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Mathematical approaches to processing scientific data

Calculations

In science, we try to quantify the natural world. In other words, we try to express the concepts in measurable terms. This allows scientists to communicate ideas, form theories and predict the outcome of systems in action.

Some problems only require basic algebraic and arithmetic calculations. You might need to add, subtract, multiply or divide quantities. These calculations could involve the following items:

- Fractions: these indicate part of a whole. For example, ½ indicates one part out of two (or a half), ¼ indicates one part out of four (or a quarter) and ¾ indicates three parts out of four (or three-quarters).
- Decimals: these are used to represent fractions of numbers when the true value does not equal a whole number. Measurements often use decimals, for example the mass of a reactant could be 10.5 g, while the mass of the product created could be 0.024 g.
- Percentages: 'per cent' means out of 100 and is represented by the symbol %. For example, 20% is 20 out of 100 or $\frac{20}{100}$. The percentage can be calculated by using the formula:

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$$\text{percentage} = \frac{\text{value}}{\text{total value}} \times 100$$

Decimals and fractions can be converted into percentages by multiplying by 100.

- Ratios: these help to compare two or more amounts and are written in the form a : b, where a and b are two numbers.
- Proportions: this is a method of comparing one component to the whole or sum value of the total amount. It is often expressed as a ratio or fraction or percentage.
- Reciprocals: Refers to the inverse of a number. If you have a number x, its reciprocal would be $\frac{1}{x}$.

Approximations

Since we want to be precise all the time, why do we need approximations?

Consider the following question: what is the world population?

A precise answer to this question can be found on the website The World Counts (https://www.theworldcounts.com/challenges/planet-earth/state-of-the-planet/world-population-clock-live).

How do you answer that question? Should you cite all the digits of the number? It also fluctuates constantly as births and deaths are registered around the world. How can you be accurate enough?

One way to answer in an efficient way is to make an approximation. For example, you can say that the world population is approximately 8 billion. This way, you can quickly communicate the order of magnitude of the quantity asked.

You make an approximation by using rounding off to reduce a number to a certain number of significant figures.

Significant figures

Here are the rules for identifying significant figures. As you read through these rules it is important to note that significant figures consider the precision that a number is recorded to.

1. All non-zero numbers are significant. For example, 23.75 has four significant figures, whereas 23 has two significant figures.

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2. All zeros between two non-zero numbers are significant. For example, 2005 has four significant figures while 200.014 has six significant figures.
3. All zeros before the first non-zero digit are not significant. For example, 0.00045 has two significant figures.
4. All trailing zeros in a number with a decimal point are significant. For example, 0.02500 has four significant figures. It is important to remember that the additional zeros after 5 show the precision of the number.
5. Trailing zeros in a number without a decimal point are generally considered as not significant. For example, 2500 has two significant figures.

Worked example 1

Determine the number of significant figures in the following:

(a) 460
(b) 63.002
(c) 9 450 000
(d) 510.260
(e) 0.03760

(a) 460 has two significant figures
(b) 63.002 has five significant figures
(c) 9 450 000 has three significant figures
(d) 510.260 has six significant figures
(e) 0.03760 has four significant figures

Some of these numbers have trailing zeros. (b) and (d) also have zeros between non-zero numbers. (e) has leading zeros and trailing zeros after a decimal point. Remember, if the number has a decimal point, the trailing zeros are significant, if not, they are not significant.

Rules for rounding off

Rounding off a number means converting it to the nearest chosen place value.

It is important to remember that while rounding off, the place value to the right of the chosen value should become zero. For example, if you want to round to the nearest tens' place, the ones' place becomes zero.

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This can be done by rounding up or rounding down following these rules.

- If the number you want to round off is 5 or greater than 5, round **up**.
- If the number you want to round off is below 5, round **down.**

For example, to round off 29 to the nearest 10 you round **up** to 30 as 9 is greater than 5. To round off 53 to the nearest 10, you round **down** to 50 as 3 is less than 5.

Worked example 2

Round off the following numbers to the underlined digit.

(a) $3\underline{4}.9$
(b) $23.\underline{9}8$
(c) $155.8\underline{6}5$
(d) $4508\underline{7}.2$
(e) $593759.3\underline{7}6$

When rounding, consider the next digit to the right of the underlined digit. If it is a 5 or greater then round up. If it is a 4 or less, then do not round up.

(a) 35
(b) 24.0
(c) 155.87
(d) 45087
(e) 593759.38

All examples except (d) have a number that is a 5 or greater so are rounded up.

Now you can combine the rules for significant figures and rounding off to make an approximation.

Worked example 3

Round off 780.58 to:

(a) 4 significant figures
(b) 3 significant figures
(c) 2 significant figures

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(d) 1 significant figure

(a) 4 significant figures = 780.6
(b) 3 significant figures = 781
(c) 2 significant figures = 780
(d) 1 significant figure = 800

Approximations help you to:

- simplify calculations
- arrive at values that are reasonably close to the actual values.

It is important to remember that while approximations help to reduce the complexity, they also reduce the accuracy. By saying that the population of the Earth is 8 billion, we underestimate it by some hundreds of millions of people!

Estimations

An estimation can be thought of as a rough guess or a rough calculation of something. The IB defines estimate as a command term as ‘Obtain an approximate value’. So here we can see that it is possible, by estimating, to obtain a value that is close to the actual value (an approximation). Rounding numbers up or down to the nearest whole number is a form of estimation.

Difference between approximation and estimation

It is important to remember that approximation means to find a value close to the correct answer to simplify calculations while estimations often involve using statistical methods or models to make informed guesses about a quantity that might not be directly measurable.

Logarithmic functions

A logarithm, or logarithmic function, is a mathematical method that uses exponents (or indices) to deal with numbers and values across a vast range in scale. A logarithmic function describes the relationship between a number, a base and an exponent, such that:

$$\log_b x = y \Leftrightarrow b^y = x$$

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where,
b is the base
y is the exponent
x is the number.

The ⇔ symbol indicates that these two equations have the same meaning and can be read backwards or forwards, such that:

$$b^y = x \Leftrightarrow \log_b x = y$$

You can also say that if two numbers are equal to each other, their logarithms are equal to each other, and vice versa. This is true independent of the base of the logarithm.

$$x = y \Leftrightarrow \log_b x = \log_b y$$

Let's apply some simple numbers into the logarithmic function:

$$10^3 = 1000 \Leftrightarrow \log_{10} 1000 = 3$$

$$10^4 = 10\ 000 \Leftrightarrow \log_{10} 10\ 000 = 4$$

In the first function, the base is 10 and the exponent is 3 and the number is 1000. This can be expressed as the 'log of base 10 of 1000 is 3'.

In the second function, the base is 10 and the exponent is 4 and the number is 10 000. This can be expressed as the 'log of base 10 of 10 000 is 4'.

Therefore, in a logarithm scale of base 10, moving from 3 to 4 is the equivalent of moving from 1000 to 10 000 in a linear number scale.

The most common uses that you are going to see are logarithms with base 10 or base e, where e is Euler's number (e = 2.718...). All logarithms, independent of their base, have some properties that make calculations easier (**Table 1**).

Table 1. Basic properties of base 10 *and* base e logarithms.

Base 10	Base e
$10^{\log_{10} x} = x$	$e^{\log_e x} = x$

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Base 10	Base *e*
$\log_{10} 1 = 0$	$\log_e 1 = 0$
$\log_{10} 10 = 1$	$\log_e e = 1$
$\log_{10} x^a = a \times \log_{10} x$	$\log_e x^a = a \times \log_e x$
$\log_{10}(x \times y) = \log_{10} x + \log_{10} y$	$\log_e(x \times y) = \log_e x + \log_e y$
$\log_{10}\left(\frac{x}{y}\right) = \log_{10} x - \log_{10} y$	$\log_e\left(\frac{x}{y}\right) = \log_e x - \log_e y$

Calculations involving exponential functions (HL)

An exponential function includes a number called the base raised to a certain exponent, for instance:

$$y = a^x$$

For this exponential function you can work out the value of *y* for any value of *x* for any given base, a. As exponential functions deal with numbers being raised to the power, then they are used to mathematically model situations that grow or decay very rapidly. If you are an HL student, exponential functions are analysed when studying radioactive decay in section E.3.6 (/study/app/math-aa-hl/sid-423-cid-762593/book/radioactive-decay-law-hl-id-46548/).

All exponential functions have some properties that make calculations easier. **Table 2** lists some of these calculations for when the base is *e* (Euler's number) and also for any generic base.

Table 2. Exponential function calculations.

Exponential functions with base, *e*	Exponential functions with generic base
$e^x \times e^y = e^{(x+y)}$	$a^x \times a^y = a^{(x+y)}$
$\frac{e^x}{e^y} = e^{(x-y)}$	$\frac{a^x}{a^y} = a^{(x-y)}$
$(e^x)^y = e^{x \times y}$	$(a^x)^y = a^{x \times y}$
$e^x = e^y \Leftrightarrow x = y$	$a^x = a^y \Leftrightarrow x = y$

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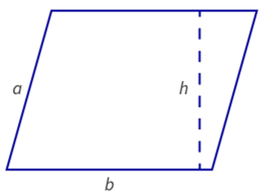
Exponential functions with base, e	Exponential functions with generic base
$e^{\ln x} = x$	$a^{\log_a x} = x$
$e^{-x} = \frac{1}{e^x}$	$x^{-n} = \frac{1}{x^n}$

Areas and volumes of simple shapes

Tables 3 and **4** give useful data on how to calculate areas and volumes of 2D and 3D shapes that appear in the syllabus. This is also summarised in section 1 section 1.6.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-equations-id-45153/) of the physics data booklet.

Table 3. Calculating circumferences and areas of common 2D shapes.

2D shape	Image	Circumference (or perimeter)	Area
Parallelogram	 More information The image shows a geometric diagram of a parallelogram outlined in blue. The parallelogram has four sides, with the left side labeled as 'a' and the base labeled as 'b'. It also has a dashed line labeled 'h' to show the height. It illustrates the geometric shape without any additional numerical values or text inside. One of the longer sides is tilted, which is typical for a parallelogram shape. This diagram focuses on the basic structure and side labeling of a parallelogram. [Generated by AI]	$C = 2a + 2b$	$A = a \times b$

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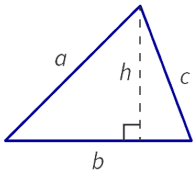
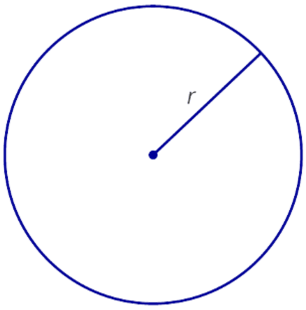
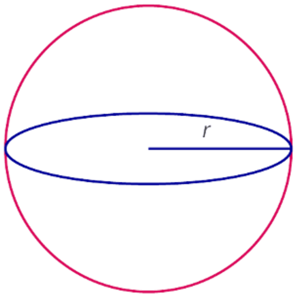
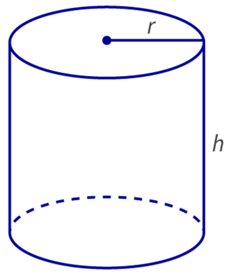
2D shape	Image	Circumference (or perimeter)	Area
Triangle		$C = a + b + c$	$A = \frac{1}{2} \times b \times h$
Circle	 More information The image depicts a circle with a 3D effect, outlined in blue. The center of the circle is marked with a small blue dot. From this center dot, there is a straight line extending to the edge of the circle, representing the radius of the circle. This line is labeled with the letter 'r'. The circle illustrates the basic geometric concept of a radius as the distance from the center to the circumference. [Generated by AI]	$C = 2\pi r$	$A = \pi r^2$

Table 4. Calculating volumes and surface areas of common 3D shapes.

3D shape	Image	Volume	Surface area
Sphere	Radius r  More information The image is a 3D outline of a sphere. It depicts a circle with a central horizontal line extending from the center to the edge, labeled as 'r' representing the sphere's radius. The circle outlines the equator of the sphere, and the radius 'r' extends outward from the center along this equatorial plane. [Generated by AI]	$V = \frac{4}{3}\pi r^3$	$SA = 4\pi r^2$

3D shape	Image	Volume	Surface area
Cylinder	Radius *r* Height *h*  More information The image shows a 3D representation of a cylinder. The cylinder is oriented vertically with a circular top and bottom base. The radius is labeled as 'r' extending from the center to the perimeter of the top base. The height of the cylinder is labeled as 'h,' indicating the distance between the top and bottom bases. The cylinder's outline and labels are shown in a simple line drawing style, with the top base depicted as a solid line and the bottom base as a dashed line to indicate it is on the back side, not directly visible from the front view. The text inside the image includes 'r' for radius and 'h' for height, consistent with the text before the image context mentioning 'RadiusrHeighth.' [Generated by AI]	$V = \pi r^2 h$	$SA = 2\pi r h + 2\pi r^2$

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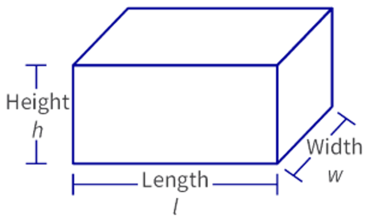
3D shape	Image	Volume	Surface area
Cuboid	 More information The image is a diagram of a 3D cuboid. It shows a rectangular box with three distinctive dimensions labeled: height, length, and width. Height is labeled as 'h' and is shown as the vertical measure. Length is labeled as 'l,' representing the longer horizontal side of the base. Width is labeled 'w,' indicating the shorter horizontal side of the base. The lines are marked in blue, clearly outlining the box and its dimensions. Each dimension is indicated with a line and arrow, showing the direction of measurement for height, length, and width. [Generated by AI]	$V = lwh$	$SA = 2(lw + wh + lh)$

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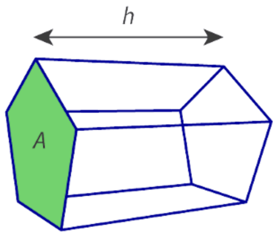
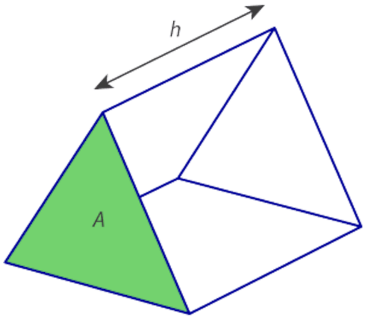
3D shape	Image	Volume	Surface area
Prism	 More information The image shows a 3D outline of a pentagonal prism. The front face, a pentagonal shape, is labeled as "A." The height of the prism is denoted by "h" with an arrow stretching from the front pentagon to the back. Dashed lines indicate the edges not directly visible. The top face of the prism is visible, connecting the pentagonal front and back faces. This image visually demonstrates the concept of the prism's volume having a cross-sectional area "A" and height "h." [Generated by AI] Area of cross-section *A* Height *h*  More information The image shows a 3D outline of a triangular prism. The triangular cross-section is shaded green and labeled as 'A'. The prism's height is indicated	$V = Ah$	$SA = 2A + (\text{base perimeter} \times$

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3D shape	Image	Volume	Surface area
	by an arrow labeled 'h' lying parallel to the prism's length. This diagram illustrates the concept of a prism with a base area 'A' and height 'h'. It emphasizes the geometric structure and labeling without additional details such as color descriptions, focusing on the area of cross-section 'A' and height 'h' as key elements of the diagram. [Generated by AI]		

Activity

Search around to find some items that have the same 2D and 3D shapes as listed in **Tables 3** and **4**.

1. Without measuring them, make an estimation of:
    a. the circumference and area for your 2D shapes
    b. the volume and surface area for your 3D shapes.
2. Measure the dimensions of your shapes and use the equations to calculate the values.
3. Compare your calculated values to your estimated values.
4. Discuss your results in your group or class.

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.6.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-equations-id-45153/) Mathematical equations.
- Section E.3.6 (/study/app/math-aa-hl/sid-423-cid-762593/book/radioactive-decay-law-hl-id-46548/) Radioactive decay law (HL).

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Scientific notation

Why do we use scientific notation in physics?

In section 1.3.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/units-symbols-and-numerical-values-id-48949/) you will learn about how scientists have worked together to standardise the units of measurement to express the values of physical quantities. For the mass of objects, the SI unit is the kilogram (kg).

The mass of the Earth is 5 972 200 000 000 000 000 000 000 kg.

This is a very large number. It is not very convenient to write and work with this number. We could also express the mass of the Earth as 5.9722×10^{24} kg.

This way of writing a number is called the scientific notation.

It's easy to see that it is much more convenient to write large numbers using scientific notation than writing them in standard notation.

Scientific notation is also used for small numbers too. Consider the mass of a single proton which is 0.000 000 000 000 000 000 000 000 001 672 622 kg.

This can be expressed in scientific notation as 1.672622×10^{-27} kg.

Once again, it's easy to see the advantage of using scientific notation to represent numbers of this magnitude. In this section, we look at how to convert numbers from standard notation to scientific notation and how it can also simplify calculations.

Writing numbers using scientific notation

A number written using scientific notation is made up of three parts – the coefficient, the base and the exponent.

$$a \times 10^b$$

where:
a = coefficient
10 = base

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b = exponent.

In scientific notation, the following rules apply:

- The coefficient should be greater than 1 but less than 10 and can be negative or positive.
- The base is always 10.
- The exponent is a positive or negative integer that represents the number of times the decimal point is moved to form the coefficient. If the decimal point is moved to the left, the exponent is positive; if moved the right, the exponent is negative.

For the mass of the Earth the coefficient is 5.9722 and the exponent is 24: 5.9722×10^{24} kg.

Worked example 1

Convert the number 1 250 000 000 into scientific notation.

First, write the number with its decimal point in place. The decimal point is placed after the last zero in the number.

1 250 000 000.

Next, move the decimal point to the left to make a number between one and ten.

In this example this is 1.25. To make this number, the decimal point is moved nine places to the left. The number of times the decimal point is moved gives the exponent, which is 9.

Finally, write the number as scientific notation. Note that all numbers in the coefficient are regarded as being significant, known as significant figures (section 1.3.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-approaches-to-processing-scientific-data-id-48947/)). The number written in scientific notation is:

1.25×10^9

Exercise

Click a question to answer

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Calculations involving numbers written using scientific notation

Next we will look at calculations involving numbers written in scientific notation. These include addition and subtraction and multiplication and division.

Addition and subtraction

To add or subtract numbers written using scientific notation, the exponent of the two numbers must be the same. If they are, you can just add or subtract the coefficients as usual. However, if the exponents are different, you will first have to make them the same and then perform the calculation. The steps involved are as follows:

1. Rewrite the numbers so that they have the same exponent (the largest one).
2. Add or subtract the coefficients.
3. Write the answer in scientific notation to the correct number of significant figures.

Worked example 2

Perform the calculation shown and write the answer in scientific notation.

$$5.62 \times 10^3 + 3.44 \times 10^5$$

First, rewrite the numbers so that they have the same exponent (the largest one).

5.62×10^3 changes to 0.0562×10^5 by moving the decimal point two places to the left.

3.44×10^5 is not changed.

Next, add the coefficients.

$$0.0562 + 3.44 = 3.4962$$

Write the answer in scientific notation.

$$3.4962 \times 10^5$$

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According to the rules of significant figures, when adding or subtracting numbers, write the answer to the least number of decimal places.

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So the answer is rounded to 3.50×10^5 (to two decimal places).

Now try this exercise question for subtraction.

Exercise

Click a question to answer

Multiplication and division

To multiply or divide numbers written in scientific notation, you don't need to have the same exponent. You just multiply the coefficients and add the exponents or divide the coefficients and subtract the exponents. The steps involved are:

1. Multiply or divide the coefficients.
2. Add or subtract the exponents.
3. Write the answer in scientific notation to the correct number of significant figures.

Worked example 3

Perform the calculation shown and write the answer in scientific notation.

$$3.419 \times 10^3 \times 2.62 \times 10^5$$

Multiply the coefficients.

$$3.419 \times 2.62 = 8.95778$$

Add the exponents.

$$10^3 + 10^5 = 10^8$$

Write the answer in scientific notation.

$$8.95778 \times 10^8$$

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According to the rules of significant figures, when multiplying or dividing numbers, write the answer to the least number of significant figures.

The answer is rounded to 8.96×10^8 (to three significant figures).

Worked example 4

Perform the calculation shown and write the answer in scientific notation.

$$6.3048 \times 10^4 \div 7.82 \times 10^2$$

Divide the coefficients.

$$6.3048 \div 7.82 = 0.806240$$

Subtract the exponents.

$$10^4 - 10^2 = 10^2$$

Write the answer in scientific notation.

$$0.8062404 \times 10^2 = 8.062404 \times 10^1$$

According to the rules of significant figures, when multiplying or dividing numbers, write the answer to the least number of significant figures.

The answer is rounded to 8.06×10^1 (to three significant figures).

Exercises

Click a question to answer

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Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) Data booklet: Fundamental constants.
- Section D.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-field-gravitational-field-lines-and-gravitational-field-strength-id-46568/) Gravitational field, gravitational field lines and gravitational field strength.
- Section E.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/atoms-and-photons-id-46593/) The nucleus and atomic energy levels.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Units, symbols and numerical values

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Application of SI and non-SI units

Making measurements is an essential part of physics. In our connected world, different cultures and countries need to share scientific ideas and collaborate on research projects. For example, scientists from over 100 countries work at the nuclear and particle physics laboratory CERN, which is run by 24 countries. Mistakes could easily happen if each country used a different unit for a particular quantity.

The International System of Units (Système International d'Unités) is a set of units used globally, giving scientists and engineers a common language for measurement. These units are called SI units.

SI units include the metre (m) for length, the second (s) for time and the joule (J) for energy.

In some situations, we also use non-SI units in physics; these include the minute, hour, day and year for time.

Values of a particular quantity can have a huge range. The radius of a proton and the distance across a galaxy are both lengths, but the difference between them is vast. Watch this video (https://www.youtube.com/watch?v=uaGEjrADGPA) which shows the enormous range of length

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scales in our universe.

To make it easier to read, write and understand quantities with different magnitudes, we can use scientific notation (see section 1.3.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/scientific-notation-id-48948/)). Alternatively (or in addition), we can use prefixes: letters written in front of units to change their sizes.

The prefixes, or metric (SI) multipliers, used in DP physics are provided in the DP data booklet (see section 1.6.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/metric-si-multipliers-id-45156/)) and shown in **Table 1**.

Table 1. Prefixes for units.

Prefix	Abbreviation	Value
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

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For example, a proton has approximate radius 1×10^{-15} m, which can be written as 1 fm.

Quantities written using the same prefix can be easier to compare. For example, the radius of a lone hydrogen atom is about 5×10^{-11} m. If we write this as 50 000 fm, we can easily see that it is about 50 000 times the radius of a proton.

Try **Interactive 1** to match the equivalent length for measurements using different prefixes.

Check

Interactive 1. Match the equivalent lengths.

Using symbols from the guide and data booklet

Humans have always used symbols to communicate. Examples include flags, traffic signs and emojis. Symbols can make information easy to understand quickly.

Many symbols are used in physics, including letters for quantities and their units, and symbols representing circuit components. Symbols we do not know can look very confusing, but after we have learned them they become easy to use. The data booklet shows the meanings of many

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symbols used in the DP physics course. However, you may find that you can work more efficiently if you memorise the common ones.

As well as symbols for quantities and units, such as *t* for time and s for the second, there are also symbols for fundamental constants. These include the acceleration of free fall on Earth, *g*, and the speed of light in a vacuum, *c*. The data booklet shows the symbols and values of constants you will need to use (see section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/)). It also shows the electrical circuit symbols used in DP physics (see section 1.6.6 (/study/app/math-aa-hl/sid-423-cid-762593/book/electrical-circuit-symbols-id-45158/)).

If you look up the speed of light in the data booklet, you will see its value is given as 3.00×10^8 m s^{-1}. The notation m s^{-1} means the same as m/s, because s^{-1} means 1/s.

Try **Interactive 2** to match quantities and their symbols to corresponding units and their symbols.

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Interactive 2. Match the units and symbols.

Using fundamental units

Many different units are used in physics. How many do we need? Is there a minimum number for describing all of the physical quantities that we know?

In the international system, there are just seven fundamental SI units: units that all other units are based on. These are shown, with the quantities they measure, in **Table 2.** You do not need to memorise this list.

Table 2. Quantities and fundamental SI units.

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Quantity	Fundamental SI unit
Time	second (s)
Length	metre (m)
Mass	kilogram (kg)
Electric current	ampere (A)
Temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Units made from combinations of fundamental units are called compound units. For example, metre per second ($m\ s^{-1}$) is a unit of speed and the kilogram per cubic metre ($kg\ m^{-3}$) is a unit of density.

Using appropriate units

We do not usually describe the age of the Sun using the SI unit of time, the second. We do not usually measure distances to other galaxies using the SI unit of distance, the metre. Why not?

To describe a measured quantity, we choose the most appropriate unit. For example, we usually describe the age of the Sun as 4.6×10^9 years instead of 1.5×10^{17} s.

To measure distances in space, astronomers often use two units called the parsec and the light year. Each of these is much, much larger than a metre. The distance to Proxima Centauri, our nearest star except for the Sun, is about 4.0×10^{16} m, which is 4.2 light years.

In the DP physics course, you will also learn to use the electron volt (eV) as a unit of energy for processes on atomic scales. For example, 2.2×10^{-18} J of energy is transferred when the electron is removed from a hydrogen atom. This equals 13.6 eV.

In **Interactive 3**, match each distance with the most suitable unit for measuring it.

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Interactive 3. Can you match the measured quantity to its unit?

Sometimes we need to convert between different units. The data booklet shows how to carry out some of these conversions (see section 1.6.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/unit-conversions-id-45157/)).

For example, the data booklet states that 1 astronomical unit (AU) = 1.50×10^{11} m.

The average distance from Earth to the farthest planet, Neptune, is 4.5×10^{12} m. What is this distance in AU?

$$
\begin{aligned}
4.5 \times 10^{12}\ \mathrm{m} &= \frac{4.5 \times 10^{12}\ \mathrm{m}}{1.50} \times \frac{10^{11}\ \mathrm{m}}{\mathrm{AU}} \\
&= 30\ \mathrm{AU}
\end{aligned}
$$

Converting between compound units is a slightly longer process. For example, to convert between $\mathrm{m\ s^{-1}}$ and $\mathrm{km\ h^{-1}}$, we need to think about both distance unit conversion and time unit conversion. Here is an example:

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A cyclist travels at $20\ \mathrm{km\ h^{-1}}$. Write this in $\mathrm{m\ s^{-1}}$.

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$$1\ \text{km} = 1000\ \text{m}$$

$$\begin{aligned}1\ \text{h} &= 1\ \text{h} \times 60\ \frac{\text{min}}{\text{h}} \times 60\ \frac{\text{s}}{\text{min}}\\ &= 3600\ \text{s}\end{aligned}$$

$$\begin{aligned}20\ \text{km h}^{-1} &= \frac{20\ \text{km}}{\text{h}} \times \frac{1000\ \frac{\text{m}}{\text{km}}}{3600\ \frac{\text{s}}{\text{h}}}\\ &= 5.6\ \text{m s}^{-1}\ (\text{to 2 s.f.})\end{aligned}$$

(We divide by the time conversion factor 3600 instead of multiplying, because speed is distance divided by time.)

Worked example 1

What is 8 years in seconds?

$$\begin{aligned}8\ \text{years} &= 8\ \text{years} \times \frac{365\,\text{day}}{\text{year}} \times \frac{24\,\text{h}}{\text{day}} \times \frac{60\,\text{min}}{\text{h}} \times \frac{60\,\text{s}}{\text{min}}\\ &= 252\,288\,000\ \text{s}\end{aligned}$$

Worked example 2

Use the unit conversions from the data booklet (see section 1.6.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/unit-conversions-id-45157/)).

1. What is the energy 18 MJ in kWh?
2. What is the temperature 293 K in °C?

1. $$\begin{aligned}18\ \text{MJ} &= 18 \times 10^6\ \text{J}\\ &= \frac{18 \times 10^6\ \text{J}}{3.60 \times 10^6\ \frac{\text{J}}{\text{kWh}}}\\ &= 5\ \text{kWh}\end{aligned}$$

2. $$\begin{aligned}293\ \text{K} &= (293 - 273)^\circ\text{C}\\ &= 20\,^\circ\text{C}\end{aligned}$$

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Expressing derived units

The fundamental SI units do not include units of speed, force or energy. How are units such as the newton and the joule derived from fundamental units?

Some derived units are simple to understand. For example, speed = distance (in m) ÷ time (in s), so the unit of speed is m s^{-1}.

However, the names of many derived units, such as the newton for force, do not reveal how they relate to fundamental units. We can work out these relationships from equations.

A well-known equation for force is:

$$\text{force (N)} = \text{mass (kg)} \times \text{acceleration } (\text{m s}^{-2})$$

Therefore, 1 N = 1 kg m s^{-2}. The kg, m and s are all fundamental SI units.

Worked example 3

Use the method above to write the unit of energy, the joule (J), in fundamental SI units.

Energy transferred is work done, where:

$$\begin{aligned}\text{work done} &= \text{force (N)} \times \text{distance (m)}\\ 1\text{ J} &= 1\text{ N m}\end{aligned}$$

The newton is not a fundamental unit, but we saw above that 1 N = 1 kg m s^{-2}.

So,

$$\begin{aligned}1\text{ J} &= 1\text{ kg m s}^{-2} \times \text{m}\\ &= 1\text{ kg m}^2\text{ s}^{-2}\end{aligned}$$

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.4.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Practical: Investigating the acceleration of free fall.

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- Section 1.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/data-analysis-id-46745/) Data analysis.
- Section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/) Describing motion.
- Section A.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/) The equations of motion.
- Section A.2.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/field-forces-id-44733/) Field forces.
- Section A.2.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/elastic-force-drag-buoyancy-id-44735/) Elastic force, drag, buoyancy.
- Section A.3.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/work-and-energy-id-43084/) Work and energy.
- Section A.3.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/power-and-efficiency-id-43086/) Power and efficiency.
- Section B.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/temperature-scales-id-44050/) Temperature scales.
- Section B.1.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/black-body-emission-id-43785/) Black body emission.
- Section E.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/atoms-and-photons-id-46593/) The nucleus and atomic energy levels.
- Section E.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-distances-in-space-id-46458/) Measuring distances in space.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Rates of change

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/rates-of-change-id-48950/print/) Assign

What does rate of change mean?

In the sciences, how much a quantity has changed is not always all we need to know; how fast or slow that change took place may also be important. An object moving 1000 metres over 1 minute is different to an object moving the same distance over 1 day.

When we talk about quantities such as velocity or acceleration, we are dealing with rates of change. A rate of change tells us how much value A changes with respect to how much value B changes.

Let’s look at a few examples:

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- Velocity is the rate of change of position. A turtle may move 1000 metres in a day, whereas a cheetah may move the same distance in 5 minutes.
- As you ascend a mountain the air will get cooler. You may observe the rate of change of temperature per unit of ascent.
- Frequency tells us how often something happens *per second*. This can have multiple applications:
  - The frequency of alternating current tells us how many times an electric current changes direction per second.
  - In simple harmonic motion, frequency tells us how many full oscillations happen per second.

In this section you will read about two different rates of change:

- The **average** rate of change, is how much change per time unit happens over a period of time. For example, car A driving between two points takes 25 minutes and moves on average 21 miles per hour. During this journey, there will be periods in which the car's speed is greater than the average, and times in which the car will be slower or even stationary, waiting at traffic lights for example.
- The **instantaneous** rate of change is how much change per time unit is happening at a particular time. For example, car A has an instantaneous velocity of 0 mph when stopped at traffic lights, or 70 mph when driving on a motorway.

Try **Interactive 1** to find the variables for these rates of change.

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Find the variables for these rates of change. Fill in as many gaps as you can using the variables in this list.

1. Velocity is the rate of change of per unit .
2. Acceleration is the rate of change of per unit .
3. A temperature gradient for a mountain climber is the rate of change of with .
4. Growth rate is the rate of change of per unit .
5. Electric frequency is the rate of change of per unit time.
6. A chemical rate of change is the rate of per unit time.

distance | time | size | height | temperature | time | time | velocity | current direction | chemical reaction

Check

Interactive 1. Rates of change.

Measuring rate of change using tabulated data

Sometimes we may need to calculate a rate of change from a data table.

Table 1. Example data for plotting distance travelled per unit time.

Time (s)	Distance travelled (m)			
	1	2	3	Mean average
0	0	0	0	0
1	7.4	7.7	7.9	7.7
2	13.2	12.6	12.9	12.9
3	14.6	14.2	14.9	14.6

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When calculating a rate of change from a data table, you are finding the difference in the dependent variable divided by the difference in the independent variable:

$$\text{rate of change} = \frac{\text{how much your dependent variables changed}}{\text{how much your independent variable changed}}$$

Calculating the rate of change in distance travelled for the data between 0–3 seconds in **Table 1**, we find:

$$\begin{aligned}\text{rate of change} &= \frac{14.6 - 0 \text{ metres}}{3 \text{ seconds}}\\ &= 4.9 \text{ m s}^{-1}\end{aligned}$$

This means for every second, the object has moved on average 4.9 metres. This is an average rate of change as you can observe that the distance travelled in the first second is greater than 4.9 metres, and less than 4.9 metres was travelled in the final second.

Worked example 1

Calculate the average rate of change of position with time from **Table 2**.

Table 2. Data for three repeated measurements for displacement of a car over a time period of 5 seconds.

Time (s)	Displacement (m)			
	1	2	3	Mean average
0	13	14	13	13
1	16	17	16	16
2	19	21	21	20
3	22	24	23	23
4	27	27	28	27
5	30	31	31	31

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To find the rate of change of position with time, we need to solve the equation:

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$$\text{rate of change} = \frac{\text{how much your dependent variable changed}}{\text{how much your independent variable changed}}$$

The dependent variable is displacement and the independent variable is time.

Over the course of 5 seconds, the average position changed from 13 m to 31 m, so the change is 18 m. Thus we can show that:

$$\begin{aligned}\text{rate of change} &= \frac{18 \text{ m}}{5 \text{ s}} \\ &= 3.6 \text{ m s}^{-1}\end{aligned}$$

This tells us that for every second that passed, the average change in displacement was 3.6 metres.

Using data tables to find rates of change is usually straightforward. However, if you need the rate of change at any time interval you don't have data for, this becomes more difficult.

Measuring rate of change using graphical data

The rate of change can be calculated from graphs too. In this case, the data is plotted with the independent variable on the x-axis and the dependent variable on the y-axis.

If there is a linear relationship between the variables, a line of best fit can be drawn between the data points (see section 1.3.11 (/study/app/math-aa-hl/sid-423-cid-762593/book/analysing-graphs-id-48958/)). By calculating the gradient of the line of best fit, you will find the average rate of change.

If there is a non-linear relationship between the variables, the rate of change at any point on the graph by using the gradient of a tangent at the point you are interested in (see section 1.3.11 (/study/app/math-aa-hl/sid-423-cid-762593/book/analysing-graphs-id-48958/)). The equation we use to find the gradient of that tangent is:

$$\text{gradient} = \frac{\text{change in } y\text{-axis of tangent line}}{\text{change in } x\text{-axis of tangent line}}$$

You can also find the instantaneous rate of change by drawing tangents and finding their gradient. As an example, examine the distance travelled vs. time graph in **Figure 1**.

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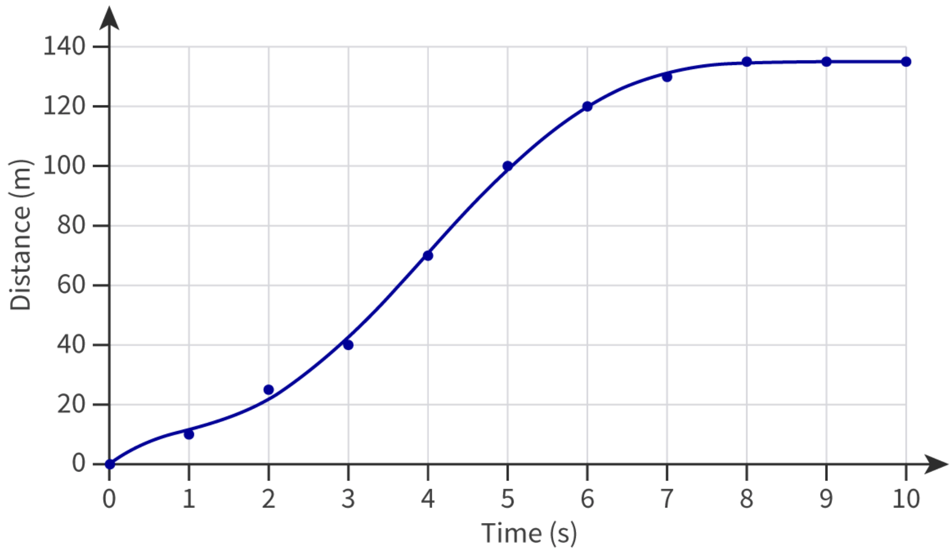

Figure 1. Distance travelled vs. time for a moving object.

More information for figure 1

Distance time graph. Along the y-axis is the distance (m) running from 0—140, along the x-axis is time (s) running from 0—10. The object moves the furthest between seconds 0—7 and slows down between seconds 7—10.

You can find the rate of change of position with time at two points: 2 seconds and 4 seconds. To begin we draw tangent lines at both points.

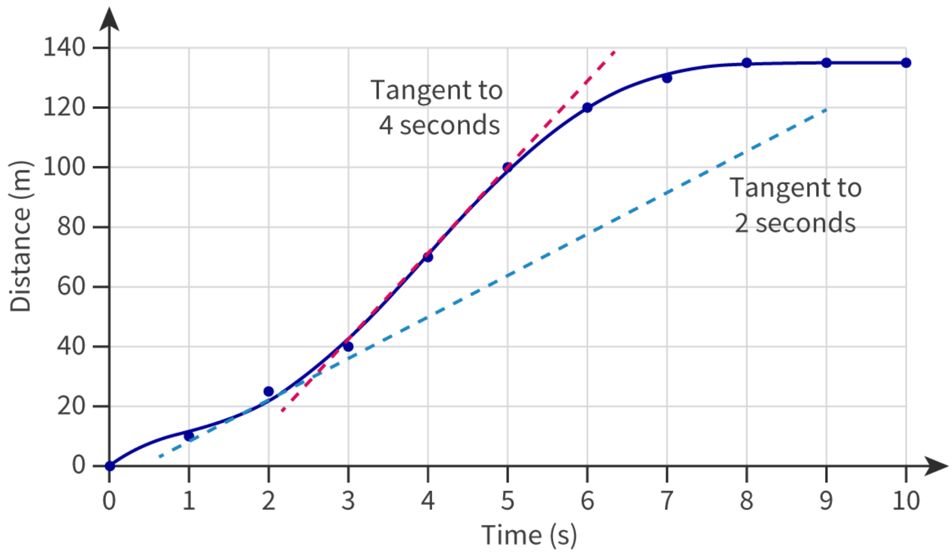

Figure 2. Tangent lines drawn for 2 seconds (blue line) and 4 seconds (red line).

More information for figure 2

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Distance time graph. A blue tangent line is drawn through the point at 2 seconds and 25 metres and a red tangent line is drawn through the point at 4 seconds and 70 metres.

Applying the equation to both red and blue tangent lines we can find the instantaneous rate of change at those points:

$$\begin{aligned}\text{gradient}_{2\text{s}} &= \frac{\text{change in } y\text{-axis of tangent line}}{\text{change in } x\text{-axis of tangent line}}\\ &= \frac{120-10}{9-1}\\ &= \frac{110}{8}\\ &= 13.75 \text{ m s}^{-1}\end{aligned}$$

$$\begin{aligned}\text{gradient}_{4\text{s}} &= \frac{\text{change in } y\text{-axis of tangent line}}{\text{change in } x\text{-axis of tangent line}}\\ &= \frac{130-20}{6-2.2}\\ &= \frac{110}{3.8}\\ &= 28.95 \text{ m s}^{-1}\end{aligned}$$

Worked example 2

Using **Figure 1**, find:

1. The average rate of change of position with time between 3 s and 5 s.
2. The instantaneous rate of change at both 3 s and 5 s.

1. To find the average rate of change of distance with time (the velocity) between 3 s and 5 s we need to find the gradient between those two points:

$$\text{gradient} = \frac{\text{change in } y\text{-axis}}{\text{change in } x\text{-axis}}$$

We can see that:

- The change in y is 100 m – 40 m = 60 m
- The change is x is 5 s – 3 s = 2 s

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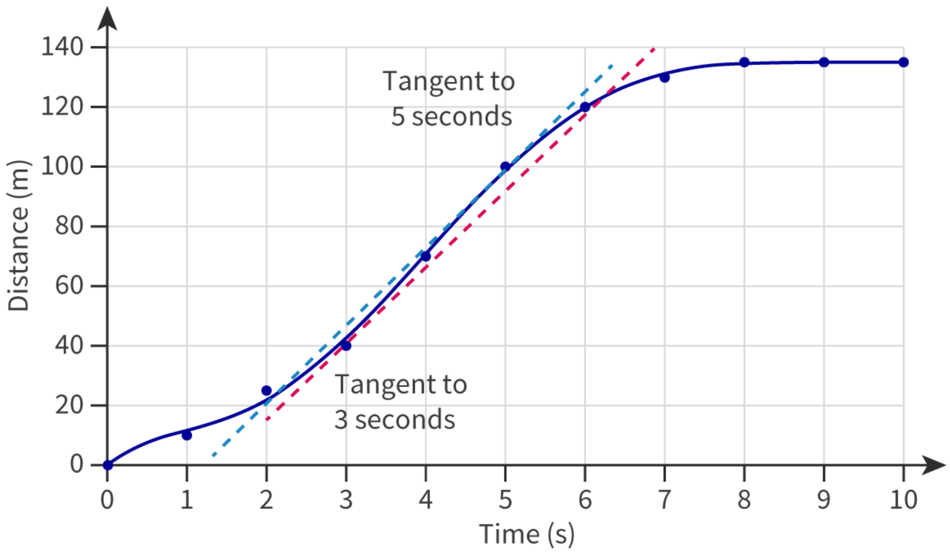
Thus:

$$\begin{aligned}\text{velocity} &= \frac{60 \text{ m}}{2 \text{ s}} \\ &= 30 \text{ m s}^{-1}\end{aligned}$$

Therefore, between 3 and 5 seconds, the average velocity was 30 metres per second.

2. To find the instantaneous rate of change at those points we need to draw a tangent line on both of the points, see the figure below.



Tangent lines drawn for 3 seconds (red line) and 5 seconds (light blue line).

You can use the equation as before to work out the instantaneous rate of change at those points:

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$$\text{gradient}_{3\text{s}} = \frac{\text{change in } y\text{-axis of tangent line}}{\text{change in } x\text{-axis of tangent line}}$$
$$= \frac{140 - 17}{6.8 - 2}$$
$$= \frac{123}{4.8}$$
$$= 25.63 \text{ m s}^{-1}$$

$$\text{gradient}_{5\text{s}} = \frac{\text{change in } y\text{-axis of tangent line}}{\text{change in } x\text{-axis of tangent line}}$$
$$= \frac{135 - 5}{6.3 - 1.4}$$
$$= \frac{130}{4.9}$$
$$= 26.53 \text{ m s}^{-1}$$

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section A.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/) Equations of motion.
- Section B.1.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/thermal-energy-transfer-id-44052/), Thermal energy transfer.
- Section D.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-potential-energy-and-gravitational-potential-hl-id-46569/) Gravitational potential energy and gravitational potential (HL).
- Section E.3.3a (/study/app/math-aa-hl/sid-423-cid-762593/book/half-life-id-46544/) Half-life.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Measures of central tendency

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/measures-of-central-tendency-id-48951/print/) Assign

Imagine someone asks you how many hours you sleep every day. The answer to this question can be complicated, since people usually sleep for a different number of hours each day. In the same way, when performing an experiment, the data that is collected from a specific measurement can

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vary due to uncertainties. However, we still like to use single values to describe parameters that vary. We do that by using a statistical concept called the mean or the average. For example, tracking a month of sleep, you can say that you slept for 7.5 hours per night.

In this section we will see four tools that you can use to summarise a set of data with a single value. These statistical tools are called measures of central tendency, because they indicate the degree at which the data tend to gather around a central value. These are the mean, the median, the mode and the range.

The mean

The mean is the arithmetic average value of a set of numbers, and it is denoted by $\bar{x}$.

Given a set of N numbers

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ \ldots,\ x_N$$

You calculate the mean by:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + \ldots + x_N}{N}$$

In other words, you add up all the numbers and divide the result with the number of values that are in the set.

Worked example 1

A student took six exams and scored the following percentages.

$$79, 82, 75, 84, 95, 89$$

What is the mean percentage score for all exams?

Step 1: Add up all the numbers in the set to find their sum:

$$79 + 82 + 75 + 84 + 95 + 89 = 504$$

Step 2: There are six numbers in the set, so you have to divide the sum you found in the previous step by 6.

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$$\bar{x} = \frac{504}{6} = 84$$

The answer is that the mean score of the student was 84%.

For any statistical tool, the mean works better when you have a greater number of values in the set. Consider an extreme example where at a sports game one player scores 99points and another player just 1 point. The set then is $\{1,99\}$. The mean is 50. So, on average, each player scored 50 points. This is not a good representation of the real situation in this case.

Range

The range of a dataset refers to the difference between the highest and lowest values. This gives an idea of how much spread there is in the data. For example, if the acceleration of several falling objects was measured at 6.5, 7.1, 8.2, 6.7 and 7.5 m s^{-2}, then the highest value is 8.2 and the lowest is 6.5. Therefore, the range is:

$$\begin{aligned}\text{Range} &= \text{maximum value} - \text{minimum value}\\ &= 8.2 - 6.5\\ &= 1.7\text{ m s}^{-2}\end{aligned}$$

The median

The median is the ‘middle’ value of a set of numbers, when this set is ordered from lower to higher values.

Use the following steps to calculate the median:

- Take all the numbers and sort them in order from lowest to highest value. For example, the set $\{4,10,5,61,2,37,18\}$ is ordered as $\{2,4,5,10,18,37,61\}$.
- If the number of values you have is odd, then pick the middle number of your ordered set. In the example set $\{2,4,5,10,18,37,61\}$ there is an odd number of values as there are seven numbers in the set. The median is 10, because it is the number exactly in the middle.
- If the number of values you have is even, then pick the middle two numbers of the ordered set and take their mean.
- Example:If you have the set $\{79,82,75,84,95,89\}$, you would order it as $\{75,79,82,84,89,95\}$. There are six (even) number values in that set. Therefore, we take the two middle values $\{82,84\}$ and calculate their mean. The median is $\frac{82+84}{2} = 83$.

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The median in a set is the value for which exactly half of the numbers of the set are smaller and exactly the other half is larger. It helps to understand the central tendency especially when the extreme values may skew the mean.

The mode

The mode in a data set is the value that appears most frequently in the set.

Look into the set and find the numbers that repeat themselves in it. The one that repeats itself the most times is the mode.

- The set $\{6,4,5,5,4,5,2,7\}$ has a mode of 5, since it appears three times and all the other numbers appear twice or once.
- If there is no number that repeats itself in the set, then there is no mode in it. The set $\{3,5,4,7,2,8,1\}$ has no mode, since no number appears more than once in the set.
- There can be more than one mode in a set. The set $\{3,7,3,9,1,7,9,9,7\}$ has two modes, since 7 and 9 appear three times in the set.

Categorical data

The mode is also useful when you have to work with qualitative data, i.e. it is not numerical. Such data can be some kind of information or label. Examples of categorical data are:

- the country that people live in
- the type of radiation emitted
- the subjects a student studies
- music genre
- the material in a shirt.

A set that contains the answers of 10 people to the question:

'What is your favourite colour?'

The set of answers given is:

$$\{\text{blue, yellow, white, blue, green, blue, purple, orange, orange, blue}\}$$

The mode of this set is 'blue' since it appears four times in the set, more frequently than any other colour.

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Exercise

Click a question to answer

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.4.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Practical: Investigating the acceleration of free fall.
- Section 1.4.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-relationship-between-velocity-id-46751/) Practical: Investigating the relationship between velocity and the horizontal distance travelled by a projectile.
- Section 1.4.9 (/study/app/math-aa-hl/sid-423-cid-762593/book/determining-the-half-life-of-random-processes-id-46753/) Practical: Determining the half-life of random processes as a simulation of radioactive decay.
- Section E.5.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/star-radius-id-46457/) Star radius.
- Section B.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/temperature-scales-id-44050/) Temperature scales.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Equations

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/equations-id-48952/print/) Assign

Selecting the right equation

When solving problems, the first challenge can be selecting the correct equation to use. This can be tough, however there are ways to ensure you make the right choice.

In DP physics you are always given a data booklet with the equations you will need for the course. This is because you are being tested on your ability to use equations, and not just remember them. If you are struggling to find an equation that works, you can list what known

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variables you have, and then identify what unknown variables you need to find.

Each of the five topics in the DP physics course has a data booklet page containing relevant equations. You should start your search by looking in the most relevant part of the data booklet.

- Section 1.6.A (/study/app/math-aa-hl/sid-423-cid-762593/book/space-time-and-motion-id-45160/) for topic A on Space, time and motion.
- Section 1.6.B (/study/app/math-aa-hl/sid-423-cid-762593/book/the-particulate-nature-of-matter-id-45161/) for topic B on The particulate nature of matter.
- Section 1.6.C (/study/app/math-aa-hl/sid-423-cid-762593/book/wave-behaviour-id-45162/) for topic C on Wave behaviour.
- Section 1.6.D (/study/app/math-aa-hl/sid-423-cid-762593/book/fields-id-45163/) for topic D on Fields.
- Section 1.6.E (/study/app/math-aa-hl/sid-423-cid-762593/book/nuclear-and-quantum-physics-id-45164/) for topic E on Nuclear and quantum physics.

Worked example 1

Calculate the work required to push a block a distance of 20 metres with a constant force of 30 newtons which acts parallel to the motion and accelerates the body?

To answer this question you need an equation with work (W) as the subject. Looking through the physics data booklet we see many equations with work in them:

- $W = Fs\cos\theta$
- $\Delta W = Fv\Delta t$
- $W = Q - \Delta U$
- $W = Vq$
- $W = P\Delta V$
- $W = m\Delta V_{\mathrm{g}}$
- $W = m\Delta V_{\mathrm{e}}$

To eliminate as many as possible, look at the context of the question and the data given:

- The force is a contact force. Therefore, equations related to field potential $(V_{\mathrm{g}}, \mathrm{V}_{\mathrm{e}})$ or pressure (P) can be discarded.
- The data we are given is a force and a distance. Look down the equations for variables that equate to this data.

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Taking these into account we can eliminate all but one of the equations:

$$W = Fs\cos\theta$$

Given our force is acting parallel to the motion of the body, our value for Θ is 0 degrees, giving a value for $\cos\theta$ of 1.

Now we can evaluate the equation to find:

$$W = Fs = 30 \times 20 = 600 \text{ J}$$

Deriving equations

Another skill that scientists need is how to derive equations. Deriving an equation means to use assumptions, estimates or other equations to make a new equation that is appropriate for the problem you are solving.

If you are studying the HL course you may be required to do a derivation combining more than two equations. On the SL course you are expected to do some basic manipulations of equations.

Worked example 2

Show that the angular frequency for a simple pendulum undergoing simple harmonic motion is given by:

$$\omega = \sqrt{\frac{g}{l}}$$

where:

l is the length of the oscillator

g is the gravitational acceleration.

From the data booklet we can see an equation giving the angular frequency, ω in the section 1.6.C (/study/app/math-aa-hl/sid-423-cid-762593/book/wave-behaviour-id-45162/) that includes simple harmonic motion:

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$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

This is a good start, and now we need to make substitutions to involve l and g in the equation. There is one extra variable we don't need in the previous equation, T. Going back to the data booklet we might find an equation for T that we can substitute in:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Now substituting this equation into the first equation:

$$2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\omega}$$

And through algebraic manipulation we find:

$$\sqrt{\frac{l}{g}} = \frac{1}{\omega}$$

$$\sqrt{\frac{g}{l}} = \omega$$

This is the equation we required.

Checking your new equation works

A powerful way of checking if derived equations are valid, is by using a method called dimensional analysis. This is a technique used to ascertain whether the units (or dimensions of the value) on either side of the equation match.

For example, imagine you derive these two equations in **Table 1**.

Table 1. The equations we want to test.

Equation	1	2
	$m = Fv$	$J = F$

Table 2 sets out a method for conducting dimensional analysis on both equations by listing the SI units for each variable, then simplifying when possible.

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Table 2. Breaking down the equations into their units.

Equation	1		2	
	$m = Fv$		$J = Ft$	
Units	kg	$\text{kg m s}^{-2} \times \text{m s}^{-1}$	kg m s^{-1}	$\text{kg m s}^{-2} \times \text{s}$
Simplified	kg	$\text{kg m}^2 \text{ s}^{-3}$	kg m s^{-1}	kg m s^{-1}

In **Equation 1** you can see the simplified units don't match on either side of the 'equal to' symbol (the equality). This equation cannot be valid. In **Equation 2** however, the simplified units do match. This is a valid equation.

Look back to **Worked example 1**. Can you see that dimensional analysis can help you check that your answer is aligned to the initial data provided?

Exercise

Click a question to answer

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.6.A—E (/study/app/math-aa-hl/sid-423-cid-762593/book/space-time-and-motion-id-45160/) Data booklet pages of equations.
- Section A.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/) The equations of motion.
- Section B.3.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/how-do-ideal-gases-behave-id-44292/) How do ideal gases behave?
- Section C.1.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/energy-hl-id-44902/) Energy (HL).
- Section D.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/keplers-laws-and-newtons-universal-law-of-gravitation-id-46566/) Kepler's laws and Newton's universal law of gravitation.

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Continuous and discrete variables

We can organise all data and variables into two categories: discrete and continuous.

Continuous variables

A continuous variable can take an infinite amount of values (all possible values) within a specific range. For example, when an object is moving straight in one direction, the distance it travels takes all the values between the initial and the final position.

You might need to collect continuous data in a laboratory. Consider recording the temperature drop for a cup of water as it cools down (**Figure 1**). The temperature takes all the values as it drops, and you can verify this by looking at the thermometer.

This type of data is usually measured and recorded to a certain number of significant figures.

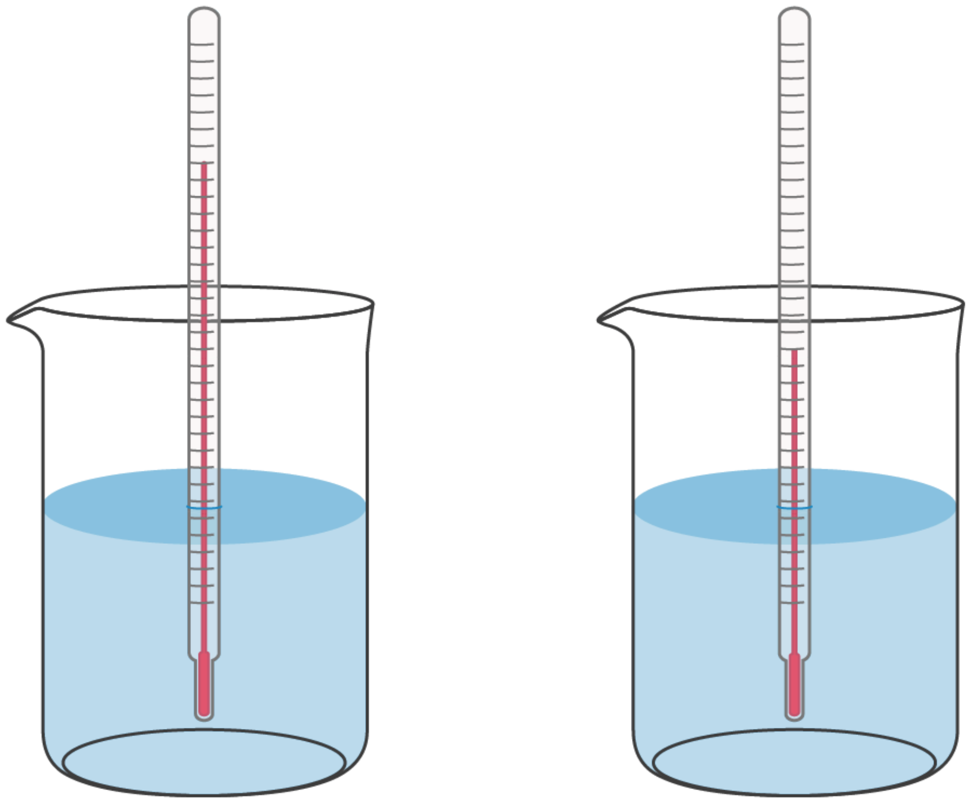

Figure 1. Recording a continuous variable such as temperature.

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Discrete variables

Discrete variables can take a finite number of values (only certain values) within a range.

An example of discrete data is the number of students in a class. There can only be an integer number of students in a class, say 16 or 21, but not 17.2 students. In this example, you count the data instead of measuring. Categorical data are also discrete. For example, the direction of a vector on a straight horizontal line can either be left or right.

In section D.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/electric-charge-id-46475/) you can see that the type of electric charge is a discrete variable – it is either positive or negative. As is typical in such variables, there is a smallest possible unit of electric charge that can exist, which is the charge of one electron. An object's charge is always an integer multiple of this elementary unit.

Figure 2 is an illustration to help you visualise the difference between the two types of data. There is one tap with running water and one dripping water. If you were to collect data about the flow of water from each tap in a set period of time, what type of data would you collect?

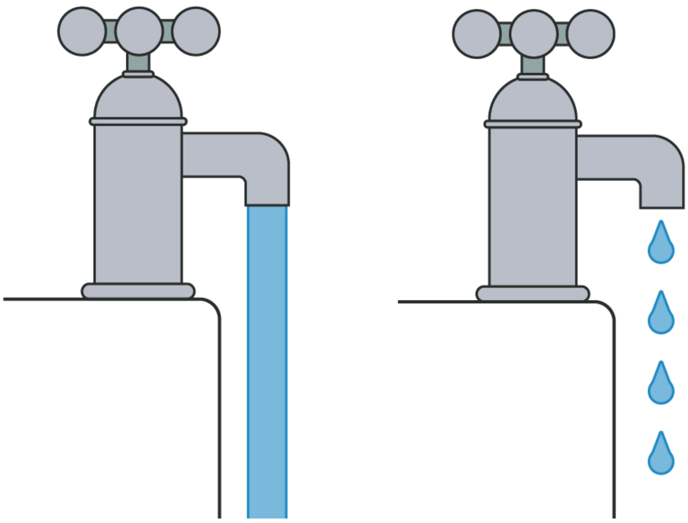

Figure 2. Which type of data (discrete or continuous) could be collected from each tap?

More information for figure 2

The illustration shows two taps mounted above a basin. The tap on the left is open, releasing a steady, continuous flow of water directly into the basin, representing continuous data collection. The tap on the right is closed, with droplets of water forming and falling at intervals, indicating discrete data collection. This visual is used to facilitate understanding of the types of data that can be collected from each tap over a given period, with focus on the contrasting flows: a continuous stream and intermittent drips.

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[Generated by AI]

If you measured the volume of water coming out of both taps, it would give you continuous data. However, you could also measure the number of drips from the dripping tap – giving you discrete data.

For example, the volume of water from the flowing tap could be 5.67 dm^3 (to three significant figures), whereas the number of drips could be 52, which is a whole number (there cannot be half a drip).

Table 1 has examples of continuous and discrete variables that you will come across in the DP physics course.

Table 1. Continuous vs. discrete variables.

Continuous variable	Discrete variable
Time	Electric charge
Temperature	Energy in atoms
Mass	Atomic number
Pressure	Mass number
Acceleration	

Continuous data and discrete data are represented using different types of graphs. Graphing is covered in detail in sections 1.3.10 (/study/app/math-aa-hl/sid-423-cid-762593/book/plotting-graphs-id-48957/) and 1.3.11 (/study/app/math-aa-hl/sid-423-cid-762593/book/analysing-graphs-id-48958/).

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/) Describing motion.
- Section D.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/electric-charge-id-46475/) Electric charge.

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- Section E.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/atoms-and-photons-id-46593/) The nucleus and atomic energy levels.

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Proportionality and percentage change

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/proportionality-and-percentage-change-id-48955/print/) Assign

Whether to generalise solutions to problems or better communicate results, data analysis is a crucial tool in a scientist's toolkit.

Proportionality

Two quantities are proportional with each other when a change in one quantity leads to a constant change in the other quantity. There are two types of proportionality, direct proportionality and inverse proportionality, as shown in **Table 1**.

Table 1. A comparison between direct and indirect proportionality.

	Direct proportionality	Inverse proportionality
Meaning	An increase in one value leads to an increase in another value and vice versa.	An increase in one value leads to a decrease in another value and vice versa.
Equation	$a \propto b$	$a \propto \frac{1}{b}$
Proportionality equation	$a = kb$ where k is the constant of proportionality.	$a = \frac{k}{b}$ where k is the constant of proportionality.

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	Direct proportionality	Inverse proportionality
Example	A building company sells bags of sand (n). Two bags cost \$6 (c). If you increase the number of bags by a factor of 4, the cost goes up by the same factor: eight bags cost \$24. The number of bags is in direct proportion to the cost. $$\begin{aligned} c &\propto n \\ c &= kn \\ 6 &= k \times 2 \\ k &= 3 \end{aligned}$$	A transport company needs to ensure its lorries arrive on time at the factory when travelling from the depot. The speed they travel (S) is inversely proportional to the time of the journey (T). A typical journey from the depot to the factory takes 2 hours at a speed of 70 km h^{-1}. $$\begin{aligned} S &\propto \frac{1}{T} \\ S &= \frac{k}{T} \\ 70 &= \frac{k}{2} \\ k &= 70 \times 2 \\ &= 140 \end{aligned}$$ The equation for this example is: $$S = \frac{140}{T}$$ where 140 is the constant of proportionality.

Worked example 1

If a pneumatic drill increases its ground impacts by 20 hits per minute (h), the sound emitted increases in direct proportion by 25 decibels (l). Find how much louder an increase of 30 hits per minute would be.

As the sound emitted increases in direct proportion to the number of ground impacts:

$$h = kl$$

We find k with the data given:

$$\begin{aligned} 20 &= k \times 25 \\ k &= \frac{20}{25} \\ &= 0.8 \end{aligned}$$

The equation is:

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$$h = 0.8 \times l$$

And our solution can be found:

$$\begin{aligned} 30 &= 0.8 \times l \\ l &= \frac{30}{0.8} \\ &= 37.5 \text{ dB} \end{aligned}$$

An increase in 30 hits per minute would result in a 37.5 dB increase in the volume of sound.

Percentage change

Sometimes it is useful to express changes as a percentage as opposed to an absolute value. For example ‘Profits increased 22% over two years’ can be more descriptive than saying ‘Profits increased by $38 000 over two years’.

To calculate the percentage change in a value, we use the equation:

$$\text{percentage change} = \frac{\text{new value} - \text{old value}}{\text{old value}} \times 100$$

For example, imagine a school has an DP physics average score of 4.15 in 2023, which increases to 4.57 in 2024. What is the percentage change?

$$\begin{aligned} \text{percentage change} &= \frac{\text{new value} - \text{old value}}{\text{old value}} \times 100 \\ &= \frac{4.57 - 4.15}{4.15} \times 100 \\ &= 10.1\% \text{ change} \end{aligned}$$

The average score has increased by 10.1% from 2023 to 2024. This can be a more useful metric than saying ‘scores increased by 0.42 points’ because a percentage is in relation to the original value. This helps anyone who does not understand what an average exam score relates to, whereas the percentage change helps them understand the scale of the achievement.

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If the new value is greater than the old value, the percentage change will be positive and there has been a percentage increase. If the new value is less than the old value, the percentage change will be negative and there has been a percentage decrease, or decline.

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Percentage difference

Percentage difference, as the term implies, compares and expresses the difference between two values as a percentage.

For two values n_1 and n_2:

$$\begin{aligned}\text{percentage difference} &= \frac{\text{difference between the values}}{\text{mean average of the values}} \times 100\\ &= \frac{n_1 - n_2}{\left(\frac{n_1 + n_2}{2}\right)} \times 100\end{aligned}$$

Expressing numbers as percentages is covered in section 1.3.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-approaches-to-processing-scientific-data-id-48947/).

Percentage uncertainty

Percentages can also be used to convey the relative size of the uncertainty in any value. The smaller the percentage uncertainty, the greater the certainty is in the measured value.

In an experiment, you will record the uncertainty as an absolute value, for example 3.0 ± 0.1 g. The absolute uncertainty of 0.1 g is a very small number. However, it can be compared to the measured value of mass to give the percentage uncertainty:

$$\begin{aligned}\text{percentage uncertainty} &= \frac{\text{uncertainty}}{\text{measured value}}\\ &= \frac{0.1}{3.0} \times 100\\ &= 3.3\%\end{aligned}$$

Now consider the situation if the mass being measured was very small. Imagine your measurement instead was 0.3 ± 0.1 g. The same absolute uncertainty of ± 0.1 g now gives a percentage uncertainty of 33.3% of the measured value.

$$\begin{aligned}\text{percentage uncertainty} &= \frac{0.1}{0.3} \times 100\\ &= 33.3\%\end{aligned}$$

Percentage uncertainty can also be calculated by comparing an experimental value to a known literature value:

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$$\text{uncertainty} = \frac{|\text{measured value} - \text{known value}|}{\text{known value}} \times 100$$

Note that this calculation is an absolute value, so even if the answer is negative, we report the positive value. Percentage uncertainty is calculated in this way because we are more interested in how far away the experimental value is from the actual value as opposed to the direction of the difference.

Worked example 2

You conduct an experiment to measure gravity, and estimate it to be $9.26\ \text{m s}^{-2}$. Calculate the percentage deviation from the accepted average value for g.

The value found through experiment is $9.26\ \text{m s}^{-2}$. Section 3 section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet gives a value for gravity as $9.8\ \text{m s}^{-2}$. You would find the percentage uncertainty as:

$$\begin{aligned}\text{percentage uncertainty} &= \frac{\text{measured value} - \text{known value}}{\text{known value}} \times 100 \\ &= \frac{9.26 - 9.8}{9.8} \times 100 \\ &= -5.5\%\end{aligned}$$

Your measured value is 5.5% less than the stated value of the acceleration of free fall at the Earth's surface.

Note that usually we do not worry about the sign in a percentage deviation as we are more interested in how far away from the actual value we are. A negative sign tells us the measured value is lower than the accepted value, whereas a positive means the measured value is greater than the accepted value.

Exercise title

Click a question to answer

Student view

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.4.7 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-resistivity-of-a-conducting-wire-id-46511/) Practical: Investigating the resistivity of a conducting wire.
- Section 1.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/data-analysis-id-46745/) Data analysis.
- Section B.3.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/how-do-ideal-gases-behave-id-44292/) How do ideal gases behave?

Recording data

Recording data and significant figures

When collecting data in the laboratory, it is important to consider the number of significant figures in the data. The number of significant figures in a measured value includes all the known figures and the last one that is estimated. See section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/) and section 1.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-digital-equipment-id-48814/) for taking measurements and section 1.3.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-approaches-to-processing-scientific-data-id-48947/) for learning about significant figures.

Significant figures and scientific notation

When representing a number using scientific notation, all the numbers to the left of the multiplication sign are significant. For example, the Avogadro constant, 6.02×10^{23}, has three significant figures. The mass of a single proton, 1.672622×10^{-27} kg, has seven significant figures.

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Calculations and significant figures

When carrying out calculations involving measured values, you need to consider the number of significant figures (or decimal places) in the recorded data. The rules differ on whether you are multiplying and dividing or adding and subtracting.

Multiplying and dividing

When multiplying or dividing, you need to consider the number of the significant figures in the measured values themselves. The value with the smallest number of significant figures will determine how many significant figures the final answer has. For example, in the calculation to determine the heat, Q, absorbed by an aqueous solution the following data is collected.

Table 1. Data for experiment into heat absorption.

Variable	Quantity	Significant figures
Mass of solution (*m*)	150.0 g	4
Specific heat capacity of solution (*c*)	4.18 J g^{-1} °C^{-1}	3
Temperature change of solution (ΔT)	12 °C	2

Using the equation $Q = mc\Delta T$, we get:

$$Q = 150.0 \text{ g} \times 4.18 \text{ J g}^{-1} \text{ °C}^{-1} \times 12 \text{ °C} = 7524 \text{ J}$$

As it is written, 7524 J has four significant figures. To determine how many significant figures we can use in the final answer, we need to look at the original values used in the calculation.

The change in temperature, ΔT, has the smallest number of significant figures (two) so it is this value that determines the number of significant figures we can use. The final answer, written to the correct number of significant figures is:

$$7.5 \times 10^3 \text{ J}$$

Note the use of scientific notation in the answer. We could also write the final answer as 7500 J, which is also two significant figures, but it is usually preferable to use scientific notation.

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Worked example 1

Calculate the following to the correct number of significant figures.

(a) 27.3×4.5
(b) 4.68×400
(c) 323×0.00024
(d) $4008 \div 2.763$
(e) $692 \div 7.041$

When multiplying or dividing, the answer should be written to the least number of significant figures in the original numbers.

(a) 1.2×10^2 (two significant figures)
(b) 2×10^3 (one significant figure)
(c) 7.8×10^{-2} (two significant figures)
(d) 1451 (four significant figures)
(e) 98.3 (three significant figures)

Addition and subtraction

When adding or subtracting measured values, it is the number of decimal places that need to be considered. For example, if the mass 20.78 g is subtracted from 34.925 g:

$$34.925 \text{ g} - 20.78 \text{ g} = 14.145 \text{ g}$$

The answer is 14.145 but we need to consider the number of decimal places in the final answer. 20.78 has two decimal places and 34.945 has three decimal places. 20.78 has the least number of decimal places so it is this number that determines the number of decimal places in the final answer. So the final answer, written to the correct number of decimal places is 14.15 g.

If we were to report the final value as 14.145, this would indicate that we knew 20.78 to the thousandths, or 20.780. Since we did not measure 20.78 to the thousandths, we cannot report the final value that level of precision.

Worked example 2

Calculate the following to the correct number of decimal places.

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(a) 5.72 + 2

(b) 500 − 79.4

(c) 0.006 + 0.04

(d) 84.3 − 0.009

(e) 66.3 + 27.008

When adding or subtracting, the answer is written to the least number of decimal places in the original numbers.

(a) 8 (zero decimal places)

(b) 421 (zero decimal places)

(c) 0.05 (two decimal places)

(d) 84.3 (one decimal place)

(e) 93.3 (one decimal place)

Exercises

Click a question to answer

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/) Measuring variables with analogue equipment.
- Section 1.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-digital-equipment-id-48814/) Measuring variables with digital equipment.
- Section 1.3.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-approaches-to-processing-scientific-data-id-48947/) Mathematical approaches to processing scientific data.

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Student view

Errors and uncertainties

Experimental uncertainties

In the DP physics course, you will conduct a number of experiments that involve the measurement of continuous variables such as time, velocity and voltage. Whenever a measurement is taken in the laboratory, there is an uncertainty associated with it. These uncertainties could have various sources. They can be associated with the conditions at the moment of measurement, the human factor, the devices you are using, or they can also have an unidentified source. In this section, we will look at the different types of experimental uncertainty and how to calculate the uncertainties associated with measured values.

Random and systematic errors

When data are recorded during an experiment, the measured values are rarely identical. These natural fluctuations are due to random uncertainties in the measured value.

Examples of causes of these fluctuations include:

- a breeze through an open door, which affects the mass measured on a mass balance
- changes in the temperature of a room between morning and afternoon
- errors of judgement when estimating the final digit on analogue equipment (see section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/)).

Figure 1 shows a piece of magnesium ribbon with a length of approximately 10 cm (100 mm). If five different students were asked to measure the length of the ribbon they might come up with different measurements, such as those shown in **Table 1**.

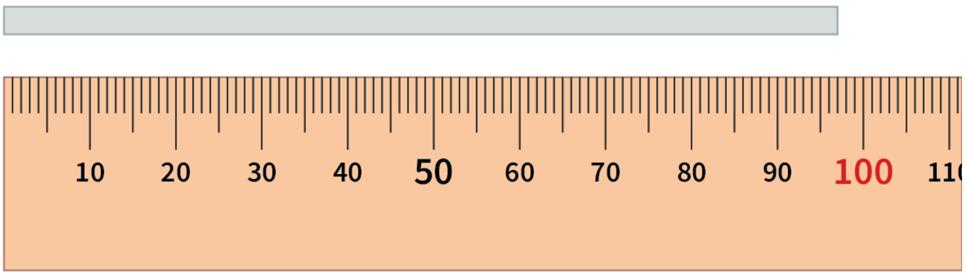

Figure 1. This length of the magnesium ribbon is approximately 10 cm (100 mm).

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Table 1. Data table for measurements.

Student	Length of magnesium ribbon (mm)
1	99
2	100
3	98
4	97
5	101

Random errors in the measured value produce readings that are likely to be both higher and lower than the actual value. Measurement uncertainty cannot be eliminated completely, however, they can be reduced by conducting repeat trials and taking an average. This is why it is good practice to take repeat measurements in the laboratory. Uncertainty and random errors can also be reduced by using more precise apparatus to take the measurements.

The second type of uncertainty encountered in the laboratory is a systematic error. Systematic errors are caused by mistakes by the person taking the measurement or apparatus that is faulty or has not been calibrated correctly. This includes the zero error explained in section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/) that reduces the accuracy of measurements.

Unlike random errors, systematic errors cause the measured value to be consistently higher or lower than the actual value, but not both. Also, systematic errors cannot be reduced by conducting repeat trials.

Examples of systematic errors include:

- not zeroing a mass balance before use (a zero error)
- heat loss during an experiment to measure an enthalpy change
- a leak in the apparatus used to measure a volume of gas.

Random and systematic errors can be identified by plotting a graph of results, as shown in **Figure 2**. Note that the line of best fit for the systematic error is consistently higher than the line of best fit for the accurate results. The line for the random errors has values both above and below.

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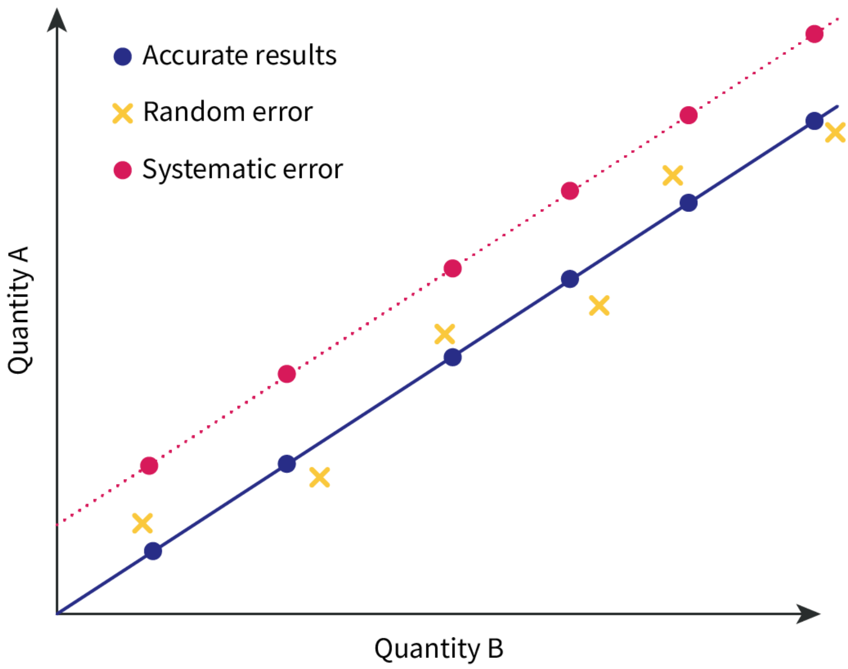

Figure 2. Different results for two proportional quantities.

More information for figure 2

An XY scatter graph with 3 data sets for the relationship between Quantity A and Quantity B. There are 6 data points from a set of accurate results. These sit exactly on a blue line of best fit. There are 6 data points from a set of measurements that contain a random error. These are distributed above and below the blue line of best fit. Another 6 data points come from a set of measurements with a systematic error. These sit on a dashed red line of best fit sits above and runs parallel to the blue line of best fit. Both lines of best fit show a strong positive linear relationship between Quantity A and Quantity B.

When comparing the two types of uncertainties, random ones are typically more difficult to completely eliminate but can be minimised due to their nature, and systematic ones can more easily be eliminated once the source is identified. However, finding the source is usually a difficult task.

Exercise

Click a question to answer

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Tool in action

To read more about the skills in this section and see examples of them being used in physics , take a look at the following:

- Section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/) Measuring variables with analogue equipment.
- Section 1.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-digital-equipment-id-48814/) Measuring variables with digital equipment.
- Sections 1.4.1—1.4.10 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Collected practicals.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Propagation of uncertainties

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/propagation-of-uncertainties-id-49161/print/) Assign

There are three ways of expressing uncertainty in measurement, no matter if this is random or systematic. You can express it as absolute, fractional or percentage uncertainty.

Absolute uncertainties

Any measurement taken in the laboratory has an uncertainty associated with it which is known as the absolute uncertainty. The magnitude of the absolute uncertainty depends on the precision of the apparatus and the type of apparatus. Absolute uncertainties are expressed with a ± sign and are written after the measured value:

$$\text{Measured value} \pm \text{absolute uncertainty}$$

For example, if the temperature in a room is 20 °C and the uncertainty is 1 °C then you can say that the temperature in this room is 20 ± 1 °C.

When taking a measurement from analogue apparatus, the absolute uncertainty is recorded as half the smallest scale division (see section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/)). For digital apparatus, the uncertainty is recorded as ± 1 the smallest scale (see section 1.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-digital-equipment-id-48814/)).

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Fractional and percentage uncertainties

Fractional and percentage uncertainty expresses the uncertainty either as a fraction or percentage of the measurement itself. To do so, divide the absolute uncertainty with the value of the measurement to take the fraction or also multiply by 100 to get the percentage.

In the same example as previously, you can find the percentage uncertainty as follows:

Measured value: 20

Absolute uncertainty: ± 1

Fractional uncertainty: $\frac{1}{20} = 0.05$

Percentage uncertainty: $0.05 \times 100 = 5\%$

You can write the temperature in the room is $20 \pm 5\%$.

Both forms of expressing uncertainty can be useful depending on the context. Absolute uncertainty gives you direct information on the fluctuation of the value. Percentage uncertainty can give you a better understanding of the importance of the uncertainty with respect to the measured value.

Uncertainty propagation refers to the calculation of the cumulative effect created by the uncertainty of each measured value in a calculation.

In other words, we try to take into account all the individual uncertainties for each measured value and combine them to find the final uncertainty on our calculated result.

To determine the number of significant figures in the final percentage uncertainty, the following protocol is applied:

- If the percentage uncertainty is less than 2%, it is expressed to not more than two significant figures.
- If the percentage uncertainty is equal to or greater than 2%, it is expressed to one significant figure.

In this section, we will look at the steps involved in this process, along with examples.

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Propagating uncertainties when adding or subtracting

When you need to add or subtract quantities that contain uncertainties, then you have to add their absolute uncertainties. The result will be an absolute uncertainty of the sum or the difference.

Worked example 1

Determine the change in mass together with the uncertainty for the data in **Table 1**.

Table 1. Data table.

	Mass of X (± 0.001 g)
Initial mass	35.387
Final mass	29.204

First subtract the final mass from the initial mass to determine the change in mass:

$$35.387 \text{ g} - 29.204 \text{ g} = 6.183 \text{ g}$$

Next, add the uncertainties and write the change in mass together with the uncertainty:

$$6.183 \pm 0.002 \text{ g}$$

Note that the uncertainty is expressed to the same number of decimal points as the measured value, which in this example, is three decimal places.

Propagating uncertainties when multiplying or dividing

When you need to multiply or divide quantities that contain uncertainties, then you have to add their fractional/percentage uncertainties. The result will be the fractional/percentage uncertainty of the product or the ratio.

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Worked example 2

We measured the sides w and h of a (rectangular) basketball court to be

$$w = 28.0 \pm 0.6 \text{ m and } h = 15.0 \pm 0.4 \text{ m respectively.}$$

What is the uncertainty in the calculated area of the court?

The area A is found by multiplying the two sides:

$$\begin{aligned} A &= wh \\ &= 28 \times 15 = 420 \text{ m}^2 \end{aligned}$$

For the propagation of uncertainty, we first find the percentage uncertainty for w and h.

$$\begin{aligned} \text{For } w\text{, percentage uncertainty} &= \frac{0.6}{28} \\ &= 0.0214 \text{ or } 2.14\% \end{aligned}$$

$$\begin{aligned} \text{For } h\text{, percentage uncertainty} &= \frac{0.4}{15} \\ &= 0.0267 \text{ or } 2.67\% \end{aligned}$$

The uncertainty in the calculated area will be:

$$\text{percentage uncertainty} = 2.14 + 2.67 = 4.81\%$$

This is expressed as 5% (to one significant figure) because the percentage uncertainty is greater than 2%. The area with the percentage uncertainty is:

$$A = 420 \text{ m}^2 \pm 5\%$$

Error propagation for averaged values

It is a common practice when collecting data in the laboratory to carry out multiple trials called repeat trials (**Table 2**). The data for each trial is then added together and divided by the number of trials to determine the mean average (see section 1.3.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/measures-of-central-tendency-id-48951/)). Here, we will look at how to express the uncertainty for the averaged value.

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Table 2. Data for the temperature change of a solution from repeat trials.

Trial	Temperature change, $\frac{\Delta T}{\pm 1.0\ ^\circ\mathrm{C}}$
1	5.3
2	5.4
3	5.2

The mean (average) temperature change , ΔT, can be calculated by adding up the values and dividing by three:

$$\begin{aligned}\text{Mean temperature change} &= \frac{(5.3+5.4+5.2)}{3}\\ &= 5.3\ ^\circ\mathrm{C}\end{aligned}$$

The uncertainty for the mean temperature change can be taken as being the same as the uncertainty for the individual values, which is $\pm 1.0\ ^\circ\mathrm{C}$.

Therefore, the average temperature change, together with the uncertainty, is expressed as:

$$5.3 \pm 1.0\ ^\circ\mathrm{C}$$

In this example, the measured values had good agreement so the uncertainty of $\pm 1.0\ ^\circ\mathrm{C}$ was reasonable. However, if there is less agreement between the measured values, then the range should be considered instead. **Table 3** shows an example where the range of data is larger.

Table 3. Data for the temperature change of a solution from repeat trials.

Trial	Temperature change, $\frac{\Delta T}{\pm 1.0\ ^\circ\mathrm{C}}$
1	8.9
2	13.1
3	10.2

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In this case, the largest value is $13.1\ ^\circ\mathrm{C}$ and the smallest is $8.9\ ^\circ\mathrm{C}$, therefore the range is:

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$$\begin{aligned} \text{range} &= 13.1 - 8.9 \\ &= 4.2\ ^\circ\text{C} \end{aligned}$$

The uncertainty for repeat trials with a large range is found by dividing this range in two:

$$\begin{aligned} \text{uncertainty} &= \frac{4.2}{2} \\ &= 2.1\ ^\circ\text{C} \end{aligned}$$

If required, the uncertainty should be rounded to one decimal place to match the precision of the measured values.

Finally, we calculate the mean temperature change and apply the uncertainty to the final result:

$$\begin{aligned} \text{Mean temperature change} &= \frac{(8.9 + 13.1 + 10.2)}{3} \\ &= 10.7 \pm 2.1\ ^\circ\text{C} \end{aligned}$$

Worked example 3

A student collects the data in **Table 4** when recording the mass change of a substance. Determine the average value together with its uncertainty.

Table 4. Data for mass change from repeat trials.

Trial	$\frac{\text{Mass change}}{\pm 0.01\ \text{g}}$
1	8.65
2	8.63
3	8.64
4	8.66

First, calculate the mean average change in mass.

$$\begin{aligned} \text{Mean change in mass} &= \frac{(8.65\ \text{g} + 8.63\ \text{g} + 8.64\ \text{g} + 8.66\ \text{g})}{4} \\ &= 8.645\ \text{g} \\ &= 8.65\ \text{g (3 significant figures)} \end{aligned}$$

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The uncertainty of the averaged value can be taken as being the same as the individual measurements since there is good agreement between the measurements. The average mass, together with the uncertainty, is expressed as:

$$8.65 \pm 0.01 \text{ g}$$

Higher level (HL)

Error propagation for exponents

In this section, we will look at how to express the uncertainties when handling values with exponents, such as:

$$x = a^{\mathrm{b}}$$

where x is the result and a is the measured value.

When propagating the uncertainties of values with exponents, you need to take the fractional/percentage uncertainty and multiply by the exponent. Please note that the square root can be also expressed an exponent $\sqrt{x} = x^{\frac{1}{2}}$.

Worked example 4

We measured the sides w, l and h of an ice cube to be:

$$w = h = l = 3.2 \pm 0.1 \text{ cm}.$$

What is the uncertainty in the calculated volume of the ice cube?

The volume of a cube is

$$V = w^3 = 32.768 \text{ cm}^3$$

For the propagation of uncertainty, we first find the percentage uncertainty for one dimension:

$$\begin{aligned} \text{percentage uncertainty} &= \frac{0.1}{3.2} \\ &= 0.031 \text{ or } 3.1\% \end{aligned}$$

Since the exponent in the formula is 3 then:

$$\begin{aligned} \text{propagated uncertainty} &= 3.1 \times 3 \\ &= 9.3\% \end{aligned}$$

This is expressed as 9% (to one significant figure) because the percentage uncertainty is greater than 2%. The volume of the cube with the percentage uncertainty is:

Student view

$$V = 33\ \text{cm}^3 \pm 9\%$$

Exercises

Click a question to answer

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-analogue-equipment-id-48813/) Measuring variables with analogue equipment.
- Section 1.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-variables-with-digital-equipment-id-48814/) Measuring variables with digital equipment.
- Section 1.3.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/measures-of-central-tendency-id-48951/) Measures of central tendency.
- Sections 1.4.1—1.4.10 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Collected practicals.

Plotting graphs

You need to be able to understand and analyse scientific data, however, it can be tricky to do this just using values in a data table. It is often easier to interpret your results if you present your data in a chart or graph.

You can create graphs digitally, using spreadsheet software such as Excel or Google Sheets. However, you still need to be able to create graphs yourself, as this software usually will not create the most appropriate graph without additional input.

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Sketch graphs

In **Figure 1**, you will notice that, although the axes of the line graphs are labelled, they are unscaled. These graphs are known as sketch graphs as they qualitatively describe how two variables relate, i.e. they describe trends.

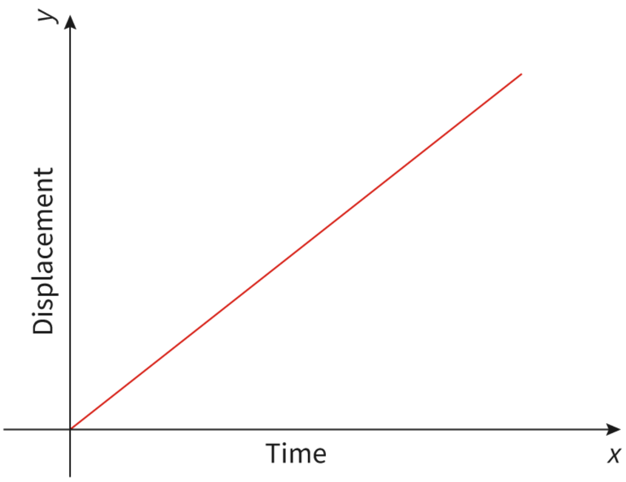

Figure 1. A displacement—time graph.

More information for figure 1

The image is an unscaled displacement-time graph. The X-axis represents Time, while the Y-axis represents Displacement. The graph shows a straight line that ascends diagonally from the origin, indicating a linear relationship between displacement and time. As time increases, displacement also increases at a consistent rate, illustrating a direct proportionality between the two variables without any intervals or specific data points marked on the axes.

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Trends in physics can be identified by plotting data and sometimes by sketching the relationship between variables (**Table 1**). In a laboratory experiment, you may be asked to carefully plot data and in the classroom you may be asked to sketch a relationship between variables.

Table 1. The difference between plots and sketches in physics.

Plot	Sketch
Data is recorded for two variables	Data is approximated for two variables with or without actual values
Data is plotted accurately	Data is plotted approximately, if at all

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Plot	Sketch
Allows for the determination of the quantitative relationship between the variables	Provides a graphical view of the pattern or trend to explain what is happening with the dataset without the need for careful plotting
Axes are labelled and scaled	Axes are labelled but unscaled

Construct and interpret tables, charts and graphs for raw and processed data

The purpose of any investigation is to collect data. Before you draw a graph, you need to create a data table, keeping in mind the following aspects.

- Raw data: enter data for the independent variable in the first column. Then, enter the values that you measure – your dependent variable – in the columns to the right of the first column.
- Processed data: enter your processed data (mean or standard deviation) in the columns to the far right.

When entering data, make sure you:

- include the units with the physical quantity in the column headers
- enter your data to the same number of decimal places
- record your raw data as per the resolution of the apparatus (see section 1.3.9a (/study/app/math-aa-hl/sid-423-cid-762593/book/recording-data-id-48956/)).

Types of graph

You can use many types of graph to represent data. How you choose your graph depends upon the type of data you are looking at. **Table 2** lists some of the main types of graphs you may use in DP physics. When you are analysing data it is up to you to decide which graph type will present your data the best.

Table 2. Main types of graphs and charts and what data types are most suitable.

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Graph type	Usage	Description	Examples
Bar charts	Used for comparison of categorical data.	Data is in the form of discontinuous rectangular bars representing different categories. Independent variable: Categorical data (for example, blood groups). Dependent variable: Numerical values often indicating total or mean values. Interpretation: The height of the bar may show the total value or the mean value of the dependent variable.	The energy density of different fuels (see **Figure 2**).
Histograms	Used to study the distribution of variables.	Data is organised into continuous rectangular bins (also called ranges or intervals), with no gaps between them. Independent variable: Numerical data grouped into bins of equal width. Dependent variable: Numerical values indicating the frequency. Interpretation: The width of the bin indicates the range or interval of values and the height of each bin shows the frequency or number of data points.	Plotting the change in the mean global temperature over time (see section B.2.2a (/study/app/math-aa-hl/sid-423-cid-762593/book/energy-balance-in-the-earth-surface-atmosphere-id-43773/)).

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Graph type	Usage	Description	Examples
Scatter graphs	Used to study the relationship between continuous variables.	Scatter graphs compare the changes in two continuous independent and dependent variables. Interpretation: The graph can indicate a positive correlation, negative correlation or no correlation between two variables based on the type of scatter plots. You can read more about this in section 1.3.11 (/study/app/math-aa-hl/sid-423-cid-762593/book/analysing-graphs-id-48958/).	Studying the emission of radiation at different temperatures (see section B.1.5 (/study/app/math-aa-hl/sid-423-cid-762593/book/black-body-emission-id-43785/)).
Line and curve graphs	Used to study a trend in the data.	The graphs are in the form of lines or curves. Both variables are continuous and have numerical values. Interpretation: Shows changes in data over a continuous range.	Studying the motion of objects (see section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/)).
Pie charts	Used to show discrete data where the proportion of each category to the whole matters.	Data is represented as sectors of a circle, where the circle represents the whole. Interpretation: The size of each sector shows its proportion to the whole.	Examining the composition of the Earth's atmosphere (see section B.2.2a (/study/app/math-aa-hl/sid-423-cid-762593/book/energy-balance-in-the-earth-surface-atmosphere-id-43773/)).

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Graph type	Usage	Description	Examples
Logarithmic graphs	Used to display data spanning large ranges.	A logarithmic scale is used in one or both the axes. The scale ensures that the high range of values of the experimental data is compressed. The values are first converted to a logarithm before being plotted on the graph. Interpretation: On a graph, the log scale is identifiable by non-linear values, say for example, values like 1,10, 100, 1000 and so on.	Studying the relationship between the luminosity of stars and their physical size (see section E.5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-hr-diagram-id-46456/)).

When you have selected your graph and built one with your data, it's important to ask yourself if it makes sense. If it doesn't make sense to you, it will not make sense to anybody else. This extra stage of reflection is often overlooked, but is a very useful process.

Plotting linear and non-linear graphs

You use graphs to show the relationship between variables.

Linear graphs are straight-line graphs, in other words, a change in one variable results in consistent changes in the other variable.

Non-linear graphs are curved-line graphs. Here, a change in one variable results in changes in the other variable which are not consistent.

Guidelines for plotting graphs:

1. Identify the independent variable (that you change) and the dependent variable (that you measure). Remember that typically the independent variable is plotted on the x-axis and the dependent variable on the y-axis.
2. Choose the scale carefully, to maximise the use of the grid provided. Use a scale that is easy to work with and helps you find intermediate values easily. For example, if using graph paper, it is easy to plot values like 0, 1, 2, 3 ... or 0, 5, 10, 15 ... on 2 cm squares.
3. Label the scales on the axes. The scale should have the same spacing throughout (linear) and be continuous (with no gaps). Show scale breaks (if used) clearly.
4. Label the origin using the starting values of each axis. Remember the axis numbering does not have to start with a zero.

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5. Label the axes and include the units of measurement.
6. Title or caption the graph.
7. Mark the data points using x or a ● (black dot) so that they are clearly visible.
8. Choose different colours or symbols if you are plotting more than one set of data.
9. Consider and use uncertainty bars (see section 1.3.11 (/study/app/math-aa-hl/sid-423-cid-762593/book/analysing-graphs-id-48958/)) if required.

Identifying trends

Once you have decided the type of graph that is appropriate for the data, we need to determine if there is a relationship between the independent and dependent variables. This can be done by applying a trend line.

Linear trends

The relationship between two variables can be described as linear when it has the equation $y = mx + c$. The linear relationship can either have a positive gradient (the blue line in **Figure 3**) or a negative gradient (the red line).

Non-linear trends

There are several types of non-linear relationships based upon different mathematical equations. Here are some common relationships you may encounter.

Figure 4 shows the trend line for an inverse relationship $y = \frac{k}{x}$, where k is a constant.

Figure 5 shows the trend line for squared relationships, either $y = kx^2$ or $y = \frac{k}{x^2}$, where k is a constant.

Figure 6 shows the trend line for square-root relationships, either $y = k\sqrt{x}$ or $y = \frac{k}{\sqrt{x}}$, where k is a constant.

Figure 7 shows the trend line for exponential relationships, either $y = e^x$ or $y = e^{-x}$.

Note: A very important distinction in science is that of the squared relationships, $y = kx^2$and $y = \frac{k}{x^2}$, and the exponential relationships, $y = e^x$ or $y = e^{-x}$. Students will often confuse the squared and exponential trend as they look very similar. Look closely at the differences between **Figures 5** and **7**.

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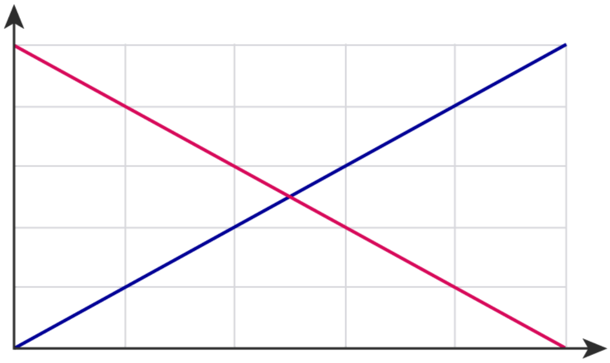

Figure 3. Linear trends for $y = mx + c$.

More information for figure 3

The graph presents two linear trends on a Cartesian plane. The X-axis represents an unspecified variable, ranging from -6 to 6. The Y-axis also represents an unspecified variable, ranging from -6 to 6. There's a blue line traversing diagonally from the bottom left corner to the top right corner indicating a positive gradient. Conversely, a red line intersects the blue line by running from the top left corner to the bottom right corner, showing a negative gradient. These lines meet at the origin (0,0). The grid is divided into equal square units, helping to visualize the slopes and intercepts of the lines.

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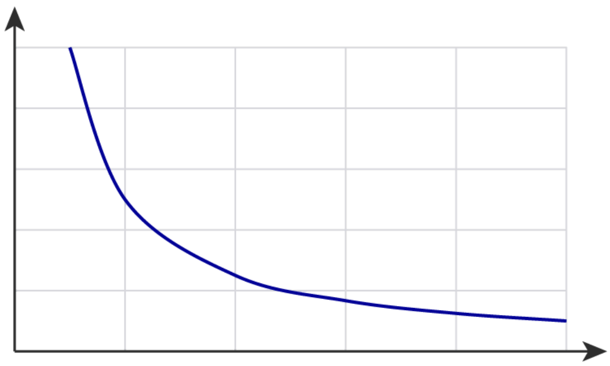

Figure 4. Non-linear trend: inverse relationship, $y = \dfrac{k}{x}$.

More information for figure 4

The image is a graph depicting a non-linear trend with an inverse relationship represented by the equation (y = \dfrac{k}{x}). The X-axis represents an independent variable and increases from left to right, but specific units or labels for the axis are not visible. The Y-axis, representing the dependent variable, increases from bottom to top with no specific scale or labels shown. A blue curve is plotted on this graph, which decreases in Y-value as the X-value increases, showcasing an inverse relationship characteristic. The curve is steep at the

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left, flattening out as it moves towards the right, indicating diminishing increases in Y as X becomes larger. This visualizes the concept that as one variable increases, the other decreases at a diminishing rate. The grid lines provide a sense of proportionality between the axes.

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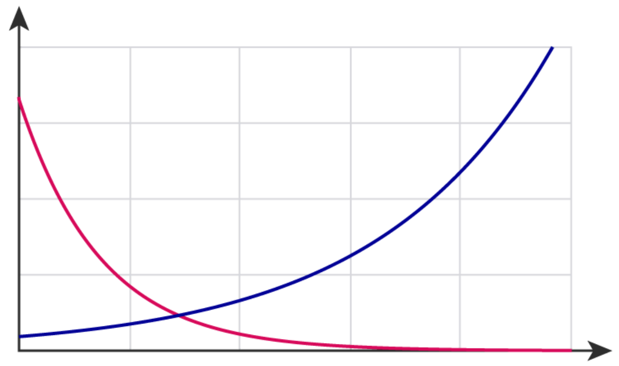

Figure 5. Non-linear trends: squared, $y = kx^2$ (blue), and inverse squared, $y = \frac{k}{x^2}$ (red).

More information for figure 5

The image is a graph that displays two non-linear curves on a Cartesian plane. The X-axis represents an unspecified variable and increases from left to right without labeled units. The Y-axis, similar to the X-axis, is not specifically labeled or numbered, but shows an increasing value upward.

Two distinct curves are plotted:

1. The blue curve represents a squared relationship, where the y-value rises at an increasing rate as the x-value increases. This curve starts at the origin and rises steeply upwards after a certain point.
2. The red curve illustrates an inverse squared relationship, where the y-value decreases at a decreasing rate as the x-value increases. It starts at a higher y-value, decreasing sharply before stabilizing and flattening out as it moves rightward.

The graph visually demonstrates the contrasting behaviors of these theoretical mathematical functions, depicting how one variable greatly impacts the rate of change in the other variable.

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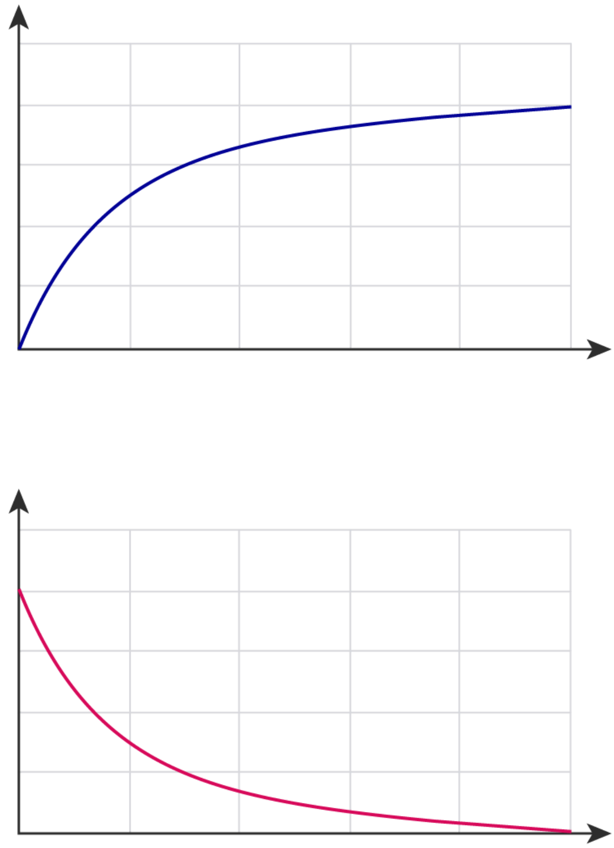

Figure 6. Non-linear trends: square root, $y = k\sqrt{x}$ (blue) and $y = \dfrac{k}{\sqrt{x}}$ (red).

More information for figure 6

The image consists of two graphs. The top graph illustrates a blue curve representing a square root function, where the y-value increases more slowly as the x-value increases. The X-axis is marked with representations starting from the origin moving right, suggesting the growth of the square root function. The Y-axis is marked at intervals, showing the gradual increase in value. The bottom graph shows a red curve depicting the inverse of a square root function, where the y-value decreases more slowly as the x-value increases. Similarly, the X-axis indicates the increase in x-values from left to right, and the Y-axis shows decreasing values as it moves downwards. Both graphs feature grid lines to facilitate measuring changes on both the X and Y axes, enhancing the understanding of the trends shown by the functions.

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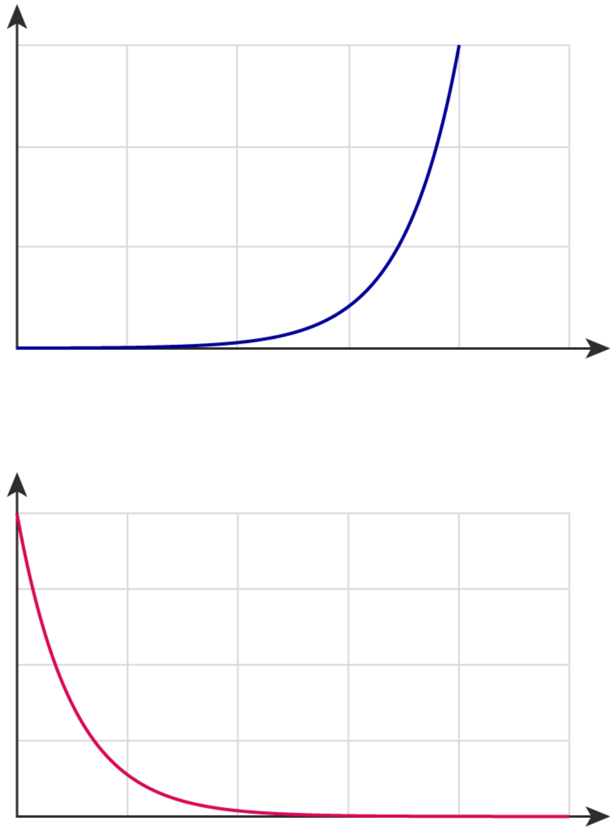

Figure 7. Non-linear trends: exponential, $y = e^x$ (blue) and $y = e^{-x}$ (red).

More information for figure 7

2 sketch graphs. A blue curve whose y-value rises slowly at first then at an increasing rate as its x-value increases and a red curve whose y-value declines rapidly at first and then at a decreasing rate as its x-value increases.

Logarithmic graphs

The logarithmic plot is different as unlike the linear axis of the other plots, one or both of its axes will be logarithmic. In **Figure 8** the x-axis has a logarithmic scale.

Student view

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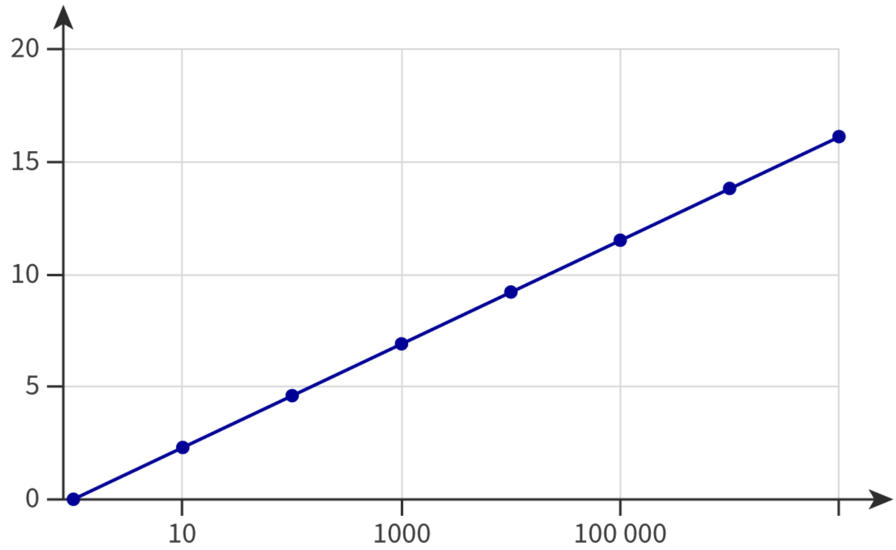

Figure 8. The logarithmic plot allows a large range of x-values on one graph.

More information for figure 8

The image depicts a line graph where the x-axis is on a logarithmic scale, ranging from 1 to 100,000, and the y-axis has a linear scale ranging from 0 to 20. There is a single line plotted, showing eight data points forming a straight line with a positive gradient. These data points are placed approximately at x-values of 1, 10, 100, 1,000, 10,000, and 100,000, demonstrating a consistent positive trend. This representation is useful for displaying data across a large range of x-values, where smaller values do not get compressed tightly together.

[Generated by AI]

Logarithmic scales are useful for showing data with a huge range of values. An example of this might be plotting how much energy different stars emit. Some stars emit only a fraction of what our Sun emits, whereas some emit over a million times more. Plotting this using any scale other than logarithmic might lead to a difficult data analysis.

Tool in action

To read more about the skills in this section and see examples of them being used in chemistry, take a look at the following:

- Section 1.4.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Practical: Investigating the acceleration of free fall.

Student view

- Section 1.4.7 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-resistivity-of-a-conducting-wire-id-46511/) Practical: Investigating the resistivity of a conducting wire.
- Section 1.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/data-analysis-id-46745/) IA Guide: Data analysis.
- Section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/) Describing motion.
- Section B.2.2a (/study/app/math-aa-hl/sid-423-cid-762593/book/energy-balance-in-the-earth-surface-atmosphere-id-43773/) Energy balance in the Earth surface—atmosphere system.
- Section E.5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-hr-diagram-id-46456/) The HR diagram.

Analysing graphs

Drawing lines or curves of best fit

When we collect real-life data, the results often look quite unlike the perfectly smooth graphs we see in maths. Real measurements are imperfect, but a line of best fit helps us see the underlying relationship between the plotted variables.

When a results graph shows a clear correlation, we can draw a line of best fit. **Interactive 1** shows data recorded by Edwin Hubble in 1929. He measured distances to galaxies beyond our own, and also their recessional velocities: the velocities at which they are moving away from us. He found that they are positively correlated and he drew a line of best fit. Move the slider to see the line of best fit.

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Interactive 1. Graph of velocity against distance for nearby galaxies.

More information for interactive 1

A scatterplot displays recessional velocity in kilometers per second on the vertical axis, ranging from 0 to 1000, and distance in megaparsecs on the horizontal axis, ranging from 0 to 2. Data points, shown as dots, are scattered across the graph, exhibiting an upward trend. A slider at the bottom controls the appearance of a best-fit line, which gradually becomes more visible and thicker as the slider moves right. The resulting best-fit line slopes upward, demonstrating a positive correlation between distance and recessional velocity, suggesting that as distance increases, recessional velocity also increases.

The approximate data for the plots are represented in a table below:

Distance	Recessional Velocity (km s superscript minus 1)
0	negative 10
0	50
0.27	80
0.3	negative 50
0.3	negative 20
0.3	0

Student view

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Distance	Recessional Velocity (km s superscript minus 1)
0.5	400
0.52	430
0.52	450
0.65	300
0.72	500
0.8	400
0.9	110
0.9	250
0.9	450
0.9	600
1	720
1.05	850
1.1	600
1.1	700
1.2	850
1.4	600
1.7	1100
2	520
2	800
2	850
2	1200

The best-fit line passes through (0.65,300), (0.8, 400), and (0.9, 450).

Student view

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You can use software to draw a line of best fit, but you should also learn to draw them 'by eye'.

When you draw a line of best fit:

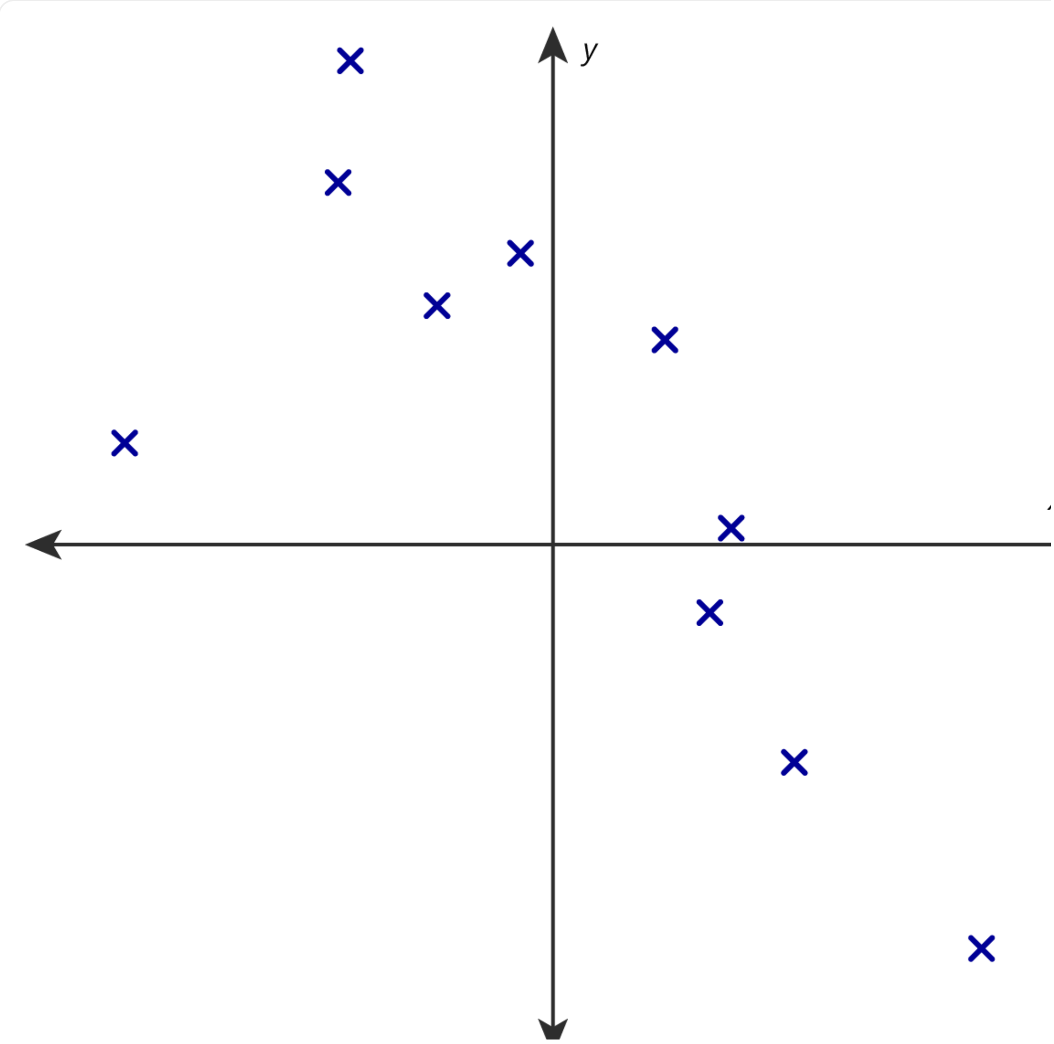
Don't: Draw a straight line if the points clearly follow a curve. A curve does not have to pass through every point, but it should be smooth.

Do: Ignore outliers when drawing the line. If most of the points lie close to a straight line or smooth curve but one or two points do not, these are outliers (probably caused by something going wrong during the measurement).

Don't: Draw the line in sections. It should be a single, thin line without any breaks in it.

Activity

Look at the scatter graphs in **Interactive 2**. In each case select the line of best fit you think best matches the data.



Interactive 2. For each row of data, choose the most appropriate line of best fit.

Student view

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Interpreting features of graphs

Scientists often draw graphs of experimental results. Why do they do this, and what can you learn from these graphs?

When we draw a line of best fit, it is a way of visually 'averaging out' the scatter in the data points. It gives a clearer picture of the relationship between the two variables.

We can describe this qualitatively. For example, 'p is directly proportional to q' or 'as x increases, y decreases at an increasing rate'. Often we can also extract quantitative information, which gives us more detail about the relationship. From a line of best fit, you can gain further information from its gradient and its intercepts. For a curve, the shape of the curve or how the gradient changes can be important.

Let's look at an example from the DP physics course: internal resistance of a battery. (Do not worry if you have not studied this yet; you will learn more about it when you work on section B.5.5a (/study/app/math-aa-hl/sid-423-cid-762593/book/cells-and-internal-resistance-id-44367/).)

When we analyse electrical circuits, we often assume that batteries have no electrical resistance. However, real batteries do have some resistance, called the internal resistance, r. This causes the electric potential difference V across the terminals of the battery to vary as the electric current I varies. The reading has its maximum value, called the electromotive force (emf) ε, when the current is zero.

The relationship between the quantities is:

$$V = -rI + \varepsilon$$

where r and ε are constants and V and I are the variables in the experiment.

Compare the above equation with the equation of a straight line:

$$y = mx + c$$

If we plot V on the y-axis and I on the x-axis, the graph will be a straight line with gradient $-r$ and y-intercept ε. **Figure 1** shows a set of example results.

Student view

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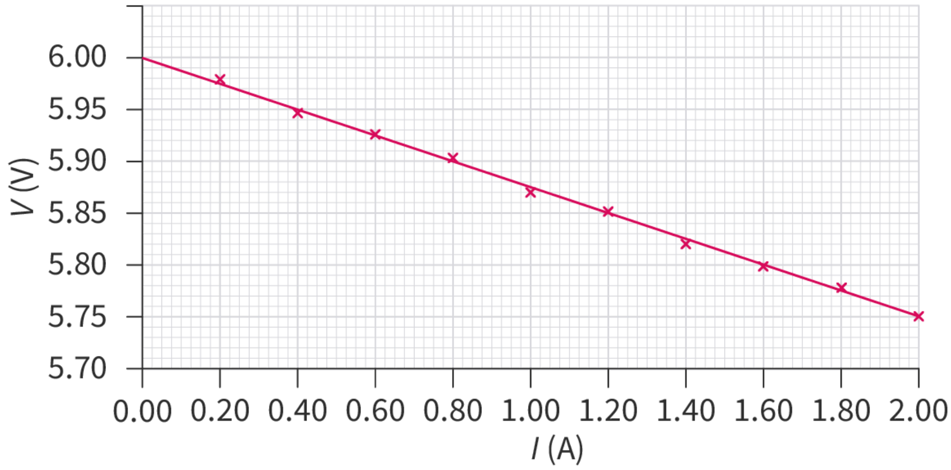

Figure 1. Graph of voltage against current for a battery.

More information for figure 1

The graph shows voltage (V) on the Y-axis and current (I) on the X-axis. The Y-axis is labeled "V (V)" representing voltage in volts, ranging from 5.70 to 6.00 volts. The X-axis is labeled "I (A)" representing current in amperes and ranges from 0.00 to 2.00 amperes. Data points are plotted with X symbols and a line of best fit is drawn through them, illustrating a downward linear trend. The line passes through the Y-axis at approximately 6.00 V, which is consistent with the described y-intercept and emf of the battery.

[Generated by AI]

The y-intercept is 6.00 V, so this is the emf ε of the battery.

The gradient is:

$$\begin{aligned}\frac{\text{change in } y}{\text{change in } x} &= \frac{5.75 - 6.00}{2.00 - 0.00} \\ &= -0.125 \\ &= -r\end{aligned}$$

The internal resistance is 0.125 Ω.

If the relationship between two variables is non-linear, it will not be possible to draw a straight line through the data points. However, we may be able to find other useful information from it.

Student
view

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Figure 2 shows measurements of a mass hanging from a spring, when the mass is oscillating (moving up and down repeatedly). The graph shows the displacement of the mass (its distance from its position when hanging still, using positive for above and negative for below) against time.

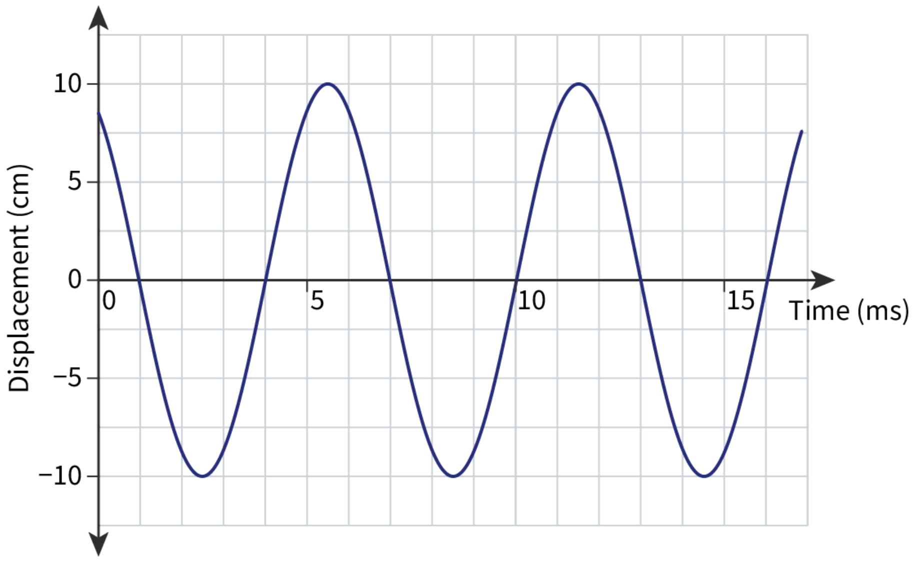

Figure 2. Graph of displacement against time for an oscillating mass on a spring.

More information for figure 2

The graph displays the relationship between displacement in centimeters and time in milliseconds for an oscillating mass on a spring. The X-axis represents time in milliseconds and ranges from 0 to 15 ms. The Y-axis indicates displacement in centimeters, ranging from -10 cm to 10 cm. The graph shows a sinusoidal curve, oscillating between -10 cm and 10 cm periodically. It crosses the Y-axis at zero displacement at intervals of approximately 5 ms each. This indicates a consistent periodic motion characteristic of a harmonic oscillator.

[Generated by AI]

From the graph in **Figure 2**, we can see that:

- the displacement changes more quickly (and so the velocity is greater) when the displacement is smaller
- the initial displacement is 8.0 cm
- the maximum displacement is 10.0 cm (in either direction).

Student view

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Extrapolating and interpolating graphs

The lowest possible temperature that can occur is called absolute zero. Even though you cannot get close to this temperature using school laboratory equipment, you can take measurements and use them to estimate the value of absolute zero. This is possible using extrapolation, which is the method used by the physicist Lord Kelvin in 1848 to find that absolute zero is –273.15 °C.

Extrapolating a graph means extending the line of best fit beyond the plotted points. It can be used to predict values of variables beyond the values measured. This is how absolute zero was determined.

Figure 3 shows the volume of a fixed mass of gas at different temperatures (and constant pressure). Absolute zero can be estimated by extrapolating the graph to the point where the volume of the gas is zero: it is about –273 °C.

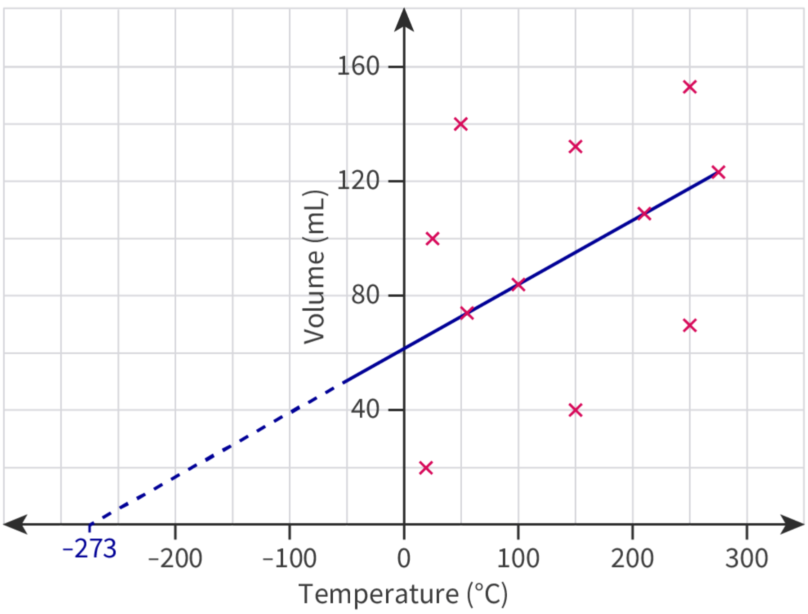

Figure 3. Volume vs. temperature for a fixed mass of gas, extrapolated to absolute zero.

More information for figure 3

The graph illustrates the relationship between volume and temperature for a fixed mass of gas, with the temperature on the X-axis ranging from -300°C to 300°C and the volume on the Y-axis ranging from 0 mL to 160 mL. Data points are scattered along the graph, and a line of best fit is drawn. This line extrapolates to a point of approximately -273°C on the temperature axis, which corresponds to the hypothetical condition where the gas volume would be zero. The overall trend shows that as temperature increases, the volume also increases consistently along the line of best fit.

Student
view

[Generated by AI]

You will not be expected to extrapolate from a curved line of best fit.

When we extrapolate an experimental graph, we are assuming that the relationship is the same for values beyond our range of measurements. This is not always true. For example, a graph of the length of a spring against the force stretching the spring is straight for smaller forces but becomes curved for larger forces.

Interpolation means using a line of best fit to find values between our measurements. We read these values from the line of best fit. To find the volume of the gas at 200 °C from the graph above, find the point on the line that has *x*-coordinate 200 and read its *y*-coordinate.

It is also possible to extrapolate or interpolate from an equation derived from a graph. The equation of the line of best fit in the above graph is:

$$y = 0.10x + 32$$

To predict the volume of the gas at temperature 400 °C, substitute this value of *x* into the equation:

$$\begin{aligned} y &= 0.10 \times 400 + 32 \\ &= 72 \text{ mL} \end{aligned}$$

Worked example 1

A student measures the average time for a leaf to fall from different heights. **Figure 4** shows the results.

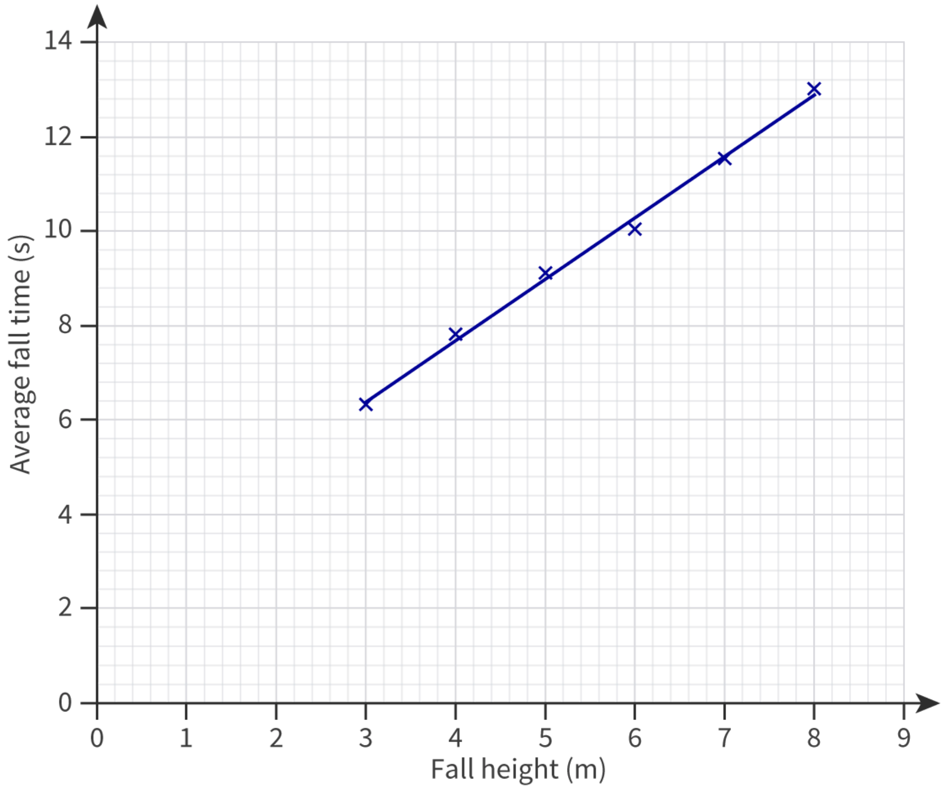

Figure 4. Graph of average fall time against height of fall for a leaf.

More information for figure 4

The graph illustrates the relationship between fall height and average fall time for a leaf. The X-axis represents fall height in meters, ranging from 0 to 9 meters. The Y-axis represents the average fall time in seconds, ranging from 0 to 14 seconds. Data points show average fall times at various heights, and a line of best fit is provided to indicate the trend.

At lower heights, such as around 2 meters, the fall time is observed to be roughly 6 seconds. As the height increases to about 8 meters, the fall time increases to approximately 13 seconds. This indicates a direct relationship, where the fall time increases with the height from which the leaf falls, indicating a positive correlation between these variables. The line of best fit running through these points confirms the steady increase in time with increasing height.

[Generated by AI]

Use **Figure 4** to answer the questions below.

1. Use interpolation to find the time taken for the leaf to fall 6.4 m.
2. Use interpolation to find the fall height for which the fall time is 12.4 s.
3. Use extrapolation to find the y-intercept of the graph. (You could hold a ruler up to the screen to do this. Why do you think the graph does not pass through (0, 0)?)

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4. Find an equation for the graph.
5. Use your equation to predict the time taken for the leaf to fall 20.0 m.

1. 10.8 s (since the graph passes through the point (6.4, 10.8))
2. 7.6 m (since the graph passes through the point (7.6, 12.4))
3. 2.5 s (either the relationship is not linear at small heights, or there is a systematic error in the measurements).
4. $y = 1.3x + 2.5$
5. When x = 20.0 m, y = 1.3 × 20.0 + 2.5 = 28.5 s

Linearise graphs

In physics, we often want to find the relationship between two variables. We have seen how to do this when the relationship is linear, but how can we find the mathematical relationship between two variables if it is non-linear and so the graph is curved?

Even when the relationship between two variables is not linear, it is still possible to draw a straight-line graph. The method depends on whether or not we know the form of the relationship. First, we will explore an example in which we do.

It is possible to determine g, the acceleration of free fall on Earth, by measuring the distance fallen by an object at different times since the start of its fall. If no significant drag acts on the object, the relationship between these quantities is $s = \frac{1}{2}gt^2$. (You will learn more about this relationship in section A.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/).)

A graph of s against t is not straight, because the equation does not have the form $y = mx + c$. **Figure 5(a)** shows this.

However, we will get a straight line if we plot s against t^2, because $s = \text{constant} \times t^2$ (which means that s is directly proportional to t^2). **Figure 5(b)** shows this.

Plotting a linear graph from a non-linear relationship is called linearising the graph. It is a technique that should only be applied where appropriate.

Student view

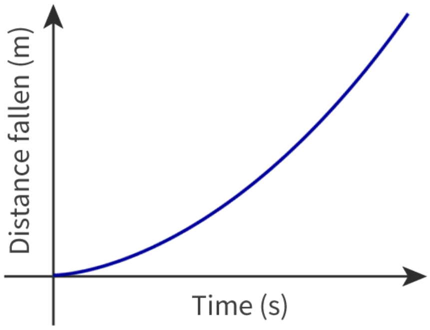

Figure 5(a). Graph for a falling object (without drag), showing the distance fallen against time.

More information for figure 5

The image is a graph depicting the distance fallen against time for an object in free fall without drag. The X-axis represents time in seconds and is labeled as "Time (s)," while the Y-axis represents distance fallen in meters and is labeled as "Distance fallen (m)." The curve on the graph is non-linear, indicating that the relationship between distance fallen and time is not proportional or linear. As time increases, the distance fallen also increases, at an increasing rate, suggesting a curved trajectory typical in free fall situations where drag is absent.

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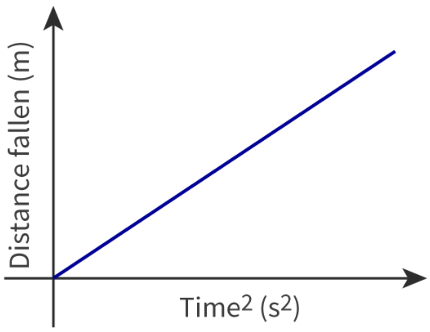

Figure 5(b). Graph for a falling object (without drag), showing the distance fallen against time squared.

More information for figure 5

This graph depicts the relationship between distance fallen and the square of time for a falling object without drag. The x-axis represents time squared (t^2), measured in seconds squared (s^2), with a linear scale. The y-axis represents the distance fallen, measured in meters (m). Both axes intersect at the origin of the graph.

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The graph shows a linear relationship, indicating that as the square of time increases, the distance fallen increases proportionally. This is represented by a straight line that starts at the origin and extends diagonally upwards to the right, suggesting a constant acceleration due to gravity without the effect of air resistance.

[Generated by AI]

Compare:

$$s = \frac{1}{2}gt^2 \text{ (where } s \text{ and } t^2 \text{ are the variables)}$$

$$y = mx + c \text{ (where } y \text{ and } x \text{ are the variables)}$$

The gradient is $\frac{1}{2}g$. To find g, we simply find the gradient and double it.

A different way to linearise this graph is to square root both sides.

$$s = \frac{1}{2}gt^2$$

$$\sqrt{s} = \sqrt{\left(\frac{1}{2}g\right)}t$$

If we plot $\sqrt{s}$ against t, the graph is a straight line with gradient $\sqrt{\left(\frac{1}{2}g\right)}$. Then $g = 2 \times (\text{gradient})^2$.

Sometimes you will not know the relationship between two variables. Imagine that you know (or suspect) that two variables p and q have a relationship of the form $p = kq^n$ where k and n are unknown constants. We may plot p against q and get a curve, but we cannot see by eye what mathematical relationship this shows.

In this situation, we can perform a 'log-log' analysis.

If we take the natural logarithm (ln) of both sides of $p = kq^n$, we get:

$$\ln p = \ln (kq^n)$$

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Using laws of logarithms (see section 1.3.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/mathematical-approaches-to-processing-scientific-data-id-48947/)), we can rewrite this as:

$$\ln p = \ln k + \ln q^n$$

$$\ln p = \ln k + n \ln q$$

or

$$\ln p = n \ln q + \ln k$$

where, $\ln p$ and $\ln q$ are variables, and $\ln k$ and n are constants.

Compare this with the general equation of a linear graph:

$$y = mx + c$$

where y and x are variables and m and c are constants.

If we plot $\ln p$ on the y-axis and $\ln q$ on the x-axis, we get a straight line with gradient n and y-intercept $\ln k$ (so $k = e^{(y-\text{intercept})}$).

Note: it is also possible to use logarithms with base 10 instead of natural logarithms in this method.

Exercises

Click a question to answer

Drawing and interpreting uncertainty (error) bars

Uncertainty is an unavoidable feature of experimental science (see section 1.3.9b (/study/app/math-aa-hl/sid-423-cid-762593/book/recording-data-id-48956/)). Except when we are simply counting objects (such as the number of marbles in a bag), we can never measure a quantity exactly – so every measurement has an uncertainty. For example, a measurement of mass might be 5.2 ± 0.1 g. This means the true value is likely to be in the range 5.1 g to 5.3 g. How should we deal with this when representing data on a graph?

Student view

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On graphs, we can show measurement uncertainties using uncertainty (error) bars. For each plotted point, a horizontal bar represents the uncertainty in the x-value and a vertical bar represents the uncertainty in the y-value. **Figure 6** shows an example.

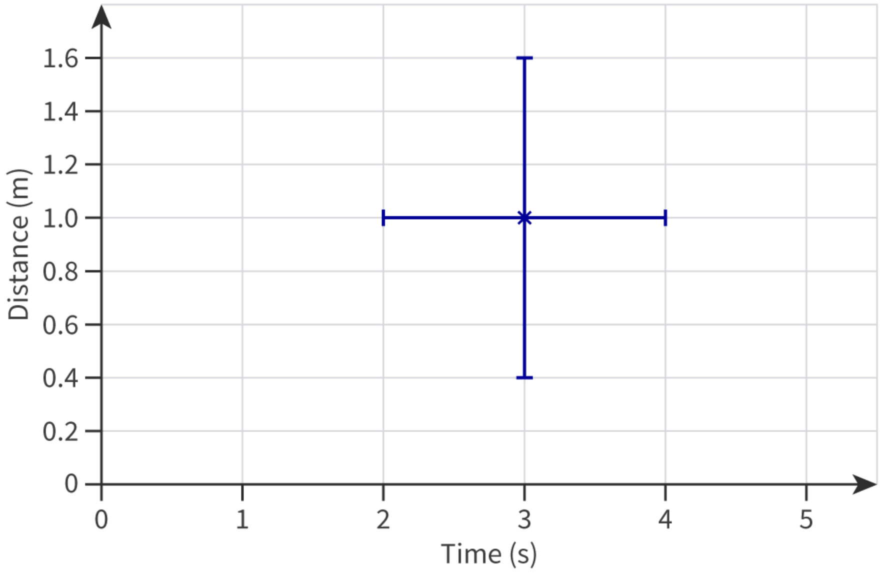

Figure 6. Plotted point with uncertainty bars.

More information for figure 6

The image is a graph displaying a single plotted point on a distance-time graph. The X-axis represents time in seconds (s), ranging from 0 to 5 seconds, with intervals of 1 second. The Y-axis represents distance in meters (m), ranging from 0 to 1.6 meters, with intervals of 0.2 meters.

At approximately 3 seconds on the X-axis and 1 meter on the Y-axis, there is a marked data point. This point is accompanied by uncertainty bars. The horizontal error bar reflects an uncertainty of ±1 second, extending from 2 seconds to 4 seconds. The vertical error bar reflects an uncertainty of ±0.6 meters, extending from 0.4 meters to 1.6 meters.

This image is an example of how measurement uncertainty can be visually represented in scientific data graphs, illustrating the range in which the actual measurement may lie.

[Generated by AI]

Student view

This shows a small part of a distance–time graph with a single plotted point. The distance measurement is 1.0 ± 0.6 m and the time measurement is 3 ± 1 s.

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When uncertainty bars are drawn on plotted points, we draw the line of best fit through all of them if possible.

Uncertainty bars may be different sizes on different points, if for some reason the uncertainties are not the same for each measurement.

Exercise

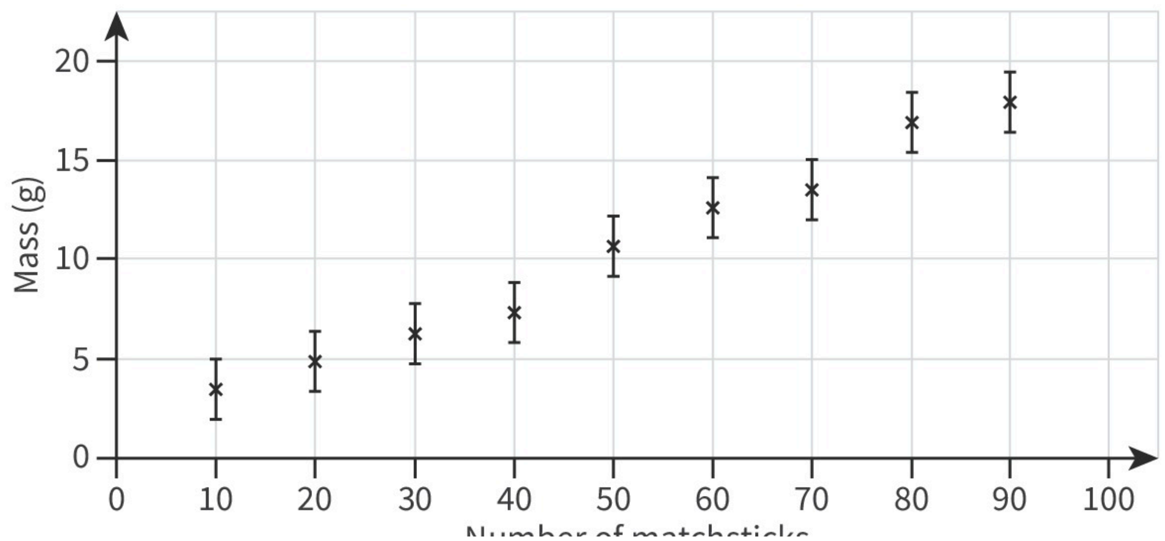
Click a question to answer

Using uncertainty bars to construct lines of maximum and minimum gradients

If the points on a graph have uncertainty bars, should we ignore them when drawing the line of best fit, or can we use them in some way?

When a graph has uncertainty bars, we can use these to draw 'extreme' lines of best fit. We draw two lines: the steepest and the shallowest lines that pass through all of the uncertainty bars. Use your eyes to judge where to place the line with maximum gradient (steepest) and minimum gradient (shallowest).

In **Interactive 3**, use the slider to explore an example.



Student view

Interactive 3. Mass vs. number of matchsticks, with uncertainty bars for mass.

More information for interactive 3

This is an interactive graph-based activity designed to help students understand the concept of uncertainty bars and how they can be used to determine the extreme lines of best fit; specifically, the lines with the maximum and minimum possible gradients.

The graph appears with several data points plotted on a Cartesian coordinate system. A slider-based interactive gallery allows users to navigate through images. The first image is a scatter plot with uncertainty bars, illustrating a dataset related to an experiment involving matchsticks. The x-axis represents the "Number of matchsticks," ranging from 0 to 100, while the y-axis shows "Mass (g)," ranging from 0 to 20 grams. Each data point, marked with a small cross, has vertical uncertainty bars extending above and below, indicating the range of possible mass values for each number of matchsticks. The points show a general trend: as the number of matchsticks increases, the mass tends to decrease, though the relationship is not perfectly linear. For example, at around 10 matchsticks, the mass is close to 5 grams, while at 90 matchsticks, it hovers around 15 grams, with some fluctuations. The uncertainty bars vary in length, suggesting differing levels of measurement precision across the data points.

The second image depicts a scatter plot with a line of best fit, showcasing a dataset involving matchsticks. A straight line labeled "Line of best fit" runs diagonally from the bottom left to the top right, passing through the center of most uncertainty bars, suggesting a linear relationship between the variables.

The third image is a scatter plot featuring a dataset related to matchsticks, with a blue line labeled "Line of maximum gradient" overlaid. The line of maximum gradient runs steeply through the upper edges of the uncertainty bars, suggesting the steepest possible linear fit that accounts for the data's variability.

The fourth image shows a scatter plot with a dataset involving matchsticks, featuring an orange line labeled "Line of minimum gradient." The line of minimum gradient slopes gently, passing through the lower edges of the uncertainty bars, representing the shallowest possible linear fit that accounts for the data's variability.

The fifth image is a scatter plot displaying a dataset related to matchsticks, with three lines overlaid: a black "Line of best fit," a blue "Line of maximum gradient," and an orange "Line of minimum gradient." The line of best fit runs centrally through the data, while the maximum gradient line slopes steeply along the upper edges of the uncertainty bars, and the minimum gradient line slopes gently along the lower edges, reflecting the range of possible linear relationships.

This interactivity provides a clear and engaging way for students to understand how experimental uncertainty influences data interpretation. By analyzing the maximum and minimum gradient lines, learners can see that a single line of best fit doesn't fully capture the possible relationships when measurement error is considered. This reinforces the importance of uncertainty in scientific data analysis and helps develop a deeper understanding of slope variability in trend lines.

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Determining the uncertainty in gradients and intercepts

Why do we draw lines of maximum and minimum gradient on a graph? How can we use these to interpret the data?

You have already seen that we can draw lines of maximum and minimum gradient on a graph. We take the mean of these gradients as the best value for the gradient.

The uncertainty in the gradient is half of the difference between the maximum and minimum gradients. (This is similar to finding the uncertainty in repeated measurements: the uncertainty is half of the range of the measurements.)

Figure 7 is an example graph, showing the line of best fit and the lines of maximum and minimum gradient.

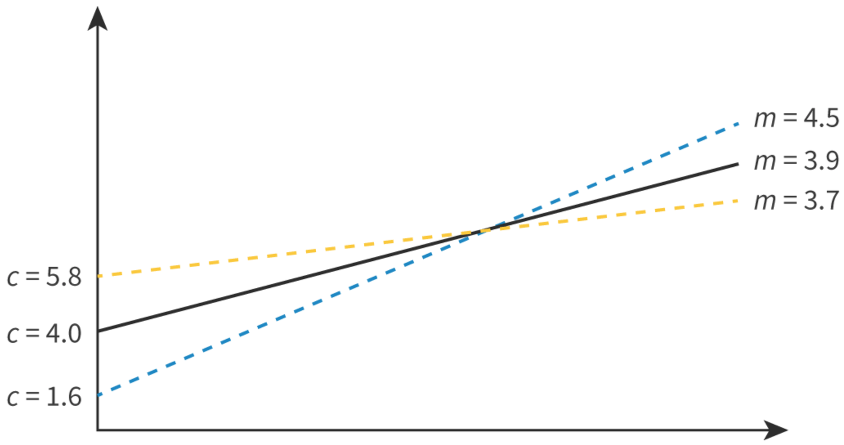

Figure 7. Line of best fit and lines of maximum and minimum gradient, showing y-intercepts and gradients.

More information for figure 7

The image is a graph showing three lines representing a line of best fit and lines of maximum and minimum gradients. The X-axis and Y-axis are present, but not explicitly labeled in the image. The main line is the line of best fit with a gradient (m) of 3.9 and a Y-intercept (c) of 4.0. A dashed line above it represents the line of maximum gradient with a gradient of 4.5 and a Y-intercept of 5.8. Another dashed line below shows the line of minimum gradient with a gradient of 3.7 and a Y-intercept of 1.6. The lines demonstrate the range in which data points might variate around the line of best fit.

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The gradient of the line of best fit is 3.9. The uncertainty in the gradient is:

$$\frac{\text{maximum gradient} - \text{minimum gradient}}{2} = \frac{4.5 - 3.7}{2}$$
$$= 0.4$$

Now, you can write the gradient as 3.9 ± 0.4.

(Note that the upper and lower limits, 3.9 + 0.4 = 4.3 and 3.9 − 0.4 = 3.5, do not have to match the maximum and minimum gradients found on the graph.)

We can calculate the percentage uncertainty in the gradient using the usual method for finding percentage uncertainty from absolute uncertainty (see section 1.3.9c (/study/app/math-aa-hl/sid-423-cid-762593/book/propagation-of-uncertainties-id-49161/)):

$$\begin{aligned}\text{percentage uncertainty} &= \frac{\text{raw uncertainty}}{\text{measured value}} \times 100\% \\ &= \frac{0.4}{3.9} \times 100\% \\ &= 10\%\end{aligned}$$

We can find the uncertainty in the y-intercept using a similar method. The y-intercept of the line of best fit is 4.0. The uncertainty in the y-intercept is:

$$\frac{\text{maximum gradient} - \text{minimum gradient}}{2} = \frac{5.8 - 1.6}{2}$$
$$= 2.1$$

Now, you can write the y-intercept as 4.0 ± 2.1.

Exercise

Click a question to answer

Activity

Use **Interactive 4** to practise drawing a best fit line, finding the equation of the line and finding the uncertainty in the gradient.

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1. Drag and drop the three lines onto the data.
2. Use the open circles to adjust the lines.
3. Move the solid circles to convenient points on the graph to measure the gradient and intercept.
4. Find the *y*-intercept and gradient of the line of best fit.
5. Use the lines of maximum and minimum gradient to find the uncertainties in the *y*-intercept and gradient.
6. Write an equation for the line of best fit.
7. When you have filled in all the values, a button will appear; you can use this to show the line of best fit and its equation.
8. Press the button at the bottom left to start again with a new set of plotted points.

Interactive 4. Practice Finding the Gradient of a Linear Graph, and its Uncertainty.

More information for interactive 4

This activity allows users to practice drawing a best-fit line and determining its equation, including the gradient and y-intercept. Users can manipulate three different fit lines, best fit, maximum fit, and minimum fit, by dragging and adjusting their angles to align with the given data points. By selecting convenient points on the graph, users calculate the gradient and y-intercept of the best-fit line and use the maximum and minimum fit lines to estimate the uncertainty in these values. Once all required values are entered, the activity provides the equation of the best-fit line and displays the uncertainty of the gradient and y-intercept.

The interactive is a cartesian plane with an x-axis and y-axis, both marked with grid lines. The plotted data points are represented by small black crosses, indicating measured values with possible uncertainties. Three fit lines named maximum fit, best fit, and minimum fit, expressed in different colors, are overlaid on the data to analyze trends and estimate uncertainties.

Each fit line has open circles that allow users to adjust their angles and solid circles that mark specific points for slope calculations. The best-fit line represents the most accurate trend through the data, while the maximum fit line and minimum fit line provide upper and lower bounds to determine uncertainty.

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Below the graph, there are text boxes where users can input numerical values for maximum fit slope, minimum fit slope, best-fit slope, uncertainty in slope, y-intercept, and uncertainty in y-intercept.

On the right, four instructions are provided which read, 1. Drag a fit line onto the graph by clicking/holding/dragging any part of the line using the mouse. 2. Change the angle of the line by clicking/holding/dragging on open circles. 3. Select line points by clicking/hold/drag on solid circles. 4. Enter your calculated values in the text boxes and press, Enter.

The maximum-fit line, minimum-fit line, and best-fit line can be drawn in the graph by adjusting the angles and dragging. The maximum fit slope can be calculated from the maximum fit line as:

$$\frac{\text{change in y}}{\text{change in x}} = \frac{29-2}{17-1} = 1.687$$

The best-fit slope can be calculated from the best-fit line as:

$$\frac{\text{change in y}}{\text{change in x}} = \frac{27.5-2{,}5}{17-1} = 1.562$$

The minimum fit slope can be calculated from the minimum fit line as:

$$\frac{\text{change in y}}{\text{change in x}} = \frac{25-3}{17-1} = 1.3125$$

The uncertainty of the slope can be found using the equation:

$$\frac{\text{maximum gradient} - \text{minimum gradient}}{2} = \frac{1.687-1.3125}{2} = 0.187$$

The y-intercept from the graph is $2.5\,\text{m}$ The equation of the best-fit line is in the form:

$y = mx + c$

Here, y is the displacement in the y-direction, m is the gradient of the line, x is the value of the x-coordinate and c is the y-intercept of the line.

The uncertainty in the y-intercept is:

$$\text{uncertainty in y intercept} = \frac{3-2}{2} = 0.5$$

After entering all the calculated values in the respective boxes press enter key, then a tab showing “show actual line of best fit” will be displayed. By clicking on this tab, the actual line of best fit (with black dotted lines) will display in the graph along with the actual equation of line, slope, and y-intercept.

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

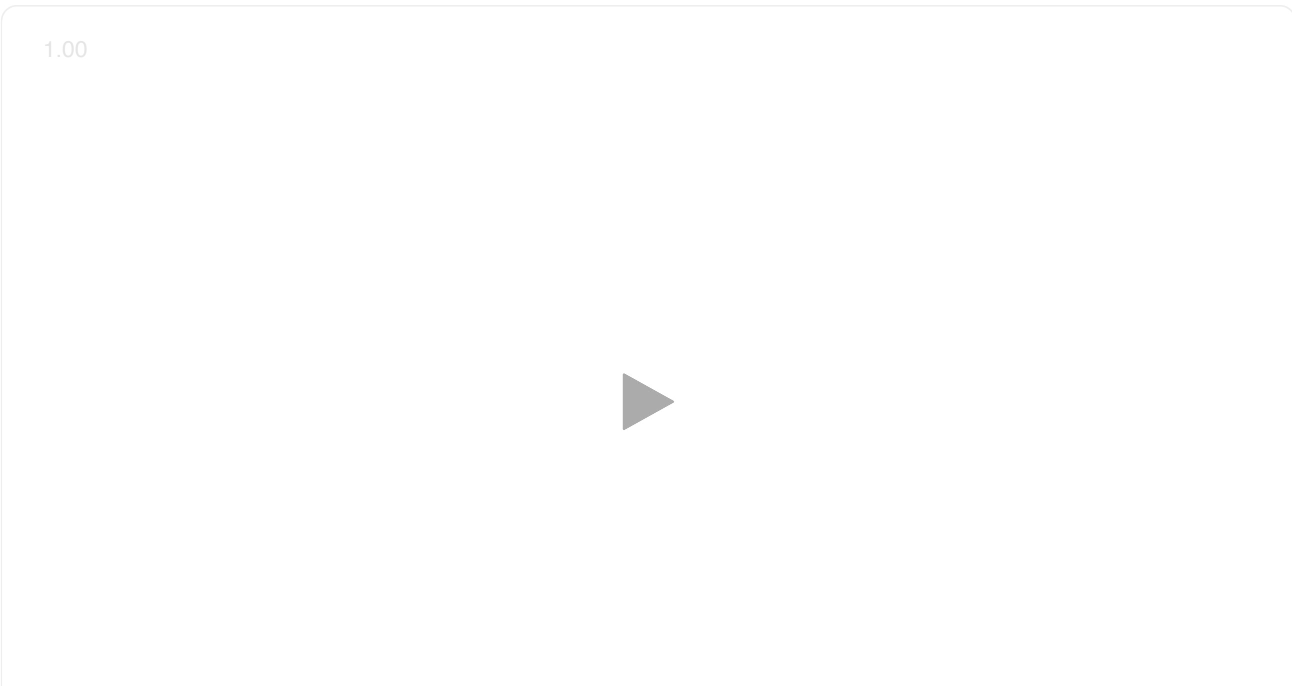
- Section 1.4.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/measuring-the-specific-latent-heat-of-vaporisation-of-water-id-46752/) Practical: Measuring the specific latent heat of vaporisation of water.

Student
view

- Section 1.4.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-an-ideal-gas-law-id-46508/) Practical: Investigating an ideal gas law.
- Section 1.4.6 (/study/app/math-aa-hl/sid-423-cid-762593/book/determining-refractive-index-id-46510/) Practical: Determining refractive index.
- Section 1.4.9 (/study/app/math-aa-hl/sid-423-cid-762593/book/determining-the-half-life-of-random-processes-id-46753/) Practical: Determining the half-life of random processes as a simulation of radioactive decay.
- Section 1.4.10 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-double-slit-and-double-id-46754/) Practical: Investigating double-slit and double-source wave interference (HL).
- Section 1.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/data-analysis-id-46745/) Data analysis.
- Section B.3.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/how-do-ideal-gases-behave-id-44292/) How do ideal gases behave?
- Section D.1.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/equipotential-surfaces-hl-id-46570/) Equipotential surfaces (HL).

Scale diagrams

Suppose you want to make a sketch of the Earth in your notebook. Obviously the Earth is too large to fit on a piece of paper. However, you can still draw a circle, label it ‘Earth’, and everyone will understand what you mean (**Interactive 1**).



Interactive 1. A sketch of the Earth.

More information for interactive 1

A progress bar shows a simple animation that presents a sketch of the Earth. The image consists of a large outlined circle with the word "Earth" centered inside it.

By doing so, you create a scale diagram. Here is a list of the most common reasons that scale diagrams are used in science.

Models of things that are too big or too small

In this case, we use scale diagrams to scale down or up objects so that we can study them. For example, a cell of an organism is too small to examine with our naked eyes, so we use microscopes to magnify it (**Figure 1**). Having that magnified image, we can take note of its features and study it.

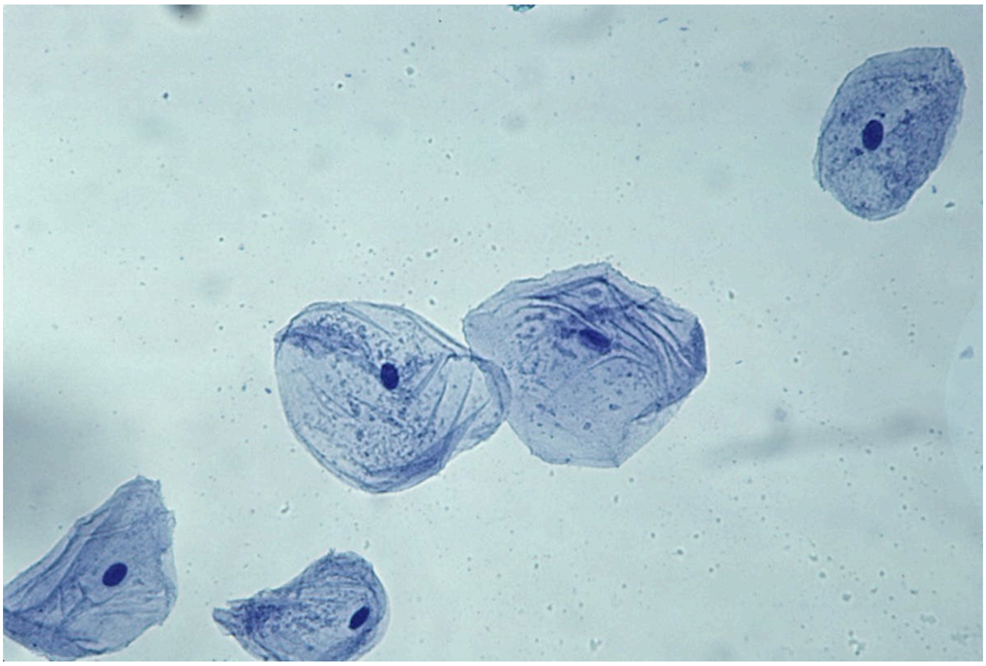


Figure 1. Cheek cells under a magnification of ×100.

Credit: Ed Reschke, Getty Images

At the other end of the scale of magnitudes, stars and galaxies are very large but they are enormous distances away so their apparent size to the unaided eye is too small to study. As with the previous example, we need to use technology to bring it to a more manageable size to study it. You can look at stars through a telescope and then do the same as before: change the scale and illustrate it in a notebook (**Figure 2**).

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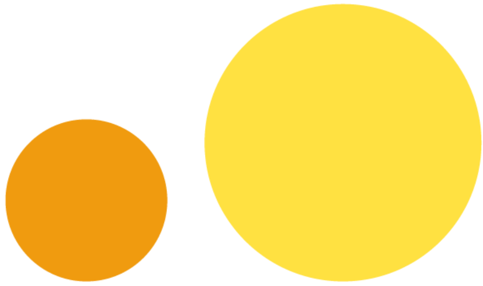

Figure 2. A scaled diagram of our Sun and Sirius A.

More information for figure 2

The image is a diagram depicting a size comparison between the Sun and Sirius A. It features two circles: an orange circle representing the Sun and a larger bright yellow circle representing Sirius A. Both circles are labeled accordingly. Below the Sirius A circle, there is additional text that states, "×1.73 diameter of the Sun," indicating that Sirius A's diameter is 1.73 times that of the Sun.

[Generated by AI]

The role of scaling

In both **Figures 1** and **2** the size of our diagram does not match the real size of the object. We have either reduced or enlarged the dimension of the object. However, all measurements on the diagram are scaled proportionally (see section 1.3.8 (/study/app/math-aa-hl/sid-423-cid-762593/book/proportionality-and-percentage-change-id-48955/)). This means that every part of the diagrams are consistently changed. A scale diagram normally includes a scale factor that has been used in the image. The scale factor is the relationship between the actual size of an object and the size used in the diagram – the constant of proportionality.

You can state a number or a ratio. In **Figure 3** the scale factor is 1 : 2000. This means that every 1 cm on the diagram corresponds to 2000 cm on the real object. This is true for any unit of measurement.

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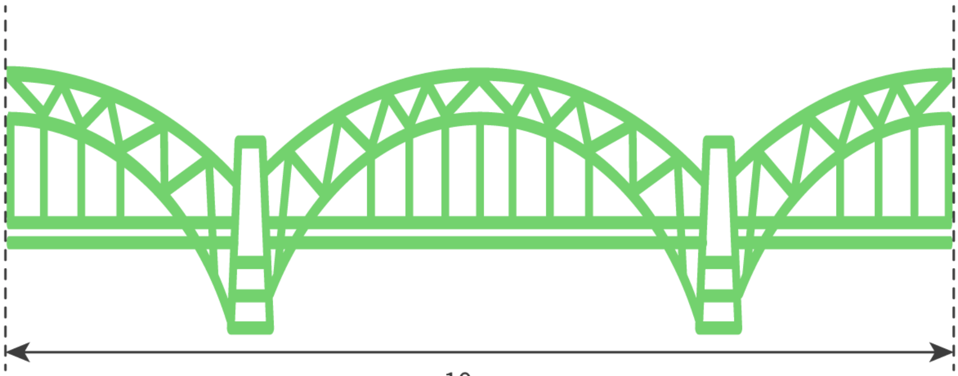

Figure 3. A scale diagram of a bridge with a scale factor of 1 : 2000.

More information for figure 3

The image depicts an illustration of a green bridge, represented within a scale diagram. The bridge is shown to measure 10 centimetres in length on the diagram. The structure features a series of arch patterns across its top, supported by vertical and diagonal struts forming a truss structure. Beneath the arches, there is a horizontal deck sectioning the bridge, complete with pillar-like supports extending to the bottom, suggesting foundational elements.

Text within the image indicates a scale factor of 1:2000, meaning that each centimetre on the diagram equates to 2000 centimetres on the actual constructed bridge. This scale helps to provide an understanding of the real-life dimensions by correlating diagrammatic measurements to potential real-world applications.

[Generated by AI]

Worked example 1

Figure 3 shows a scale diagram of a bridge. The scale factor is 1 : 2000. What is the real length of the bridge?

The bridge has a length of 10 cm on the scale diagram. Every 1 cm of the image corresponds to 2000 cm in real life.

You need to multiply the length of the bridge by 2000 to calculate the real-life length of the bridge.

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$$\begin{aligned}\text{Length of bridge} &= \text{length of bridge in diagram} \times \text{scale factor}\\ &= 10 \text{ cm} \times 2000\\ &= 20\,000 \text{ cm}\end{aligned}$$

To express the length in metres, divided by 100:

$$\frac{20\,000}{100} = 200 \text{ m}$$

An alternative way of displaying the scale factor is to indicate a specific distance on the scale diagram and state the distance in real-life. In **Figure 4** the real-life length of the string is 50 cm as noted, although if you measure it on the scale diagram, it is obviously drawn significantly shorter.

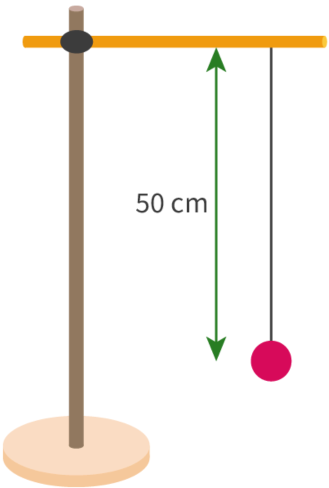

Figure 4. Adding a measured scale to a scaled diagram.

More information for figure 4

The image depicts a diagram of a simple pendulum. It consists of a vertical stand with a horizontal bar, and to the right of the bar hangs the pendulum. The pendulum is illustrated by a thin line, representing the string, and a small sphere at the bottom, representing the bob. Next to the pendulum is an arrow pointing from the bar to the bob, indicating the pendulum's length, which is labeled as 50 cm. This diagram shows the pendulum at rest in a vertical position, emphasizing the relationship between the stand and the pendulum's length, which corresponds to a real-life measurement of 50 cm. The arrangement clearly specifies the dimensions and components of the pendulum.

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Not to scale — the exception

Sometimes, you might be given a diagram which will explicitly mention that it is ‘not to scale’. This means that you cannot perform any measurements on it. However, you can get some numerical information from it and you can still use it to visualise the set up and find a way to work on it.

Worked example 2

Can you explain why the diagram in **Figure 5** is not to scale?

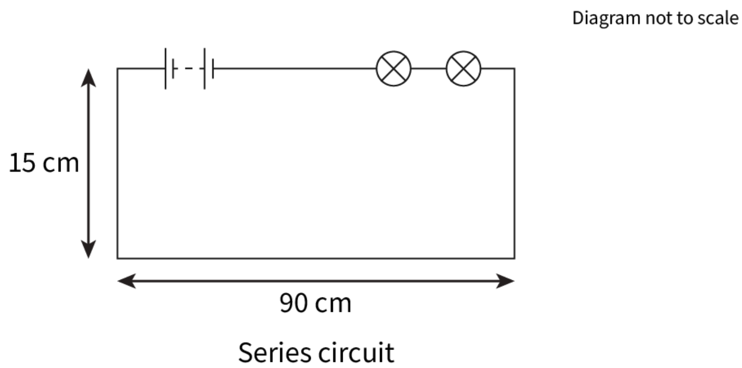

Figure 5. A circuit diagram not drawn to scale.

More information for figure 5

The image is a diagram of a series circuit labeled "Series circuit," which is marked as "not to scale."

The diagram consists of a simple rectangle representing the circuit's structure, with the longer side labeled as 90 cm and the shorter side labeled as 15 cm. Inside the rectangle, there is a battery symbol on the left side, depicted by two parallel lines of differing lengths, indicating the positive and negative terminals. Two circular symbols representing light bulbs are placed along the top horizontal line of the rectangle, connected in series with each other and with the battery. Arrows are pointing inwards to indicate the 15 cm measurement on the left side and the 90 cm measurement along the bottom line of the rectangle.

[Generated by AI]

Figure 5 is not to scale because, according to the given dimensions, the horizontal side should be drawn six times longer than the vertical side.

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Exercise

Click a question to answer

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section 1.4.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Practical: Investigating the acceleration of free fall.
- Section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/) Vectors.
- Section A.1.4a (/study/app/math-aa-hl/sid-423-cid-762593/book/projectile-motion-id-44300/) Projectile motion.
- Section A.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-forces-id-44732/) The forces.
- Section A.2.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/normal-force-friction-tension-id-44734/) Normal force, friction, tension.
- Section E.5.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/star-radius-id-46457/) Star radius.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Scalars and vectors

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/scalars-and-vectors-id-48960/print/) Assign

If you are talking about the time or temperature, you tend to say a number followed by some units. For instance, you may say:

‘*There are 5 minutes left before the end of the class*’ or ‘*The temperature in Stockholm today is 7 °C*’.

In these examples, the numbers 5 and 7 are the magnitudes, and minutes and degrees Celsius are the units. Both sentences have a clear meaning, and neither of them need direction.

Student view

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Physical quantities that can be described using a single number and a unit are called scalar quantities.

However, a number and a unit are not enough to describe some other physical quantities. A direction must be given with them. These are called vector quantities. For example:

'The car accelerated at 5 m s^{-2} to the south-west.'

5 is the magnitude, m s^{-2} is the unit and south-west is the direction.

Table 1 lists some examples of scalar and vector quantities. Section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/) gives a full explanation of what are vectors and scalars and how to identify if a physical quantity is a vector or a scalar.

Table 1. Scalar and vector quantities.

Scalar	Vector
time	velocity
temperature	acceleration
density	force
mass	weight
distance	displacement

How to draw a vector

A vector can be represented by a straight arrow. The length of the arrow represents the magnitude of the vector, and the direction of the vector is shown by the direction the arrow is pointing towards. **Figure 1** shows a force vector for a person pushing a box with force *F*. The arrow starts at the point where the force is being applied to the box. This is known as the point of application.

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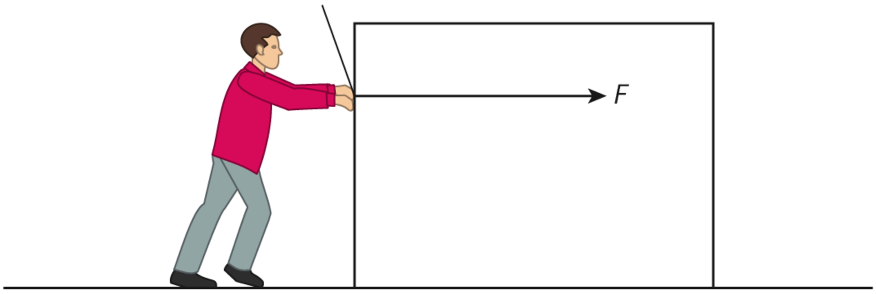

Figure 1. A force vector for a person pushing a box.

More information for figure 1

The image depicts a man pushing a large box across a flat surface. An arrow, representing a force vector labeled 'F,' extends horizontally from where the man's hands contact the box, indicating the direction and point of application of the force. This illustration is used to demonstrate the concept of vectors, where the arrow's length indicates force magnitude and direction denotes the vector's action.

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When drawing vectors, it is important to draw them as a scale diagram (see section 1.3.12 (/study/app/math-aa-hl/sid-423-cid-762593/book/scale-diagrams-id-48959/)) as the relative size determines the magnitude of the vector.

Free-body diagrams

Imagine there is an object at rest on a ramp (**Figure 2**). How many forces are acting on that box?

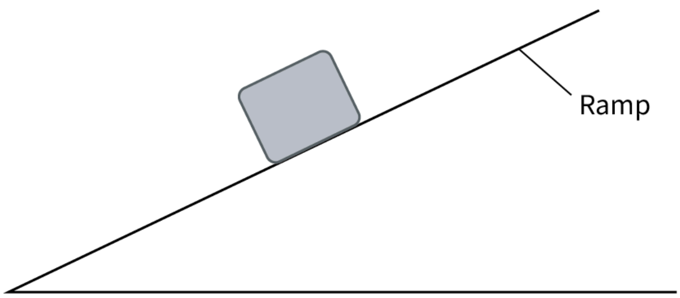

Figure 2. An object at rest on a ramp.

There are three forces on the object.

Student
view

1. The gravitational force or weight (W) is pulling the object downwards.
2. The friction force (F_f) between the object and the ramp is directed up the ramp.

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3. The normal reaction force (F_N) representing the contact force between the object and the ramp.

Section A.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-forces-id-44732/) contains more information about forces and section A.2.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/normal-force-friction-tension-id-44734/) contains information on normal forces and frictional forces.

You can represent all the forces acting on an object using a free-body diagram in which each force acting on the object is drawn as a vector line showing magnitude and direction with their points of application coming from the object. **Figure 3** shows the free-body diagram for **Figure 2.**

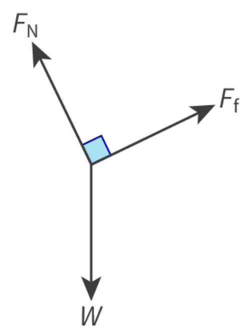

Figure 3. Free-body diagram for an object at rest on a ramp.

More information for figure 3

Diagram with three vector arrows coming out from a central point. One arrow represents weight and points vertically downwards. Another arrow represents the frictional force between the object and the ramp and points directly up the slope of the ramp. The final arrow represents the normal force between the object and the ramp and points perpendicularly upwards from the frictional force.

Adding and subtracting vectors

A body can experience more than one vector. When this happens, it is often important to calculate what is the overall effect of the vectors on the body to find the resultant vector.

For scalar quantities, basic mathematical rules can be used, such as sum or difference. For example, mass is a scalar quantity. Consider a bag containing 2 kg of apples. When you add 1 kg of apples to the bag, the total mass of the apples can be found by addition:

$$2 + 1 = 3 \text{ kg}$$

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Alternatively, if you remove 0.5 kg of apples from the bag, the remaining mass of the apples can be found by subtraction:

$$2 - 0.5 = 1.5 \text{ kg}$$

However, these simple rules are not enough for adding and subtracting two or more vectors. See section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/) for how to add or subtract vectors.

Worked example 1

Imagine a circular path with a radius of 100 m. The circumference of the path is:

$$\begin{aligned} C &= 2\pi r \\ &= 2\times\pi\times100 \\ &= 628 \text{ m} \end{aligned}$$

When you start to walk along the path and get to a distance of 314 m, you will be halfway around the circumference. Calculate your displacement at this point.

You have travelled a distance of 314 m. Distance is a scalar quantity. However, your displacement is not equal to 314 m because displacement is a vector quantity.

Displacement equals the length of the vector from the initial starting point to your final position. As you have walked halfway around the circumference, you will be on exactly the opposite side of the circle. Hence, your displacement will be the diameter of the circular path, which is:

$$\begin{aligned} \text{Displacement} &= 2 \times 100 \\ &= 200 \text{ m} \end{aligned}$$

Multiplying vectors by a scalar

A vector can be multiplied by a scalar using these rules:

- If the scalar is a positive number, the direction of the vector remains the same but the magnitude will change.
- If the scalar is a negative number, the direction of the vector will be reversed and the magnitude will change.

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For example, a table is pulled by a force of $F = 15$ N to $+x$ direction.

When it is multiplied by 2, the table is moved by $2F = 30$ N to $+x$ direction.

However, when it is multiplied by -3, the table is moved by $-3F = -45$ N to $-x$ direction.

Resolving a vector

A vector can be expressed by two other vectors named component vectors. These component vectors must be perpendicular to each other and when combined the addition of the two components is equivalent to the original vector. **Figure 4** shows how the original vector A can be resolved into two components that are perpendicular to each other, A_y and A_x.

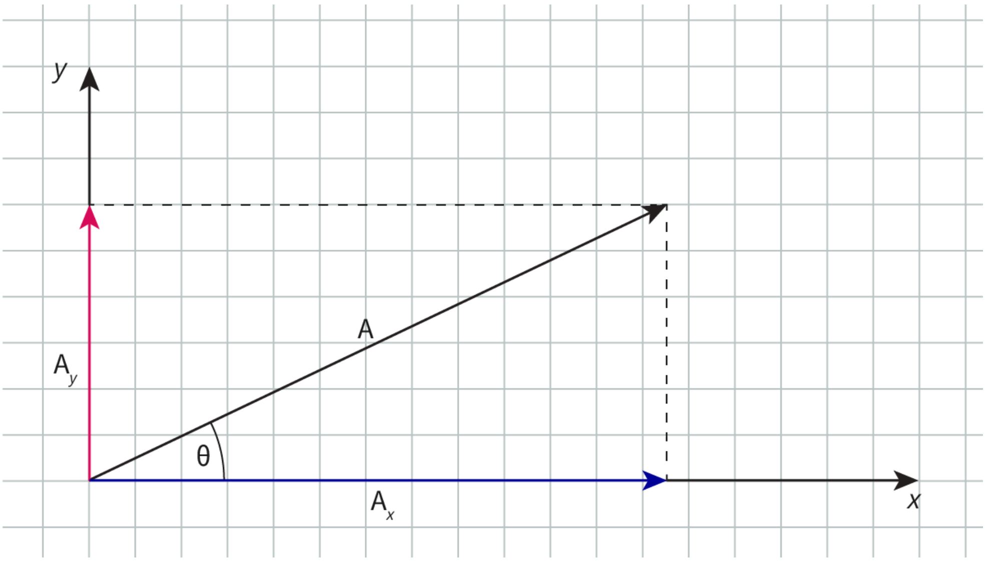

Figure 4. Resolving a vector.

More information for figure 4

Sketch on graph paper showing how a vector can be resolved into a horizontal component vector and a vertical component vector that lie perpendicular to each other and the original vector lies at an angle theta to the horizontal.

The magnitudes of A_y and A_x can be calculated using sine and cosine functions, and you can see those calculations in section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/).

Exercises

Click a question to answer

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Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

- Section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/) Vectors.
- Section A.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-forces-id-44732/) The forces.
- Section A.1.4a (/study/app/math-aa-hl/sid-423-cid-762593/book/projectile-motion-id-44300/) Projectile motion.
- Section A.2.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/normal-force-friction-tension-id-44734/) Normal force, friction, tension.
- Section C.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/) Simple harmonic motion (SHM).
- Section 1.4.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) Practical: Investigating the acceleration of free fall.
- Section 1.4.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-relationship-between-velocity-id-46751/) Practical: Investigating the relationship between velocity and the horizontal distance travelled by a projectile.

1. Essential skills and support guides / 1.3 Tool 3: Mathematics

Radians

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/radians-id-48961/print/) Assign

When we discuss curves, corners or inclinations, there are two different ways to quantify the magnitude of an angle: radians and degrees. When learning about angles for the first time, it is more common for us to learn about degrees. As a reminder, 360 degrees makes up the central angle of one whole circle, 180 degrees is the angle for half a circle, 90 degrees for a quarter, etc.

Degrees and radians are interchangeable, however by using radians more in specific situations we can make our calculations, understandings or communications better. For example:

- With radians you can use small angle approximations to simplify equations.

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- Degrees need converting to radians when used in trigonometric equations, potentially introducing systematic uncertainty. Radians do not need to be converted.

What is a radian?

The radian is defined using a circle. Consider a circle with a radius *r*. Take an arc on the circle with the same length as the radius *r*. Now you have an angle of one radian that intercepts this arc (**Figure 1**).

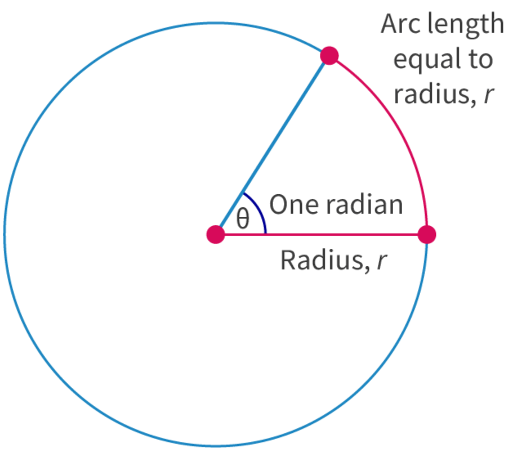

Figure 1. Defining the radian using a circle.

More information for figure 1

The diagram illustrates a circle with a radius marked from the center to the circumference. Two points are indicated on the circumference, forming an arc equal in length to the radius. The angle subtending this arc at the center of the circle is labeled as one radian (θ). Lines are drawn from the center to the two points on the circumference, creating an enclosed area with the labeled angle. The arc is highlighted, and the relevant text annotations in the image indicate the arc length as equal to the radius (r) and the angle as one radian.

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The arc of length r corresponds to an angle θ of one radian.

You can convert degrees to radians and vice versa if you remember that a whole circle is 360 degrees or 2π radians:

$$\frac{\theta_{\text{rad}}}{\theta_{\text{deg}}} = \frac{2\pi}{360}$$

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Overview
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An angle of 90° corresponds to $\frac{\pi}{2}$ radians and 180° corresponds to π radians.

When we present values for an angle in radians we often express them in multiples of π. This is because a radian is defined as a ratio of the arc length subtended by the angle to the radius of the circle.

Worked example 1

Convert the following values from radians to degrees, or vice versa.

1. 1.8 π radians
2. $\frac{\pi}{3}$ radians
3. 276 degrees
4. 225 degrees

The first two conversions of radians to degrees will use the equation:

$$\frac{\theta_{\text{rad}}}{\theta_{\text{deg}}} = \frac{2\pi}{360}$$

To get the subject as the angle in degrees, we invert the equations and rearrange:

$$\frac{\theta_{\text{rad}}}{\theta_{\text{deg}}} = \frac{360}{2\pi}$$

$$\theta_{\text{deg}} = \theta_{\text{rad}} \times \left(\frac{360}{2\pi}\right)$$

Thus

$$1.\ \theta_{\text{deg}} = 1.8 \times \pi \times \left(\frac{360}{2\pi}\right)$$
$$= 324^0$$

$$2.\ \theta_{\text{deg}} = \frac{\pi}{3} \times \left(\frac{360}{2\pi}\right)$$
$$= 60^0$$

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Note, here we have a common relationship, $\frac{\pi}{3} = 60^0$.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

For the conversions from degrees to radians we use the equation:

$$\frac{\theta_{\text{rad}}}{\theta_{\text{deg}}} = \frac{2\pi}{360}$$

To get the subject as the angle in radians, we just need to multiply by the angle in degrees:

$$\theta_{\text{rad}} = \theta_{\text{deg}} \times \left(\frac{2\pi}{360}\right)$$

Thus:

3. $$\begin{aligned}\theta_{\text{rad}} &= 276 \times \left(\frac{2\pi}{360}\right)\\ &= \frac{276 \times 2}{360}\pi\\ &= \frac{23}{15}\pi\end{aligned}$$

4. $$\begin{aligned}\theta_{\text{rad}} &= 225 \times \left(\frac{2\pi}{360}\right)\\ &= \frac{225 \times 2}{360}\pi\\ &= \frac{5}{4}\pi\end{aligned}$$

Note, we usually express radians in terms of π, so as to keep that relationship with the circle.

Exercise

Click a question to answer

Tool in action

To read more about the skills in this section and see examples of them being used in physics, take a look at the following:

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Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

- Section A.2.8 (/study/app/math-aa-hl/sid-423-cid-762593/book/circular-motion-id-44739/) Circular motion.
- Section A.4.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/uniformly-angularly-accelerated-motion-hl-id-43163/) Uniformly angularly accelerated motion (HL).
- Section A.4.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/angular-momentum-hl-id-43165/) Angular momentum (HL).
- Section C.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/) Simple harmonic motion (SHM).

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