Index

The big picture

? Guiding question(s)

- How can forces acting on a system be represented both visually and algebraically?
- How can Newton’s laws be modelled mathematically?
- How can knowledge of forces and momentum be used to predict the behaviour of interacting bodies?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

Have you ever wondered why a ball will bounce on the floor but an egg will not? Look at **Video 1** and **Video 2**.

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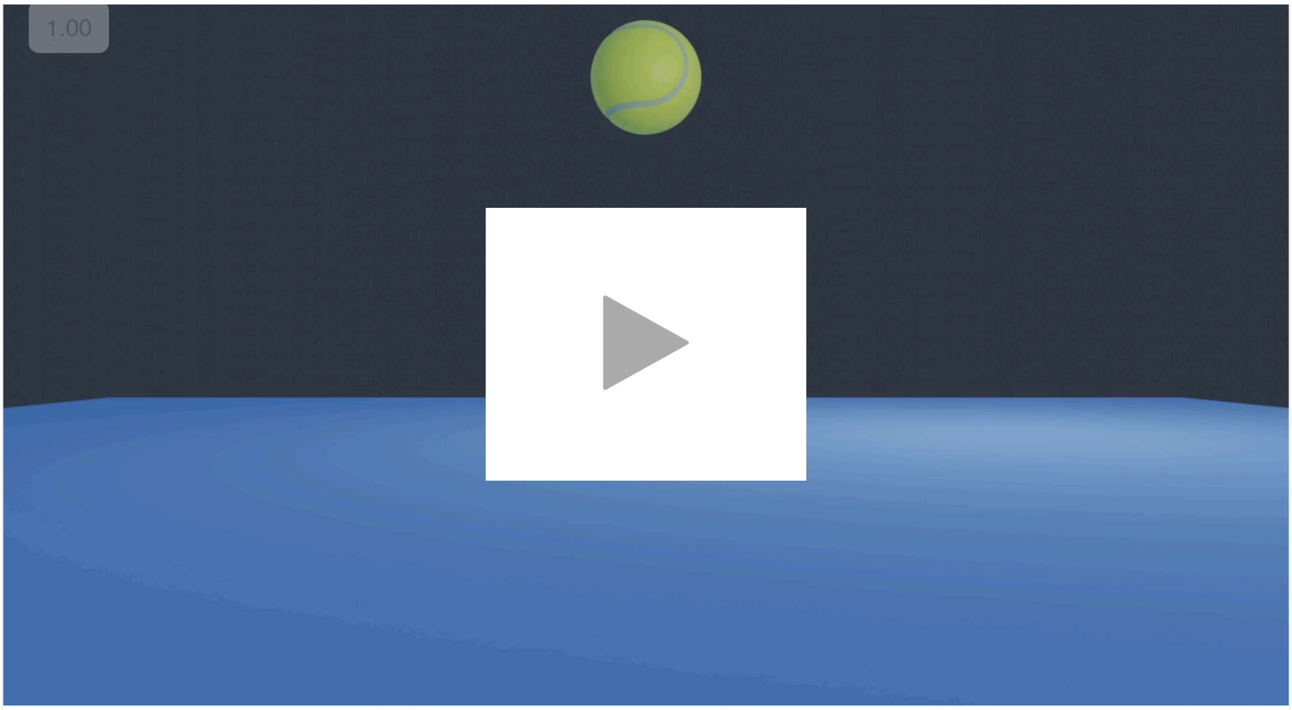

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Rights of use

Video 1. A Ball Bouncing.

More information for video 1

The interactive is a simple animation of a green-blue tennis ball bouncing on a flat blue surface. The ball starts bouncing off at one end of the table and comes towards the user with each bounce. Finally, the ball fills in the screen and the user can see only a part of the ball.

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Video 2. An Egg Cracking.

More information for video 2

The video interactive shows a brown egg falling onto a hard surface and breaking, demonstrating the concept of inelastic collisions and material fragility. It includes the moment of impact where the eggshell cracks and the contents spread out, illustrating how brittle materials absorb force differently than elastic ones. The animation explains why certain objects do not bounce but instead shatter, reinforcing key concepts in physics related to impact forces and material properties.

Video 3 shows a Newton's cradle. How does a Newton's cradle work? One ball hits another ball and something is transferred, but what? How does the motion of one ball affect another ball? And why do the balls in the middle not move?

Video 3. Newton's cradle.

At the moment, you are probably sitting on a chair and looking at a screen. How fast do you think you are moving through space? You may not feel like you are moving at all, but you are actually moving at hundreds of kilometres per hour.

All the examples above are about bodies interacting. In this subtopic, you are going to see how bodies interact and why bodies move in the first place.

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Nature of Science

Aspect: Experiments

Motion can be studied with a kinematics approach, using quantities such as position, velocity and acceleration. It can also be studied from a dynamics approach, using quantities such as force and momentum.

This is common in physics. One phenomenon can be looked at from different perspectives, which supplement each other to give us a complete picture. Every new physics tool gives us a better insight into known problems.

Prior learning

Before you study this subtopic make sure that you understand the following:

- Understanding, drawing and adding vectors (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).
- Determining the area under the line on a graph (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).

Practical skills

Once you have completed this subtopic, go to Practical 2: Investigating projectile motion (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-relationship-between-velocity-id-46751/).

A. Space, time and motion / A.2 Forces and momentum

The forces

A.2.1: Newton’s three laws of motion A.2.2: Forces as interactions between bodies A.2.3: Free-body diagrams
A.2.4: Analysis of free-body diagrams to find resultant force

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Learning outcomes

By the end of this section you should be able to:

- Describe what forces are.
- Represent the forces acting on a body using free-body diagrams.
- Determine the resultant force acting on a body.
- Understand and apply Newton's third law of motion.

The Ever Given is one of the largest container ships in the world (**Figure 1**). In 2021, the Ever Given ran aground, blocking the Suez Canal. The ship has a mass of over 200 million kg. Why does such a massive object not sink?

Figure 1. The Ever Given container ship.

Imagine throwing a ball of modelling clay onto the floor. What happens to the ball of clay? Does the clay stay the same shape?

Forces

A force is a physical quantity that describes the **interaction** between bodies.

Student view

- When bodies interact, forces are experienced by the bodies.

- When bodies stop interacting, the forces stop.

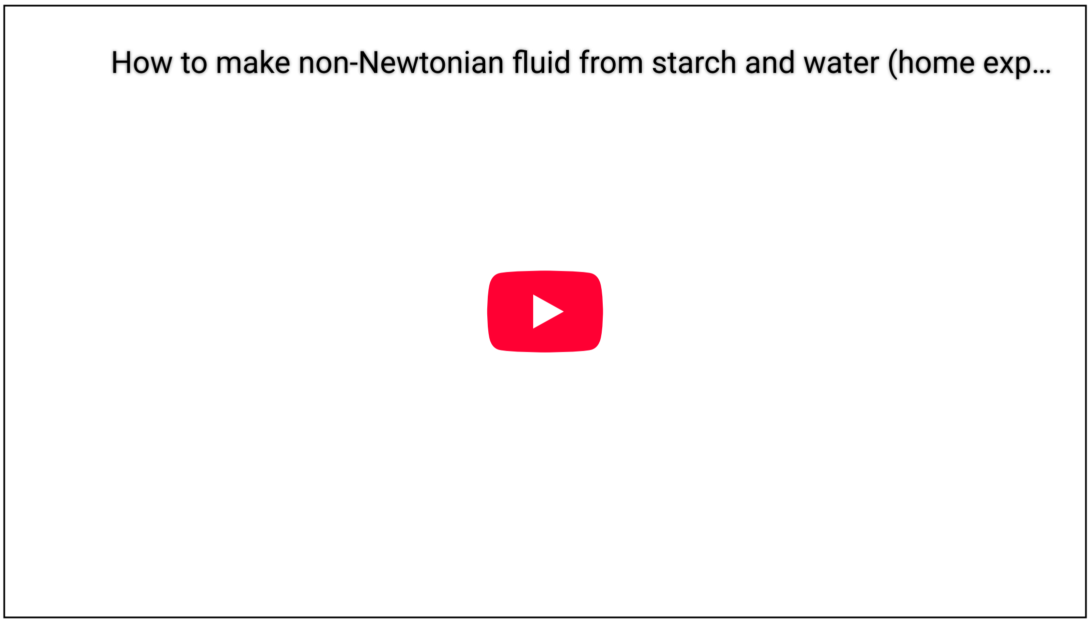
A force can:

- change the motion of an object – set it in motion or stop it from moving
- change the shape or size of a body.

Creativity, activity, service

Strand: Activity
Learning outcome: Demonstrate that challenges have been undertaken, developing new skills in the process

In certain very unusual materials, increasing the rate at which force is applied makes it harder to change the shape of an object. These objects are typically made from materials called non-Newtonian fluids. You can even make some at home with some simple ingredients.



Video 1. Making non-Newtonian fluids.

More information for video 1

The video opens with a visually engaging demonstration in which a pair of bare feet run across a white, opaque liquid contained in a bright red plastic tub. This instantly piques curiosity as the feet remain supported by the surface, defying the typical expectations of how liquids behave. A text appears onscreen: “Non-Newtonian fluid”. Then, a question appears onscreen: "How can you run on a liquid and not drown?" This serves as the central inquiry of the video, prompting an exploration into the science behind non-Newtonian fluids—a type of material that does not conform to standard fluid dynamics.
The video transitions into the step-by-step preparation of the fluid using simple household

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materials: cornstarch and water. Cornstarch is poured from a glass beaker into a clear casserole dish, during which a text appears onscreen: “Cornstarch”. Further, water is poured into the casserole dish from a red measuring cup, for which a text appears onscreen: “Water”. The ingredients are mixed by hand until they form a consistent, thick, white paste. When the person starts mixing, a text appears onscreen: “Stir the mixture until it forms a thick homogenous mass”, followed by another text: “The starch particles swell in the water and bond in the form of chaotically interlaced molecules”. As more water is added, the mixture transforms into a milky and opaque substance. This behavior is linked to the molecular structure of the starch and water blend. The cornstarch particles absorb water and form a chaotic and disordered network. This structure is key to the unique physical properties of the fluid.

To illustrate its dual nature, the video demonstrates how the mixture responds to different types of force. When a hand punches the surface, the fluid behaves like a solid, during which a text is displayed onscreen: “At a higher shear rate, the tight bonds do not let the molecules move and the mixture behaves like a solid body”. The force applied quickly is met with resistance because the starch particles lock together under sudden pressure. Conversely, when the hand is moved slowly through the mixture, it flows freely like a typical liquid. A text appears onscreen: “At a lower shear rate, the molecules spread out and untangle, and the mixture behaves like a liquid”. This phenomenon is attributed to shear rate—the rate at which force is applied to a material. onscreen, the behavior is demonstrated visually with a hammer hitting the surface of the liquid, during which a text is displayed onscreen: “This liquid is called non-Newtonian, because it does not obey the usual law of physics”. Rather than sinking, the hammer bounces slightly and stands upright, reinforcing the material’s solid-like behavior when subjected to quick, strong forces. Conversely, when the hammer is dropped into the liquid slowly, it sinks into the liquid, showing liquid-like behavior when subjected to a lower sheer rate.

The visual narrative returns to the feet running across the tub, with a text displayed onscreen: “You can even run on it!”. Rapid motion keeps the person above the surface because the high-speed impact forces the particles to lock together, creating temporary solidity. When the feet stop moving, they slowly begin to sink, and a text is displayed onscreen: “But if you stop for a second, you’ll sink to the bottom”. Sinking indicates a shift back to liquid behavior as the pressure lessens and is applied more gradually. The visual narrative again returns to the feet running across the tub, with a text displayed onscreen: "So, let’s keep moving!" emphasizing that continuous, rapid movement is essential for staying on the surface of the fluid.

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This demonstration serves as an accessible introduction to the core principles of non-Newtonian fluids, with a specific focus on shear-thickening behavior. The learning outcomes center around understanding how certain materials can change their properties based on how force is applied. Viewers are encouraged to consider practical applications for such materials. Non-Newtonian fluids could be beneficial in scenarios requiring adaptive resistance to impact—such as protective sports gear, shock-absorbing footwear, or even roadway materials that harden under sudden pressure.

What products can you think of that would make good use of a material like this? Can you design something that would be of benefit to an individual or community? How could your design help them?

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In this section, you are going to look at how forces affect the motion of objects. We say that a force is **applied to, acts on**, or is **exerted on** or **by** a body.

Force, F, is a **vector** quantity (see section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/)). This means force has a magnitude (how strong the force is) and a direction in which the force is applied. Force is measured in newtons (N).

Study skills

You need to know the difference between scalars (magnitude only) and vectors (magnitude and direction) (section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/)). If we talk about the temperature of an object, it does not have a direction. If we talk about the velocity of the object, it needs a direction to fully describe the motion.

For a body to experience a force, the body needs to interact with another body. This means that whenever there is a force, two or more bodies are involved. Several forces can act on each of the interacting bodies.

A force cannot be experienced by an isolated body. However, to simplify problems in physics, we draw a diagram that shows the forces on the body we are interested in. This is called a free-body diagram. **Figure 2** shows a free-body diagram showing the forces acting on an object.

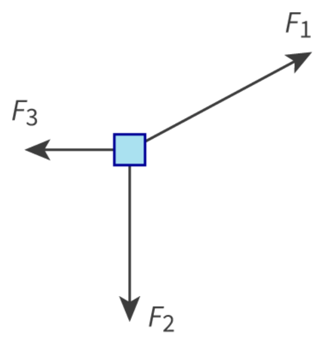

Figure 2. A free-body diagram showing the forces acting on an object.

More information for figure 2

The image depicts a free-body diagram showing a square representing an object at its center. Three forces are acting on the object: F1, F2, and F3.

- F1 is represented by an arrow pointing upwards and to the right.

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- F2 is depicted with an arrow pointing directly downward.
- F3 is shown with an arrow pointing to the left.

Each force is labeled with the corresponding text (F1, F2, F3) near the arrow's tip to show the direction. The overall structure illustrates how the forces act simultaneously on the object, demonstrating the principles of force interactions in physics.

[Generated by AI]

Study skills

You need to know how to draw and interpret free-body diagrams. When drawing free-body diagrams only include the forces that act on the body you are investigating.

Forces are vectors. When you analyse forces, you need to be able to (section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/)):

- add forces as vectors
- resolve force vectors into components.

Adding forces

The sum of all the forces acting on a body is called the resultant force. The resultant force has the same result on the body as all the individual forces acting together on the body. In other words, the resultant force is the single force that can replace all the individual forces acting on a body.

Table 1 shows the resultant force acting on each body.

Table 1. Resultant forces.

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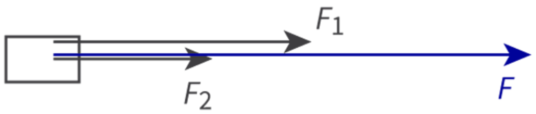
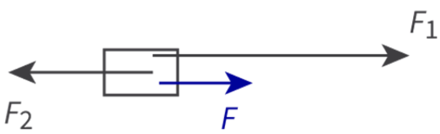
Forces	Resultant force
 More information The image is a diagram illustrating the concept of forces acting on an object. It shows a rectangular object on the left, with two forces acting on it. F1 is represented by a gray arrow pointing to the right, F2 is another gray arrow also pointing to the right but smaller, and the resultant force, F, is a larger blue arrow pointing to the right. The diagram effectively explains how different forces combine, with F being the net force resulting from F1 and F2 working in the same direction on the object. [Generated by AI]	$F = F_1 + F_2$
 More information The diagram illustrates forces acting on an object. The object is represented as a rectangle, and three arrows indicate forces interacting with it. The force labeled 'F1' is directed to the right, 'F2' is directed to the left, and 'F' is directed to the right but shorter in length, indicating a lesser magnitude than 'F1'. This suggests that 'F' is an additional force supplementing the net force but not as strong as 'F1'. The object appears to be experiencing some form of equilibrium or motion as influenced by these forces. The diagram visualizes the relationships and directions of these forces. [Generated by AI]	$F = F_1 – F_2$

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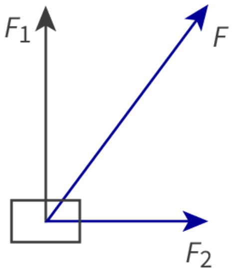
Forces	Resultant force
 More information This diagram illustrates three vectors originating from a box. Vector (F_1) points directly upward from the top of the box. Vector (F_2) extends horizontally to the right from the right side of the box. The resultant force (F) is depicted as a diagonal arrow originating from the center of the box, pointing upwards and to the right, indicating the resultant of vectors (F_1) and (F_2). The lines representing the vectors are labeled accordingly near their tips with (F_1), (F_2), and (F). [Generated by AI]	$F = \sqrt{(F_1^2 + F_2^2)}$

Resolving forces

We can resolve a force into components at right angles to each other (see section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/)). **Figure 3** shows how a force F can be resolved into two forces, F_x and F_y.

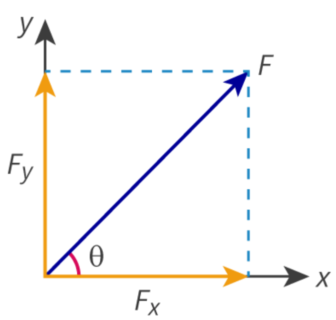


Figure 3. Resolving a force into its components.

More information for figure 3

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The diagram illustrates how a force F can be resolved into two perpendicular components, Fx and Fy. The force F is represented as a diagonal vector pointing upwards and to the right.

- Fx is the horizontal component of the force, directed along the x-axis. It is depicted with an arrow pointing to the right.
- Fy is the vertical component of the force, directed along the y-axis. It is shown with an arrow pointing upwards.

The angle θ is formed between the force F and the horizontal component Fx. The force F is shown as a blue line, and the components Fx and Fy are shown as orange lines. This arrangement forms a right-angled triangle, with F being the hypotenuse and Fx and Fy being the two legs. The force components form a dotted path to help visualize their vector relation.

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Worked example 1

The free-body diagram shows the forces acting on a body. Determine magnitude of the resultant force acting on the body.

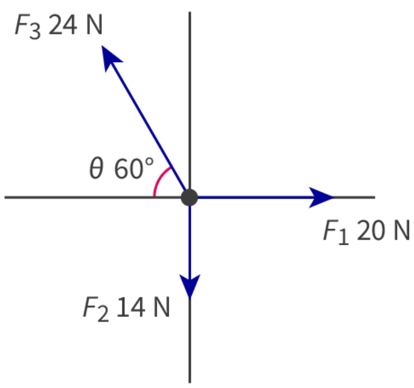

Figure 4. The forces acting on a body.

More information for figure 4

The diagram illustrates a free-body diagram with three forces acting on a point. The forces are represented by arrows originating from a central point. The force labeled F1 is horizontal and points to the right, with a magnitude of 20 N. The force labeled F2 points downward, with a magnitude of 14 N. The force labeled F3 forms a 60-degree angle with the horizontal axis, pointing upwards and to the left, with a magnitude of 24 N. The angles between the forces are marked on the diagram with the symbol θ.

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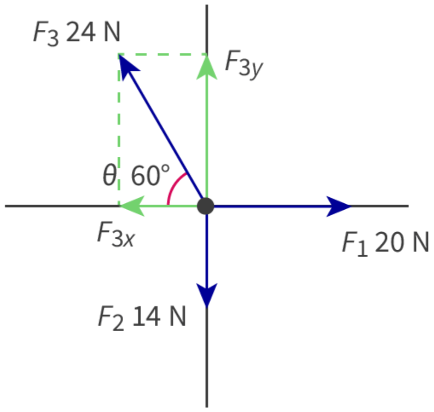
Solution steps	Calculations
Step 1: Identify a strategy.	Forces are vectors, so work on each axis separately. F_1 and F_2 do not need to be resolve but F_3 does.
Step 2: Resolve F_3 into its components and calculate the components of F_3 using the known angle.	 $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin 60 = \frac{F_{3\text{y}}}{F_3}$ $F_{3\text{y}} = F_3 \sin 60$ $= 24\times \sin 60$ $= 20.78\text{ N}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos 60 = \frac{F_{3\text{x}}}{F_3}$ $F_{3\text{x}} = F_3 \cos 60$ $= 24\times \cos 60$ $= 12\text{ N}$

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Solution steps	Calculations
Step 3: Calculate the sums of the forces on the two axes.	On the *x*-axis: $$\begin{aligned} F_\text{x} &= F_1 – F_{3\text{x}} \\ &= 20–12 \\ &= 8\text{ N} \end{aligned}$$ On the *y*-axis: $$\begin{aligned} F_\text{y} &= F_{3\text{y}} - F_2 \\ &= 20.78–14 \\ &= 6.78\text{ N} \end{aligned}$$
Step 4: Determine the resultant force and state the answer with appropriate units and the number of significant figures used in rounding.	$$\begin{aligned} F &= \sqrt{(F\text{x}^2 + F_\text{y}^2)} \\ &= \sqrt{(8^2 + 6.78^2)} \\ &= 10.49\text{ N} \\ &= 10\text{ N (1 s.f.)} \end{aligned}$$

Newton’s third law of motion

Forces never occur on their own – there is always a pair of forces in nature. This is something you experience every time you touch an object. Take a pen and press your finger against the tip. You are pressing against the pen, so why is your finger feeling a force from the pen?

This is an example of **action** and **reaction**, also known as Newton’s third law of motion. Newton’s third law states that any two bodies interact by applying forces on each other. These forces are equal in magnitude and opposite in direction. So, we can say that every action has an equal and opposite reaction.

This means that when you press against the pen with a force *F*, the pen is also pressing against your finger with an equal force *F*, in the opposite direction.

Study skills

It is very important to remember that action and reaction are forces that act on different bodies. This means that you cannot add them together.

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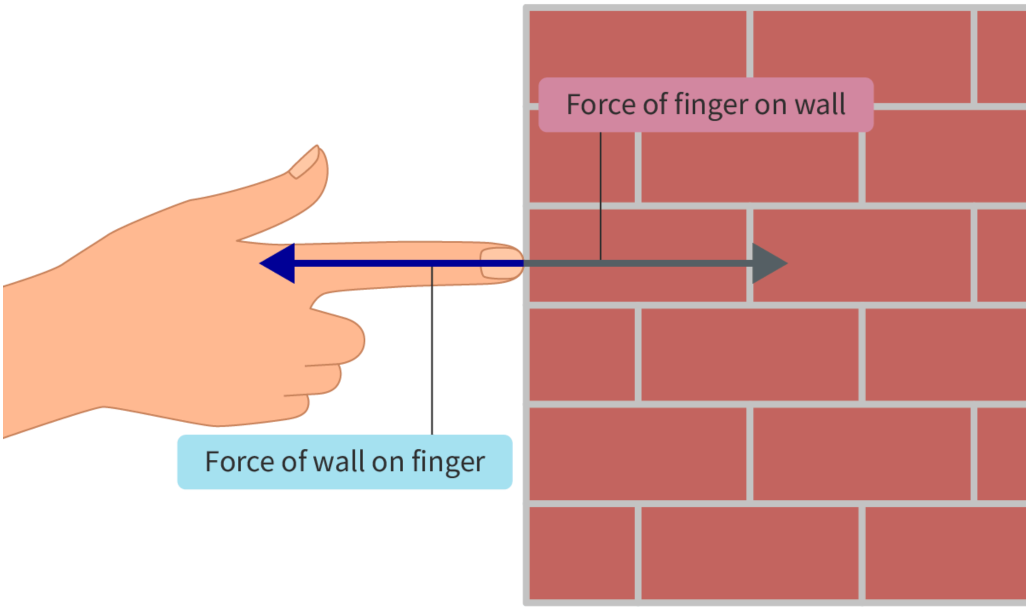

Figure 5. A force pair.

More information for figure 5

The image depicts a finger pressing against a brick wall. Two arrows are shown, one pointing away from the finger towards the wall labeled "Force of finger on wall," and another pointing back towards the finger labeled "Force of wall on finger." This illustrates Newton's third law of motion, where every action has an equal and opposite reaction, emphasizing the interaction between the two bodies, the finger, and the wall.

[Generated by AI]

You may be given a force and asked to find its force pair. Remember to identify the two bodies interacting, for example, body A and body B. Body A will exert a force on body B, and body B will exert a force on body A.

Imagine a car travelling on a road. The car hits a fly, and the fly is smashed on the windshield. Does the car exert a greater force on the fly, or does the fly exert a greater force on the car? According to Newton’s third law, the two forces are equal in magnitude. The magnitude of the force is enough to smash the fly on the windshield, but not enough to cause any damage to the car.

Theory of Knowledge

Forces occur in force pairs and act simultaneously on the interacting bodies. Can we say which force is the action and which force is the reaction?

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There is no time delay between the ‘acting’ force and the ‘reacting’ force, so we cannot distinguish between action and reaction using time. There is also no other characteristic that can separate one from the other.

However, we can say that, for some force pairs, if there isn’t an action, then there is no reaction. For example, if you don’t press your finger against your pen, you won’t experience the force of the pen on your finger.

Work through the activity to check your understanding of Newton’s third law and forces.

Activity

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills — Asking questions and framing hypotheses based upon sensible scientific rationale
- **Time required to complete activity:** 10 minutes
- **Activity type:** Group activity

Imagine a heavy box resting on the floor in your room.

You start pushing on the box. According to Newton’s third law, the box is pushing back with an equal force.

You push harder. The box is responding with an equal force. No matter how hard you push, the box will push back with an equal force.

Discuss in your group why the box will move if you push hard enough.

Newton's third law.

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5 section questions

A. Space, time and motion / A.2 Forces and momentum

Field forces

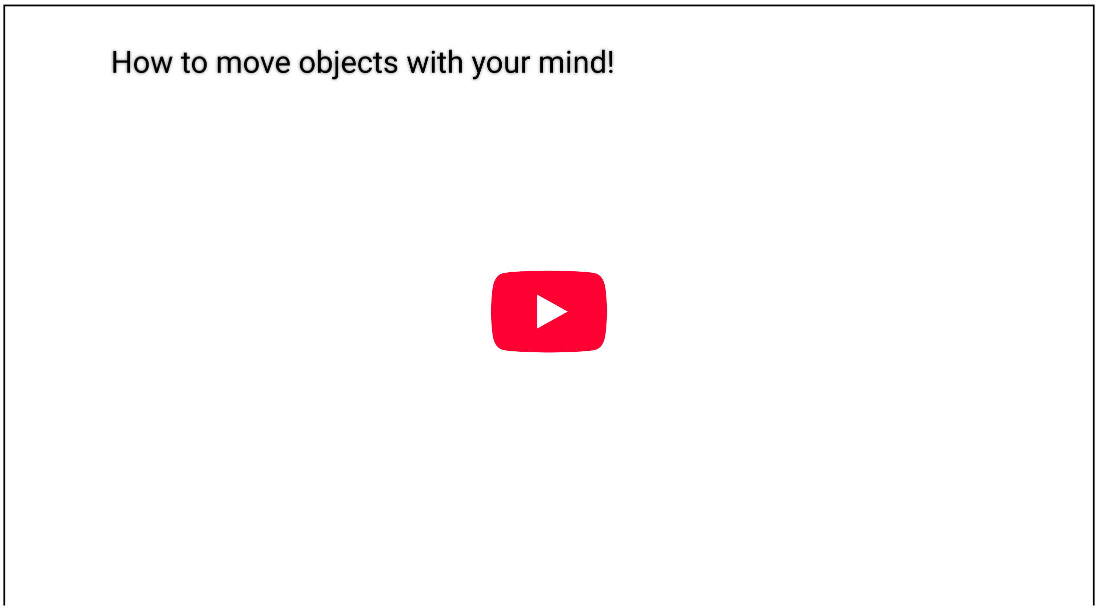
A.2.6: The nature and use of the field forces

Learning outcomes

By the end of this section you should be able to:

- Understand the concept of gravitational force, and apply the equation $F_\mathrm{g} = mg$.
- Understand the concept of electric force.
- Understand the concept of magnetic force.

Magicians sometimes perform tricks where they seem to move objects from a distance with their minds (**Video 1**). Can objects be moved without touching them?



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Video 1. Moving objects with the 'mind'.

There are forces in nature that act at a distance. Objects can be very far away from each other but still attracted to or repulsed from each other. How can this happen?

Fields

The concept of a field is used to explain how forces can act at a distance.

If you have ever experimented with a magnet, you have experienced the effects of a field. Particular objects (including other magnets, and items made of iron or steel) experience attractive or repulsive forces when they are near a magnet. The magnet is affecting the space around it, which is now said to contain a magnetic field. This field (or force field) is not directly visible, but its existence can be deduced from its effects on other objects.

Because field forces act at a distance, the bodies that interact do not need to be touching. Field forces include gravitational force, electric force and magnetic force.

Making connections

The field is a very important concept in physics. Fields are also covered in subtopics D.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44096/) and D.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44743/).

Gravitational force

The athlete in **Figure 1** uses skill and technique to jump high and pass over the bar. But no matter how high she jumps, she always lands back on the mat. There is a force that pulls the athlete back down toward the Earth. It is the same force that keeps you sitting on a chair or lying in bed.

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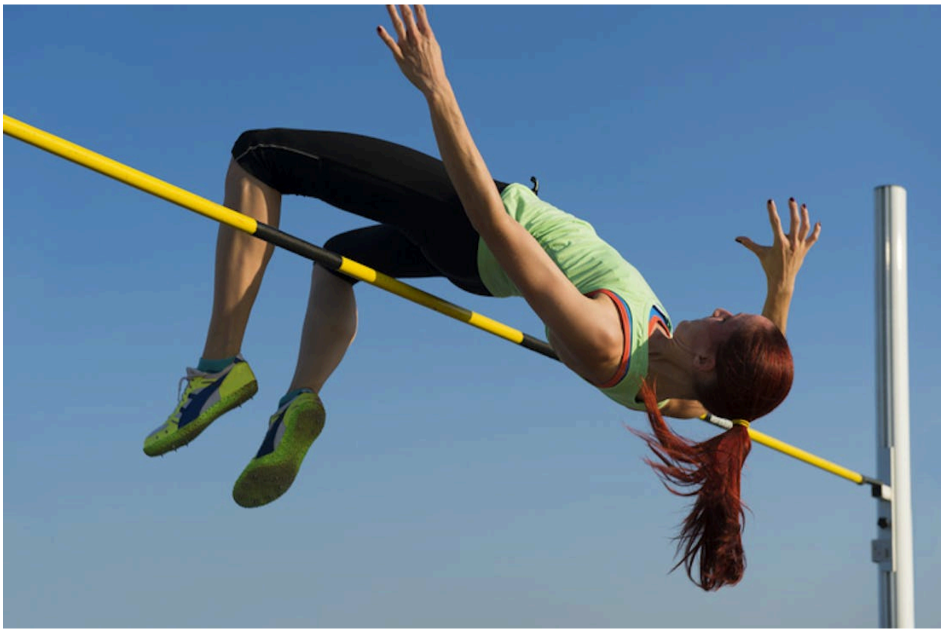

Figure 1. An athlete performing the high jump.

Credit: technotr, Getty Images (https://www.gettyimages.com/detail/photo/young-woman-at-high-jump-royalty-free-image/161931066)

This force is called the gravitational force, F_g. It is also known as the weight of a body. You experience it when you are standing on the ground, jumping, or in an aeroplane in the sky.

Every object that has mass experiences the gravitational force. The direction of the gravitational force is toward the centre of the Earth. For objects near the surface of the Earth, we can calculate the magnitude of this force using the equation in **Table 1**.

Table 1. Equation for gravitational force.

Equation	Symbols	Units
$F_g = mg$	F_g = gravitational force (or weight)	newtons (N)
	m = mass	kilograms (kg)
	g = acceleration of free fall (9.8 m s^{-2} at the Earth's surface)	Given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/)of the DP physics data booklet

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Worked example 1

Determine the gravitational force exerted on an object of mass 15 kg on the surface of the Earth.

$$m = 15\ \text{kg}$$

$$g = 9.8\ \text{m s}^{-2}$$

$$\begin{aligned} F_{\text{g}} &= mg \\ &= 15 \times 9.8 \\ &= 147\ \text{N} \\ &= 150\ \text{N}\ (2\ \text{s.f.}) \end{aligned}$$

Study skills

You need to understand the difference between the mass of an object and its weight. Mass is a measure of how much matter there is in an object. Mass is a scalar quantity, measured in kilograms (kg). Weight is the gravitational force that bodies experience near the Earth, or other massive objects. Weight is a vector quantity, measured in newtons (N).

Gravitational force (weight) is proportional to the mass of the object and the acceleration of free fall:

- The greater the mass, the greater the gravitational force.
- The greater the acceleration of free fall, the greater the gravitational force.

Acceleration of free fall is considered to be a constant close to the surface of the Earth. In reality, the value of acceleration of free fall varies depending on where you are on the Earth.

Acceleration of free fall is different on other planets. For example, on Jupiter, it is 2.5 times larger than on the Earth. This means that if you were standing on the surface of Jupiter, you would weigh 2.5 times more. On the Moon, the acceleration of free fall is about $\frac{1}{6}$ of that on the Earth. On the Moon, you would weigh only $\frac{1}{6}$ of what you weigh on the Earth.

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Study skills

As with all interactions between bodies, Newton's third law applies to the gravitational force. This means that when the Earth exerts a force F_g on an apple, the apple is also exerting a force F_g on the Earth of the same magnitude.

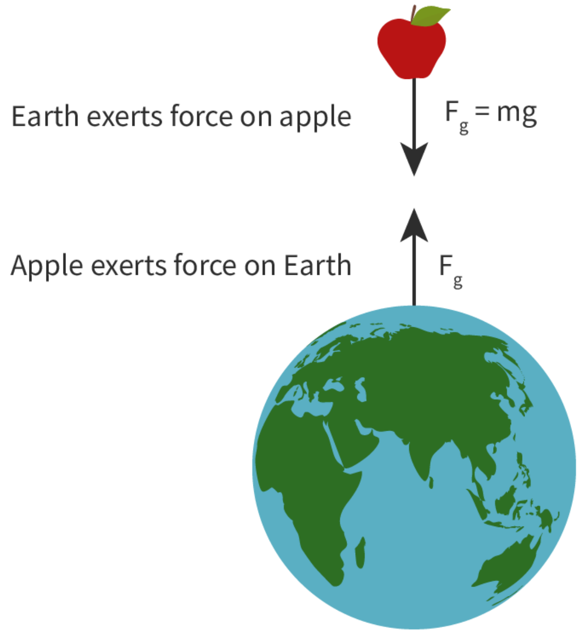

Figure 2. Force exerted by the Earth on an apple, and force exerted by an apple on the Earth.

More information for figure 2

The diagram illustrates Newton's third law with an apple and Earth. At the top, there's an apple with a downward arrow labeled 'F_g = mg' indicating the force it experiences due to gravity. Below it, the Earth is shown with an upward arrow labeled 'F_g' illustrating the equal and opposite force exerted by the apple on Earth. Text on the left explains the interactions: 'Earth exerts force on apple' and 'Apple exerts force on Earth.' This visual representation demonstrates the concept of equal and opposite forces.

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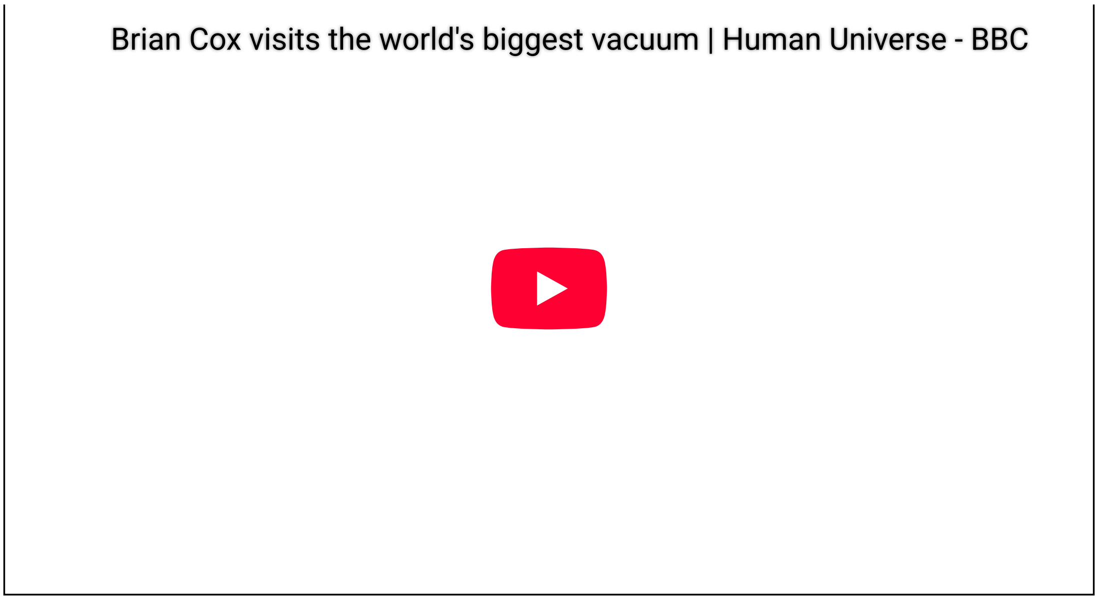
During free fall, a body accelerates with a constant acceleration equal to the acceleration of free fall, g (see section A.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/)). The only force on the object is weight. This is only true when there is no (or negligible) air resistance. When there is air resistance, a heavy object will fall faster than a light object. When objects with very different masses fall in a vacuum chamber (air has been removed), the objects fall simultaneously (**Video 2**).

Student view



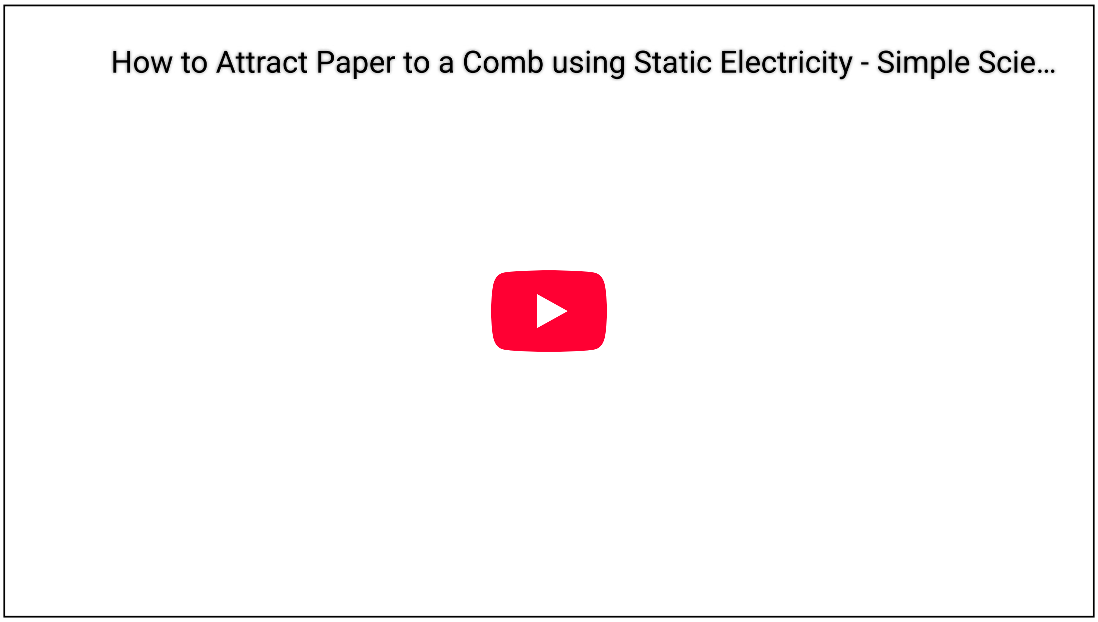
Video 2. Objects falling in a vacuum chamber.

Exercise 1

Click a question to answer

Electric force

Video 3 shows what happens when you run a comb through your hair then bring it close to some pieces of paper – the paper is attracted to the comb.



Video 3. A comb attracting pieces of paper.

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This happens because when you run the comb through your hair, you **charge** the comb.

Electric charge is a property of matter and depends on the matter's microscopic structure. There are two types of charge: positive charge (+) and negative charge (−) (subtopic D.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44743/)).

Charged bodies interact with a force called the electric force, F_e. The forces between charges can be attractive or repelling (**Figure 3**).

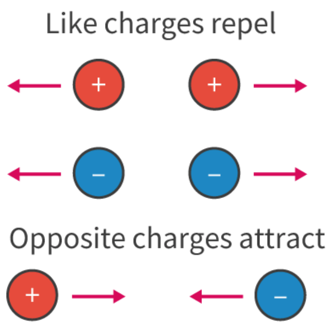

Figure 3. The electric force between charges.

More information for figure 3

The diagram shows the interactions between charged particles. The top section depicts two like charges, each marked with a plus sign, positioned side by side with arrows pointing away from each other, labeled "Like charges repel." Below, another set of two like charges, each with a minus sign, also has arrows pointing away from each other. In the lower section, there are two opposite charges: one with a plus sign and the other with a minus sign, with arrows pointing towards each other. This is labeled "Opposite charges attract." The diagram visually represents the basic principle of electrostatic force where like charges repel, and opposite charges attract.

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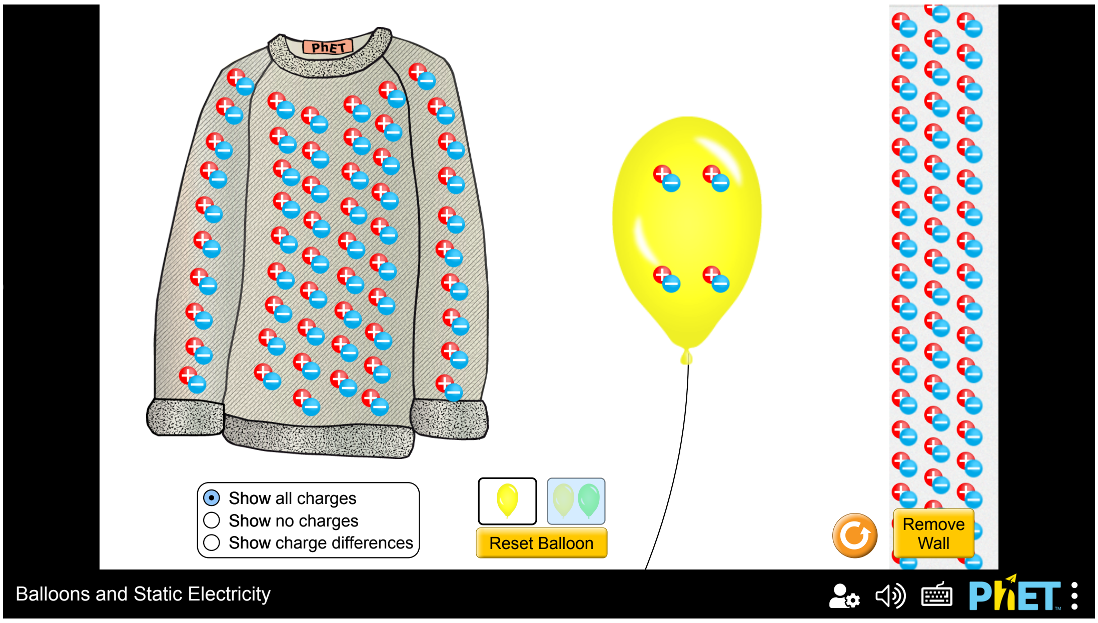
- Like charges (two positive charges or two negative charges) repel each other.
- Opposite charges (positive charge and negative charge) attract each other.
- The greater the charge, the greater the electric force.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Investigate the effect of charge using the simulation in **Interactive 1**. Rub the balloon on the jumper. What type of charge does the balloon now have? Use the idea of positive and negative charge to explain why the balloon now sticks to the wall. (Note that in the simulation the balloon sticks to the wall even when it is not charged, which would not happen in real life.)



Interactive 1. Charging a balloon.

More information for interactive 1

Charging a Balloon is an interactive simulation that explores the principles of static electricity through the interaction of a balloon, a jumper, and a wall. Initially, the balloon and jumper have equal numbers of positive and negative charges, represented by red and blue symbols, evenly distributed to indicate electrical neutrality.

Users can manipulate the balloon by rubbing it against the jumper, and observing how negative charges transfer from the jumper to the balloon. This charge transfer leaves the jumper positively charged while the balloon becomes negatively charged. As a result, an electrostatic force emerges—opposite charges attract, drawing the balloon toward the jumper.

Users can also bring the balloon near a neutral wall to explore how charged objects influence their surroundings. The negative charges on the balloon repel the negative charges in the wall's surface, causing the wall to become slightly positively charged. This charge redistribution creates an attractive force, allowing the balloon to stick to the wall. The simulation visually represents charge movement, helping users understand how electrostatic forces shape interactions between objects. While the simulation allows the balloon to stick to the wall even when uncharged (a limitation noted in the instructions), it accurately demonstrates key electrostatic principles.

Throughout the interactive experience, users can experiment with different settings to deepen their understanding. Options include visualizing all charges, hiding charges, or displaying only charge differences. A reset button allows users to restart the process, and the ability to remove the wall enables focused exploration of the balloon-jumper interaction.

Student view

By engaging with the simulation, users gain a deeper understanding of static electricity, including charge transfer, attraction, and repulsion. The hands-on nature of the simulation reinforces key physics concepts, making abstract ideas more tangible and intuitive.

Magnetic force

Some trains travel without touching the rails. They are called magnetic levitation (maglev) trains (**Figure 4**). They float approximately 10 cm above the rails, and they can reach over 400 km h^{-1}. How is this possible?

Figure 4. A maglev train.

Maglev trains make use of another force that acts at a distance, the magnetic force, F_m. This force occurs between bodies that behave as magnets.

Whether a body will behave as a magnet depends on its microscopic structure, and some objects can be magnetised. All magnets have a north pole (N) and a south pole (S). Like poles repel each other, and opposite poles attract each other. **Figure 5** shows the magnetic force between bar magnets.

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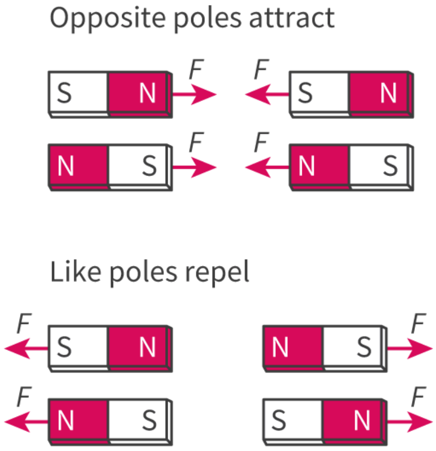

Figure 5. The magnetic force between bar magnets.

More information for figure 5

The diagram illustrates the magnetic forces between bar magnets. It shows two main scenarios. In the first scenario, labeled "Opposite poles attract," two sets of bar magnets are displayed with opposite poles facing each other (S facing N and N facing S). Arrows labeled 'F' indicate the attractive force pulling the poles together.

In the second scenario, labeled "Like poles repel," two sets of magnets are positioned with like poles facing each other (S facing S and N facing N). Arrows labeled 'F' illustrate the repulsive force pushing the poles apart.

[Generated by AI]

Maglev train engineers use magnets that repel each other to make the train levitate and travel without touching the rails.

Bodies can have a positive charge or a negative charge, but a body cannot only have a north pole or a south pole. If you cut a magnet in half, you get two smaller magnets, each with a north pole and a south pole.

Drag and drop the 'attract' and 'repel' labels into the correct positions in **Interactive 2**.

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Interactive 2. Do these poles attract or repel each other?

Carry out the activity to check your understanding of gravitational force.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

The value of gravitational force, *g*, tells you the magnitude of the force per unit mass exerted on an object by the Earth, or another object with mass.

The value of *g* is generally taken to be constant at the surface of the Earth, but it actually varies slightly in different places on the Earth. It can be very different on other planets. The value of *g* near the surface of the Earth is given in IB physics as 9.8 m s^{-2}.

Another way of calculating *g* is using the following equation (subtopic D.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44096/)):

$$g = G\left(\frac{M}{r^2}\right)$$

where G is the gravitational constant, *M* is the mass of the Earth (or other planet), and *r* is the distance from the centre of the Earth (or planet).

Try to answer the following questions. Click on 'Show or hide solution' to see the answers.

1. Using the equation, determine the value of *g* for a point mass at a distance of 7500 km from the centre of the Earth.

   Mass of the Earth is 6.0×10^{24} kg

Student
view

G is 6.67×10^{-11} $\text{Nm}^2\ \text{kg}^{-2}$

$$\begin{aligned} r &= 7500 \text{ km} \\ &= 7\,500\,000 \text{ m} \end{aligned}$$

$$\begin{aligned} g &= G\left(\frac{M}{r^2}\right) \\ &= 6.67\times10^{-11}\times\left(\frac{6.0\times10^{24}}{7\,500\,000^2}\right) \\ &= 7.1 \text{ m s}^{-2} \end{aligned}$$

Why is the gravitational force different for different places on the Earth? Can you use the equation above to explain why?

Different places on Earth have slightly different distances from the centre of the Earth, so if r in the equation changes, so does g.

Also, the Earth is not perfectly uniform, and changes in density of the Earth in different areas could affect g.

You can calculate g where you are now using the sensorsone (https://www.sensorsone.com/local-gravity-calculator/) website.

1. **Table 2** below shows the approximate mass and radius of bodies in our solar system. Use the equation to calculate the gravitational force acting on a 1 kg object on the surface of each body.

Table 2. Mass and radius of planets.

Planet	Mass (kg)	Radius (km)
Pluto	1.3×10^{22}	1190
Mars	6.4×10^{23}	3400
Jupiter	1.9×10^{27}	71 500
Venus	4.9×10^{24}	6050

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Pluto: 0.61 N

Mars: 3.7 N

Jupiter: 25 N

Venus: 8.9 N

5 section questions

A. Space, time and motion / A.2 Forces and momentum

Normal force, friction, tension

A.2.5: The nature and use of the contact forces

Learning outcomes

By the end of this section you should be able to:

- Understand the concept of the normal force.
- Understand the concept of surface frictional force, and apply the equations for friction on a stationary body and a moving body:

$$F_{\text{f}} \leqslant \mu_{\text{s}} F_{\text{N}} \text{ and } F_{\text{f}} = \mu_{\text{d}} F_{\text{N}}$$

Put your foot lightly on the ground and try to slide it sideways (**Figure 1**). Think about how hard you have to push it. Next, do the same thing but this time push down harder with your foot when you are sliding it sideways. Is it easier or harder to slide? What forces are acting here? What changes would make it easier or harder to slide your foot?

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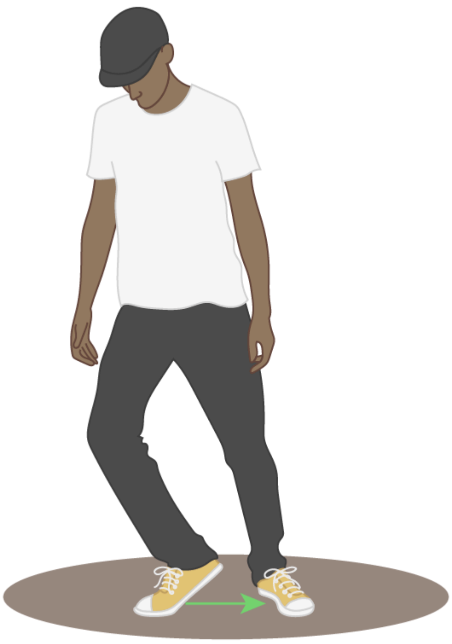

Figure 1. How difficult is it to slide your foot sideways?

Normal force

Field forces are **non-contact forces** – the two bodies do not need to be touching for them to exert a force on each other (see section A.2.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/field-forces-id-44733/)). If the two bodies need to be touching for a force to act, then the force is a **contact force**.

Imagine a mug resting on a desk. The Earth exerts a downwards pull on the mug (which we call the weight of the mug). This makes the mug push downwards against the desk. As a result, the desk pushes upwards on the mug with a force of the same size. When two objects in contact push against each other, these forces are called normal forces, F_N. **Figure 2** shows the forces acting on the mug. It is a free-body diagram of the mug and so it does not show the normal force acting on the desk.

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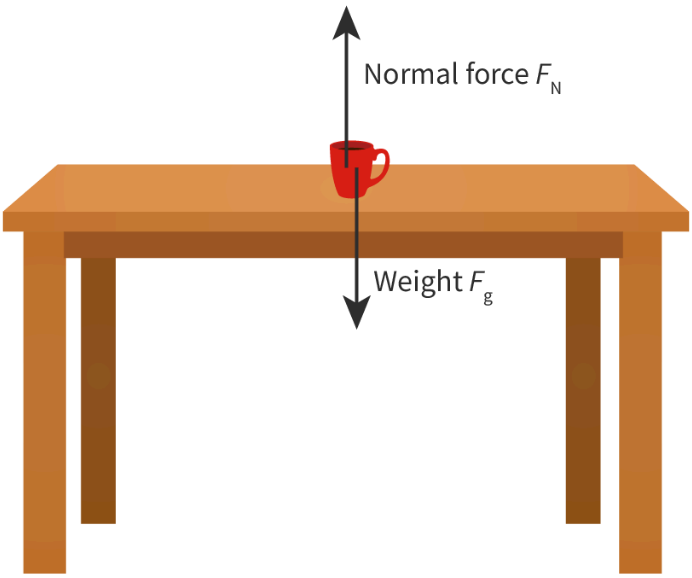

Figure 2. The normal force.

More information for figure 2

The diagram shows a wooden table with a red mug placed on its surface. The mug is subject to two forces. An upwards arrow labeled "Normal force F_N" is shown extending from the top of the mug pointing vertically upwards, representing the force exerted by the table on the mug. A downwards arrow labeled "Weight F_g" extends from the bottom of the mug pointing vertically downwards, indicating the gravitational force acting on the mug. These two forces are equal in magnitude and opposite in direction, illustrating the concept of normal force as a reactionary force to gravity. The arrows emphasize the perpendicular nature of the normal force with respect to the table surface.

[Generated by AI]

The normal force always acts perpendicular to the surface that pushes against the body (**Figure 3**).

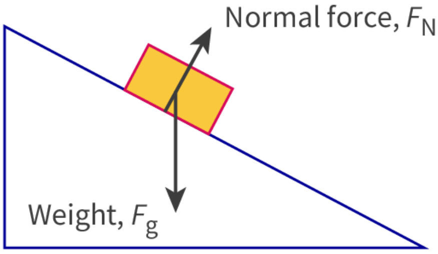

Figure 3. The normal force acts perpendicular to the surface.

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More information for figure 3

The diagram illustrates a rectangular block placed on an inclined plane. The block is positioned near the top right side of the incline. Two arrows depict forces acting on the block. The first arrow, labeled 'Normal force, F_N', is perpendicular to the surface of the incline, pointing upwards. The second arrow, labeled 'Weight, F_g', points downwards vertically, illustrating the gravitational force acting on the block.

The inclined plane is shown with a blue outline forming a right-angle triangle, with the base on the ground and the hypotenuse making the inclined surface. The block is outlined in yellow on the plane, emphasizing the position where the forces act.

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Frictional force

Take a book and place an eraser on it. Lift up one end of the book slowly and observe what happens (**Figure 4**). Does the eraser slide down immediately? Can you find the angle at which the eraser starts sliding?

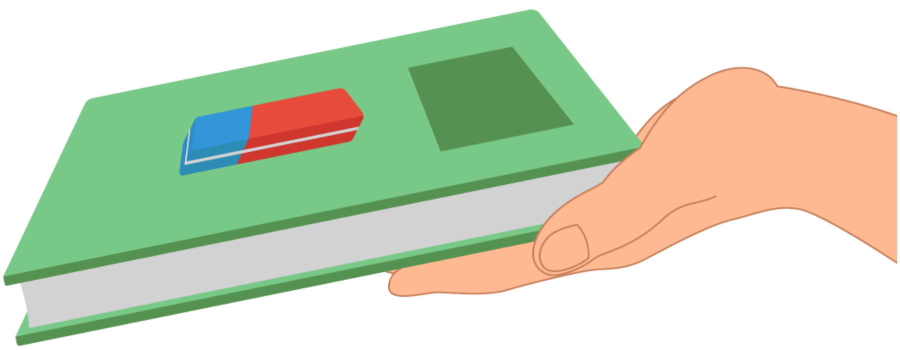

Figure 4. Why does the eraser not slide down the book immediately?

It is the frictional force that prevents the eraser from sliding down when you lift the book at a small angle. The frictional force acts when two bodies are in contact. It acts in a direction parallel to the plane of contact between the body and the surface.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Imagine you have a heavy box on the floor in your room. You start pushing lightly on the box but it does not move – why?

The reason is because of the frictional force between the box and the floor. Static friction acts between two bodies at rest. It prevents the box from moving until you push hard enough. Then the force you exert overcomes static friction and the box starts moving along the floor.

The frictional force acting on a stationary body can be determined using the equation in **Table 1**.

Table 1. Equation for frictional force acting on a stationary body.

Equation	Symbols	Units
$F_f \leqslant \mu_s F_N$	F_f = frictional force	newtons (N)
	μ_s = coefficient of static friction	unitless
	F_N = normal force	newtons (N)

Worked example 1

A 45 kg box is at rest on a flat surface. The coefficient of friction between the surface and the box is 0.73. Determine the minimum force needed to move the box.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$\text{mass} = 45\text{ kg}$ $\text{weight} = 45 \times 9.8$ $= 441\text{ N}$ The box is on a horizontal surface so the normal force must also equal 441 N.
Step 2: Write out the equation.	$F_f \leqslant \mu_s F_N$
Step 3: Substitute the values given.	$\leqslant 0.73 \times 441$

Student view

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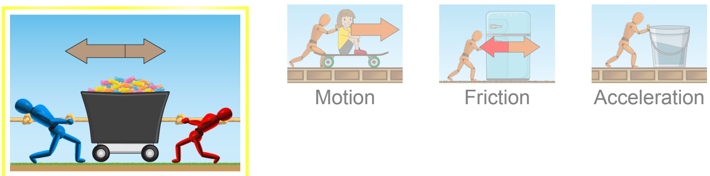
Solution steps	Calculations
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$\leqslant 321.9\text{ N} = 320\text{ N (2 s.f.)}$

Use the simulation in **Interactive 1** to investigate friction. Without making any other changes, move the ‘Friction’ slider (top right) to ‘Lots’. Slowly increase the value on the ‘Applied force’ slider. Determine the maximum value of the static friction acting on the box. What happens, and why, when the applied force is less than / equal to / greater than the maximum value of static friction?



Interactive 1. Investigating friction.

More information for interactive 1

A simulation demonstrates the basics of forces and motion. It features four panels labeled net force, motion, friction, and acceleration. The friction panel is selected. A robotic figure pushes a wooden crate on a horizontal surface. The user can adjust the applied force, the friction, and the mass by adding objects on the crate.

Here are 2 situations. When the applied force equals 25 Newtons, the friction force is 25 Newtons in the opposite direction and the sum of forces is 0. When the applied force equals 150 Newtons, the sum of forces is greater than 0 and the crate moves.

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While the box is moving on the floor, there is still a frictional force between the box and floor that resists the motion. Dynamic friction acts between two bodies moving relative to each other.

The frictional force acting on a moving body can be determined using the equation in **Table 2**.

Table 2. Equation for frictional force acting on a moving body.

Equation	Symbols	Units
$F_{\mathrm{f}} = \mu_{\mathrm{d}} F_{\mathrm{N}}$	F_{f} = frictional force	newtons (N)
	μ_{d} = coefficient of dynamic friction	unitless
	F_{N} = normal force	newtons (N)

Worked example 2

A child is dragging a toy at a constant speed along a flat floor. The child is exerting a horizontal force of $8.8\,\mathrm{N}$ and the mass of the toy is $950\,\mathrm{g}$. Calculate the coefficient of dynamic friction between the toy and the floor.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$\text{mass} = 950\text{ g}$ $= 0.95\text{ kg}$ $\text{weight} = 0.95 \times 9.8$ $= 9.31\text{ N}$ The weight is equal in magnitude to F_{N} because the floor is flat so: $F_{\mathrm{N}} = 9.31\text{ N}$ The force that the child is exerting must equal the frictional force because the toy is moving at a constant speed, so: $F_{\mathrm{f}} = 8.8\text{ N}$

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Solution steps	Calculations
Step 2: Write out the equation and rearrange to find μ_{d}.	$F_{\mathrm{f}} = \mu_{\mathrm{d}} F_{\mathrm{N}}$ $\mu_{\mathrm{d}} = \dfrac{F_{\mathrm{f}}}{F_{\mathrm{N}}}$
Step 3: Substitute the values given.	$= \dfrac{8.8}{9.31}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 0.945 = 0.95\ (2\ \text{s.f.})$

Figure 5 shows static friction and dynamic friction.

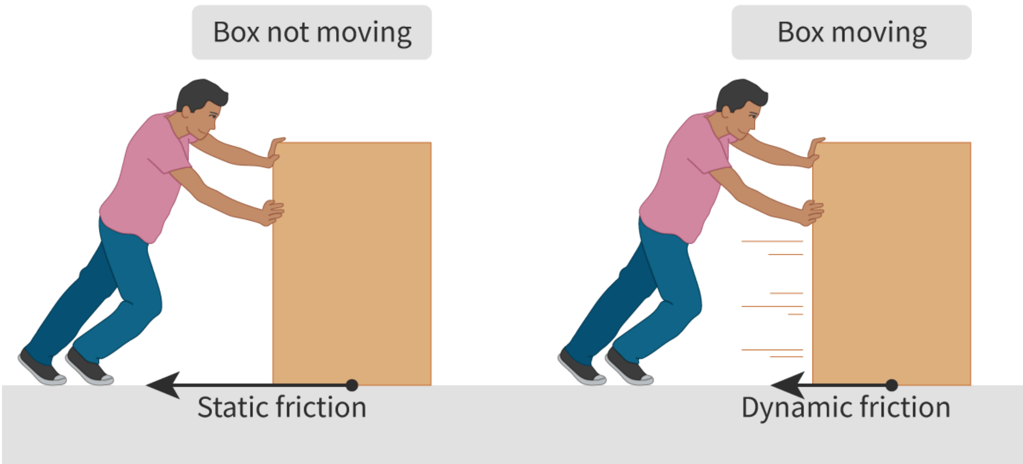

Figure 5. Static friction and dynamic friction.

More information for figure 5

The image shows two scenarios of friction involving a person and a box. On the left side, a person is pushing a box labeled "Box not moving" with the term "Static friction" written below it, indicating that the force is being applied but the box remains stationary due to the static frictional force. On the right side, the same person is pushing a box labeled "Box moving" with the term "Dynamic friction" below it, suggesting that the box is moving because the applied force has overcome the static friction, and now dynamic friction is acting on the moving box. The dynamic friction is typically less than the static friction, allowing the box to move.

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The coefficient of static friction and the coefficient of dynamic friction depend on the type of surface. The greater the coefficient, the greater the frictional force.

Frictional force is also proportional to the normal force, so heavier objects experience greater frictional force.

Figure 6 shows a graph of frictional force versus applied force for an object.

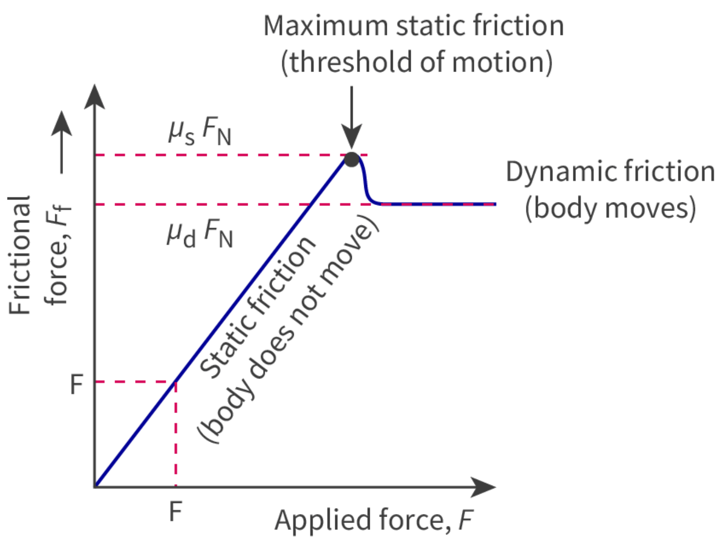

Figure 6. Graph of frictional force versus applied force.

More information for figure 6

The graph depicts the relationship between frictional force (Ff) and applied force (F) for an object. The X-axis represents the applied force (F), and the Y-axis represents the frictional force (Ff).

Initially, as the applied force increases, the frictional force also increases linearly, indicating static friction where the body does not move. This continues up to a point labeled as the maximum static friction or threshold of motion, marked by a peak on the graph. At this point, the static friction reaches its maximum value (μsFN), where the object begins to move.

Once motion starts, the static friction decreases and stabilizes at a lower constant value, indicated by a horizontal line representing dynamic friction (μdFN), where the body moves.

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The object is initially stationary. Static friction increases as the applied force increases, up to maximum value. This is called the **maximum static friction** or threshold of motion. At this point, the object starts to move. The static friction decreases and stabilises at a fixed value of dynamic friction.

The frictional force needed to keep a body at rest is greater than the frictional force needed to oppose the motion of the body.

Static friction depends on the magnitude of the applied force, while dynamic friction does not depend on the magnitude of the applied force.

The frictional force:

- acts on stationary bodies and moving bodies
- depends on the types of surfaces in contact
- depends on the weight of the body.

Tension force

Imagine that a climber is hanging from the roof of a cave using a rope (**Figure 7**).

The climber is applying a force to the rope, and the rope exerts a force on the climber. The force that the rope experiences is called the tension force, F_T. It has the same magnitude but opposite direction to the applied force – in this case, the weight of the climber.

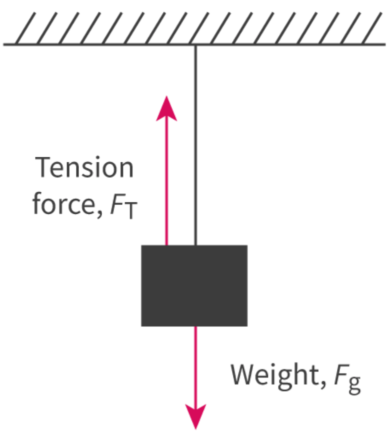

Figure 7. Tension force.

Student view

More information for figure 7

The image is a diagram showing a box suspended by a rope from a ceiling. There is an arrow pointing upwards labeled 'Tension force, F_T', indicating the direction and location of the tension force in the rope. The rope connects the top of the diagram (representing the ceiling) with the box. Below the box, another arrow is pointing downward labeled 'Weight, F_g', representing the gravitational force or weight of the box. The arrows illustrate how tension in the rope works in opposition to the weight of the object, maintaining equilibrium if the forces are balanced. This diagram visually represents the concept of tension and weight forces acting along the vertical axis of the rope and the suspended object.

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The tension force acts along the length of the rope – it pulls the roof of the cave down and the climber up.

If we are interested in studying the climber, or any body connected to a fixed point by a rope or wire, then we should include the tension force F_T in the body's free body diagram and take it into account.

A pulley is used with a rope to change the direction of a force, without affecting the magnitude of the force. **Figure 8** shows a pulley being used to raise a mass.

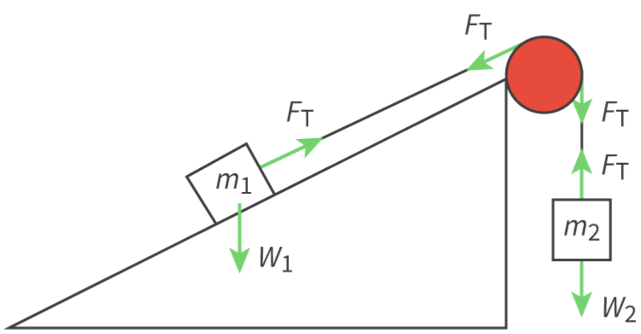

Figure 8. A pulley being used to raise a mass.

More information for figure 8

The diagram illustrates a pulley system on an inclined plane. There are two masses: m1 is on the incline, and m2 is hanging vertically. The incline is shown at an angle with m1 positioned on it, connected to the rope that goes over the pulley. The pulley itself is depicted as a circular object.

Force vectors are shown with green arrows. The tension force, labeled as F_T, acts along the rope on both sides of the pulley. For m1, the tension and its weight (W1) are shown. For m2, the tension and its weight (W2) are also depicted.

There is a smooth flow of forces indicating how the pulley functions to lift or balance the masses, depending on the system's configuration. The arrows imply the direction of forces applied due to gravity and tension in the system.

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Work through the activity to check your understanding of the normal force and frictional force.

Activity

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills — Asking questions and framing hypotheses based upon sensible scientific rationale
- **Time required to complete activity:** 10 minutes
- **Activity type:** Individual activity

Investigate the relationship between the normal force and the maximum value of static friction for an object on a horizontal surface. Determine the coefficient of static friction between the object and the surface.

You could use a block of known mass, and change the mass (and hence the normal force) by placing more blocks on top. If you use a force meter (spring balance) to measure the applied force, you could attach the meter to the block using string, or using a hook screwed into the block.

Consider the following questions when planning your investigation.

- What safety precautions should you take?
- What are the control variables and how will you control them?
- How will you measure the independent and dependent variables?
- What values of the independent variable will you use?
- Will you take repeat measurements, and why?

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- What graph will you plot, and how will you use it to determine the coefficient of static friction?

The coefficient of static friction is always greater than the coefficient of dynamic friction. In other words, it is harder to start something moving than to keep it moving. Is this what you found?

5 section questions

A. Space, time and motion / A.2 Forces and momentum

Elastic force, drag, buoyancy

A.2.5: The nature and use of the contact forces

Learning outcomes

By the end of this section you should be able to:

- Understand the concept of elastic restoring force, and use the equation $F_{\mathrm{H}} = -kx$.
- Understand the concept of viscous force drag, and use the equation $F_{\mathrm{d}} = 6\pi\eta rv$.
- Understand the concept of buoyancy, and use the equation $F_{\mathrm{b}} = \rho Vg$.

Video 1 shows bungee jumpers. Bungee jumping is a type of extreme sport where a person jumps from a high place while secured by an elastic rope.

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Video 1. A Bungee Jumper.

More information for video 1

A video illustrates a person performing a bungee jump, demonstrating the effects of elastic restoring force and gravity. It includes the jumper leaping from a high platform, free-falling until the elastic cord stretches to its maximum length, and then recoiling upward. The video explains how forces such as tension, gravity, and air resistance interact to control motion, reinforcing key concepts in physics related to oscillatory motion and energy transformation.

How does the rope stretch, then contract to stop the jumper hitting the ground or water? What forces are involved?

In this section, you are going to learn about three more **contact forces** (see section A.2.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/normal-force-friction-tension-id-44734/)): elastic restoring force, viscous drag force and buoyancy, and how these forces affect the motion of bodies.

Elastic restoring force

The bungee jumper in **Video 1** almost reaches the water then is pulled back up. This happens because the rope is **elastic**. This means that it extends then returns to its original length.

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The rope acts as a spring. Springs can change their length then go back to their original length. Imagine a spring on a table (**Figure 1**):

- The **natural length** of the spring is the length of the spring when you are not pushing or pulling the spring.
- If you push the spring, the spring gets shorter (compression).
- If you pull the spring, the spring gets longer (elongation).

When the spring is pushed or pulled, the spring exerts an opposing force that tries to bring itself back to its natural length.

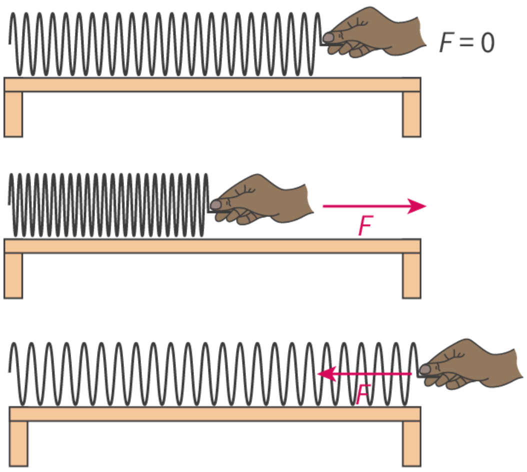

Figure 1. A spring on a table.

More information for figure 1

The diagram shows three states of a spring positioned horizontally on a table.

1. **Natural Length:** The top image depicts a spring at its natural length without any force applied. A hand is shown near the spring, and the label reads "F = 0 Natural length."
2. **Compression:** The middle image illustrates the spring being compressed. A hand pushes the spring to the left, and the label reads "F" with an arrow indicating the direction of the push. The word "Compression" is on the right.
3. **Elongation:** The bottom image displays the spring being pulled to the right. A hand is applying force to the right, annotated with "F" and an arrow indicating the direction of the force. The word "Elongation" is written on the right.

In each state, the direction of force and the state of the spring are clearly depicted.

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The force that returns a spring to its natural length is called the elastic restoring force, F_H. The direction of the force is always toward the natural length.

The elastic restoring force follows Hooke's law. Hooke's law states that the restoring force acting to return a spring to its length is proportional to the extension of the spring. Remember, it is the **extension** of the spring (how much it has been stretched by), not its total length.

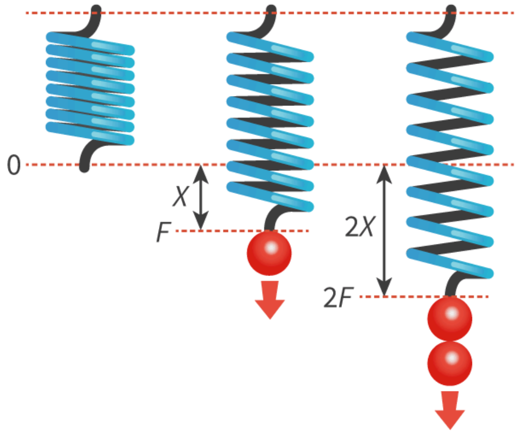

Figure 2. Hooke's law.

More information for figure 2

The diagram illustrates Hooke's law through three stages of spring compression and extension.

1. The first spring on the left is at its natural length with a label '0' indicating no force applied.
2. The middle spring shows an extension marked as 'X' with a force 'F' acting downward, illustrated by a red ball and an arrow indicating the direction. This demonstrates the proportional extension caused by an applied force.
3. The final spring on the right exhibits a greater extension labeled '2X' due to a doubled force of '2F'. This is shown using two red balls joined and a longer downward arrow. The extensions are aligned within horizontal dashed lines to depict the starting and various extended positions of each spring, showing the linear relationship between force applied and spring extension.

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The elastic restoring force can be determined using the equation in **Table 1**.

Table 1. Equation for elastic restoring force.

Equation	Symbols	Units
$F_H = -kx$	F_H = elastic restoring force	newtons (N)
	x = displacement of spring	metres (m)
	k = spring constant	newtons per metre (N m^{-1})

The spring constant is a property of the spring. The higher the value, the stiffer the spring.

The minus sign in the equation above tells us that the direction of the elastic restoring force, F_H, is opposite to the direction of the displacement, x.

It is the elastic restoring force that pulls a bungee jumper up, so that they do not hit the water or the ground.

Figure 3 shows a graph of applied force, F, against displacement, x. The force is directly proportional to the displacement (extension or compression). The gradient of the graph line gives you the spring constant, k.

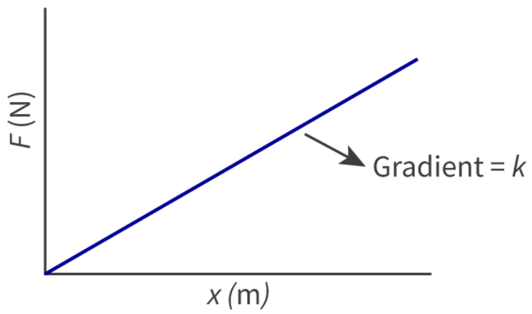

Figure 3. A graph of applied force against displacement.

More information for figure 3

The image is a graph representing applied force (F) against displacement (x). The X-axis is labeled as 'x (m)' which denotes displacement, measured in meters. The Y-axis is labeled 'F (N)', indicating force in Newtons. The graph features a straight line indicating a direct proportionality between force and displacement, meaning as displacement increases, force increases linearly. The gradient of the line is referred to as k, denoting the spring constant, which is

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highlighted on the graph with an arrow pointing to the line. This visual representation illustrates Hooke's Law, where the force applied to a spring is proportional to the displacement from its original position, and the constant of proportionality is the spring constant k.

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Exercise 1

Click a question to answer

Making connections

Linking question: How does the application of a restoring force acting on a particle result in simple harmonic motion? (see subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/))

Viscous drag force

A fluid is a gas or a liquid. When a body is moving inside a fluid, it experiences a force from the fluid that opposes its motion. This is a resistive force, just like air resistance (see section A.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/)). The particles of the fluid exert a force on the surface of the body, opposing its motion. This force is called the viscous drag force (**Figure 4**).

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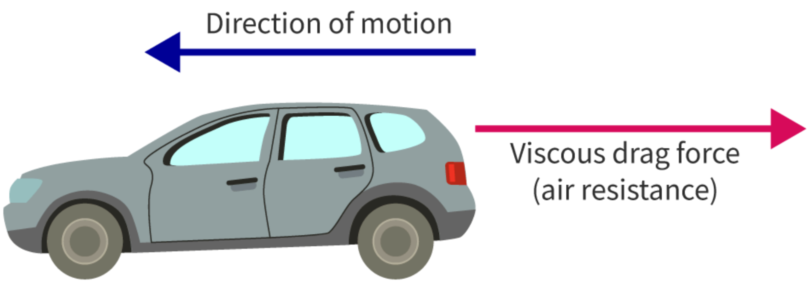

Figure 4. Viscous drag force.

More information for figure 4

The image is an illustration of a car demonstrating the concept of viscous drag force, commonly referred to as air resistance. The car is depicted in motion, indicated by a blue arrow pointing to the left labeled "Direction of motion." Opposing this, a red arrow points to the right and is labeled "Viscous drag force (air resistance)." This visually represents the concept that as the car moves in one direction, the force of air resistance acts in the opposite direction. The illustration is used to explain how fluids like air exert a resistive force on moving bodies, opposing their motion.

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You will have experienced this force, when you run and feel the air pushing on your face and body.

Making connections

The viscous drag force acts because the particles of the fluid hit the surface of an object, while the object is moving inside the fluid. How is this related to the pressure of a gas inside a container?

Take an empty plastic bottle. Put the cap on and squeeze it. You will feel that the bottle pushes back, because its pressure increases. The reason for this is that the air particles hit the walls of the bottle and you feel that as pressure. This is covered in more detail in subtopic B.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44289/).

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Viscous drag force depends on the **viscosity** of a fluid. Viscosity is the property of a fluid that describes the fluid's resistance to flow. A lower viscosity means it is easier for the fluid to flow – water (low viscosity) is easier to pour than syrup (high viscosity).

Calculating viscous drag force is complicated. The force depends on the shape of the object that moves inside the fluid. Some shapes result in less resistance.

In IB physics, we only look at the force experienced by a small spherical body that moves with a relatively small velocity inside the fluid.

The viscous drag force can be determined using the equation in **Table 2**.

Table 2. Equation for viscous drag force.

Equation	Symbols	Units
$F_{\mathrm{d}} = 6\pi\eta r v$	F_{d} = viscous drag force	newtons (N)
	η = fluid viscosity	kilograms per metre per second (kg $\mathrm{m}^{-1}\,\mathrm{s}^{-1}$)
	r = radius of the sphere	metres (m)
	v = velocity of sphere	metres per second (m s^{-1})

The viscous drag force is proportional to the velocity of the body, so the faster a body travels in a fluid, the greater the resistance it experiences. If you stick your hand out of the window of a slow-moving car, and then out the window of a fast-moving car, you will feel a different force on your hand.

Worked example 1

A sphere of radius 3.3 cm travels through a fluid of viscosity 5.6 kg $\mathrm{m}^{-1}\,\mathrm{s}^{-1}$. The viscous drag force is 15 N. Calculate the velocity of the sphere.

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Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$r = 3.3\ \text{cm}$ $= 0.033\ \text{m}$ $\eta = 5.6\ \text{kg m}^{-1}\ \text{s}^{-1}$ $F_\text{d} = 15\ \text{N}$
Step 2: Write out the equation and rearrange to make v the subject.	$F_\text{d} = 6\pi\eta r v$ $v = \frac{F_\text{d}}{6\pi\eta r}$
Step 3: Substitute the values given.	$= \frac{15}{(6\pi \times 5.6 \times 0.033)}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 4.3061\ \text{m s}^{-1} = 4.3\ \text{m s}^{-1}$ (2

Buoyancy

A large container ship has a mass of about 200 000 000 kg. How can such a massive ship float in the ocean?

Buoyancy, F_b, is the force experienced by a body when it is partly or fully immersed in a fluid. (It is sometimes called 'upthrust'.) Buoyancy is what makes bodies, such as container ship, float on the surface of a fluid, such as water (**Figure 5**).

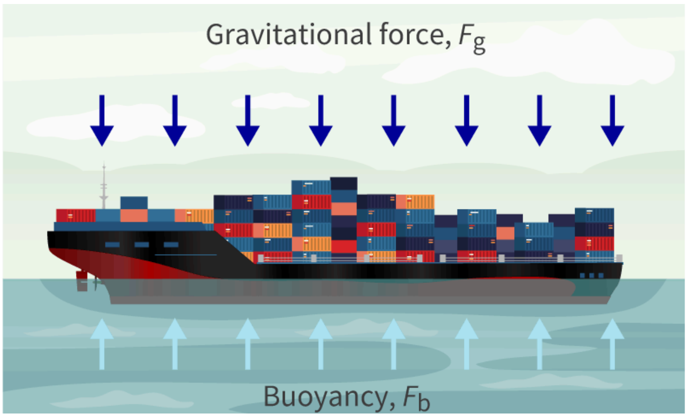

Figure 5. Buoyancy.

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More information for figure 5

The image is an illustration depicting a container ship floating on a body of water. It visually explains the concept of buoyancy and gravitational force. The ship is loaded with containers and is shown in the center of the image. Above the ship, there are downward-pointing blue arrows labeled 'Gravitational force, Fg,' indicating the direction and nature of gravitational force acting downwards on the ship. Below the ship, upward-pointing lighter blue arrows are labeled 'Buoyancy, Fb,' illustrating the buoyant force acting upwards against gravity, enabling the ship to float. The image is designed to provide a visual representation of the balance of forces that allow objects to float by illustrating the opposing forces of gravity and buoyancy. There are clouds in the sky, and the water is gently rippled, giving a sense of calm sea conditions.

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Some bodies float and other bodies do not float. In both cases, the bodies experience buoyancy.

Buoyancy is caused by the difference in pressure between the top and bottom of a body in a fluid (**Figure 6**).

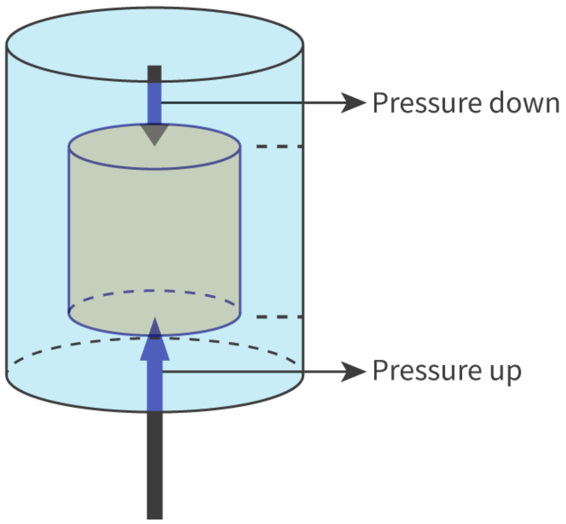

Figure 6. The pressure on an object in a fluid.

More information for figure 6

The diagram illustrates a vertical cylinder submerged in a larger fluid-filled cylinder. The smaller inner cylinder represents an object in a fluid. An arrow labeled "Pressure down" points downward from the top of the fluid, intersecting the upper surface of the inner cylinder, indicating the exerted pressure from above. Another arrow

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labeled "Pressure up" points upward from below the inner cylinder's base, indicating the pressure exerted from below. This setup demonstrates the concept of buoyancy, highlighting the different pressure levels acting on an immersed object's top and bottom surfaces.

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Imagine that a rock is fully immersed in water. Since the rock occupies a certain volume, it displaces the water in all directions. The Ancient Greek inventor and mathematician Archimedes noticed and developed a principle that helps us calculate buoyancy:

The buoyancy experienced by a body immersed in a fluid is equal to the weight of the fluid displaced.

Figure 7 shows the volume of water displaced, V, when a rock is dropped into a measuring cylinder of water.

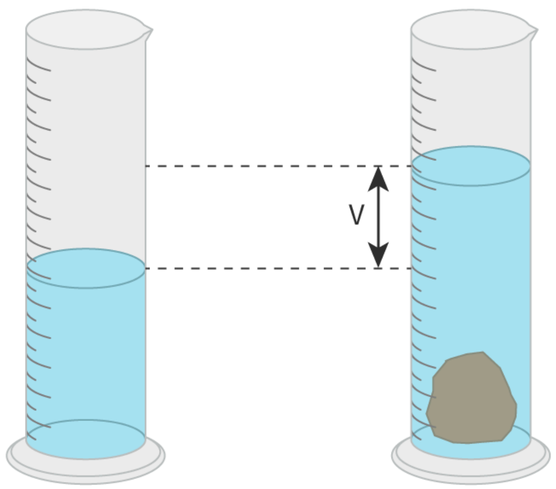

Figure 7. The volume of water displaced by a rock.

More information for figure 7

Illustration showing two measuring cylinders side by side. The cylinder on the right contains a submerged rock near the base, with the water level significantly raised compared to the initial level in the left cylinder. A dashed line marks the height difference labeled 'V', indicating the volume of water displaced. The scale on both cylinders shows the change in volume, demonstrating the rock's effect on water displacement.

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Buoyancy can be determined using the equation in **Table 3**.

Table 3. Equation for buoyancy.

Equation	Symbols	Units
$F_{\mathrm{b}} = \rho V g$	F_{b} = buoyancy	newtons (N)
	ρ = density of the fluid	kilograms per cubic metre (kg m^{-3})
	V = volume of the fluid displaced	cubic metres (m^3)
	g = acceleration of free fall (9.8 m s^{-2} at the Earth's surface)	Given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet

The density of a fluid is determined using the equation (subtopic B.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43777/)):

$$\rho = \frac{m}{V}$$

A body does not need to be fully immersed in a fluid to experience buoyancy. Even if the body is partly immersed, like a ship, it will still displace water. So, according to Archimedes' principle, the body will experience buoyancy.

Worked example 2

Fish 1 with mass m and volume V is at rest in water of density X. Fish 2 with mass $4m$ is at rest in water of density $1.2X$.

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Determine the volume of the second fish in terms of V.

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Solution steps	Calculations
Step 1: Write out the appropriate equations.	$F_b = \rho V g$ $F_{b1} = XV_1g$ $F_{b2} = 1.2XV_2g$
Step 2: Equate the forces.	The fish is at rest, so all vertical forces mus be balanced. $\text{Vertically downwards} = \text{weight} \\ = mg$ $\text{Vertically upwards} = F_b$ $mg = XV_1g$ $4mg = 1.2XV_2g$
Step 3: Make both equations equal to the same constants and cancel where possible.	$V_1 = \frac{m}{X}$ $\frac{1.2V_2}{4} = \frac{m}{X}$
Step 4: Equate the two equations and rearrange to find V_2.	$V_1 = \frac{1.2V_2}{4}$ $V_2 = \frac{4V_1}{1.2} \\ = 3.3V_1$

It is possible to derive the formula $F_b = \varrho \text{Vg}$ by considering the pressure in a fluid. The pressure at a depth d in a fluid of density ρ on a planet with gravitational field strength g is given by $P = \rho gd$. You will not need to use this equation in the DP physics course, but you can deduce from it that fluid pressure increases with depth (which is why stronger submarines are needed for deeper diving) and with the density of the fluid.

Consider an object immersed in a fluid of density ρ_f, as shown in **Figure 8**.

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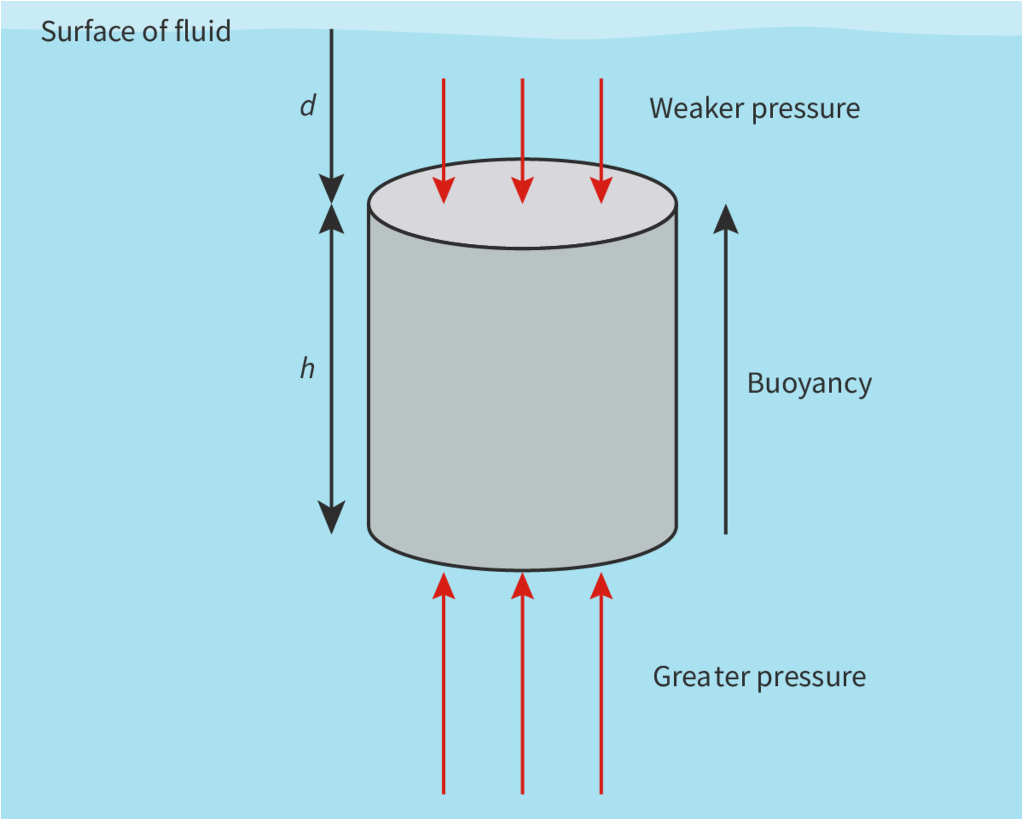

Figure 8. An object immersed in a fluid.

More information for figure 8

This diagram illustrates a cylindrical object submerged in a fluid. The surface of the fluid is labeled at the top. The object is depicted with its top surface labeled with 'Weaker pressure' and its bottom surface labeled with 'Greater pressure,' indicating pressure differences due to the fluid's depth. An upward arrow labeled 'Buoyancy' is shown on the right side of the object, opposing the downward arrows of pressure. The diagram includes vertical arrows on the object's sides, with 'd' representing the depth of the upper surface and 'h' representing the height of the object, both marked along the left side of the image.

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If the depth of the upper surface of the object is d and the height of the object is h, then the pressures P_{U} and P_{L} on the upper and lower surfaces are:

$$P_{\text{U}} = \rho_{\text{f}} g d$$
$$P_{\text{L}} = \rho_{\text{f}} g (d + h)$$

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If the cross-sectional area of the object is A then the forces F_{U} and F_{L} on the upper and lower surfaces are:

$$F_{\mathrm{U}} = P_{\mathrm{U}}A = \rho_{\mathrm{f}}gdA \text{ (downwards)}$$
$$F_{\mathrm{L}} = P_{\mathrm{L}}A = \rho_{\mathrm{f}}g(d+h)A \text{ (upwards)}$$

The buoyancy F_b is the resultant of these two forces:

$$\begin{aligned} F_{\mathrm{b}} &= \rho_{\mathrm{f}}g(d+h)A - \rho_{\mathrm{f}}gdA \\ &= \rho_{\mathrm{f}}ghA \end{aligned}$$

The volume of the object equals hA, which also equals the volume of fluid displaced by the object. So:

$$F_{\mathrm{b}} = \rho_{\mathrm{f}}gV_{\mathrm{f}}$$

However, ρ_{f} V_{f} equals the mass of displaced fluid, and multiplying it by g gives the weight of displaced fluid. Therefore the buoyancy is equal in magnitude to the weight of fluid displaced, as stated in Archimedes' Principle.

Work through the activity to check your understanding of buoyancy.

Activity

- **IB learner profile attribute:**
  - Thinker
  - Communicator
- **Approaches to learning:**
  - Thinking skills — Being curious about the natural world
  - Communication skills — Clearly communicating complex ideas in response to open-ended questions
- **Time required to complete activity:** 15 minutes
- **Activity type:** Individual activity

Many magic tricks use physics. The Cartesian diver works because of buoyancy. Try making your own Cartesian diver. You will need a large plastic bottle (such as a water bottle), a bendable plastic straw, and some paper clips. Watch the video to see how to make a Cartesian diver.

Video 2. How to make a Cartesian diver.

More information for video 2

The video presents a hands-on demonstration of the Cartesian diver, a classic science experiment that vividly illustrates the principles of buoyancy and air pressure. It begins with a visual demonstration: a clear plastic bottle filled with water contains a small orange and yellow object floating near the surface. As the bottle is squeezed by a pair of hands, the object sinks to the bottom; when the pressure is released, it rises again. This simple yet captivating motion introduces the concept behind the Cartesian diver. The screen then fades to black with the bold white text "CARTESIAN DIVER," signaling the start of a step-by-step guide on how to create the diver using everyday materials.

On a wooden table, the materials are clearly laid out one by one and labeled as they appear: a bendy straw, scissors, paper clips, and a cup of water. The video then presents a person's hands using scissors to cut the long end of the bendy straw so that both ends of the straw are of equal length and have a "V" shape, where there are two openings on one side of the straw and the other end is closed. The video then proceeds to show a paper clip being divided forming a "V" shape. The paperclip is then inserted into the open ends of the straw, with one end of the paperclip in one opening of the straw and the other end of the paperclip in another end of the straw, thus closing the straw from both ends. The hands then take another paperclip and join it with the paperclip on the end of the straw.

The screen then fades to black with the bold white text " This is the important part," followed by another text "If it doesn't float vertically, the diver won't work." The video then proceeds to show a pair of hands taking a glass of water and placing it in the middle of the wooden table. The bendy straw with paper clips (straw segment) is then added to the glass of water, with the part having paper clips facing downwards. When placed in the cup of water, the straw segment initially floats horizontally, which the video indicates as incorrect with a large black "X" on the right side of the screen.

The text "Add more paper clips for weight" appears as the hands attach additional paper clips. After adding enough weight, the straw floats vertically in the water—this time a large checkmark appears, indicating the correct setup. This stage emphasizes the importance of adjusting the diver's buoyancy by modifying its weight until it is just buoyant enough to float upright without sinking.

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Next, a clear plastic water bottle is filled with water, and the weighted straw segment is dropped into the water bottle. After sealing the bottle with a cap, the hands demonstrate how squeezing the sides of the bottle makes the straw sink, and releasing the pressure causes it to rise. This visual reinforces the core scientific principle at play: when the bottle is squeezed, the pressure compresses the air inside the diver, reducing its volume and increasing its density, making it less buoyant and causing it to sink. When the pressure is released, the air expands, making the diver more buoyant and allowing it to float back to the top. This illustrates how changes in volume affect an object's ability to float due to buoyant force acting on it relative to its density.

The screen then fades to black with the bold white text "You can make your diver out of all sorts of materials," followed by another text "Here is another example..." On a wooden table, the materials are clearly laid out one by one and labeled as they appear: Eye dropper, scissors, and plasticine. The video continues by showcasing a second method for building a diver using different materials. A medicine dropper, labeled "Eye Dropper," is modified by cutting off its thin top. A small ball of orange plasticine is rolled into a thin strand and coiled around the dropper for weight. After testing it in water to ensure vertical floating (confirmed by a checkmark), the screen then fades to black with the bold white text "Now to dress it up." The hands then proceed to decorate the diver. A yellow balloon is introduced, and the end is cut off using scissors before being sliced into fringes. This decorative balloon piece is attached to the plasticine, transforming the dropper into a colorful, whimsical diver. Once inserted into a filled water bottle, the diver again demonstrates the same sink-and-float behavior when pressure is applied and released.

This activity provides a practical exploration of buoyancy—a force that determines whether an object will float or sink—by asking learners to adjust the weight of their diver until it just barely floats. It also brings attention to the behavior of gases under pressure, demonstrating how compressing the air inside the diver affects its density and, consequently, its buoyancy. By constructing their own Cartesian diver and experimenting with different materials and designs, learners gain a tangible understanding of how fluid pressure and object density influence floating and sinking. This demonstration effectively ties together physics concepts with accessible, hands-on experimentation.

When you squeeze the bottle, the diver goes down. When you let go of the bottle, the diver goes up.

But why does this happen? Why does the diver float when the bottle is not squeezed, then sink when you squeeze the sides of the bottle? Try to explain this using buoyancy and the weight of the object.

When the weight of an object is greater than the buoyancy, the object sinks.

When you squeeze the bottle, it compresses the air inside the diver, causing water to move into the diver. This increases the mass, and therefore the weight, of the diver whilst its volume remains the same. The weight is then greater than the buoyancy force and the diver sinks. When you release it, the opposite happens and it floats again.

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5 section questions

A. Space, time and motion / A.2 Forces and momentum

Newton’s laws

A.2.1: Newton’s three laws of motion

Learning outcomes

By the end of this section you should be able to:

- Understand Newton’s first law of motion, and use Newton’s first law to determine resultant force.
- Understand Newton’s second law of motion, and use the equation $F = ma$.

Voyager 1 is the most distant human-made object. Voyager 1 was launched in 1977 and is still moving in space, away from the Earth. Use this NASA interactive (https://eyes.nasa.gov/apps/orrery/#/sc_voyager_1) to find out more about Voyager 1.

For many years, the thrusters of Voyager 1 have not been fired. This means that no force is acting on Voyager 1 (there is actually still a small force from the sun, but this is negligable). How is Voyager 1 still moving, and what kind of motion is it experiencing?

Subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/) covers how to describe the motion of bodies, and section A.2.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/elastic-force-drag-buoyancy-id-44735/) covers how forces can set bodies in motion. It is time to connect these two ideas. What will help us do this are the three statements by Isaac Newton published in 1687. Today, they are called Newton’s three laws of motion.

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In this section, you are going to look at Newton's first and second laws. Newton's third law is covered in section A.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-forces-id-44732/).

Newton's first law

Consider a chair resting on the floor. Why is it not moving? Is it because there are no forces acting on the chair? No – the Earth is exerting a gravitational force downwards on it, while the floor is exerting a normal force upwards. The reason why the chair remains stationary is that the resultant force on it is zero.

Is it always true that an object is stationary if no resultant force acts on it?

Voyager 1 is moving in empty space. There is no air and no bodies to collide with, so there is nothing to slow Voyager 1 down or accelerate it. The result is that Voyager 1 keeps moving with a constant speed away from the Earth.

These two examples show that when you do not apply a force to an object, there are two possible outcomes. Newton's first law of motion states that:

A body that experiences zero resultant force will remain at rest or continue to move at a constant velocity.

Another way of saying this is that a resultant force is required to change the velocity of an object.

A body is in **equilibrium** if there is a zero resultant force acting on it. Note that we refer to **resultant force**. There can be forces acting on the body, but if the body is in equilibrium, then the sum of the forces is zero (the forces are balanced). According to Newton's first law, a body in equilibrium can either be at rest or moving at a constant velocity.

If a body is in equilibrium, it is experiencing a zero resultant force. Whenever you see a body remaining at rest or moving at a constant velocity, you can say that the resultant force on it is zero.

Worked example 1

Student view

1. A box of mass 23 kg is resting on the ground. Determine the normal force, F_N.

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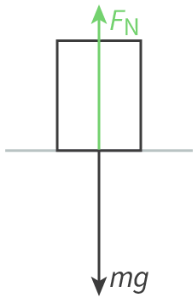
2. A force of 110 N is now exerted on the top of the box. Determine the new normal force.
3. The 110 N force is removed, and then you try to lift the box by applying a force of 150 N, but the box does not move. Determine the normal force now.

Solution steps	Calculations
Step 1: Draw a free body diagram to answer question 1.	
Step 2: Apply Newton's first law.	The box is not moving (it is in equilibrium) so: $\text{resultant force, } F = 0$ $F_N - mg = 0$ $F_N = mg$ $F_N = 23 \times 9.8 = 225.4\text{ N} = 230\text{ N (2 s.f.)}$ The normal force is balanced by the weight of the box.

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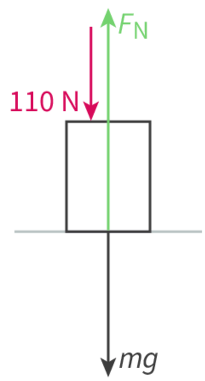
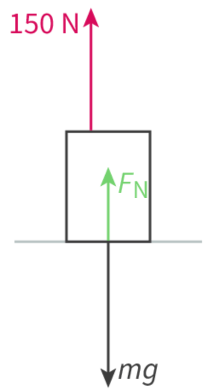
Solution steps	**Calculations**
Step 3: Draw a free body diagram to answer question 2.	
Step 4: Apply Newton's first law.	$\text{resultant force, } F = 0$ $F_N = mg + 110$ $F_N = 225.4 + 110 = 335.4\text{ N} = 340\text{ N (2 s.f.)}$ The normal force is greater than the weight of the box.
Step 5: Draw a free body diagram to answer question 3.	

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Solution steps	Calculations
Step 6: Apply Newton's first law.	$\text{resultant force, } F = 0$ $F_N + 150 = mg$ $F_N = mg - 150$ $= 225.4 - 150$ $= 75.4 \text{ N}$ $= 75 \text{ N (2 s.f.)}$ The normal force is smaller than the weight of the box.

Making connections

Linking question: How are the concepts of equilibrium and conservation applied to understand matter and motion from the smallest atom to the whole universe? (see subtopics B.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-44324/) and E.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44745/)).

A body that remains at rest or is moving with a constant velocity experiences zero resultant force. What does it take to change this state of motion?

When you are in a car that starts from rest and accelerates, you are pushed against the back of your seat. Why does this happen? Can you relate this to Newton's first law?

When there is zero resultant force, a body will keep its original state of motion (at rest or constant velocity). But a car that accelerates changes its velocity. This means that your velocity must also change. The force acting on you to accelerate you is the push of the seat against your back, and this is the force you feel.

When a car is moving at a constant velocity and starts slowing down, your body tends to go forwards (**Figure 1**). (That's why it's important to wear a seatbelt in a car.) Can you explain this effect using the ideas above?

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Figure 1. When the brakes are applied in a car, your body keeps moving forwards.

The tendency of bodies to keep their original state of motion is called **inertia**.

Newton's second law

Newton's second law of motion states that the acceleration of a body depends on the mass of the body and the resultant force acting on the body. The resultant force acting on a body can be determined using the equation in **Table 1**.

Table 1. Equation for resultant force.

Equation	Symbols	Units
$F = ma$	F = resultant force	newtons (N)
	m = mass	kilograms (kg)
	a = acceleration	metres per second per second (m s^{-2})

The acceleration of a body is proportional to the resultant force acting on it. For a particular mass, if the force increases, the acceleration increases.

Study skills

Newton's second law refers to the **resultant force** acting on a body, just like Newton's first law.

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Worked example 2

An aeroplane takes off at a velocity of 75 m s^{-1}. To reach this velocity, it accelerates from rest along a runway of length 1.6 km. If the mass of the aeroplane is 72 000 kg, determine the force required from the engines for take-off.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$m = 72\,000 \text{ kg}$ $s = 1.6 \text{ km}$ $= 1600 \text{ m}$ $u = 0 \text{ m s}^{-1}$ $v = 75 \text{ m s}^{-1}$
Step 2: Find *a* using the appropriate *suvat* equation.	$v^2 = u^2 + 2as$ $a = \frac{(v^2 – u^2)}{2s}$ $= \frac{(75^2 – 0^2)}{(2 \times 1600)}$ $= 1.76 \text{ m s}^{-2}$
Step 3: Write out the equation.	$F = ma$
Step 4: Substitute the values given.	$= 72\,000 \times 1.76$
Step 5: State the answer with appropriate units and the number of significant figures used in rounding	$= 126\,720 \text{ N} = 130\,000 \text{ N}$ (2

Worked example 3

A metal ball of mass 8.0 g is falling through a liquid. The volume of the ball is 1.0 cm^3. The density of the liquid is 1300 kg m^{-3}.

At one moment during its fall, the instantaneous acceleration of the ball is 5.0 m s^{-2}. Calculate the drag on the ball at that moment.

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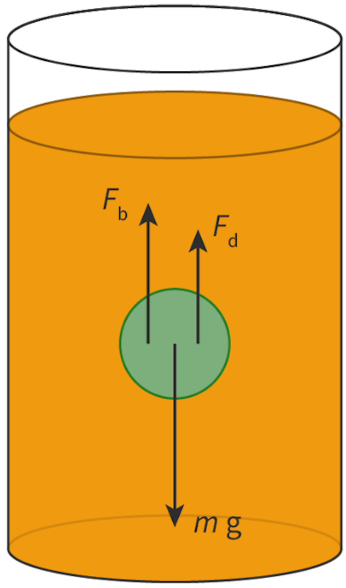
Solution steps	Calculations
Step 1: Consider the forces acting on the ball (as shown in this free-body diagram).	
Step 2: Write out the values given in the question and convert to suitable units for the calculations.	$\text{mass} = 8.0 \text{ g} = 8.0\times10^{-3} \text{ kgvolume} = 1.0 \text{ cm}^3 = 1.0\times10^{-6} \text{ m}^3\text{liquid}$
Step 3: Write an equation relating the acceleration to the resultant of the forces.	$ma = mg - F_b - F_d$ $= mg - \rho_f V g - F_d$

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Solution steps	Calculations
Step 4: Calculate the drag and state it with appropriate units and the number of significant figures used in rounding.	$F_d = mg - \rho_f Vg - ma$ $= (8.0\times10^{-3}\times9.8)-(1300\times1.0\times10^{-6}\times9.8) - (8.0\times10^{-3}\times5.0)$ $= 0.0784 - 0.01274 - 0.040$ $= 0.026\text{ N (2 s.f.)}$

We can now use Newton's second law to show that in free fall (see section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/)), the acceleration, a, is equal to the value of acceleration of free fall, g.

A body is in free fall when the only force acting on it is its weight (air resistance is negligible):

$$F_g = mg = W$$

We know that $F = ma$, so:

$$ma = mg$$

Cancelling the masses, we get:

$$a = g$$

Look at the video in **Video 1**. It shows the forces on a skydiver when air resistance is not negligible.

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Video 1. The forces on a skydiver.

The two forces acting on the skydiver throughout the motion are gravitational force (weight) and viscous drag force (air resistance). The weight is a constant force, but the air resistance is proportional to the velocity of the skydiver (see section A.2.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/elastic-force-drag-buoyancy-id-44735/)).

The motion of the skydiver from the moment they start their dive until the parachute opens can be modelled as shown in **Figure 2**.

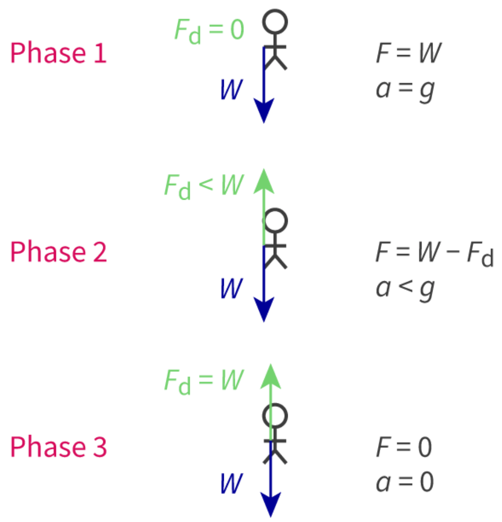

Figure 2. The forces on a skydiver in three phases.

More information for figure 2

The diagram depicts a representation of the forces acting on a skydiver during three phases of descent. Each phase shows a stick figure to represent the diver and involves symbols and arrows indicating different forces.

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- **Phase 1:**
- A single downward arrow represents the weight (W) of the skydiver.
- The text beside the figure shows: "F = W" and "a = g", indicating that the only force acting is the weight, with acceleration equal to gravity.
- **Phase 2:**
- Two arrows: a downward arrow representing weight (W) and an upward arrow showing drag (Fd), which is less than the weight.
- Accompanying text shows: "F = W - Fd" and "a < g", meaning the skydiver's drag force has started countering the weight, reducing net force and acceleration.
- **Phase 3:**
- Both upward drag and downward weight are displayed with equal-length arrows, representing equilibrium where drag equals weight (Fd = W).
- The text beside shows: "F = 0" and "a = 0", indicating that the skydiver reaches terminal velocity where net force and acceleration are zero.

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Phase 1: The skydiver jumps out of the aeroplane and starts falling. The only force acting on them is their weight because their velocity is still zero. The acceleration is $a = g$.

Phase 2: The weight stays constant, but the air resistance is increasing, since the skydiver is accelerating, thus increasing the velocity. The resultant force decreases, and so does the acceleration, which is now smaller than g.

Phase 3: As the skydiver accelerates, the viscous drag force (air resistance) increases. At some point, it becomes equal to the weight. The resultant force acting on the body is zero. According to Newton's first law, the skydiver continues to move with a constant velocity. This is called the **terminal speed**.

Video 1 also shows why the skydiver does not hit the ground moving at terminal speed. Try to answer the following questions. Click 'Show or hide solution' to see the answers.

Why does air resistance become larger than weight?

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The shape of the parachute increases air resistance, so when it opens, air resistance increases suddenly.

Why does air resistance reduce again?

For the period that air resistance is greater than weight, the skydiver is decelerating, reducing their speed. Air resistance is proportional to speed, so it reduces, until it equals weight again.

Work through the activity to check your understanding of Newton's laws.

Activity

- **IB learner profile attribute:** Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 30 minutes
- **Activity type:** Group activity

Imagine that a person (m_1 = 76 kg) is standing on a measuring scale (m_2 = 2 kg) in an elevator box. The measuring scale is calibrated in newtons. The person is holding onto a massless rope that is connected to the elevator box (m_3 = 30 kg). The whole system is connected to a pulley hanging from the ceiling. Try to answer the following questions. Click on 'Show or hide solution' to see the answers. (Take g as 10 m s^{-2} to make the calculations easier.)

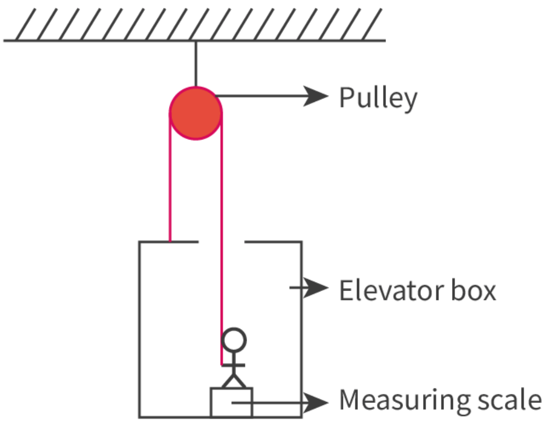

Figure 3. Person on scales in elevator.

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More information for figure 3

The diagram illustrates a person standing on a measuring scale inside an elevator box. The person is holding onto a rope that is connected to the elevator box. The rope goes over a pulley attached to the ceiling. The components are labeled as follows: "Pulley" at the top-center, indicating the circular mechanism attached to the ceiling; "Elevator box," showing the enclosure containing the person and the scale; and "Measuring scale" at the bottom-right, indicating where the person is standing. The diagram represents a physics scenario involving masses and a pulley system, associated with static and dynamic calculations in an educational context.

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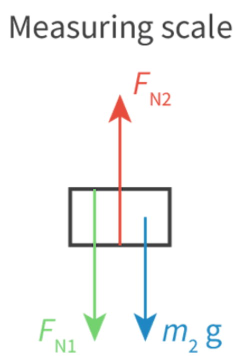
1. The elevator is not moving. What is the reading on the scale?

Here is a free-body diagram of the measuring scale.



The reading on the scale equals the downwards normal force exerted on it by the person, F_{N1}. The diagram shows that this equals $F_{N2} - m_2g$, but this cannot be calculated because F_{N2} is also unknown.

One way to find F_{N1} is to write a force equation (of the form: resultant force = 0) for each of the three objects. There are three unknowns (F_{N1}, F_{N2} and T) and the three equations can be solved simultaneously to find F_{N1}. An alternative method, which does not involve simultaneous equations, is shown here.

Consider the forces acting on the person:

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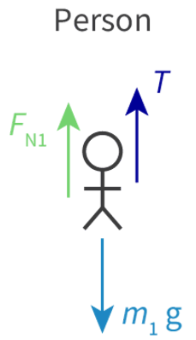
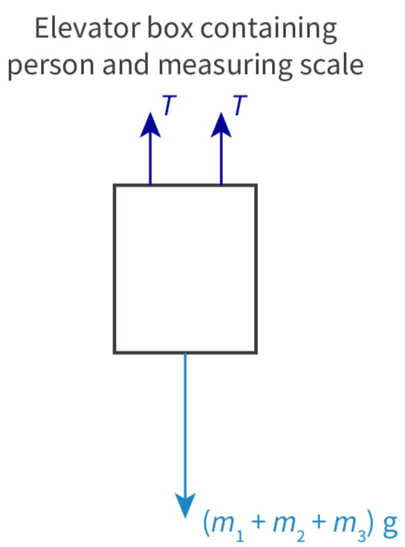


According to Newton's third law, the scale exerts an normal force F_{N1} upwards on the person. T is unknown but can be calculated by treating the three objects (person, scale and elevator box) as one object.

Connected objects that are not moving relative to each other can be treated as a single object (or 'system'), and only the external forces need to be considered. The diagram below shows the external forces in this example. (The normal forces acting between the person and the scale, and between the scale and the elevator box, are internal force pairs that can be ignored.)

Note that the tension has the same magnitude at all points along a rope.



Since there is no movement, the system is in equilibrium and the resultant force is zero, so:

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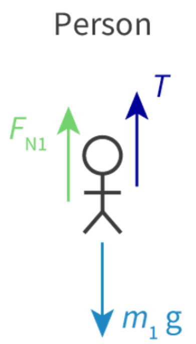
$$
\begin{aligned}
2T &= (m_1 + m_2 + m_3)g \\
T &= \frac{(m_1 + m_2 + m_3)g}{2} \\
&= \frac{(76 + 2 + 30) \times 9.8}{2} \\
&= 529.2 \text{ N}
\end{aligned}
$$

Since the person is in equilibrium, the free-body diagram shows that:

$$T + F_{N1} = m_1 g$$



So the normal force of the scale on the person, which is equal to the reading on the scale, is:

$$
\begin{aligned}
F_{N1} &= m_1 g - T \\
&= 76 \times 9.8 - 529.2 \\
&= 744.8 - 529.2 \\
&= 215.6 \text{ N} \\
&= 220 \text{ N } (2 \text{ s.f.})
\end{aligned}
$$

(You could try the method of writing a force equation for each of the three objects. Start by drawing a free-body diagram of the elevator box.)

1. Would the reading on the measuring scale change if the elevator is moving upwards or downwards with a constant velocity? Why or why not?

No. The forces are balanced and so the resultant force acting on the person is zero. The person's weight is supported entirely by the measuring scale and there is no resultant force making the forces unbalanced. The scale will show exactly the same value as if the elevator was not moving.

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1. The cable snaps and the box, scale and person begin to fall downwards. What is the reading on the scale now?

The reading is zero. The system is in free fall, and so there is no upwards force from the scale on the person, and no downwards force from the person on the scale.

5 section questions

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/elastic-force-drag-buoyancy-id-44735/print/) Assign

A. Space, time and motion / A.2 Forces and momentum

Momentum and impulse

A.2.7: Linear momentum A.2.8: Impulse A.2.9: Impulse and the change in momentum A.2.10: Force and the change of momentum over time

Learning outcomes

By the end of this section you should be able to:

- Understand the principle of the conservation of momentum.
- Understand linear momentum, and use the equation $p = mv$.
- Understand what an impulse is, and use the equation $J = F\Delta t$.
- Distinguish between Newton's second law equations $F = ma$ and $F = \frac{\Delta p}{\Delta t}$.

Imagine you are playing catch with a friend and they throw a tennis ball to you at a low speed. You can stop it without any problem. Now imagine that they throw a steel ball of the same size to you at the same speed as before. The steel ball will be harder to stop – why?

Video 1 shows someone throwing a medicine ball to their partner at different speeds. What happens when the speed is faster?

Student view

Video 1. Throwing a medicine ball at different speeds.

More information for video 1

In the video, two men are in a gym setting, one about to toss the medicine ball, and the other prepared to catch it. The man on the left has the medicine ball, raises it over his head, throws it between his legs and tosses it to the wall on the left. The ball then falls on the floor. He lifts the ball and throws it to the other man on the right. The other man catches the ball and throws it back to the first man. They continue to throw and catch the ball for four rounds at different speeds.
Then they come to half-kneeling position and continue to throw and catch the medicine ball back and forth at different speeds. The man on the left mentions that it is easier to catch the ball while standing than half-kneeling. It is demonstrated through this video that when the ball is thrown at greater speed, it is more difficult to catch the ball.

Linear momentum

The tennis ball and the steel ball have the same velocity, but it is harder for you to stop the steel ball while it is moving because the steel ball has a greater mass than the tennis ball.

If a ball is thrown to you several times, each time with a greater velocity, the greater the velocity, the more difficult it will be to stop the ball.

The linear momentum, p, of a body is the product of the mass of the body and its velocity, and can be determined using the equation in **Table 1**.

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Table 1. Equation for linear momentum.

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Equation	Symbols	Units
$p = mv$	p = momentum	kilogram metres per second (kg m s^{-1})
	m = mass	kilograms (kg)
	v = velocity	metres per second (m s^{-1})

Exercise 1

Click a question to answer

Linear momentum is a vector quantity. The direction of linear momentum is the same as the direction of the velocity of the body.

Linear momentum remains constant unless the system is acted on by a resultant external force. The system is the body or bodies, we are interested in. Everything else around them is the surroundings. Systems are covered in more detail in subtopic B.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-44324/).

Figure 1 shows two ice skaters pushing each other.

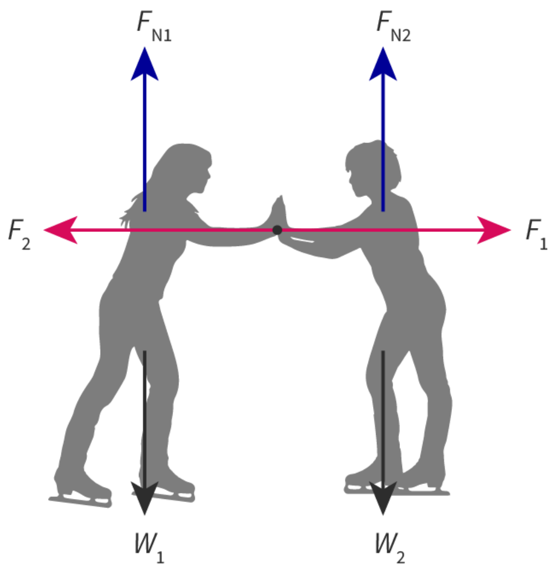

Figure 1. Two ice skaters pushing each other.

More information for figure 1

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The diagram depicts two ice skaters facing each other. Each skater is exerting equal and opposite horizontal forces on the other, labeled F1 and F2. Skater on the left pushes to the right with force F2, while the skater on the right pushes to the left with force F1. Each skater also experiences their own weight, W1 and W2, acting downwards. Normal forces from the ice, FN1 and FN2, act upwards on each skater to balance their weights. The diagram is a visual representation of Newton's third law of motion, illustrating that for every action, there is an equal and opposite reaction.

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According to Newton's third law, the two skaters exert equal and opposite forces F_1 and F_2 on each other. They also experience their own weights, W_1 and W_2, as well as the normal forces from the ice, F_{N1} and F_{N2}.

- Forces F_1 and F_2 act between the two skaters, which are the system. F_1 and F_2 are called **internal** forces.
- Weights W_1 and W_2, are exerted by the Earth on the skaters. Since the Earth is not part of the system, these forces are **external**.
- The ice pushes the skaters up, and since the ice is not part of the system, the normal forces are also **external**.

Study skills

The bodies within a system interact with forces, which are called internal forces. When the environment interacts with the system, the forces in action are called external forces.

Momentum is conserved when the resultant external force equals zero. That means that there are no external forces, or the external forces sum to zero. We call this type of a system an **isolated system** (subtopic B.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-44324/)).

If the resultant external force equals zero, then the change in linear momentum also equals zero. Change in linear momentum can be found using the equation:

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$$\Delta p = p_{\text{final}} - p_{\text{initial}}$$

The skaters in **Figure 1** are an isolated system, so their total momentum will not change: it will still be zero after they start moving apart. This is possible because their momentums will be of equal magnitude but in opposite directions.

Worked example 1

A ball of mass *m* hits a wall with a velocity *v* and bounces back with the same velocity *v* as shown in **Figure 2**. State whether the momentum of the ball changes.

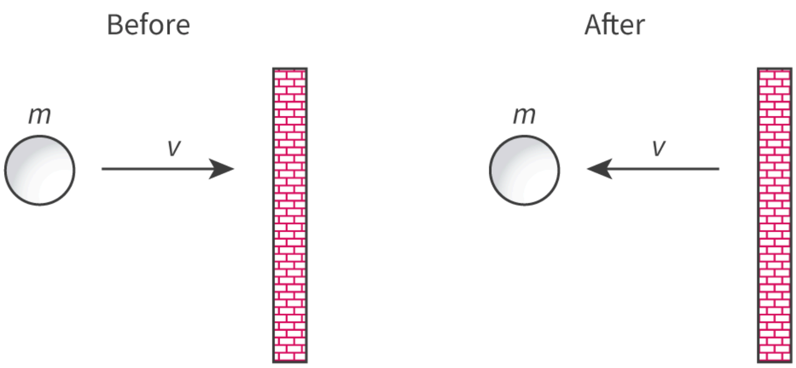

Figure 2. A ball hits a wall and bounces back.

More information for figure 2

The diagram illustrates a ball of mass m moving towards a wall with velocity v and then reversing direction with the same velocity after impact. On the left, labeled 'Before,' the ball is moving right towards a brick wall with velocity vector v. On the right, labeled 'After,' the ball, now on the opposite side, moves left away from the wall with the same velocity vector v. The mass m is marked on both instances of the ball.

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Momentum is a vector, so you need to assign directions to the velocities.

Let velocity of the ball before it hits the wall be positive.

Let velocity of the ball after it hits the wall be negative.

The mass of the ball does not change.

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$$\Delta p = p_{\text{final}} - p_{\text{initial}}$$

$$p_{\text{initial}} = mv$$

and

$$p_{\text{final}} = -mv$$

$$\begin{aligned} \Delta p &= -mv - mv \\ &= -2mv \end{aligned}$$

The momentum of the ball has changed by $\Delta p = -2mv$.

The magnitude of the momentum is the same before and after hitting the wall, but the direction has changed.

How we define a system is important. Look at the ball bouncing off a wall in **Worked example 1**.

If we look at the ball on its own as the system, momentum is not conserved. This is not a problem because the system is not isolated. During the collision with the wall, there is an external force, the normal force from the wall to the ball.

If the system is the ball and the wall, then momentum is conserved. The wall does not appear to move, although it has gained some momentum. This is because the mass of the wall is huge compared to the mass of the ball, making the wall's increase in velocity too small to notice.

Changing momentum

Objects we encounter every day undergo changes in momentum all the time. How can we change the momentum of a body?

Newton's second law of motion states that (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)):

$$F = ma$$

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Another equation for determining force is shown in **Table 2.**

Table 2. Equation for force.

Equation	Symbols	Units
$F = \dfrac{\Delta p}{\Delta t}$	F = force	newtons (N)
	Δp = change in momentum	kilogram metres per second (kg m s^{-1})
	Δt = change in time	seconds (s)

Worked example 2

A car of mass 1500 kg is initially moving at 13 m s^{-1} and is accelerated by a driving force of 3.6 kN for 2.4 s. Determine the final velocity of the car.

Solution steps	Calculations
Step 1: Determine the initial momentum of the car.	$p = mv = 1500 \times 13$ $= 19\,500 \text{ kg m s}^{-1}$
Step 2: Determine the change in momentum.	$F = \dfrac{\Delta p}{\Delta t}$ $F\Delta t = \Delta p$ $= 3600 \times 2.4$ $= 8640 \text{ kg m s}^{-1}$
Step 3: Determine the final momentum of the car.	initial momentum + change in momentu final momentum $19\,500 + 8640 = 28\,140 \text{ kg m s}^{-1}$
Step 4: Determine the final velocity and state the answer with appropriate units and the number of significant figures used in rounding	$p = mv$ $v_{\text{final}} = \dfrac{p_{\text{final}}}{m}$ $= \dfrac{28\,140}{1500}$ $18.76 \text{ m s}^{-1} = 19 \text{ m s}^{-1}$ (2 s.f.)

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Change in momentum, Δp, is the change of the product of mass and velocity: $\Delta(mv)$:

$$\begin{aligned} F &= \frac{\Delta p}{\Delta t} \\ &= \frac{\Delta(mv)}{\Delta t} \end{aligned}$$

If we assume the mass of the body does not change:

$$F = \frac{m(\Delta v)}{\Delta t}$$

Acceleration is rate of change of velocity $\left(a = \dfrac{\Delta v}{\Delta t}\right)$, so:

$$F = ma$$

Impulse

To change the momentum of a body, you need to apply a force to the body. Use **Interactive 1** to investigate kicking a block. Try changing the force and time it is applied. How do these changes affect the momentum of the block?

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Interactive 1. Force, Time, and Momentum.

More information for interactive 1

The "**Impulse and Momentum Interactive**" helps users explore how **force and time affect momentum** through the concept of **impulse.**

The layout is divided into key sections. The graph section has two graphs. The top graph represents momentum versus time, showing how the block's momentum changes due to the applied force. A dot marks the block's momentum at a given moment. The bottom graph represents force versus time, depicting the force applied over time. A dot moves to indicate the applied force. The controls include buttons labeled Start, Pause, Reset, allowing users to run, pause, reset, or step through the simulation frame by frame. The sliders and input fields enable adjustments to key parameters like maximum force, time of impact, and mass of the block. A block represents the object being kicked, and an image of a shoe symbolizes the applied force. An arrow next to the block labeled, v block, displays the block's speed change after impact.

The interface includes real-time calculations for impulse delta p, showing the relationship between force, time, and velocity change. The impulse equation delta p is dynamically updated based on user inputs. This interactive tool provides an engaging way to explore Newton's second law and impulse-momentum principles through direct

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manipulation and visualization.

From the interactive, the block initially had zero velocity, and after experiencing a force of 398 Newton for 0.1 seconds, it gained a velocity of 14 meters per second, as shown in the velocity equation at the bottom. The impulse, p = 39.8 Newton seconds, matches the area under the force-time graph, confirming the consistency of the impulse-momentum theorem. By adjusting the applied force and the duration of impact, users can observe how changes in these variables affect the momentum gained by the block. A greater force or a longer application time results in a larger change in momentum, demonstrating the direct proportionality of impulse to momentum change.

Rearranging the equation for force to find change in momentum, Δp:

$$\Delta p = F\Delta t$$

This change of momentum is called the impulse, J. When we deal with a system of bodies, the only forces that can change the total momentum of the whole system are the external forces, so we define impulse as shown in **Table 3**.

Table 3. Equation for impulse.

Equation	Symbols	Units
$J = F\Delta t$	J = impulse	kilogram metres per second (kg m s^{-1})
	F = average resultant force	newtons (N)
	Δt = time of contact	seconds (s)

Worked example 3

Calculate the change in momentum experienced by a car when a constant braking force of 550 N is applied for 4.8 s.

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$$\begin{aligned}\Delta p &= J\\ &= F\Delta t\\ &= 550\times 4.8\\ &= 2640\ \text{kg m s}^{-1}\\ &= 2600\ \text{kg m s}^{-1}\ (2\ \text{s.f.})\end{aligned}$$

The equation above shows that impulse is proportional to the force applied and the time for which it is applied. If you want the momentum of an object to change a lot, then you need to either apply a large force, or apply a force for a longer time.

Impulse can be represented by a force against time graph. The top graph in **Figure 3** shows a constant force and the bottom graph shows a force that is changing.

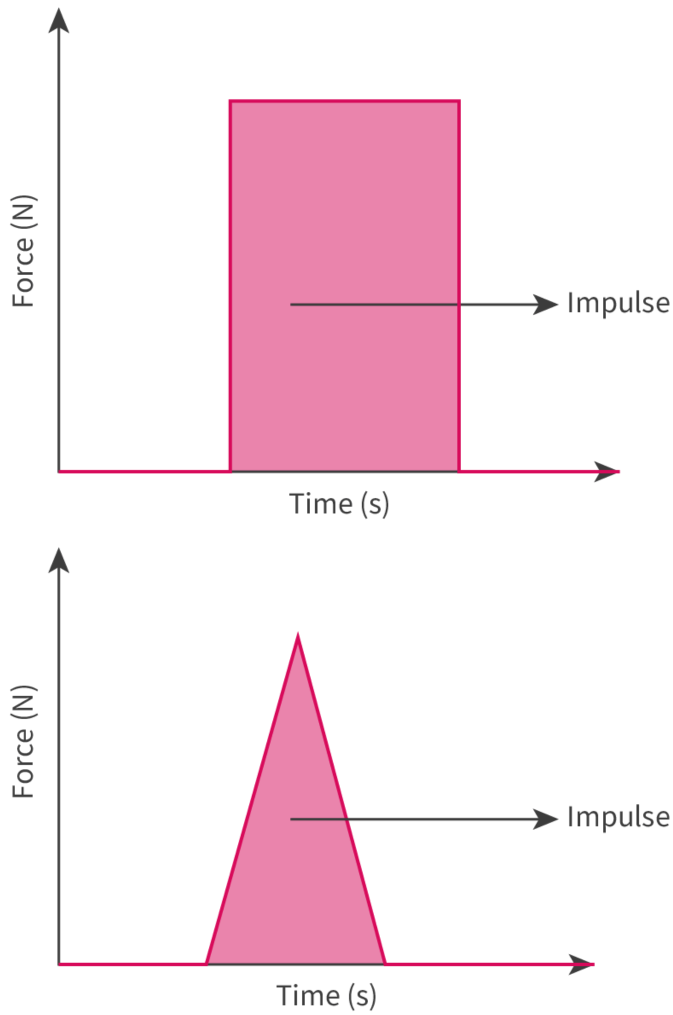

Figure 3. Impulse shown on force—time graphs.

More information for figure 3

The image consists of two graphs representing impulse on force-time graphs. Each graph has an X-axis labeled as "Time (s)" and a Y-axis labeled as "Force (N)."

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- The top graph displays a constant force, visualized as a red rectangle, indicating that the force remains steady over time.
- The bottom graph shows a red triangular shape, depicting a changing force - starting low, peaking, and then returning to the baseline.

Both graphs have arrows pointing to the area under the curves, labeled "Impulse," indicating that the impulse is calculated as the area beneath each curve.

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We can find the impulse of each force by calculating the area under the graph line.

Making connections

Finding the area between a graph line and the *x*-axis is covered in subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/). For example, the area of a velocity—time diagram gives you the displacement of an object in motion. The shape formed by the line does not matter.

Worked example 4

A tennis ball of mass 55 g is approaching a racket at a velocity of 8.5 m s^{-1}. The force on the racket is shown in **Figure 4**.

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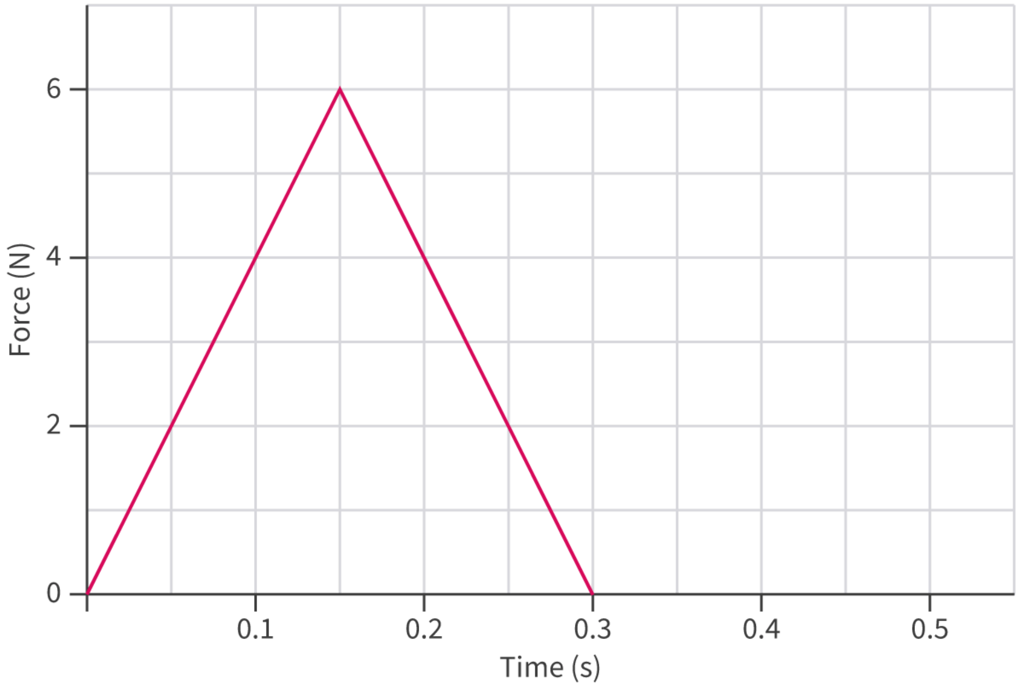

Figure 4. Force—time graph for tennis racket on a ball.

More information for figure 4

This is a line graph illustrating the force applied by a tennis racket on a ball over a period of time. The X-axis represents time in seconds, ranging from 0 to 0.5 seconds, and the Y-axis represents force in Newtons, ranging from 0 to 7 Newtons. The graph starts at 0, quickly ascends to reach a peak force of approximately 6.5 Newtons at 0.15 seconds, and then descends back to 0 by 0.3 seconds, forming a triangular shape.

[Generated by AI]

Determine the velocity of the ball as it leaves the racket. Assume that the ball is travelling horizontally both before and after the collision.

Solution steps	Calculations
Step 1: Determine the initial momentum of the ball.	$p = mv$ $= 0.055 \times 8.5$ $= 0.4675 \text{ kg m s}^{-1}$

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Solution steps	Calculations
Step 2: Determine the change in momentum.	The change in momentum is the impulse, which is the area under the F–t graph: $$\begin{aligned} J &= 0.5\times0.3\times6 \\ &= 0.9\ \text{kg m s}^{-1} \end{aligned}$$
Step 3: Determine the final momentum of the ball.	The change in momentum is in the opposite direction t the initial momentum, so the final momentum (away from the racket) is: $$0.4675 - 0.9 = -0.4325\ \text{kg m s}^{-1}$$
Step 4: Determine the final velocity and state the answer with appropriate units and the number of significant figures used in rounding.	$p = mv$ $$v = \frac{p}{m}$$ $$= -\frac{0.4325}{0.055}$$ $-7.86\ \text{m s}^{-1} = -7.9\ \text{m s}^{-1}$ (2 s.f.)

Forces can change the motion of a body, or change the shape or size of a body (see section A.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-forces-id-44732/)). The larger the force, the greater the impact.

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There are many cases in which we want to minimise a force, and we can do this by increasing the time of contact, Δt. **Video 2** shows the effect of a car collision on passengers without and with a seatbelt.

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Video 2. A car collision showing the effect on passengers without and with a seatbelt.

More information for video 2

The video presents a controlled crash test involving a silver four-door sedan to demonstrate the effects of a high-speed collision on vehicle occupants and the importance of restraint systems like seatbelts and airbags. The vehicle is staged inside a professional crash test facility with doors removed on both the driver and passenger sides to provide clear visibility of the interior and occupants. There are four crash test dummies seated inside—two in the front seats and two in the rear—positioned to simulate a realistic carload of passengers. All dummies are equipped with color-coded markers, likely used to track impact points and body movement throughout the collision.

The on-screen text “Crashed at 40km/h” appears in the upper left corner, clearly indicating the speed at which the crash is simulated. The video is shown from two distinct camera angles—first from the passenger’s side and then from the driver’s side—providing a full view of how the crash affects all four occupants and allowing for a thorough visual comparison of body movement and restraint function from both perspectives. The dummies in the front and rear seat of the driver’s side are wearing seatbelts, while the dummies in the front and rear seat of the passenger’s side are not wearing seatbelts.

As the test begins, the sedan collides head-on at 40 km/h. At first, the video shows the dummies seated on the passenger side. Upon impact, the front airbags deploy rapidly in a burst of compressed gas, creating a puff of smoke as they inflate. The dummy in the passenger seat moves forward due to force but is restrained from hitting the hard surface due to the deployed airbag. The dummy in the rear seat of the passenger side also moves forward due to force and collides with the backside of the front seat, and then with the roof of the car.

The collision sequence is shown again from the opposite side, where the dummies are seated in the driver's seat and the rear seat behind the driver. These two dummies are wearing seatbelts. As the collision happens, the front airbags deploy rapidly in a burst of compressed gas, creating a puff of smoke as they inflate. The dummies move forward due to force. However, the seatbelt restrained the dummies from hitting any hard surfaces and then pushed the dummies backward. The airbag provides additional protection to the dummy in the driver's seat. The dummies bend forward naturally at the waist but stay restrained and upright, showing minimal violent movement due to the seatbelt.

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.This crash test vividly illustrates fundamental physics concepts, especially those relating to force, momentum, and time. If a passenger is unrestrained, the time of deceleration is extremely short because it's the hard surface of the dashboard or seat in front that stops the body. This short interaction time produces a large force, greatly increasing the likelihood of injury.

Seatbelts are engineered to stretch slightly upon impact, thereby increasing the time over which the body comes to a stop and reducing the magnitude of the force. Airbags enhance this protection further by providing a cushioned barrier that both absorbs energy and distributes it over a larger surface area, further increasing Δt. The rear dummies, also restrained, demonstrate the importance of seatbelt use in all seating positions, not just the front seats. The video effectively conveys how increasing the time during which a force acts is essential to reducing the risk of injury during a collision. Through clear visuals and multiple perspectives, it communicates the life-saving role of seatbelts and airbags while reinforcing key physics principles. The takeaway is not only to understand the science behind these systems but also to recognize their critical importance in everyday life.

If the car you are travelling in comes to a sudden stop, you are also brought to a sudden stop (your momentum changes to zero). If you are not wearing a seatbelt, the force that decelerates you will likely be the normal force from the seatback or dashboard in front of you. Because seatbacks and dashboards are hard objects and not movable, the time in which you are decelerated is short. As $J = F\Delta t$, a short time means a large force for a given change in momentum.

A seatbelt stretches and so it decelerates you over a longer period of time: if Δt is larger, F is smaller. This could save your life.

Another real-life example of extending the time of contact is when an egg breaks when it falls onto hard concrete but not when it falls onto a soft cushion.

Imagine that two identical eggs are dropped from the same height. Air resistance is negligible. Both eggs have the same acceleration (g). Since they fall from the same height, they have the same velocity just before they land. Since they have the same mass, they also have the same momentum just before they land.

During the landing, both eggs decelerate to zero velocity, losing all of their momentum.

According to Newton's second law: $F = \frac{\Delta p}{\Delta t}$. The change in momentum for the two eggs is the same. However, the time, Δt, during which decelerating forces act will be different.

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The egg that lands on the cushion takes longer to stop, because the cushion deforms as the egg pushes against it, allowing the egg to sink down into it. The concrete deforms much less, so the egg that lands on concrete stops moving in a much shorter time.

Since $\Delta p = F\Delta t$ is the same for both eggs, the egg that lands on the cushion experiences a smaller force over a longer time, so it is less likely to break.

Changing mass

The mass of a body or system does not always remain the same. For example, an aeroplane uses fuel while flying, so the mass of the fuel reduces during the flight.

$F = ma$ assumes that the mass remains constant, while $F = \frac{\Delta p}{\Delta t}$ allows for mass to be changing.

For example, a conveyor belt in an airport luggage claim travels at a constant speed (**Video 3**). Bags are being added to the belt, and removed from the belt by passengers. The mass on the belt is constantly changing.

0:00

Rights of use

Video 3. Luggage on a Conveyor Belt.

More information for video 3

Student view

A video featuring luggage moving on a conveyor belt at an airport, demonstrating the concept of changing mass in motion. It includes a continuous flow of suitcases of different sizes being added to the conveyor belt. The video illustrates how the internal mechanism of the conveyor belt provides force as the mass of the suitcases varies over time to keep the constant velocity of the belt, reinforcing key concepts in dynamics and force analysis.

Even if the belt is moving at a constant velocity, the resultant force is not zero. A force is provided by the internal mechanism of the conveyor belt to keep the velocity of the belt constant.

This is not an application of Newton's first law of motion: the body is moving at a constant velocity, but its mass is varying.

Worked example 5

A conveyor belt is moving at a constant velocity of 1.5 m s^{-1}. One end of the belt is being loaded at a rate of 3.7 kg s^{-1}. Determine the force required to keep the belt moving at a constant velocity.

Using Newton's second law:

$$\begin{aligned} F &= \frac{p}{\Delta t} \\ &= \frac{\Delta(mv)}{\Delta t} \end{aligned}$$

The velocity is constant and not changing, so:

$$F = v\left(\frac{\Delta m}{\Delta t}\right)$$

$\dfrac{\Delta m}{\Delta t}$ is the rate at which mass is changing: 3.7 kg s^{-1}

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$$F = v\left(\frac{\Delta m}{\Delta t}\right)$$
$$= 1.5 \times 3.7$$
$$= 5.55\text{ N}$$
$$= 5.6\text{ N (2 s.f.)}$$

Work through the activity to check your understanding of momentum.

Activity

- **IB learner profile attribute:** Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

A rocket is travelling through space on its way to a distant planet. The diagram shows the rocket at an instant in time as seen from Earth using a powerful telescope. Try to answer the questions. Click on 'Show or hide solution' to see the answers.

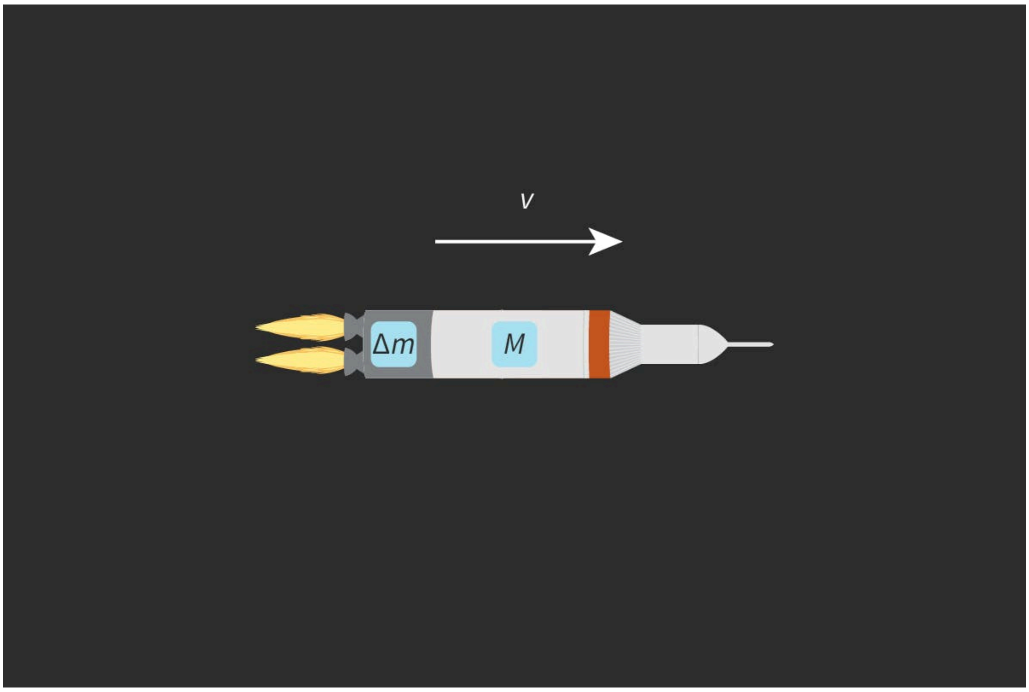

Figure 5. A rocket travelling through space.

More information for figure 5

Student
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The image is a diagram illustrating a rocket traveling through space. The rocket is shown horizontally with exhaust flames emitting from the back, indicating movement. The body of the rocket is divided into sections labeled 'Δm' and 'M'. An arrow labeled 'v' points in the direction of the rocket's travel, signifying velocity. This diagram is intended to depict the rocket's motion dynamics as observed from Earth with a telescope.

[Generated by AI]

1. In order to increase the velocity of the rocket, an empty fuel tank is ejected. Why does ejecting the tank result in the rocket increasing its velocity? What assumption do you need to make in order for this to be true?

   Momentum must be conserved. The empty fuel tank is detached from the main rocket and separates when the main rocket fires its engines. The exhaust gases impact upon the empty fuel tank causing a deceleration and reduction in velocity. Because of conservation of momentum, the rocket's momentum must increase by the same amount in the forwards direction, increasing its velocity. We assume that there are no external forces acting.

2. The mass of the empty fuel tank is Δm and the mass of the remainder of the rocket is M. The whole system is moving with a velocity v. What is the momentum of the system (rocket and tank)?

$$\begin{aligned} p_{\text{initial}} &= \text{mass} \times \text{velocity} \\ &= (M + \Delta m)v \end{aligned}$$

3. The tank with mass Δm is ejected. The rocket moves with a velocity $v + \Delta v$ and the tank moves with a velocity v_{tank}. What is the momentum of the system now?

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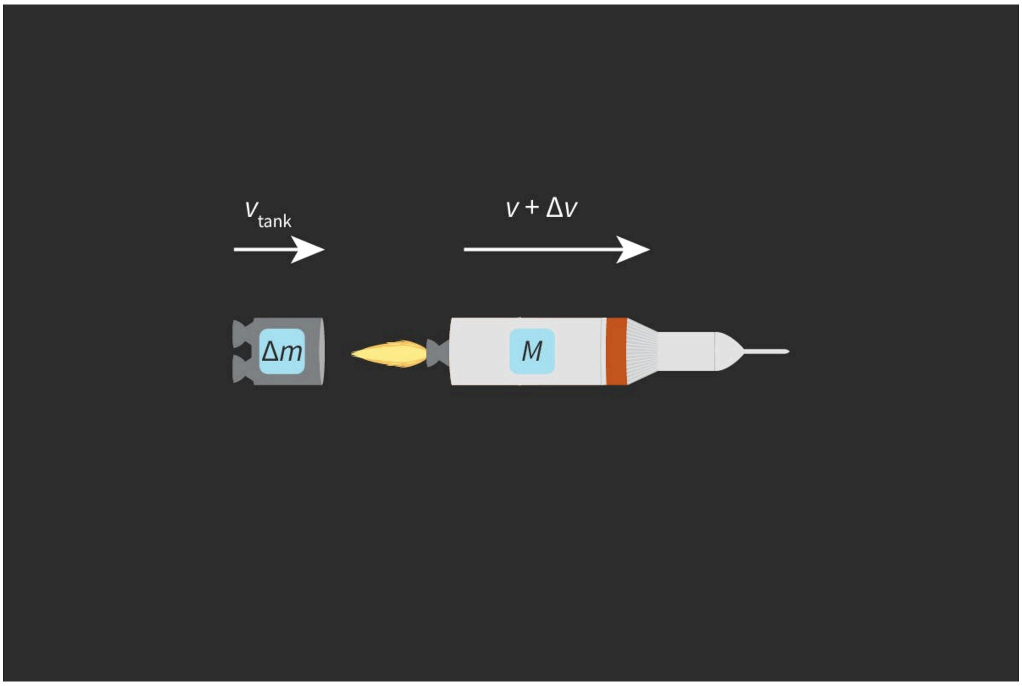

Figure 6. A rocket travelling through space.

More information for figure 6

The image is a diagram illustrating a rocket in motion through space. It features two main components: the main rocket body with a label 'M', which is moving to the right at a velocity labeled 'v + Δv', indicating an increase in its velocity. Behind the main body is a separate tank labeled 'Δm', moving to the left with a velocity labeled 'v_tank'. A visual representation of rocket exhaust between the two components suggests propulsion. The diagram demonstrates the physics concept of changing momentum in a system where a part of mass (the tank) is ejected from the main body, causing a change in velocity.

[Generated by AI]

$$\begin{aligned} p_{\text{final}} &= \text{mass} \times \text{velocity} \\ &= M(v + \Delta v) + \Delta m v_{\text{tank}} \end{aligned}$$

4. Apply conservation of momentum to the system and find the change in velocity, Δv, of the rocket.

$$\begin{aligned} p_{\text{initial}} &= p_{\text{final}} \\ (M + \Delta m)v &= M(v + \Delta v) + \Delta m v_{\text{tank}} \\ Mv + \Delta m v &= Mv + M\Delta v + \Delta m v_{\text{tank}} \\ \Delta v &= \frac{\Delta m(v - v_{\text{tank}})}{M} \end{aligned}$$

Extension

So far, all of the velocities have been expressed relative to an observer on Earth. You can also state the velocity of the ejected fuel tank relative to the

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rocket (see section A.5.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/reference-frames-and-galilean-relativity-hl-id-46604/)). (That means the tank's velocity according to an observer in the rocket.) This velocity is the difference between the velocity of the rocket and the velocity of the tank:

$$u = v – v_{\text{tank}}$$

The increase of the velocity of the rocket after ejecting the tank can be expressed as:

$$\Delta v = \frac{\Delta m u}{M}$$

Imagine that instead of ejecting a fuel tank, the rocket ejects fuel of mass Δm at a steady rate during time Δt (because the rocket has more fuel than it needs). The rate of change (decrease) of mass is then:

$$\frac{\Delta m}{\Delta t}$$

5. Write a formula relating the acceleration of the rocket to the rate of decrease of its mass.

$$\Delta v = \frac{\Delta m u}{M}$$

$$a = \frac{\Delta v}{\Delta t} = \left(\frac{u}{M}\right)\left(\frac{\Delta m}{\Delta t}\right)$$

5 section questions

A. Space, time and motion / A.2 Forces and momentum

Collisions and explosions

A.2.11: Elastic and inelastic collisions of two bodies A.2.12: Explosions A.2.13: Energy considerations in collisions and explosions

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Learning outcomes

By the end of this section you should be able to:

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- Understand collisions and conservation of momentum.
- Describe kinetic energy in elastic and inelastic collisions.
- Understand explosions in terms of momentum.
- Demonstrate a quantitative approach for two-dimensional collisions and explosions (HL only).

The Space Shuttle Endeavour was a NASA space shuttle that operated between 1992 and 2011, completing 25 missions, 7179 hours of flight and 4671 orbits around the Earth. In **Video 1**, you can see the final launch of Space Shuttle Endeavour.

Video 1. The final launch of Space Shuttle Endeavour.

Figure 1 shows fireworks that are used in celebrations around the world.

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Figure 1. Fireworks.

Credit: Katsumi Murouchi, Getty Images

What links the Space Shuttle Endeavor and a firework rocket?

Collisions

Imagine two balls hitting each other. The balls interact with each other and exert a force on each other, which changes the way the balls move. This is called a collision. We can predict the motion of the balls after their collision. **Video 2** shows two balls colliding.

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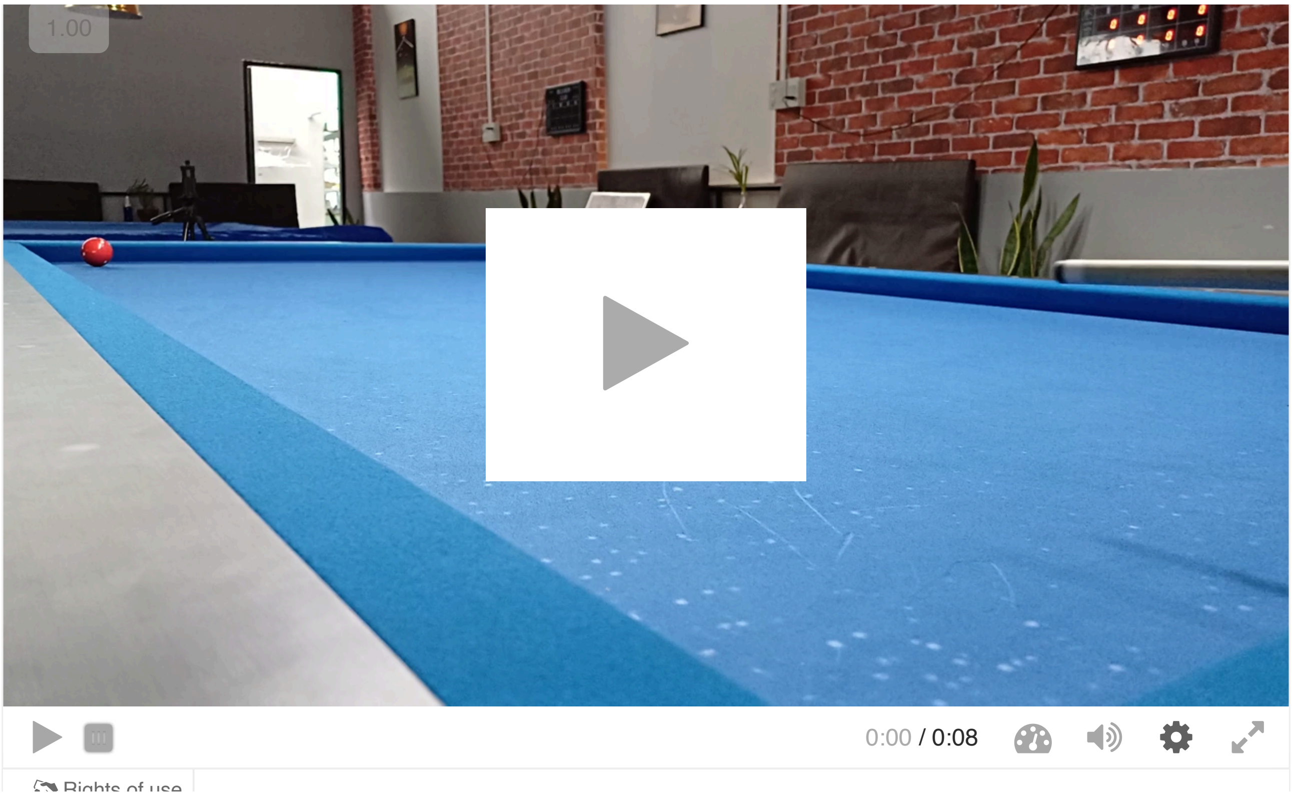

Video 2. Balls Colliding.

More information for video 2

A video featuring two billiard balls colliding on a pool table demonstrates the principles of momentum conservation. There are three billiard balls on the table, one in the center and the other two balls on the left and right corners of the table. When the player hits the ball in the center of the table, it includes an initial movement, where one ball strikes another, causing both to move in different directions. The video explains how momentum is transferred between the balls and how their velocities change due to the collision.

The **system** (see section A.2.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/newtons-laws-id-44736/)) is the bodies that collide. In this case, the two balls.

If the system is **isolated**, the total momentum of the system is **conserved.** The total momentum of the balls (momentum of ball 1 plus momentum of ball 2) before the collision is equal to the total momentum of the balls (momentum of ball 1 plus momentum of ball 2) after the collision:

$$\text{total } p_{\text{initial}} = \text{total } p_{\text{final}}$$

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Momentum is a vector quantity, so we need to take into account the direction of momentum for each body.

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Study skills

When we study a collision using conservation of momentum, we look at the system just **before** the collision and just **after** the collision. We don't examine the microscopic behaviour of the bodies **during** the collision.

Worked example 1

A blue ball with mass 0.3 kg moving at 2 m s^{-1} hits a stationary green ball with mass 0.6 kg. The blue ball is stationary after the collision. Determine the velocity of the green ball after the collision.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	Use u for initial speed and v for final speed. Deal with the two balls as an isolated system. Before the collision: Blue ball: $m_{\mathrm{b}} = 0.3\ \mathrm{kg}$ $u_{\mathrm{b}} = 2\ \mathrm{m\ s^{-1}}$ Green ball: $m_{\mathrm{g}} = 0.6\ \mathrm{kg}$ $u_{\mathrm{g}} = 0\ \mathrm{m\ s^{-1}}$ After the collision: Blue ball: $m_{\mathrm{b}} = 0.3\ \mathrm{kg}$ $v_{\mathrm{b}} = 0\ \mathrm{m\ s^{-1}}$ Green ball $mg = 0.6\ \mathrm{kg}$ $v_{\mathrm{g}} = ?\ \mathrm{m\ s^{-1}}$

Student view

Solution steps	Calculations
Step 2: Write out the equation.	total momentum before collision = total momentum after collision: total p_{initial} = total p_{final} $p = mv$ $(m_b u_b + m_g u_g) = (m_b v_b + m_g v_g)$
Step 3: Substitute the values given and rearrange to find v_g.	$(0.3 \times 2 + 0.6 \times 0) = (0.3 \times 0 + 0.6 v_g)$ $v_g = \frac{0.6}{0.6}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 1 \text{ m s}^{-1}$ (1 s.f.)

Worked example 2

Two balls (ball 1 and ball 2) are moving towards each other as shown in the diagram. After the collision, ball 1 moves at 1 m s^{-1} to the right.

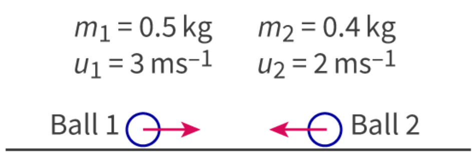

Figure 2. Two balls moving towards each other.

More information for figure 2

The diagram illustrates two balls moving towards each other horizontally. Ball 1 is labeled with a mass of 0.5 kg and an initial velocity of 3 m/s to the right. Ball 2 is labeled with a mass of 0.4 kg and an initial velocity of 2 m/s to the left. Both balls are represented as circles with arrows indicating their direction of movement, positioned on a straight line. The labels for each ball and their respective velocities and masses are positioned above them in the diagram.

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Calculate the velocity of ball 2 after the collision.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	Since you know that ball 1 moves to the right after the collision, take its velocity to be positive. Before the collision: Ball 1: $m_1 = 0.5\text{ kg}$ $u_1 = 3\text{ m s}^{-1}$ Ball 2: $m_2 = 0.4\text{ kg}$ $u_2 = -2\text{ m s}^{-1}$ After the collision: Ball 1: $m_1 = 0.5\text{ kg}$ $v_1 = 1\text{ m s}^{-1}$ Ball 2: $m_2 = 0.4\text{ kg}$ $v_2 = ?\text{ m s}^{-1}$
Step 2: Write out the equation.	$\text{total } p_{\text{initial}} = \text{total } p_{\text{final}}$ $p = mv$ $(m_1u_1 + m_2u_2) = (m_1v_1 + m_2v_2)$
Step 3: Substitute the values given and rearrange to find v_2.	$(0.5\times3 + 0.4\times -2) = (0.5\times1 + 0.4v_2)$ $v_2 = \frac{(0.7–0.5)}{0.4}$

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Solution steps	Calculations
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 0.5\ \text{m s}^{-1}$ (1 s.f.) Ball 2 moves $0.5\ \text{m s}^{-1}$ to the right after the collision. (If the result had been negative, you would need to say that the ball moves to the left after the collision.)

Kinetic energy in collisions

For an isolated system, the momentum of the system is conserved. This is always true, no matter how the bodies collide,

However, this is not true for the total kinetic energy of a system during a collision. (Kinetic energy is covered in more detail in subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)). Kinetic energy is conserved in some collisions and not conserved in others.

- In an elastic collision, the total kinetic energy of the system is conserved – the total kinetic energy before the collision is the same as the total kinetic energy after the collision.
- In an inelastic collision, the total kinetic energy of the system is not conserved – the total kinetic energy after the collision is less than the total kinetic energy before the collision.

In some inelastic collisions, the bodies stick together after the collision and become stationary. **Interactive 1** shows this type of inelastic collision.

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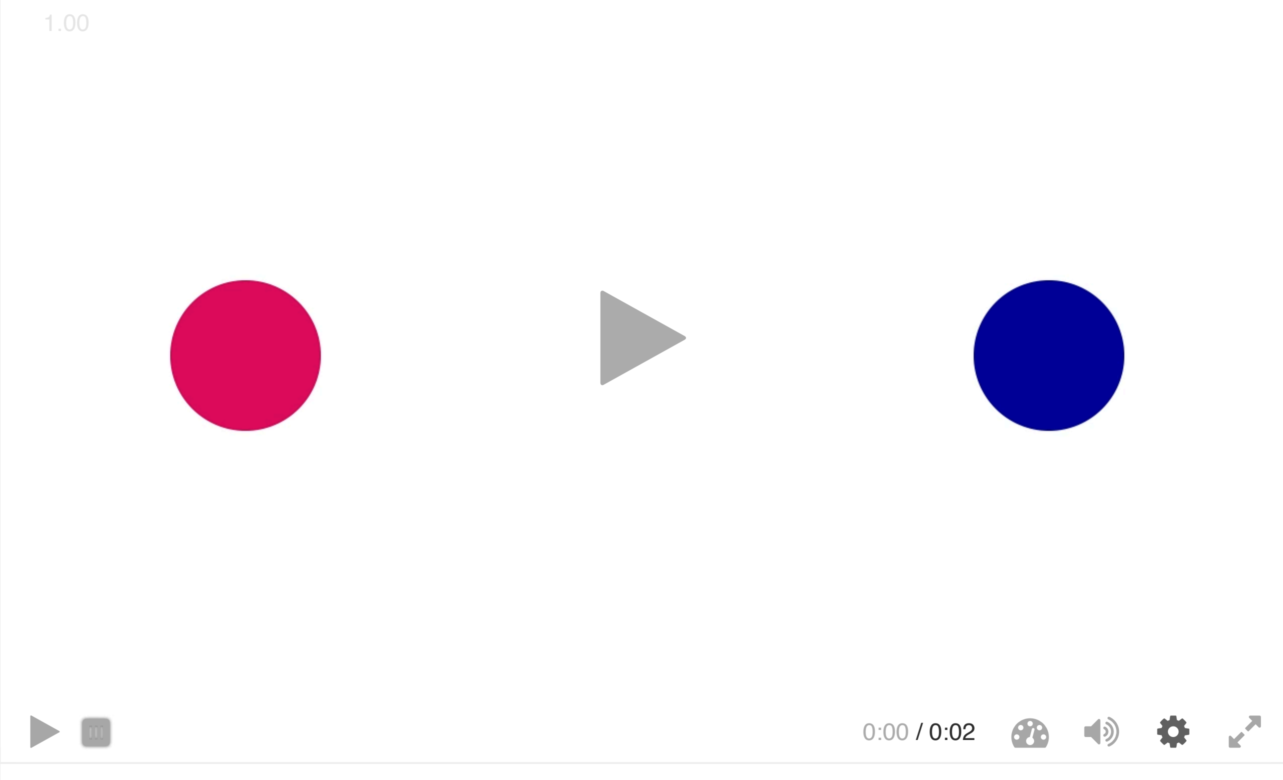

Interactive 1. An Inelastic Collision Where the Balls Stick Together and Stop Moving.

More information for interactive 1

A simple animation of two balls colliding in an inelastic collision. It includes a pink (on the left) and a blue (on the right) ball moving toward each other, colliding, and sticking together. The animation explains how kinetic energy is not conserved in inelastic collisions, and how the combined mass moves as a single unit, eventually stopping due to external forces. It reinforces key concepts in momentum conservation and energy loss.

Worked example 3

A ball of mass 2.0 kg moving horizontally to the right at 3.0 m s^{-1} collides with a stationary ball of mass 4.0 kg. After the collision, the 4 kg ball moves at 2.0 m s^{-1} to the right. Determine whether the collision is elastic or inelastic..

Student view

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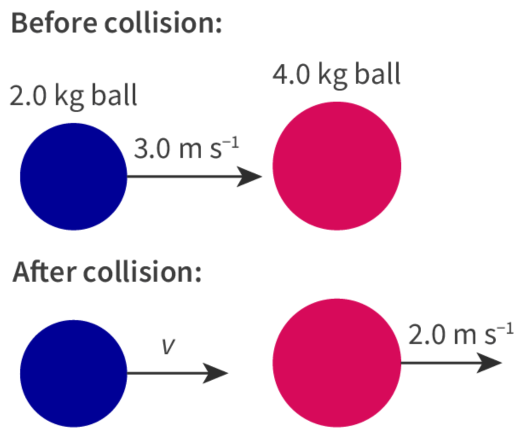
Solution steps	Calculations
Step 1: Sketch the situation before and after the collision.	
Step 2: Determine the initial momentum of the system.	For the moving ball: $p = mv = 2.0 \times 3.0 = 6.0 \text{ kg m s}^{-1}$ For the stationary ball: $p = 0$ Total momentum $= 6.0 \text{ kg m s}^{-1}$ (to the right)
Step 3: Use conservation of momentum to determine the velocity of the 2 kg ball after the collision.	$p_{\text{initial}} = p_{\text{final}}$ $6.0 = 2.0v + 4.0 \times 2.0$ $v = \frac{6.0 - 8.0}{2.0}$ $= -1.0 \text{ m s}^{-1}$ (The negative sign means that after the collision, thi ball moves to the left.)
Step 4: Calculate the total kinetic energy before the collision.	$E_{\text{initial}} = \frac{1}{2} \times 2.0 \times 3.0^2 = 9.0 \text{ J}$
Step 5: Calculate the total kinetic energy after the collision.	$E_{\text{final}} = \frac{1}{2} \times 2.0 \times 1.0^2 + \frac{1}{2} \times 4.0 \times 2.0^2$ $= 1.0 + 8.0$ $= 9.0 \text{ J}$
Step 6: State what this shows.	The total kinetic energy is the same before and afte the collision. Therefore the collision is elastic.

Student view

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Worked example 4

A trolley of mass m moves at speed u and hits a stationary trolley of mass m. The two trolleys then stick together and move on at speed v.

Show that this collision is inelastic, and determine the fraction of the initial kinetic energy that is transferred to other forms of energy during this collision.

Solution steps	Calculations
Step 1: Express the velocity after the collision in terms of the given quantities *m* and *u*.	$p_{\text{initial}} = p_{\text{final}}$ Mass of combined trolleys is *m*, so $mu = 2mv$ $v = \frac{u}{2}$
Step 2: Express the initial kinetic energy in terms of the given quantities.	$E_{\text{initial}} = \frac{1}{2}mu^2$
Step 3: Express the final kinetic energy in terms of the given quantities.	$E_{\text{final}} = \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{1}{4}mu^2$ This is less than the initial kinetic energy, so the collision is inelastic.
Step 4: Express the lost kinetic energy as a fraction of the initial kinetic energy.	$E_{\text{lost}} = E_{\text{initial}} - E_{\text{final}} = \frac{1}{2}mu^2 - \frac{1}{4}mu^2 = \frac{1}{4}mu^2$ $\frac{E_{\text{lost}}}{E_{\text{initial}}} = \frac{\frac{1}{4}mu^2}{\frac{1}{2}mu^2} = \frac{1}{2}$

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As with forces and other vector quantities, a momentum can be expressed as the sum of two perpendicular momentums. Since motions in perpendicular directions are independent of each other (as you learned in section A.1.4a (/study/app/math-aa-hl/sid-423-cid-762593/book/projectile-motion-id-44300/) on projectile motion), conservation of momentum applies separately in perpendicular directions.

Higher level (HL)

If you are taking the higher level (HL) course, you need to be able to complete a quantitative approach to apply conservation of momentum in two dimensions.

Worked example 5

A bowling ball of mass 5.0 kg moving along a horizontal surface with velocity u collides with a stationary bowling ball of mass 5.0 kg. After the collision, the balls move as shown in the diagram.

Determine the values of u and θ.

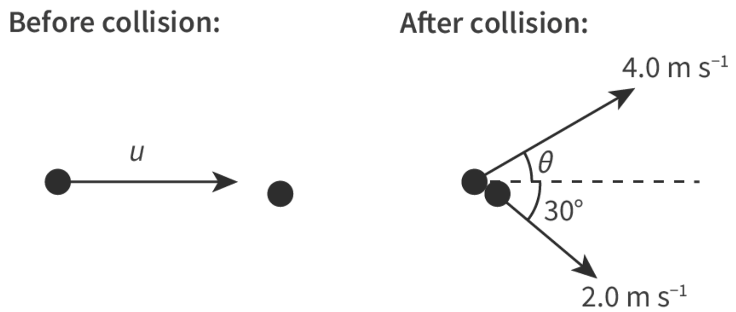

Figure 3. Before and after a collision between two balls in two dimensions.

More information for figure 3

The diagram illustrates a two-dimensional collision between two balls. On the left, before the collision, there is a single ball with an arrow labeled 'u' pointing to the right, indicating its velocity.

On the right, after the collision, there are two balls positioned close together. One ball moves at an angle of 30 degrees below the horizontal, with a velocity of 2.0 m/s, represented by a downward arrow. The second ball moves at an angle θ, with a velocity of 4.0 m/s, represented by an upward arrow. The angle θ is located between the horizontal dashed line and the 4.0 m/s velocity arrow.

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Solution steps	Calculations
Step 1: Find θ by considering conservation of momentum vertically (up the page).	Vertically: $p_{\text{final}} = p_{\text{initial}} = 0$ Taking up the page to be the positive directi $p_{\text{final}} = 0 = 5.0 \times 4.0 \sin\theta - 5.0 \times 2.0 \sin 30$ Rearranging and solving for θ: $\sin\theta = \frac{5.0 \times 2.0 \times \sin 30^{\circ}}{5.0 \times 4.0} = 0.25$ $\theta \approx 14.47^{\circ}$ $= 14.5^{\circ}$ (1 d.p.)
Step 2: Find *u* by considering conservation of momentum horizontally (across the page).	Horizontally: $p_{\text{final}} = p_{\text{initial}} = 5.0u$ Taking the positive direction as to the right: $p_{\text{final}} = 5.0u = 5.0 \times 4.0 \cos 14.47^{\circ} + 5.0 \times$ Rearranging and solving for u: $u = \frac{5.0 \times 4.0 \cos 14.47^{\circ} + 5.0 \times 2.0 \cos 30^{\circ}}{5.0}$ $= 5.6 \text{ m s}^{-1} (2 \text{ s.f.})$

Explosions

We can study an explosion as if the explosion is an inelastic collision happening backwards.

The body that explodes is stationary at the beginning, so the total momentum before the collision is zero. The momentum of a system is always conserved, so if the momentum is zero before an explosion, it will also be zero after an explosion.

Video 3 shows a firework exploding in the sky.

Student
view

Video 3. A Firework Exploding.

More information for video 3

A simple animation of a firework exploding in the sky. The animation initially shows spots of lights produced by fireworks in the night sky in between which a firework is traveling high illuminating its path. The firework, upon reaching its peak height, momentarily stays still and then bursts into multiple bright curved fragments that spread outward in different directions. Consecutively, four more fireworks burst out brightly in the night sky at different heights. The animation explains how momentum is conserved, with individual fragments balancing each other's motion, while kinetic energy is not conserved due to the explosion. It reinforces key concepts in momentum conservation and energy transformation.

The firework is shot into the sky. Just before it explodes, it reaches a maximum height and stays there for a moment, so its momentum is zero.

Immediately after the explosion, if we determine the total momentum of all the pieces of firework, they add up to zero. Momentum is a vector quantity, so momentums in opposite directions cancel each other out.

An explosion is the 'opposite' of an inelastic collision, so the total kinetic energy of the system is **not conserved** during an explosion.

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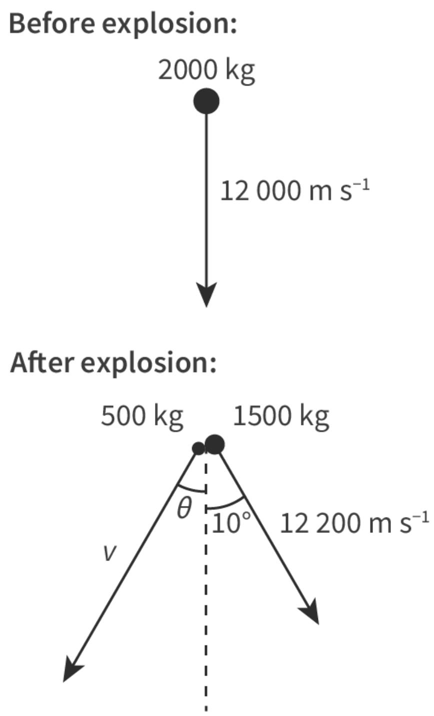
Higher level (HL)

If you are taking the higher level (HL) course, you need to be able to complete a quantitative approach to apply conservation of momentum in two dimensions.

Worked example 6

A meteor of mass 2000 kg falls vertically through the Earth's atmosphere. While it is falling, it explodes into two pieces of mass 500 kg and 1500 kg.

Immediately before the explosion, the speed of the meteor is 12 000 m s^{-1}. Immediately after the explosion (before enough time has passed for gravity to significantly affect the velocities), the two pieces travel at angles to the vertical, as shown in the diagram:



The meteor before and after the explosion

More information

The diagram shows a meteor of 2000 kg traveling straight downwards at a speed of 12,000 m/s before an explosion. After the explosion, the meteor splits into two pieces. The larger piece weighs 1500 kg and moves to the right at an angle of 10° to the vertical with a speed of 12,200 m/s. The smaller piece, with a mass of 500 kg, moves to the left at an unknown angle θ to the vertical and with an unknown speed denoted as 'v'. Arrows indicate the direction of each piece's movement, and angles are labeled relative to a dashed vertical line.

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Determine the values of v (the speed of the smaller piece) and θ (the angle of the smaller piece to the vertical).

Solution steps	Calculations
Step 1: Sketch the situation before and after the explosion.	Before the explosion, there is no horiz component of momentum. Therefore collision, one piece must move towarc one towards the right, so that there is horizontal component of the total mor
Step 2: Determine v by considering conservation of momentum vertically.	$p_{\text{final}} = p_{\text{initial}} = 2000 \times 12\,000$ $= 2.4 \times 10^7 \text{ kg m s}^{-1}$ (downwa So $2.4 \times 10^7 = 500v\cos\theta + 1500 \times 12\,2\text{C}$ $v\cos\theta = \frac{2.4 \times 10^7 - 1500 \times 12\,20\text{(}}{500}$ $\approx 11\,956$
Step 3: Consider conservation of momentum horizontally.	Horizontally: $p_{\text{final}} = p_{\text{initial}} = 0$ So $0 = 500v\sin\theta - 1500 \times 12\,200\,\text{s}$ $v\sin\theta = 36\,600\sin 10^\circ$ ≈ 6356
Step 4: Use the equations from Step 2 and Step 3 to solve for θ.	$\frac{v\sin\theta}{v\cos\theta} = \frac{\sin\theta}{\cos\theta}$ $\approx 6356/11956 \approx 0.5316$ Using the trigonometric identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\theta \approx \tan^{-1} 0.5316 \approx 27.99$ $= 28.0^\circ (1 \text{ d.p.})$

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Solution steps	Calculations
Step 5: Use the value of θ either the equation from Step 2 or the equation from Step 3 to solve for *v*.	$v\cos\theta \approx 11\,956$ $v \approx \frac{11\,956}{\cos 27.99^\circ} = 13\,540$ $= 13\,500\ \mathrm{m\ s^{-1}} (3\ \mathrm{s.f.})$

International Mindedness

Space doesn't 'belong' to any one country, and any country can launch satellites into orbit. There are international regulations about launching satellites, but only quite vague guidelines about removing debris from space after a satellite is decommissioned or destroyed. The Outer Space Treaty of 1967 only says that countries or organisations should carry out activities in space 'with due regard to the corresponding interests of all other States Parties'. Does that mean removing satellites from space?

You can see how much space debris and active satellites there are using this program (https://wayfinder.privateer.com/).

If we keep launching satellites and rockets into space, what should we do with the trash left over? What will happen when there is more and more space junk? Whose responsibility should it be to clear it up, and who should pay?

Work through the activity to check your understanding of collisions.

Activity

- **IB learner profile attribute:** Thinker
- **Approaches to learning:** Thinking skills — Providing a reasoned argument to support conclusions
- **Time required to complete activity:** 10 minutes
- **Activity type:** Pair activity

Is momentum always conserved? **Figure 4** shows firefighters all holding tight to a hose. Can you explain why?

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Figure 4. Firefighters holding a water hose.
Credit: shaunl, Getty Images (https://www.gettyimages.com/detail/photo/fire-fighting-royalty-free-image/157163784)

Does the water have momentum? Do the firefighters? Where are there changes in momentum? Is the momentum of the system conserved? What is the system?

Click on 'Show or hide solution' for an explanation.

This question is about defining what the system is. Imagine the firefighters in space, with water shooting out the hose, with nothing around them. The firefighters would accelerate backwards, like in the explosions examples above.

However, this is not happening, so we are obviously not considering the full system. The full system includes the Earth, because the firefighters' feet are experiencing friction with the Earth. Momentum is still conserved. The magnitude of the momentum gained by the water is equal to the magnitude of momentum gained by the Earth and firefighters combined. The mass of the Earth is so large, there is no noticeable change in its velocity.

If the hose is ejecting 15 kg of water every 0.40 s, increasing its speed by 10 m s^{-1}, what is the force that accelerates this water?

Click on 'Show or hide solution' to see the answer.

$F = ma$, and acceleration is change in velocity per unit time, so:

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$$\begin{aligned} F &= m\left(\frac{\Delta v}{\Delta t}\right) \\ &= 15\times\left(\frac{10}{0.4}\right) \\ &= 375 \text{ N} \end{aligned}$$

5 section questions

A. Space, time and motion / A.2 Forces and momentum

Circular motion

A.2.14: Bodies moving along a circular trajectory A.2.15: Circular motion and centripetal force

A.2.16: Centripetal force and change of direction A.2.17: Angular velocity

Learning outcomes

By the end of this section you should be able to:

- Understand angular velocity, and relate linear speed and angular velocity using the equations:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \omega r \end{aligned}$$

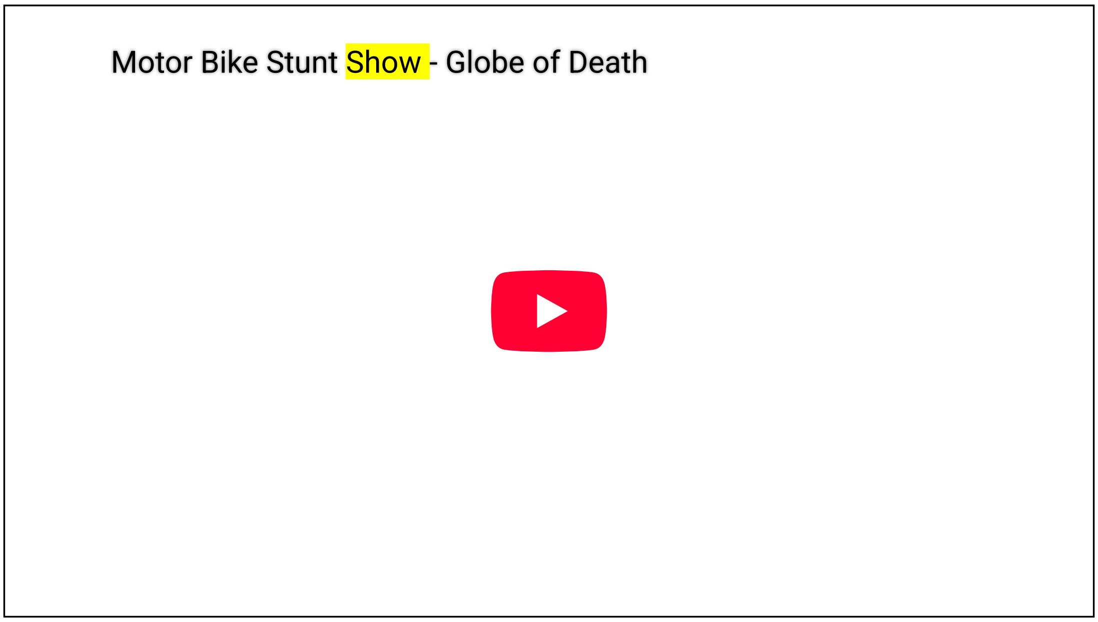
- Understand centripetal acceleration, and use the equations:

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \omega^2 r \\ &= \frac{4\pi^2 r}{T^2} \end{aligned}$$

- Understand that circular motion is caused by a centripetal force.
- Describe the effect of centripetal force on the motion of a body.

Student
view

The ‘Globe of death’ is a stunt done by skilled motorcyclists. Motorbikes are ridden on the inner surface of a steel mesh sphere, and they seem to defy gravity (**Video 1**).



Video 1. The ‘Globe of death’.

More information for video 1

The video features a thrilling live demonstration of the "Globe of Death," a gravity-defying motorbike stunt performed within a large spherical metal cage. The video opens with the logo being displayed against a plain white background, establishing the identity of the performing group. This branding then transitions into the bold title "Motor Bike Wheel Of Death Show," which appears at the top center of the screen, setting the stage for the high-energy display to follow.

Below the text, the video presents a wide outdoor setting, showcasing a large, spherical steel mesh cage—often referred to as the "Globe of Death." The cage is mounted securely on a round platform, with several strong metal poles extending outward for additional stability. A sizable crowd surrounds the structure, watching with growing anticipation.

Inside the metal sphere, two motorcyclists, both wearing bold red and black outfits, begin their performance. Initially, one rider starts circling along the inner walls of the sphere. Shortly afterward, the second rider joins in, and the performance intensifies as both motorbikes simultaneously ride inside the cage. They loop around each other, moving in opposite or intersecting paths—both horizontally and vertically—demonstrating precise timing, incredible coordination, and unwavering control. At various moments, both motorcyclists appear to narrowly miss each other while crisscrossing paths within the sphere, increasing the visual drama and thrilling the crowd.

The action inside the cage continues with both riders maintaining enough speed to ride completely around the inside surface, including upside down at the top of the sphere. This defiance of gravity creates a stunning display of centripetal force and dynamic balance.

Through this high-adrenaline demonstration, essential physics principles are brought vividly to life. The expectation is to observe and interpret how continuous, fast circular motion allows the riders to remain suspended within the globe, not by escaping gravity, but by leveraging it through coordinated motion and centripetal force.

But these motorbikes and their riders are subject to the gravitational force. So how do they manage not to fall over? What keeps them moving in a circle?

Period and frequency

Interactive 1 shows a motorbike moving in a circle.

Interactive 1. A Motorbike Moving in a Circle.

More information for interactive 1

The interactive shows a red dot at the center and a red motorbike moving around it. The red dot at the center represents the axis of rotation, while the motorbike follows a smooth circular trajectory around it. The animation explains the concepts of period and frequency by showing how the motorbike completes one full revolution in a fixed time. It reinforces key concepts in circular motion, demonstrating the inverse relationship between period and frequency.

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The motorbike covers one complete circle in a certain amount of time. This is called the **period**, T. Period is measured in seconds (s).

The **frequency**, f, is a measure of how many complete revolutions the motorbike makes in one second. Frequency is measured in hertz (Hz).

Period and frequency are inversely related – the longer a body takes to complete one circle, the fewer circles it will complete in one second. So these quantities are related mathematically by:

$$f = \frac{1}{T}$$

$$T = \frac{1}{f}$$

Worked example 1

During the 'Globe of death' stunt, a motorcyclist completes 42 revolutions inside the sphere in one minute. What is the period of the motion?

The number of revolutions completed in one minute, corresponds to the frequency:

$$1 \text{ min} = 60 \text{ s}$$

$$\begin{aligned} f &= \frac{42}{60} \\ &= 0.7 \text{ Hz} \end{aligned}$$

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{0.7} \\ &= 1.43 \text{ s} \\ &= 1.4 \text{ s } (2 \text{ s.f.}) \end{aligned}$$

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Position

As the body is moving along a circular path, we need to find a way of determining its position.

There are two ways of doing this:

- Find the length of the circular path travelled, s, starting from a given point.
- Find the angle the body has covered, θ, starting from a given point.

These are shown in **Figure 1**.

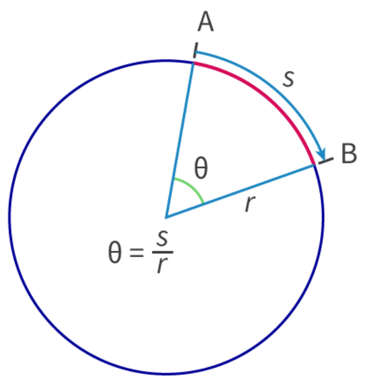

Figure 1. Determining the position of an object.

More information for figure 1

The image depicts a circle with a central angle labeled as theta (θ), the angle subtended by the arc at the center of the circle. The circle has two radii labeled as lines OA and OB, with the letter 'r' denoting the length of the radii. The arc between points A and B on the circumference of the circle is labeled 's' and represents the arc length. There is a mathematical formula shown in the image, θ = s/r, which relates the central angle θ in radians to the arc length 's' and the circle's radius 'r'. The position of the angle θ, arc length 's', and radius 'r' are shown in relation to each other within the circle, illustrating the geometric relationship for determining the position of an object based on these parameters.

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The body in **Figure 1** is moving clockwise. (The results are the same if the body is moving counter-clockwise.)

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- The body starts from A and moves along an **arc** of length s until the body reaches B. The unit of s is metres (m).

The body starts from A and covers an angle θ, with respect to the centre of the circle, until the body reaches B. The unit of angle θ is degrees (°) or radians (rad).

Study skills

The radian is defined using a circle. Consider a circle with a radius *r*. Take an arc on the circle with the same length as the radius *r*. Now you have an angle of one radian that intercepts this arc.

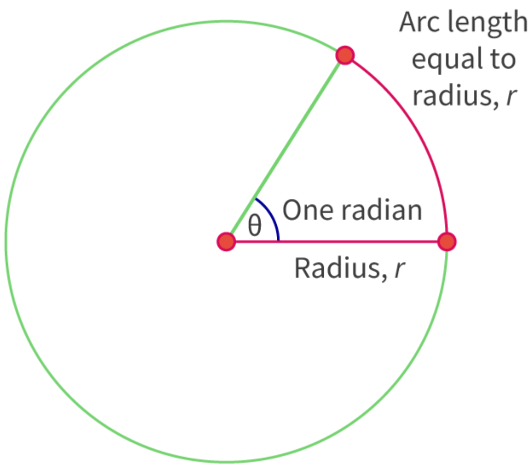

Figure 2. Defining the radian using a circle.

More information for figure 2

This diagram illustrates the concept of a radian using a circle. The circle is shown with a central angle marked as θ, which is labeled 'One radian'. Two radii, both equal in length 'r', form the angle θ at the center of the circle. An arc on the circumference of the circle is highlighted, and this arc has a length also equal to the radius 'r'. The arc and the radii form a sector of the circle, visually demonstrating that one radian is the angle subtended at the center by an arc equal in length to the radius of the circle. The diagram also includes text labels: 'Arc length equal to radius, r', 'One radian', and 'Radius, r'.

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The arc of length r corresponds to an angle θ of one radian. You can determine the size of arc, s, using the equation:

$$s = r\theta$$

The angle in radians is:

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$$\theta = \frac{s}{r}$$

You can convert degrees to radians and vice versa if you remember that a whole circle is 360 degrees or 2π radians:

$$\frac{\theta_{\text{rad}}}{\theta_{\text{deg}}} = \frac{2\pi}{360}$$

An angle of 90° corresponds to $\frac{\pi}{2}$ radians and 180° corresponds to π radians.

It is important to know how to use radians in calculations, such as those in subtopics A.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-43153/) and C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/).

Exercise 1

Click a question to answer

Speed and angular velocity

The Earth is moving around the Sun. If we assume the Earth moves in a perfect circle, then we can track this motion by saying that the Earth moves a distance s on the circle, or an angle θ with respect to the Sun (**Figure 3**).

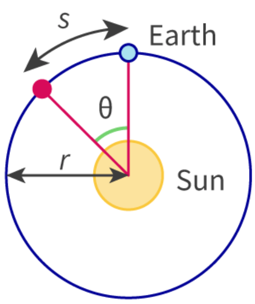

Figure 3. The Earth moving around the Sun.

More information for figure 3

Student view

The image is a diagram illustrating the circular motion of Earth around the Sun. The Sun is at the center with a yellow circle representing it. The Earth is on the circumference of a blue circle, which represents its orbit. Labels on the diagram include 's' for the arc length, 'θ' for the central angle, and 'r' for the radius from the Sun to the Earth. Arrows indicate the direction of these measurements, showing the relationship between the Earth, its orbit, and the Sun.

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There are two ways of describing the position of the Earth on the circle, so there are two ways of describing the speed of the Earth: speed and angular velocity.

Speed is the rate of change of position on the circle. In other words, speed is the length of the arc covered in a specific amount of time. **Uniform circular motion** is circular motion with a constant speed.

The equation for speed (subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)) is:

$$v = \frac{s}{\Delta t}$$

where s is the length of the arc and Δt is change in time.

When the body completes one circle ($s = 2\pi r$), the change in time is one period ($\Delta t = T$). Speed can then be determined using the equation in **Table 1**.

Table 1. Equation for speed.

Equation	Symbols	Units
$v = \frac{2\pi r}{T}$	v = speed	metres per second (m s^{-1})
	r = radius of circle	metres (m)
	T = period	seconds (s)

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Worked example 2

The Moon orbits the Earth at a distance of 3.48×10^5 km. It completes one orbit once every 28 days. Calculate the (linear) speed of the Moon.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$r = 3.48 \times 10^5 \text{ km} = 3.48 \times 10^8 \text{ m}$ $T = 28 \text{ days} = 28 \times 24 \times 60 \times 60 = 2\,419\,200 \text{ s}$
Step 2: Write out the equation.	$v = \frac{2\pi r}{T}$
Step 3: Substitute the values given.	$= \frac{(2\pi \times 3.48 \times 10^8)}{2\,419\,200}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 903.8 \text{ m s}^{-1} = 900 \text{ m s}^{-1}$ (2

The velocity, v, of a body moving in a circle is a vector quantity. The direction of the vector is always at a tangent to the circle (**Figure 4**).

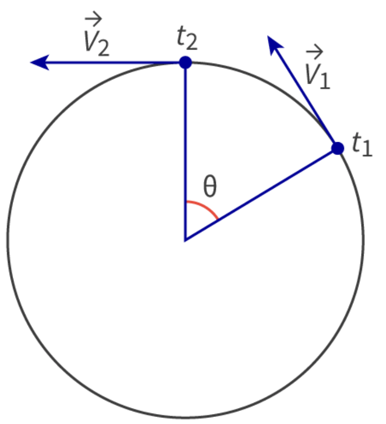

Figure 4. Velocity vectors for a body moving in a circle.

More information for figure 4

Student view

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The diagram depicts a circle representing the path of a body in circular motion. Two points on the circle, labeled t1 and t2, are marked. At point t1, a vector labeled v1 is tangent to the circle, indicating the velocity direction at that point. Similarly, at point t2, another vector v2 is tangent to the circle, showing the velocity direction at t2. An angle θ is formed between the radii connecting the center of the circle to the points t1 and t2. This angle represents the angular displacement as the body moves along the circle. The diagram emphasizes that velocity vectors are always tangent to the path of motion in circular motion.

[Generated by AI]

Angular velocity, ω, is the rate of change of the angle covered by the body, and is measured in radians per second (rad s^{-1}). In other words, angular velocity describes the angle that a body covers in a certain amount of time:

$$\omega = \frac{\theta}{\Delta t}$$

where θ is angle covered and Δt is change in time.

When the body completes one circle ($\theta = 2\pi$), the change in time is one period ($\Delta t = T$). Angular velocity can then be determined using:

$$\omega = \frac{2\pi}{T}$$

Rearranging the equation for angular velocity gives:

$$T = \frac{2\pi}{\omega}$$

Substituting for T into the equation for linear speed $\left(v = \frac{2\pi r}{T}\right)$ gives:

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$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi r}{\left(\frac{2\pi}{\omega}\right)}$$

$$= \frac{2\pi r\omega}{2\pi}$$

$$= \omega r$$

This means that another equation for speed is as shown in **Table 2**.

Table 2. Equation for speed using angular velocity.

Equation	Symbols	Units
$v = \omega r$	v = speed	metres per second (m s^{-1})
	ω = angular velocity	radians per second (rad s^{-1})
	r = radius of circle	metres (m)

For a body travelling in a circle, the angular velocity is related to the linear speed by:

$$v = \frac{2\pi r}{T}$$
$$= \omega r$$

Centripetal acceleration

As a body moves around a circle, the direction of velocity is constantly changing. This means the velocity is constantly changing.

Acceleration is the rate of change of velocity, so the body has acceleration. This acceleration is called the centripetal acceleration. Centripetal acceleration can be determined using linear speed as shown in the equation in **Table 3**.

Student view

Table 3. Equation for centripetal acceleration using speed.

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Equation	Symbols	Units
$a = \frac{v^2}{r}$	a = centripetal acceleration	metres per second per second (m s^{-2})
	v = speed	metres per second (m s^{-1})
	r = radius of circle	metres (m)

Worked example 3

A particle moving in a circle at a constant speed of 3.5 m s^{-1} experiences an acceleration of 0.53 m s^{-2}. Determine the radius of the circle.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$v = 3.5\ \text{m s}^{-1}$ $a = 0.53\ \text{m s}^{-2}$
Step 2: Write out the equation and rearrange to find *r*.	$a = \frac{v^2}{r}$ $r = \frac{v^2}{a}$
Step 3: Substitute the values given.	$= \frac{3.5^2}{0.53}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 23.1\ \text{m} = 23\ \text{m}$ (

Centripetal acceleration can also be expressed using angular velocity using the equation in **Table 4**.

Table 4. Equation for centripetal acceleration using angular velocity.

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Equation	Symbols	Units
$a = \omega^2 r$	a = centripetal acceleration	metres per second per second (m s^{-2})
	ω = angular velocity	radians per second (rad s^{-1})
	r = radius of circle	metres (m)

Finally, centripetal acceleration can be expressed using period using the equation in **Table 5.**

Table 5. Equation for centripetal acceleration using period.

Equation	Symbols	Units
$a = \frac{4\pi^2 r}{T^2}$	a = centripetal acceleration	metres per second per second (m s^{-2})
	r = radius of circle	metres (m)
	T = period	seconds (s)

Worked example 4

Calculate the centripetal acceleration of a roller coaster that completes a circular loop of diameter 18 m in 4.6 s.

Combine the equation for angular velocity with the equation for centripetal acceleration to give:

$$\omega = \frac{2\pi}{T}$$

$d = 2r$, so:

$$a = \frac{4\pi^2}{T^2} r$$

$$= \frac{4\pi^2}{4.6^2} \times 9$$

Student view

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$$= 17 \text{ m s}^{-2}$$

In uniform circular motion, the magnitude of velocity (speed) is constant, but the direction of velocity is constantly changing. In other words, every circular motion is an accelerated motion.

Centripetal acceleration is also a vector. **Figure 5** shows how change in velocity is determined using a vector triangle.

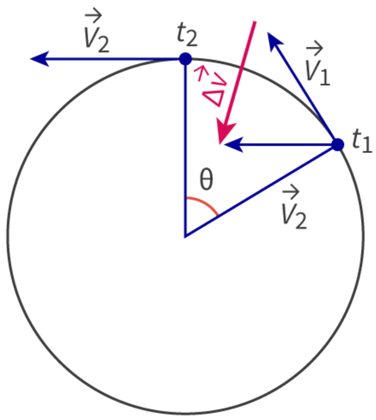

Figure 5. Determining change in velocity.

More information for figure 5

The diagram illustrates a vector triangle used to determine the change in velocity in circular motion. It features a circle with two points labeled as t1 and t2 on its circumference. These points represent two different instances in time for the moving object. Velocity vectors v1 and v2 are positioned to indicate the object's direction and speed at these points. Vector v1 is drawn from t1 and points along the tangent to the circle. Vector v2 originates from t2 and also points along the tangent. The angle between these two vectors at the circle's center is denoted by theta (θ). Additionally, a third vector, representing the change in velocity (Δv), is drawn from the end of v1 to the end of v2, forming the base of the triangle inside the circle. This triangle highlights how velocity changes direction, pointing towards the circle's center. Each vector is labeled, and the diagram includes dotted lines connecting the circle's center to the points where the vectors are drawn, emphasizing the radial direction of centripetal acceleration.

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The change in velocity vector, Δv, points towards the centre of the circle. Acceleration is the rate of change of velocity, so centripetal acceleration also points towards the centre of the circle.

Centripetal force

A car moves in a circle. Because the direction of velocity changes, there is an acceleration pointing towards the centre of the circle.

Newton's second law ($F = ma$) states that if there is an acceleration, then there must be a force. Look at **Figure 6.** What force is responsible for accelerating the car, the ball and the satellite? Click on 'Show or hide solution' to see the answers.

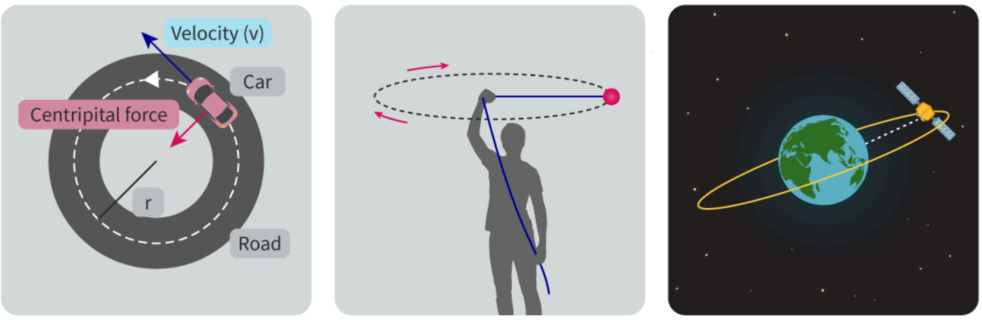

Figure 6. Bodies moving in circles.

More information for figure 6

The image consists of three illustrations depicting centripetal force in different scenarios.

1. The first illustration shows a car moving in a circular path on a road. The car is labeled and arrows indicate the direction of velocity and centripetal force. The term 'Centripetal force' is highlighted, pointing towards the center of the circle, while 'Velocity (v)' points tangentially to the circle.
2. The second illustration depicts a person holding a string attached to a ball, moving it in a circular path above their head. Arrows indicate the direction of the ball's motion and the force applied.
3. The third illustration represents a satellite orbiting Earth in space. The satellite follows a curved path around Earth, illustrating the concept of centripetal force in gravitational orbits.

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Car: The frictional force between the tyres and the road is the force responsible for keeping the car travelling in a circle.

Ball: The person exerts a force on the string. The tension force experienced by the string keeps the ball moving in a circle.

Satellite: The gravitational force of the Earth on the satellite keeps the satellite in a circular orbit.

A body moving in a circle experiences a centripetal acceleration. The force responsible for this acceleration is called the centripetal force. Centripetal force can be determined using the equation:

$$F = \frac{mv^2}{r}$$

Worked example 5

A student swings a ball attached to a string in a circle above her head. The string has a length of 52 cm and the ball has a mass of 56 g. The ball has a linear speed of 25 m s^{-1}. Determine the centripetal force.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$\text{length of string} = r = 52 \text{ cm} = 0.52 \text{ m}$ $m = 56 \text{ g} = 0.056 \text{ kg}$ $v = 25 \text{ m s}^{-1}$
Step 2: Write out the equation.	$F = \frac{mv^2}{r}$
Step 3: Substitute the values given.	$= \frac{(0.056 \times 25^2)}{0.52}$

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Solution steps	Calculations
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 67.3\text{ N} = 67\text{ N}\ (2\text{ s.f.})$ The centripetal force is provided by the tension force, which in turn is provided by the force of th student.

Study skills

Newton's second law relates **resultant** force to acceleration. So, **centripetal force** is the resultant force acting on the body directed towards the centre of the circle. It is not a new force acting on the body.

Bodies can move in vertical circles as well as horizontal circles. For example, imagine a roller coaster does a 'loop-the-loop'. **Figure 7** shows a roller coaster car in three positions, A, B and C, around the loop.

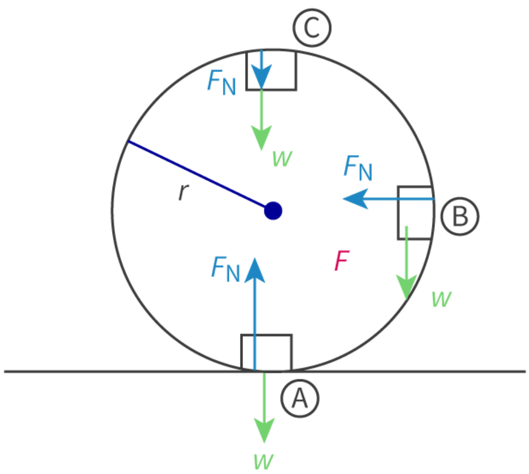

Figure 7. Forces acting on a roller coaster car.

More information for figure 7

The diagram illustrates a roller coaster car moving in a vertical loop with three distinct positions labeled A, B, and C. It shows forces acting on the car at each position: the weight (w) directed downward, and the normal force (FN) perpendicular to the track surface.

- At position A, at the bottom of the loop, the weight w points downwards, and the normal force FN points upwards.

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- At position B, on the side of the loop, the weight w points vertically downward, while the normal force FN points horizontally inward towards the center of the loop.
- At position C, at the top of the loop, the weight w points downwards, aligning with the direction of the normal force FN which also points downward.

The circle is labeled with its radius r, and the central point is denoted where all radii converge. Each section displays the relative direction of forces, demonstrating how they change throughout the circular path of the roller coaster.

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The two forces acting on the car are the weight, w, directed downwards, and the normal force, F_N, acting perpendicular to the surface.

The centripetal force is the resultant force directed towards the centre of the circle, so we can find the centripetal force for each position:

$$\text{Position A: } F = F_\text{N} - w = \frac{m{v_\text{A}}^2}{r}$$

$$\text{Position B: } F = F_\text{N} = \frac{m{v_\text{B}}^2}{r}$$

$$\text{Position C: } F = F_\text{N} + w = \frac{m{v_\text{C}}^2}{r}$$

Figure 8 shows the direction of the centripetal force at each position.

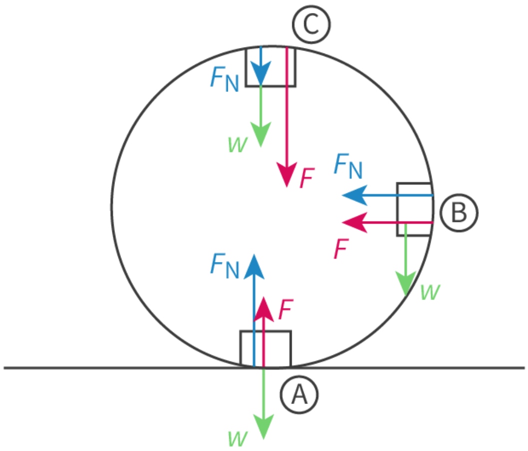

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Figure 8. Centripetal force acting on a roller coaster car.

More information for figure 8

The diagram illustrates the forces acting on a roller coaster car at three positions within a loop labeled A, B, and C. Each position shows vectors for the normal force (F_N), gravitational force (w), and centripetal force (F).

- At Position A (bottom of the loop), the normal force (F_N) and centripetal force (F) point upward, while the gravitational force (w) points downward.
- At Position B (side of the loop), the normal force (F_N) points toward the center, the gravitational force (w) points downward vertically, and the centripetal force (F) points horizontally toward the center.
- At Position C (top of the loop), the normal force (F_N) and gravitational force (w) both point downward, with the centripetal force (F) also pointing downward toward the center.

[Generated by AI]

The weight is the same magnitude in all positions, but the normal force is not constant during the motion. This is why the vector F_N has a different length in every position. The same is true for the centripetal force.

For a mass on a string rotating in a vertical circle, the varying normal force is replaced by the varying tension in the string.

Worked example 6

A mass of 6.5 kg is swung on a rigid spoke of negligable mass, in a vertical circle of radius 1.1 m at a constant speed. The time for 10 revolutions is 55 s.

Find the tension in the string when the mass is at its highest and lowest points.

Student
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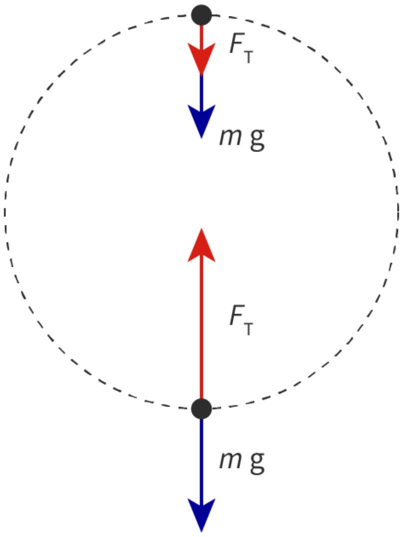
Solution steps	Calculations
Step 1: Sketch free-body diagrams of the mass at the two positions.	 Forces acting at the highest and lowest positions of a mass swur on a string in uniform vertical circular motion.
Step 2: Write equations for the centripetal force, F, acting on the mass at the two positions.	Top: $F_c = mg + F_T$ Bottom: $F_c = F_T - mg$
Step 3: Calculate the centripetal force from the centripetal acceleration.	Period, $T = 5.5\text{ s}$ Since the speed and radius are constant, the centripetal force is constant. $F = ma = m \times \frac{4\pi^2 r}{T^2}$ $= 6.5 \times \frac{4\pi^2 \times 1.1}{5.5^2}$ $= 9.33\text{ N (3 s.f.)}$

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Solution steps	Calculations
Step 4: Calculate the tension at the two positions.	Top: $F_T = F_c - mg$ $= 9.33 - (6.5 \times 9.8)$ $= -54 \text{ N (2 s.f.)}$ Bottom: $F_T = mg + F_c$ $= (6.5 \times 9.8) + 9.33$ $= 73 \text{ N (2 s.f.)}$

Worked example 7

A stunt pilot of mass 65 kg flies a plane in a vertical circle of radius 1000 m. The speed of the plane is not constant during this motion.

1. At the highest point, the speed of the plane is 100 m s^{-1}. Calculate the normal force acting on the pilot.
2. The greatest acceleration the pilot can undergo without losing consciousness is 7.0g. Determine the maximum speed at the lowest point in the circle so the pilot does not lose consciousness.

Solution steps	Calculations
Step 1: Calculate the centripetal force at the highest point.	At the top of the circle, the centripetal force is: $F = \frac{mv^2}{r}$ $= \frac{65 \times 100^2}{1000}$ $= 650 \text{ N}$
Step 2: Calculate the normal force on the pilot at the highest point.	The normal force and the weight both act downwards, and the centripetal force is the resultant, so: $F_N = F - mg$ $= 650 - 65 \times 9.8$ $= 13 \text{ N (2 s.f.)}$

Student view

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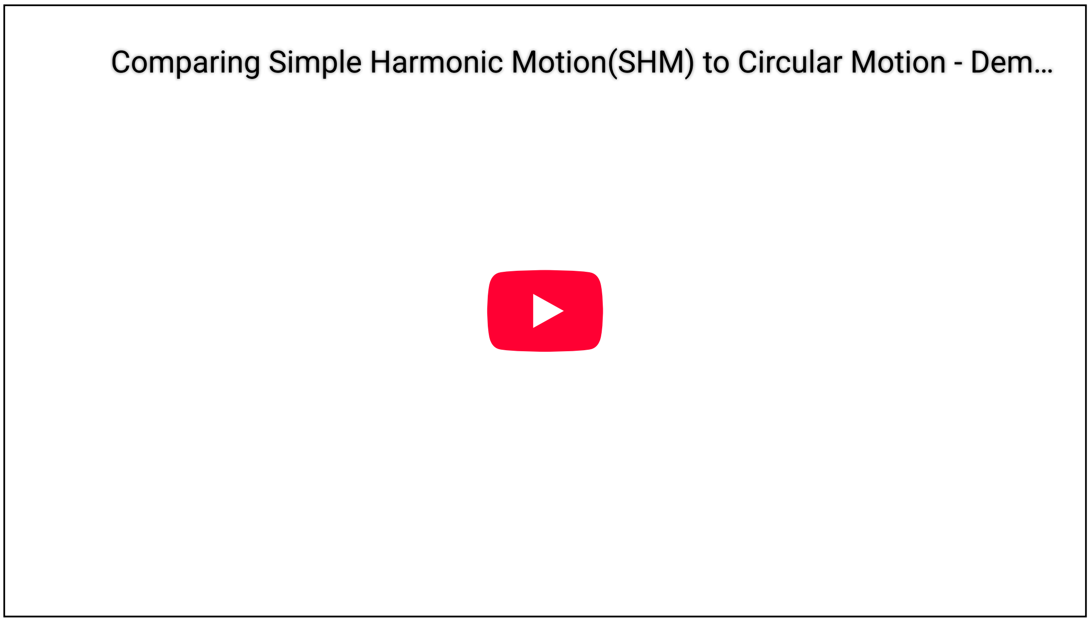
Solution steps	Calculations
Step 3: Equate the centripetal acceleration to the maximum acceleration, and solve to find *v*.	$\frac{v^2}{r} = 7.0g$ $v = \sqrt{7.0gr}$ $v = \sqrt{7.0 \times 9.8 \times 1000}$ $= 260 \text{ m s}^{-1} \text{ (2 s.f.)}$

The centripetal force:

- is the resultant force directed radially towards the centre of the circle that keeps the body moving in a circle
- acts perpendicular to the velocity
- causes the body to change direction and accelerate even though the speed is constant
- is always provided by another force, such as tension force, frictional force or normal force.

Making connections

There are many similarities between the motion of bodies moving in circular motion and the motion of bodies moving in simple harmonic motion (see subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/)). If you look at uniform circular motion from the side, the motion shows simple harmonic motion (**Video 2**).



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Video 2. Uniform circular motion and simple harmonic motion.

Work through the activity to apply your understanding of circular motion to the motion of a satellite orbiting the Earth.

Activity

- **IB learner profile attribute:** Risk-taker
- **Approaches to learning:**
  - Thinking skills — Applying key ideas and facts in new contexts
  - Research skills — Using search engines and libraries effectively
- **Time required to complete activity:** 15 minutes
- **Activity type:** Individual activity

Try and answer the following questions about the motion of a satellite orbiting the Earth with a circular orbit.

Click on 'Show or hide solution' to see the answer to each question.

1. What is the centripetal force? Can you calculate it? Look up Newton's law of gravitation to answer this question, or look at subtopic D.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44096/).

The centripetal force is provided by gravity. The gravitational force of attraction is calculated by:

$F=\frac{GMm}{r^2}$, where G is the gravitational constant, M is the mass of the Earth, m is the mass of the satellite and r is the distance between the centre of the Earth and the centre of the satellite.

1. Find an expression for the period and the speed/angular velocity of the satellite.

The period of the satellite is given by $T^2=\frac{4\pi^2r^3}{GM}$ where T is the period, r is the radius of the orbit, G is the gravitational constant and M is the mass of the Earth.

Student view

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The orbital speed is distance per unit time, so a complete orbit is a full circle and the time taken for this is the time period, so:

$$v = \frac{2\pi r}{T}$$

$\frac{2\pi}{T}$ is the angular velocity, ω, so $v = \omega r$.

1. Apply this to the International Space Station, which is in orbit around 420 km above the surface of the Earth and completes a full circle approximately every 93 minutes. How fast is it moving?

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi \times 420\,000}{93 \times 60} \\ &= 470 \text{ m s}^{-1} \end{aligned}$$

5 section questions

A. Space, time and motion / A.2 Forces and momentum

Summary and key terms

- Forces describe interaction between bodies, either in contact or at a distance.
- Free-body diagrams help us represent the forces on a body and determine the resultant force acting on the body.
- Gravitational, electric and magnetic forces are field forces which can act at a distance. These are also known as non-contact forces.
- Normal force, friction and tension are contact forces.
- Static friction acts on a stationary body, and can vary in size up to a maximum value. Dynamic friction acts on a moving body, and has a constant value.
- Newton's first law describes the equilibrium of bodies and states that unless an external force acts, a body will continue its current state of motion.

Student view

- Newton's second law connects force and acceleration. In the case of variable mass, it is better to describe resultant force as the rate of change of momentum.
- Newton's third law states that forces appear in action–reaction pairs.
- Momentum is conserved in isolated systems. Impulse describes changes in momentum and is the product of the applied force and the time for which the force is applied. Motion can occur even if the total momentum of a system is zero, for example, through explosions.
- A body performing circular motion can be described using the acceleration, radius and speed of the body.
- Circular motion is always accelerated towards the centre of the circle, perpendicular to the velocity of the object.
- The centripetal force is the resultant force along the radius, and it is required to keep a body moving in a circular orbit. It is always provided by another force, such as weight or tension force, and cannot be found in isolation.

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Key terms

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1. Forces
2. A force
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6. Elastic
7.
8. Linear
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Interactive 1. Forces and Momentum.

Figure 1 shows a concept diagram for the forces covered in this subtopic.

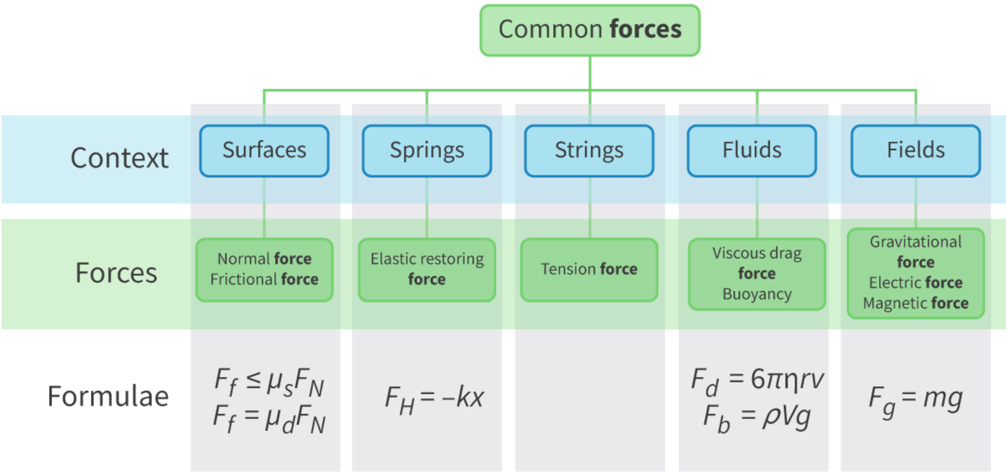

Figure 1. Concept diagram for forces.

More information for figure 1

The diagram provides a conceptual overview of different common forces as related to their contexts. It is organized in a tabular format with three rows labeled 'Context,' 'Forces,' and 'Formulae'.

1. **Context:**
2. Surfaces
3. Springs
4. Strings
5. Fluids
6. Fields

7. **Forces:**

8. For surfaces: Normal force, Frictional force
9. For springs: Elastic restoring force
10. For strings: Tension force
11. For fluids: Viscous drag force, Buoyancy
12. For fields: Gravitational force, Electric force, Magnetic force

13. **Formulae:**

Student
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14. Surfaces: (F_f \leq \mu_S F_N) and (F_f = \mu_d F_N)
15. Springs: (F_H = -kx)
16. Strings: No specific formula displayed
17. Fluids: (F_d = 6 \pi \eta r v), (F_b = \rho V g)
18. Fields: (F_g = mg)

The diagram uses colored boxes to delineate the areas for clarity, although specific colors are not essential for understanding the concept. Arrows indicate the relation of 'Common forces' to each context, enhancing the flow of information across the diagram.

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A. Space, time and motion / A.2 Forces and momentum

Checklist

What you should know

After studying this subtopic, you should be able to:

- Describe what forces are.
- Represent the forces acting on a body using free-body diagrams.
- Determine the resultant force acting on a body.
- Understand and apply Newton's third law of motion.
- Understand the concept of gravitational force, and apply the equation:

$$F_{\mathrm{g}} = mg$$

- Understand the concept of electric force.
- Understand the concept of magnetic force.
- Understand the concept of the normal force.
- Understand the concept of surface frictional force, and apply the equations for friction on a stationary body and a moving body:

$$F_{\mathrm{f}} \leqslant \mu_{\mathrm{s}} F_{\mathrm{N}} \text{ and } F_{\mathrm{f}} = \mu_{\mathrm{d}} F_{\mathrm{N}}$$

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- Understand the concept of elastic restoring force, and use the equation:

$$F_{\mathrm{H}} = -kx$$

- Understand the concept of viscous force drag, and use the equation:

$$F_{\mathrm{d}} = 6\pi\eta rv$$

- Understand the concept of buoyancy, and use the equation:

$$F_{\mathrm{b}} = \rho Vg$$

- Understand Newton's first law of motion, and use Newton's first law to determine resultant force.
- Understand Newton's second law of motion, and use the equation:

$$F = ma$$

- Understand the principle of the conservation of momentum.
- Understand linear momentum, and use the equation:

$$p = mv$$

- Understand what an impulse is, and use the equation:

$$J = F\Delta t$$

- Distinguish between Newton's second law equations:

$F = ma$ and $F = \dfrac{\Delta p}{\Delta t}$

- Understand collisions and conservation of momentum.
- Describe kinetic energy in elastic and inelastic collisions.
- Understand explosions in terms of momentum.
- Understand collisions and conservation of momentum.
- Describe kinetic energy in elastic and inelastic collisions.
- Understand explosions in terms of momentum.
- Understand angular velocity, and relate linear speed and angular velocity using the equations:

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$$v = \frac{2\pi r}{T} = \omega r$$

- Understand centripetal acceleration, and use the equations:

$$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$$

- Understand that circular motion is caused by a centripetal force.
- Describe the effect of centripetal force on the motion of a body.

Practical skills

Once you have completed this subtopic, go to Practical 2: Investigating projectile motion (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-relationship-between-velocity-id-46751/).

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/summary-and-key-terms-id-44740/print/) Assign

A. Space, time and motion / A.2 Forces and momentum

Investigation

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills – Applying key ideas and facts in new contexts
- **Time to complete activity:** 60 minutes
- **Activity type:** Individual activity

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Your task

The Earth is rotating around its axis, and you are moving with the Earth (**Figure 1**). Your linear speed depends on your position on the Earth. The radius of the Earth is 6371 km.

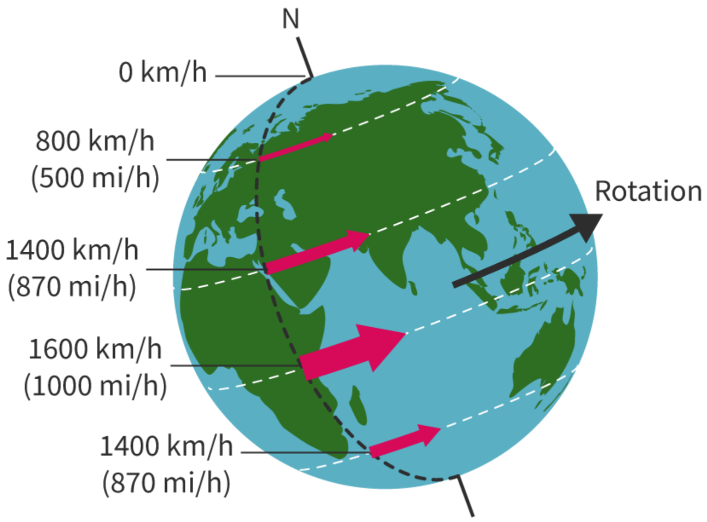

Figure 1. The Earth rotating about its axis.

More information for figure 1

This diagram illustrates the Earth with a focus on its rotation about its axis. A large arrow labeled "Rotation" shows the direction of rotation. Several latitudinal lines are marked with different linear speeds indicating how fast points on the Earth move due to rotation, depending on their latitude. At the North Pole, marked "N", the speed is 0 km/h. Near the mid-latitudes, the speed increases to 800 km/h (500 mi/h). Closer to the equator, the speed reaches 1400 km/h (870 mi/h), and at the equator, the speed is highest at 1600 km/h (1000 mi/h). This demonstrates how linear speed varies with latitude due to the Earth's rotation.

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1. Calculate the speed for your village or town. Assume that the Earth is perfectly spherical. You will need to look up the latitude for your area.

Student view

Overview
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You will need to draw a sketch and look up the latitude of your position as well as the radius of the Earth. Then use some geometry (or this website (https://rechneronline.de/earth-radius/)) and the equations of circular motion.

1. Using your value for speed, determine the centripetal acceleration and the centripetal force needed for you to stay on the Earth without flying off the Earth.
2. What force(s) provide this centripetal force? Draw a free-body diagram of yourself, showing all the relevant forces.
3. Think about forces on objects on the Earth. Are objects on the Earth at equilibrium in reality? Does this contradict the assumptions that you make when solving problems?
4. Use the same processes as above to calculate the speed of the Earth as it moves around the Sun. What is the momentum of the Earth in its orbit?
5. The force of SpaceX Starship's thrusters at full power is around 76 MN. If a force of the same magnitude could be applied to the Earth in a way that produced the greatest possible torque, what would be the value of the torque? How long would it take for this torque to reduce the Earth's momentum to zero?

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A. Space, time and motion / A.2 Forces and momentum

Reflection

Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.

Reflection

Student view

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Now that you've completed this subtopic, let's come back to the guiding questions introduced in The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/).

- How can forces acting on a system be represented both visually and algebraically?
- How can Newton's laws be modelled mathematically?
- How can knowledge of forces and momentum be used to predict the behaviour of interacting bodies?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

- What main points have you learned from this subtopic?
- Is anything unclear? What questions do you still have?
- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

Once you submit your response, you won't be able to edit it.

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Submit

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/reflection-id-47868/print/) Assign

Rate subtopic A.2 Forces and momentum

Help us improve the content and user experience.

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