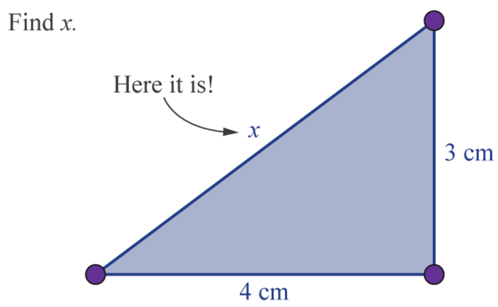
Index

The big picture

It is the quintessential question in mathematics: ‘Find x.’



More information

The image shows a right triangle labeled to solve for 'x.' The hypotenuse is labeled 'Find x' with an arrow pointing to a vertex where 'Here it is!' and 'x' are marked and this is opposite to the right angle formed by the triangle. The horizontal base of the triangle is labeled '4 cm,' and the vertical side, perpendicular to the base, is labeled '3 cm.' The vertices of the triangle have circular dots, indicating these key points. It's a classic representation of the problem of finding a missing side in a right triangle, usually solved using the Pythagorean theorem.

[Generated by AI]

In answering such questions, invariably you end up with an equation to solve.

An equation is made up of two parts:

- an equal sign $=$
- two expressions, one on the left-hand side (LHS) and one on the right-hand side (RHS) of the equal sign.

The = sign is a strong condition: it demands that the two sides be equal to each other. So solving an equation usually boils down to finding the value(s) of the variable x that makes the statement of equality true.

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Making connections

In subtopic 1.6 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26479/) you also met the equivalence sign, $\equiv$, which demands that LHS and RHS always be the same, regardless of the value(s) of any variable(s). However, in this subtopic you will be looking for one or two specific values of the variable for which the equality holds.

There are two general approaches to finding solutions to equations:

- the analytical approach, which basically involves you, a pen and a piece of paper
- the graphical method, which often makes use of technology such as your graphical display calculator (GDC).

In this subtopic, you will learn how to solve quadratic equations using both analytical and graphical methods.

Concept

While learning how to solve quadratic equations using analytical and graphical methods, think about the advantages and disadvantages of analytical and graphical **representations** of solutions. Is one kind of representation more **valid** or useful than the other in particular contexts? Reflect on what the solutions of quadratic equations might represent in models of real-life applications. What information from a quadratic equation can help you determine whether the equation has a solution at all and, if it does, how many solutions there are?

2. Functions / 2.7 Quadratic equations and quadratic inequalities

Solving quadratic equations by factorisation

Solving quadratic equations analytically

Solution of a quadratic equation

Important

A quadratic equation has the form

$$ax^2 + bx + c = 0$$

where $a, b, c \in \mathbb{R}$ with $a \neq 0$.

A solution of the quadratic equation is a value of x for which the equation is true.

Student view

For example, consider the equation

$$x^2 - 5x + 6 = 0$$

When $x = 3$:

$$\begin{aligned} x^2 - 5x + 6 &= 3^2 - 5(3) + 6 \\ &= 9 - 15 + 6 \\ &= 0 \end{aligned}$$

Also, when $x = 2$:

$$\begin{aligned} x^2 - 5x + 6 &= 2^2 - 5(2) + 6 \\ &= 4 - 10 + 6 \\ &= 0 \end{aligned}$$

Therefore, $x = 3$ and $x = 2$ are solutions of the quadratic equation $x^2 - 5x + 6 = 0$.

Be aware

A specific value of the input variable x that makes a function $f(x)$ equal to zero is called a zero, or root, of $f(x)$. In other words, that value of x is a solution of the equation $f(x) = 0$. The words 'solution', 'root' and 'zero' are often used interchangeably in the context of solving equations.

In this subtopic, you will learn the following analytical methods of solving quadratic equations:

- by factorising the quadratic
- by completing the square
- by using the quadratic formula.

This section focuses on the first of these methods.

Solving by factorising

To solve a quadratic equation $ax^2 + bx + c = 0$ by factorising the quadratic $ax^2 + bx + c$, you apply the null factor law.

The null factor law states that if the product of two factors equals 0, then at least one of the factors is 0 .

Important

Null factor law: if $a \times b = 0$, then either $a = 0$ or $b = 0$ or both a and b are 0.

When one of the coefficients b or c in $ax^2 + bx + c = 0$ is equal to zero, the quadratic equation is easier to solve, so we will look at one of these cases first.

Consider the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0,\ b \neq 0$ and $c = 0$.

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So the equation is

$$ax^2 + bx = 0$$

Factorising the LHS of the equation gives

$$x\left(ax + b\right) = 0$$

Applying the null factor law, we deduce that

$$x = 0 \quad \text{or} \quad ax + b = 0$$

Therefore, the solutions are

$$x = 0 \quad \text{or} \quad x = -\frac{b}{a}$$

Be aware

For a quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$, $b \neq 0$ and $c = 0$, the solutions are always real and distinct, and one solution is always zero.

For the quadratic equation $4x^2 = 2x$, the solutions can be found by factorisation as follows:

Steps	Explanation
$4x^2 = 2x$	Do **not** divide by x to get $4x = 2$.
$4x^2 - 2x = 0$	Rearrange the equation so that one side is 0.
$2x\left(2x - 1\right) = 0$	Factorise the quadratic expression.
$2x = 0$ or $2x - 1 = 0$	Apply the null factor law.
$x = 0$ or $x = \frac{1}{2}$	Solve for x.

Therefore, the solutions of the quadratic equation $4x^2 = 2x$ are $x = 0$ and $x = \frac{1}{2}$.

Think about why you should not divide both sides of the original equation by x.

Be aware

Student view

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In quadratic equations where there is no constant term, **do not divide through by x**. This is because you do not yet know what the value of x is — it could be zero, and in that case you will miss the solution $x = 0$ if you divide by x.

Exam tip

When solving a quadratic equation by factorisation:

1. Rearrange the equation so that one side is zero.
2. Factorise the quadratic expression into two linear factors.
3. Use the null factor law to deduce that one or both of the factors must equal zero.
4. Solve the resulting linear equations.

You have seen this procedure in action for quadratic equations with no constant term.

Now let's look at more general quadratic equations. You might want to review the method of factorisation learned in subtopic 2.6 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26462/).

Making connections

Every time you factorise an expression, ask yourself the following questions:

- Is there a common factor?
- Can you factorise by grouping?
- Does the expression contain a perfect square?
- Can you spot a quadratic expression that is the difference of two squares?

Example 1

★☆☆

Find the solutions of the quadratic equation $x^2 = -7x - 12$.

Steps	Explanation
$x^2 = -7x - 12$	Write the given equation.
$x^2 + 7x + 12 = 0$	Rearrange so that one side is 0 .
$(x + 3)(x + 4) = 0$	Factorise the quadratic expression.
$x + 3 = 0$ or $x + 4 = 0$	Use the null factor law.

Student view

Steps	Explanation
$x = -3$ or $x = -4$	Solve the resulting linear equations.
Therefore, the solutions of $x^2 = -7x - 12$ are $x = -3$ and $x = -4$.	

Example 2

★☆☆

Find the solutions of the quadratic equation $x^2 = x + 30$.

Steps	Explanation
$x^2 = x + 30$	Write the given equation.
$x^2 - x - 30 = 0$	Rearrange so that one side is 0 .
$(x - 6)(x + 5) = 0$	Factorise the quadratic expression.
$x - 6 = 0$ or $x + 5 = 0$	Use the null factor law.
$x = 6$ or $x = -5$	Solve the resulting linear equations.
Therefore, the solutions of $x^2 = x + 30$ are $x = 6$ and $x = -5$.	

Example 3

★☆☆

Find the solutions of the quadratic equation $(2x + 1)^2 = 3x^2 - x - 3$.

Steps	Explanation
$4x^2 + 4x + 1 = 3x^2 - x - 3$	Expand the LHS of the given equation.
$x^2 + 5x + 4 = 0$	Rearrange so that one side is 0.
$(x + 1)(x + 4) = 0$	Factorise the LHS.
$x + 1 = 0$ or $x + 4 = 0$	Use the null factor law.
$x = -1$ or $x = -4$	Solve the resulting linear equations.

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Steps	Explanation
Therefore, the solutions of $(2x+1)^2 = 3x^2 - x - 3$ are $x = -1$ and $x = -4$.	

Example 4

★★☆

Find the solutions of the equation $x + \frac{28}{x} = -11$.

Steps	Explanation
$x + \frac{28}{x} = -11$	Write the given equation.
$x + \frac{28}{x} + 11 = 0$	Rearrange so that one side is 0.
$x \cdot x + x \cdot \frac{28}{x} + 11x = 0$	Multiply the equation through by x to cancel out the denominator.
$x^2 + 11x + 28 = 0$	Simplify and rearrange the quadratic to standard form.
$(x+7)(x+4) = 0$	Factorise the LHS.
$x + 7 = 0$ or $x + 4 = 0$	Use the null factor law.
$x = -7$ or $x = -4$	Solve the resulting linear equations.
Therefore, the solutions of $x + \frac{28}{x} = -11$ are $x = -7$ and $x = -4$.	

Example 5

★★☆

Find the solutions of the equation $x^3 = x$.

Steps	Explanation
$x^3 = x$	Write the given equation.
$x^3 - x = 0$	Rearrange so that one side is 0.

Student view

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Steps	Explanation
$x\left(x^2-1\right)=0$	Factorise the LHS: take out the obvious factor x .
$x\left(x-1\right)\left(x+1\right)=0$	Further factorise the difference of squares.
$x=0$ or $x-1=0$ or $x+1=0$	Use the null factor law; it applies to products of more than two factors as well.
$x=0$ or $x=1$ or $x=-1$	Solve the resulting linear equations.
Therefore, the solutions of $x^3=x$ are $x=0$ and $x=1$ and $x=-1$.	

5 section questions

2. Functions / 2.7 Quadratic equations and quadratic inequalities

Solving quadratic equations by completing the square

Further analytical methods for solving quadratic equations

In the previous section (/study/app/math-aa-sl/sid-177-cid-761925/book/solving-quadratic-equations-by-factorisation-id-26472/) you started by looking at how to solve quadratic equations $ax^2+bx+c=0$ where $a\neq 0,\ b\neq 0$ and $c=0$.

Activity

Investigate how to solve a quadratic equation $ax^2+bx+c=0$ where $a\neq 0, b=0$ and $c\neq 0$. Formalise your findings by writing a general formula for the solutions.

From the activity you will have discovered that quadratic equations with just a squared term and a constant term are easy to solve. This forms the basis of the next analytical technique for solving quadratic equations.

Completing the square

In subtopic 2.6 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26462/) you learned the method of ‘completing the square’ to transform a quadratic function from general form to vertex form. In this section, the same method is used to solve quadratic equations that cannot be easily factorised.

Student view

The basic idea is to convert an equation $ax^2+bx+c=0$ to the form $a(x+p)^2=q$, which contains just a squared term and a constant term. The solutions are then easy to obtain.

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For example, consider the quadratic equation

$$x^2 + 6x + 2 = 0$$

First, rearrange the equation so that the constant term is on the RHS:

$$x^2 + 6x = -2$$

Next, add $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2$ to both sides of the equation to complete the square:

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = \left(\frac{6}{2}\right)^2 - 2$$

Rewrite the expressions on both sides of the equation:

$$x^2 + 2\left(3\right)x + 3^2 = 9 - 2$$

Now you can recognise the LHS as a perfect square:

$$\left(x + 3\right)^2 = 7$$

Take the square root of both sides of the equation (remember to include both positive and negative square roots):

$$x + 3 = \pm\sqrt{7}$$

Finally, solve for x:

$$x = -3 \pm \sqrt{7}$$

Therefore, the solutions of $x^2 + 6x + 2 = 0$ are $-3 + \sqrt{7}$ and $-3 - \sqrt{7}$.

Exam tip

When solving a quadratic equation $ax^2 + bx + c = 0$ by completing the square:

1. If $a \neq 1$, divide the equation through by a: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.
2. Rearrange so that all terms containing x are one side and the constant is on the other side of the equal sign: $x^2 + \frac{b}{a}x = -\frac{c}{a}$
3. Add $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.
4. Write the side containing x-terms as a perfect square $\left(x + \frac{b}{2a}\right)^2$.
5. Take the square root of both sides, then rearrange and solve for x.

Student view

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Example 1

★★☆

Solve the quadratic equation $x^2 + 6x - 2 = 0$ by completing the square.

Steps	Explanation
$x^2 + 6x = 2$	The coefficient of x^2 is 1. Rearrange so that all x -terms are on one side of the equation and the constant is on the other side.
$x^2 + 6x + 9 = 2 + 9$	Add $\left(\frac{b}{2a}\right)^2 = \left(\frac{6}{2}\right)^2$ to both sides of the equation.
$(x+3)^2 = 11$	Write the LHS as a perfect square.
$x + 3 = \pm\sqrt{11}$	Take the square root of both sides. Don't forget to include both positive and negative roots.
$x = -3 \pm \sqrt{11}$	Solve for x.
The solutions are $-3 + \sqrt{11}$ and $-3 - \sqrt{11}$.	

Example 2

★★★

Solve the quadratic equation $4x^2 + 8x + 10 = 0$ by completing the square.

Steps	Explanation
$4x^2 + 8x + 10 = 0$	The given equation has $a = 4$.
$x^2 + 2x + \frac{10}{4} = 0$	First divide through by 4 .
$x^2 + 2x = -\frac{10}{4}$	Rearrange so that all x -terms are on one side of the equation and the constant is on the other side.
$x^2 + 2x + \left(\frac{2}{2}\right)^2 = \left(\frac{2}{2}\right)^2 - \frac{10}{4}$ $x^2 + 2x + 1 = 1 - \frac{10}{4}$	Add $\left(\frac{2}{2}\right)^2$ to both sides of the equation.

Student view

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Steps	Explanation
$(x+1)^2 = -\frac{6}{4}$	Express the LHS as a perfect square and simplify the RHS.
The equation has no real solutions since the square $(x+1)^2$ cannot be negative.	

The quadratic formula

There are many instances in which factorising a quadratic or completing the square can be time-consuming or difficult. We now derive a general formula for the solutions of the quadratic equation $ax^2 + bx + c = 0$, by generalising the method of completing the square.

Steps	Explanation
$ax^2 + bx + c = 0$	
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide through by a . This is valid because we know $a \neq 0$.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Rearrange so that all x -terms are on the LHS and the constant is on the RHS.
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Add $\left(\frac{b}{2a}\right)^2$ to both sides.
$x^2 + 2 \cdot \frac{b}{2a} \cdot x + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	Express the LHS as a perfect square using the formula $(A+B)^2 = A^2 + 2AB + B^2$.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Put the RHS over a common denominator.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Take the square root of both sides.
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	Solve for x .
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Put the expression over a common denominator.
The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.	

Student view

✓ **Important**

Overview
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The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by the quadratic formula :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Activity

In this activity you will derive the quadratic formula in a different way. Copy and complete the proof by filling in the missing steps or explanations.

Steps	Explanations
$ax^2 + bx + c = 0$	
	Multiply the equation through by a.
	Add and subtract $\left(\frac{b}{2}\right)^2$ on the LHS.
$\left(ax + \frac{b}{2}\right)^2 - \left(\frac{b^2 - 4ac}{4}\right) = 0$	
$\left(ax + \frac{b}{2}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2}\right)^2$	
	Apply the identity $A^2 - B^2 = (A - B)(A + B)$.
Either $ax + \frac{b}{2} - \frac{\sqrt{b^2 - 4ac}}{2} = 0$ or $ax + \frac{b}{2} + \frac{\sqrt{b^2 - 4ac}}{2} = 0$	
	Solve for x to find the solutions of the quadratic equation.

Example 3

★★☆

Use the quadratic formula to solve the equation $2x^2 - 4x + 1 = 0$.

Steps	Explanation
$2x^2 - 4x + 1 = 0$	$a = 2,\ b = -4,\ c = 1$.

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Steps	Explanation
$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$	Apply the quadratic formula.
$x = \dfrac{4 \pm \sqrt{16-8}}{4}$ $x = \dfrac{4 \pm \sqrt{8}}{4}$ $x = \dfrac{4 \pm 2\sqrt{2}}{4}$ $x = 1 \pm \dfrac{\sqrt{2}}{2}$	Simplify.

Example 4

★★☆

Use the quadratic formula to solve the equation $\dfrac{3x-2}{x-1} = 3x - 1$.

Steps	Explanation
$\dfrac{3x-2}{x-1} = 3x - 1$	
$3x - 2 = (3x-1)(x-1)$ $3x - 2 = 3x^2 - 3x - x + 1$ $3x^2 - 7x + 3 = 0$	Rearrange the equation to the standard form $ax^2 + bx + c = 0$.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(3)}}{2(3)}$ $x = \dfrac{7 \pm \sqrt{49-36}}{6}$ $x = \dfrac{7 \pm \sqrt{13}}{6}$	Apply the quadratic formula with $a = 3,\ b = -7,\ c = 3$

Example 5

★★★

Use the quadratic formula to solve the equation $(p-1)x^2 + p = 2px + 2x$, where $p \neq 1$. Express x in terms of the real parameter p.

Student view

Steps	Explanation
$(p-1)x^2+p=2px+2x$	
$(p-1)x^2+p=2(p+1)x$ $(p-1)x^2-2(p+1)x+p=0$	Rearrange the equation to the standard form $ax^2+bx+c=0$.
Here $a=(p-1)\,,\ b=-2(p+1)\,,\ c=p$	Identify the coefficients.
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{2(p+1)\pm\sqrt{(-2(p+1))^2-4(p-1)p}}{2(p-1)}$ $=\frac{2(p+1)\pm\sqrt{4(p+1)^2-4(p^2-p)}}{2(p-1)}$ $=\frac{2(p+1)\pm\sqrt{4p^2+8p+4-4p^2+4p}}{2(p-1)}$ $=\frac{2(p+1)\pm\sqrt{12p+4}}{2(p-1)}$ $=\frac{2(p+1)\pm\sqrt{4(3p+1)}}{2(p-1)}$ $=\frac{2(p+1)\pm2\sqrt{3p+1}}{2(p-1)}$ The solutions are $x=\frac{p+1}{p-1}+\frac{\sqrt{3p+1}}{p-1}$ and $x=\frac{p+1}{p-1}-\frac{\sqrt{3p+1}}{p-1}$	Apply the quadratic formula and simplify.

4 section questions

2. Functions / 2.7 Quadratic equations and quadratic inequalities

Solving quadratic equations by the graphical method

Solving quadratic equations graphically using a GDC

What do the solutions of a quadratic equation $ax^2+bx+c=0$ represent graphically? Why do you think the x-intercepts of a graph such as a parabola are also often called ‘zeros’ or ‘roots’?

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The most straightforward method for solving a quadratic equation graphically is to use your GDC. There are various ways of doing this depending on the form of the quadratic equation.

Example 1

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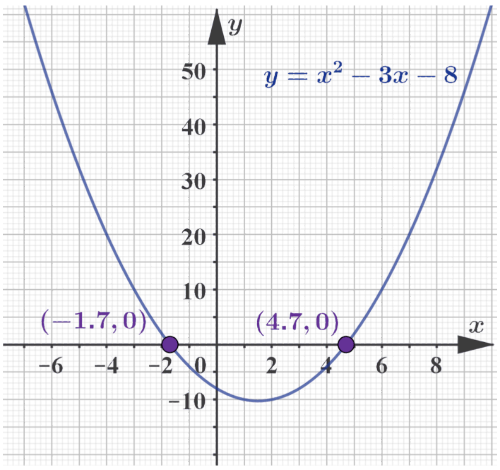
Use a GDC to solve the equation $x^2 - 3x - 8 = 0$.

Steps	Explanation
 Graph of $y = x^2 - 3x - 8$	The LHS of the equation is a quadratic function. Use your GDC to graph the parabola.
The solutions of $x^2 - 3x - 8 = 0$ are $x = -1.70$ and $x = 4.70$.	Determine the x -intercepts.

Example 2

★☆☆

Use a GDC to solve the equation $x^2 - 4x = 2x - 3$.

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view

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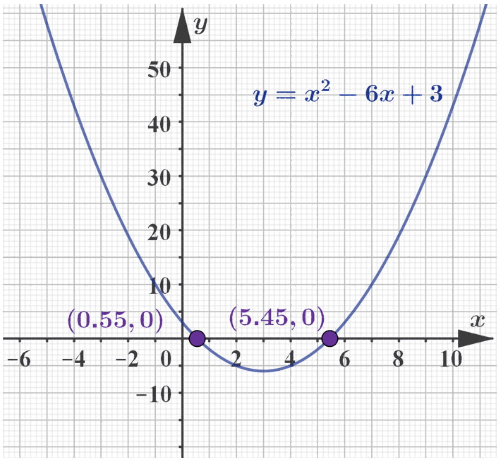
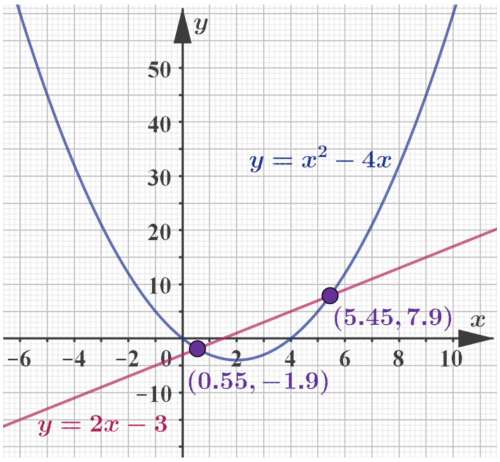
Steps	Explanation
$x^2 - 6x + 3 = 0$  Graph of $y = x^2 - 6x + 3$	**Method 1:** rearrange the equation to the standard form $ax^2 + bx + c = 0$ and then apply the method used in **Example 1.**
The solutions of $x^2 - 6x + 3 = 0$ are $x = 0.551$ and $x = 5.45.$	Determine the x -intercepts of $y = x^2 - 6x + 3$.
 The parabola $y = x^2 - 4x$ and the straight line $y = 2x - 3$	**Method 2:** consider the LHS of the equation as a quadratic function $f(x) = x^2 - 4x$ and the RHS as a linear function $g(x) = 2x - 3$. Use your GDC to graph both functions, $y = f(x)$ and $y = g(x)$, on the same set of axes.
The solutions of $x^2 - 6x + 3 = 0$ are $x = 0.551$ and $x = 5.45.$	Find the x -coordinates of the points of intersection between the parabola and the straight line. These correspond to the solutions of $x^2 - 4x = 2x - 3$.

Refer to the calculator instructions in section 2.4.3 (/study/app/math-aa-sl/sid-177-cid-761925/book/intersection-points-id-25871/) for instruction on how to find the intersection points of two graphs.

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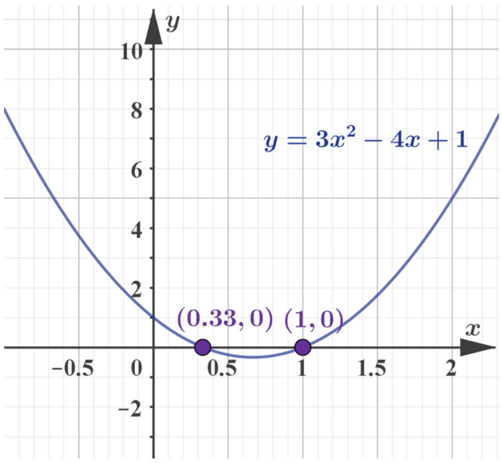
Example 3

★☆☆

Use a GDC to solve the quadratic equation $3x^2 - 4x + 1 = 0$.

Steps	Explanation
 Graph of $y = 3x^2 - 4x + 1$	**Method 1:** use your GDC to graph the parabola $y = 3x^2 - 4x + 1$.
The solutions of $3x^2 - 4x + 1 = 0$ are $x = 1$ and $x = \frac{1}{3} = 0.333\ldots$.	Determine the x -intercepts.
	Method 2: Your calculator has application that can find the solutions of a quadratic equation without graphing it. See the guide below on how to access this option.

Your GDC also has an option to solve quadratic equation without drawing the graph. The instructions below show you how to do this on different models.

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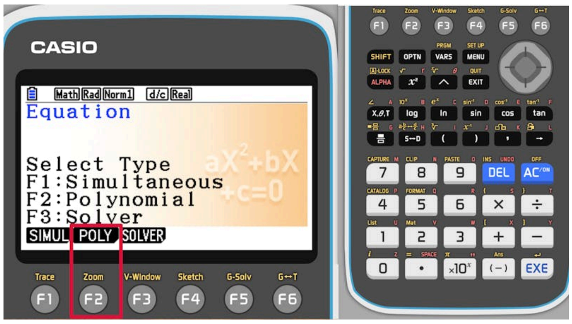
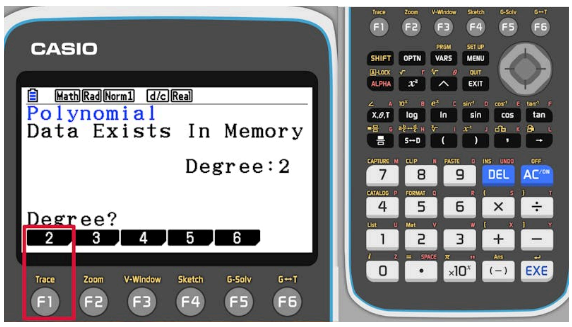
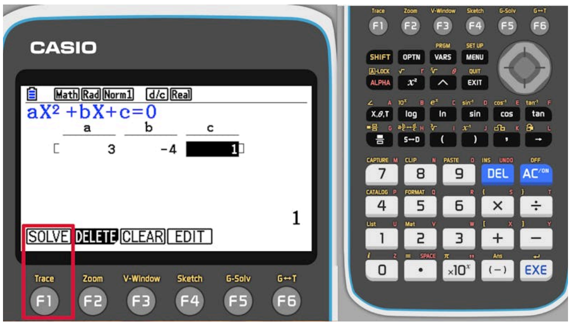
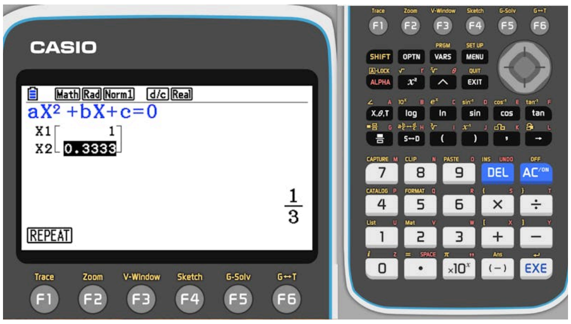
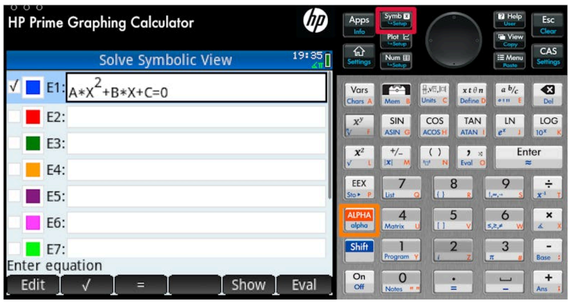
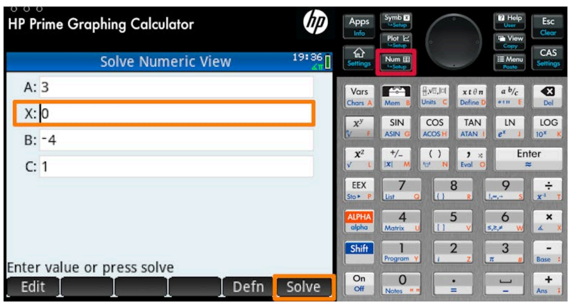
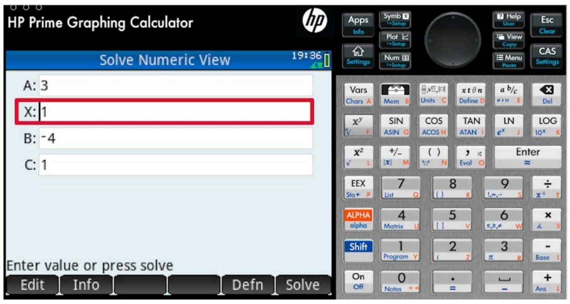
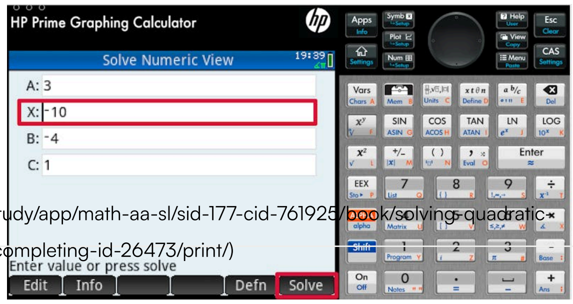
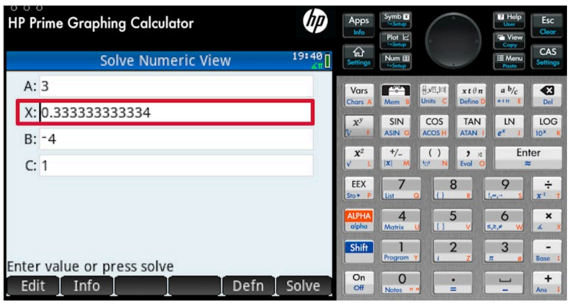
Steps	Explanation
These instructions show you how to use the polynomial equation solver option of your calculator to find the solutions of the quadratic equation $$3x^2 - 4x + 1 = 0.$$ On the main screen choose the equation solver option.	
Press F2 to select the polynomial root finder option.	
You want to solve a quadratic equation, so press F1 to select degree 2.	

Student view

Steps	Explanation
Enter the coefficients and when done, press F1 to find the solutions.	
The calculator shows you the two solutions, x_1 and x_2.	

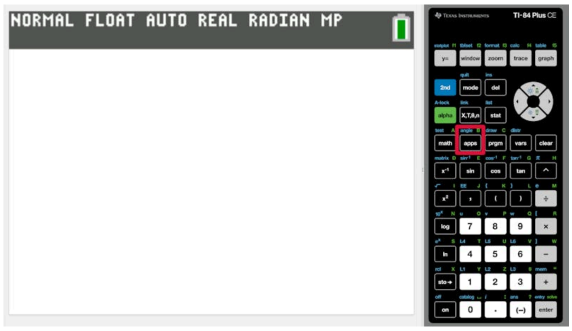
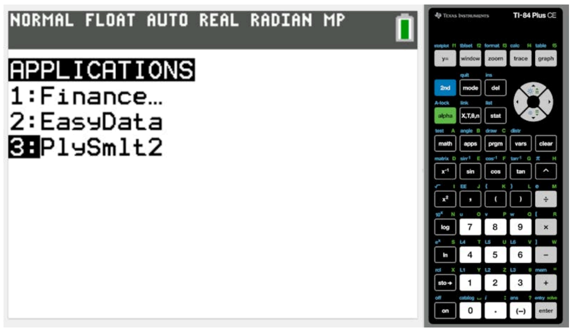
Steps	Explanation
These instructions show you how to use the equation solver application of your calculator to find the solutions of the quadratic equation $$3x^2 - 4x + 1 = 0.$$ On the main screen choose the equation solver application.	
In symbolic view type in the general form of a quadratic equation.	
In the current operating system there is no dedicated polynomial root finder option, but this may change in the future. At the moment you need to find the two solutions in two steps. Change to numeric view. Enter the coefficients of the quadratic equation, move to the line with x (telling the calculator that you would like to solve the equation for x), type in any value as your guess of the solution and tap on solve.	

Overview
(/study/app/math-aa-sl/sid-177-cid-761925/o

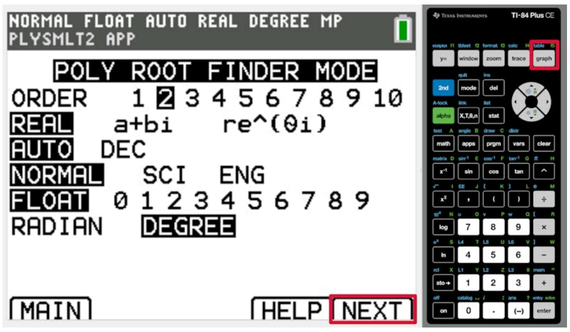
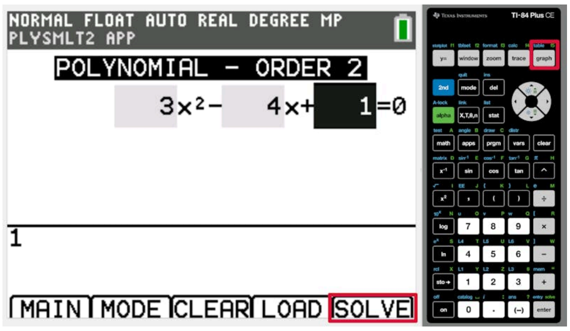
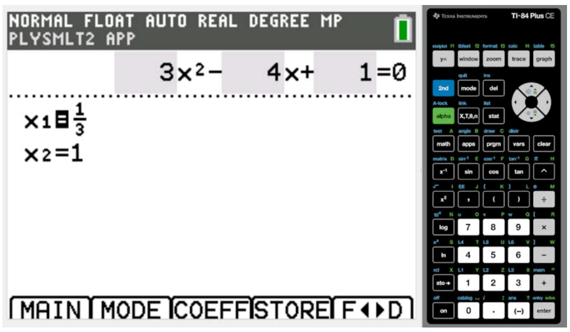
Steps	Explanation
The calculator replaces your guess with one of the solutions.	
To find the other solution, you need to start with another guess. You may need several tries. It can happen that with an other guess the calculator still finds the previous solution.	
This is the second solution. Since a quadratic equation has only two solutions, you do not need to try again. You found both solutions.	

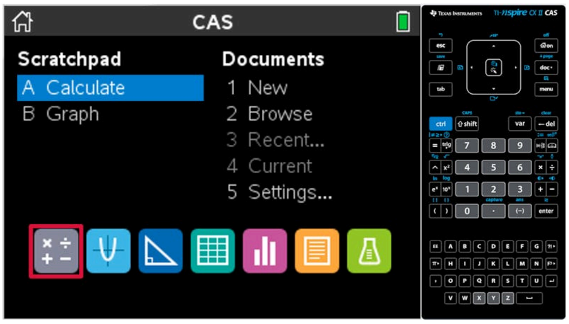
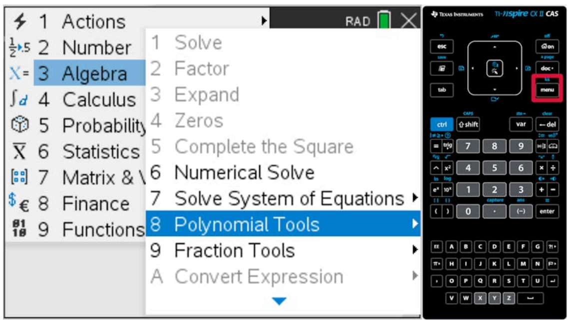
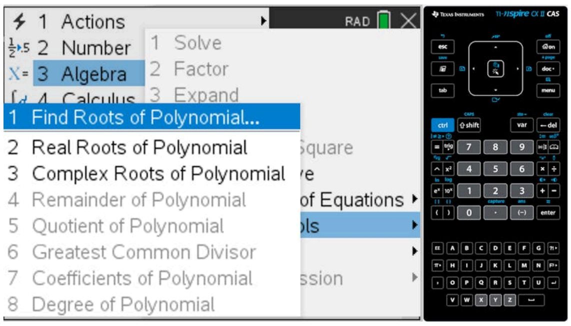
Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/solving-quadratic-equations-by-completing-id-26473/print/) Assign

Student view

Steps	Explanation
These instructions show you how to use the polynomial equation solver option of your calculator to find the solutions of the quadratic equation $$3x^2 - 4x + 1 = 0.$$ On the main screen open the application list ...	
... and choose PlySmlt2, the polynomial root finder and simultaneous equation solver application.	
In the application, choose the polynomial root finder option.	

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>You want to solve a quadratic equation, so select order 2.

Once done, press the graph button that corresponds to NEXT written on the screen.</td><td>
</td></tr>
<tr><td>Enter the coefficients and when done, press the graph button again to find the solutions.</td><td>
</td></tr>
<tr><td>The calculator shows you the two solutions, x_1 and x_2.</td><td>
</td></tr>
</table>

Steps	Explanation
These instructions show you how to use the polynomial equation solver option of your calculator to find the solutions of the quadratic equation $$3x^2 - 4x + 1 = 0.$$ On the main screen open a calculator page ...	
Open the menu and search for the polynomial tools ...	
... and choose the option to find the roots of a polynomial.	

Overview
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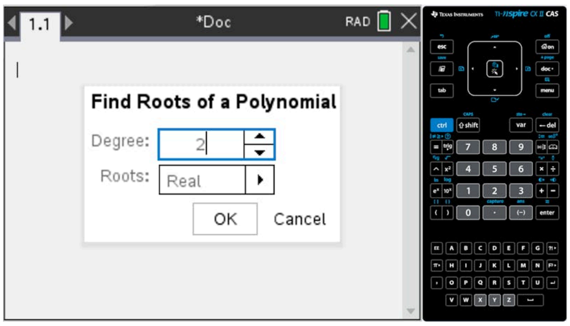
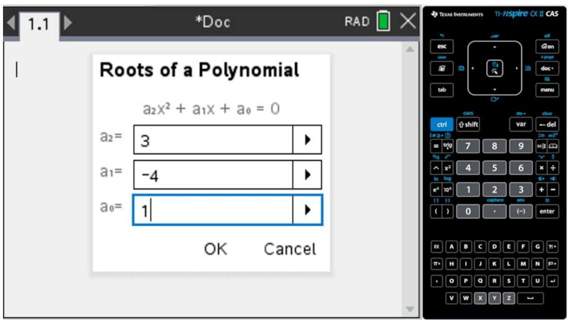
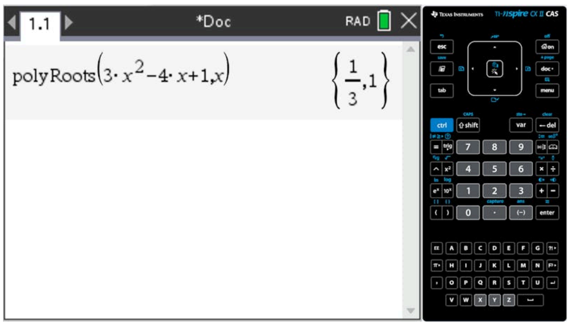
Steps	Explanation
You want to solve a quadratic equation, so select degree 2.	
Enter the coefficients and when done, click on OK to find the solutions.	
The calculator shows you the two solutions.	

Student
view

The discriminant of a quadratic

Nature of roots of a quadratic equation

For a quadratic equation $ax^2 + bx + c = 0$, the roots (or solutions) are given by the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Important

The expression $b^2 - 4ac$ under the square root in the quadratic formula is called the discriminant. Usually the Greek capital letter Δ is used to represent the discriminant.

Hence, the quadratic formula can be written as $x = -\frac{b \pm \sqrt{\Delta}}{2a}$ where $\Delta = b^2 - 4ac$.

Activity

What do the roots of a quadratic equation $ax^2 + bx + c = 0$ represent graphically?

In the applet below, use the sliders to change the values of a, b and c so that the discriminant Δ becomes:

- positive
- zero
- negative.

Overview
(/study/app/math-aa-sl/sid-177-cid-761925/o

Interactive 1. Roots of a Quadratic Equation and Discriminant.

More information for interactive 1

This interactive tool allows users to investigate root of quadratic equations $ax^2 + bx + c = 0$ in the form $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The expression $b^2 - 4ac$ under the square root in the quadratic formula is called the discriminant denoted by Δ.

The screen is divided in two parts. The top part consists of a graph of X and Y axes, with three sliders a, b and c. Slider a ranges from 1 to 10, b and c ranges from −5 to 5.

Changing the value shows how different discriminant values - positive, zero, or negative - correspond to distinct graphical behaviors of the parabola. By experimenting with various coefficient combinations, users can discover for themselves how the discriminant relates to the number and type of solutions, without being explicitly told the rules.

At the bottom portion of the screen the equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ also changes accordingly to give the output.

For example,

Discriminant

$\Delta = b^2 - 4ac = 1^2 - 4(2)(-2) = 17$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{17}}{4}$$

Solution 1 $x = -1.28$

Solution 2 $x = 0.78$

The applet encourages exploration, prompting users to formulate their own conclusions about the relationship between the discriminant and the roots of quadratic equations.

Student view

What do you notice about the value of the discriminant and the number of roots of the quadratic equation? Try to formalise your observations into a general rule.

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(/study/app/math-aa-sl/sid-177-cid-761925/o

Important

The nature of the roots of a quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$ and $a \neq 0$, is determined by the value of the discriminant Δ, as follows:

- If $\Delta > 0$, the quadratic equation has **two distinct roots** $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.
- If $\Delta = 0$, the quadratic equation has a **repeated** (or **double**) **root** $x = \dfrac{-b}{2a}$.
- If $\Delta < 0$, the quadratic equation has **no real roots.**

Example 1

★☆☆

Without solving the equation $x^2 + x + 2 = 0$, determine the nature of the roots.

Steps	Explanation
$x^2 + x + 2 = 0$	$a = 1, b = 1, c = 2$
$\Delta = b^2 - 4ac = (1)^2 - 4(1)(2) = 1 - 8 = -7 < 0$	Calculate the discriminant.
$\Delta < 0$ so the equation has no real roots.	

Example 2

★☆☆

Without solving the equation $x^2 - 6x + 8 = 0$, determine the nature of the roots.

Steps	Explanation
$x^2 - 6x + 8 = 0$	$a = 1, b = -6, c = 8$
$\Delta = b^2 - 4ac = (-6)^2 - 4(1)(8) = 36 - 32 = 4 > 0$	Calculate the discriminant.
$\Delta > 0$ so the equation has two distinct real roots.	

Student view

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Example 3

★☆☆

Without solving the equation $4x^2 + 4x + 1 = 0$, determine the nature of the roots.

Steps	Explanation
$4x^2 + 4x + 1 = 0$	$a = 4, b = 4, c = 1$
$\Delta = b^2 - 4ac = (4)^2 - 4(4)(1) = 16 - 16 = 0$	Calculate the discriminant.
$\Delta = 0$ so the equation has one repeated real root.	

Example 4

★★☆

Consider the quadratic equation $3x^2 - 4x + k + 1 = 0$. Find the values of k for which the equation has:

a) two distinct real roots
b) no real roots.

	Steps	Explanation
	$3x^2 - 4x + k + 1 = 0$	
	$a = 3,\ b = -4,\ c = k + 1$	Identify the coefficients $a,\ b$ and c .
	$\Delta = b^2 - 4ac = (-4)^2 - 4(3)(k+1)$	Find the discriminant.
	$= 16 - 12k - 12 = -12k + 4$	Simplify as much as possible.
a)	$\Delta > 0$ $-12k + 4 > 0$ $-12k > -4$ $k < \frac{-4}{-12}$ $k < \frac{1}{3}$	For two distinct real roots the discriminant must be positive, so set $\Delta > 0$ and solve the inequality for k .

Student
view

	Steps	Explanation
b)	$\Delta < 0$ $-12k + 4 < 0$ $-12k < -4$ $k > \frac{-4}{-12}$ $k > \frac{1}{3}$	For no real roots the discriminant must be negative, so set $\Delta > 0$ and solve the inequality for k .

Example 5

★★★

Consider the quadratic equation $x^2 - 2(k-2)x + k = 0$. Find the values of k such that the quadratic equation has one repeated root.

Steps	Explanation
$x^2 - (k-2)x + k = 0$	
$a = 1,\ b = -2(k-2),\ c = k$	Identify the coefficients $a,\ b$ and c .
$\Delta = b^2 - 4ac = (-2(k-2))^2 - 4(1)k$ $= 4(k-2)^2 - 4k = 4k^2 - 16k + 16 - 4k$ $= 4k^2 - 20k + 16$	Find the discriminant.
$\Delta = 0$ $4k^2 - 20k + 16 = 0$ $4(k^2 - 5k + 4) = 0$ $4(k-1)(k-4) = 0$ $k - 1 = 0$ or $k - 4 = 0$ $k = 1$ or $k = 4$	For one repeated root the discriminant should be equal to zero, so set $\Delta = 0$ and solve the equation for k .

Theory of Knowledge

The Bakhshali Manuscript

The Bakhshali Manuscript was discovered in 1881 in what is now Pakistan, by a villager who was excavating for stones. As a result of his digging, the manuscript was badly damaged, however the remaining fragments offer a tantalising glimpse into early Indian mathematics.

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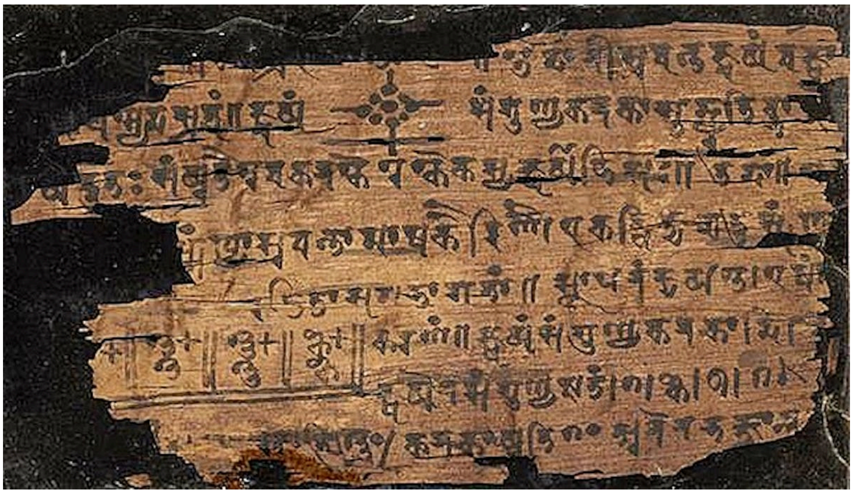
(/study/app/math-aa-sl/sid-177-cid-761925/o

It contains a set of rules governing arithmetic and an early form of algebra. It also provides a method for approximating square roots to a high degree of accuracy, and shows evidence of decimal notation.

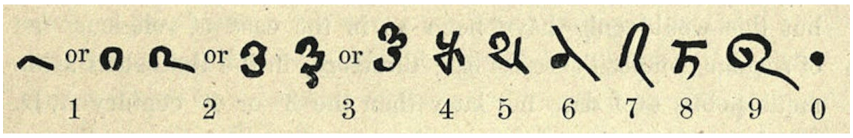
There is much argument surrounding the date that the Bakhshali manuscript was first written, ranging from around 300AD to around 1200AD. Many believe that its origins are closer to the earlier date, which would place it just before what is regarded as the classical era of Indian mathematics, thus implying that it could have been influential in shaping the development of this period.

The figure below shows a page from the Bakhshali Manuscript.



A page from the Bakhshali Manuscript
Source: " Bakhshali manuscript (https://commons.wikimedia.org/wiki/File:Bakhshali_manuscript.jpg) " by National Geographic is in public domain.

If you look at the page carefully, you will see that certain symbols are repeated frequently. In fact, the page is essentially covered with 10 symbols .



The 10 symbols from the Bakhshali Manuscript
Source: " Bakhshali numerals 2 (https://commons.wikimedia.org/wiki/File:Bakhshali_numerals_2.jpg) " by Augustus Hoernle is in public domain.

More information

The image shows the Bakhshali numerals, with symbols assigned to each number from 1 to 9, and also the number 0. Underneath each symbol, the corresponding numbers are labeled starting from the left: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Some numbers have symbols with variations, shown by the use of 'or' between different symbol versions of the same numeral. For example, the numeral 1 can be represented by a single symbol followed by the word 'or' and another similar symbol. The visual layout shows all ten numerical symbols in a linear arrangement with text labels beneath them.

[Generated by AI]

The Bakhshali Manuscript also contains an algebraic formula for solving quadratic equations. In subtopic 2.6 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26462/) you saw an image of a Babylonian clay tablet showing solutions to quadratic equations, and you learned that the Babylonians and a Persian mathematician called al-Khwarizmi had developed the method of completing the square for quadratics.

Student view

Overview
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Bearing in mind that different civilisations and cultures appear to have come up with the same methods for solving common mathematical problems — even though the symbols they used to express the solutions look very different — reflect on the following questions:

- Is mathematics invented or discovered?
- Is mathematics a kind of universal ‘language’, or simply the manipulation of symbols following a set of rules?
- Is there a set of key concepts that provide the building blocks for mathematical knowledge? If so, what are these concepts?

3 section questions

2. Functions / 2.7 Quadratic equations and quadratic inequalities

Quadratic inequalities

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/quadratic-inequalities-id-26476/print/) Assign

Solving quadratic inequalities

So far you have looked at how to solve quadratic equations $ax^2 + bx + c = 0$ or, more generally, equations of the form $f(x) = g(x)$ where one of $f(x)$ or $g(x)$ is a quadratic function and the other is a constant or linear function.

In this section, you will extend the techniques you have learned to solve quadratic inequalities: statements of the form $f(x) < g(x)$, $f(x) \leqslant g(x)$, $f(x) > g(x)$ or $f(x) \geqslant g(x)$. The video clip below gives an introduction to this.

Student view

Overview
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Video 1. Understanding Quadratic Inequalities: Solving and Graphing.

More information for video 1

1

00:00:00,300 --> 00:00:02,433

narrator: In this video, we're going

to look at inequalities

2

00:00:02,500 --> 00:00:03,767

when it comes to functions.

3

00:00:04,000 --> 00:00:06,633

So here we've got two functions,

$y = f(x)$,

4

00:00:06,700 --> 00:00:09,100

the blue one,

and $y = g(x)$, the reddish one.

5

00:00:09,633 --> 00:00:10,567

Now we've already seen

6

00:00:10,633 --> 00:00:12,067

that we can ask ourselves the question,

7

00:00:12,133 --> 00:00:14,633

what are the points of intersection

between those two functions?

8

Student view

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00:00:14,800 --> 00:00:18,433

And of course, those are the places

where for a certain x,

9

00:00:18,500 --> 00:00:20,967

they evaluate to the same y value.

10

00:00:21,033 --> 00:00:23,200

In other words, $f(x) = g(x)$,

11

00:00:23,267 --> 00:00:25,633

and this can happen in one point

or more than one point.

12

00:00:25,700 --> 00:00:28,467

In this case, we see two points

of intersection.

13

00:00:29,233 --> 00:00:31,367

But of course, we can also

ask ourselves a question

14

00:00:31,433 --> 00:00:34,800

instead of an equality,

what about an inequality, for example?

15

00:00:34,933 --> 00:00:37,633

Whereas $f(x) < g(x)$.

16

00:00:38,067 --> 00:00:41,633

Now the most important thing to realize

is that in this case,

17

00:00:41,700 --> 00:00:44,133

you are looking for a region of x.

18

00:00:44,300 --> 00:00:47,667

So don't be surprised

when you have one or more regions.

19

00:00:47,733 --> 00:00:49,133

So for example, here, the fact

20

00:00:49,200 --> 00:00:51,600

that we have a regions

indicated by the shaded area,

Student view

21

00:00:52,333 --> 00:00:56,233

among all of which that statement is true,

f (x) < g(x).

22

00:00:56,300 --> 00:00:58,900

Or you can do it with a number line

as indicated here,

23

00:00:58,967 --> 00:01:02,667

where the open end,

the open circle indicates

24

00:01:02,733 --> 00:01:06,000

that it is an inequality,

so not an equality.

25

00:01:06,067 --> 00:01:08,167

So that is very important to realize

26

00:01:08,233 --> 00:01:10,433

and keep in mind that

when you solve inequalities,

27

00:01:10,500 --> 00:01:15,333

you're looking for regions

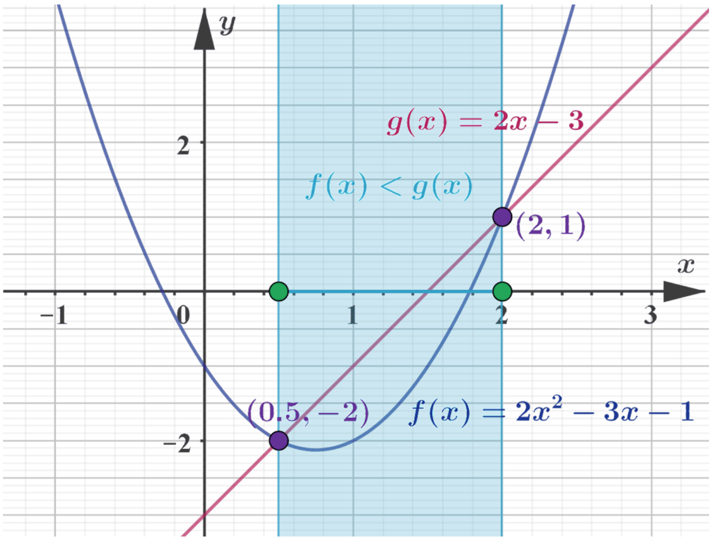
where the inequality is true.

28

00:01:15,400 --> 00:01:17,000

That is where it holds.

Just as with solving equations, you can solve quadratic inequalities using both analytical and graphical methods. In either case, it is important to remember that the solution to a quadratic inequality will be a **set of values** of x for which the inequality holds. This can be seen in the figure below, which shows the solution of the inequality $f(x) < g(x)$. The line segment on the x-axis in the shaded area represents all the x values for which the y values of $f(x)$ are smaller than the y values of $g(x)$. Because the inequality $<$ is strict (or non-inclusive), there are open circles on both ends of the line segment and dotted lines on both sides of the shaded region.



More information

This graph illustrates the solution of the inequality (f(x) < g(x)). The x-axis represents the variable (x), while the y-axis represents the function values. The blue curve represents (f(x) = 2x^2 - 3x - 1) and the red curve represents (g(x) = 2x - 3). The shaded blue region on the graph indicates the range of (x) values for which the inequality (f(x) < g(x)) holds true. This shaded segment is delimited by open circles, indicating the exclusion of the boundary values for (x). The graph shows the curves intersecting at points labeled ((0.5, -2)) and ((2, 1)), with the vertical lines and grid in the background helping to delineate the coordinate system. Along the x-axis, the line segment representing the solution does not include the endpoint values, reflecting the strict inequality.

[Generated by AI]

The solution of the inequality $f(x) < g(x)$ is the segment on the x-axis in the shaded region, representing the set of values of x for which the blue curve $y = f(x)$ lies below the green curve $y = g(x)$.

The analytical method

Important

A quadratic inequality in one variable is an inequality that can be written in the form

$$ax^2 + bx + c > 0$$

where a, b and c are real numbers with $a \neq 0$. The symbols $\geqslant$, $\leqslant$ or $<$ may be used instead of $>$.

To solve a quadratic inequality analytically, factorise the quadratic expression $ax^2 + bx + c$ into linear factors and consider the sign of the product of the linear factors for different values of x. One way of doing this in an organised way is to construct a 'sign table'.

Be aware

Remember that when you multiply two numbers A and B :

Overview (/study/app/math-aa-sl/sid-177-cid-761925/o

- The product AB is positive when both A and B are positive or when both A and B are negative.
- The product AB is negative when A and B have opposite signs.

For example, to solve the quadratic inequality $x^2 > x + 2$, we construct a 'sign table' as follows:

Steps	Explanation
$x^2 > x + 2$	The given equation is not in the form $ax^2 + bx + c > 0$.
$x^2 - x - 2 > 0$	Rearrange so that the RHS of the inequality is 0.
$(x - 2)(x + 1) > 0$	Factorise the quadratic expression on the LHS.
The corresponding equation is $(x - 2)(x + 1) = 0$	Write the correspond quadratic **equation.**
The roots of the corresponding quadratic equation are $x = 2$ and $x = -1$.	

Student view

Overview (/study/app/math-aa-sl/sid-177-cid-761925/o

Steps	Explanation
Construct a sign table like the one shown below:	Use the roots of the quadratic equation to divide the number line into three regions. Then consider the signs of the linear factors and the sign of their product in each region of the number line.

x	- ∞	- 1		2		+ ∞
x - 2	-		-	●	+	
x + 1	-	●	+		+	
(x - 2)(x + 1)	+	●	-	●	+	

More information

The image is a sign table used to evaluate the signs of a polynomial expression at various intervals. The table includes the following columns and rows:

Columns: - Label 'x' with ranges: '-∞', '-1', '2', and '+∞'.

Rows: 1. "x - 2": - Negative ('-') for intervals '-∞ to 2'. - Positive ('+') for intervals '2 to +∞'.

1. "x + 1":
2. Negative ('-') for intervals '-∞ to -1'.
3. Positive ('+') for intervals '-1 to +∞'.

4. "(x - 2)(x + 1)": Product of two expressions:

5. Positive ('+') for intervals '-∞ to -1'.
6. Negative ('-') for intervals '-1 to 2'.
7. Positive ('+') for intervals '2 to +∞'.

Points: At '-1', '2' marked with purple dots indicating specific values where expressions change signs.

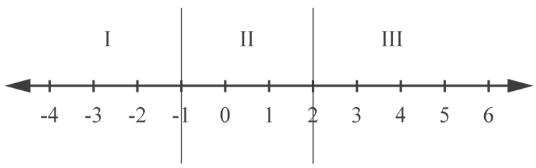
[Generated by AI]

Therefore, the solution of the inequality $(x-2)(x+1) > 0$

is the set of x values with $x < -1$ or $x > 2$.

In interval notation, $x \in (-\infty, -1) \cup (2, +\infty)$.

Alternatively, after using the roots of the corresponding quadratic equation to divide the number line into regions, as shown below, test one number in each region to determine the sign of the quadratic expression.



More information

Student view

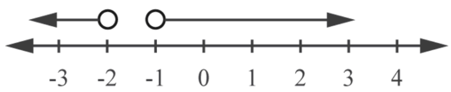
The image is a diagram of a number line with arrows at both ends, indicating it extends indefinitely in both directions. The number line is divided into three regions labeled I, II, and III. The divisions are marked at -1 and 2. Region I extends from negative infinity to -1, region II is between -1 and 2, and region III extends from 2 to positive infinity. Below the line, tick marks represent the integers from -4 to 6, indicating the position of each integer relative to the division points. The number line is used to analyze the sign of a quadratic expression by dividing it into regions and testing a number from each region.

[Generated by AI]

Region	Test point	Inequality	Status of inequality
I	$x = -2$	$(-2-2)(-2+1) = (-4)(-1) = 4 > 0$	True
II	$x = 0$	$(0-2)(0+1) = (-2)(1) = -2 < 0$	False
III	$x = 3$	$(3-2)(3+1) = 1(4) = 4 > 0$	True

This gives the same result as the previous method above: the solution is the set of x values with $x < -1$ or $x > 2$.

The solution of the inequality $(x-2)(x+1) > 0$ can be represented on the following number line. Note the open circles at -1 and 2.



More information

The image is a number line representing the solution of the inequality ((x - 2)(x + 1) > 0). It contains integers marked from -3 to 4. There are open circles at -1 and 2, indicating these values are not included in the solution set. From the open circle at -1, an arrow points to the left, extending indefinitely. Similarly, from the open circle at 2, another arrow extends to the right, also indicating an indefinite extension. This signifies the solution set is (x < -1) or (x > 2).

[Generated by AI]

Exam tip

Overview
(/study/app/math-aa-sl/sid-177-cid-761925/o

To solve a quadratic inequality analytically:

1. Rearrange the inequality so that the RHS is 0 and the LHS is a quadratic expression.
2. Solve the corresponding quadratic equation (where the inequality symbol is replaced with the equal sign).
3. Use the roots of the quadratic equation to divide the number line into regions. Then consider the sign of the product of the linear factors in each region. Put this information in a sign table.
4. Use the sign table to write the solutions of the quadratic inequality.

Example 1

★★☆

Solve the quadratic inequality $2x^2 + 22x + 56 > 0$.

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>$2x^2 + 22x + 56 > 0$</td><td></td></tr>
<tr><td>$2\left(x^2 + 11x + 28\right) > 0$</td><td>Factor 2 out.</td></tr>
<tr><td>$2\left(x+7\right)\left(x+4\right) > 0$</td><td>Factorise the quadratic expression.</td></tr>
<tr><td>$2\left(x+7\right)\left(x+4\right) = 0$</td><td>Write the corresponding quadratic equation.</td></tr>
<tr><td>$x + 7 = 0$ or $x + 4 = 0$</td><td>Apply the null factor law.</td></tr>
<tr><td>The roots of the quadratic equation are
$x = -7$ or $x = -4$</td><td>Solve the quadratic equation.</td></tr>
<tr><td>Construct a sign table:
<table>
<tr><th>x</th><th>- ∞</th><th>- 7</th><th>- 4</th><th>+ ∞</th></tr>
<tr><td>x + 7</td><td>-</td><td>+</td><td>+</td><td></td></tr>
<tr><td>x + 4</td><td>-</td><td>-</td><td>+</td><td></td></tr>
<tr><td>(x + 7)(x + 4)</td><td>+</td><td>-</td><td>+</td><td></td></tr>
</table>
</td><td>Use the roots to divide the number line into three regions.

Consider the signs of the linear factors and the sign of their product in each region.</td></tr>
</table>

Student view

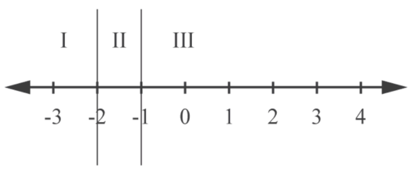
Overview (/study/app/math-aa-sl/sid-177-cid-761925/o

Steps	Explanation
Therefore, the solution of the inequality $2(x+7)(x+4) > 0$ is the set of x values with $x < -7$ or $x > -4$. In interval notation, $x \in (-\infty, -7) \cup (-4, +\infty)$.	

Example 2

★★☆

Solve the quadratic inequality $x^2 + 3x \geqslant -2$.

Steps	Explan
$x^2 + 3x \geqslant -2$	
$x^2 + 3x + 2 \geqslant 0$	Rearrang that the l of the inequalit 0.
To find the roots of the equation $$x^2 + 3x + 2 = 0$$	Find the of the correspo quadrati equation
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)} = \frac{-3 \pm 1}{2}$ So the roots are -1 and -2, and thus the quadratic can be factorised as $(x+1)(x+2)$.	You can factorise quadrati use the quadrati formula find the
	Use the to divide number into regi II and III.

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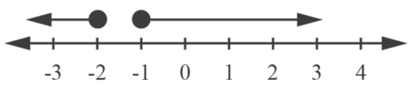
Steps	Explana
Make a sign table: <table><tr><th>Region</th><th>Test point</th><th>Inequality</th><th>Status</th></tr><tr><td>I</td><td>$x=-3$</td><td>$(-3+1)(-3+2)=(-2)(-1)$ $=2>0$</td><td>True</td></tr><tr><td>II</td><td>$x=-\frac{3}{2}$</td><td>$\left(-\frac{3}{2}+1\right)\left(-\frac{3}{2}+2\right)=\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $=-\frac{1}{4}<0$</td><td>False</td></tr><tr><td>III</td><td>$x=0$</td><td>$(0+1)(0+2)=(1)(2)=2>0$</td><td>True</td></tr></table>	Test one in each region.
Therefore, the solution of $x^2+3x\geqslant -2$ is $x\leqslant -2$ or $x\geqslant -1$. 	In the number-represen of the solution, the filled circles at and -1, reflecting fact that inequalit inclusive strict).

Example 3

★★★

For which values of k does the quadratic function $y=x^2+kx+k$ have no real roots.

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Steps	Explana
The quadratic equation $x^2 + kx + k = 0$ has no real solutions if $\Delta < 0$.	Recall th number roots of a quadrati equation determir the sign discrimir
$\Delta < 0$ $k^2 - 4(1)(k) < 0$ $k^2 - 4k < 0$ $k(k-4) < 0$	Find the discrimir and write down th inequalit you need solve.
The corresponding equation $k(k-4) = 0$ has roots $k = 0$ and $k = 4$.	Solve the correspo quadrati equation
Make a sign table: (see table below)	Use the r to divide number into regi and test number each reg
The discriminant is negative for $0 < k < 4$, so the equation $y = x^2 + kx + k$ has no real roots for $0 < k < 4$.	

Region	Test point	Inequality	Status
I: $k < 0$	$k = -1$	$(-1)(-1-4) = (-1)(-5) > 0$	False
II: $0 < k < 4$	$k = 1$	$(1)(1-4) = -3 < 0$	True
III: $k > 4$	$k = 5$	$(5)(5-4) = 5 > 0$	False

Example 4

★★★

Consider the quadratic equation $x^2 + (k+2)x + 4 = 0$. Find the value of k for which the equations has:

a) two distinct real roots

b) one repeated root

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Overview (/study/app/math-aa-sl/sid-177-cid-761925/o

c) no real roots.

	Steps	E
	$x^2+(k+2)x+4=0$ $a=1,\ b=k+2,\ c=4$	
	$\Delta=b^2-4ac=(k+2)^2-4(1)(4)=k^2+4k-12=(k-2)(k+6)$	Fir dis an it.
	The quadratic equation $(k-2)(k+6)=0$ has roots $k=2$ and $k=-6$.	Sc co qu eq
	(see table below)	Us to nu int an sig
a)	$(k-2)(k+6)>0$ $k>2$ or $k<-6$	Fo dis ro
b)	$(k-2)(k+6)=0$ $k=2$ or $k=-6$	Fo rep ro
c)	$(k-2)(k+6)<0$ $-6<k<2$	Fo ro .

Region	Test point	$(k-2)(k+6)$	Sign
I: $k<-6$	$k=-7$	$(-7-2)(-7+6)=(-9)(-1)$ $=9>0$	$+$
II: $-6<k<2$	$k=0$	$(0-2)(0+6)=-12<0$	$-$
III: $k>2$	$k=3$	$(3-2)(3+6)=9>0$	$+$

The graphical method

Solving a quadratic inequality of the form $ax^2+bx+c>0$ is equivalent to identifying the parts of the parabola $y=ax^2+bx+c$ that are above the x-axis. Likewise, the solution of a quadratic inequality of the form $ax^2+bx+c\leqslant 0$ corresponds to the x intervals where the parabola is on or below the x-axis.

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Consider the quadratic inequality $x^2-x-2>0$.

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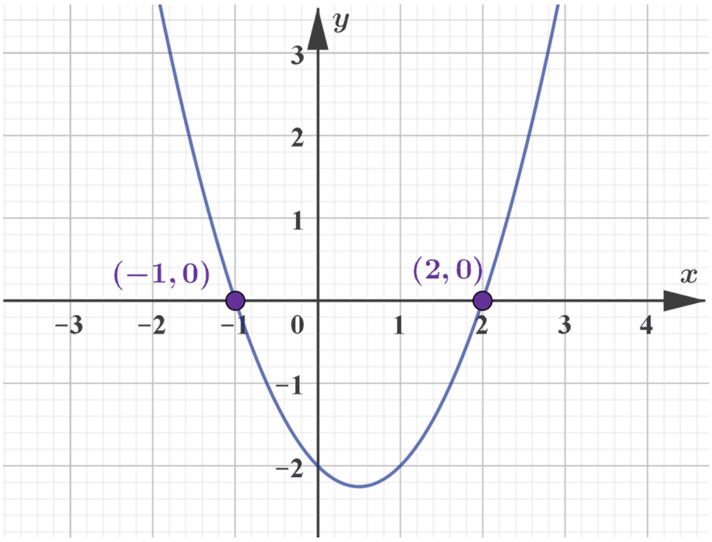
Use your GDC to plot the graph of $y = x^2 - x - 2$ (see figure below).

Solve the inequality by identifying the x intervals where the graph of the parabola lies above the x-axis.



More information

The image shows a graph of a parabola plotted on a grid. The x-axis is labeled and has marks from -4 to 4, and the y-axis has marks up to 4. The curve of the parabola crosses the x-axis at the points (-1, 0) and (2, 0). The parabola opens upwards and reaches a minimum point below the x-axis between the points (-1, 0) and (2, 0). This indicates that between these x-values, the parabola lies below the x-axis. Outside this interval, the parabola lies above the x-axis for x < -1 and x > 2. This is consistent with the solution text provided after the image, which states that the parabola lies above the x-axis for x < -1 or x > 2.

[Generated by AI]

The parabola lies above the x-axis for $x < -1$ or $x > 2$.

Therefore, the solution of the inequality $x^2 - x - 2 > 0$ is $x \in (-\infty, -1) \cup (2, +\infty)$.

Example 5

★★☆

Solve the inequality $2x^2 - 3x - 1 \leqslant 0$.

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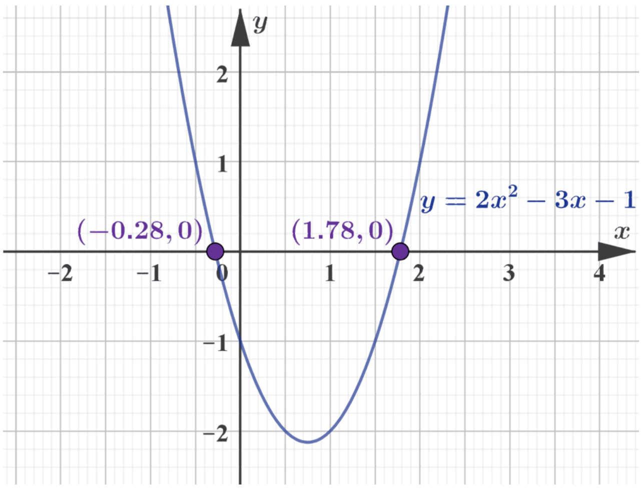
Steps	Explanation
Graph of $y = 2x^2 - 3x - 1$ from GDC: 	Use your GDC to plot the graph of the quadratic function and find the x -intercepts of the parabola.
The parabola lies on or below the x -axis for all $x \in [-0.28,\ 1.78]$. So the solutions to the quadratic inequality $2x^2 - 3x - 1 \leqslant 0$ are $-0.28 \leqslant x \leqslant 1.78$.	The solution of the quadratic inequality is the x interval where the parabola lies below the x -axis, including the points where the quadratic function equals zero (x -intercepts).

You can also use the graphical method to solve more general inequalities of the form $f(x) < g(x)$, $f(x) \leqslant g(x)$, $f(x) > g(x)$ or $f(x) \geqslant g(x)$, where one of $f(x)$ and $g(x)$ is a quadratic function and the other is a constant or linear function.

This involves plotting the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes and finding the intersection points of the graphs.

Example 6

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Let $f(x) = x^2 + 3x$ and $g(x) = 10x - 4$. Solve the inequality $f(x) < g(x)$.

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view

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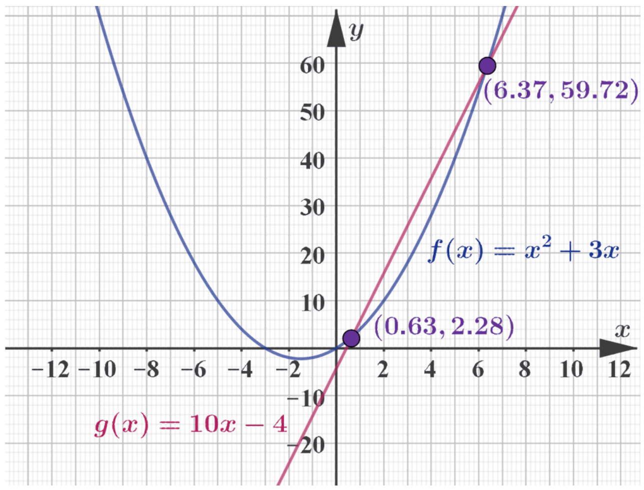
Steps	Explanation
	Use your GDC to obtain the graphs of $y = f(x)$ and $y = g(x)$.
Points of intersection $(0.63,\ 2.28)$ and $(6.37,\ 59.72)$.	Use the GDC to obtain the points of intersection between the two graphs.
$f(x) < g(x)$ for $0.63 < x < 6.37$	Identify the x values for which the parabola lies below the line.

4 section questions

2. Functions / 2.7 Quadratic equations and quadratic inequalities

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/checklist-id-26477/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- solve quadratic equations using several analytical methods, including factorisation, completing the square and applying the quadratic formula
- solve quadratic equations using graphical methods with the aid of a GDC
- determine the nature of the roots of a quadratic expression, by using the discriminant of the quadratic

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- solve quadratic inequalities using both analytical and graphical methods.

Investigation

The applet below allows you to visualise a geometric construction of completing the square to solve a quadratic equation of the form

$$x^2 - bx = c$$

It is based on the method described by the great Persian mathematician al-Khwarizmi (ca. 780–850 CE). Al-Khwarizmi's beautiful method of completing the square lies at the heart of the quadratic formula.

Interactive 1. Solving a Quadratic Equation.

More information for interactive 1

This interactive allows users to visualise a geometric construction of completing the square to solve a quadratic equation of the form $x^2 - bx = c$ using geometric principles. The visualization begins by representing x^2 as a square with side length x, while the bx term appears as a rectangle attached to this square.

The interactive allows users to drag sliders that animate this geometric process, showing how the algebraic manipulation corresponds to physical rearrangement of shapes. Users can experiment with different coefficients to see how the required completion changes, observing how the method consistently transforms the original equation into a perfect square that can be easily solved.

For example After entering positive coefficients 4 in first box and 12 in second box we get an quadrating equation $x^2 - 4x = 12$. By dragging

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the slider below we get the transition of solution using geometric progression. The animation first shows an x^2 square, then removes four 1-unit wide strips representing $-4x$. This reveals a missing area that's completed by adding four 1×1 squares. After balancing the equation by adding 4 to both sides, it forms the perfect square $(x - 2)^2 = 16$. The solution reveals two roots: $x = 6$ (when $x - 2 = 4$) and $x = -2$ (when $x - 2 = -4$). The detailed solution can be viewed using the check box next to “Quadratic equation solution”.

- In the input boxes, enter your choice of positive coefficients for the quadratic equation.
- Drag the slider to visualise the geometric construction of completing the square.
- Complete the solution for the resulting quadratic equation.
- Describe the steps of using completing the square to solve the quadratic equation.
- How does the area model help you understand the process of completing the square?
- Why is this process called completing the square?

Research how quadratic equations were derived and solved throughout history. Find out who first discussed quadratic equations and what they were used for at the time.

Rate subtopic 2.7 Quadratic equations and quadratic inequalities

Help us improve the content and user experience.

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