Index

1. Number and algebra / 1.3 Geometric sequences and series

The big picture

Have you ever wondered how scientists work out the age of a fossil, or how they estimate when Stonehenge was created?

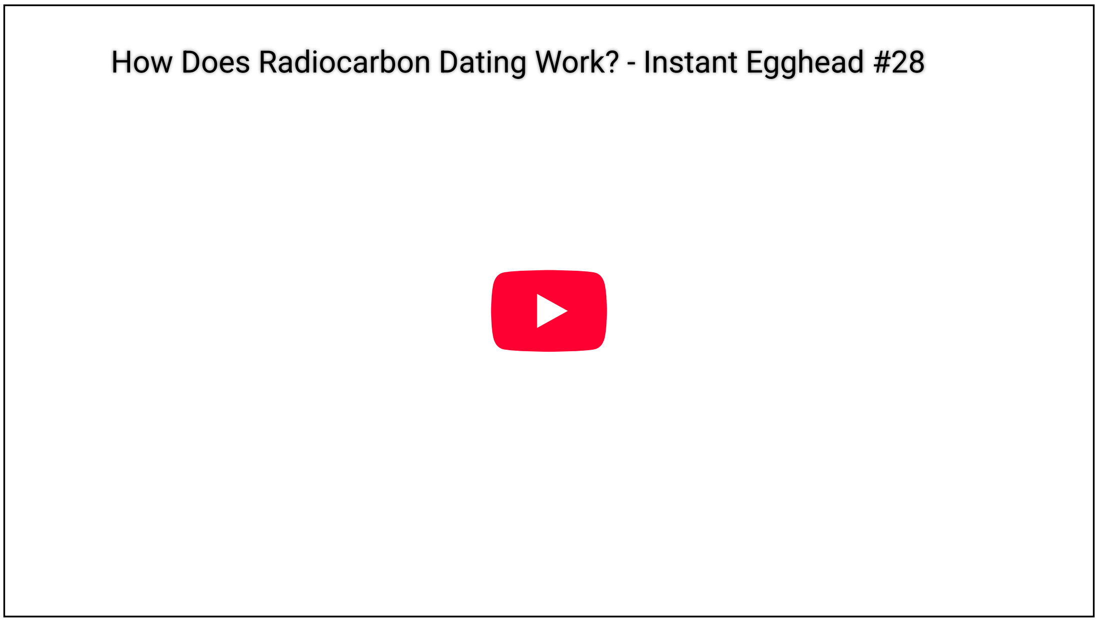
Example of a fossil

Credit: GoodLifeStudio Getty Images

Stonehenge

Credit: Ahrys Art Getty Images

In both cases a technique called carbon dating is used. Watch the video to see how it works.



Carbon dating is based on the fact that half of the amount of carbon-14 present in a sample or artefact decays within a predicable time period of 5700 years. This period is called the half-life. If you start with 200.0 grams of carbon-14 you will observe the pattern shown in the following table.

Overview
(/study/app/sid-122-cid-754029/

Time (years)	Amount of carbon-14 (grams)
5700	100.0
$11\ 400$	50.0
$17\ 100$	25.0
$22\ 800$	12.5

This pattern is an example of a geometric sequence which you will study in this subtopic.

Concept

In this subtopic, you will learn how to use geometric sequences to model growth and decay of populations, salaries and chemical compounds. Reflect on whether these models are always valid when applied to these scenarios, or whether there are limitations such as the time period within which the models are applicable.

1. Number and algebra / 1.3 Geometric sequences and series

Geometric sequences

Activity

Draw a square with an area of $16\ \text{cm}^2$ and label its vertices as A, B, C, and D.

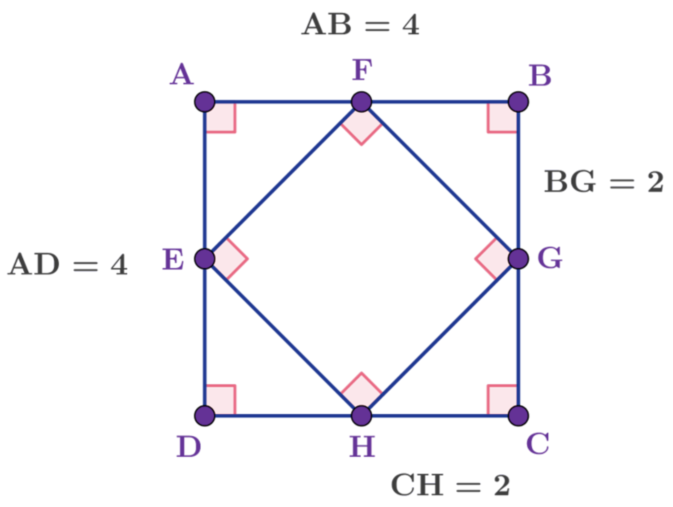
Draw a new square by connecting the midpoints of the sides of square ABCD, as shown in the diagram below. Continue this pattern until you have drawn a total of six squares.

Find the area of each square and examine your results; comment on any patterns that you notice.

Predict the area of the 10th square and check your prediction.

Student view

Overview
(/study/app/sid-122-cid-754029/



More information

The image is a geometric diagram showing a square with side length labeled as AB = 4, AD = 4, BG = 2, and CH = 2. Points A, B, C, D, E, F, G, and H are marked at each corner and along the sides. Inside the square, diagonal lines form a diamond shape connecting points E, F, G, and H. Red squares are placed in each corner between the diagonal and square sides. The structure illustrates the division of the square into smaller sections.

[Generated by AI]

In the previous Activity, each time a new square is drawn, the area of the bigger square is divided into two equal parts. Thus, the areas of the squares form the following sequence $16,\ 8,\ 4,\ 2,\ 1,\ \ldots$, which can be described as $u_n = \frac{1}{2} \times u_{n-1}$.

This is an example of a geometric sequence .

Important

In a geometric sequence, each new term is generated by multiplying the previous term by a constant called the common ratio and represented by the letter r.

$$r = \frac{u_n}{u_{n-1}}$$

Student view

Example 1

★☆☆

Determine whether or not each of the following sequences is geometric.

Find the common ratio if the sequence is geometric.

a) $-2,\ 6,\ -18,\ 54,\ \ldots$

b) $1,\ 2,\ 8,\ 64, \ldots$

c) $3,\ \frac{3}{4},\ \frac{3}{16},\ \frac{3}{64},\ldots$

d) $5x^2,\ 5x^4,\ 5x^6,\ 5x^8, \ldots$

	Steps	Explanation
a)	$\frac{54}{-18} = -3$ $-\frac{18}{6} = -3$ $\frac{6}{-2} = -3$	The ratio is the same for each pair of terms. To get each new term you multiply the previous one by -3.
	Geometric sequence $r = -3$	
b)	$\frac{64}{8} = 8$ $\frac{8}{2} = 4$ $\frac{2}{1} = 2$	There is no common ratio.
	Not geometric.	

Overview (/study/app/sid-122-cid-754029/

	Steps	Explanation
c)	$\dfrac{\frac{3}{64}}{\frac{3}{16}} = \dfrac{3}{64} \times \dfrac{16}{3} = \dfrac{1}{4}$ $\dfrac{\frac{3}{16}}{\frac{3}{4}} = \dfrac{3}{16} \times \dfrac{4}{3} = \dfrac{1}{4}$ $\dfrac{\frac{3}{4}}{3} = \dfrac{1}{4}$	The ratio is the same for each pair of terms. To get each new term you multiply the previous one by $\frac{1}{4}$.
	Geometric sequence $r = \frac{1}{4}$	
d)	$\dfrac{5x^8}{5x^6} = x^2$ $\dfrac{5x^6}{5x^4} = x^2$ $\dfrac{5x^4}{5x^2} = x^2$	The ratio is the same for each pair of terms. To get each new term you multiply the previous one by x^2.
	Geometric sequence $r = x^2$	

Section Student... (0/0) Feedback Print (/study/app/preview-p/sid-122-cid-754029/book/geometric-sequences-id-26120/print/) Assign

Activity

Let the first term of a geometric sequence be u_1 and the common ratio r.

List the first six terms of this sequence in terms of $u_1$and r.

1. Describe any patterns that you notice.
2. Generalise your findings by writing an expression for u_n in terms of $u_1$and r.
3. Test your generalisation by using the geometric sequences from **Example 1.**

The Activity above shows that the general term of a geometric sequence is given by

Student view

$$u_n = u_1 \times r^{n-1}$$

Overview
(/study/app/sid-122-cid-754029/

Exam tip

You do not need to memorise $u_n = u_1 r^{n-1}$.

The formula for the general term of a geometric sequence is given in the IB formula booklet.

Example 2

★★☆

A geometric sequence has a first term of 81 and a fourth term of 3. Find the common ratio and an expression for the general term.

Steps	Explanation
$u_1 = 81$ $u_4 = 81r^3 = 3$	Using $u_n = u_1 r^{n-1}$.
$81r^3 = 3 \quad \Leftrightarrow \quad r^3 = \frac{3}{81} \quad \Leftrightarrow \quad r = \sqrt[3]{\frac{3}{81}}$ $r = \frac{1}{3}$	Solve for r.
$u_n = 81\left(\frac{1}{3}\right)^{n-1}$	Use $u_1 = 81$ and $r = \frac{1}{3}$.

Example 3

★☆☆

Show that a sequence with three consecutive terms $75,\ 375,\ 1875$ is geometric.

Student view

Overview (/study/app/sid-122-cid-754029/

Steps	Explanation
$r = \frac{1875}{375} = 5$ $r = \frac{375}{75} = 5$	Check that $r = \frac{u_n}{u_{n-1}}$ gives the same common ratio for all consecutive terms.
The r value is equal for all pairs of consecutive terms, so the sequence is geometric.	

Example 4

★★★

A geometric sequence has a second term of 30 and a fifth term of -468.75 .

Find the common ratio, first term and general term.

Steps	Explanation
$u_2 = u_1 \times r = 30$ $u_5 = u_1 \times r^4 = -468.75$	Write equations for u_2 and u_5.
$\frac{u_1 \times r^4}{u_1 \times r} = \frac{-468.75}{30} \quad \Leftrightarrow \quad r^3 = -\frac{468.75}{30}$ $r = \sqrt[3]{-\frac{468.75}{30}} = -2.5$	Solve the system of equations by dividing u_5 by u_2.
$u_1 \times (-2.5) = 30$ $u_1 = \frac{30}{-2.5} = -12$	Use $r = -2.5$ in the equation for u_2 to solve for u_1.

Student view

Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
$u_n = -12 \times (-2.5)^{n-1}$	Do not forget to put negative r values in brackets.

Example 5

★★★

A geometric sequence has a second term of 14 and a sixth term of 224.

Find the possible values of the common ratio and the first term.

Steps	Explanation
$u_2 = u_1 \times r = 14$ $u_6 = u_1 \times r^5 = 224$	Write equations for u_2 and u_6.
$\frac{u_1 \times r^5}{u_1 \times r} = \frac{224}{14} \quad \Leftrightarrow \quad r^4 = \frac{224}{14} = 16$ $r = \pm\sqrt[4]{16} = \pm 2$	Solve the system of equations by dividing u_6 by u_2. There are two possible values of r.
$u_1 \times r = 14 \quad \Leftrightarrow \quad u_1 = \frac{14}{r}$ $u_1 = \frac{14}{-2} = -7 \; or \; u_1 = \frac{14}{2} = 7$	There are two possible first terms.

In the following applet, you can practise questions similar to the examples above with numbers given up to 2 decimal places. However, in the exams you may be given the numbers as fractions and be asked for answers in the form of fractions (exact values).

Student view

Overview
(/study/ap
p/sid-
122-
cid-
754029/

Interactive 2. Geometric Sequence Practise Questions.

More information for interactive 2

This interactive is designed to help users practice solving problems related to geometric sequences. Random questions are generated by providing two specific terms of a geometric sequence with only positive terms. Users are tasked with finding the common ratio and the first term based on the given terms.

Users can generate new practice questions by clicking the "New Question" button, which presents a different geometric sequence problem based on two given terms. After attempting to solve for the common ratio and first term, they can click "Show Answer" to reveal the correct solution.

For example, when a user clicks New Question, they might see:

“A geometric sequence has only positive terms, and

$u_2 = 4, u_9 = 4.7$

Find the common ratio and the first term.”

Solution:

Using the formula for the n^{th} term of a geometric sequence:

$$u_n = u_1 r^{n-1}$$

Step 1: Set Up the Equations

For $u_2 = 4$: $u_1 r^{2-1} = 4 \implies u_1 r = 4$........................(1)

For $u_9 = 4.7$: $u_1 r^{9-1} = 4.7 \implies u_1 r^8 = 4.7$............................(2)

Step 2: Solve for the Common Ratio r

Divide equation (2) by equation (1):

$$\frac{u_1 r^8}{u_1 r} = \frac{4.7}{4} \implies r^7 = 1.175 \implies r \approx 1.023$$

Student
view

Overview
(/study/app/sid-122-cid-754029/

Step 3: Solve for the First Term (u_1)

Substitute $r \approx 1.023$ into equation (1):

$$u_1 \cdot 1.023 = 4 \implies u_1 \approx 3.91$$

Final Answer:

$r \approx 1.023$ and $u_1 \approx 3.91$

After solving, clicking "Show Answer" will display:

“The common ratio is $r = 1.023$.

The first term is $u_1 = 3.91$.”

If the user clicks “New Question” again, another example might appear with different values.

By generating new problems and providing immediate feedback, the tool allows users to repeatedly practice finding these key sequence properties, reinforcing their understanding through varied examples while developing problem-solving skills for geometric sequences. The restriction to positive terms ensures all calculations remain within real number solutions, making it particularly suitable for introductory practice with geometric progressions.

Example 6

★★★

The general term of a sequence is given by $u_n = 5^{3-4n}$.

Determine whether or not this is a geometric sequence.

Steps	Explanation
$r = \frac{5^{3-4n}}{5^{(3-4(n-1))}} = \frac{5^{3-4n}}{5^{7-4n}}$	Use $r = \frac{u_n}{u_{n-1}}$.
$r = \frac{5^{3-4n}}{5^{7-4n}} = 5^{3-4n-(7-4n)} = 5^{-4}$	Use the exponent rule for division.
Since $r = 0.0016$ is constant for all values of n, the sequence is geometric.	

Student view

Overview
(/study/app/sid-122-cid-754029/

Example 7

★★★

Given that the sequence $x-1,\ x+2,\ 3x,...$ is geometric, find the possible values of x.

Steps	Explanation
$\frac{3x}{x+2}=\frac{x+2}{x-1}$	Since the sequence is geometric $\frac{u_3}{u_2}=\frac{u_2}{u_1}=r$
$\frac{3x}{x+2}=\frac{x+2}{x-1}$ $3x\left(x-1\right)=\left(x+2\right)\left(x+2\right)$ $3x^2-3x=x^2+4x+4$ $2x^2-7x-4=0$	Solve for x. Cross multiply so that your equation no longer has fractions. Solve the quadratic equation by using the quadrat formula or your calculator.
$x=-\frac{1}{2}$ or $x=4$	There are two possible geometric sequences. When $x=-\frac{1}{2}$, the sequence is $-\frac{3}{2},\frac{3}{2},-\frac{3}{2},\ldots$ where $r=-1$. When $x=4$, the sequence is $3,6,12,\ldots$where $r=2$.

Be aware

If you are given consecutive terms of a sequence and told that the sequence is geometric, then you can use $\frac{u_n}{u_{n-1}}=\frac{u_{n-1}}{u_{n-2}}$, as seen in **Example 7.**

5 section questions

Student view

1. Number and algebra / 1.3 Geometric sequences and series

Geometric series

A geometric series is the sum of the terms in a geometric sequence.

The sum of the first n terms of a geometric sequence, S_n, is given by two formulae, each of them useful in different circumstances.

Important

The sum of a geometric sequence is given by

$$S_n = \frac{u_1\left(r^n - 1\right)}{r - 1} = \frac{u_1\left(1 - r^n\right)}{1 - r},\ r \neq 1.$$

The proof for the formulae of the sum of n terms of a geometric series is explored in the following video.

This proof is often described as being elegant. Why do you think that is?

Overview
(/study/app/sid-122-cid-754029/

Video 1. Understanding Geometric Series: A Complete Guide.

More information for video 1

1

00:00:00,067 --> 00:00:01,433

narrator: In this video,

we're going to prove

2

00:00:01,500 --> 00:00:03,700

the finite sum of geometric series,

3

00:00:03,767 --> 00:00:09,233

which was given by $s_n 2 \frac{u_1(r^n-1)}{(r-1)}$.

4

00:00:09,833 --> 00:00:13,333

Now in general, of course,

the sum of finite series

5

00:00:13,400 --> 00:00:18,333

is given by $S_n = u_1 + u_2 +$

all the way up to $u_{n-1} + u_n$.

Student view

Overview (/study/app/sid-122-cid-754029/

6

00:00:19,100 --> 00:00:21,867

So for our geometric series,

let's write this down,

7

00:00:21,933 --> 00:00:26,533

what this means, $S_n = u_1 + u_1 r$,

8

00:00:26,600 --> 00:00:30,667

which is $u_2 + \ldots + u_1 r^{(n-2)}$

9

00:00:30,733 --> 00:00:33,167

to the $n - 2$,

which is the $n - 1$th terms,

10

00:00:33,233 --> 00:00:36,900

and $u_1 r^{n-1}$,

which is the nth term.

11

00:00:37,033 --> 00:00:39,533

Now, let's write this again,

well multiplied by r

12

00:00:39,633 --> 00:00:41,033

and let's displace everything.

13

00:00:41,100 --> 00:00:45,667

So we've displaced the series

while multiplying it

14

00:00:45,733 --> 00:00:50,500

by a factor of r,

so that we end up with this

15

00:00:50,667 --> 00:00:52,567

series above it

16

00:00:52,633 --> 00:00:55,167

Student view

Overview (/study/app/sid-122-cid-754029/

and you can see some

of the alignments happening.

17

00:00:55,233 --> 00:00:58,967

So the first term

in this case is $u_1 \times r$,

18

00:00:59,033 --> 00:01:01,433

all the way up to u_1

times r through power of n,

19

00:01:01,500 --> 00:01:04,467

because we've multiplied the series by r.

20

00:01:04,933 --> 00:01:07,000

And now let's take a difference

of those two series.

21

00:01:07,067 --> 00:01:09,867

So we've got $rS_n - S_n$,

22

00:01:09,933 --> 00:01:12,200

and now you can see that

because we line them up,

23

00:01:12,267 --> 00:01:16,600

it's easy to see that those cancellations

are going to occur everywhere

24

00:01:17,600 --> 00:01:20,233

from the second to the four last one.

25

00:01:20,533 --> 00:01:25,500

The only ones remaining as

$u_1 r^n - u_1$.

26

00:01:26,100 --> 00:01:31,167

So in other words, $S_n(r - 1)$

Student view

Overview
(/study/app/sid-122-cid-754029/

is $u_1(r^n - 1)$.

27

00:01:31,233 --> 00:01:37,200

In other words, $S_n = \frac{u_1(r^n-1)}{(r-1)}$,

28

00:01:37,667 --> 00:01:39,500

which is what we needed to prove.

29

00:01:39,700 --> 00:01:43,700

And that is then the proof

of the finite sum of a geometric series,

30

00:01:43,767 --> 00:01:46,733

A rather pleasing method to prove it.

Exam tip

Both formulae for the sum of the first n terms of a geometric series are in the formula booklet.

Example 1

★★☆

Find the sum of the first eight terms of the geometric sequence defined by $u_n = \frac{24}{5} \times \left(-\frac{5}{2}\right)^n$.

Student view

Overview (/study/app/sid-122-cid-754029/

Steps	Explanation
$u_1 = \frac{24}{5} \times \left(-\frac{5}{2}\right)^1 = -12$	Note, that u_1 is not $\frac{24}{5}$, because in $u_n = \frac{24}{5} \times \left(-\frac{5}{2}\right)^n$ the exponent is n and not $n-1$.
$r = -\frac{5}{2}$ and $n = 8$ $S_8 = \frac{-12\left(1-\left(-\frac{5}{2}\right)^8\right)}{1-\left(-\frac{5}{2}\right)} = 5228.16\ldots = 5230$ (to 3 significant figures)	Use $S_n = \frac{u_1(1-r^n)}{1-r}$.

Activity

$4,\ 4,\ 4,\ \ldots$ is a geometric sequence. Find the common ratio and explain why this is a geometric sequence.

Find the sum of the first ten terms of the sequence. What do you notice about the application of the formula for the sum of a geometric series to this question?

Can the sum of the first ten terms be found without using $S_n = \frac{u_1(1-r^n)}{1-r}$ or $S_n = \frac{u_1(r^n-1)}{r-1}$?

Example 2

★★☆

The sum of the first ten terms of a geometric sequence is 1364, and two consecutive terms are 32 and -64. Calculate the value of the first term.

Student view

Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
$r = -\frac{64}{32} = -2$	Use $r = \frac{u_n}{u_{n-1}}$.
$S_{10} = \frac{u_1\left(1-(-2)^{10}\right)}{1-(-2)} = 1364$	Write an equation for the sum of the first ten terms.
$\frac{u_1(1-1024)}{3} = 1364$ $\frac{-1023}{3} \times u_1 = 1364$ $-341 \times u_1 = 1364$ $u_1 = \frac{1364}{-341} = -4$	Solve for u_1.

Exam tip

Remember that $u_1 = S_1$ and $u_2 = S_2 - S_1$.

This can be generalised to $u_n = S_n - S_{n-1}$.

You used this result before in the section on arithmetic series.

3 section questions

1. Number and algebra / 1.3 Geometric sequences and series

Applications

Section Student... (0/0) Feedback Print (/study/app/preview-p/sid-122-cid-754029/book/applications-id-26122/print/) Assign

Student view

Overview (/study/app/sid-122-cid-754029/

Applications of geometric sequences and series

Population growth or decline

When a population grows by a fixed percentage it can be modelled using a geometric sequence.

If you start with a population of $150\ 000$ people, that grows at 12% each year, then after 1 year the population will be 112% of the original, so it is given by

$$150\ 000 \times 1.12 = 168\ 000.$$

After 2 years the population will be 112% of the population in the previous year, so it will be

$$168\ 000 \times 1.12 = 188\ 160.$$

When you continue this pattern, you see that it follows the pattern of a geometric progression with $r = 1.12$.

How would you need to modify this working if instead of growing by 12% the population decreased by 12% each year?

Example 1

★★☆

The population of a city is $3\ 456\ 000$ at the end of 2019.

Find the population at the end of 2021, if it is decreasing by 5% each year.

Hint: The original population is 100%.

Student view

Overview
(/study/ap
p/sid-
122-
cid-
754029/

Steps	Explanation
End of 2020: $3\ 456\ 000 \times 0.95 = 3\ 283\ 200$ End of 2021: $3\ 283\ 200 \times 0.95 = 3\ 119\ 040$	Decrease in population means that you will multip by $1 - 0.05 = 0.95$ for each year.
The population at the end of 2021 is $3\ 119\ 040$.	

Since population growth follows a geometric progression, you can use the expression for the general term as your model.

However, you are usually given an initial population. If you use that number for u_1 in $u_n = u_1 r^{n-1}$ then the number of each term in the sequence will not be the same as the amount of time that has passed – it will be one more. For instance, $u_3$will correspond to the population after 2 years, and u_{10} to the population after 9 years. This can prove inconvenient and confusing but can be easily fixed, as shown below.

✓ **Important**

When you use $u_n = u_1 r^{n-1}$ to model population changes, you can modify this equation to

$$u_n = u_0 r^n$$

where

- u_n is the population after n time periods (years, weeks, etc.)
- u_0 is the initial population
- r is common ratio
- n is the number of time periods that passed (years, weeks, etc.)

Student
view

Overview
(/study/app/sid-122-cid-754029/

Example 2

★★☆

At the start of an experiment there are 251 song birds in a state park. Scientists observe that the population of these birds grows by 4% each year.

a) Use an appropriate model to predict how many birds will be in the park after 8 years.

b) Use your model to predict the number of birds that will be in the park after 100 years.

	Steps	Explanation
a)	$u_n = u_0 r^n$ $u_n = 251 \times (1.04)^n$	Geometric model for the population increase.
	$u_8 = 251 \times (1.04)^8 = 343.511 \approx 344$	You are asked for the population after 8 yea so $n = 8$.
b)	$u_{100} = 251 \times (1.04)^{100} \approx 12\ 700$	

Making connections

You will study exponential growth and decay models in one of the future topics. These models are written in the form of $y = a\ b^x$ which is very similar to $u_n = u_0\ r^n$.

Growth and decay in chemistry

In the big picture section, you were introduced to the idea of carbon dating using the half-life of carbon-14.

Here is how you can model this situation using geometric sequences:

Student view

Overview
(/study/app/sid-122-cid-754029/

Be aware

The time period for the rate of growth or decay will vary in application questions.

Read each question carefully and note any information related to time.

Example 3

★★☆

The half-life of carbon-14 is $5\ 700$ years. This is how long it takes for half of the amount of carbon-14 present in a sample to decay. A sample initially contains 127 grams of carbon-14. Calculate how much carbon-14 remains in the sample after $22\ 800$ years.

Steps	Explanation
$n = \frac{22\ 800}{5700} = 4$ $r = \frac{1}{2}$	n corresponds to the number of half lives. The amount is multiplied by $\frac{1}{2}$ for each half-life.
$u_n = u_0 \times \left(\frac{1}{2}\right)^n$ $u_4 = 127 \times \left(\frac{1}{2}\right)^4 = 7.94$	Using the population model with initial amount, $u_0 = 127$. Evaluate and round the final answer to significant figures.
After $22\ 800$ years there are 7.94 grams of carbon-14 remaining.	

Activity

Use the model $u_n = 127 \times \left(\frac{1}{2}\right)^n$ from **Example 3** to calculate the amount of carbon-14 remaining after

a) $28\ 500$ years

Student view

Overview (/study/app/sid-122-cid-754029/

b) $57\ 000$ years

c) $114\ 000$ years

d) $570\ 000$ years.

Is it possible to find that there is no carbon-14 remaining using this model?

How can this model be used to approximate how many years it will take for all of the carbon-14 to decay?

Growth and decay of salaries

Example 4

★★☆

A British company offers an employee a salary that starts at $\text{GBP } 45\ 000$ and increases by 5% after each year of employment.

a) Calculate the annual salary after working in the company for 7 years. Give your answer correct to the nearest pound.

b) Calculate the total amount of money earned by an employee if they work at the company for 7 years. Give your answer correct to the nearest pound.

c) Hence, determine whether a worker would earn more money if they accept this job for 7 years or another offer that initially pays $\text{GBP } 50\ 000$ and increases annually by 2%.

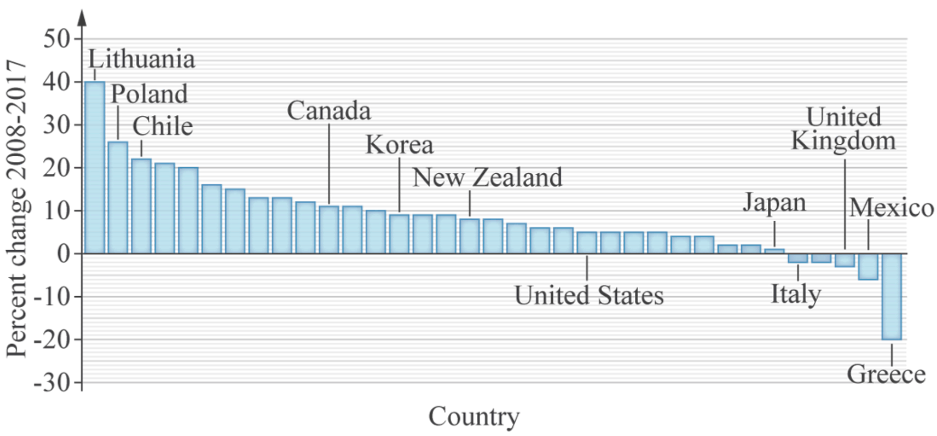
	Steps	Explanation
a)	$u_8 = 45\ 000\ (1.05)^7 = 63\ 319.51902 \approx 63\ 320$	$u_1 = 45\ 000$ is the sala the first year. $r = 1.05$ is the commo ratio of the sequence o salaries. After working for 7 year the question asks the sa in the 8th year.

Student view

Overview
(/study/ap
p/sid-
122-
cid-
754029/

	Steps	Explanation
	After working for 7 years the salary will be $63\ 320$.	
b)	$S_n = \frac{u_1(r^n - 1)}{r - 1}$ $S_7 = \frac{45\ 000(1.05^7 - 1)}{1.05 - 1} = 366\ 390.3804 \approx 366\ 390$	You are asked for the to amount of money. You n to add $u_1 + u_2 + \ldots + u_7 =$
	The total amount earned over 7 years is $366\ 390$.	
c)	$S_7 = \frac{50\ 000(1.02^7 - 1)}{1.02 - 1} = 371\ 714.1691 \approx 371\ 714$	You are comparing both over the 7 year period s you need to compare th total amount earned for each.
	The alternative job would pay $371\ 714$ over the 7 years which is more than the original job considered. A worker would earn more money in total if they take the alternative job.	

International Mindedness



Annual percentage change in wages for OECD countries.
Source: OECD (https://www.oecd-ilibrary.org/social-issues-migration-health/data/oecd-employment-and-labour-market-statistics/average-annual-wages_data-00571-en)

More information

Student
view

The bar chart displays the annual percentage change in wages from 2008 to 2017 for various OECD countries. The X-axis represents different countries, and the Y-axis shows the percent change in wages, ranging from -30% to 50%. Lithuania shows the highest increase, close to 40%, followed by Ireland and Chile with increases over 20%. Canada, Korea, and New Zealand also depict notable wage growths, all above 10%. In contrast, countries like Japan, Italy, Mexico, and Greece illustrate wage declines, with Greece experiencing a significant drop near -30%. The data highlights wage disparities and trends across these countries during the specified period.

[Generated by AI]

Although annual wages tend to increase, this is not always the case. Examine the above graph to see how wages changed in several countries from 2017 to 2018. Think about how you would have to approach planning for the future in a country where wages are declining.

Many application questions require you to find an unknown time. To do this you need to solve an equation in which the unknown is in the exponent position. In a later subtopic, you will learn how to solve this kind of equation algebraically using logarithms, but for now you will need to use your graphic display calculator.

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>

These are steps you can follow to answer an extension of **Example 2**. You will see how to use the calculator to find the time it takes until the number of birds in the state park reach 500.

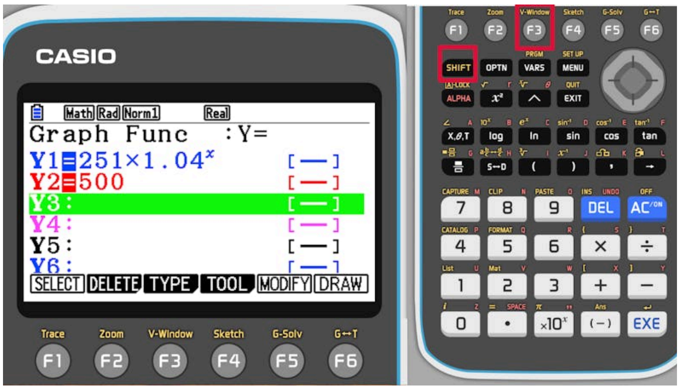
To start, enter the graphics mode of your calculator.

</td><td>

</td></tr>
<tr><td>

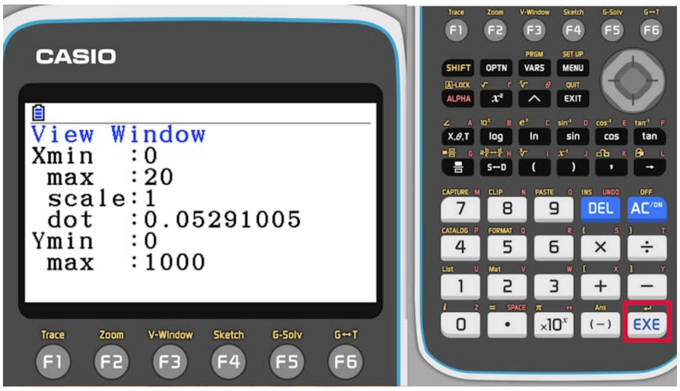
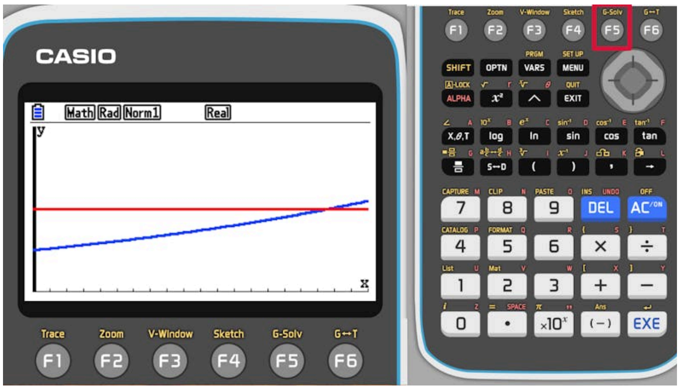
After entering the function that describes the increase of birds and the target number, choose the option (V-Window) to set the viewing window.

</td><td>



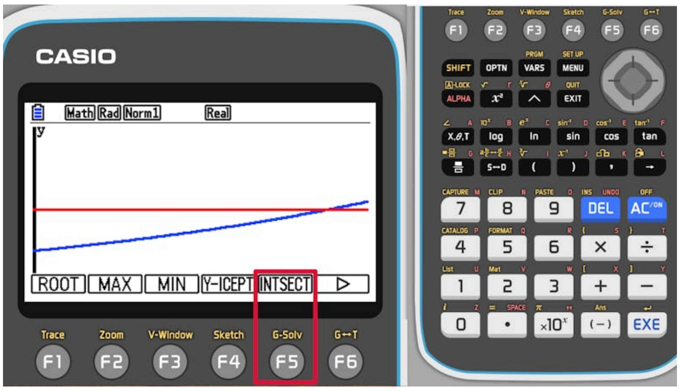
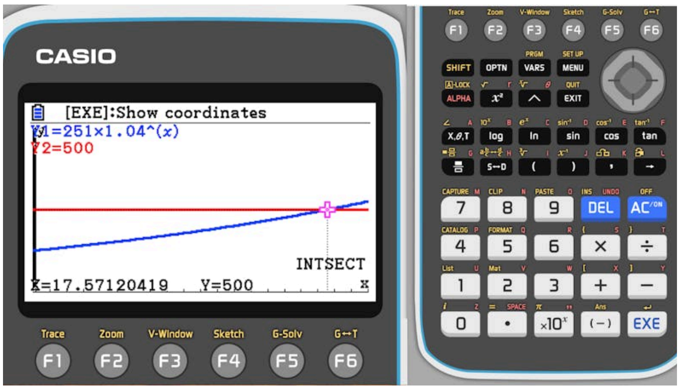
</td></tr>
</table>

Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
For the calculator, x represents the number of years. Since in **Example 2** you found that in 8 years the number of birds is not yet 500, you can try to see the increase in the first 20 years. For the calculator, y represents the number of birds, you can choose to see the y-range with 500 in the middle.	
When you see the graphs in the window you set, use F5 (G-Solv) to enter the menu where you can analyse the graphs.	

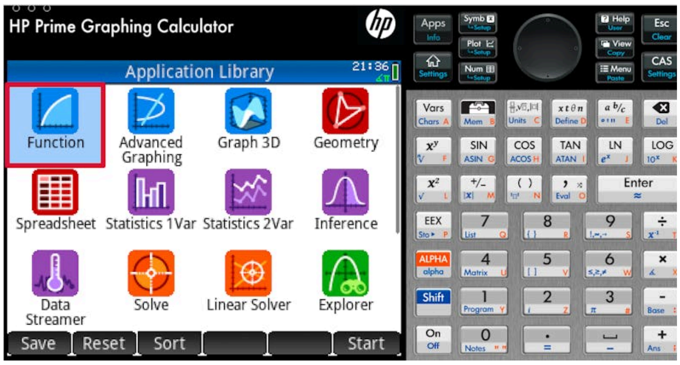
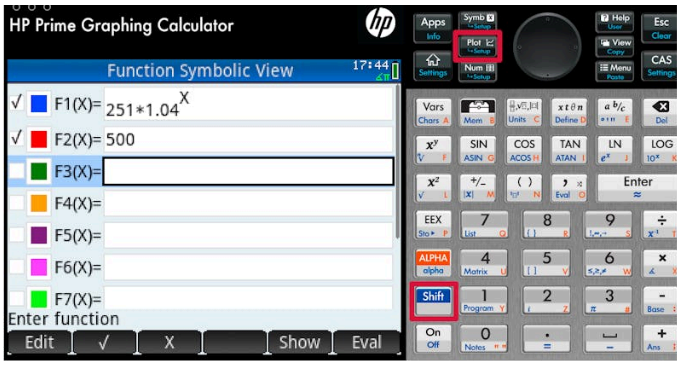
Student view

Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
Choose the option to find the intersection point of the graphs.	
Since the x-coordinate of the intersection point is 17.6, it takes 18 full years for the number of birds to increase above 500.	

Student view

Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
These are steps you can follow to answer an extension of **Example 2**. You will see how to use the calculator to find the time it takes until the number of birds in the state park reach 500. To start, choose the function graphing application.	
After entering the function that describes the increase of birds and the target number, choose the option to set the viewing window (plot settings).	

Student view

Overview
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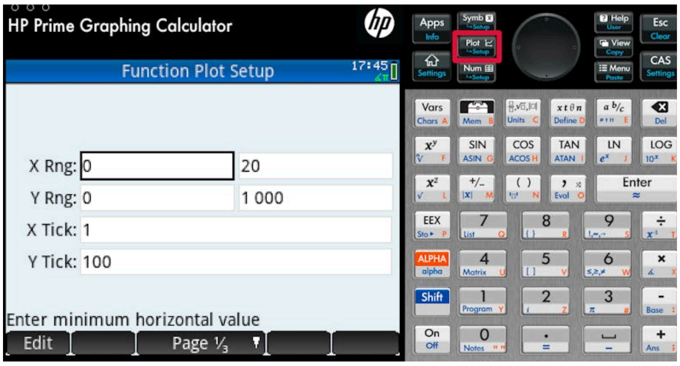
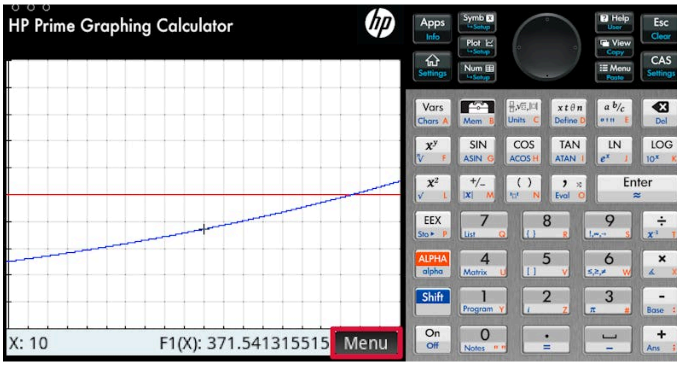
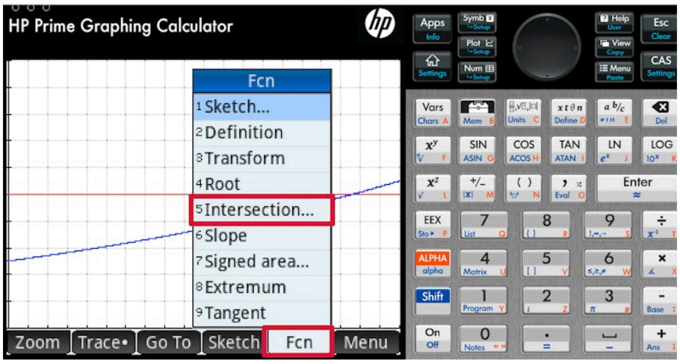
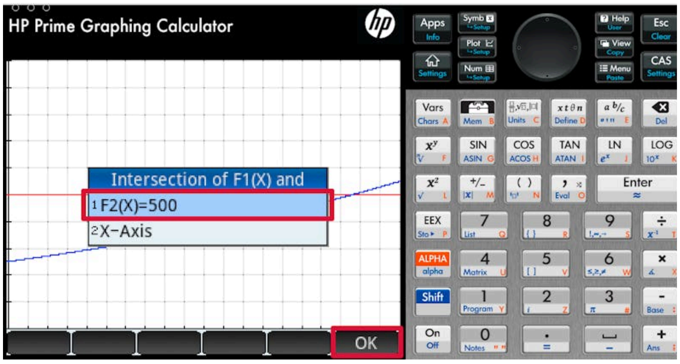
cid-
754029/

Steps	Explanation
For the calculator, x represents the number of years. Since in **Example 2** you found that in 8 years the number of birds is not yet 500, you can try to see the increase in the first 20 years. For the calculator, y represents the number of birds, you can choose to see the y-range with 500 in the middle. Once done setting the view, enter the plot view.	
Once you see the graphs, choose the menu option.	

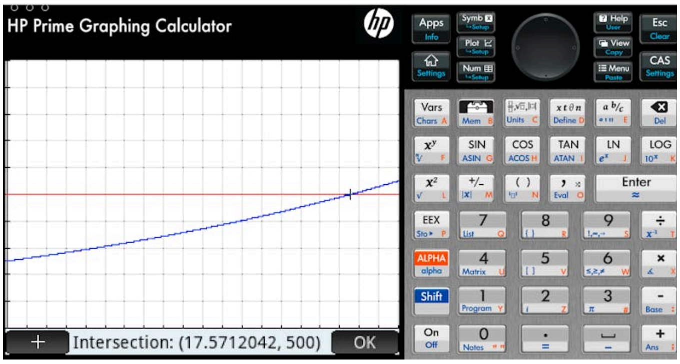
Student
view

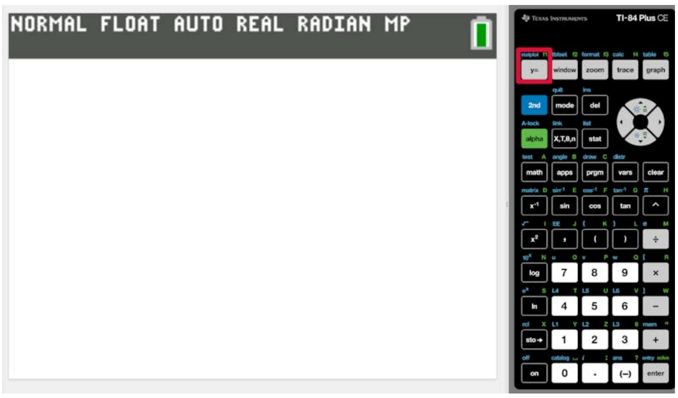
Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
Choose the option to find the intersection point of the graphs.	
Choose the second function.	

Student view

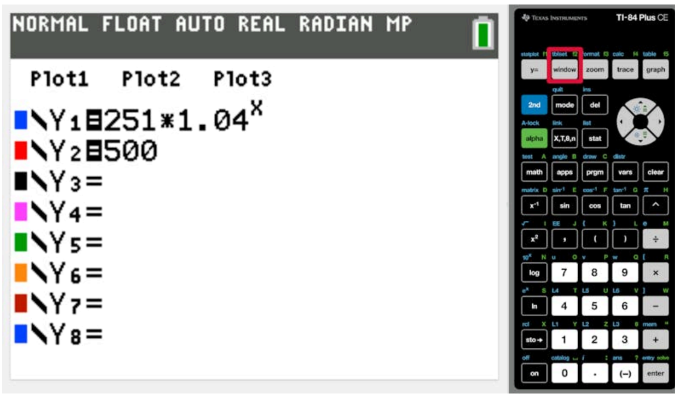
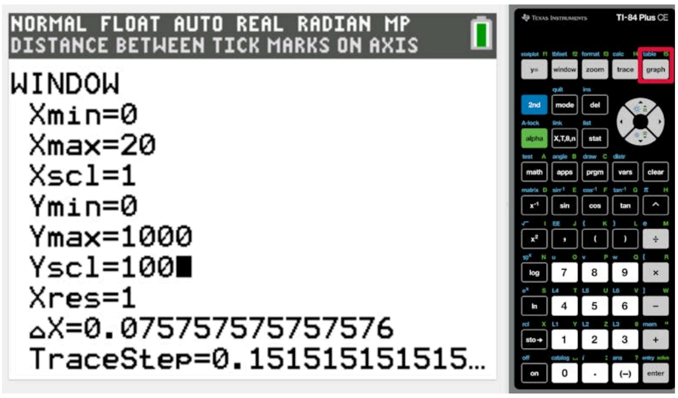
Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
Since the x-coordinate of the intersection point is 17.6, it takes 18 full years for the number of birds to increase above 500.	

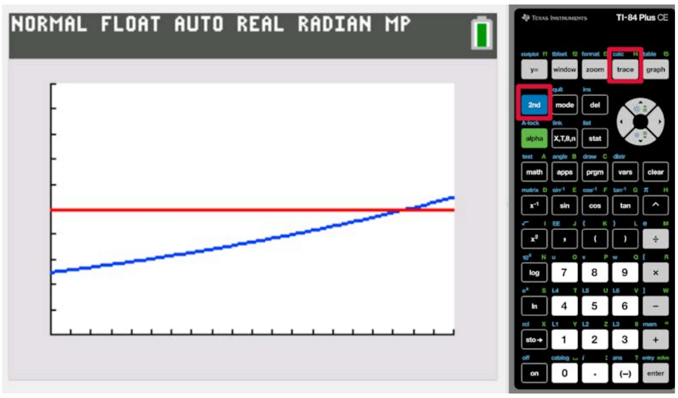
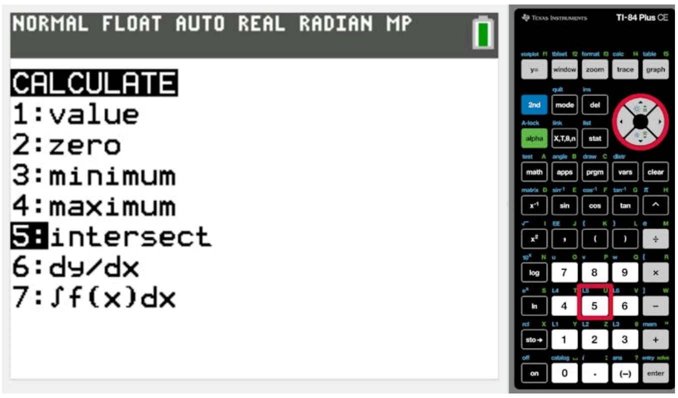
Steps	Explanation
These are steps you can follow to answer an extension of **Example 2.** You will see how to use the calculator to find the time it takes until the number of birds in the state park reach 500. To start, bring up the function entry screen of your calculator.	

Student view

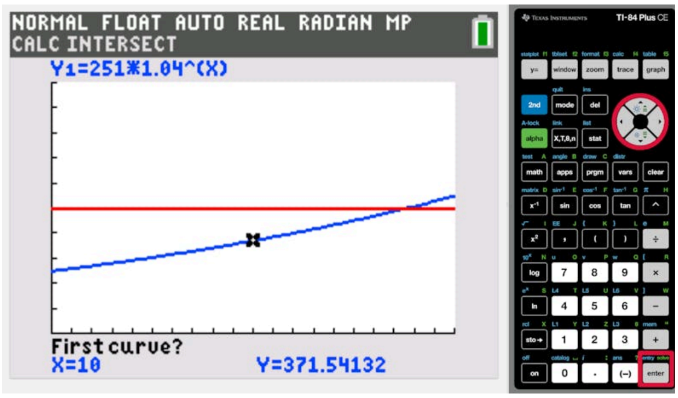
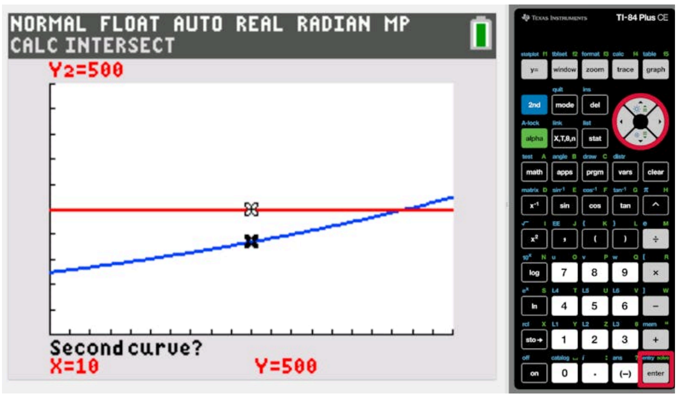
Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
After entering the function that describes the increase of birds and the target number, choose the option to set the viewing window.	
For the calculator, x represents the number of years. Since in **Example 2** you found that in 8 years the number of birds is not yet 500, you can try to see the increase in the first 20 years. For the calculator, y represents the number of birds, you can choose to see the y-range with 500 in the middle. Once done setting the view, enter the plot view.	

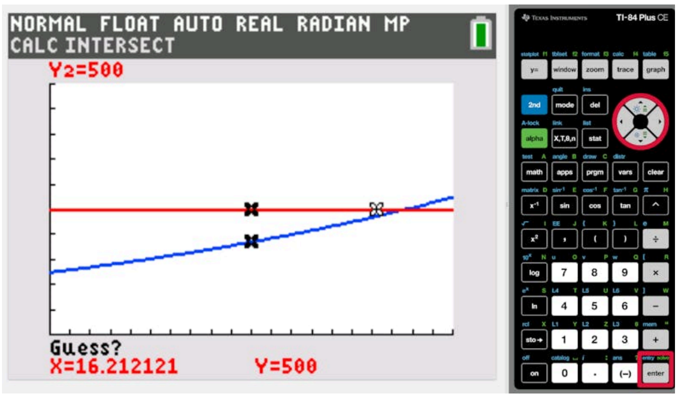
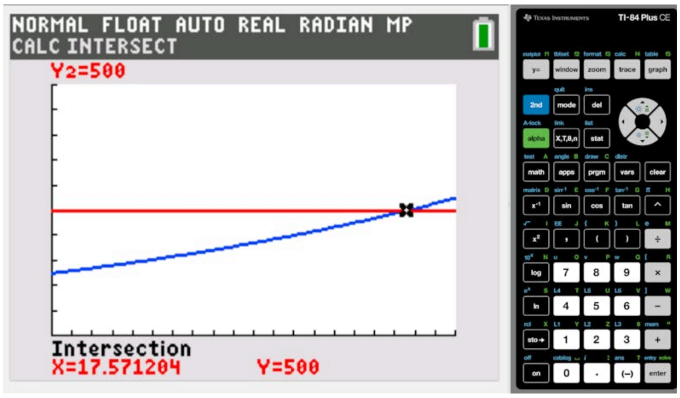
Student view

Steps	Explanation
Select the option (calc) that lets you analyse the graphs.	
Choose the option to find the intersection point of the graphs.	

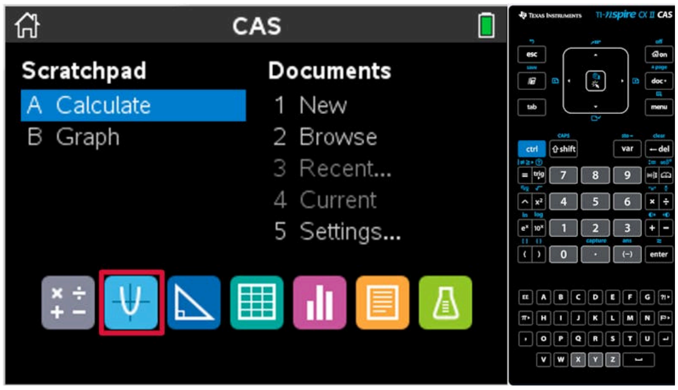
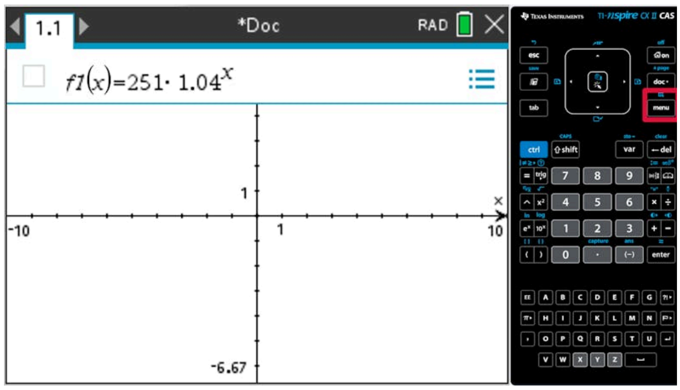
Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
Before the calculator can find the intersection point, it needs more information. Use the arrow key to choose one of the curves and confirm your choice.	
Use the arrow key to choose the other curve and confirm your choice.	

Student view

Steps	Explanation
Use the arrow key to move close to the intersection point and confirm your choice.	
Since the x-coordinate of the intersection point is 17.6, it takes 18 full years for the number of birds to increase above 500.	

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>These are steps you can follow to answer an extension of Example 2. You will see how to use the calculator to find the time it takes until the number of birds in the state park reach 500.

To start, open a graph page on your calculator.</td><td>
</td></tr>
<tr><td>Enter the function that describes the increase of birds and then bring up the menu.</td><td>
</td></tr>
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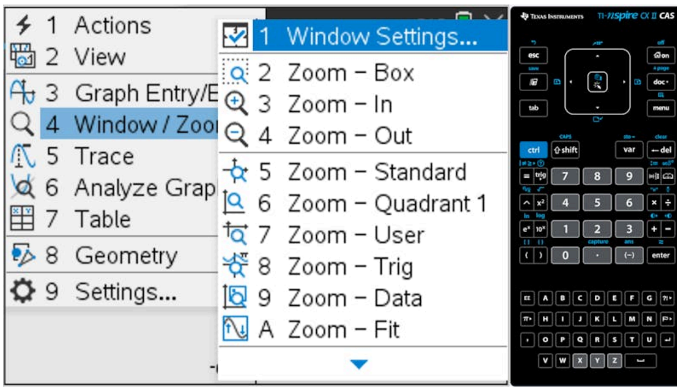
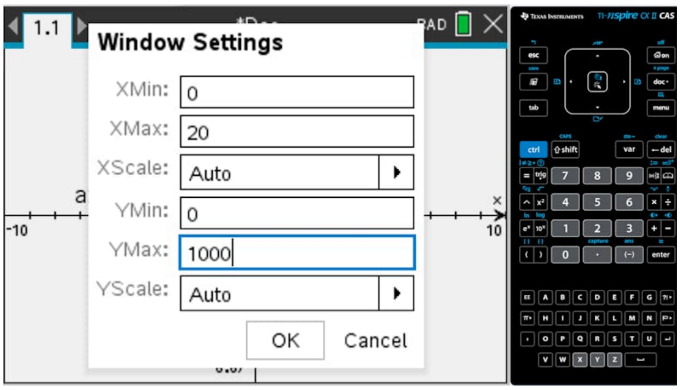
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>One way of setting the view window is to manually adjust the settings.</td><td>
</td></tr>
<tr><td>For the calculator, x represents the number of years. Since in Example 2 you found that in 8 years the number of birds is not yet 500, you can try to see the increase in the first 20 years. For the calculator, y represents the number of birds, you can choose to see the y-range with 500 in the middle.

Press OK when you are done.</td><td>
</td></tr>
</table>

Overview
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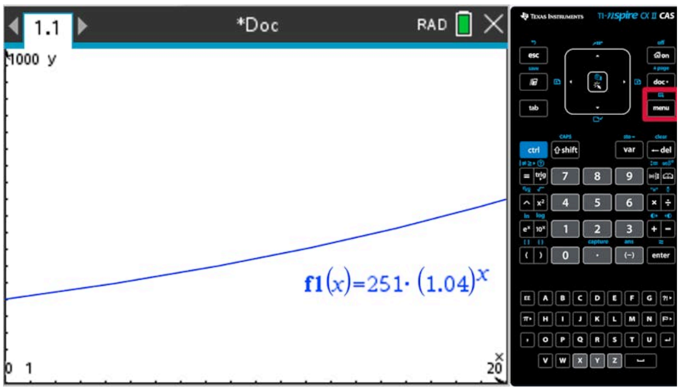
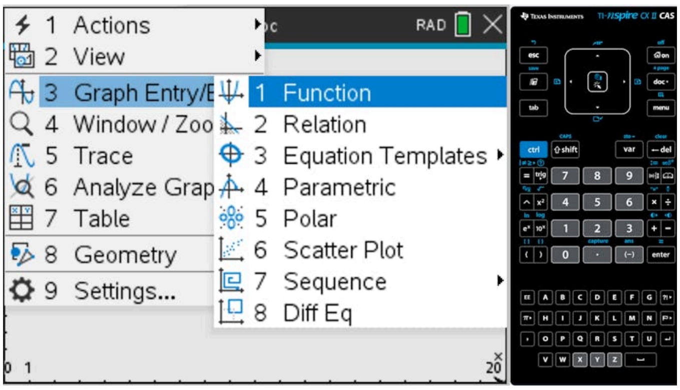
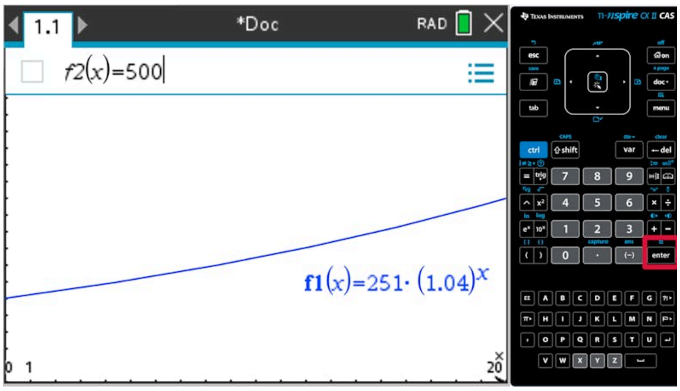
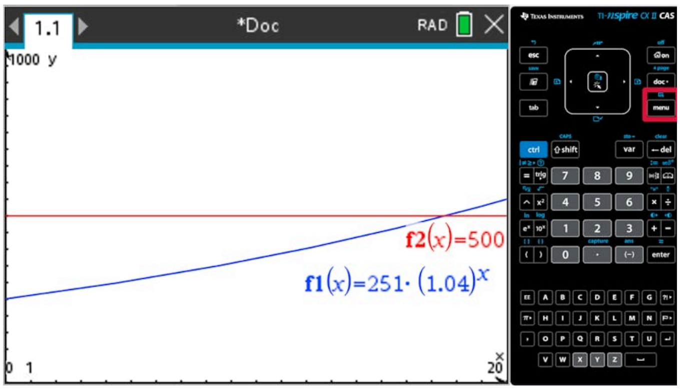
Steps	Explanation
Choose the menu again to enter 500, the target number of birds.	
Chose the option to define a new function.	

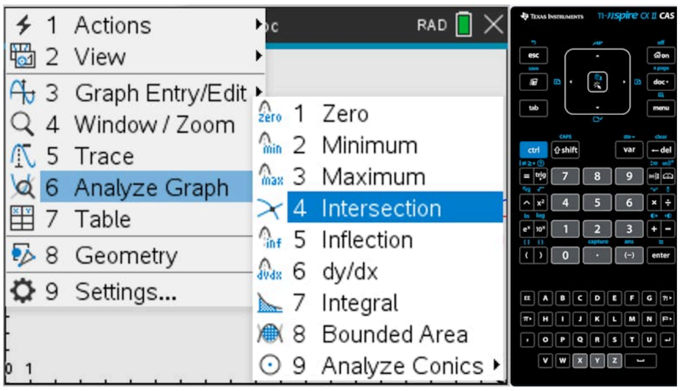
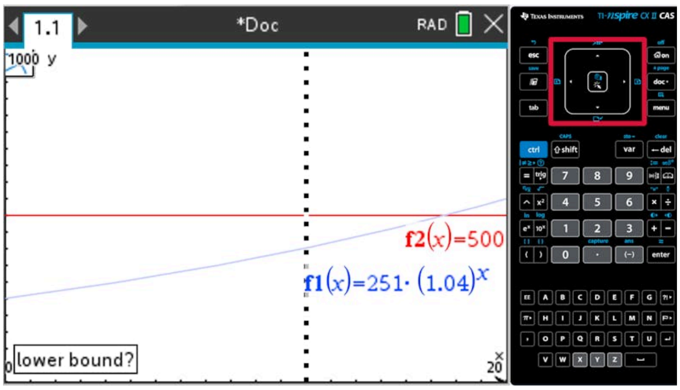
Student
view

Overview
(/study/app/sid-122-cid-754029/

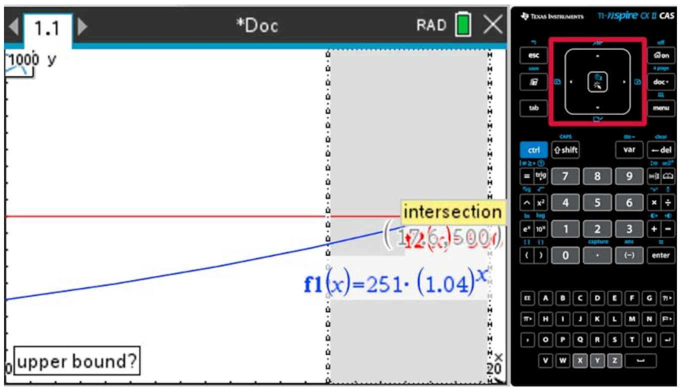
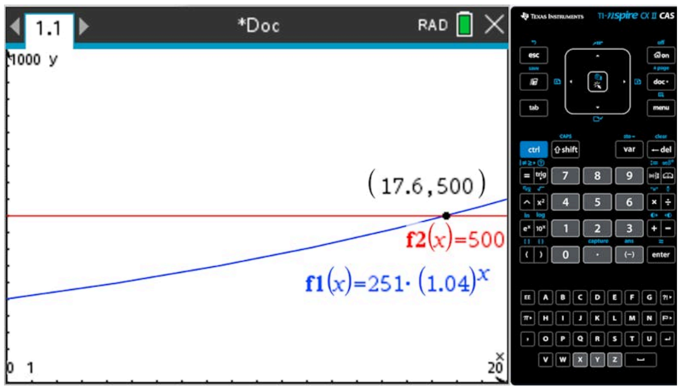
Steps	Explanation
Enter the target number of birds and press enter.	
Once you see both curves, bring up the menu yet again.	

Student view

Overview
(/study/app/sid-122-cid-754029/

Steps	Explanation
Choose the option to find the intersection point of the graphs.	
Before the calculator can find the intersection point, it needs more information. Drag the vertical line to the left of the intersection point and press enter to confirm your choice.	

Student view

Steps	Explanation
Drag the second vertical line to the right of the intersection point. The coordinates of the intersection point will be displayed automatically.	
Since the x-coordinate of the intersection point is 17.6, it takes 18 full years for the number of birds to increase above 500.	

Example 5

★★☆

The starting salary of an electrical engineer is 61 000 USD. Their salary increases at an average rate of 3.7% per year. Find the number of complete years that must pass before the engineer's salary reaches 78 000 USD.

Overview (/study/app/sid-122-cid-754029/

Steps	Explanation
$78\ 000 = 61\ 000 \times (1.037)^n$	Use $u_n = u_0 r^n$.
$n = 6.77$ so it will take 7 complete years.	Use your calculator to graph $y = 78\ 000$ and $y = 61\ 000 \times (1.037)^x$ and find the value of x where the curves intersect.
After working 7 complete years, in the eights year the engineer will have an annual salary that is more than $78\,000$ USD.	$u_7 = 78665$

Be aware

There are many more examples of geometric sequences in maths that are associated with finance, such as those involving investment and interest rates. These will be examined separately in subtopic 1.4 (/study/app/preview-p/sid-122-cid-754029/book/the-big-picture-id-26132/).

Theory of Knowledge

Mathematics' scope and application are far-reaching. Every Area of Knowledge utilises mathematics in some way. Consider the role of mathematics in economic and sociological modeling, such as the population model in **Example 1**, that follow a geometric sequence. A knowledge question that emerges from such an application is, 'To what extent can current political decisions be made based on future predictions?'

Discuss with a partner the role of certainty in answering the knowledge question above; and while doing so, consider a second knowledge question, 'What are the limits of mathematical certainty?'

4 section questions

Student view

Checklist

What you should know

By the end of this subtopic you should be able to:

- use $r = \frac{u_n}{u_{n-1}}$ to find the common ratio and show that a sequence is geometric
- identify u_1 and r and use them to write the general term of a geometric sequence in the form $u_n = u_1 r^{n-1}$
- apply $\frac{u_n}{u_{n-1}} = \frac{u_{n-1}}{u_{n-2}}$ to questions that give three consecutive terms of a geometric sequence
- solve questions that give specific terms of a geometric sequence by using $u_n = u_1 r^{n-1}$ to write equations for these terms
- find sums of geometric series using $S_n = \frac{u_1 \left(r^n - 1\right)}{r - 1}$ or $S_n = \frac{u_1 \left(1 - r^n\right)}{1 - r}$ for $r \neq 1$
- recognise that $u_1 = S_1, u_2 = S_2 - S_1$ and $u_n = S_n - S_{n-1}$ and apply to questions where sums are given
- recognise that any application question where a quantity grows or decays by a constant percentage rate can be solved by using a geometric sequence model
- use $u_n = u_0 r^n$ to model growth and decay in application questions, where u_0 is the initial amount and r is $1 \pm$ rate.

Investigation

The video below explores how folding a piece of paper can create a paper stack thick enough to reach from the Earth to the Moon.

Overview
(/study/app/sid-122-cid-754029/

Exponential growth: How folding paper can get you to the ...

Carry out your own investigation into paper folding.

- Start by thinking about a theoretical piece of paper just like the one in the video. Create a model for the thickness of this paper folded n times.
- Discuss the assumptions that you made for your theoretical model and think of its limitations.
- Predict how well this model would work for a real piece of paper folded n times.
- Collect data from an actual piece of paper. Compare your theoretical model with your data and comment on the results.

Student view