Problem Set #8

ECON 815, Prof. Jason DeBacker
Due Monday, December 4, 5:00 p.m.

This problem set requires you to solve for the stationary general equilibrium of the economy with dynamic firms and households.

Assume that the firms face stochastic productivity, z, and quadratic adjustment costs. Productivity shocks will follow an AR(1) process:

$$\ln(z_{t+1}) = \rho \ln(z_t) + (1-\rho)\mu + u_t, \tag{1}$$

where $u_t \sim N(0, \sigma_z)$ Further, assume that prices for output and new capital can both be normalized to one: $p = p^k = 1$. The parameterization of the firm's problem you should use is summarized in the table below.

Table 1: Parameterization

Parameter	Description	Value
α_k	Capital's share of output	0.297
α_l	Labor's share of output	0.650
δ	Depreciation rate	0.154
ψ	Coefficient on quadratic adjustment costs	1.080
w	Wage rate	0.700
r	Interest rate	0.040
σ_z	Std. deviation of disturbances to z	0.213
μ	Mean of $ln(z)$ process	0.000
ρ	Persistence of z process	0.7605
`sizez`	Number of grid points in z space	9

The household sector will be represented by a single, infinitely-lived household wit the follwoing objective function:

$$\max_{\{C_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{2}$$

Assume the per period utility function is separable and given by:

$$u(C, L) = \ln(c) - \frac{hL^2}{2}, \tag{3}$$

where h is a scaling parameter on the disutility of labor. Assume that the household parameters are given by $\beta = 0.96$ and $h = 6.616$. The household's per-period budget constraint is thus:

$$C_t + \int P_t \theta_{t+1} d\Gamma_t + B_{t+1} = \int \left[d_t + P_t^0\right] \theta_t d\Gamma_t + (1 + r_t)B_t + w_t L_t, \tag{4}$$

where B_{t+1} are household claims on risk free bonds that earn interest r_{t+1}. These bonds are in zero net supply.

In general equilibrium, all markets will clear. That is $B_t = 0$ (since zero net supply of these bonds), labor demand from the firms equals labor supply from the representative household, and goods demand (for consumption by households and investment by firms) equals the supply of goods (i.e., total output produced). You can uses these market clearing conditions to help determine the factor prices in general equilibrium (in the steady-state, these will be $\bar{r}$ and $\bar{w}$). Remember that Walras' Law means that you only need to solve for market clearing in two of the three markets since if those clear, the third will as well.

You need to do the following:

1. Solve the model and report the equilibrium wage rate
2. Plot the stationary distribution and describe it.
3. Plot the policy function for k'. How does it vary with productivity?

You will submit your problem set by pushing the `*.py` and `*.pdf` files to your GitHub repository that you created from forking the repository for this class. You will put this and all other problem sets in the path `/CompEcon_Fall2017/ProblemSets/ProblemSet8`.

Tips:

1. Pay attention to grid sizes. What kind of range do want for the values of k? Of z?
2. You'll need to approximate the continuous AR(1) process with something discrete.
3. You can solve for $\bar{r}$ quite easily from the household's necessary conditions.
4. Your general equilibrium will be found by some fixed point process. Be sure that your grid space for capital is not binding as you iterate over the factor price(s) to find a fixed point.
5. You'll want to modularize your code because functions will be called repeatedly through the solution algorithm.
6. `Numba` will be your friend here.
7. Recall the national accounting identify: $Y = C + I$ (where Y is output, C is consumption, and I invesmtent).
8. To find the zero for the market clearing condition(s) you'll want to use a root finder from `Scipy` or write your own algorithm.