3.13 Products of vectors

Checklist

What you should know

By the end of this subtopic you will be able to:

- define the scalar product as directional multiplication of vectors: $\boldsymbol{v} \cdot \boldsymbol{w} = v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3$, where $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$
- define the scalar product as the projection of one vector onto another: $\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}|\,|\boldsymbol{w}|\cos\theta$, where θ is the angle between the vectors $\boldsymbol{v}$ and $\boldsymbol{w}$
- recall that if $\boldsymbol{a} \cdot \boldsymbol{b} = 0$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular and, conversely, if $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, then $\boldsymbol{a} \cdot \boldsymbol{b} = 0$
- recall that if $\boldsymbol{u}$ and $\boldsymbol{v}$ are parallel, then $\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}|\,|\boldsymbol{w}|$
- recall that the angle between two straight lines is given by the angle between their direction vectors:
  - if $\boldsymbol{b}$ and $\boldsymbol{d}$ are the direction vectors of two straight lines, then the angle θ between these lines is given using the scalar product as $\theta = \cos^{-1}\left(\frac{\boldsymbol{b} \cdot \boldsymbol{d}}{|\boldsymbol{b}||\boldsymbol{d}|}\right)$
  - recall that the angle between two straight lines is usually given as the acute angle not the obtuse angle
- recall that the vector product of vectors $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ is denoted by $\boldsymbol{v} \times \boldsymbol{w}$
- calculate the vector product from the components of $\boldsymbol{u}$ and $\boldsymbol{v}$ using the formula

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$$

- recall that the vector product of two vectors $\boldsymbol{v} \times \boldsymbol{w}$ is oriented perpendicular to the plane containing $\boldsymbol{v}$ and $\boldsymbol{w}$
- define the vector product as $|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}|\,|\boldsymbol{w}|\sin\theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
- recall that a unit vector that is perpendicular to vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ is given by

$$\widehat{\boldsymbol{n}} = \frac{\boldsymbol{v} \times \boldsymbol{w}}{|\boldsymbol{v} \times \boldsymbol{w}|}$$

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- recall that the area of a parallelogram can be calculated using

$$|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}|\,|\boldsymbol{w}| \sin\theta$$

- recall that the area of a triangle can be calculated using $\frac{1}{2}\,|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}|\,|\boldsymbol{w}| \sin\theta$
- recall that the component of vector $\boldsymbol{a}$ acting in the direction of vector $\boldsymbol{b}$ is given by

  $\frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{b}|} = |\boldsymbol{a}| \cos\theta$, where θ is the acute angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

- recall that the component of vector $\boldsymbol{a}$ acting perpendicular to vector $\boldsymbol{b}$ in the plane formed by the two vectors is given by

  $\frac{|\boldsymbol{a} \times \boldsymbol{b}|}{|\boldsymbol{b}|} = |\boldsymbol{a}| \sin\theta$, where θ is the acute angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

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