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3. Geometry and trigonometry / 3.6 Trigonometric identities



Notebook



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# The big picture

## Identity or equation

In mathematics, an equation is a statement that equates two statements. They can be true for some values of the variable(s), or not true, which means there is no solution.

For example, you can solve the equation  $2x - 4 = 6$  and find that it is true for  $x = 5$ .

But  $\cos \theta = 5$  has no solution, because  $-1 \leq \cos \theta \leq 1$  for all values of  $\theta$ , and so it is not true.

When the equation holds true for all values of the variable(s) then it is called an identity.

The equation  $x^2 - y^2 = (x - y)(x + y)$  is true for all the values of  $x$  and  $y$ .

Therefore, it is an identity. In identities, instead of  $=$  you could use  $\equiv$  to say it is an identity. But in the IB formula booklet,  $=$  is used for identities.



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- Overview  
(/study/ap) A familiar example of a trigonometric identity is  $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$  as it is true for all values of the angle  $\theta$ .
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761926/o In this section, you will be extending your knowledge about trigonometric identities.



Identical twins

Credit: Image Source Getty Images

## 💡 Concept

In this subtopic, you will look at the relationships between trigonometric ratios and identify equivalent expressions to form identities.

How do identities help us to prove mathematical results?



3. Geometry and trigonometry / 3.6 Trigonometric identities

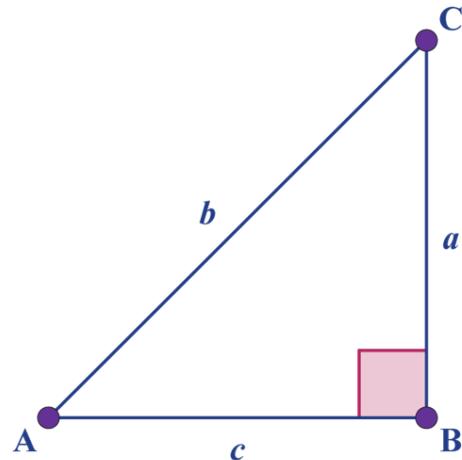
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# Pythagorean identity



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# Revisiting right-angled triangles



More information

The image is a diagram of a right-angled triangle labeled as ( $\triangle ABC$ ). In this triangle, ( $\angle ABC$ ) is the right angle. The side opposite the right angle, ( $\overline{AC}$ ), is the longest side and is labeled as ( $b$ ). The side ( $\overline{AB}$ ) is labeled as ( $c$ ), and the side ( $\overline{BC}$ ) is labeled as ( $a$ ). Each vertex (A, B, C) is marked with a purple circle. There's a small square at angle ( $\angle ABC$ ), indicating it's a right angle.

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In the diagram above, ABC is a right-angled triangle. For this triangle you can write Pythagoras' theorem as



$$c^2 + a^2 = b^2$$

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If you divide both sides of the identity by  $b^2$ , you get

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**Assign**

$$\frac{c^2}{b^2} + \frac{a^2}{b^2} = 1$$

which is also

$$\left(\frac{c}{b}\right)^2 + \left(\frac{a}{b}\right)^2 = 1 \quad (1)$$

The trigonometric ratios are

$$\cos A = \frac{c}{b} \text{ and } \sin A = \frac{a}{b}$$

so you can substitute them into identity (1) to get

$$(\cos A)^2 + (\sin A)^2 = 1$$

Therefore, in any right-angled triangle with an acute angle  $A$ ,

$$\cos^2 A + \sin^2 A = 1$$

### Be aware

Squares of trigonometric ratios can be written as  $(\cos A)^2$  or  $\cos^2 A$ .

If it is written as  $\cos A^2$  without the brackets it means the square of the angle  $A$ .

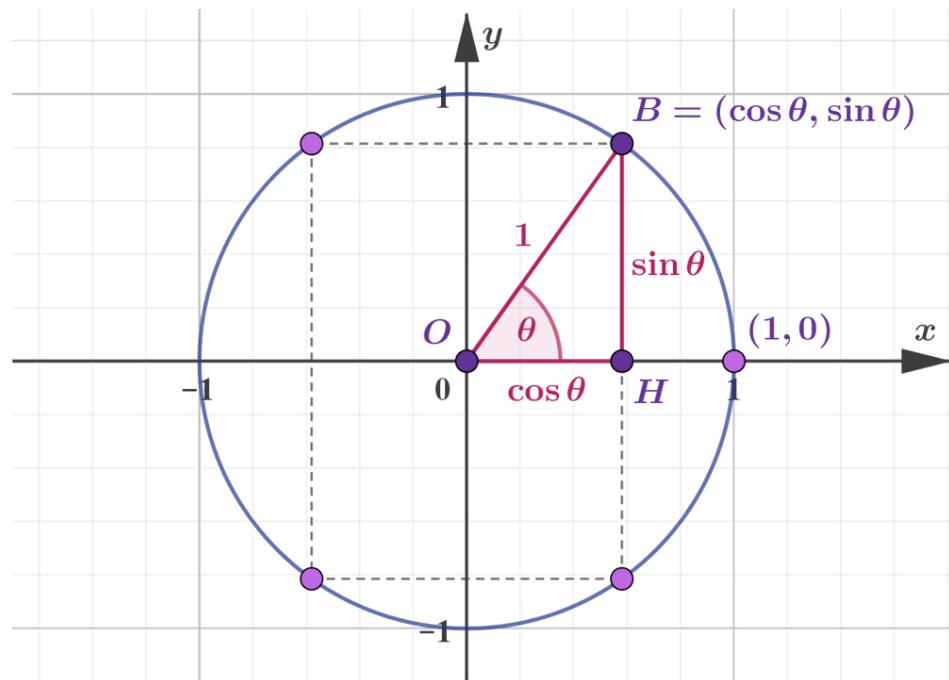
## Revisiting the unit circle

After defining the relation between sine and cosine of an acute angle, the natural progression is to see whether this identity holds true for any angle.



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More information

The image is a diagram of a unit circle on a coordinate grid. A right-angled triangle, labeled OBH, is inscribed in the circle. Point O is at the origin (0, 0), point H is on the x-axis at  $(\cos \theta, 0)$ , and point B is located on the circle at  $(\cos \theta, \sin \theta)$ , forming angle  $\theta$  with the x-axis. The hypotenuse OB equals 1, representing the radius of the circle. The length OH is  $\cos \theta$ , and the vertical side BH is  $\sin \theta$ . The axes are labeled x and y, with tick marks at intervals of 0.5 from -1 to 1. A purple curve represents the circumference of the circle, and the right triangle is shaded pink with annotations for angle  $\theta$ ,  $\cos \theta$ , and  $\sin \theta$ .

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In [subtopic 3.5](#) (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27741/), you studied the unit circle and defined trigonometric ratios of any angle using the unit circle. In the diagram above, triangle OBH is a right-angled triangle so you can use Pythagoras' theorem:

$$OH^2 + BH^2 = OB^2$$



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If you substitute the values of each length you get

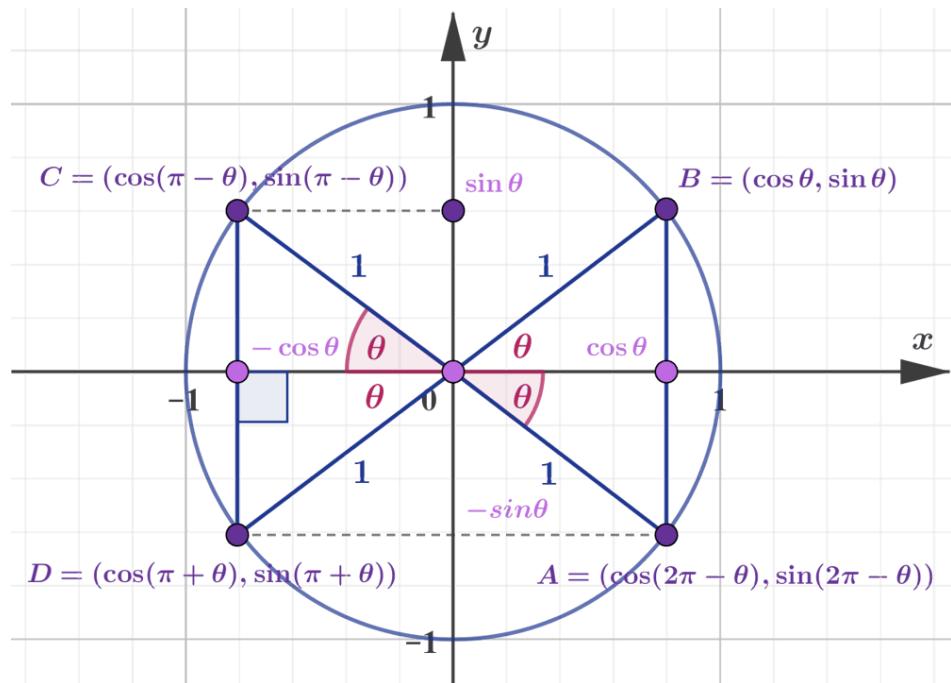


$$\cos^2\theta + \sin^2\theta = 1$$

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which is also known as the Pythagorean identity.

If you extend the idea to other quadrants using a unit circle and a rectangle, as in the diagram below, points on the circumference of the unit circle are symmetrical. As the right-angled triangles formed in each quadrant are congruent, each will have base length  $\cos \theta$  and height length  $\sin \theta$ .



More information

The image depicts a unit circle on a coordinate grid. The circle is divided into four quadrants with angles and trigonometric functions labeled. The x-axis and y-axis intersect at the center of the circle. Points on the circle are labeled as A, B, C, and D with specific coordinates. The coordinates correspond to cosine and sine functions of angles ( $\theta$ ), ( $\pi - \theta$ ), ( $\pi + \theta$ ), and ( $2\pi - \theta$ ). Right-angled triangles are formed in each quadrant, showing base and height as ( $\cos \theta$ ) and ( $\sin \theta$ ). The central point is marked with 0, and the circle has a radius of 1, with intersections on the axes at coordinates 1, -1, and 0. Angles ( $\theta$ ), ( $\pi - \theta$ ), ( $\pi + \theta$ ) are highlighted in red within the triangles.



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For example, point D represents the angle  $\pi + \theta$ .

$$\cos(\pi + \theta) = -\cos\theta \text{ and } \sin(\pi + \theta) = -\sin\theta$$

Therefore,

$$\cos^2(\pi + \theta) + \sin^2(\pi + \theta) = (-\cos\theta)^2 + (-\sin\theta)^2 = \cos^2\theta + \sin^2\theta$$

Using the Pythagorean identity,

$$\cos^2(\pi + \theta) + \sin^2(\pi + \theta) = 1.$$

You can show the identity for other quadrants as well following a similar approach.

Thus, the Pythagorean identity holds true for angles in quadrants II, III and IV.

✓ **Important**

An identity is always true.

Therefore,  $\cos^2\theta + \sin^2\theta = 1$  holds true for all values of  $\theta$ .

⌚ **Exam tip**

In the IB examination, the Pythagorean identity will be in the formula booklet as



$$\cos^2\theta + \sin^2\theta = 1$$

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## ① Exam tip

In the IB examination, if you are asked to find exact values you should not approximate square roots.

Instead, you should leave the answer in **surd form**. For example,  
 $\tan 60^\circ = \sqrt{3}$ .

## Example 1



Find the exact values of  $\cos \alpha$  if  $\sin \alpha = \frac{1}{3}$ .

Steps	Explanation
$\cos^2 \alpha + \left(\frac{1}{3}\right)^2 = 1$	Using Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$
$\cos^2 \alpha = 1 - \frac{1}{9}$ $\cos \alpha = \sqrt{\frac{8}{9}}$ or $\cos \alpha = -\sqrt{\frac{8}{9}}$ Therefore, $\cos \alpha = \frac{2\sqrt{2}}{3}$ or $\cos \alpha = -\frac{2\sqrt{2}}{3}$	Solving for $\cos \alpha$ . Make sure to have both positive and negative $\cos \alpha$ as the angle is not specified. You should leave your answer in surd form as the question asks for the exact value.



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## Example 2

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Simplify the expression  $\frac{1 - \cos^2\theta}{1 - \sin^2\theta}$ .

Steps	Explanation
$\frac{1 - \cos^2\theta}{1 - \sin^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \left(\frac{\sin \theta}{\cos \theta}\right)^2$	Use the Pythagorean identity in the form $\cos^2\theta = 1 - \sin^2\theta$ and $\sin^2\theta = 1 - \cos^2\theta$
Therefore $\frac{1 - \cos^2\theta}{1 - \sin^2\theta} = \tan^2\theta$	Using $\tan \theta = \frac{\sin \theta}{\cos \theta}$

### 🌐 International Mindedness

Pythagorean identity is named after Pythagoras. There are many other famous identities which are named after famous mathematicians like Euler's identity (which is also known as Euler's equation)

$$e^{i\pi} + 1 = 0 ,$$

where  $e$  is Euler's number,  $i$  is the imaginary number which satisfies the equation  $i^2 = -1$ ,  $\pi$  is the ratio of the circumference of a circle to its diameter.

Although this identity is considered to be a monumental work of Euler, there are some historians claiming he might have acquired the knowledge from another mathematician Bernoulli. Similar to Pythagoras

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he might have taken someone else's work and published under his name. What do you think the consequences of such an action would be in modern days?

## 3 section questions ^

### Question 1



If  $\cos \theta = -\frac{3}{5}$ , where  $\pi \leq \theta \leq \frac{3\pi}{2}$ , find the **exact** value of  $\sin \theta$ .

1      $\sin \theta = -\frac{4}{5}$



2      $\sin \theta = \frac{4}{5}$

3      $\sin \theta = -\frac{16}{25}$

4      $\sin \theta = \frac{16}{25}$

### Explanation

Using  $\cos^2 \theta + \sin^2 \theta = 1$  and the value for  $\cos \theta = -\frac{3}{5}$ :

$$\left(-\frac{3}{5}\right)^2 + \sin^2 \theta = 1 \Leftrightarrow \frac{9}{25} + \sin^2 \theta = 1 \Leftrightarrow \sin^2 \theta = \frac{16}{25} \therefore \sin \theta = \pm \frac{4}{5}$$

However, given the interval, namely the third quadrant, we reject the positive solution and thus, the solution is  $\sin \theta = -\frac{4}{5}$ .

### Question 2



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If  $\tan x = \frac{5}{12}$ , where  $\pi \leq x \leq \frac{3\pi}{2}$ , find the **exact** value of  $\cos x$ .

1  $-\frac{12}{13}$



2  $\frac{12}{13}$

3  $-\frac{13}{12}$

4  $\frac{13}{12}$

### Explanation

Using  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  and the value of  $\tan \theta$ , we obtain

$$\sin \theta = \frac{5}{12} \cos \theta.$$

Now using the Pythagorean identity and solving for  $\cos \theta$ :

$$\begin{aligned} \cos^2 \theta + \sin^2 \theta &= 1 \\ \Leftrightarrow \cos^2 \theta + \left( \frac{5}{12} \cos \theta \right)^2 &= 1 \\ \Leftrightarrow \cos^2 \theta + \frac{25}{144} \cos^2 \theta &= 1 \\ \Leftrightarrow \frac{169}{144} \cos^2 \theta &= 1 \\ \Leftrightarrow \cos^2 \theta &= \frac{144}{169} \\ \therefore \cos \theta &= \pm \frac{12}{13}. \end{aligned}$$

However, we reject the positive solution as we are in the third quadrant where  $\cos \theta$  is negative. Thus, the solution is  $\cos \theta = -\frac{12}{13}$ .

### Question 3



Simplify  $\cos \alpha + \tan \alpha \sin \alpha$ .

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1  $\frac{1}{\cos \alpha}$



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2  $\cos \alpha$

3  $\frac{1}{\sin \alpha}$

4  $\sin \alpha$

### Explanation

$$\begin{aligned}\cos \alpha + \tan \alpha \sin \alpha &= \cos \alpha + \frac{\sin \alpha}{\cos \alpha} \sin \alpha \\&= \cos \alpha + \frac{\sin^2 \alpha}{\cos \alpha} \\&= \frac{\cos^2 \alpha + \sin^2 \alpha}{\cos \alpha} \\&= \frac{1}{\cos \alpha}\end{aligned}$$

3. Geometry and trigonometry / 3.6 Trigonometric identities

## Double-angle identities

### Double-angle identity for cosine

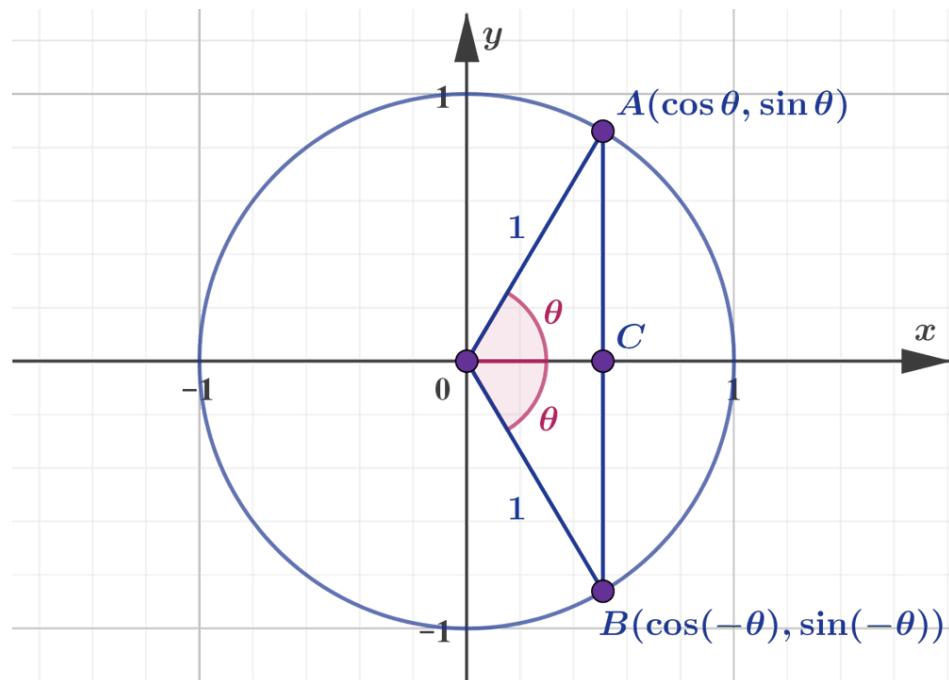
In the diagram below, angles  $\theta$  and  $-\theta$  are drawn in standard positions on a unit circle. Therefore, the coordinates of point A are  $(\cos \theta, \sin \theta)$  and the coordinates of point B are  $(\cos(-\theta), \sin(-\theta))$ .



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[More information](#)

This is a unit circle diagram on a Cartesian coordinate system. The circle is centered at the origin (0, 0) with a radius of 1. Two angles,  $\theta$  and  $-\theta$ , are illustrated in standard position. The angle  $\theta$  extends counterclockwise from the positive x-axis, and  $-\theta$  extends clockwise. There are two key points on the circumference: Point A with coordinates  $(\cos \theta, \sin \theta)$  is located in the first quadrant, above the x-axis. Point B with coordinates  $(\cos(-\theta), \sin(-\theta))$  is in the fourth quadrant. The diagram includes a triangle, AOB, where O is the origin, and the line segments OA and OB represent the radius of the circle. The segment OC represents  $\cos(\theta)$  and lies along the x-axis. The y-axis represents  $\sin(\theta)$ . The angle  $\theta$  is shaded between these lines, highlighting the symmetrical properties of sine and cosine for positive and negative angles on the unit circle.

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In triangle AOB ,

$$OC = \cos \theta \text{ and } AC = \sin \theta.$$

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Point C is the midpoint of AB  $\Rightarrow AC = CB = \sin \theta$  and  $AB = 2 \sin \theta$



$$AOB = 2\theta.$$

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Use the cosine rule in triangle OAB to find the length of the side AB :

$$AB^2 = OA^2 + OB^2 - 2(OA) \cdot (OB) \cdot \cos 2\theta$$

and substitute the lengths of each side:

$$(2 \sin \theta)^2 = 1^2 + 1^2 - 2(1) \cdot (1) \cdot \cos 2\theta$$

$$4(\sin \theta)^2 = 2 - 2 \cos 2\theta.$$

Rearranging gives

$$2 \cos (2\theta) = 2 - 4 \sin^2 \theta.$$

Dividing both sides by 2 you get the first double-angle identity for cosine as

$$\cos (2\theta) = 1 - 2 \sin^2 \theta.$$

Using the Pythagorean identity  $\cos^2 \theta + \sin^2 \theta = 1$  you can write variations of the double-angle cosine identity.

Substitute  $1 = \cos^2 \theta + \sin^2 \theta$  to get

$$\cos (2\theta) = (\cos^2 \theta + \sin^2 \theta) - 2 \sin^2 \theta$$

then simplify to get the second version of the double-angle cosine identity:

$$\cos (2\theta) = \cos^2 \theta - \sin^2 \theta.$$

Rearrange the Pythagorean identity as



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$$\sin^2 \theta = 1 - \cos^2 \theta$$



then substitute into the double-angle identity for cosine

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$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta = 1 - 2(1 - \cos^2\theta).$$

Then simplify to get the third version of the double-angle identity for cosine:

$$\cos 2\theta = 2\cos^2\theta - 1.$$

### ① Exam tip

In the IB examination, all three versions of the cosine double-angle identity will be in your formula booklet.

$$\text{Double-angle identity } \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$$



### Activity

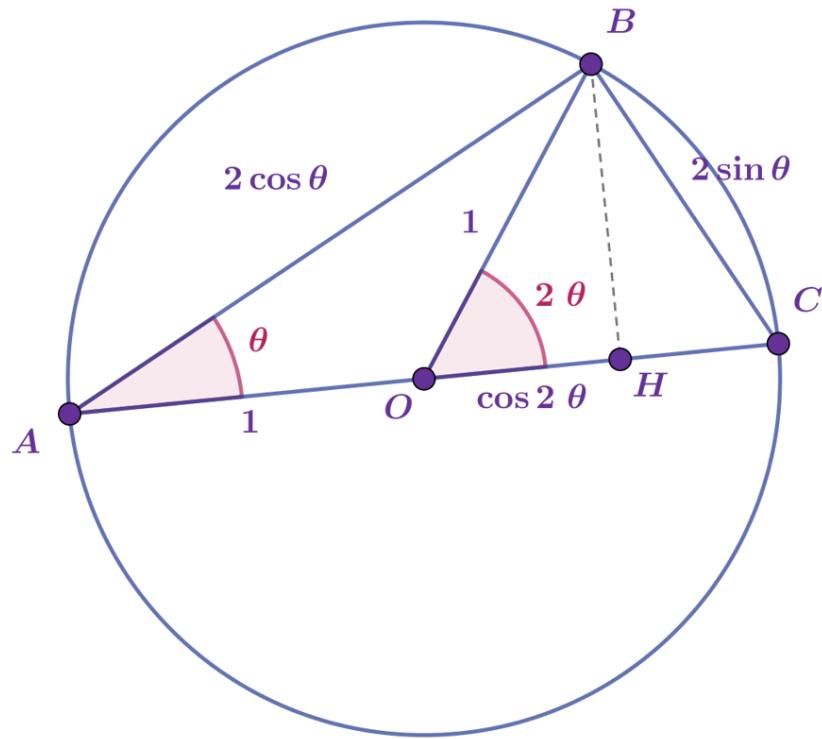
In the diagram below, circle with radius 1 is centred at point O. Points A, B and C are on the circumference of the circle. Points A, O and C are colinear. BH is perpendicular to AC.



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The diagram shows a circle with a radius of 1 and is centered at point O. Points A, B, and C lie on the circumference of the circle. The points A, O, and C are collinear, forming a straight line through the center of the circle. There is a line segment BH that is perpendicular to AC.

The triangle BOC is labeled with the angles and lengths of the sides. The angle BOC is marked as  $2\theta$ . The distance AB is labeled as  $2\cos(\theta)$ , BC as  $2\sin(\theta)$ , and OH as  $\cos(2\theta)$ . The distance AO is labeled as 1.

The diagram illustrates the relationships between these geometric elements and aids in explaining the derivation of the trigonometric identities provided in the surrounding text.

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Section Explain why angle BOC =  $2\theta$ ;  $AB = 2\cos(\theta)$ ,  $BC = 2\sin(\theta)$  and  $OH = \cos(2\theta)$ .

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Assign

Hence, show that  $\cos 2\theta = 2\cos^2\theta - 1$  and  $\cos 2\theta = 1 - 2\sin^2\theta$ .



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## Example 1

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If  $\cos \theta = \frac{1}{3}$ , find the exact value of  $\cos 2\theta$ .

Steps	Explanation
$\cos 2\theta = 2 \cdot \left(\frac{1}{3}\right)^2 - 1$	Using $\cos 2\theta = 2 \cos^2 \theta - 1$
Therefore $\cos 2\theta = -\frac{7}{9}$	Simplify.

## Example 2



Show that  $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2 \theta$ .



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Steps	Explanation
$\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \frac{1 - (1 - 2\sin^2\theta)}{1 + (2\cos^2\theta - 1)}$	Using $\cos 2\theta = 1 - 2\sin^2\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$ to eliminate the 1s in both numerator and denominator.
$\frac{1 - (1 - 2\sin^2\theta)}{1 + (2\cos^2\theta - 1)} = \frac{2\sin^2\theta}{2\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$	Open brackets and simplify.
$\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$	Using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ .
Therefore $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2\theta$	

## Double-angle identity for sine

Using the Pythagorean identity for  $2\theta$  gives

$$\cos^2 2\theta + \sin^2 2\theta = 1 \Rightarrow \cos^2 2\theta = 1 - \sin^2 2\theta \quad (1)$$

The cosine double-angle identity is

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$$\cos 2\theta = 1 - 2\sin^2\theta.$$



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Square both sides



$$\cos^2 2\theta = (1 - 2\sin^2 \theta)^2 \quad (2)$$

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Equate (1) and (2)

$$1 - \sin^2 2\theta = (1 - 2\sin^2 \theta)^2.$$

Expand brackets and simplify:

$$1 - \sin^2 2\theta = 1 - 4\sin^2 \theta + 4\sin^4 \theta$$

$$\sin^2 2\theta = 4\sin^2 \theta - 4\sin^4 \theta$$

$$\sin^2 2\theta = 4\sin^2 \theta (1 - \sin^2 \theta).$$

Substitute  $1 - \sin^2 \theta = \cos^2 \theta$

$$\sin^2 2\theta = 4\sin^2 \theta \cos^2 \theta.$$

Take the square root of both sides

$$\sin 2\theta = \pm 2 \sin \theta \cos \theta.$$

When you consider the four quadrants for angle  $\theta$ ,  $2\theta$  and the signs of the sine and cosine values;

$$0 < \theta < \frac{\pi}{2} \Rightarrow 0 < 2\theta < \pi \Rightarrow \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\frac{\pi}{2} < \theta < \pi \Rightarrow \pi < 2\theta < 2\pi \Rightarrow \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\pi < \theta < \frac{3\pi}{2} \Rightarrow 2\pi < 2\theta < 3\pi \Rightarrow \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\frac{3\pi}{2} < \theta < 2\pi \Rightarrow 3\pi < 2\theta < 4\pi \Rightarrow \sin 2\theta = 2 \sin \theta \cos \theta.$$

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Therefore, the double-angle identity for sine is  $\sin 2\theta = 2 \sin \theta \cos \theta$ .

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### ⓘ Exam tip

In the IB examination, the sine double-angle identity will be in your formula booklet.

Double-angle identity  $\sin 2\theta = 2 \sin \theta \cos \theta$ .



### Activity

Use the applet below to demonstrate that the left-hand and right-hand sides of the identity always have the same value.

### Interactive 1. The double angle formulae.

Credit: GeoGebra  (<https://www.geogebra.org/m/joLKGc3a>) Integral Resources



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More information for interactive 1



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This interactive visually demonstrates the sine double-angle identity

$\sin 2\theta = 2 \sin \theta \cos \theta$  through direct geometric manipulation.

A circle is present on screen with the center at O. The diameter BC crosses through O. Another point D on the circumference when joined to B and C creates a triangle BCD. A line from point D drops perpendicular to line BC making point E. A line joins point O and point D making an angle  $2\alpha$ . Line BD is denoted by  $2 \cos \alpha$  and line DC is denoted by  $2 \sin \alpha$ . Point D is denoted by a red dot and can be moved along the circumference of the circle. As user move the red point D around the unit circle, its coordinates  $(\sin 2\alpha, \cos 2\alpha)$  dynamically illustrate this fundamental trigonometric identity. Angle DOB is  $180 - 2\alpha$

The construction clearly shows two equivalent expressions for the same quantity: the x-coordinate of point D is  $\sin 2\alpha$ , while the length DE in triangle BDE calculates as  $2\sin\alpha \times \cos\alpha$ .

For example :

When you position D is at  $2\alpha = 60^\circ$  ( $\alpha = 30^\circ$ ), angle DBO is  $30^\circ$  then the length DE can be obtained from 2 methods.

$$DE = \sin 2\alpha = \sin 60^\circ = 0.87$$

$$DE = 2 \sin \alpha * \cos \alpha = 2 * .5 * .87 = 0.87$$

This perfect match holds for all angles, proving the identity geometrically.

This hands-on verification complements the algebraic proof, showing why the double-angle formula works and how it relates to the unit circle's geometry.

## Example 3



Simplify the expression  $10 \sin 3x \cos 3x$ .

Steps	Explanation
$10 \sin 3x \cos 3x = 5 (2 \sin 3x \cos 3x)$	Factorise.
$5 (2 \sin 3x \cos (3x)) = 5 \sin (2 \times 3x)$	Using $2 \sin \theta \cos \theta = \sin$



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Steps	Explanation
<p>Therefore</p> $10 \sin 3x \cos 3x = 5 \sin 6x$	

## Example 4



Given that  $\sin x = \frac{3}{5}$  and  $\frac{\pi}{2} < x < \pi$ , find the exact value of  $\sin(2x)$ .

Steps	Explanation
<p>Using the unit circle diagram, we can see that the vertical coordinate of point A is <math>\frac{3}{5}</math>. Since the radius OA is 1, we can use Pythagoras' theorem to find the horizontal coordinate <math>a</math>:</p> $1 = \left(\frac{3}{5}\right)^2 + a^2$ $a = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$	<p>Using the unit circle diagram, we can see that the vertical coordinate of point A is <math>\frac{3}{5}</math>. Since the radius OA is 1, we can use Pythagoras' theorem to find the horizontal coordinate <math>a</math>:</p> $1 = \left(\frac{3}{5}\right)^2 + a^2$ $a = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

Steps	Explanation
$\cos x = -\frac{4}{5}$	As the point is in quadrant, $\cos x$
$\sin 2x = 2 \left(\frac{3}{5}\right) \left(-\frac{4}{5}\right)$	Using $\sin 2\theta = 2 \sin \theta$
Therefore	
$\sin 2x = -\frac{24}{25}$	

## ⌚ Making connections

Euler's identity  $e^{i\pi} + 1 = 0$  is a special case of Euler's formula which states that for any real number  $x$

$$e^{ix} = \cos x + i \sin x$$

where  $i$  is the imaginary number which satisfies the equation  $i^2 = -1$ ,  $e$  is Euler's number which is an irrational number  $e = 2.718\dots$

Both complex numbers and Euler's number have many applications in, for example, compound interest, half-life of radioactive atoms and for phasors in electrical engineering.

All of these combine various areas of mathematics, such as vectors and complex numbers, which are topics in the higher level of this course .

## 3 section questions ^



### Question 1



Given that  $\cos \theta = -\frac{1}{2}$ , find the exact value of  $\cos 2\theta$ .

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1  $-\frac{1}{2}$



2  $-\frac{3}{4}$

3  $-\frac{3}{2}$

4 0

### Explanation

$$\begin{aligned}\cos 2\theta &= 2 \cos^2 \theta - 1 \\&= 2 \times \left(-\frac{1}{2}\right)^2 - 1 \\&= 2 \times \frac{1}{4} - 1 \\&= \frac{1}{2} - 1 \\&= -\frac{1}{2}\end{aligned}$$

### Question 2



Which of the following is equivalent to  $4 \sin 5\theta \cos 5\theta$ ?

1  $2 \sin 10\theta$



2  $\sin 10\theta$

3  $\sin 20\theta$

4  $4 \sin 10\theta$



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### Explanation

Since in the expression we have the product of  $\sin 5\theta$  and  $\cos 5\theta$ , we let  $\alpha = 5\theta$  and use the double angle formula  $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ .



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**Question 3**

$\cos \theta = \frac{2}{3}$  and  $\theta$  is an acute angle. Find the exact value of  $\tan 2\theta$ .

1  $-4\sqrt{5}$



2  $4\sqrt{5}$

3  $\frac{-4\sqrt{5}}{9}$

4  $\frac{4\sqrt{5}}{9}$

**Explanation**

$$\cos \theta = \frac{2}{3} \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\frac{2}{3}\right)^2$$

$$\sin^2 \theta = \frac{5}{9} \Rightarrow \sin \theta = \pm \sqrt{\frac{5}{9}}$$

$$\text{as } \theta \text{ is an acute angle } \sin \theta = \frac{\sqrt{5}}{3}$$

Using double-angle formulae

$$\sin 2\theta = 2 \left( \frac{\sqrt{5}}{3} \right) \left( \frac{2}{3} \right) \Rightarrow \sin 2\theta = \frac{4\sqrt{5}}{9}$$

$$\cos 2\theta = 2\cos^2 \theta - 1 = 2 \left( \frac{2}{3} \right)^2 - 1 \Rightarrow \cos 2\theta = -\frac{1}{9}$$



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$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{4\sqrt{5}}{9}}{-\frac{1}{9}}$$

Therefore  $\tan 2\theta = -4\sqrt{5}$ .

### 3. Geometry and trigonometry / 3.6 Trigonometric identities

## Checklist

### Section

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Feedback

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### What you should know

By the end of this subtopic you should be able to:

- use the trigonometric identity:  $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- use the Pythagorean identity:  $\cos^2 \theta + \sin^2 \theta = 1$ 
  - to find the value of  $\sin \theta$  and  $\tan \theta$  if  $\cos \theta$  is given
  - to find the value of  $\cos \theta$  and  $\tan \theta$  if  $\sin \theta$  is given
  - to find the value of  $\sin \theta$  and  $\cos \theta$  if  $\tan \theta$  is given
  - to prove other trigonometric identities
- use the double-angle identities:  $\sin 2\theta = 2 \sin \theta \cos \theta$ ,

$$\cos 2\theta = \begin{cases} \cos^2 \theta - \sin^2 \theta, \\ 2\cos^2 \theta - 1, \\ 1 - 2\sin^2 \theta \end{cases}$$

- to find the value of  $\sin 2\theta$ ,  $\sin 4\theta$ ,  $\cos 2\theta$ ,  $\cos 4\theta$  or  $\tan 2\theta$ ,  $\tan 4\theta$  , ... if  $\sin \theta$ ,  $\cos \theta$  or  $\tan \theta$  is given
- to prove identities involving double angles
- to find the exact values of trigonometric ratios involving half of special angles (for example,  $\sin 15^\circ$  or  $\cos 75^\circ$ ).





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# Investigation

## Section

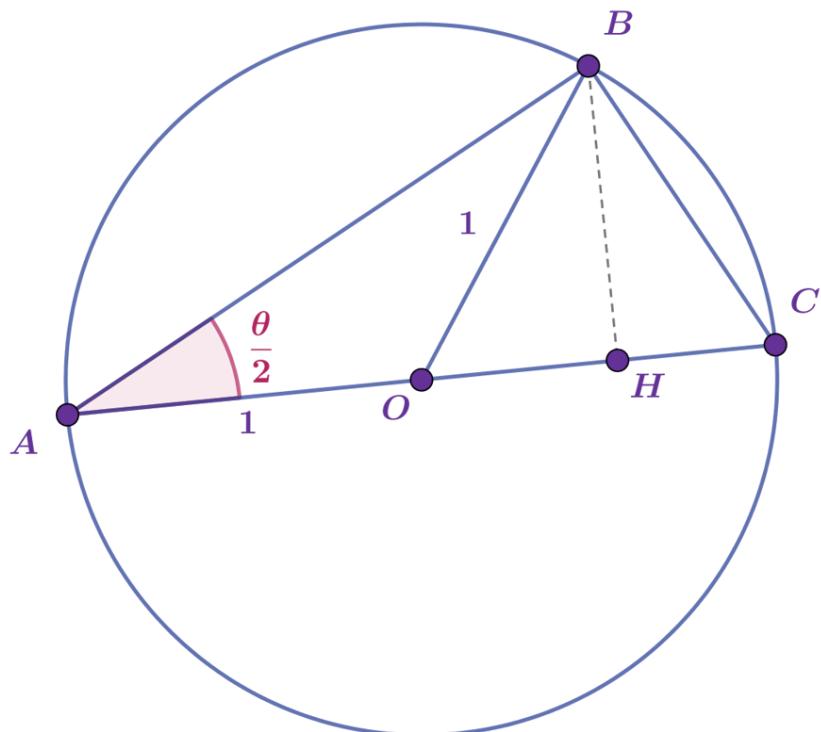
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Assign

Draw a unit circle as in the diagram below, where points A, B and C are on the circumference of the circle. The angle  $BAC = \frac{\theta}{2}$ , BH is perpendicular to AC, and points A, O, H and C are colinear.



More information

The image is a diagram of a unit circle with a triangle inscribed. The circle is centered at point O, and the points A, B, and C lie on the circle's circumference. Point A is to the left, B at the top, and C to the right, forming a triangle ABC. The angle ( $\angle BAC$ ) is given as ( $\frac{\theta}{2}$ ), marked by a red sector. Point H is on segment AC such that BH is perpendicular to AC, forming a right angle at point H.



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AO and OC are colinear with a length of 1 each, meaning AO + OC covers the diameter of the circle.

Segments AB and BC are also part of the circle, with lengths that need to be determined in relation to the angle.

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Find the lengths of AB and BC in terms of trigonometric ratios of  $\frac{\theta}{2}$ .

What is the measure of the angle  $BOC$  in terms of  $\frac{\theta}{2}$ ?

Find the lengths of OH, BH and HC in terms of trigonometric ratios of the angle  $BOC$ .

Hence, show that  $\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$  and  $\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$

You proved the half angle identities for  $0 < \frac{\theta}{2} < \frac{\pi}{2}$  or  $0 < \theta < \pi$ . Can you extend the half angle identities for any angle  $\theta$ ?

### Rate subtopic 3.6 Trigonometric identities

Help us improve the content and user experience.



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