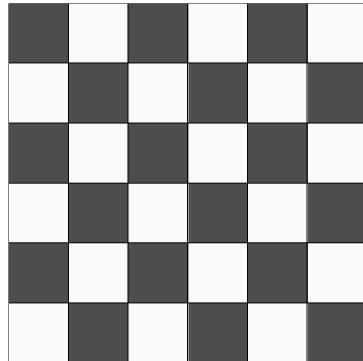


# Spatial autocorrelation

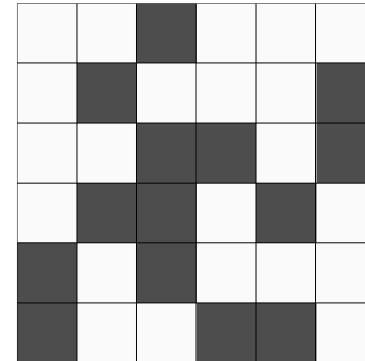
Everything is related to everything else, but near things are more related than distant things.

—Tobler, W. R.

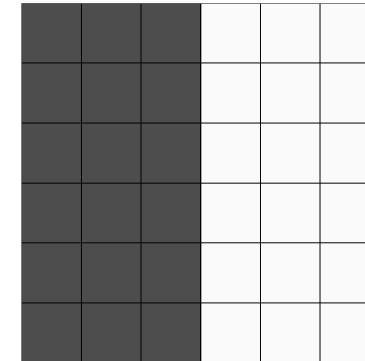
**Negative spatial autocorrelation**



**No spatial autocorrelation**



**Positive spatial autocorrelation**



# Global Moran's $I$

$$I = \frac{n \sum_i \sum_j w_{ij} (Y_i - \bar{Y})(Y_j - \bar{Y})}{(\sum_{i \neq j} w_{ij}) \sum_i (Y_i - \bar{Y})^2}$$

When we use a row-standardized spatial weight matrix,  $W^{st}$ , and denote  $z_i \equiv (Y_i - \bar{Y})$ :

$$I = \frac{\sum_{i=1}^n z_i \sum_{j=1}^n w_{ij}^{st} z_j}{\sum_{i=1}^n z_i^2}$$

# Global Moran's $I$ interpretation

- Moran's  $I$  values usually range from  $-1$  to  $1$ .
- Values significantly above the expected value indicate positive spatial autocorrelation or clustering.
- Values significantly below the expected value indicate negative spatial autocorrelation or dispersion.

# Local Moran's $I$

$$I_i = \frac{n(Y_i - \bar{Y})}{\sum_j (Y_j - \bar{Y})^2} \sum_j w_{ij}(Y_j - \bar{Y})$$

Noting that

$$I = \frac{1}{\sum_{i \neq j} w_{ij}} \sum_i I_i$$

When we use a row-standardized spatial weight matrix,  $W^{st}$ , we have

$$I = \frac{\sum_i I_i}{n} = \bar{I}_i.$$

# Multiple testing

The  $\alpha$  in hypothesis testing is the probability of making a type I error (false positive).

In our context, it means concluding that a truly random spatial pattern is non-random, thus rejecting the null hypothesis.

When running multiple tests simultaneously using overlapping information, we are more likely to encounter these types of errors purely due to chance.

Therefore, we must adjust  $p$ -values (downward) to ensure that we maintain the desired  $\alpha$  level.

One way to do this is Bonferroni correction:  $\alpha_{adjusted} = \frac{\alpha}{m}$ , where  $m$  is the number of simultaneous tests.

# plotly package

The `plotly` package in R is a binding to the open-source JavaScript graphing library, `plotly.js`. It excels at creating interactive web graphics, including maps and figures.

However, it does **not** support knitting the `Rmd` file into a PDF since PDFs are static files, unlike HTML outputs.

To include `plotly` graphics in a knitted PDF, take a screenshot of the output and insert the screenshot as a static image in the PDF file:

```
1  ```{r, eval=FALSE}
2  Some R codes that you do not want to evaluate during knitting.
3  ```
4
5  
```

# Functions in R recap

Functions in R use the following syntax. Note that any thing wrapped in <...> indicates that it is just a name and can be called anything you like.

```
1  ````{r}
2  <function_name> <- function(<parameter1>, <parameter2>) {
3      Some R codes to execute based on the inputs
4      and at last return an output
5  }
6
7  function_name(<object1>,
8                 parameter2 = <object2>)
9  ````
```

You only need to define functions once in your code. Then, you can call it by its name.

# Activities for today

- We will work on the following chapter from the textbook:
  - Chapter 24: Activity 11: Area Data III
  - Chapter 26: Activity 12: Area Data IV
- The hard deadline is Tuesday, March 3.

# Reference

- <https://www.paulamoraga.com/book-spatial/spatial-autocorrelation.html>