2. Functions / 2.10 Solving equations

# The big picture

Many people leave school thinking that algebra is all about solving equations. Hopefully during the course of your IB studies, you have already begun to realise that there is more to the subject than that – not least in providing a mathematical way of describing patterns and rules, and in modelling the real world. You may have also started to draw connections between the rules of arithmetic and the rules of algebra. Indeed, much of algebra is about generalising arithmetic ('what you do to numbers, you can also do to letters').

However, solving equations remains an important aspect of algebra, and the more algebra that you learn, the harder the equations that you will be able to solve. As you will discover, some equations can be solved analytically, whereas others can only be solved approximately using graphical, numerical or technological means. In this section you will study analytical and graphical methods to solve exponential, polynomial and other types of equations that arise when modelling with functions.

In the following applet you can drag around either of the graphs and observe the respective equation. What do the solutions of the equation represent?

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**Interactive 1** . Intersection of Quadratic and Exponential Functions

More information for interactive 1

This interactive tool allows users to investigate graphical solutions to equations by analyzing intersections between an exponential function (in red) and a quadratic function (in blue). Both the functions are movable on the screen. For instance, the fixed equation might be shown as: $(x - 0.07)^2 - 5.45 = 1.3^{x-0.57} + 0.64$,
The solutions are represented as the points of intersection between the curves, which are dynamically labeled on the graph (e.g., $(-2.49, 1.09)$ and $(2.89, 2.48)$) and also displayed in set notation at the bottom right (e.g., Solutions: $\{-2.49, 2.89\}$ ). Users can click and drag either the quadratic (blue) or exponential (red) curve. As the shapes shift, the interactive shows how the number of solutions may change (zero, one, or two intersections) and the solutions themselves move along the x-axis
This hands-on visualization helps learners understand why equations involving polynomial and exponential terms often require graphical methods to solve. It also illustrates key concepts like how the graphs' behavior and intersection points relate to real solution sets.

## Concept

While learning how to solve various types of equations, reflect on the concept of **equivalence** in analytical and graphical methods of solving equations. What does a solution of an equation **represent** ?

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# Analytical methods

## Solving by substitution

There is a certain creativity that comes into play in solving equations analytically, but, as with any creative pursuit, practice is key. The word 'analytically' comes from the same root as 'analysis', which in mathematics means the study of the properties of objects. In this case, solving an equation analytically means finding a solution simply by exploiting known rules: addition and subtraction, associativity , commutativity, exponent rules, and so on. The method of **substitution** that you will study in this section is useful for quadratic equations 'in disguise', and it therefore draws on the analytical methods of solving quadratic equations.

### Making connections

In subtopic 2.7 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26471/)you studied the following analytical methods of solving quadratic equations:

- factorisation (null factor law)
- completing the square
- quadratic formula.

### Exam tip

The formula booklet will contain the formula for the solutions of a quadratic equation:

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0.$$

In the following two examples, you will practise solving equations analytically using exponent rules. These are really important in the application of the substitution method.

## Example 1



Solve the equation $9^{2x-1} = 3^{2x+5}$.

| Steps | Explanation |
|---|---|
| $3^{2x+5} = 9^{2x-1}$ | Use exponent rules and the relationship between exponential and logarithmic functions. |
| $3^{2x+5} = \left(3^2\right)^{2x-1}$ | Express $9$ as a power of $3$. |
| $3^{2x+5} = 3^{2(2x-1)}$ | Use exponent rules and simplify. |
| $3^{2x+5} = 3^{4x-2}$ | |
| $2x + 5 = 4x - 2$ | Equate the exponents, as the exponential function is one-to-one. |
| $-2x = -7$ | Solve for $x$. |
| $x = \frac{7}{2}$ | |

In **Example 1**, notice that while it may not look as if you have made a substitution, you actually have. By equating the exponents, you have taken the logarithm of both sides, substituting a different equation for the original.

### Important

You must remember that: $\left(\frac{1}{a}\right)^m = \left(a^{-1}\right)^m = a^{-m}$.

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### Exam tip

The formula booklet will contain the following formulae for exponential and logarithmic functions:

$$a^x = e^{x \ln a}; \quad \log_a a^x = x = a^{\log_a x} \quad \text{where } a,\ x > 0,\ \ a \neq 1$$

## Example 2

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Solve the equation $\left(\frac{1}{3}\right)^x = 27^{x-1}$.

| Steps | Explanation |
|---|---|
| $\left(\frac{1}{3}\right)^x = 27^{x-1}$ | Express $\frac{1}{3}$ and 27 as powers of 3. |
| $\left(3^{-1}\right)^x = \left(3^3\right)^{x-1}$ | Use exponent laws. |
| $3^{-x} = 3^{3x-3}$ | Simplify. |
| $-x = 3x - 3$ | Equate the exponents. |
| $-4x = -3$ | Solve for $x$. |
| $x = \frac{3}{4}$ . | |

### Be aware

For the method of substitution, make sure that you know the exponent laws really well. A technique that is often used in the method of substitution is shown below:

$$36^x = \left(6^2\right)^x = (6 \times 6)^x = 6^x \times 6^x = (6^x)^2$$

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# Example 3

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Solve the equation $25^x - 4 \times 5^x - 5 = 0$.

| Steps | Explanation |
|---|---|
| $25^x - 4 \times 5^x - 5 = 0$ . | |
| $(5^2)^x - 4 \times 5^x - 5 = 0$ | Express 25 as power of 5. |
| $(5^x)^2 - 4 \times 5^x - 5 = 0$ | Use exponent laws. |
| $y^2 - 4y - 5 = 0$ | Substitute $5^x = y$. |
| $(y-5)(y+1) = 0$ | Factorise quadratic. |
| $y - 5 = 0$ or $y + 1 = 0$ | Use null factor law. |
| $y = 5$ , $y = -1$ (rejected as $y > 0$) | Solve for $y$. |
| $5^x = 5$ | Substitute $y$ back to $5^x = y$. |
| $5^x = 5^1$ | |
| $x = 1$ | Equate exponents. |

In **Example 3**, note that that the substitution $y = 5^x$ led to a quadratic equation in $y$ whose solutions were 5 and $-1$. The negative solution was rejected, as there is no solution $5^x = -1$ for any $x \in \mathbb{R}$.

There are a few points to consider when solving an equation using the method of substitution.

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✓ **Important**

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- Choose a suitable substitution to transform the equation into a quadratic equation.
- Solve the quadratic equation using any of the methods discussed in subtopic 2.7 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26471/).
- Substitute back and solve for the initial variable.
- Check that none of the solutions to the transformed problem cause an issue with the original equation.

## Example 4

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Solve the equation $e^{2x} - 3e^{x} + 2 = 0$.

| Steps | Explanation |
|---|---|
| $\mathrm{e}^{2x} - 3\mathrm{e}^{x} + 2 = 0$ | |
| $(\mathrm{e}^{x})^2 - 3\mathrm{e}^{x} + 2 = 0$ | Express $\mathrm{e}^{2x}$ as $(\mathrm{e}^{x})^2$ |
| $y^2 - 3y + 2 = 0$ | Substitute $y = \mathrm{e}^{x}$. |
| $(y-2)(y-1) = 0$ | Factorise the quadratic. |
| $y - 2 = 0 , y - 1 = 0$ | Use null factor law. |
| $y = 2 , y = 1$ | Solve for $y$. |
| $\mathrm{e}^{x} = 2 , \mathrm{e}^{x} = 1$ | Substitute back to $y = \mathrm{e}^{x}$. |
| $x = \ln(2) , x = 0$ | Solve for $x$. |

## Example 5

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Solve the equation $x^4 - 2 = x^2$.

| Steps | Explanation |
|---|---|
| $x^4 - x^2 - 2 = 0$ | Rearrange the equation so the RHS equal to $0$. |
| $\left(x^2\right)^2 - x^2 - 2 = 0$ | See the quadratic in disguise by expressir $x^4$ as $\left(x^2\right)^2$. |
| $y^2 - y - 2 = 0$ | Substitute $y = x^2$. |
| $(y-2)(y+1) = 0$ | Factorise. |
| $y - 2 = 0$ or $y + 1 = 0$ | Use null factor law. |
| $y = 2$ or $y = -1$ | |
| Now you must recall that you need to solve for $x$, not just $y$. | |
| $x^2 = 2$ and $x^2 = -1$ (rejected as $x^2 > 0$) | Substitute back $x^2 = y$. |
| $x = \pm\sqrt{2}$ | |
| Thus, the solutions are $x = \sqrt{2}$ or $x = -\sqrt{2}$. | |

## International Mindedness

For hundreds of years, $x$ has been the go-to symbol for the unknown quantity in mathematical equations. In algebra, we are often asked to solve for $x$, and, in the English language, the letter $x$ is often used to signify the unknown: $X$ marks the spot, $X$-ray, $X$-files. But how did this particular letter become associated with so much mystery, and who started this practice?

In the following video, the director of *The Radius Foundation* , Terry Moore, provides an explanation about the history of the letter $x$ in mathematics.

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Why is 'x' the unknown? | Terry Moore

Reflect on the video and find out whether there is direct evidence for the theory that Terry Moore provides.

Who was the first mathematician that used the notation of $x, y$ and $z$ to represent unknown quantities?

## Theory of Knowledge

The quest for finding exact solutions to equations is balanced by the real-life contexts to which those equations are applied. For example, consider the forces calculated by a structural engineer, with a view to building a large suspension bridge across water. The stakes are high, particularly when it is expected that a high volume of traffic will use the bridge on a daily basis. Any significant error could have catastrophic results.

The key phrase in the last sentence is *significant error* . The equations used by the structural engineer may be so complicated that they can only be solved numerically, using technology, and accurate to a given number of significant figures. It is also highly likely that the mathematical model behind those equations contains assumptions that neglect some of the factors behind the forces on the bridge, perhaps because they are seen to be negligible. However, it is almost certainly the case that a particular degree of error will be acceptable, and will not compromise the safety of the structure. It is therefore imperative that the structural engineer knows with great confidence the degree of accuracy needed for this particular construction.

Student view

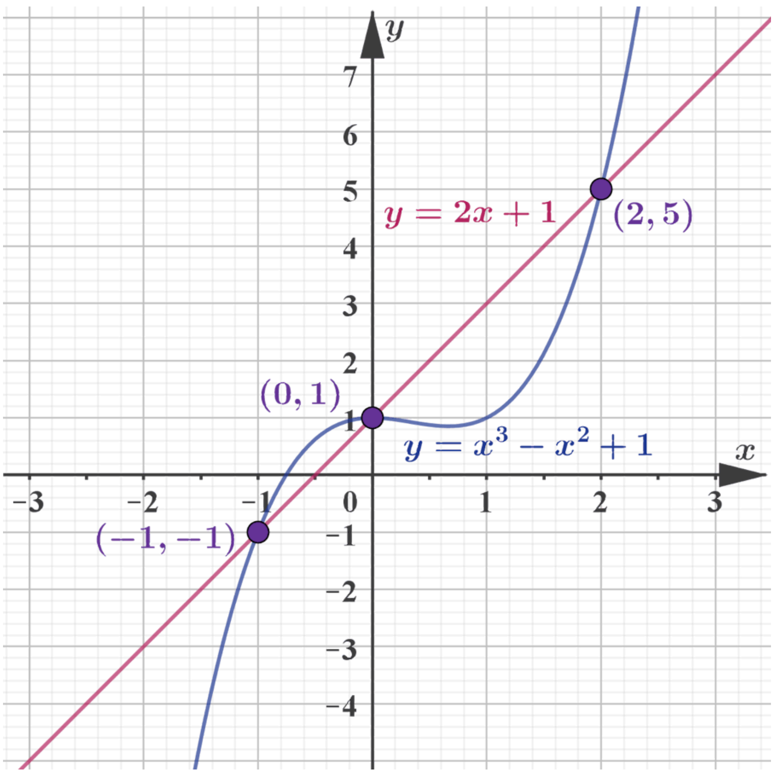
## 3 section questions

# Graphical methods

## Solving equations using technology

Solving an equation means to find all the values that make the statement true. To solve an equation **graphically**, you draw the graph for each side of the equation and see where the curves cross. The $x$-values of these intersection points are the solutions to the equation.

For example, consider the equation $x^3 - x^2 + 1 = 2x + 1$, where the left-hand side (LHS) is a function $f(x) = x^3 - x^2 + 1$ and the right-hand side (RHS) is a function $g(x) = 2x + 1$. Use your GDC to graph both functions and find the $x$-values of the points of intersection. The graphs of the functions are shown below.




More information

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This graph displays two functions: (y = x^3 - x^2 + 1) and (y = 2x + 1). The x-axis ranges from -3 to 4, and the y-axis ranges from -2 to 6. The curve for (y = x^3 - x^2 + 1) is shown, presenting a cubic behavior with inflection and curves at multiple points. The line for (y = 2x + 1) is a straight line with an upward slope. The two functions intersect at three points: (-1, -1), (0, 1), and (2, 5). These points of intersection are marked by dots where the two functions have equal values. The trend of the cubic function shows a decline from left reaching a minimum inflection before increasing again, whereas the linear function increases continuously with a steep positive slope.

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Therefore, the solutions to the equation $x^3 - x^2 + 1 = 2x + 1$ are$x = -1,\ 0,\ 2$.

Alternatively, for the equation $x^3 - x^2 + 1 = 2x + 1$, you can make the RHS of the equation equal to zero and then consider the LHS of the equation as a function for which you can find the $x$-intercepts. That is:

| Steps | Explanation |
|---|---|
| $x^3 - x^2 + 1 = 2x + 1$ | |
| $x^3 - x^2 + 1 - (2x + 1) = 0$ | Make the RHS equal to zero. |
| $x^3 - x^2 - 2x = 0$ | Simplify. |
| | Now consider the LHS as a function and use the GDC to find the $x$-intercepts. |

The graph of the function $y = x^3 - x^2 - 2x$ is shown below.

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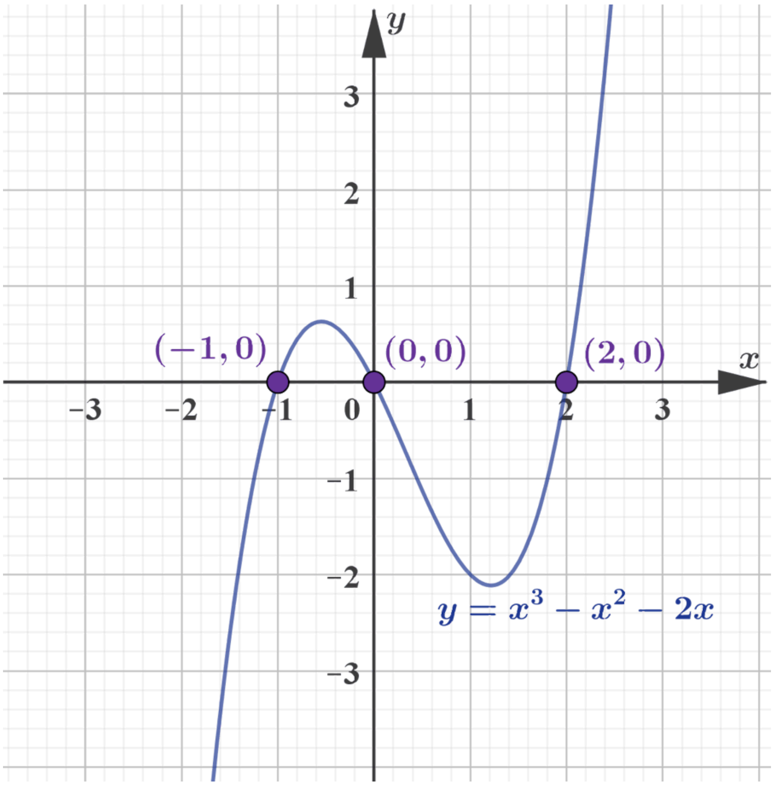


More information

The graph represents the function y = x^3 - x^2 - 2x. It plots the function along the Cartesian plane, showing the relationship between x and y values.

X-axis: Unlabeled, measures the values of x, ranging from -3 to 3. Y-axis: Unlabeled, measures the values of y, ranging from -3 to 3.

The curve passes through significant points (x-intercepts) at (-1, 0), (0, 0), and (2, 0), indicating these are the roots of the equation. The function decreases from negative infinity, rises after x = -1, dips slightly, and then increases sharply after x = 2.

Overall, the graph shows an inflection at x = 0, with curvature changing from concave up to concave down as it moves from left to right. The plot illustrates the polynomial's behavior across the given range, with smooth transitions and notable inflection points.

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The solutions to the equation $x^3 - x^2 - 2x = 0$ are the $x$-values of the $x$-intercepts, and thus the solutions are $x = -1,\ 0,\ 2$.

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## Example 1

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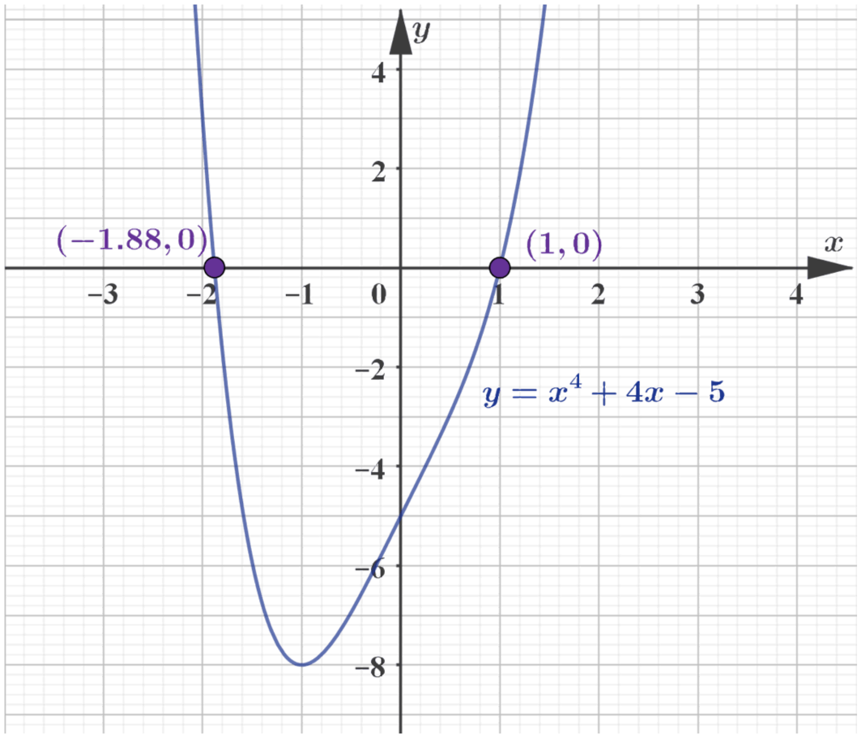
Solve the equation $x^4 + 4x - 5 = 0$.

Use the GDC to obtain the graph of $y = x^4 + 4x - 5$ and find the $x$-values of the $x$-intercepts of the graph. The graph with its $x$-intercepts is shown below.




The solutions to the equation $x^4 + 4x - 5 = 0$ are $x = -1.88$ (to 3 significant figures) and $x = 1$.

## Example 2

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Solve the equation $x^3 - 2x + 1 = e^{2x} - 1$

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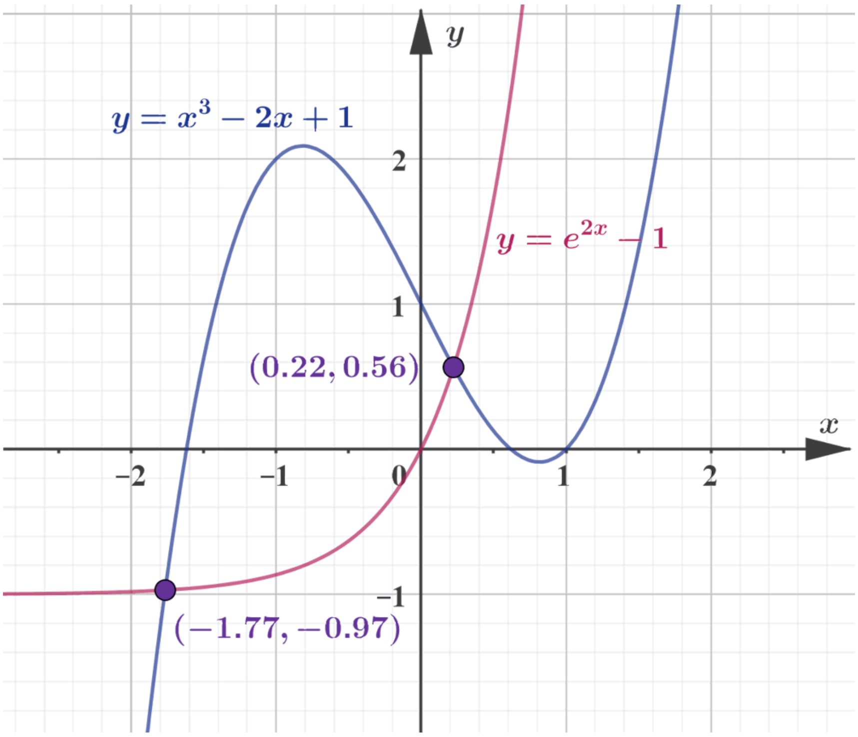
view

Use your GDC to graph the functions $y = x^3 - 2x + 1$ and $y = \mathrm{e}^{2x} - 1$ and find the $x$-values of the points of intersection of the graphs. The graphs of the functions are shown below.




The $x$-values of the points of intersection of the graphs are $x = -1.77$ and $x = 0.22$ (to 2 decimal places). These are the solutions to the equation $x^3 - 2x + 1 = \mathrm{e}^{2x} - 1$
.

### Be aware

The Equation Solver on your GDC is another great tool for solving one-variable equations. Keep in mind that the Solver can only produce real-number solutions.

## Example 3

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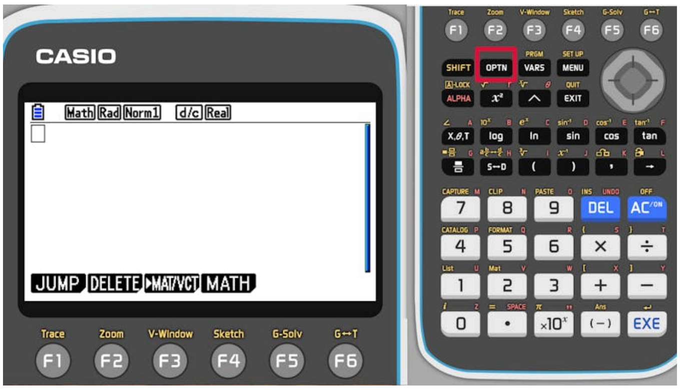
Solve the equation $-2\sin x - \mathrm{e}^x + x^2 = 0$.

Use your GDC (Solver utility) to solve the equation. The solution is $x = -0.318$ (correct to 3 significant figures).

| Steps | Explanation |
|---|---|
| These instructions show you a way to find the solution of<br>$$-2\sin x - \mathrm{e}^x + x^2 = 0$$<br>using the equation solver capability of the calculator.<br>Open the calculator application. |  |
| Press OPTN to access several options ... |  |

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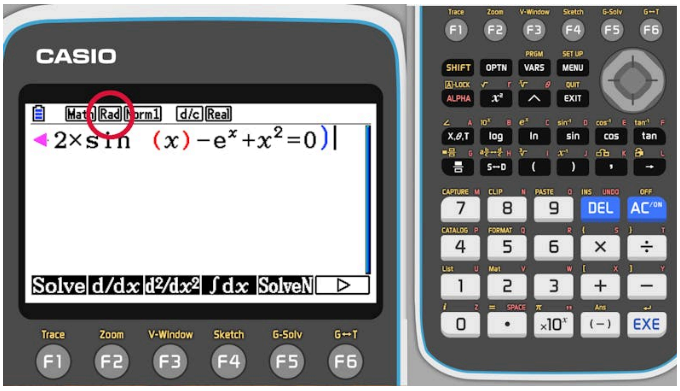
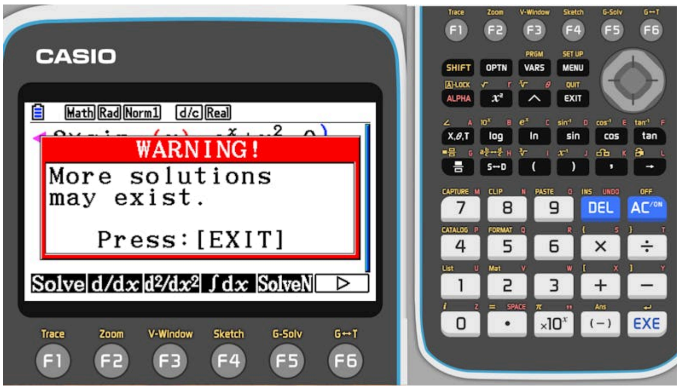
| Steps | Explanation |
|---|---|
| ... press F4 for the calculus related options ... |  |
| ... and F5 for the numerical equation solver. |  |

| Steps | Explanation |
|---|---|
| Enter the equation and press EXE.<br><br>Since a trigonometric function is involved, make sure your calculator is in radian mode. |  |
| You will see a warning message, that the calculator may not give you all solutions. This solver can give a maximum of ten solutions. Best to only use it when you have some information about the number of solutions. |  |

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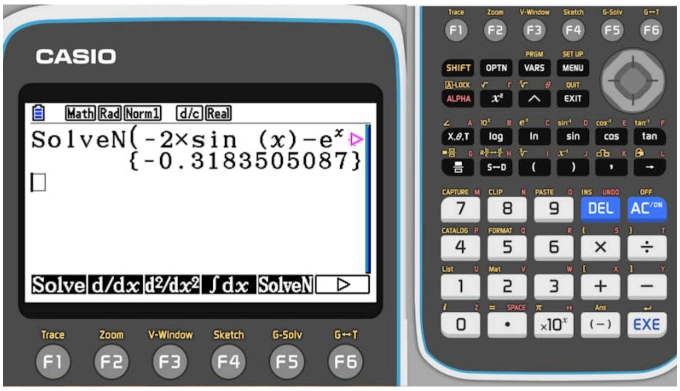
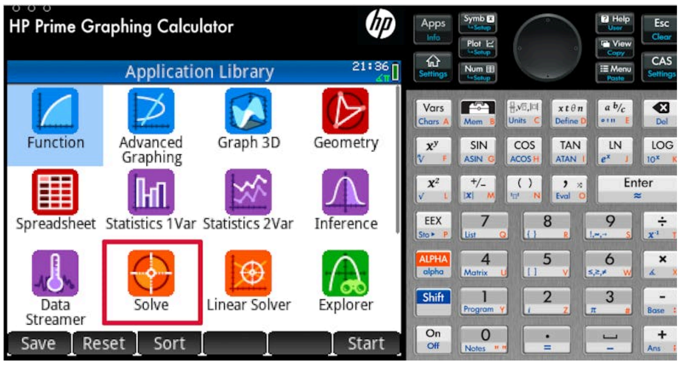
| Steps | Explanation |
|---|---|
| This equation has a unique solution, which the calculator displays. |  |



| Steps | Explanation |
|---|---|
| These instructions show you a way to find the solution of $$-2\sin x - e^x + x^2 = 0$$ using the equation solver capability of the calculator. Open the equation solver application. |  |



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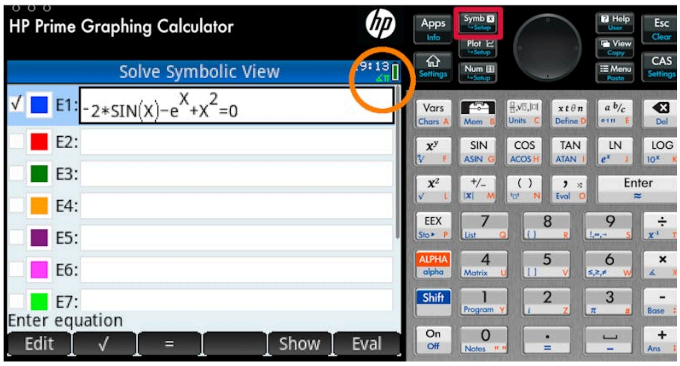
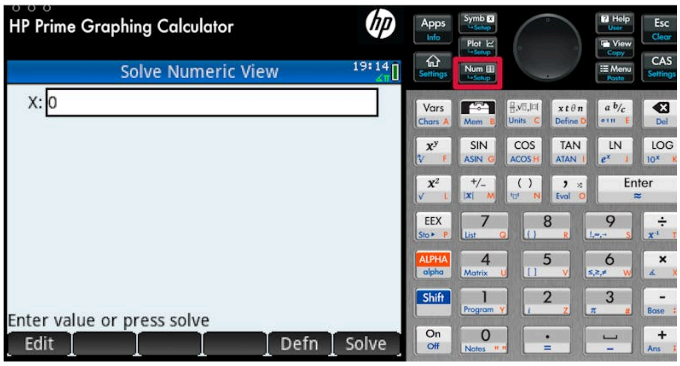
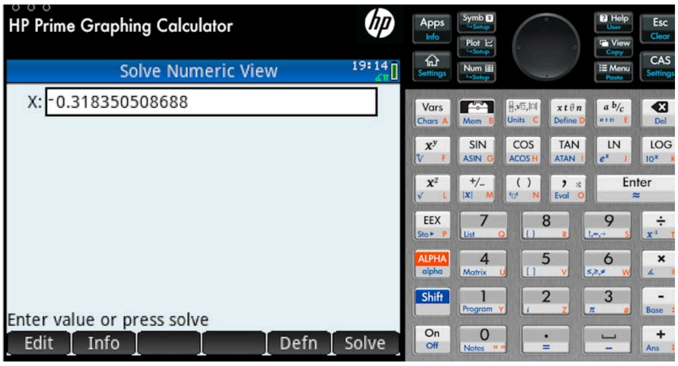
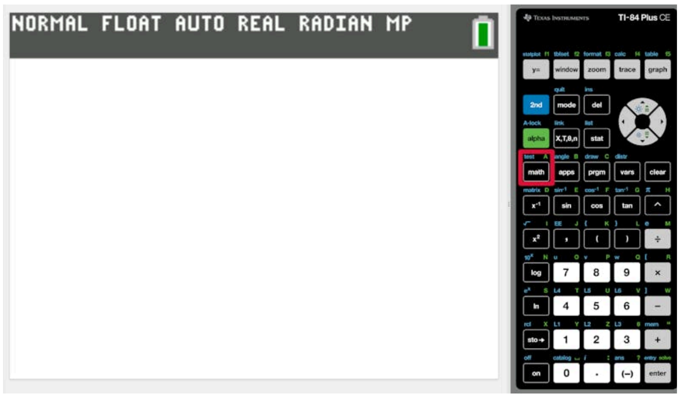
| Steps | Explanation |
|---|---|
| In symbolic view, enter the equation.<br><br>Since a trigonometric function is involved, make sure your calculator is in radian mode. |  |





| Steps | Explanation |
|---|---|
| In numeric view give a starting value for the algorithm. The calculator will find the solutions one at a time. The starting value determines which solution it finds. It is best to use this application when you have some approximate knowledge of the solution, or when you know that there is only one solution. The calculator will only give one solution without a warning that there might be other possibilities. |  |

| Steps | Explanation |
|---|---|
| This equation has a unique solution, which the calculator displays (replacing your starting value). |   |

| Steps | Explanation |
|---|---|
| These instructions show you a way to find the solution of $$-2\sin x - e^{x} + x^{2} = 0$$ using the equation solver capability of the calculator.<br><br>Press math to find the equation solver application. |   |

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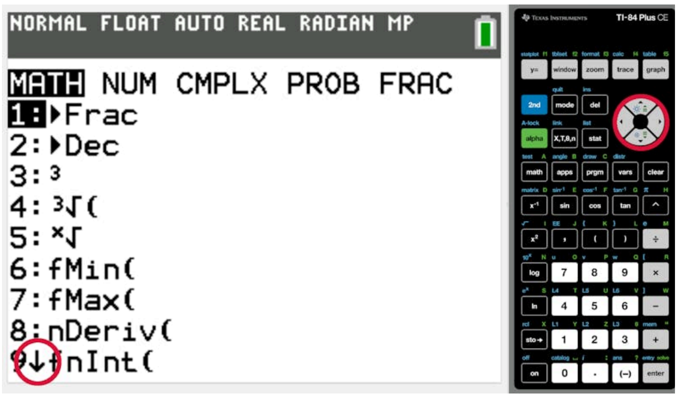
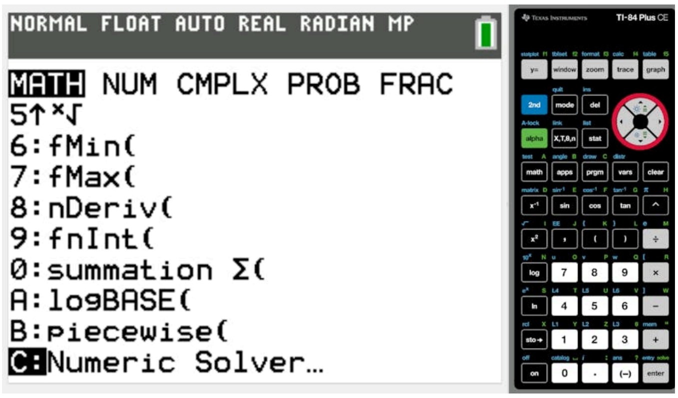
| Steps | Explanation |
| --- | --- |
| Scroll down ... |  |
| ... and select the numerical equation solver. |  |

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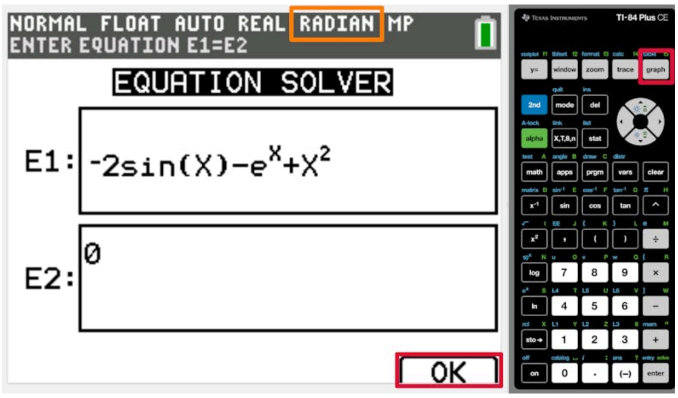
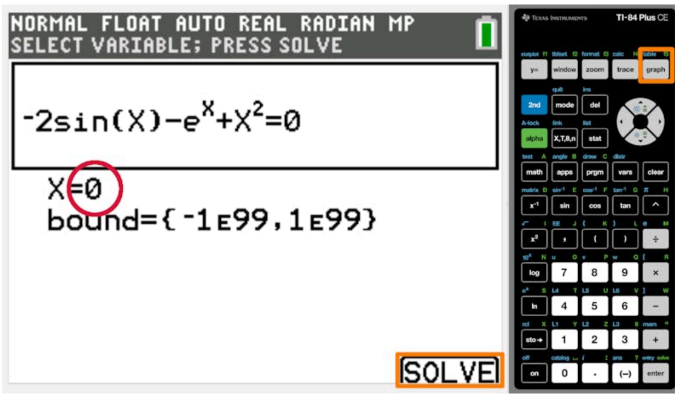
| Steps | Explanation |
|---|---|
| Enter the two sides of your equation and press graph to confirm.<br><br>Since a trigonometric function is involved, make sure your calculator is in radian mode. |  |
| Give a starting value for the algorithm. The calculator will find the solutions one at a time. The starting value determines which solution it finds. It is best to use this application when you have some approximate knowledge of the solution, or when you know that there is only one solution. The calculator will only give one solution without a warning that there might be other possibilities.<br><br>Once done, press graph to solve the equation. |  |

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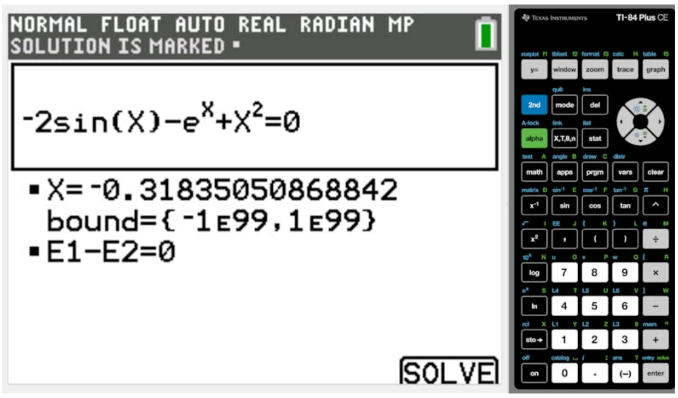
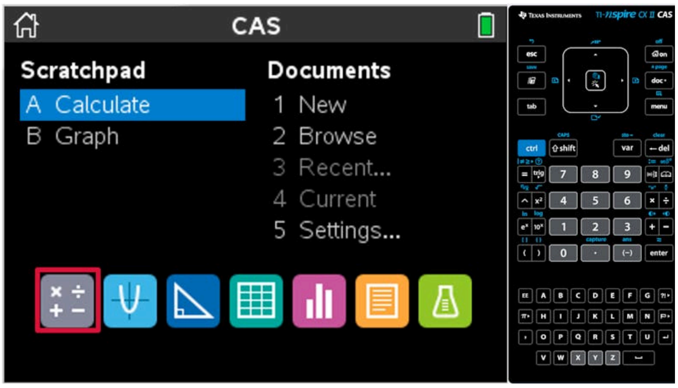
| Steps | Explanation |
|---|---|
| This equation has a unique solution, which the calculator displays (replacing your starting value). |   |

| Steps | Explanation |
|---|---|
| These instructions show you a way to find the solution of $$-2\sin x - \mathrm{e}^x + x^2 = 0$$ using the equation solver capability of the calculator.<br><br>Open a calculator page. |   |

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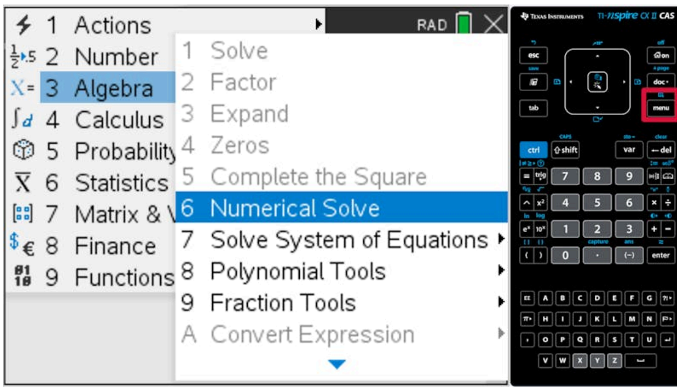
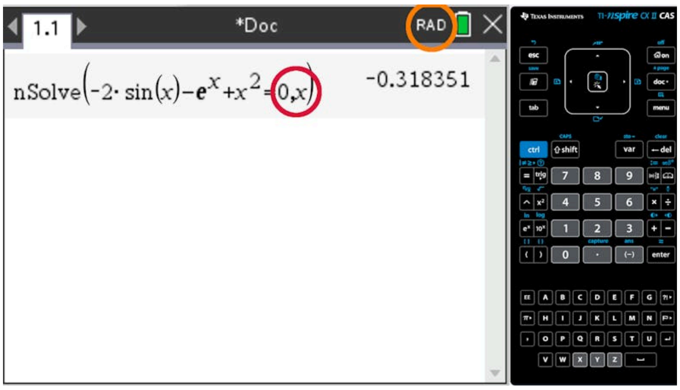
| Steps | Explanation |
|---|---|
| Open the menu and find the numerical equation solver. |  |
| Type in the equation. You also have to tell the calculator the variable name you used.<br><br>Since a trigonometric function is involved, make sure your calculator is in radian mode.<br><br>This equation has a unique solution, which the calculator displays.<br><br>It is best to use this application when you know that there is only one solution. The calculator will only give one solution without a warning that there might be other possibilities. |  |

## Activity

Solve the equation $2\cos^2 x - 4\cos x + 1 = 0$ both analytically and graphically.

Do both methods result the same solution?

Which method do you think is more precise?

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# Applications of equations in real life

You have already seen that when modelling phenomena with functions, it is natural to end up with an equation that needs to be solved.

### Making connections

In subtopic 2.9 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26559/)you studied **compound interest** and used the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to model the balance $A$ in an account after $t$ years with principal $P$ and annual interest $r\%$, where $n$ is the number of times that the interest is compounded per year.

## Example 4

★★☆

Sofia deposits $\$900$ into an account that pays $4.55\%$ interest per annum, compounded monthly. How long will it take Sofia to triple her money?

| Steps | Explanation |
|---|---|
| $x = nt$ | Let $x = nt$ be the number of monthly instalments until the initial capital is doubled. |
| $2700 = 900\left(1 + \frac{0.0455}{12}\right)^x$ | Set an equation for the balance of the account to be $\$2700$ (triple the initial amount) and simplify. |
| $3 = \left(1 + \frac{0.0455}{12}\right)^x$ | |
| $x = 290.29\ldots$ | Use GDC to find the $x$ value of the intersection betwee the graphs $y = 3$ and $y = \left(1 + \frac{0.0455}{12}\right)^x$. |

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| Steps | Explanation |
|---|---|
| Hence, after $290$ months Sofia will have a bit less than $\$2700$, and after $291$ months, which is $24$ years and $3$ months, she will have a bit more than triple the initial investment. | |

## Making connections

In subtopic 2.9 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26559/)you used exponential functions to describe **radioactive decay:** a phenomenon that is widely used in dating techniques by scientists to determine the age of an artefact.

# Example 5

★★☆

An amount of radioactive material, $M$, in grams, is modelled according to the function $M(t) = 250e^{-kt}$, where $k > 0$ and $t$ is time measured in years. It is determined that after $20$ years, the amount of radioactive material present is $50$ grams. How long does it take for half of the material to disintegrate (i.e. the half-life of the radioactive material)?

| Steps | Explanation |
|---|---|
| $M(0) = 250\left(\frac{1}{\mathrm{e}}\right)^0 = 250$ and thus, the initial amount, regardless the value of $k$ is $250$ grams | Find the initial amount of the material present by substituting $t = 0$ into the function. |
| $50 = 250\mathrm{e}^{-20k}$<br>$\ln 50 = \ln 250 + \ln \mathrm{e}^{-20k}$<br>$k = \frac{\ln 50 - \ln 250}{-20}$ | Find $k$ by using $M(20) = 50$ at $t = 20$ . |

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| Steps | Explanation |
|---|---|
| $k = 0.08047$ | Round to 5 decimal places during computation. |
| $125 = 250\mathrm{e}^{-0.08047t}$ | Set an equation for the amount of the material to be half of its initial amount. |
| $0.5 = \mathrm{e}^{-0.08047t}$ | Simplify. |
| $x = 8.61373$ | Use your GDC to find the $x$ value of the intersection point between the two graph $y = 0.5$ and $y = \mathrm{e}^{-0.08047x}$ . |
| $t = 8.61$<br>Thus, the half-life of the material is $8.61$ years. | Round answer to 3 significant figures. |

## Example 6

★★★

A drug is developed to suppress the temperature of a patient during a fever. The model for the effect of the drug is given by the equation $T(t) = t \times 0.84^t,\ t \geq 0$, where $T$ is the temperature of the patient above $37^\circ\mathrm{C}$, and $t$ is the time in hours after the drug has been given.

( *Note that this is not an example that follows the general exponential model* ***exactly*** *as there is no constant p to be multiplied with the power but rather the independent variable* $t$ *is in its place. However, you can still solve it.* )

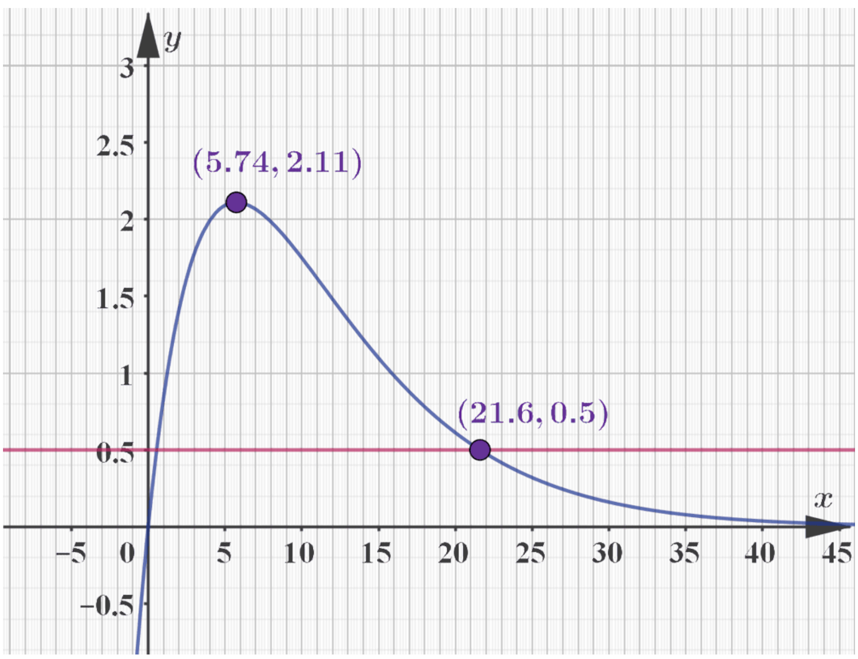
a) Determine the maximum temperature and the time when it occurs.

b) How long does it take for a patient to return to within $0.5^\circ$ of $37^\circ\mathrm{C}$?

Student view

| | Steps | Explanation |
|---|---|---|
| a) | The maximum point is $(5.74,\ 2.11)$.<br><br>Thus, the maximum temperature of the patient is $37 + 2.11 = 39.1°\text{C}$ and occurs $5.74$ hours after taking the drug. | Use GDC to find the maximum point o the graph of the function $y = x \times 0.8$<br>.<br><br>Round to 3 significant figures. |
| b) | $0.5 = x \times 0.84^x$ | The answer is the solution to the equation $0.5 = x \times 0.84^x$ , where you accept the furthest point from $x = 0$. |
| | $x = 21.5985$ | Use GDC to obtain the point of intersection for the graphs $y = 0.5$ an $y = x \times 0.84^x$ . |
| | $x = 21.6$<br><br>Thus, it takes $21.6$ hours to return to within $0.5°$ of $37°\text{C}$. | Give answer correct to 3 significant figures. |
| | | |

The solutions are shown as points in the figure below.

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Overview
(/study/ap
aa-
sl/sid-
177-
cid-
761925/o

# 4 section questions

2. Functions / 2.10 Solving equations

# Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/checklist-id-26730/print/) Assign

## What you should know

By the end of this subtopic you should be able to:

- solve equations using the method of substitution
- solve equations using a graphical method
- use technology to solve equations.

2. Functions / 2.10 Solving equations

# Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/investigation-id-26731/print/) Assign

In the following applet you can practise solving equations using a graphical method. When solving equations graphically, you sometimes need to manipulate and rearrange the equation. On either the LHS or the RHS, you must have the exact function of the graph. On the other side of the equation you should arrange all other terms, then look for the point(s) of intersection on the graph, which will be your approximate solution(s) to the equation.

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Overview
(/study/ap
aa-
sl/sid-
177-
cid-
761925/o

**Interactive 1.** Solving an Equation Using a Graphical Method.

More information for interactive 1

This interactive tool provides an intuitive way to solve equations graphically by allowing users to manipulate lines on a coordinate plane. The interface displays a fixed function graph (such as a parabola for quadratic equations) while users can draw and adjust linear functions to find $-2x^2 - 4x + 6 = 0$ intersection points. The equation being solved is shown in the top right corner of the interface. For instance, when solving an equation like using the function $f(x) = -2x^2 - 3x + 2$, users first rearrange the equation to match the given function's form, then plot the resulting linear component (in this case, $y = x + 4$). As users move the line, the tool dynamically calculates and displays intersection points with the fixed graph, providing immediate visual feedback about potential solutions.

The interactive includes two buttons: "New graph" to generate a different function and challenge. and "Show answer" to reveal the exact solutions after attempting to find them manually. The system includes helpful features like coordinate readouts and solution verification to confirm results. This hands-on approach effectively demonstrates the relationship between algebraic equations and their graphical representations while highlighting the practical considerations of graphical problem-solving - including how factors like scale and resolution affect solution accuracy.

- Click 'new graph' to generate a new equation example.
- Adjust the position of the red line to represent the equation graphically.
- Solve the equation and verify your answer by clicking the 'Show answer' button.
- Try solving a few examples by looking for the points of intersection of the graphs.

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Reflect on this method of solving equations. How accurate are graphical solutions? How could you make them more accurate?

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## Rate subtopic 2.10 Solving equations

Help us improve the content and user experience.

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