# AI, Headquarters and Guijie

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## 1 Model

**Information.** The state is $\theta \in \{0, 1\}$ with prior $\Pr(\theta = 1) = p \in (0, 1)$. The principal observes a binary signal $s$ with $\Pr(s = 1 \mid \theta = 1) = 1$ and $\Pr(s = 1 \mid \theta = 0) = q \in (0, 1)$; the agent observes $\theta$ perfectly.

**Timing and contracts.** (i) The principal observes $s$. If $s = 1$, she initiates a campaign and offers an outcome-contingent wage scheme $w = (w_0, w_1)$; if $s = 0$, no campaign occurs. (ii) The agent observes $\theta$ and $w$ and chooses effort $a \in [0, 1)$. (iii) Outcome $y \in \{0, 1\}$ realizes with $\Pr(y = 1 \mid \theta, a) = \theta a$. Effort is unobservable and non-verifiable; only the outcome is contractible. Without loss of generality set $w_0 = 0$. Both parties are risk neutral.

**Technology and payoffs.** The agent's cost is $g(a)$ with $g(0) = 0$, $\lim_{a \to 1} g(a) = \infty$, $g'(0) = 0$, $g' > 0$, $g'' > 0$. The principal's gross value from success is $r_p > 0$. With $w_0 = 0$, the expected payoffs are

$$\mathbb{E}[U_P \mid \theta, a] = \theta a \,(r_p - w_1), \qquad \mathbb{E}[U_A \mid \theta, a] = \theta a \, w_1 - g(a).$$

After $s = 1$, letting $\pi \equiv \Pr(\theta = 1 \mid s = 1) = \frac{p}{p + (1-p)q}$,

$$\mathbb{E}[U_P \mid s = 1, a] = \pi a \,(r_p - w_1), \qquad \mathbb{E}[U_A \mid s = 1, a] = \pi a \, w_1 - g(a).$$

## 2 Results

**First-best.** With contractible effort and full information, a planner chooses $a(\theta)$ to maximize $r_p \, \theta \, a - g(a)$. Using $g'(0) = 0$ and $g'' > 0$, the unique solution is $a^{FB}(0) = 0$ and, for $\theta = 1$, a strictly positive interior effort $a^{FB} \in (0,1)$ defined by $g'(a^{FB}) = r_p$ (since $g'(0) = 0 < r_p$). After $s = 1$, expected surplus is $\pi\,[r_p a^{FB} - g(a^{FB})]$.

**Second-best.** With hidden effort and outcome-only contracts, the agent's best response to $w_1$ is $a^*(0) = 0$ and, for $\theta = 1$, $a^*(1) = a$ satisfying $g'(a) = w_1$. The principal equivalently chooses $a$ via $w_1 = g'(a)$ to maximize $\pi\,[r_p a - g'(a)\,a]$. The unique optimum $a^{SB}$ is characterized by

$$r_p = g'(a^{SB}) + a^{SB} g''(a^{SB}), \qquad w_1^{SB} = g'(a^{SB}).$$

Note that $a^{SB} < a^{FB}$ since $g'' > 0$.