2.12 Polynomial functions

Checklist

What you should know

By the end of this subtopic you should be able to:

- know that a polynomial of degree n are real numbers and can be written as:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

- know how to add, subtract, multiply and divide polynomials
- apply the Factor and Remainder theorems:
  - when a polynomial $P(x)$ is divided by $(x - a)$
    - $P(a)$ is the remainder if $(x - a)$ is not a factor
    - $P(a) = 0$ if $(x - a)$ is a factor
  - when a polynomial $P(x)$ is divided by $(ax + b)$
    - $P(-\frac{b}{a})$ is the remainder if $(ax + b)$ is not a factor
    - $P(-\frac{b}{a}) = 0$ if $(ax + b)$ is a factor
- know that a polynomial of degree n can be written as a product of n factors, where $a_1,\ a_2,\ a_3 \ldots a_n$ are the zeros of the polynomial

$$\mathrm{P}(x) = (x - a_1)(x - a_2)\ldots.(x - a_n)$$

- use the sum and product of polynomial equations:

  For $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

$$\text{Sum} = -\frac{a_{(n-1)}}{a_n}, \quad \text{Product} = \frac{(-1)^n (a_0)}{a_n}$$

- know that roots of a polynomial equation could be real or complex. If a polynomial has complex roots, they always occur in conjugate pairs $(a \pm \mathrm{i}b)$.
- be able to sketch polynomial functions
- be able to find equations from graphs of polynomial functions.