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# The big picture



Think about an animated movie or a computer game. Why do the characters appear to be moving? A film sequence is made up of still pictures or frames. Each frame is slightly different from the previous one. Do they draw each and every one by hand? Well, in the early years of animated movies, they did.

Nowadays, not only cartoons but also many live action movies, use vector geometry. Scenes with real actors are shot in front of blue or black screens and the background is developed by computer animations using vectors. Vectors are used to represent forces, acceleration, velocity and momentum and enable the motion of an object to be predicted and described.

This video describes how hand drawings and vector animation can be combined.

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## Concept

Vectors help you to quantify positions and movements. They provide the tools for enhancing your spatial awareness in two and three dimensions. This topic provides you with the tools for analysis, measurement and transformation of quantities, movements and relationships.

## Theory of Knowledge

Knowledge and Technology: The comprehension of vectors plays a large role in the development and innovation in regard to mechanised motion. Mathematics, perhaps more than any other area of knowledge, has a very diverse application array. Given that mathematics is the foundation for so many other technologies and sets of knowledge, does this imply that mathematics is the 'best' area of knowledge?

Knowledge Question: Should society place a greater emphasis on some areas of knowledge over others?

3. Geometry and trigonometry / 3.10 Vectors

# The concept of a vector

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/the-concept-of-a-vector-id-28352/print/) Assign

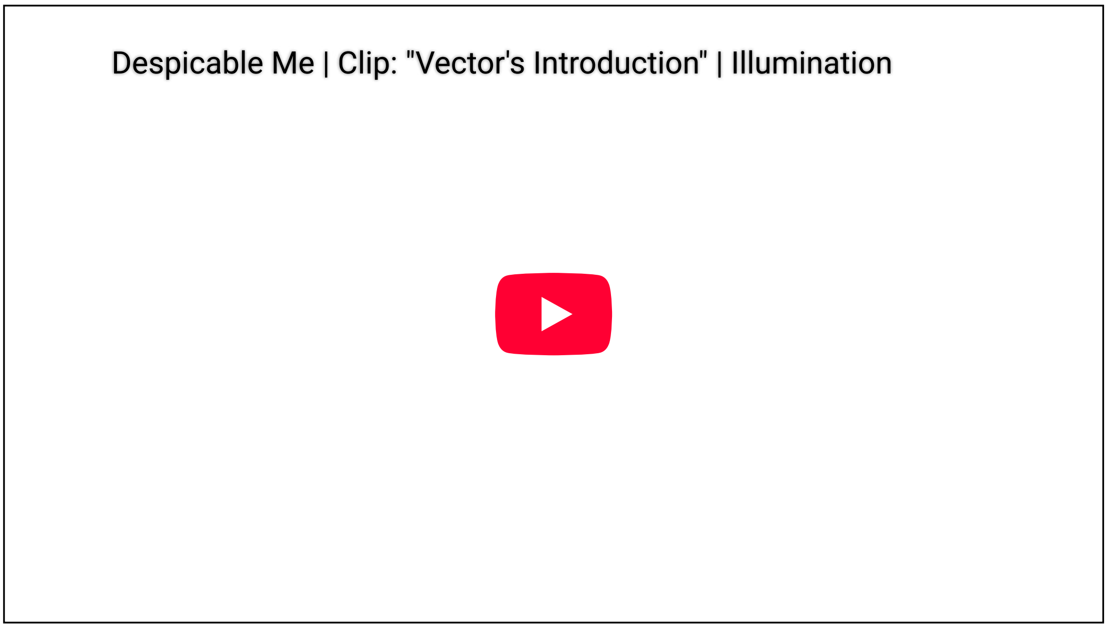
## Scalar and vector quantities

In daily life, you regularly use quantities such as temperature, measured in degrees Fahrenheit or Celsius, mass, in kilograms or pounds, and distances, in kilometres or miles. All these quantities are scalar quantities. They have magnitude but no direction. In contrast, other measurements such as velocities, displacements and forces need both a magnitude and a direction to describe them fully. There is a difference between walking $2$ km and walking $2$ km north. The first case represents a distance, which is a scalar

Student view

quantity because it only gives the magnitude (size) of how far you walked. The second case gives both a magnitude and a direction and is an example of a displacement. A quantity that has both direction and magnitude is called a vector quantity.

Take a look at the villain in the video below.




## Example 1

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Categorise each quantity as a scalar or vector

a) 3 km north east

b) 2000 calories

c) 80 km per hour

d) $-15\,^{\circ}\mathrm{C}$

e) Running towards the finishing line at 7 km per hour.

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| Part | Quantity | Type | Explanation |
|---|---|---|---|
| a) | $3\,\text{km}$ north east | vector | This has both magnitude and direction. |
| b) | $2000$ calories | scalar | This has magnitude only. |
| c) | $80\,\text{km}$ per hour | scalar | This has magnitude only. |
| d) | $-15°\text{C}$ | scalar | This has magnitude only. |
| e) | Running towards the finishing line at $7\,\text{km}$ per hour. | vector | This has both magnitude and direction. |

## Directed line segments, vectors and displacement vectors

As vectors have both magnitude and direction, both quantities need to be represented in the notation used. When you write a line segment you only need to use end points such as $\text{AB}$ or $\text{BA}$ as the length of both segments are equal. But when you represent a directed line segment it needs to have a direction, which means a starting point and an end point.

The usual notation for a **directed** line segment which starts at $\text{A}$ and ends at point $\text{B}$ is $\overrightarrow{\text{AB}}$. The arrow above the letters is used to distinguish it from the segment $\text{AB}$.

What would $\overrightarrow{\text{BA}}$ represent?

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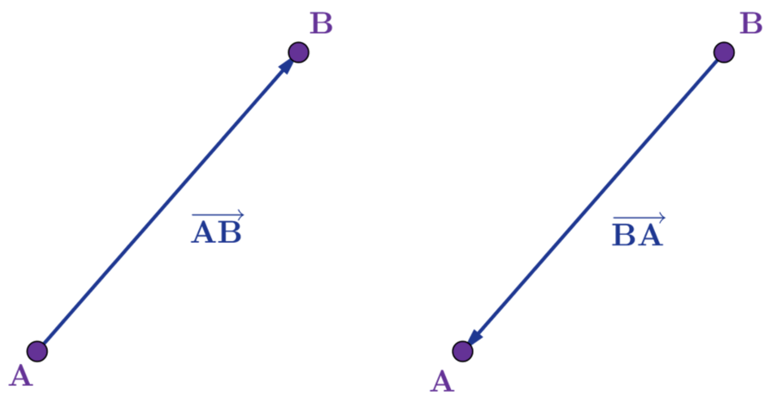


More information

The image shows two diagrams comparing two different vectors. The first diagram on the left displays a vector represented by an arrow from point A to point B, labeled as ( \overrightarrow{\text{AB}} ). The second diagram on the right shows a reversed vector represented by an arrow from point B to point A, labeled as ( \overrightarrow{\text{BA}} ). Each point is marked by a dot with corresponding labels, illustrating the directional difference between the two vectors.

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## Important

$\overrightarrow{\text{AB}} \neq \overrightarrow{\text{BA}}$ as the starting and end points are different.

## Making connections

You might need to revisit some of the definitions and notations of Euclidean geometry:

**Point**: This does not have any dimensions. It is used to represent a unique location in Euclidian space. It is usually represented by a capital letter and a dot, e.g. $\cdot\text{A}$ or by an ordered pair, e.g. $(x, y)$.

**Line**: A line is one dimensional and consists of infinitely many points. It is either represented by a lower case italic letter, e.g. $l$, a capital letter, $\text{L}$, or by two points, e.g. $\text{AB}$, that are on the line.


More information

The image is a diagram illustrating a straight line labeled as AB, denoting the passage through two points, A and B. Point A is marked on the left side, while point B is on the right. The line is depicted as an arrow extending from point A towards point B, emphasizing the directionality and representation through these two points. The diagram visually supports the concept that there exists one and only one unique straight line passing through any two distinct points.

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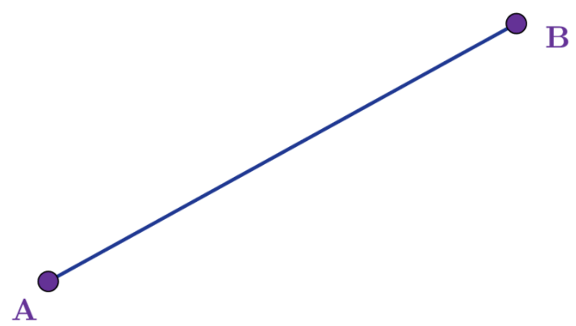
There is one and only one straight line passing through two points.

**Line segment:** This is any straight segment joining two points.




**Plane:** This is a flat two dimensional surface.

Three non-collinearpoints or a point and a line determine a plane.

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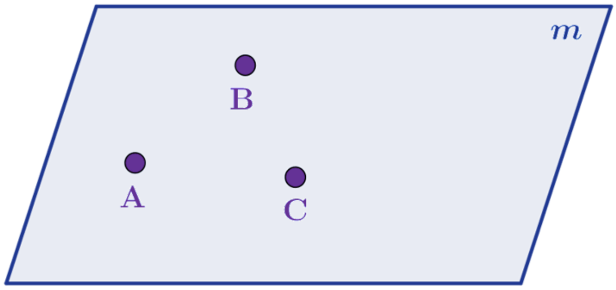


More information

The diagram illustrates a plane denoted by 'M', which is a blue parallelogram shape. Three points labeled A, B, and C are marked on this plane. Point A is located towards the lower left region, point B is centrally positioned, and point C is on the lower right side. These are depicted as small circles with labels beside them, demonstrating how these three non-collinear points lie on the same plane, thereby determining it.

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Two directed line segments are congruent if they have the same direction and magnitude. In the diagram below, all the directed line segments are congruent as each starts from some initial point and moves $3$ units to the right and $2$ units up.

This relationship is represented by the symbol $\cong$. In the diagram below, $\overrightarrow{\mathrm{AB}} \cong \overrightarrow{\mathrm{EF}} \cong \overrightarrow{\mathrm{CD}} \cong \overrightarrow{\mathrm{IJ}}$.

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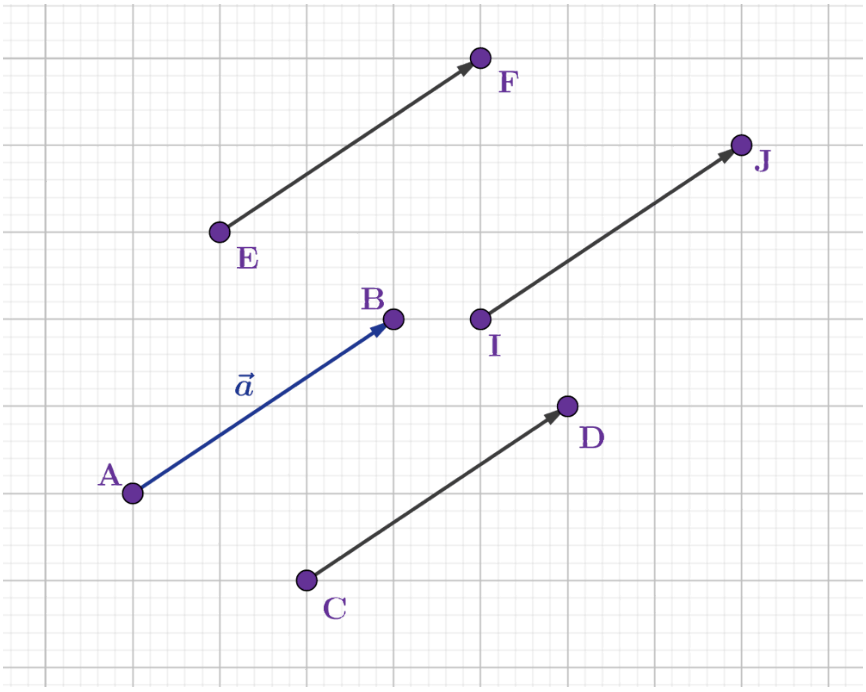


More information

The diagram illustrates several vectors positioned on a grid. There are four vectors labeled as follows: (\overrightarrow {{\rm{AB}}} ), (\overrightarrow {{\rm{EF}}} ), (\overrightarrow {{\rm{CD}}} ), and (\overrightarrow {{\rm{IJ}}} ). Each vector is shown using points on a grid, indicating that they are congruent by the symbol (\cong). The points are marked with capital letters corresponding to the vector labels. The grid provides a background for reference, and purple dots highlight the endpoints of the vectors, connected by lines indicating direction and magnitude. The vectors appear to have equal length and parallel orientation.

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## Important

A vectoris represented by a directed line segment. All line segments which have the same magnitude (length) and direction are congruent. A vector can be denoted by a bold lower case letter, e.g. $\boldsymbol{a}$, or by upper case letters with an arrow on top:

$$\boldsymbol{a} = \overrightarrow{\mathrm{AB}} \cong \overrightarrow{\mathrm{EF}} \cong \overrightarrow{\mathrm{CD}} \cong \overrightarrow{\mathrm{IJ}}$$

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## Exam tip

The IB examination papers and textbooks use the notation $\boldsymbol{a}$, to denote a vector. But it is not possible to write in bold by hand, so when you are writing vectors you **must** identify them as such by underlining, $\underline{\boldsymbol{a}}$ , or by writing an arrow on top, $\vec{a}$, otherwise you might be penalised for not using the correct notation.

## Important

One application of vectors is to show displacements. The directed line segment $\overrightarrow{\mathrm{AB}}$ could represent a displacement vector where $\mathrm{A}$ is the initial point and $\mathrm{B}$ is the terminal point.

## International Mindedness

There are various forms of notation for vectors.

- In a printed text, bold type is used to denote a vector, e.g. $\boldsymbol{v}$ or $|\boldsymbol{v}|$ .
- Vectors may be written in component form, e.g. $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{k}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors parallel to the $x$ and $y$ axes, or column vector format, e.g. $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ .
- When you are writing a vector by hand, you can use $\vec{v} = \vec{i} + 2\vec{k}$, or column vector format $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ .
- You may also see <1,2>
- When you are writing the magnitude of a vector by hand, you can write $|\vec{v}|$, or $||\vec{v}||$.

Why do these different forms of notation exist in the language of mathematics if it is a universal language?

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# Magnitude of a vector

The magnitude of a vector is given by the length of the directed line segment. The magnitude of the displacement vector $\overrightarrow{\mathrm{AB}}$ is denoted by $\left|\overrightarrow{\mathrm{AB}}\right|$ and the magnitude of vector $\boldsymbol{a}$ is written as $|\boldsymbol{a}|$.

## Example 2

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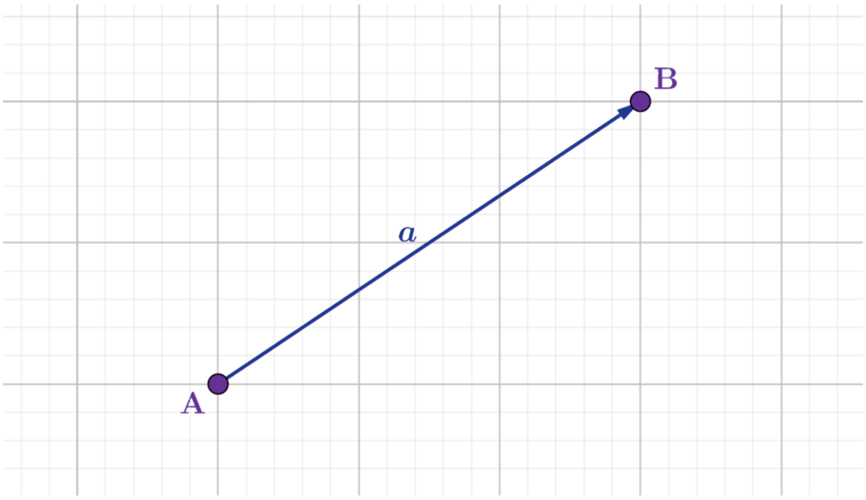
Find the magnitude of vector $\boldsymbol{a}$ in the diagram below.




More information

The diagram shows a vector on a grid, with the vector labeled from point A to point B. The grid serves as the background, providing a reference for measuring the magnitude and direction of the vector. The vector is a straight line segment that connects point A, located at the bottom left of the grid, to point B at the top right. The line is marked with an arrow indicating direction from A to B. There may be numerical values along the axes or other labels inside the grid, providing exact coordinates or measurements for each point, though these are not specified here.

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Draw the right-angled triangle.

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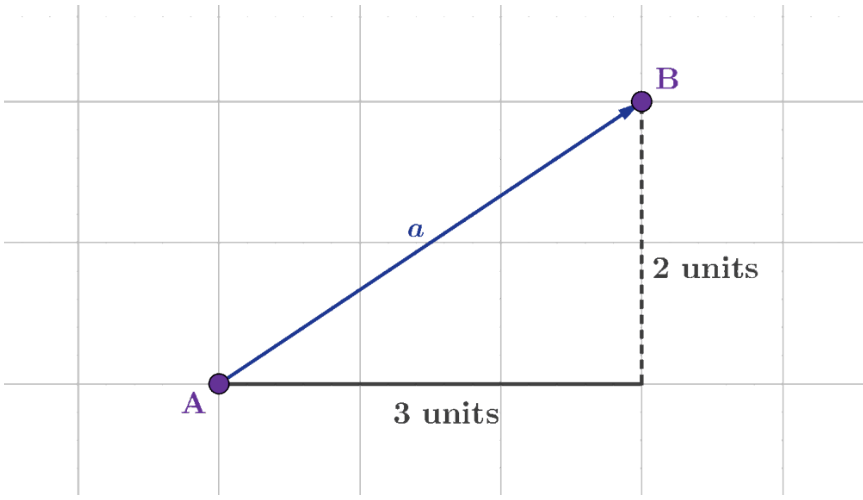
Vector $\overrightarrow{\mathrm{AB}}$ represents a movement of 3 units to the right and 2 units up.

These are the components of the vector.




Use Pythagoras' theorem

$$|\boldsymbol{a}| = \sqrt{3^2 + 2^2}$$

Therefore the magnitude of the vector is $\sqrt{13}$ units.

## Zero vector

If the magnitude of a vector is 0, then it is called the zero vector. Geometrically it is the vector which has the same starting point and end point. Therefore the displacement is zero. Its direction is undefined.

## Equal vectors

Student view

If two vectors have the same direction and magnitude, then they are equal vectors regardless of their starting point. In the diagram below, although $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ start and finish at different points they are all equal because their magnitudes and directions are equal

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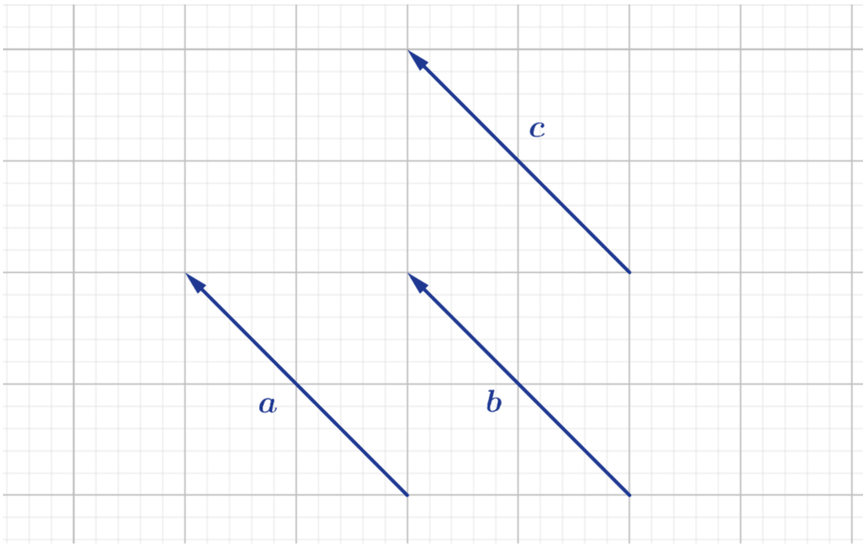
$$\boldsymbol{a} = \boldsymbol{b} = \boldsymbol{c}$$




More information

The image shows three vectors labeled a, b, and c, placed diagonally on a grid. Each vector points in a downward-right direction at consistent angles, creating parallel lines. The vectors are equally spaced apart. The grid provides a reference for their direction and length but has no additional labels or numbers. The vectors illustrate the condition where the vectors a, b, and c are equal in some respect, likely in magnitude or a specific characteristic.

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## Example 3

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Which of the following vectors, shown in the diagram below, are equal?

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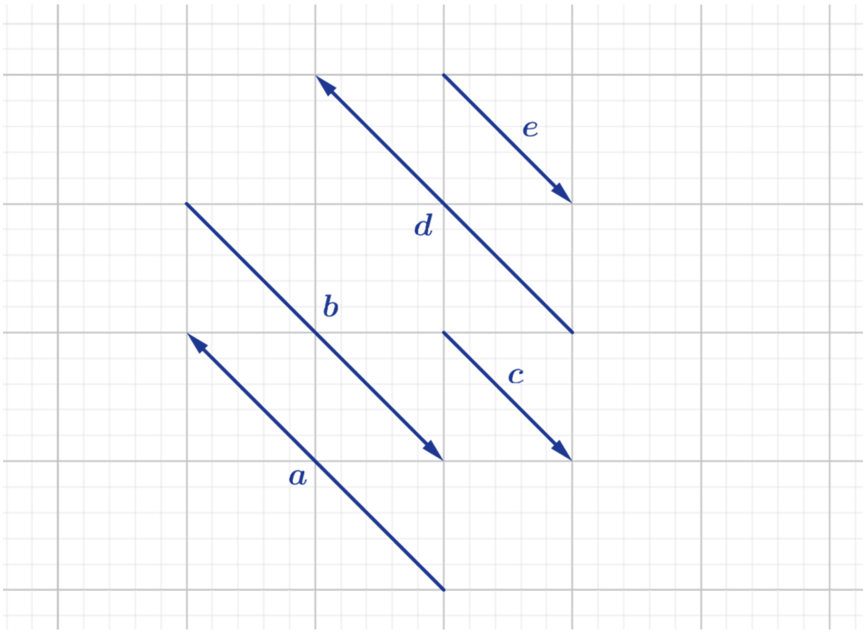


More information

The image is a diagram depicting five vectors labeled as a, b, c, d, and e, positioned on a square grid background.

- Vector a points towards the bottom left. It's located near the bottom-left corner but angled slightly upwards parallel to vector d.
- Vector b starts in the middle-left of the diagram and points upwards, running nearly parallel to vector a.
- Vector c is situated below the center and angled downwards, slightly to the right, intersecting the path between vectors d and e.
- Vector d is positioned diagonally across the grid from the top-right to bottom-left, appearing longest and parallel with vector a.
- Vector e extends diagonally downwards from the top-middle in a rightward direction, shorter than the others and parallel to vector c.

These vectors demonstrate parallel relationships as well as differing lengths and directions.

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| Steps | Explanation |
|---|---|
| $\boldsymbol{a} = \boldsymbol{d}$ | $\|\boldsymbol{a}\| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$<br>$\|\boldsymbol{d}\| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$<br>Both vectors have the same direction and magnitude, $2\sqrt{2}$.<br>Why is $\boldsymbol{a} \neq \boldsymbol{b}$ ? |
| $\boldsymbol{c} = \boldsymbol{e}$ | $\|\boldsymbol{c}\| = \sqrt{1^2 + 1^2} = \sqrt{2}$<br>$\|\boldsymbol{e}\| = \sqrt{1^2 + 1^2} = \sqrt{2}$<br>Both vectors have the same direction and magnitude, $\sqrt{2}$. |

## Opposite vectors

If two vectors are parallel, have the same magnitude but not point in the same directions, then one vector is **opposite** to the other. These pairs are represented by $\boldsymbol{a}$ and $-\boldsymbol{a}$.

### 3 section questions

3. Geometry and trigonometry / 3.10 Vectors

# Vector algebra and geometrical applications

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## Journeys and the triangle rule

Suppose you are travelling from Istanbul in Turkey to Milan in Italy. You start in Istanbul, go to Sofia in Bulgaria, then go to Athens in Greece and finally end your journey in Milan.

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How would you represent this journey using displacement vectors?

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How could you describe your starting and finishing points?


More information

The image depicts a map outlining a journey across four European cities. Starting in Istanbul, Turkey, the journey proceeds to Sofia, Bulgaria; then to Athens, Greece; and finally reaches Milan, Italy. Each city is labeled with its name and marked with a point on the map. Lines connect the cities in the order of the journey, illustrating the displacement vectors: Istanbul to Sofia, Sofia to Athens, and Athens to Milan. The map provides a geographical context showing parts of Turkey, Greece, Bulgaria, and Italy, with some country names labeled as well. This depiction visually represents the path and direction of the described trip, in alignment with the text explanation of vector summation.

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First, assign a letter to each city: Istanbul–I, Sofia–S, Athens–A and Milan–M. You can represent each leg of the journey by a displacement vector. So, in order, the journey would be the sum of vectors $\overrightarrow{\mathrm{IS}} + \overrightarrow{\mathrm{SA}} + \overrightarrow{\mathrm{AM}}$. The total displacement is from Istanbul to Milan so $\overrightarrow{\mathrm{IS}} + \overrightarrow{\mathrm{SA}} + \overrightarrow{\mathrm{AM}} = \overrightarrow{\mathrm{IM}}$, as seen in the diagram below.

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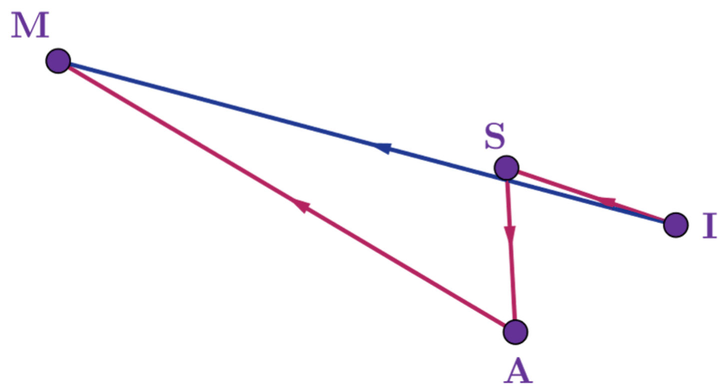


More information

The diagram represents a journey between four cities labeled as I (Istanbul), S (Sofia), A (Athens), and M (Milan). It visually depicts displacement vectors where each leg of the journey is shown by an arrow between the cities. The paths include ( \overrightarrow{IS} ), ( \overrightarrow{SA} ), and ( \overrightarrow{AM} ). The total displacement is depicted by ( \overrightarrow{IM} ), showing the journey from Istanbul to Milan. The lines are displayed with directional arrows from one city point to another, indicating the sequence and direction of travel.

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If you were to fly back directly from Milan to Istanbul after your trip, you would end up where you started. Therefore the resultant displacement would be zero. You could represent this by

$$\overrightarrow{\mathrm{IS}} + \overrightarrow{\mathrm{SA}} + \overrightarrow{\mathrm{AM}} + \overrightarrow{\mathrm{MI}} = \overrightarrow{\mathrm{II}} = 0$$

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## Important

**Triangle rule for addition:**

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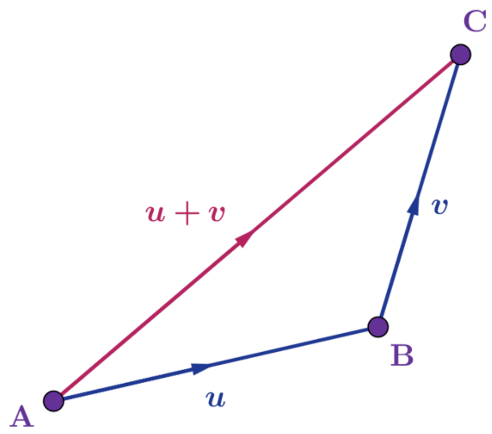
Vector addition can be represented by joining the initial point of the second vector to the end of the first vector, as seen in the diagram below.

$$\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}=\overrightarrow{\mathrm{AC}}$$

This is often referred to as drawing the vectors 'nose to tail'.

Vector addition is commutative, i.e. it does not matter in which order the vectors are added.




More information

The diagram depicts three vectors forming a triangle in a plane. It includes points labeled A, B, and C. The vector from A to B is labeled 'u', the vector from B to C is labeled 'v', and the direct vector from A to C is labeled 'u+v'. This illustrates the concept of vector addition. The arrangement shows that the sum of vectors u and v results in the vector u+v, following the triangle law of vector addition. Text on the diagram reads 'u', 'v', and 'u+v' corresponding to each vector, demonstrating how adding vectors from a point can lead back to the starting point if the sum forms a closed shape with another vector, resulting in a zero vector.

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If you add three or more vectors and end up where you started, the resulting vector would be zero. For example,

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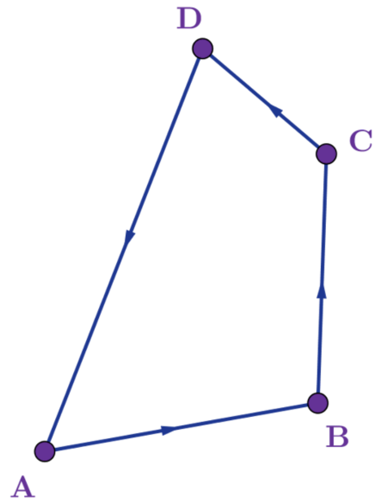
$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0$$

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The image is a diagram showing a quadrilateral formed by points labeled A, B, C, and D. Vectors are drawn connecting these points, forming the closed shape of the quadrilateral, and are labeled as (\overrightarrow{AB}), (\overrightarrow{BC}), (\overrightarrow{CD}), and (\overrightarrow{DA}). Each vector has an arrow indicating its direction, and all vectors together form a closed loop, showcasing the equation (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0). The vectors depict that their sum results in zero, illustrating the concept of vector equilibrium in a closed shape.

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## Making connections

When solving problems related to position and movement, a displacement vector can be used to represent the position of one object relative to another. If the magnitude of the displacement vector is zero, the objects will have the same position.

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Vectors can also be used to represent forces. If the sum of two or more forces is zero, then the forces are in equilibrium.

## Example 1

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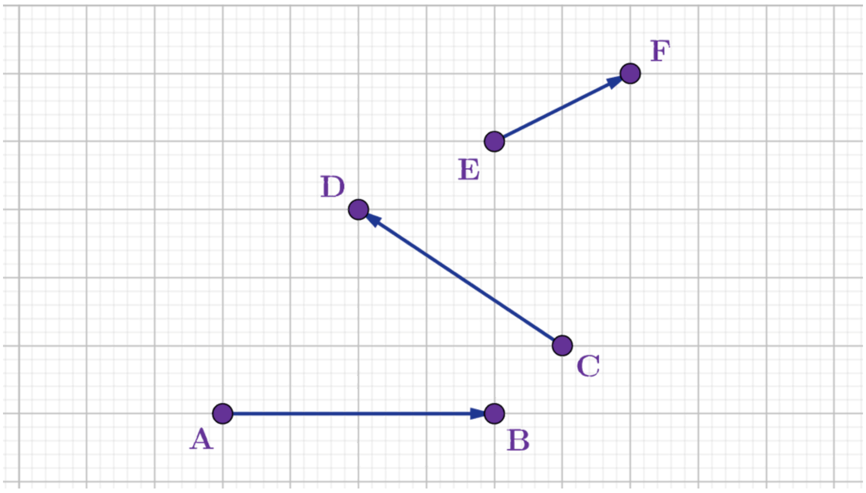
Draw the vector representing the sum $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF}$.




More information

The image shows a grid with three vectors, each represented as line segments with arrows. The labels for the points of these vectors are A, B, C, D, E, and F. Vector (\overrightarrow{AB}) runs horizontally from point A to point B, Vector (\overrightarrow{CD}) runs diagonally downward from point C to point D, and Vector (\overrightarrow{EF}) runs diagonally upward from point E to point F. The task is to draw the resultant vector of the sum (\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF}).

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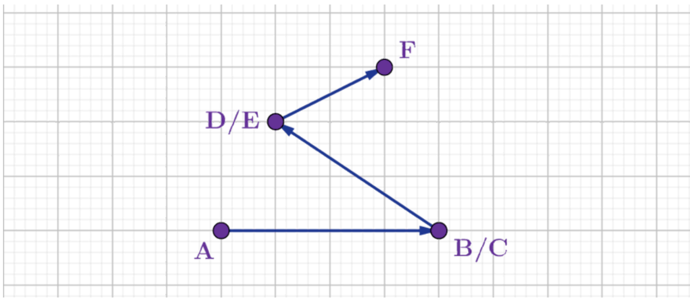
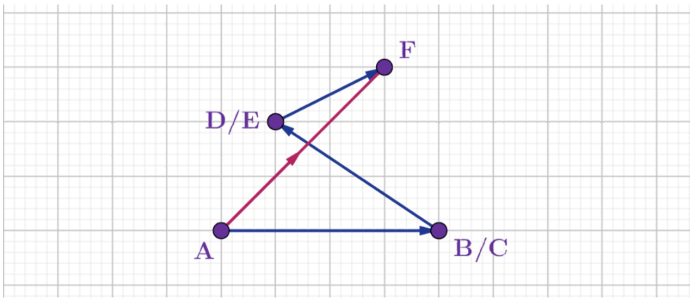
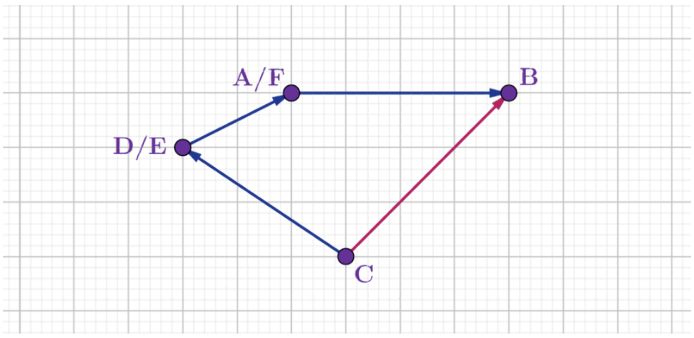
| Steps | Explanation |
| --- | --- |
|   | Draw the vector representing $\overrightarrow{\mathrm{AB}}$<br>Translate vector $\overrightarrow{\mathrm{CD}}$ so that it foll on from $\overrightarrow{\mathrm{AB}}$.<br>Then translate $\overrightarrow{\mathrm{EF}}$ so that it follow on from $\overrightarrow{\mathrm{CD}}$.<br>The vector from $\mathrm{A}$ to $\mathrm{F}$ represent the sum $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{CD}}+\overrightarrow{\mathrm{EF}}$. |
|   | $\overrightarrow{\mathrm{AF}}$ is the resultant of the vectors $\overrightarrow{\mathrm{AB}}$, $\overrightarrow{\mathrm{CD}}$ and $\overrightarrow{\mathrm{EF}}$. |
|   | Note that there are many possible ways to answer the question.<br>It does not matter which vector yo start with and in which order you draw the vectors. The resultant wi have the same magnitude and direction. |

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### Important

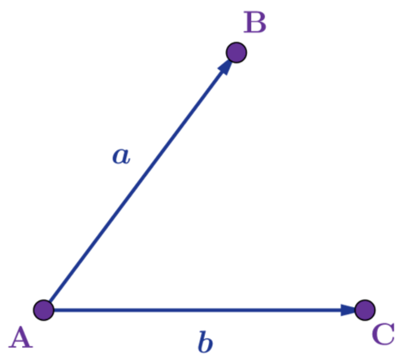
The sum of two or more vectors is called the resultant.

Always draw the vectors 'nose to tail' when finding the resultant. You may need to use a scale drawing when adding vectors. The resultant is often indicated by a double arrowhead on the vector.

## Using parallelograms for adding and subtracting vectors

Two vectors may have the same starting point.

Consider the diagram below. How would you add $\overrightarrow{AB}$ and $\overrightarrow{AC}$?




More information

The diagram illustrates a geometric setup with three labeled points: A, B, and C. Point A is at the origin, with vector (\overrightarrow{AB}) pointing towards B at an angle above the horizontal axis. The vector (\overrightarrow{AC}) extends from A to C horizontally. The length of vector (\overrightarrow{AB}) is labeled as (a) and the length of vector (\overrightarrow{AC}) is labeled as (b). The vectors form a right-angled triangle with the origin at point A, illustrating the addition of vectors (\overrightarrow{AB}) and (\overrightarrow{AC}).

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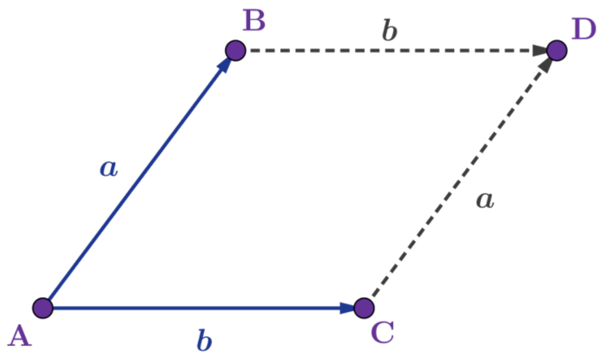
Draw the parallelogram as shown below:

$$\overrightarrow{\rm AB} \cong \overrightarrow{\rm CD} \text{ so } \overrightarrow{\rm CD} = \boldsymbol{a}$$
$$\overrightarrow{\rm AC} \cong \overrightarrow{\rm BD} \text{ so } \overrightarrow{\rm BD} = \boldsymbol{b}$$

Draw $\rm BD$ parallel to $\rm AC$ and $\rm DC$ parallel to $\rm CD$, forming a parallelogram.




More information

The image shows a geometric diagram illustrating a parallelogram. The corners of the parallelogram are labeled A, B, C, and D. Point A is at the lower left, B is above A, C is to the right of A, and D is to the right of B and above C. Vectors are drawn between the points: from A to B labeled as 'a', from B to D labeled as 'b', from A to C labeled as 'b', and from C to D labeled as 'a'. The labels indicate vectors laid out to form the parallelogram. The diagram visually represents the equation: (\overrightarrow{\rm{AB}} + \overrightarrow{\rm{AC}} \cong \overrightarrow{\rm{AB}} + \overrightarrow{\rm{BD}}) so (\overrightarrow{\rm{AB}} + \overrightarrow{\rm{BD}} = \overrightarrow{\rm{AD}}).

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Using the parallelogram, $\overrightarrow{\rm AB} + \overrightarrow{\rm AC} \cong \overrightarrow{\rm AB} + \overrightarrow{\rm BD}$ so $\overrightarrow{\rm AB} + \overrightarrow{\rm BD} = \overrightarrow{\rm AD}$.

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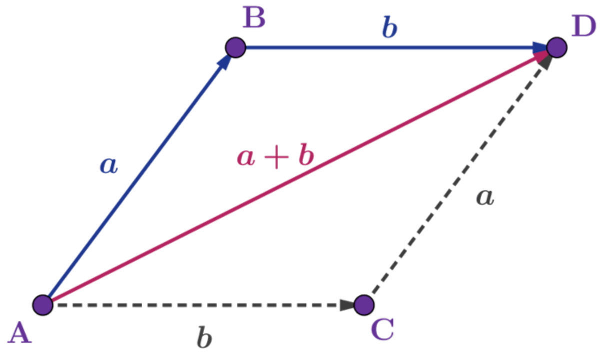
$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$ which is the diagonal of the parallelogram $ABCD$.




More information

The image is a diagram of a parallelogram labeled ABCD. The vector (\overrightarrow{AB}) is labeled with (a), (\overrightarrow{AD}) is labeled with (a + b), (\overrightarrow{BC}) is labeled with (b), and (\overrightarrow{AC}) is labeled with (b). Points A, B, C, and D are marked at the corners of the parallelogram with purple dots. The vectors (\overrightarrow{AB}) and (\overrightarrow{BC}) are shown in blue, while (\overrightarrow{AD}) is shown in pink, and the vector (\overrightarrow{AC}) is depicted with a grey dashed line. Arrows indicate the direction of each vector, illustrating how (\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}), the diagonal of the parallelogram ABCD.

[Generated by AI]

## Example 2

★★☆

Use the diagram below, where $ABCD$ is a parallelogram, to represent $\overrightarrow{AB} - \overrightarrow{AC}$

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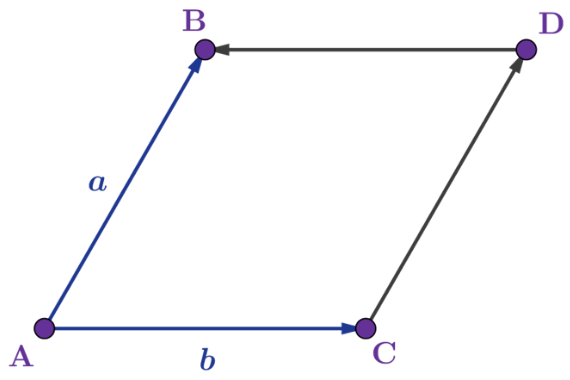
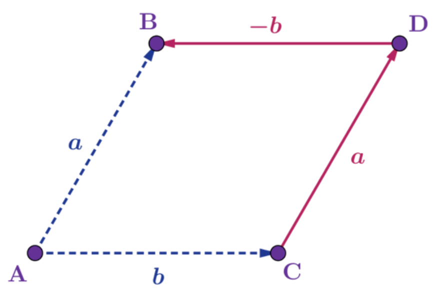



More information

The diagram depicts a parallelogram labeled ABCD. Point A is at the bottom left, point B at the top left, point C at the bottom right, and point D at the top right, forming the parallelogram. A vector labeled 'a' points from A to B, while another vector labeled 'b' points from A to C. The arrows illustrate the direction of the vectors, with AB and AC forming part of the parallelogram's sides. The figure represents vectors ( \overrightarrow{AB} - \overrightarrow{AC} ) in relation to the parallelogram shape.

[Generated by AI]

| Steps | Explanation |
| --- | --- |
|   $\overrightarrow{AB} - \overrightarrow{AC} \cong \overrightarrow{CD} + \overrightarrow{DB}$ | $\overrightarrow{CD} \cong \overrightarrow{AB}$ as they have same magnitu and direction.<br><br>$\overrightarrow{DB} \cong -\overrightarrow{AC}$ as they have the same magnitude but opposite directions. |

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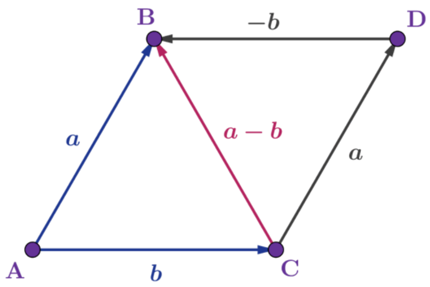
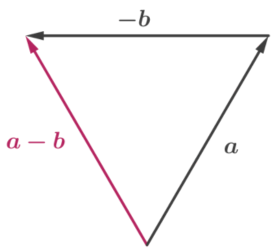
| Steps | Explanation |
|---|---|
| $\overrightarrow{AB} - \overrightarrow{AC} \cong \overrightarrow{CD} + \overrightarrow{DB} = \overrightarrow{CB}$ | Add vectors $\overrightarrow{CD}$ and $\overrightarrow{DB}$. |
|  | |
|  | There are other ways of drawing the diagram. |

## Be aware

Make sure the vectors follow on ‘nose to tail’ when subtracting one vector from another.

Subtraction of vectors is not commutative.

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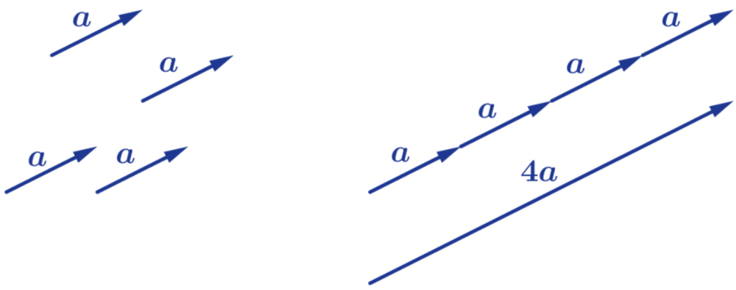
# Multiplying vectors by scalars: parallel vectors

Consider the sum of the vectors in the diagram below: $\boldsymbol{a}+\boldsymbol{a}+\boldsymbol{a}+\boldsymbol{a}$.

The resultant vector is in the same direction and four times as long as $\boldsymbol{a}$.

Thus, $\boldsymbol{a}+\boldsymbol{a}+\boldsymbol{a}+\boldsymbol{a}=4\boldsymbol{a}$




More information

The image consists of two diagrams. On the left, there are four vectors each labeled "a" and pointing towards the top right. They are scattered, representing separate vectors. On the right, the vectors are aligned end-to-end in a straight line along the same direction. Below this sequence, there is a larger vector labeled "4a." This arrangement visually demonstrates that the sum of the four individual vectors "a" equals the larger vector "4a." The structure shows vector addition, illustrating that combining these vectors results in the same vector pointing in the same direction but scaled up four times.

[Generated by AI]

The sum of the vectors in the first diagram below can be written as $4\boldsymbol{a}$ as shown in the diagram on the right, that is, $\boldsymbol{a}+\boldsymbol{a}+\boldsymbol{a}+\boldsymbol{a}=4\boldsymbol{a}$.

Student view

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## Important

When a vector $\boldsymbol{a}$ is multiplied by a scalar $k$, the resultant vector is $\boldsymbol{v} = k\boldsymbol{a}$ which is parallel to $\boldsymbol{a}$ and has a magnitude $|\boldsymbol{v}| = k\,|a|$. If $k < 0$, then the direction of $\boldsymbol{v}$ is opposite to $\boldsymbol{a}$.

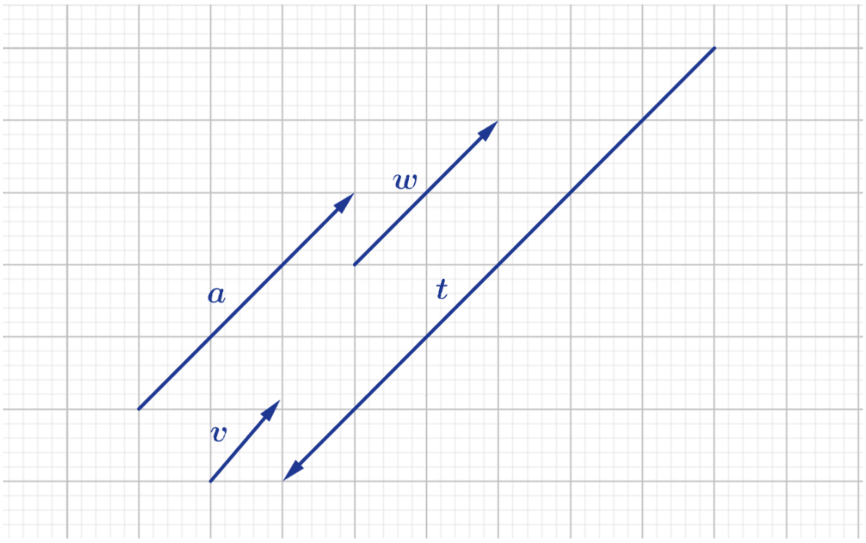
If $\boldsymbol{v} = k\boldsymbol{a}$ then $\boldsymbol{v}$ and $\boldsymbol{a}$ are parallel vectors, that is $\boldsymbol{v} \parallel \boldsymbol{a}$.

Two vectors are parallel if they are scalar multiples of the same vector.

# Example 3

★☆☆

Write the following vectors in terms of $\boldsymbol{a}$.




More information

The image is a grid diagram displaying vectors labeled as (\boldsymbol{a}), (\boldsymbol{b}), and (\boldsymbol{c}). Each vector is represented by an arrow on the grid. (\boldsymbol{a}) is positioned diagonally and appears to be the main vector. (\boldsymbol{b}) is smaller and directed slightly downwards compared to (\boldsymbol{a}). (\boldsymbol{c}) is parallel to (\boldsymbol{a}) and extends further than (\boldsymbol{b}). These vectors are depicted as lying on a coordinate plane grid that allows measurement and comparison of their relative lengths and orientations. The task is to write the following vectors in terms of (\boldsymbol{a}).

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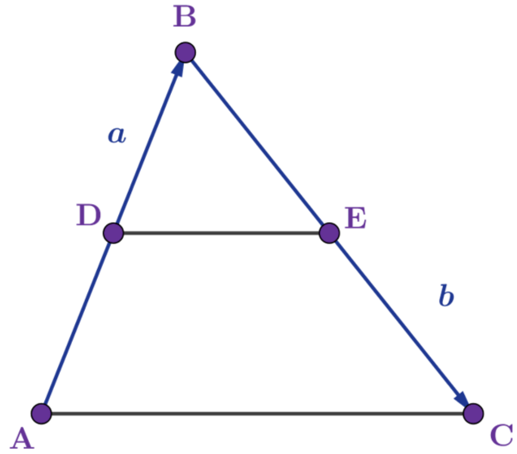
| Steps | Explanation |
|---|---|
| $\boldsymbol{v} = \frac{1}{3}\boldsymbol{a}$ | $\boldsymbol{v}$ has the same direction as $\boldsymbol{a}$ and its magnitude is $\frac{1}{3}$ the magnitude o . |
| $\boldsymbol{w} = \frac{2}{3}\boldsymbol{a}$ | $\boldsymbol{w}$ has the same direction as $\boldsymbol{a}$ and its magnitude is $\frac{2}{3}$ of the magnitud of $\boldsymbol{a}$. |
| $\boldsymbol{t} = -2\boldsymbol{a}$ | $\boldsymbol{t}$ is in the opposite direction to $\boldsymbol{a}$ and its magnitude is twice that of $\boldsymbol{a}$. |

## Example 4

★★☆

In triangle $\text{ABC}$, points $\text{D}$ and $\text{E}$ are midpoints of $\text{AB}$ and $\text{BC}$ respectively.

If $\overrightarrow{\text{AB}} = \boldsymbol{a}$ and $\overrightarrow{\text{BC}} = \boldsymbol{b}$, show that $\text{DE}$ is parallel to $\text{AC}$.




More information

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The image depicts a geometric diagram with points labeled A, B, C, D, and E, forming a triangle. The triangle's vertices are labeled: ( \overrightarrow{AB} = \boldsymbol{a} ) and ( \overrightarrow{BC} = \boldsymbol{b} ). Additionally, line DE is drawn parallel to line AC, with D positioned on AB and E on BC, dividing the sides proportionally. The diagram is used to demonstrate a geometric property linking parallel lines within the triangle.

[Generated by AI]

| Steps | Explanation |
|---|---|
| $\overrightarrow{\mathrm{AC}} = \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \boldsymbol{a} + \boldsymbol{b}$ | Triangle rule |
| $\overrightarrow{\mathrm{DE}} = \overrightarrow{\mathrm{DB}} + \overrightarrow{\mathrm{BE}}$ | Triangle rule |
| $\overrightarrow{\mathrm{DE}} = \frac{1}{2}\overrightarrow{\mathrm{AB}} + \frac{1}{2}\overrightarrow{\mathrm{BC}} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$ | As $\mathrm{D}$ and $\mathrm{E}$ are midpoints, $$\overrightarrow{\mathrm{DB}} = \frac{1}{2}\overrightarrow{\mathrm{AB}} \text{ and } \overrightarrow{\mathrm{BE}} = \frac{1}{2}\overrightarrow{\mathrm{BC}}$$ |
| $\overrightarrow{\mathrm{DE}} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$ | Factorise |
| $\overrightarrow{\mathrm{DE}} = \frac{1}{2}\left(\overrightarrow{\mathrm{AC}}\right)$ | $\overrightarrow{\mathrm{DE}}$ is a scalar multiple of $\overrightarrow{\mathrm{AC}}$ so they are parallel. |
| Therefore $$\mathrm{DE} \parallel \mathrm{AC}$$ | |

## Example 5

★★☆

Student view

In triangle $\mathrm{ABC}$, point $\mathrm{D}$ lies on $\mathrm{BC}$.

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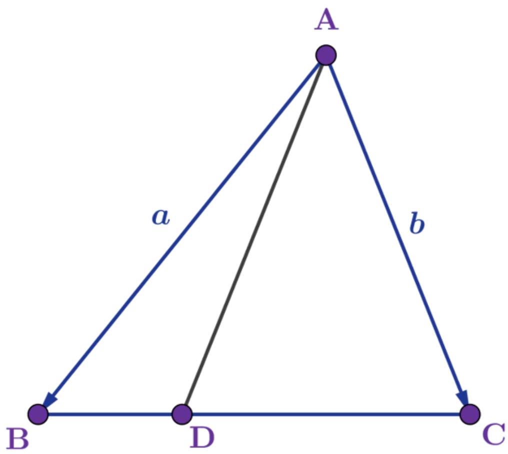
If $3\text{BD} = 2\text{DC}$, $\overrightarrow{\text{AB}} = \boldsymbol{a}$ and $\overrightarrow{\text{AC}} = \boldsymbol{b}$, write the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

a) $\overrightarrow{\text{BC}}$

b) $\overrightarrow{\text{AD}}$




More information

The image is a diagram of a triangle labeled ABC with an additional point, D, on side BC. There are markings on the sides of the triangle: AB is labeled 'a', AC is labeled 'b', BD and DC are part of BC. There are arrows indicating the direction from B to A and from C to A. Points A, B, C, and D are marked with purple dots, and all side labels are in blue. This diagram could represent a geometrical problem involving the vectors or distances between these points.

[Generated by AI]

| | Steps | Explanation |
|---|---|---|
| a) | $\overrightarrow{\text{BC}} = \overrightarrow{\text{BA}} + \overrightarrow{\text{AC}}$ | Triangle rule |
| | $\overrightarrow{\text{BC}} = -\boldsymbol{a} + \boldsymbol{b}$ | $\overrightarrow{\text{BA}} = -\overrightarrow{\text{AB}}$ |

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| | Steps | Explanation |
|---|---|---|
| | So, the answer is $-\boldsymbol{a}+\boldsymbol{b}$ | |
| b) | $\overrightarrow{\mathrm{AD}}=\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BD}}$ | Triangle rule |
| | $\overrightarrow{\mathrm{AD}}=\boldsymbol{a}+\frac{2}{5}(-\boldsymbol{a}+\boldsymbol{b})$ | $3BD=2DC$ so $BD:DC=2:3$<br>$\overrightarrow{\mathrm{BD}}=\frac{2}{5}\overrightarrow{\mathrm{BC}}$ |
| | $\overrightarrow{\mathrm{AD}}=\frac{3}{5}\boldsymbol{a}+\frac{2}{5}\boldsymbol{b}$ | Simplify |

## 3 section questions

3. Geometry and trigonometry / 3.10 Vectors

# Component form

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/component-form-id-28354/print/) Assign

## Linear combination of vectors

Any vector in a plane can be written as a combination of two non-parallel vectors. Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non-parallel vectors. Then the linear combination of the two vectors would be

$$\boldsymbol{c}=\lambda\boldsymbol{a}+\mu\boldsymbol{b}$$

where $\lambda$ and $\mu$ are scalars.

### Example 1

★☆☆

Write $\overrightarrow{\mathrm{PQ}}$ as linear combination of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

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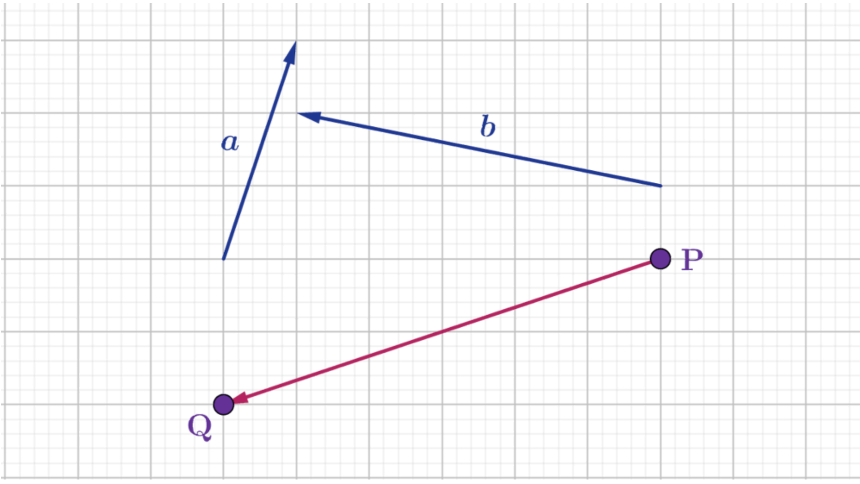


More information

The image shows a vector illustration on a grid. There are two points, labeled P and Q, connected by a vector path, depicted in red. Vector (\overrightarrow{PQ}) originates from Q and points toward P. Two additional vectors, (\boldsymbol{a}) and (\boldsymbol{b}), are depicted in blue and represent different directions. Vector (\boldsymbol{a}) is positioned vertically, while vector (\boldsymbol{b}) is diagonal. The goal is to represent vector (\overrightarrow{PQ}) as a linear combination of vectors (\boldsymbol{a}) and (\boldsymbol{b}). The grid helps visualize and calculate the relationship between these vectors in a mathematical space.

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Draw the vectors:

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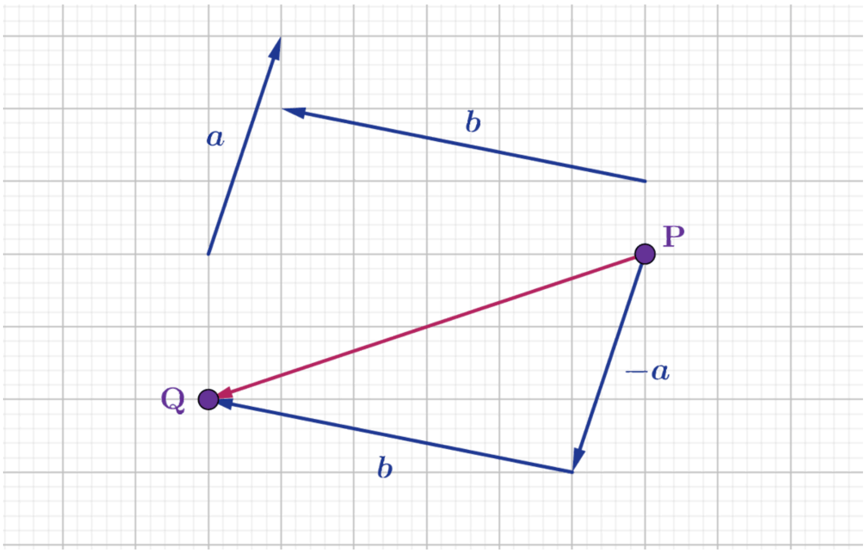


The scalars are $-1$ and $1$.

Therefore

$$\overrightarrow{PQ} = -\boldsymbol{a} + \boldsymbol{b}$$

# $\boldsymbol{i}$ and $\boldsymbol{j}$ unit vectors

Any vector in a 2D plane can be written as a linear combination of a vector parallel to the positive $x$-axis and a vector parallel to the positive $y$-axis.

A vector one unit long parallel to the $x$-axis is denoted by $\boldsymbol{i}$ and a vector one unit long parallel to the $y$-axis is denoted by $\boldsymbol{j}$.

The diagram below shows these vectors. All the other vectors in the Cartesian plane can be written as linear combinations of $\boldsymbol{i}$ and $\boldsymbol{j}$, for example, $\overrightarrow{OB} = 2\boldsymbol{i} + \boldsymbol{j}$ and $\overrightarrow{OE} = -1.5\boldsymbol{i} - \boldsymbol{j}$.

The multiples of $\boldsymbol{i}$ and $\boldsymbol{j}$ are called the components of the vector.

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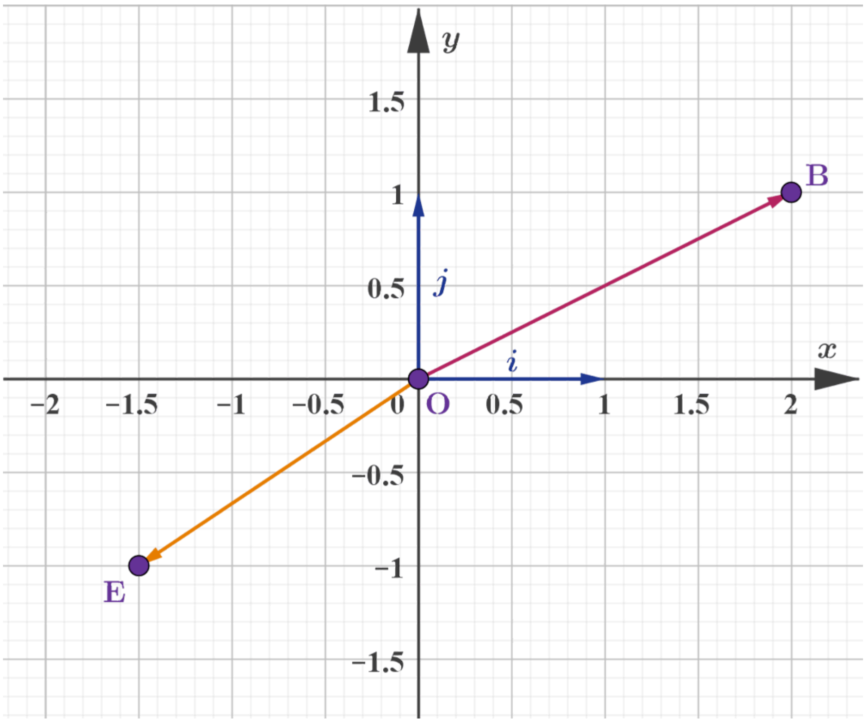


More information

The image is a graph depicting vectors in a two-dimensional coordinate system. The background is a grid with both X and Y axes labeled as 'i' and 'j', respectively. The graph shows several vectors originating from the origin (0,0).

There is a prominent pink vector labeled 'OB' extending from the origin to the point (2,1) on the grid. Additionally, an orange vector labeled 'OA' spans from the origin to the point (-1,1).

Lines and vectors are drawn with arrows indicating direction. The grid has markings at each unit interval. The graph highlights how these vectors can be represented in terms of their components along the axes labeled 'i' and 'j'.

[Generated by AI]

## Important

Although any vector can be written as a linear combination of any two other non-parallel vectors, by convention, vectors are written in terms of the two perpendicular unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$. These vectors are often called base vectors . They are unit vectors because they are one unit long.

Vectors can be represented in variety of ways:

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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$$\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\boldsymbol{i} + \boldsymbol{j}$$

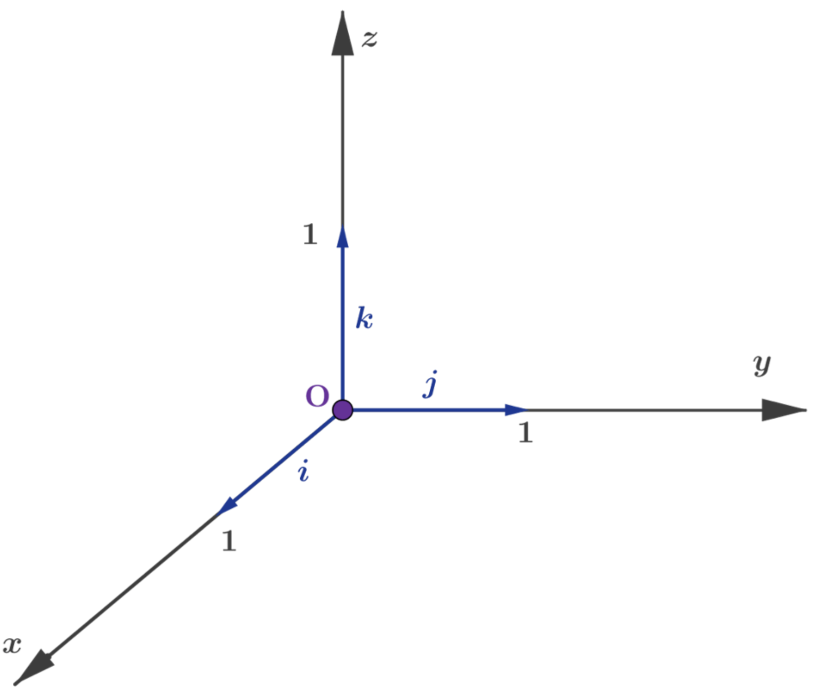
Depending on the question, you can choose the most suitable notation.

When a vector is written in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ , you do not write $\boldsymbol{i}$ and $\boldsymbol{j}$ with the scalars to indicate the components of the vector. The vector bracket is sufficient to denote a vector.

The top number represents the horizontal component and the bottom number gives the vertical component.

The vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ can also be used to denote unit vectors due east and due north, respectively. The context of the question will make this clear.

In 3D, you would define the base vectors as $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ as shown below. These vectors are parallel to the $x$, $y$ and $z$ coordinate axes, respectively, and are therefore perpendicular.




More information

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This image is a diagram of the 3D Cartesian coordinate system, illustrating the base vectors. It shows three arrows originating from the origin (O): (\boldsymbol{i}) vector along the x-axis, (\boldsymbol{j}) vector along the y-axis, and (\boldsymbol{k}) vector along the z-axis. Each vector is labeled at its endpoint with its respective letter. The vectors are depicted in blue and are perpendicular to each other, denoting the orthogonal nature of the 3D coordinate system. The directions and unit length of each vector are marked by numbers: '1' is indicated on each axis away from the origin, representing the unit vectors in their respective directions.

[Generated by AI]

✓ **Important**

In 3D, base vectors are $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

All the other vectors in 3D can be written as linear combinations of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

## Example 2

★☆☆

Points $\mathrm{A}$ and $\mathrm{B}$ have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, relative to a fixed origin $\mathrm{O}$.

Write vector $\overrightarrow{\mathrm{AB}}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Draw a diagram showing the origin, points $\mathrm{A}$ and $\mathrm{B}$ and the position vectors:

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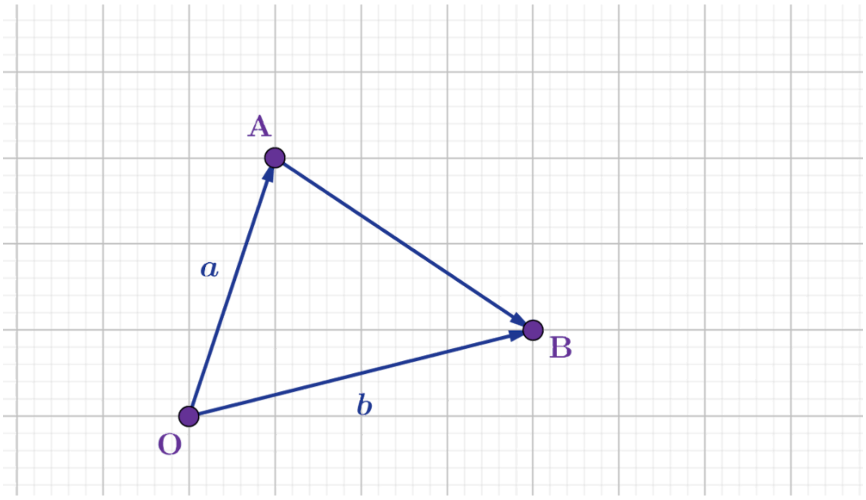
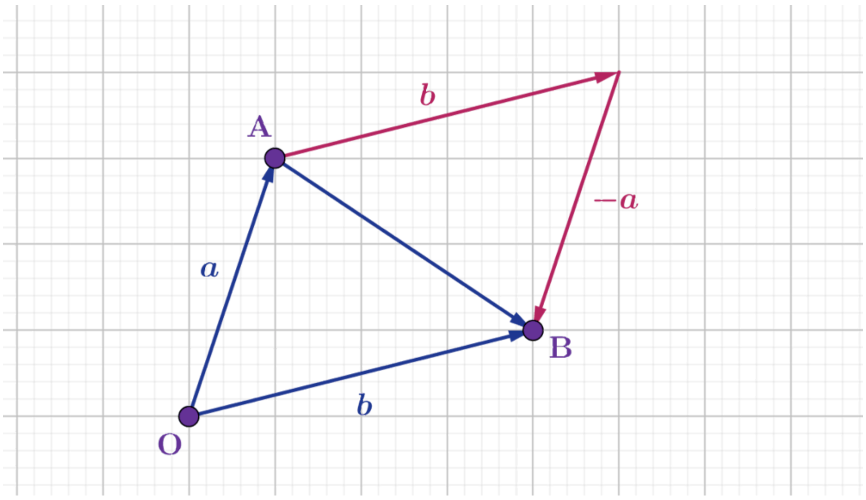



Use the parallelogram rule to find $\overrightarrow{\mathrm{AB}}$:




Therefore,

$$\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a}$$

Student view

✓ **Important**

Overview
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If points $\mathrm{A}$ and $\mathrm{B}$ have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, then

$$\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a} \text{ or } \overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{AO}}$$

### Activity

In this activity, you will investigate the algebra of vectors written in component form using GeoGebra.

- Create two vectors using the vector command for example $\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.
- Use the 'algebra input' to insert $\boldsymbol{u} + \boldsymbol{v}, \boldsymbol{u} - \boldsymbol{v}, 2\boldsymbol{u}, -2\boldsymbol{v}$.
- What do you notice about the components of each vector?

# Algebra of vectors in component form

In 2D: For two vectors $\boldsymbol{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} c \\ d \end{pmatrix}$

$$\boldsymbol{u} \pm \boldsymbol{v} = \begin{pmatrix} a \pm c \\ b \pm d \end{pmatrix}$$

$$k\boldsymbol{u} = k \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}, \text{ where } k \in \mathbb{R}$$

In 3D: $\boldsymbol{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

$$\boldsymbol{u} \pm \boldsymbol{v} = \begin{pmatrix} a \pm d \\ b \pm e \\ c \pm f \end{pmatrix}$$

$$k\boldsymbol{u} = k \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}, \text{ where } k \in \mathbb{R}$$

Student view

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## Example 3

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If $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, find

a) $\boldsymbol{u} + \boldsymbol{v}$

b) $\boldsymbol{u} - 2\boldsymbol{v}$

| | Steps | Explanation |
|---|---|---|
| a) | $\boldsymbol{u} + \boldsymbol{v} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ | $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ It is easier to work with vectors when they are written in column notation. |
| | Therefore $\boldsymbol{u} + \boldsymbol{v} = 3\boldsymbol{i} + 2\boldsymbol{j}$ | |
| b) | $\boldsymbol{u} - 2\boldsymbol{v} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -3 \end{pmatrix}$ | $2\boldsymbol{v} = 2\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ |
| | $\boldsymbol{u} - 2\boldsymbol{v} = 5\boldsymbol{j} - 3\boldsymbol{k}$ | |

## Example 4

★★☆

Points A and B have position vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$ respectively.

Point C lies on the line segment AB. If $AC : CB = 1 : 2$, find the position vector of C.

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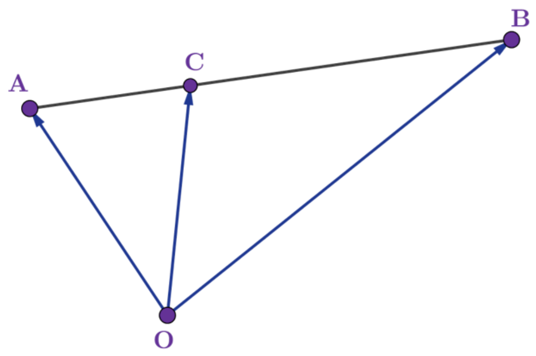
| Steps | Explanation |
|---|---|
|   | Draw a diagram. |
| $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$ | $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$ |
| $\overrightarrow{\mathrm{AC}} = \frac{1}{3}\overrightarrow{\mathrm{AB}} = \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$ | $AC : CB = 1 : 2$<br>so $\overrightarrow{\mathrm{AC}} = \frac{1}{3}\overrightarrow{\mathrm{AB}}$ |
| $\overrightarrow{\mathrm{OC}} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$ | $\overrightarrow{\mathrm{AC}} = \overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OA}}$ |
| $\overrightarrow{\mathrm{OC}} = \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ | Rearrange |
| $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} \frac{4}{3} \\ \frac{7}{3} \\ \frac{-2}{3} \end{pmatrix}$ | Simplify |

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| Steps | Explanation |
|---|---|
| Therefore the position vector of point $\text{C}$ is $\begin{pmatrix} \frac{4}{3} \\ \frac{7}{3} \\ \frac{-2}{3} \end{pmatrix}$ | |

## Example 5

★☆☆

If the vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ a \\ b \end{pmatrix}$ are parallel, find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$.

As the two vectors are parallel $\boldsymbol{v} = k\boldsymbol{a}$:

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = k \begin{pmatrix} -6 \\ a \\ b \end{pmatrix} = \begin{pmatrix} -6k \\ ak \\ bk \end{pmatrix}$$

Respective components will be equal:

$$-6k = 3 \Rightarrow k = -\frac{1}{2}$$

$$ak = 2 \Rightarrow \boldsymbol{a} = -4$$

$$bk = 1 \Rightarrow \boldsymbol{b} = -2$$

Therefore,

$$\boldsymbol{a} = -4 \text{ and } \boldsymbol{b} = -2$$

Student view

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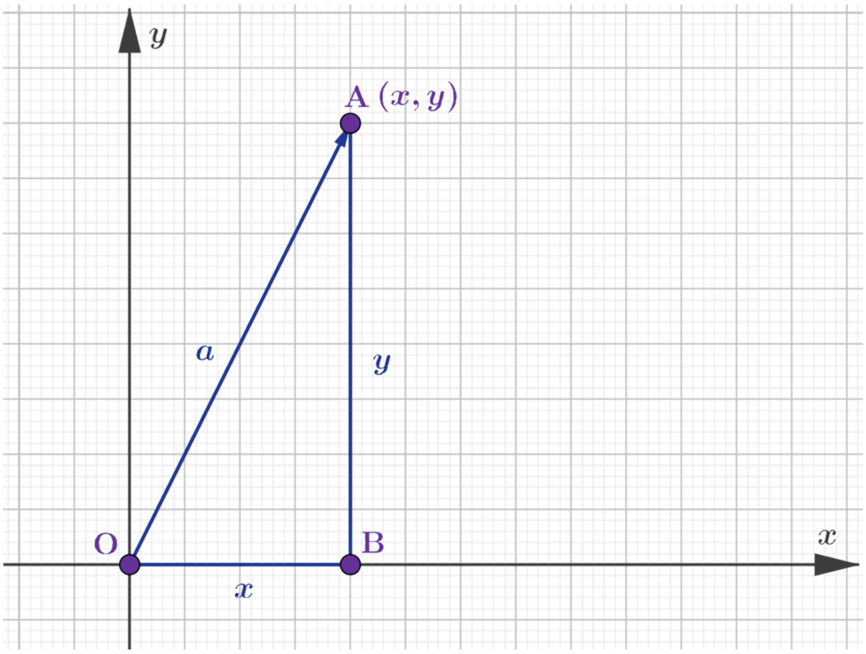
# Magnitude of vectors in component form

Point $(x, y)$ in the diagram below, has position vector $\boldsymbol{a}$.

You can use Pythagoras' theorem to find the magnitude: $|\boldsymbol{a}| = \sqrt{x^2 + y^2}$.




More information

The image shows a 3D Cartesian coordinate system with axes labeled x, y, and z. The plane visible is the x-y plane, with a triangle formed by three points: O at the origin (0,0), B at point (x,0), and A at point (x,y). Line segments OB, OA, and AB form a right triangle. The length of OB is labeled x, and the length of AB is labeled y. The hypotenuse OA represents the vector a with the magnitude (|\boldsymbol{a}|). The labeled point A at (x,y) shows the coordinates and the endpoint of the vector.

[Generated by AI]

You can find the distance between two points in 3D using Pythagoras' theorem twice as shown in the diagram below. You can apply this to finding the magnitude of a vector in 3D.

Point $(x, y, z)$ in the diagram below has position vector $\boldsymbol{a}$.

Student view

Overview
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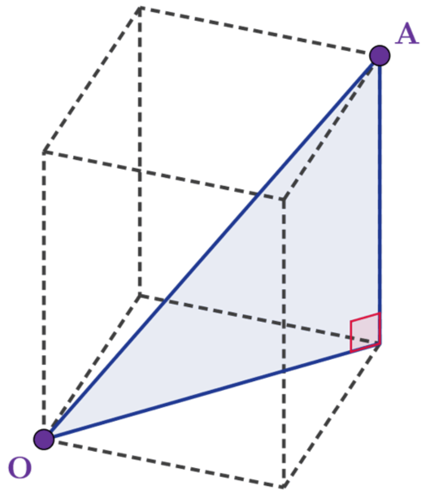
Using Pythagoras' theorem, $|a| = \sqrt{x^2 + y^2 + z^2}$.




More information

The image is a 3D diagram illustrating a right triangle within a cube. The cube is shown with dashed lines to indicate its edges and corners. The vertices of the triangle are labeled 'O', 'A', and a point on the cube representing the right angle. The triangle is in a blue plane, which is partially shaded to distinguish it from the rest of the cube. The label 'a' is used to represent the hypotenuse of the triangle, which extends diagonally from vertex 'O' at the bottom left to vertex 'A' at the top right. The diagram serves to visually explain Pythagoras' theorem in three-dimensional space, showing that the length of the hypotenuse 'a' can be calculated using the formula ( a = \sqrt{x^2 + y^2 + z^2} ). Here, 'x', 'y', and 'z' represent the side lengths of the triangle along the edges of the cube.

[Generated by AI]

## Important

$$\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow |\boldsymbol{a}| = \sqrt{x^2 + y^2}$$

$$\boldsymbol{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow |\boldsymbol{b}| = \sqrt{x^2 + y^2 + z^2}$$

Student view

Overview
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# Example 6

★☆☆

Find the magnitude of the vector $\boldsymbol{w} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$.

Use Pythagoras' theorem:

$$|\boldsymbol{w}| = \sqrt{(-2)^2 + 3^2 + 6^2} = \sqrt{49} = 7$$

### Exam tip

In the IB formula booklet, the formula for the magnitude of a vector is given as

$$|\boldsymbol{v}| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}\text{, where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

### Important

Vectors can be used to represent any quantity that has both magnitude and direction.

What is the difference between speed and velocity ?

In everyday language the words can be interchanged, but in maths and physics they have different meanings.

- Speed is a scalar quantity — it measures how fast an object is moving.
- Velocity is a vector quantity. It not only measures how fast an object is moving, but it also gives the direction, e.g. $4\,\mathrm{m\,s^{-1}}$ west.
- The magnitude of a velocity vector gives the speed.

# Example 7

Student
view

★★☆

Overview
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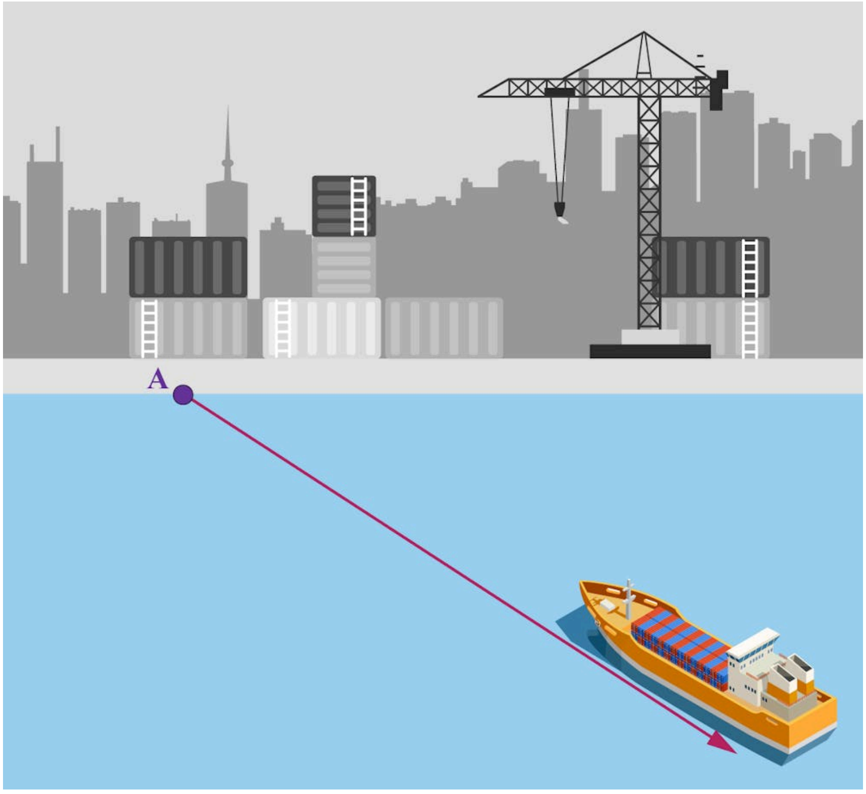
A ship travelling from port $\text{A}$, the origin, travels with velocity $\begin{pmatrix} 30 \\ 40 \end{pmatrix}$ kilometres per hour.

a) How far will the ship be from the port after 10 hours?

b) Write the coordinates of the ship relative to the port.




More information

The image shows a port with various stacked shipping containers and a large crane in the background, indicating a working dock. In the foreground, there is a body of water on which a cargo ship is depicted. The ship is heading towards the open sea. A line extends from a purple point labeled "A" on the dock out towards the ship, which may represent a directional coordinate in a nautical context. The image serves to explain how to write the coordinates of the ship relative to the port, based on its geometric positioning in relation to point A.

[Generated by AI]

Student view

Overview
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| | Steps | Explanation |
|---|---|---|
| a) | $\boldsymbol{v} = \begin{pmatrix} 30 \\ 40 \end{pmatrix} \Rightarrow \lvert \boldsymbol{v} \rvert = \sqrt{30^2 + 40^2} = 50\,\mathrm{km\,h^{-1}}$ | The magnitude of the velocity is the speed of the ship. |
| | Distance = $10 \times 50 = 500$ km | The speed is constant so $$\text{distance} = \text{speed} \times \text{tim}$$ |
| b) | $\boldsymbol{p} = 10 \times \begin{pmatrix} 30 \\ 40 \end{pmatrix}$ | $\text{displacement} = \text{velocity} \times \text{t}$ So the position vector of the sl after $10 \text{ hours} = \text{velocity vector} \times$ |
| | $\boldsymbol{p} = \begin{pmatrix} 300 \\ 400 \end{pmatrix}$ | |
| | So the coordinates are $(300, 400)$. | $300$ km east and $400$ km north the port. |

## International Mindedness

There is a global decline in the number of bees. Bees are very important for pollination of food crops — without them, many food crops would not grow.

Around the globe, wild flower gardens are being built in cities to provide suitable habitats for bees to help them to survive and to halt the decline. You could help by planting some wild flowers in a pot or in your garden.

Find out what kinds of plants are likely to attract bees. Try to observe their behaviour.

But how do bees know where the flowers are?

Watch the following video to find out the answer.

Student
view

e Waggle Dance of the Honeybee

# 6 section questions



# Unit vectors



You have already seen the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ which are used as base vectors.

A unit vector is a vector that is 1 unit long in a specified direction.

To find a unit vector you need to divide the components of a vector by its magnitude.

Consider the vector $\boldsymbol{v} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

This has magnitude $|\boldsymbol{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$.

To find a unit vector parallel to $\boldsymbol{v}$ you need to divide each component of $\boldsymbol{v}$ by $\sqrt{5}$.

Unit vectors are denoted by $\widehat{\boldsymbol{v}}$.

Student view

Overview
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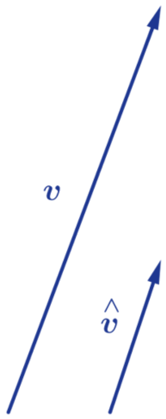
A unit vector parallel to $\widehat{\boldsymbol{v}} = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\0\\2\end{pmatrix}$ or $\begin{pmatrix}\frac{1}{\sqrt{5}}\\0\\\frac{2}{\sqrt{5}}\end{pmatrix}$

$$\widehat{\boldsymbol{v}} = \begin{pmatrix}\frac{1}{\sqrt{5}}\\0\\\frac{2}{\sqrt{5}}\end{pmatrix}$$




More information

The image depicts two vectors labeled (v) and (\hat{v}). The vector (v) is shown as a longer arrow pointing upwards and slightly to the right, while the norm of (v), denoted (\hat{v}), is depicted as a shorter arrow also pointing upwards and to the right. Both vectors originate from the same point at the bottom, suggesting a relationship in direction, but differing in magnitude.

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Student view

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✓ **Important**

For a vector $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, the unit vector in the same direction as $\boldsymbol{v}$ is

$$\widehat{\boldsymbol{v}} = \frac{1}{|\boldsymbol{v}|} \times \boldsymbol{v} = \frac{1}{|\boldsymbol{v}|} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

# Example 1

★☆☆

Find the unit vector in the direction of the vector $\boldsymbol{w} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$

This is given by

$$\widehat{\boldsymbol{w}} = \frac{1}{|\boldsymbol{w}|}\boldsymbol{w} = \frac{1}{\sqrt{(-2)^2 + 3^2 + 6^2}} \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} \frac{-2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix}$$

# Example 2

★★☆

Consider the points $\mathrm{A}(1, -3, 2)$ and $\mathrm{B}(-4, 0, 3)$.

Find

a) the vector $\overrightarrow{\mathrm{AB}}$

b) the distance between the points $\mathrm{A}$ and $\mathrm{B}$

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

c) the unit vector in the direction $\overrightarrow{\mathrm{AB}}$.

| Options | Explanation |
|---|---|
| a) | The position vectors of the two points are $$\overrightarrow{\mathrm{OA}} = \boldsymbol{a} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \text{ and}$$ $$\overrightarrow{\mathrm{OB}} = \boldsymbol{b} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$$ Then, $\overrightarrow{\mathrm{AB}} = -\boldsymbol{a} + \boldsymbol{b} = -\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$ |
| b) | The distance between points $\mathrm{A}$ and $\mathrm{B}$ is the magnitude of vector $\overrightarrow{\mathrm{AB}}$, i.e. $$\left\|\overrightarrow{\mathrm{AB}}\right\| = \left\|\begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}\right\| = \sqrt{(-5)^2 + 3^2 + 1^2} = \sqrt{35}$$ |
| c) | The unit vector in the direction of $\overrightarrow{\mathrm{AB}}$ is given by $$\widehat{\mathrm{AB}} = \frac{1}{\sqrt{35}}\begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$$ |

# Example 3

★★☆

Consider the points $\mathrm{A}(1, k, 2)$ and $\mathrm{B}(4, -2, 1)$.

If the length of the vector $\overrightarrow{\mathrm{AB}}$ is $\sqrt{46}$, find $k$ and then the unit vector in the direction $\overrightarrow{\mathrm{AB}}$.

Student view

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The position vectors of the two points are

$$\overrightarrow{\mathrm{OA}} = \boldsymbol{a} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} \text{ and } \overrightarrow{\mathrm{OB}} = \boldsymbol{b} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

Then,

$$\overrightarrow{\mathrm{AB}} = -\boldsymbol{a} + \boldsymbol{b} = -\begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2-k \\ -1 \end{pmatrix}$$

Thus, the distance between points A and B is the magnitude of vector $\overrightarrow{\mathrm{AB}}$, i.e.

$$\left|\overrightarrow{\mathrm{AB}}\right| = \left|\begin{pmatrix} 3 \\ -2-k \\ -1 \end{pmatrix}\right| = \sqrt{3^2 + (-2-k)^2 + (-1)^2} = \sqrt{10 + (2+k)^2}$$

And as it is given that $\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{46}$,we have

$$\begin{aligned} \sqrt{10 + (2+k)^2} &= \sqrt{46} \\ 10 + (2+k)^2 &= 46 \\ (2+k)^2 &= 36 \\ 2+k &= \pm 6 \\ k &= 4 \text{ or k} = -8 \end{aligned}$$

Thus, either

$$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ -6 \\ -1 \end{pmatrix} \text{ or } \overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix}$$

with relative unit vectors

Student view

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$$\widehat{\mathrm{AB}} = \frac{1}{\sqrt{46}}\begin{pmatrix}3\\-6\\-1\end{pmatrix} \text{ or } \widehat{\mathrm{AB}} = \frac{1}{\sqrt{46}}\begin{pmatrix}3\\6\\-1\end{pmatrix}$$

# Scaling

When you multiply a vector by a scale factor you create a new vector parallel to the original one. These vectors will have the same direction but different magnitudes. They will be scalar multiples of each other. Multiplying a vector by a scalar is called scaling.

✓ **Important**

When a vector $\boldsymbol{a}$ is multiplied by a scalar $k$, the resultant vector is $\boldsymbol{v} = k\boldsymbol{a}$ which is parallel to $\boldsymbol{a}$ and has a magnitude $|\boldsymbol{v}| = k\,|\boldsymbol{a}|$. If $k < 0$, then the direction of $\boldsymbol{v}$ is opposite to $\boldsymbol{a}$.

If you need to find a vector in the same direction as a given vector but with a different magnitude, you can find a unit vector in the required direction and multiply it by a suitable scale factor. This process is called rescaling.

# Example 4

★★☆

Find a vector $2$ units long in the direction of $\begin{pmatrix}1\\1\\1\end{pmatrix}$

| Steps | Explanation |
|---|---|
| $\boldsymbol{v} = \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \lvert\boldsymbol{v}\rvert = \sqrt{3}$ | |
| $\hat{\boldsymbol{v}}$ | This is the unit vector in the direction of $\boldsymbol{v}$. |

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

| Steps | Explanation |
|---|---|
| $\boldsymbol{w} = 2 \times \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ | This is a vector in the same direction as the unit vector with a magnitude of 2. |
| The required vector is $\begin{pmatrix} \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix}$ | |

## 3 section questions

3. Geometry and trigonometry / 3.10 Vectors

# Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-28356/print/) Assign

### What you should know

By the end of this subtopic you should be able to:

- recall that vectors have a size (magnitude) and a direction
- represent a displacement between two points as a vector
- decompose a 2D vector into its components in the $x$- and $y$-directions and a 3D vector into its components in the $x$-, $y$- and $z$-directions
- write a vector in column form, e.g. $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ or base vector form $\boldsymbol{v} = v_2\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit base vectors in the $x$-, $y$- and $z$-directions, respectively:
$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Student view

- recall that two vectors are equal if and only if all their components are equal
- add vectors by adding their components: e.g. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\boldsymbol{v} + \boldsymbol{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_3 + w_3 \end{pmatrix}$
- recall that addition of vectors is commutative: $\boldsymbol{v} + \boldsymbol{w} = \boldsymbol{w} + \boldsymbol{v}$
- recall that if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, then $-\boldsymbol{v} = -\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \boldsymbol{v} = \begin{pmatrix} -v_1 \\ -v_2 \\ -v_3 \end{pmatrix}$
- recall that $\boldsymbol{v} + (-\boldsymbol{v}) = \mathbf{0}$, where $\mathbf{0}$ is the zero vector
- recall how to subtract one vector from another by adding the negative components of the vector being subtracted: i.e. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w}) \begin{pmatrix} v_1 - w_1 \\ v_2 - w_2 \\ v_3 - w_3 \end{pmatrix}$
- recall that subtraction of vectors is not commutative: i.e. $\boldsymbol{v} - \boldsymbol{w} \neq \boldsymbol{w} - \boldsymbol{v}$
- multiply a vector by a scalar, e.g. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is multiplied by scalar $k \in \mathbb{R}$ the result is $k\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
- recall that $k\boldsymbol{v}$ is a vector in the same direction as $\boldsymbol{v}$ but with a different magnitude (unless $k = \pm 1$ )
- recall that two vectors are parallel if they are scalar multiples of the same vector: i.e. $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel if there exist $m \in \mathbb{R}$ and $n \in \mathbb{R}$ such that $\boldsymbol{v} = m\boldsymbol{u}$ and $\boldsymbol{w} = n\boldsymbol{u}$
- find the magnitude of a vector using Pythagoras' theorem: if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, the magnitude of $\boldsymbol{v}$ is denoted by $|\boldsymbol{v}|$ and is given by $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- recall the properties of the magnitude of a vector:
  - $|\boldsymbol{v}| \geqslant 0$, i.e. it is never negative
  - $|k\boldsymbol{v}| = |k||\boldsymbol{v}|, k \in \mathbb{R}$
  - $|\boldsymbol{v}| = |-\boldsymbol{v}|$
  - in general, $|\boldsymbol{v} + \boldsymbol{w}| \neq |\boldsymbol{v}| + |\boldsymbol{w}|$
- recall that a unit vector in a specified direction is denoted by $\hat{\boldsymbol{v}}$

Overview
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- calculate a unit vector in the direction of a given vector using $\hat{\boldsymbol{v}} = \frac{1}{|\boldsymbol{v}|}\boldsymbol{v}$
- write the position vector of point $\mathrm{A}$ relative to a fixed origin $\mathrm{O}$: if point $\mathrm{A}$ has coordinates $(x, y, z)$ then the position vector of $\mathrm{A}$ is denoted by $\overrightarrow{\mathrm{OA}} = \boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$
- recall that the displacement vector between two points $\mathrm{A}$ and $\mathrm{B}$ is given in terms of their position vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$ and $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ by $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = -\boldsymbol{a} + \boldsymbol{b}$
- calculate displacement vectors between two points: e.g. if $\mathrm{A}$ has coordinates $(x_A, y_{A,} z_A)$ and $\mathrm{B}$ has coordinates $(x_B, y_{B,} z_B)$ then $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix}$.

3. Geometry and trigonometry / 3.10 Vectors

# Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-28357/print/) Assign

In a chess game, the knight is one of the powerful pieces as it can hop over other pieces and move freely from black squares to white squares.

However, its moves follow a certain rule: They must be L shaped. A knight can move two squares right/left and one square up/down, or one square right/left followed by two squares up/down.

The knight in the diagram below, could move two squares to the right and one square up to reach square A, or two squares left and one square up to reach square B.

Student view

Overview
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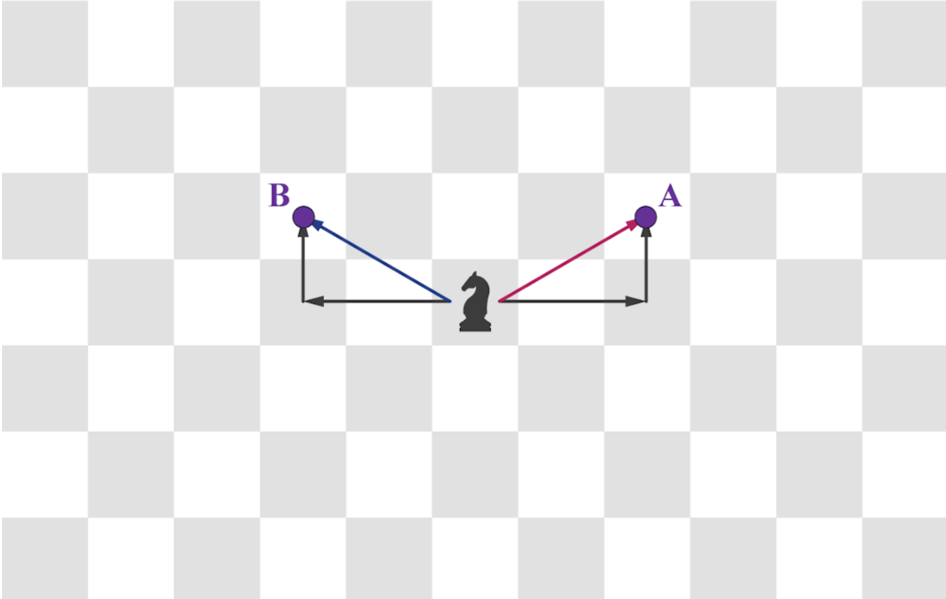


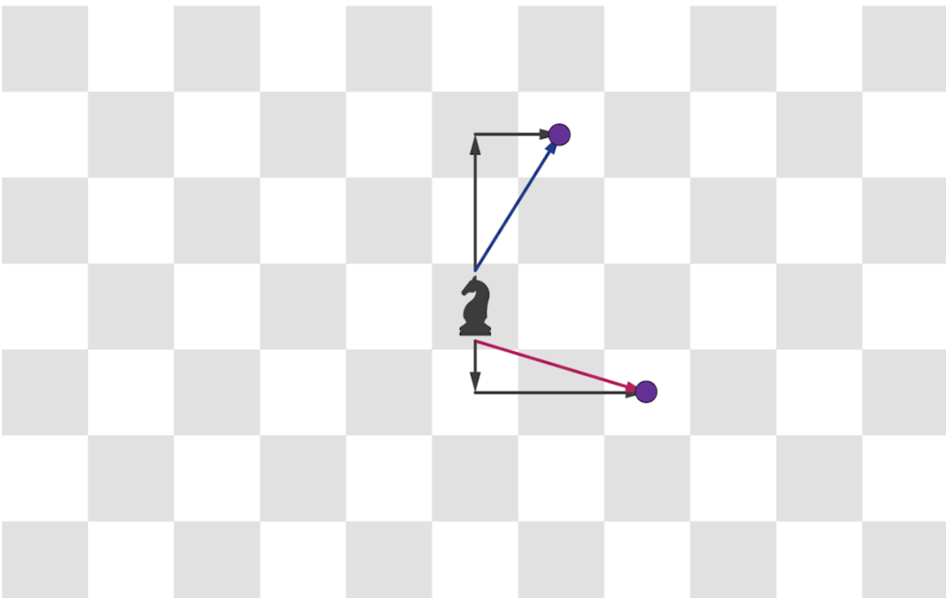

More information

The image is a diagram of a chessboard with light and dark squares. In the center of the board is a knight piece. Two vectors are illustrated: one leading to square A and another to square B. Square A is reached by moving two squares to the right and one square up, shown by a red arrow with directions annotated. Square B is reached by moving two squares left and one square up, indicated by a blue arrow with directions annotated. Each vector has labeled points and arrows to show the knight's movement paths.

[Generated by AI]

How can you represent these two moves using vectors?

How can you represent the moves shown in the diagram below?



Student view

Overview
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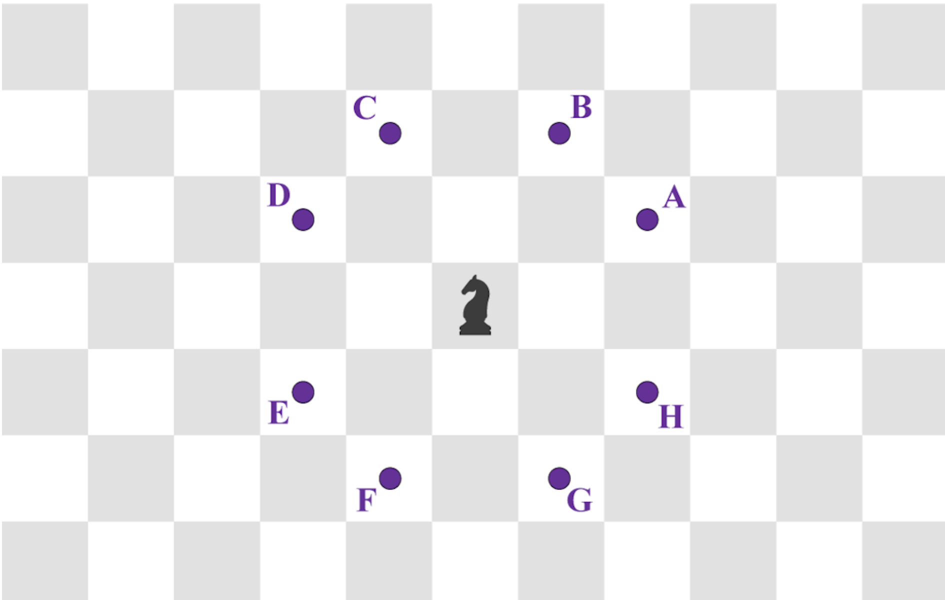
More information

The diagram shows a chessboard with a knight piece in the center. Two possible moves of the knight are represented with arrows. The knight can move to two squares away horizontally and one square vertically, or two squares away vertically and one square horizontally. The paths are indicated with colored arrows demonstrating the L-shape movements typical of a knight in chess. The board is a standard 8x8 grid, and the knight stands on one square with arrows pointing to two possible destination squares.

[Generated by AI]

All the moves a knight can make are shown in the diagram below.




More information

The diagram illustrates a chessboard section with a knight positioned at the center. The board consists of alternating white and gray squares. The knight's traditional potential moves are marked with purple dots labeled A through H. These marks are placed in an 'L' shaped pattern around the knight, demonstrating the two squares by one square movement that knight pieces make in chess. The potential move squares are positioned at various points surrounding the knight: A is to the right, B is to the top-right, C is directly above, D is to the top-left, E is to the left, F is directly below, G is to the bottom-right, and H is to the bottom-left of the knight's central position. This pattern demonstrates the unique mobility of the knight piece in a chess game.

[Generated by AI]

Student
view

Overview
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In a chess board, there are $8 \times 8 = 64$ black and white squares. If a knight starts at one of the corners of the board, what is the smallest number of moves it needs to make to reach the corner which is diagonally opposite?

**Rate subtopic 3.10 Vectors**

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