The big picture

Early Egyptians and Babylonians knew about the ratios of sides of similar triangles but they did not use the concept of an angle; they only studied the side lengths of triangles.

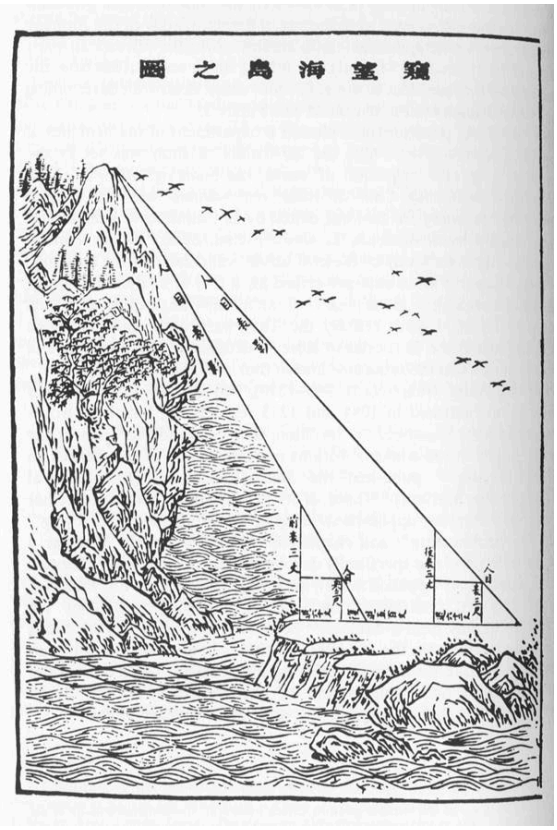
Hipparchus of Nicaea of Hellenistic Anatolia (now Iznik, Turkey), a Greek astronomer and mathematician, also known as ' *Father of the trigonometry* ', was the first to use arithmetic techniques to calculate the trigonometric ratios for angles.

Once the concepts of angles and trigonometric ratios of angles had been introduced, people were able to apply them in many practical contexts, many of which are still used today.

For example, forest surveyors use trigonometry to find the length of a river or boundaries of a forest; bearings are used to calculate the distances between locations on maps; and in oceanography the height of tides and waves are calculated by using angles of elevation and depression. An up-to-date application of trigonometry is the global positioning system (GPS) in a smartphone.

In this subtopic, you will be looking into the practical uses of trigonometry.

Overview (/study/ap 122- cid- 754029/



Sea Island survey diagram , first written by the Chinese mathematician Liu Hui during the Three Kingdoms era (220—280 CE).

Source: " Sea island survey (https://commons.wikimedia.org/wiki/File:Sea_island_survey.jpg) " by Liu Hui is in public domain.

More information

This is a diagram illustrating an ancient Chinese method of using trigonometry to survey a sea island. The image features a detailed drawing of a rocky island with a triangular shape overlaying it, representing the trigonometric calculations. The base of the triangle is along the shore, while one side extends up to the peak of the island. Several small annotations in Chinese script are placed along the sides of the triangle, indicating measurements or calculations. At the bottom left, waves are illustrated, suggesting the sea surrounding the island. Additionally, there are birds flying in the area above the triangle, providing a sense of perspective and scale.

[Generated by AI]

Student view

Overview (/study/ap 122- cid- 754029/

Concept

There are many real-world applications of trigonometry including land surveys, bearings, angles of elevation and depression and GPS.

How could you use trigonometry to define the space around us?

How could the height of a distant object be estimated without physically measuring it?

How accurate are our calculations when the surface of Earth is not flat and the geometry you are using is for flat surfaces?

When making calculations and predictions based on the real world, it is often necessary to use modelling to reduce a complex shape into a system of triangles to enable trigonometry to be used.

Constructing diagrams

Labelling and drawing diagrams

In examinations, you will often be given a diagram of the question together with an explanation. Although these diagrams are not drawn to scale, they are helpful when solving a geometry problem.

In some cases, if you are not given a diagram you would benefit from sketching one as it would help you to spot the relationships.

Example 1

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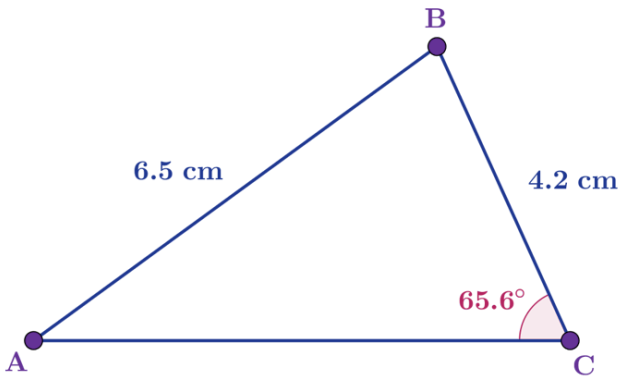
Student view

In triangle ABC, $AB = 6.5$ cm, $BC = 4.2$ cm and angle $\mathrm{BCA} = 65.6^\circ$.

Overview (/study/ap 122-cid-754029/

Calculate the area of triangle ABC.

Solution

Steps	Explanation
 More information The image is a diagram of a triangle labeled ABC. The triangle has three sides: one side from A to B measuring 6.5 cm, one side from B to C measuring 4.2 cm, and the last side from A to C. The angle at point C is marked as 65.6 degrees. Each vertex is labeled with a letter and represented by a purple dot. The lengths of the sides are labeled in blue, and the angle at C is highlighted in red. [Generated by AI]	You can use the formula for th of a triangle using the sine of included angle, $\text{area of triangle} = \frac{1}{2}ab$ When you sketch triangle AB can see that the angle given i the included angle, so you wil to find the length of AC to be to use this area formula.
$c^2 = a^2 + b^2 - 2ab\cos C$ $6.5^2 = 4.2^2 + b^2 - 2 \times 4.2 \times b \times \cos 65.6^\circ$	As two sides and an angle are you can use the cosine rule to the missing side length.
$b = 6.990\ldots \text{ cm}$	Use the GDC instructions pro below this table. Don't round this value yet as y need it in the next calculation

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Overview (/study/ap 122- cid- 754029/

Steps	Explanation
$\text{area of triangle} = \frac{1}{2}ab\sin C$ $\text{area} = \frac{1}{2} \times 4.2 \times 6.990\ldots \times \sin 65.6^\circ.$ $\text{area} = 13.368\ldots$ $\approx 13.4\,\text{cm}^2\ (3\ \text{significant figures})$	Now that you have the side le needed, you can use the form the area.
	Note that you can solve this p by drawing the perpendicular from vertex B to the side AC finding the height using the g angle and side AC. The formula $\text{area} = \frac{1}{2}\text{base} \times \text{height}$ could then be used.

GDC instructions for finding b:

Student view

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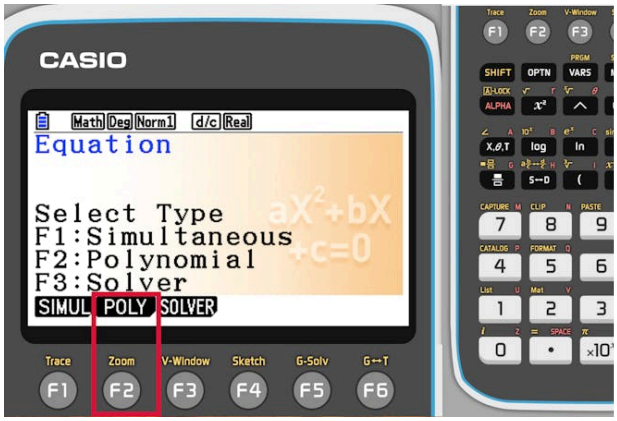
Step	Explanation
In these instructions you will see how to use the polynomial equation solver to find the roots of the quadratic equation $$ax^2 + bx + c = 0$$ for $$a = 1$$ $$b = -2 \times 4.2 \times \cos(65.6^\circ)$$ $$c = 4.2^2 - 6.5^2$$ Choose the equation solving mode.	
Press F2 to choose the polynomial root finder.	

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Student view

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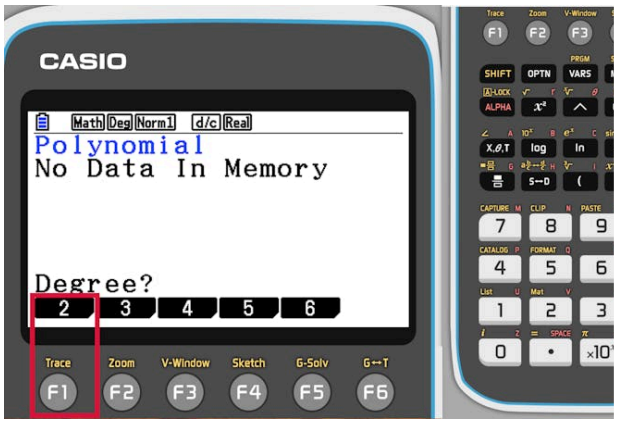
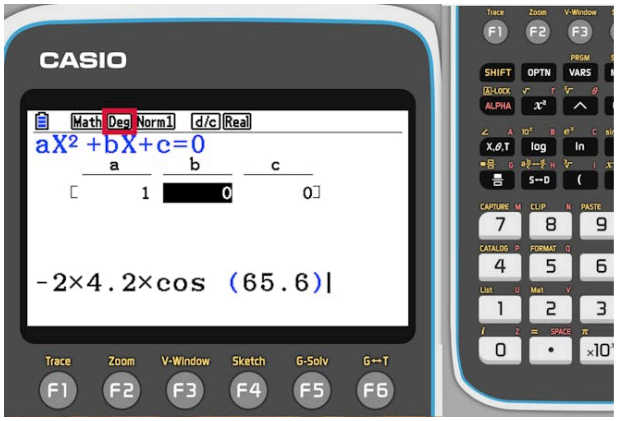
Step	Explanation
This is a quadratic equation, so press F1 to choose degree 2.	
You can enter an expression as the coefficient. Make sure your calculator is in degree mode so that you get the correct cosine value.	

Student view

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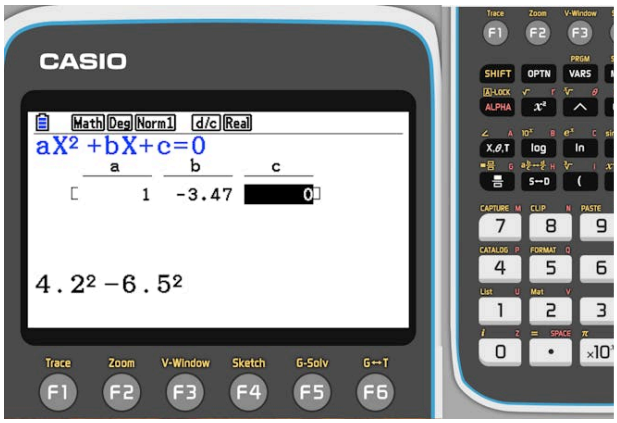
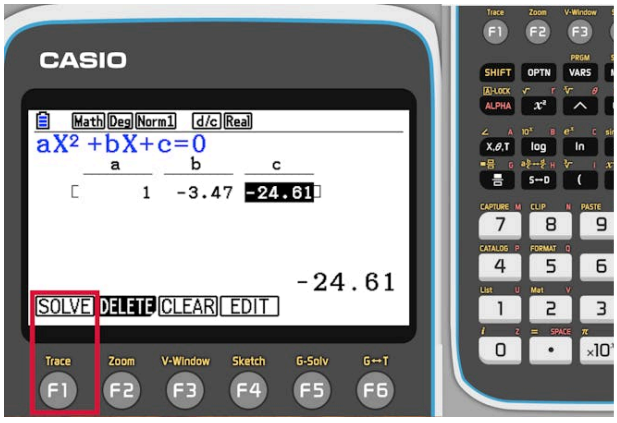
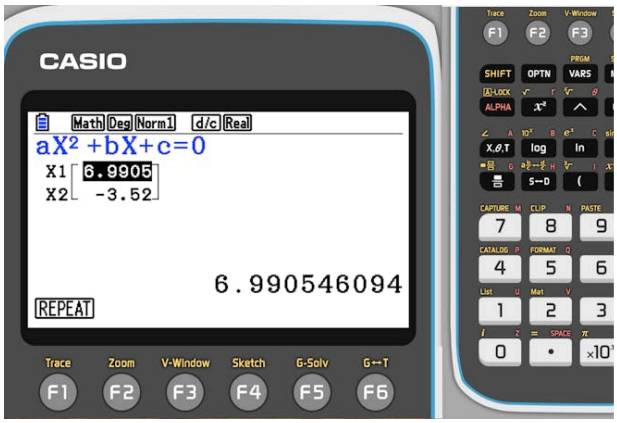
Step	Explanation
Note, that the expression you entered for b is converted to a number.	
Once you entered all the coefficients, press F1 to solve the equation.	

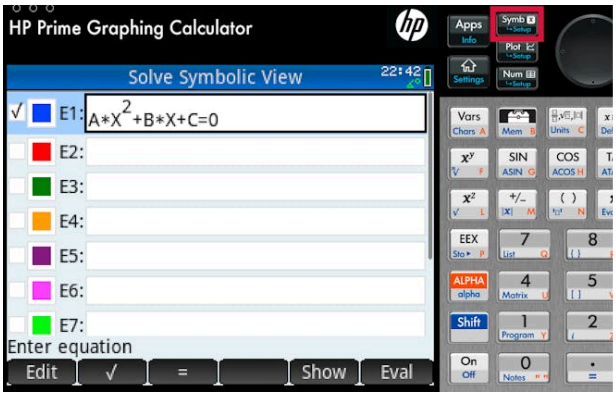
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
There are two solutions. In the context (x is the side length of a triangle), only the positive solution is meaningful.	

Student view

Overview (/study/ap 122- cid- 754029/(

Step	Explanation
In these instructions you will see how to use the polynomial equation solver to find the roots of the quadratic equation $$ax^2 + bx + c = 0$$ for $$a = 1$$ $$b = -2 \times 4.2 \times \cos(65.6^\circ)$$ $$c = 4.2^2 - 6.5^2$$ Choose the solver application. Some parts of the explorer application is also available in exam mode, you can experiment with it.	
In the symbolic view, enter the general form of a quadratic equation.	

Student view

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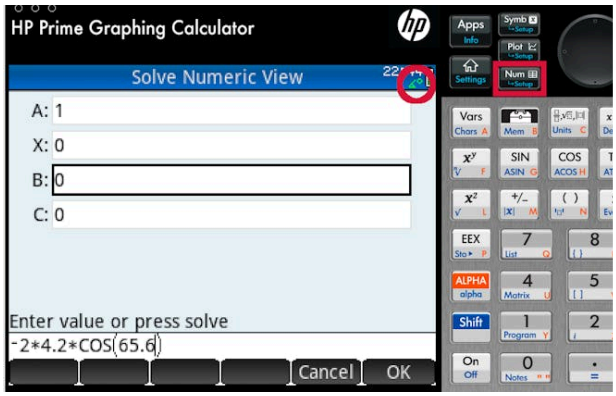
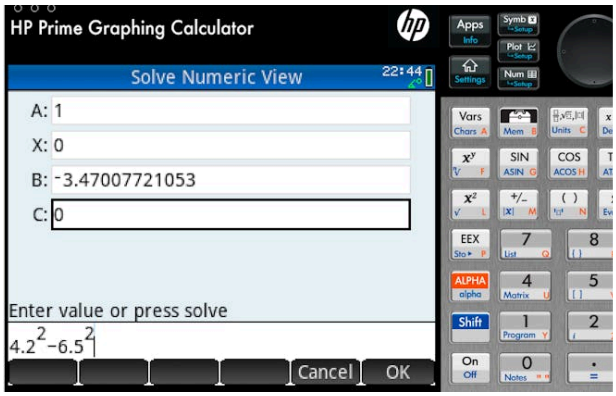
Step	Explanation
Move to the numeric view to enter the coefficients. You can enter an expression as the coefficient. Make sure your calculator is in degree mode so that you get the correct cosine value.	
Note, that the expression you entered for b is converted to a number.	

Student view

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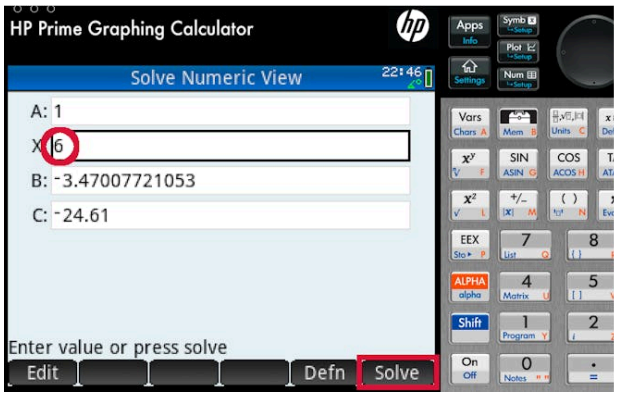
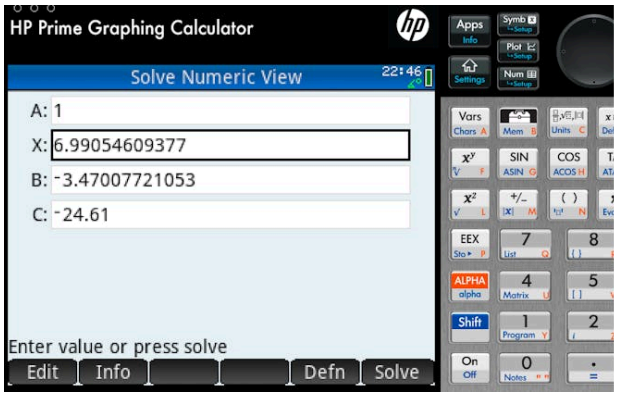
Step	Explanation
Once the coefficients are entered, move to the row for x to tell the calculator that you would like to solve this equation for x. The calculator finds the solutions one at a time. Enter an x-value that you think is close to the solution. In this example, the diagram indicates, that the side length you are looking for is somewhere around th elength of the longer given side. Once done, press solve.	
The calculator gives you a solution.	

Student view

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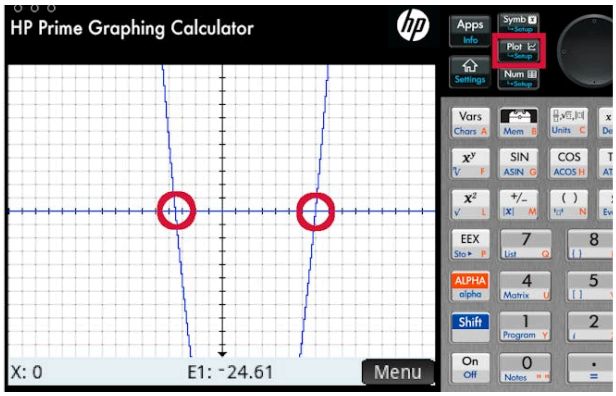
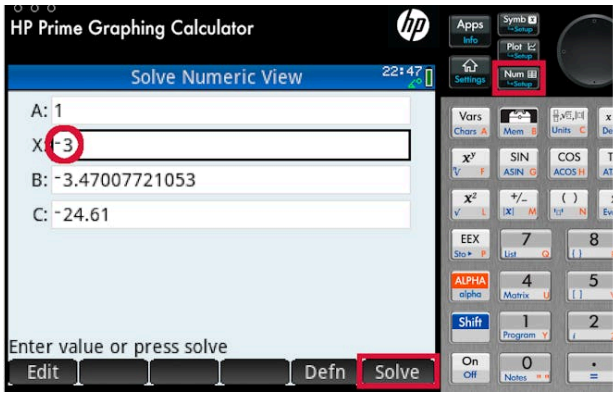
Step	Explanation
To find the other solution, open the plot view. You will see the graph of the quadratic. You can see that the solution you did not get earlier is negative, so it is not meaningful in this context (you are looking for the length of a side of a triangle). If you are still interested, make a note that this other solution is around -3 and go back to the numeric view.	
In the numeric view, enter the new approximation for the solution and press solve.	

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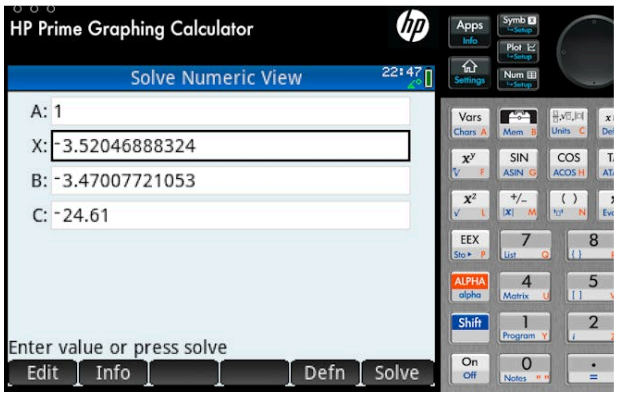
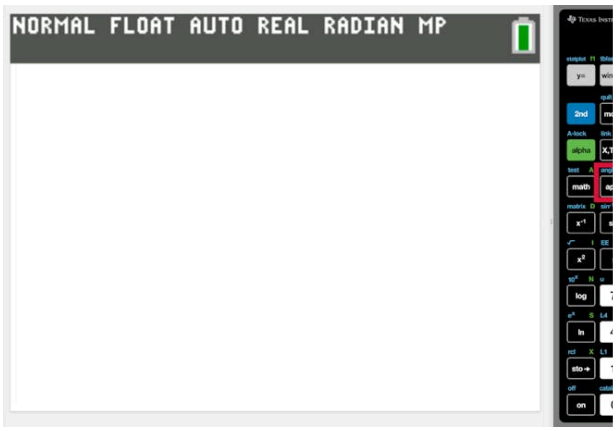
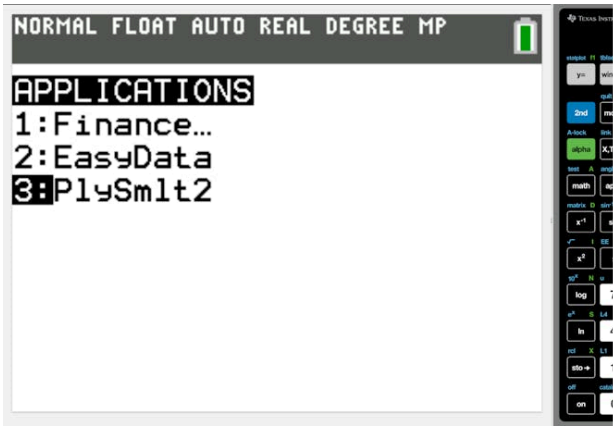
Step	Explanation
The second solution is now displayed.	

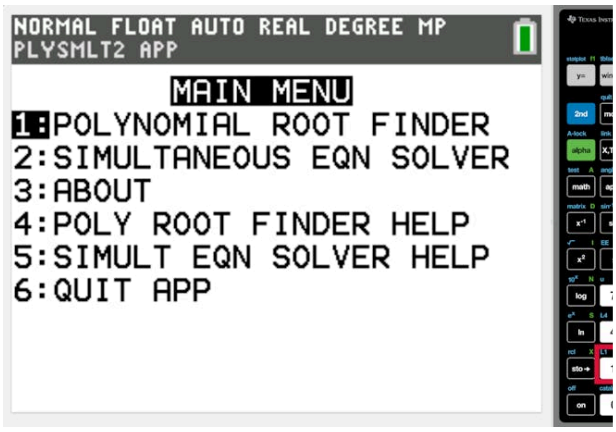
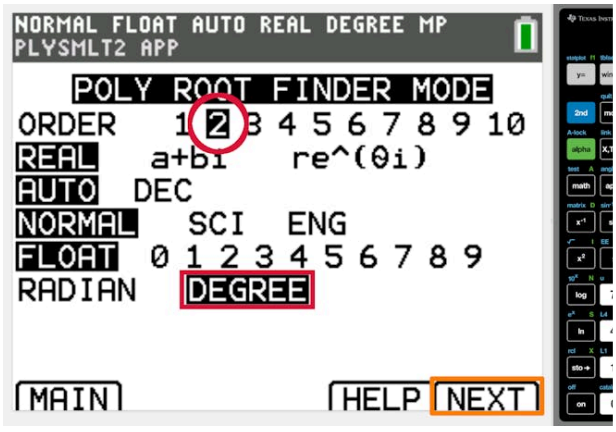
Student view

Overview (/study/ap 122- cid- 754029/(

Step	Explanation
In these instructions you will see how to use the polynomial equation solver to find the roots of the quadratic equation $ax^2 + bx + c = 0$ for $a = 1$ $b = -2 \times 4.2 \times \cos(65.6^\circ)$ $c = 4.2^2 - 6.5^2$ Open the list of applications.	
Open PlySmlt2, the polynomial equation solver applicaton.	

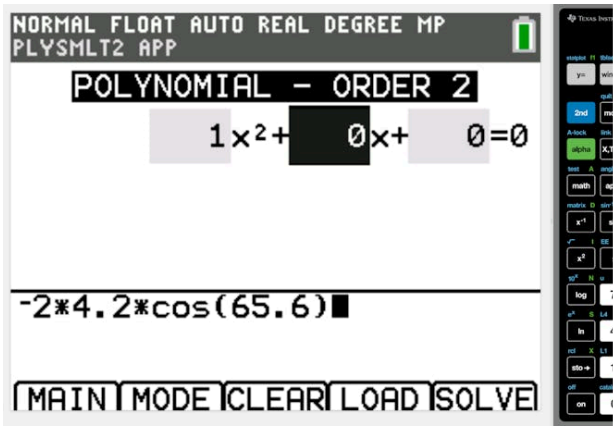
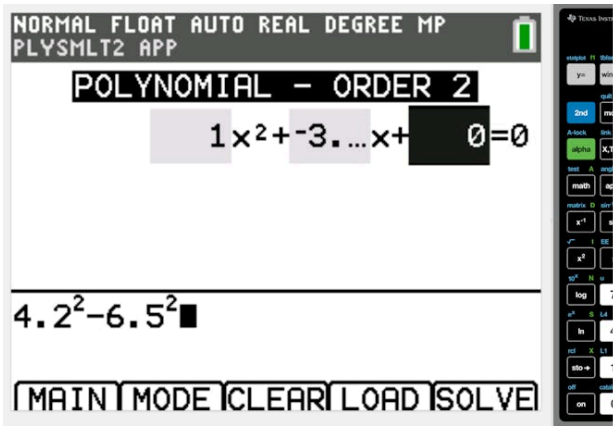
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
You will need the polynomial root finder option.	
This is a quadratic equation, so choose order 2. Make sure you choose degree mode so that you get the correct cosine value in the coefficient. Once done setting the options, move to the next step by pressing the corresponding button under the screen.	

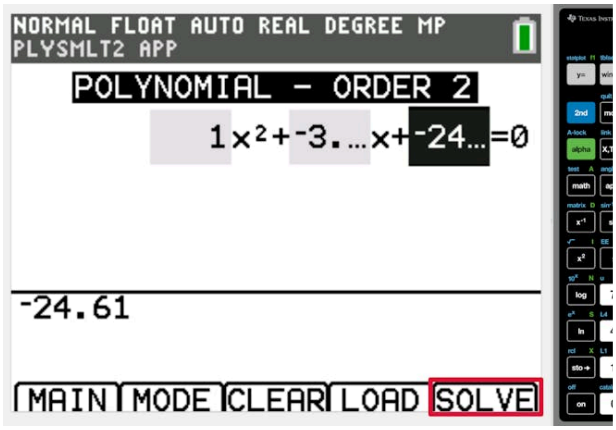
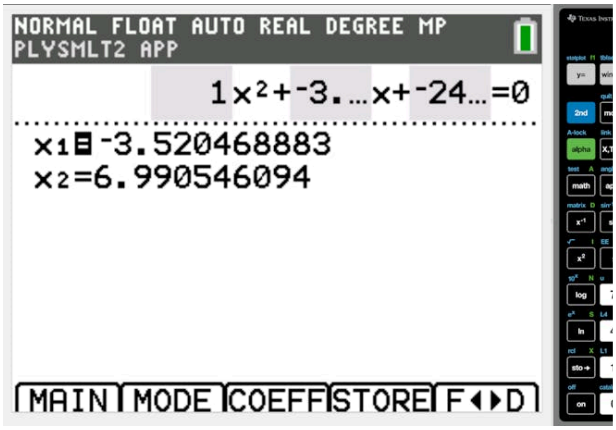
Student view

Overview (/study/ap 122- cid- 754029/(

Step	Explanation
You can enter an expression as the coefficient.	
	

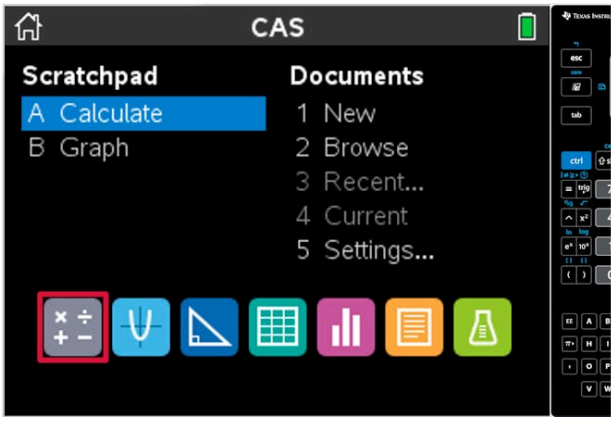
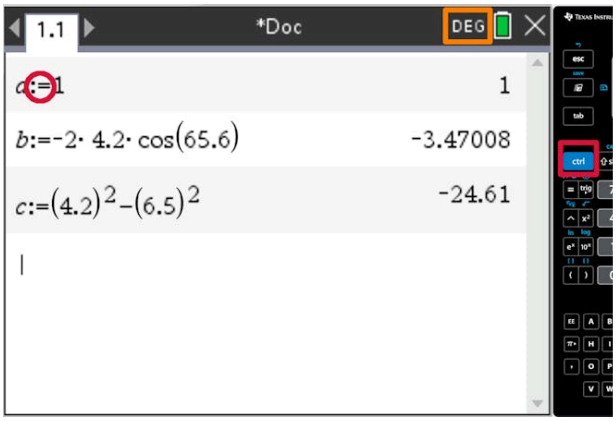
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
Once you entered all the coefficients, press the button under the screen that corresponds to the solve option.	
There are two solutions. In the context (x is the side length of a triangle), only the positive solution is meaningful.	

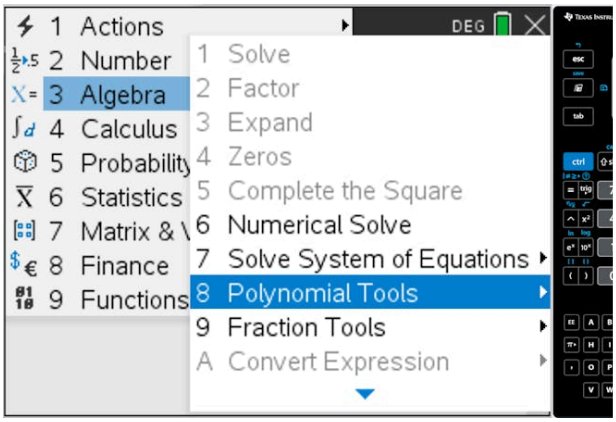
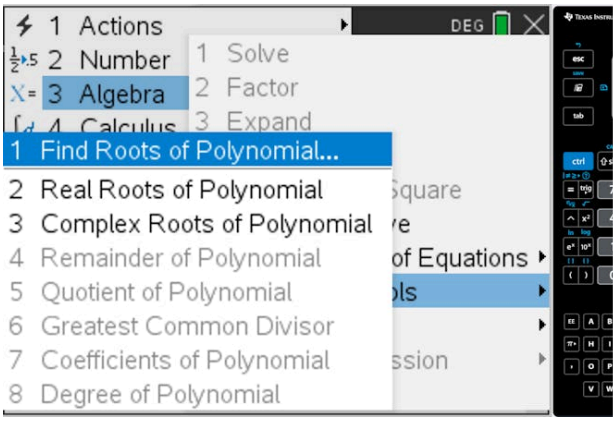
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
In these instructions you will see how to use the polynomial equation solver to find the roots of the quadratic equation $$ax^2 + bx + c = 0$$ for $$a = 1$$ $$b = -2 \times 4.2 \times \cos(65.6^\circ)$$ $$c = 4.2^2 - 6.5^2$$ Start with opening a calculator page.	
You can use variables to store the coefficients. Make sure you use the assigned equal. Make sure your calculator is in degree mode so that you get the correct cosine value in the definition of b.	

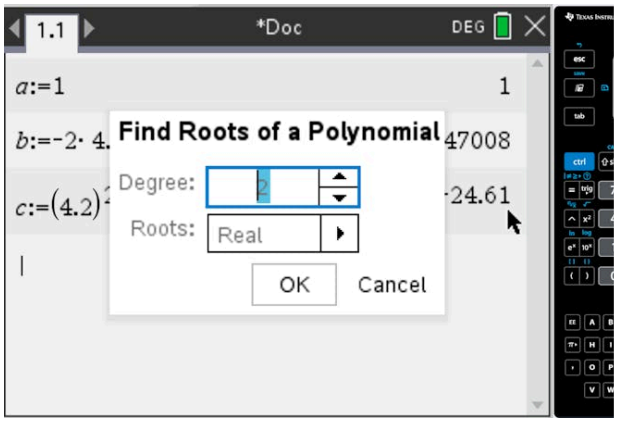
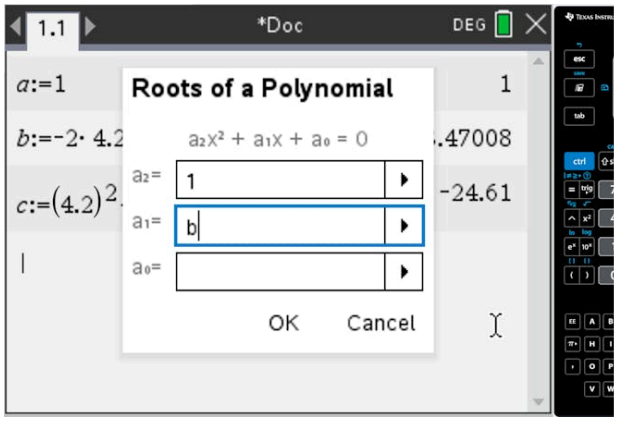
Student view

Overview (/study/ap 122- cid- 754029/(

Step	Explanation
You can access the polynomial root finder through the menu system ...	
... choose the polynomial root finder option.	

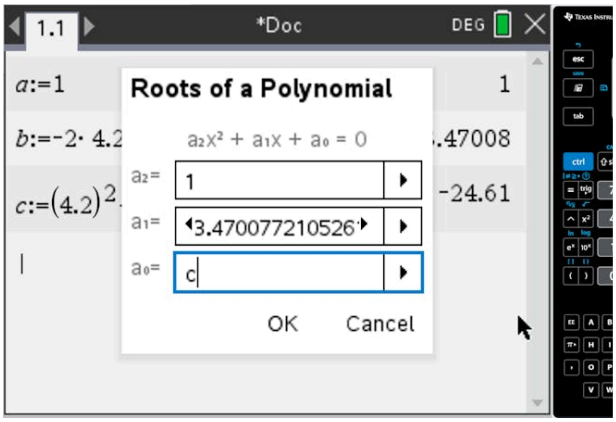
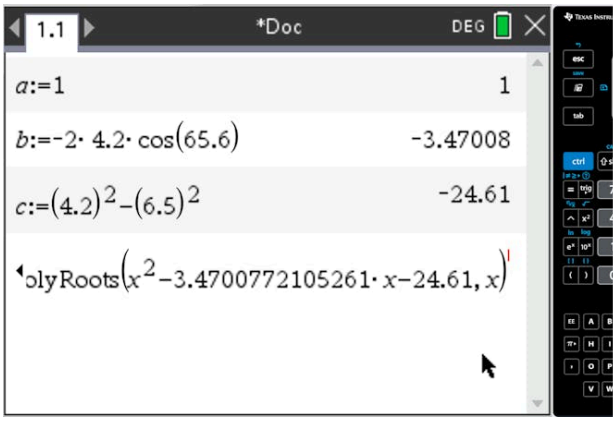
Student view

Overview (/study/ap 122-cid-754029/

Step	Explanation
This is a quadratic equation, so choose degree 2.	
You can enter the names of the coefficients instead of typing in the full axpression again.	

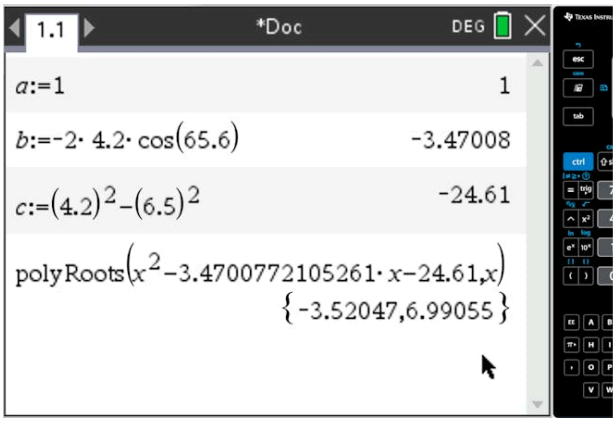
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
Note, that the expression you entered for b is converted to a number.	
The command is now put on the calculation screen. If you remember the format, you can type this in directly. If you do this, you can use the names a, b and c instead of the values. Press enter to see the solutions.	

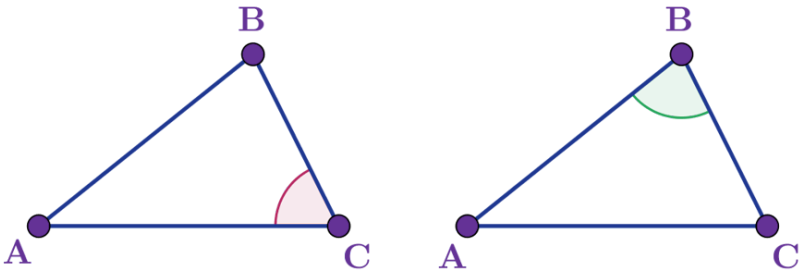
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
There are two solutions. In the context (x is the side length of a triangle), only the positive solution is meaningful.	

✓ Important

When labelling angles using three letters, the letter in the middle is the vertex of the angle. For example the vertex of angle ABC is B.



Student view

More information

Overview (/study/ap 122- cid- 754029/

The image shows two side-by-side triangles labeled ABC. In the first triangle on the left, there is an emphasis on angle C, highlighted with a pink area. The vertices are marked A, B, and C from the bottom left to bottom right. In the second triangle on the right, angle B is highlighted with a green area, indicating the angle at vertex B. The orientation and labels remain consistent across both triangles, emphasizing the relation between the internal angles of a triangle and showing the highlighted angles at each specific vertex. This description aligns with the text provided beneath the image that denotes angle equivalencies: (\text{angle\;} C = \text{angle}\;\text{BCA}) = (\text{angle\;ACB}) and (\angle B = \text{angle}\;\text{ABC}) = (\;\text{angle\;CBA}).

[Generated by AI]

$\text{angle } C = \text{angle BCA} = \text{angle ACB} \quad \angle B = \text{angle ABC} = \text{angle CBA}$

Exam tip

Make sure that your calculator is set to working in degrees when solving triangles and check that your answer looks sensible.

Example 2

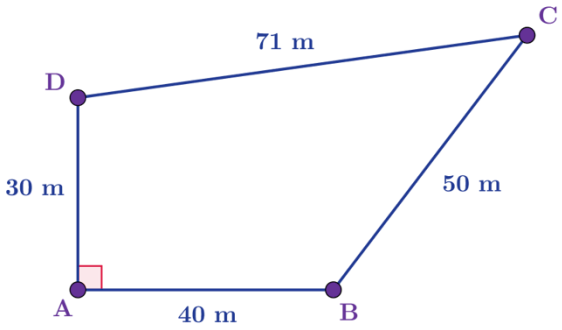
★★☆

ABCD is a quadrilateral $\text{AB} = 40\,\text{m},\ \text{BC} = 50\,\text{m},\ \text{CD} = 71\,\text{m},\ \text{AD} = 30\,\text{m}$ and the angle $\text{DAB} = 90°$.

Student view

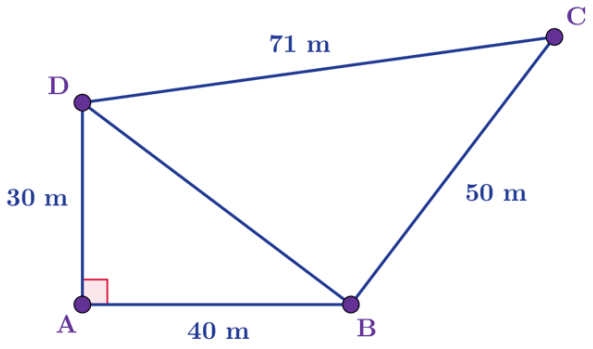
Find the area of quadrilateral ABCD. Give your answer correct to 1 decimal place.

Overview (/study/ap 122-cid-754029/

Step	Explanation
 The image is a diagram of a quadrilateral labeled ABCD. It consists of four sides and four vertices: • Vertex A is at the bottom left with a right angle. From A to B, the side is horizontal measuring 40 meters. • From B, there is a side extending upwards and diagonally to the right towards vertex C, measuring 50 meters. • From C, a side extends horizontally to the left towards D, measuring 71 meters. • From D, a vertical side goes straight down back to A, measuring 30 meters. The quadrilateral is not a rectangle or square, due to the different side lengths and the slanted angle from B to C. [Generated by AI]	Draw the diagram of the quadrilateral. You can sta any point and follow eithe clockwise or anticlockwis to mark the vertices.

Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
 The image is a geometric diagram of a quadrilateral labeled ABCD. Points A, B, C, and D are connected by lines to form two triangles within the quadrilateral. • Segment AB is horizontal and measures 40 meters. • Segment AD is vertical and measures 30 meters, forming a right angle with AB at point A. • Segment BD connects points B and D, forming a diagonal within the quadrilateral. • Segment BC measures 50 meters. • Segment DC measures 71 meters. The diagram illustrates the relationship between the areas of the triangles formed within quadrilateral ABCD and is labeled as follows: "area ABCD = area DAB + area DBC." [Generated by AI] $\text{area } ABCD = \text{area } (DAB) + \text{area } (DBC)$	Draw the diagonal DB tc quadrilateral into two tria
$\text{area } (DAB) = \frac{1}{2} \times 40 \times 30$ $\text{area } (DAB) = 600 \text{ m}^2$	Using area $= \frac{1}{2}$base $\times$

Student view

Overview (/study/ap 122- cid- 754029/

Step	Explanation
$DB^2 = 30^2 + 40^2$ $DB = 50 \text{ m}$	You will need to find mor information about triangl order to find angle C. Yo Pythagoras' theorem to fi length DB.
$\cos C = \frac{50^2 + 71^2 - 50^2}{2 \times 50 \times 71}$ $\angle C = \cos^{-1}\left(\frac{50^2 + 71^2 - 50^2}{2 \times 50 \times 71}\right) = 44.765\ldots^\circ$	Use this length and the c in triangle DBC.
$\text{area } (DBC) = \frac{1}{2} \times 50 \times 71 \times \sin 44.765\ldots^\circ$ $\text{area } (DBC) = 1249.95\ldots\ldots \text{ m}^2$	Using the formula $\text{area of triangle } =$.
$\text{area } ABCD = 600 + 1249.95\ldots$ $= 1850.0 \text{ m}^2 \text{ (1 d. p.)}$	Therefore, adding both tr areas, $\text{area } ABCD = \text{area } (D$ $\text{area } (D$

Important

When labelling polygons, start from any point and follow either a clockwise or anticlockwise direction.

2 section questions

Student view

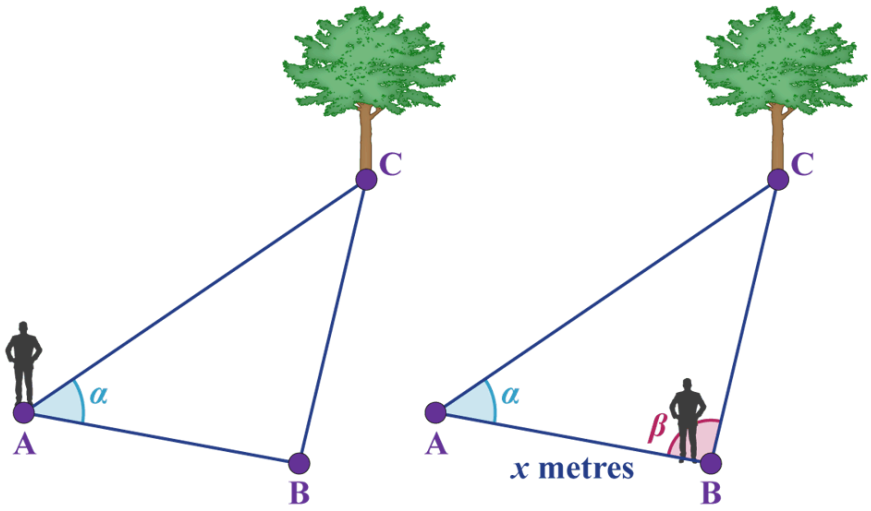
Solving problems by sketching diagrams

Finding the distance without actually measuring

Have you ever noticed a building, a hill or a landmark at a distance and wondered how far away it is? Did you know that you can use trigonometry to find the distance to the landmark without actually having to walk there?

The method for calculating the distance without physically measuring it is called triangulation, which is a method widely used by land surveyors.

A surveyor measures the viewing angle to a landmark from a distance, then walks along a straight line and measures the distance walked and the new viewing angle.



More information

The image shows a diagram illustrating how a surveyor measures angles to a landmark from two different positions. On the left, a surveyor at point A measures angle α to a tree at point C. On the right, the surveyor has moved to point B, a distance x meters away from point A, and measures a new angle β to the same

Student view

Overview (/study/ap 122- cid- 754029/

tree at point C. The diagram visually represents how these measurements relate to the positions of the surveyor and the tree, helping in calculating distances and angles through triangulation.

[Generated by AI]

Measure the viewing angle to the tree. Measure the distance walked and the new viewing angle.

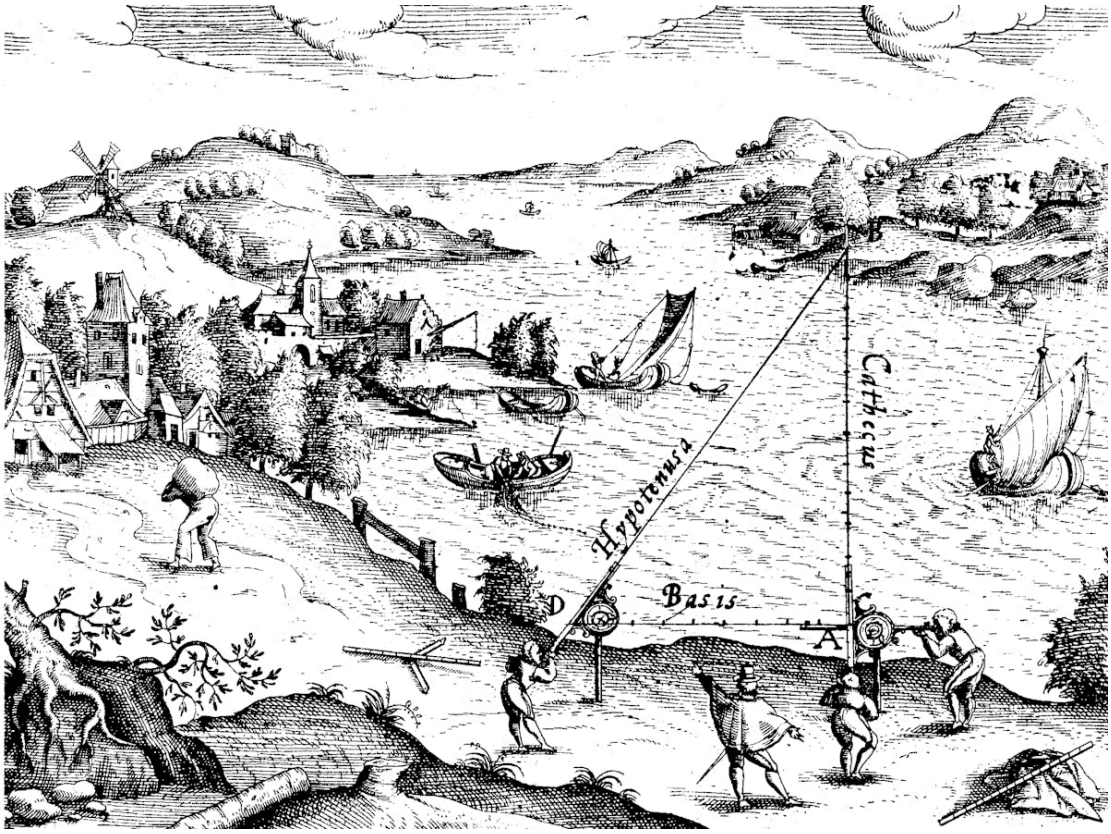
Once you have the two base angles and the length of the base, you can solve the triangle.

☆ Definition

In surveying, the tracing and measurement of a series of triangles in order to work out how far away distant objects are is called triangulation. This is usually done by measuring the base angles of a triangle and the distance between the vertices of the base. Trigonometry can then be used to work out the required distances.

Student view

Overview (/study/ap 122- cid- 754029/



16th century triangulation

Source: " CD006-Triangulation 16th century (https://commons.wikimedia.org/wiki/File:CD006-Triangulation_16th_century.png) " by Hulsius is in public domain.

More information

The image is an illustration from the 16th century, depicting a triangulation process used for measurement. The landscape features hills, trees, buildings, and a body of water. In the foreground, there are several figures positioned strategically with measurement tools. Two lines labeled 'Hypotenusia' and 'Cathetus' are drawn between the figures, showing the angles and sides of a triangle used in the measurement. Another line, labeled 'Basis,' forms the base of the triangle. The illustration includes various details such as boats on the water, distant hills, and small windmills on the far-left hill.

[Generated by AI]

Example 1

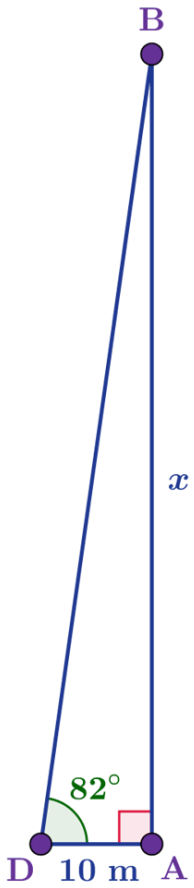
Student view

★★☆

Overview (/study/ap 122-cid-754029/

In the picture above, river surveyors in France are using triangulation to find the width of the river Seine. Angle $D = 82°$, the length of the base $DA = 10$ m and the angle $A = 90°$. What is the width of the Seine to the nearest metre?

The diagram below shows what you know about triangle DAT.



The diagram illustrates a triangle labeled as DAT with a right angle at point A. The side DA is labeled as 10 meters. The angle at D is marked as 82 degrees. The hypotenuse extends from D to B, labeled as x, and another line extends vertically from point A to B. A square is drawn adjacent to line DA at point A, indicating a right angle.

[Generated by AI]

Student view

Overview (/study/ap 122- cid- 754029/

$$\tan 82 = \frac{x}{10}$$

$$x = 10 \times \tan 82 = 71.15\ldots = 71 \text{ m to the nearest metre}$$

Be aware

The width of the river is the perpendicular distance from D to A , not the hypotenuse of the triangle.

International Mindedness

Between 1816 and 1855, the astronomer F.G.W. Struve used triangulation to map the region of land between the Arctic Sea and the Black Sea — a distance of over 2800 km. The chain includes 265 main station points and 258 main triangles. This study was the first accurate measurement of a long segment of a meridian which then helped with the development of Earth sciences and topographic mapping. You can find more information about this path, which was added to the UNESCO world heritage list in 2005, here in this link. (https://www.maanmittauslaitos.fi/en/about-nls/themes/struve-geodetic-arc)

Example 2

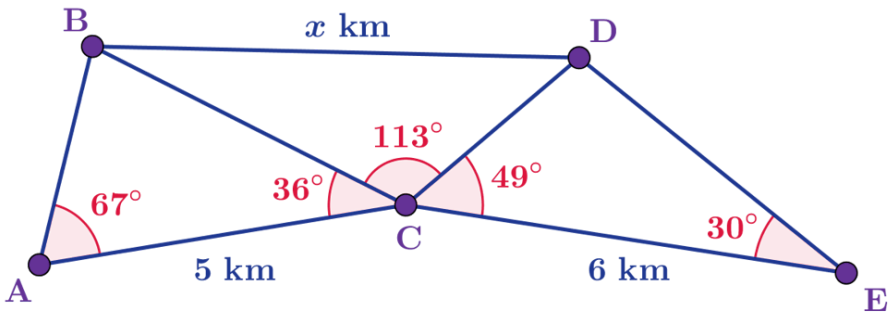
★★★

A land surveyor, measures the initial angle to a distant tree as $67°$. She walks 5 km and measures the angle to the same tree from the base line to be $36°$. She measures the angle between the viewing lines to both of the landmarks as $113°$. She then walks 6 km along the base line CE, where the base angles are $49°$ and $30°$. You can see her sketches in the diagram below.

Student view

Find the distance between the two landmarks B and D to the nearest km .

Overview (/study/ap 122- cid- 754029/



More information

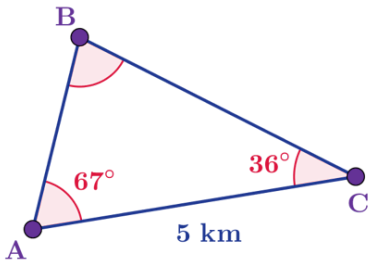
This diagram illustrates a geometric shape consisting of lines connecting five points: A, B, C, D, and E. The lines form two triangles: ABC and CDE. The line segment AB measures 5 km, BD is represented as x km, and DE measures 6 km. The angles of triangle ABC at A and B are 67° and 36° respectively, with the angle at C being 113°. Triangle CDE has angles at C and E of 49° and 30° respectively. The diagram is used for calculating distances and angles between marked points.

[Generated by AI]

Steps	Explanation
Triangle BAC: $\text{angle B} = 180 - (67 + 36) = 77$	There are three triangles one of the side lengths c BCD in which only the o angle to side BD is given can find the sides BC an the length of BD can be using the cosine rule.

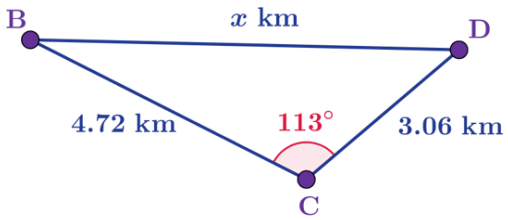
Student view

Overview (/study/ap 122- cid- 754029/

Steps	Explanation
$\frac{\mathrm{BC}}{\sin 67} = \frac{5}{\sin 77} = \frac{\mathrm{BC}}{\sin 67}$ $\mathrm{BC} = \frac{5 \times \sin 67}{\sin 77}$ $\mathrm{BC} = 4.7235\ldots$	You can find BC using th rule. Don't round your answer in the memory of your ca
 $\text{angle D} = 180 - (30 + 49) = 101^\circ$	
$\frac{\mathrm{DC}}{\sin 30} = \frac{6}{\sin 101}$ $\mathrm{DC} = \frac{6 \times \sin 30}{\sin 101}$ $\mathrm{DC} = 3.056\ldots$	You can find DC using th rule. Don't round your answer in the memory of your ca

Student view

Overview (/study/ap 122-cid-754029/

Steps	Explanation
 A triangle is shown with points labeled as B, C, and D. The distance between B and C is labeled as 4.72 km, and the distance between C and D is labeled as 3.06 km. The triangle has an angle at point C measuring 113 degrees. The third side of the triangle, BD, is labeled as x km. This is a geometric diagram displaying a triangle with given sides and angle measurements, commonly used for calculations or problem-solving in geometry. [Generated by AI]	In triangle BCD: As you have two sides ar angle, you can use the c to find the missing lengtl
$x^2 = 4.7235\ldots^2 + 3.056\ldots^2 - 2 \times 4.7235\ldots \times 3.056\ldots \times \cos 113^\circ$ $x = 6.552\ldots = 7\,\text{km}\ (\text{to the nearest km})$	Using the cosine rule $c^2 = a^2 + b^2 -$

Exam tip

Do not round your answers for *BC* and *DC* to the nearest km. Only round the final answer, the length of *DB* , to the nearest km. Otherwise you might lose A (accuracy) marks because of early rounding.

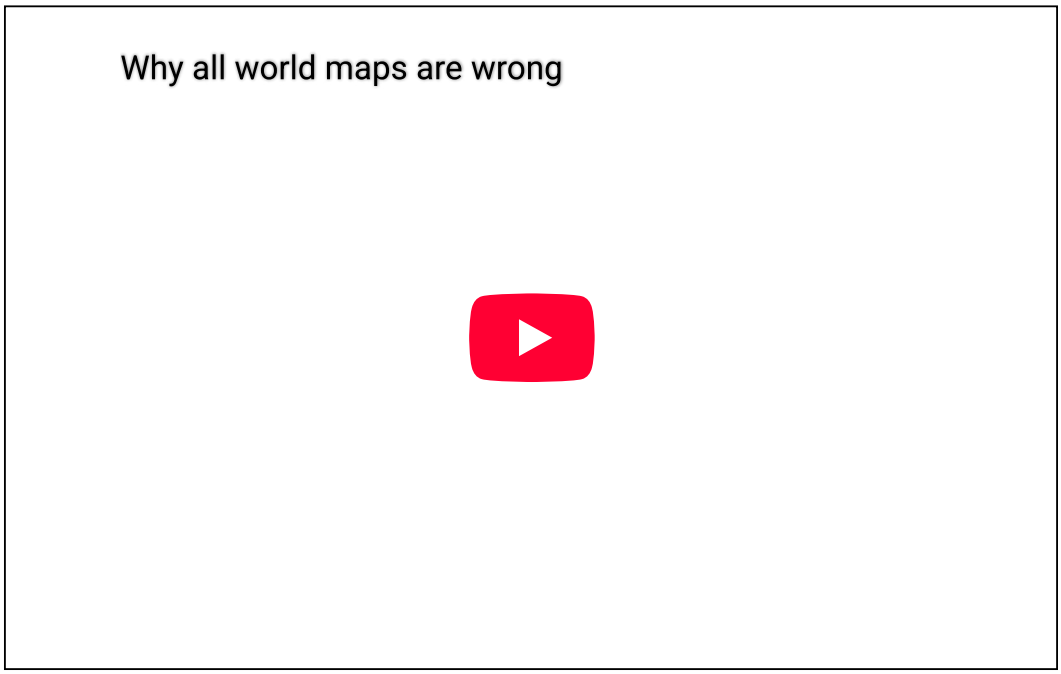
Theory of Knowledge

Trigonometry and geometry rely a great deal on representation. How accurate are visual representations in regard to the knowledge they provide? Watch the video below on the problem of maps and representing

Student view

Overview (/study/ap 122- cid- 754029/

a three-dimensional object in a two-dimensional space.



1 section question

3. Geometry and trigonometry / 3.3 Applications of trigonometry

Bearings

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/bearings-id-26218/print/) Assign

Compass directions

There are four main compass directions: north, south, east and west. The north–south line is perpendicular to the west–east line. The directions half way between are named north–east, south–east, south–west and north–west.

Compass angles are always measured clockwise from north and they are written as 3-digit bearings. For example, north–east is 045° and north–west is 315°.

Student view

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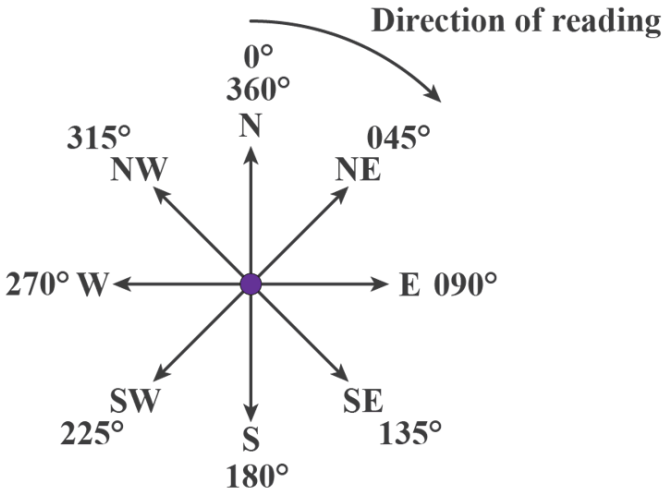


More information

This image is a diagram of a compass rose showing the cardinal and intercardinal directions with their corresponding angles. At the center is a purple circle. Eight arrows emanate from the center pointing towards north, north-east, east, south-east, south, south-west, west, and north-west. Each direction is marked with its bearing angle: North (N) is labeled as 0°/360°, North-East (NE) as 045°, East (E) as 090°, South-East (SE) as 135°, South (S) as 180°, South-West (SW) as 225°, West (W) as 270°, and North-West (NW) as 315°. There is also an arrow labeled 'Direction of reading' indicating clockwise movement starting from the north.

[Generated by AI]

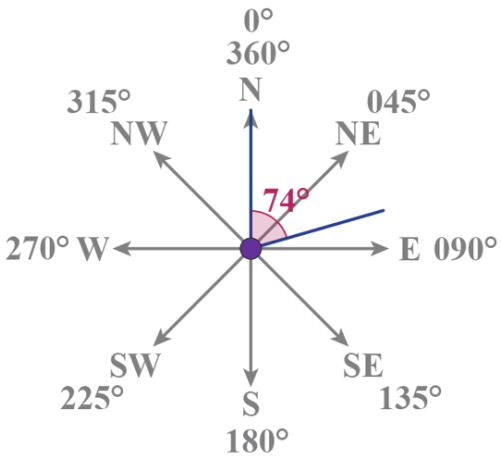
☆ Definition

A bearing is the angle between the north line and the direction of a movement from the centre of the compass, measured in a clockwise direction. Bearings always have three figures.

On the compass shown below you can see a bearing of 074°.

Student view

Overview (/study/ap 122- cid- 754029/



More information

The image is a compass rose diagram displaying cardinal directions along with intercardinal directions and corresponding degrees. At the top of the compass is North marked as both 0° and 360°, and directly opposite is South marked as 180°. East is on the right side marked as 090°, and West is on the left marked as 270°.

Intercardinal directions include: - Northeast (NE) at 045° - Southeast (SE) at 135° - Southwest (SW) at 225° - Northwest (NW) at 315°

In the center of the compass is a circle that serves as the origin point for all direction vectors. A highlighted bearing line extends from the center illustrating a bearing of 074°, which falls between North and East, slightly closer to East. This bearing is shown as a distinct line on the compass rose along with a marked arc to indicate the angle from the north direction.

[Generated by AI]

Example 1

★★☆

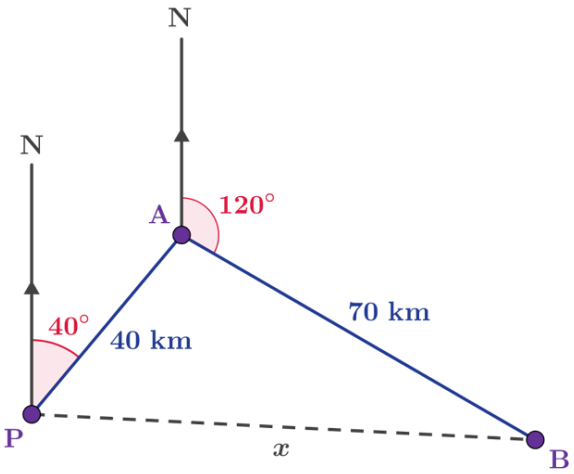
Student view

A ship leaves a port and travels 40 km on a bearing of 040°.

After changing direction, it continues to travel another 70 km on a bearing of 120°.

How far is the ship from the port?

Overview (/study/ap 122- cid- 754029/

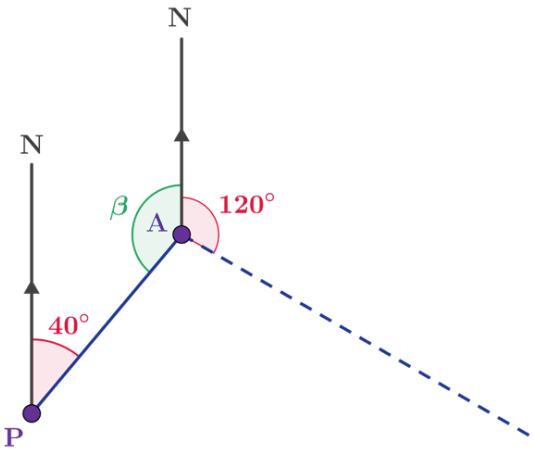
Step	Explana
	First, sketch th information. In triangle PAB the lengths of t and need to fir length of the th can find the siz PAB, you can u cosine rule to c the distance fr port.

This is a diagram of a triangle labeled with distances and angles. The triangle is formed between three points labeled as P, A, and B. Starting from point P, a line extends vertically labeled with "N" indicating north direction. This line forms a 40° angle with line PA, which is 40 km long. At point A, another vertical line labeled with "N" is shown. The angle between lines PA and AB is marked as 120°. Line AB is 70 km long. Another line extends horizontally from point P to B, forming a base with length x.

[Generated by AI]

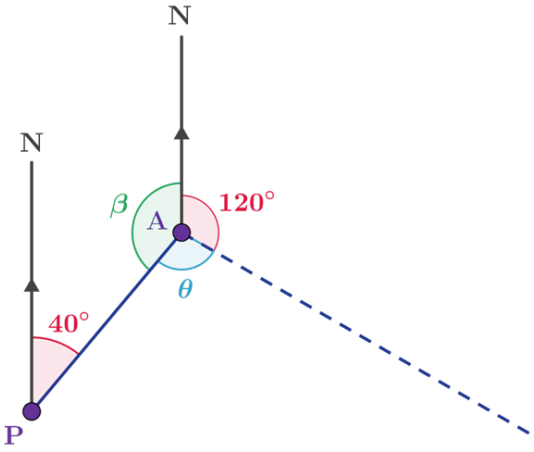
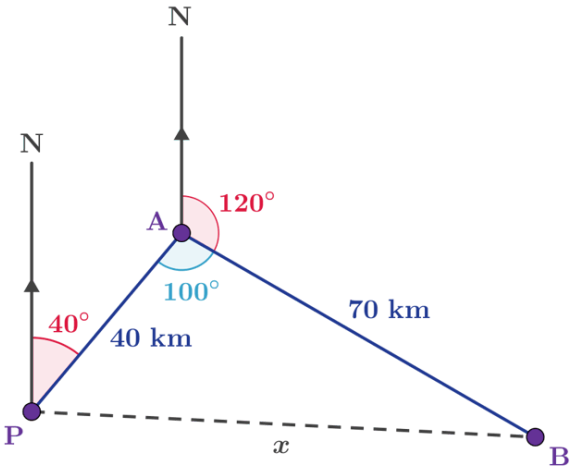
Student view

Overview (/study/ap 122- cid- 754029/

Step	Explana
 The diagram illustrates a geometric configuration involving a triangle with specific angles and labeled points and lines. The triangle is marked with point A at the top, representing the vertex where angle β and 120° meet. There is a line segment from point P to A with an angle of 40° at point P. The line segment extends from A onward and the horizontal dashed line represents an extension helping to define angle measurements with 120° marked inside a red shaded area. The vertical arrows labeled N suggest direction or indicative forces around the triangle. [Generated by AI]	Angles *P* and β interior angles 180°.
$40 + \beta = 180$ $\beta = 140^\circ$	

Student view

Overview (/study/ap 122- cid- 754029/

Step	Explana
	Angles at a poi 360°.
$\theta + 140 + 120 = 360$ $\theta = 100^\circ$	
	Now you can u cosine rule in t PAB.

Student view

Overview (/study/ap 122- cid- 754029/

Step	Explana
$x^2 = 40^2 + 70^2 - 2 \times 40 \times 70 \times \cos 100$ $x = 86.44... = 86.4$ km (3 significant figures)	

Example 2

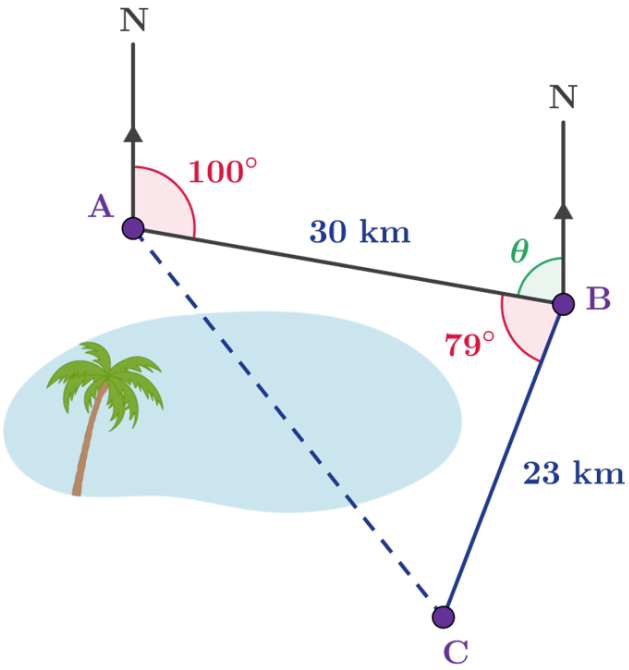
★★☆

A ship sailing around an island, travels 30 km on a bearing of 100° to reach point B, as shown in the diagram below.

From B, the ship changes direction and travels 23 km to point C.

a) If angle $\mathrm{ABC} = 79^\circ$, find the bearing of C from B.

b) Find the distance AC.



Student view

Overview (/study/ap 122- cid- 754029/(

More information

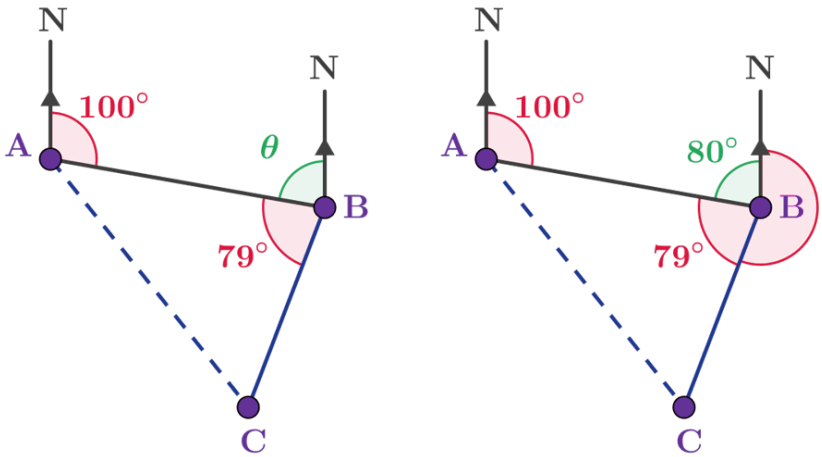
The image is a diagram illustrating a triangle labeled with points A, B, and C. Point A is connected to B with a distance of 30 km and an angle of 100° measured counterclockwise from North. Point B to C is 23 km, forming an angle of 79° with the line AB. There is an angle θ labeled between the line AB and vertical at point B. The triangle is positioned with some geographical elements like a palm tree and a body of water depicted below the triangle, suggesting a real-world context for the distances.

[Generated by AI]

a) Angles A and θ are co-interior angles; therefore $\theta = 180° - 100° = 80°$.

Angles around point B add up to $360°$.

Therefore, the bearing of C from B is $360° - 80° - 79° = 201°$.



Student view

Overview (/study/ap 122- cid- 754029/(

The image consists of two diagrams illustrating angles related to points A, B, and C with respect to north directions.

On the left, there is a diagram showing point A connected to point B and C, forming a triangle (triangle ABC). The angle at point A with respect to the north is marked as 100°, at point B as an angle θ and 79°, and point C is part of the triangle but not labeled with an angle.

On the right, a similar setup is shown, with angles at point B highlighted. The north direction is indicated with arrows for both sides. Lines indicating segments AB, BC, and AC are drawn, with the AC line appearing as a dashed line in both diagrams. The angles are marked in red and green colors, showing co-interior and other angles around the points.

This setup demonstrates the calculations related to interior angles, the concept of co-interior angles, and angles around a central point, helping visualize the mathematical explanation given in the text before and after the image.

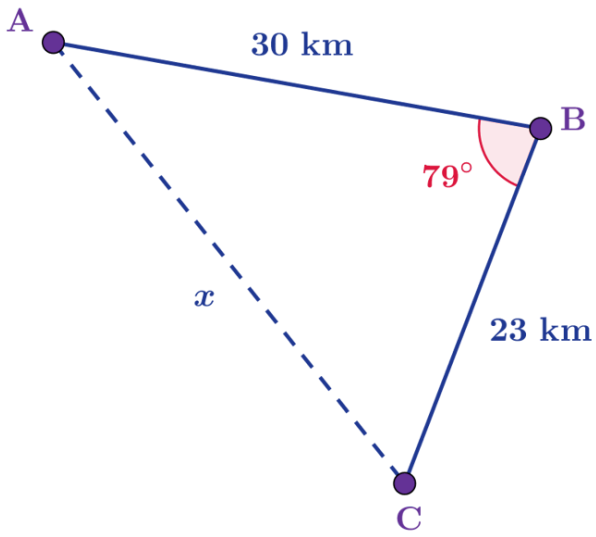
[Generated by AI]

Angles θ and A are co-interior angles Angles around point B add up to 360°

b) I n triangle ABC , two sides and the included angle are given and you are asked to find the third side. So you can use the cosine rule to find the length of AC.

Student view

Overview (/study/ap 122- cid- 754029/(



The image depicts triangle ABC. Side AB is 30 km, side BC is 23 km, and angle ABC is 79 degrees. The third side, AC, is labeled as 'x' and is not yet determined. The triangle is clearly marked to illustrate the elements necessary to apply the cosine rule to find the unknown side AC using the formula: ({x^2} = {30^2} + {23^2} - 2 \times 30 \times 23 \times \cos 79^\circ).

[Generated by AI]

$$x^2 = 30^2 + 23^2 - 2 \times 30 \times 23 \times \cos 79^\circ = 1165.68\ldots$$

$$x = 34.142\ldots = 34.1 \text{ km} \text{ (3 significant figures)}$$

2 section questions

Student view

3. Geometry and trigonometry / 3.3 Applications of trigonometry

Angles of elevation and depression

Defining the heights

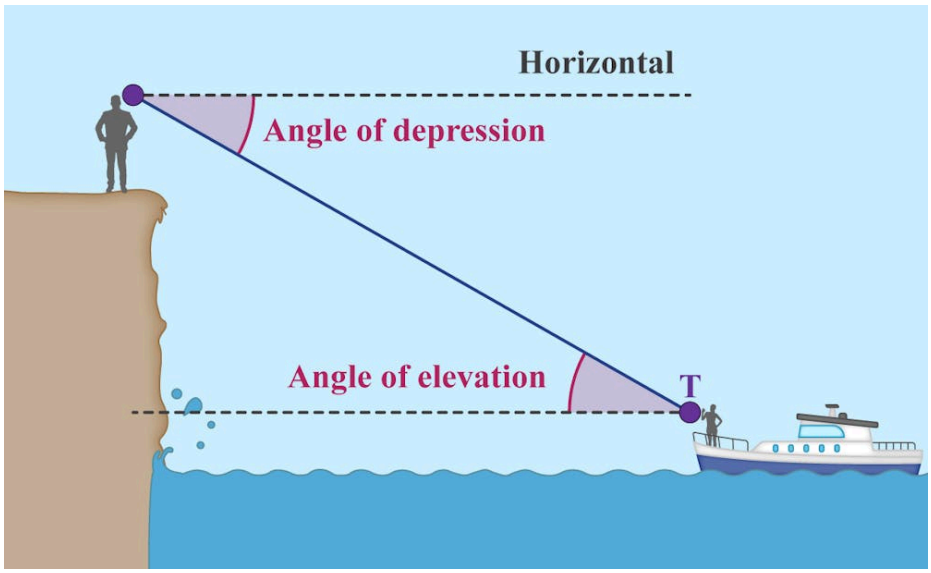
If you are standing on a tower or a high object and looking downwards, your perspective will be different from that of a person who is standing at a distance and looking upward to where you are standing.

Definition

The upward angle from a horizontal line of sight from an observer to a landmark on a high location is called **the angle of elevation.**

The downward angle from a horizontal line of sight from an observer to a landmark on a lower location is called **the angle of depression.**

The diagram shows an angle of elevation and an angle of depression. These two angles are used to calculate the distances and heights in navigation and surveying.



Overview (/study/ap 122- cid- 754029/(

The image is a diagram illustrating the angles of elevation and depression used in navigation and surveying. A person is standing on a cliff edge observing a boat on the water level. A horizontal line intersects at the observer's eye level, extending towards the boat. The angle of depression is marked from the horizontal line downwards to the boat. Conversely, the angle of elevation is measured upwards from the water level position of the boat to the same horizontal line. The observer's line of sight towards the boat is depicted as a solid line, creating a triangular shape with the horizontal. The boat is labeled 'T'.

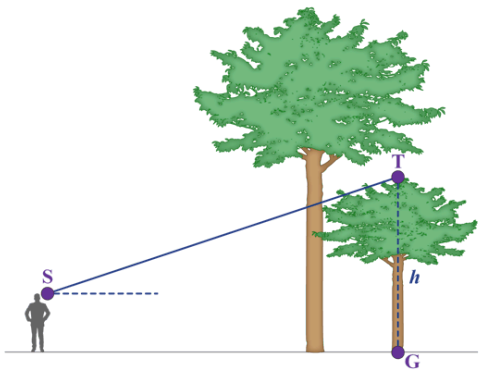
[Generated by AI]

Example 1

★★☆

The world’s second tallest tree is a centurion, in Arve Valley Tasmania, Australia. It towers above the trees around it as it is 99.82 m tall. A forest surveyor is measuring the heights of other trees around the region. He measures the angle of elevation from his eye level to be 81° and the distance to the foot of the tree to be 8 m.

If his eye level is 1.58 m above the ground, find the height of the tree to the nearest centimetre.



More information

Student view

Overview (/study/ap 122-cid-754029/

The diagram shows a comparison between two trees in terms of height. The larger tree is positioned in the background, and the smaller tree is in the foreground. A person is standing to the left of the smaller tree, marked by the letter 'S'. From 'S', a diagonal line is drawn towards the top of the smaller tree marked 'T'. From 'T', a vertical line labeled 'h' extends down to the base point 'G' of the smaller tree. This represents the tree's height. The larger tree does not have any specific labels or measurements shown. The diagram illustrates a method of measuring tree height using proportional relationships, with visual lines showing horizontal and vertical alignments.

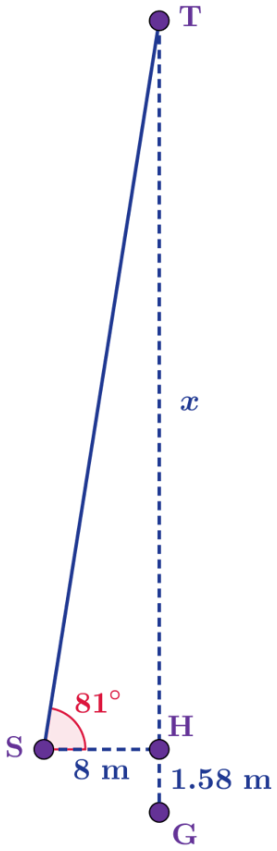
[Generated by AI]

Centurion tree, Arve Valley Tasmania, Australia

Source: " Centurion tree 99.82 (https://commons.wikimedia.org/wiki/File:Centurion_tree_99.82.jpg) " by Eucalyptus 99 is licensed under CC BY-SA 4.0 (https://creativecommons.org/licenses/by-sa/4.0/)

Student view

Overview (/study/ap 122- cid- 754029/

Step	Explana
	Sketch the diag showing all the information you given.

This is a geometric diagram illustrating a right triangle. The triangle is formed by three points: S, H, and T. Point S is at the left, with an angle labeled 81° next to it. The line segment between S and H is horizontal and measures 8 meters. From H, a vertical dash extends downward to point G, labeled as 1.58 meters. The vertical line from H to T is labeled x, indicating an unknown length. A dashed line connects S to T. The triangle illustrates the relationship between these points and lines, likely to solve for the unknown variable x or to explain trigonometric relationships.

[Generated by AI]

Student view

Overview (/study/ap 122- cid- 754029/

Step	Explana
$\tan 81 = \frac{x}{8}$ $x = 8 \tan 81$ $= 50.510\ldots$	
$TG = 50.510\ldots + 1.58 = 52.09\ldots$ So the height of the tree is 52.09 m (to the nearest cm).	

Example 2

★★☆

An eagle sits at the top of a tree at the edge of the ocean watching a fishing boat.

The angle of depression is $24°$ and the tree is 47 m tall.

Bonelli's eagle

Credit: AndreAnita Getty Images

Student view

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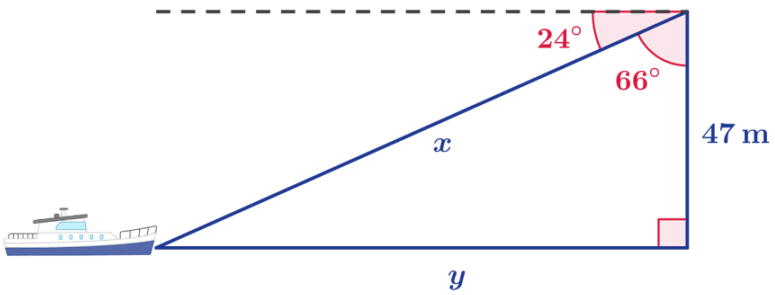
a) How far is the fishing boat from the eagle?

b) How far is the fishing boat from the base of the tree?

Give your answers to nearest metre.

Step	Explanation
 The image depicts a right triangle formed by a boat, with the hypotenuse being the line of sight from the boat's position to a point on a vertical line. The base of the triangle is a horizontal line labeled 'y,' and the height is labeled as 47 m. The hypotenuse, labeled 'x,' creates an angle of 24° with the horizontal line 'y,' and an angle of 66° with the vertical line of 47 m. There is a 90° angle indicated at the intersection of the height and the base. This diagram visually represents the trigonometric relationship between the angles and the sides of the triangle formed by the boat's position related to a vertical and horizontal reference. [Generated by AI]	First, sketch a diagram. Notice that the angle of depression is measured horizontal, not from the

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Step	Explanation
$\cos 66 = \frac{\text{adj}}{\text{hyp}}$ $= \frac{47}{x}$ $x \cos 66 = 47$ $x = \frac{47}{\cos 66} = 115.55\ldots$ So the distance of the fishing boat to the eagle is 116 m (to 3 significant figures).	To find the distance fro eagle to the boat, you n find the length of the hy You have the side adjac angle $66°$, so you can u cosine ratio.
$\tan 66 = \frac{\text{opp}}{\text{adj}}$ $= \frac{y}{47}$ $47 \tan 66 = y$ $y = 105.56\ldots$ So the distance of the fishing boat to the base of the tree is 106 m (to 3 significant figures).	To find the distance fro of the tree to the boat, to find the length of the opposite the angle of 6 you have been given th side, you can use the ta ratio.

Making connections

Have you ever wondered how ships find their way in the ocean where there are no visible landmarks on the surface of earth? This section of the historic Piri Reis map (1513)shows South America and Antarctica in great detail even before they were claimed to have been discovered. How is this possible?

How does maths guide our ships at sea?

Student view

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How does math guide our ships at sea? - George Christoph

Example 3

★★★

The 3D diagram below shows the air traffic control tower (CD) and the starting point of two runways. The start of Runway 1 is at point A and the start of Runway 2 is at point B. Points A, B and C are on horizontal ground and the control tower is vertical.

AB = 230 m, AC = 340 m, the angle of elevation of point D from point A is 12° and angle CAB =108°.

Student view

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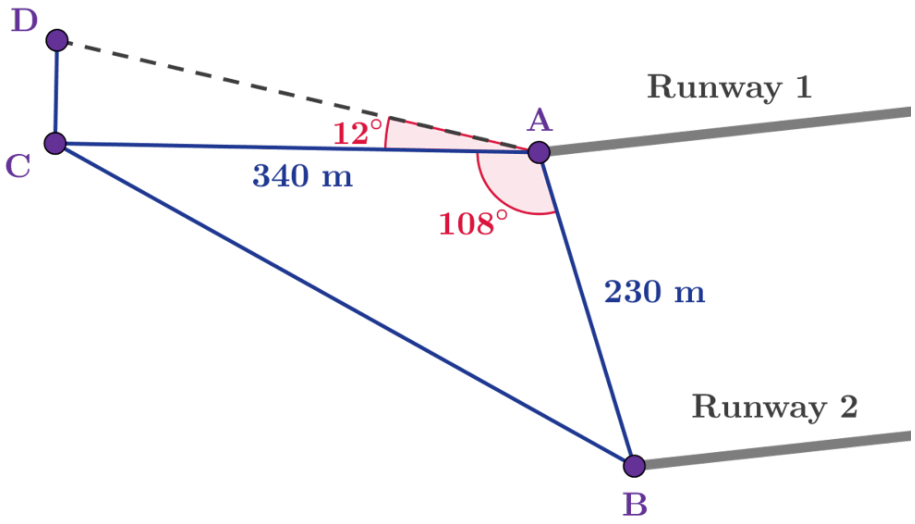

More information

The image is a geometric diagram depicting a quadrilateral with labeled vertices A, B, C, and D. Points B and C are connected by a line segment labeled 230 m, while points C and A are connected by another segment labeled 340 m. The angle at vertex A between lines AB and AC is marked as 108 degrees. Above this angle, the angle of elevation from point A to point D is noted as 12 degrees. The diagram also includes lines representing runway paths labeled as "Runway 1" along AC and "Runway 2" along AB.

[Generated by AI]

Calculate

a) the distance BC

b) the height of the control desk above the ground

c) the angle of depression of the start of Runway 2 from the control desk.

The airport management is planting more grass to make the airport feel more welcoming to passengers. The triangle ABC is to be seeded with grass. A bag of grass seed weighing 10 kg covers 1156 m^2 and costs 320 Norwegian krone (NOK).

Student view

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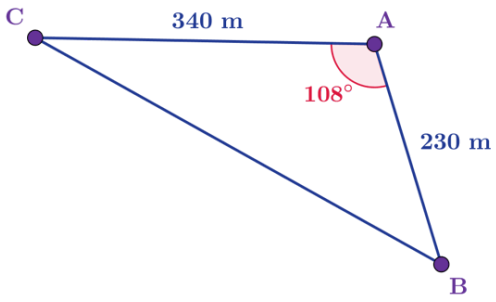
d) How much will it cost to seed the triangular area ABC? Give your answer to the nearest 100 NOK.

	Step	Explanation
a)	 The image is a diagram of a triangle labeled A, B, and C. The line segment AC is labeled as 340 meters, and the line segment AB is labeled as 230 meters. There is an angle marker at point A indicating an angle of 108 degrees between lines AC and AB. [Generated by AI]	First, sketch the triangle ABC information you are given.
	$\mathrm{BC}^2 = 340^2 + 230^2 - 2(340)(230)\cos 108$ $\mathrm{BC}^2 = 216830.257\ldots$ $\mathrm{BC} = 465.65036\ldots$ So $BC = 466\,\mathrm{m}$ (to 3 significant figures).	You have two sides and the enclosed angle. Therefore, y use the cosine rule $c^2 = a^2 + b^2 - 2a$ This is a question given in cc you need to include units in answer to score full marks.

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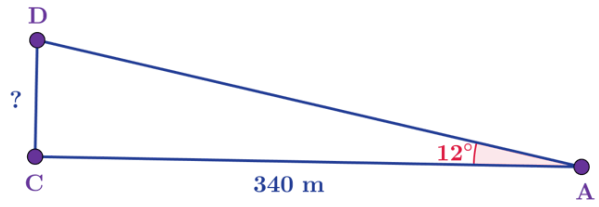
	Step	Explanation
b)	 The image is a diagram of a right triangle with the following points: D, C, and A. Point D is at the top left, directly above point C, which is at the bottom left. Point A is at the bottom right. The triangle is marked with a 340-meter distance between C and A. At point A, there is a marked angle of 12 degrees between side AC and the hypotenuse AD. The vertical side DC is marked with a question mark indicating an unknown length. This diagram illustrates the components of a right triangle, including an angle and a side length, to demonstrate trigonometric calculations. [Generated by AI]	You can use the right-angle *DCA* to work out the height control desk above the grou is the length *CD* on the diag Since you have the sides op adjacent to the angle of 12° use the tangent ratio.
	$\tan 12 = \frac{\mathrm{CD}}{340}$ $\mathrm{CD} = 340 \tan 12 = 72.2692\ldots$	Rearrange and solve for *CD*.
	Therefore, the height of the control desk above the ground is $72.3\ \mathrm{m}$ (to 3 significant figures.).	

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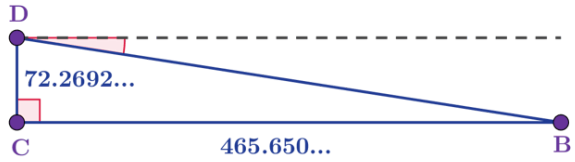
	Step	Explanation
c)	 The image depicts a geometric diagram involving a right triangle and a smaller triangle. The points D, C, and B form a right triangle. Point D is directly above point C, forming the right angle at point C. The horizontal line from C to B is labeled as 465.650... units long. The vertical line from D to C is labeled as 72.2692... units long. A smaller red right triangle is visible at the top of the diagram, overlaying the main triangle. The main components of the diagram are: - Points D, C, and B, forming the vertices of the right triangle. - The base CB of the right triangle, which is horizontal. - The height CD of the right triangle, which is vertical. The diagram uses dashed and solid lines to differentiate various elements and employs a purple color scheme for labels and points. [Generated by AI]	To find the angle of depress need to look down from poi point B. The angle you want measured downwards from horizontal, as shown in the d Using alternate angles, the a depression is equal to angle
	$\tan B = \frac{72.2692}{465.650}$	Again, you have opposite ar sides in a right-angled trian can use the tangent ratio.

Student view

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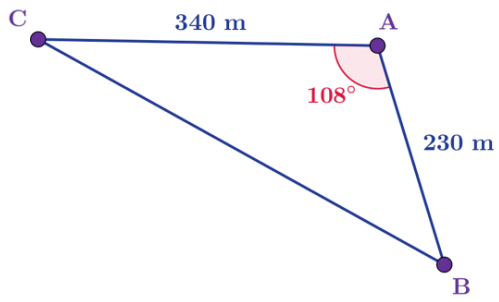
	Step	Explanation
	$\tan^{-1}\left(\frac{72.2692}{465.650}\right) = 8.82196\ldots^{\circ}.$ The angle of depression of B from D is 8.82° (to 3 significant figures).	To find the angle, you need inverse tangent. Remember: on the calculatc either type $\tan^{-1}\left(\frac{72.2692}{465.650}\right)$ out $\frac{72.2692}{465.650}$ and then enter $\tan^{-1}(ANS)$ rather than re decimal.
d)	 The image is a diagram of triangle ABC. It shows the distance between points C and A is 340 meters, and between points A and B is 230 meters. The angle at vertex A is marked as 108 degrees. The triangle appears to be drawn with solid lines connecting points C, A, and B, forming a closed shape. The angle at A is visually highlighted with a red arc, denoting its measurement, and labeled explicitly with the degree value. This diagram provides a visual representation of a triangle with specific measurements and angle. [Generated by AI]	To find the amount of grass need to know the area of th ABC. The easiest approach i the formula $\text{area} = \frac{1}{2}ab\ \sin($ because you are given two s the included angle and the t does not have a right angle.

Student view

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	Step	Explanation
	Number of bags of grass seed = $37186.3\ldots \div 1156 = 32.1680\ldots$ so 33 bags are needed.	To find out how many bags required, you need to divide $37\,186.3\ldots\ \mathrm{m}^2$ by $1156\,\mathrm{m}^2$ You don't need to use the fa each bag weighs 10 kg. You need more than 32 bag cannot buy exactly 32.168 b will need to buy 33 bags.
	This costs $33 \times 320 = 10\,560\mathrm{NOK}$ To the nearest 100 NOK, the answer is $10\,600$ NOK.	Each bag costs 320 NOK. Round your final answer to t 100 NOK.

Exam tip

It is essential to include the line of work that demonstrates correct substitution into the correct formula. When the IB exam papers say 'you are advised to show all working' they definitely mean to include this particular line of working.

Examiners consistently advise students to show their unrounded answers in their work, before attempting to round correctly. Do not round until you give your final answer.

2 section questions

3. Geometry and trigonometry / 3.3 Applications of trigonometry

Checklist

Student view

What you should know

By the end of this subtopic you should be able to:

- sketch diagrams in order to solve problems involving angles and lengths
- select and use the correct formula to solve a problem from the information given in the question
- use triangulation to find distances and areas
- find a bearing and use it to calculate distances
- identify and calculate angles of elevation and depression.

Investigation

Parallax

Have you wondered how astronomers measure the distance to stars without actually having to travel there? Or why some objects seem to be moving faster than others when you are in a moving car and watching the view outside. Or why, when you hold your thumb up in front of your eye and close only your left eye looking at an object in the distance, and then close only your right eye looking at the same object without moving your thumb, the object moves? All this illusion is created by parallax, an apparent displacement of an object because of the change in the observer's point of view.

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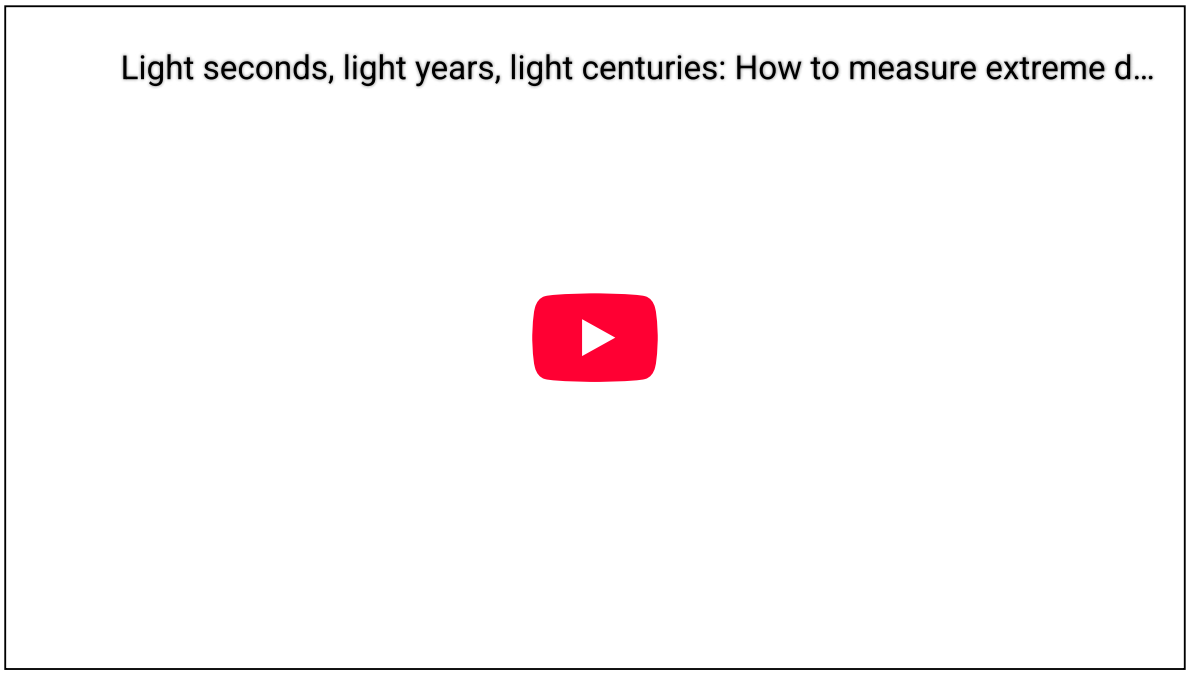
Stellar parallax is used to determine the distance of nearby stars but it is not reliable because of the errors involved in measurements. See the video below if you would like to learn more about the use of parallax in astronomy.



In this investigation, you will be using the idea of parallax to measure the distance to a distant object.

Materials needed:

- A protractor or a compass
- A tape measure or a trundle wheel

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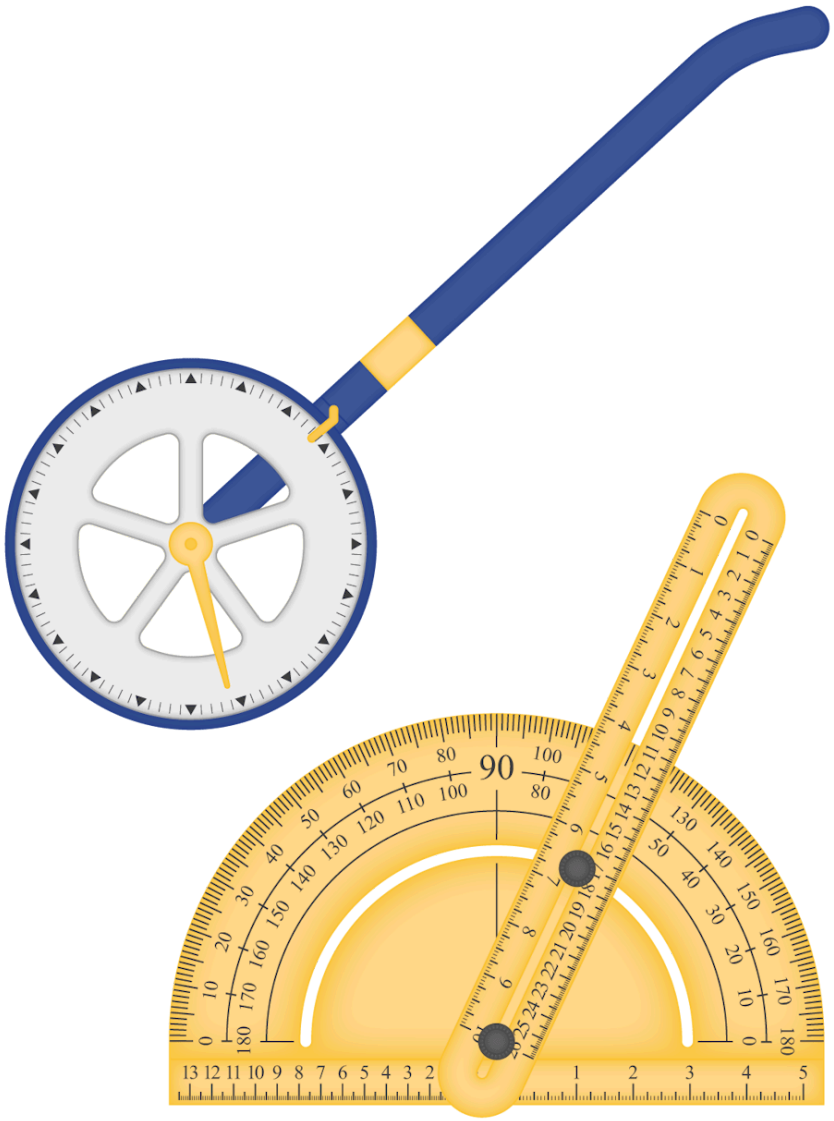

Before you go out and start measuring

Parallax:

- Why do you think there is a difference in the observed and measured distance to distant objects?
- Does the distance to the distant object have effect on parallax measurements? Explain.

Measurements:

- What are the measurements that you can take to calculate to distance? Sketch a diagram of measurements that you will need to take.
- What do you need to be careful about when measuring?

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- Which materials will you need?

Outside

- Identify a distant object and measure the distance and angles. (Remember you are not going to measure the actual distance.)
- Label your initial sketch with the measurements you take.

Calculations and reflection

- Calculate the distance to the object.
- Consider whether the solutions make sense in real life.
- Consider the accuracy of your solutions – justify the accuracy and explain any reasons why inaccuracy exists.
- Write your report showing how you solved the problem.
- Write a conclusion.

Rate subtopic 3.3 Applications of trigonometry

Help us improve the content and user experience.

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