

[?\(https://intercom.help/kognity\)](https://intercom.help/kognity)

Overview  
(/study/ap...)  
122-  
cid-  
754029/k

---

Teacher view

**Index**

- [The big picture](#)
- [Graphing using a GDC](#)
- [Operations with functions](#)
- [Checklist](#)
- [Investigation](#)



Table of  
contents

2. Functions / 2.3 Graphs of functions



Notebook



Glossary



Reading  
assistance

# The big picture

This set of ordered pairs can be plotted on a graph, which is a useful representation to visualise relevant properties of functions.

To determine the properties of a function you can either use analytical methods, which you will learn in the topic of Calculus, or you can use a graphic display calculator (GDC).

In this subtopic, you will learn how to use a graphic display calculator (GDC) to:

- graph functions
- determine the axes-intercepts of graphs.

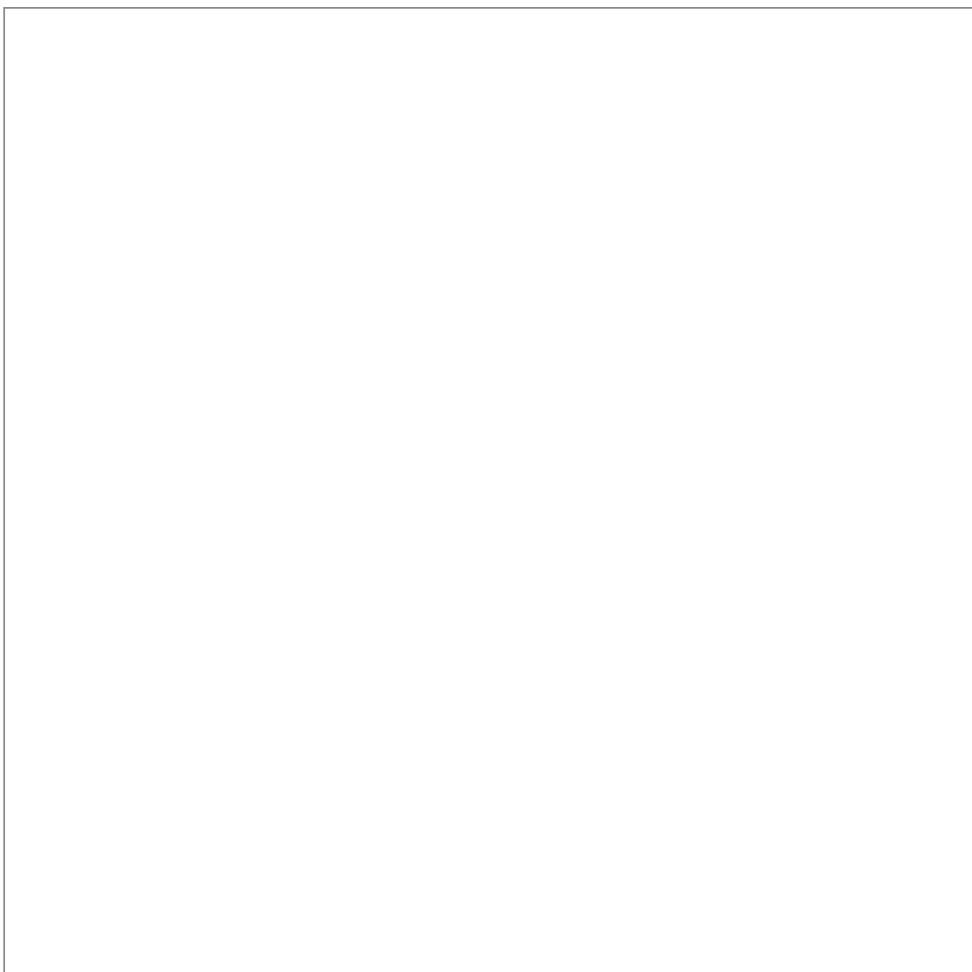
In the applet below, use the slider ‘Change function’ to visualise some of the graphs that you will encounter in this subtopic.



Student  
view



Overview  
(/study/ap  
122-  
cid-  
754029/k



## Interactive 1. Graphing Functions: The Big Picture.

More information for interactive 1

This interactive tool allows users to explore and visualize various functions, such as  $f(x) = 2$ ,  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = x^3$ ,  $f(x) = x^4$ ,  $f(x) = \frac{1}{x}$ ,  $f(x) = |x|$ ,  $f(x) = e^x$  and  $f(x) = \ln(x)$ , by adjusting a slider (“Change function”) to switch between different graph types. Users can observe how these functions map inputs  $x$  to outputs  $y = f(x)$  generating ordered pairs  $(x, y)$  that are plotted on the graph.

Users can study key features of the tool include:

1. Identifying  $x$ - and  $y$ -intercepts of functions.
2. Observing how the shape and position of a graph change with different function types.
3. Comparing the behavior of linear, quadratic, exponential, and logarithmic functions.

Designed to build conceptual understanding, this tool provides an intuitive way to analyze how functions behave and to explore the relationships between their inputs and outputs. Through hands-on interaction, users gain deeper insight into the graphical and algebraic characteristics of different mathematical functions.



Student  
view

 **Concept**

Functions represent relationships between quantities, and graphs allow us to visualise these relationships. While learning how to use a GDC to graph functions and find the axes-intercepts, reflect on whether studying the graph of a function contains the same level of mathematical rigour as studying the function algebraically. Think of the advantages and disadvantages of the various representational forms of functions.

2. Functions / 2.3 Graphs of functions

## Graphing using a GDC

### The graph of a function

 **Important**

The graph of a function  $y = f(x)$  is the set of all points of the form  $(x, y)$  that satisfy the equation of the function  $y = f(x)$ .

### From screen to paper

There will be times when you must use technology to obtain the graph of a function and then make a sketch of it. In this section, you will use a GDC to graph functions and determine the  $x$ -intercepts and  $y$ -intercept of a graph.

 **Exam tip**

It is important to realise that if you are asked to **sketch** a graph, the examiner expects you to give a general idea of the required shape of the graph and should include the  $x$ -intercepts and  $y$ -intercept of the graph.

However, when you are asked to **draw** a graph, the examiner expects you to use pencil and ruler for drawing straight lines, and the graphs should be drawn to scale. Points should be plotted accurately and joined in a straight line or with a



Overview  
(/study/app/m/sid-122-cid-754029/k)

smooth curve.

## x -intercepts and y -intercepts

### ✓ Important

The  $x$  -intercepts of a graph are the points where the graph of a function crosses the  $x$ -axis.

The  $y$ -intercept of a graph is the point where the graph of a function crosses the  $y$ -axis.

For example, consider the function  $f(x) = 2x^3 + 7x^2 + 2x - 1$ .

To obtain the graph and determine the  $x$ - and  $y$ -intercepts of the function  $f(x)$ , use a GDC as shown below.

### Section

Student... (0/0)

Feedback

Print

(/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26185/print/)

Assign

### Steps

### Explanation

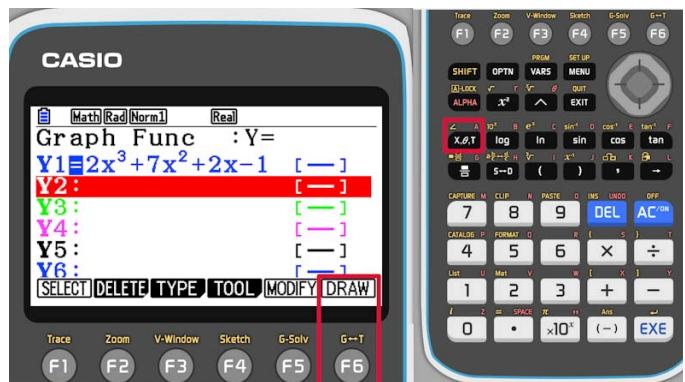
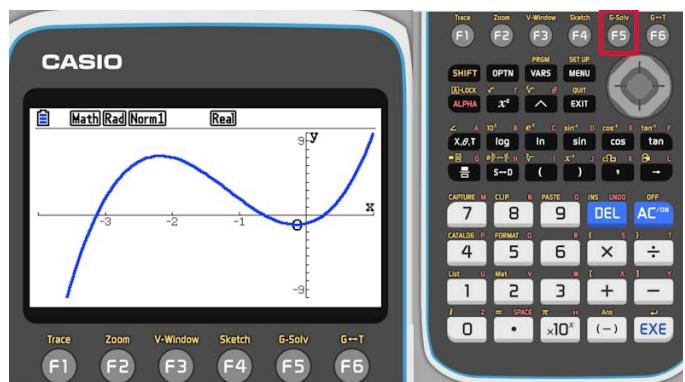
Choose the graph mode.



Student view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

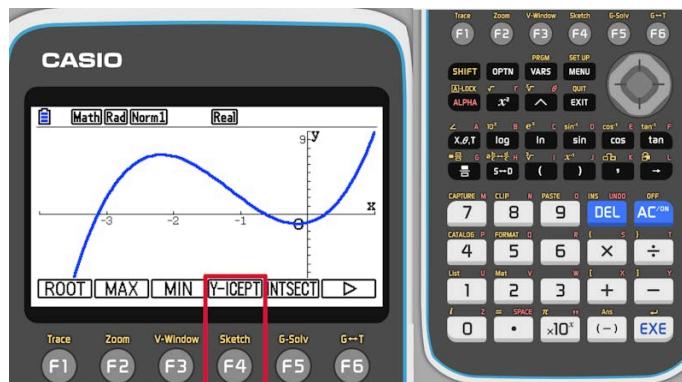
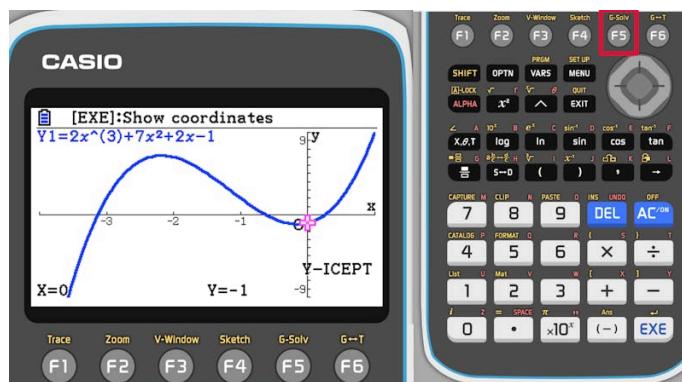
---

Steps	Explanation
<p>Enter the formula (remember to use the variable button for <math>x</math>) and press F6 to view the graph (before viewing the graph you may want to set an appropriate viewing window using the V-Window option).</p>	
<p>Once you see the graph, pressing F5 (G-Solv) will bring up options to analyse the graph.</p>	



Student  
view

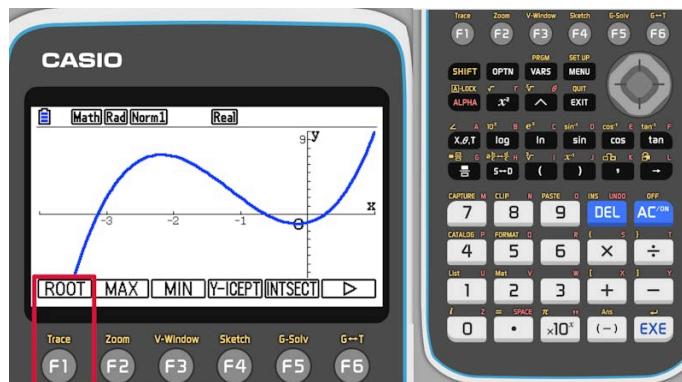
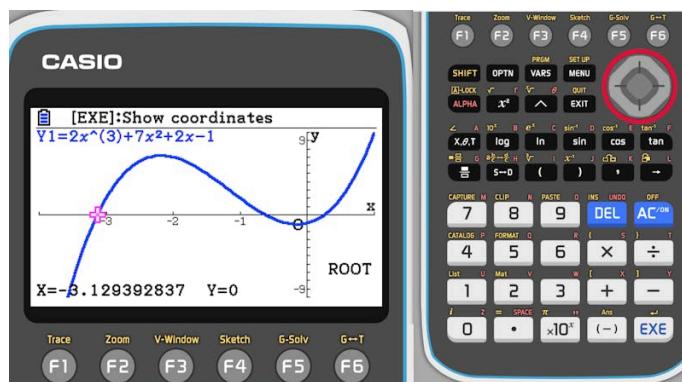
Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

Steps	Explanation
<p>One of the options is to find the <math>y</math>-intercept.</p>	
<p>The cursor moves to the <math>y</math>-intercept and the coordinates are displayed. Press F5 again to bring back the options to analyse the graph.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

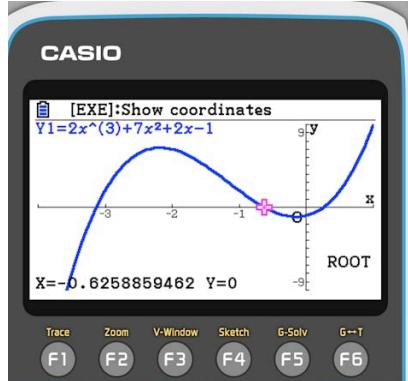
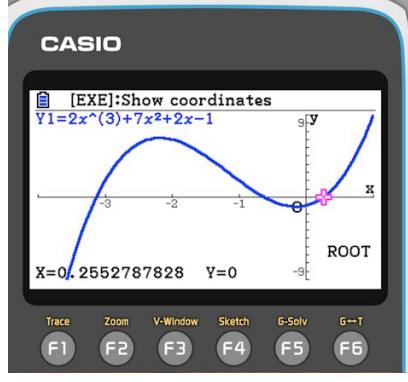
Steps	Explanation
<p>To find the <math>x</math>-intercepts, you can look for the roots.</p>	
<p>The calculator gives you the roots one at a time. To find the next one, simply move to the right.</p>	



Student view



Overview  
(/study/ap/  
122-  
cid-  
754029/k)

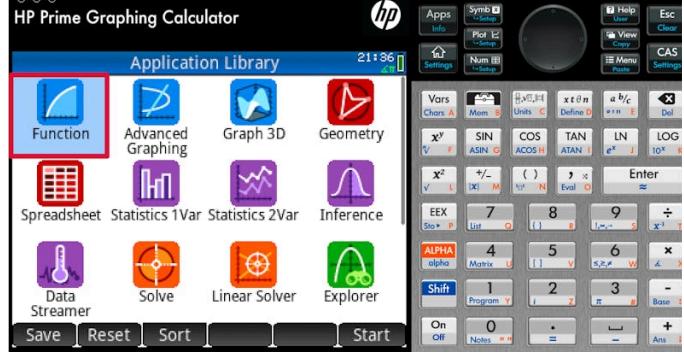
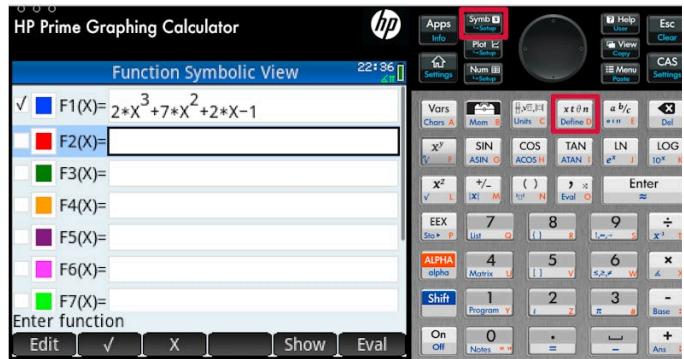
Steps	Explanation
<p>Move to the right again to find the next root.</p>	 
	 



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

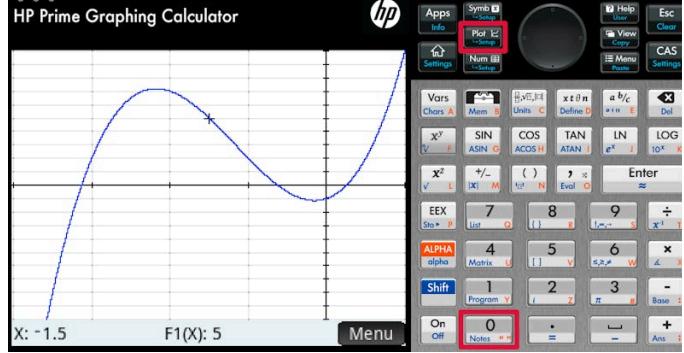
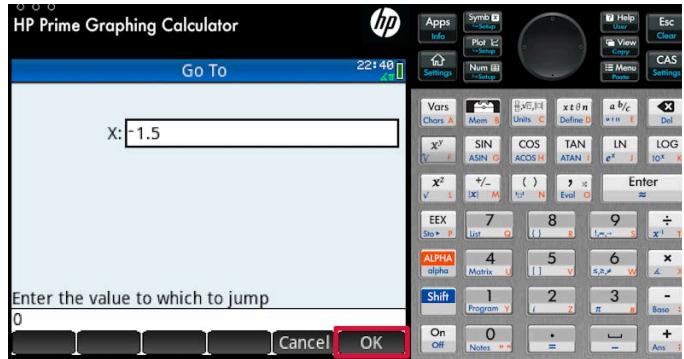
Steps	Explanation
<p>Choose the graph mode.</p>	 <p>The screenshot shows the HP Prime Graphing Calculator's Application Library. The 'Function' icon is highlighted with a red box. Other icons include Advanced Graphing, Graph 3D, Geometry, Spreadsheet, Statistics 1Var, Statistics 2Var, Inference, Data Streamer, Solve, Linear Solver, and Explorer. At the bottom are Save, Reset, Sort, and Start buttons.</p>
<p>In the symbolic view you can enter the formula (remember to use the variable button for <math>x</math> ).</p>	 <p>The screenshot shows the HP Prime Graphing Calculator in Function Symbolic View. The formula <math>F1(X) = 2*X^3 + 7*X^2 + 2*X - 1</math> is entered. Below it, there is a field for <math>F2(X) =</math> which is currently empty. There are also fields for <math>F3(X)</math>, <math>F4(X)</math>, <math>F5(X)</math>, <math>F6(X)</math>, and <math>F7(X)</math>. At the bottom are Edit, Show, and Eval buttons. The 'Symbol' button on the top right is highlighted with a red box.</p>



Student  
view

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k

---

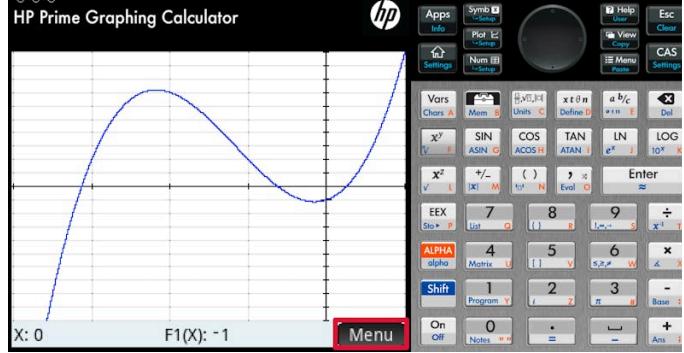
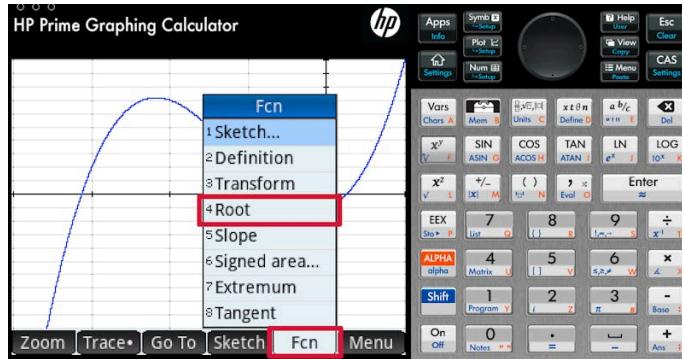
Steps	Explanation
<p>In plot view you can see the graph (before viewing the graph you may want to set an appropriate viewing window using the plot setup option).</p> <p>There is no dedicated option to find the <math>y</math>-intercept, but you can move the cursor there by selecting <math>x = 0</math> as the first coordinate. Simply press 0.</p>	
<p>You can specify any <math>x</math>-value. The value <math>x = 0</math> gives the <math>y</math>-intercept.</p>	



Student  
view

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k

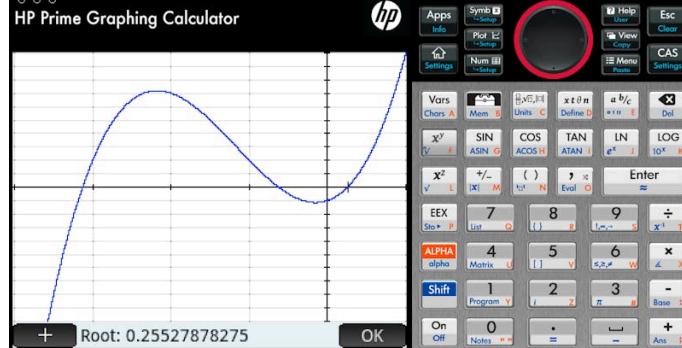
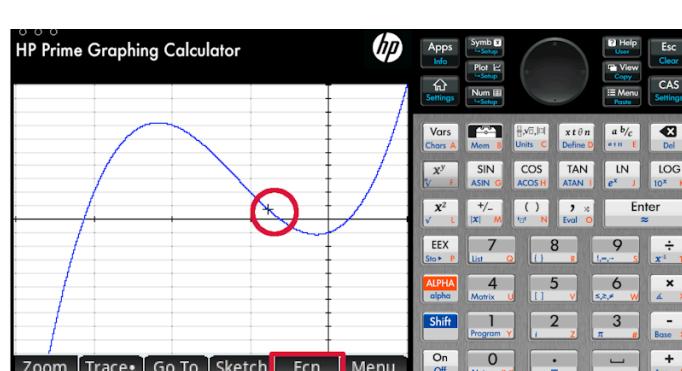
---

Steps	Explanation
<p>It is not very visible now, but the cursor is on the <math>y</math>-intercept and the coordinates are displayed in the bottom row.</p> <p>To find the <math>x</math>-intercept, press menu to bring up the options to analyse the graph.</p>	
<p>To find the <math>x</math>-intercepts, you can look for the roots.</p>	



Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k

---

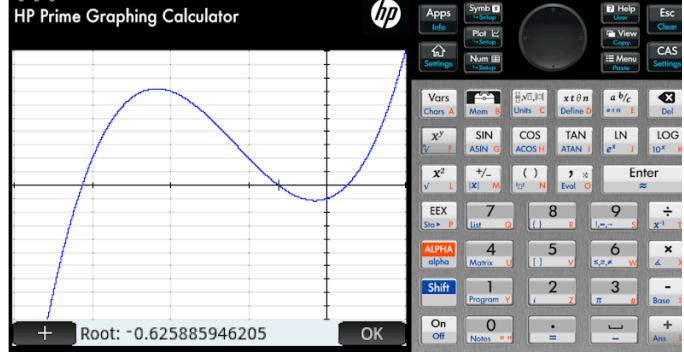
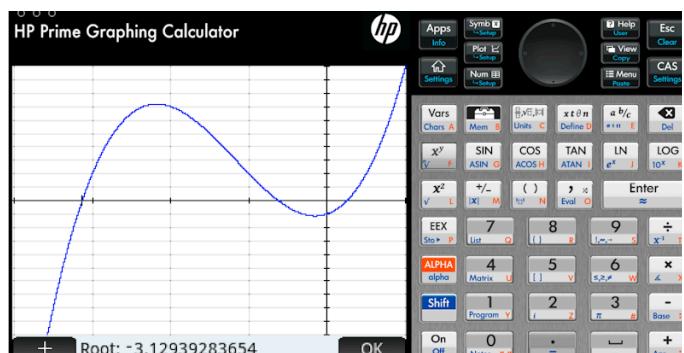
Steps	Explanation
<p>The calculator will move the cursor to one of the roots.</p> <p>To find another root, you need to move the cursor close to the one you are interested in.</p>	
<p>The calculator will find the root closest to the cursor.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

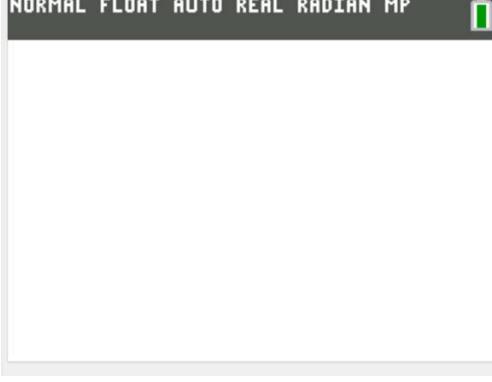
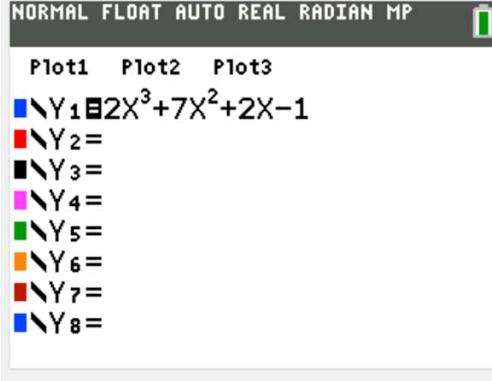
Steps	Explanation
<p>The cursor is not very visible, but it is on the second <math>x</math>-intercept now.</p>	
<p>If you start the root search process close the the leftmost <math>x</math>-intercept, the calculator will find that one.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

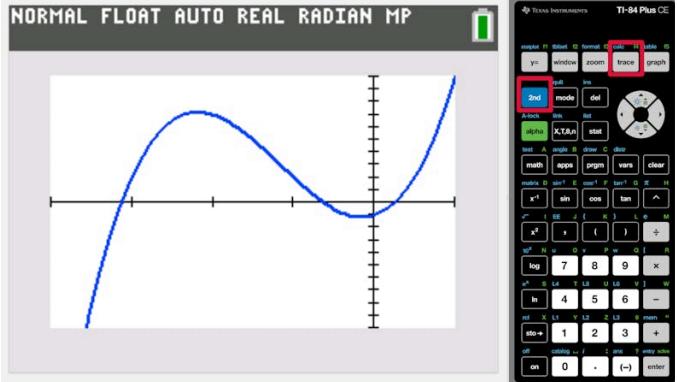
---

Steps	Explanation
Choose the option to define a function.	
Enter the formula (remember to use the variable button for $x$ ) and press the button that brings up the graph (before viewing the graph you may want to set an appropriate viewing window using the window option).	



Student  
view

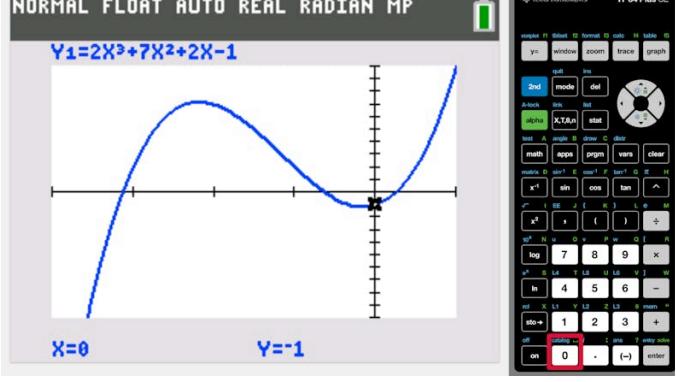
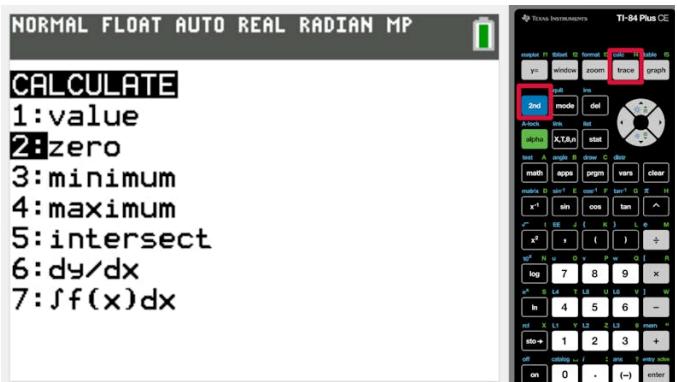
Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k

Steps	Explanation
<p>You can analyse a graph through the calculate menu.</p>	
<p>To find the <math>y</math> intercept you can search for the value at <math>x = 0</math>.</p>	



Student  
view

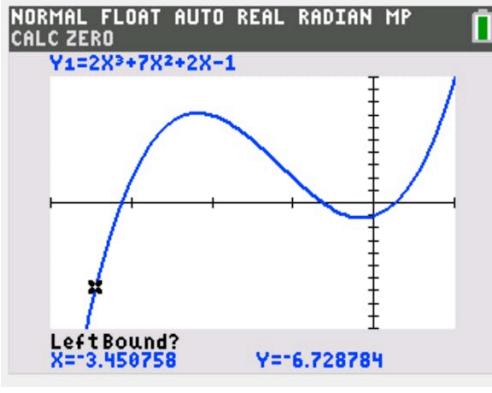
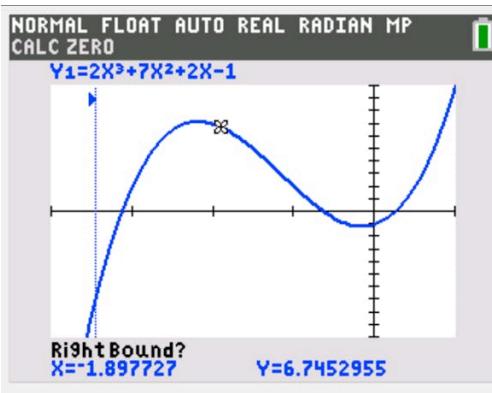
Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

Steps	Explanation
<p>After entering 0 the cursor will move to the <math>y</math>-intercept.</p>	
<p>To find the <math>x</math>-intercepts, look for the zeroes in the calculate menu.</p>	



Student  
view

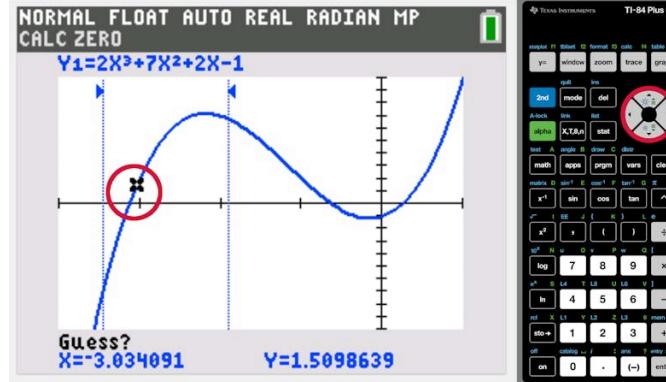
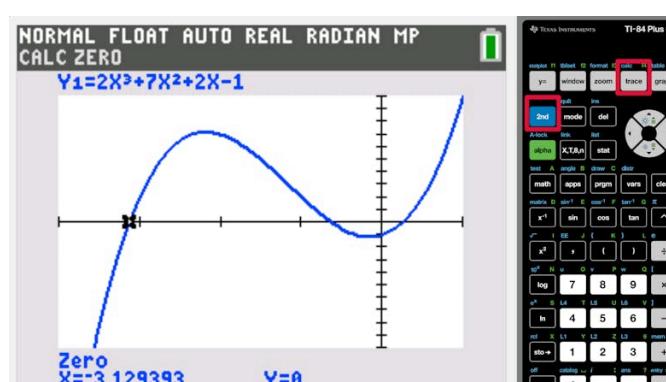
Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k  
—

Steps	Explanation
<p>Before it displays the <math>x</math>-intercept, the calculator needs more information. Move the cursor to the left of the <math>x</math>-intercept you are interested in and press enter.</p>	
<p>Next, the calculator asks for an upper bound for the <math>x</math>-intercept.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k  
—

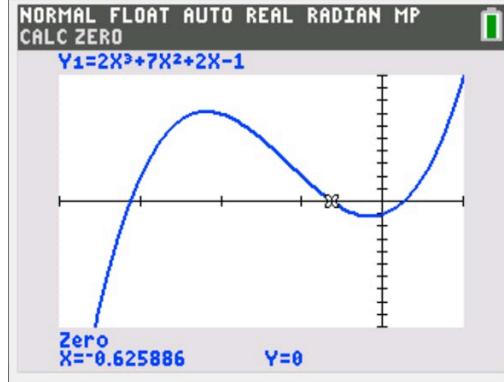
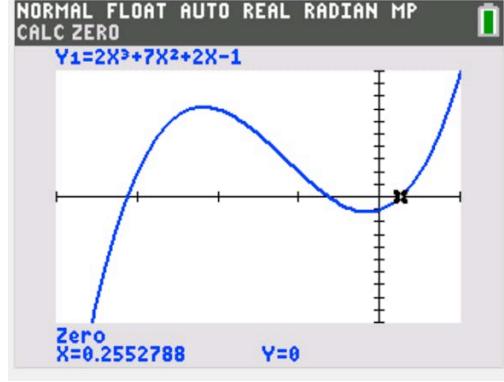
Steps	Explanation
<p>Next you need to move close to the <math>x</math>-intercept. The calculator will find the one closest to your guess within the bounds you specified. Of course, if there is only one intercept, the guess is not important (as long as it is within the bounds).</p>	
<p>The calculator will find the <math>x</math>-intercepts one at a time. To find the other <math>x</math>-intercepts, you need to repeat the process through the calculate menu.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

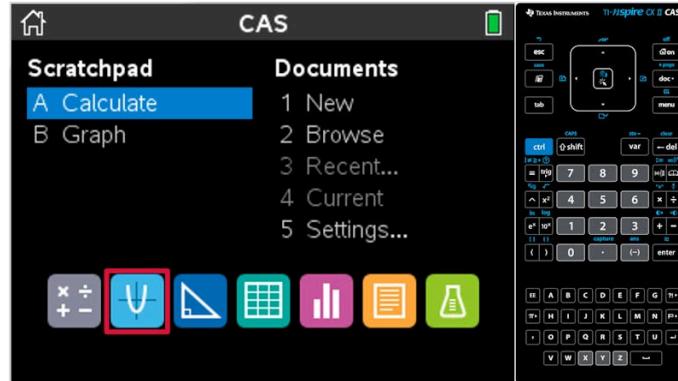
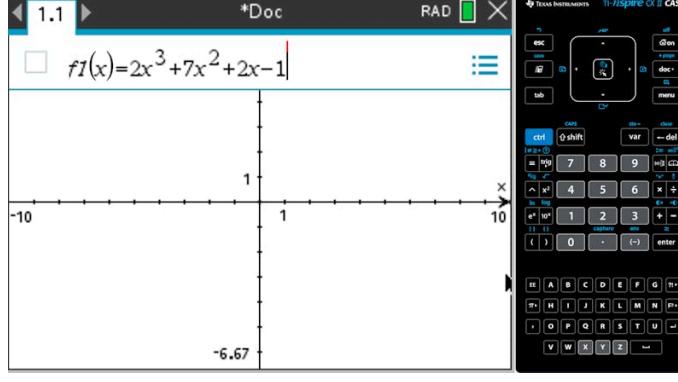
Steps	Explanation
	
	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

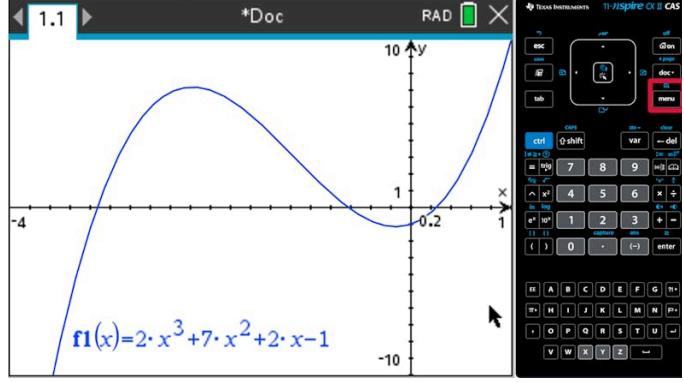
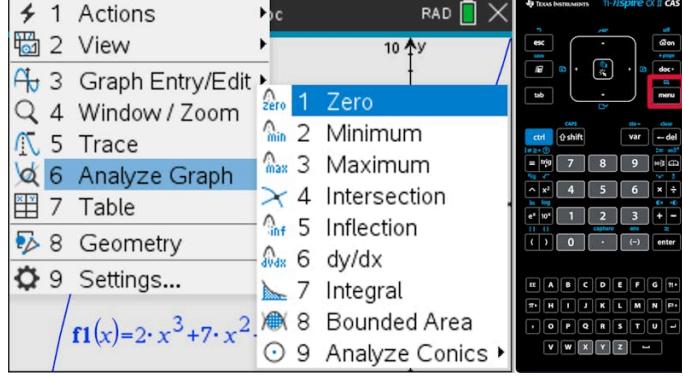
Steps	Explanation
<p>Open a graph page.</p>	
<p>Enter the formula.  Before viewing the graph you may want to set an appropriate viewing window either by zooming or by manually setting the bound through the menu.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

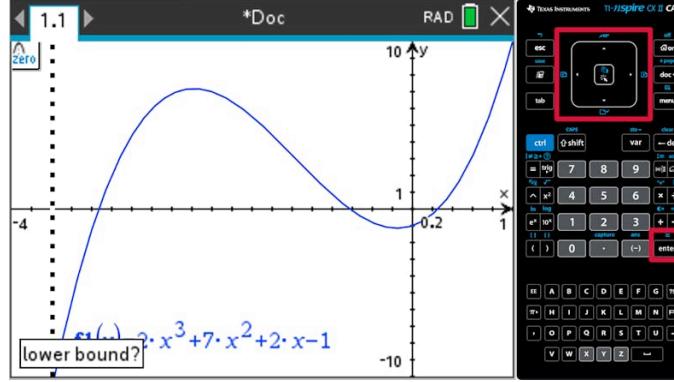
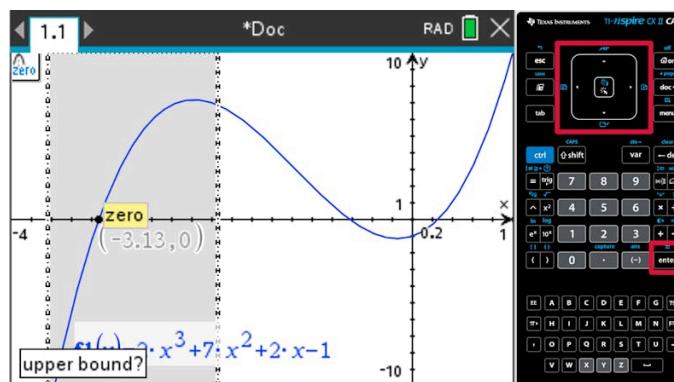
---

Steps	Explanation
<p>You can analyse the graph through the menu.</p>	
<p>You can find the <math>x</math>-intercepts by searching for the zeroes of the function.</p>	



Student  
view

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k  
—

Steps	Explanation
<p>Before it displays the <math>x</math>-intercept, the calculator needs more information. Move the cursor to the left of the <math>x</math>-intercept you are interested in and press enter.</p>	
<p>Next, the calculator asks for an upper bound for the <math>x</math>-intercept.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

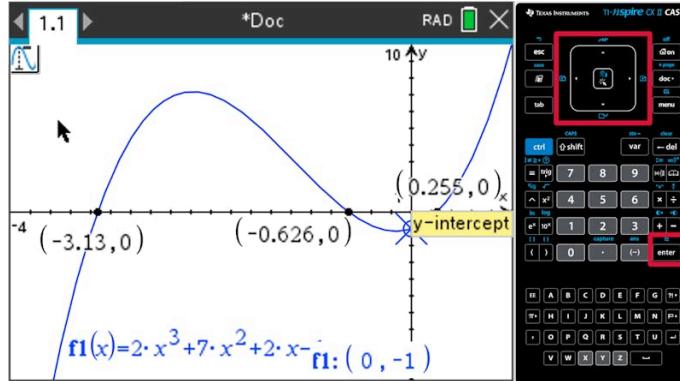
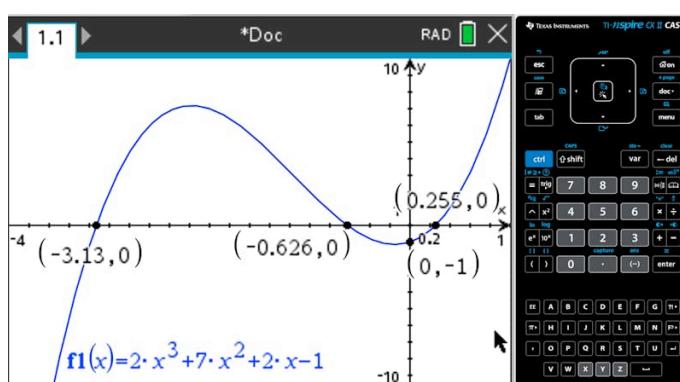
Steps	Explanation
<p>If you repeat this process three times, all three <math>x</math>-intercepts will be displayed on the screen.</p> <p>To find the <math>y</math>-intercept, bring up the menu again.</p>	

This time, choose to trace the graph.	
---------------------------------------	--



Student  
view

Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k  
—

Steps	Explanation
<p>While moving the cursor along the graph, the calculator will find important points and alerts you. By pressing enter, the coordinates will be displayed on the screen.</p> <p>Actually, the <math>x</math>-intercepts can also be found using trace instead of the method shown above.</p>	
<p>This is the screen you will see with all intercepts marked and the coordinates displayed.</p>	

## Example 1



X  
Student view

Sketch the function  $y = \frac{1}{2}x - 2$  by showing the intercepts of the graph.

Home  
Overview  
(/study/app-  
122-  
cid-  
754029/k)

---

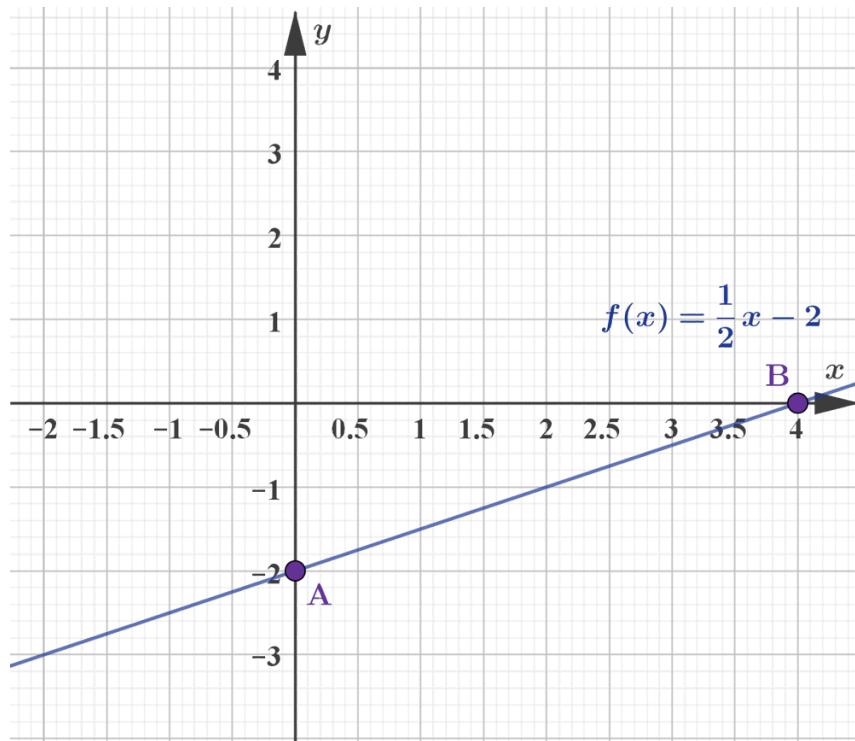
This is a linear function expressed in gradient-intercept form and hence the  $y$ -intercept of the function is the point  $(0, -2)$ .

The line has  $x$ -intercept when  $y = 0$

$$\text{so } 0 = \frac{1}{2}x - 2 \Leftrightarrow x = 4.$$

Therefore the  $x$ -intercept is the point  $(4, 0)$ .

The graph of the function is shown below.



## Example 2



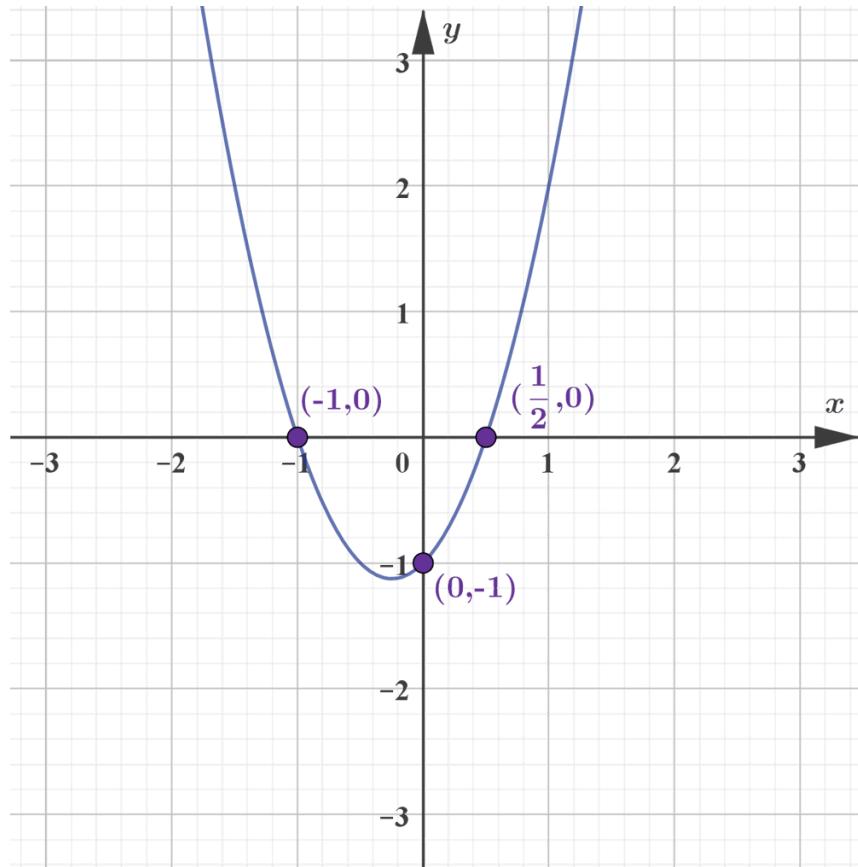
Student view

Sketch the function  $f(x) = 2x^2 + x - 1$  by showing the intercepts of the graph.

Home  
Overview  
(/study/app-  
122-  
cid-  
754029/k

---

The graph of the function  $f(x) = 2x^2 + x - 1$  is shown below, including the  $x$ -intercepts and the  $y$ -intercept.



### Example 3



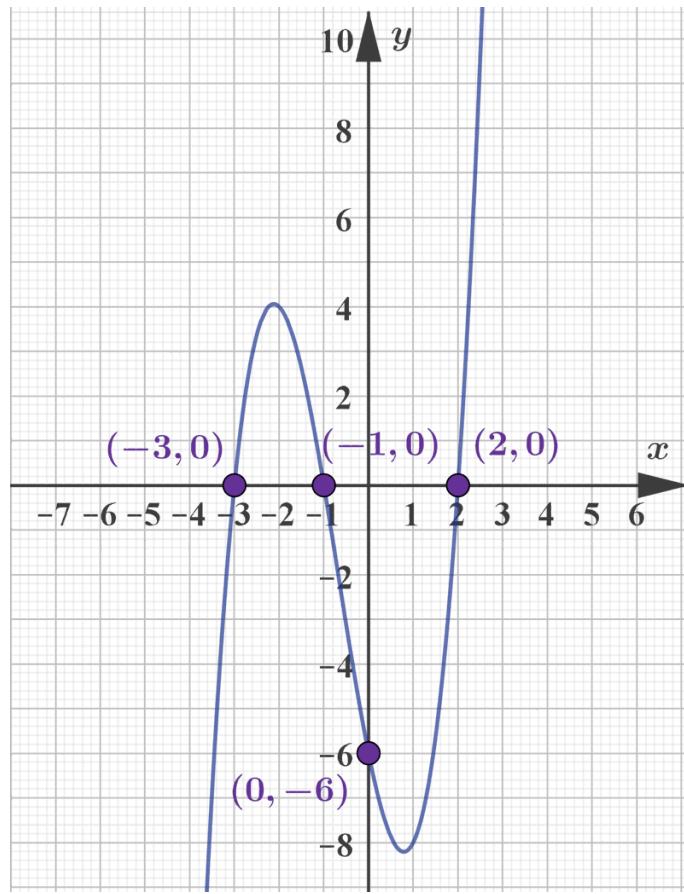
Sketch the function  $f(x) = x^3 + 2x^2 - 5x - 6$  by showing the intercepts of the graph.

The graph of the function is shown below, including the  $x$ -intercepts and  $y$ -intercept of the graph.

Home  
Student  
view

---

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k  
—



### ⚠ Be aware

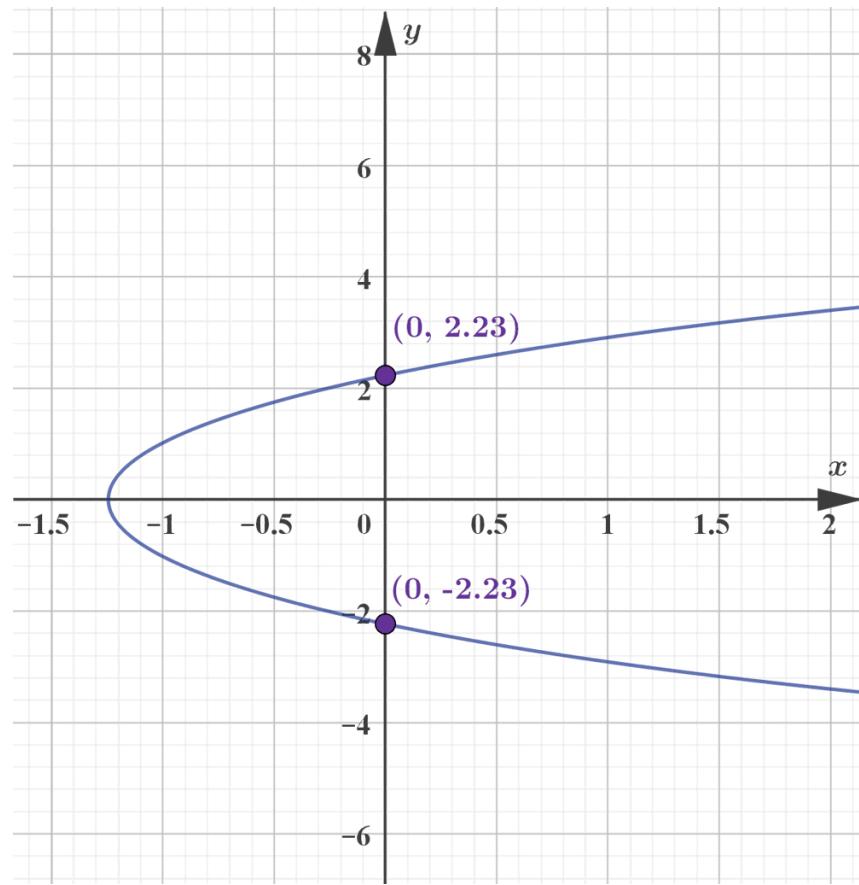
- At the  $x$ -intercepts, the  $y$ -coordinate of the points is equal to zero and thus the points have the form  $(x, 0)$ .
- At the  $y$ -intercept, the  $x$ -coordinate of the point is equal to zero and thus the point has the form  $(0, y)$ .

### ଓ Making connections

Recall that the graph of a relationship should pass the vertical line test to represent a function, and thus the graph of a function should not have more than one  $y$ -intercept.

✖  
Student  
view

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k  
—



More information

The image is a graph depicting a mathematical curve plotted against a grid. The X-axis represents values ranging from -2 to 4, while the Y-axis ranges from -3 to 3. Two key points are highlighted and labeled on the curve: (0, 2.28) and (0, -2.28), indicating the y-intercepts of the function. The curve shows a shape consistent with a sideways parabola, illustrating that it is not a function as it fails the vertical line test by having more than one y-intercept. Both axes have arrowheads indicating positive direction, and grid lines are evenly spaced throughout the graph, providing a reference for the plotted curve. The vertical symmetry of the graph causes it to cross the X-axis at multiple Y-values at x=0.

[Generated by AI]

## Theory of Knowledge

Calculators and computers are a huge part of our lives, and inevitably, this technology has affected the way in which we think and construct knowledge.

What are the advantages and disadvantages of using technology for knowledge construction? Think about the issues of power, justification, certainty and truth as you discuss the implications of using technology to construct knowledge. While

Student view

doing so, it may be useful to watch this TED Talk by mathematician Conrad Wolfram in which he advocates reducing the role of mental paper and pencil maths in favour of calculators and large-scale computing.

Conrad Wolfram: Teaching kids real math with computers



## 3 section questions ▾

2. Functions / 2.3 Graphs of functions

# Operations with functions

## Basic operations

### Notation

You know how to add, subtract, multiply and divide real numbers as well as algebraic expressions. So, you will be able to add, subtract, multiply and divide two functions.

Consider the functions  $f(x) = 3x - 2$  and  $g(x) = x^2 + 5$ .

You can find the sum of the functions  $f(x)$  and  $g(x)$  using



$$(f + g)(x) = f(x) + g(x) \text{ so,}$$

$$(f + g)(x) = (3x - 2) + (x^2 + 5)$$

$$= x^2 + 3x + 3.$$

That's it! The sum of the two functions is the sum of the two polynomials.

The notation used for the four basic operations between two functions  $f(x)$  and  $g(x)$  is shown below:

<b>Addition</b>	$f(x) + g(x)$	$(f + g)(x)$
<b>Subtraction</b>	$f(x) - g(x)$	$(f - g)(x)$
<b>Multiplication</b>	$f(x) \times g(x)$	$(f \times g)(x)$
<b>Division</b>	$\frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x)$

Now consider the domain.

As an example, take the linear function  $f(x) = x + 2$  and the square root function  $g(x) = \sqrt{x}$ . The domain of function  $f$  is  $D_f = \{x|x \in \mathbb{R}\}$  and the domain of function  $g$  is

$$D_g = \{x|x \in \mathbb{R}, x \geq 0\}.$$

The domain of the sum, difference and product of  $f(x)$  and  $g(x)$  is the intersection of their domains  $D_f$  and  $D_g$ . Both functions must be well defined for a combination of functions to be defined and hence the domain of the sum, difference and product of  $f(x)$  and  $g(x)$  is

$$D_f \cap D_g = \{x|x \in \mathbb{R}, x \geq 0\}.$$

## ② Making connections

The intersection of two sets  $A$  and  $B$  is the set that contains all elements that belong to  $A$  and  $B$  at the same time. In other words, the intersection set contains the elements that the two set have in common.



Overview  
 (/study/app/122-cid-754029/k)

An additional requirement for the quotient function  $\frac{f(x)}{g(x)}$  is that the denominator cannot be zero and thus you need to exclude the values of  $x$  where function  $g(x)$  is equal to zero from the domain.

## ✓ Important

For a function  $f(x)$  with domain  $D_f$  and function  $g(x)$  with domain  $D_g$ , the domain of the sum, difference and product of  $f$  and  $g$  is  $D_f \cap D_g$ .

The domain of the quotient of  $f$  and  $g$  is  $D_f \cap D_g - \{x | g(x) = 0\}$ .

Below you can find help on how to draw the graph of the quotient of two functions.

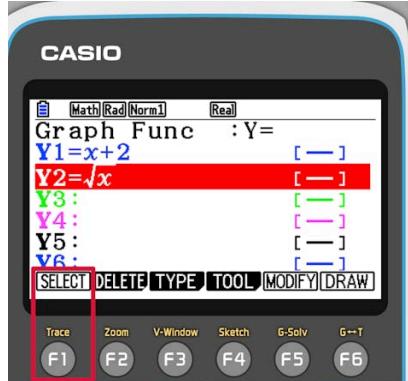
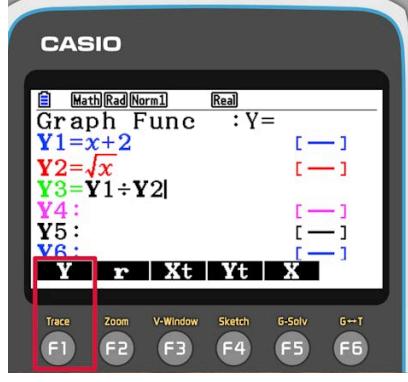
Steps	Explanation
Choose the graph mode.	 



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

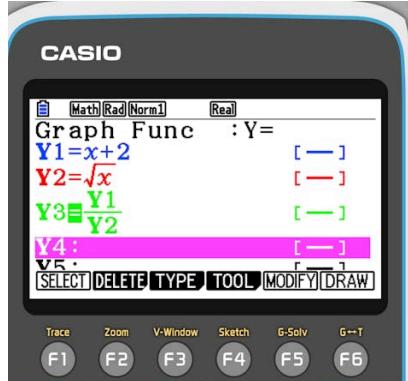
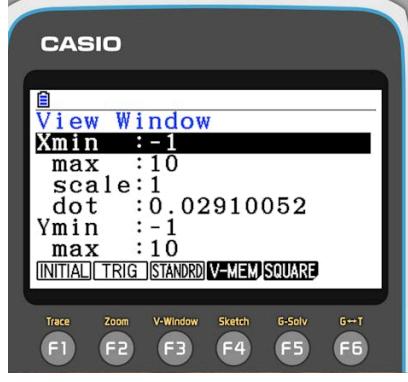
---

Steps	Explanation
<p>Use the variable button for the unknown <math>x</math> and use F1 to deselect the function. These expressions are only used as building blocks, no need to display the graphs.</p>	 
<p>In entering the quotient, there is no need to type in the two expressions again. Use F1 to insert the names of the already defined functions.</p>	 



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

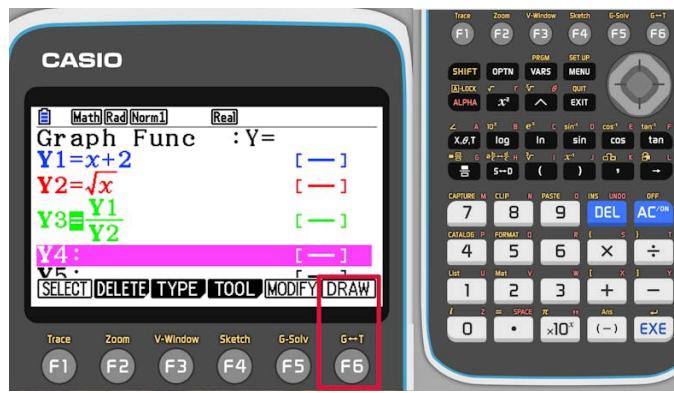
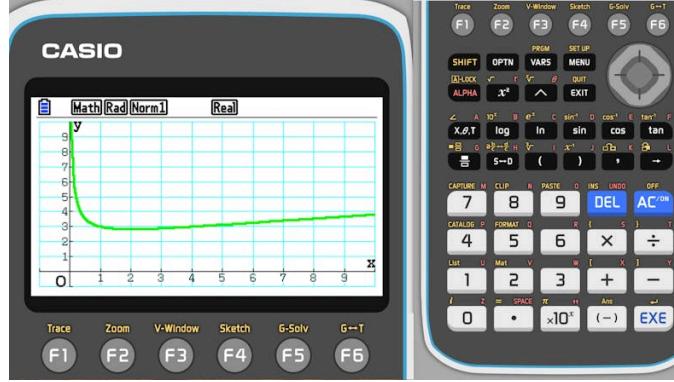
Steps	Explanation
<p>Before graphing the functions, you need to set the viewing window.</p>	 
	 



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

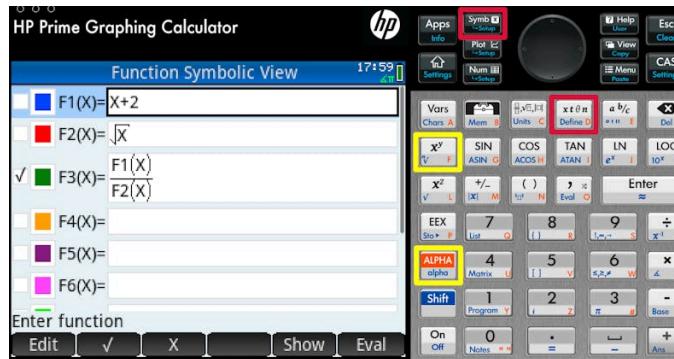
Steps	Explanation
<p>Once the viewing window is set, you are ready to graph the function. A highlighted equality sign shows which functions are selected for the graph.</p>	
<p>This is the graph of the quotient function. Using similar steps you can also graph the sum, the difference and the product.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

---

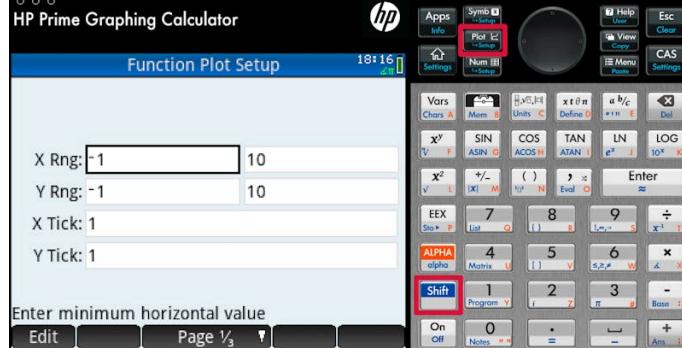
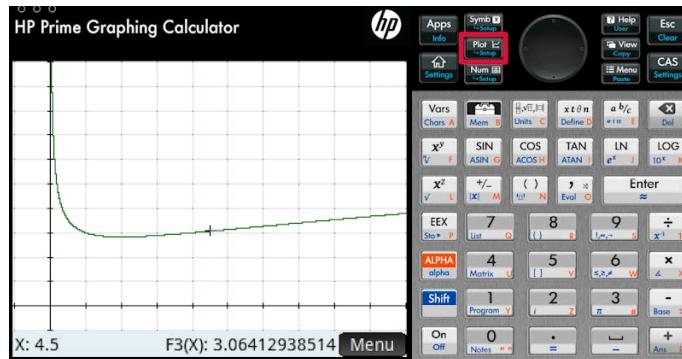
Steps	Explanation
<p>Choose the function application.</p>	 <p>The screenshot shows the HP Prime Graphing Calculator's Application Library. The 'Function' icon, which is blue with a white graph symbol, is highlighted with a red box. Other icons include Advanced Graphing, Graph 3D, Geometry, Spreadsheet, Statistics 1Var, Statistics 2Var, Inference, Data Streamer, Solve, Linear Solver, and Explorer. The calculator's numeric keypad and function keys are visible on the right.</p>
<p>In the symbolic view you can define the functions. Use the variable button for the variable <math>x</math> and use capital F to refer to the already defined functions. The tickmark in front of the function shows whether it will be displayed on the graph or not.</p>	 <p>The screenshot shows the HP Prime Graphing Calculator's Function Symbolic View. It lists six functions: F1(X)=X+2 (blue), F2(X)=<math>\sqrt{X}</math> (red), F3(X)=<math>\frac{F1(X)}{F2(X)}</math> (green), F4(X)= (orange), F5(X)= (purple), and F6(X)= (pink). The first three have a tickmark in front of them. The variable <math>x</math> is highlighted with a yellow box on the numeric keypad. The calculator's numeric keypad and function keys are visible on the right.</p>



Student  
view

Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k

---

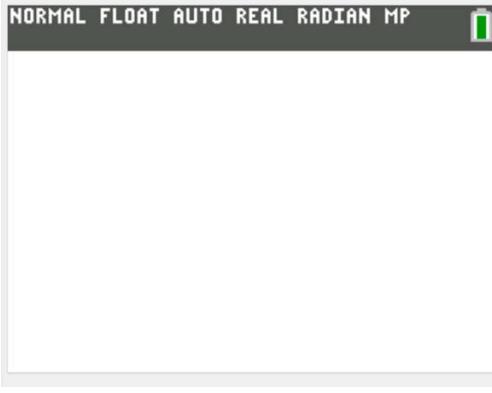
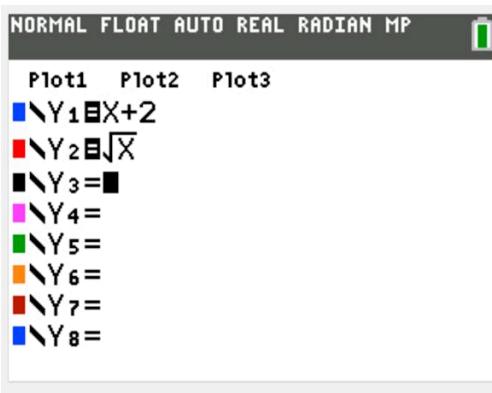
Steps	Explanation
In the plot setup view you can set the viewing window.	
In the plot view you can see the graph.  This is the graph of the quotient function. Using similar steps you can also graph the sum, the difference and the product.	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k

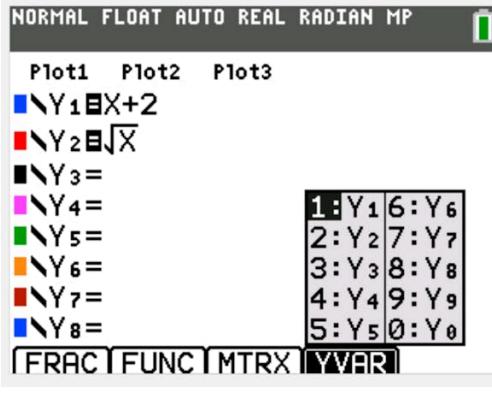
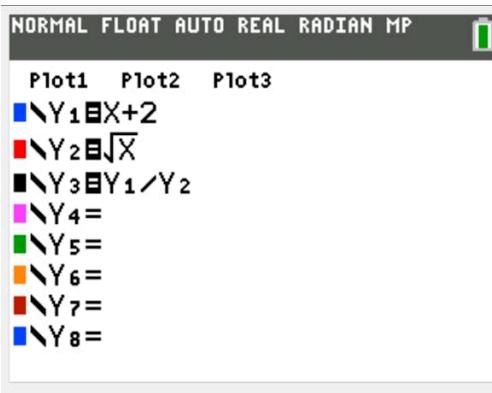
---

Steps	Explanation
<p>You need to bring up the screen where you can enter expressions for the functions.</p>	
<p>Use the variable button for the unknown <math>x</math>.</p>	



Student  
view

Home  
Overview  
(/study/ap/  
122-  
cid-  
754029/k  
—

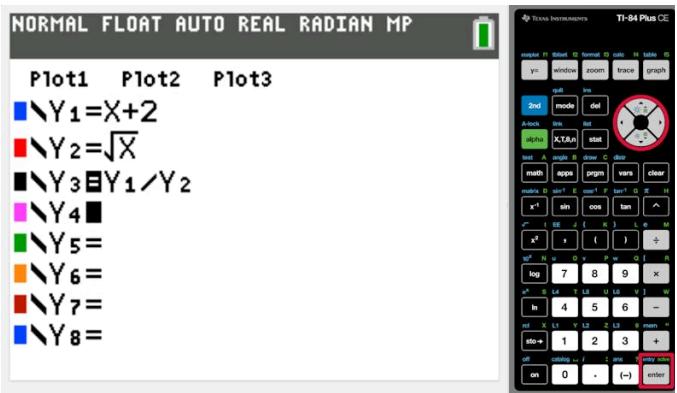
Steps	Explanation
<p>To enter the quotient function without typing in the expressions again, you can use the names of the already defined functions.</p>	 <p>The TI-84 Plus CE calculator screen shows the Y= editor. The functions Y1, Y2, and Y3 are defined. Y3 is defined as Y1/Y2. The cursor is on the Y3 line. The top menu bar shows 'NORMAL FLOAT AUTO REAL RADIAN MP'. The bottom menu bar shows 'Plot1 Plot2 Plot3', 'FRAC FUNC MTRX', and 'YVAR'.</p>
<p>Before graphing the functions, you need to set the viewing window.</p>	 <p>The TI-84 Plus CE calculator screen shows the MODE menu. The 'window' option is highlighted with a red box. The top menu bar shows 'NORMAL FLOAT AUTO REAL RADIAN MP'. The bottom menu bar shows 'Plot1 Plot2 Plot3', 'FRAC FUNC MTRX', and 'YVAR'.</p>



Student  
view

Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k

---

Steps	Explanation
	
<p>You can select which graph you want to see by pressing enter above the equality sign in front of the definition. A graph will not be displayed if the equality sign is not highlighted.</p>	



Student  
view

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k

---

Steps	Explanation
<p>You can bring up the graph by pressing the graph button.</p> <p>This is the graph of the quotient function. Using similar steps you can also graph the sum, the difference and the product.</p>	



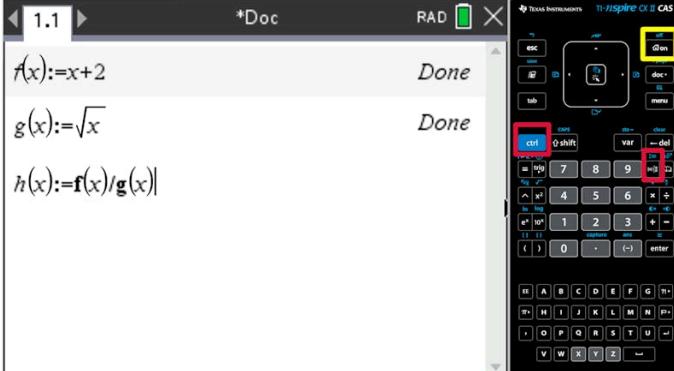
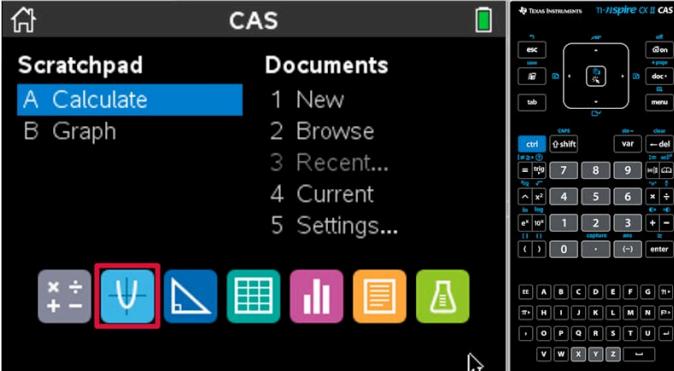
Section	Steps	Explanation
<p>Student... (0/0)</p> <p>Feedback</p>	<p>Open up a calculator page to define the functions.</p>	<p>Print (/study/app/m/sid-122-cid-754029/book/operations-with-functions-id-26187/print/)</p> <p>Assign</p>



X  
Student view

Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k

---

Steps	Explanation
<p>You can define functions and give any names. You can use previously defined functions in new definitions. It is important that you need to use the assigned equality symbol for the definitions.</p> <p>Once you are done with the definitions, press the home button so that you can open a graphics page.</p>	
<p>Open a graphing page.</p>	



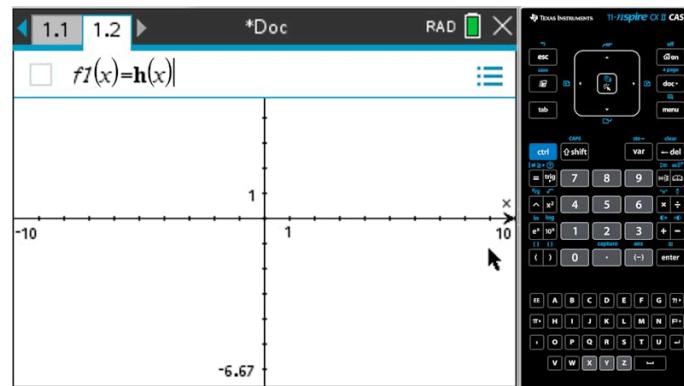
Student  
view

Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k  
—

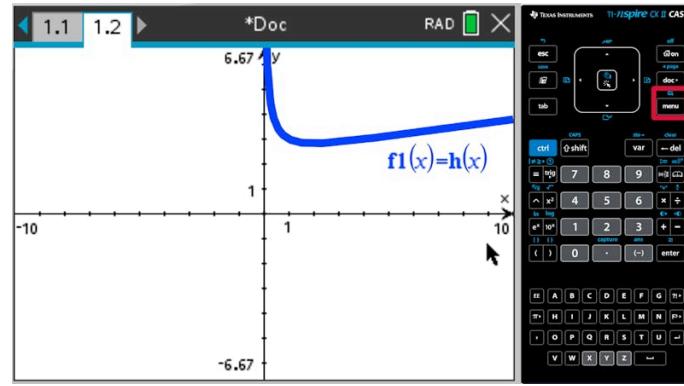
**Steps**

You want to see the graph of the quotient function. The name for the quotient in the calculator page was  $h$ .

**Explanation**



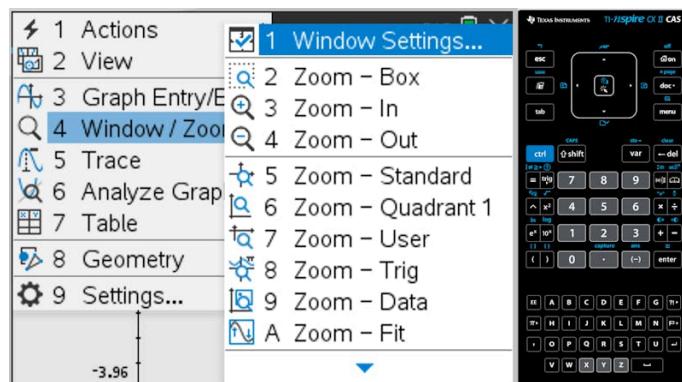
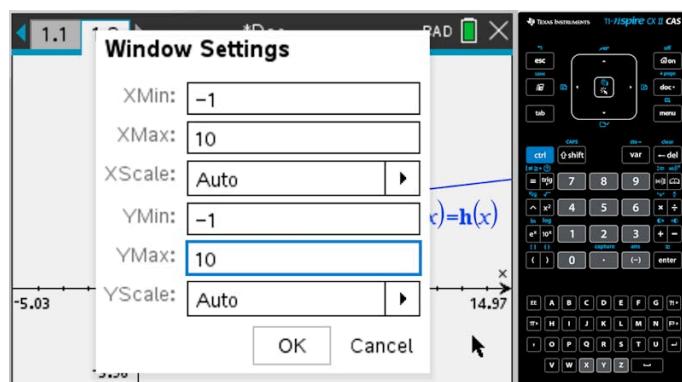
There are several ways of changing the viewing window. You can set specific values through the menu system.



Student  
view

Home  
Overview  
(/study/app  
122-  
cid-  
754029/k

---

Steps	Explanation
	
	



Student  
view

Home  
Overview  
(/study/ap  
122-  
cid-  
754029/k  
—

Steps	Explanation
<p>This is the graph of the quotient function. Using similar steps you can also graph the sum, the difference and the product.</p>	

## Activity

The instructions above show you how to use the calculator to find the graph of the quotient of  $f(x) = x + 2$  and  $g(x) = \sqrt{x}$ . Following similar steps, obtain the graphs of the sum, difference, product and quotient of the quadratic function  $f(x) = x^2$  and the rational function  $g(x) = \frac{x - 2}{x + 1}$ . You can compare your graphs with the ones below. Did you get the same graphs?

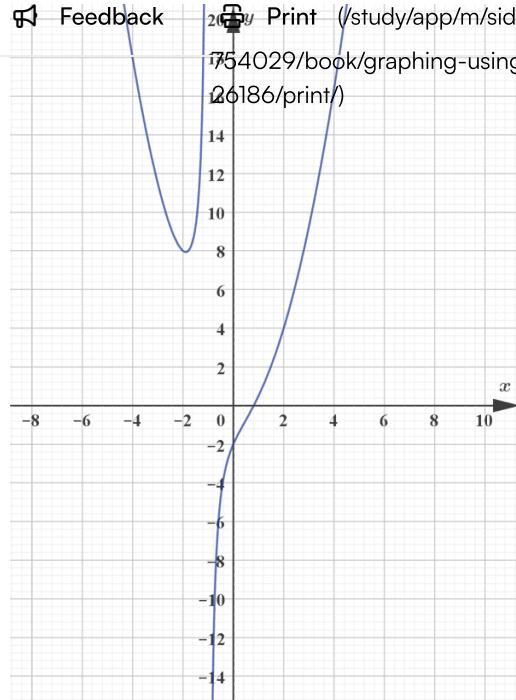
Basic operations with functions	Domain
$(f + g)(x) = f(x) + g(x) = x^2 + \frac{x - 2}{x + 1}$	$\{x   x \in \mathbb{R}, x \neq -1\}$
$(f - g)(x) = f(x) - g(x) = x^2 - \frac{x - 2}{x + 1}$	$\{x   x \in \mathbb{R}, x \neq -1\}$
$(f \times g)(x) = f(x) \times g(x) = x^2 \cdot \left(\frac{x - 2}{x + 1}\right)$	$\{x   x \in \mathbb{R}, x \neq -1\}$

Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k  
—

Basic operations with functions		Domain
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{\frac{x-2}{x+1}} = \frac{x^2(x+1)}{x-2}$		$\{x x \in \mathbb{R}, x \neq -1, x \neq 2\}$

**Section**

Student... (0/0)

**Feedback****Print**(/study/app/m/sid-122-cid-  
754029/book/graphing-using-a-gdc-id-  
26186/print/)**Assign**

[More information](#)

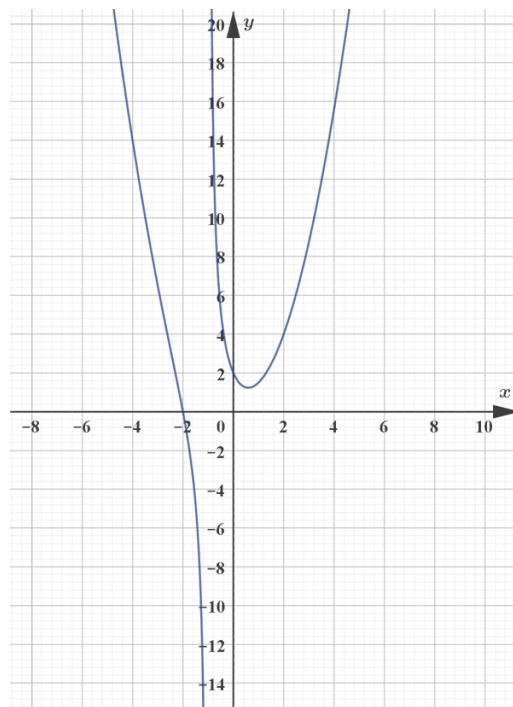
The image is a graph showing a curving blue line plotted on a grid. The X-axis ranges from -10 to 10, while the Y-axis ranges from -10 to 18. The graph shows a curve with an initial descent, reaching a low point around the X-axis, followed by an upward movement. The curve continues upwards as it moves to the right, passing through different sections of the grid.

[Generated by AI]

Student  
view



Overview  
 (/study/app/  
 122-  
 cid-  
 754029/k  
 —



More information

The graph depicts the function  $((f + g))(x) = x^2 + \frac{x-2}{x+1}$ , with the constraint  $(x \neq -1)$ . The X-axis represents the input ( $x$ ) values, and ranges from negative to positive with labeled intervals, centered around zero. The Y-axis represents the function output  $((f+g)(x))$ , also ranging from negative to positive numbers with similar interval labeling.

Key features of the graph include: - A distinct vertical asymptote at  $(x = -1)$ , meaning the graph approaches but never touches or crosses the line where  $(x = -1)$ . - The graph shows a parabola behavior in its curvature, indicative of the quadratic component  $(x^2)$  of the function. - The curve passes through key points including the origin, with an upward and downward trend depending on the region relative to the asymptote. - Overall, the function shows different behaviors in segments, transitioning sharply at the x-value exclusion  $(x = -1)$ .

[Generated by AI]

The graph of function

$$(f + g)(x) = x^2 + \frac{x-2}{x+1}, \quad x \neq -1$$

The graph of function

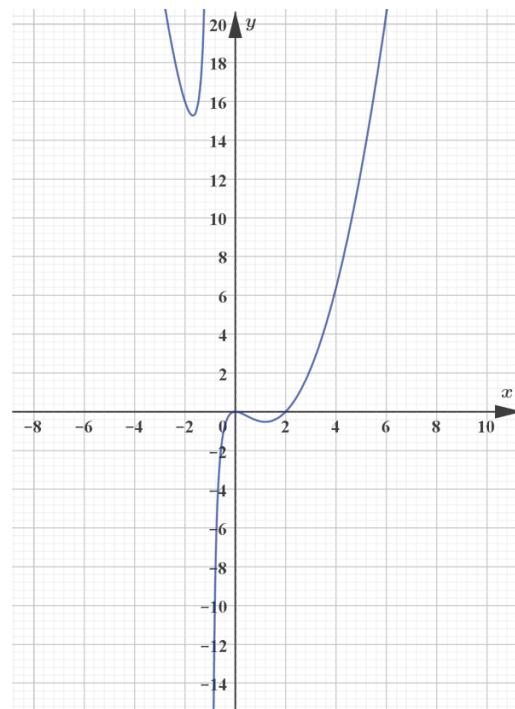
$$(f - g)(x) = x^2 - \frac{x-2}{x+1}, \quad x \neq -1$$



Student  
view



Overview  
(/study/app/  
122-  
cid-  
754029/k  
—



More information

The image is a graph of the function  $((f-g)(x)) = x^2 - \frac{x-2}{x+1}$ . The graph is plotted on a Cartesian coordinate system with the X-axis representing the variable ( $x$ ) and the Y-axis representing the function value ( $f-g(x)$ ). The graph shows the behavior of the function, displaying a curve that passes through several points. The Y-axis has labeled intervals from -10 to 10, while the X-axis ranges from -10 to 10 as well. The graph shows a significant inflection point at ( $x = -1$ ), where the function is not defined. The curve dips sharply and then rises, indicating changes in the slope of the graph.

[Generated by AI]

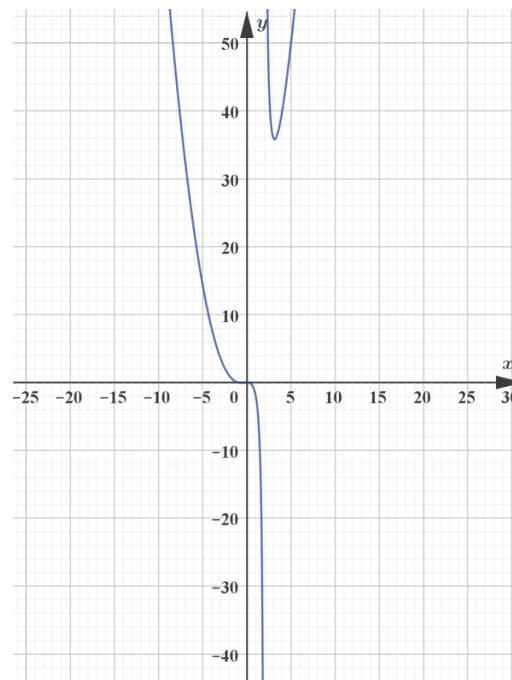


Student  
view



Overview  
 (/study/app/  
 122-  
 cid-  
 754029/k)

---



More information

The image displays the graph of the function  $(f \times g)(x) = x^2 * ((x - 2)/(x + 1))$ , defined for  $x \neq -1$ . The graph is plotted on a coordinate plane with x-axis and y-axis both marked with increments of 1. The x-axis ranges from -2 to 2, while the y-axis ranges from -30 to 30. The function illustrates a curve passing through various points, showing different types of behavior such as increasing, decreasing, and asymptotic behavior near  $x = -1$ . The graph's main feature is the pronounced drop and rise as  $x$  approaches  $-1$  from both sides, indicating a vertical asymptote. There are also parabolic features near the origin and towards positive x-values, highlighting the polynomial and rational nature of the function. The graph is drawn in a blue line against a grid of equal squares.

[Generated by AI]

The graph of function

$$(f \times g)(x) = x^2 \cdot \left( \frac{x - 2}{x + 1} \right), \quad x \neq -1$$

The graph of function

$$\left( \frac{f}{g} \right)(x) = \frac{x^2}{\frac{x - 2}{x + 1}} = \frac{x^2(x + 1)}{x - 2}, \quad x \neq -1, 2$$



Student  
view

## Example 1

Overview  
(/study/app/122-cid-754029/k)

★★☆

Consider function  $f(x) = x + 3$  and  $g(x) = \frac{1}{x - 2}$ .

Find the domain of the sum function  $(f + g)(x)$ .

The domain of  $f$  is  $D_f = \{x|x \in \mathbb{R}\}$  and the domain of  $g$  is  $D_g = \{x|x \in \mathbb{R}, x \neq 2\}$ .

The domain of the sum function is  $D_{f+g} = \{x|x \in \mathbb{R}, x \neq 2\}$ .

## Example 2

★★☆

Consider function  $f(x) = x + 3$  and  $g(x) = \frac{1}{x - 2}$ .

Find the domain of the difference function  $(f - g)(x)$ .

The domain of  $f$  is  $D_f = \{x|x \in \mathbb{R}\}$  and the domain of  $D_g = \{x|x \in \mathbb{R}, x \neq 2\}$ .

The domain of the difference function is  $D_{f-g} = \{x|x \in \mathbb{R}, x \neq 2\}$ .

## Example 3

★★☆

A small retail shop owner knows by experience that they can sell 500 t-shirts each year. In addition to the price of the t-shirts, they need to pay a fixed 5 euros whenever they order a new supply. To store a t-shirt for a year costs the company 2 euros, and it costs proportionally less if they do not have to keep it in the store for the full year. Graph the annual cost of ordering and storing the t-shirts in terms of the quantity they order in each shipment.

If they order  $x$  t-shirts at a time, then in a year they need to order  $\frac{500}{x}$  times. Each time they pay 5 euros for shipping, so the annual ordering cost is:

$$O(x) = 5 \times \frac{500}{x} = \frac{2500}{x} \text{ euros.}$$

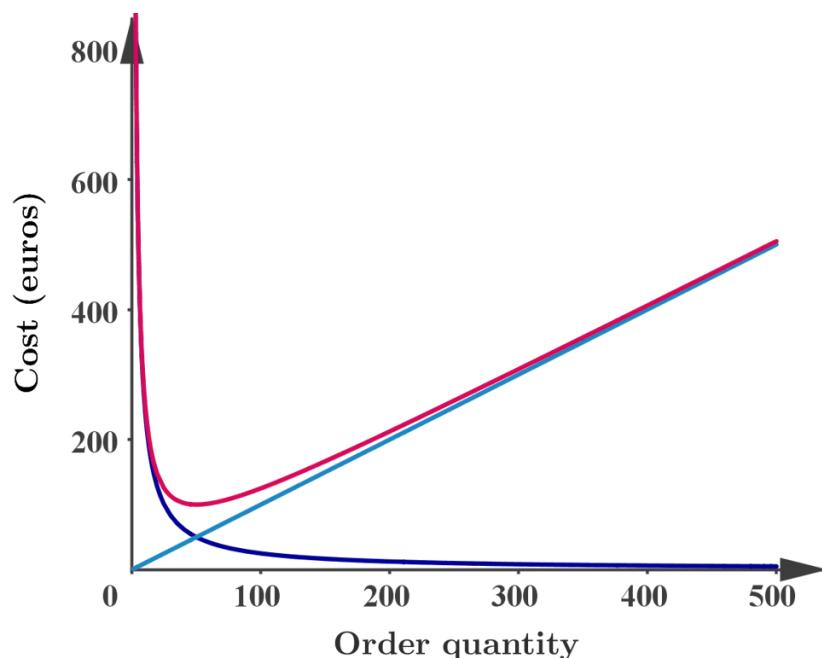
If they order  $x$  t-shirts in each shipment and they only order the next shipment when they are running low on the amount they have, then on average they have  $\frac{x}{2}$  t-shirts in stock on each day. The annual storage cost of this many t-shirts is:

$$S(x) = 2 \times \frac{x}{2} = x \text{ euros.}$$

The annual cost of ordering and storage (in addition to the cost of the t-shirts) is the sum of these functions.

$$C(x) = \frac{2500}{x} + x$$

The graph below shows the ordering and storing cost in blue and the sum in red.



Home  
Overview  
(/study/app/  
122-  
cid-  
754029/k  
—

From this graph you can see that ordering too few t-shirts in each shipment results in high ordering cost. On the other hand, ordering too many results in large storage cost. In the next sections you will see how to find the optimal ordering quantity to minimize the total of the ordering and storing costs. This optimal quantity is called the economic order quantity.

## 3 section questions ▾

2. Functions / 2.3 Graphs of functions

# Checklist

### Section

Student... (0/0)

Feedback

Print (/study/app/m/sid-122-cid-  
754029/book/checklist-id-26188/print/)

Assign

### What you should know

By the end of this subtopic you should be able to:

- graph a function using a GDC
- determine the  $x$ -intercepts and  $y$ -intercept of a function using a GDC
- sketch the graph of a function showing the  $x$ -intercepts and  $y$ -intercept
- find the sum, difference, product and quotient of two functions  $f(x)$  and  $g(x)$
- determine the domain of the sum, difference, product and quotient function of two functions  $f(x)$  and  $g(x)$ .

2. Functions / 2.3 Graphs of functions

# Investigation

Student view



Overview  
(/study/ap-  
122-  
cid-  
754029/k

In the following applet, you can simulate adding water to different types of vase and observe graphs of the depth of the water over time.



### Interactive 1. Investigating Water Depth and Graphs.

More information for interactive 1

This interactive allows users to simulate adding water to different types of containers and observe how the depth of the water changes over time. Users can move points or click buttons to select various shapes of containers, such as rectangular vases, and use a slider to adjust the water depth. The interactive generates graphs showing the relationship between water depth and time as water is added at a constant rate.

Users can explore this relationship and predict the depth–time graphs for different container shapes. By adjusting the container shapes and observing the resulting graphs, users can check their predictions and understand how the shape of a container influences the graph's gradient and overall behavior.

- When water is added at a constant rate to a rectangular container, the depth of water in the container is a linear function of time. Explain why this is true?
- Choose the rectangular vase from the shapes shown. Use the slider to change the depth of water in the container. Given that water is added to the container at a constant rate, the graph shows the depth as time increases. Moving the slider slowly will give the smoothest version of the graph.
- By examining the shapes of the containers provided, predict the depth–time graph when water is added at a constant rate.



Student  
view

- Use the applet to check your answers to the previous question.
- Work in pairs, adjusting the points provided in the vases to create different shapes and ask your peer to predict the graph of the water depth over time. Use the slider as before to check your predictions.
- Write a report on the connection between the shape of a container and the corresponding depth–time graph. Consider different shapes of container and discuss the gradient of the curves produced in the graphs.

**Rate subtopic 2.3 Graphs of functions**

Help us improve the content and user experience.

