3.9 Planar transformations

Checklist

What you should know

By the end of this subtopic you should be able to:

- represent the transformation from x, y to x', y' under transformation $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ as $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
- represent an anticlockwise (counter-clockwise) rotation of angle θ about the origin $(\theta > 0)$ using $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
- represent a horizontal stretch parallel to the x-axis with a scale factor of k as $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$
- represent a vertical stretch parallel to the y-axis with a scale factor of k as $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$
- represent an enlargement, with a scale factor k , centre $(0, 0)$, as $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$
- represent a translation of a units parallel to the x-axis and b units parallel to

  the y-axis using vector $\begin{pmatrix} a \\ b \end{pmatrix}$
- represent affine transformations as $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$
- find the area of a triangle with vertices $(a, b), (c, d)$, and (e, f) using

$$A = \frac{1}{2} \begin{vmatrix} a & b \\ c & d \\ e & f \\ a & b \end{vmatrix} = \frac{1}{2} \left| (ad + cf + eb) - (af + ed + cb) \right|$$

- find the area of an image using

$$\text{area of image} = |\det A| \times \text{area of object} .$$