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5. Calculus / 5.2 Increasing and decreasing functions

The big picture

You saw in the previous subtopic how to define the derivative of a function. In this subtopic, you will explore how the derivative can give you information about the shape of the graph of the function. In particular, you will see how to identify points where the graph of a function has turning points. Later, this will help you to use algebraic methods to find optimum values of certain expressions, for example the minimal surface area of a container of fixed volume.

⚙⚙   **Activity**

Calculus is not the only tool you can use to find optimum solutions to certain problems. Take a look at the video below. You will see an interesting problem and a solution to the problem using an experiment. The same problem can also be solved using the techniques of calculus. Would you consider both methods valid?

Maths Problem: Connect the towns solution (Motorway Pro...

▶ (YouTube)

🔑 **Concept**

While investigating graphs, think about the relationship between the sign of the derivative and the direction of the graph of a function.

🧩 **Theory of Knowledge**

Calculus was a set of principles, equations and axioms designed to answer problems of continuous change. Interestingly, both Isaac Newton and Gottfried Leibniz are both credited with 'inventing' it. This relates to a common question posed within Theory of Knowledge: 'To what extent is the development of knowledge within the area of knowledge of mathematics a result of collaborative work as opposed to individual?'

5. Calculus / 5.2 Increasing and decreasing functions

Graph properties

⚙⚙ **Activity**

On the applet below move the red points and change the ticks in the boxes.

The applet will show you a graph of a function that corresponds to the properties you set for the derivative.

- Formulate a statement that connects the sign of the derivative with some properties of the graph of the function.

- What do you notice about the graph at the position of the red points?
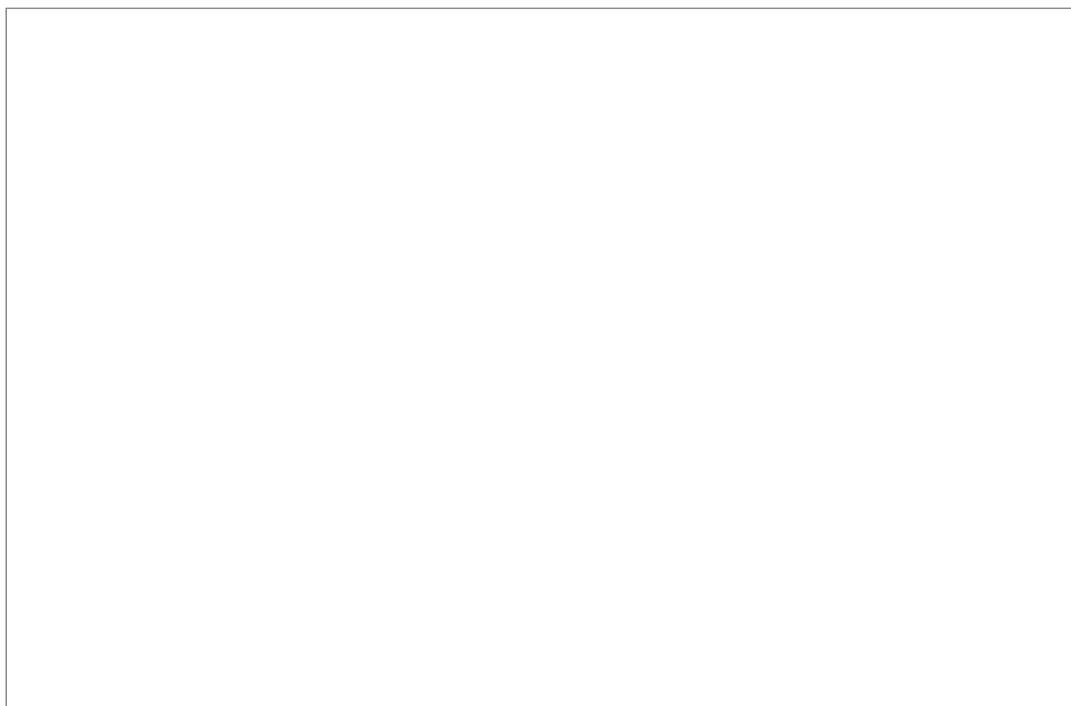
Interactive 1. Graph Properties.

| Section | Student... (0/0) | 📢 Feedback | 🖨 Print | 👁 More information for interactive 1 | Assign |

This interactive graph helps users explore the relationship between a function and its derivative by allowing them to analyze how the function's behavior determines the sign of $f'(x)$. The tool visually represents a function $f(x)$ and enables users to investigate where it is increasing or decreasing by adjusting red points along the curve. The graph is divided into colored regions, representing intervals where the function changes its behavior.

Users can move the red point along the curve to observe how the derivative behaves at different positions. By selecting the appropriate checkboxes—either $f'(x) > 0$ or $f'(x) < 0$ they can classify

each region based on whether the function is increasing or decreasing.

When f $'$ (x) > 0, the function is increasing, meaning its graph slopes upward, indicating that as x increases, f $'$ (x) also increases. This happens because the tangent lines at those points have positive slopes.

Conversely, when f $'$ (x) < 0, the function is decreasing, meaning its graph slopes downward, showing that as x increases, f (x) decreases. This is because the tangent lines have negative slopes.

This interactive helps users visually understand how the function's behavior determines the sign of the derivative, reinforcing the fundamental relationship between differentiation and the shape of a graph.

Based on the activity above, you can make the following observations.

✓ **Important**

- If $f'(x) > 0$ on an interval $]a, b[$, then f is increasing on $]a, b[$.
- If $f'(x) < 0$ on an interval $]a, b[$, then f is decreasing on $]a, b[$.
- If the function f is differentiable and $(a, f(a))$ is a turning point of the graph of f, then $f'(a) = 0$.

Example 1

★★☆

For a function, f, you are given that $f'(x) = (x + 1)(x - 3)$.

- Identify the intervals where f is increasing and where f is decreasing.
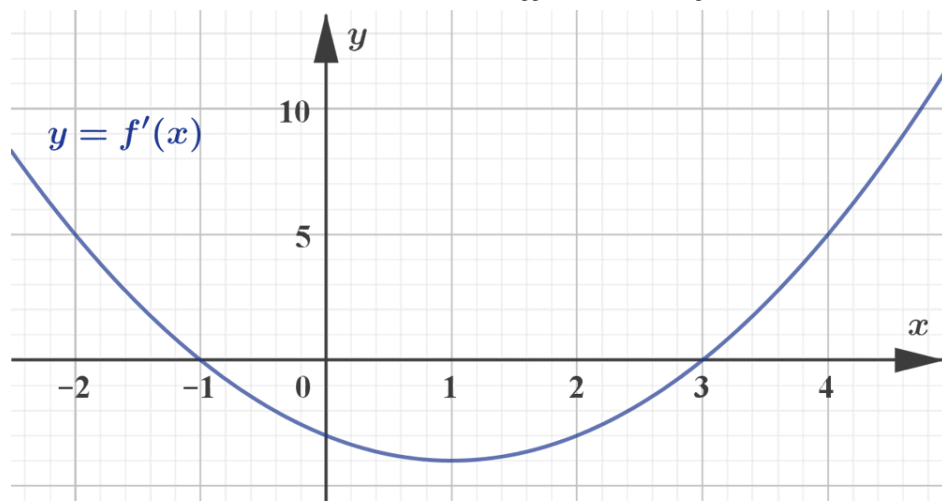- Find the x-coordinate of the turning points of the graph of f.

The diagram below shows part of the graph of $y = f'(x)$.

From the graph of f' you can see that:

- for $x < -1$, $f'(x)$ is positive, so f is increasing on the interval $]-\infty, -1[$;
- for $-1 < x < 3$, $f'(x)$ is negative, so f is decreasing on the interval $]-1, 3[$;
- for $3 < x$, $f'(x)$ is positive, so f is increasing on the interval $]3, \infty[$.

Using these observations you can conclude, that

- the graph of f arrives at the point where $x = -1$ as an increasing curve and leaves this point as a decreasing curve. Hence the graph of f has a turning point at $(-1, f(-1))$;
- the graph of f arrives at the point where $x = 3$ as a decreasing curve and leaves this point as an increasing curve. Hence the graph of f has a turning point at $(3, f(3))$.

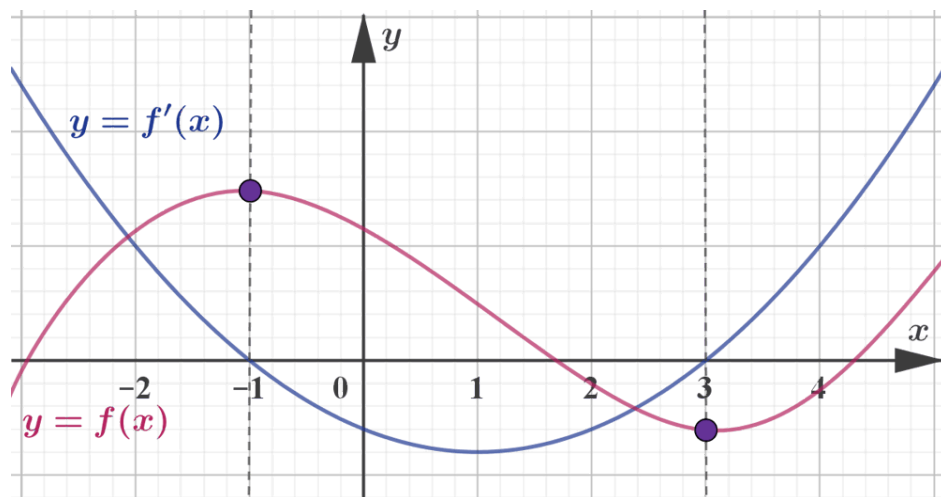The diagram below shows a possible graph of f illustrating the turning points you found.

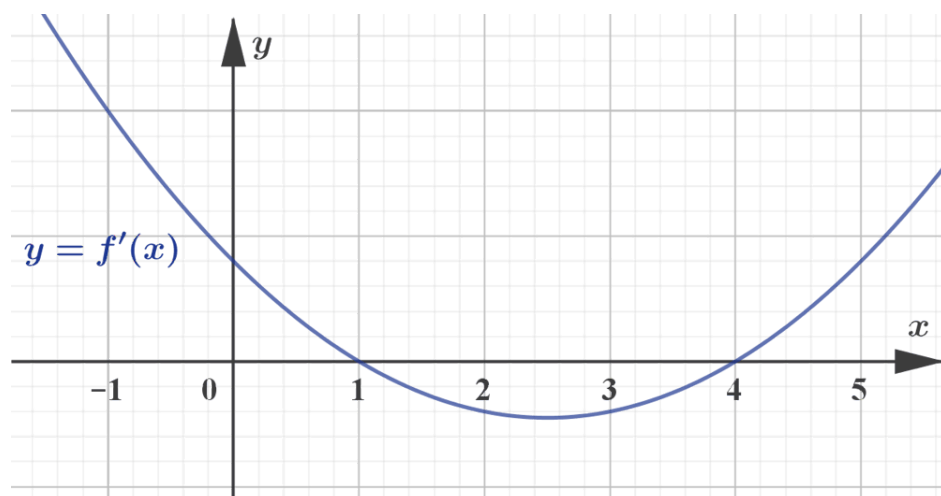How could you describe the difference between these two turning points?

Student
view

Example 2

★★☆

The diagram below shows the graph of $y = f'(x)$.

◈ More information

The graph is plotting the function y = f'(x). The X-axis represents the variable 'x' with values ranging from -1 to 5, marked at regular intervals. The Y-axis represents the function's derivative, y = f'(x). The curve starts from a high value on the Y-axis at the left, decreases sharply through the origin where x = 0, reaches a minimum and then rises back as x increases further. This creates a concave upwards parabola shape, suggesting the roots and turning points of the derivative function. At x = 0, the graph cuts through the Y-axis, indicating a zero crossing.

[Generated by AI]

- Which value is larger, $f(0)$ or $f(1)$?
- Which value is larger, $f(1)$ or $f(4)$?
- Which value is larger, $f(4)$ or $f(5)$?
- Draw a possible sketch of the graph of f.

- From the diagram you can see that f' is positive on $]0, 1[$. This means that f is increasing on $]0, 1[$, so $f(0) < f(1)$.
- The diagram shows that f' is negative on $]1, 4[$. This means that f is decreasing on $]1, 4[$, so $f(1) > f(4)$.
- Similarly, from the diagram, f' is positive on $]4, 5[$. This means that f is increasing on $]4, 5[$, so $f(4) < f(5)$.

The diagram below shows a possible sketch of the graph of f. The points on the graph indicate the relationship between the values found above. Note that since we do not know the actual values of the function at these points, we cannot draw a scaled sketch of the graph of f. Nevertheless, a sketch should indicate that f is increasing on $]-\infty, 1]$ and $[4, \infty[$ and decreasing on $[1, 4]$.
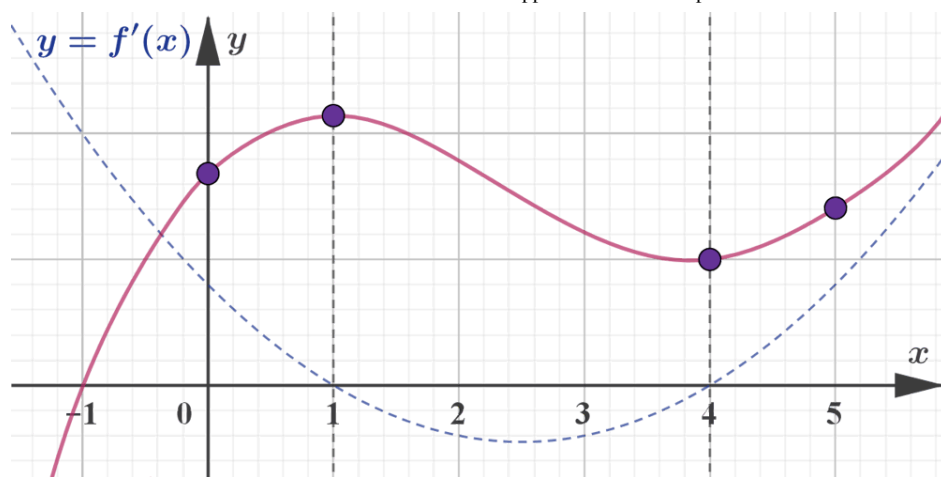
Example 3

★★☆

Assume, that the caffeine content (in milligrams) of the blood of a certain person t hours after drinking a cup of coffee is modelled by $c(t)$ for some function c. The rate of change of caffeine in the blood (in milligrams per hour) of this person is given by

$$c'(t) = 614.95\mathrm{e}^{-2.51t} - 36.75\mathrm{e}^{-0.15t}.$$

- Find how fast the caffeine level is changing initially.
- Describe the change of caffeine level in the blood.
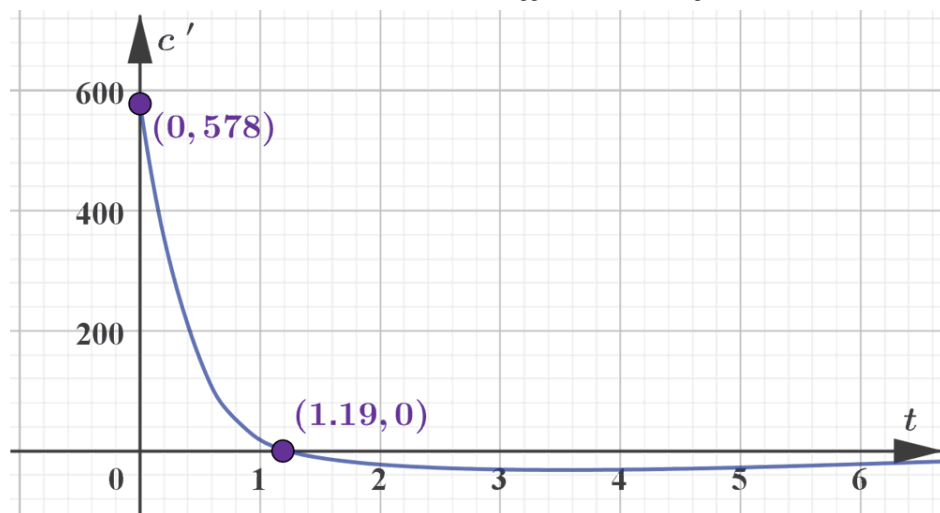- How long does it take for the caffeine level to reach its peak?

The diagram below shows the graph of the change of the level of caffeine in the first 7 hours. The diagram also shows the coordinates of the axis intercepts of the graph.

- The initial rate of change of caffeine level is $c'(0) = 578$ milligrams per hour.
- Since $c'(t) > 0$ for $0 < t < 1.19$ and $c'(t) < 0$ for $1.19 < t$, the caffeine level is first increasing, then decreasing.
- The caffeine level reaches its peak when it stops increasing, so 1.19 hours after the intake.

3 section questions ⌄

5. Calculus / 5.2 Increasing and decreasing functions

Checklist

| Section | Student... (0/0) | 📢 Feedback | 🖶 Print (/study/app/m/sid-122-cid-754029/book/checklist-id-26279/print/) | Assign |

📋 **What you should know**

By the end of this subtopic you should be able to:

- identify properties of graphs based on information given by the derivative:
  - identify intervals where the function is increasing

○ identify intervals where the function is decreasing

○ identify the x-coordinates of turning points.

5. Calculus / 5.2 Increasing and decreasing functions

Investigation

Section   Student... (0/0)   📢 Feedback   🖶 Print   (/study/app/m/sid-122-cid-   **Assign**
                                                                      754029/book/investigation-id-
                                                                      26280/print/)

In this investigation you return to the problem presented in the video of section 5.2.0 (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26277/). In the applet below, you can find an approximate position of the point with smallest total distance from three given points.

The applet shows three points and you can move a fourth point around. The applet tells you the total distance of this fourth point from the other three. The applet also illustrates this distance by graphing the distance as a (two-variable) function of the position of the movable point. In this syllabus, you will not learn about finding the optimum value of a two-variable function, but you can still use the applet to approximately find this minimum.

While working with the applet, think about

- what is the normal line that the applet shows?
- what is the tangent plane that the applet shows?
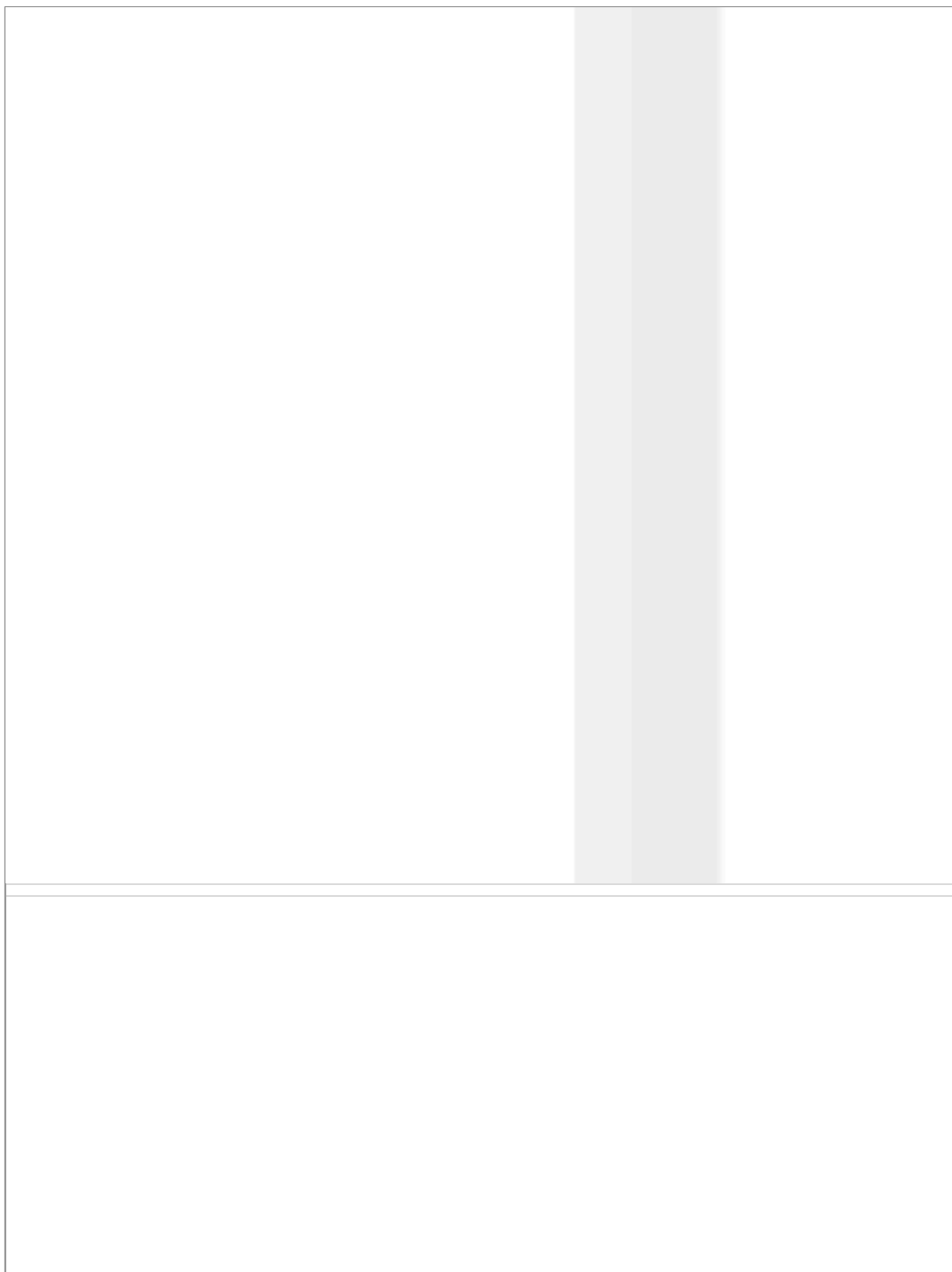- how do this line and plane help you to find the optimum point?

Interactive 1. Exploring Distances on a 3D Surface.

More information for interactive 1

This interactive allows users to manipulate and analyze 3D geometric elements through intuitive controls.

By checking "Adjust triangle," users can modify the triangle's shape by dragging its red corner points.

When "Move point" is selected, a red point becomes movable within the plane, with the tool continuously

calculating and displaying its distance to the upper surface (e.g., showing "Total distance: 11.4169").

Additional visualization options include "Show normal " and "Show tangent plane," displaying a black

dashed normal line perpendicular to the surface and showing the tangent plane at the selected point,

enabling users to explore the fundamental geometric relationships between points, normal vectors, and

tangent planes in three-dimensional space.

Student
view

The normal line appears to extend perpendicularly from the 3D surface, while the tangent plane lies flat against the surface at the point's location. These visual elements change as the user adjusts the position of the fourth point, providing real-time feedback on how the distance and surface behavior are affected. The user can use these observations to guide the movement of the fourth point toward a position where the total distance appears minimized.

Rate subtopic 5.2 Increasing and decreasing functions

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