Kinematics

The big picture

? Guiding question(s)

- How can the motion of a body be described quantitatively and qualitatively?
- How can the position of a body in space and time be predicted?
- How can the analysis of motion in one and two dimensions be used to solve real-life problems?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

The word kinematics comes from the Greek word ‘kinesi’, which means motion. Walking in a park, cars on the road, trains on a railway and aeroplanes flying in the sky are examples of motion. Other examples include a firework exploding, with particles being ejected into the sky, someone bungee-jumping and experiencing an acceleration and a motorbike taking off from a ramp (**Figure 1**).

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Figure 1. The motion of a motorbike as it jumps.

Everything in the Universe is in constant motion through space and time – all the particles in every object are moving. If you think you are not moving while sitting on your chair, you are wrong. The Earth is moving in many different ways. It is spinning on its axis and also orbiting the Sun. The Sun itself is moving. The Solar System is moving within the Milky Way galaxy, and the Milky Way galaxy is also moving.

Why is it important to study motion? How can we describe all these types of motion?

Theory of Knowledge

You will be introduced to terms regarding the motion of bodies. Terminology is the 'sum' of the terms used in a specific subject.

Think about the following questions about terminology:

- Why do we need to use specific terms when we acquire or communicate knowledge?
- How does terminology affect the way we justify or explain something?
- How does terminology help in reconciling different perspectives on a topic?
- The terminology of a specific field of studies can change through time. Does this mean that knowledge is dependent on the terms used? If so, how?

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- Do you use terms in your everyday life? How does this influence the way you communicate your thoughts?
- Suggest one example of a term that reflects knowledge in an effective manner and another term that does not.

Prior learning

Before you study this subtopic make sure that you understand the following:

- Basics of algebra.
- Fundamentals of interpreting data for problem solving.
- Basic drawing and understanding of diagrams.

Practical skills

Once you have completed this subtopic, you can apply the equations of motion by going to Practical 1: Investigating the acceleration of free fall (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) and you can apply your knowledge of projectile motion by going to Practical 2: Investigating projectile motion (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-relationship-between-velocity-id-46751/).

A. Space, time and motion / A.1 Kinematics

Vectors

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Why, in science, do we use the concept of physical quantities? For example, instead of saying something ‘is cold’ or ‘it is hot’, we use a physical quantity called the temperature of a body.

Every physical quantity is determined by a number called the value or magnitude of the quantity, and a unit of measurement. For example: ‘The height of the building is 15 metres’ or ‘h = 15 m’. Why do physical quantities have units? Do all scientists use the same units for the same quantities?

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Nature of Science

Aspect: Measurement

When we call something, for example, a distance, ‘large’ or ‘small’, we are actually comparing it to something. But what are we comparing it to? The idea that we subconsciously view the magnitudes of the Universe in comparison to human scales is called anthropocentrism. This idea expands into interpreting and understanding the world through the lens of human experience.

In a scientific context, these expressions have less value. What is a ‘small’ distance or a ‘large’ distance in physics? The average distance between the atoms in an object is 10^{-10} m, but the average distance between two stars in the Milky Way galaxy is about 10^{16} m. If you walked a long way, what should you compare this distance to?

We can use the concept of a dimension scale. There are many different scales, which you can see in this video animation.

Video 1. The scale of the Universe.

Understanding the scale in which a phenomenon takes place is crucial for our understanding of nature.

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Scalars and vectors

Physical quantities help us determine various properties of nature in a measurable way. The value means we can refer to a number that is independent of how we feel about it, and communicate it in a scientifically meaningful way.

There are some quantities that are not fully determined without including a direction. For example, velocity is a quantity that tells you how fast a body is moving. However, two bodies can have exactly the same value (magnitude) of velocity, but are moving in different directions (**Figure 1**).

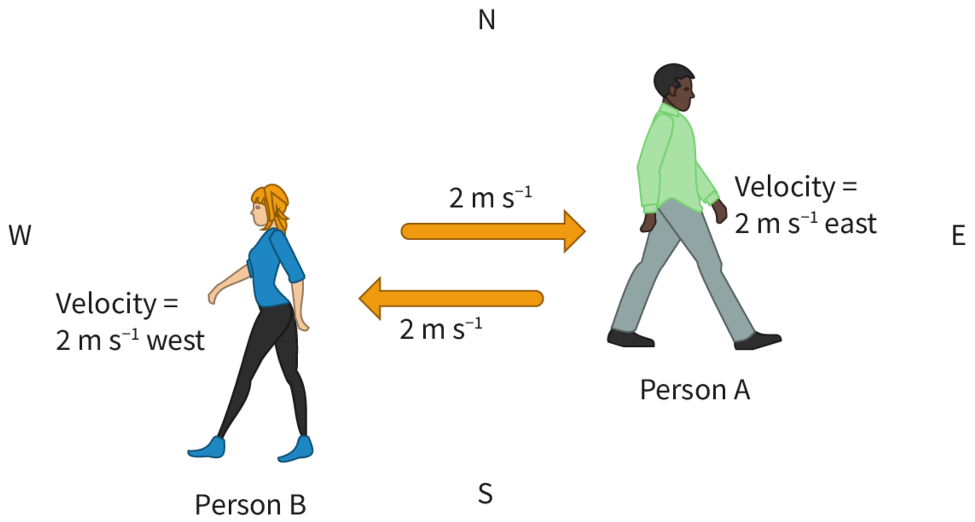

Figure 1. Two people walking in opposite directions.

More information for figure 1

The image shows two individuals walking in opposite directions, accompanied by arrows illustrating their velocity. Person A is walking to the right, labeled as east, with a velocity of 2 meters per second (m/s) east. Person B is walking to the left, labeled as west, with a velocity of 2 meters per second (m/s) west. Between them is a double-headed arrow indicating the same magnitude of 2 m/s for both directions. The cardinal directions are marked as N (north), S (south), E (east), and W (west), providing context for their movements. The image visually represents the concept of velocity having both magnitude and direction.

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We need to specify the magnitude of the velocity and also the direction. For person A, the velocity is $2\ \text{m s}^{-1}$ **to the right** (east). For person B, the velocity is $2\ \text{m s}^{-1}$ **to the left** (west).

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There are two categories of physical quantities:

- Scalars are quantities that are fully described by just a magnitude.
- Vectors are quantities that are fully described by a magnitude and a direction.

Table 1 shows some scalar and vector quantities.

Table 1. Scalar and vector quantities.

Scalar	Vector
Mass	Velocity
Temperature	Displacement
Distance	Acceleration
Time	Force

Every time a physical quantity is introduced, you will be told whether it is a scalar or a vector.

Representing a vector quantity

To graphically represent a vector, we draw an arrow. The length of the arrow shows the magnitude of the vector. The longer the arrow, the greater the magnitude. The direction of the vector is shown by the direction of the arrow. **Figure 2** shows a vector with magnitude and direction.

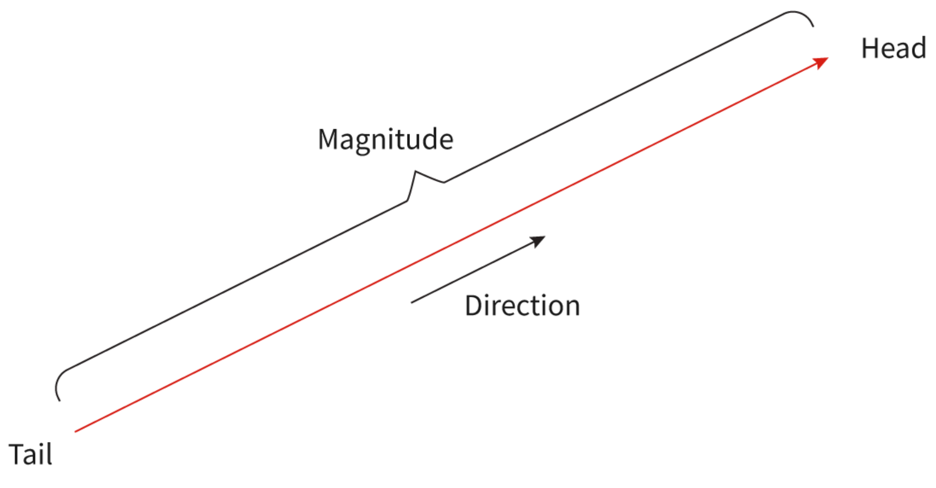

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Figure 2. A vector showing magnitude and direction.

More information for figure 2

The image shows an arrow diagram representing a vector. The arrow points from the 'Tail' on the left to the 'Head' on the right, with a label "Magnitude" adjacent to the arrow indicating the strength of the vector. Another label, "Direction," is placed along the path of the arrow, showing the vector's direction from left to right. This illustration visually depicts how vectors are represented using arrows with specific lengths and directions.

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Figure 3 shows a person pushing a box with force F. Force is a vector quantity. We start the force arrow at the point where the force is applied on the left side of the box. The force acts to the right, so the arrow points to the right.

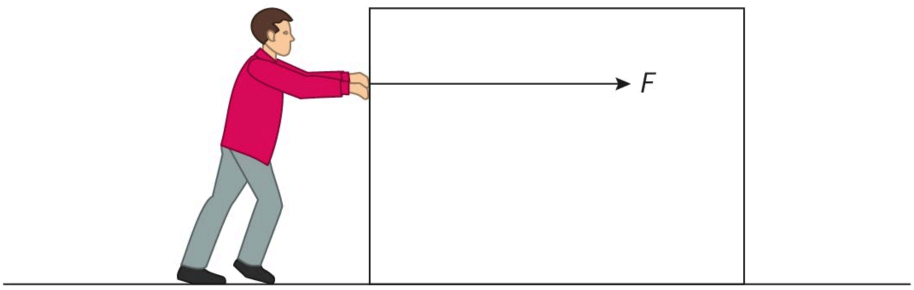

Figure 3. A person pushing a box with force F.

More information for figure 3

The image shows a person in a red shirt and gray pants pushing a tall box to the right. An arrow labeled "F" starts from the person's hands touching the box and points horizontally to the right, indicating the direction of the applied force. The arrow represents a vector quantity, emphasizing that it has both magnitude and direction. The scene is set on a flat, horizontal surface.

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Multiplying a vector by a scalar

You can multiply a vector by a scalar. A scalar is just a number (magnitude). When you multiply a vector quantity by a scalar, you change the length of the vector (its magnitude).

Figure 4 shows some examples of the products of vectors and scalars.

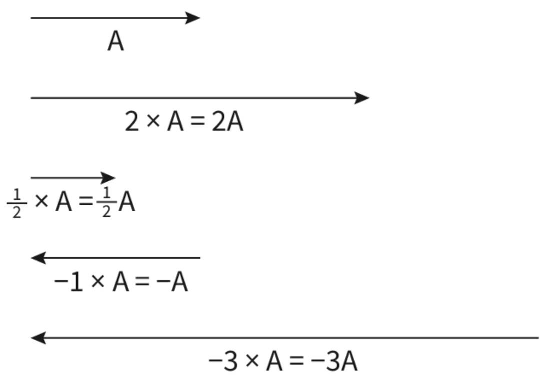

Figure 4. Multiplying a vector by a scalar.

More information for figure 4

The image illustrates the multiplication of a vector, labeled as A, by various scalar values. Five arrows are depicted horizontally, each representing a different result of the multiplication. The original vector, A, points to the right and is a medium length.

1. The first arrow with the label '2 x A = 2A' is twice the length of the original and points in the same direction, indicating the vector A multiplied by 2.
2. The second arrow is labeled '1/2 x A = 1/2 A' and is shorter than A, representing the vector A multiplied by 1/2.
3. The third arrow, labeled '-1 x A = -A,' is of similar length to A but points to the left, indicating the vector A multiplied by -1.
4. The fourth arrow, labeled '-3 x A = -3A,' is three times the length of A and points to the left, representing the vector A multiplied by -3.

This visual explains how multiplying a vector by positive and negative scalars affects its direction and magnitude.

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Note the following:

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- Vector 2A is twice the length of vector A, vector ½A is half the length of vector A, and so on.
- If the scalar has a negative value, reverse the vector to point in the opposite direction. For example, vector –3A is three times as long as vector A and in the opposite direction.
- Dividing by a scalar is the same as multiplying by the reciprocal, for example, $\frac{A}{2}$ is the same as ½A.

Adding vectors

It is important to know how to add two or more vectors together. You can only add vectors that represent the same physical quantity. The sum of two or more vectors is a vector, with a magnitude and direction.

Use the simulation in **Interactive 1** to explore how vectors can be added. You can vary the magnitude and direction of the vectors. The sliders show you the sum of the vectors.

Interactive 1. Adding vectors.

More information for interactive 1

The interactive, titled Adding Vectors, visually demonstrates vector addition using two sliders and graphical representations. It allows users to explore the commutative property of vector addition, which states that u + v is equal to v + u. Moving a slider fully aligns the corresponding vector alongside the other, showing the resultant vector

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as the sum of both. Despite the order of addition, the resultant vector remains the same, reinforcing the concept that vector addition is independent of order.

The vectors **u** and **v** are color-coded for distinction, while the **resultant vector**, sum of both vectors, is displayed in **black**. A reset button allows users to return to the initial state and experiment again. This interactive helps learners understand vector addition conceptually and geometrically. By manipulating the sliders, users can understand how the resultant vector remains unchanged regardless of the sequence.

Resultant vectors are found by adding together the vector components. When the vectors have the same direction, you add the magnitudes. When the vectors have opposite directions, you subtract the magnitudes. This is shown in **Figure 5**.

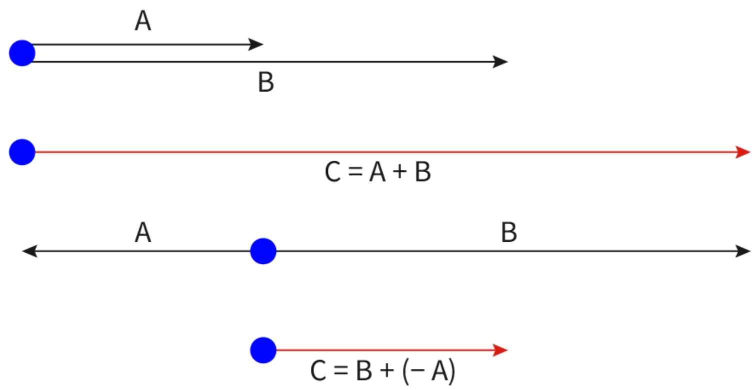

Figure 5. Adding vectors in the same and opposite directions.

More information for figure 5

This diagram illustrates the concept of vector addition using arrows to represent vectors.

1. **Same Direction:**
2. Two vectors are shown with arrows labeled A and B, both pointing to the right.
3. A larger red arrow labeled C = A + B represents the resultant vector when vectors A and B are combined in the same direction.
4. **Opposite Direction:**
5. Vector A is shown with an arrow pointing to the left, and vector B is shown with an arrow pointing to the right.
6. A smaller red arrow labeled C = B + (-A) represents the resultant vector when vector A is subtracted from vector B, indicating opposite directions.

Blue circles mark the starting points of each set of arrows, demonstrating the concept visually.

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Worked example 1

The vectors in **Figure 6** represent the horizontal forces acting on a box. Calculate the resultant horizontal force.

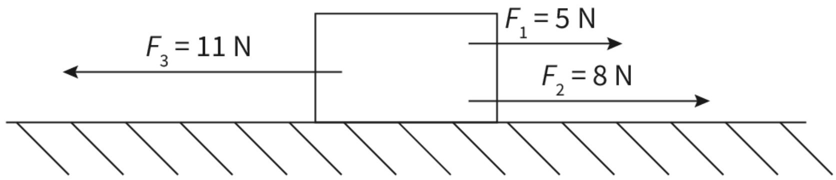

Figure 6. Forces on a box.

More information for figure 6

The diagram shows a box on a surface with three horizontal vectors indicating forces acting on it. There is a vector pointing to the right labeled F1 with a force of 5 N and another vector pointing to the right labeled F2 with a force of 8 N. Additionally, there is a vector pointing to the left labeled F3 with a force of 11 N. The vectors represent the horizontal forces acting on the box.

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Solution steps	Calculations
Step 1: Identify the direction of the vectors.	F_1 and F_2 are in the same direction F_3 is in the opposite direction
Step 2: Write out the values given in the question.	$F_1 = 5\text{ N}$ $F_2 = 8\text{ N}$ $F_3 = 11\text{ N}$
Step 3: Add the magnitudes in the same direction.	$F_1 + F_2 = 5 + 8 = 13\text{ N}$

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Solution steps	Calculations
Step 4: Subtract the magnitudes in the opposite direction.	$(F_1 + F_2) + (-F_3) = 13 - 11 = 2\ \text{N}\ (1\ \text{s.f.})$
Step 5: State the answer with appropriate units.	The resultant force is $2\ \text{N}$ to the right.

Another method of adding vectors is the head-to-tail method. To add vectors using this method (**Figure 7**):

1. Draw the first vector.
2. Draw the beginning (tail) of the second vector at the end (head) of the first vector.
3. Continue to draw all the vectors in this way.
4. Draw an arrow to connect the tail of the first vector to the head of the last vector. The vector that is formed is the sum of the vectors.

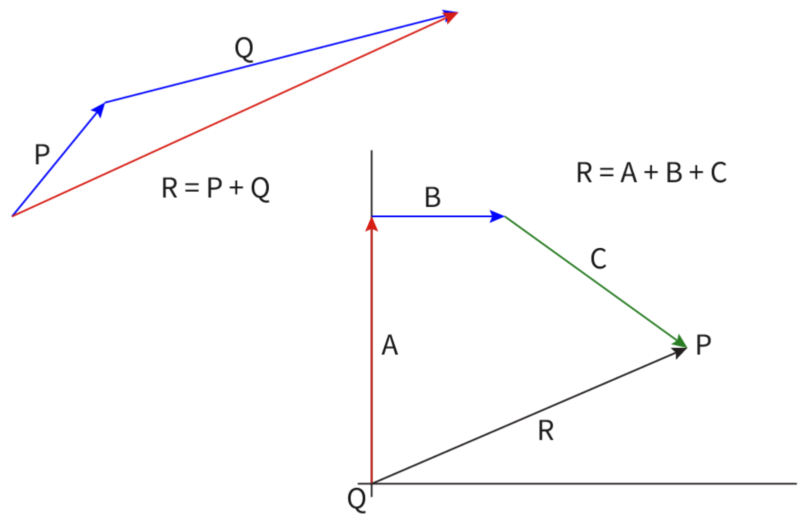

Figure 7. Adding two and three vectors together.

More information for figure 7

The image shows two separate diagrams illustrating vector addition. On the left side, vector P is drawn as an arrow pointing up and to the right. Vector Q is drawn continuing from the head of P, going further to the right. Together, these vectors form a red resultant vector R, labeled as 'R = P + Q'.

On the right side, a second set of vectors is shown. Vector A points upwards, then vector B continues horizontally to the right. From B, vector C starts and extends to the right and downward direction, forming a green line. The resultant vector R starts from the origin point and ends at the tip of vector C, marked as 'R = A + B + C'. Each vector

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is color-coded and labeled accordingly.

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If the beginning of the first vector coincides with the end of the last vector added, the sum is zero (**Figure 8**).

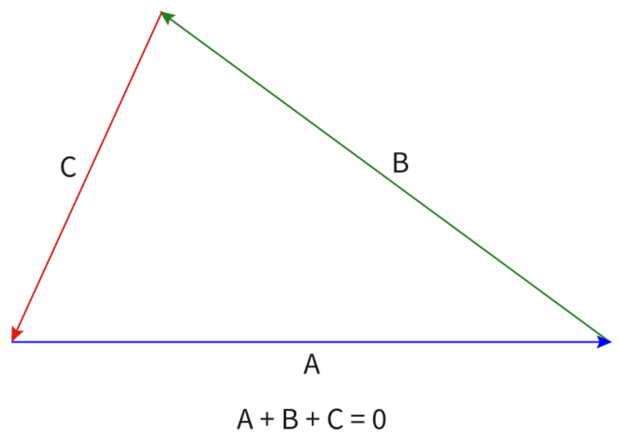

Figure 8. The sum of the vectors can be zero.

More information for figure 8

The image is a vector diagram showing three arrows labeled A, B, and C forming a closed triangle. Vector A is a horizontal blue arrow pointing right, Vector B is a green arrow originating from the tip of Vector A, and extending upwards and to the left, and Vector C is a red arrow originating from the tip of Vector B, pointing downwards completing the triangle back to the origin of Vector A. There is a text below the diagram stating "A + B + C = 0," illustrating that the sum of the vectors results in zero, which means they form a closed loop.

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Study skills

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When drawing diagrams, you can move a vector to a different location (for example, to connect with other vectors), as long as you do not change its orientation or its length.

You also need to draw vectors to scale and have some way of measuring the length of each vector. For example, in **Figure 9**, vector A has a length of 4 'boxes' and vector B has a length of 6 'boxes'. The sum, vector C, should have a length of 10 'boxes'.

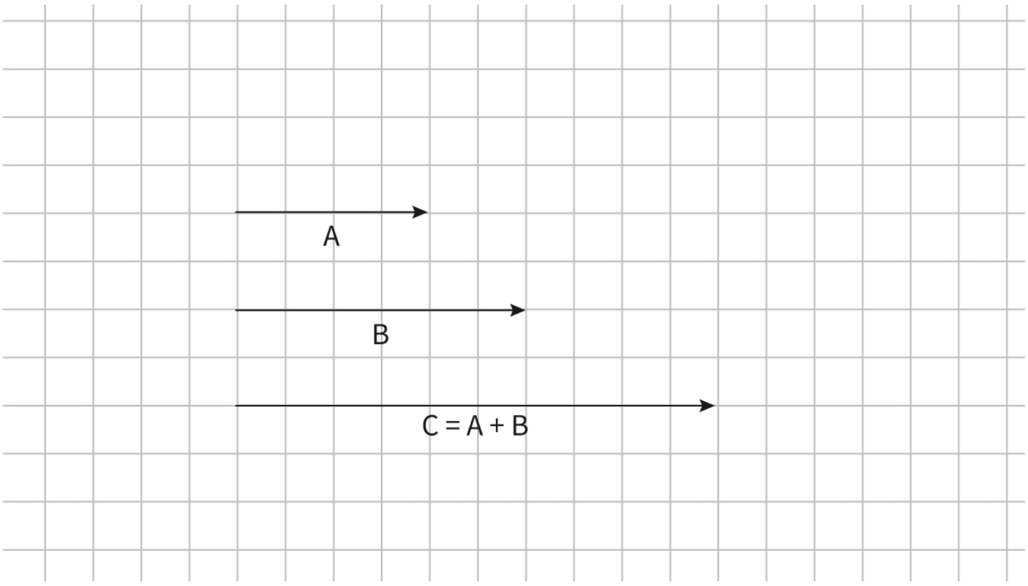

Figure 9. Drawing vectors to scale.

More information for figure 9

The image is a vector diagram displayed on a grid. There are three vectors: A, B, and C. Vector A is horizontal and spans 4 grid boxes to the right. Vector B starts from the endpoint of vector A, extends further to the right, and covers an additional 6 grid boxes. The resultant vector C starts from the origin of vector A and extends continuously to the endpoint of vector B, covering a total of 10 grid boxes. The relationship is expressed as C = A + B, indicating vector addition.

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Another method of adding vectors is the parallelogram method. To add vectors using this method (**Figure 10**):

1. Draw the first vector.
2. Draw the second vector, starting at the same point as the first vector's origin.
3. Complete the parallelogram by drawing dotted lines parallel to each vector.
4. Draw the diagonal of the parallelogram.
5. For two vectors that are at an angle of 90° to one another, find the length of the diagonal (the sum of the vectors) by applying the Pythagorean theorem:

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$$C^2 = A^2 + B^2$$
$$C = \sqrt{A^2 + B^2}$$

6. Find the direction of the vector (in this case, the angle from the horizontal):

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{B}{A}$$
$$\theta = \tan^{-1}\frac{B}{A}$$

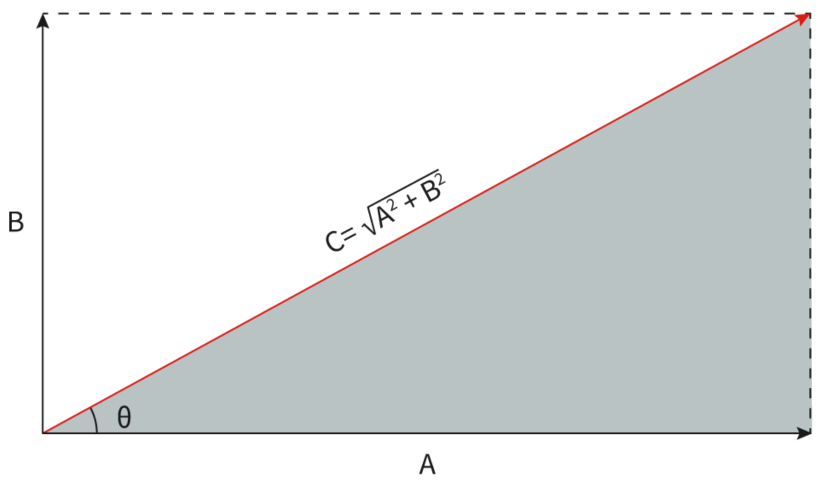

Figure 10. The parallelogram method of adding vectors.

More information for figure 10

The image is a diagram of a right triangle illustrating vector addition using the parallelogram method. The triangle includes:

- A horizontal side labeled as 'A' at the bottom, indicating one vector's magnitude.
- A vertical side labeled as 'B' on the left side, representing the other vector's magnitude.
- The hypotenuse labeled as 'C = √(A² + B²)', showing the resultant vector calculated using the Pythagorean theorem.
- An angle marked as θ between side A and the hypotenuse C.

The diagram uses arrows to show vector direction and a shaded area to indicate the triangle formed by the vectors and their resultant being added.

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Study skills

In order to determine the angles of right-angled triangles, use the sine, cosine and tangent functions.

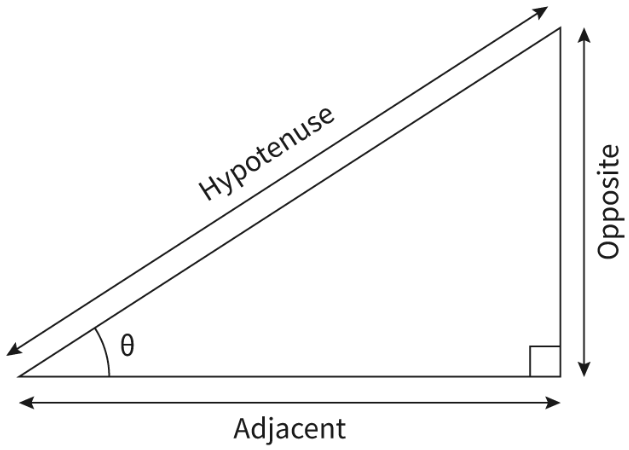

Figure 11. Determining the angles in a right-angled triangle.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

More information for figure 11

This image is a diagram of a right-angled triangle. The triangle includes labels for the sides and an angle, θ (theta). The longest side, labeled as the 'Hypotenuse,' slopes diagonally upward from the bottom left to the top right of the image. The bottom side is labeled 'Adjacent,' and the vertical side is labeled 'Opposite.' The right angle is at the bottom right. Angle θ is at the base of the hypotenuse and adjacent sides. The diagram illustrates the sides involved in calculating the trigonometric functions sine, cosine, and tangent for angle θ.

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Worked example 2

A boat crosses a river from one bank to the other. The captain initially aims the boat towards point A but arrives at point B. Determine the distance the boat travels and its direction.

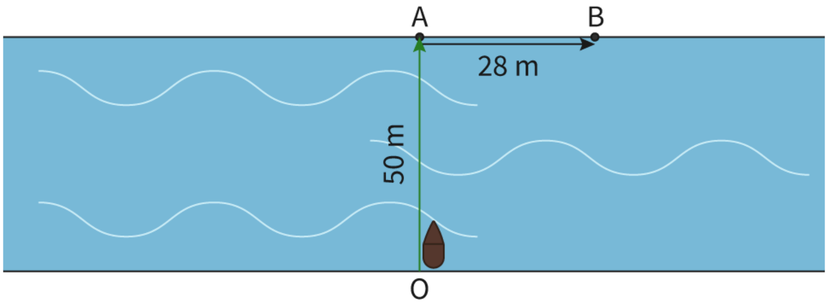

Figure 12. Boat crossing a river.

More information for figure 12

The diagram shows a river with a boat crossing from point O to point B. Initially, the boat is aimed towards point A. A green line labeled '50m' represents the direct path from O to A. The boat instead travels a diagonal path and arrives at point B, which is 28 meters horizontally from A, forming a right triangle with the riverbank. The river is depicted with blue waves, and the boat travels across the water.

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Solution steps	Calculations
Step 1: Identify the vectors and their direction.	$\text{Vector OA} = 50 \text{ m}$ $\text{Vector AB} = 28 \text{ m}$

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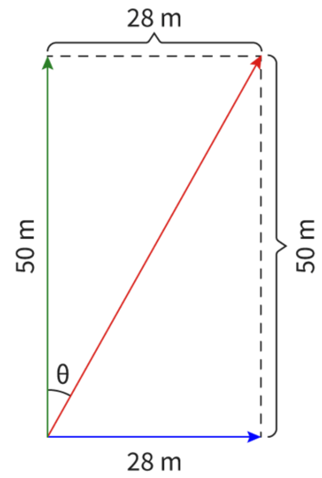
Solution steps	Calculations
Step 2: Use the parallelogram method to add the vectors.	 $x = \sqrt{50^2 + 28^2}$ $= \sqrt{3284}$ $= 57.3\text{ m}$ $= 57\text{ m (2 s.f.)}$
Step 3: Calculate the angle of the vector.	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $= \frac{28}{50}$ $\theta = \tan^{-1}\frac{28}{50}$ $= 29.2^\circ$ $= 29^\circ\text{ (2 s.f.)}$
Step 4: State the answer with appropriate units.	The boat travelled 57 m at an angle of 29° to the vertical.

Use **Interactive 2** to investigate a boat crossing a river as in Worked example 2. Adjust the variables using the sliders and click ‘Run’ to start.

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Interactive 2. Investigate a boat crossing a river.

More information for interactive 2

This interactive simulation, Investigate a boat crossing a river, allows users to explore the motion of a boat crossing a river while considering velocity vectors and the influence of the river's current. The boat moves from one bank to the other, with its trajectory affected by both its own velocity and the velocity of the river. Users can control the boat's speed relative to the river and the direction in which it is aimed. The river's speed can also be adjusted, demonstrating how external currents impact navigation. By toggling the velocity vector display, users can visualize the boat's velocity relative to both the river and the Earth.

The interface includes a velocity diagram showing how the river's velocity ($V_{r \to e}$) and the boat's velocity relative to the river ($V_{b \to r}$) combine to produce the boat's velocity relative to Earth ($V_{b \to e}$). This demonstrates vector addition in motion. The boat's aim direction is measured relative to the north, and users can set it to any value between −90 degrees west and 90 degrees east. The animation runs in real time, showing how the boat drifts downstream when not aimed directly opposite to the river's flow. Users can pause, reset, or adjust parameters dynamically.

A sample calculation follows a worked example of a boat crossing a river. Suppose the river's velocity relative to the Earth is 2.5 meters per second eastward, and the boat's velocity relative to the river is 3.4 meters per second directed northward. The boat's velocity relative to Earth is found using the Pythagorean theorem,

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$$V_{b\to e} = \sqrt{(V_{b\to r})^2 + (V_{b\to e})^2} V_{b\to e} = \sqrt{(3.4)^2 + (2.5)^2} = \sqrt{11.56 + 6.25} = \sqrt{17.81} = 4.22 \text{m/s}$$

Next, the boat's actual direction relative to north is determined using the inverse tangent function,

$$\theta = \tan^{-1}\left(\frac{v_{r\to e}}{v_{b\to r}}\right) \theta = \tan^{-1}\left(\frac{2.5}{3.4}\right) = \tan^{-1}(0.735) = 36.4^\circ$$

Thus, the boat moves at 4.22 meters per second at an angle of 36.4 degrees east of north. This interactive visually reinforces these calculations, allowing users to see how different parameters affect the boat's trajectory.

This interactive helps users understand relative velocity by visualizing how a boat's motion is affected by both its own velocity and the current of a river. It reinforces vector addition principles, showing how different velocity components combine to determine the boat's actual path.

Resolving a vector

We have seen how two perpendicular vectors can be expressed as one resultant vector. Resolving a vector is the reverse process of adding two perpendicular vectors. We can replace one vector with two component vectors at right angles to each other. The combined component vectors are equivalent to the original vector.

To resolve a vector (**Figure 13**):

1. Draw a coordinate system, with an x-axis and y-axis.
2. Draw the vector.
3. Starting from the tail of the vector, draw lines parallel to each of the axes. These are the two components of vector A, the x-component A_x and the y-component A_y.

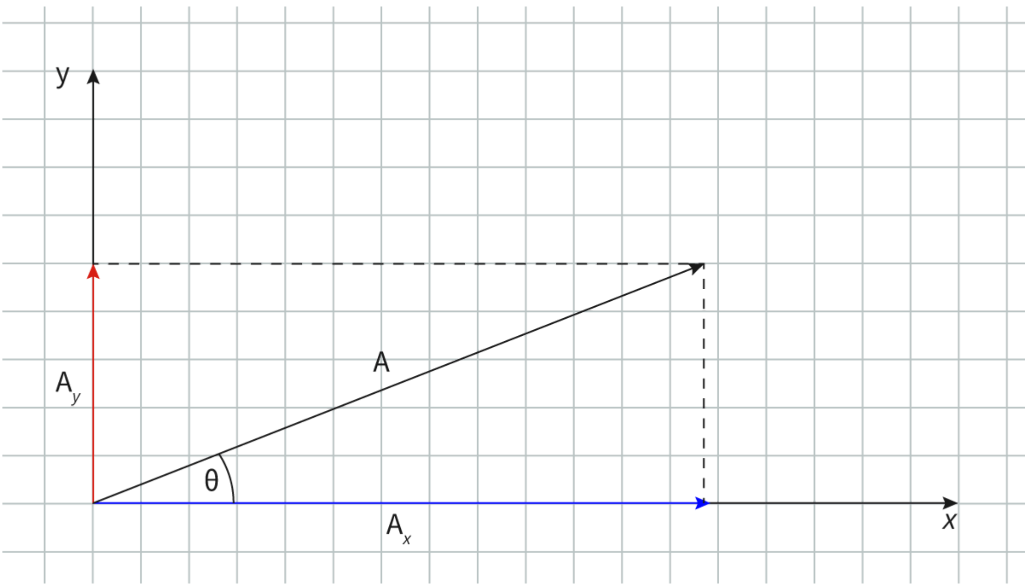

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Figure 13. Resolving a vector.

More information for figure 13

The image is a diagram of a coordinate system with a grid background. It features a right triangle formed within the grid. The horizontal axis is labeled as 'x' with a blue arrow pointing to the right, while the vertical axis is labeled as 'y' with a red arrow pointing up. The hypotenuse of the triangle represents a vector and is marked with a dashed line. The horizontal component of the vector is labeled as 'A_x' and the vertical component is labeled as 'A_y'. An angle θ is shown between the vector and the x-axis. This diagram visually explains the concept of resolving a vector into its components along the x and y axes using sine and cosine functions associated with the angle θ.

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Using the definitions of sine and cosine for an angle:

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\ &= \frac{\mathrm{A}_y}{\mathrm{A}}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ &= \frac{\mathrm{A}_x}{\mathrm{A}}\end{aligned}$$

So, solving for the components of vector A, we get:

$$\mathrm{A}_x = \mathrm{A}\cos\theta \text{ and } \mathrm{A}_y = \mathrm{A}\sin\theta$$

Use the simulation in **Interactive 3** to explore resolving a vector. Tick the box 'Show vector' to see the vector. Tick the box 'Show directions for components' to create the axes. Tick the box 'Show components' to see the components.

Interactive 3. Resolving a vector simulation.

More information for interactive 3

The **"Resolving a Vector Simulation"** interactive visually demonstrates how a **vector can be broken down into its horizontal and vertical components** using **trigonometry.** This tool allows users to explore **vector decomposition** dynamically, helping them understand how a single vector can be expressed as the sum of two perpendicular component vectors.

A **dark arrow** represents the original vector, which has a given **magnitude and direction.**

Users can **adjust the vector** by **dragging its endpoint**, changing its magnitude and direction. As the vector changes, two **pink arrows** dynamically adjust to represent the **horizontal and vertical components** of the vector.

For example, if a vector has a magnitude of **10 units** and is directed **at 30° from the horizontal**, the interactive will show its horizontal component as **10 cos(30°) ≈ 8.66** and its vertical component as **10 sin(30°) = 5.** As users adjust the vector, these values update dynamically, reinforcing the mathematical relationship.

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The interactive includes options to display the vector, component directions, and numerical values, reinforcing the concept of vector resolution. The values of horizontal and vertical components are updated in real time based on the vector's orientation. The users develop an intuitive understanding of vector decomposition.

Study skills

We can rewrite the vector C = A — B as C = A + (—B).

You can transform any subtraction of vectors into an addition of vectors. All you have to do is add the reversed vector.

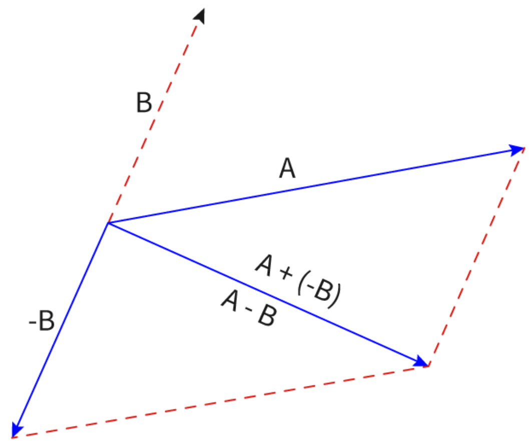

Figure 14. Subtracting vectors.

More information for figure 14

The diagram illustrates the concept of vector subtraction by showing how vector A minus vector B is equivalent to vector A plus the reverse of vector B (-B). It shows three vectors: A, B, and -B, with arrows indicating their directions. Vector A is shown pointing in its original direction, vector B points in a different direction, and -B is depicted in the opposite direction to B. The parallelogram is formed by completing the vectors A and B on a 2D plane. An arrow labeled A - B is drawn from the tail of B to the head of A, representing the result of vector subtraction. The diagram visually demonstrates that subtracting a vector is equivalent to adding its opposite.

[Generated by AI]

Work through the activity to check your understanding of vectors.

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Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Reflecting at all stages of the assessment and learning cycle
- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

Download the worksheet and complete the practice question.

Worksheet (https://d3vrb2m3yrmyfi.cloudfront.net/media/edusys_2/content_uploac
A.1.1 ACTIVITY Vectors.0eeefa285bd9d3b6a28c.pdf)

Remember to show all your working. You can check your answer on the last page.

5 section questions

A. Space, time and motion / A.1 Kinematics

Describing motion

A.1.1: The motion of bodies through space and time A.1.2: Velocity and acceleration A.1.3: Displacement
A.1.4: The difference between distance and displacement A.1.5: The difference between instantaneous and average values
Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/print/) Assign

Learning outcomes

At the end of this section you should be able to:

- Explain the motion of an object in terms of position, displacement, distance, velocity and acceleration.
- Understand the differences between instantaneous and average values for velocity, speed and acceleration and be able to determine them.

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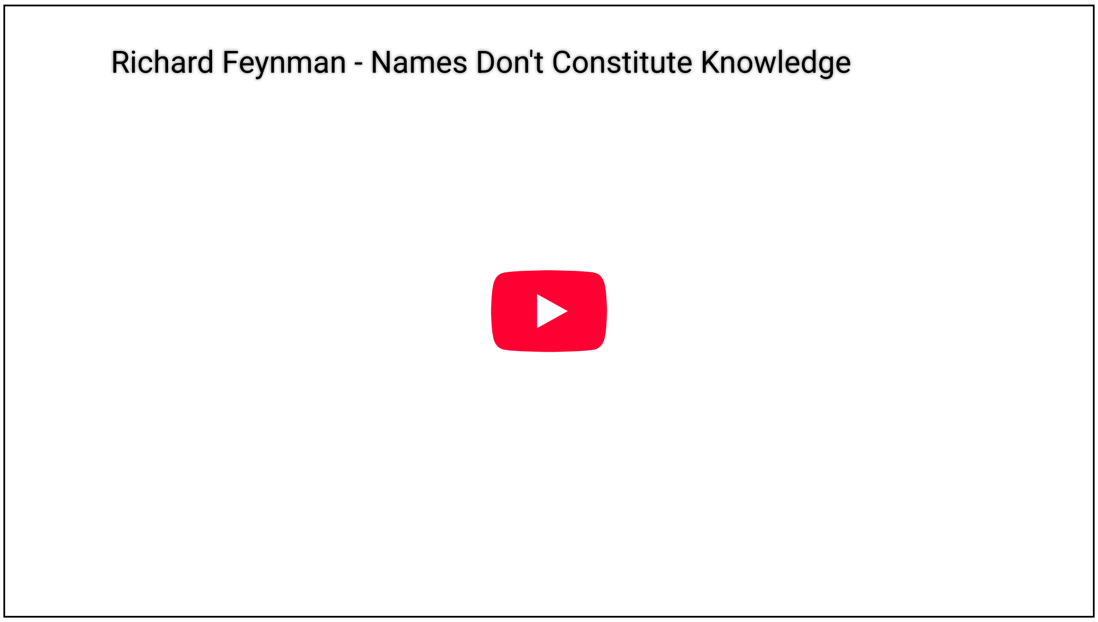
Richard Feynmann, a physicist awarded the Nobel prize in 1965, said that knowing the name of something is different to actually knowing something. Watch **Video 1** to see him discuss this.



Video 1. Richard Feynmann.

In science, we need to understand concepts and be able to express them in words.

For example, velocity and acceleration are two important concepts in the study of motion. How would you explain them in words? Try to answer this question now and then again at the end of the section, to see how your answer has changed.

International Mindedness

Different cultures throughout history have used different measurements to communicate distance and speed. In ancient Egypt, for example, some measurements were based on human dimensions (hand sizes, foot sizes, etc.). Horses are still measured in hands in most of the world.

We have more sophisticated technology today to make more accurate measurements, but we are essentially measuring the same thing. To what extent is our understanding of motion limited to the measurements we can make? If you can make more accurate measurements, does that mean you can understand more?

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Position, displacement and distance

In order to describe how objects move, we need to know where they are at each moment – we need to know their position.

Figure 1 shows a horizontal axis. The position of an object can be given in relation to the reference point, 0. For example, the object at position A is –3 from 0, while an object at position D is 1 from 0.

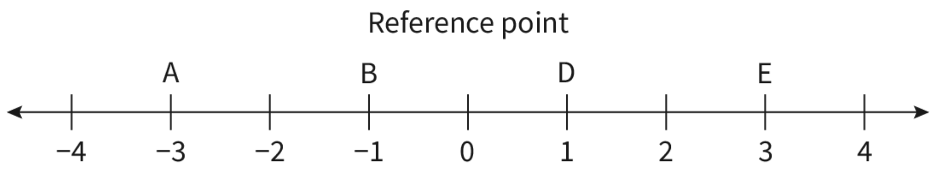

Figure 1. Indicating position.

More information for figure 1

The image is a number line diagram representing various positions relative to a reference point labeled "0." The line extends horizontally with arrows on both ends to indicate direction. Along the number line, positions are marked with labels: A at -3, B at -1, D at 1, and E at 3. Each position is marked relative to the central reference point 0. The extremes of the number line are labeled -4 to 4, progressing by increments of one unit. This visual representation shows how positions are described as deviations from the central reference point 0.

[Generated by AI]

Once we know how to describe the position of a body, we can discuss how this position changes over time. There are two different physical quantities that can help us do this.

The first quantity is called displacement. It is defined as the change in position:

$$\text{displacement} = \text{final position} - \text{initial position}$$

Displacement is a vector quantity and can be positive, negative or zero. The sign of the displacement gives information on the direction of the change in position relative to the starting position.

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The second quantity is called distance and it shows the length of the path followed.

Distance is a scalar quantity and can only be positive or zero, never negative. The direction of motion is not important.

Figure 2 shows the difference between displacement and distance.

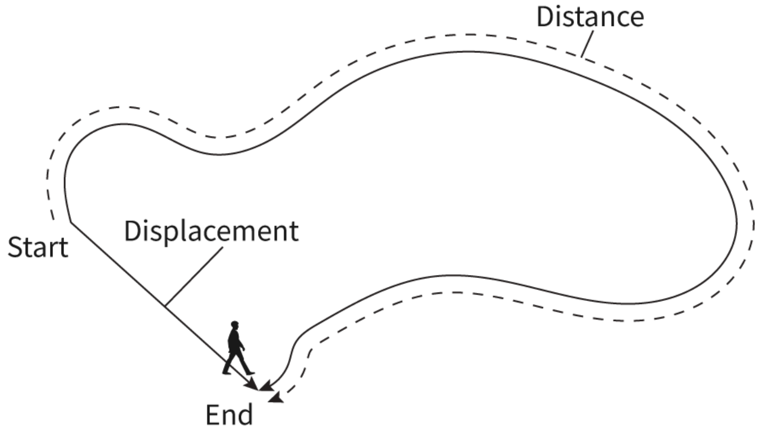

Figure 2. The difference between displacement and distance.

More information for figure 2

The image is a diagram that illustrates the difference between displacement and distance using a path with start and end points. The path shows a wavy, curving line labeled 'Distance' that connects the 'Start' position to the 'End' position, highlighting the actual distance traveled. In contrast, a straight line labeled 'Displacement' connects the 'Start' and 'End' points directly, representing the shortest path between these two points. The person walking is near the end point, emphasizing the concept of movement along a path. The diagram effectively visualizes the concept, showing how distance may differ from displacement due to the path's nature.

[Generated by AI]

Think about the following questions then click 'Show or hide solution'.

1. What should the path of a body be in order for distance to be equal to displacement?
2. Can the distance ever be smaller than the displacement?
3. What is the displacement if a body takes a path that takes it back to its starting point?

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1. A line
2. No
3. Zero

Velocity and speed

Two people cover the same distance, one walking and the other riding a bike. Although they cover the same distance, they do so in different amounts of time. They have different velocities.

Velocity is defined as the rate of change of position. It can be calculated using the equation in **Table 1**.

Table 1. Equation for velocity.

Equation	Symbols	Units
$v = \frac{\Delta s}{\Delta t}$	v = velocity	metres per second (m s^{-1})
	Δs = change in position	metres (m)
	Δt = change in time	seconds (s)

Velocity is a vector quantity, which means it has a magnitude and a direction.

As described earlier, the change in position is equal to the displacement. Therefore, we could also describe velocity as the rate of change of displacement.

Speed is a scalar quantity, which means it only has magnitude. To calculate the speed of the object, use distance instead of displacement as in **Table 2**.

Table 2. Equation for speed.

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Overview
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Equation	Symbols	Units
$v = \frac{d}{\Delta t}$	v = average speed	metres per second (m s^{-1})
	d = distance travelled	metres (m)
	Δt = change in time	seconds (s)

Worked example 1

1. A swimming pool is 50 m long. A swimmer swims 4 lengths during a race. Calculate the total distance and the total displacement.
2. Another swimmer swims 400 m across a lake in 4 minutes and 20 seconds. What is the average speed of that swimmer?

1. $\text{total distance} = 4 \times 50 = 200\text{ m}$
   $\text{total displacement} = 0\text{ m}$ as the swimmer ends the race where they started
2. $$\begin{aligned} d &= 400\text{ m} \\ t &= 4\,\text{min}\ 20\text{ s} \\ &= 260\,\text{s} \\ v &= \frac{d}{\Delta t} \\ &= \frac{400}{260} \\ &= 1.5\,\text{m s}^{-1}\ (2\text{ s.f.}) \end{aligned}$$

Aspect: Measurement

When we measure a physical quantity, we express it with a unit. Scientists all over the world use the standard SI system of units to communicate measurements — for example, the SI unit for displacement is the metre (m). When non-standard units are used, this can cause problems.

NASA's Mars Climate Orbiter (https://solarsystem.nasa.gov/missions/mars-climate-orbiter/in-depth) was launched in December 1998 with the mission of orbiting Mars to collect information about the planet and facilitating

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communication with other spacecraft. In September 1999, the spacecraft had reached Mars. When it received commands from Earth to enter a particular orbit, the outcome was a failure. Two pieces of software used to steer the Orbiter were using different units. One of them was using the SI units of newtons × seconds while the other was using pound-force × seconds. The result was that the numbers were interpreted incorrectly and the spacecraft approached Mars more closely than was intended, and communication was lost.

Acceleration

Bodies do not always move at the same rate. They might change their motion throughout the journey, speeding up or slowing down.

Acceleration is defined as the rate of change of velocity. It can be calculated using the equation in **Table 3**.

Table 3. Equation for acceleration.

Equation	Symbols	Units
$a = \frac{\Delta v}{\Delta t}$	a = acceleration	metres per second per second (m s^{-2})
	Δv = change in velocity	metres per second (m s^{-1})
	Δt = change in time	seconds (s)

Acceleration is a vector quantity. Its direction is the same as the direction of the change in velocity Δv.

Acceleration can be positive or negative, depending on the change in velocity:

- A positive acceleration means a positive change in velocity.
- A negative acceleration means a negative change in velocity.
- An object with decreasing velocity in the positive direction has a negative acceleration.
- An object with increasing velocity in the negative direction has a negative acceleration.

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Graphs of motion

We can draw graphs to describe the motion of an object. A displacement–time graph shows how the displacement of an object changes over time (**Figure 3**).

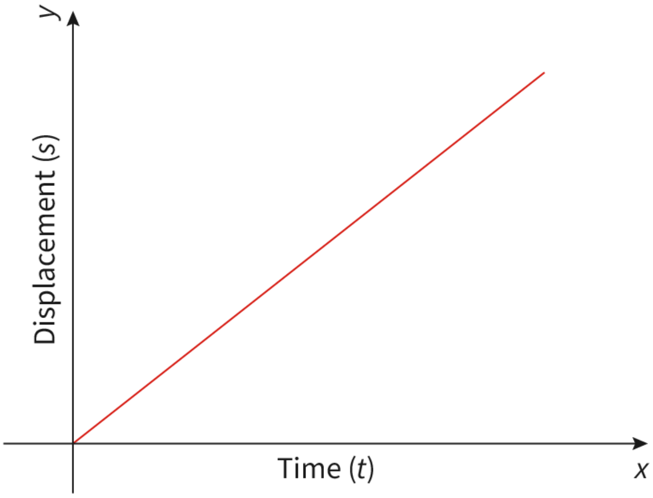

Figure 3. A displacement—time graph.

More information for figure 3

The image is a displacement-time graph illustrating the motion of an object. The X-axis represents time, while the Y-axis represents displacement. The graph features a single red line beginning at the origin, moving diagonally upwards to the right, indicating a constant rate of change of displacement over time. This suggests a uniform motion where the rate of displacement is constant, demonstrating a linear relationship between time and displacement. No specific units or numerical values are shown on the axes, focusing instead on the overall trend of the object's movement.

[Generated by AI]

The gradient of a line gives you the rate of change of the quantity on the y-axis with respect to the quantity on the x-axis.

For a displacement–time graph, the gradient is:

$$\frac{\Delta s}{\Delta t} = v$$

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The gradient of a displacement–time graph is the velocity. The steeper the line, the larger the gradient, and the faster the body is moving.

Exercise 1

Click a question to answer

A velocity–time graph shows how the velocity of an object changes over time (**Figure 4**).

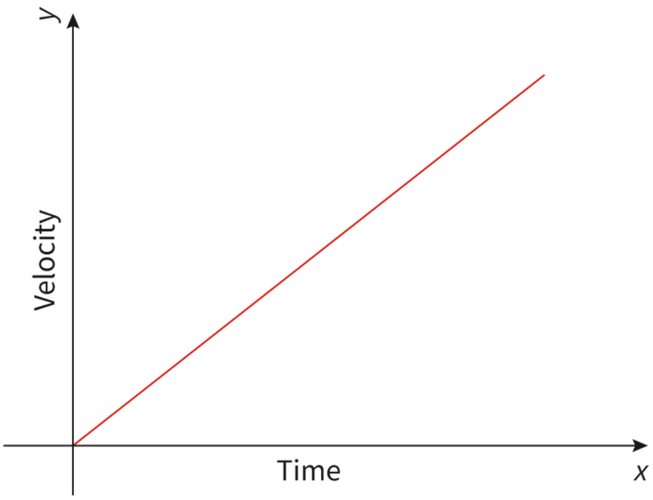

Figure 4. A velocity—time graph.

More information for figure 4

The graph is a velocity-time plot depicting how velocity changes over time. The X-axis is labeled 'Time' and the Y-axis is labeled 'Velocity.' The graph displays a straight line with a positive slope, indicating a steady increase in velocity as time progresses. The line starts from the origin, suggesting that the initial velocity is zero and increases at a constant rate, reflecting constant acceleration.

[Generated by AI]

For a velocity–time graph, the gradient is:

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$$\frac{\Delta v}{\Delta t} = a$$

The gradient of a velocity–time graph is the acceleration. The steeper the line, the larger the gradient, and the greater the acceleration.

Figure 5 shows a velocity–time graph of an object. The velocity of the body is constant. What does the shaded area under the graph represent?

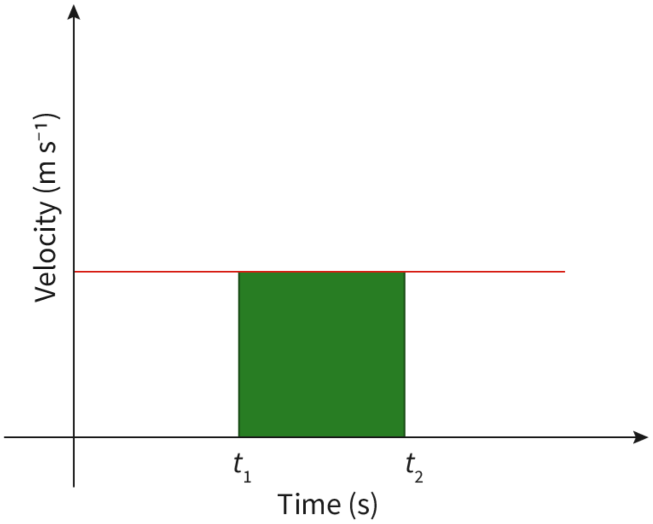

Figure 5. The area under a velocity—time graph.

More information for figure 5

The image is a velocity-time graph depicting an object's constant velocity. The graph has a horizontal line parallel to the time axis, indicating that velocity remains constant over time. The X-axis is labeled 'Time (s)' and has two points indicated as t1 and t2. The Y-axis is labeled 'Velocity (m s^-1)'. There is a shaded area under the horizontal line between t1 and t2, representing the area under the curve. This area indicates the object's displacement, calculated by multiplying velocity by the time interval (t2 - t1).

[Generated by AI]

You can calculate the area under the graph by multiplying the lengths of the two sides. In this case:

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$$\begin{aligned} \text{area} &= v \times (t_2 - t_1) \\ &= v \times \Delta t \\ &= \Delta s \end{aligned}$$

The area under the graph represents the displacement of the body. The displacement can be positive or negative:

- If the area is above the x-axis, it represents a positive displacement.
- If the area is below the x-axis, it represents a negative displacement.

Instantaneous and average values

Imagine a bus driving on a certain route. The bus makes stops so that people can get on and off. Between the stops, it speeds up and slows down.

The average velocity during its journey can be calculated by dividing the total change in displacement by the total time taken for the journey:

$$\text{average velocity} = \frac{\Delta s}{\Delta t}$$

The instantaneous velocity of the bus varies from moment to moment. The instantaneous velocity is how fast a body is moving at a specific instant in time.

Figure 6 shows the displacement–time graph for an object. You can see from the graph that the gradient is not constant. This means that the velocity is not constant.

To find the instantaneous velocity of the object at a particular moment in time, you need to draw a tangent to the curve as shown, then calculate the gradient of the line.

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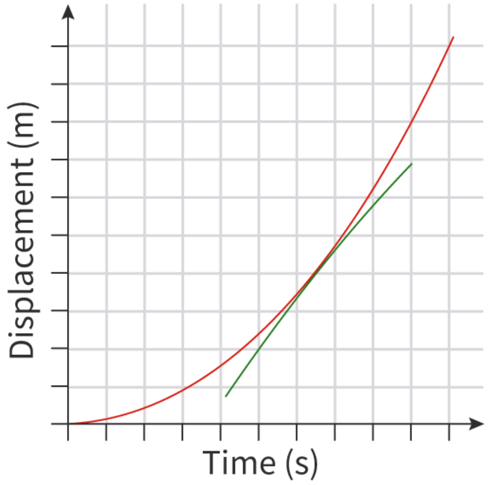

Figure 6. Determining instantaneous velocity from a displacement—time graph.

More information for figure 6

The image is a displacement-time graph used to determine instantaneous velocity. The X-axis represents time in seconds (s) and is marked at regular intervals. The Y-axis shows displacement in meters (m) and is also marked at regular intervals. The graph displays a red curve that starts close to the origin and curves upwards in a quadratic manner, indicating increasing displacement over time. Superimposed on the curve is a tangent line in green, touching the curve at a specific point, which illustrates how to calculate the instantaneous velocity by determining the gradient of this tangent line.

[Generated by AI]

Worked example 2

The diagram shows a displacement–time graph for the motion of a ball. Determine the instantaneous velocity at 6.5 s.

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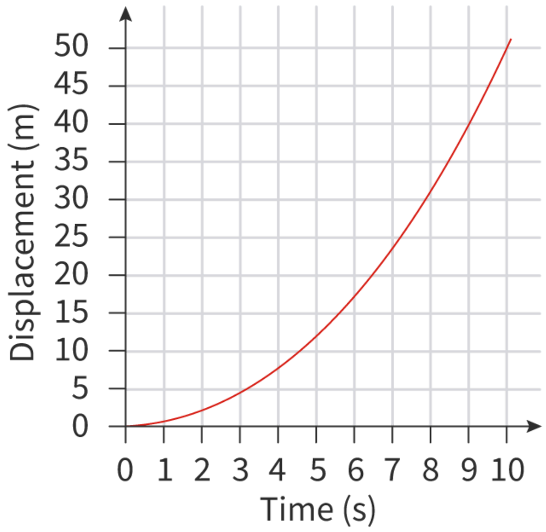

Figure 7. Displacement—time graph for the motion of a ball.

More information for figure 7

The image is a graph illustrating the relationship between displacement and time for a moving ball. The X-axis represents time measured in seconds, ranging from 0 to 10 seconds. The Y-axis represents displacement in meters, ranging from 0 to 50 meters. Each axis is labeled appropriately with division marks indicating equal intervals. The plot is a curve starting from (0,0), gradually rising and forming an upward concave shape, indicating an increase in displacement over time. The overall trend shows a consistent acceleration of the ball's movement.

[Generated by AI]

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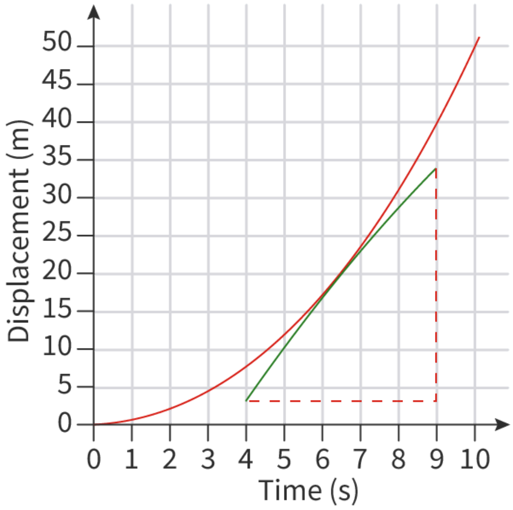
Solution steps	Calculations
Step 1: Draw the tangent to the graph line at 6.5 s.	
Step 2: Determine the gradient of the line.	$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{(33-3)}{(9-4)} = 6\,\text{m}\,\text{s}^{-1}$
Step 3: State the answer with appropriate units and the number of significant figures used in rounding.	The instantaneous velocity is $6\ \text{m s}^{-1}$

Average speed and average acceleration can be determined using the equations for speed and acceleration. You can determine instantaneous speed from a distance–time graph and instantaneous acceleration from a velocity–time graph.

Concept

The rate of change of a quantity at one instant in time is called the instantaneous rate of change.

In physics it is common to draw and use graphs that show time on the *x*-axis and another quantity on the *y*-axis. The gradient of a tangent to the graph at any particular time tells you the instantaneous rate of change of the quantity at that time.

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Consider the displacement—time graph in **Interactive 1.** The velocity equals the gradient, which varies throughout the time interval shown.

Use the tangent (the line with one moveable point) to find the instantaneous velocity (to 2 d.p.) at about 1.5 s.

Use the other line (which has two moveable points) to find the average velocity (to 2 d.p.) between about 1 s and 2 s. (Move one point to 1 s and the other point to 2 s.) Why is this average velocity different from the instantaneous velocity at 1.5 s?

Now gradually move each of the two points closer to the point where the tangent touches the graph. As you do this, compare the average and instantaneous velocities. What do you notice?

Interactive 1. Velocity Analysis from a Position-Time Graph.

More information for interactive 1

The "Velocity Analysis from a Position-Time Graph" interactive helps users understand the concepts of average and instantaneous velocity by exploring how motion is represented on a position-time graph.
The graph plots displacement in meters on the y-axis against time in seconds on the x-axis, allowing users to analyze how an object's position changes over time.
Two key velocity concepts are explored.
Instantaneous Velocity — Represented by a tangent line, which users can adjust by moving a single point along the curve. The slope of the tangent at any moment gives the object's velocity at that specific instant. For example, at t = 1.5s, the instantaneous velocity is approximately 0.51 m/s.
Average Velocity — Represented by a secant line, which passes through two selected points on the curve. The slope of this line represents the object's velocity over a time interval. Users can move these points to see how the average velocity changes. For instance, between t = 1s and t = 2s, the average velocity is -0.7 m/s, calculated using Δx = -1.4m over Δt = 2s.

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The key features of the interactive includes, a tangent line which measures instantaneous velocity at a specific time, a secant line which calculates average velocity over a time interval. The adjustable points allow users to interactively change time values to explore velocity changes.

This interactive helps users differentiate average velocity (displacement over time) from instantaneous velocity (velocity at a specific moment). By analyzing position-time graphs, users learn to determine velocity using secant lines for average velocity and tangent lines for instantaneous velocity. They observe how reducing the secant interval makes average velocity approach instantaneous velocity.

Other areas of physics where the average and instantaneous rates of change of a quantity are not the same include:

- simple harmonic motion, a type of repeating movement in which the position, velocity and acceleration of an object are continuously changing, at changing rates (subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/)).
- the decay of a radioactive sample, for which the instantaneous rate of decay decreases over time (subtopic E.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44319/)).

If the relationship between the quantity and the time can be expressed algebraically, then instantaneous rates of change can be determined without drawing graphs or tangents, using a method called differential calculus. You will not need to use this technique in the DP physics course.

Work through the activity to check your understanding of motion.

Activity

- **IB learner profile:**
  - Knowledgeable
  - Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity**: 20 minutes
- **Activity type**: Individual activity

Student view

A child crosses a playground in a straight line to get to the exit gate. **Figure 8** shows their displacement with respect to time.

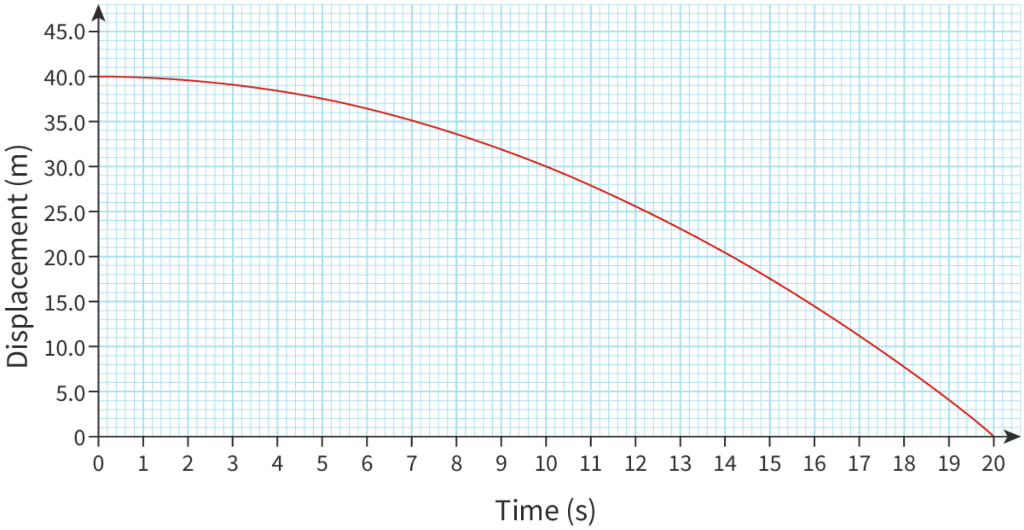


Figure 8. Displacement—time graph for the motion of a child.

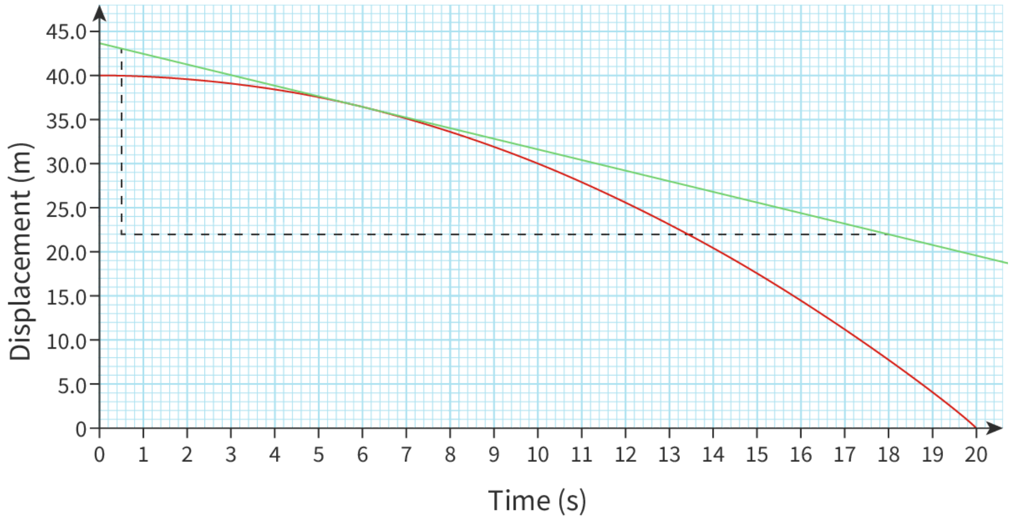
More information for figure 8

The graph displays a line representing the displacement of a child over time. The x-axis is labeled 'Time (s)' and ranges from 0 to 20 seconds. The y-axis is labeled 'Displacement (m)' and ranges from 0 to 45 meters. The line begins at approximately 40 meters on the y-axis at time 0, then gradually decreases, creating a downward curve until reaching 0 meters at 20 seconds. The data suggests a consistent decrease in displacement over the given time period, indicating continuous backward motion towards the starting point.

[Generated by AI]

Use the graph to answer the questions.

1. State whether the velocity is zero at any time during this journey, and explain your answer.
2. Describe the motion of the child during the time interval shown. (You are not required to calculate any velocities or accelerations.)
3. Calculate the average velocity of the child during the time interval shown.
4. Determine the instantaneous velocity of the child at 6 s.



1. The velocity is zero at $t = 0$. The gradient of the graph is zero at this point.
2. The child is initially stationary, and then increases speed throughout the motion, travelling a total distance of 40 m in 20 s to reach the playground's exit gate. (The graph takes the exit gate as the position of zero displacement in the graph, and the child's displacement from it is shown as positive.)

3. $$\begin{aligned} v &= \frac{\Delta s}{\Delta t} \\ &= \frac{-40}{20} \\ &= -2\ \mathrm{m\ s^{-1}} \end{aligned}$$

4. $-1.2\ \mathrm{m\ s^{-1}}$

   (Draw a tangent to the graph at $t = 6\ \mathrm{s}$. The instantaneous velocity is the gradient of the tangent. The diagram shows one way of finding this, with dashed lines drawn from points on the tangent that are easy to read.

   $$\begin{aligned} \text{Gradient} &= \frac{43 - 22}{18 - 0.5} \\ &= \frac{21}{17.5} \\ &= 1.2 \end{aligned}$$

   Note that in the DP physics exam, a small range of possible values would be accepted, to allow for slight variations in tangents estimated by eye.)

The equations of motion

Learning outcomes

At the end of this section you should be able to:

- Explain the difference between motion with uniform and non-uniform acceleration.
- Apply the equations of motion to solve problems of motion with uniform acceleration.

It is common nowadays to use navigation systems in our daily lives. You might have one on your smartphone that you use to reach a destination. They show you the route, the journey time and an estimated time of arrival.

On an aeroplane, screens tell you when you are expected to reach your destination, and at a bus stop, you can see in real time when the next bus is due.

Figure 1. A GPS system.

Credit: Image Source, Getty Images

More information for figure 1

The image shows a GPS navigation device mounted on a car dashboard. The screen displays a map with a highlighted route in yellow and an arrow indicating a direction change. The map includes road names such as A5 and Seymour Place. Information on the screen shows navigation details: '45 yards', '6.7M', '0:20 hours', and '5:53 pm', along with the road name 'A5 Edgware Road' at the bottom. The screen also displays some icons and a progress bar.

[Generated by AI]

All of these systems need to do some 'behind the scenes' calculations to come up with a result and present it to you. So, how exactly do these systems calculate these things?

Uniformly accelerated motion and non-uniformly accelerated motion

Figure 2 shows three velocity–time graphs for a body moving in a constant direction:

- **A** shows a constant velocity – there is no acceleration. This is known as uniform motion .
- **B** shows the velocity changing at a constant rate. This is known as uniformly accelerated motion and the body has uniform acceleration.
- **C** shows the velocity changing at a rate that is not constant. This is known as non-uniformly accelerated motion and the body has non-uniform acceleration.

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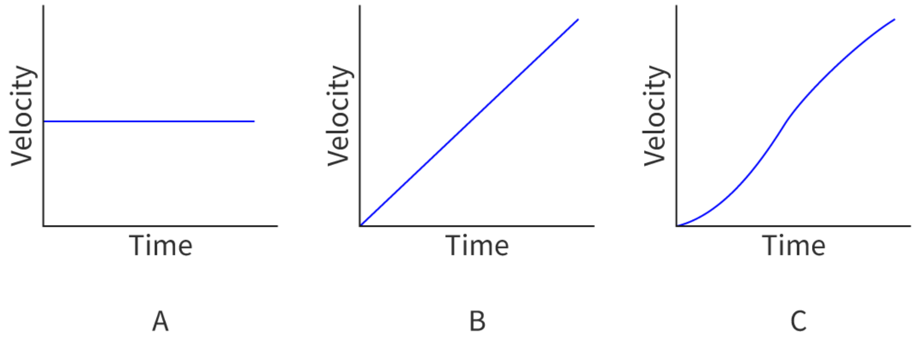

Figure 2. A: uniform motion, B: uniformly accelerated motion, and C: non-uniformly accelerated motion.

More information for figure 2

The image consists of three graphs labeled A, B, and C, each showing a different type of motion with velocity on the Y-axis and time on the X-axis.

- **Graph A** represents uniform motion. The graph shows a horizontal line, indicating constant velocity over time. The X-axis is labeled as Time, and the Y-axis is labeled as Velocity.
- **Graph B** depicts uniformly accelerated motion. This graph features a straight line with a positive slope, which illustrates that the velocity is increasing linearly with time.
- **Graph C** shows non-uniformly accelerated motion. The curve is upward and concave, indicating that the velocity increases at a non-constant rate over time.

All three graphs have similar axes with time and velocity labels but demonstrate different motion patterns through their respective lines.

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Equations of motion

We can use equations to determine the motion of uniformly accelerated objects. These equations are often known as the 'suvat' equations.

Student
view

First, we need to define some quantities. **Table 1** shows each quantity, its symbol and its unit.

Overview
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Table 1. Quantities in suvat equations.

Quantity	Symbol	Unit
Displacement	s	m
Initial velocity	u	m s^{-1}
Final velocity	v	m s^{-1}
Acceleration	a	m s^{-2}
Time	t	s

The equation for velocity is (see section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/)):

$$v = \frac{\Delta s}{\Delta t}$$

Acceleration is rate of change of velocity:

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v - u}{t}$$

This rearranges to give the equation in **Table 2.**

Table 2. Equation for final velocity.

Equation	Symbols	Units
$v = u + at$	v = final velocity	metres per second (m s^{-1})
	u = initial velocity	metres per second (m s^{-1})
	a = acceleration	metres per second per second (m s^{-2})
	t = time	seconds (s)

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Worked example 1

An object starts from rest and accelerates uniformly at 5.3 m s^{-2} for 2.0 minutes. Calculate its final velocity.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$u = 0\text{ m s}^{-1}$ $a = 5.3\text{ m s}^{-2}$ $t = 2.0\text{ minutes}$ $= 2.0 \times 60$ $= 120\text{ s}$
Step 2: Write out the equation.	$v = u + at$
Step 3: Substitute the values given.	$= 0 + (5.3 \times 120)$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 636\text{ m s}^{-1} = 640\text{ m s}^{-1}$ (2 s

The average velocity of an object (if the acceleration is constant) is:

$$\frac{(\text{initial velocity} + \text{final velocity})}{2} = \frac{(u+v)}{2}$$

The equation for velocity can be rearranged to give the equation for displacement:

$$s = vt$$

Substituting in the equation for average velocity, $\frac{(u+v)}{2}$, gives the equation in **Table 3**.

Table 3. Equation for displacement using average velocity.

Student view

Equation	Symbols	Units
$s = \frac{u+v}{2}t$	s = displacement	metres (m)
	u = initial velocity	metres per second (m s^{-1})
	v = final velocity	metres per second (m s^{-1})
	t = time	seconds (s)

Worked example 2

A cyclist is cycling at 2.4 m s^{-1}. They accelerate for 7.0 s and reach a velocity of 3.9 m s^{-1}. Calculate the distance travelled in the 7.0 s.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$u = 2.4 \text{ m s}^{-1}$ $v = 3.9 \text{ m s}^{-2}$ $t = 7.0 \text{ s}$
Step 2: Write out the equation.	$s = \left[\frac{(u+v)}{2}\right]t$
Step 3: Substitute the values given.	$= \left[\frac{(2.4+3.9)}{2}\right] \times 7.0$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding	$= 22.05 \text{ m} = 22 \text{ m (2 s.f.)}$

Substituting the expression for $v = u + at$, into the equation $s = \frac{u+v}{2}t$, gives:

$$\begin{aligned} s &= \frac{(u+u+at)\times t}{2} \\ &= \frac{(2ut + at^2)}{2} \end{aligned}$$

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This gives the equation in **Table 4**.

Table 4. Equation for displacement using acceleration.

Equation	Symbols	Units
$s = ut + \frac{1}{2}at^2$	s = displacement	metres (m)
	u = initial velocity	metres per second (m s^{-1})
	t = time	seconds (s)
	a = acceleration	metres per second per second (m s^{-2})

Worked example 3

A ball is dropped from rest from the top of a 16 m building. How long does it take to hit the ground?

The acceleration due to gravity is 9.8 m s^{-2}

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$s = 16 \text{ m}$ $u = 0 \text{ m s}^{-1}$ $a = 9.8 \text{ m s}^{-2}$
Step 2: Write out the equation and rearrange the equation to make *t* the subject.	$s = ut + \frac{1}{2}at2$ $u = 0$ so: $s = \frac{1}{2}at^2$ $t = \sqrt{\left(\frac{2s}{a}\right)}$

Student view

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Solution steps	Calculations
Step 3: Substitute the values given.	$= \sqrt{\left[\frac{(2\times 16)}{9.8}\right]}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 1.8\text{ s }(2\text{ s.f.})$

Rearranging $s = \frac{u+v}{2}t$ to make t the subject gives:

$$t = \frac{2s}{(u+v)}$$

Combining this with $v = u + at$ gives:

$$v = u + a\left[\frac{2s}{(u+v)}\right]$$

Multiplying by $(u+v)$ gives:

$$v(u+v) = u(u+v) + 2as$$

Multiplying out the brackets gives:

$$v^2 + uv = u^2 + uv + 2as$$

Cancelling the uv on each side gives the final equation shown in **Table 5**.

Table 5. Equation for final velocity.

Student
view

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Equation	Symbols	Units
$v^2 = u^2 + 2as$	v = final velocity	metres per second (m s^{-1})
	u = initial velocity	metres per second (m s^{-1})
	a = acceleration	metres per second per second (m s^{-2})
	s = displacement	metres (m)

Worked example 4

A car accelerates at a rate of 2.1 m s^{-2} over a distance of 8.5 m. Its final velocity is 19 m s^{-1}. What was its initial velocity?

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$a = 2.1\ \text{m s}^{-2}$ $s = 8.5\ \text{m}$ $v = 19\ \text{m s}^{-1}$
Step 2: Write out the equation and rearrange to make u^2 the subject.	$v^2 = u^2 + 2as$ $u^2 = v^2 - 2as$
Step 3: Substitute the values given.	$u^2 = 19^2 - (2\times2.1\times8.5)$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding	$u = 18\ \text{m s}^{-1}\ (2\ \text{s. f.})$

Study skills

The four equations of uniformly accelerated motion are:

$$v = u + at$$

$$s = \left[\frac{(u+v)}{2}\right] t$$

Student view

Overview
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$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

These equations will be given to you in section 1.6.A (/study/app/math-aa-hl/sid-423-cid-762593/book/space-time-and-motion-id-45160/) of the DP physics data booklet. So how do you know which equation to use?

List all the quantities given to you in the question, and the quantity that you need to find, then choose the equation that has these quantities.

Work through **Worked example 5** to see how to approach a question and choose the correct equations.

Worked example 5

A car starts from rest and accelerates at 3.0 m s^{-2} for 8.0 seconds. Then the driver fully presses the brakes for 6.0 seconds and the car negatively accelerates uniformly until it comes to a stop. Calculate the total distance that the car travels.

Student view

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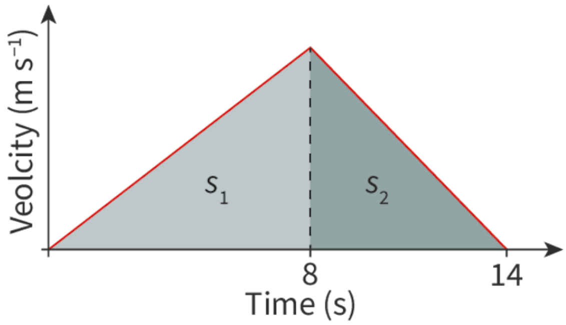
Solution steps	Calculations
Step 1: Sketch a graph of the motion.	 The area between the graph and the *x*-axis equals the displacement (which equals the distance, since the car travels in a straight line in one direction only). The car travels a distance in part 1 of the journey (while accelerating uniformly), and distance in part 2 of the journey (while decelerating to a stop The total distance is $s_1 + s_2$.

Student
view

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Solution steps	Calculations
Step 2: Calculate the velocity, , at the end of the first part of the journey.	One method is to calculate $s_1$and s_2, given by the areas of the shaded triangles, separately and then add them together. (Before finding the distanc travelled in the second part, you would need to calculate the velocity at 8.0 s A quicker method is to see that the total distance equals the area of the whc triangle, which has base 14 and height equal to the velocity at 8.0 s. In part 1 of the journey: $s_1 = \text{unknown}$ $u_1 = 0$ $a_1 = 3.0 \text{ m s}^{-2}$ $t_1 = 8.0 \text{ s}$ So: $a_1 = \frac{\Delta v_1}{\Delta t_1} = \frac{v_1 - u_1}{\Delta t_1}$ $v_1 = u_1 + a_1 \Delta t_1 = 0 + 3.0 \times 8.0 = 24 \text{ m s}^{-1}$
Step 3: Calculate the area of the triangle	$s = \text{area of triangle} = \frac{1}{2} \times 14 \times 24 = 168 \text{ m} = 170 \text{ m (2 s.f.)}$

Free fall

When a body is dropped near the surface of the Earth, it falls downwards. This motion is called free fall, and it is uniformly accelerated motion. During free fall, the acceleration of free fall, g, is the same for all objects: $g = 9.8 \text{ m s}^{-2}$ (this is the value that will be given to you in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet).

You can use the equations of motion to solve problems of free fall, just substitute g for a.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Worked example 6

Two students are standing on a cliff overlooking the sea. They wonder how high the cliff is, so they drop a small stone. The time it takes for the stone to reach the sea is 1.8 seconds. Calculate the height of the cliff and the velocity of the stone when it hits the surface of the sea.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$a = g$ $= 9.8\ \text{m s}^{-2}$ $u = 0\ \text{m s}^{-1}$ $t = 1.8\ \text{s}$
Step 2: Calculate the height of the cliff.	$s = ut + \frac{1}{2}at^2$ $= 0\times1.8 + \frac{1}{2}\times9.8\times$ $= 15.88\ \text{m}$ $= 16\ \text{m (2 s.f.)}$
Step 3: Calculate the velocity of the stone.	$v = u + at$ $= 0 + 9.8\times1.8$ $= 17.64\ \text{m s}^{-1}$ $= 18\ \text{m s}^{-1}\ \text{(2 s.f.)}$

Carry out the activity to check your understanding of the equations of motion.

Activity

- **IB learner profile:**
  - Knowledgeable
  - Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 15 minutes

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- **Activity type**: Individual activity

Instructions

There are three balls in a row on a table. They are equal distances from three holes, as shown in the diagram.

When you hit any ball, it travels across the table, slowing down at a rate of $0.5\,\text{m}\,\text{s}^{-2}$.

You want to hit all three balls, one after the other, so that they fall into their holes at the same time.

You start with ball A, then one second later you hit ball B, then one second later you hit ball C.

If you give ball A a speed of $1.5\ \text{m s}^{-1}$ and it takes 3.0 s to fall in the hole, what is the speed you need to give to ball C?

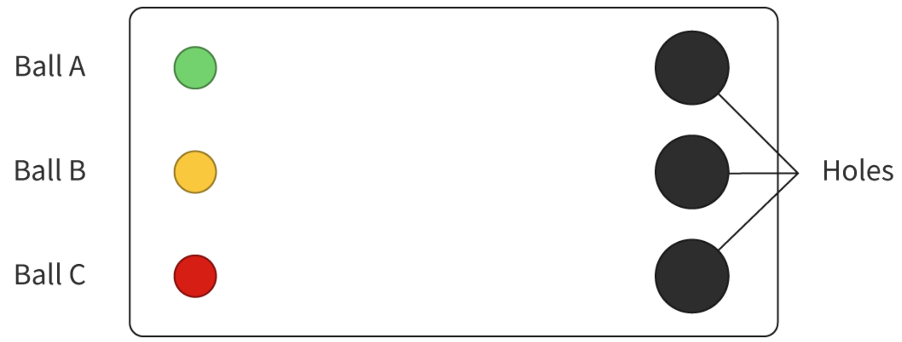

Figure 3. Table set-up.

More information for figure 3

The image is a diagram showing three colored balls labeled Ball A, Ball B, and Ball C, each aligned horizontally in a column on the left. Ball A is green, Ball B is yellow, and Ball C is red. To the right of each ball, there is a line connecting to three holes aligned vertically in a column labeled 'Holes.' The diagram visually represents the setup described in a problem context where different speeds might be needed for each ball to reach a specific hole.

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Student
view

Consider the time that each ball takes to fall into its hole:

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- Ball A takes 3.0 s.
- Ball B is hit 1.0 s after ball A, so ball B has to cover the same distance in 2.0 s.
- Ball C is hit 1.0 s after ball B, so ball C has to cover the same distance in 1.0 s.

The acceleration is a constant —0.5 m s^{-2} for all balls.

Use ball A to calculate the distance between the balls and the holes:

$$u = 1.5 \text{ m s}^{-1}$$

$$t = 3.0 \text{ s}$$

$$a = -0.5 \text{ m s}^{-2}$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= 1.5 \times 3.0 + 0.5 \times -0.5 \times 3^2 \\ &= 4.5 - 2.25 \\ &= 2.25 \text{ m} \end{aligned}$$

Calculate the initial velocity of ball C:

$$s = 2.25 \text{ m}$$

$$a = -0.5 \text{ m s}^{-2}$$

$$t = 1.0 \text{ s}$$

$$s = ut + \frac{1}{2}at^2$$

$$\begin{aligned} u &= \frac{(s - \frac{1}{2}at^2)}{t} \\ &= \frac{(2.25 - 0.5 \times -0.5 \times 1.0^2)}{1.0} \\ &= 2.5 \text{ m s}^{-1} \end{aligned}$$

5 section questions ⌄

A. Space, time and motion / A.1 Kinematics

Projectile motion

Student view

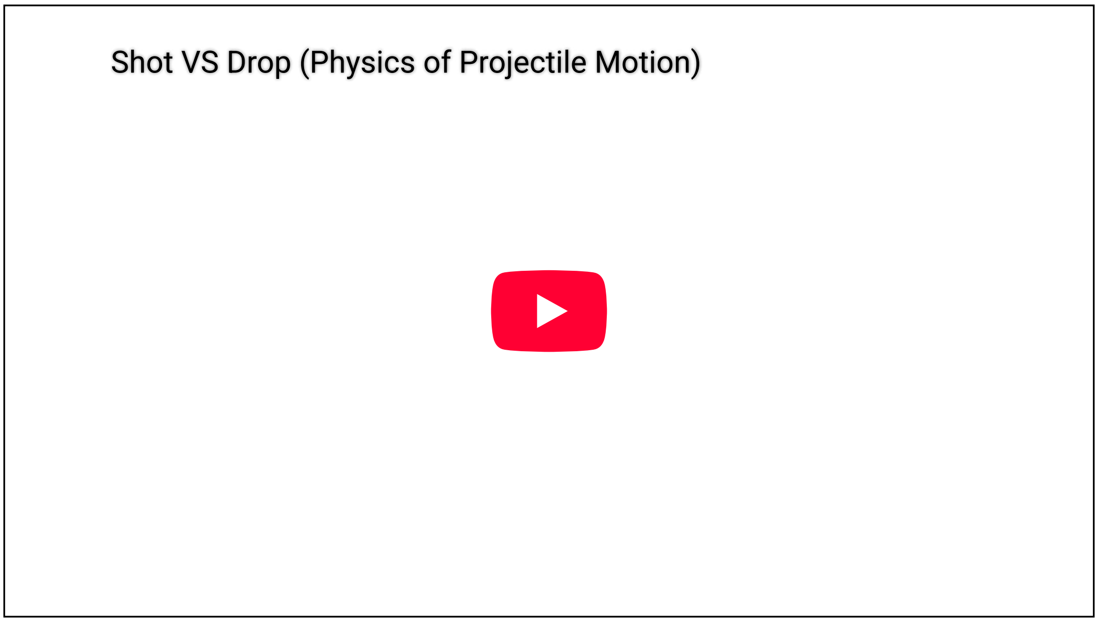
Learning outcomes

At the end of this section you should be able to:

- Describe the motion of a projectile.
- Apply the equations of motion to projectile motion in the absence of fluid resistance.
- Qualitatively describe the effects of fluid resistance on the motion of a projectile.

Look at **Video 1**. You can see a mechanism that ejects a ball horizontally. This is similar to when you are throwing a ball to a friend. How would you describe this motion?

At the same time, it lets another (identical) ball fall freely. Why do both balls reach the ground at the same time?



Video 1. Projectile motion and free fall.

More information for video 1

In this video demonstration titled "Shot vs Drop," a powerful visual explains a foundational concept in physics: the independence of horizontal and vertical motion. The video begins with a simple black background and the word "Shot" in white text. This is followed by two synchronized video clips placed side by side. The left video shows a small metal ball resting on a flat surface, poised to fall freely. In contrast, the right video shows a hand preparing to flick an

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identical ball horizontally off the same surface. Both videos focus on the same moment—the instant one ball is dropped straight down while the other is ejected with a horizontal force.

As the scene shifts, a new title appears: "Shot vs Drop." The visual setup emphasizes that the only difference between the two scenarios is the presence of horizontal motion. The ball on the left begins to fall vertically with no initial horizontal velocity, while the ball on the right follows a curved trajectory due to its combined horizontal and vertical motion.

Crucial explanatory text appears on the screen to guide understanding. A question is posed: "How does the vertical part of a launched object's motion compare to a freely falling object with a vertical acceleration of -9.8 m/s^2?" A red arrow labeled "y-direction" points straight downward, emphasizing that gravity acts in the vertical direction only. This visual cue directs attention to the fact that both balls are subject to the same vertical force—gravity—regardless of any horizontal motion present in one of them.

To reinforce this key point, evenly spaced red horizontal lines are introduced just below the tabletop, representing equal vertical intervals. As both balls descend, they cross each line at the same time, clearly illustrating that their vertical displacements are identical at every moment. Onscreen text reinforces the observation: "The shot ball moves vertically the SAME as the dropped ball." Additional text appears below this explanation, stating: "$a_Y = -9.8 \mathrm{m/s^2}$" and "This means both balls have the SAME vertical acceleration!" These annotations emphasize that gravity exerts the same influence on both balls, making their vertical motions indistinguishable.

This demonstration visually confirms that the vertical motion of a projectile is unaffected by any horizontal force applied to it. The horizontally ejected ball does not fall slower or faster than the one simply dropped; both hit the ground at the same time. This is because horizontal motion and vertical motion are independent of each other in the absence of air resistance. The only force acting on both balls in the vertical direction is gravity, pulling them downward with an acceleration of -9.8 meters per second squared.

The key takeaway is that when a ball is thrown horizontally, it still experiences the same vertical acceleration as a ball that is dropped straight down. The horizontal velocity does not affect how fast it falls. This concept is crucial in understanding two-dimensional motion, especially in projectile motion scenarios where objects follow curved paths yet accelerate downward due to gravity alone. The video reinforces that the horizontal and vertical components of motion operate independently, and gravity's effect on vertical acceleration remains constant regardless of horizontal movement.

Projectile motion

Section A.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/describing-motion-id-44298/) looked at linear motion, which is motion in a straight line. In this section, you will look at motion that happens in two dimensions.

Student view

Imagine that you kick a ball with your foot, at a certain angle to the ground. There are four questions that you need to answer:

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- How high will the ball rise?
- How far will the ball travel?
- How long is the ball in the air?
- How does air resistance affect the motion?

Figure 1 shows the trajectory that the ball will follow. This type of motion is called projectile motion. You will only look at projectile motion which happens close to the ground (Earth).

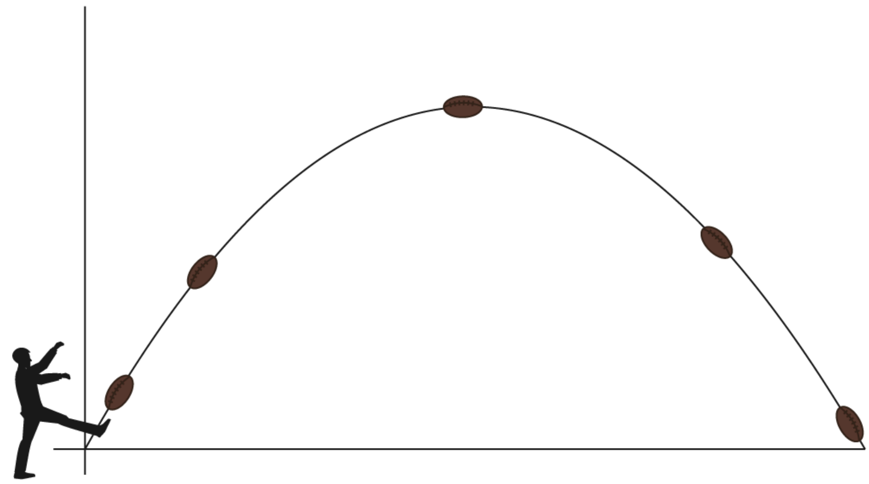

Figure 1. The trajectory of a kicked football.

More information for figure 1

The image is a diagram illustrating the trajectory or arc of a football that has been kicked. Beginning from the left, a figure kicks a football represented at multiple points along a curved path. The path is an upward arc demonstrating projectile motion, descending back towards the ground after reaching its peak. The start and end points are at the same horizontal level, indicating the initial and final positions on the ground. This visual depiction complements the text's reference to analyzing projectile motion close to the ground. The diagram is important for understanding the concepts of vertical and horizontal motion resolved through the equations of motion, as mentioned in the surrounding text.

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In order to learn everything we can about this motion, we must resolve it in both the vertical and horizontal direction (see section A.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/vectors-id-44297/)) using the equations of motion (see section A.1.3

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

(/study/app/math-aa-hl/sid-423-cid-762593/book/the-equations-of-motion-id-44299/)). Let's look at a series of questions about the flight of a ball. Try to answer each question before looking at the solution.

Worked example 1

A ball is kicked at 60° to the horizontal (**Figure 2**). Its initial speed, u, is 20 m s^{-1}.

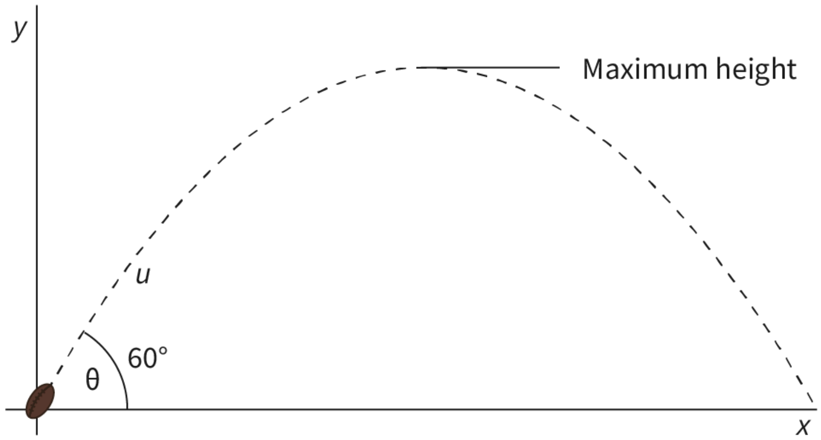

Figure 2. A ball is kicked at 60° to the horizontal.

More information for figure 2

The image is a diagram depicting the trajectory of a ball kicked at an angle of 60° to the horizontal axis, labeled as 'x'. The vertical axis is labeled 'y'. The path of the ball is represented by a dashed curve, starting from the bottom left where the ball is initially at rest. The initial speed is denoted as 'u'. The curve rises to a peak, which is labeled as 'Maximum height', before descending back towards the horizontal axis at the far right.

Key elements of this diagram include: - The angle of 60° is marked between the ball's initial position and the horizontal axis. - The trajectory illustrates a classic projectile motion curve, showing the ball rising and falling symmetrically. - The maximum height point is indicated on the peak of the curve, signifying the highest point reached during the ball's flight.

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Student view

What maximum height does the ball reach?

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The **vertical** plane determines the maximum height:

$$u = 20\sin 60 \text{ m s}^{-1}$$
(vertical component of the initial speed)

$$a = -9.8 \text{ m s}^{-2}$$

$$v = 0 \text{ m s}^{-1}$$
(when the ball reaches its maximum height, its vertical speed is zero)

$$v^2 = u^2 + 2as$$

$$\begin{aligned} s &= \frac{(v^2 - u^2)}{2a} \\ &= \frac{(0^2 - (20\sin 60)^2)}{(2\times -9.8)} \\ &= 15.3 \text{ m} \end{aligned}$$

How long is the ball in the air in total? (This is known as the time of flight.)

The **vertical** plane determines the time of flight:

$$u = 20\sin 60 \text{ m s}^{-1}$$
(vertical component of the initial speed)

$$a = -9.8 \text{ m s}^{-2}$$

$$v = -20\sin 60 \text{ m s}^{-1}$$
(when the ball comes back down, it is travelling at the same speed downwards as it was going upwards when it was kicked)

Rearrange for t,

Student view

Overview
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$$v = u + at$$

$$t = \frac{(v-u)}{a}$$

$$= \frac{(-20\sin 60 - 20\sin 60)}{-9.8}$$

$$= 3.535 \text{ s}$$

$$t = 3.5 \text{ s (2 s.f.)}$$

How far away, horizontally, does the ball land? (This is known as the range.)

The **horizontal** plane determines the range:

$$u = 20\cos 60 \text{ m s}^{-1}$$

(horizontal component of the initial speed)

$$a = 0 \text{ m s}^{-2}$$

$$v = 20\cos 60 \text{ m s}^{-1}$$

(horizontal component of the final speed)

$$t = 3.5 \text{ s}$$

$$s = \frac{(u+v)t}{2}$$

$$= \frac{[(20\cos 60 + 20\cos 60)\times 3.5]}{2}$$

$$= 35 \text{ m}$$

Study skills

You have seen a multi-step approach to answering the questions about the ball, and at the end, you found the horizontal range. You may have to combine all these steps into one if you are asked to calculate the range given the initial speed. No matter the situation, the same steps apply.

Student view

Sometimes a projectile does not land at the same height as it was launched from, as in the worked example below.

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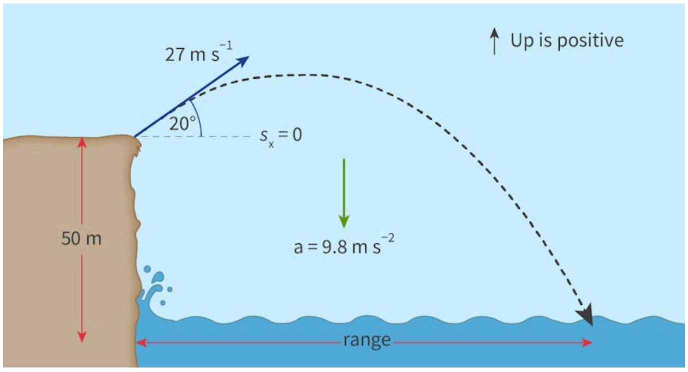
Worked example 2

A boy throws a stone from the edge of a cliff at an angle of 20.0° above the horizontal with a speed of 27 m s^{-1}.

The top of the cliff is 50 m above the sea surface. Air resistance is negligible, and the height of the thrower can be ignored.

1. Calculate the maximum height of the stone above the sea during its flight.
2. Calculate the range of the stone's motion.

Solution steps	Calculations
Step 1: Draw a sketch, and choose a positive *y*-direction and a zero point for vertical displacement.	 Upwards is positive. Vertical displacement = 0 at the top of the cliff. (Different choices could be made here, which would not affect the answers as long as they were applied consistently.)

Student view

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Solution steps	Calculations
Step 2: Decide whether to consider all or only part of the stone's journey, and write down the suvat quantities.	Consider the part of the stone's journey from its release to its highest point. Notice that the vertical component velocity is zero at the maximum height. Using y to represent vertical components: $u_\mathrm{y} = 27 \times \sin 20.0^\circ = 9.23\ \mathrm{m\ s^{-1}}$ (positive because it is upwards) $v_\mathrm{y} = 0$ $a_\mathrm{y} = -9.8\ \mathrm{m\ s^{-1}}$ (negative because it is downwards) $t = \mathrm{unknown}$
Step 3: Choose a suitable equation and use it to calculate s_y.	${v_\mathrm{y}}^2 = {u_\mathrm{y}}^2 + 2a_\mathrm{y}s_\mathrm{y}$ $s_\mathrm{y} = \dfrac{{v_\mathrm{y}}^2 - {u_\mathrm{y}}^2}{2a_\mathrm{y}} = \dfrac{0 - 9.23^2}{2(-9.8)} = 4.35\ \mathrm{m}$ (positive means above the clifftop)
Step 4: Calculate the maximum height above the sea — this is the answer to question 1.	$\text{Height above the sea} = 50 + 4.35 = 54\ \mathrm{m}\ (2\ \mathrm{s.f.})$
Step 5: To find the range, first find the time of flight, t, which is determined by the vertical motion.	Consider the entire journey. Note that the values of suva quantities are different from those shown in Step 2, whi only considered part of the journey. $s_\mathrm{y} = -50\ \mathrm{m}$ (negative because it is below the clifftop) $u_\mathrm{y} = 27 \times \sin 20.0^\circ = 9.23\ \mathrm{m\ s^{-1}}$ $v_\mathrm{y} = \mathrm{unknown}$ $a_\mathrm{y} = -9.8\ \mathrm{m\ s^{-1}}$

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Solution steps	Calculations
Step 6: Choose a suitable equation (or equations) and calculate t.	One method is to write a quadratic equation using $s_y = u_y t + \frac{1}{2} a_y t^2$ and solve it to find t. An alternative method is to use ${v_y}^2 = {u_y}^2 + 2a_y s_y$ to find v and then use $v_y = u_y + a_y t$ to find t: $v_y = \sqrt{{u_y}^2 + 2a_y s_y} = \sqrt{9.23^2 + 2(-9.8)(-50)} = -32.6\ \text{m s}^{-1}$ (taking the negative value of the square root because th vertical component of velocity is downwards at the end of the journey) $t = \frac{v_y - u_y}{a_y} = \frac{-32.6 - 9.23}{-9.8} = 4.27\ \text{s}$
Step 7: Use t to calculate the horizontal distance travelled.	$v_x = 27 \times \cos 20.0° = 25.4\ \text{m s}^{-1}$ $t = 4.27\ \text{s}$ $v_x = \frac{d}{t}$ (since there is no horizontal acceleration) $d = v_x t = 25.4 \times 4.27 = 108\ \text{m} = 110\ \text{m}\ (2\ \text{s.f.})$

Creativity, activity, service

Strand: Creativity

Learning outcome: Demonstrate how to initiate and plan a CAS experience

Student view

Many sports involve projectile motion. If we have suvat equations to describe motion, why can’t everyone score in basketball every time, or place a football in exactly the right spot in the goal?

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Design a CAS activity to test how knowledge of physics and projectile motion affects the ability of a student to score a goal/basket. Does knowing about physics help? How could you coach a team to improve their accuracy based on your physics understanding?

The effect of fluid resistance on a projectile

A fluid is a substance that does not have a fixed shape and can flow easily. Air and water are both fluids. Fluid resistance can affect the motion of a body. For example, when you run, you can feel the air pushing back on your face and body. This is called air resistance.

The effect of air resistance on a projectile is shown in **Figure 3**.

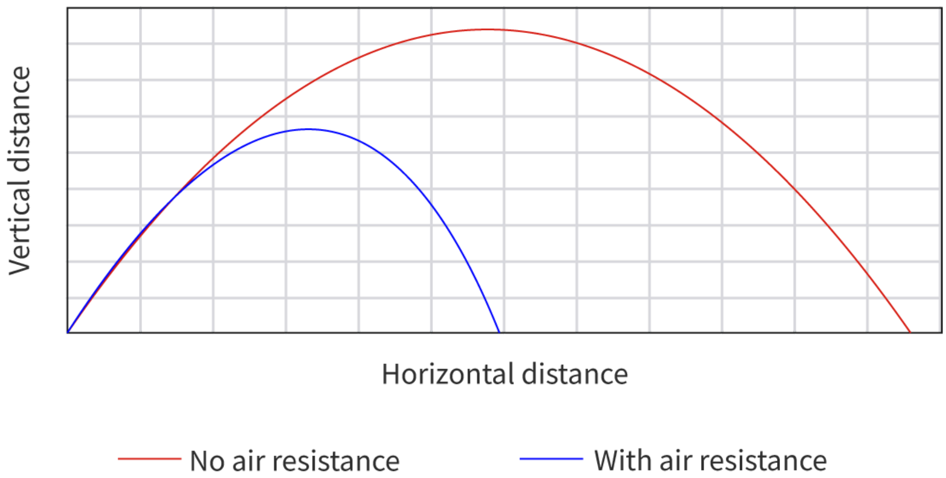

Figure 3. The trajectory of a ball with and without air resistance.

More information for figure 3

The image is a graph depicting the trajectory of a ball with and without air resistance. The X-axis is labeled "Horizontal distance" with grid lines indicating intervals, and the Y-axis is labeled "Vertical distance". Two curves are shown on the graph: one red curve labeled "No air resistance" and one blue curve labeled "With air resistance". The red curve forms a larger, symmetrical arc, indicating a parabolic trajectory without air resistance. In contrast, the blue curve forms a smaller arc, demonstrating a decreased range and height due to air resistance. The graph visually indicates the impact of air resistance on the projectile's range and trajectory.

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We can see from **Figure 3** that when there is no air resistance, the shape of trajectory is a parabola. (You do not need to know the equation for a parabola.)

When there is air resistance:

- The trajectory is a different shape (it is not parabolic).
- The maximum height is lower.
- The range is shorter.

We can also deduce that:

- The velocity of the object will be lower due to the effect of air resistance decreasing the acceleration of the object.
- An object experiencing air resistance travelling upwards will reach its maximum height sooner, so the time of flight when going up is less.
- An object experiencing air resistance travelling downwards will experience an upwards air resistance, so the time of flight when going down is greater.

Study skills

In projectile motion, air resistance will be considered only qualitatively. This means that you will not be asked to calculate quantities like maximum height, time of flight or range in a question that includes air resistance.

All problems with projectile motion will include a constant value of g. This means that the motion takes place near the Earth's surface.

Air resistance also affects an object in free fall. **Figure 4** shows the velocity–time graph for a skydiver.

Student view

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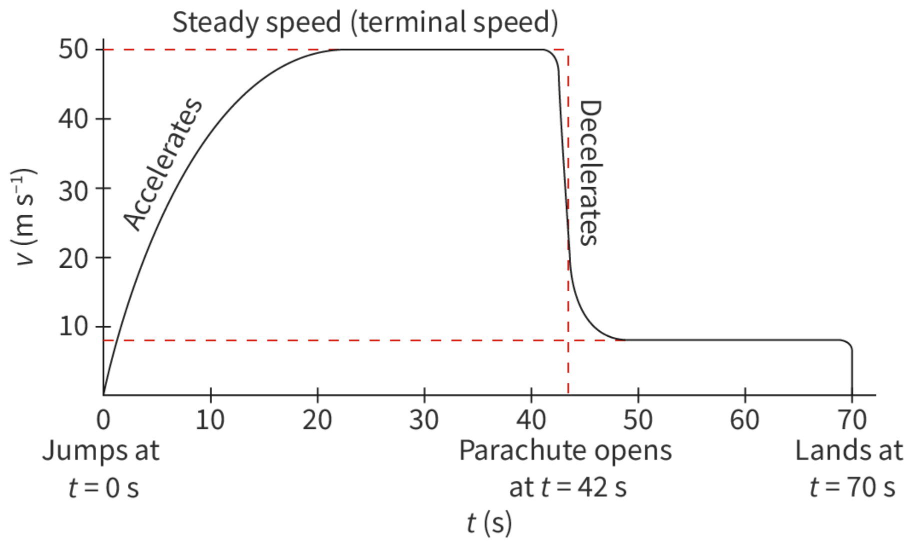

Figure 4. A velocity—time graph for a skydiver.

More information for figure 4

This is a velocity-time graph for a skydiver. The X-axis represents time in seconds (t), ranging from 0 to 70 seconds, with key points marked at 0, 42, and 70 seconds. The Y-axis represents velocity in meters per second (v), ranging from 0 to 50 m/s. The curve indicates the skydiver's journey. Initially, the velocity increases rapidly as the skydiver accelerates after the jump at t=0s. At about 10 seconds, the velocity starts to stabilize, reaching a steady speed known as terminal speed at approximately 50 m/s. This steady speed continues till the parachute opens at t=42s, causing a sudden deceleration and reduction in velocity. The skydiver then reaches a slower, consistent speed before landing at t=70s. The graph helps illustrate how air resistance and the parachute affect the skydiver's speed during descent.

[Generated by AI]

As the skydiver is moving downwards, air is pushing upwards. The faster the person falls, the greater the air resistance, and the smaller the acceleration. Eventually, the acceleration becomes zero and the speed is constant. This speed is called the terminal speed.

At 42 seconds on the graph, the parachute opens. The air resistance is now much greater than the weight, so the skydiver decelerates sharply until they reach a new terminal speed, which they maintain until they land.

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Nature of Science

Aspect: Models

Physics tries to describe natural phenomena by observation, experiments and analysis.

To achieve that, scientists develop models, which are a set of ideas that try to reconstruct what is observed. All models are simplifications of real-life phenomena. This means that every model has limitations.

In physics, we first use a simple model to explain a phenomenon, then develop a more complex one.

In this section, you looked at the motion of a projectile, assuming there is no air resistance. Air is everywhere in the atmosphere of the Earth, so why don't we take it into account? Think about the following questions:

- How does the effect of air resistance change our findings? Does it affect all our findings? Does it affect them every time or under specific conditions?
- How much would including air resistance complicate the model? Would it make projectile motion impossible to study? Would we need complex mathematics to study it?

Work through the activity in the next section to check your understanding of projectile motion.

5 section questions

A. Space, time and motion / A.1 Kinematics

Activity: Projectile motion

A.1.8: The behaviour of projectiles A.1.9: The qualitative effect of fluid resistance on projectiles

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/activity-projectile-motion-id-44301/print/) Assign

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Activity

- **Learner profile:**
  - Inquirer
  - Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

Instructions:

You are going to use the simulation to investigate projectile motion.

1. On the front page, select the 'Lab' option.
2. Keep air resistance switched off. Change the following settings only:

- change the type of projectile to a golf ball (in the drop-down menu near the top right)
- change the launch angle to 50° (by dragging the cannon to rotate it)
- change the initial speed to 15 m s^{-1} (using the slider at the bottom left).

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view

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3. Launch the golf ball by pressing the ‘Fire cannon’ button.
4. Experiment with changing the launch angle until the ball hits the target on the ground. (Note that you can use the yellow eraser button at any time to delete all previous trajectories.)
5. After you have hit the target, use the suvat equations and the ball’s initial speed and angle to calculate the time of flight, range and maximum height of the ball.
6. Use the ‘Time, Range and Height’ tool (at the top of the screen) to check the results of your calculations. (Drag the tool down and place its crosshair over any point on the trajectory to see the time, range (horizontal displacement from the start) and height at that point.)
7. Predict whether the trajectory of a cannonball will be different from the trajectory of the golf ball (under the same conditions as before: no air resistance, launch angle 50°, initial speed 15 m s^{-1}), and explain your prediction. Use the simulation to check your prediction by selecting cannonball from the drop-down menu.
8. Erase all previous trajectories. Change the projectile back to a golf ball and fire it. Predict and explain the shape of the trajectory in the presence of air resistance. Check your prediction by ticking the ‘Air resistance’ box and firing the golf ball again.
9. Predict and explain whether the effect of air resistance on the trajectory will be more or less significant for the cannonball than for the golf ball. Check your prediction by firing the cannonball with and without air resistance.
10. Switch off air resistance. Now choose an independent variable (launch angle, gravitational field strength, or initial speed) and a dependent variable (maximum height, flight time, or range). Use the simulation to investigate the relationship between the two variables. Write predictions, results and conclusions.

A. Space, time and motion / A.1 Kinematics

Summary and key terms

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/summary-and-key-terms-id-44302/print/) Assign

- The motion of bodies can be described and analysed using position, displacement, velocity and acceleration, where displacement is the change in position, velocity is the rate of change of position and acceleration is the rate of change of velocity.
- Distance is the length of the path travelled, while displacement is the change in position.
- The average value for velocity (or acceleration) is the total change in displacement (or change in velocity) divided by the total time. The instantaneous value changes from moment to moment and can be calculated by drawing a tangent to a displacement–time graph (or velocity–time graph) and calculating the gradient.
- Constant velocity with no acceleration is uniform motion. Velocity changing at a constant rate is uniformly accelerated motion. Velocity changing at a non-constant rate is non-uniformly accelerated motion.

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- The equations of motion ('suvat' equations) can be used to analyse linear motion for uniformly accelerated motion, including free fall.
- Projectile motion is motion in two dimensions and the equations of motion can be used to determine maximum height, time of flight and range.
- Air resistance affects projectile motion, and the motion of free fall. In projectile motion, the range and maximum height reached both decrease in the presence of air resistance. In free fall, the maximum velocity decreases.

Key terms

Review these key terms. Do you know them all? Fill in as many you can using the terms in this list.

1. is the change in position.
2. is the length of the path travelled.
3. is the rate of change of position.
4. is the rate of change of velocity.
5. motion is motion with a constant velocity.
6. is motion with a constant acceleration.
7. The maximum horizontal distance travelled by a projectile is called .
8. During free fall, when there is air resistance, an object's velocity wil due to the effect of air resistance its acceleration.

Acceleration | Velocity | decreasing | Uniform | Uniformly accelerated motion | lower | range | Displacement | Distance

Check

Interactive 1. Understanding Key Terms of Kinematics.

Student view

Checklist

What you should know

After studying this subtopic, you should be able to:

- Explain the motion of an object in terms of position, displacement, distance, velocity and acceleration.
- Understand the differences between instantaneous and average values for velocity, speed and acceleration and be able to determine them.
- Explain the difference between motion with uniform and non-uniform acceleration.
- Apply the equations of motion to solve problems of motion with uniform acceleration.
- Describe the motion of a projectile.
- Apply the equations of motion to projectile motion in the absence of fluid resistance.
- Qualitatively describe the effects of fluid resistance on the motion of a projectile.

Practical skills

Once you have completed this subtopic, go to:

- Practical 1: Investigating the acceleration of free fall (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-acceleration-of-free-fall-id-43210/) in which you will apply the equations of motion.
- Practical 2: Investigating projectile motion (/study/app/math-aa-hl/sid-423-cid-762593/book/investigating-the-relationship-between-velocity-id-46751/) in which you will analyse and describe the motion of a projectile.

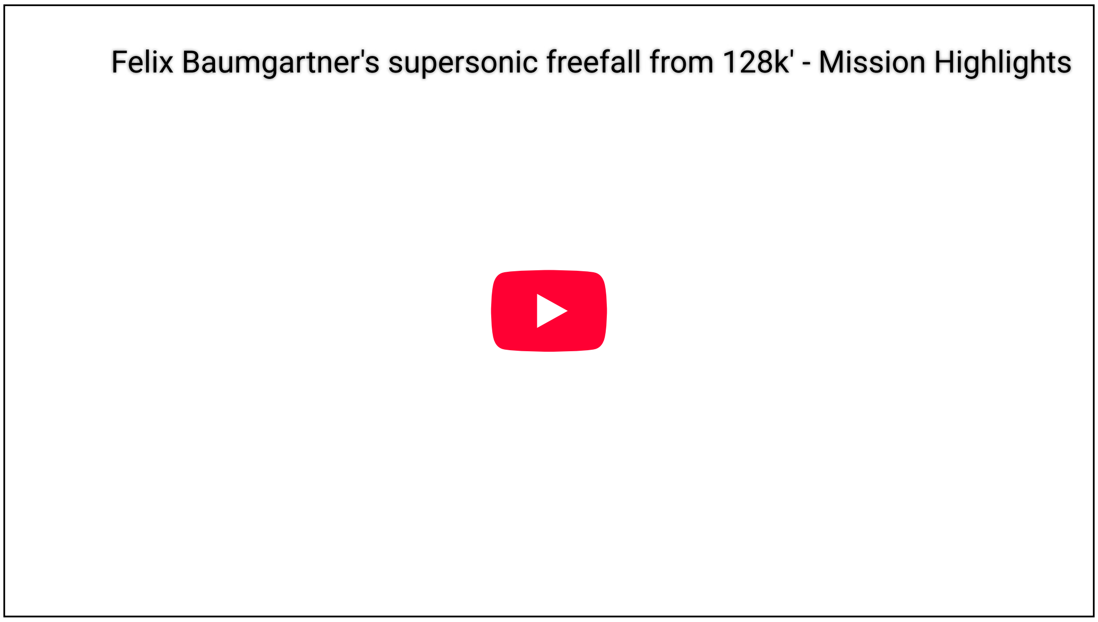
A. Space, time and motion / A.1 Kinematics

Investigation

- **Learner profile attribute**:
  - Inquirer
  - Thinker
- **Approaches to learning**:
  - Research skills – Using search engines and libraries effectively
  - Thinking skills – Applying key ideas and facts in new contexts
- **Time to complete activity**: 60 minutes
- **Activity type**: Group activity

Your task

Work in a group. Watch the video of the Felix Baumgartner jump, which set various world records for skydiving.



Video 1. Felix Baumgartner's record-breaking sky dive.

Research the jump:

- What was the altitude of the jump?
- How long did it take for Felix to reach the ground?
- What was the maximum speed reached?
- What was the speed reaching the ground?

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Apply a simple model of free fall (without air resistance) to the jump and calculate the following based on the height Felix Baumgartner left his capsule:

- time of flight
- maximum speed reached
- speed at the ground.

Compare your calculations to the values found in the research and answer the following questions:

- How realistic is the simple model of free fall? Does it apply at all stages of the jump?
- How could you improve this model?

Try to improve your model, by taking into account the effect of air resistance:

- At what altitude does air resistance start playing a role?
- What is the effect of air resistance qualitatively?

Try to calculate the terminal speed:

- Find an equation to calculate terminal speed in a fluid. (You could look for one online, or in subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/).)
- Identify the values you need for the calculation and substitute the values from the actual jump.

Compare the terminal speed you found with the maximum speed reached by Felix:

- How close are the two values?
- What are the problems with the model that you used for the effect of air resistance?
- Can one formula give the right values for every part of the jump?

Evaluate your findings and discuss what the main issues are. Try to think of a way to model this jump in a more accurate, realistic way. Think about the density of the atmosphere, and how it changes as Felix Baumgartner gets closer to the Earth.

Student view

Reflection

Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.

Reflection

Now that you've completed this subtopic, let's come back to the guiding questions introduced in The big (https://app.kognity.com/study/app/class/sid-423-cid-0/book/the-big-picture-id-43128/)picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/).

- How can the motion of a body be described quantitatively and qualitatively?
- How can the position of a body in space and time be predicted?
- How can the analysis of motion in one and two dimensions be used to solve real-life problems?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

- What main points have you learned from this subtopic?
- Is anything unclear? What questions do you still have?
- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

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Once you submit your response, you won't be able to edit it.

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Help us improve the content and user experience.

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