Sine of the Students

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1 Model

Students. There are two students x and y. Both have to attend two courses (denoted by $j \in \{1, 2\}$). Each student chooses an intensity $r_i \in [0, 1]$ and an angle $\varphi_i \in [0, \frac{\pi}{2}]$, but due to energy constraint, the efforts are parameterized by the unit quarter-circle

$$(e_{i1}, e_{i2}) \;=\; r_i\,(\cos\varphi_i,\, \sin\varphi_i), \qquad e_{i1}^2 + e_{i2}^2 \le 1.$$

The outcome in the courses are normalized to equal effort, $y_{ij} = e_{ij}$. The student x only cares about how far he can reach in the $x-$axis, and the student y only cares about how far he can reach in the $y-$axis. However, they try their best to avoid a policy penalty (defined below) is triggered.

Without loss of generality, we can assume the budget binds in the cases of interest ($r_i = 1$).

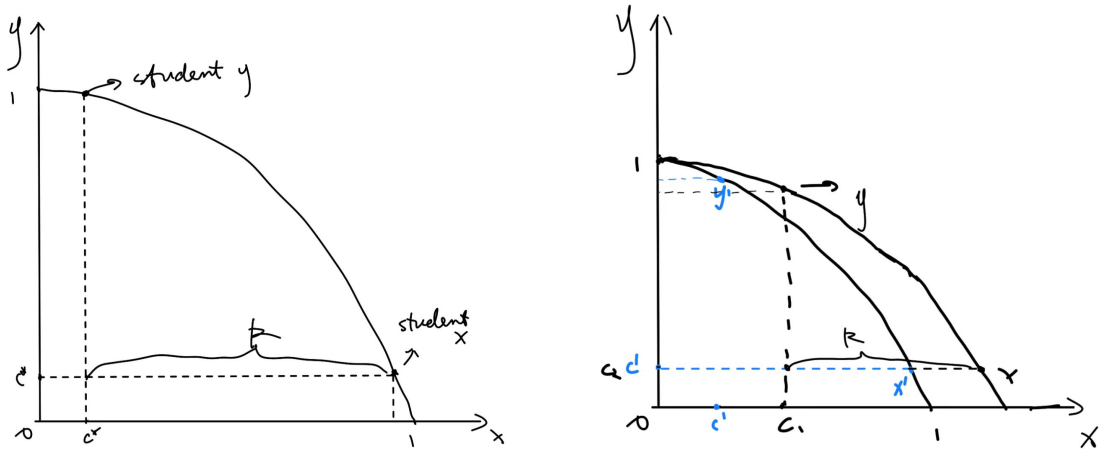
Lecturers. There are two lecturers, one for each course $j \in \{1, 2\}$. The outcome gap across students is $\Delta_j \equiv \max_{i\in\{x,y\}} y_{ij} - \min_{i\in\{x,y\}} y_{ij}$. The lecturers will be very upset if she sees a large gap $k \in (0, 1)$ between the highest and lowest outcomes. Formally, the lecturer's utility is 0 if $\Delta_j < k$ and $-\infty$ otherwise.

Director. A director cares about lecturers' welfare, whereas she uses a minimal policy to minimize the distortion to students' efforts. To ensure that the outcome gap is not too large, she requires each student to reach at least a certain outcome in each course. Given $c_j \ge 0$, if any student i attains $y_{ij} < c_j$ in any course j, then that student's utility is $-\infty$.

2 Analysis

Pre-AI age. The students' preferred outcomes are $(1, 0)$ for x and $(0, 1)$ for y, but this will greatly upset the lecturers. Because the property of our setup, it can be found a minimal policy that satisfies the lecturers' requirement, illustrated below.

Post-AI age. Suppose that AI greatly but only improves the productivity in course 1 (efforts in the $x-$ axis are more efficient). The blue colored denotes the efforts and outcomes prior to AI. If the director only raises the minimum requirement c_1 on the $x-$axis, AI will lead to better results for the student x, but to the detriment of the student y. The student y has no choice but to allocate fewer efforts in course 2 which he likes. (Implication: the director can raise c_2 instead.)



(a) pre-AI

(b) post-AI

Figure 1: Policies and Outcomes