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The big picture

There are two important questions you may ask while learning about differential calculus:

- Why is finding derivatives useful?
- How do you find derivatives?

These questions were partially addressed in [subtopics 5.1](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25542/) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25542/), [5.2](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25549/) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25549/), [5.3](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/) and [5.4](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25560/) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25560/). In this and the following subtopics you will build on these foundations. In this subtopic, you will learn about some tools that will help you to find the derivatives of the types of functions you encountered in [topics 2](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-24414/) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-24414/) and [3](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25407/) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25407/), including combinations of these. In the following subtopics, you will apply these tools to solving problems.

The process of finding derivatives is based on a set of rules. The derivatives of complex functions can be built up using basic derivatives. The formula booklet contains these basic derivatives and the rules for combining them; you will learn how to apply these in this subtopic.

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In some sense, this process is similar to any complicated system. If you understand the building blocks and the connections, you understand the whole system. The mechanical watch in the following video is a good example.

How a Mechanical Watch Works | Explained in 5 Minutes



Video 1. How a Mechanical Watch Works | Explained in 5 Minutes.

More information for video 1

The title "HOW A MECHANICAL WATCH WORKS" is displayed in over a blurred image of a partially disassembled mechanical watch. The watch's intricate components, including black plates, silver and gold gears, and small ruby-colored jewels, are visible. In the background, watchmaking tools such as tweezers and a movement holder can be seen, setting the stage for a detailed exploration of the watch's inner workings.

The camera zooms in on the watch movement, focusing on the gear train. The interplay of gears is highlighted, with the words "MADE SWISS" etched on a prominent silver gear. The focus then shifts to the balance wheel, and the text "EXPLAINED IN 5 MINUTES" appears on the right side of the frame. The screen transitions to black, introducing the first chapter with the text: "CHAPTER 1: THE POWERSOURCE."

A quartz watch movement is shown, secured in a holder. A gloved hand uses tweezers to hold a small silver button cell battery, labeled "BATTERY." Below the movement, text explains that this is the power source of a quartz watch. The scene transitions to a coiled mainspring, gleaming in its relaxed state, held by tweezers. The text "MAINSRING" identifies it as the power source of a mechanical watch. The mainspring is wound into a cylindrical mainspring barrel by gloved hands using a specialized tool. Social media handles for "/reddeadrestoration" on Facebook and "@reddeadrestoration" on Instagram appear in the upper left corner, along with a note about a Patreon page. The camera focuses on the barrel as the mainspring is inserted and secured with a flat metal disc. A transparent barrel is used to demonstrate the process, offering a clear view of the mainspring inside. The text "MAINSRING BARREL" identifies the component.

The screen fades to black, introducing "CHAPTER 2: THE GEAR TRAIN." Tweezers place gold-colored gears onto the mainplate, each labeled with text: "ESCAPE WHEEL," "SECOND WHEEL," and "THIRD WHEEL." A black plate is



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secured over the gear train with screws, and the “CENTER WHEEL” is positioned and labeled. Additional plates and components are added, with small ruby jewels identified as “BEARINGS.” A toothpick gently touches the mainspring barrel, causing it and the gear train to move. Text explains that the movement of the mainspring barrel drives the gear train.

The screen transitions to “CHAPTER 3: WINDING THE MAINSPRING.” A winding stem is inserted into the movement, and the watch is gently wound, engaging the gears. The “CROWN WHEEL” is labeled as it interacts with the winding stem. The “RATCHET WHEEL” is shown rotating during the winding process, while the “CLICK SPRING” and “CLICK LEVER” are added with precision. The movement is wound further, and text explains that winding the mainspring drives the gear train. A gold watch face with hour and minute hands is displayed, but the hands move rapidly, demonstrating uncontrolled motion. Text notes that the gears and hands are still turning without regulation.

The screen fades to black, introducing “CHAPTER 4: THE ESCAPEMENT.” A small, intricate part with ruby jewels, the pallet fork, is held by tweezers and labeled. The pallet fork is carefully inserted into the movement and secured. A balance cock is added and screwed into place. Text explains that the mainspring is no longer driving the gear train, allowing it to save energy. The pallet fork is manually moved with tweezers, demonstrating its interaction with the escape wheel. The watch face is shown again, with the second hand advancing in distinct steps, illustrating the controlled release of energy.

The screen transitions to “CHAPTER 5: THE BALANCE.” The balance wheel and hairspring assembly are shown spinning. Key components are labeled: “IMPACT JEWEL,” “BALANCE WHEEL,” and “HAIRSPRING.” The balance wheel assembly is inserted into the movement and secured. A close-up reveals the impact jewel resting between the forks of the pallet fork. Slow-motion footage highlights the interaction between the escape wheel, pallet fork, and balance wheel. The balance wheel oscillates, driving the gear train and regulating the watch's motion. The gold watch face is shown again, with the second hand ticking smoothly. Text notes that the balance wheel rotates at 21,600 vibrations per hour, or six steps per second.

The fully assembled watch movement is displayed, and the screen fades to black before showing the movement again. The text “SUBSCRIBE” appears with an arrow pointing down, followed by a message thanking patrons and subscribers, with special mentions for Diego Macias and Jim Janson. The balance wheel continues to oscillate, and the gears turn, bringing the demonstration to a close.



Concept

Don't be put off by what seems to be a complicated list of rules. The best way to learn to apply them is with a lot of repetitive practice, rather than simply trying to memorise them. This will help you to see a common pattern in seemingly different examples. Look for the inner structure of a complicated looking function, and look for the relationship between the simpler functions that are used as the building blocks.



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Theory of Knowledge

Differentiation rules are used to find derivatives (rates of change) in mathematics. This is done so we have knowledge about a function and can thus make predictions.

The fact that maths has predictive validity is a strength of mathematics as a way of knowing; however, not all areas of knowledge have high degrees of predictive validity. History, for example, has zero predictive validity and religious knowledge systems have very little predictive validity.

Knowledge Question: To what extent does predictive validity demonstrate quality of knowledge?

5. Calculus / 5.6 Differentiation rules

Power rule revisited

In [subtopic 5.3 \(/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/\)](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/) you encountered the sum rule and the constant factor rule for differentiation. These rules will be used, without quotation, from now on. Here is a combination of these rules to refresh your memory.

✓ Important

If $h(x) = af(x) \pm bg(x)$, then the derivative is $h'(x) = af'(x) \pm bg'(x)$.

You have also seen the formula for the derivatives of power functions.

✓ Important

- The derivative of $f(x) = c$ is $f'(x) = 0$.
- The derivative of $f(x) = x$ is $f'(x) = 1$.
- The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$.



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In [subtopic 5.3 \(/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/\)](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/)

you only used this formula for integer (positive or negative) exponents, but the formula is true for any real number exponent. This is what you will practise now.

Example 1



Find the derivatives of $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$.

| Steps | Explanation |
|--|--|
| $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ | You can use the formula if $f(x)$ is in the form x^n . |
| $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1}$ $= \frac{1}{2}x^{-\frac{1}{2}}$ $= \frac{1}{2\sqrt{x}}$ | The formula can be used for $n = \frac{1}{2}$. |
| $g(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ | You can use the formula if $g(x)$ is in the form x^n . |
| $g'(x) = -\frac{1}{2}x^{-1/2-1}$ $= -\frac{1}{2}x^{-3/2}$ $= -\frac{1}{2\sqrt{x^3}}$ $= -\frac{1}{2x\sqrt{x}}$ | The formula can be used for $n = -\frac{1}{2}$. |

Example 2



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Find the derivative of $y = \frac{3x^2 - 7}{\sqrt[3]{x}}$.



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| Steps | Explanation |
|---|---|
| $y = \frac{3x^2 - 7}{\sqrt[3]{x}}$ $= \frac{3x^2}{\sqrt[3]{x}} - \frac{7}{\sqrt[3]{x}}$ $= 3x^{2-\frac{1}{3}} - 7x^{-\frac{1}{3}}$ $= 3x^{\frac{5}{3}} - 7x^{-\frac{1}{3}}$ | To use the formula, the expression needs to be rewritten as a combination of expressions of the form ax^n . |
| $\frac{dy}{dx} = 3 \times \frac{5}{3} x^{\frac{5}{3}-1} - 7 \times -\frac{1}{3} x^{-\frac{1}{3}-1}$ $= 5x^{\frac{2}{3}} + \frac{7}{3} x^{-\frac{4}{3}}$ $= 5\sqrt[3]{x^2} + \frac{7}{3\sqrt[3]{x^4}}$ | |

Example 3



Find the derivatives of the following expressions.

a) $y = 5\sqrt[3]{x^4}$

b) $y = \frac{3}{7\sqrt{x}}$

c) $y = \sqrt{x}(3x^2 - 9)$

d) $y = \frac{2x^3}{\sqrt[4]{x}} - \frac{\sqrt[4]{x}}{2x^3}$

e) $y = \frac{(\sqrt{x} - 3)^2}{\sqrt{x}}$



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| | Steps | Explanation |
|----|---|--|
| a) | $y = 5\sqrt[3]{x^4} = 5x^{\frac{4}{3}}$ | $\begin{aligned}\frac{dy}{dx} &= 5 \times \frac{4}{3} x^{\frac{4}{3}-1} \\ &= \frac{20}{3} x^{\frac{1}{3}} \\ &= \frac{20\sqrt[3]{x}}{3}\end{aligned}$ |
| b) | $y = \frac{3}{7\sqrt{x}} = \frac{3}{7} x^{-\frac{1}{2}}$ | $\begin{aligned}\frac{dy}{dx} &= \frac{3}{7} \times -\frac{1}{2} x^{-\frac{1}{2}-1} \\ &= -\frac{3}{14} x^{-\frac{3}{2}} \\ &= -\frac{3}{14x\sqrt{x}}\end{aligned}$ |
| c) | $\begin{aligned}y &= \sqrt{x} (3x^2 - 9) \\ &= 3x^{\frac{1}{2}+2} - 9x^{\frac{1}{2}} \\ &= 3x^{\frac{5}{2}} - 9x^{\frac{1}{2}}\end{aligned}$ | $\begin{aligned}\frac{dy}{dx} &= 3 \times \frac{5}{2} x^{\frac{5}{2}-1} - 9 \times \frac{1}{2} x^{\frac{1}{2}-1} \\ &= \frac{15}{2} x^{\frac{3}{2}} - \frac{9}{2} x^{-\frac{1}{2}} \\ &= \frac{15x\sqrt{x}}{2} - \frac{9}{2\sqrt{x}}\end{aligned}$ |
| d) | $\begin{aligned}y &= \frac{2x^3}{\sqrt[4]{x}} - \frac{\sqrt[4]{x}}{2x^3} \\ &= 2x^{3-\frac{1}{4}} - \frac{1}{2} x^{\frac{1}{4}-3} \\ &= 2x^{\frac{11}{4}} - \frac{1}{2} x^{-\frac{11}{4}}\end{aligned}$ | $\begin{aligned}\frac{dy}{dx} &= 2 \times \frac{11}{4} x^{\frac{11}{4}-1} - \frac{1}{2} \times \frac{-11}{4} x^{\frac{-11}{4}-1} \\ &= \frac{11}{2} x^{\frac{7}{4}} + \frac{11}{8} x^{\frac{-15}{4}} \\ &= \frac{11x\sqrt[4]{x^3}}{2} + \frac{11}{8x^3\sqrt[4]{x^3}}\end{aligned}$ |
| e) | $\begin{aligned}y &= \frac{(\sqrt{x} - 3)^2}{\sqrt{x}} \\ &= \frac{(x - 6\sqrt{x} + 9)}{\sqrt{x}} \\ &= x^{\frac{1}{2}} - 6 + 9x^{-\frac{1}{2}}\end{aligned}$ | $\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} x^{\frac{1}{2}-1} - 0 + 9 \times \frac{-1}{2} x^{-\frac{1}{2}-1} \\ &= \frac{1}{2} x^{-\frac{1}{2}} - \frac{9}{2} x^{-\frac{3}{2}} \\ &= \frac{1}{2\sqrt{x}} - \frac{9}{2x\sqrt{x}}\end{aligned}$ |

3 section questions ▾

5. Calculus / 5.6 Differentiation rules



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Derivative of the exponential function



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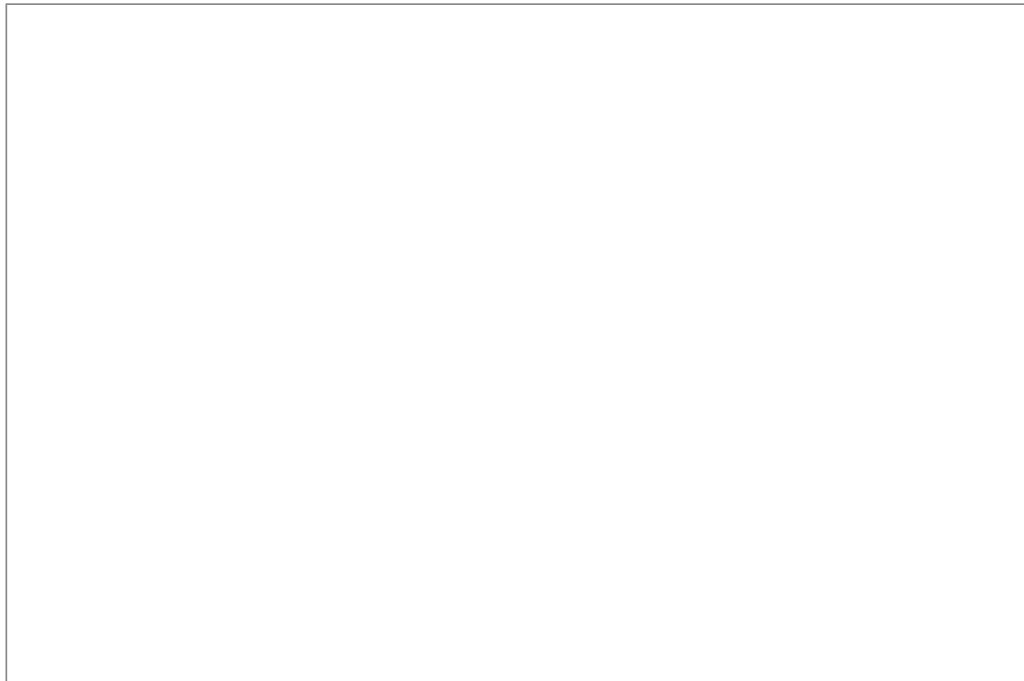
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The next goal is to learn about derivatives of functions other than power functions. This section concentrates on the exponential function which you learned about in subtopic 2.9 (</study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27900/>).



Activity

On the applet below you can see the graph of $y = a^x$ and the derivative for different values of the base a . Move the slider to change the base. What do you notice?



Interactive 1. Derivative of Exponential Functions.

More information for interactive 1

This interactive allows users to practice finding the derivatives of functions of the form for $y = k^{ax+b}$ randomly generated parameters a , b , and k . To differentiate such functions, users will first apply the chain rule, where $u = ax + b$ and then differentiate accordingly:

On applying chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$.

$$\frac{dy}{dx} = a k^{(ax+b)}$$

For example, find the derivative of $y = (4x + 3)^2$

Here, $\frac{dy}{dx} = 2(4x + 3)4$



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$$\frac{dy}{dx} = 8(4x + 3)$$

Users can generate new questions by clicking on **New Question**, allowing them to practice different levels of problems based on the same concept. They can also verify their solutions by selecting the **Show Answers** option. By using this interactive, users will develop a strong grasp of the chain rule in differentiation and improve their problem-solving skills.

From the applet you may have noticed that the derivative of $y = a^x$ is $\frac{dy}{dx} = ka^x$ for some value of k , which depends on the base a . The applet also shows you that for a specific value of the base the derivative is the same as the function. To find the value of this special base, you can type

$$\text{solve } y' = y$$

in the [WolframAlpha](http://www.wolframalpha.com/)  (<http://www.wolframalpha.com/>) command line to search for function with the property that the derivative is the same as the function itself.

In the answer WolframAlpha gives you can see that the functions with this property are constant multiples of the exponential function $y = e^x$, where the base is Euler's number, $e \approx 2.71828$. You learned about this number in [subtopic 1.5 \(/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25055/\)](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25055/) and [subtopic 2.9 \(/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27900/\)](/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27900/).

The formula booklet contains this information.

✓ Important

If $f(x) = e^x$, then the derivative is $f'(x) = e^x$.

You can investigate a generalisation of this claim with the following applet.



Activity

The applet below gives the derivatives of functions of the form $y = ke^{ax+b}$ for randomly generated parameters a , b and k .

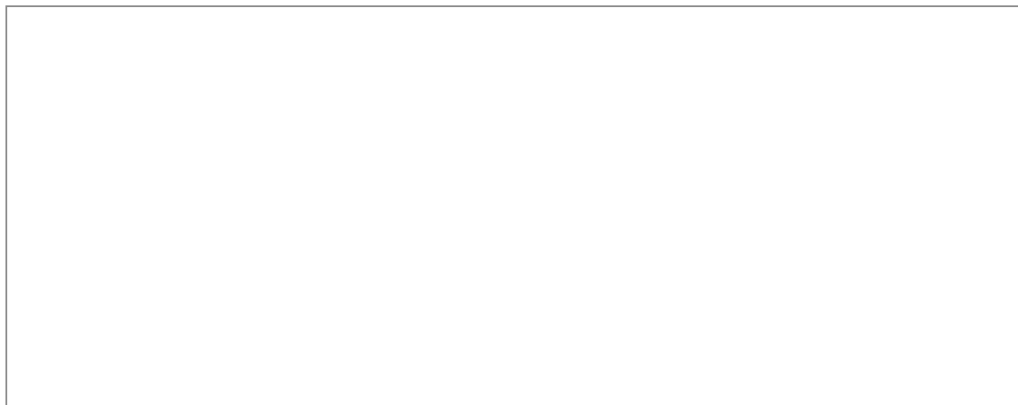


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Keep generating examples until you can predict the derivative without looking at the answer.



Interactive 2. Finding the Derivative of Exponential Function.

More information for interactive 2

This interactive allows users to practice differentiating functions of the form $y = c \cdot e^{(ax+b)}$ for randomly generated parameters a , b , and c . Users can click on [Click here for new questions](#) to generate different functions of this type and apply differentiation rules to find their derivatives. To verify their solutions, they can use the [Show answers](#) option.

To differentiate $y = c \cdot e^{(ax+b)}$, users will apply the chain rule:

$$\frac{d}{dx}(c \cdot e^{(ax+b)}) = cae^{(ax+b)}$$

For example, for the function $y = 7 \cdot e^{(-3x+9)}$ the derivative is:

$$\frac{dy}{dx} = 7 \cdot (-3)e^{(ax+b)} = -21e^{(ax+b)}$$

Through this interactive, users will develop a strong understanding of differentiating exponential functions and applying the chain rule effectively.

Did you notice the following relationship?

✓ Important

If $f(x) = ke^{ax+b}$, then the derivative is $f'(x) = kae^{ax+b}$.

In the following examples you can see how this formula is used to find derivatives of certain types of functions.



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Example 1



Find the derivative of $f(x) = \frac{2e^{3x}}{5e^4}$

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Assign

| Steps | Explanation |
|--|--|
| $f(x) = \frac{2e^{3x}}{5e^4} = \frac{2}{5}e^{3x-4}$ | You can apply the formula if the expression is in the form ke^{ax+b} . |
| $f'(x) = \frac{2}{5} \times 3e^{3x-4} = \frac{6}{5}e^{3x-4}$ | |

Example 2



Find the derivative of $y = \frac{7}{6e^{2x}}$.

| Steps | Explanation |
|---|--|
| $y = \frac{7}{6e^{2x}} = \frac{7}{6}e^{-2x}$ | You can apply the formula if the expression is in the form ke^{ax+b} . |
| $\frac{dy}{dx} = \frac{7}{6} \times (-2)e^{-2x}$ $= -\frac{7}{3}e^{-2x}$ $= -\frac{7}{3e^{2x}}$ | |



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5. Calculus / 5.6 Differentiation rules

Derivative of the natural logarithm function

In this section, you will see how to differentiate functions of the form $y = k \ln(ax + b)$, where k , a and b are real parameters.



Activity

To start, take a look at the following argument.

$$\begin{aligned} y &= \ln x \\ x &= e^y \\ \frac{dx}{dy} &= e^y \\ \frac{dx}{dy} &= x \end{aligned}$$

- Do you agree that each line is the consequence of the line before it?
- Did you notice that in the differentiation step x is the dependent and y is the independent variable?
- Suggest a formula for $\frac{dy}{dx}$, the derivative of $y = \ln x$.

The activity above gives an informal justification of the formula for the derivative of $y = \ln x$, which you can find in the formula booklet.



Important

If $f(x) = \ln x$, then the derivative is $f'(x) = \frac{1}{x}$.

You can investigate a generalisation of this claim with the following applet.



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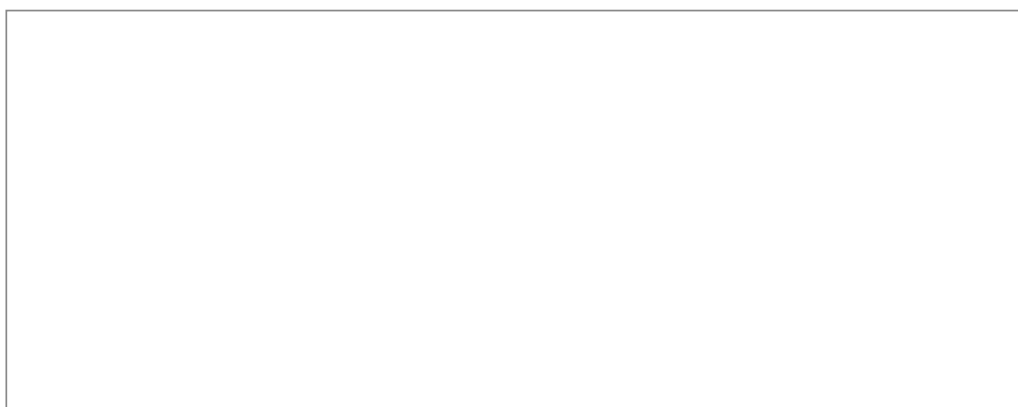
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Activity

The applet below gives the derivatives of functions of the form $y = k \ln(ax + b)$ for randomly generated parameters a , b and k .

Keep generating examples until you can predict the derivative without looking at the answer.



Interactive 1. Derivatives of Natural Logarithm Functions.

More information for interactive 1

This interactive allows the user to find the derivatives of natural logarithm functions of the form $y = k \ln(ax + b)$ for randomly generated parameters a , b and k . Users can generate new differentiation problems by clicking on 'Click for a new question'. After attempting the solution, they can click "Show Answer" to reveal the correct answer. To find the derivative users will first apply the formula $\frac{d}{dx} \ln(x) = \frac{1}{x}$

For example, find the derivative of $y = [11 \ln(5x - 5)]$.

$$\Rightarrow \frac{dy}{dx} = \frac{11}{(5x-5)} \frac{d(5x-5)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{55}{5x-5}$$

The users will develop a better understanding of the concept of natural logarithm. This technique will also help them in finding derivatives of complicated functions. Eventually, helping users to understand how it has applications in various fields like physics, economics and finance.

Did you notice the following relationship?



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**Important**

If $f(x) = k \ln(ax + b)$, then the derivative is $f'(x) = \frac{ka}{ax + b}$.

In the following examples you can see how this formula is used to find derivatives of certain types of functions.

Example 1

Find the derivative of $f(x) = 2 \ln(3x - 4)^5$.

| Steps | Explanation |
|---|---|
| $\begin{aligned} f(x) &= 2 \ln(3x - 4)^5 \\ &= 2 \times 5 \ln(3x - 4) \\ &= 10 \ln(3x - 4) \end{aligned}$ | You can apply the formula if the expression is in the form $f(x) = k \ln(ax + b)$. |
| $f'(x) = \frac{10 \times 3}{3x - 4} = \frac{30}{3x - 4}$ | |

Example 2

Find the derivative of $y = \frac{\ln \sqrt{7 - 12x}}{3}$.



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| Steps | Explanation |
|---|---|
| $y = \frac{\ln \sqrt{7-12x}}{3}$ $= \frac{1}{3} \ln (7-12x)^{\frac{1}{2}}$ $= \frac{1}{3} \times \frac{1}{2} \ln(7-12x)$ $= \frac{1}{6} \ln(7-12x)$ | <p>You can apply the formula if the expression is in the form $y = k \ln(ax + b)$.</p> |
| $\frac{dy}{dx} = \frac{\frac{1}{6} \times (-12)}{7-12x} = \frac{2}{12x-7}.$ | |

3 section questions ▾

5. Calculus / 5.6 Differentiation rules

Derivative of trigonometric functions

The next goal is to find the formulae for the derivatives of $y = \sin x$ and $y = \cos x$.

Section

Activity

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Feedback



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In the applet below move the red point on the graph. The applet shows the tangent line at the given point and also shows a trace of the gradient function.

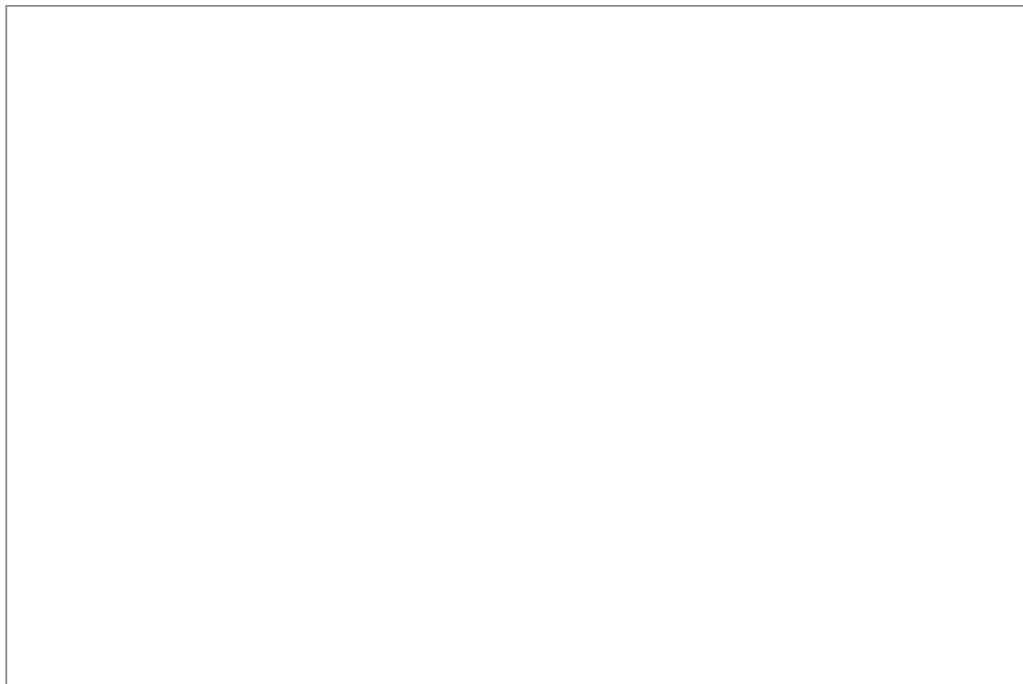
- Can you formulate a conjecture for the derivatives of $y = \sin x$ and $y = \cos x$?
- Does it matter whether the angles are measured in degrees or radians?



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Interactive 1. Exploring the Derivatives of Sine and Cosine Functions.

More information for interactive 1

This interactive allows the users to find the formula for the derivative of trigonometric functions $y = \sin x$ and $y = \cos x$. Users can drag the red point along the curve to observe how the slope of the tangent line changes at different points. The slope value is displayed in real time. Users can also choose whether the angle is measured in degrees or radians using the appropriate checkbox. The “sine” and “cosine” buttons allow users to switch between viewing the sine and cosine curves.

The interactive also shows a trace of the gradient function. The users will gain the knowledge that when x is measured in radians:

the derivative of $f(x) = \sin x$ is $f'(x) = \cos x$

the derivative of $f(x) = \cos x$ is $f'(x) = -\sin x$

For instance, if $f(x) = \cos x$ then $f'(x) = -\sin x$ and gradient $m = f'(x)$.

At $x = 0$ the gradient will be $m = f'(0) = -\sin(0) = 0$

You may have noticed the following information.

✓ Important

When x is measured in radians:

- the derivative of $f(x) = \sin x$ is $f'(x) = \cos x$
- the derivative of $f(x) = \cos x$ is $f'(x) = -\sin x$.



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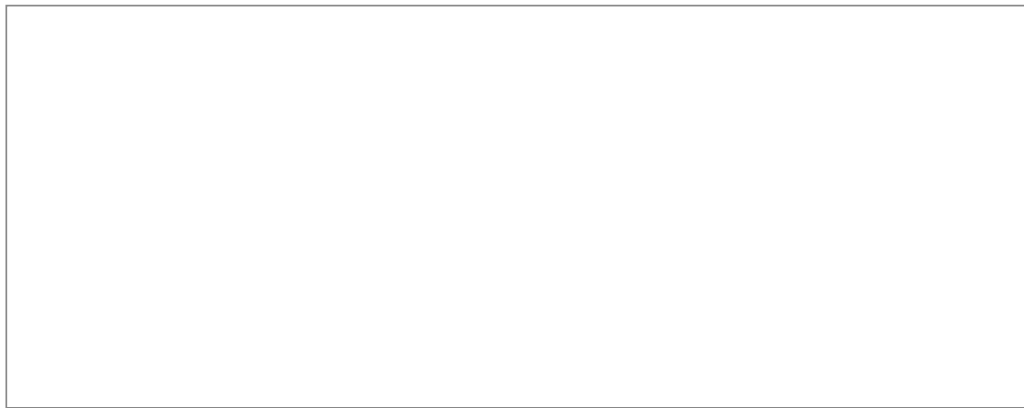
You can investigate a generalisation of this claim with the following applet.



Activity

The applet below gives the derivatives of functions of the form $y = k \sin(ax + b)$ and $y = k \cos(ax + b)$ for randomly generated parameters a , b and k .

Keep generating examples until you can predict the derivative without looking at the answer.



Interactive 2. Derivative of Inverse Trigonometric Functions.

More information for interactive 2

This interactive allows the users to apply the formula for the derivative of trigonometric functions

$y = \sin x$ and $y = \cos x$. The interactive helps in understanding the derivatives of functions of the form

$y = k \sin(ax + b)$ and $y = k \cos(ax + b)$ for randomly generated parameters a , b and k . Users can click the "

Click here of a new question" to solve different problems, allowing them to practice and analyze the results. By

clicking on the 'Show answers' users can verify their result as well. The derivative of $f(x) = k \sin(ax + b)$ is

$$f'(x) = k a \cos(ax + b)$$

the derivative of $f(x) = k \cos(ax + b)$ is $f'(x) = -k a \sin(ax + b)$

For example, if $f(x) = 4 \sin(2x + 3)$

$$\text{Therefore, } f'(x) = 8 \cos(2x + 3)$$

The users will deepen their knowledge of the concept through practice. The concept is crucial for understanding the rate of change (derivative) of these functions and how it is useful in the field of physics, engineering and modelling periodic formulas.



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Did you notice the following relationship?



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Important

When x is measured in radians:

- the derivative of $f(x) = k \sin(ax + b)$ is $f'(x) = ka \cos(ax + b)$
- the derivative of $f(x) = k \cos(ax + b)$ is $f'(x) = -ka \sin(ax + b)$

Section

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In the following examples you can see how this formula is used to find derivatives of certain types of functions.

Example 1



- Write $10 \sin 3x \cos 3x$ in the form $k \sin ax$.
- Find the derivative of $f(x) = 10 \sin 3x \cos 3x$.

| Steps | Explanation |
|---|---|
| $10 \sin 3x \cos 3x = 5 \times 2 \sin 3x \cos 3x$ $= 5 \sin(2 \times 3x)$ $= 5 \sin 6x$ | You can use the double angle formula $\sin 2\theta = 2 \sin \theta \cos \theta$ for $\theta = 3x$. |
| $f(x) = 5 \sin 6x$ $f'(x) = 5 \times 6 \cos 6x$ $= 30 \cos 6x$ | You can use $f'(x) = ka \cos(ax + b)$, where $k = 5$, $a = 6$ and $b = 0$. |

Example 2



Find the derivative of $y = \sin^2 x$.



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| Steps | Explanation |
|--|---|
| $y = \sin^2 x = \frac{1}{2} \times 2 \sin^2 x$ $= \frac{1}{2} (1 - \cos 2x)$ $= \frac{1}{2} - \frac{1}{2} \cos 2x$ | You can use the double angle formula $\cos 2x = 1 - 2 \sin^2 x$ to express $\sin^2 x$ in terms of $\cos 2x$ |
| $\frac{dy}{dx} = 0 - \frac{1}{2} \times 2 \times (-\sin 2x)$ $= \sin 2x$ | You can use $f'(x) = -ka \sin(ax + b)$, where $k = \frac{1}{2}$, $a = 2$ and $b = 0$. |

Example 3



Find the derivative of $y = \sin^3 x \cos x$.

| Steps | Explanation |
|---|--|
| $y = \sin^3 x \cos x$ $= \sin^2 x (\sin x \cos x)$ $= \frac{1}{4} \times 2 \sin^2 x (2 \sin x \cos x)$ $= \frac{1}{4} (1 - \cos 2x) \sin 2x$ $= \frac{1}{4} \sin 2x - \frac{1}{8} \times 2 \cos 2x \sin 2x$ $= \frac{1}{4} \sin 2x - \frac{1}{8} \sin 4x$ | You can start by writing the product differently and using the double angle formulae to find an equivalent form as a sum of trigonometric expressions. |
| $\frac{dy}{dx} = \frac{1}{4} \times 2 \cos 2x - \frac{1}{8} \times 4 \cos 4x$ $= \frac{\cos 2x - \cos 4x}{2}$ | |



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5. Calculus / 5.6 Differentiation rules

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Chain rule

In this section, you will learn about differentiating composite functions.

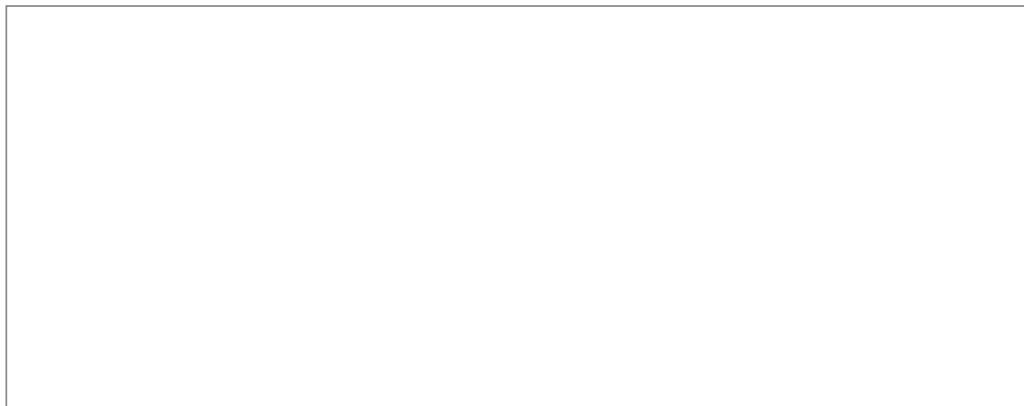


Activity

The applet below gives the derivatives of functions of the form $y = g(f(x))$.

- Identify f and g in the examples.
- State the derivatives, f' and g' .
- Can you see how $\frac{dy}{dx}$ is formed using f , g , f' and g' ?

Keep generating examples until you can predict the derivative without looking at the answer.



Interactive 1. Application of the Chain Rule in Differentiation.

More information for interactive 1

This interactive allows users to explore the differentiation of simple and composite functions, focusing on the application of the chain rule in finding derivatives of functions of the form $y = g(f(x))$. Users can click on **Click here for a new question** to generate different problems, enabling them to practice and analyze their results. They can also verify their answers by selecting the **Show Answers** option, reinforcing their understanding of the differentiation process.

The chain rule states that if $f(x) = g(h(x))$, then the derivative is given by $f'(x) = g'(h(x))f'(x)$.

For example, if $y = \sin(5x)$ Applying the chain rule helps users understand how the derivative is

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computed step by step.

$$\Rightarrow y' = \cos(5x) \frac{d(5x)}{dx}$$

$$\Rightarrow y' = 5\cos(5x)$$

Through this interactive, users will strengthen their knowledge of the chain rule and its application in differentiating composite functions. They will develop a deeper understanding of how the rate of change of one function influences the rate of change of another, enhancing their problem-solving skills in calculus.

You may have noticed the following rule for the derivative of composite functions. This is called the chain rule.

✓ Important

- If $h(x) = g(f(x))$, then $h'(x) = g'(f(x)) \times f'(x)$.

The formula booklet gives this rule in the following form:

- If $y = g(u)$, where $u = f(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

Example 1



- Identify the two functions that are used to form the composite functions in the first column of the table below.
- Find the derivatives missing from the table.

| $h(x) = g(f(x))$ | $g(u)$ | $u = f(x)$ | $g'(u)$ | $f'(x)$ | $h'(x)$ |
|------------------------|--------|------------|---------|---------|---------|
| $(x^2 - 2x)^5$ | | | | | |
| $\frac{1}{x^3 - 4x^2}$ | | | | | |
| e^{-x^2} | | | | | |



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| | | | | | |
|---------------------|--|--|--|--|--|
| $\ln(x^4 + 2x - 3)$ | | | | | |
| $\sin x^2$ | | | | | |
| $\sin^2 x$ | | | | | |
| $\cos(2x^3 - 3x)$ | | | | | |
| $e^{\cos x}$ | | | | | |
| $\cos e^x$ | | | | | |
| $\ln \sin x$ | | | | | |
| $\sin \ln x$ | | | | | |

- For $h(x) = (x^2 - 2x)^5$
 - $g(u) = u^5$
 - $u = f(x) = x^2 - 2x$
 - $g'(u) = 5u^4$
 - $f'(x) = 2x - 2$
 - $h'(x) = g'(f(x))f'(x) = 5(x^2 - 2x)^4(2x - 2)$
- For $h(x) = \frac{1}{x^3 - 4x^2}$
 - $g(u) = \frac{1}{u} = u^{-1}$
 - $u = f(x) = x^3 - 4x^2$
 - $g'(u) = -u^{-2} = \frac{-1}{u^2}$
 - $f'(x) = 3x^2 - 8x$
 - $h'(x) = g'(f(x))f'(x) = \frac{-(3x^2 - 8x)}{(x^3 - 4x^2)^2}$
- For $h(x) = e^{-x^2}$
 - $g(u) = e^u$
 - $u = f(x) = -x^2$
 - $g'(u) = e^u$
 - $f'(x) = -2x$
 - $h'(x) = g'(f(x))f'(x) = -2xe^{-x^2}$
- For $h(x) = \ln(x^4 + 2x - 3)$
 - $g(u) = \ln u$



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$$\circ u = f(x) = x^4 + 2x - 3$$

$$\circ g'(u) = \frac{1}{u}$$

$$\circ f'(x) = 4x^3 + 2$$

$$\circ h'(x) = g'(f(x))f'(x) = \frac{4x^3 + 2}{x^4 + 2x - 3}$$

$$\bullet \text{ For } h(x) = \sin x^2$$

$$\circ g(u) = \sin u$$

$$\circ u = f(x) = x^2$$

$$\circ g'(u) = \cos u$$

$$\circ f'(x) = 2x$$

$$\circ h'(x) = g'(f(x))f'(x) = 2x \cos x^2$$

$$\bullet \text{ For } h(x) = \sin^2 x$$

$$\circ g(u) = u^2$$

$$\circ u = f(x) = \sin x$$

$$\circ g'(u) = 2u$$

$$\circ f'(x) = \cos x$$

$$\circ h'(x) = g'(f(x))f'(x) = 2 \sin x \cos x$$

$$\bullet \text{ For } h(x) = \cos(2x^3 - 3x)$$

$$\circ g(u) = \cos u$$

$$\circ u = f(x) = 2x^3 - 3x$$

$$\circ g'(u) = -\sin u$$

$$\circ f'(x) = 6x^2 - 3$$

$$\circ h'(x) = g'(f(x))f'(x) = -(6x^2 - 3) \sin(2x^3 - 3x)$$

$$\bullet \text{ For } h(x) = e^{\cos x}$$

$$\circ g(u) = e^u$$

$$\circ u = f(x) = \cos x$$

$$\circ g'(u) = e^u$$

$$\circ f'(x) = -\sin x$$

$$\circ h'(x) = g'(f(x))f'(x) = -\sin x e^{\cos x}$$

$$\bullet \text{ For } h(x) = \cos e^x$$

$$\circ g(u) = \cos u$$

$$\circ u = f(x) = e^x$$

$$\circ g'(u) = -\sin u$$

$$\circ f'(x) = e^x$$

$$\circ h'(x) = g'(f(x))f'(x) = -e^x \sin e^x$$

$$\bullet \text{ For } h(x) = \ln \sin x$$

$$\circ g(u) = \ln u$$

$$\circ u = f(x) = \sin x$$



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$$\begin{aligned} \circ g'(u) &= \frac{1}{u} \\ \circ f'(x) &= \cos x \\ \circ h'(x) &= g'(f(x))f'(x) = \frac{\cos x}{\sin x} \end{aligned}$$

• For $h(x) = \sin \ln x$

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$$\circ g(u) = \sin u$$

Feedback



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$$\circ u = f(x) = \ln x$$

$$\circ g'(u) = \cos u$$

$$\circ f'(x) = \frac{1}{x}$$

$$\circ h'(x) = g'(f(x))f'(x) = \frac{\cos \ln x}{x}$$

Example 2



Find the derivative of $f(x) = 2 \ln (3x - 4)^5$.

You encountered this function in section 5.6.3. This solution shows you how to find the derivative using the chain rule, rather than the laws of logarithms.

| Steps | Explanation |
|---|---|
| $f(x) = g(u(x))$, where $g(u) = 2 \ln u$ and $u(x) = (3x - 4)^5$ | You can write $f(x)$ as a composition. |
| $g'(u) = \frac{2}{u}$ $u'(x) = 3 \times 5(3x - 4)^4 = 15(3x - 4)^4$ | To use the chain rule, you need the derivatives of g and u . |
| $f'(x) = \frac{2}{(3x - 4)^5} \times 15(3x - 4)^4$ $= \frac{30}{3x - 4}$ | According to the chain rule, $f'(x) = g'(u(x)) \times u'(x)$. |



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In the examples above you found derivatives of compositions of two functions. The function in the next example is built as a composition of more than two functions.



Example 3

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Consider the functions defined by $f(x) = e^x$, $g(x) = \sqrt{1+x^2}$ and $h(x) = \sin x$.

- Find $g'(x)$.
- Find $l(x) = (g \circ h)(x)$ and $l'(x)$.
- Find $m(x) = (f \circ g \circ h)(x)$ and $m'(x)$.

Recall, that $p \circ q$ is the composition of the functions p and q , so $(p \circ q)(x) = p(q(x))$.



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| Steps | Explanation |
|---|---|
| $g(x) = s(u(x))$, where $s(u) = \sqrt{u} = u^{\frac{1}{2}}$, and $u(x) = 1 + x^2$ | You can write $g(x)$ as a composition. |
| $s'(u) = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$ $u'(x) = 2x$ | To use the chain rule, you need the derivatives of s and u . |
| $g'(x) = \frac{1}{2\sqrt{1+x^2}} \times 2x = \frac{x}{\sqrt{1+x^2}}$ | According to the chain rule, $g'(x) = s'(u(x)) \times u'(x)$. |
| $l(x) = g(h(x)) = g(\sin x)$ $= \sqrt{1 + \sin^2 x}$ | |
| $l'(x) = \frac{\sin x}{\sqrt{1 + \sin^2 x}} \times \cos x$ $= \frac{\sin x \cos x}{\sqrt{1 + \sin^2 x}}$ | According to the chain rule, $l'(x) = g'(h(x)) \times h'(x)$. |
| $m(x) = f(g(h(x))) = f(l(x))$ $= f(\sqrt{1 + \sin^2 x})$ $= e^{\sqrt{1 + \sin^2 x}}$ | |
| $m'(x) = e^{\sqrt{1 + \sin^2 x}} \times \frac{\sin x \cos x}{\sqrt{1 + \sin^2 x}}$ | According to the chain rule, $m'(x) = f'(l(x)) \times l'(x)$. |



Making connections

It was not emphasised in the previous example, but if you put together the pieces you can notice that the derivative of $m(x) = f(g(h(x)))$ is in fact

$$m'(x) = f'(g(h(x))) \times g'(h(x)) \times h'(x).$$

This type of pattern is the reason why this rule is called the chain rule.

Can you find the similar formula for the derivative of the composition of four functions?



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The following example illustrates a situation where, instead of being given a formula, you are given only partial information about a function.

Example 4



For some function, f , you are given that $f'(10) = 7$.

Let $g(x) = f(x^2 - 15)$.

Find $g'(5)$.

| Steps | Explanation |
|---|---|
| $u'(x) = 2x$ | $g(x)$ is the composition of $f(u)$ and $u(x) = x^2 - 15$. To use the chain rule, you need the derivative of u . |
| $g'(x) = f'(u(x)) \times u'(x) = 2x f'(x^2 - 15)$ | |
| $ \begin{aligned} g'(5) &= 2 \times 5 \times f'(5^2 - 15) \\ &= 10 \times f'(10) \\ &= 10 \times 7 = 70 \end{aligned} $ | You can get $g'(5)$ by substituting $x = 5$ and using the information given in the question. |

The table below summarises information about some basic derivatives and also shows the chain rule applied to these functions.

The information in the first two columns is given in the formula booklet.

$$f(x)$$

$$f'(x)$$

$$g(x) = f(ax + b)$$



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$$g'(x)$$

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$$h(x) = f(u(x))$$

$$h'(x)$$

$$x^n$$

$$nx^{n-1}$$

$$(ax + b)^n$$

$$an(ax + b)^{n-1}$$

$$(u(x))^n$$

$$n(u(x))^{n-1}u'(x)$$

$$\sin x$$

$$\cos x$$

$$\sin(ax + b)$$

$$a \cos(ax + b)$$

$$\sin u(x)$$

$$u'(x) \cos u(x)$$

$$\cos x$$

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$$- \sin x$$



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$$\cos(ax + b)$$

$$-a \sin(ax + b)$$

$$\cos u(x)$$

$$-u'(x) \sin u(x)$$

$$e^x$$

$$e^x$$

$$e^{ax+b}$$

$$ae^{ax+b}$$

$$e^{u(x)}$$

$$e^{u(x)} u'(x)$$

$$\ln x$$

$$\frac{1}{x}$$

$$\ln(ax + b)$$

$$\frac{a}{ax + b}$$

$$\ln u(x)$$



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$$\frac{u'(x)}{u(x)}$$



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5 section questions ▾

5. Calculus / 5.6 Differentiation rules

Product rule

In this section, you will learn about differentiating products of functions.

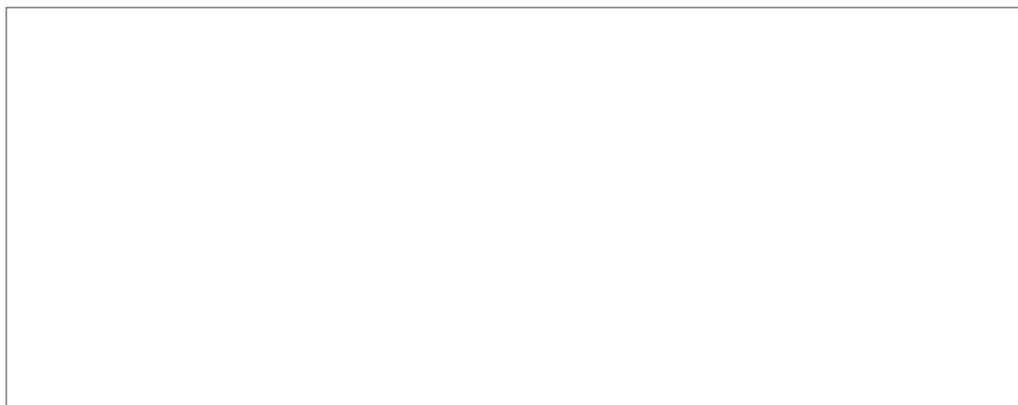


Activity

The applet below gives the derivatives of functions of the form $y = u(x)v(x)$.

- Identify u and v in the examples.
- State the derivatives, $\frac{du}{dx}$ and $\frac{dv}{dx}$.
- Can you see how $\frac{dy}{dx}$ is formed using u , v , $\frac{du}{dx}$ and $\frac{dv}{dx}$?

Keep generating examples until you can predict the derivative without looking at the answer.

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Interactive 1. Application of the Product Rule in Differentiation.

More information for interactive 1

This interactive tool helps users practice differentiating product functions of the form

$y = u(x)v(x)$. Each time users generate a new question using the "click here for a new question", they'll see a different product of functions to work with. The tool guides users through a clear process: first identifying the two functions $u(x)$ and $v(x)$, then finding their separate derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$, and finally combining them using the product rule formula $\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$.

Example: The given question is $y = \sin(x) \cdot x^3$

The Answer is - $\frac{dy}{dx} = \sin(x) \cdot 3x^2 + x^3 \cdot \cos(x)$

Users can work with various function combinations, from simpler products like polynomials and trigonometric functions to more complex pairings. The "Show answer" feature lets users check their work against the correct solution, helping them identify any mistakes and learn from them. By repeatedly solving different examples, users will naturally recognize the pattern of the product rule and gain confidence in applying it.

Through consistent practice with these interactive exercises, users will develop the ability to quickly and accurately differentiate any product functions they encounter in their calculus studies.

You may have noticed the following rule for the derivative of the product of two functions. This is called the product rule.

✓ Important

- If $h(x) = u(x)v(x)$, then $h'(x) = u(x)v'(x) + v(x)u'(x)$.

The formula booklet gives this rule in the following form:

- If $y = uv$, then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$.

Example 1



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- The expressions in the first column of the table below are products of two simpler expressions. Identify these two expressions.
- Find the derivatives missing from the table.

| $h(x) = u(x)v(x)$ | $u(x)$ | $v(x)$ | $u'(x)$ | $v'(x)$ | $h'(x)$ |
|-----------------------|--------|--------|---------|---------|---------|
| $(x^3 + 7)e^{-x}$ | | | | | |
| $(x^2 + 1)^3 \ln(2x)$ | | | | | |
| $e^{x^2} \cos x$ | | | | | |
| $\sin(5 - 2x) \ln x$ | | | | | |

- For $h(x) = (x^3 + 7)e^{-x}$
 - $u(x) = x^3 + 7$
 - $v(x) = e^{-x}$
 - $u'(x) = 3x^2$
 - $v'(x) = -e^{-x}$
 - $h'(x) = u(x)v'(x) + v(x)u'(x)$
 - $= (x^3 + 7) \times (-e^{-x}) + e^{-x} \times 3x^2$
 - $= -e^{-x}(x^3 - 3x^2 + 7)$
- For $h(x) = (x^2 + 1)^3 \ln(2x)$
 - $u(x) = (x^2 + 1)^3$
 - $v(x) = \ln(2x)$
 - $u'(x) = 3(x^2 + 1)^2 \times 2x = 6x(x^2 + 1)^2$
 - $v'(x) = \frac{1}{2x} \times 2 = \frac{1}{x}$
 - $h'(x) = u(x)v'(x) + v(x)u'(x)$
 - $= (x^2 + 1)^3 \times \frac{1}{x} + \ln(2x) \times 6x(x^2 + 1)^2$
 - $= (x^2 + 1)^2 \left(\frac{x^2 + 1}{x} + 6x \ln(2x) \right)$
- For $h(x) = e^{x^2} \cos x$
 - $u(x) = e^{x^2}$
 - $v(x) = \cos x$
 - $u'(x) = 2xe^{x^2}$
 - $v'(x) = -\sin x$



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$$h'(x) = u(x)v'(x) + v(x)u'(x)$$

$$\begin{aligned} &= e^{x^2} \times (-\sin x) + \cos x \times 2xe^{x^2} \\ &= e^{x^2} (2x \cos x - \sin x) \end{aligned}$$

- For $h(x) = \sin(5 - 2x) \ln x$

$$\circ u(x) = \sin(5 - 2x)$$

$$\circ v(x) = \ln x$$

$$\circ u'(x) = \cos(5 - 2x) \times (-2) = -2 \cos(5 - 2x)$$

$$\circ v'(x) = \frac{1}{x}$$

$$h'(x) = u(x)v'(x) + v(x)u'(x)$$

$$\circ = \sin(5 - 2x) \times \frac{1}{x} + \ln x \times (-2 \cos(5 - 2x))$$

$$= \frac{\sin(5 - 2x)}{x} - 2 \ln x \cos(5 - 2x)$$

Example 2



Find the derivative of $y = \sin^3 x \cos x$.

| Steps | Explanation |
|---|--|
| $y = uv$ for $u = \sin^3 x$ and $v = \cos x$ | You can write the expression as a product. |
| $\frac{du}{dx} = 3 \sin^2 x \cos x$ $\frac{dv}{dx} = -\sin x$ | To use the product rule, you need the derivatives of u and v . |
| $\begin{aligned} \frac{dy}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ &= \sin^3 x \times (-\sin x) + \cos x \times 3 \sin^2 x \cos x \\ &= 3 \sin^2 x \cos^2 x - \sin^4 x \end{aligned}$ | |



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Note that you have already seen the derivative of the function from **Example 2** in [section 5.6.4 \(/study/app/math-aa-hl/sid-134-cid-761926/book/derivative-of-trigonometric-functions-id-27782/\)](#). There, the answer for the derivative was $\frac{\cos 2x - \cos 4x}{2}$. Can you prove (without using differentiation) that these two expressions are equivalent?

In the examples above, you found derivatives of products of two functions. The function in the next example is built as a product of more than two functions.

Example 3



Find the derivative of $f(x) = \sqrt{x}e^x \sin x$.

| Steps | Explanation |
|--|---|
| $f(x) = u(x)v(x)$ with $u(x) = \sqrt{x}$ and $v(x) = e^x \sin x$ | You can write the expression as a product. |
| $v(x) = g(x)h(x)$ with $g(x) = e^x$ and $h(x) = \sin x$ | To find the derivative of v , it can also be written as a product. |
| $g'(x) = e^x$ $h'(x) = \cos x$ | To find v' using the product rule, you need the derivatives of g and h . |
| $v'(x) = g(x)h'(x) + h(x)g'(x)$ $= e^x \times \cos x + \sin x \times e^x$ $= e^x(\sin x + \cos x)$ | |
| $u'(x) = \frac{1}{2\sqrt{x}}$ | To find f' using the product rule, in addition to v' , you also need u' . |
| $f'(x) = u(x)v'(x) + v(x)u'(x)$ $= \sqrt{x} \times e^x(\sin x + \cos x) + e^x \sin x \times \frac{1}{2\sqrt{x}}$ $= \frac{e^x(2x \sin x + 2x \cos x + \sin x)}{2\sqrt{x}}$ | |



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Making connections

It was not emphasised in the previous example, but if you put together the pieces you may notice that the derivative of $f = ugh$ is

$$f' = u'gh + ug'h + ugh'.$$

Do you see the pattern?

Can you find the similar formula for the derivative of the product of four functions?

The following example illustrates a situation where, instead of being given a formula, you are given only partial information about a function.

Example 4

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Assign

For some function, f , you are given that $f(5) = -3$ and $f'(5) = 2$.

Let $g(x) = (x^2 - 15)f(x)$.

Find $g'(5)$.

| Steps | Explanation |
|---|---|
| $u'(x) = 2x$ | $g(x)$ is the product of $f(x)$ and $u(x) = x^2 - 15$. To use the product rule, you need the derivative of u . |
| $g'(x) = u(x)f'(x) + f(x)u'(x)$ $= (x^2 - 15)f'(x) + 2xf(x)$ | |
| $g'(5) = (5^2 - 15) \times f'(5) + 2 \times 5 \times f(5)$ $= 10 \times 2 + 10 \times (-3) = -10$ | You can get $g'(5)$ by substituting $x = 5$ and using the information given in the question. |



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5. Calculus / 5.6 Differentiation rules

3 section questions ▾

Quotient rule

In this section, you will learn about differentiating quotients of functions.

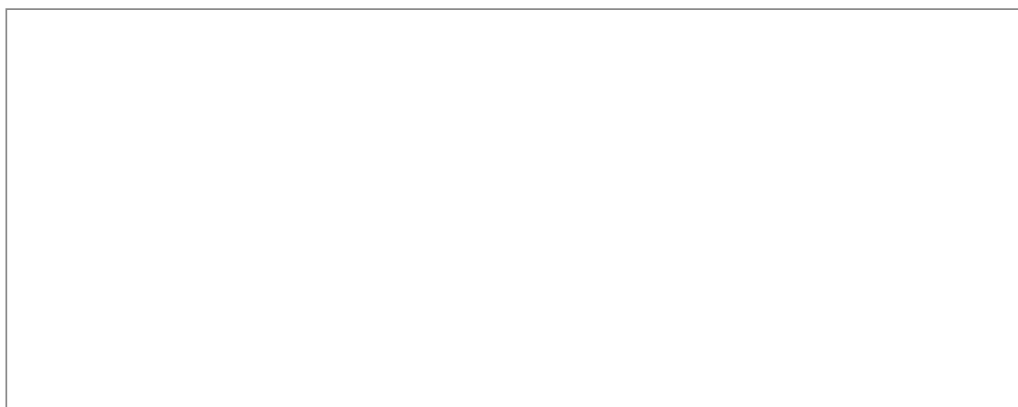


Activity

The applet below gives the derivatives of functions of the form $y = \frac{u(x)}{v(x)}$.

- Identify u and v in the examples.
- State the derivatives, $\frac{du}{dx}$ and $\frac{dv}{dx}$.
- Can you see how $\frac{dy}{dx}$ is formed using u , v , $\frac{du}{dx}$ and $\frac{dv}{dx}$?

Keep generating examples until you can predict the derivative without looking at the answer.



Interactive 1. Application of the Quotient Rule in Differentiation.

🕒 More information for interactive 1



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You may have noticed the following rule for the derivative of the quotient of two functions. This is called the quotient rule.



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**Important**

- If $h(x) = \frac{u(x)}{v(x)}$, then $h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$.

The formula booklet gives this rule in the following form:

- If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$.

Example 1



- The expressions in the first column of the table below are quotients of two simpler expressions. Identify these two expressions.
- Find the derivatives missing from the table.

| $h(x) = \frac{u(x)}{v(x)}$ | $u(x)$ | $v(x)$ | $u'(x)$ | $v'(x)$ | $h'(x)$ |
|-------------------------------|--------|--------|---------|---------|---------|
| $\frac{x^3 + 7}{e^{-x}}$ | | | | | |
| $\frac{(x^2 + 1)^3}{\ln(2x)}$ | | | | | |
| $\frac{e^{x^2}}{\cos x}$ | | | | | |
| $\frac{\sin(5 - 2x)}{\ln x}$ | | | | | |

- For $h(x) = \frac{x^3 + 7}{e^{-x}}$
 - $u(x) = x^3 + 7$
 - $v(x) = e^{-x}$
 - $u'(x) = 3x^2$
 - $v'(x) = -e^{-x}$



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Assign

$$h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

$$= \frac{e^{-x} \times 3x^2 - (x^3 + 7) \times (-e^{-x})}{(e^{-x})^2}$$

$$= \frac{e^{-x}(x^3 + 3x^2 + 7)}{(e^{-x})^2}$$

$$= e^x(x^3 + 3x^2 + 7)$$

• For $h(x) = \frac{(x^2 + 1)^3}{\ln(2x)}$

◦ $u(x) = (x^2 + 1)^3$

◦ $v(x) = \ln(2x)$

◦ $u'(x) = 3(x^2 + 1)^2 \times 2x = 6x(x^2 + 1)^2$

◦ $v'(x) = \frac{1}{2x} \times 2 = \frac{1}{x}$

$$h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

◦ $= \frac{\ln(2x) \times 6x(x^2 + 1)^2 - (x^2 + 1)^3 \times \frac{1}{x}}{(\ln(2x))^2}$

$$= \frac{(x^2 + 1)^2 \left(6x \ln(2x) - \frac{x^2 + 1}{x} \right)}{(\ln(2x))^2}$$

• For $h(x) = \frac{e^{x^2}}{\cos x}$

◦ $u(x) = e^{x^2}$

◦ $v(x) = \cos x$

◦ $u'(x) = 2xe^{x^2}$

◦ $v'(x) = -\sin x$

$$h'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

◦ $= \frac{\cos x \times 2xe^{x^2} - e^{x^2} \times (-\sin x)}{\cos^2 x}$

$$= \frac{e^{x^2}(2x + \tan x)}{\cos x}$$

• For $h(x) = \frac{\sin(5 - 2x)}{\ln x}$

◦ $u(x) = \sin(5 - 2x)$

◦ $v(x) = \ln x$

◦ $u'(x) = \cos(5 - 2x) \times (-2) = -2 \cos(5 - 2x)$

◦ $v'(x) = \frac{1}{x}$



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$$\begin{aligned}
 h'(x) &= \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2} \\
 &= \frac{\ln x \times (-2 \cos(5 - 2x)) - \sin(5 - 2x) \times \frac{1}{x}}{(\ln x)^2} \\
 &= -\frac{2x \ln x \cos(5 - 2x) + \sin(5 - 2x)}{x(\ln x)^2}
 \end{aligned}$$

Example 2



Express the derivative of $g = \frac{1}{f}$ in terms of f and f' .

Method 1 (using the quotient rule)

| Steps | Explanation |
|--|---|
| $g = \frac{u}{v}$ with $u = 1$ and $v = f$. | You can think of g as a quotient. |
| $u' = 0$ $v' = f'$ | To find the derivative using the quotient rule, you need the derivatives of u and v . |
| $ \begin{aligned} g' &= \frac{vu' - uv'}{v^2} \\ &= \frac{f \times 0 - 1 \times f'}{f^2} \\ &= -\frac{f'}{f^2} \end{aligned} $ | |

Method 2 (using the chain rule)



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| Steps | Explanation |
|--|---|
| For $h(u) = \frac{1}{u} = u^{-1}$, $g(x) = (h \circ f)(x) = h(f(x))$ | You can think of g as a composition. |
| $h'(u) = -u^{-2} = -\frac{1}{u^2}$ | To find the derivative using the chain rule, you need the derivative of h . |
| $g'(x) = h'(f(x)) \times f'(x)$ $= -\frac{1}{(f(x))^2} \times f'(x)$ $= -\frac{f'(x)}{(f(x))^2}$ | |

The following example illustrates a situation where, instead of being given a formula, you are given only partial information about a function.

Example 3



For some function, f , you are given that $f(5) = -3$ and $f'(5) = 2$.

Let $g(x) = \frac{x^2 - 15}{f(x)}$ and $h(x) = \frac{f(x)}{x^2 - 15}$.

Find $g'(5)$ and $h'(5)$.

| Steps | Explanation |
|--|---|
| $u'(x) = 2x$ | Both $g(x)$ and $h(x)$ are quotients of $f(x)$ and $u(x) = x^2 - 15$. To use the quotient rule , you need the derivative of u . |
| $g'(x) = \frac{f(x)u'(x) - u(x)f'(x)}{(f(x))^2}$ $= \frac{2xf(x) - (x^2 - 15)f'(x)}{(f(x))^2}$ | |



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| Steps | Explanation |
|---|--|
| $g'(5) = \frac{2 \times 5 \times f(5) - (5^2 - 15) \times f'(5)}{(f(5))^2}$ $= \frac{2 \times 5 \times (-3) - (5^2 - 15) \times 2}{(-3)^2}$ $= -\frac{50}{9}$ | You can get $g'(5)$ by substituting $x = 5$ and using the information given in the question. |
| $h'(x) = \frac{u(x)f'(x) - f(x)u'(x)}{(u(x))^2}$ $= \frac{(x^2 - 15)f'(x) - 2xf(x)}{(x^2 - 15)^2}$ | |
| $h'(5) = \frac{(5^2 - 15) \times f'(5) - 2 \times 5 \times f(5)}{(5^2 - 15)^2}$ $= \frac{(5^2 - 15) \times 2 - 2 \times 5 \times (-3)}{(5^2 - 15)^2}$ $= \frac{20 + 30}{100} = \frac{1}{2}$ | You can get $h'(5)$ by substituting $x = 5$ and using the information given in the question. |

3 section questions ▾

5. Calculus / 5.6 Differentiation rules

Checklist

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Assign

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What you should know

By the end of this subtopic you should be able to:

- find the derivatives of functions built from the basic functions defined by x^n , $\sin x$, $\cos x$, e^x and $\ln x$
 - recall the derivatives of these basic functions; these are given in the formula booklet
- use the constant factor rule, the sum rule, the chain rule, the product rule and the **quotient rule** when appropriate.



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5. Calculus / 5.6 Differentiation rules

Investigations

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Assign

In this subtopic, you were shown some tools that will help you to find the derivatives of a wide range of functions.

You learned how to work with

- linear combinations ($af(x) + bg(x)$)
- compositions ($f(g(x))$)
- products ($f(x)g(x)$)
- quotients $\left(\frac{f(x)}{g(x)}\right)$.



Activity

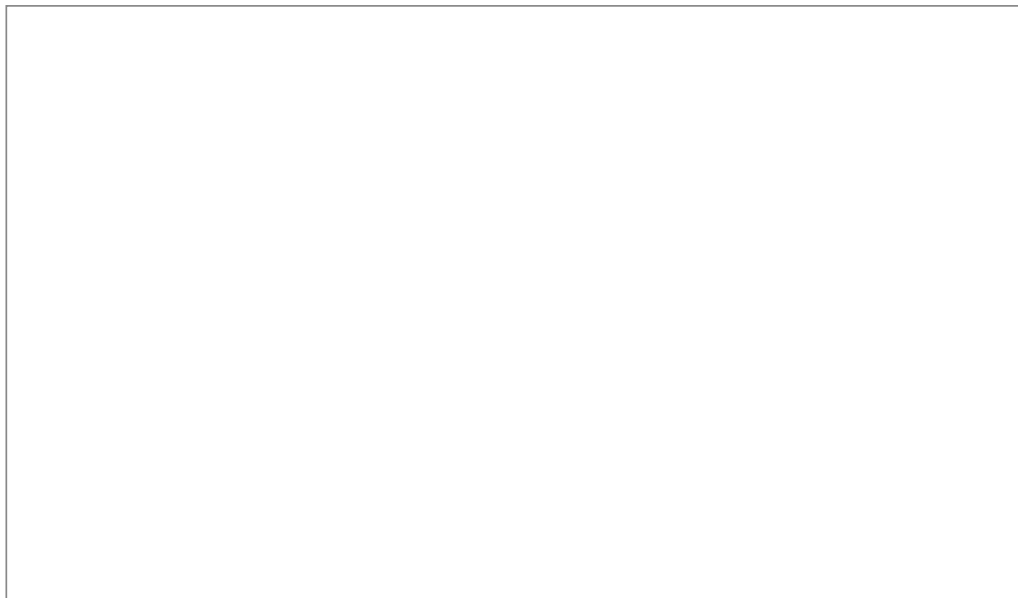
In the applet below you can investigate the derivatives of functions of the form $y = f(x)^{g(x)}$.

- Change the base and the exponent and take a look at the derivative.
 - The default question of the applet asks about the derivative of $y = x^x$. Can you find this without looking at the answer?
 - If you got the correct derivative, well done.
 - If the formula you suggested does not match the actual derivative, can you explain why?
- Can you find a general formula for the derivative of $y = f(x)^{g(x)}$?

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Interactive 1. Differentiation Rules Investigation.

More information for interactive 1

This interactive helps users understand the derivatives of functions of the form $y = f(x)^{g(x)}$. It allows users to explore how changes in the base and exponent affect the derivative by selecting different options from the dropdown menu. Users can choose from various base functions and exponents, including 2, e, x, x^2 , x^3 , $\ln(x)$, e^x , $\sin(x)$ and $\cos(x)$.

For example, when selecting $y = 2^x$, where the base is 2 and the exponent is x,

- $\log(y) = \log(2^x)$
- $y' = \log(2) \cdot 2^x$

Users can observe how the derivative is computed using logarithmic differentiation. This method helps in differentiating complex exponentials and power functions efficiently.

Additionally, the interactive allows users to practice multiple questions and verify their solutions by clicking on the “Show Answers” option. Through this hands-on approach, users will strengthen their understanding of logarithmic differentiation, product rule, and their applications in calculus.

Rate subtopic 5.6 Differentiation rules

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