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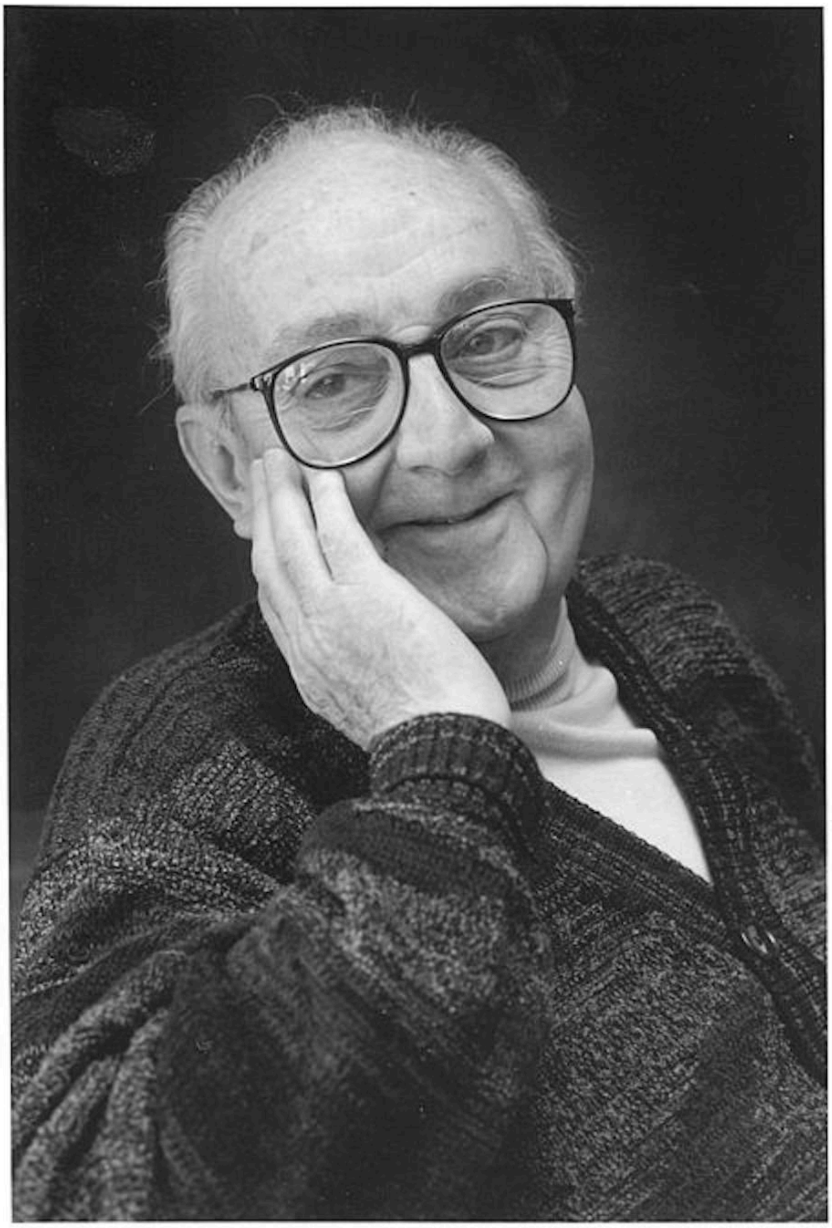
The big picture

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27513/print/) Assign

So far, you have focused your attention on first-order differential equations, both individually and as coupled systems. Revisiting George Box's principle that ‘all models are wrong, but some are useful', how simple can a model be before it is no longer useful? Are there times you need to expand your horizons and use a second-order differential equation? How would you analyse that? Will the analytical and numerical techniques still work?

Student view



George Box, who began his career as a chemist and taught himself statistics, is now considered one of the greatest statisticians of modern times.

One way to handle second-order equations is to convert them to a system of first-order equations, and then use the techniques already covered in previous subtopics. This could be taken further to analyse even more challenging problems, but, for this

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course, second-order equations will suffice.

Concept

In science and engineering, there are many applications that cannot be **modelled** with a simple linear first-order differential equation. A few examples include mass—spring systems, harmonic resonance and electrical circuitry. As you go through this subtopic, think about how you can use second-order differential equations to study the world around you.

5. Calculus / 5.18 Second order differential equations

Second-order differential equations

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/secondorder-differential-equations-id-27943/print/) Assign

The need for second-order differential equations

In the last four subtopics, this course has focused on first-order differential equations. In these equations you investigated the relationship between a function and its derivative. However, there are context where the model also involves the second derivative of the function.

Important

A second-order differential equation is a relationship between a variable, a function and its first and second derivative.

Student view

An example taken from basic physics involves a mass attached to a spring that is able to slide on a frictionless table.

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More information

The image is a diagram illustrating a spring-mass system on a frictionless table. It shows a rectangular block representing the mass on the right, attached to a zigzag line representing the spring. The left end of the spring is fixed to a vertical line, representing a wall or support, indicating the spring's point of attachment. The spring is shown in a straight line, symbolizing that it is neither compressed nor elongated beyond its equilibrium position. This setup is an example of harmonic motion, often studied in basic physics, where the mass can slide back and forth due to the spring's restorative force.

[Generated by AI]

If the mass is displaced, it will oscillate about the equilibrium point over time. If no energy loss occurs due to friction with the table or deformation of the spring, the acceleration is proportional to the stretch of the spring. This system can be modelled by the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{k}{m}x = 0$$

where k is the spring constant and m is the mass. Without friction, this results in a periodic function with constant amplitude.

Student view

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If the assumption of a frictionless table is relaxed, an additional term is added to account for the damping of the system. Damping is the result of energy being dissipated to the surroundings through friction, which is proportional to the speed of the object. The new model looks like

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{k}{m}x = 0$$

where b represents the friction from the table as well as air resistance. If you are also studying physics, this is a different model for dynamic friction than you will see in physics. Neither model is perfect, but both have their place.

In dynamics, the Van der Pol oscillator is a non-conservative oscillator with non-linear damping. This oscillator can be modelled with the Van der Pol equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - \mu\left(1 - x^2\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

where μ represents the strength of the damping. Although you will not see something this advanced in this course, you can certainly read more about it online.

Another field of physics and engineering that involves the solving of second-order differential equations is the study of vibrations.

Example 1

★★☆

Find a model for the swaying motion of a skyscraper. Use $x(t)$ for the displacement of the top of the building relative to the upright position. Build a relationship between the acceleration, $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$,the velocity, $\frac{\mathrm{d}x}{\mathrm{d}t}$,and the displacement, x based on the following.

- The acceleration is effected by the force applied to the building (for example wind).

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- The stiffness of the building slows down the sway. This effect is proportional to the displacement.
- Engineers build in dampers to further slow the sway. The effect of these dampers are proportional to the speed of the sway.

The differential equation modelling the sway is of the following form.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = p - qx - r\frac{\mathrm{d}x}{\mathrm{d}t}$$

In this equation:

- The constant p represents the constant force (wind).
- The constant q depends on the stiffness of the building. The negative sign indicates that this factor is working against the displacement.
- The constant r depends on the characteristics of the damper. The negative sign indicates that this factor is working againts the movement.

Exam tip

Conversion of information from a physical example to the second-order equation, such as a mass—spring system or the Van der Pol equation, will not be required. In these cases, the second-order differential equation will be provided on exams.

In all of these examples, second derivatives are present in the equations.

Exam tip

The second order differential equations discussed in this course are of the form

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = f(x, \frac{\mathrm{d}x}{\mathrm{d}t}, t).$$

Student view

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Converting second-order differential equations

One way of solving certain second-order differential equations is to rewrite the equation as a system of first-order equations and then solving through techniques covered in subtopic 5.17 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27938/).

Consider the first example above, the spring–mass system with an equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\frac{k}{m}x=0$$

or

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\frac{k}{m}x$$

From kinematics, you know that:

$$v=\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$$

Therefore, by substitution, the second-order differential equation can be represented by the following coupled system:

$$\frac{\mathrm{d}x}{\mathrm{d}t}=v$$

$$\frac{\mathrm{d}v}{\mathrm{d}t}=-\frac{k}{m}x$$

Example 2

Student view

★★☆

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Consider the differential equation:

$$3\frac{\mathrm{d}x^2}{\mathrm{d}t^2}+\frac{\mathrm{d}x}{\mathrm{d}t}+x=0$$

Convert the equation to a system of two first-order differential equations.

$$3\frac{\mathrm{d}x^2}{\mathrm{d}t^2}+\frac{\mathrm{d}x}{\mathrm{d}t}+x=0$$

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\frac{1}{3}\frac{\mathrm{d}x}{\mathrm{d}t}-\frac{x}{3}$$

Set $y=\frac{\mathrm{d}x}{\mathrm{d}t}$

Therefore $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$

Substitute

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\frac{1}{3}\frac{\mathrm{d}x}{\mathrm{d}t}-\frac{x}{3}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{1}{3}y-\frac{x}{3}$$

Coupled system:

$$\frac{\mathrm{d}x}{\mathrm{d}t}=y$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{1}{3}y-\frac{1}{3}x$$

On the applet below you can move the red point on a trajectory of the coupled system you found in Example 2 to see how this trajectory is related to the corresponding solution curve of the second order equation.

Student view

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Interactive 1. Visualizing Solutions of Second-Order Differential Equations.

More information for interactive 1

This interactive is a graph that allows the user to understand solutions of Second-Order Differential Equation $3\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$. This equation describes a damped harmonic oscillator, where the term $3\frac{d^2x}{dt^2}$ represents inertia, $\frac{dx}{dt}$ corresponds to damping (resistance proportional to velocity), and x represents a restoring force. The coefficients (3, 1, 1) determine the system's behavior, leading to damped oscillations as seen in the trajectory spiraling inward toward the origin.

The graph displays a phase portrait on top and a corresponding time series plot at the bottom for a system involving a variable x and its derivative x' (denoted as 'y' in the phase portrait). The grey arrows indicate the direction and relative magnitude of the flow in the phase plane at different points (x, x'). The blue curve shows the trajectory of the system in the phase plane, starting from the purple dot. The red dot marks a later state of the system along the trajectory which users can move to see the change in the time series plot.

In the bottom Time Series Plot (x vs. t), the horizontal axis represents time t and the vertical axis represents the value of the variable x. The pink curve shows how the value of x changes over time, corresponding to the blue trajectory in the phase portrait. Each point on the blue phase trajectory corresponds to a (x, x') state at a specific time t, which is reflected vertically in the time series plot as (t, x). Moving the red dot along the phase trajectory updates the time series to highlight how $x(t)$ evolves quantitatively over time.

Student view

2 section questions

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5. Calculus / 5.18 Second order differential equations

Solving second-order differential equations

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/solving-secondorder-differential-equations-id-27944/print/) Assign

In subtopics 5.16 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27933/) and 5.17 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27938/), you learned how to find the solution to a system of two differential equations numerically through Euler's method, analytically through separation of variables, and qualitatively through the use of phase portraits. In section 5.18.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/secondorder-differential-equations-id-27943/), you learned how to convert a second-order differential equation into a system of two first-order differential equations. Now, the final step is to put all of this together. When you need to solve a second-order differential equation, one of the most straightforward techniques is to first convert it to a system of first-order equations and then solve the system.

Exam tip

Conversion from a second-order differential equation to a system of first-order differential equations may be required on the exam.

Example 1

★★★

Consider the differential equation:

$$2\frac{\mathrm{d}x^2}{\mathrm{d}t^2} + \frac{\mathrm{d}x}{\mathrm{d}t} + 6x = 0$$

Student view

(a) Convert the equation to a system of two first-order differential equations.

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(b) U sing Euler's method with an initial value of $(x_0, x'_0) = (2, 0)$ and $\Delta t = 0.1$, find an approximate value of x at $t = 2.0$.

(c) In a coordinate system, plot the points you used in the steps in the previous part. Using a phase portrait plotter with an initial value of $(x_0, y_0) = (2, 0)$, classify and comment on the solution.

(a) Convert to system of first-order differential equations:

$$2\frac{\mathrm{d}x^2}{\mathrm{d}t^2} + \frac{\mathrm{d}x}{\mathrm{d}t} + 6x = 0$$

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}t} - 3x$$

Set $y = \dfrac{\mathrm{d}x}{\mathrm{d}t}$

Therefore $\dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$

Substitute

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}t} - 3x$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{1}{2}y - 3x$$

Coupled system:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{1}{2}y - 3x$$

Student view

(b) Euler's method:

Using the methods from **subtopic 5.16** with either a calculator or a computer:

k	t	x_k	y_k	x'_k	
0	0	2	0	0	
1	0.1	2	-0.6	-0.6	–
2	0.2	1.94	-1.17	-1.17	$-$!
3	0.3	1.823	-1.6935	-1.6935	$-4.$
4	0.4	1.65365	-2.15573	-2.15573	$-3.$
5	0.5	1.438078	-2.54403	-2.54403	$-3.$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
19	1.9	-1.66788	0.473349	0.473349	4.7
20	2	-1.62055	0.950046	0.950046	4.3

At $t = 2.0, (x, y) = (-1.62055, 0.950046)$

$x = -1.62$

(c) Phase portrait:

- The diagram below shows the points that are used in the Euler method to approximate a solution. You can use a graphing calcultor to draw a scatter plot like this. You are expected to be able to do this in an exam.
- The diagram also shows the phase portrait and a trajectory. You graphing calculator does not draw these for you. However, based on the scatter plot, you are expected to be able to sketch trajectories to show the phase portrait.
- The trajectory appears to be spiralling inwards. This would be classified as a spiral sink and the origin would be considered a stable equilibrium

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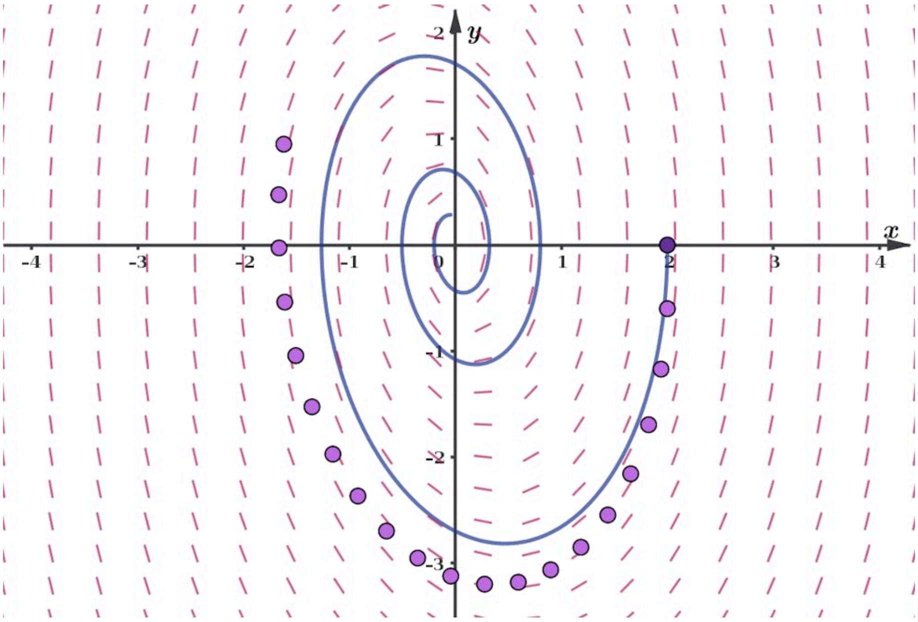
point.

- The first coordinates of the points on the trajectory give the solution to the original second order differential equation. From the diagram it looks that this solution is oscillating and approaching 0.



Activity

Let's now revisit the Van der Pol oscillator equation from the previous section.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - \mu(1-x^2)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

On the applet below you can see a trajectory in the $x, y = x'$ plane, the phase portrait and the first 50 steps of the approximation of the trajectory using Euler's method.

- Move the red point to adjust the initial condition.
- Move the slider to adjust the parameter in the equation.

Student view

- Comment on what you see.

Interactive 1. Phase Portrait of a Second-Order Differential Equation Solution.

More information for interactive 1

This interactive visualizes the phase portrait of a second-order differential equation, specifically modeling the Van der Pol oscillator. The graph is plotted in the xy-plane, where the horizontal axis represents the variable x (ranging approximately from -6 to 6), and the vertical axis represents y (ranging from -4 to 4). The background contains numerous small gray arrows forming a vector field, representing the direction of change for the system at each point in the plane.

A blue curve shows a specific trajectory (solution) of the system over time, determined by an initial condition. A red dot marks this initial point and can be dragged to change the starting position. Along the blue curve, a sequence of purple circles marks the system's state at discrete time steps, helping users visualize how the solution evolves over time. Regions with more densely packed dots indicate slower movement, while sparser regions show faster evolution. In the top-right corner of the screen, there is a horizontal slider labeled μ (mu). This slider allows users to adjust the value of μ, a key parameter in the Van der Pol equation. μ controls the nonlinearity and damping of the oscillator:

- When μ is small, the system behaves more like a simple harmonic oscillator.
- As μ increases, the system exhibits nonlinear oscillations and converges to a limit cycle, regardless of the starting point (as long as it is not at the origin, the unstable equilibrium).

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For example, when $\mu = 0.8$ and the initial condition is placed away from the origin, the trajectory initially spirals outward but is gradually attracted to a stable closed loop. This closed trajectory, or limit cycle, is characteristic of self-sustained oscillations. The purple dots track the progression, showing how the system evolves toward and eventually follows this repeating path. Users can explore the system's dynamics by moving the red initial condition dot to different locations and also by adjusting the μ-slider to observe how the shape and stability of the trajectory change. This interactive helps users understand the Van der Pol oscillator, a nonlinear second-order differential equation used to model real-world systems with self-sustained oscillations, such as electrical circuits, biological rhythms, and cardiac behavior.

So far you saw methods to find approximate solutions. In the next example you see the method when you are expected to be able to find exact solution.

Exam tip

You need to know how to find exact solutions when the equation is of the form

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + a\frac{\mathrm{d}x}{\mathrm{d}t} + bx = 0.$$

Example 2

★★★

A mass is attached to a spring and the system is immersed in a thick liquid. The spring is stretched 4 centimetres from its natural length and released from rest.

The differential equation modelling the stretch of the spring is the following:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = 0$$

In this equation the $x = x(t)$ represents the stretch of the spring from its natural position in centimetres, t seconds after the release of the mass.

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(a) The natural length of the spring is 8 centimetres. How long is the spring after 2 seconds?

(b) How fast is the mass moving after 2 seconds?

(c) How long does it take before the spring is less than 8.1 centimetres long?

You can answer these questions by finding the solution to the differential equation given in the question. Let's write the corresponding coupled system first.

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= y\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -3x-4y\end{aligned}$$

The matrix corresponding to this system contains the coefficients.

$$\begin{pmatrix}0 & 1\\ -3 & -4\end{pmatrix}$$

The characteristic polynomial is
$\det(B-\lambda I)=(-\lambda)(-4-\lambda)-1(-3)=\lambda^2+4\lambda+3$

The eigenvalues are given by setting this equal to 0 and solving the equation.

$$\begin{aligned}\lambda^2+4\lambda+3&=0\\ \lambda&=-1,-3\end{aligned}$$

The corresponding eigenvectors are multiples of $\begin{pmatrix}1\\ -1\end{pmatrix}$ and $\begin{pmatrix}1\\ -3\end{pmatrix}$.

This gives the general solution to the system.

$$\begin{aligned}x&=A\mathrm{e}^{-t}+B\mathrm{e}^{-3t}\\ y&=-A\mathrm{e}^{-t}-3B\mathrm{e}^{-3t}\end{aligned}$$

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You can use the given initial conditions to set up a system of equations for the coefficients.

The initial stretch is 4 centimetres.	$A\mathrm{e}^{-0}+B\mathrm{e}^{-3\times 0}=A+B=4$
The starting velocity is $0\ \mathrm{cm/s}$.	$-A\mathrm{e}^{-0}-3B\mathrm{e}^{-3\times 0}=-A-3B=0$

The solution of this system is $A=6$ and $B=-2$, so the equation that models the motion of the mass is

$$x(t)=6\mathrm{e}^{-t}-2\mathrm{e}^{-3t}.$$

Let's use this equation to answer the questions.

(a) After 2 seconds the stretch is $x(2)=0.8070542\ldots$ centimetres, so the string is 8.81 centimetres long.

(b) The velocity is $y(t)=-6\mathrm{e}^{-t}+6\mathrm{e}^{-3t}$, so after 2 seconds the speed of the mass is $|y(2)|=0.797$ centimetres per second.

(c) The length drops below 8.1 centimetres when the stretch drops below 0.1 centimetres.

$$x(t)=6\mathrm{e}^{-t}-2\mathrm{e}^{-3t}<0.1$$

You can use a calculator to solve this inequality. The solution is $t>4.09$. It takes 4.09 seconds for the length of the spring drop below 8.1 centimetres.

3 section questions

Student view

5. Calculus / 5.18 Second order differential equations

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Checklist

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What you should know

By the end of this subtopic you should be able to:

- understand second-order differential equation models in context
- find approximate solutions to second-order differential equations of the form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = f(x, \frac{\mathrm{d}x}{\mathrm{d}t}, t)$
  - convert second-order differential equations into a coupled system of first-order differential equations $\frac{\mathrm{d}x}{\mathrm{d}t} = y$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = f(x, y, t)$.
  - use Euler's method to find approximate solutions.
- use phase portrait method to investigate solutions when the equation has the special form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + a\frac{\mathrm{d}x}{\mathrm{d}t} + bx = 0$.

5. Calculus / 5.18 Second order differential equations

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-27946/print/) Assign

In this course you learn about second order differential equations, but the methods you see can be applied in other situations. For example, let's look at the Clohessy-Wiltshire equations.

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$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 3n^2x + 2n\frac{\mathrm{d}y}{\mathrm{d}t}$$
$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -2n\frac{\mathrm{d}x}{\mathrm{d}t}$$
$$\frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -n^2z$$

This is a second order differential equation system describing the change of the three coordinates of a point in space. Using appropriate parameters, these equations are helpful in planning the docking of spaceships.

Let's look at a simplified form with parameter $n = 1$ and restricting the equation to a plane where only the x and y coordinates are involved.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 3x + 2\frac{\mathrm{d}y}{\mathrm{d}t}$$
$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -2\frac{\mathrm{d}x}{\mathrm{d}t}$$

- Introduce new functions $u = \frac{\mathrm{d}x}{\mathrm{d}t}$ and $v = \frac{\mathrm{d}y}{\mathrm{d}t}$ and set up a differential equation system consisting of four first order equations.
- Use Euler's method to approximate the trajectories of the equation system in two dimensions. Your calculator will not be able to do this, since it cannot handle four sequences at a time, so you will need to use a computer.

The applet below shows the (x_n, y_n) points Euler's method gives when the solution starts at $(0, 0)$. Moving the red point allows you to change the starting velocity. Can you get a similar plot?

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Interactive 1. Exploring Trajectories: A Study of Second Order Differential Equations.

More information for interactive 1

This interactive provides a dynamic visualization of the Clohessy-Wiltshire equations, specifically focusing on the motion of a point in a two-dimensional plane using a simplified version of the equations. By transforming the original second-order differential equations into a system of four first-order equations and applying Euler's method for numerical approximation, the applet helps illustrate the trajectory of an object (such as a spacecraft) influenced by relative orbital dynamics.

A graph is displayed with the xy-plane where x-axis ranging from -2 to 2 and y-axis ranging from -5 to 1. At the origin (0,0), where the simulation begins, a red movable point represents the initial velocity vector. The trajectory points generated by Euler's method are shown as a series of closely spaced purple dots, forming a spiral or looping pattern depending on the velocity input. These dots represent the successive (x_n, y_n) positions calculated step-by-step, demonstrating how the motion evolves over time according to the governing equations.

Users can drag the red point to change the initial velocity. As the red point is moved, the trajectory dynamically updates, revealing how different initial conditions affect the overall motion. For instance, a slight adjustment of the red point towards the origin might show a tight inward spiral, while taking it away from the origin, a more significant change could cause the pattern to stretch out or shift direction, helping users intuitively explore the system's behavior. This interactivity highlights the sensitivity of the trajectory to initial velocities and provides instant visual feedback on those changes.

By engaging with this tool, users gain a deeper understanding of both the mathematical structure and

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view

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practical implications of differential equation systems in modeling physical phenomena, particularly in astrodynamics. They also develop insights into numerical methods like Euler's method and how they approximate continuous systems step-by-step.

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