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The big picture

Quadratic functions are a special class of functions that are very useful in modelling real-life phenomena. The graph of a quadratic function is a type of curve called a parabola, which is of great interest and utility in mathematics and many other fields. Parabolic curves and their special features appear in nature as well as in design and engineering.

Credit: visualspace Getty Images

Source: " L'Oceanografic, Valencia, Spain 1 - Jan 07 (https://fr.wikipedia.org/wiki/Fichier:L%27Oceanografic,_Valencia,_Spain_1_-_Jan_07.jpg) " by Diliff

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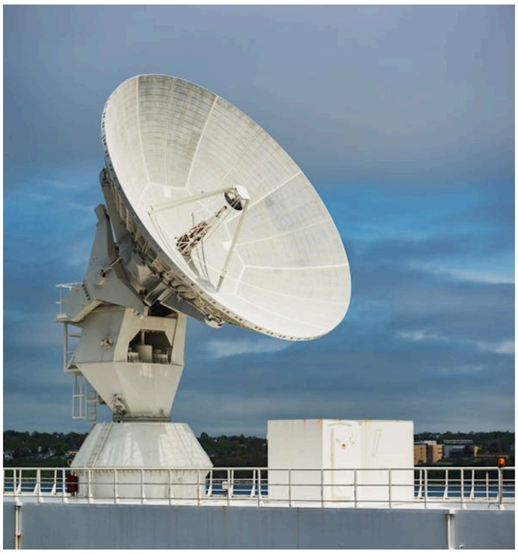
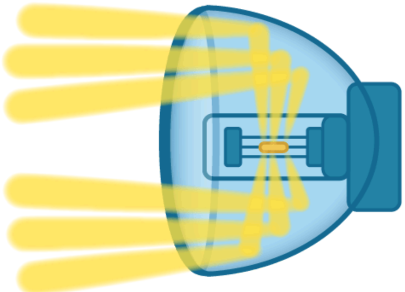


Credit: shaunl Getty Images



More information

The image is a diagram illustrating a car headlight. It shows the headlight emitting yellow beams of light, radiating outward from the bulb inside the housing. The headlight is depicted as a semi-transparent blue structure, allowing visibility of the internal components. The bulb, centrally located, is highlighted as the source of light. The beams are shown as parallel lines extending from the bulb towards the left of the image, indicating the direction of the emitted light. The housing around the bulb is shaped like a dome, focusing and directing the light beams forward. This diagram illustrates the basic functionality and structure of a car headlight assembly.

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Parabolas in the real world: trajectory of a ball, architectural arches, satellite dish and headlight

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Parabolas are a widely used shape for the reflectors in headlights and satellite dishes. Using the applet below, you can explore how parabolic curves reflect light from a point source. Line segment [AB] represents a light ray sent from the light source A. Adjust the position of point B on the parabola to visualise the reflection of the light rays by the parabola. What do you notice?

Interactive 1. Parabolas in the Real World: Reflection of Light.

More information for interactive 1

This interactive allows users to explore the concept of parabolic reflection by moving point B along the parabola and observing how light rays behave after reflecting off the surface. A point source at A emits multiple rays of light, which strike different points on the parabolic mirror. As users adjust point B, the interactive shows how each light ray reflects off the curved surface and travels parallel to the axis of symmetry.

This dynamic visualization reinforces a key geometric property of parabolas: all light rays emitted from the focus reflect off the parabolic surface and become parallel to the axis of symmetry. This principle is fundamental in real-world applications such as satellite dishes, car headlights, and telescopes, where focused energy or light must be directed efficiently. By interacting with this tool, users gain a deeper understanding of how parabolas are used to control the direction of reflected rays through precise geometric properties.

This reflection property of parabolic curves can be used in light, sound and other forms of signal transmission and was first applied to telescopes in the 17th century. Nowadays, satellite dishes use the reflection property of parabolas to receive signals and direct them to

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the receiver, where they are interpreted and transmitted to our TVs. You will discover more about the reflection principle of parabolic curves in the Investigations section.

In this subtopic, you will study quadratic functions and parabolas in the context of coordinate geometry and learn about:

- the various algebraic forms that represent quadratic functions
- the algebraic and graphical features of parabolic curves
- modelling with quadratic functions.

Concept

While learning how to move between the different forms of **representation** of quadratic functions, reflect on what information each form gives you. Think about how the **modelling** techniques you learned earlier can be extended to quadratic functions and parabolic curves.

Theory of Knowledge

You can test the validity of a quadratic function by graphing it and seeing if it ‘works’. This provides one key element of knowledge — falsifiability — and brings up an important knowledge question with ramifications both in mathematics and other areas of knowledge.

Knowledge Question: Must knowledge be falsifiable in order to be considered valid?

2. Functions / 2.6 Quadratic functions

Standard form of quadratic functions

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Standard form and features of a quadratic

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function

The standard form of a quadratic function f is

$$f(x) = ax^2 + bx + c\,,$$

where a , b and c are real numbers and $a \neq 0$. The numbers $a,$ b and c are called the **parameters** of the quadratic function. The graph $y = f(x)$ of a quadratic function is called a parabola, and its shape is determined by the parameters $a,$ b and c. In the applet below, you can visualise the graph of a quadratic function in its standard form.

Interactive 1. Standard Form and Features of Quadratic Functions.

More information for interactive 1

This interactive enables users to explore the concept of quadratic functions in their standard form, $y = ax^2 + bx + c$, by manipulating the parameters a, b, and c using sliders that range from −5 to 5. By adjusting a, users can observe how the parabola's direction and width change—positive values make it open upward, while negative values make it

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open downward. Higher values of |a| make the parabola narrower, whereas smaller values make it wider. Changing c shifts the parabola up or down, affecting the y-intercept. Modifying b alters the position of the vertex and the axis of symmetry, influencing the overall shape and orientation of the parabola. This interactive helps users develop a deeper understanding of how quadratic equations behave graphically, reinforcing key concepts in coordinate geometry and function transformations.

Activity

In the applet above, you see the graph and equation of a quadratic function in its standard form, $y = ax^2 + bx + c$. Using the sliders 'a', 'b' and 'c', you can change the values of the parameters a, b and c, leading to changes in the graph of the function.

- Using slider 'a', observe the graph of the function for positive and negative values of a. What do you notice?
- Compare the parabolas for different positive values of a. What do you notice?
- Using slider 'c', observe the graph of the function for various values of c. What do you notice?
- Use slider 'b' to explore how the parameter b affects the parabola.

Important

A quadratic function has standard form $f(x) = ax^2 + bx + c, \quad a \neq 0$. Its graph $y = ax^2 + bx + c, \quad a \neq 0$ is a parabola, a symmetric curve with the following features:

- If $a > 0$, the parabola opens upwards; its vertex is the **minimum turning point** and the curve is concave up .
- If $a < 0$, the parabola opens downwards; the **vertex** is the **maximum turning point** and the curve is concave down .
- The parabola crosses the y-axis at the point $(0, c)$, called the y **-intercept** of the parabola.
- The vertex of the parabola is $\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$.
- The **axis of symmetry** of the parabola is the vertical line $x = -\frac{b}{2a}$, which passes through the vertex.

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For example, the quadratic function

$$f(x) = -x^2 + x + 4$$

has $a = -1,\ b = 1$ and $c = 4$. The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{1}{2(-1)} = \frac{1}{2}$$

and the corresponding y-coordinate is

$$f\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 4 = -\frac{1}{4} + \frac{1}{2} + 4 = \frac{1}{4} + 4 = \frac{17}{4}.$$

Since $a < 0$ the parabola is concave down and the vertex $\left(\frac{1}{2}, \frac{17}{4}\right)$ is the maximum turning point. Thus, the range of the function is $\left\{y \mid y \le \frac{17}{4}\right\}$.

The axis of symmetry of the parabola passes through the vertex and therefore has equation $x = \frac{1}{2}$.

Finally, $c = 4$ means that the y-intercept of the graph is the point $(0,\ 4)$. You can graph the function on a GDC and verify these properties. The parabola for the quadratic function $f(x) = -x^2 + x + 4$ is shown in the figure below.

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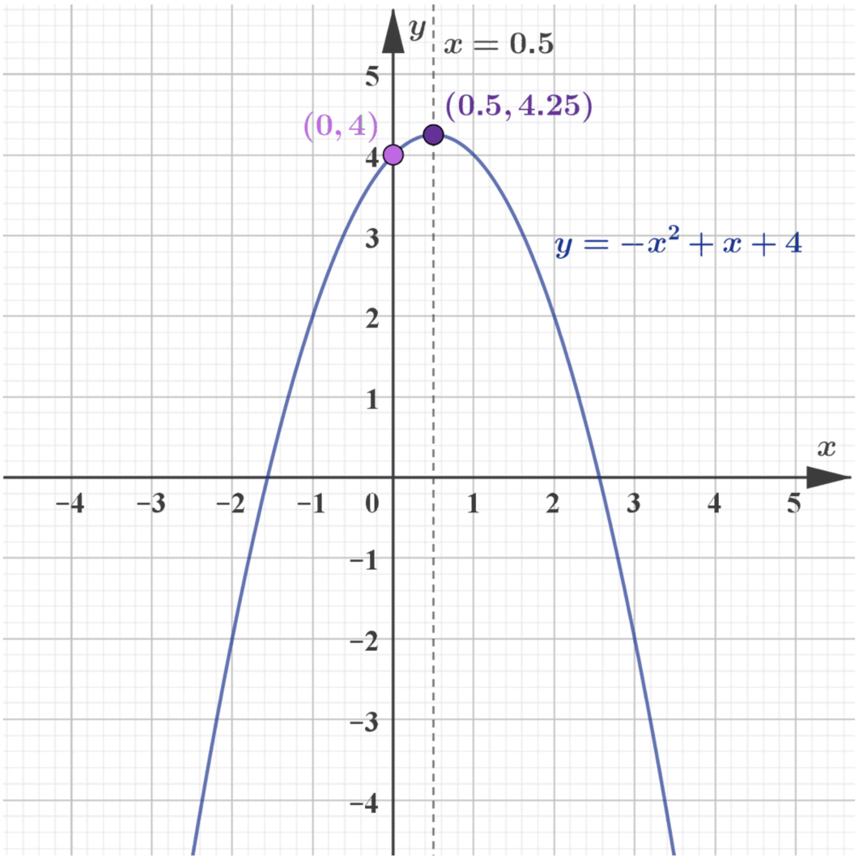

More information

The image is a graph representing the quadratic function f(x) = -x^2 + x + 4. It is plotted on a Cartesian coordinate system with a grid background. The X-axis represents the input values, ranging from -2 to 5, marked with integer intervals. The Y-axis represents the output values, ranging from -2 to 5, also marked with integer intervals.

The graph is a downward-opening parabola. Key points on the graph include the y-intercept at (0, 4), where the parabola crosses the Y-axis. Another important point visible is (0.5, 4.25), slightly above the y-intercept on the graph.

Overall, the graph starts high on the Y-axis, peaks near (0.5, 4.25), and then descends symmetrically as x increases or decreases, displaying the characteristics of an inverted parabola with its vertex at approximately x = 0.5 and y = 4.25.

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Be aware

In general, the domain of a quadratic function is the set of real numbers $\mathbb{R}$, unless otherwise stated.

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Example 1

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Consider the quadratic function $f(x) = x^2 - 4x$. Without using a GDC:

a) Find the domain and range of the function.

b) Find the y-intercept.

c) State the equation of the axis of symmetry.

d) Sketch the parabola.

	Steps	Explanation
a)	The domain of the quadratic function is the set of real numbers $\mathbb{R}$.	
	$\text{Vertex is } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ $-\frac{b}{2a} = -\frac{-4}{2(1)} = 2$ $f(2) = 2^2 - 4(2) = -4$ The vertex is the point $(2, -4)$.	To find the range of the function, first find the vertex This quadratic function has $a = 1,\ b = -4$ and $c = 0$.
	So the range of the function is $\{y \mid y \geq -4\}$.	The vertex is $(2, -4)$, and since $a > 0$ the parabola is concave up. Hence, the vert is the minimum point.
b)	The y -intercept is $(0,\ 0)$.	The y -intercept is determin by value of the parameter c which equals zero.
c)	The equation of the axis of symmetry is $x = 2$	The axis of symmetry passes through the vertex, which ha x -coordinate 2 .

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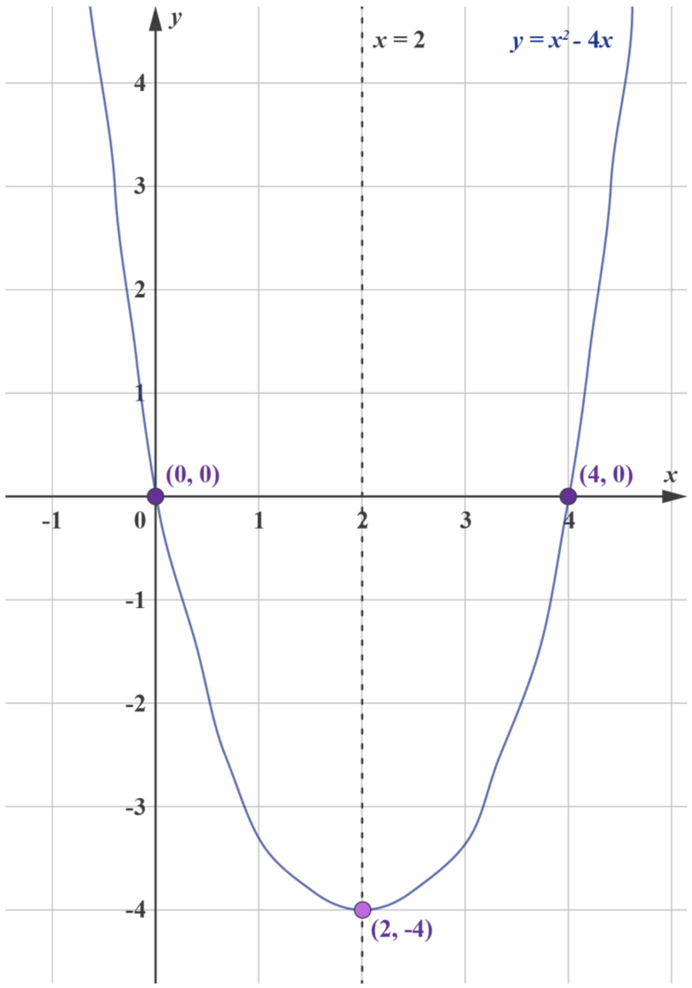
	Steps	Explanation
d)	The sketch of the parabola is shown below. 	Since the y -intercept $(0,\ 0)$ is also one of the x -intercepts, to locate the second x-intercept use the symmetry of the parabola.

Exam tip

Even if an exam question does not ask you for a graph, it is often useful to sketch one anyway, as it may help you answer other parts of the question. You can start with a general parabolic curve, opening upwards or downwards depending on the sign of a, and then add other features as you go along.

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Example 2

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Consider the quadratic function $y = 2x^2 - 6x + 1$.

a) Find the domain and range of the function.

b) State the equation of the axis of symmetry.

c) Find the y-intercept and use your GDC to find the x-intercepts.

d) Hence, sketch the parabola.

	Steps	Explanation
a)	The domain of the quadratic function is the set of real numbers.	
	$\text{Vertex is } \left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$ $-\frac{b}{2a} = -\frac{-6}{2\,(2)} = \frac{6}{4} = \frac{3}{2}$ $f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 1 = -\frac{7}{2}$ The vertex is $\left(\frac{3}{2}, -\frac{7}{2}\right)$	To find the range of the functio first find its vertex. This quadratic function has $a = 2,\ b = -6$ and $c = 1$.
	The range of the function is $\left\{y \middle\vert\ y \geq -\frac{7}{2}\right\}$.	The vertex is $\left(\frac{3}{2}, -\frac{7}{2}\right)$, and since $a > 0$ the parabola is concave up. Hence, the vertex the minimum point.
b)	The axis of symmetry has equation $x = \frac{3}{2}$.	The axis of symmetry is the vertical line that passes througl the vertex, which has x-coordinate $\frac{3}{2}$.
c)	The y-intercept is $(0,\ 1)$.	The y-intercept is determined k the parameter c.

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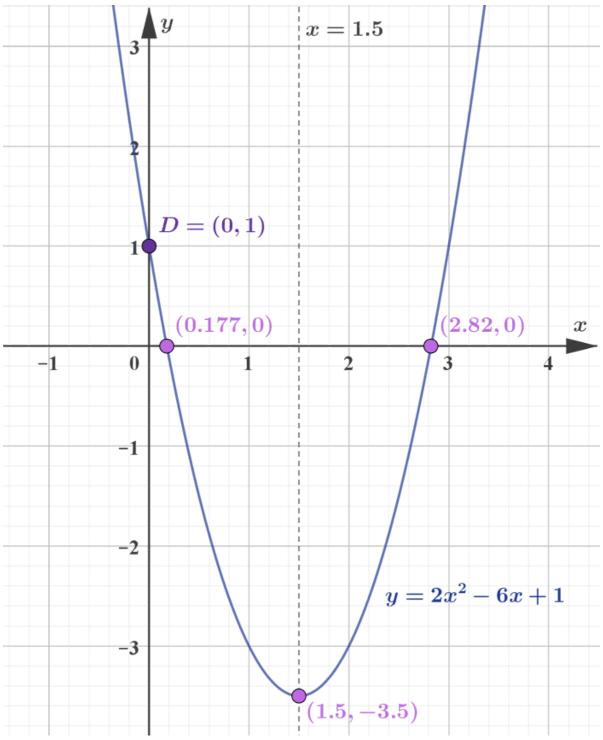
	Steps	Explanation
	The x-intercepts are the points $(0.177,\ 0)$ and $(2.82,\ 0)$.	Use your GDC and round to three significant figures.
d)	The graph of the parabola with all relevant featu res is shown below. 	Show all the features found in parts (a)—(c) on your graph.

4 section questions

2. Functions / 2.6 Quadratic functions

Vertex form of quadratic functions

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In this section and the next, you will learn about different forms of representing quadratic functions and how to transform a quadratic function from one form to another.

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The vertex form of a quadratic function f is

$$f(x) = a(x - h)^2 + k$$

The applet below allows you to visualise parabolas when their functions are expressed in vertex form.

Activity

Move around the red point labelled 'Change parabola' to adjust the position and shape of the parabola, and observe the formula displayed for the corresponding quadratic function. What do you notice? Based on your observations, can you formulate a rule about the vertex form of a quadratic function? Why do you think this is called the vertex form?

Interactive 1. Understanding Quadratic Functions in Vertex Form.

More information for interactive 1

This interactive tool allows users to explore quadratic functions in vertex form by manipulating a parabola and analyzing its equation.
In the interactive, users can adjust the parabola by moving the red point labeled "Change parabola," which updates the vertex (h, k) and the coefficient (a) in real-time. For example, the

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displayed equation ($y = 0.11(x - 4)^2 + 1.79$) shows a parabola with its vertex at (4, 1.79). By experimenting with different positions, users observe how changes in (h) and (k) shift the vertex horizontally and vertically, while modifying (a) alters the parabola's steepness or width.
This hands-on approach helps users deduce that the vertex form explicitly reveals the parabola's vertex, making it easy to identify transformations. The interactive reinforces why this is called the "vertex form" — because the vertex (h, k) is directly visible in the equation, allowing for intuitive graphing and analysis of quadratic functions. Users can test their understanding by predicting the vertex from the equation and verifying it by moving the parabola's vertex point.

Important

A parabola representing a quadratic function in vertex form has equation

$$y = a(x - h)^2 + k$$

- The **vertex** of the parabola is the point (h, k).
- If $a > 0$ the parabola is concave up, and if $a < 0$ the parabola is concave down.
- The **axis of symmetry** has equation $x = h$.
- The y **-intercept** is $(0,\ ah^2 + k)$.

For example, the quadratic function

$$y = -3(x - 2)^2 - 1$$

has vertex $(2, -1)$. Because $a = -3 < 0$, the parabola is concave down and so the vertex is the maximum turning point. The axis of symmetry passes through the vertex and has equation $x = 2$. The y-intercept has y-coordinate $-3 \times 2^2 - 1 = -13$. These features are shown in the figure below.

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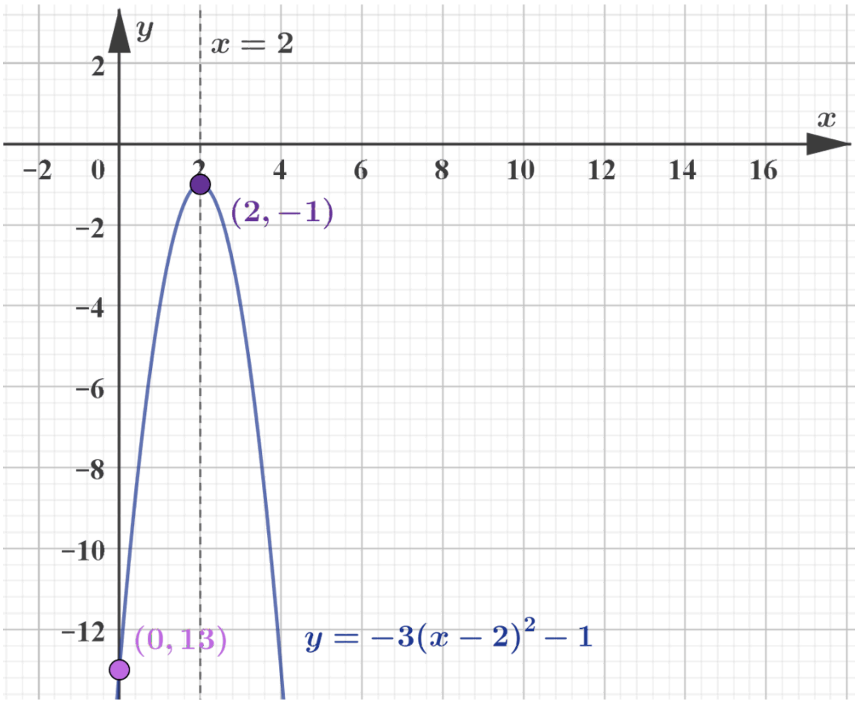

More information

The image is a graph showing a downward-opening parabola. The parabola is defined by the equation (y = -3(x - 2)^2 - 1). The X-axis spans from 0 to 16, marked at intervals of 2, and the Y-axis spans from -13 to 0, marked at intervals of 1. The vertex of the parabola is at (2, -1), which is also the maximum turning point, given that (a = -3 < 0). A purple point marks the vertex. The axis of symmetry is vertical and passes through (x = 2). The Y-intercept is noted at the point ((0, -13)), also marked with a purple point. The overall trend of the graph shows the parabola reaching its maximum point at the vertex before curving downward on both sides.

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Example 1

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Use the vertex, axis of symmetry and y-intercept to sketch the graph of

$$y = -\frac{1}{2}(x+1)^2 - 2.$$

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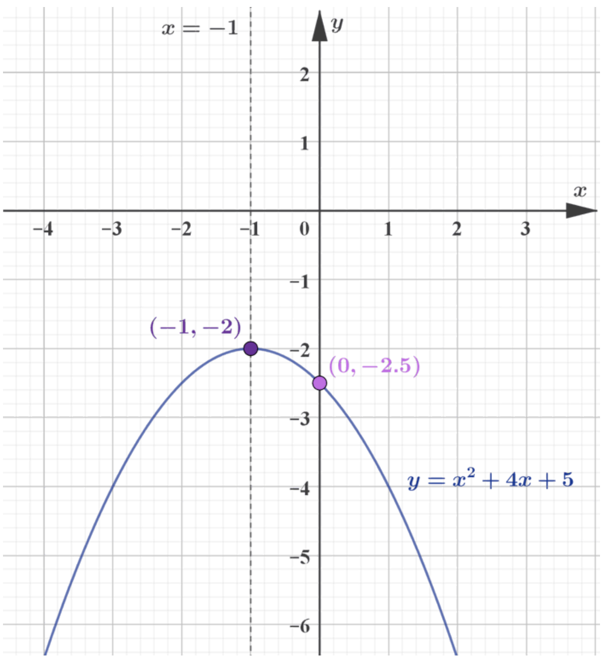
Steps	Explanation
The vertex is $(-1, -2)$. The axis of symmetry is $x = -1$. $a < 0$ so the parabola is concave down and the vertex is the maximum turning point.	The parameters are $a = -\frac{1}{2}$, $h = -1$ and $k = -2$. Be careful with the sign of h : the square term can be written a $(x - (-1))^2$.
For the y -intercept, when $x = 0$, $$y = -\frac{1}{2}(0+1)^2 - 2 = -\frac{5}{2}$$ So the y -intercept is $\left(0, -\frac{5}{2}\right)$.	You can also use the formula $ah^2 + k$ to find the y-intercept.
The graph of the function is as follows. 	Show the vertex, axis of symmetry and y-intercept clearly on your graph.

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Example 2

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A parabola has vertex $(2,\ 3)$ and passes through the point $(-2,\ -1)$.

a) Find the equation of the quadratic function in vertex form.

b) Express the quadratic function in standard form.

c) Find the y-intercept of the parabola using the standard form.

d) Hence, sketch the parabola.

	Steps	Explanation
a)	$y = a(x-2)^2 + 3$	Substitute the coordinates of the vertex into the vertex form of a quadratic function
	$\begin{aligned} -1 &= a(-2-2)^2 + 3 \\ -1 &= 16a + 3 \\ 16a &= -4 \\ a &= -\frac{4}{16} \\ a &= -\frac{1}{4} \end{aligned}$	Substitute the coordinates $(-2,\ -1)$ into the equation. Then sol for a.
	Therefore, the quadratic function has vertex form $y = -\frac{1}{4}(x-2)^2 + 3$.	
b)	$\begin{aligned} y &= -\frac{1}{4}(x-2)^2 + 3 \\ &= -\frac{1}{4}\left(x^2 - 4x + 4\right) + 3 \\ &= -\frac{1}{4}x^2 + x + 2 \end{aligned}$	Transform the vert form to standard form by expanding the brackets and simplifying.
c)	The y-intercept is $(0,\ 2)$.	In the standard for $c = 2$.

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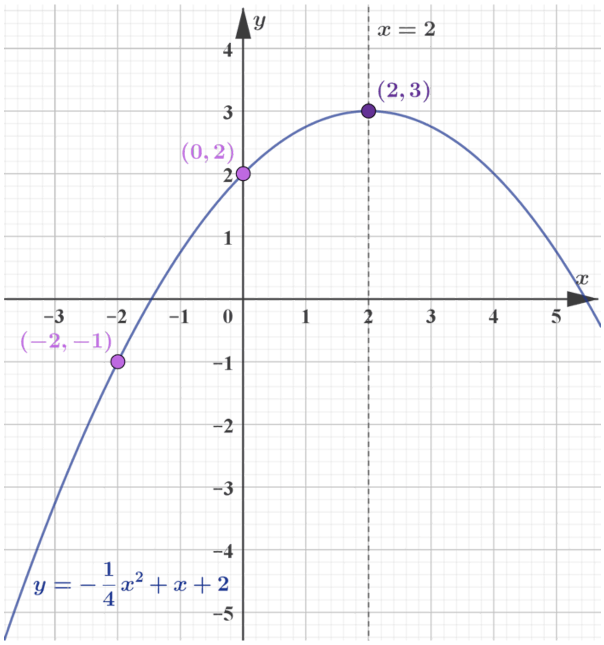
	Steps	Explanation
d)	The graph of the function is shown below. 	Show the vertex, y intercept and poin $(-2,\ -1)$ on your graph.

Completing the square

In **Example 2**, you saw that to transform a quadratic function from vertex form to standard form, you just expand the brackets and simplify. In the opposite direction, to transform from standard form to vertex form, one approach is **completing the square**.

Completing the square is not the only way to convert from standard form to vertex form. Before reading further, skip down to Example 3 and Example 4 and think about how you would solve those questions.

International Mindedness

The method of completing the square was described extensively by the great mathematician al-Khwarizmi (ca. 780—850 CE) in his book *The Compendious Book on Calculation by Completion and Balancing.* Al-Khwarizmi was a Persian mathematician and astronomer who lived in Baghdad and is considered to be the

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father of algebra. His name, formerly Latinized as *Algorithmi* , is the origin of the term 'algorithm'. Some of al-Khwarizmi's work was based on Babylonian, Indian and Greek mathematics.

The method of completing the square gets its name from the fact that the vertex form of a quadratic function contains a perfect square.

Activity

The following applet demonstrates a geometric interpretation of completing the square, as described by al-Khwarizmi. Al-Khwarizmi showed how a quadratic expression of the form $x^2 + bx$ can be transformed into a perfect square, by using a geometrical approach.

Interactive 2. Geometric Interpretation of Competing a Square.

More information for interactive 2

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This interactive tool visually demonstrates the ancient geometric method of completing the square, as developed by the Persian mathematician al-Khwarizmi. The process begins with representing the quadratic expression $x^2 + bx$ as two distinct shapes: a square with sides of length x (representing the x^2 term) and a rectangle with dimensions $b \times x$ (representing the bx term). User can modify the numerical in b by typing in the box provided.
The key insight comes when we split the bx rectangle into two equal smaller rectangles, each measuring $(\frac{b}{2}) * x$. These two rectangles are then arranged adjacent to two sides of the original x^2 square. This arrangement reveals a missing corner - precisely a smaller square with sides of length $(\frac{b}{2})$. By adding this missing square, we complete a larger perfect square with sides measuring $(x + \frac{b}{2})$. However, since we've added this new area $(\frac{b}{2})^2$ to complete the shape, we must subtract it again to maintain equality with our original expression. This elegant geometric transformation gives us the algebraic identity $x^2 + bx = (x + \frac{b}{2})^2 - (\frac{b}{2})^2$.
For example, consider the expression $x^2 + 8x$. Following this method: we split the 8x rectangle into two 4x rectangles, attach them to the x^2 square, and see we need to add a 4×4 square (area 16) to complete the larger square. This gives us the transformed expression$(x + 4)^2 - 16$. The interactive allows users to manipulate these shapes and values, providing a tangible understanding of how the algebraic process of completing the square corresponds to geometric rearrangement, making abstract mathematical concepts concrete and visually intuitive.

In the input box, enter a positive value for coefficient b.

- Drag the slider to visualise al-Khwarizmi's method of completing the square.
- Try different values of b and find the value of c that should be added to $x^2 + bx$ to make a perfect square.
- Generalise your observation by formulating a rule on how to complete the square for $x^2 + bx$.
- Explain al-Khwarizmi's interpretation of x and b in his method.
- What are the steps of the completing the square?

The method described by al-Khwarizmi deals only with quadratic functions $ax^2 + bx + c$ where $a = 1$ and $c = 0$. So how do you complete the square for more general quadratic functions? Let's look at an example: completing the square for the quadratic function $y = 2x^2 - 4x + 7$.

The first step is to factor out any coefficient of x^2 (namely, a) that is not 1. The second step is to add and subtract the square of half the coefficient of x (namely, b) within the brackets.

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Steps	Explanation
$y = 2x^2 - 4x + 7$ $= 2\left(x^2 - 2x\right) + 7$	Write function in standard form and factor 2 out of any terms containing x.
$= 2\left(x^2 - 2x + 1 - 1\right) + 7$	Add and subtract $\left(-\frac{2}{2}\right)^2 = 1$ within the brackets.

Next, regroup the terms so that the three terms remaining within the brackets form a perfect square. In this case the -1 is moved outside the brackets; note that it needs to be multiplied by the factor 2 in front of the brackets.

Steps	Explanation
$y = 2\left(x^2 - 2x + 1\right) - 2\left(1\right) + 7$	Regroup terms and move -1 outside the brackets, remembering to multiply it by 2.
$= 2\left(x^2 - 2x + 1\right) - 2 + 7$ $= 2(x-1)^2 + 5$	Write the expression in the parentheses as a perfect square and simplify the constant terms.

From the vertex form it is easy to see that the parabola $y = 2(x-1)^2 + 5$ opens upwards as $a = 2 > 0$ and its vertex is $(1,\ 5)$. The axis of symmetry has equation $x = 1$ and the y-intercept is $(0,\ 7)$. These features are shown in the figure below.

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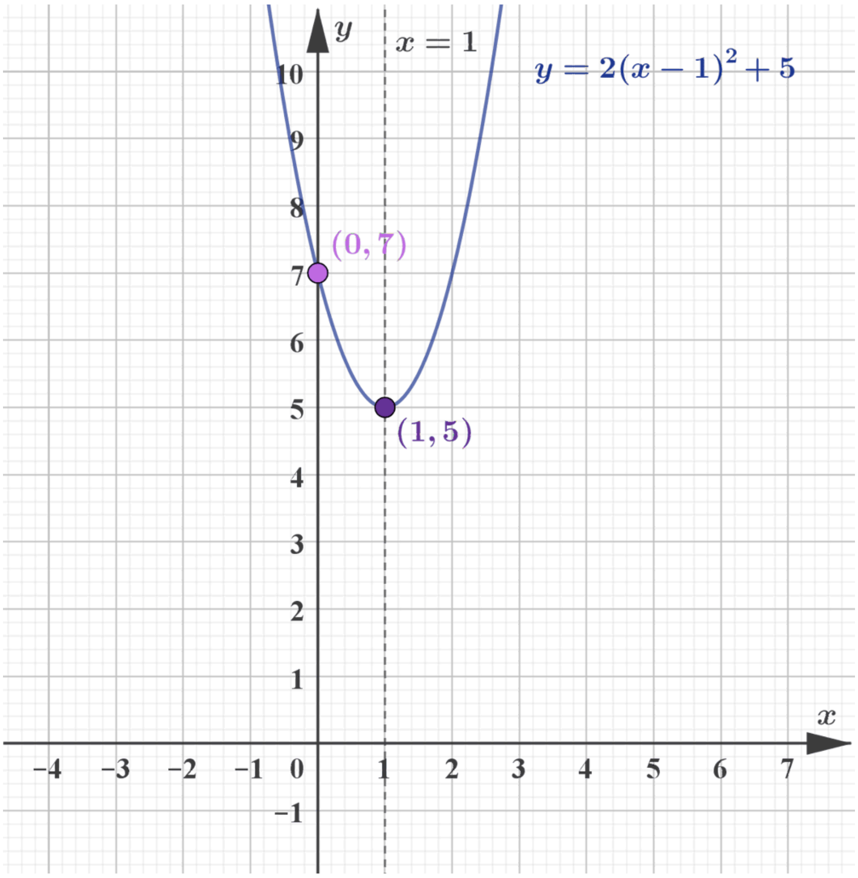

More information

The image is a graph displaying a parabola described by the equation (y = 2(x - 1)^2 + 5). The graph is plotted on a grid with x and y-axes marked. The vertex of the parabola is at the point (1, 5), which is labeled. The parabola opens upwards as indicated by its positive coefficient (2) in the equation. The line of symmetry is shown on the graph as a vertical line passing through x = 1. Another labeled point, the y-intercept, is at (0, 7) on the graph. This illustrates the parabola's height at which it intersects the y-axis. The overall shape suggests the typical characteristics of upward-opening parabolas, and both labeled points are marked with purple dots for clarity.

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Example 3

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Write $y = x^2 + 4x + 5$ in the form $y = (x-h)^2 + k$ and state the coordinates of the vertex.

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Method 1 (completing the square)

Steps	Explanation
$y = x^2 + 4x + 5$	Write the function in standard form. Check whether the coefficient of x^2 is 1. In this case it is, so yc don't need to take out any factor.
$y = x^2 + 4x + 2^2 - 2^2 + 5$	Add and subtract $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2$.
$y = x^2 + 4x + 4 - 4 + 5$	Evaluate the squared constants.
$y = (x+2)^2 + 1$	Write the first three terms as a perfect square, and combine tl remaining constant terms.
$y = (x - (-2))^2 + 1$	Express the perfect square in the vertex form of a quadratic function.
The vertex is $(-2,\ 1)$.	

Method 2 (using the axis of symmetry)

Steps	Explanation
Axis of symmetry: $x = \frac{-4}{2 \times 1} = -2$	The axis of symmetry of $y = ax^2 + bx + c$ is $x = \frac{-b}{2a}$
The first coordinate of the vertex is -2 .	The vertex is on the axis of symmetry.
The second coordinate of the vertex is $y = (-2)^2 + 4(-2) + 5 = 1$	The vertex is on the graph.
The vertex is $(-2,\ 1)$.	
$x^2 + 4x + 5 = a(x - (-2))^2 + 1$	In the vertex form, $a(x-h)^2 + k$, the vertex is (h, k).

Student view

Overview (/study/ap aa-hl/sid-134-cid-761926/o

Steps	Explanation
$a=1$ The vertex form is: $y=(x+2)^2+1$	The main coefficient is the same on both sides.

Example 4

★★★

Write $f\left(x\right)=4x^2-2x+1$ in vertex form and state the coordinates of the vertex.

Method 1 (using the axis of symmetry)

Steps	Explanation
Axis of symmetry: $x=\frac{-(-2)}{2\times 4}=\frac{1}{4}$	The axis of symmetry of $y=ax^2+bx+c$ is $x=\frac{-b}{2a}$
The first coordinate of the vertex is $\frac{1}{4}$.	The vertex is on the axis of symmetry.
The second coordinate of the vertex is $y=4\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)+1=\frac{3}{4}$	The vertex is on the graph.
The vertex is $\left(\frac{1}{4},\ \frac{3}{4}\right)$.	
$4x^2-2x+1=a\left(x-\frac{1}{4}\right)^2+\frac{3}{4}$	In the vertex form, $a(x-h)^2+k$, the vertex is (h,k).

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
$a = 4$ The vertex form is: $f(x) = 4\left(x - \frac{1}{4}\right)^2 + \frac{3}{4}$	The main coefficient is the same on both sides.

Method 2 (completing the square)

Steps	Explanation
$f(x) = 4x^2 - 2x + 1$	Write the function in standard form.
$= 4\left(x^2 - \frac{1}{2}x\right) + 1$	Factor 4 out of the x -terms.
$= 4\left(x^2 - \frac{1}{2}x + \left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right)^2\right) + 1$	Add and subtract $\left(-\frac{\frac{1}{2}}{2}\right)^2 = \left(-\frac{1}{4}\right)$ within the brackets.
$= 4\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) + 1$	Evaluate the squared constants.
$= 4\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) - 4\left(\frac{1}{16}\right) + 1$	Move $-\frac{1}{16}$ outside the brackets, multiplying it by 4.
$= 4\left(x - \frac{1}{4}\right)^2 - \frac{1}{4} + 1$	Write the expression in brackets as a perfect square.
$= 4\left(x - \frac{1}{4}\right)^2 + \frac{3}{4}$	Combine the constant terms.
The vertex is $\left(\frac{1}{4}, \frac{3}{4}\right)$.	

Student view

Exam tip

Overview
(/study/ap
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On exams you can use any method to change standard form of a quadratic to vertex form.

- You can complete the square.
- You can use the axis of symmetry to identify the vertex and use this vertex to write the vertex form

Unless the question specifically tells you to use a particular method, choose the one you are more comfortable using.

Making connections

The method of completing the square is also discussed in section 2.7.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/solving-quadratic-equations-by-factorisation-id-27706/) as a method for solving quadratic equations.

4 section questions

2. Functions / 2.6 Quadratic functions

Factorised form of quadratic functions

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/factorised-form-of-quadratic-functions-id-27700/print/) Assign

The factorised form of a quadratic function is

$$f(x) = a(x-p)(x-q)$$

Activity

The following applet allows you to visualise a parabola whose equation is given in the factorised form $y = a(x-p)(x-q)$.

- Use the sliders to adjust the values of a, p and q. What do you notice about the resulting parabola?
- What is the effect of the parameter a on the graph?
- What do the parameters p and q represent on the graph?

Student view

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- Can you formulate a rule involving a, p and q about the y-intercept of the parabola?
- Can you formulate a rule involving a, p and q about the axis of symmetry or the vertex of the parabola?

Interactive 1. Visualizing a Parabola in the form of a Factorized Equation.

More information for interactive 1

This interactive allows users to visualize a parabola represented in its factored form, $y = a(x - p)(x - q)$, by adjusting the parameters a, p and q using sliders ranging from −5 to 5. The parameter a controls the parabola's direction and width: positive values open it upwards, negative values open it downwards, and larger absolute values make it narrower. The parameters p and q represent the x-intercepts (zeros) of the parabola, where the graph crosses the x-axis at $(p, 0)$ and $(q, 0)$. By manipulating the sliders, users can observe how changes in a, p and q affect the parabola's shape, orientation, and zeros, providing adeeper understanding of the relationship between the factored form of a quadratic function and its graphical representation.

Example: The slider for a, p, and q are set to 0.4, 0.7, and 2.8.

This results in the equation: $y = 0.4(x - 0.7)(x - 2.8)$

The parabola's x-intercepts are at $(0.7, 0)$ and $(2.8, 0)$.

Important

A parabola representing a quadratic function in factorised form has equation

$$y = a\,(x - p)\,(x - q)$$

Student view

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- The x **-intercepts** of the parabola are $(p, 0)$ and $(q, 0)$.
- The y **-intercept** of the parabola is $(0, apq)$.
- The **axis of symmetry** has equation $x = \dfrac{p+q}{2}$.
- The **vertex** of the parabola is $\left(\dfrac{p+q}{2}, f\left(\dfrac{p+q}{2}\right)\right)$,
  where $f\left(\dfrac{p+q}{2}\right) = -\dfrac{a(p-q)^2}{4}$

Example 1

★★☆

For the graph of the quadratic function $f(x) = 2(x+1)(x-2)$, find the x-intercepts, y-intercept and coordinates of the vertex. Hence, sketch the graph.

Steps	Explanation
The x-intercepts are $(-1,\ 0)$ and $(2,\ 0)$.	The function can be expressed as $f(x) = 2(x-(-1))(x-2)$, so $p=-1$ and $q=2$ (or vice versa).
$2(-1)(2) = -4$ The y -intercept is $(0, -4)$.	The y-coordinate of the y-intercept is $apq = 2(-1)(2)$.
$\dfrac{p+q}{2} = \dfrac{-1+2}{2} = \dfrac{1}{2}$ $f\left(\dfrac{p+q}{2}\right) = f\left(\dfrac{1}{2}\right) = 2\left(\dfrac{1}{2}+1\right)\left(\dfrac{1}{2}-2\right)$ $= 2\left(\dfrac{3}{2}\right)\left(-\dfrac{3}{2}\right) = -\dfrac{9}{2} = -4.5$ Thus, the vertex is $\left(\dfrac{1}{2}, -\dfrac{9}{2}\right)$.	The x-coordinate of the vertex is $\dfrac{p+}{2}$, and the y-coordinate is $f\left(\dfrac{p+q}{2}\right) = -\dfrac{a(p-q)^2}{4}$

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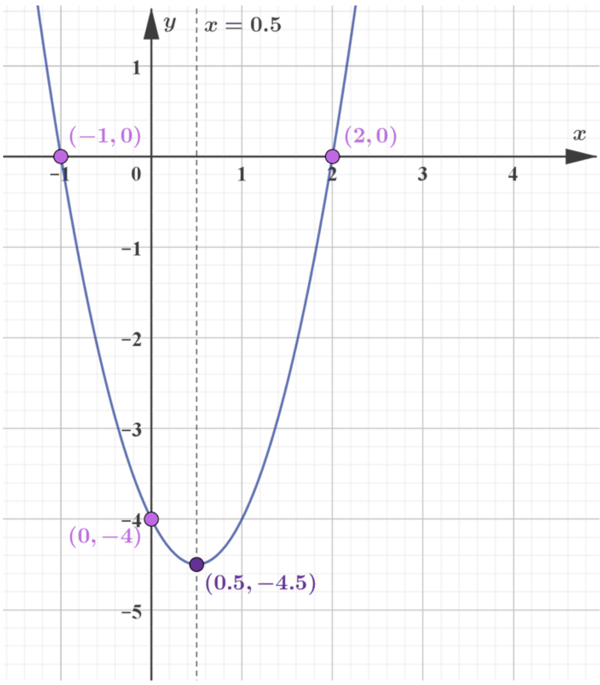
Steps	Explanation
	The axis of symmetry has equation $x = \frac{1}{2}$. Use all the information you have founc to sketch the parabola.

To transform a quadratic function from factorised form to standard form, just multiply out the brackets.

The video clip below shows how to transform a quadratic function from standard form to factorised form.

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Video 1. The Factorized Form of Quadratics: A Key to Solving Equations.

More information for video 1

1

00:00:00,267 --> 00:00:03,733

narrator: In this video we're gonna look

at factorization of quadratic functions

2

00:00:04,133 --> 00:00:05,633

and the method is trial by error.

3

00:00:05,733 --> 00:00:07,733

So it really only works

if the numbers are nice,

4

00:00:07,800 --> 00:00:10,533

so don't spend too much time

if the numbers don't seem to work out.

5

00:00:10,900 --> 00:00:12,633

Let's look at a few examples.

Student
view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

6-7

00:00:12,900 --> 00:00:19,433

The first one is $y = x^2 - x - 6$

8

00:00:19,600 --> 00:00:23,233

So let's start by putting the factors

in and then let's look at the 8a is 1.

9

00:00:23,300 --> 00:00:25,567

So I'm gonna have

an one x and then one x there.

10

00:00:26,033 --> 00:00:28,233

And the last one is minus six.

11

00:00:28,300 --> 00:00:32,467

So nice numbers integers

that multiply out to six is two and three,

12

00:00:32,533 --> 00:00:34,967

but then when you worry

about what signs they are,

13

00:00:35,033 --> 00:00:39,700

so it's either plus 2 and minus 3

or minus 2 and plus 3.

14

00:00:39,800 --> 00:00:42,967

But then we realized

that the middle term is minus x,

15

00:00:43,033 --> 00:00:44,367

so I need a minus 3

16

00:00:44,433 --> 00:00:47,500

and a plus 2 to give me the fully

factorized form

17

Student view

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00:00:47,567 --> 00:00:51,167

of this quadratic $x + 2$

times $x - 3$.

18

00:00:51,400 --> 00:00:52,600

Now this is useful of course

19

00:00:52,667 --> 00:00:54,967

because now we can sketch

the graph straight away.

20

00:00:55,033 --> 00:00:58,333

We have x intercepts

at minus 2 and plus 3,

21

00:00:58,400 --> 00:01:02,167

and of course we also have the y intercept

from the original function minus 6.

22

00:01:02,800 --> 00:01:04,067

Let's look at another example.

23

00:01:04,133 --> 00:01:08,800

$y = 2x^2 + 11x + 5$.

24

00:01:08,867 --> 00:01:10,400

So we see the $2x^2$ there.

25

00:01:10,500 --> 00:01:12,967

So the two factors, one's gonna be $2x$,

26

00:01:13,033 --> 00:01:15,133

the other one's gonna be x

because there better be integers

27

00:01:15,300 --> 00:01:17,733

and then the other numbers

need to multiply out to 5.

Student view

28

00:01:17,833 --> 00:01:20,967

So it's either one times

five or five times one.

29

00:01:21,033 --> 00:01:25,800

But then the middle term is 11,

so 2 times 5 plus 1 times 1 gives me 11.

30

00:01:25,900 --> 00:01:29,033

So the factors are $2x + 1$ and $x + 5$.

31

00:01:29,100 --> 00:01:31,033

And again, this is very useful

to draw the graphs.

32

00:01:31,100 --> 00:01:33,633

Let's quickly sketch it y, x axis

33

00:01:34,100 --> 00:01:35,833

and then we have a concave up again

34

00:01:36,167 --> 00:01:39,333

'cause a is positive, the intercepts

are minus 5 and minus a half

35

00:01:39,400 --> 00:01:40,767

and the y intercept is 5.

36

00:01:40,900 --> 00:01:42,167

Now remember it's a sketch only,

37

00:01:42,233 --> 00:01:45,033

but you do want to indicate

the points of interest.

38

00:01:45,100 --> 00:01:50,233

One last example, $y = -5x^2 + 6x - 1$.

Overview
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39

00:01:50,300 --> 00:01:51,567

Now I don't like the minus signs.

40

00:01:51,633 --> 00:01:55,233

I wanna take it out leaving me

with $5x^2 - 6x + 1$

41

00:01:55,300 --> 00:01:56,267

to worry about.

42

00:01:56,333 --> 00:01:59,033

So I'm gonna write the factors $5x$ and x.

43

00:01:59,200 --> 00:02:01,400

Now the other two factors

I have to multiply to plus 1,

44

00:02:01,467 --> 00:02:03,733

so it's plus 1 or minus 1, both of them.

45

00:02:04,067 --> 00:02:06,533

But I have a minus $6x$ in the middle,

46

00:02:06,600 --> 00:02:10,400

so it becomes $5x - 1$ into $x - 5$

47

00:02:10,467 --> 00:02:13,100

all multiplied

by minus 1 that's out in front.

48

00:02:13,400 --> 00:02:15,367

Of course we can now easily sketch it,

49

00:02:16,033 --> 00:02:18,133

and it's an upside down.

50

00:02:18,400 --> 00:02:22,667

Student view

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A concave down quadratic

for a while y intercept minus 1,

51

00:02:23,000 --> 00:02:26,333

roots are fifth

and 1 and that is factorization.

Example 2

★★☆

A parabola has x-intercepts $(1,\ 0)$ and $(4,\ 0)$ and passes through point $(6, -12)$. Find the equation of the quadratic function and express it in standard form.

Steps	Explanation
$y = a\,(x-1)\,(x-4)$	You are given the x -intercepts, so first write the function in factorised form with $p=1$ and $q=4$ (o vice versa).
	To find the value of a, use the fact that the point $(6, -12)$ lies on the graph.
$-12 = a\,(6-1)\,(6-4)$ $-12 = a\,(5)\,(2)$ $10a = -12$ $a = -\dfrac{12}{10}$ $a = -\dfrac{6}{5}$	Substitute the x- and y-coordinates of the given point into the equation. Then solve for a.
The equation of the quadratic function in factorised form is $$y = -\frac{6}{5}\,(x-1)\,(x-4)$$	

Student
view

Steps	Explanation
$$y = -\frac{6}{5}\left(x^2 - 5x + 4\right)$$ $$= -\frac{6}{5}x^2 + 6x - \frac{24}{5}$$ So in standard form the quadratic function is $$y = -\frac{6}{5}x^2 + 6x - \frac{24}{5}$$	Expand the brackets to get the quadratic in standa form.

Making connections

Now that you've seen some different ways of representing quadratic functions, reflect on the types of information that each form of representation gives you about features of the function and its graph. Do you think that one form would be more useful than the others when modelling specific real-life situations with quadratic functions?

3 section questions

2. Functions / 2.6 Quadratic functions

Modelling with quadratic functions: change and motion

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/modelling-with-quadratic-functions-id-27701/print/) Assign

Applications of quadratic functions

Various real-life situations, such as a rocket launch or the design of a product, can be modelled with quadratic functions. In this section, you will use what you learned in the previous sections and your GDC to graph quadratic functions and answer questions about quadratic models.

Overview
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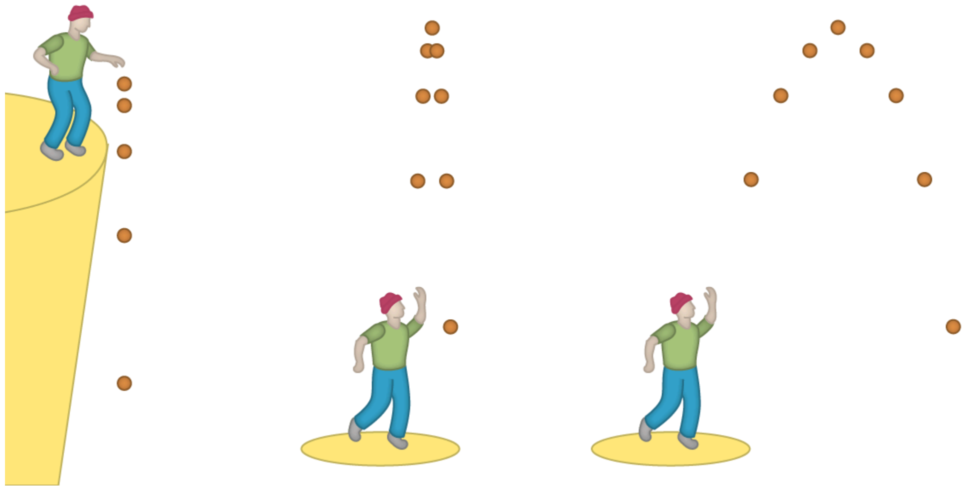
Projectile motion

A projectile is an object that is launched up in the air and falls under the influence of gravity. If an object is thrown straight up or dropped down from a certain height, a quadratic function can be used to describe the height of the object as a function of time. Factors such as the height from which the object was launched, the initial velocity of the object and the force of gravity are incorporated into the quadratic model.



More information

The image is an illustration showing three scenarios of a projectile's trajectory. On the left, a person is standing on a raised platform, dropping a series of balls straight down. In the center, another person stands on the ground throwing balls directly upwards, depicting the vertical motion of the projectile. On the right, a third person is also on the ground, throwing balls in a curved trajectory, illustrating the parabolic path characteristic of a projectile.

Each person's actions represent different paths a projectile can take, influenced by initial velocity and gravity. The balls in the image are shown at consistent intervals to highlight the changes in motion, displaying a visual representation of projectile motion as it travels up and comes back down due to gravitational forces.

[Generated by AI]

For example, suppose that the height h, in metres above the ground, of a projectile at time t seconds after its launch is given by the quadratic function $h(t) = 2.25 + 6.55t - 1.75t^2$.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Consider the following questions:

- From what height was the projectile launched?

This is the initial height of the projectile at time $t = 0$. It is given by $h(0) = 2.25\text{m}$.

- What is the maximum height reached by the projectile?

Using a GDC, you can find the y-coordinate of the maximum point (vertex). It is 8.38 correct to three significant figures. Hence, the maximum height reached by the projectile is 8.38m.

- At what time will the projectile be at its initial height again?

Using a GDC, find the x value of the intersection point between the parabola $y = 2.25 + 6.55x - 1.75x^2$ and the line $y = 2.25$. Rejecting the $x = 0$ solution (wh ich corresponds to the initial time $t = 0$), we are left with the solution $x = 3.74$. Thus, the projectile reaches its initial height again at 3.74s after launch.

- At what time after launch does the projectile reach ground level?

Using the GDC you can find the positive x-intercept of the parabola $y = 2.25 + 6.55x - 1.75x^2$ to be 4.06. So the projectile reaches ground level at 4.06s.

The graph of $y = 2.25 + 6.55x - 1.75x^2$ and the answers to the above questions are shown below.

Student view

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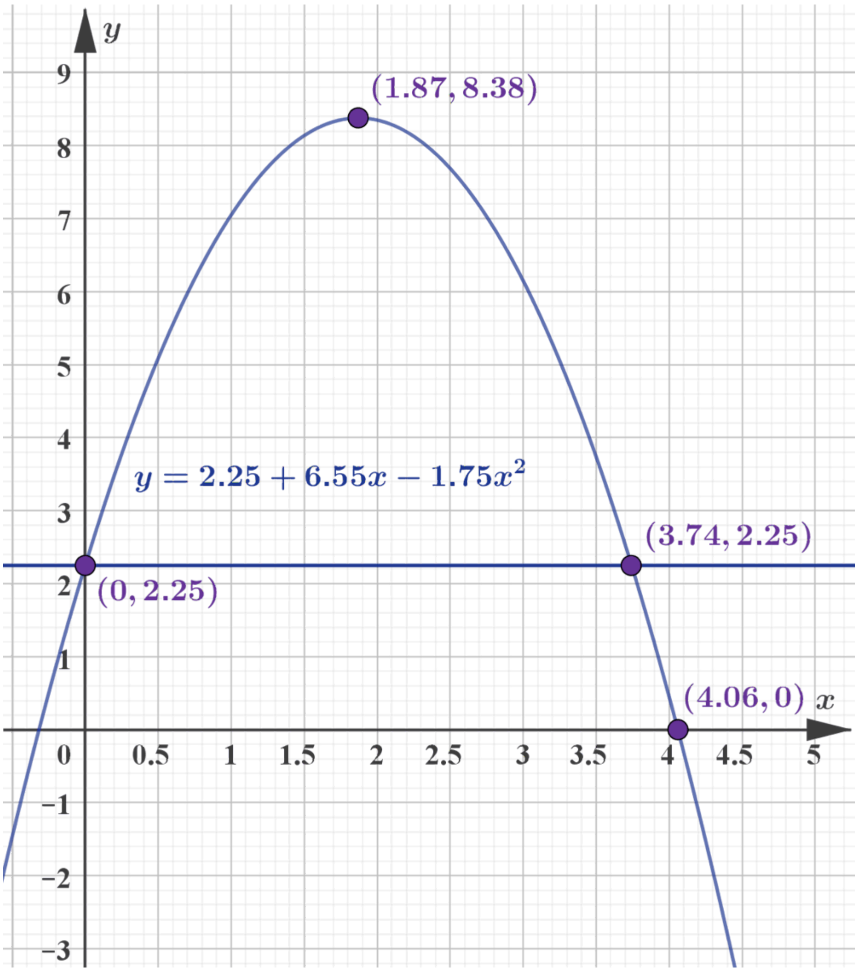

More information

The image depicts a graph of the quadratic function (y = 2.25 + 6.55x - 1.75{x^2}).

The X-axis ranges from 0 to 5, and the Y-axis from 0 to 9. The graph line forms a parabola opening downwards with the following key points labeled:

1. Point at the origin ((0, 2.25)) where the graph intercepts the Y-axis.
2. The vertex of the parabola is at approximately ((1.87, 8.33)), showing the maximum value of the graph.
3. The curve intersects the X-axis at ((0, 2.25)) and ((4.06, 0)).
4. Another notable point is ((3.74, 2.25)) where the curve returns to the same Y-value as the origin.

The curve shows a consistent change in direction, starting with a gradual rise, reaching a peak at the vertex, and then descending to intersect the X-axis again.

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Overview
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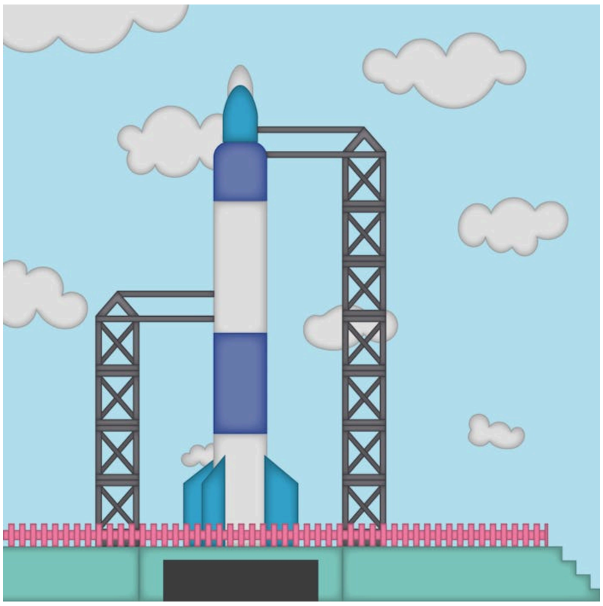
Be aware

The graph of the function might give the impression that the projectile moves in the horizontal direction, but this is not the case as the graph of the function is actually height against time.

Example 1

★★☆

A model rocket is launched straight upward from a platform 10 feet above ground level. The rocket pulls off the platform at an initial velocity of 64 feet per second. The model that describes the height h, in feet above ground level, of the rocket as a function of time t, in seconds after launch, is given by the quadratic function $h\left(t\right) = -16t^2 + 64t + 10$.



Use the graph of the parabola to determine:

a) The maximum height reached by the rocket.

b) The amount of time it took the rocket to reach the maximum height.

c) The amount of time it took the rocket to come back to its initial height again.

d) The total amount of time the rocket was in the air.

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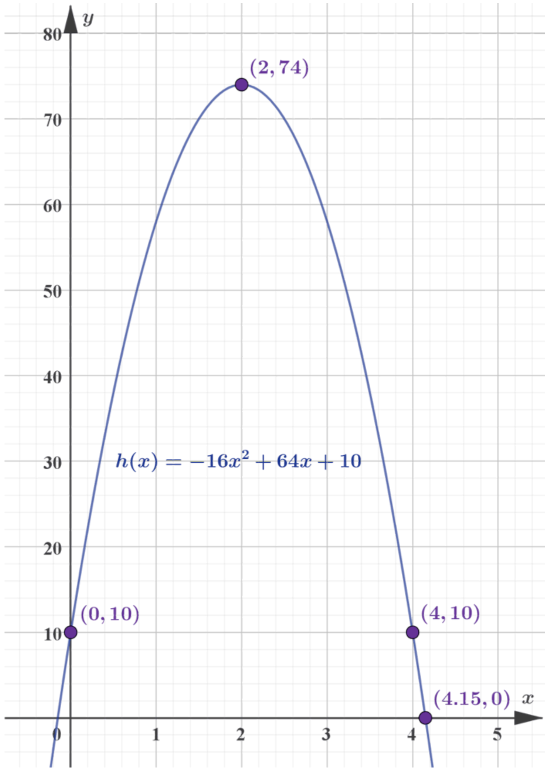
	Steps	Explanation
		Use your GDC to obtain the graph of $y = -16x^2 + 64x + 10$.
a)	The rocket's maximum height is 74 feet above ground level.	The maximum height is the y-coordinate (the vertex (maximum turning point).
b)	The amount of time it took the rocket to reach its maximum height is 2 seconds.	The time taken to reach the maximum is tl x-coordinate of the vertex.
c)	It took 4 seconds to the rocket to reach its initial height again.	This is the positive x-coordinate of the point of intersection between the parabol and the line $y = 10$.
d)	The rocket was in the air for 4.15 seconds.	The time at which the rocket reached the ground is the positive x-intercept of the parabola, and this is the total amount of time that the rocket spent in the air.

Student view

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Example 2

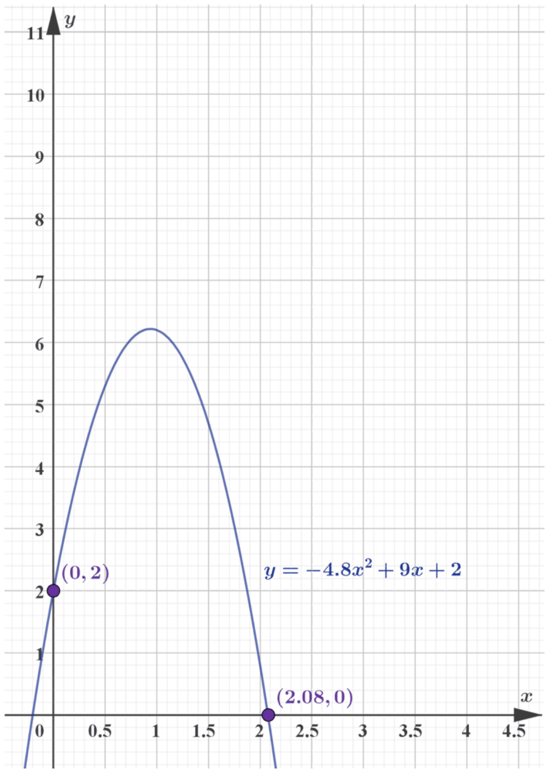
★★☆

A man who is 2 metres tall throws a ball straight up in the air, and the ball's height above the ground at time t seconds is $h = 2 + 9t - 4.8t^2$ m.

a) Find the time (in seconds) when the ball reaches its maximum height.

b) Find the maximum height of the ball.

c) After how much time does the ball hit the ground?

	Steps	Explanation
		Use a GDC to obtain the graph of $y = 2 + 9x - 4.8x^2$.
a)	The ball reaches its maximum height 0.938 s after it was thrown.	The time at which the ball reaches its maximum height is the x-coordinate of th vertex.

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	Steps	Explanation
b)	The maximum height that the ball reached is $y = 6.22\text{ m}$ above the ground.	The maximum height is the y-coordinate of the vertex.
c)	The ball hits the ground after 2.08 s.	The time at which the ball reaches the ground is the positive x-intercept of the parabola.

International Mindedness

Galileo Galilei (1564—1642), who was born in Pisa, Tuscany, was a philosopher and mathematician who a played significant role in the scientific revolution of that period. Galileo experimented with paths of projectiles and attempted to describe falling objects using mathematics.

A portrait of Galileo
Credit: ilbusca Getty Images

Galileo took various measurements of rolling a ball off the surface of a table and marking where it lands according to how fast the ball was going. Carry out research to answer the following questions:

- What experiments did Galileo perform to model projectile motion?

Student
view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

- What did Galileo discover by comparing the motion of a rolling ball that falls off the surface of a table to the motion of a ball that is just dropped vertically from the same height?
- What quadratic models did Galileo form?

Fluid mechanics

Fluids and their motion play a huge role in our lives. The areas of application range from industrial liquid transport, aircraft control systems and transport ship elevation to everyday uses such as hidden underground car parking with hydraulic lifting. The various ways in which fluids flow are influenced by many factors, and quadratic models are often used to model the behaviour of fluids.

Example 3

★★☆

A tank is filled with water. A valve at its base is opened and the tank drains in such a way that the height h, in centimetres, of water remaining in the tank at time t, in hours after the draining began, is given by the function $h(t) = (0.12t - 8.25)^2$.

a) What is the initial water level?

b) What is the water level 10 hours after draining began?

c) When, to the nearest hour, will the tank be empty?

d) When, to the nearest tenth of an hour, will the tank be one-third full?

	Steps	Explanation
a)	T he initial water level correct to three significant figures is $$h(0) = (0.12(0) - 8.25)^2 = 68.1$$ cm.	The initial water level is the height $t = 0$.

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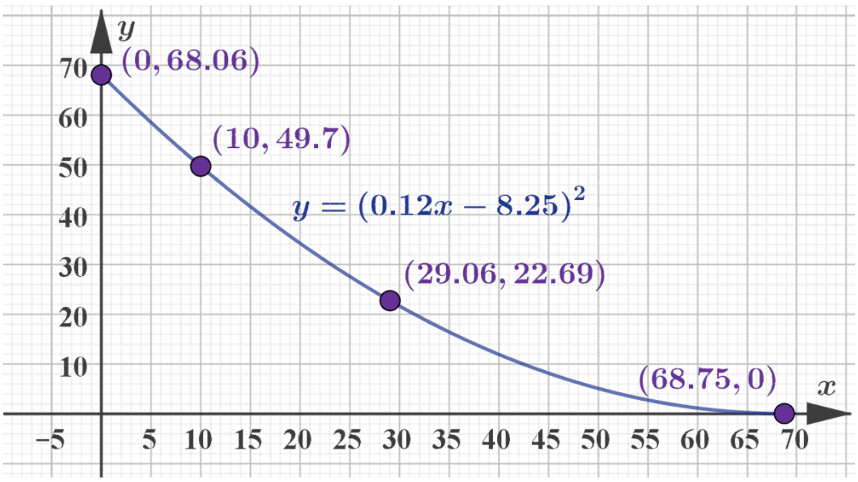
	Steps	Explanation
b)	The water level after 10 hours is $$h\,(10) = (0.12\,(10) - 8.25)^2 = 49.7$$ cm.	Evaluate function $h\,(t)$ at $t = 10$.
c)	To the nearest hour, it takes 69 hours for the tank to drain.	The tank will be empty when the water level, h, is equal to zero. Tha corresponds to an x-intercept of t parabola. Use your GDC to find the x-intercept, which is at $(68.75,\ 0)$.
d)	One-third of the initial height is $\frac{1}{3} \times 8.25^2 = 22.69$. From GDC, to the nearest tenth of an hour, it takes 29.1 hours for the tank to drain to $\frac{1}{3}$ of its initial level.	Use your GDC to find the x-coordinate of the point of intersection between the graphs $y = (0.12x - 8.25)^2$ and $y = 22.($.
	The graph of this parabola and the answers to parts a)—d) are shown below. 	

3 section questions

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Modelling with quadratic functions: optimisation

Maximising or minimising quadratic models

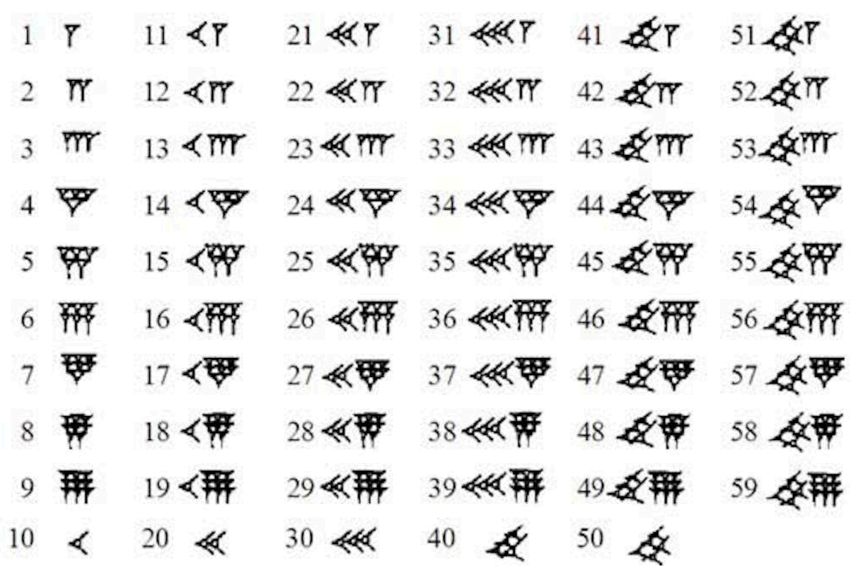
In modelling with quadratic functions, we are often interested in finding the 'optimal' solution (optimisation), which means finding the maximum or minimum point of a parabola.

Area division

Quadratic models have been extensively used in area division problems since ancient times.

International Mindedness

The Sumerians and the Babylonians were early advanced civilisations based in Mesopotamia, a historical region of Western Asia. Sumerian and Babylonian mathematicians used a sexagesimal (base-60) counting system and inscribed notation on clay tablets that were baked in the sun.



Sexagesimal numbers in value notation

Source: (https://commons.wikimedia.org/wiki/File:Babylonian_numerals.jpg

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This image is a grid displaying Babylonian numerals from 1 to 60. The numerals are arranged in rows and columns with each numeral accompanied by its corresponding Arabic number. The numerals have unique cuneiform shapes, showing the sexagesimal (base-60) system used by the Babylonians. For example, numbers 1 to 9 are displayed in a vertical line in the grid, each represented by unique cuneiform symbols; number 10 starts a new row with a different symbol, representing a grouping system. The pattern continues with numerals reflecting combinations to denote higher numbers up to 60. Each row appears to follow a systematic representation pattern, providing insight into the Babylonian numerical notation style.

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The clay tablet shown in the photo below depicts a model for dividing the area of a ‘Temen’ (a large quadrangle that was staked out to approximate the area of a field). The mathematician Otto E. Neugebauer (1899—1990) discovered in the late 1920s that the notation on this clay tablet appears to give solutions to a series of quadratic equations.

Clay tablet showing a juridical field division of the area of a Temen
Credit: Atypeek Getty images

More information

This is a photo of an ancient clay tablet covered with lines of cuneiform script. The tablet appears to be rectangular in shape, with some wear and tear visible on its surface, typical of artifacts from ancient times. The script is made up of wedge-shaped marks and characters arranged in horizontal lines across the face of the tablet. This particular tablet is used to illustrate a legal or mathematical concept, historically related to the field division of a 'Temen', as discovered by mathematician Otto E. Neugebauer in the late 1920s. It is significant because the notation on the

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tablet seems to provide solutions to quadratic equations, demonstrating advanced mathematical understanding and record-keeping in ancient cultures. The background is a muted, dark color, which contrasts with the lighter color of the clay tablet, making the inscribed text stand out more prominently.

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Example 1

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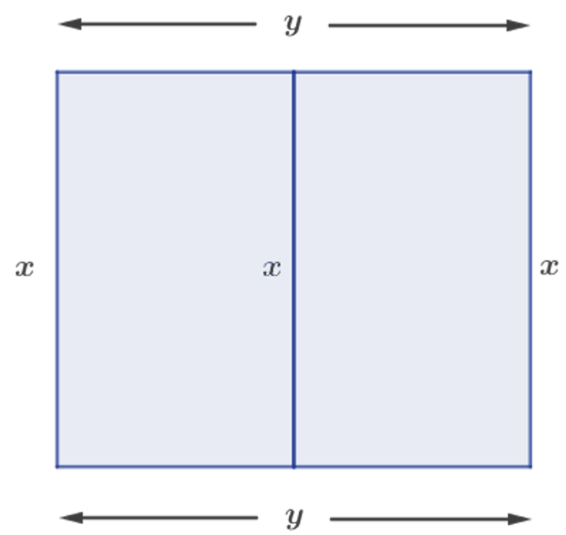
Two rectangular fields are to be enclosed with 100 m of fence, as shown in the following figure



More information

The diagram illustrates two adjoining rectangular fields that are enclosed with a total of 100 meters of fencing. The fields are placed side by side. The width of each field is labeled as 'x,' and the total length of both fields together is labeled as 'y.' The diagram shows these labels clearly, indicating the placement and dimensions relevant for calculating the total length 'y' based on the width 'x.' The labeled arrows pointing to 'x' and 'y' help in understanding how the dimensions relate to each other for the problem scenario described in the text.

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a) Express the total length, y, of the two rectangular fields in terms of the width, x.

b) Create a function that models the total area of the fields in terms of the width x.

c) Determine the value of the width x that results in the maximum area of the two fields.

d) Find the maximum area of the two fields.

	Steps	Explanation
a)	$3x + 2y = 100$	The total length of fence equals 100 m, so you can an equation linking x and
	$2y = 100 - 3x$ $y = \dfrac{100 - 3x}{2}$	Solve the equation for y.
b)	$A = xy$	The total area of the two f is given by $A = \text{length} \times \text{width} = y$
	$A\left(x\right) = x\left(\dfrac{100 - 3x}{2}\right)$	Express the area function terms of x only, by substit the expression for y founc part a).
	$A\left(x\right) = x\left(50 - \dfrac{3}{2}x\right)$ $A\left(x\right) = -\dfrac{3}{2}x^2 + 50x$	Simplify and expand the brackets.

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	Steps	Explanation
c)	$A(x) = -\frac{3}{2}x^2 + 50x$	The area function is a quadratic function. First w in standard form.
	The x-coordinate of the vertex is $$-\frac{b}{2a} = -\frac{50}{2\left(-\frac{3}{2}\right)} = \frac{50}{3}$$ So $x = \frac{50}{3}$ will result in the maximum area.	Because $a = -\frac{3}{2} < 0$, th quadratic function is conc down. Thus, the vertex wil the maximum point.
d)	The maximum area is $$A\left(\frac{50}{3}\right) = \frac{50}{3}\left(\frac{100 - 3\left(\frac{50}{3}\right)}{2}\right)$$ $$= \frac{50}{3}\left(\frac{100-50}{2}\right) = \frac{2500}{6} = \frac{1250}{3}\ \mathrm{m}^2$$	The y-coordinate of the v is the maximum area. You find it easier to use the factorised form of $A(x)$ t evaluate this.

Optimising product design

In subtopic 2.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25390/) you created a model to describe the area of a window satisfying the following design requirements:

- The window has the shape of a rectangle with a semicircle placed on top of it.
- The window should have a perimeter of 30 feet.

The model that describes the area of the window is a quadratic function

$$A = x\left[15 - \frac{(2+\pi)x}{4}\right] + \frac{\pi x^2}{8},$$

where x is the width of the window.

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Example 2

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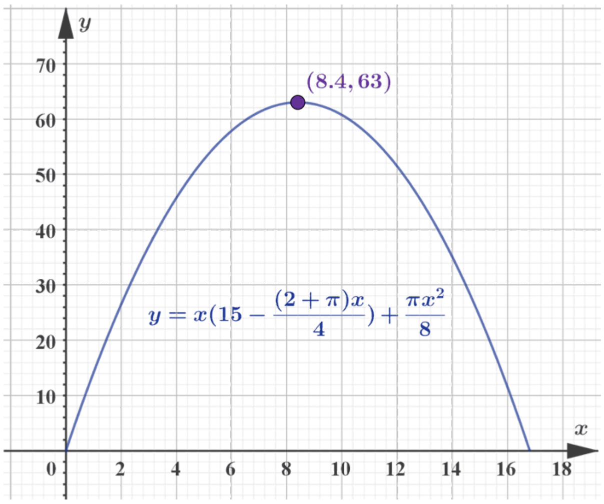
Use the quadratic function $A = x\left[15 - \frac{(2+\pi)\,x}{4}\right] + \frac{\pi x^2}{8}$ to find the width x that gives the maximum area for the window.

Steps	Explanation
	Use a GDC to obtain the graph the function $y = x\left[15 - \frac{(2+\pi)\,x}{4}\right] + \frac{\pi}{}$ The parabola is concave down so the vertex is the maximum point. The maximum area of the winc is 63 square feet and the width 8.4 feet.

3 section questions

2. Functions / 2.6 Quadratic functions

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27703/print/) Assign

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What you should know

By the end of this subtopic you should be able to:

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- find the vertex, axis of symmetry and y-intercept of a quadratic function from its standard form
- find the vertex, axis of symmetry and y-intercept of a quadratic function from its vertex form
- find the vertex, axis of symmetry and y-intercept of a quadratic function from its factorised form
- determine the domain and range of a quadratic function
- use the method of completing the square to transform a quadratic function from standard form to vertex form
- transform quadratic functions from one form to another
- use a GDC to find the x-intercepts of a parabola
- sketch a parabola, showing all relevant features such as the vertex, axis of symmetry, y-intercept and x-intercepts if there are any.

2. Functions / 2.6 Quadratic functions

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-27704/print/) Assign

Important

A parabola can be defined as the locus of all points that are equidistant from a fixed point called the **focus** and a fixed line called the **directrix**.

In the applet below you can visualise two types of parabolas:

- the parabola with equation $y^2 = 4ax$, which has directrix parallel to the y-axis and whose focus is the point $F\left(a, 0\right)$
- the parabola $x^2 = 4ay$, which has directrix parallel to the x-axis and whose focus is $F\left(0, a\right)$.

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Interactive 1. Two Types of Parabolas.

More information for interactive 1

This interactive tool allows users to explore the geometric properties of standard parabolas through hands-on manipulation. Users can select between two fundamental forms: the horizontal parabola $y^2 = 4ax$ with its directrix parallel to the y-axis and focus at $(a, 0)$, or the vertical parabola $x^2 = 4ay$ with directrix parallel to the x-axis and focus at $(0, a)$. By dragging point M along the directrix, users observe the key parabolic property that the distance from any point P on the curve to the focus (PF) always equals its distance to the directrix (PM). The applet dynamically updates all elements as users adjust parameters, showing how changes to the focus position affect the parabola's shape and equation.

For example- When user selects the option $x^2 = 4ay$ the vertical parabola comes. Users can select the focus and move it along the y axis. Focus comes to F(0,3.01) while $y = -3.01$. The line projected from the focus to the parabola gives point P and the point projected on the x-axis gives point M. User can move Point M and observe PM = PF at all given points.

The same can be seen if user selects $y^2 = 4ax$ just the orientation of the parabola changes to the x-axis.

For the following questions, click the button $y^2 = 4ax$.

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1. Drag point M along the directrix and observe the distances PM and PF. What do you notice?
2. Adjust the focus point F and observe the changes happening. What do you notice?

Now consider the general case where a parabola has focus at $(a, 0)$ and the $x = -a$ line is the directrix.

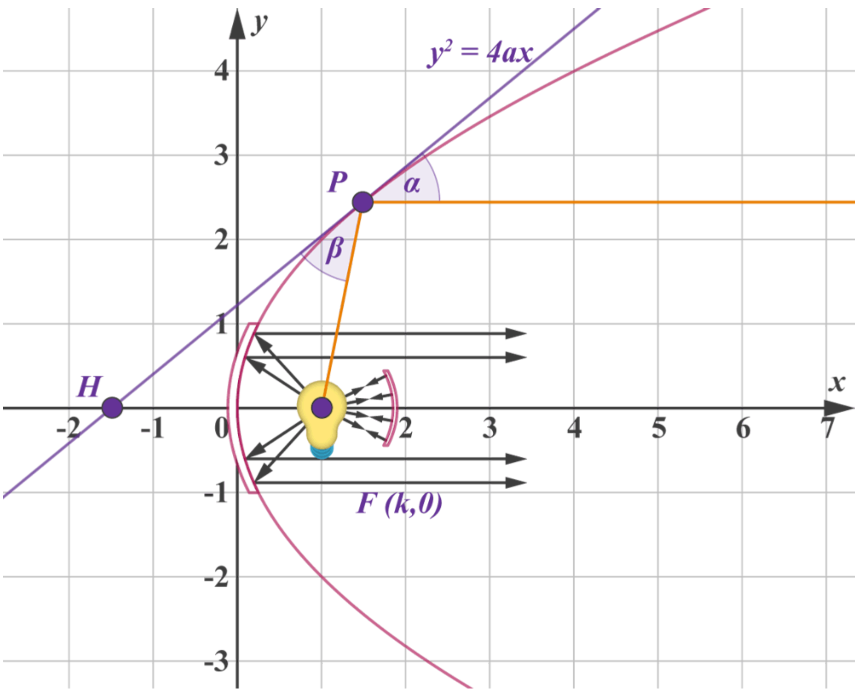
3. Use the definition of the parabola given above to derive the formula $y^2 = 4ax$ for the parabola.

4. Imagine that the curve $y^2 = 4ax$ is a parabolic mirror, as shown in the figure below. According to part 3, the focus is at $F\,(a, 0)$. A ray of light from the source at the focus strikes the parabola at P and is reflected.

a) Explain why the triangle FPH is isosceles.

b) What does this tell you about the relationship between the reflected ray and the x-axis? Note that according to the reflection principle, $\alpha = \beta$.

c) Explain how parabolic mirrors can direct light from a source in a desired direction.



More information

The image is a diagram illustrating the reflection principle on a parabola. The grid shows both x and y axes with labels and increments. The parabola is displayed with the equation $y^2 = 4ax$. There are several marked points: H, P, and F, where F is at (k,0) on the x-axis.

Line segments are drawn from F to P with an orange line, indicating a light path reflecting off the parabola at point P. Point P is marked on the parabola, and rays are drawn reflecting horizontally, towards the right. The diagram also highlights the isosceles triangle FPH formed by these points.

The angles alpha and beta are shown as being equal at the point of reflection, consistent with the reflection principle which states that the angle of incidence equals the angle of reflection. This implies that the reflected ray runs parallel to the x-axis.

The diagram showcases how parabolic mirrors direct incoming rays parallel to the axis of symmetry through the focus, demonstrating the reflective properties of parabolic shapes.

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Rate subtopic 2.6 Quadratic functions

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