Semicircle problem with simulation

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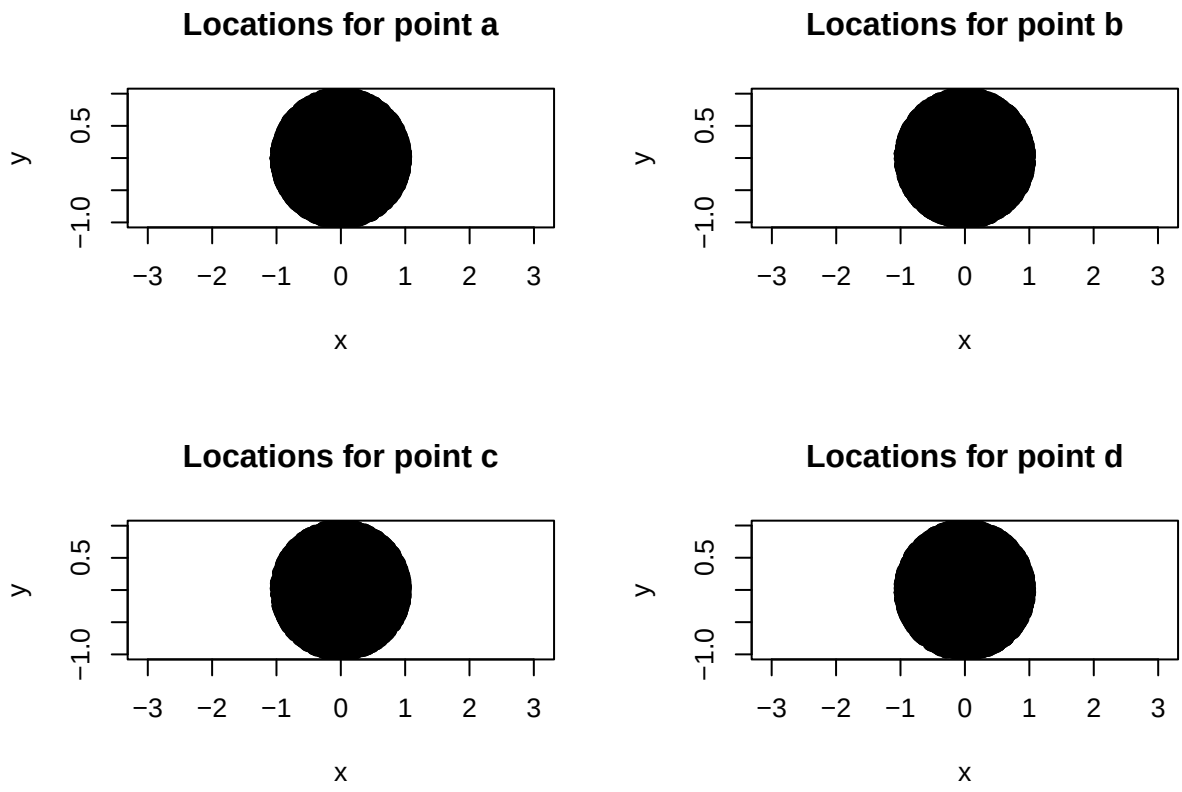
If we have four ducks swimming in a circle and their locations are random and independent, what is the probability that all four of the ducks are in the same half of the circle?

We first generate the duck locations

```
# create a function to genereate random location in a circle
generate_point_in_circle <- function(n, radius){
  output <- data.frame()
  while (nrow(output) < n) {
    iteration <- runif(2, min = -radius, max = radius)
    if (iteration[1]^2 + iteration[2]^2 <= radius^2) {
      output <- rbind(output, iteration)
    }
  }
  colnames(output) <- c("x", "y")
  return(output)
}

# generate 10000 samples of set of four points
n <- 10000
a <- generate_point_in_circle(n, 1)
b <- generate_point_in_circle(n, 1)
c <- generate_point_in_circle(n, 1)
d <- generate_point_in_circle(n, 1)

par(mfrow = c(2,2))
plot(a[, "x"], a[, "y"], asp=1,
     main = "Locations for point a", xlab = "x", ylab = "y")
plot(b[, "x"], b[, "y"], asp=1,
     main = "Locations for point b", xlab = "x", ylab = "y")
plot(c[, "x"], c[, "y"], asp=1,
     main = "Locations for point c", xlab = "x", ylab = "y")
plot(d[, "x"], d[, "y"], asp=1,
     main = "Locations for point d", xlab = "x", ylab = "y")
```



Then we check for each set of points whether they fall within the same semicircle or not.

```
# create variable to record the number of TRUE happens
count_ture <- 0
for (i in 1:n) {
  cond <- 0
  # check whether b, c, d falls within the semicircle created by a and origin (0,0)
  if (b[i,"y"]*a[i,"x"]-b[i,"x"]*a[i,"y"] > 0 &
      c[i,"y"]*a[i,"x"]-c[i,"x"]*a[i,"y"] > 0 &
      d[i,"y"]*a[i,"x"]-d[i,"x"]*a[i,"y"] > 0) {
    cond <- cond + 1
  }
  if (b[i,"y"]*a[i,"x"]-b[i,"x"]*a[i,"y"] < 0 &
      c[i,"y"]*a[i,"x"]-c[i,"x"]*a[i,"y"] < 0 &
      d[i,"y"]*a[i,"x"]-d[i,"x"]*a[i,"y"] < 0) {
    cond <- cond + 1
  }
  # check whether a, c, d falls within the semicircle created by b and origin (0,0)
  if (a[i,"y"]*b[i,"x"]-a[i,"x"]*b[i,"y"] > 0 &
      c[i,"y"]*b[i,"x"]-c[i,"x"]*b[i,"y"] > 0 &
      d[i,"y"]*b[i,"x"]-d[i,"x"]*b[i,"y"] > 0) {
    cond <- cond + 1
  }
  if (a[i,"y"]*b[i,"x"]-a[i,"x"]*b[i,"y"] < 0 &
      c[i,"y"]*b[i,"x"]-c[i,"x"]*b[i,"y"] < 0 &
      d[i,"y"]*b[i,"x"]-d[i,"x"]*b[i,"y"] < 0) {
    cond <- cond + 1
```

```
  }
  # check whether a, b, d falls within the semicircle created by c and origin (0,0)
  if (a[i,"y"]*c[i,"x"]-a[i,"x"]*c[i,"y"] > 0 &
      b[i,"y"]*c[i,"x"]-b[i,"x"]*c[i,"y"] > 0 &
      d[i,"y"]*c[i,"x"]-d[i,"x"]*c[i,"y"] > 0) {
    cond <- cond + 1
  }
  if (a[i,"y"]*c[i,"x"]-a[i,"x"]*c[i,"y"] < 0 &
      b[i,"y"]*c[i,"x"]-b[i,"x"]*c[i,"y"] < 0 &
      d[i,"y"]*c[i,"x"]-d[i,"x"]*c[i,"y"] < 0) {
    cond <- cond + 1
  }
  # check whether a, b, c falls within the semicircle created by d and origin (0,0)
  if (a[i,"y"]*d[i,"x"]-a[i,"x"]*d[i,"y"] > 0 &
      b[i,"y"]*d[i,"x"]-b[i,"x"]*d[i,"y"] > 0 &
      c[i,"y"]*d[i,"x"]-c[i,"x"]*d[i,"y"] > 0) {
    cond <- cond + 1
  }
  if (a[i,"y"]*d[i,"x"]-a[i,"x"]*d[i,"y"] < 0 &
      b[i,"y"]*d[i,"x"]-b[i,"x"]*d[i,"y"] < 0 &
      c[i,"y"]*d[i,"x"]-c[i,"x"]*d[i,"y"] < 0) {
    cond <- cond + 1
  }
  # if any of the 4 conditions is met, then the four points are in the same semicircle
  if (cond >= 1) {
    count_ture <- count_ture + 1
  }
}

# calculate the proportion
cat("Proportion of four ducks fall within the same semicircle \nwith", n,
    "simulations equals =", count_ture*100/n, "%")
```

```
## Proportion of four ducks fall within the same semicircle
## with 10000 simulations equals = 50.11 %
```

Based on the simulation we can see that the proportion is 50.11% which is around 50%.