ECON 815 - Firm Dynamics

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Notes for the General Equilibrium Firm Problem

General Equilibrium Analysis

- In many instances, partial equilibrium analysis of policy changes is not enough.
- E.g., suppose we lower taxes on corporate income
  - Looking at the firm's FOC for it's choice of investment:

$$\underbrace{p^k(r+\delta) + (1+r)c_1(k',k) + c_2(k'',k')}_{\text{Cost of investment}} = \underbrace{(1-\tau^c)E_{z'|z}\pi_2(z',k')}_{\text{Benefit of investment}} \quad (1)$$

  - We can see that as τ_c increases, the benefits of investment go down $\implies$ less investment
  - We know the sign. But what about the quantitative effect?
  - If the corporate income tax goes from 35% to 15%, how much does corporate investment increase?
- Partial equilibrium:
  - $\downarrow$ corporate tax $\rightarrow$ $\uparrow$ investment
- General equilibrium:
  - $\downarrow$ corporate tax $\rightarrow$ $\uparrow$ invest $\rightarrow$ $\uparrow$ labor demand $\rightarrow$ $\uparrow$ wages $\rightarrow$ $\downarrow$ profits per unit of output $\rightarrow$ $\downarrow$ investment
- Want full effect of policy after all price changed - the general equilibrium effect.
- E.g., Gourio and Miao [2010]
  - Use investment model similar to what we've worked with
  - Consider 2003 Bush tax cuts - lowered rates on dividends and long term capital gains to 15%
  - PE effect on investment = 6× the GE effect

External Financing

- Thus far, we haven't talked about corporate finance.
- That is, how are these investments financed?
- Recall our definition of per period firm cash flow, $e(z,k,k')$
  - If operating profits exceeded the amount of investment plus adjustment costs, cash flows were positive: $e(z,k,k) = \pi(z,k) - I - c(k,k') > 0$
  - If operating investment plus adjustment costs exceed operating profits, cash flows were negative: $e(z,k,k) = \pi(z,k) - I - c(k,k') < 0$
- In our PE model (without costly external finance), we could just let this be.
- Now, as we move to a GE model, we need to be more explicit about what is going on when cash flows are positive and negative because we will be modeling the other sectors of the economy, where these cash flows go.

- In particular, let's assume that households are the owners of these firms.
- Firm's distribute positive cash flows (i.e., per period profits net of investment costs) back to households through dividend distributions, d
- Further, if the firm cannot finance it's optimal investment through it's earnings (and thus cash flows are negative), it raises the remaining financing by issuing new shares, s which are purchased by households.
  - This is a bit of a simplification. In reality, the firm might repurchase shares rather than issue dividends. Or it might retain those earnings.
  - Since this model does not feature taxes or costly external finance, we'll abstract from these options.
- With our explicit notation for dividends and new share issues, we have:

$$e(z,k,k') = \pi(z,k) + s - d - I - c(k,k') \tag{2}$$

- The firm's Bellman equation can be written as:

$$V(z,k) = \max_{k',s,d} \underbrace{\pi(z,k) + s - d - I - c(k',k)}_{e(z,k,k')} + \frac{1}{1+r'} E_{z'|z} V(z',k') \tag{3}$$

  subject to $d \geq 0$ and $s \geq 0$
  - We can use Equation (2) to solve for d, leaving two choice variables, k' and s
  - Then an interior solution has the following two necessary conditions:
    1. FOC k' $\implies$ $1 + c_1(k',k) = \frac{1}{1+r'} E_{z'|z} V_2(z',k')$
    2. FOC s $\implies$ $1 - 1 = 0$
  - The second condition means that at an interior solution, d and s are indeterminant. Any $s - d$ that solves Equation (2) with the optimal k' works.
    * This is an example of the Modigliani-Miller Theorem on the irrelevance of corporate finance when there are no taxes or costs to external finance (see Modigliani and Miller [1958])
    * In this case, firm is indifferent between using internal or external funds to finance investment.
  - If the firm is able to finance k' with internal funds and has resources left over, those are distributed via dividends and we have the corner solution, $d > 0, s = 0$
    * Here, $d = \pi(z,k) - I - c(k',k)$
  - If the firm needs external funds to finance k', the firm issues new equity and we have the corner solution, $s > 0, d = 0$
    * Here, $s = -1 \times (\pi(z,k) - I - c(k',k))$

Adding a household sector

- In our PE model of firms, firms demand capital at a given price. At that price, all the capital the firms demand is supplied and all the labor firms demand is supplied.
- All other prices remain constant: r, w, p, p^k
- In general equilibrium, prices will not remain constant.
- To determine how prices change, we need to model both demand (e.g., firms' demand for capital and labor) and supply
- Thus we'll add households who supply labor, supply savings, and demand consumption goods.

- Households:
  - Demand consumption goods, supply labor, purchase a risk free bond, and own and trade equity shares in all firms to solve:

$$\max_{\{C_t,L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{4}$$

  - With per period utility:

$$u(C, L) = \ln(c) - \frac{hL^2}{2} \tag{5}$$

  - and budget constraint:

$$C_t + \int P_t \theta_{t+1} d\Gamma_t + B_{t+1} = \int \left[d_t + P_t^0\right] \theta_t d\Gamma_t + (1 + r_t)B_t + w_t L_t \tag{6}$$

    - θ_t denotes the shares held by the household
    - Γ_t is the distribution of firms at time t
    - P_t is the price of the firm at time t
    - P_t^0 is the time t price of the shares outstanding at time $t-1$ (i.e. $P_t = P_t^0 + s_t$)
    - B_t are bond holdings

- There are four choice variables in any period t: C_t, L_t, B_{t+1}, and θ_{t+1}.
- Using the budget constraint to substitute in for C_t leaves 3 choice variables.
- The three necessary conditions (for an interior solution) are:
  - For bond purchases:

$$u_1(C_t, L_t) = \beta(1 + r_{t+1})u_1(C_{t+1}, L_{t+1}) \tag{7}$$

  - For labor supply:

$$-u_2(C_t, L_t) = w_t u_1(C_t, L_t) \tag{8}$$

  - For equity holdings in each firm:

$$P_t u_1(C_t, L_t) = \beta E_t \left[d_{t+1} + P_{t+1}^0\right] u_1(C_{t+1}, L_{t+1}) \tag{9}$$

- We are going to be analyzing the model in the steady state.
  - i.e, where $C_t = C_{t+1} = \bar{C}$, $B_t = B_{t+1} = \bar{B}$, etc.
  - In the steady-state the distribution of firms will be *stationary*
    - i.e., $\Gamma_t = \Gamma_{t+1} = \Gamma^*$
- Our 3 conditions become:
  - For bond purchases:

$$\begin{aligned} u_1(\bar{C}, \bar{L}) &= \beta(1 + \bar{r})u_1(\bar{C}, \bar{L}) \\ \implies \frac{1}{\beta} - 1 &= \bar{r} \end{aligned} \tag{10}$$

  - For labor supply:

$$\begin{aligned} -u_2(\bar{C}, \bar{L}) &= \bar{w} u_1(\bar{C}, \bar{L}) \\ \implies \frac{-u_2(\bar{C}, \bar{L})}{u_1(\bar{C}, \bar{L})} &= \bar{w} \end{aligned} \tag{11}$$

- For equity holdings:

$$\begin{aligned} P_t u_1(\bar{C}, \bar{L}) &= \beta E_t \left[d_{t+1} + P^0_{t+1} \right] u_1(\bar{C}, \bar{L}) \\ \implies P_t &= \beta E_t \left[d_{t+1} + P^0_t \right] \\ \implies \frac{1}{\beta} &= \frac{E_t \left[d_{t+1} + P^0_{t+1} \right]}{P_t} \\ \implies 1 + \bar{r} &= \frac{E_t \left[d_{t+1} + P^0_{t+1} \right]}{P_t} \end{aligned} \tag{12}$$

  - This is a standard asset pricing eq'm condition - if the household is to hold more than one risk free financial asset, they must yield the same rate of return
  - Here, with no aggregate uncertainty, neither the bond and the portfolio of equity in all firms have any risk, thus they must both yield the same rate of return in eq'm

- In equilibrium, we know the $\bar{B} = 0$ since the risk free bonds are in zero net supply.
- We also know that all shares in firms must be held by the household, so $\theta_t = 1 \ \forall t$.
- Equation (10) implies the risk free interest rate.
- Equation (12) says that the expected rate of return on equity is equal to the risk free rate.
- With this, we can write the household's equilibrium budget constraint as:

$$\bar{C} = \int \left[d(z, k, k') - s(z, k, k') \right] d\Gamma^* + \bar{w}\bar{L} \tag{13}$$

Definition of the Stationary Recursive Competitive Equilibrium

- A Stationary Recursive Competitive Equilibrium (SRCE) consists of a wage rate w^*, a distribution of firms $\Gamma^*(z, k; w^*)$, and functions $V(z, k; w^*)$, $l(z, k; w^*)$, $k'(z, k; w^*)$, $d(z, k; w^*)$, and $s(z, k; w^*)$ such that:
  - Given w^* (and r^*), $V(z, k; w^*)$, $l(z, k; w^*)$, $k'(z, k; w^*)$, $d(z, k; w^*)$, and $s(z, k; w^*)$ solve the firm's problem.
  - The stationary distribution is such that $\Gamma^*(z, k; w^*) = \mathcal{H}^*(\Gamma^*(z, k; w^*)$
  - Given w^*, the household maximizes utility subject to its budget constraint.
  - The labor market clears: $L = \int l(z, k; w^*)\Gamma^*(dz, dk; w^*)$
  - The goods market clears: $Y(\Gamma^*; w^*) = C(\Gamma^*; w^*) + I(\Gamma^*; w^*) + \Upsilon(\Gamma^*; w^*)$
  - Asset markets clear: $B = 0$, $\theta = 1$

Solving for the Stationary General Equilibrium

- We've made the output/consumption/investment good the numeraire and normalized it's price to 1.
- We are able to pin down $\bar{r}$ from the household's FOC for bond purchases: $\bar{r} = (\frac{1}{\beta} - 1)$
- The remaining price in the model is the wage rate, $\bar{w}$
- This is what we'll solve for in our GE solution algorithm.
- The GE solution algorithm will thus work as follows:
  1. Guess $\bar{w}_i$

2. Given $\bar{w}_i$, solve the firms' problems to obtain:
   - Earnings: $e(z, k, h(z, k; \bar{w}_i); \bar{w}_i)$
   - Labor demand: $l^d(z, k; \bar{w}_i)$
   - Investment demand: $h(z, k; \bar{w}_i) - (1 - \delta)k$
3. Use the policy function, $h(z, k)$, found from the solution to the firms' problems to solve for the stationary distribution of firms, $\Gamma^*(z, k; \bar{w}_i)$
   - See this Jupyter Notebook on stationary distributions.
4. Use the stationary distribution to find aggregate labor and investment demand, output, and adjustment costs:
   - $\bar{L}^d = \int l^d(z, k; \bar{w}_i) d\Gamma^*(z, k, ; \bar{w}_i)$
   - $\bar{I} = \int [h(z, k; \bar{w}_i) - (1 - \delta)k] \, d\Gamma^*(z, k, ; \bar{w}_i)$
   - $\bar{\Psi} = \int c(h(z, k), k) d\Gamma^*(z, k, ; \bar{w}_i)$
   - $\bar{Y} = \int z k^{\alpha_k} \left(l^d(z, k; \bar{w}_i)\right)^{\alpha_l} d\Gamma^*(z, k, ; \bar{w}_i)$
5. Use the aggregate resource constraint, to solve for household consumption:
   - Aggregate resource constraint: $\bar{Y} = \bar{C} + \bar{I} + \bar{\Psi}$
   - $\implies \bar{C} = \bar{Y} - \bar{I} - \bar{\Psi}$
6. Use aggregate consumption in the household's FOC for it's choice of labor supply (Equation (11)) together with $\bar{w}_i$ to find $\bar{L}^s$
7. Check the labor market clearing condition: $\bar{L}^s = \bar{L}^d$
   - If this condition is met to the specified tolerance, stop.
   - If not, then update the guess at the equilibrium wage to $\bar{w}_{i+1}$ and go back to step 2.

- Note that to update the wage rate at each iteration through the algorithm, you can use any root finding algorithm.
  - A bisection method is the most robust, but slowest
  - Newton-type method are faster, but less robust to the initial guess of $\bar{w}$
  - Tradeoffs!
  - Also, note that the wage rate is bounded below since it's strictly positive.

References

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