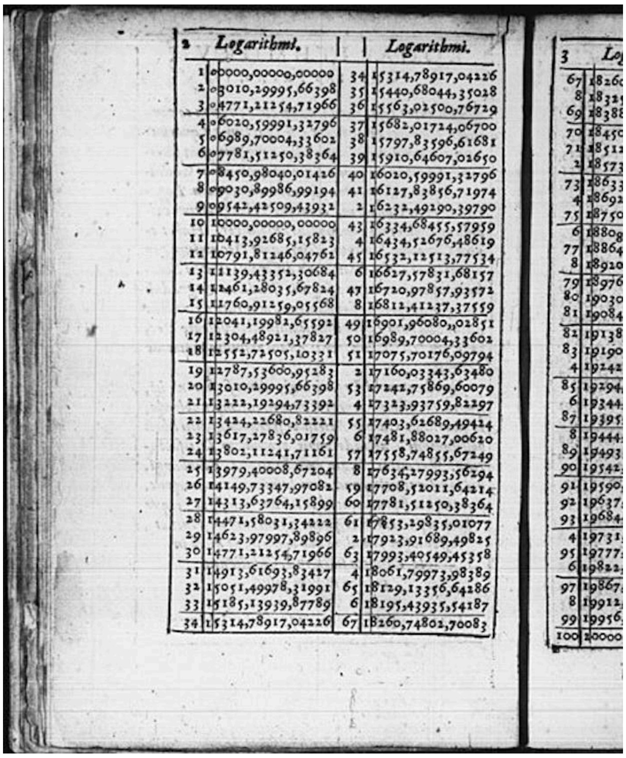
The big picture

When you work with exponents you will inevitably arrive at a question where the exponent is the unknown.

That question might be simple, such as $2^x = 4$ where you can easily see that $x = 2$.

Often, this type of question is more difficult, such as $2^x = 1.37$.

John Napier, a Scottish mathematician who lived in the 16th century, defined the concept of logarithms which we now use to solve $2^x = 1.37$.



2	Logarithmi.		Logarithmi.
1	00000,00000,00000	34	15314,78917,04226
2	03010,29995,66398	35	15440,68044,35028
3	04771,21254,71966	36	15563,02500,76729
4	06020,59991,32796	37	15682,01724,06700
5	06989,70004,33602	38	15797,83596,61681
6	07781,51250,38364	39	15910,64607,02650
7	08450,98040,01426	40	16020,59991,32796
8	09030,89986,99194	41	16127,83856,71974
9	09542,42509,43932	2	16232,49290,39790
10	10000,00000,00000	43	16334,68455,57959
11	10413,92685,15823	4	16434,52676,48619
12	10791,81246,04762	45	16532,12513,77534
13	11139,43352,30684	6	16627,57831,68157
14	11461,28035,67824	47	16720,97857,93572
15	11760,91259,05568	8	16812,41237,37559
16	12041,19982,65592	49	16901,96080,02851
17	12304,48921,37827	50	16989,70004,33602
18	12552,72505,10331	51	17075,70176,09794
19	12787,53600,95283	2	17160,03343,63480
20	13010,29995,66398	53	17242,75869,60079
21	13222,19294,73392	4	17323,93759,82297
22	13424,22680,82221	55	17403,62689,49424
23	13617,27836,01759	6	17481,88027,00620
24	13802,11241,71161	57	17558,74855,67249
25	13979,40008,67204	8	17634,27993,56294
26	14149,73347,97082	59	17708,52011,64214
27	14313,63764,15899	60	17781,51250,38364
28	14471,58031,34222	61	17853,29835,01077
29	14623,97997,89896	2	17923,91689,49825
30	14771,21254,71966	63	17993,40549,45358
31	14913,61693,83427	4	18061,79973,98389
32	15051,49978,31991	65	18129,13356,64286
33	15185,13939,87789	6	18195,43935,54187
34	15314,78917,04226	67	18260,74802,70083

Example of a logarithmic table from the 1600—1700 time period.

Source: " Logarithmorum Chilias Prima page 0-67 (https://commons.wikimedia.org/wiki/File:Logarithmorum_Chilias_Prima_page_0-67.jpg) " by Henry Briggs is in public domain.

More information

The image shows a page from an old logarithmic table from the 1600—1700 time period. The page is divided into two main columns labeled "Logarithmi." Each column contains several rows, numbered sequentially from 1 to 34 and 34 to 67, respectively. Alongside each row number, there is a corresponding logarithmic value, which is displayed in a series of ten digits with several decimal places. The layout has a structured, grid-like appearance, typical of tables used for mathematical calculations. The tables provide logarithmic values crucial for calculations before the advent of electronic calculators.

[Generated by AI]

An interesting application of logarithms is that they allow you to reliably multiply and divide numbers. This was a very important application given that handheld scientific calculators were only invented in the 1970s (see the example shown below).

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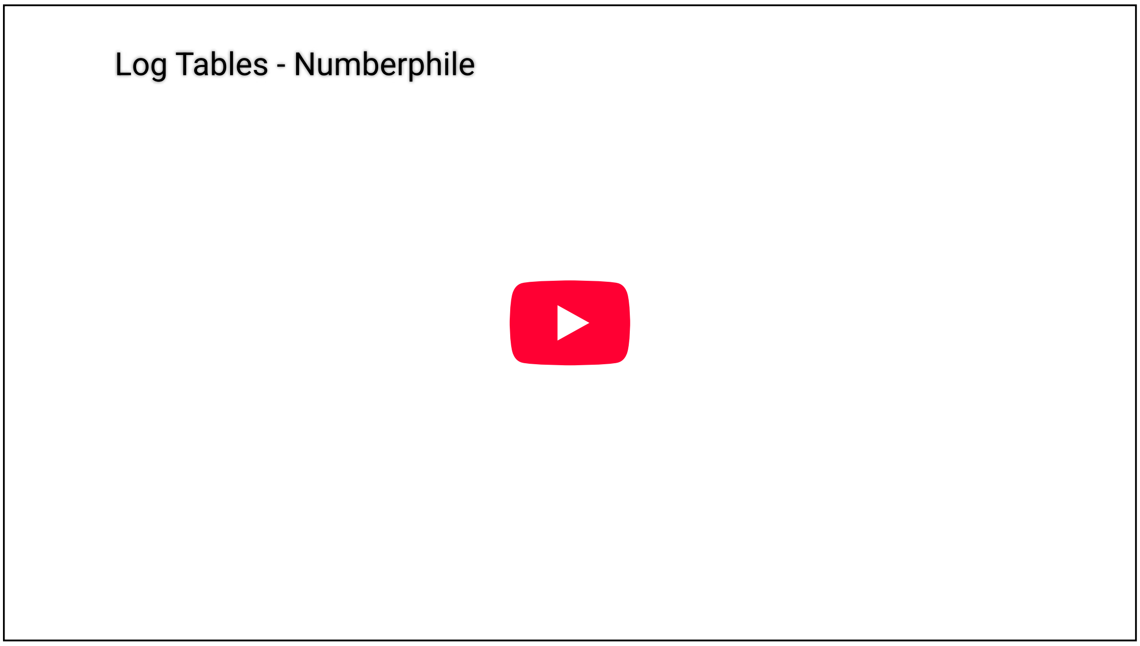
Hewlett-Packard's first handheld scientific calculator.

Watch the video to see how logarithms can be used for multiplication and division.



Concept

Logarithms and exponents allow you to write equivalent mathematical expressions using an alternative form. Think about what new information is revealed or hidden about numbers when you write them in these equivalent but different formats.

1. Number and algebra / 1.5 Exponents and logarithms

Laws of exponents

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Exponents

Exponents and exponent laws enable you to write long multiplication expressions in condensed and more efficient format such as

$$a \times a \times a \times a \times a \times a = a^6$$

Generally, a number written in exponential form such as a^n has two named parts:

the exponent n , which may also be called the power or the index

the base a

Interactive 1. Examples of Exponents.

More information for interactive 1

This interactive allows users to explore and understand exponents and exponent laws by changing the base and exponent values. Users can switch the base between numbers and letters using the first button, and adjust the exponent within a range of -10 to 10 (by using a slider at the bottom). Users can change the base as a number or letter using the first button, and users can change the base into a new letter or new number using the button at the top right. The interactive demonstrates how exponents condense long multiplication expressions into a more efficient format.

For example:

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With a base of 2 and an exponent of 5, it displays:

$2^5 = 2\times 2\times 2\times 2\times 2$ (The exponent 5 indicates that the base 2 should be multiplied by itself 5 times)

With a base of 2 and an exponent of -5, it illustrates the concept of negative exponents by showing:

$2^{-5} = \frac{1}{2^5} = \frac{1}{2\times 2\times 2\times 2\times 2}$ (The negative exponent indicates the reciprocal of the positive exponent value)

The interactive clearly shows the relationship between the exponent value and the number of times the base is multiplied by itself. For positive exponents, this means directly repeated multiplication, while negative exponents represent the reciprocal of that multiplication.

By experimenting with different values, users can observe how changes in the base and exponent affect the result, enhancing their understanding of exponential notation and its applications.

Exam tip

Below you will find rules for working with exponents. None of these rules are given in the IB formula booklet. You will need to remember them for the exam.

Multiplication

Activity

Use the applet below to explore the relationship between exponents m and n in a^m and a^n when you find the result of $a^m \times a^n$.

You can generate a variety of examples by changing the base and by using the sliders to change the exponents.

Generalise your observations to write a rule for $a^m \times a^n$.

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Interactive 2. Practice Exercise on Multiplying Exponents.

More information for interactive 2

This interactive allows users to explore the multiplication of exponents with the same base. Users can switch the base between numbers and letters. It visually demonstrates the exponent rule: $r^m \times r^n = r^{m+n}$. Users can see how exponents are added when multiplying terms with the same base, making it easier to understand exponent laws.

The interactive contains two adjustable sliders at the bottom of the interface, each corresponding to an exponent. The blue slider on the left side controls the first exponent, while the pink slider on the right side controls the second exponent. As users drag the sliders, the exponents change from 1 to 9. Users can change the base as a number or letter using the “letter” button, and users can change the base into a new letter or new number using the “New number” button. The equation updates in real-time to show the multiplication process step; it displays the expanded form of the base being multiplied. Then, it groups the repeated multiplications. Finally, it presents the simplified result by adding the exponents.

For example, if the first exponent is 5 and the second is 3 for a base of 9, the interactive expands the expression $9^5 \times 9^3 = 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^8$., If the user increases the second exponent by 2 using the slider, the new expression becomes $9^5 \times 9^5 = 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^{10}$.

Through this interactive, users can develop a clear understanding of the product of powers property in exponents. They can observe how exponents are added when multiplying terms with the same base. By adjusting the sliders, users can see real-time updates to the calculation, reinforcing the concept of exponent rules in an engaging and interactive way.

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Example 1

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Simplify $2^4 \times 2^6$.

Steps	Explanation
$2^4 \times 2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$	You can simply write ou full multiplication and c the number of twos.
$2^4 \times 2^6 = 2^{4+6} = 2^{10}$	The faster method is to the observation $a^m \times a^n = a^{m+n}$ (from Activity above).

✓ **Important**

The rule for multiplication is

$$a^m \times a^n = a^{m+n}.$$

Example 2

★★☆

Given that $3x^3 \times 2x^5 = 6x^b$, find the value of b.

Steps	Explanation
$\begin{aligned} 3x^3 \times 2x^5 &= 3 \times 2 \times x^3 \times x^5 \\ &= 6x^8 \end{aligned}$ So, $b = 8$.	The $a^m \times a^n = a^{m+n}$ rule can be extended to $ca^m \times da^n = (cd) \times a^m \times a^n = (cd) \times a^{m+n}$ by rewriting the multiplication in a different order.

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Example 3

★★★

Simplify $-2x^3y^2 \times 4x^5y^8z^2$.

Steps	Explanation
$-2x^3y^2 \times 4x^5y^8z^2$ $= -8x^{3+5}y^{2+8}z^2$ $= -8x^8y^{10}z^2$	Only add exponents for multiplication of the sa base (shown here colour coded with same colo

Division

Activity

Use the applet below to explore division and to write a general rule for $\frac{a^m}{a^n}$.

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Interactive 3. Practice Exercise on Dividing Exponents.

More information for interactive 3

This interactive allows users to explore the division of exponents with the same base. Users can switch the base between numbers and letters. It visually demonstrates the exponent rule:

$$\frac{r^m}{r^n} = r^{m-n}$$

Users can see how exponents are subtracted when dividing terms with the same base, making it easier to understand exponent laws.

The interactive contains two adjustable sliders, each corresponding to an exponent. The blue slider controls the exponent in the numerator, while the pink slider controls the exponent in the denominator. As users drag the sliders, the exponents change dynamically. Users can change the base as a number or letter using the “letter” button, and users can change the base into a new letter or new number using the "New number” button at the top right. The equation updates in real time to show the division process step by step. It first displays the expanded form of the base being divided. Then, it cancels out matching factors in the numerator and denominator. Finally, it presents the simplified result by subtracting the exponents.

For example, if the exponent in the numerator is 6 and the exponent in the denominator is 9, the interactive expands the expression:

$$\frac{6^6}{6^9} = \frac{6\times 6\times 6\times 6\times 6\times 6}{6\times 6\times 6\times 6\times 6\times 6\times 6\times 6\times 6}$$

By canceling out the common factors, it simplifies to:

$$\frac{1}{6^3} = 6^{-3} = 6^{6-9}$$

If the user increases the exponent in the denominator by 2 using the slider, the new expression becomes:

$$\frac{6^6}{6^{11}} = 6^{-5}$$

Through this interactive, users can develop a clear understanding of the quotient of powers property in

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exponents. They can observe how exponents are subtracted when dividing terms with the same base. By adjusting the sliders, users can see real-time updates to the calculation, reinforcing the concept of exponent rules in an engaging and interactive way.

Example 4

★★☆

Simplify $\frac{8x^4}{2x^2}$.

Steps	Explanation
$\frac{8x^4}{2x^2} = \frac{8 \times x \times x \times x \times x}{2 \times x \times x} = 4 \times x \times x = 4x^2$	
$\frac{8x^4}{2x^2} = 4x^{4-2} = 4x^2$	The faster way to do this question use the observation that $\frac{a^m}{a^n} = a^{}$

✓ **Important**

The rule for division is

$$\frac{a^m}{a^n} = a^{m-n}.$$

Example 5

★★★

Simplify $\frac{9a^7b^3c}{27a^{-2}b^5}$.

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Steps	Explanation
$\frac{9a^7b^3c}{27a^{-2}b^5} = \frac{1}{3}a^{7-(-2)}b^{3-5}c = \frac{1}{3}a^9b^{-2}c$	You will learn about negative exponents a littl For now, you just need to know that they follo same rules as positive exponents.

Powers

Activity

Use what you already know about exponents to deduce a general rule for

1. $(a^m)^n$
2. $(ab)^m$
3. $\left(\frac{a}{b}\right)^m$.

Example 6

★★☆

Show that each of the following is true.

a) $\left(x^3\right)^4 = x^{3\times 4}$

b) $(xy)^3 = x^3y^3$

c) $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$

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	Steps	Explanatio
a)	$\left(x^3\right)^4 = x^3 \times x^3 \times x^3 \times x^3 = x^{3+3+3+3} = x^{3\times 4}$	Expand and use $a^m \times a^n = a^{m+n}$.
b)	$(xy)^3 = xy \times xy \times xy = x \times x \times x \times y \times y \times y = x^3y^3$	Expand, reorder a use $a^m \times a^n = a$
c)	$\left(\frac{x}{y}\right)^5 = \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} = \frac{x^5}{y^5}$	Expand and use $a^m \times a^n = a^{m+n}$.

✓ **Important**

The rules for powers are:

$$(a^m)^n = a^{m\times n}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example 7

★★☆

Simplify $\left(5a^2b\right)^3$.

Steps	Explanation
$\left(5a^2b\right)^3 = 5^{1\times 3} \times \left(a^2\right)^3 \times b^3 = 125a^6b^3$	Do not forget that $5 = 5^1$. In your working yc not need to write $5^{1\times 3}$. You can go straight t

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Zero and negative exponents

Activity

1. Use the rule for division to simplify $\frac{5^3}{5^3}$. Evaluate $\frac{125}{125}$.
   Hence, deduce the rule for a^0.
2. Use the rule for division to simplify $\frac{3^2}{3^5}$. Rewrite $\frac{3^2}{3^5}$ in $\frac{a \times a \times \ldots}{a \times a \times a \ldots}$ form.
   Hence, deduce the rule for a^{-m}.

Example 8

★★☆

Simplify $\frac{x^4}{x^8}$. Hence, show that $x^{-4} = \frac{1}{x^4}$.

Steps	Explanation
$\frac{x^4}{x^8} = x^{-4}$	Using the rule for division.
$\frac{x^4}{x^8} = \frac{x \times x \times x \times x}{x \times x \times x \times x \times x \times x \times x \times x} = \frac{1}{x^4}$ So, $x^{-4} = \frac{1}{x^4}$.	

Important

The rules for zero and negative exponents are:

$$a^0 = 1$$

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$$a^{-m} = \frac{1}{a^m}$$

Example 9

★★★

Simplify $\left(2x^2\right)^3 \times \left(\frac{4y^3}{x^4z^2}\right)^5$.

Steps	Explanation
$\left(2x^2\right)^3 = 2^3x^{2\times3} = 2^3x^6 = 8x^6$	Simplify each expression separately to make the working easier.
$\left(\frac{4y^3}{x^4z^2}\right)^5 = \frac{4^5y^{3\times5}}{x^{4\times5}z^{2\times5}} = \frac{1024y^{15}}{x^{20}z^{10}}$	
$8x^6 \times \frac{1024y^{15}}{x^{20}z^{10}} = \frac{8192x^6y^{15}}{x^{20}z^{10}} = \frac{8192\ y^{15}}{x^{20-6}z^{10}} = \frac{8192y^{15}}{x^{14}z^{10}}$	Combine and simplify further.

Example 10

★★☆

Simplify $\left(2a^{-5}b^{10}\right)^0 \times \left(a^2b^{-4}\right)^5$ using only positive exponents in your answer.

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Steps	Explanation
$\left(2a^{-5}b^{10}\right)^0 = 1$	Using $a^0 = 1$.

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Steps	Explanation
$\left(a^2b^{-4}\right)^5 = a^{10}b^{-20} = \dfrac{a^{10}}{b^{20}}$	Use $a^{-m} = \dfrac{1}{a^m}$ to rewrite any nega exponents.
$\left(2a^{-5}b^{10}\right)^0 \times \left(a^2b^{-4}\right)^5 = 1 \times \dfrac{a^{10}}{b^{20}} = \dfrac{a^{10}}{b^{20}}$	

8 section questions

1. Number and algebra / 1.5 Exponents and logarithms

Logarithms with base 10

Consider $10^x = 100$. Can you find the value of x that makes this equation true?

Now, consider $10^x = 50$. Why is this equation more difficult to solve?

Solving each of these equations requires you to ask the following question:

'What power should 10 *be raised to in order to get ...?'*

Logarithms allow you to write this question using symbols. So the power to which you need to raise 10 in order to get 50 is written as $\log_{10}50$, for which you would say: 'log to the base 10 of 50'.

The numbers that you enter into a logarithm are named.

For $\log_a b = x$, a is the base, b is the argument and x is the answer.

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Steps	Explanation
In these instructions you will see how to find the approximate value of $\log_{10} 896$. Choose the calculator application.	
Press F4 to get in the math menu.	

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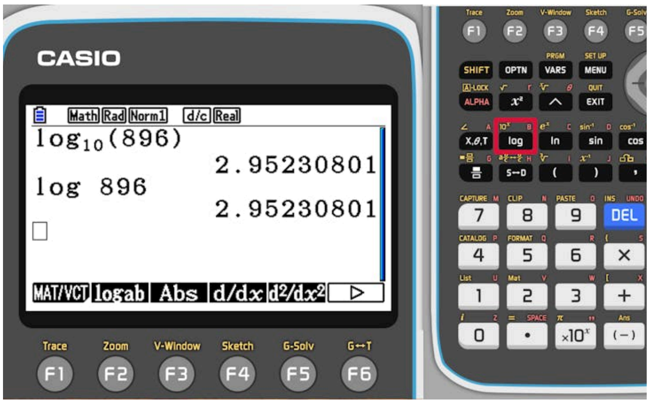
Steps	Explanation
Press F2 to choose the option to enter a logarithmic expression.	
After entering the base and the argument, the calculator will give you the approximate answer. Note that there is also a button on the calculator for logarithm without a base specified. Using this button you get the same answer as before. Without a base specified, the calculator will assume that the base is 10.	

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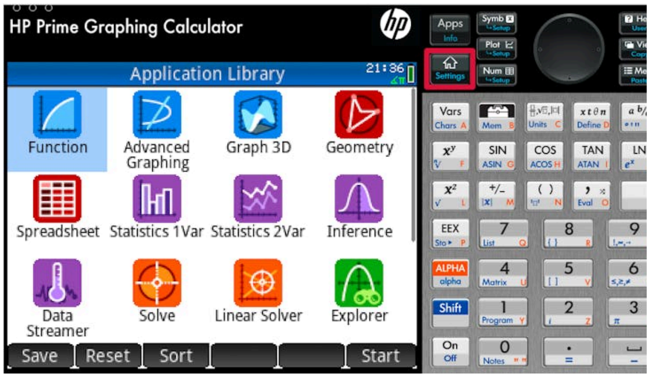
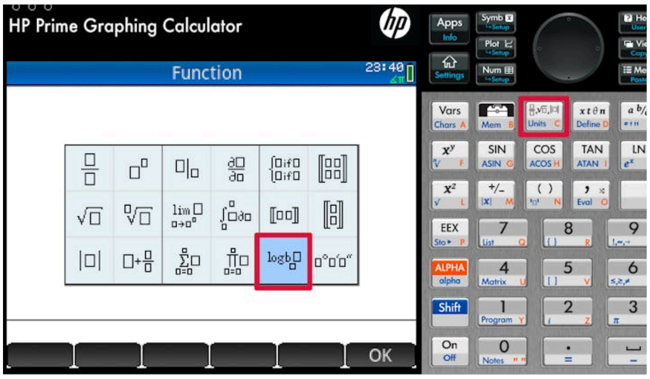
Steps	Explanation
In these instructions you will see how to find the approximate value of $\log_{10} 896$. Enter the home screen of any application.	
Bring up the formatting options and choose the one to enter logarithmic expressions.	

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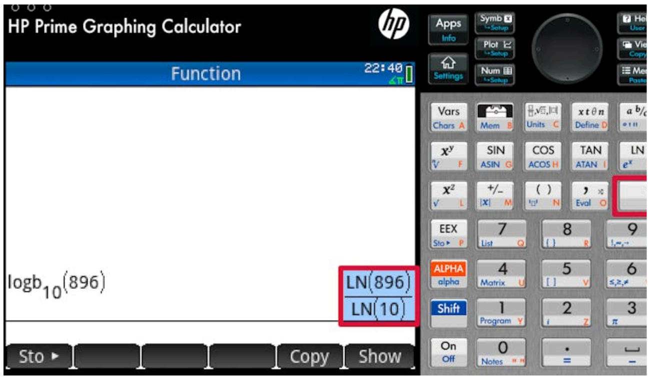
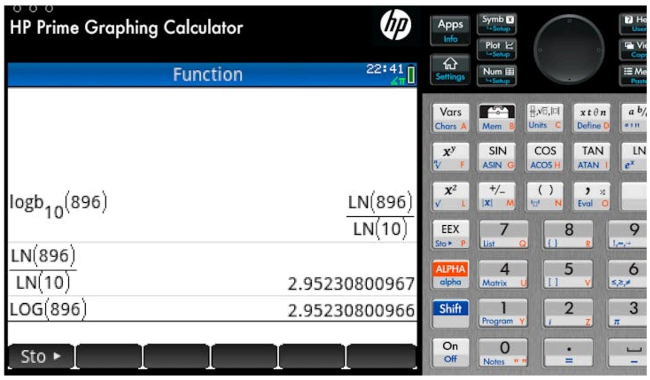
Steps	Explanation
After entering the base and the argument, you can notice that the calculator gives another logarithmic expression instead of an approximate number. Choose this result and evaluate it again.	
The evaluation of this second expression is indeed an approximate number. Note that there is also a button on the calculator for logarithm without a base specified. Using this button you get the same answer as before. Without a base specified, the calculator will assume that the base is 10.	

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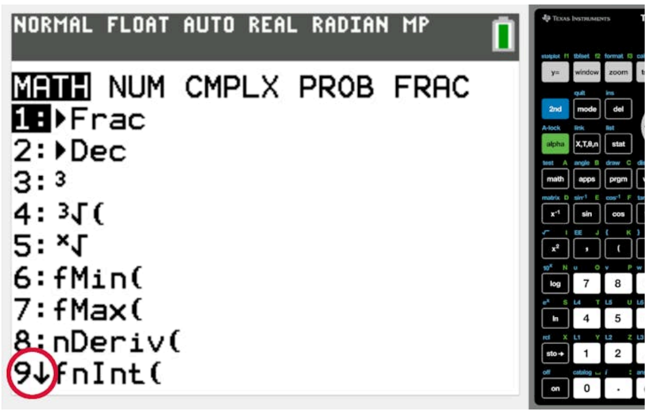
Steps	Explanation
In these instructions you will see how to find the approximate value of $\log_{10} 896$. Bring up the list of mathematical options.	
You will need to scroll down to find the option you need here.	

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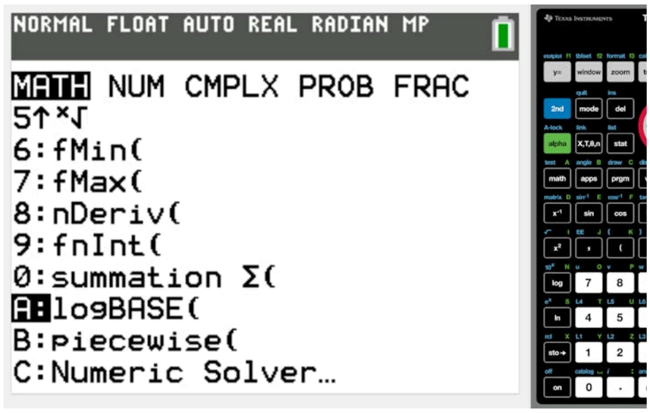
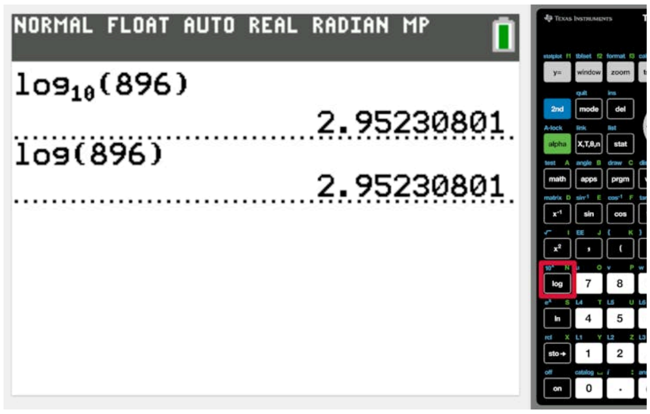
Steps	Explanation
Scroll down to choose the option to enter a logarithmic expression.	
After entering the base and the argument, the calculator will give you the approximate answer. Note that there is also a button on the calculator for logarithm without a base specified. Using this button you get the same answer as before. Without a base specified, the calculator will assume that the base is 10.	

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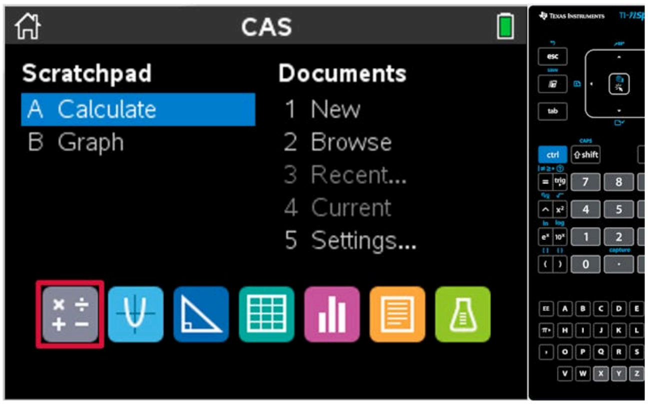
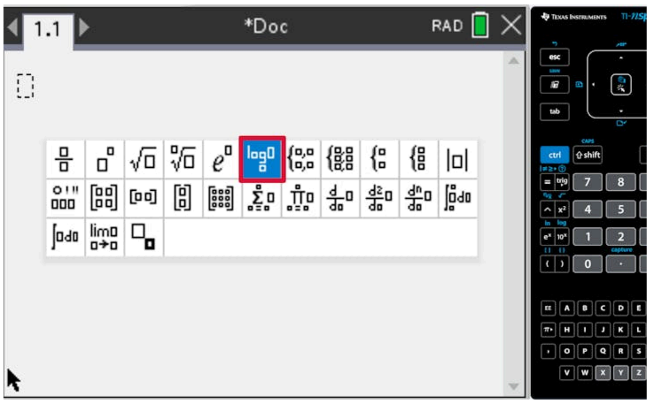
Steps	Explanation
In these instructions you will see how to find the approximate value of $\log_{10} 896$. Open a calculator document.	
Choose the form of the logarithmic expression from the template palette.	

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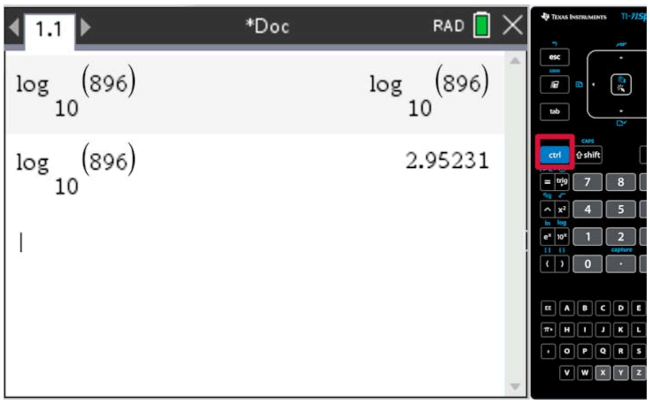
Steps	Explanation
Note that when you press enter, nothing seems to happen.	
You can evaluate the same expression approximately. This way you get an approximate decimal answer.	

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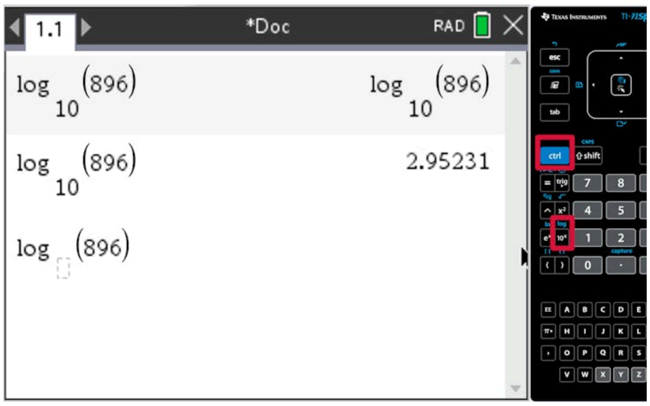
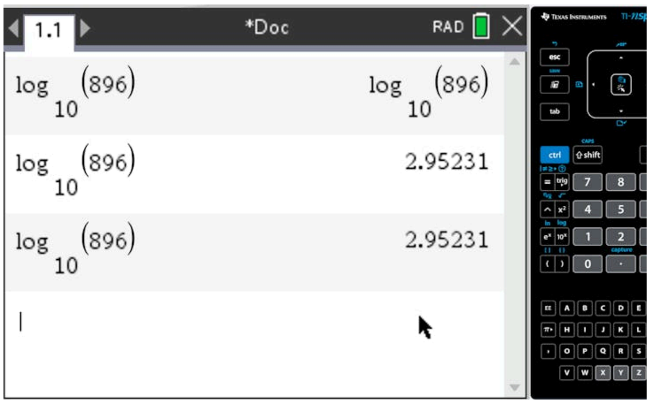
Steps	Explanation
You can also enter logarithmic expression directly using the keyboard. Try to enter the expression without specifying the base.	
Without a base specified, the calculator will assume that the base is 10.	

Activity

Use your calculator to evaluate $\log_{10}50$. Determine how your answer is related to $10^x = 50$.

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Repeat this process for:

1. $\log_{10}4$
2. $\log_{10}137$

Hence, deduce how $\log_a b = x$ can be rewritten in exponential form.

Important

Exponential equations in the form $a^x = b$ are equivalent to $\log_a b = x$ in logarithmic form. This holds true for $a > 0$, $b > 0$, $a \neq 1$.

Logarithms can be evaluated using a calculator.

Exam tip

The IB formula booklet includes this observation in the following form:

$$a^x = b \quad \Leftrightarrow \quad x = \log_a b, \text{ where } a > 0,\ b > 0,\ a \neq 1$$

Example 1

★☆☆

a) Evaluate $\log_{10}0.01$.

b) Hence, solve $10^x = 0.01$.

	Steps	Explanation
a)	$\log_{10}0.01 = -2$	Use a graphic display calculator.
b)	If $\log_{10}0.01 = -2$, then $10^{-2} = 0.01$. Hence, in $10^x = 0.01$, $x = -2$.	Using, $a^x = b$ is equivalent to $\log_a b =$

Student view

International Mindedness

The word logarithm is composed of two Greek words *logos* and *arithmos* and the symbol for logarithms, $\log_{10}a$, uses an abbreviation of the Latin word *logarithmus* . If you want to discuss logarithms with a Spanish speaker you would say *logaritmo,* in German you would say *logarithmus* , in Croatian *logaritam* , in Estonian *logarithm* , in Uzbek *logaritma* and in Swahili *logarithm* , amongst other languages. What do these facts tell you about the nature of mathematical ideas and how they spread throughout the world?

Logarithms can have a variety of bases. Some examples are:

$$\log_2 7 = 2.807\ldots$$

$$\log_{1.35} 9 = 7.321\ldots$$

$$\log_{\sqrt{2}} 4 = 4$$

Base 10 is a very common base because our counting system uses base 10. Base 10 is so common that it has its own simplified notation. You can write $\log_{10}5$ as just $\log 5$.

Making connections

- If you are studying the Applications and interpretation course at Standard Level, you only need to know about logarithms base 10 and the natural logarithm, which you will learn about in the next section.
- If you are studying one of the other courses, you will learn about logarithms to other bases too.

Be aware

If you see $\log b$, you should interpret it as $\log_{10} b$.

Activity

Rewrite the following exponential equations in the equivalent logarithmic form $\log_a b = x$.

1. $2^3 = 8$
2. $4^2 = 16$
3. $3^{-2} = \frac{1}{9}$
4. $a^x = b$

Explain why $b > 0$ in $\log_{10} b$.

Example 2

★★☆

a) Show that $\log b$ is only defined for $b > 0$.

b) Evaluate $\log 0$ on your calculator. Explain the result obtained.

	Steps	Explanation
a)	Let $\log b = c$. $10^c = b$	Rewrite in exponential form.
	$10^c = 10 \times 10 \times \ldots \times 10 \ \ (c \text{ times})$ So, $10^c > 0$ and therefore $b > 0$	

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	Steps	Explanation
b)	Let $\log 0 = c$. $$10^c = 0$$ The calculator can not evaluate this result because you can not raise 10 to a power to get 0 .	The calculator gives you an error or you that the expression is undefined

Example 3

★☆☆

Rewrite $\log 12 = c$ in exponential form.

Steps	Explanation
$10^c = 12$	Remember that $\log 12 = \log_{10} 12$.

Example 4

★☆☆

Evaluate $\log 17$.

Steps	Explanation
$\log 17 = 1.23$ (3 significant figures)	Use a graphic display calculator.

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Example 5

★★☆

The pH scale is used to determine how acidic or basic an aqueous solution is. The pH of a substance is a measure of the concentration of hydrogen ions in solution. It is defined as follows:

$$\mathrm{pH} = -\log[\mathrm{H}^+]$$

In this formula $[\mathrm{H}^+]$ refers to the concentration of hydrogen ions in the solution and measured in $\mathrm{mol\ dm}^{-3}$..

a) A solution has hydrogen ion concentration of $5.7 \times 10^{-4} \mathrm{mol\ dm}^{-3}$. Calculate its pH value.

b) Calculate the hydrogen ion concentration of a solution with a pH value of 5.78.

	Steps	Explanation
a)	$\mathrm{pH} = -\log\left(5.7 \times 10^{-4}\right)$ $\mathrm{pH} \approx 3.24$ The pH value of the solution is 3.24.	Substitute 5.7×10^{-4} for $[\mathrm{H}^+]$ in th formula.
b)	$5.78 = -\log[\mathrm{H}^+]$ $\log[\mathrm{H}^+] = -5.78$ $[\mathrm{H}^+] = 10^{-5.78} \approx 1.66 \times 10^{-6}$ The hydrogen ion concentration of the solution is $1.66 \times 10^{-6} \mathrm{mol\ dm}^{-3}$.	Substitute 5.78 for pH in the formul solve it for $[\mathrm{H}^+]$.

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3 section questions

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1. Number and algebra / 1.5 Exponents and logarithms

Natural logarithms

Base e

In addition to base 10 there is another base that is commonly used. This base is the number e ≈ 2.71828.

Although you might not yet be familiar with e, it is a number that occurs naturally in a surprising number of places. Watch the video to learn more.

Natural logarithms

A natural logarithm is a logarithm with base e.

Important

You can use $a^x = b \quad \Leftrightarrow \quad x = \log_a b$ to rewrite an exponential equation with base e as a logarithm:

$$e^x = b \quad \Leftrightarrow \quad x = \log_e b.$$

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Alternative notation for $\log_e b$ is $\ln b$.

To read $\ln b$ you say: ‘the natural logarithm of b ’ .

Example 1

★☆☆

Rewrite $\ln 2 = c$ in exponential form.

Steps	Explanation
$e^c = 2$	Use $e^x = b \quad \Leftrightarrow \quad \log_e b$.

Example 2

★☆☆

Evaluate $\ln 5$.

Steps	Explanation
$\ln 5 = 1.61$ (3 significant figures)	Use a graphic display calculator. You can enter $\log_e 5$ or us dedicated $\ln 5$ button.

Example 3

★★☆

Student view

Determine which of $\ln 2$ or $\ln 15$ has the smallest value:

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Steps	Explanation
$\ln 2 = 0.693$ $\ln 15 = 2.71$ $\ln 2$ is smaller	Use your graphic display calculator to evaluate both numbers.

Example 4

★★☆

In computer industry the strength of a password is usually measured in terms of its information entropy, H, which is given by the following formula:

$$H = \frac{L \ln N}{\ln 2},$$

where L is the length of the password and N is the number of possible symbols that can be used in the password.

a) What is the information entropy of a password of length 8, where each symbol can be either a number digit or a letter from the English alphabet? The letters can be lower or upper case.

b) Using the same symbols as in part a), how long should the password be to have an information entropy greater than 64?

	Steps	Explanation
a)	$N = 10 + 2 \times 26 = 62$	There are 10 number digits. There are 26 letters in the English alphabet and all have both a lower-upper-case form.

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	Steps	Explanation
	$H = \dfrac{8 \times \ln 62}{\ln 2} \approx 47.6$ The information entropy of the password is 47.6.	Substitute $L = 8$ and $N = 62$ in th formula.
b)	$64 < \dfrac{L \times \ln 62}{\ln 2}$ $L > \dfrac{64 \times \ln 2}{\ln 62} = 10.74872\ldots$ The length of the password needs to be at least 11 to exceed information entropy 64.	Substitute $H = 64$ and $N = 62$ in t formula.

Theory of Knowledge

As discussed in this section, logarithms are mathematical knowledge created that then is used to create future knowledge. When using a log table, one does not check for the validity of the mathematical knowledge within the log table, it is simply trusted.

This trust in previously established knowledge results in a system of knowledge that is said to be *a priori* — free from first-hand experience.

This leads to the knowledge question, ‘To what extent is the validity of knowledge predicated upon sensory perception?’

2 section questions

1. Number and algebra / 1.5 Exponents and logarithms

Checklist

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What you should know

By the end of this subtopic you should be able to:

- know and apply the exponent rules:

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^0 = 1$$

- use exponent rules to simplify expressions and rewrite them with positive exponents
- recognise that exponential equations in the form $a^x = b$ are equivalent to $\log_a b = x$ in logarithmic form for $a > 0$, $b > 0$, $a \neq 1$
- rewrite exponential equations in logarithmic form and logarithmic equations in exponential form
- interpret $\log a$ as $\log_{10} a$ and $\ln a$ as $\log_e a$
- evaluate logarithms to base 10 and base e on the calculator.

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1. Number and algebra / 1.5 Exponents and logarithms

Investigation

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Student view

Part 1

To get started, begin the simulation and select the option 'Micro'.

Interactive 1. Simulation: Connection Between the Concentration of the Hydronium Ion and PH for a Variety of Substances.

More information for interactive 1

This interactive simulation allows users to explore the relationship between hydronium ion concentration H_3O^+ and *pH* levels for various substances, such as coffee, milk, or hand soap.

In the "macro" tab, users can mix liquids with water using adjustable taps—one to add water and another to empty the beaker—while dragging a pH sensor into the solution to measure its exact pH value. Also the tab highlights whether the substance is acidic or basic. For example Mixing $0.3L$ orange juice ($pH \sim 3.5$) with $0.7L$ water produces pH ~ 4.1 showing how dilution affects acidity logarithmically rather than linearly.

In the "micro" tab, users can examine the molecular-level composition of substances, including the concentrations of hydronium (H_3O^+) and hydroxide (OH^-) ions, displayed in both logarithmic and linear scales. The interface provides precise measurements, such as the concentration of (H_2O) molecules (e.g., $5.7 \times 10^{-8} mol/L$) and pH values (e.g., 7.25 for blood), allowing users to observe how these values correlate with acidity or alkalinity. The logarithmic scale compresses wide-ranging data into a manageable format, while the linear scale offers a direct view of molecular quantities.

For example, for pure water, the micro tab displays equal concentrations of H_3O^+ and OH^- ions 1.0×10^{-7} mol/L each), resulting in a neutral pH of 7.0. If a small amount of acid is added, the H_3O^+ concentration increases (e.g., to 1.0×10^{-4} mol/L), causing the pH to drop to 4.0, visually demonstrating the inverse logarithmic relationship between ion concentration and pH. and When analyzing blood (pH 7.25), the

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micro tab reveals $H3O^+$ concentrations around 5.6×10^{-8} mol/L, highlighting how slight deviations from neutral *pH* reflect significant biological balance.

In the “My Solution tab” simulation, users can directly manipulate the concentrations of hydronium ($H3O^+$) and hydroxide (OH^-) ions by dragging their respective indicators along the logarithmic scale. As users adjust these values, the interface dynamically updates to display both the resulting $H3O^+/OH^-$ ratio and the actual particle counts in the solution. The display shows concentrations in mol/L alongside absolute quantities in moles, with the pH value automatically calculated and displayed based on the current ion balance e.g., pH 8.66 shown). The logarithmic scale, ranging from 10^0 *to* 10^{-16} allows users to work with the enormous span of possible ion concentrations while the particle counts (visible in quantities like 6.63×10^{14}) provide concrete molecular-scale understanding of the solution's composition.

When creating a basic solution with pH 8.66, the simulation shows OH^- ions dominating at 4.5×10^{-9} mol/L *concentration*, while ($H3O^+$) ions are nearly absent at 2.2×10^{-9} mol/L. The particle counts reveal this imbalance dramatically - with hydroxide particles outnumbering hydronium particles by factors of 10^5 or more. If the user then drags the ($H3O^+$) indicator to increase its concentration to 1×10^{-4} mol/L (pH 4), they'll immediately see the OH^- concentration drop to 1×10^{-10} mol/L and watch the particle counts flip to show ($H3O^+$) dominance, visually demonstrating the reciprocal relationship between these ions in water.

Use the simulation to explore the connection between the concentration of H_3O^+ (the hydronium ion) and pH for a variety of substances such as chicken soup, hand soap, spit, milk, etc.

The pH scale is a logarithmic scale. Deduce how logarithms are used to find the value of pH from the concentration of the acid which is given by H_3O^+.

Discuss the advantages and disadvantages in using a logarithmic scale instead of simply stating the concentration of H_3O^+.

Part 2

The Richter scale is another logarithmic scale. Do some research on your own to learn about this scale and when it is used.

Student view

Consider the following questions:

- How is the Richter scale similar to the pH scale?
- In what ways does a logarithmic scale enhance our understanding of the reported values?
- What kinds of quantities are best described using a logarithmic scale?
- Can you find some other real-world examples where a logarithmic scale is used?