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Teacher view

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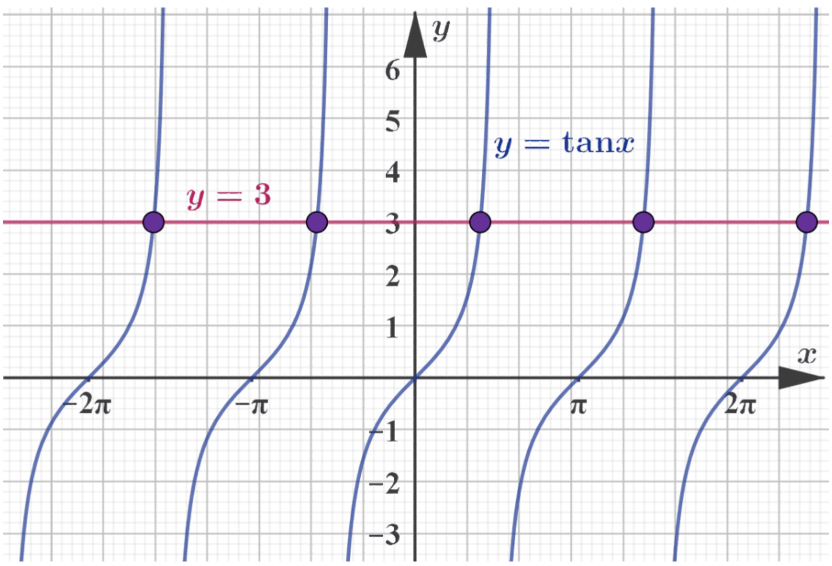
Glossary

Reading assistance

3. Geometry and trigonometry / 3.8 Trigonometric equations

The big picture

As you learned in previous sections, trigonometric functions are periodic and they repeat the same values of y in certain intervals. As you saw in subtopic 3.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27752/) , water tides follow the period of the moon and therefore will reach the same height over and over again. A swinging pendulum will reach the same height, as long as it continues to swing without any loss of energy. Therefore, when a trigonometric function is equal to a certain value, there could be infinitely many solutions to the equation.



More information

Student view

The graph displays the function y = tan(x) as a repeating series of curves between vertical asymptotes. The x-axis is labeled with multiples of π/2, specifically -3π/2, -π/2, π/2, and 3π/2. The y-axis ranges from -6 to 6, marked in unit intervals. A horizontal line, y = 3, intersects the tan(x) curves at periodic points. These intersection points are highlighted with purple markers, showing the solutions to the equation tan(x) = 3 within the given window. Both the tangent function and the line are drawn to illustrate the periodic nature and the infinite number of solutions in a repeated cycle.

[Generated by AI]

The diagram above shows the solutions of the trigonometric equation $\tan x = 3$. Although only a finite number of solutions can be seen in the given window, there are infinitely many solutions to this equation.

In this subtopic, you will find solutions of equations in a finite interval using analytical approaches and technology.

Concept

Trigonometric functions follow predictable recurring patterns.

How can you use these patterns to help you predict and approximate solutions to mathematical equations?

Theory of Knowledge

Solving equations can sometimes seem formulaic (pun intended!) and void of creativity. Does mathematics lack creativity?

More importantly, a key knowledge question is, ‘Is the degree of creative freedom within an area of knowledge positively correlated with the expansion of knowledge within that AOK?’

Trigonometric equations in finite intervals

Solving trigonometric equations analytically is no different from solving any other equation. You should balance and simplify the equation until you are left with a single trigonometric function on one side and a value on the other side of the equation. Then you can use inverse trigonometric functions to find the angle. During this process you might need to use some of the trigonometric identities and values of trigonometric ratios of special angles you have learned in previous sections of this topic.

Important

Here are some identities you might find useful when dealing with equations.

These will be given in the IB formula booklet.

- Pythagorean identity:
  $\cos^2\theta + \sin^2\theta = 1$
- Double-angle identities:
  $\sin 2\theta = 2\sin\theta\cos\theta$
  $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- Identity for $\tan\theta$:
  $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

The trigonometric ratios of special angles are also useful. Below are the special triangles you could use to find these ratios.

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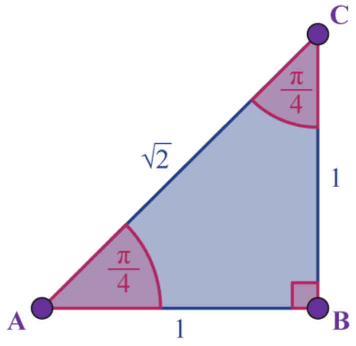

More information

The image shows a right triangle, labeled as triangle ABC. The right angle is at point B. The side AB is labeled as 1, the side BC is also labeled as 1, and the hypotenuse AC is labeled as the square root of 2. The angle at point A is labeled as π/4, and the angle at point C is also labeled as π/4. These angles suggest the triangle is a 45-45-90 special right triangle.

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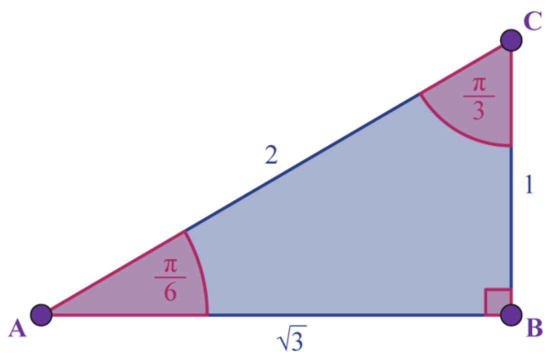

More information

This image is a diagram of a right triangle labeled with points A, B, and C. The triangle is oriented with angle B as the right angle. The side opposite angle B is labeled with a length of 2, the adjacent side is labeled with a length of √3, and the hypotenuse, opposite angle A, is labeled with a length of 1.

Angles are labeled inside the triangle: the angle at point A is π/6, and the angle at point C is π/3. A small square at point B indicates a 90-degree angle, confirming it as a right triangle.

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Example 1

★★☆

Solve the equation $\cos\left(\theta - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$ for $-\pi \leq \theta \leq \pi$.

Steps	Explanation
$\left(\theta - \frac{\pi}{2}\right) = \frac{\pi}{6}$ or $\left(\theta - \frac{\pi}{2}\right) = \frac{-\pi}{6}$	Using the unit circle, $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ or $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.
$\theta = \frac{\pi}{6} + \frac{\pi}{2}$ or $\theta = -\frac{\pi}{6} + \frac{\pi}{2}$ $\theta = \frac{4\pi}{6} = \frac{2\pi}{3}$ or $\theta = \frac{2\pi}{6} = \frac{\pi}{3}$	Balancing the equations. Both solutions are in the given interval $-\pi \leq \theta \leq \pi$.
Therefore, $\theta = \frac{2\pi}{3}$ or $\theta = \frac{\pi}{3}$	

Example 2

★★★

Solve the equation $3\cos\theta - 4\sin\theta = 0$ for $0 \leq \theta \leq \frac{\pi}{2}$.

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Steps	Explanation
$3\cos\theta - 4\sin\theta = 0$ $3\cos\theta = 4\sin\theta$ $\frac{\sin\theta}{\cos\theta} = \frac{3}{4}$	Rearrange. Since there is no angle for which both $\cos\theta = 0$ and $\sin\theta = 0$, the case cos does not give a solution. If $\cos\theta \neq 0$, you can divide the equa
$\tan\theta = \frac{3}{4}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\theta = \tan^{-1}\left(\frac{3}{4}\right)$	Using the inverse function of $\tan\theta$.
Therefore, $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ or $\theta = \arctan\left(\frac{3}{4}\right)$	As $0 \leq \theta \leq \frac{\pi}{2}$, there is only one ans

Example 3

★★★

Find the solution to the equation $2\sin\theta\cos\theta = \sin\theta$ for $-\pi < \theta < \pi$.

Steps	Explanation
$2\sin\theta\cos\theta = \sin\theta$ $2\sin\theta\cos\theta - \sin\theta = 0$	Do not simplify $\sin\theta$. If you do so, you will lose one of solutions, as $\sin\theta$ might be equal to 0. Rearrange the equation.

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Steps	Explanation
$\sin\theta(2\cos\theta - 1) = 0$	Factorise $\sin\theta$.
$\sin\theta = 0$ or $2\cos\theta - 1 = 0$	Using the null factor theorem .
$\sin\theta = 0 \Rightarrow \theta = 0$	Using the unit circle.
$2\cos\theta - 1 = 0$ $\cos\theta = \frac{1}{2}$	Rearrange the equation.
$\theta = \frac{\pi}{3}$ or $\theta = -\frac{\pi}{3}$	Using special angles and $-\pi < \theta < \pi$.
So, $\theta = \frac{\pi}{3}$, $-\frac{\pi}{3}$ or 0	

Exam tip

In Paper 1 of the IB examination, you cannot use your GDC, so all your answers should be exact values. For special angles, write the exact value; otherwise, use the inverse trigonometric functions arcsine, arccosine and arctangent. Do not round π; leave your answer as a multiple of π .

Example 4

★★☆

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Solve $\tan^2 x = \frac{1}{3}$ for $0 \le x \le 2\pi$.

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Steps	Explanation
$\tan^2 x = \frac{1}{3} \Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$	Take the square root of both sides and do nc the negative values.
$\tan x = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6} \text{ or } \frac{7\pi}{6}$	Using special triangles and the unit circle.
$\tan x = -\frac{1}{\sqrt{3}} \Rightarrow x = \frac{5\pi}{6} \text{ or } \frac{11\pi}{6}$	Using special triangles and the unit circle.
So, $$x = \frac{\pi}{6}\ ,\ \frac{7\pi}{6}, \frac{5\pi}{6} \text{ or } \frac{11\pi}{6}$$	

Example 5

★★☆

Find all the solutions of the equation $2\sin^2 x - 1 = 0$ for $x \in [0,\ 2\pi]$.

Steps	Explanation
$$\begin{aligned} 2\sin^2 x - 1 &= 0 \\ 2\sin^2 x &= 1 \\ \sin^2 x &= \frac{1}{2} \\ \sin x &= \pm\frac{\sqrt{2}}{2} \end{aligned}$$	Rearrange to get a single trigonometric r to a number.

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Steps	Explanation
These angles are $$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ and } \frac{7\pi}{4}.$$	Look for the angles that correspond to p the unit circle with y -coordinate equal to
Thus, $$x \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ and } \frac{7\pi}{4}\right\}.$$	

Example 6

★★☆

Find all the values of θ such that $2\sin 2\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$.

Steps	Explanation
$$\begin{aligned} 2\sin 2\theta &= \cos\theta \\ 2\times 2\sin\theta\cos\theta &= \cos\theta \\ 4\sin\theta\cos\theta - \cos\theta &= 0 \\ \cos\theta(4\sin\theta - 1) &= 0 \\ &\text{so} \\ \cos\theta &= 0 \text{ or} \\ 4\sin\theta - 1 &= 0 \\ &\text{so} \\ \cos\theta &= 0 \text{ or} \\ \sin\theta &= \frac{1}{4} \end{aligned}$$	Rearrange to get a single trigonometric ratio equal number.

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Steps	Explanation
$\cos\theta = 0 \;\Rightarrow\; \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\sin\theta = \frac{1}{4} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{4}\right)$ or $\theta = \pi - \sin^{-1}\left(\frac{1}{4}\right)$	Using the unit circle.
Thus, $\theta \in \left\{\sin^{-1}\left(\frac{1}{4}\right), \frac{\pi}{2}, \pi - \sin^{-1}\left(\frac{1}{4}\right), \frac{3\pi}{2}\right\}$	

3 section questions

Question 1

★★☆

Solve $\sin 2\theta = 1 - \cos^2\theta, 0 \leq \theta < \pi$

1 $0, \tan^{-1} 2$ ✓

2 0

3 $\tan^{-1} 2$

4 $0,\ \pi$

Explanation

Using the identities $\sin(2\theta) = 2\sin\theta\cos\theta$ and $\cos^2\theta + \sin^2\theta = 1$

$\sin\, 2\theta = 1 - \text{co s}^2\theta$

$2\sin\theta\cos\theta = \sin^2\theta$

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Rearranging:

$$2 \sin \theta \cos \theta - \sin^2\theta = 0.$$

Factorising:

$$\sin \theta(2 \cos \theta - \sin \theta) = 0.$$

The product is 0 if one of the factors is 0.

$$\sin \theta = 0 \text{ or } 2 \cos \theta - \sin \theta = 0$$

- $\sin \theta = 0$ gives $\theta = 0$
- The rearrangement of $2 \cos \theta - \sin \theta = 0$ is $2 \cos \theta = \sin \theta$. Dividing this equation by $\cos \theta$ gives $\tan \theta = 2$, so the corresponding solution between 0 and π is $\theta = \tan^{-1} 2$.

Question 2

★★★

Solve $\sin \theta \tan \theta = \sin^2 \theta$, $0 \leq \theta \leq 2\pi$

1 $0,\ \pi,\ 2\pi$ ✓

2 $\pi,\ 2\pi$

3 $0,\ \pi$

4 $0,\ 2\pi$

Explanation

Using $\tan \theta \equiv \dfrac{\sin \theta}{\cos \theta}$

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$$\sin\theta\frac{\sin\theta}{\cos\theta} = \sin^2\theta$$
$$\frac{\sin^2\theta}{\cos\theta} = \sin^2\theta$$
$$\frac{\sin^2\theta}{\cos\theta} - \sin^2\theta = 0$$
$$\frac{\sin^2\theta}{\cos\theta} - \frac{\cos\theta\sin^2\theta}{\cos\theta} = 0$$
$$\sin^2\theta - \cos\theta\sin^2\theta = 0$$
$$\sin^2\theta(1-\cos\theta) = 0$$

Using the null factor theorem

$$\sin^2\theta = 0 \Rightarrow \theta = 0,\ \pi,\ 2\pi$$

$$1-\cos\theta = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0,\ 2\pi.$$

Therefore,

$$\theta = 0,\ \pi,\ 2\pi.$$

Question 3

★★☆

What are the solutions to $\cos 2x = -\frac{1}{\sqrt{2}},\ 0 \leq x \leq \pi$?

1 $\left\{\frac{3\pi}{8}, \frac{5\pi}{8}\right\}$ ✓

2 $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

3 $\left\{\frac{3\pi}{2}, \frac{5\pi}{2}\right\}$

4 $\left\{\frac{\pi}{8}, \frac{7\pi}{8}\right\}$

Explanation

If $0 \leq x \leq \pi$, then $0 \leq 2x \leq 2\pi$.

We know that $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

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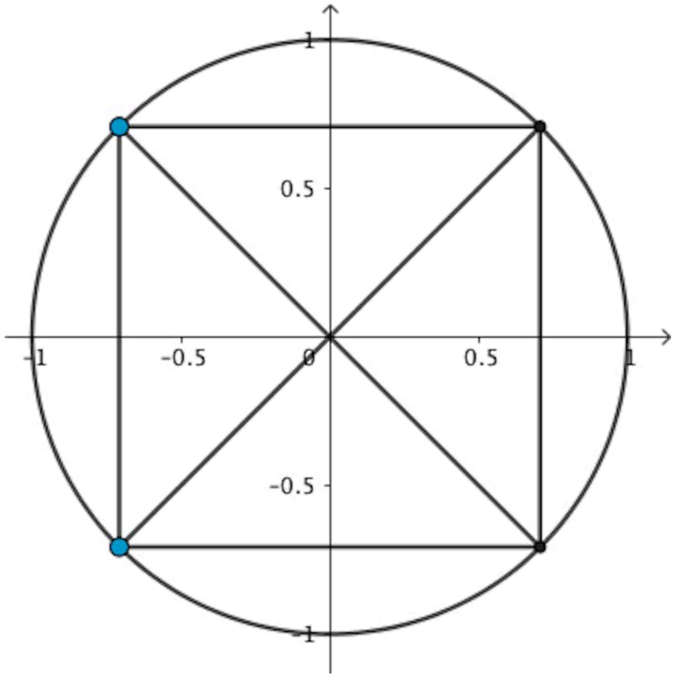
Using the symmetries of the unit circle (see the diagram below), this implies that the solutions of $\cos\theta = -\frac{1}{\sqrt{2}}$ between 0 and 2π are $\theta = \frac{3\pi}{4}$ and $\theta = \frac{5\pi}{4}$.

Hence, $2x = \frac{3\pi}{4}$ or $2x = \frac{5\pi}{4}$, so $x = \frac{3\pi}{8}$ or $x = \frac{5\pi}{8}$.



More information

3. Geometry and trigonometry / 3.8 Trigonometric equations

Using technology

Finite interval equations: graphical method

As we have seen, the solutions to trigonometric equations may not correspond to nice values, i.e. the values on the unit circle corresponding to the special angles. In such cases, you will need to use your graphic display calculator. The graphical method for solving trigonometric equations often requires the use of your graphic display calculator.

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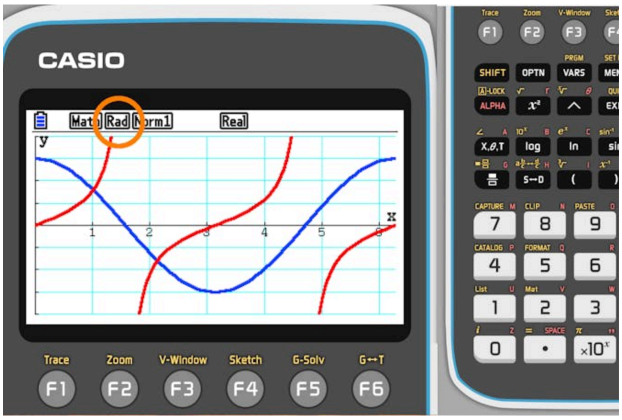
Let us consider the equation

$$3\cos x = \tan x\ ,\ \ x \in [0, 2\pi].$$

From our knowledge of the signs of cosines and tangents, we know that there are two solutions, which are, moreover, in the first (both positive) and second (both negative) quadrants.

In the following, we show how to solve this equation with different graphic display calculator models.

Steps	Explanation
In these instructions you will see how to find the solutions of the equation $3\cos x = \tan x$ for $0 \leq x \leq 2\pi$. It is assumed, that you have the graph of $y = 3\cos x$ and $y = \tan x$ on the screen for $0 \leq x \leq 2\pi$. Make sure that the calculator is in radian mode. Press F5 (G_Solve) to bring up the options to analyze the graphs ...	

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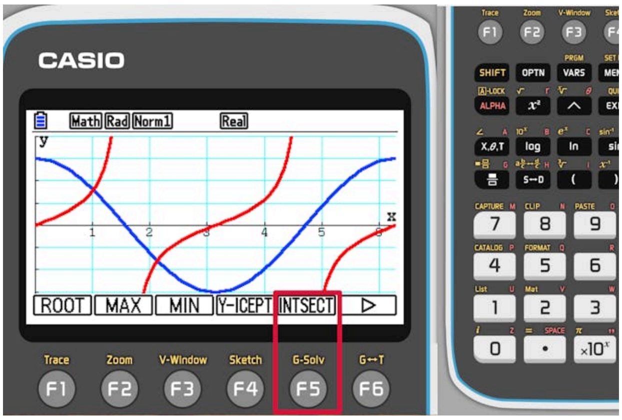
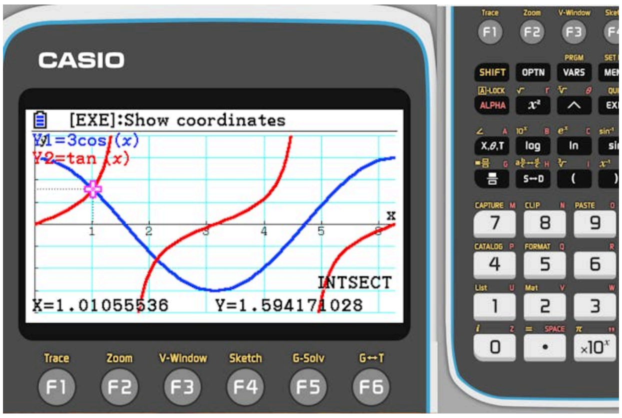
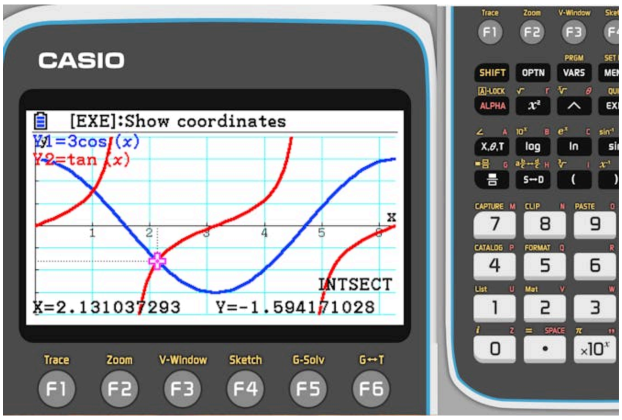
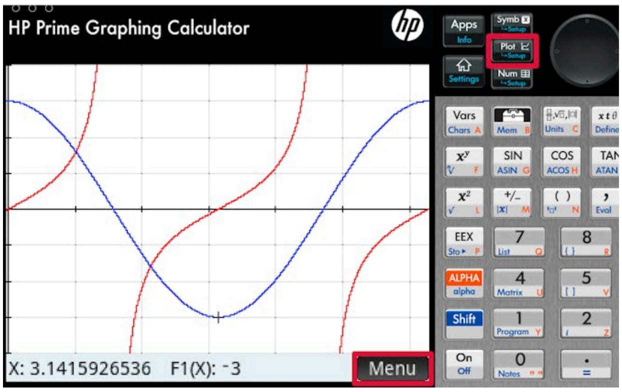
Steps	Explanation
... and F5 again to look for the intersection points of the graphs.	
The calculator moves the cursor to the first intersection point and displays its coordinates. The x-coordinate is the first solution of the equation. To find the other intersection point, move to the right.	

Steps	Explanation
The cursor is moved to the second intersection point. The x-coordinate is the second solution of the equation.	

Steps	Explanation
In these instructions you will see how to find the solutions of the equation $3\cos x = \tan x$ for $0 \leq x \leq 2\pi$. It is assumed, that you have the graph of $y = 3\cos x$ and $y = \tan x$ on the screen for $0 \leq x \leq 2\pi$. Make sure that the calculator is in radian mode. In the plot view of the function application tap on menu ...	

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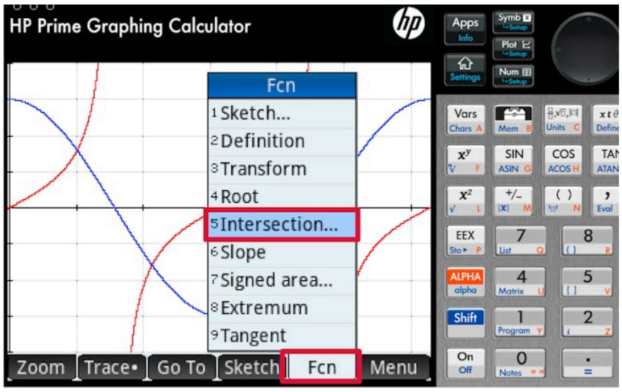
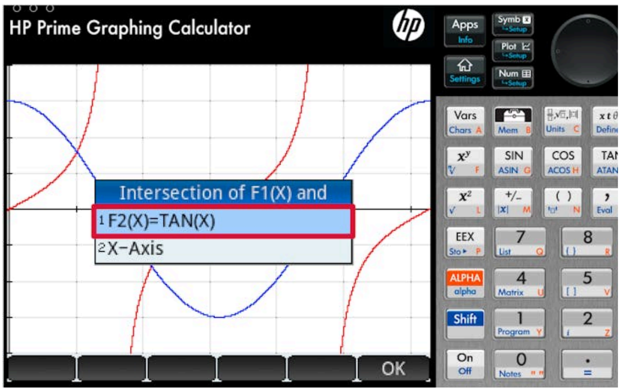
Steps	Explanation
... and choose the option to find the intersection points of the two graphs.	
Choose the two graphs.	

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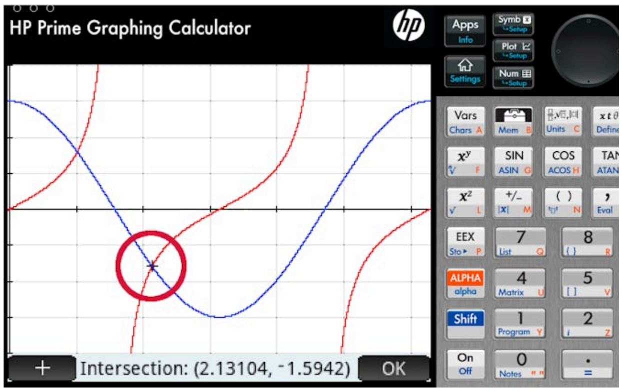
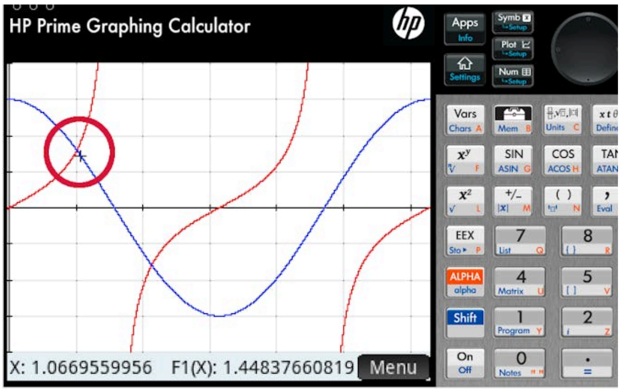
Steps	Explanation
The calculator moves the cursor to one of the intersection points and displays its coordinates. The x-coordinate is the first solution of the equation.	
To find the other intersection point, move the cursor close to it and repeat the process (menu ...).	

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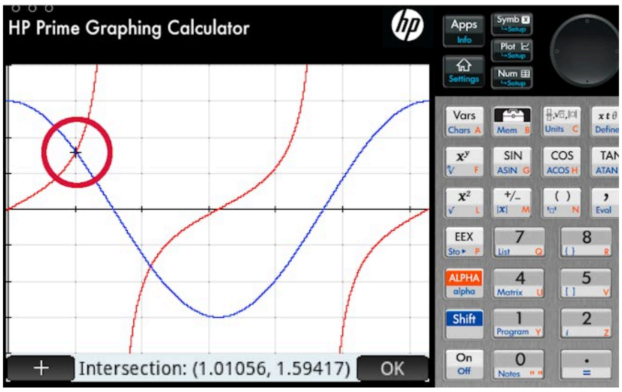
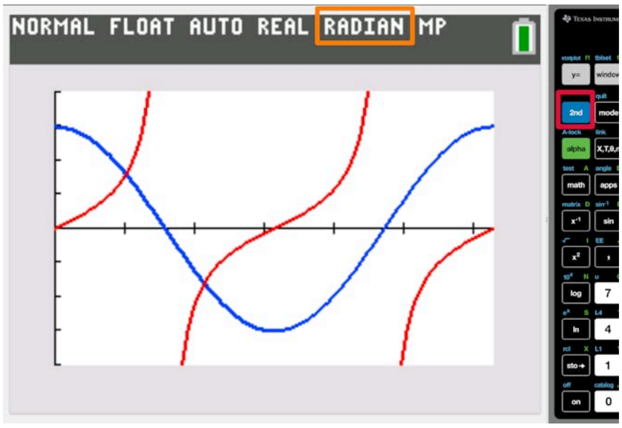
Steps	Explanation
The cursor is moved to the second intersection point. The x-coordinate is the second solution of the equation.	

Steps	Explanation
In these instructions you will see how to find the solutions of the equation $3\cos x = \tan x$ for $0 \leq x \leq 2\pi$. It is assumed, that you have the graph of $y = 3\cos x$ and $y = \tan x$ on the screen for $0 \leq x \leq 2\pi$. Make sure that the calculator is in radian mode. Press 2nd/calc to bring up the options to analyze the graphs ...	

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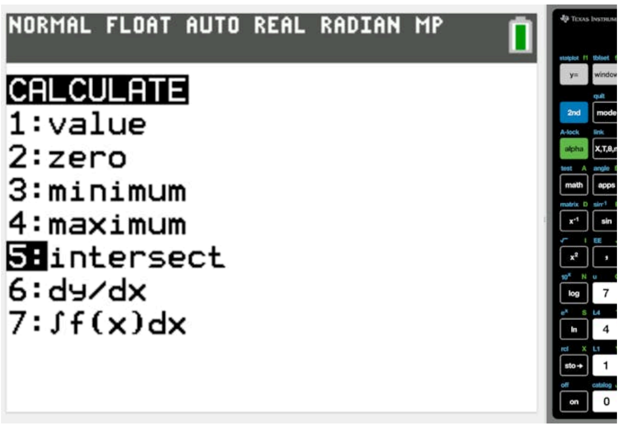
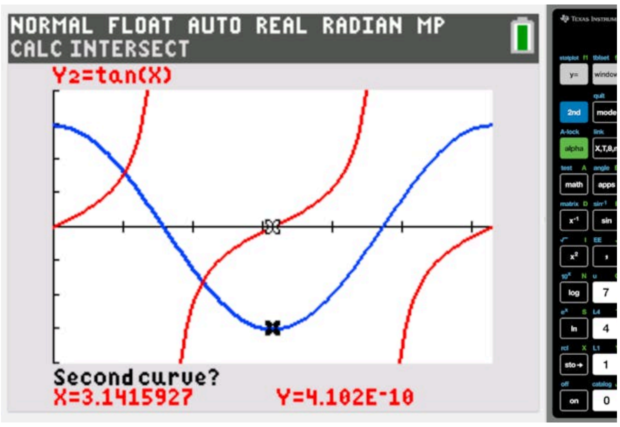
Steps	Explanation
... and choose the option to find the intersection points of the two graphs.	
The calculator will ask you to identify the first and the second curve (move the cursor to the curve and press enter to confirm), ...	

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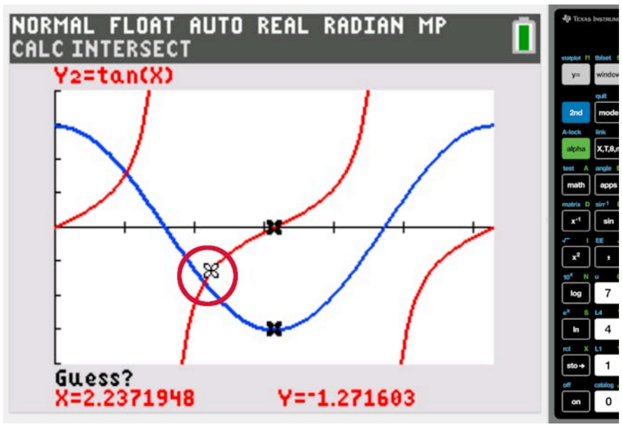
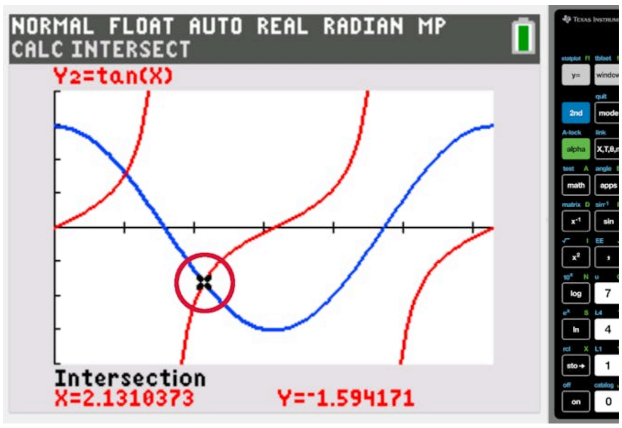
Steps	Explanation
... and the calculator asks you to make a guess. Move the cursor close to the intersection point.	
The calculator moves the cursor to the intersection point and displays its coordinates. The x-coordinate is the first solution of the equation.	

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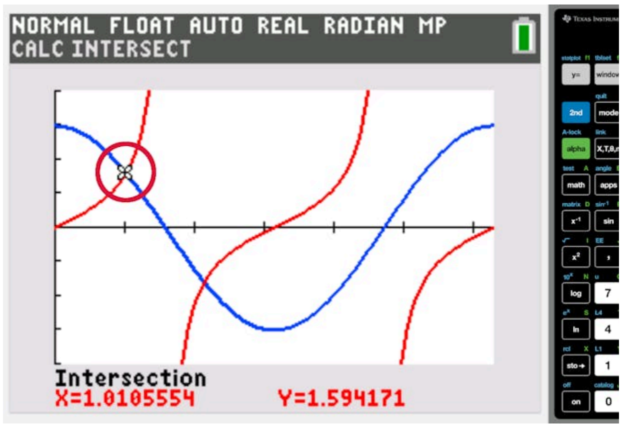
Steps	Explanation
To find the other intersection point, repeat the process (2nd/calc, ...). This time type $x = 1$ when the calculator asks for a guess (since from the graphs you can see, that the intersection point is close to that value). The cursor is moved to the second intersection point. The x-coordinate is the second solution of the equation.	

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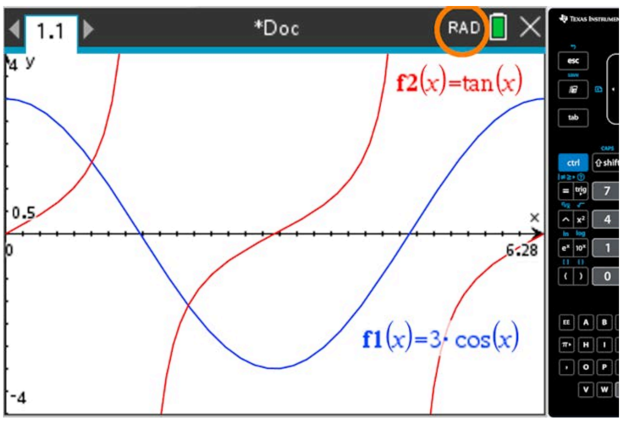
In these instructions you will see how to find the solutions of the equation $3\cos x = \tan x$ for $0 \leq x \leq 2\pi$. It is assumed, that you have the graph of $y = 3\cos x$ and $y = \tan x$ on the screen for $0 \leq x \leq 2\pi$. Make sure that the calculator is in radian mode. Press menu to bring up the options to analyze the graphs ...	

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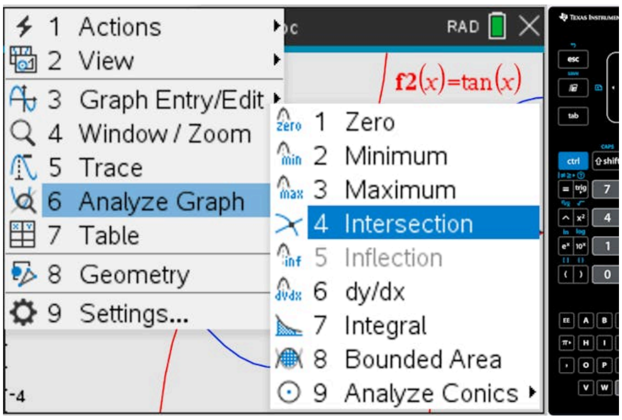
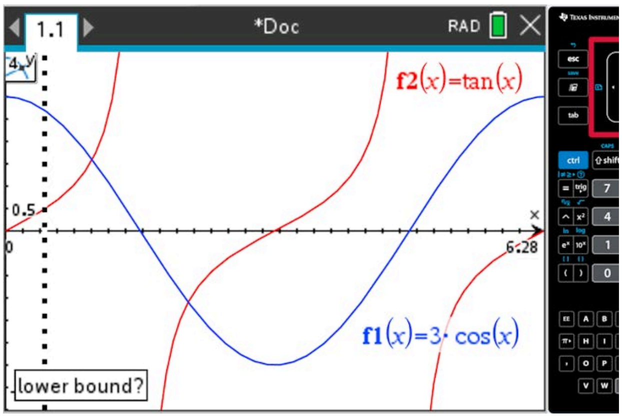
Steps	Explanation
... and choose the option to find the intersection points of the two graphs.	
The calculator needs to know which intersection point you are looking for. Move to the left of the point and press enter to confirm the position ...	

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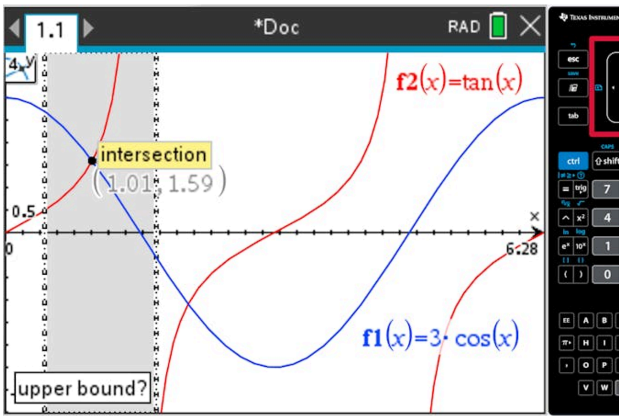
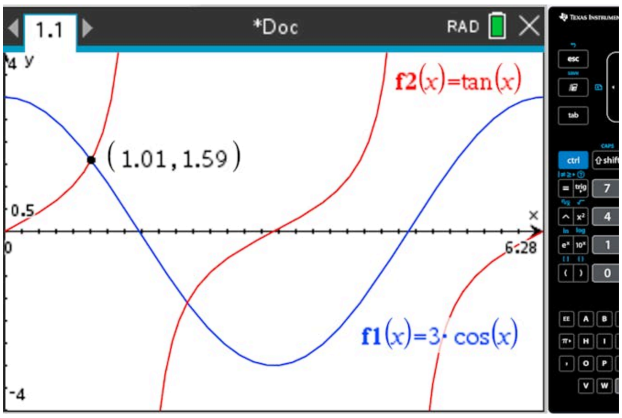
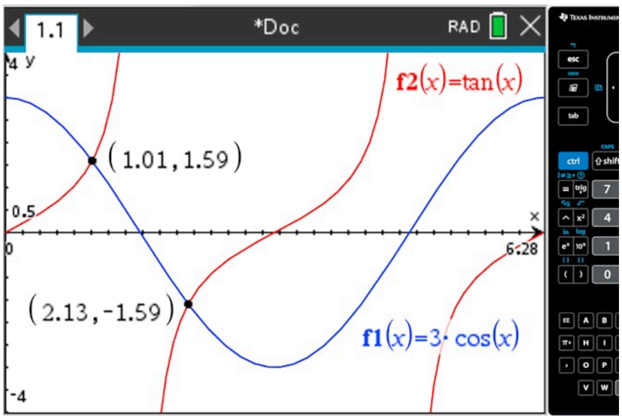
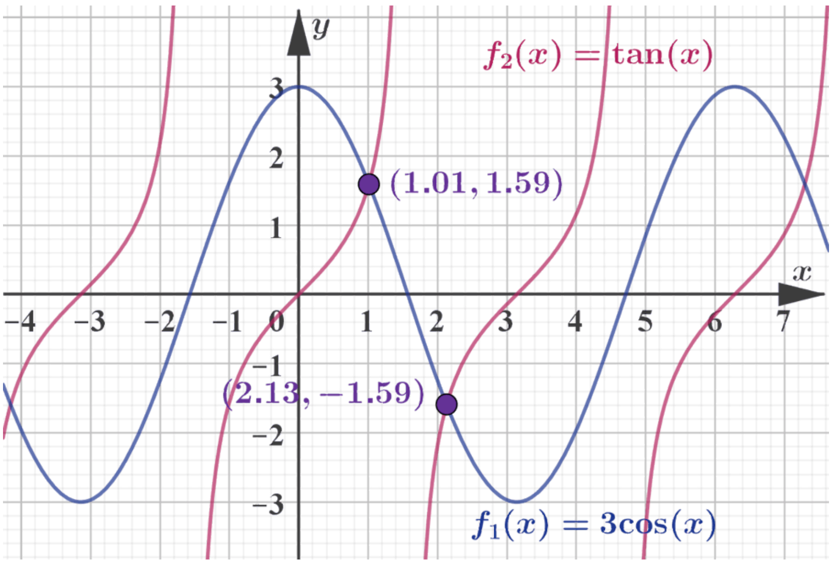
Steps	Explanation
... then move to the right of the intersection point and press enter.	
The calculator marks the intersection points and displays its coordinates. The x-coordinate is the first solution of the equation.	

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
Repeat the process (menu ...) to find the second intersection point. The x-coordinate is the second solution of the equation.	

In the above, we graphed both $3\cos x$ and $\tan x$ over an interval that included $[0, 2\pi]$, and found the points of intersection with the graphic display calculator . The graph is reproduced in the diagram below .



More information

Student view

This image is a graph that displays the functions f1(x) = 3cos(x) and f2(x) = tan(x). The graph is plotted on a grid over the interval [0, 2π], with the x-axis labeled from 0 to 7 and the y-axis labeled from -3 to 3. The function 3cos(x) is depicted as a blue wave-like curve, with peaks and troughs between these intervals. The function tan(x) is shown as a red curve with vertical asymptotes and swift changes that repeat in regular intervals.

There are two marked intersection points on the graph: one at approximately (1.01, 1.59) and another at (2.13, -1.59), representing where the two functions intersect. These points are highlighted with purple dots. The y-axis shows the range of -3 to 3, capturing the behavior of both functions within this interval.

[Generated by AI]

As you will have experienced, finding intersection points with your graphic display calculator though easy, is time-consuming. Thus, if you are not required to find the values of the intersection points, but merely find the number of intersection points, all you need to do is plot the two functions and count the points of intersection.

Thus, if the question states:

‘Find the number of solutions to $\cos^2 x = x^5 + 4x^4 - x^3 - 6x^2 + x + 1.5$’,

you need not find the values of the solutions. Hence, plotting these two functions and counting the points of intersection, we see that there are five solutions (shown in the diagram below).

Student
view

Overview
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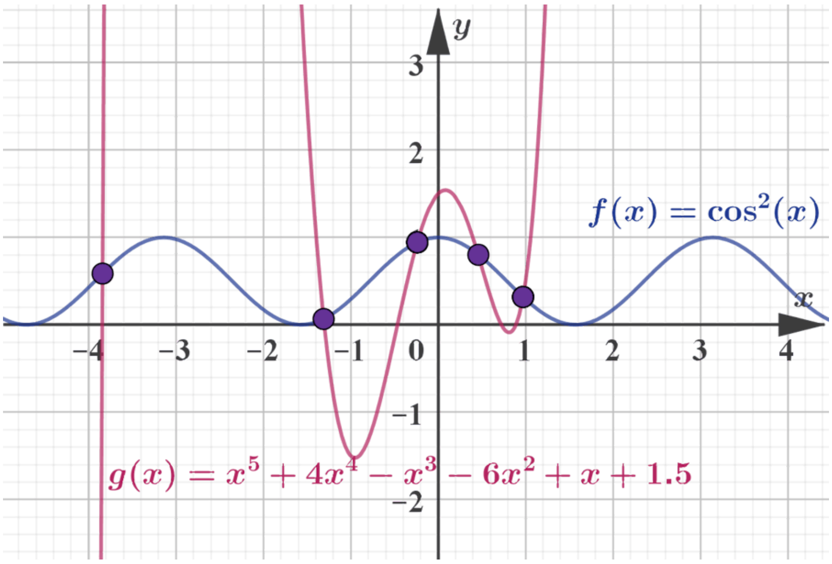

More information

The graph displays two functions, (f(x) = \cos^2(x)) and (g(x) = x^5 + 4x^3 - x - 6x^2 + x + 1.5), plotted on a grid. The X-axis ranges from -4 to 4, and the Y-axis ranges from -3 to 3. The functions intersect at five points, marked by purple circles. The first function, (f(x)), is a blue wave-like curve, while the second function, (g(x)), is a more complex magenta curve with multiple peaks and valleys. The intersections appear between x-values of approximately -4 to 4.

[Generated by AI]

Be aware

When solving trigonometric equations graphically with your graphic display calculator , make sure your graphic display calculator is set to the correct unit of angle, i.e. degrees or, usually, radians.

Important

When solving trigonometric equations, make sure to check your answers using graphical methods as well. Often, your calculator will give you one of the answers but there might be more than one solution.

Student view

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Activity

Using software, graph the function $f(x) = 3 + 2\sin\left(\frac{\pi x}{6}\right), 0 \leqslant x \leqslant 3\pi$.

Create a slider $0 \leqslant a \leqslant 6$.

Move the slider to investigate when $f(x) = a$ has:

- no solution
- one solution
- two or more solutions.

Explain algebraically when and why $f(x) = a$ has a solution and identify the nature of the roots.

Making connections

There are many real-life applications of trigonometry. It is used in heart rate monitors to follow patients' heart rates, in video games to control smooth movement and in construction.

Trigonometry is also used in sciences such as physics and chemistry.

Chemists use Bragg's law which states that when the x-ray is incident onto a crystal surface, its angle of incidence, θ , will reflect back with a same angle of scattering, θ .

$$n\lambda = 2d\sin\theta \text{, where}$$

- λ is the wavelength of the x-ray
- d is the spacing of the crystal layers (path difference)
- θ is the incident angle (the angle between incident ray and the scatter plane)
- n is an integer.

They use the law and a Bragg spectrometer to study the structure of crystals and molecules.

Student view

Crystal surface
Credit: Obradovic Getty Images

2 section questions

Question 1

★☆☆

What are the solutions to the equation $5\sin^2(2x) = 4\cos^2(x-1)$ where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$?

Give your answer rounded to three significant figures.

1 $\{-1.27, -0.176, 0.424, 1.02\}$ ✓

2 $\{-1.26583, -0.175718, 0.424180, 1.01737\}$

3 There are no solutions

4 $\{0.424, 1.02\}$

Explanation

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Using our GDC, this is best done using the graphical method, and stating the values to 3 significant figures.

Question 2

★☆☆

How many solutions are there to the equation $\tan x \sin^2 2x = \cos x$, $-2\pi \leq x \leq 2\pi$?

1 4 ✓

2 6

3 8

4 2

Explanation

Use your GDC to graph the two functions $y = \tan x \sin^2 2x$ and $y = \cos x$ and count the intersection points over the interval $[-2\pi, 2\pi]$.

Note that the graphs seem to intersect eight times, but at four of these intersection points $\cos\ x = 0$, so $\tan\ x$, and hence the left hand side, is not defined.

3. Geometry and trigonometry / 3.8 Trigonometric equations

Equations leading to quadratics

Some of the trigonometric equations will require use of solutions of quadratic equations, which you studied in subtopic 2.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27705/). You can solve quadratic equations either by factorising or using the quadratic formula.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Example 1

★★☆

Solve $2\cos^2 x + \cos x - 1 = 0\,,\,0 \leq x \leq 2\pi$

Steps	Explanation
Let $a = \cos x$ $2a^2 + a - 1 = 0$	You can solve the equation for $\cos x$ by defining it a $a = \cos x$.
$(2a - 1)\,(a + 1) = 0$	Factorise the equation.
$2a - 1 = 0$ or $a + 1 = 0$	Using the null factor theorem.
$a = \frac{1}{2}$ or $a = -1$	Solve both equations for a.
$\cos x = \frac{1}{2}$ or $\cos x = -1$	Substitute $a = \cos x$.
$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}\ or\ \frac{5\pi}{3}$	Using special triangles, solve in $0 \leq \theta \leq 2\pi$.
$\cos x = -1 \Rightarrow x = \pi$	Using the unit circle.
So, $x = \frac{\pi}{3}\,,\ \frac{5\pi}{3}\ or\ \pi$	

Example 2

Student view

★★★

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Solve $\sin^2 2\theta + 2\sin\theta\cos\theta - 2 = 0\,,\, 0 \leq \theta \leq \pi$

Steps	Explanation
$\sin^2 2\theta + \sin 2\theta - 2 = 0$ $a^2 + a - 2 = 0$	Substitute $\sin 2\theta = a$.
$(a-1)(a+2) = 0$	Factorise the equation.
$a - 1 = 0 \text{ or } a + 2 = 0$	Using the null factor theorem.
$a = 1 \text{ or } a = -2$	Solve each equation.
$\sin 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$	Substitute $a = \sin 2\theta$.
$\sin 2\theta = -2 \text{ has no solution}$	As $\lvert\sin x\rvert \leq 1$.
Therefore, $\theta = \frac{\pi}{4}$	

International Mindedness

Global positioning systems, or GPS, have become widely available to the public with the use of smartphones. GPS uses trigonometry to track the movement of individuals using their phone signals. This system has been used to find criminals and also to help victims in natural disasters. There are various debates considering whether it is ethical to collect GPS data from mobile phone users, as often users are not aware that their movements are being followed. What do you think?

Student view

GPS
Credit: Sam Bianchini Getty Images

3 section questions

Question 1

★★☆

Solve $4 - \cos 2\theta + 5\sin\theta = 0,\ 0 \leq \theta \leq 2\pi.$

1 $\frac{3\pi}{2}$ ✓

2 $\frac{\pi}{2}$

3 $\frac{3}{2}$

4 $\frac{-3}{2}$

Explanation

Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$

$$4 - (1 - 2\sin^2\theta) + 5\sin\theta = 0.$$

Expand brackets and simplify

$$2\sin^2\theta + 5\sin\theta + 3 = 0.$$

Solving the quadratic equation

$$\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2} \text{ or } \sin\theta = -\frac{3}{2} \text{ has no solution.}$$

Therefore, $\theta = \frac{3\pi}{2}$.

Question 2

★★★

Solve $2\sin^2 x - 1 = \cos x$, $0 \leq x \leq 360°$.

1 $60°,\ 180°,\ 300°$ ✓

2 $30°,\ 180°,\ 330°$

3 $60°,\ 180°,\ 330°$

4 $60°,\ 270°,\ 330°$

Explanation

Using $\sin^2 x = 1 - \cos^2 x$

$$2\left(1 - \cos^2 x\right) - 1 = \cos x.$$

Rearranging,

$$2\cos^2 x + \cos x - 1 = 0.$$

Factorise:

$$(2\cos x - 1)(\cos x + 1) = 0$$

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$$2\cos x - 1 = 0 \text{ or } \cos x + 1 = 0$$

$$\cos x = \frac{1}{2} \text{ or } \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi.$$

Question 3

★★★

Solve $2\tan^2 x - 3\tan x = 2$, $0 \leq x \leq 360°$ to the nearest degree.

1 $63°,\ 153°,\ 243°,\ 333°$ ✓

2 $63°,\ 53°,\ 117°,\ 333°$

3 $27°,\ 153°,\ 243°,\ 333°$

4 $27°,\ 53°,\ 207°,\ 333°$

Explanation

Rearrange:

$$2\tan^2 x - 3\tan x - 2 = 0$$

Factorise:

$$(2\tan x + 1)(\tan x - 2) = 0$$

$$2\tan x + 1 = 0 \text{ or } \tan x - 2 = 0$$

$$\tan x = -\frac{1}{2} \text{ or } \tan x = 2$$

Using a GDC:

$$x = \tan^{-1}\left(-\frac{1}{2}\right) = -26.57$$

Student
view

In the given domain, $x = 180° - 27°$ or $360° - 27°$.

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$$x = \tan^{-1}(2) = 63.43$$

In the given domain, $x = 63°$ or $180° + 63°$.

So the solution set is

$$63°,\ 153°,\ 243°,\ 333°$$

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27763/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- solve trigonometric equations analytically using trigonometric identities when necessary
- identify whether a trigonometric equation leads to a quadratic equation and solve the equation using solutions of quadratic equations
- use your GDC to solve trigonometric equations approximately
- identify the difference between exact values of solutions and approximate values.

Student view

Investigation

Use the applet below to explore the number of solutions for some simple trigonometrical equations.

Interactive 1. Solutions for Simple Trigonometrical Equations.

Credit: GeoGebra (https://www.geogebra.org/m/NmnpVUmd) Paul Walter

More information for interactive 1

This interactive applet enables users to explore and solve fundamental trigonometric equations of the form $\sin x = a$, $\cos x = a$, and $\tan x = a$. Users can select their desired trigonometric function using checkboxes (sine, cosine, or tangent) and adjust the constant value ‘a’ using a slider at top. the interval for

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

solving the equation is set using two horizontal sliders labeled Start and End, which define the domain in degrees.

The applet provides a graphical representation of the selected trigonometric function along with the dashed horizontal line $y = a$. By clicking "Show Solution," users can instantly see all intersection points between the curve and the line, which correspond to the solutions of the equation within the specified domain. This visual approach helps users understand how the number and location of solutions depend on both the chosen function and the value of a.

For example, when solving $\cos x = 0.3$ within the default domain of $-180^\circ \leq x \leq 180^\circ$, the applet displays two solutions: $x = -72.54^\circ$ and $x = 72.54^\circ$. This demonstrates how the cosine function intersects the line $y = 0.3$ at symmetric points about the y-axis, illustrating the even nature of the cosine function and its periodic behavior. Users can further experiment by changing the function to sine or tangent, or by adjusting the domain to observe how these modifications affect the solutions.

Create some more complex trigonometrical equations and use your GDC to graph them. Can you tell from the graph how many solutions they have? Can you find the values of these solutions from the graph?

Rate subtopic 3.8 Trigonometric equations

Help us improve the content and user experience.

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