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The big picture

When you send a message on your mobile phone, the predictive text feature may offer suggestions for words based on the letters you enter and the context of the message. In 2018, Google added a feature to its email program that offers suggestions on how to end your sentence as you type. How do these features work and how do they predict what you want to say?

How likely is that computers will reach a level of sophistication that allows them to think for themselves? Is it possible for a computer to create a new thought on its own, or can it only follow the programming that it has been given?

Concept

The representation of probabilities within transition matrices allows us to use current conditions to efficiently predict long-term outcomes.

Theory of Knowledge

As you have inevitably noticed, mathematics is built upon itself conceptually. Lower-level axioms provide the basis upon which more complex mathematics and subsequent axioms are built. This methodology gives maths its epistemic clout.

To what extent are other areas of knowledge built in a similar fashion? If you were to rank the AOKs in terms of validity of knowledge produced, would the most axiomatic AOKs be at the top of your list?

Knowledge Question: To what extent is new knowledge a result of prior knowledge?

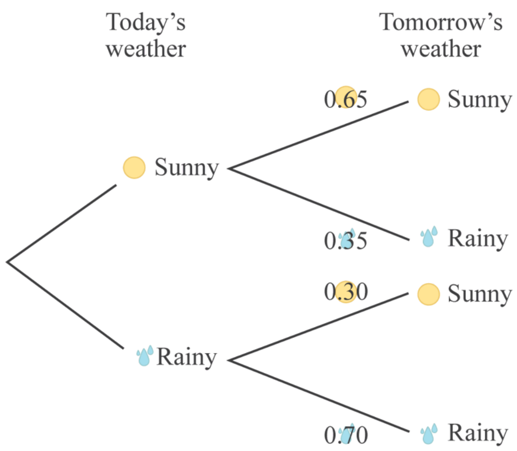
Transition matrices

Conditional probability revisited

Making connections

Recall from section 4.6.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/probabilities-of-related-events-id-26098/) that conditional probability is the probability of an event occurring given that another event occurs.

Suriya is in charge of developing the scenery and environment for a video game. For the weather, he wants a mixture of sunny days and rainy days. To help create a realistic change between the different types of days, he programs in the probabilities shown in a tree diagram, as below.



More information

The image is a tree diagram representing the probabilities of weather transitions from today's weather to tomorrow's weather. The diagram has two main branches stemming from "Today's weather": one branch for "Sunny" and another for "Rainy." Each branch is further split into two branches, indicating the probability of "Tomorrow's weather" being either "Sunny" or "Rainy."

For the "Sunny" branch: - A branch leads to "Sunny" tomorrow with a probability of 0.65. - Another branch leads to "Rainy" tomorrow with a probability of 0.35.

For the "Rainy" branch: - A branch leads to "Sunny" tomorrow with a probability of 0.30. - Another branch leads to "Rainy" tomorrow with a probability of 0.70.

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These probabilities demonstrate the dependency on the current state of weather: a 65% chance of sunshine following a sunny day and only a 30% chance of sunshine following a rainy day, and vice versa for rain probabilities. The diagram helps in understanding the transitional likelihood between weather conditions for game development purposes.

[Generated by AI]

Note that the probabilities are dependent on the current state of the weather. For example, there is a 65% chance of it being sunny the day after a sunny day while there is only a 30% chance of it being sunny the day after a rainy day. Using the conditional probability notation from section 4.6.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/probabilities-of-related-events-id-26098/), these statements can be expressed as follows:

$$\mathrm{P}\left(\text{sunny tomorrow}|\text{sunny today}\right) = 0.65$$

$$\mathrm{P}\left(\text{sunny tomorrow}|\text{rainy today}\right) = 0.30.$$

Activity

1. Write down the probabilities for it being rainy tomorrow using conditional probability notation.
2. Copy the tree diagram onto a piece of paper. Add another column (another set of branches) to represent the weather for day after tomorrow.
3. Using your tree diagram, calculate the following probabilities:
   a) $\mathrm{P}\left(\text{sunny the day after tomorrow}|\text{sunny today}\right)$
   b) $\mathrm{P}\left(\text{rainy the day after tomorrow}|\text{sunny today}\right)$
   c) $\mathrm{P}\left(\text{sunny the day after tomorrow}|\text{rainy today}\right)$
   d) $\mathrm{P}\left(\text{rainy the day after tomorrow}|\text{rainy today}\right)$.

Transition matrices

The information from the tree diagram below can also be represented as a transition matrix as seen below.

Weather today

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sunny
rainy

Weather tomorrow

sunny
rainy

$$\begin{bmatrix} 0.65 & 0.30 \\ 0.35 & 0.70 \end{bmatrix}$$

Important

Transition matrices are square matrices that show the probabilities of current states becoming future states. Each column represents one current state, while each row represents one future state. The probabilities in each column should sum to 1.

Making connections

You learned how to multiply matrices in section 1.14.3 (/study/app/math-ai-hl/sid-132-cid-761618/book/matrix-multiplication-id-27431/).

Activity

Consider the transition matrix:

$$T = \begin{bmatrix} 0.65 & 0.30 \\ 0.35 & 0.70 \end{bmatrix}$$

1. Without using your graphic display calculator, find T^2 .
2. Compare your answer for T^2 to the probabilities you found in Step 3 of the previous activity. How does your working for T^2 compare to the tree diagram that you created for the previous activity?
3. Using your graphic display calculator, find T^3 .
4. Describe the meaning of the values in the solution matrix of T^3 .
5. Describe the meaning of n in the expression T^n .

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Example 1

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In designing a new game, Suriya programs the weather to change based on the transition matrix shown below.

Weather today

sunny
rainy

Weather tomorrow

sunny
rainy

$$\begin{bmatrix} 0.75 & 0.45 \\ 0.25 & 0.55 \end{bmatrix}$$

Given that it is sunny today, find the probability that it will be rainy 3 days from now. You can use your graphic display calculator to multiply matrices and calculate powers of matrices (see section 1.15.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/powers-of-matrices-id-27438/)).

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Steps	Explanation
$T = \begin{bmatrix} 0.75 & 0.45 \\ 0.25 & 0.55 \end{bmatrix}$	Begin by writing down the transition matrix.
$T = \begin{bmatrix} 0.6525 & 0.6255 \\ 0.3475 & 0.3745 \end{bmatrix}$	Use your graphic display calculator to find T^3
$\therefore$ there is a 34.75% chance of it being rainy 3 days from now, if it is sunny today.	Select the appropriate value from the solution matrix.

Transition state diagrams

The transitions that are shown within a transition matrix can also be visualised using a transition state diagram. In the applet below, you can change the various probabilities to see how they affect the transitioning between states A, B and C.

Interactive 1. Markov Chains: Transition State Diagrams.

More information for interactive 1

This interactive enables users to explore the concept of state transitions through dynamic visualization.

Users can modify transition probabilities by dragging interactive controls, with changes instantly reflected in the visualization.The diagram represents states as points and as probabilities change, the transition state diagram automatically updates, with connecting curves between states adjusting their thickness proportionally to reflect the new transition probabilities. Green moving points demonstrate the system's evolution over time based on the current

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transition rules.

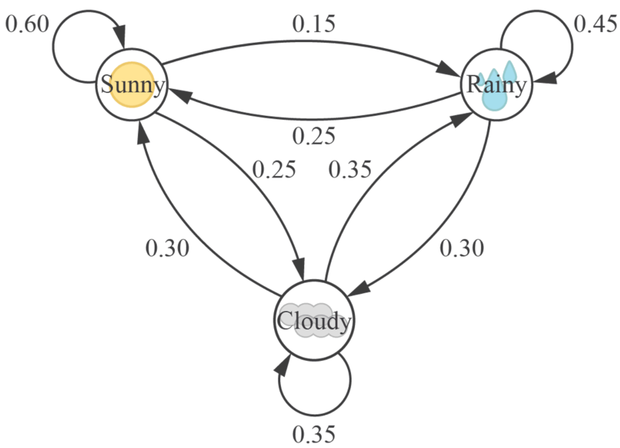
The tool provides real-time updates between the numerical matrix and graphical representation, helping users understand how probability adjustments affect state transitions. Users can experiment with different configurations to observe their impact on the system's behavior.

For example, when user sets the transition probabilities as $(P(A|A) = 0.3, P(B|A) = 0.5, P(C|A) = 0.2)$, they understand how a system starting in state A evolves. The visualization shows that after one transition, there's a 50% chance to move to state B (the most likely outcome), while staying in A or moving to C are less probable. Over multiple transitions, users can watch how the green points gradually distribute across all states according to these probabilities, with state B becoming increasingly dominant due to its high self-transition probability $(P(B|B) = 0.5)$ and strong incoming transitions. This demonstrates how steady-state probabilities emerge from repeated transitions.

Example 2

★★☆

Suriya decides to add a layer of complexity to the weather patterns in his video game. The new probabilities that he plans to use are shown in the transition state diagram below.



More information

This is a transition state diagram illustrating the probability of weather conditions transitioning from one state to another. The diagram consists of three main states, each represented by circles labeled 'Sunny', 'Rainy', and 'Cloudy'. Arrows between these states show the probabilities of transitioning from one state to another.

From 'Sunny', there's a 0.60 probability of remaining 'Sunny', a 0.25 probability of transitioning to 'Rainy', and a 0.15 probability of transitioning to 'Cloudy'. From 'Rainy', there's a 0.45 probability of remaining 'Rainy', a 0.25 probability of transitioning to 'Sunny', and a 0.30 probability of transitioning to 'Cloudy'. From 'Cloudy', there's a 0.35 probability of transitioning to 'Sunny', a 0.30 probability of transitioning to 'Rainy', and a 0.35 probability of remaining 'Cloudy'. The direction of transition is indicated by arrows, and the values on each arrow represent the transition probability.

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Given that it is rainy on a certain day, find the probability that it will be sunny three days later.

Steps	Explanation
Weather today sunny cloudy rainy **Weather tomorrow** sunny cloudy rainy $\begin{bmatrix} 0.60 & 0.30 & 0.25 \\ 0.25 & 0.35 & 0.30 \\ 0.15 & 0.35 & 0.45 \end{bmatrix}$	Begin by putting the information from the diagram into matrix form.
$T = \begin{bmatrix} 0.60 & 0.30 & 0.25 \\ 0.25 & 0.35 & 0.30 \\ 0.15 & 0.35 & 0.45 \end{bmatrix}$	Write down the transition matrix.
$T^3 = \begin{bmatrix} 0.4295 & 0.3955 & 0.3885 \\ 0.2905 & 0.2965 & 0.2975 \\ 0.2800 & 0.3080 & 0.3140 \end{bmatrix}$	Use your graphic display calculator to find T^3 .
There is a 38.85% chance of it being sunny three days after a rainy day.	Select the appropriate value from the solution matrix.

Exam tip

Checking to see that the values in each column of your solution matrix sum to 1 is a quick way to check your work during exams.

3 section questions

4. Probability and statistics / 4.19 Markov chains

Markov chains

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/markov-chains-id-28005/print/) Assign

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Making connections

Recall the matrix notation you learned in section 1.14.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/definitions-and-equality-id-27429/). An $m \times n$ matrix contains m rows and n columns. The specific element u_{ij} within a matrix is found in the i^{th} row and the j^{th} column.

Important

A transition matrix T_{ij} represents the probability of moving from state j to state i .

The cafeteria at a certain large company uses statistical analysis when ordering supplies for each day. Using the historical data it collected, the following transition matrix was created.

Eats at cafeteria today?

yes no

Eats at cafeteria tomorrow?

yes

no

$$\begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}$$

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It is known that 792 out of the company's 964 employees ate at the cafeteria today.

How can you use the transition matrix to predict how many of the employees will eat in the cafeteria tomorrow?

Important

Markov chains are very useful in predicting the future states of a system when the current state is known. The future state, s_n , after n transitions can be found using the formula $s_n = T^n s_0$, where s_0 is the initial state matrix .

Activity

Let $s_0 = \begin{bmatrix} 792 \\ 172 \end{bmatrix}$ represent how the employees chose to eat today. Substituting into the formula for s_1 gives you $s_1 = \begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix} \begin{bmatrix} 792 \\ 172 \end{bmatrix}$.

1. The first step in multiplying the two matrices involves the calculation $0.85 \times 792 + 0.10 \times 172$. In reference to the context of the problem, describe the meaning of each part of this calculation.
2. Write down the next step in multiplying the two matrices. Describe the meaning of each part of what you have written down.
3. The result of the multiplication is $s_1 = \begin{bmatrix} 690.4 \\ 273.6 \end{bmatrix}$. State what information this gives the cafeteria.
4. Imagine that the initial state matrix describes where employees chose to eat on Friday. Employees at the company do not work on the weekend. Explain how the cafeteria could use this system to predict the total number of employees who will eat at the cafeteria during the next work week.
5. What are the limitations of using Markov chains as described in Step 4?

Example 1

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On a certain Monday, it was recorded that only 589 of the 964 employees ate at the cafeteria. Using the transition matrix shown below, calculate a prediction for how many employees will eat at the cafeteria on Friday.

$$T = \begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}$$

Use the information given to create the initial state matrix.

$$s_0 = \begin{bmatrix} 589 \\ 375 \end{bmatrix}$$

Substitute the information into the formula, noting that Friday is 4 days after Monday.

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$$s_n = T^n s_0$$
$$s_4 = \begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}^4 \begin{bmatrix} 589 \\ 375 \end{bmatrix}$$

Use your graphic display calculator to solve for the solution matrix.

$$s_4 = \begin{bmatrix} 449.957\ldots \\ 514.042\ldots \end{bmatrix} \approx \begin{bmatrix} 450 \\ 514 \end{bmatrix}$$

Select the appropriate value from the solution matrix.

$\therefore$ it is predicted that 450 employees will eat at the cafeteria on Friday.

International Mindedness

What do you eat for lunch each day? School and work lunches around the world have a great deal of variation that depends on the country you are in. This article (https://www.thedailymeal.com/travel/what-office-lunches-look-around-world-slideshow/slide-2) discusses some of the typical meals that students eat in various countries around the world.

3 section questions

4. Probability and statistics / 4.19 Markov chains

Long-term probabilities

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/longterm-probabilities-id-28006/print/) Assign

Steady state vector

Let us consider again the example in section 4.19.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/markov-chains-id-28005/), the cafeteria at a large company. The transition matrix the cafeteria used to predict how many employees would eat at the cafeteria on future days is shown below.

Eats at cafeteria today?

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yes no

Eats at cafeteria tomorrow?

yes
no

$$\begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}$$

In **Example 1** from the previous section you found that if 589 out of the 964 employees at the cafeteria on Monday, a Markov chain using the above transition matrix predicts that only 450 would eat at the cafeteria on Friday. What would happen if you continued the Markov chain? Would the number of employees continue to decrease?

Activity

Recall the formula $s_n = T_n s_0$.

1. Using your graphic display calculator, calculate s_{10}, s_{20}, s_{30}, s_{40} and s_{50} where $T = \begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}$ and $s_0 = \begin{bmatrix} 589 \\ 375 \end{bmatrix}$.
2. Is the solution matrix tending towards certain values? If so, what are the values?
3. Calculate the value of s_{51} by multiplying the transition matrix to your solution matrix for s_{50} . What do you notice about the values of s_{50} and s_{51} ?
4. Repeat Step 1 using the initial state matrix $s_0 = \begin{bmatrix} 792 \\ 172 \end{bmatrix}$.
5. What values is the solution matrix tending towards now?
6. Do the values in the initial state matrix affect the values that the solution matrix tends towards?
7. If you were the manager in charge of sales at the cafeteria, what significance would these values have for you?
8. Consider other possible factors affecting sales, and thus discuss how valid this model would be in predicting long-term trends in the number of employees eating at the cafeteria.

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In Step 3 of the above activity, you found that s_{50} and s_{51} contained values identical to the third decimal place. If you continued finding further future values, consecutive results would tend towards being identical. The solution matrix the values would tend towards is known as the steady state vector.

✓ **Important**

The steady state vector of a square transition matrix, T , is the solution matrix, q , that satisfies the equation $Tq = q$.

Example 1

★☆☆

A large company has 964 employees. The number of employees that eat at the cafeteria each day can be predicted using the transition matrix shown below.

Eats at cafeteria today?

yes no

Eats at cafeteria tomorrow?

yes

no

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$$\begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}$$

Show that the steady state vector for this system is $q = \begin{bmatrix} 385.6 \\ 578.4 \end{bmatrix}$

As the command term here is 'show that', you must show your work in deriving the answer. Begin by substituting the transition matrix into the equation:

$$Tq = q$$
$$\begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix} \begin{bmatrix} y \\ n \end{bmatrix} = \begin{bmatrix} y \\ n \end{bmatrix}$$

Multiply the matrices to create simultaneous equations:

$$0.85y + 0.10n = y$$

$$0.15y + 0.90n = n$$

Rearrange each equation to be equal to zero:

$$-0.15y + 0.10n = 0$$

$$0.15y - 0.10n = 0$$

Use the fact that there are 964 employees to create a third equation:

$$y + n = 964$$

You have now three equations for the two unknowns y and n. One way of solving this system of equations is using matrices. Write all three equations together as a matrix of the coefficients:

$$\begin{bmatrix} 1 & 1 & 964 \\ -0.15 & 0.10 & 0 \\ 0.15 & -0.10 & 0 \end{bmatrix}$$

Use your graphic display calculator to row-reduce the matrix (you can find help on how to access this option on your calculator below this exercise) :

$$\begin{bmatrix} 1 & 0 & 385.6 \\ 0 & 1 & 578.4 \\ 0 & 0 & 0 \end{bmatrix}$$

The first line of this reduced matrix gives $y = 385.6$ and the second line gives $n = 578.4$.

Write final steady state vector:

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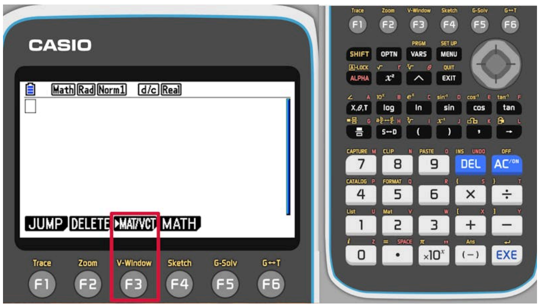
$$q = \begin{bmatrix} 385.6 \\ 578.4 \end{bmatrix}$$

In **Example 1** the solution used the row-reduced echelon form of a matrix to solve an equation system. In the instructions below you can find help on how to access this feature on your calculator.

Steps	Explanation
To find the reduced row echelon form of the matrix $\begin{pmatrix} 2 & -1 & 1 & 3 \\ 5 & 4 & -1 & 0 \\ 6 & 2 & 7 & -5 \end{pmatrix}$, open the calculator mode, ...	
... and press F3 to open the page to work with matrices.	

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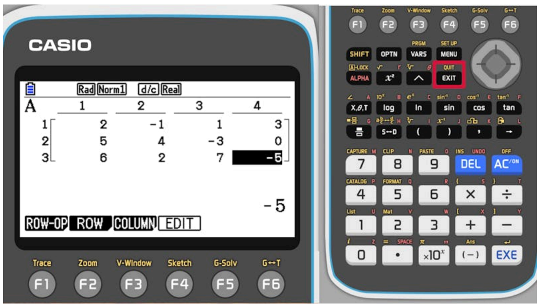
Steps	Explanation
The calculator needs to know the matrix. You can store it in any of the available matrix memory. First press F3 to tell the dimensions.	
The matrix has three rows and four columns.	
Enter the entries of the matrix. When you are done, press EXIT until you get back to the main calulation screen. The matrix is now stored in matrix memory A.	

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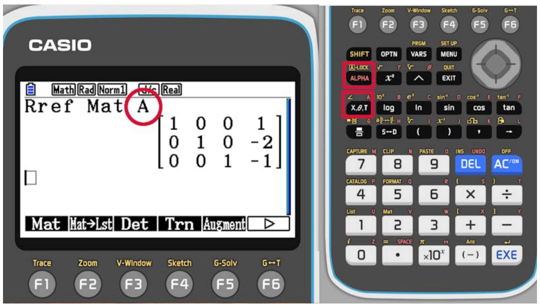
Steps	Explanation
You now need to find the option that calculates the reduced row echelon form. Press OPTN, ...	
... press F2 for the matrix related options, ...	
... press F6 to scroll to see the next options, ...	

Student view

Steps	Explanation
... and finally F5 to choose the tool that calculates the reduced row echelon form (Rref).	
Scroll back to the previous options and press F1 to choose a matrix ...	
... and choose the letter corresponding to the memory where you stored the matrix. After pressing EXE, the reduced matrix is displayed.	

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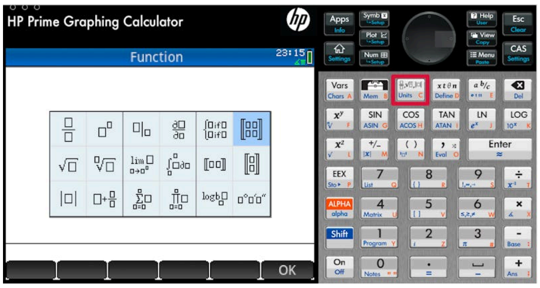
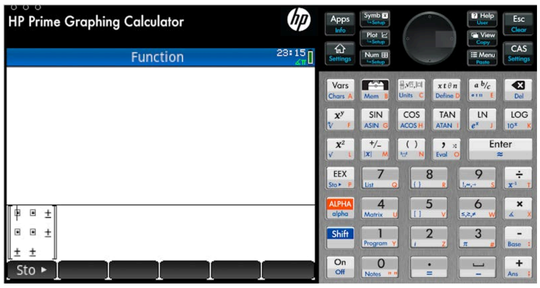
Steps	Explanation
To find the reduced row echelon form of the matrix $\begin{pmatrix} 2 & -1 & 1 & 3 \\ 5 & 4 & -1 & 0 \\ 6 & 2 & 7 & -5 \end{pmatrix}$, enter the home screen of any application.	
The calculator needs to know the matrix, so open the template menu and choose the matrix template.	
Originally you are offered to fill a 2×2 matrix template. Start entering the entries, the template will automatically expand.	

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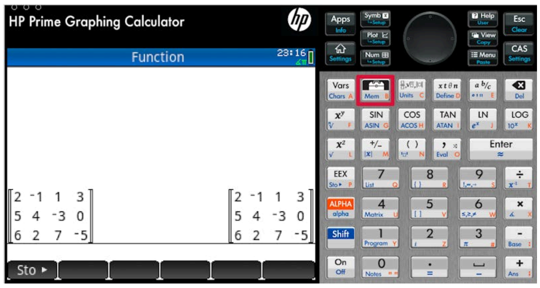
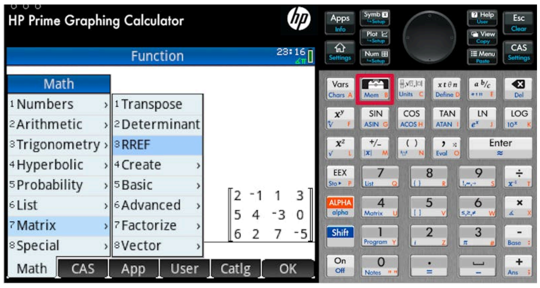
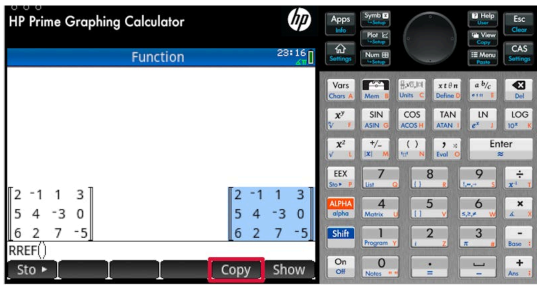
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>
When the matrix is entered, you need to find the option that calculates the reduced row echelon form.

Open the toolbox ...
</td><td>


</td></tr>
<tr><td>
... and find the tool that calculates the reduced row echelon form (RREF).
</td><td>


</td></tr>
<tr><td>
One way of copying the matrix inside the brackets after RREF is to tap on the matrix and then choose the copy option.
</td><td>


</td></tr>
</table>

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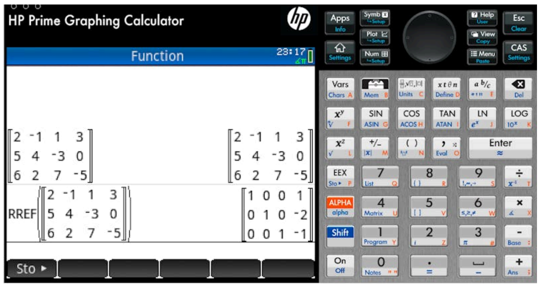
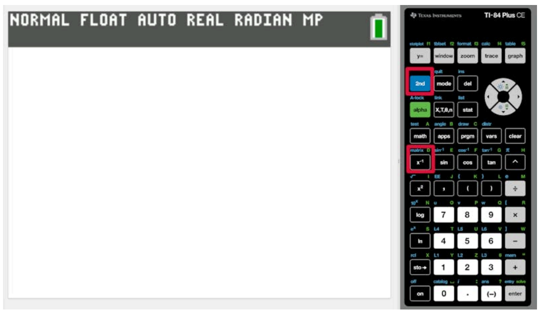
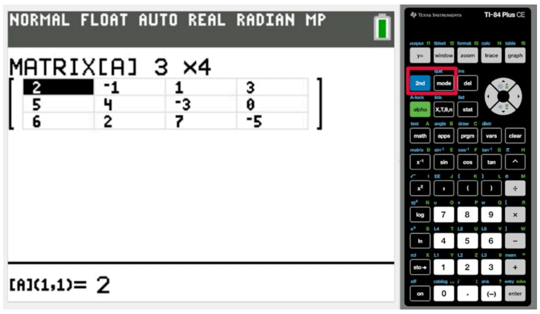
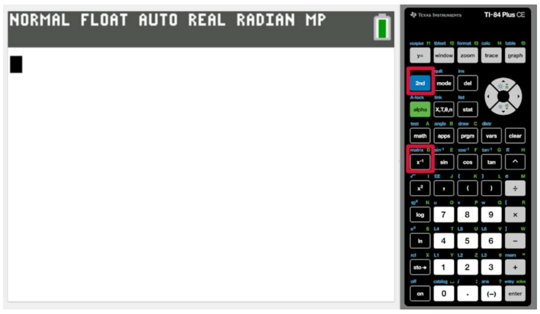
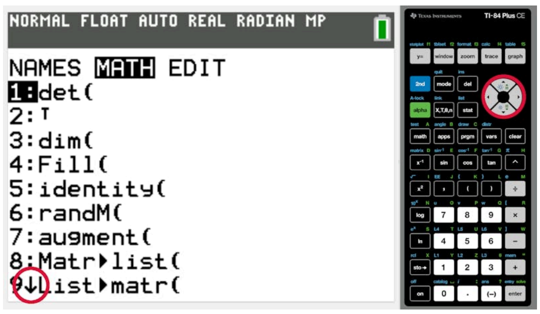
Steps	Explanation
After pressing enter, the reduced matrix is displayed.	

Steps	Explanation
To find the reduced row echelon form of the matrix $\begin{pmatrix} 2 & -1 & 1 & 3 \\ 5 & 4 & -1 & 0 \\ 6 & 2 & 7 & -5 \end{pmatrix}$, open the options to work with matrices ...	
... and choose to edit any of the matrices.	

Steps	Explanation
Enter the entries of the matrix. When you are done, quite the editing mode to get back to the main calulation screen. The matrix is now stored in matrix memory A.	
You now need to find the option that calculates the reduced row echelon form. Open the options to work with matrices again ...	
... choose the math options and scroll down ...	

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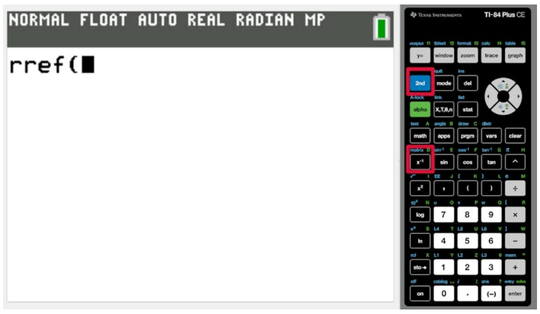
Steps	Explanation
... until you find the tool that calculates the reduced row echelon form (rref).	
You need to tell the calculator which matrix you want to work with, so open the matrix menu yet again...	
... and choose the name of the matrix where you stored the entries.	

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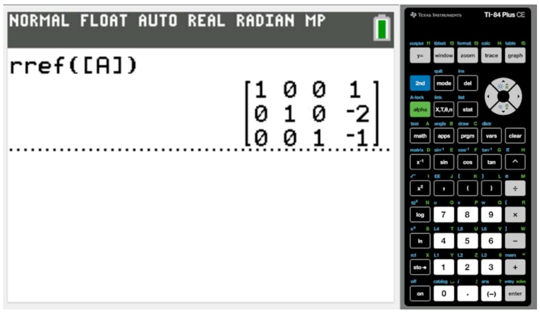
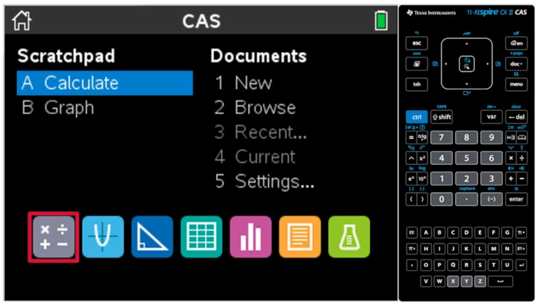
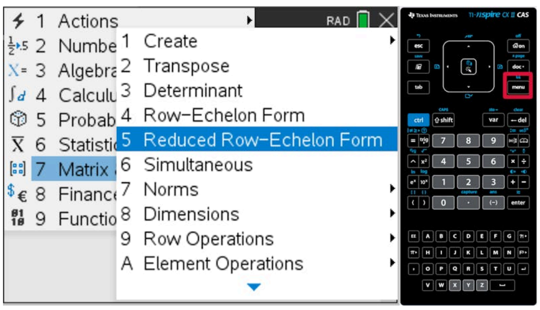
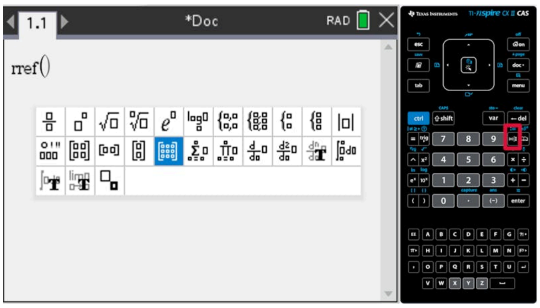
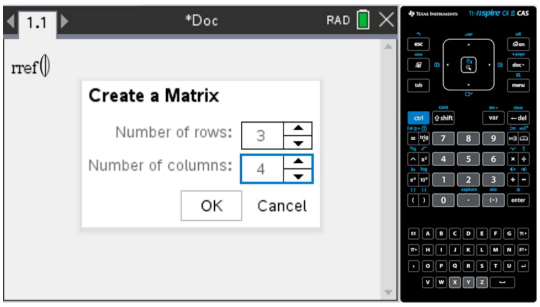
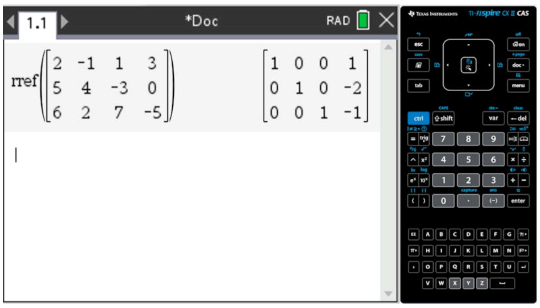
Steps	Explanation
After pressing enter, the reduced matrix is displayed.	

Steps	Explanation
To find the reduced row echelon form of the matrix $\begin{pmatrix} 2 & -1 & 1 & 3 \\ 5 & 4 & -1 & 0 \\ 6 & 2 & 7 & -5 \end{pmatrix}$, open a calculator page.	
Open the menu and find the tool that calculates the reduced row echelon form of a matrix.	

Student view

Steps	Explanation
To enter the matrix, open the template menu and choose the matrix template.	
The matrix has three rows and four columns.	
Enter the entries of the matrix. After pressing enter, the reduced matrix is displayed.	

Making connections

Recall, from subtopic 1.15 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27436/), the equation $Ax = \lambda x$, where A is an $n \times n$ matrix, x is an eigenvector, and λ is an eigenvalue.

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

The solution for the steady state vector, q, in the equation $Tq = q$ is the eigenvector corresponding to the eigenvalue equal to 1.

Long-term probabilities

The steady state vector you found in the example above gave you an estimate of how many employees would eat (and not eat) in the cafeteria if the probabilities within the transition matrix held true over time. As you saw, you will arrive at this estimate independent of the values within your initial state matrix. If the values in the initial state matrix do not affect the outcome of the steady state vector, then what does determine its outcome?

To answer this, let us look again at the transition matrix $T = \begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix}$ and substitute it into the equation $Tq = q$:

$$\begin{bmatrix} 0.85 & 0.10 \\ 0.15 & 0.90 \end{bmatrix} \begin{bmatrix} y \\ n \end{bmatrix} = \begin{bmatrix} y \\ n \end{bmatrix}$$

Activity

In **Example 1**, you solved for the expected **number** of employees that will eat at the cafeteria. For this activity, you will solve for the expected **percentage** of employees that will eat at the cafeteria.

Let $y + n = 1$ since y and n have to include all employees at the company.

1. Following a similar method as in **Example 1**, solve the systems of equations for y and n.
2. Multiply your values for y and n to 964. How do the numbers compare to the result of **Example 1**?
3. Evaluate T^n for $n = 5, 10, 15, 20$ and 25. What are the values tending towards?

The values that you found in the above activity are the long-term probabilities of the cafeteria system. Over time, if the system continues to follow the transition matrix, the number of employees eating in the cafeteria will trend towards 40% of the total number of employees.

Exam tip

There are many changes that could occur within the system that could alter the trends of the numbers. For example, more people may eat in the cafeteria in winter, when they want a hot meal, than in summer, when they like to bring food from home into work, which can be eaten outside. Be sure to keep this in mind when working on problems, since exam questions could ask you to reflect on possible limitations of the system.

Example 2

★★☆

The transition matrix for the game Suriya was developing in **Example 1** from section 4.19.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/transition-matrices-id-28004/) is shown below.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Weather today

sunny
rainy

Weather tomorrow

sunny
rainy

$$\begin{bmatrix} 0.75 & 0.45 \\ 0.25 & 0.55 \end{bmatrix}$$

Find the long-term probability of a day being sunny in Suriya's game.

Give your answer as an exact fraction.

Begin by substituting the transition matrix into the equation $Tq = q$:

$$Tq = q$$
$$\begin{bmatrix} 0.75 & 0.45 \\ 0.25 & 0.55 \end{bmatrix} \begin{bmatrix} S \\ R \end{bmatrix} = \begin{bmatrix} S \\ R \end{bmatrix}$$

Multiply the matrices to create simultaneous equations:

$$0.75S + 0.45R = S$$

$$0.25S + 0.55R = R$$

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Rearrange each equation to be equal to zero:

$$-0.25S + 0.45R = 0$$

$$0.25S - 0.45R = 0$$

Write the equations together with $S + R = 1$ as a matrix of the coefficients:

$$\begin{bmatrix} 1 & 1 & 1 \\ -0.25 & 0.45 & 0 \\ 0.25 & -0.45 & 0 \end{bmatrix}$$

Use your graphic display calculator to row-reduce the matrix:

$$\begin{bmatrix} 1 & 0 & 0.642857\ldots \\ 0 & 1 & 0.357142\ldots \\ 0 & 0 & 0 \end{bmatrix}$$

Use your graphic display calculator to transform the numbers to fractions to obtain the exact answer:

$$\begin{bmatrix} 1 & 0 & \frac{9}{14} \\ 0 & 1 & \frac{5}{14} \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore S = \frac{9}{14}$ and $R = \frac{5}{14}$.

Thus the long-term probability of it being a sunny day in Suriya's game is $\frac{9}{14}$.

You may be asked to write the long-term probability matrix instead of finding the long-term probability of a specific event happening.

✓ **Important**

The long-term probability matrix includes the long-term probabilities and has the same dimensions as the transition matrix.

The long-term probability matrix for Suriya's game in **Example 2** is $\begin{bmatrix} \frac{9}{14} & \frac{9}{14} \\ \frac{5}{14} & \frac{5}{14} \end{bmatrix}$

⚠ **Be aware**

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

It is also possible to find the long-term probabilities and long-term matrix by raising the transition matrix to a sufficiently large value of n and then having your calculator convert the decimals to fractions. However, be sure you also understand how to manually calculate the probabilities as you may be asked to show your work in the exams.

3 section questions

4. Probability and statistics / 4.19 Markov chains

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-28007/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- calculate the probability of the next state of a system by using a transition matrix
- predict future states of a system by finding different powers of a transition matrix
- interpret transition state diagrams
- calculate the steady state vector and long-term probabilities of a Markov chain.

4. Probability and statistics / 4.19 Markov chains

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-28008/print/) Assign

House finches, native to North America, can be infected and sometimes killed by a bacterial pathogen

Credit: Jerry Deutsch Getty Images

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

1. Click here (https://besjournals.onlinelibrary.wiley.com/doi/10.1111/j.2041-210X.2010.00018.x) to read the paper *A primer on the application of Markov chains to the study of wildlife disease dynamics* .
2. According to the paper, how are Markov chains being used to analyse the infection and recovery of the house finch.
3. Consider the graphs from the paper that are shown here (https://besjournals.onlinelibrary.wiley.com/doi/10.1111/j.2041-210X.2010.00018.x#f1). How were the graphs generated?
4. Think back to some of the examples within this section. How could you generate similar graphs for the examples?
5. Now click here (https://vlab.amrita.edu/?sub=3&brch=65&sim=183&cnt=1) to see an example of a Leslie matrix.
6. How are Leslie matrices similar to Markov chains and transition matrices?

Rate subtopic 4.19 Markov chains

Help us improve the content and user experience.

Student view