## AI, Headquarters and Guijie: A Principal-Agent Analysis

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## 1 Model

Consider a principal P (headquarters) and an agent A (local manager, "guijie"). HQ deploys an AI system that both detects manipulation and provides guidance; the dashboard can be strategically gamed, and AI has dual-use effects.

**Technology and KPIs with AI** The campaign outcome is  $y \in \{0, 1\}$  (success or failure). The agent allocates effort across two activities

$$e = (e_c, e_q) \in \mathbb{R}^2_+,$$

where  $e_c$  raises true commercial impact ("core") and  $e_g$  is gaming/manipulation that inflates the dashboard without creating real value.

AI has two design knobs chosen by HQ: guidance  $g \ge 0$  and detection intensity  $\ell \ge 0$ . Guidance directly raises the productivity of core and (leakily) of gaming; detection directly raises the sensitivity of manipulation detection. Formally:

- Success arrives with probability  $f(e_c; g)$  with  $\partial f/\partial e_c > 0$ ,  $\partial^2 f/\partial e_c^2 < 0$ , and  $\partial^2 f/(\partial e_c \partial g) > 0$  (AI-complementarity).
- The measured KPI is  $m(e_c, e_g; g) = \delta(g) e_c + \psi(g) e_g$ , where  $\delta'(g) > 0$  and  $\psi'(g) = \sigma \delta'(g)$  with leakage parameter  $\sigma \in [0, 1]$  (dual-use guidance).
- Manipulation is detected with probability  $d(e_g, \ell, g)$ , increasing in  $e_g$  and  $\ell$ , convex in  $e_g$ , and with detection synergy  $\partial^2 d/(\partial e_g \partial g) > 0$ .

The agent bears a separable convex cost  $G(e) = G_c(e_c) + G_g(e_g)$  with  $G'_i > 0$ ,  $G''_i > 0$ .

**Dashboard design and timing** HQ commits to a linear dashboard and AI design before effort choices. Let  $b \geq 0$  be the incentive slope (bonus sensitivity),  $\lambda \in [0,1]$  the weight on true success versus the measured KPI,  $g \geq 0$  the guidance intensity, and  $\ell \geq 0$  the detection intensity, with convex costs K(g) and  $C(\ell)$ .

Payment rule: the realized wage is

$$w = w_0 + b(\lambda y + (1 - \lambda) m(e)) - \mathbb{1}\{\text{detected}\} P,$$

where P > 0 is a clawback/penalty applied if gaming is detected. In expectations,

$$\mathbb{E}[w] = w_0 + b\left(\lambda f(e_c; g) + (1 - \lambda) \left(\delta(g)e_c + \psi(g)e_g\right)\right) - P d(e_g, \ell, g).$$

Timing: (i) HQ chooses  $(b, \lambda, \ell, g)$ ; (ii) the agent chooses e; (iii) y is realized, m is measured, detection occurs with probability  $d(e_g, \ell, g)$ , and payments are made.

Payoffs HQ's expected payoff is

$$v_P = r f(e_c; g) - \mathbb{E}[w] - C(\ell) - K(g),$$

where r > 0 is the revenue from a success. The agent's expected payoff is

$$v_A = \mathbb{E}[w] - G(e).$$

**Strategic interaction** For an interior agent optimum  $e^*(b, \lambda, \ell, g)$ , first-order conditions satisfy

$$G'_{c}(e_{c}) = b(\lambda \partial f/\partial e_{c}(e_{c}; g) + (1 - \lambda) \delta(g)),$$
  

$$G'_{q}(e_{g}) = b(1 - \lambda) \psi(g) - P \partial_{e_{q}} d(e_{g}, \ell, g).$$

These equations highlight: (i) a direct AI effect via  $\ell$  that raises detection and lowers  $e_g$ ; (ii) an indirect AI effect via g that raises core productivity and dashboard sensitivity to core (through  $\delta(g)$ ) but—with leakage  $\sigma$ —also raises returns to gaming, while detection synergy in g steepens  $\partial_{e_g} d$ .

## 2 Equilibrium and Comparative Statics with Dual-Use AI

Given  $(b, \lambda, \ell, g)$ , the agent chooses  $e^*(b, \lambda, \ell, g)$  that solves the conditions above. Anticipating this, HQ chooses  $(b, \lambda, \ell, g)$  to maximize  $v_P$  subject to incentive compatibility.

**Proposition 1** (Dual-use AI: direct vs indirect effects). Suppose f is increasing and concave with positive AI complementarity, d is increasing in  $e_g$  and  $\ell$  and convex in  $e_g$  with detection synergy in g, K and C are convex, and G is separable and convex. Then any interior equilibrium exhibits:

- 1. (Direct effect)  $e_g$  is decreasing in  $\ell$  and P, with a corner  $e_g^* = 0$  whenever  $P \partial_{e_g} d \ge b(1-\lambda) \psi(g)$ .
- 2. (Indirect guidance)  $e_c$  is increasing in g (AI complements core). With leakage  $\sigma > 0$ ,  $e_g$  is locally increasing in g via  $\psi'(g)$  but decreasing in g via detection synergy if  $\partial^2 d/(\partial e_g \partial g)$  is large; thus  $e_g$  can be non-monotone in g.
- 3. (Design) If leakage is moderate (small  $\sigma$ ) or detection synergy is strong, the optimal  $g^*$  crowds back from gaming:  $\partial e_q^*/\partial g < 0$  at  $g^*$  while  $\partial e_c^*/\partial g > 0$ .
- 4. Optimal detection  $\ell^*$  is (weakly) increasing in the gaming productivity slope  $\psi'(g)$  and decreasing in synergy strength.

Sketch. Follows from the agent FOCs and the envelope theorem: g shifts both marginal benefits and the detection slope; signs depend on leakage  $\sigma$  and synergy  $\partial^2 d/(\partial e_q \partial g)$ .

Example (linear-quadratic). Let  $G_c(e_c) = \frac{1}{2}c_ce_c^2$ ,  $G_g(e_g) = \frac{1}{2}c_ge_g^2$ ,  $f(e_c;g) = (1 + \eta g)e_c$ ,  $\delta(g) = \delta_0 + \alpha g$ ,  $\psi(g) = \psi_0 + \sigma \alpha g$ , and  $d(e_g, \ell, g) = \ell(1 + \kappa g)e_g$ . Then

$$e_c^* = \frac{b\left(\lambda(1+\eta g) + (1-\lambda)\,\delta(g)\right)}{c_c}, \quad e_g^* = \max\left\{0, \, \frac{b(1-\lambda)\,\psi(g) - P\,\ell(1+\kappa g)}{c_g}\right\}.$$

Hence  $\partial e_g^*/\partial g < 0$  when  $P \ell \kappa > b(1-\lambda) \sigma \alpha$  (guidance-induced detection dominates dual-use leakage), generating a U-shaped gaming response in g.