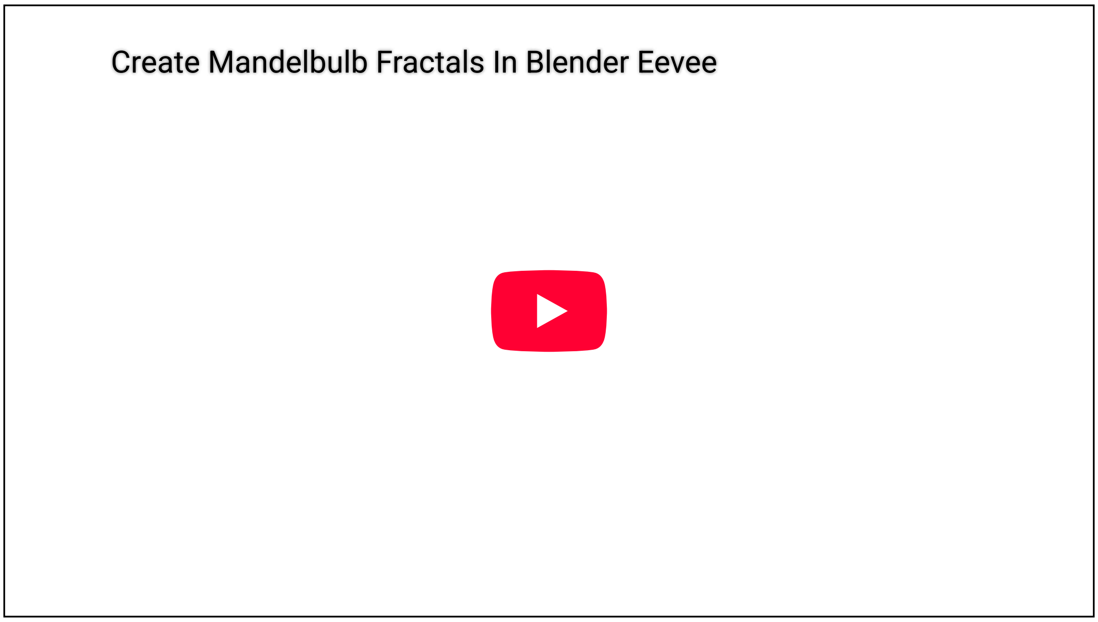
Index

The big picture

In this subtopic you will revisit the trigonometric ratios and the graphs of trigonometric functions and explore their applications in real-life contexts. You will define some new trigonometric ratios and find out about inverse trigonometric functions and their graphs.

Inverse trigonometric functions have many applications, including in computer graphics. This video shows a three-dimensional shape called a Mandelbulb, which can be generated from trigonometric functions using graphics software.



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Concept

In this subtopic you will define new relationships between sides of polygons and trigonometric ratios of angles. You will also describe the relationship between trigonometric functions and their inverses.

Theory of Knowledge

Solving equations can sometimes seem formulaic (pun intended!) and void of creativity. Does mathematics lack creativity?

More importantly, a key knowledge question is, ‘Is the degree of creative freedom within an area of knowledge positively correlated with the expansion of knowledge within that AOK?’

3. Geometry and trigonometry / 3.9 Reciprocal trigonometric ratios and inverse trigonometric functions

Reciprocal trigonometric ratios

The diagram below shows the relationship between the three primary trigonometric ratios of an angle and line segments defined on a unit circle.

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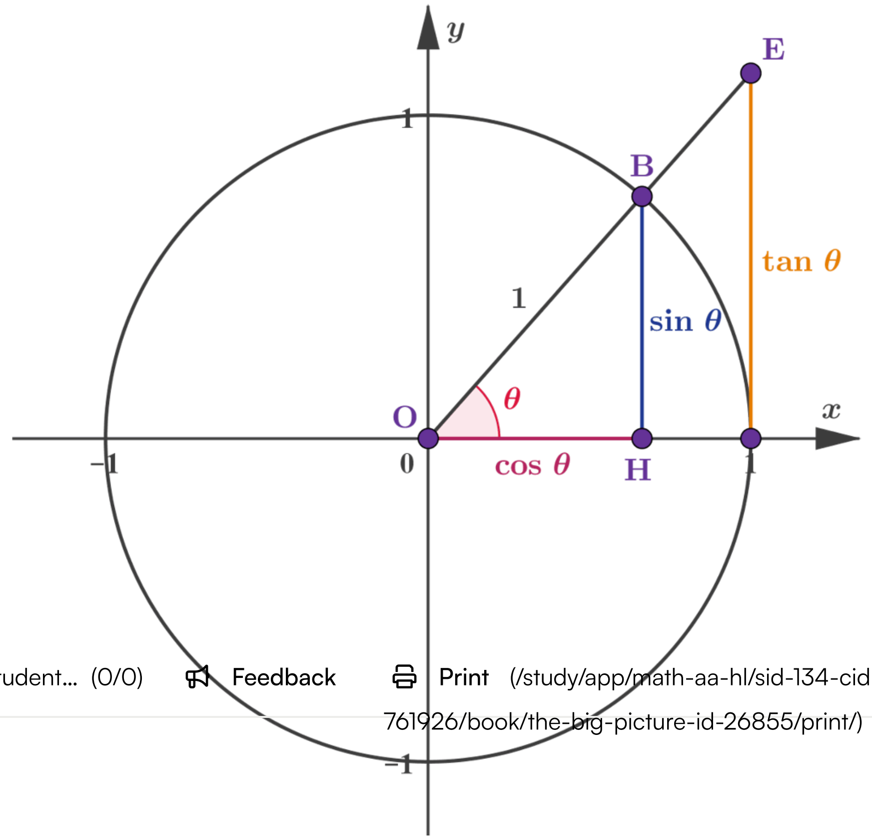

More information

The diagram depicts the relationship between the three primary trigonometric ratios—sine, cosine, and tangent—in a unit circle. Centered at the origin, point O(0,0), the circle has a radius of 1 unit. The angle theta (θ) is marked in red within the first quadrant.

The line segment from O to H represents the cosine (cos θ) of the angle, extending horizontally along the x-axis. The sine (sin θ) of the angle is the vertical line segment from H to B, indicating the height above the x-axis. The tangent (tan θ) of the angle is the line segment from H extended to the line joining point E above B, which represents the line extending infinitely (or externally tangent to the unit circle at point B).

Points are labeled as O (origin), H (intersection on x-axis), B (intersection of the radius on the unit circle above H), and E (extension above B). The y-axis and x-axis are labeled with values ranging from -1 to 1, reflecting the unit circle's diameter and positioning in the Cartesian plane.

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The new trigonometric ratios that you will learn about are based on these three primary ratios.

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The reciprocal of a fraction $\frac{a}{b}$, where $a \neq 0$ and $b \neq 0$, is $\frac{b}{a}$. In this section you will be learning about the reciprocals of the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$.

Reciprocal of $\cos\theta$

Similar triangles OEH and OFG on a unit circle, are shown in the diagram below.

If you write the similarity ratios for both triangles

$$\frac{\mathrm{OE}}{\mathrm{OF}} = \frac{\mathrm{EH}}{\mathrm{FG}}$$

$$\frac{1}{\mathrm{OF}} = \frac{\sin\theta}{\tan\theta}$$

$$\mathrm{OF} = \frac{\tan\theta}{\sin\theta}$$

As $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\mathrm{OF} = \frac{\sin\theta}{\cos\theta \cdot \sin\theta} = \frac{1}{\cos\theta}$$

$$\mathrm{OF} = \frac{1}{\cos\theta}$$

This ratio, $\frac{1}{\cos\theta}$, is called secant of the angle θ, and is denoted by $\sec\theta$.

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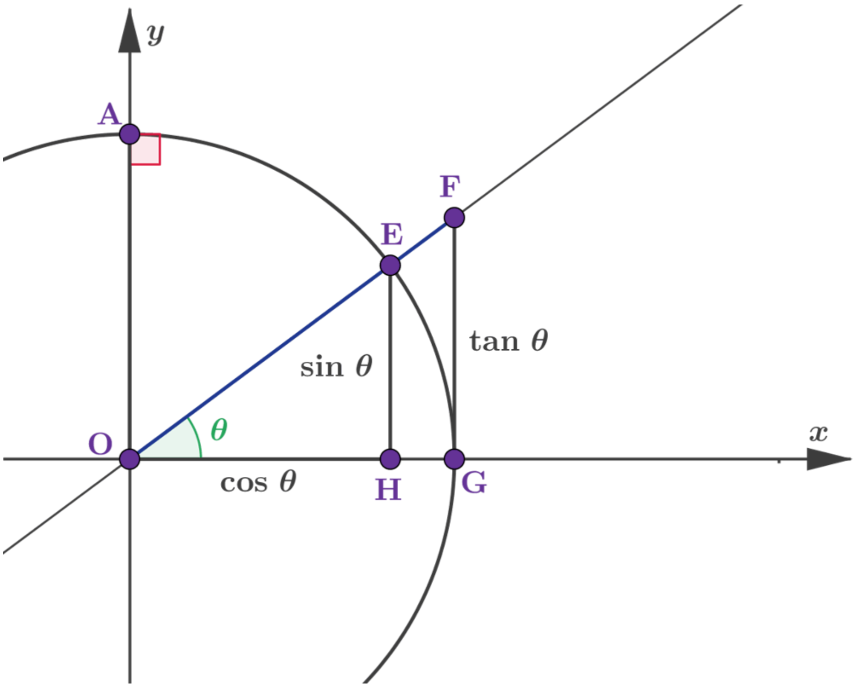

More information

The image is a diagram of a unit circle with trigonometric relationships displayed, primarily focusing on an angle θ. The unit circle is centered at point O. Several points and lines are marked on or outside the circle:

- Point A is on the y-axis perpendicular above O on the circumference.
- Point E is on the circumference to the right of O making a triangle with F on the x-axis line. Line OF represents the secant of angle θ.
- Points along the x-axis, from left to right, are marked as O, H, and G.
- The right triangle formed includes:
- The radius (line OA) of the circle, representing the hypotenuse.
- Line OH is the adjacent side and corresponds to cos θ.
- Line AE represents sin θ.
- Line FG represents tan θ, extending from point F on the tangent line to the intersection of FG and the x-axis (point G).

The diagram illustrates the sine, cosine, tangent, and secant values based on the constructed angle θ created with central point O and radii lines intersecting with the x and y axes.

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Reciprocal of $\sin\theta$

Triangles OEH and BOA are similar, as shown in the diagram below.

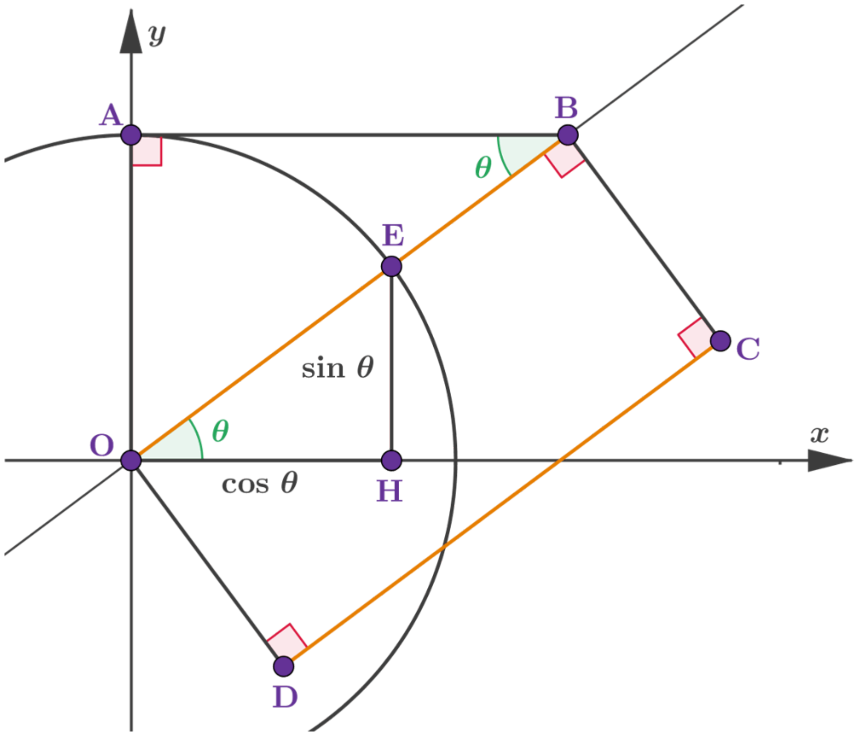
If you write the similarity ratios for both triangles

$$\frac{\mathrm{OE}}{\mathrm{OB}} = \frac{\mathrm{EH}}{\mathrm{OA}}$$

$$\frac{1}{\mathrm{OB}} = \frac{\sin\theta}{1}$$

$$\mathrm{OB} = \frac{1}{\sin\theta}$$

This ratio, $\frac{1}{\sin\theta}$, is called cosecant of the angle θ, and is denoted by $\operatorname{cosec}\theta$.



More information

The diagram illustrates the geometric representation of trigonometric ratios in a unit circle. It contains a coordinate plane with x and y axes intersecting at the origin labeled as O. There are several labeled points (A, B, C, D, E, H) and lines connecting these points.\n\nPoint A is positioned on the y-axis. Points B and C lie on a line that extends diagonally through the coordinate plane. A notable point B is connected to the origin O, forming an angle labeled θ. Point E is positioned on the line between O and C. The segment OB forms an angle θ at point O, and adjacent to OB

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is the perpendicular OC, identifying triangle OBC. \n\nThe segment OE represents the adjacent side (cos θ), and the segment AE represents the opposite side (sin θ) of the right triangle OAE within the circle. The different colored sections and right angles are marked with small squares, indicating 90-degree angles at specific points (A, B, C, and D).\n\nThe circle itself helps visualize the relationship between the angles and sides, showing how the sine and cosine relate to the x and y coordinates.

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Reciprocal of $\tan\theta$

Triangles OEH and BOA are similar, as shown in the diagram below.

If you write the similarity ratios for both triangles

$$\frac{\text{OH}}{\text{AB}}=\frac{\text{EH}}{\text{AO}}$$

$$\frac{\cos\theta}{\text{AB}}=\frac{\sin\theta}{1}$$

$$\text{AB}=\frac{\cos\theta}{\sin\theta}$$

The ratio $\frac{\cos\theta}{\sin\theta}$ is the reciprocal of $\tan\theta$. It is called the cotangent of angle θ and is denoted by $\cot\theta$.

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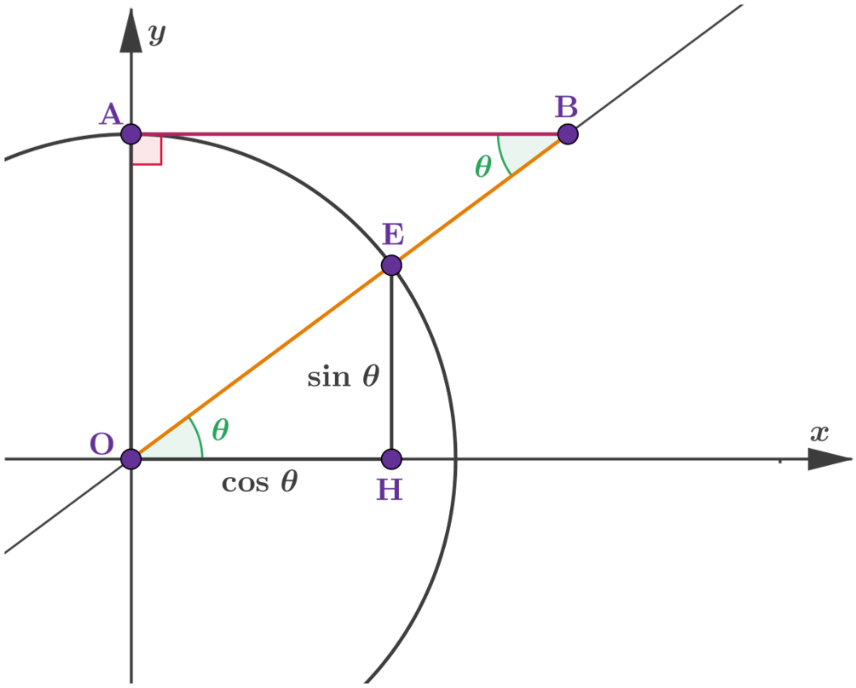

More information

The diagram represents a unit circle with a central angle θ. The main components include points O, A, B, E, and H. Point O is the center of the circle, and the radius is equal to 1. Point A is the northernmost point on the circle, lying on the y-axis. The line segment OA is perpendicular to the line segment OB, forming a right triangle OAB inside the unit circle.

The angle ∠AOB is marked as θ. Line segment OE represents the radius of the circle and is labeled as the hypotenuse. Line OH is the adjacent side of angle θ along the x-axis, labeled as "cos θ," and extends from O to H. Line AH is the length along the y-axis from A to H labeled as "sin θ." The line BE extends horizontally, intersecting the hypotenuse OE at point E, marking the opposite side of angle θ.

This visual representation of the unit circle shows the relationships between the sine, cosine, and associated ratios, including cotangent and tangent, following the placement and length of segments derived from right triangle relationships within the unit circle.

Arrows indicate the positive directions of the x and y axes. The segment extending from E along the radius intersection adjacent to the unit circle creates a right angle with the x-axis. The angles and triangles depict how trigonometric functions relate to the unit circle.

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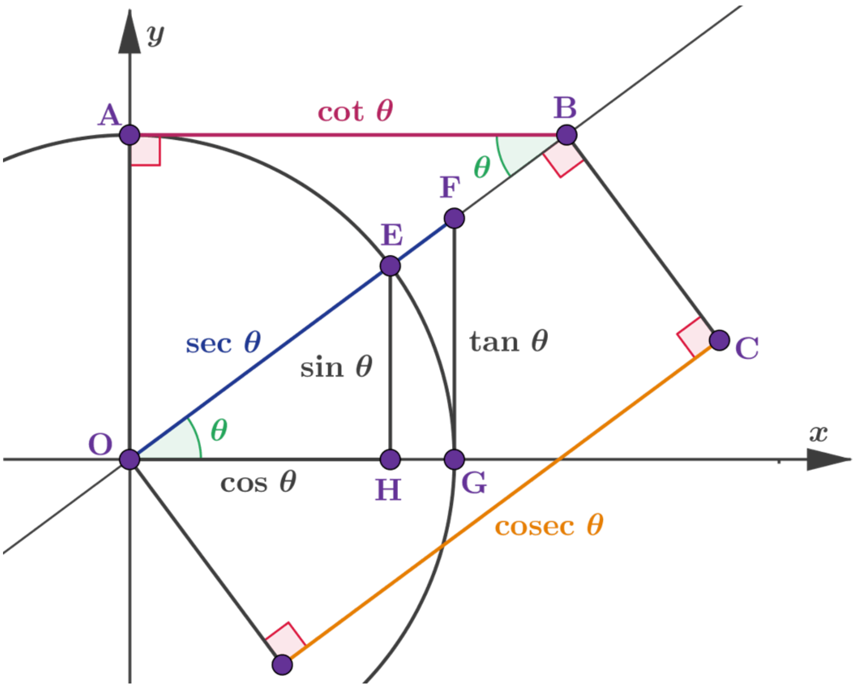
In the diagram below, you can see all six trigonometric ratios on a unit circle for a central angle θ .



More information

The image is a diagram illustrating the six trigonometric ratios on a unit circle for a central angle θ. It depicts a circle centered at point O, intersecting the x-axis at point H and the y-axis at point A. A right triangle is formed at the origin O with angle θ. Various key points such as B, E, F, G, and C are marked along the lines representing the trigonometric ratios.

Lines and arcs are labeled with different trigonometric functions: - Point A to B is horizontal, labeled as 'cot θ'. - Line OB is the radius, and forms an angle θ with the positive x-axis. - Segment OF is labeled 'sec θ'. - Segment OC is the extension of OB, labeled 'cosec θ'. - Horizontal projection OH is marked 'cos θ'. - Vertical projection OE is labeled 'sin θ'. - Line CF is diagonal, and the projection from F to G on the x-axis is labeled 'tan θ'.

The diagram shows key relationships of these trigonometric ratios involving the angle θ with respect to the unit circle, depicting how each ratio is represented geometrically.

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Activity

Consider the given unit circle below where right angles and the intersection of the line segments are marked.

Interactive 1. Reciprocal Trigonometric Ratios and Inverse Functions.

More information for interactive 1

This interactive visual representation provides a comprehensive overview of how the six trigonometric ratios—sine, cosine, tangent, cosecant, secant, and cotangent—relate to one another geometrically on the unit circle.

A graph is displayed on the xy-axis. The x-axis ranges from -2 to 3, and the y-axis ranges from -2 to 2, respectively. A circle centered at the origin of the graph O has a radius of 1 unit. The circle intersects the x-axis at point G and the y-axis at point A. A line passing through the center at an angle theta with the positive x-axis has a movable red point E on the circumference of the circle. A line parallel to the y-axis from point G on the x-axis is extended to line OE to meet at point F. A perpendicular dropped from point E to the x-axis creates a right-angled triangle with angle EOG as θ and the line opposite to it as $\sin\ \theta$, the line from the origin to the point where the perpendicular intersects the x-axis is represented as $\cos\ \theta$, and the line FG represents $\tan\ \theta$, denoted in blue.

A line parallel to the x-axis extending from point A cuts the extended line OE at point B. OBCD forms a rectangle where point D is movable. The rectangle OBCD offers further insight into how secant, cosecant, and cotangent fit into this system. The diagram projected on the graph helps to bridge the understanding between basic trigonometric

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ratios and their reciprocal counterparts through clearly marked line segments and points of intersection.

Users can move point E on the circumference of the circle, thereby changing the value of angle θ. As point E moves, point D is dynamically adjusted so that the line segment OD remains perpendicular to OB, maintaining the geometric integrity of the trigonometric relationships. For example, as θ increases from 30 to 60, a corresponding change in the length of the tangent and cotangent segments deepens their intuitive understanding of these ratios.

Through this activity, users will gain an enriched understanding of how all six trigonometric ratios are interconnected in terms of coordinates and segment lengths on the unit circle. The diagram enhances comprehension by linking algebraic expressions with geometric representation, reinforcing the meaning of each trigonometric function and its reciprocal.

Find a triangle similar to triangle OAB . Hence, find the length of line segments AB , DC and OB in terms of trigonometric ratios of angle θ.

Find a triangle that is similar to triangle OFG . Hence find the length of line segment OF in terms of trigonometric ratios of angle θ.

Exam tip

In IB examinations, the following reciprocal trigonometric ratios will be given in the formula booklet.

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

Important

The reciprocal of the tangent ratio will not be given in the formula booklet.

You can use this right-angled triangle.

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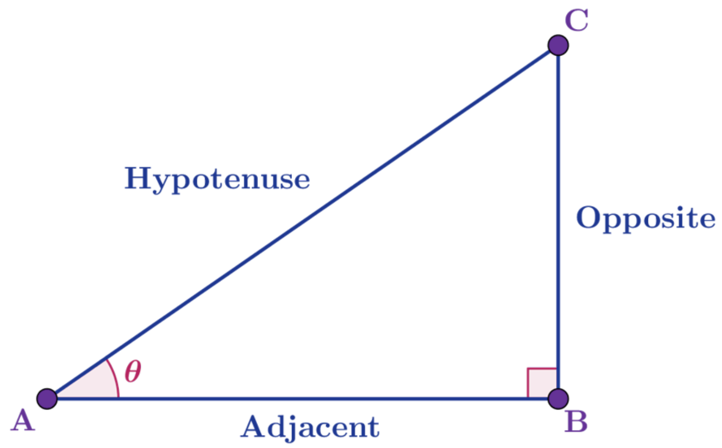

More information

The image is a diagram of a right-angled triangle labeled at each corner with points A, B, and C. The side labeled 'Hypotenuse' runs from point A at the bottom left to point C at the top. The 'Adjacent' side is from point A to point B at the bottom right. The 'Opposite' side goes from point B to point C vertically on the right side. The right angle is marked at point B. An angle θ (theta) is shown at point A, indicating the angle opposite the 'Opposite' side. Each side's name ('Hypotenuse', 'Adjacent', 'Opposite') helps in understanding trigonometric relationships in the triangle, relevant for the formula provided: (\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}).

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$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}} = \frac{\cos\theta}{\sin\theta}$$

Be aware

The notation a^{-1} is not used for reciprocal trigonometric ratios, so that there is no confusion between the inverse trigonometric ratios and reciprocals.

Example 1

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Given that $\cos\theta = a$ for $0 \leqslant \theta < \frac{\pi}{2}$, find the following in terms of a.

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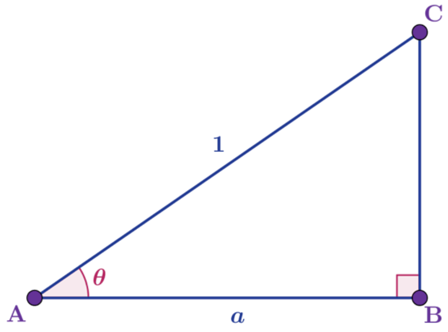
a) $\sec\theta$

b) $\cot\theta$

	Steps	Explanation
a)	$\sec\theta = \frac{1}{a}$	$\sec\theta$ is the reciprocal of $\cos\theta$.
b)	$\cot\theta = \frac{1}{\tan\theta} = \frac{a}{\sqrt{1-a^2}}$ Therefore, $\cot\theta = \frac{1}{\tan\theta} = \frac{a}{\sqrt{1-a^2}}$	Using the right-angled triangle:  and Pythagoras' theorem, $CB = \sqrt{1-a^2}$

Example 2

★★☆

Find the exact value of cosec $\frac{2\pi}{3}$.

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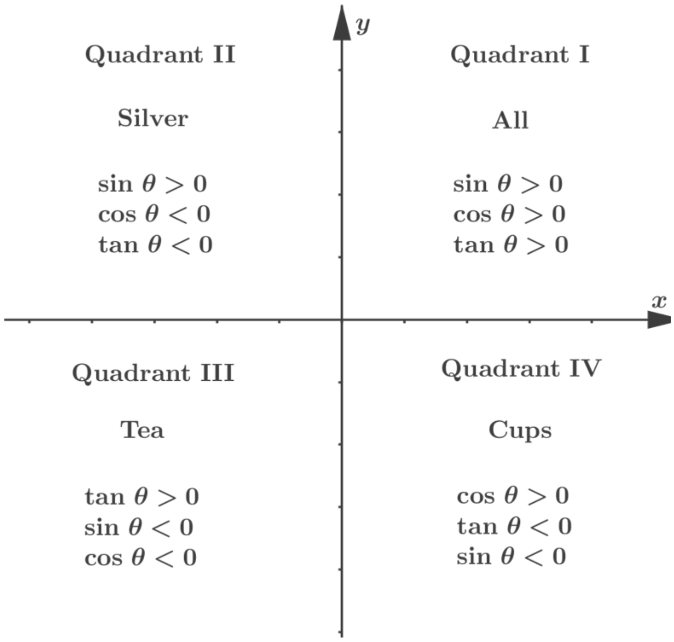
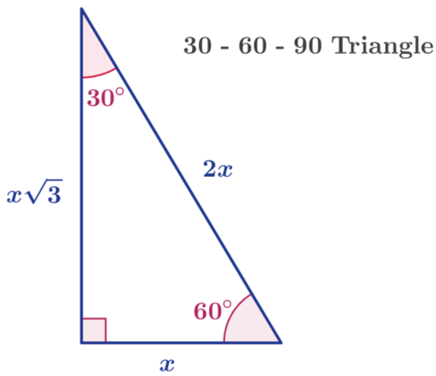
Steps	Explanation
$\text{cosec}\frac{2\pi}{3} = \frac{1}{\sin\frac{2\pi}{3}} = \frac{1}{\sin\frac{\pi}{3}}$	 $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$ which is in the second quadrant.
$\frac{1}{\sin\frac{\pi}{3}} = \frac{2}{\sqrt{3}}$	Use the special triangle: 

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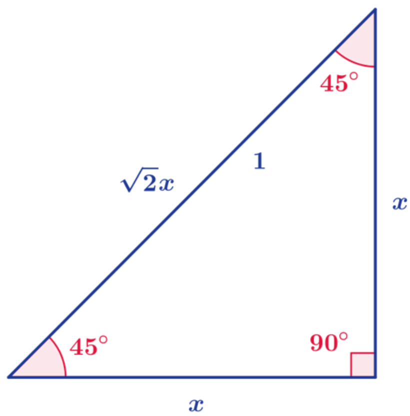
Steps	Explanation
Therefore, $$\text{cosec}\frac{2\pi}{3}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$$	

Example 3

★★☆

Find the exact value of $\sec\frac{5\pi}{4}$.

Steps	Explanation
$\sec\frac{5\pi}{4}=\frac{1}{\cos\frac{5\pi}{4}}=-\frac{1}{\cos\frac{\pi}{4}}$	$\frac{5\pi}{4}=\pi+\frac{\pi}{4}$ which is in the third quadrant where the cosine is negative.
$-\frac{1}{\cos\frac{\pi}{4}}=-\sqrt{2}$	Using the special triangle 

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Steps	Explanation
Therefore, $$\sec\frac{5\pi}{4}=-\sqrt{2}$$	

2 section questions

Question 1

★★☆

Given that $\tan\theta=m$ for $0\leqslant\theta<\frac{\pi}{2}$, find $\sec\theta$ in terms of m.

1 $\sqrt{m^2+1}$ ✓

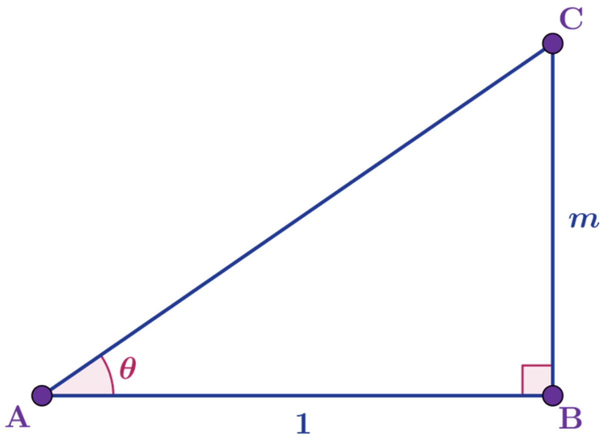
2 $\sqrt{1-m^2}$

3 $\frac{1}{\sqrt{m^2+1}}$

4 $\frac{1}{\sqrt{m^2-1}}$

Explanation

Using the given ratio and Pythagoras:



More information

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$$AC = \sqrt{m^2+1}$$

So $\cos\theta = \dfrac{1}{\sqrt{m^2+1}}$

$$\sec\theta = \frac{1}{\cos\theta} = \sqrt{m^2+1}$$

Therefore, the correct answer is $\sqrt{m^2+1}$

Question 2

★☆☆

Find the exact value of cot $\dfrac{7\pi}{3}$

1 $\dfrac{1}{\sqrt{3}}$ ✓

2 $\sqrt{3}$

3 $-\sqrt{3}$

4 $-\dfrac{1}{\sqrt{3}}$

Explanation

$$\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$$

so it is in the first quadrant where the cosine is positive.

$$\cot\frac{7\pi}{3} = \frac{1}{\tan\frac{\pi}{3}}$$

Using the special triangle:

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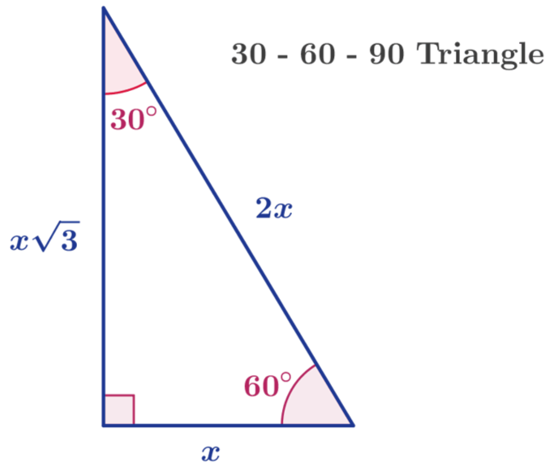

More information

$$\tan\frac{\pi}{3} = \sqrt{3}$$

$$\cot\frac{7\pi}{3} = \frac{1}{\tan\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$$

So, the correct answer is $\frac{1}{\sqrt{3}}$

3. Geometry and trigonometry / 3.9 Reciprocal trigonometric ratios and inverse trigonometric functions

Pythagorean identities

In this section, you will be extending the Pythagorean identity for the reciprocal trigonometric ratios.

Making connections

In subtopic 3.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27747/), you used the Pythagorean identity:

$$\cos^2\theta + \sin^2\theta = 1$$

along with several other identities. You may wish to refresh your memory about these before you continue.

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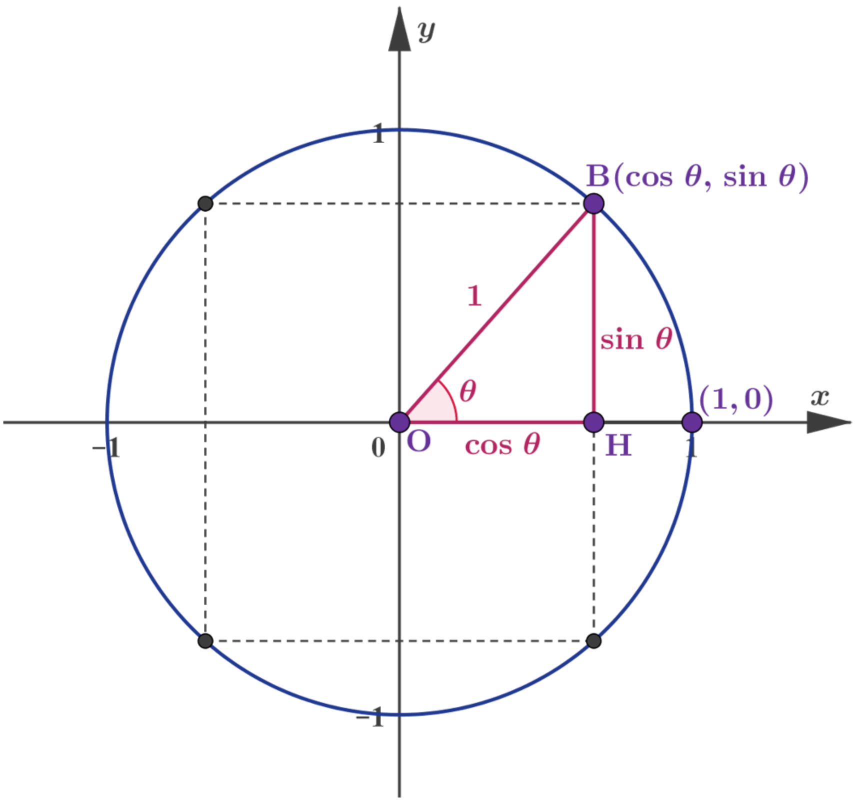

More information

The image displays a unit circle on a coordinate plane with both x and y axes ranging from -1 to 1. Inside the unit circle, a right triangle labeled (\triangle OHB) is drawn. The point (O) is the origin ((0, 0)), point (H) is at ((\cos \theta, 0)), and point (B) is at ((\cos \theta, \sin \theta)). The hypotenuse (OB) of the triangle measures 1 unit. The angle (\theta) is marked at the origin (O), and adjacent to (OH), with (\cos \theta) and (\sin \theta) being the lengths of the adjacent and opposite sides to (\theta) respectively. The diagram illustrates the relationship between the angle (\theta), the cosine and sine values, and the coordinates on the unit circle.

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In the diagram above, triangle OHB has the hypotenuse 1 unit.

Therefore, the Pythagorean identity states that, for any angle θ,

$$\cos^2\theta + \sin^2\theta = 1$$

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If you divide both sides of the Pythagorean identity by $\cos^2\theta$ you get:

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$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

Simplifying gives:

$$1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}\right)^2$$

Thus,

$$1 + \tan^2\theta = \sec^2\theta$$

What if you divide both sides of $\cos^2\theta + \sin^2\theta = 1$ by $\sin^2\theta$; which identity would you get?

Exam tip

In IB examinations, Pythagorean identities will be given in the formula booklet:

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \mathrm{cosec}^2\theta$$

Example 1

★★☆

Given that $\sec\alpha = n$ for $\pi < \alpha < \frac{3\pi}{2}$, find $\tan\alpha$ in terms of n.

Steps	Explanation
$1 + \tan^2\alpha = n^2$	Use the identity $1 + \tan^2\alpha = \sec^2\alpha$
$\tan^2\alpha = n^2 - 1$	Rearrange
$\tan\alpha = \pm\sqrt{n^2 - 1}$	Take the square root of both sides

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Steps	Explanation
Therefore, the answer is $$\tan\alpha = \sqrt{n^2-1}$$	As the angle is in the third quadrant, the tangent will be positive.

Example 2

★★★

Simplify $\dfrac{1-\sec^2\theta}{(1-\cos\theta)(1+\cos\theta)}$

Steps	Explanation
~~mfrac~~ $\dfrac{1-\left(1+\tan^2\theta\right)}{1-\cos^2\theta}$	Use $\sec^2\theta = 1+\tan^2\theta$
$\Rightarrow \dfrac{-\tan^2\theta}{\sin^2\theta}$	Expand the brackets and simplify. Use $\sin^2\theta = 1-\cos^2\theta$
$\Rightarrow \dfrac{-\dfrac{\sin^2\theta}{\cos^2\theta}}{\sin^2\theta} = -\dfrac{1}{\cos^2\theta}$	Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and simplify.
$\Rightarrow -\dfrac{1}{\cos^2\theta} = -\sec^2\theta$	
Therefore $$\frac{1-\sec^2\theta}{(1-\cos\theta)(1+\cos\theta)} = -\sec^2\theta$$	

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Example 3

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Show that $\dfrac{\cos\theta}{1-\operatorname{cosec}\theta}+\dfrac{\cos\theta}{1+\operatorname{cosec}\theta}=-2\sin\theta\tan\theta$.

Steps	Explanation
LHS: $\cos\theta\left(\dfrac{1}{1-\operatorname{cosec}\theta}+\dfrac{1}{1+\operatorname{cosec}\theta}\right)$	Factorise $\cos\theta$
$=\cos\theta\left(\dfrac{(1+\operatorname{cosec}\theta)+(1-\operatorname{cosec}\theta)}{(1-\operatorname{cosec}\theta)(1+\operatorname{cosec}\theta)}\right)$	Add the fractions by putting over a common denominator.
$=\cos\theta\left(\dfrac{2}{(1-\operatorname{cosec}^2\theta)}\right)$	Expand the brackets and simplify.
$=\cos\theta\left(\dfrac{2}{-\cot^2\theta}\right)$	Use: $$1+\cot^2\theta=\operatorname{cosec}^2\theta$$ which gives: $$1-\operatorname{cosec}^2\theta=-\cot^2\theta$$
$=\cos\theta\left(\dfrac{2}{-\dfrac{\cos^2\theta}{\sin^2\theta}}\right)=\cos\theta\left(-\dfrac{2\sin^2\theta}{\cos^2\theta}\right)$	Substitute $$-\cot^2\theta=-\frac{\cos^2\theta}{\sin^2\theta}$$
$=-\dfrac{2\sin^2\theta}{\cos\theta}=-2\sin\theta\tan\theta=\text{RHS}$	Simplify using $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$
Therefore, $$\frac{\cos\theta}{1-\operatorname{cosec}\theta}+\frac{\cos\theta}{1+\operatorname{cosec}\theta}=-2\sin\theta\tan\theta$$	

⚠ Be aware

When $a^2=b^2$ and you take square root of both sides of the equation, you get

$a=b$ or $a=-b$ (or $a=\pm b$)

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Depending on the context of the question, the answer you give could be both a and b, either a or b or neither a nor b if they are not in the given domain.

3 section questions

Question 1

★★☆

Simplify $(1 + \cot a)^2 + (1 - \cot a)^2$.

1 $2\text{cosec}^2 a$ ✓

2 $2\sec^2 a$

3 $\text{cosec}^2 a$

4 $-2\text{cosec}^2 a$

Explanation

Expand the brackets and simplify:

$$1 + 2\cot a + \cot^2 a + 1 - 2\cot a + \cot^2 a = 2 + 2\cot^2 a$$

Substitute:

$$\cot^2 a = \text{cosec}^2 a - 1$$

$$2 + 2\cot^2 a = 2 + 2\left(\text{cosec}^2 \theta - 1\right) = 2\,\text{cosec}^2 a$$

Therefore, the correct answer is $2\,\text{cosec}^2 a$

Question 2

★★☆

Simplify $(\sec\theta + 1)(\sec\theta - 1)$.

1 $\tan^2\theta$ ✓

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2 $\cot^2\theta$

3 $-\tan^2\theta$

4 $-\cot^2\theta$

Explanation

Multiply out the brackets:

$$(\sec\theta + 1)(\sec\theta - 1) = \sec^2\theta - 1$$

Rearranging the Pythagorean identity:

$$\sec^2\theta = 1 + \tan^2\theta$$

gives

$$\sec^2\theta - 1 = \tan^2\theta$$

So the correct answer is $\tan^2\theta$

Question 3

★★☆

Simplify $(1 - \operatorname{cosec}\theta)(\operatorname{cosec}\theta + 1)$.

1 $-\cot^2\theta$ ✓

2 $\cot^2\theta$

3 $\tan^2\theta$

4 $-\tan^2\theta$

Explanation

This is the difference of two squares.

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Expand the brackets:

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$$(1 - \operatorname{cosec}\theta)(\operatorname{cosec}\theta + 1) = 1 - \operatorname{cosec}^2 a$$

Rearrange the Pythagorean identity:

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

which gives

$$1 - \operatorname{cosec}^2\theta = -\cot^2\theta$$

So the correct answer is $-\cot^2\theta$

3. Geometry and trigonometry / 3.9 Reciprocal trigonometric ratios and inverse trigonometric functions

Inverse trigonometric functions

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/inverse-trigonometric-functions-id-27601/print/) Assign

Revisiting inverse functions

The inverse of a relation f could be obtained by reflecting it through the line $y = x$. However, remember that not all relations are functions, and not all the inverse relations are inverse functions. If a function is to have an inverse function, it needs to satisfy certain properties.

In the following activity, you will explore the properties necessary for a function to have inverse.

Activity

Use the following applet to explore inverse relations and functions.

Write a function and choose the natural domain.

Is the inverse relation a function? Explain why or why not?How can you ensure that the inverse relation is also function?

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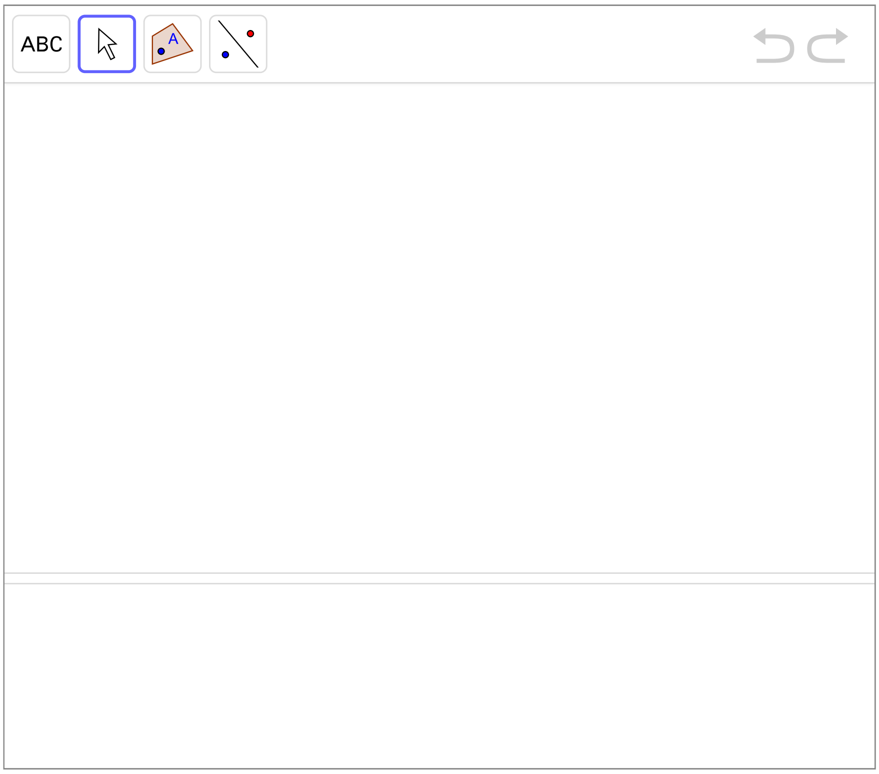

Interactive 1. Exploring Inverse Relations and Functions.
Credit: GeoGebra (https://www.geogebra.org/m/kArSVk59) Tim Brzezinski

More information for interactive 1

This interactive graph allows users to explore inverse relations and functions dynamically.
The top portion of the screen displays a coordinate plane with a visible range from approximately -16 to 20 on the x-axis and from 8 to -12 on the y-axis. A dashed line representing the identity function $y = x$ runs diagonally through the origin (0, 0), serving as a reference for inverse relationships. A “Centre” button is located at the top left of the interface. Clicking this button recenters the graph on the origin.
Below the graph, users can enter a function in the input box labeled $f(x)$. For example, entering $f(x) = 0.2x^2$ displays a blue parabolic curve that opens upward with its vertex at the origin. Two horizontal sliders labeled x start and x final allow users to interactively adjust the domain of the function. The domain can be modified between -10 and 10 using the sliders or by typing values directly into the input fields.
There are also two checkboxes:

- Show inverse relation: When selected, this displays the inverse of the current function as a red curve, reflected across the line $y = x$. This visual helps determine whether the inverse is also a function.
- Default to natural domain of f: Selecting this box resets the domain to the natural domain of the function. For example, the function $f(x) = 0.2x^2$ has a natural domain of all real numbers $(-\infty, \infty)$, and checking this option expands the visible range accordingly.

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Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/pythagorean-identities-id-27600/print/) Assign

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For instance, if the domain is set to $x = -5$ to $x = 6.6$, only that portion of the graph is displayed. Clicking Show inverse relation will then generate the inverse relation within the same restricted domain. Since $f(x) = 0.2x^2$ is not one-to-one over its full domain, the inverse is not a function—this can be visually confirmed using the horizontal line test.

This interactive activity helps users to understand the distinction between inverse relations and inverse functions, to explore how domain restrictions affect invertibility and to visualize symmetry across $y = x$.

Making connections

In section 2.14.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/inverse-of-a-function-id-26758/) , you studied functions and inverse functions. If a function f is one-to-one and onto in a given domain and range, then function g which satisfies

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/reciprocal-trigonometric-ratios-id-27599/print/) Assign

$$g \circ f(x) = f \circ g(x) = x$$

is called the inverse function of f and is written as:

$$g(x) = f^{-1}(x)$$

Consider the following graph, where $f(x) = x^2$. As $f(x)$ is not one-to-one, each output has more than one input, so its reflection in the line $y = x$ will not be a function unless you restrict the domain.

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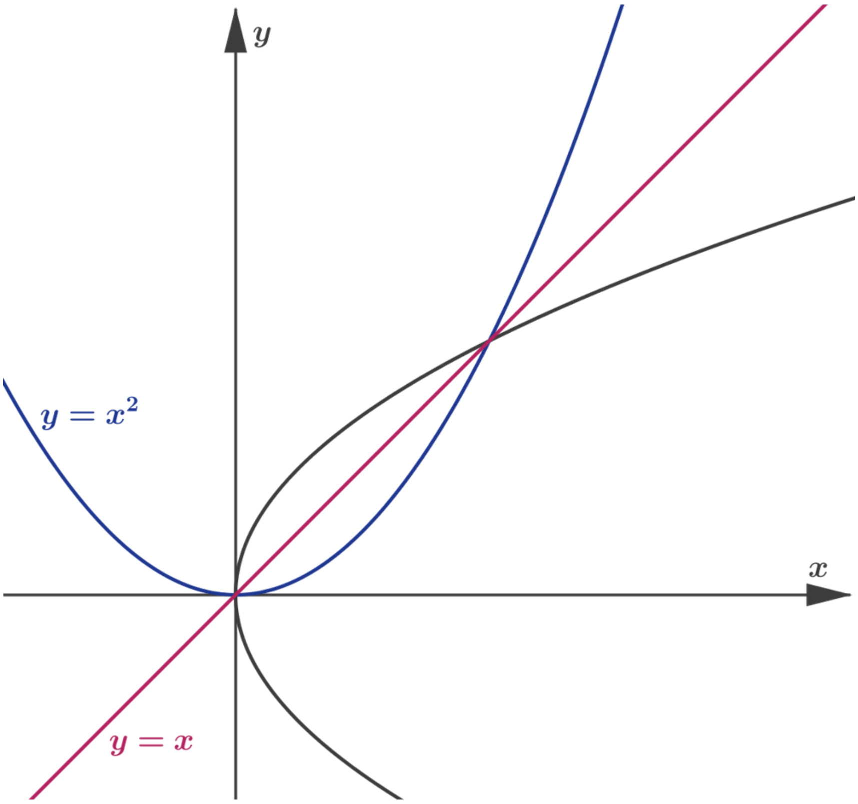

More information

The image is a graph illustrating three functions: y = x², y = x, and y = √x. The horizontal and vertical axes are labeled x and y, respectively. The parabola y = x² is shown in blue, curving upwards from the origin and only plotted for x > 0, emphasizing the restriction of the domain. The line y = x is red, running diagonally through the origin and acting as a reflection line for inverse functions. The curve y = √x is gray, starting from the origin and moving upwards, also only defined for x ≥ 0. The point where y = x² and y = √x intersect y = x is at the origin, showing where these two functions meet the line of reflection. Additional arrows on axes denote direction, and tick marks are included for scale but not numerically labeled.

[Generated by AI]

But if you restrict the domain so that for each output there is only one input, then the reflection will be a function. In the graph below, $f(x) = x^2$ for $x > 0$; therefore the inverse function is $f^{-1}(x) = \sqrt{x}$ and you can write $f^{-1} \circ f(x) = f \circ f^{-1}(x) = x$.

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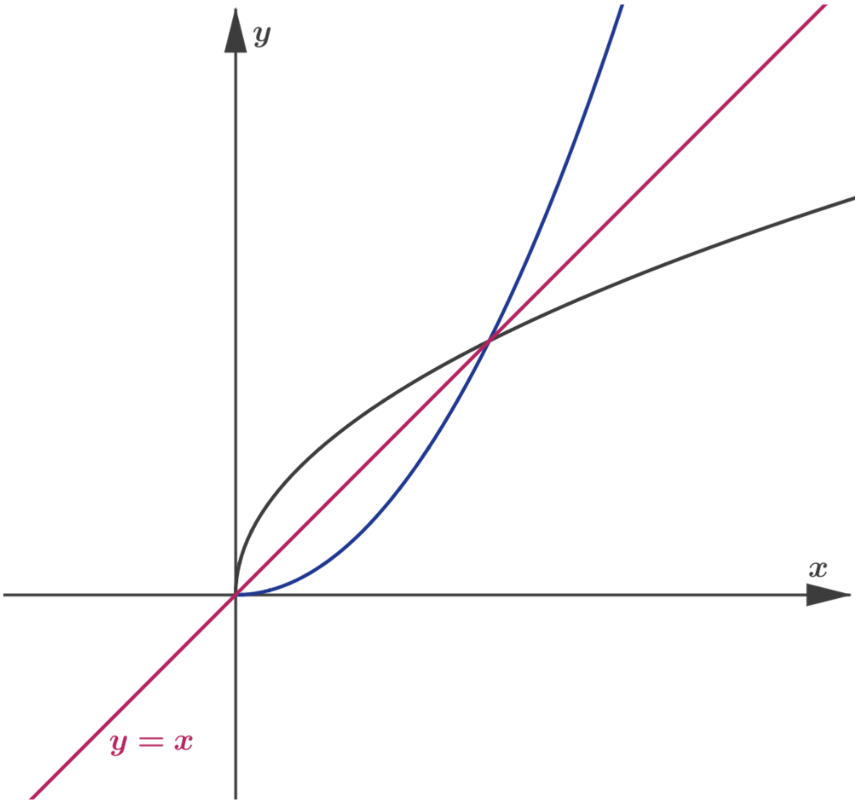

More information

The graph illustrates three distinct functions plotted on a Cartesian plane. The x-axis represents the independent variable, while the y-axis represents the dependent variable.

1. The first function, indicated in pink, is the line y = x. It is a straight line that passes through the origin and has a slope of 1.
2. The second function, represented in blue, is y = x^2. This is a parabolic curve that starts at the origin (0,0) and curves upward, becoming steeper as x increases.
3. The third function, shown in gray, is y = sqrt(x), which is the inverse of y = x^2 for x > 0. This curve starts from the origin, moving gradually upwards with a decreasing slope as x increases.

All three functions intersected at the origin (0,0). The graph visually demonstrates the relationships between these functions, highlighting how y = sqrt(x) is the inverse of y = x^2 for x > 0.

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Be aware

- If a function is not one-to-one and onto, then there is no inverse function, which means that even if $g \circ f(x) = f \circ g(x) = x$ for some values of x, $g(x)$ is not the inverse of $f(x)$.
- $f^{-1}(x)$ is not the reciprocal of the function $f(x)$, i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$. Can you find an example where $f^{-1}(x) = \frac{1}{f(x)}$?
- $f(x)$ and $f^{-1}(x)$ are reflections of each other in the line $y = x$ for a given domain.

Inverse trigonometric functions

Trigonometric functions are continuous and periodic. Unless you restrict the domain, they are not one-to-one. Therefore, to be able to define inverse trigonometric functions you need to restrict the domain so the function is one-to-one.

Activity

Use the applet below to explore the domain restrictions for inverse trigonometric functions.

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Interactive 2. Exploring Inverse Trigonometric Functions: Domain Restrictions.
Credit: GeoGebra (https://www.geogebra.org/m/c2mpbtrf) BHNmath

More information for interactive 2

This interactive graph allows users to explore the relationship between trigonometric functions and their inverses, with a focus on understanding the necessary domain restrictions for inverse functions to be valid.

The screen is divided into two sections. The top represents a graph of xy-axis, with x-axis measured in radians ranging from $-\frac{3\pi}{2}$ to $\frac{3\pi}{2}$ and the y-axis ranging from -3 to 3. A straight dashed line $y = x$ passes through the origin.

In the bottom section, users can select a function—sine, cosine, or tangent—by clicking on the respective buttons, which updates the graph with the corresponding curves.

Four checkboxes control the display: "Original function" projects the selected trigonometric graph, "Inverse" displays its inverse function in yellow, "Domain restriction" highlights the critical domain limitation in blue, and "Points" reveals two movable red points that can track corresponding values between a function and its inverse. A reset button restores all settings to their default state.

When selecting the sine function and checking all boxes, the original sine wave $(y = \sin x)$, its restricted domain portion (blue highlight between $-\frac{\pi}{2}$ to $\frac{\pi}{2}$), and the inverse arcsine function $(y = \sin^{-1} x)$ reflected across $y = x$. Dragging the red points shows how $(\frac{\pi}{6}, 0.5)$ on the sine

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function corresponds to $(0.5, \frac{\pi}{6})$ on its inverse. Switching to cosine reveals the different domain restriction (0 to π) needed for arccosine, while tangent shows its vertical asymptotes and the corresponding horizontal asymptotes in arctangent.

The visualization demonstrates how restricting the domain of sine, cosine, and tangent functions enables their inverses to become proper functions, while also showing the original and inverse graphs simultaneously for comparison. Users can manipulate various elements to gain insight into these fundamental trigonometric concepts.

Through this exploration, users develop a concrete understanding of why domain restrictions are essential for inverse trigonometric functions and how these restrictions affect the graphs. They learn to identify the standard restricted domains for arcsine, arccosine, and arctangent, and recognize the graphical relationship between functions and their inverses across the line $y = x$.

Although you can use the notation $f^{-1}(x)$ for inverse trigonometric functions, the prefix arc is commonly used to represent them: arcsin, arccos and arctan. So, if

$$f(x) = \sin x \Rightarrow f^{-1}(x) = \arcsin x$$

for a defined domain.

Because the function $f(x) = \sin x$ has the range $[-1, 1]$, the domain of the inverse function will be $[-1,\ 1]$. In the activity above, when the domain of $f(x) = \sin x$ is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ it becomes a one-to-one function.
Therefore, the range of the $f^{-1}(x) = \arcsin x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

You can find the domain and range of inverse trigonometric functions in **Table 1.** Note that, in the range of $y = \arctan x$, the end points $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ are not included because $\tan\left(-\frac{\pi}{2}\right)$ and $\tan\left(\frac{\pi}{2}\right)$ are not defined.

Table 1. The domains and ranges of the three inverse trigonometric functions.

Function	Domain	Range
$y = \arcsin x = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \arccos x = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$

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Function	Domain	Range
$y = \arctan x = \tan^{-1} x$	$\mathbb{R}$	$\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

Example 1

★☆☆

Find the **exact** value of $\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

Note that $-\frac{\sqrt{3}}{2} \in [-1, 1]$ and thus $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ exists. Then,

$$
\begin{aligned}
y &= \arccos\left(-\frac{\sqrt{3}}{2}\right) \\
\cos y &= -\frac{\sqrt{3}}{2}, \quad 0 \leqslant y \leqslant \pi \\
y &= \pi - \frac{\pi}{6} && \text{[using the unit circle or otherwise]} \\
y &= \frac{5\pi}{6}
\end{aligned}
$$

The solution is shown below.

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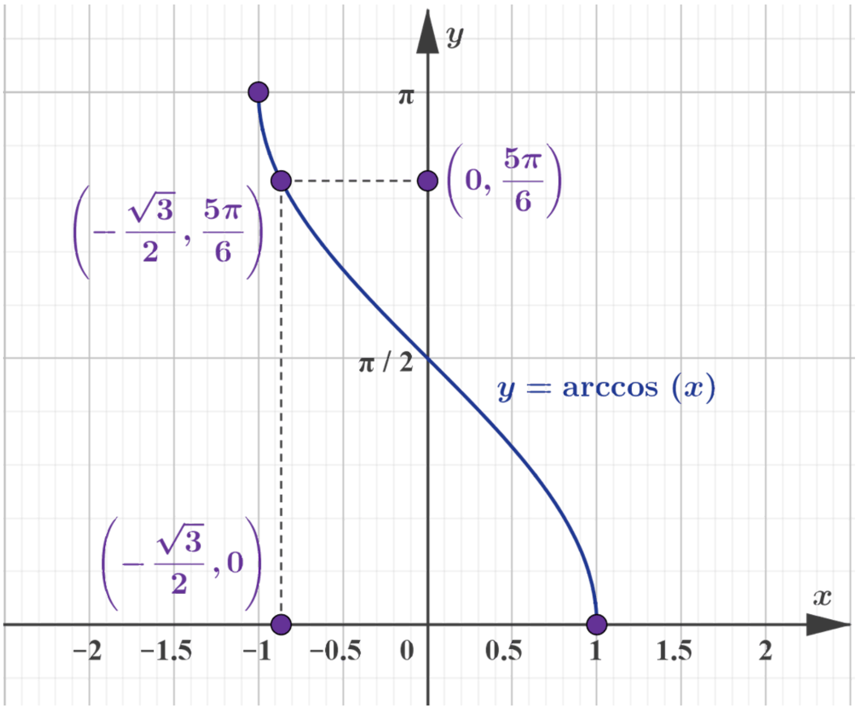

Example 2

★☆☆

Find the **exact** value of $\sin\left(\arccos\left(\frac{1}{4}\right)\right)$.

As $\frac{1}{4} \in [-1, 1]$, then $\arccos\left(\frac{1}{4}\right)$ exists. We let $y = \arccos\left(\frac{1}{4}\right)$ such that $\cos y = \frac{1}{4}$. Hence, y can be represented as the angle in a triangle whose adjacent side is of value 1 and whose hypotenuse has value 4. Thus, the opposite side to angle y has value $\sqrt{4^2 - 1} = \sqrt{15}$. Hence, the exact value of $\sin\left(\arccos\left(\frac{1}{4}\right)\right) = \sin y = \frac{\sqrt{15}}{4}$.

The use of the triangle is shown below.

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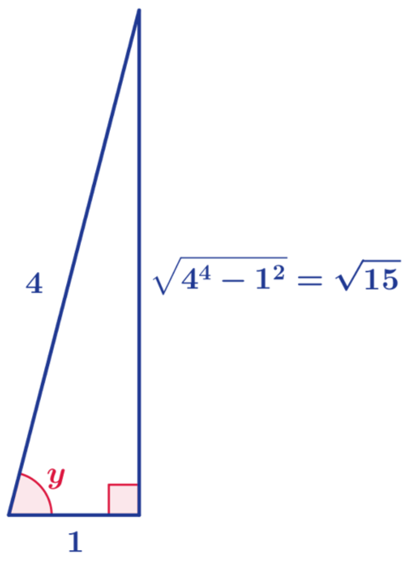

Example 3

★★☆

Find the **exact** solution to $3\arcsin(2x) = \arcsin\frac{\sqrt{2}}{2} + \arccos\frac{\sqrt{2}}{2}$.

Use the exact solutions for the trigonometric ratios and the domain and range restrictions on the inverse ratios. Then,

$$\begin{aligned}
3\arcsin(2x) &= \arcsin\frac{\sqrt{2}}{2} + \arccos\frac{\sqrt{2}}{2} \\
3\arcsin(2x) &= \frac{\pi}{4} + \frac{\pi}{4} \\
3\arcsin(2x) &= \frac{\pi}{2} \\
\arcsin(2x) &= \frac{\pi}{6} \\
\sin(\arcsin(2x)) &= \sin\frac{\pi}{6} \\
2x &= \frac{1}{2} \\
x &= \frac{1}{4}
\end{aligned}$$

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Be aware

1. Always keep in mind the restrictions on the domain of the inverse trigonometric functions. Thus, $\arcsin\left(3\sin\frac{\pi}{3}\right)$ does not exist as $3\sin\frac{\pi}{3} = 3\sqrt{\frac{3}{2}} > 1$.
2. Note that if you allow arguments for trigonometric ratios beyond what they correspond to in terms of their inverse, you may not find the general result for inverse functions, namely that $f^{-1}(f(x)) = x$. For example, $\arccos\left(\cos\frac{3\pi}{2}\right) = \arccos 0 = \frac{\pi}{2} \neq \frac{3\pi}{2}$.
3. In terms of functions, note that:
   - $\arcsin x$ is odd, i.e. $\arcsin(-x) = -\arcsin x$
   - $\arccos x$ is neither even nor odd
   - $\arctan x$ is odd, i.e. $\arctan(-x) = -\arctan x$.

Example 4

★★☆

Function $f(x) = 3\sin x - 1$, for $a \leqslant x \leqslant \pi$.

a) Find the smallest value of a so that $f(x)$ has an inverse function, $f^{-1}(x)$.

b) Hence, find $f^{-1}(x)$.

c) Write the domain and range of $f^{-1}(x)$.

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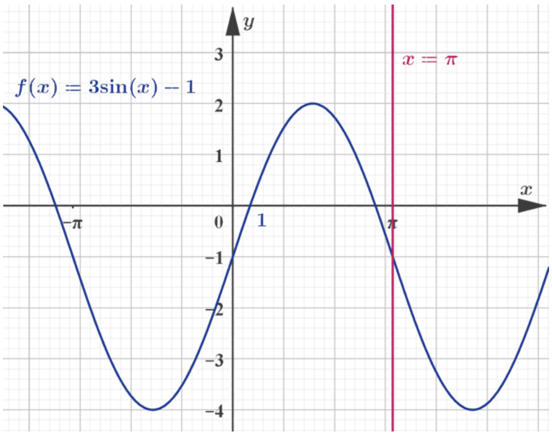
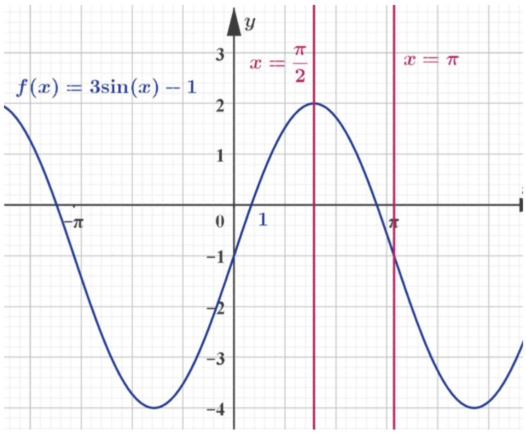
	Steps	Explanation
a)		Sketch the graph of $$f(x) = 3\sin x - 1$$
	$a = \dfrac{\pi}{2}$	 Between $\dfrac{\pi}{2}$ and π the function is one-to one.
b)	$x = 3\sin y - 1$	$f^{-1}(x) = y$
	$\sin y = \dfrac{x+1}{3}$	Rearrange by balancing.

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	Steps	Explanation
	$y=\sin^{-1}\left(\frac{x+1}{3}\right)+2k\pi$ or $y=\pi-\sin^{-1}\left(\frac{x+1}{3}\right)+2k\pi$ for some integer k.	There are infinitely many y with the same $\sin y$.
	$y=\pi-\sin^{-1}\left(\frac{x+1}{3}\right)$	The inverse sine function picks the angle that is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Considering the restricted domain of the function, you need the y-value between $\frac{\pi}{2}$ and π.
	$f^{-1}(x)=\pi-\arcsin\left(\frac{x+1}{3}\right)$ or $f^{-1}(x)=\pi-\sin^{-1}\left(\frac{x+1}{3}\right)$	Write $f^{-1}(x)$
c)	Range of $f^{-1}(x)=\left[\frac{\pi}{2},\pi\right]$ Domain of $f^{-1}(x)=[-1,2]$	The range of the inverse is the (restricted) domain of the original function. The domain of the inverse is the range of the original function.

4 section questions ^

Question 1

★☆☆

What is the **exact** value of $\arctan\left(-\sqrt{3}\right)$?

1 $-\frac{\pi}{3}$ ✓

2 $\frac{\pi}{3}$

3 $-\frac{\pi}{3},\frac{2\pi}{3}$

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4 $\frac{2\pi}{3}, \frac{5\pi}{3}$

Explanation

Let $\arctan\left(-\sqrt{3}\right) = y$, then $-\sqrt{3} = \tan y,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$. Thus, $y = -\frac{\pi}{3}$.

Question 2

★★☆

What is the **exact** value of $\cos\left(\arcsin\frac{2}{3}\right)$?

1 $\frac{\sqrt{5}}{3}$ ✓

2 $\frac{2}{3}$

3 $-\frac{\sqrt{5}}{3}$

4 0.745

Explanation

Let $\arcsin\frac{2}{3} = y$, then $\sin y = \frac{2}{3},\ -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$. Thus, in a right-angled triangle, angle y defines the opposite to be 2, the hypothenuse to be 3, such that the adjacent becomes $\sqrt{3^2 - 2^2} = \sqrt{5}$. Thus,

$\cos\left(\arcsin\frac{2}{3}\right) = \cos y = \frac{\sqrt{5}}{3}$.

Question 3

Difficulty:

★★☆

What is the **exact** solution to $2\arccos\left(\frac{1}{2}x\right) = \arcsin\frac{1}{2} + \arccos\frac{1}{2}$?

1 $\sqrt{2}$ ✓

2 $\sqrt{3}$

3 2

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4 $\frac{\sqrt{3}}{2}$

Explanation

$$
\begin{aligned}
& 2\arccos\left(\frac{1}{2}x\right) = \arcsin\frac{1}{2} + \arccos\frac{1}{2} \\
\Leftrightarrow \quad & = \frac{\pi}{3} + \frac{\pi}{6} \\
\Leftrightarrow \quad & = \frac{\pi}{2} \\
\Longrightarrow \quad & \arccos\left(\frac{1}{2}x\right) = \frac{\pi}{4} \\
\Longrightarrow \quad & \cos\left(\arccos\left(\frac{1}{2}x\right)\right) = \cos\frac{\pi}{4} \\
\Longrightarrow \quad & \frac{1}{2}x = \frac{\sqrt{2}}{2} \\
\Longrightarrow \quad & x = \sqrt{2}
\end{aligned}
$$

Question 4

★★☆

What is the **exact** solution to $\frac{5}{4}\arcsin\left(\frac{1}{3}x\right) = \arccos\frac{\sqrt{3}}{2} + \arccos\frac{\sqrt{2}}{2}$?

1 $\frac{3\sqrt{3}}{2}$ ✓

2 $\frac{\sqrt{3}}{2}$

3 $\frac{3}{2}$

4 $\frac{\sqrt{3}}{4}$

Explanation

$$
\begin{aligned}
\frac{5}{4}\arcsin\left(\frac{1}{3}x\right) &= \arccos\frac{\sqrt{3}}{2} + \arccos\frac{\sqrt{2}}{2} \\
\frac{5}{4}\arcsin\left(\frac{1}{3}x\right) &= \frac{\pi}{6} + \frac{\pi}{4} \\
\frac{5}{4}\arcsin\left(\frac{1}{3}x\right) &= \frac{5\pi}{12} \\
\arcsin\left(\frac{1}{3}x\right) &= \frac{\pi}{3} \\
\sin\left(\arcsin\left(\frac{1}{3}x\right)\right) &= \sin\frac{\pi}{3} \\
\frac{1}{3}x &= \frac{\sqrt{3}}{2} \\
x &= \frac{3\sqrt{3}}{2}
\end{aligned}
$$

Student view

Further trigonometric identities

In subtopic 3.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27747/) and subtopic 3.8 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27759/), you studied trigonometric identities and equations. In this section, you will solve problems involving the reciprocal trigonometric ratios and inverse trigonometric functions.

Example 1

★☆☆

Prove the identity $\text{cosec } x \cdot \cos x \equiv \cot x$.

Steps	Explanation
$\text{cosec } x \cdot \cos x = \frac{1}{\sin x} \cdot \cos x$	Using $\text{cosec } x = \frac{1}{\sin x}$.
$\frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$ So $\text{cosec } x \cdot \cos x \equiv \cot x$	From the definition of $\cot x$.

Example 2

★☆☆

Show that $\frac{1}{1-\cos x} - \frac{1}{1+\cos x} = 2\cot x \cdot \text{cosec } x$.

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Steps	Explanation
LHS $$\frac{1}{1-\cos x}-\frac{1}{1+\cos x}=\frac{1+\cos x-(1-\cos x)}{1-\cos^2 x}$$	Add the fractions by putting over common denominator.
$$\frac{1+\cos x-(1-\cos x)}{1-\cos^2 x}=\frac{2\cos x}{1-\cos^2 x}$$	Expand the brackets and simplify.
$$\frac{2\cos x}{1-\cos^2 x}=\frac{2\cos x}{\sin^2 x}$$	Rearranging the Pythagorean identity $$\sin^2 x+\cos^2 x=1$$ gives $$1-\cos^2 x=\sin^2 x$$
RHS $$2\cot x\cdot\ \operatorname{cosec} x=2\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$$	Use the definitions $$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$$ $$\cot\theta=\frac{\cos\theta}{\sin\theta}$$
$$2\cot x\cdot\operatorname{cosec} x=\frac{2\cos x}{\sin^2 x}$$	
As $\text{LHS}=\text{RHS}$, $$\frac{1}{1-\cos x}-\frac{1}{1+\cos x}=2\cot x\cdot\ \operatorname{cosec} x$$	

Be aware

When a question asks you to ‘show that’ or ‘prove that’, you cannot treat it as an equation where you would balance both sides. You need to simplify either the LHS or the RHS and show that it is equivalent to the other side.

Student view

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Example 3

★★☆

If $2\,\text{cosec}^2\,x - 7\,\text{cosec}\,x - 4 = 0$, find the exact value of $\sin x$.

Steps	Explanation
Let $\text{cosec}x = a$ $2\text{a}^2 - 7a - 4 = 0$	
$2\text{a}^2 - 7a - 4 = (2a+1)(a-4) = 0$	Factorise.
$2a + 1 = 0$ or $a - 4 = 0$	Use the null factor theorem.
$a = -\frac{1}{2}$ or $a = 4$	
$\text{cosec}\,x = -\frac{1}{2}$ or $\text{cosec}\,x = 4$	
$\frac{1}{\sin x} = -\frac{1}{2}$ or $\frac{1}{\sin x} = 4$	$\text{cosec}x = \frac{1}{\sin x}$
$\sin x = -2$ or $\sin x = \frac{1}{4}$	
Reject -2. Therefore, the answer is $\frac{1}{4}$	Because $-1 \leqslant \sin x \leqslant 1$, by definition.

Exam tip

In IB examinations, angles should be assumed to be given in radians, unless stated otherwise.

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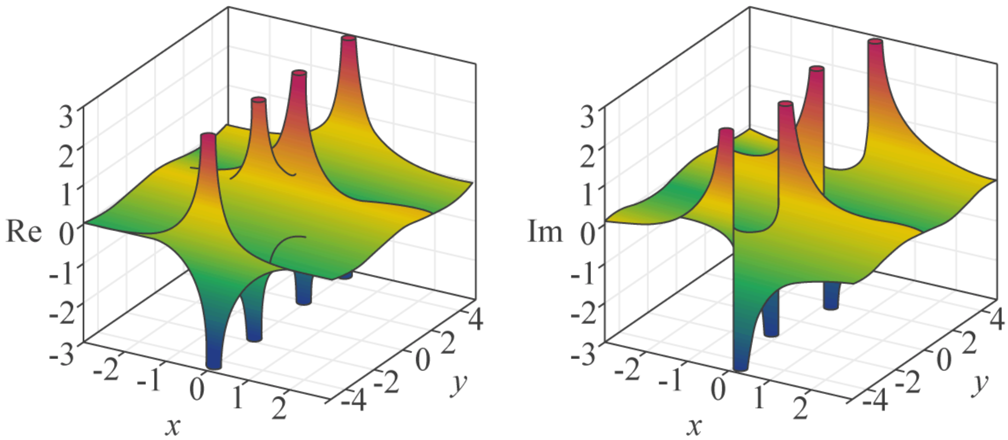
International Mindedness

The words secant and cotangent were first introduced by Persian mathematician, astronomer and geographer, Habash al-Hasib al-Marwazi, around the year 800. Euler also used these two ratios in his work. Euler numbers, or zig numbers, can be defined using the Maclauren series of secant.

In the complex z -plane, you can define $\sec z = \frac{1}{\cos z} = \frac{1}{e^{iz} + e^{-iz}}$.

Below are the graphs of the real and imaginary parts of $\sec z$ over the complex plane.



More information

The image shows two 3D graphs side by side. The left graph represents the real part, labeled 'Re', and the right graph represents the imaginary part, labeled 'Im', of the function (\sec z) over the complex plane. Each graph has three axes: x, y, and the respective 'Re' or 'Im' axis, with scales ranging from -3 to 4. The surfaces resemble a landscape with peaks and valleys, featuring bright colors to indicate variations in the function's value. The real part graph shows peaks prominently above the x and y-plane center, while the imaginary part graph differs in shape and position, depicting how both components vary across the plane.

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3 section questions

Student view

Question 1

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Find the exact solutions of the equation

$$4\cot^2 x + 7\operatorname{cosec} x + 2 = 0,\ 0 \leqslant x \leqslant 2\pi$$

1 $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ ✓

2 $\frac{7\pi}{3}$ and $\frac{11\pi}{3}$

3 $-\frac{\pi}{6}$ and $\frac{5\pi}{6}$

4 $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

Explanation

Rearrange the Pythagorean identity 1 + $\cot^2 x = \operatorname{cosec}^2 x$ to give

$$\cot^2 x = \operatorname{cose} \mathrm{c}^2 x - 1,$$

and substitute

$$4(\operatorname{cose} \mathrm{c}^2 \theta - 1) + 7 \operatorname{cosec} x + 2 = 0.$$

Expand the brackets and simplify:

$$4 \operatorname{cosec}^2 \theta + 7 \operatorname{cosec} x - 2 = 0$$

Factorise:

$$(4 \operatorname{cosec} x - 1)(\operatorname{cosec} x + 2) = 0$$

Using the null factor theorem:

$$\operatorname{cosec} x = \frac{1}{4} \text{ or } \operatorname{cosec} x = -2$$

$$\sin x = 4 \text{ or } \sin x = -\frac{1}{2}$$

$\sin x = 4$ has no solution, so reject.

Solve $\sin x = -\frac{1}{2}$ for $0 \leqslant x \leqslant 2\pi$

$$x = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}$$

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Therefore, the correct answer is

$$\frac{7\pi}{6} \text{ and } \frac{11\pi}{6}.$$

Question 2

★☆☆

Simplify $\frac{\cos^2\theta + \sin^2\theta}{1 - \sin^2\theta}$

1 $\sec^2\theta$ ✓

2 $\operatorname{cosec}^2\theta$

3 $\tan^2\theta$

4 $\cot^2\theta$

Explanation

Using Pythagorean identities:

$$\frac{\cos^2\theta + \sin^2\theta}{1 - \sin^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{1}{\cos^2\theta} = \sec^2\theta$$

Therefore, the correct answer is $\sec^2\theta$.

Question 3

★★★

Which of the following is equivalent to $(\operatorname{cosec} x + \cot x)^2$.

1 $\frac{1 + \cos x}{1 - \cos x}$ ✓

2 $\frac{1 - \cos x}{1 + \cos x}$

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3 $\dfrac{1-\cot x}{1+\cot x}$

4 $\dfrac{1+\cot x}{1-\cot x}$

Explanation

$$(\operatorname{cosec} x+\cot x)^2=\left(\frac{1}{\sin x}+\frac{\cos x}{\sin x}\right)^2=\left(\frac{1+\cos x}{\sin x}\right)^2$$

Expand the brackets and simplify:

$$\left(\frac{1+\cos x}{\sin x}\right)^2=\frac{(1+\cos x)(1+\cos x)}{\sin^2 x}$$

Using the $\text{Pythagorean identity}$:

$$\frac{(1+\cos x)(1+\cos x)}{\sin^2 x}=\frac{(1+\cos x)(1+\cos x)}{1-\cos^2 x}$$

Factorise the denominator as a difference of two squares and simplify:

$$\frac{(1+\cos x)(1+\cos x)}{1-\cos^2 x}=\frac{(1+\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}=\frac{(1+\cos x)}{(1-\cos x)}$$

Therefore, the correct answer is

$$\frac{1+\cos x}{1-\cos x}$$

3. Geometry and trigonometry / 3.9 Reciprocal trigonometric ratios and inverse trigonometric functions

Checklist

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What you should know

By the end of this subtopic you should be able to:

- define the difference between inverse trigonometric functions and reciprocals of trigonometric functions
- use the reciprocal trigonometric ratios

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- $\sec\theta = \dfrac{1}{\cos\theta}$
- $\text{cosec}\theta = \dfrac{1}{\sin\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

- The domains and ranges of the inverse trigonometric functions:

1. $y = \arcsin x,\ -1 \leqslant x \leqslant 1,\ -\dfrac{\pi}{2} \leqslant y \leqslant \dfrac{\pi}{2}$
2. $y = \arccos x,\ -1 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant \pi$
3. $y = \arctan x,\ x \in \mathbb{R},\ -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$.

3. Geometry and trigonometry / 3.9 Reciprocal trigonometric ratios and inverse trigonometric functions

Investigation

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Defining the domain of a relation could also identify whether it is a function or not. You might be able to say something similar for identities as well. Some identities are only true for certain values of the variable.

Consider the following identity.

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$

- Is it true for all values of θ?
- What are the restrictions for this identity to hold true?
- For which values of θ does this identity holds true?

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Now see if you can find any other identities that are only valid over a restricted domain.

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