dF(s) + \mu = \int_{s_\mu^*}^{\infty} s\, dF(s),$$

or equivalently,

$$\int_{s_\mu^*}^{\infty} (s+1) dF(s) = 1 + \mu.$$

The equilibrium wage is given by $w_\mu \equiv \frac{s_\mu^*}{s_\mu^*+1} < \frac{s^*}{s^*+1} < w = \frac{s^*}{s^*+1}$*. The profit of an employer with thinking skill* s *is given by* $\pi_\mu(s) = \frac{s}{s_\mu^*+1}$*, which has a larger slope than that in the pre-AI equilibrium.*

Proof. Please see the appendix. □

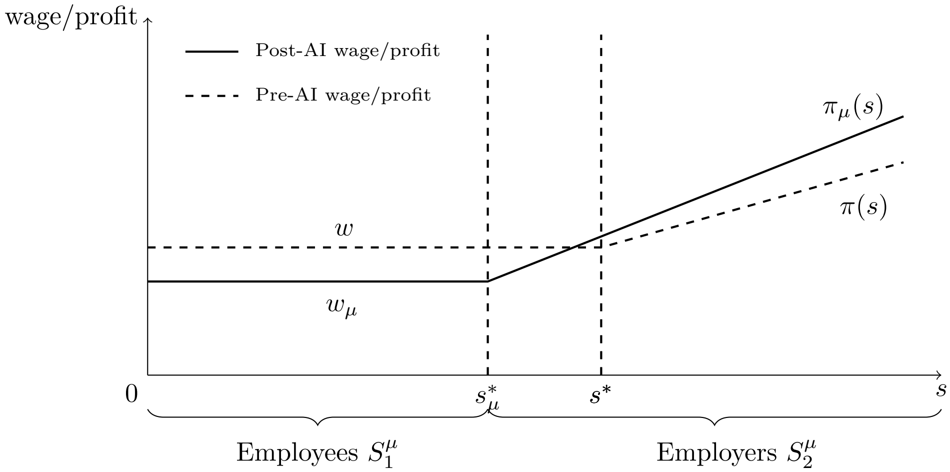

Figure 3: Equilibrium with substitution AI

5 General Distribution of Execution

We now allow both thinking and execution to be heterogeneous across individuals. Types are $z = (e, s) \in \mathbb{R}_+^2$ drawn from a joint distribution F with independent marginals F_s and F_e. We assume F_s and F_e are absolutely continuous with densities f_s and f_e, have full support on $[0, \infty)$, and finite expectations $\mathbb{E}[s] < \infty$ and $\mathbb{E}[e] < \infty$.

We summarize the results of the previous section in the following proposition 4.

Proposition 4. *The competitive equilibrium is characterized by what follows. There always exists a unique cutoff $\alpha^* \in [0, \infty)$ such that:*

- $S_1 = \{(e, s) : s \leq \alpha^* e\}$, $S_2 = \{(e, s) : s \geq \alpha^* e\}$. *Either* $S_{solo} = \emptyset$ *or* $S_{solo} = \{(e, s) : s = \alpha^* e\}$.
- *all employees are paid a wage proportional to their execution level, which is $w^* e$, where $w^* \equiv \frac{\alpha^*}{\alpha^*+1}$;*
- *the profit of an employer with thinking level s equals $s - w^* s = \frac{s}{\alpha^*+1}$, which is strictly increasing in s.*

The α^* is pinned down by market clearing (the measure of execution from employees should be equal *to the measure of thinking from employers),*

$$\iint_{S_1} e\, dF_s\, dF_e \;=\; \iint_{S_2} s\, dF_s\, dF_e.$$

Proof. See the appendix. $\square$

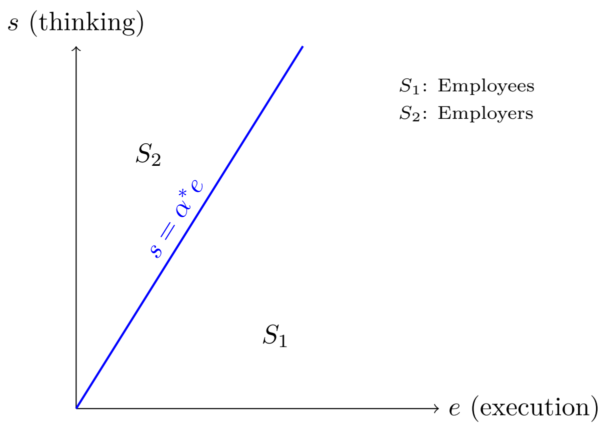

Figure 4: Equilibrium partition with heterogeneous abilities

Proposition 4 establishes that the competitive equilibrium with heterogeneous abilities exhibits a linear partition structure that mirrors the homogeneous case analyzed in Section **??**. The equilibrium partitions the population along the ray $s = \alpha^* e$, where α^* is the endogenous market-clearing ratio that equates the marginal value of thinking to execution. This linear boundary emerges from the indifference condition: individuals with abilities (e, s) are indifferent between employment roles when $s(1 - w^*) = w^* e$, which reduces to $s = \alpha^* e$ with $\alpha^* = w^*/(1 - w^*)$.

The equilibrium allocation reflects efficient sorting based on comparative advantage. Individuals with high thinking-to-execution ratios $(s/e > \alpha^*)$ optimally choose to become employers, while those with low ratios $(s/e < \alpha^*)$ become employees. The endogenous cutoff α^* adjusts to satisfy market clearing, ensuring that aggregate thinking supply from employers equals aggregate execution demand. The linear wage schedule $w^* e$ implements uniform pricing of execution across employees, consistent with firms' valuation of aggregate execution inputs rather than individual worker characteristics. This structure generates a clear occupational hierarchy: high-ability thinkers earn profits proportional to their thinking capacity, while execution specialists receive wages proportional to their execution productivity.

A Appendix

A.1 Proof of Proposition 1

Proof. We proceed with some lemmas characterizing the equilibrium properties.

Lemma 1. *$\forall s \in S_1$, $w(s) = \frac{\overline{s}_1}{\overline{s}_1+1}$.*

Proof. Let s be an employee hired by some employer $i \in S_2$. For any other employer $j \in S_2$ with $j \neq i$, the equilibrium wage offered by j cannot be strictly below $w(s)$; otherwise i could offer a slightly higher wage to poach some player from j, and strictly increase profit, a contradiction.

Let $\overline{w} \equiv \sup\{\, w(s) : s \in S_1 \,\}$. Fix any $\varepsilon > 0$. By the definition of supremum, there exists $s_\varepsilon \in S_1$ with $w(s_\varepsilon) > \overline{w} - \varepsilon$. Let the employer hiring s_ε be i. For any other employer $j \in S_2$ with $j \neq i$, the equilibrium wage offered by j cannot be strictly below $w(s_\varepsilon)$; otherwise i could offer a slightly higher wage to poach some player from j and strictly increase profit. Hence every employer offers a wage weakly at least $w(s_\varepsilon) > \overline{w} - \varepsilon$. Because ε is arbitrary, every equilibrium wage is weakly at least $\overline{w}$. However, by the definition of $\overline{w}$ as a supremum, no equilibrium wage can exceed $\overline{w}$. Therefore, for all $s \in S_1$, $w(s) = \overline{w}$. This establishes that $\forall s, s' \in S_1$, $w(s) = w(s') \equiv w$.

We next show that $w = \frac{\overline{s}_1}{\overline{s}_1+1}$. First, to show $w \leq \frac{\overline{s}_1}{\overline{s}_1+1}$, suppose instead that $w > \frac{\overline{s}_1}{\overline{s}_1+1}$. Equivalently, $\overline{s}_1 < \frac{w}{1-w}$ (note that $w < 1$ since the marginal contribution of an employee with execution 1 is always less than 1.). Pick some $s \in (\overline{s}_1, \frac{w}{1-w})$ where s is either a solo worker or an employer. Notice that $s < \frac{w}{1-w}$ is equivalent to $w > \frac{s}{s+1}$. Now consider a new wage offer $w' = \max\left\{\frac{\frac{s}{s+1}+w}{2}, \frac{s(1-w)+w}{2}\right\}$ by some employer $s' \neq s$. This new wage offer can strictly save s' some employment cost as $w - \frac{\frac{s}{s+1}+w}{2} = \frac{w-\frac{s}{s+1}}{2} > 0$ and $w - \frac{s(1-w)+w}{2} = \frac{w-s(1-w)}{2} > \frac{w-\frac{w}{1-w}(1-w)}{2} = 0$. For s, if he is a solo worker, he would strictly prefer to be employed under this new wage offer, as $w' - \frac{s}{s+1} \geq \frac{\frac{s}{s+1}+w}{2} - \frac{s}{s+1} > \frac{w-\frac{s}{s+1}}{2} > 0$; if he is an employer, he would also be strictly better off, as $w' - s(1-w) \geq \frac{s(1-w)+w}{2} - s(1-w) = \frac{w-s(1-w)}{2} > 0$. Therefore, $w \leq \frac{\overline{s}_1}{\overline{s}_1+1}$. Finally, because each employee must weakly prefer employment to working solo, we have $w \geq \frac{\overline{s}_1}{\overline{s}_1+1}$, which is the outside option value of $\overline{s}_1$ as a solo worker. To conclude, $w = \frac{\overline{s}_1}{\overline{s}_1+1}$. □

We introduce a notation $S \preceq S'$ to mean that set S is at the left-hand side of set S'. Formally, $S \preceq S'$ if $\sup S \leq \inf S'$. In addition, according to lemma 1, write the equilibrium wage as w.

Lemma 2. *$S_1 \preceq S_{solo} \preceq S_2$.*

Proof. (a) $S_1 \preceq S_{solo}$: Suppose that there exists some $s \in S_{solo}$ such that $s < \overline{s}_1$. The profit for s is $\frac{s}{s+1}$, then we have $\frac{s}{s+1} < \frac{\overline{s}_1}{\overline{s}_1+1} = w$. Then some employer hiring $\overline{s}_1$ can strictly benefit by hiring s instead, and s would strictly prefer to be employed. Therefore, $S_1 \preceq S_{solo}$.

(b) $S_{solo} \preceq S_2$: Suppose that there exists some $s \in S_{solo}$ such that $s > \underline{s}_2$. Since $\underline{s}_2$ weakly prefers to become an employer, we have $\underline{s}_2 - \underline{s}_2 \cdot w \geq \frac{\underline{s}_2}{\underline{s}_2+1}$, i.e., $1 - w \geq \frac{1}{\underline{s}_2+1}$. But it will be strictly profitable for s to become an employer, as $s(1-w) \geq \frac{s}{\underline{s}_2+1} > \frac{s}{s+1}$, a contradiction. Therefore, $S_{solo} \preceq S_2$. $\square$

Lemma 3. *$S_{solo} = \{s^*\}$ or $S_{solo} = \emptyset$.*

Proof. From lemma 2, we have $\overline{s}_1 \leq \inf S_{solo}$. Suppose that there exists an $s \in S_{solo}$ such that $s > \overline{s}_1$. Then s will find himself strictly better off by becoming an employer and offering a slightly higher wage than w to some employee. To see this, notice that $s(1-w) = s(1 - \frac{\overline{s}_1}{\overline{s}_1+1}) > s(1 - \frac{s}{s+1}) = \frac{s}{s+1}$, which is his profit as a solo worker, a contradiction. Therefore, either $S_{solo} = \{s^*\}$ or $S_{solo} = \emptyset$. $\square$

Finally, the market clearing condition determines the cutoff of employers as thinkers and employees as executors. We write s^* as the cutoff thinking level, which is also equal to $\overline{s}_1$. In our economy, the total measure of execution from employees should be equal to the total measure of thinking from employers, which is given by

$$\int_0^{s^*} dF(s) = \int_{s^*}^{\infty} s dF(s),$$

which is equivalent to

$$\int_{s^*}^{\infty} (s+1) dF(s) = 1.$$

Such an s^* indeed always exists. To see this, define $G(s) \equiv \int_0^s dF(\tilde{s}) - \int_s^{\infty} \tilde{s}\, dF(\tilde{s})$. Because F is supported on $[0, \infty)$ with a density and $\mathbb{E}[s] < \infty$, G is continuous on $[0, \infty)$, $G(0) = -\mathbb{E}[s] < 0$, and $\lim_{s \to \infty} G(s) = 1$. From $\lim_{s \to \infty} G(s) = 1$, there exists a finite $k > 0$ with $G(k) > 0$. By the intermediate value theorem on $[0, k]$, there is some $s^* \in [0, k]$ such that $G(s^*) = 0$. Moreover, because G is strictly increasing, s^* is unique.

As a result, in equilibrium, the employee set $S_1 = [0, s^*]$. There may be some solo workers at $s = s^*$, but they have a zero measure. The employer set $S_2 = [s^*, \infty)$. All employees receive a wage of $w = \frac{s^*}{s^*+1}$. The profit of an employer with a thinking level s is given by $s - ws = \frac{s}{s^*+1}$, strictly increasing with s.

Finally, what we characterized above indeed suffices to be an equilibrium. It can be easily checked that no employees will find it strictly profitable to instead be a solo worker or an employer. The employees also cannot do better as they have fully utilized their thinking resources, and they cannot hire with a strictly lower wage unilaterally as well.

$\square$

A.2 Proof of Proposition 2

Proof. For $\gamma > 1$, let s_γ^* and s^* solve

$$\gamma \int_0^{s_\gamma^*} dF(s) = \int_{s_\gamma^*}^{\infty} s\, dF(s), \qquad \int_0^{s^*} dF(s) = \int_{s^*}^{\infty} s\, dF(s).$$

Let $M(s) \equiv \int_s^\infty x\, dF(x)$, $m_1(s) \equiv M(s)/F(s)$. By construction, $m_1(s_\gamma^*) = \gamma$, $m_1(s^*) = 1$. Since $M'(s) = -sf(s)$ and $F'(s) = f(s)$, we have

$$m_1'(s) = \frac{M'(s)F(s) - M(s)F'(s)}{F(s)^2} = -\frac{f(s)}{F(s)^2}\big(sF(s) + M(s)\big) < 0,$$

so m_1 is strictly decreasing and hence $s_\gamma^* < s^*$ for $\gamma > 1$. $\square$

A.3 Proof of Observation 1

Proof. For the cumulative distribution function $F(x) = 1 - (1+x)^{-2}$ for $x \geq 0$, its probability density function is $f(x) = 2(1+x)^{-3}$, which is monotonically decreasing on $[0, \infty)$. The equation giving us the cutoff level s is:

$$\gamma \int_0^s dF(x) = \int_s^\infty x dF(x)$$

where $\gamma > 1$.

Simplify the equation we can yield equation:

$$\gamma \left[1 - (1+s)^{-2}\right] = \frac{2}{1+s} - \frac{1}{(1+s)^2}.$$

Solving for s in terms of γ, we have $s = \dfrac{1-\gamma+\sqrt{\gamma^2-\gamma+1}}{\gamma} \equiv s^*_\gamma$. Now let's derive γ/s^*_γ:

$$\frac{\gamma}{s^*_\gamma} = \frac{\gamma^2}{1-\gamma+\sqrt{\gamma^2-\gamma+1}} = \gamma(\gamma-1+\sqrt{\gamma^2-\gamma+1}),$$

which is strictly increasing in $\gamma \geq 1$. Observe that the equilibrium wage $w_\gamma \equiv \frac{\gamma s^*_\gamma}{s^*_\gamma+\gamma}$ can be expressed as $\frac{\frac{\gamma}{s^*_\gamma}}{\frac{\gamma}{s^*_\gamma}+1}$, which is also strictly increasing in $\gamma \geq 1$. As a result, the augmentation AI leads to an increase in the equilibrium wage. □

A.4 Proof of Proposition 3

Proof. Again, write $M(s) \equiv \int_s^\infty x\,dF(x)$ and $m_2(s) \equiv M(s) - F(s)$. By construction, $m_2(s^*_\mu) = \mu > 0$, $m_2(s^*) = 0$. Since $M'(s) = -sf(s)$ and $F'(s) = f(s)$, we have

$$m_2'(s) = -(s+1)f(s) < 0,$$

so m_2 is strictly decreasing and hence $s^*_\mu < s^*$ for $\mu > 0$.

The equilibrium wage is given by $w_\mu \equiv \frac{s^*_\mu}{s^*_\mu+1} < \frac{s^*}{s^*+1} = w$ as the substitution does not affect the outside option value of employees. As a result, the substitution AI leads to a decrease in the equilibrium wage. □

A.5 Proof of Proposition 4

Proof. We establish the competitive equilibrium through three sequential steps.

Step 1: Linear wage structure from arbitrage-free pricing.

Define the per-unit wage $p(e) \equiv w(e)/e$ for $e > 0$. We claim that $p(e)$ must be constant across all execution levels in equilibrium.

Suppose, for contradiction, that there exist $e_1, e_2 > 0$ with $p(e_1) < p(e_2)$. Consider any firm operating at the execution margin, meaning its aggregate execution equals its thinking level, so the marginal value of execution is exactly 1.

This firm can execute the following arbitrage: replace one employee with execution e_2 by exactly e_2/e_1 employees each with execution e_1. This substitution maintains total execution since $(e_2/e_1)\cdot$

$e_1 = e_2$, reduces wage costs from $w(e_2) = e_2 p(e_2)$ to $(e_2/e_1)w(e_1) = e_2 p(e_1)$, and preserves output since total execution is unchanged.

Since $p(e_1) < p(e_2)$, this arbitrage yields strictly positive profit, contradicting competitive equilibrium. Therefore, $p(e)$ must be constant on $\text{supp}(F_e)$. Let $p(e) \equiv w \in [0, 1)$ denote this constant per-unit wage.

By absolute continuity of F_e, we have $w(e) = w \cdot e$ for all e almost everywhere. The constraint $w < 1$ follows because the marginal product of execution cannot exceed 1 in equilibrium.

Step 2: Role partition and boundary characterization.

Under the linear wage schedule $w(e) = w \cdot e$, individual payoffs are:

$$\pi_E(s) = s(1 - w) \quad \text{(employer)} \tag{1}$$

$$\pi_W(e) = w \cdot e \quad \text{(employee)} \tag{2}$$

$$\pi_S(e, s) = \frac{se}{s + e} \quad \text{(solo)} \tag{3}$$

To determine optimal role choice, we compare all three options pairwise. Define $\alpha \equiv \frac{w}{1-w} > 0$.

Employee vs. Solo: An individual prefers employment over solo work if and only if:

$$\begin{aligned}
\pi_W(e) \geq \pi_S(e, s) &\iff w \cdot e \geq \frac{se}{s + e} &(4)\\
&\iff w(s + e) \geq s &(5)\\
&\iff ws + we \geq s &(6)\\
&\iff we \geq s(1 - w) &(7)\\
&\iff s \leq \frac{w}{1 - w} \cdot e = \alpha e &(8)
\end{aligned}$$

Employer vs. Solo: An individual prefers being an employer over solo work if and only if:

$$\pi_E(s) \geq \pi_S(e,s) \iff s(1-w) \geq \frac{se}{s+e} \tag{9}$$

$$\iff (1-w)(s+e) \geq e \tag{10}$$

$$\iff s + e - ws - we \geq e \tag{11}$$

$$\iff s(1-w) \geq we \tag{12}$$

$$\iff s \geq \frac{w}{1-w} \cdot e = \alpha e \tag{13}$$

Employee vs. Employer: An individual prefers employment over being an employer if and only if:

$$\pi_W(e) \geq \pi_E(s) \iff w \cdot e \geq s(1-w) \tag{14}$$

$$\iff s \leq \alpha e \tag{15}$$

These comparisons yield a consistent partition: individuals with $s < \alpha e$ become employees, those with $s > \alpha e$ become employers, and those with $s = \alpha e$ are indifferent between all three roles (solo work yields the same payoff).

Therefore, the optimal role partition is:

$$S_2(\alpha) = \{(e,s) : s \geq \alpha e\} \quad \text{(employers)} \tag{16}$$

$$S_1(\alpha) = \{(e,s) : s \leq \alpha e\} \quad \text{(employees)} \tag{17}$$

Solo workers lie precisely on the boundary $\{(e,s) : s = \alpha e\}$. Since F is absolutely continuous, this one-dimensional boundary has Lebesgue measure zero, implying S_{solo} has measure zero.

Step 3: Existence and uniqueness of equilibrium.

Market clearing requires that total thinking supplied by employers equals total execution demanded by employees:

$$\iint_{S_2(\alpha)} s\, dF_s\, dF_e = \iint_{S_1(\alpha)} e\, dF_s\, dF_e \tag{18}$$

Using independence of s and e, this simplifies to:

$$H(\alpha) \equiv \int_0^\infty \left[\int_{\alpha e}^\infty s\, dF_s(s) - e \int_0^{\alpha e} dF_s(s) \right] dF_e(e) = 0 \tag{19}$$

To establish existence, we show that $H(\alpha)$ crosses zero exactly once. At $\alpha = 0$, we have $H(0) = \mathbb{E}[s] > 0$ since everyone becomes an employer and total thinking exceeds total execution. As $\alpha \to \infty$, we have $H(\alpha) \to -\mathbb{E}[e] < 0$ since everyone becomes an employee and total execution exceeds total thinking.

For uniqueness, since $H(\alpha)$ is continuous and strictly decreasing in α (higher α means fewer employers and more employees), the intermediate value theorem guarantees a unique $\alpha^* > 0$ such that $H(\alpha^*) = 0$.

The equilibrium wage is $w^* = \frac{\alpha^*}{1+\alpha^*} \in (0, 1)$, the role sets are $S_1(\alpha^*)$ and $S_2(\alpha^*)$, and payoffs follow from Step 2. Under w^*, no individual can profitably deviate: employers optimally hire execution up to $e = s$ and earn $\pi_E(s) = s(1 - w^*)$, employees receive $w^* e$, and solo payoffs are weakly dominated except on the boundary. □