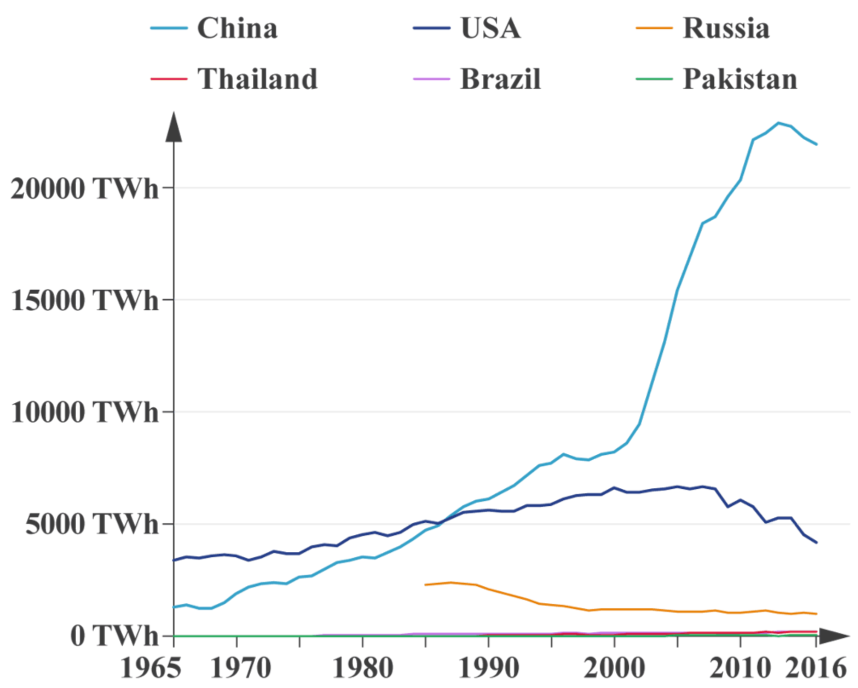
Index

The big picture

Functions presented as expressions are used to model many important phenomena and drawing the graph of a function gives you a better understanding of what the function 'does' and how mathematical models behave.

Of course, everything you see with your eyes on a graph can also been seen by doing calculations but it is hard to do the calculations when you do not know what you are looking for.

The following graph shows coal consumption by country from 1965 to 2016. Can you describe some relevant features of the graph that would inform decision makers and stakeholders in the coal production industry?



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Source: " Fossil fuels (https://ourworldindata.org/fossil-fuels) " by Our World in Data

More information

The line graph depicts coal consumption by country from 1965 to 2016, measured in terawatt-hours (TWh). The X-axis represents years, ranging from 1965 to 2016, while the Y-axis shows coal consumption in TWh, from 0 to 20,000 TWh.

Key trends in the graph: - **China:** Shows a significant increase in coal consumption, especially after 2000, peaking around 2014 at close to 20,000 TWh before a slight decline. - **USA:** Has a steady consumption pattern with a peak around the early 2000s at about 6,000 TWh followed by a gradual decline. - **Russia:** Maintains a consistent level around 1,000 to 2,000 TWh throughout the years, with minor fluctuations. - **Thailand, Brazil, Pakistan:** These countries display minimal coal consumption compared to the others, staying mostly below 1,000 TWh throughout the timeframe.

The graph illustrates significant disparities in coal consumption growth among these countries, with China experiencing dramatic growth, while others show stability or decline.

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In subtopic 2.3 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25396/), you saw how to graph a function and how to determine the axes-intercepts of a graph using a graphic display calculator (GDC). In this section, you will look at features of graphs such as:

- turning points as maxima and minima of a curve
- asymptotic behaviour of functions
- the intersection of two curves.

The diagram below shows some of the graphs that you will encounter in this subtopic. Your GDC is a great asset in the visualisation and analysis of functions.

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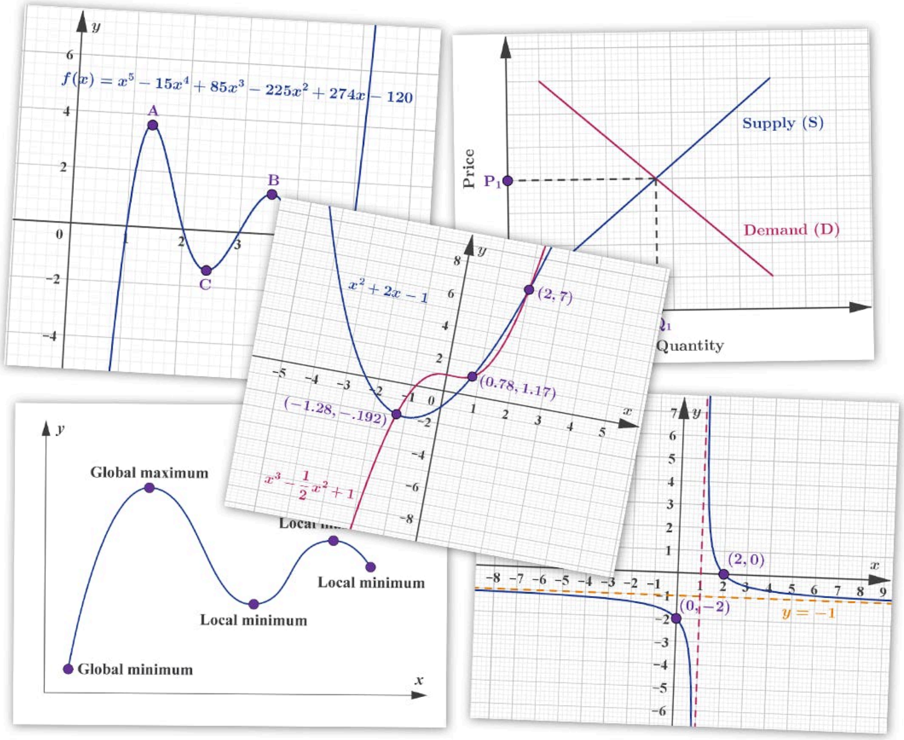

More information

The image displays various mathematical graphs and diagrams on grid-paper backgrounds. Each graph displays distinct functions and economic models, marked with key points and mathematical annotations:

1. **Upper Left Graph**: A polynomial function graph showing a curve labeled as f(x) = x^5 - 15x^4 + 85x^3 - 225x^2 + 274x - 120, with points A, B, and C identified, marked on the x and y coordinate plane.

2. **Upper Right Graph**: An economic diagram illustrating supply and demand curves on a grid, labeled 'Supply (S)' and 'Demand (D)' intersecting to mark point P1 with price and quantity axes.

3. **Central Graph**: A plot with the equation x^2 + 2x - 1 and another x^3 / 2 + x^2 + 1, showing intersections and notable points at (2,7) and negative numbers.

4. **Lower Left Graph**: A curve showing global and local extrema labeled as 'Global maximum', 'Local minimum', reflecting undulating behavior on the x, y grid.

5. **Lower Right Graph**: A graph showing an asymptote with points at (0,-2) and (2,0), and a horizontal line y=-1 suggesting behavior near horizontal asymptotes.

Each graph depicts different aspects of mathematical concepts through various line curvatures and annotations, illustrating significant data and trends.

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Concept

Functions are used to model real-life phenomena, and visualising the graph of a function allows us to describe the behaviour of mathematical models. Learning how to use a GDC to will help you in determining relevant features of graphs such as:

- axes-intercepts
- maximum or minimum points
- asymptotes of curves
- points of intersection between graphs.

Consider how you might tackle the above without the aid of a GDC to help you to sketch the graph of a function.

2. Functions / 2.4 Key features of graphs

Turning points

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Describing the graph of a function

Increasing and decreasing functions

Important

A function is increasing when the y value increases as the x value increases. The graph of an increasing function goes up as you move in the positive direction of the x -axis (from left to right).

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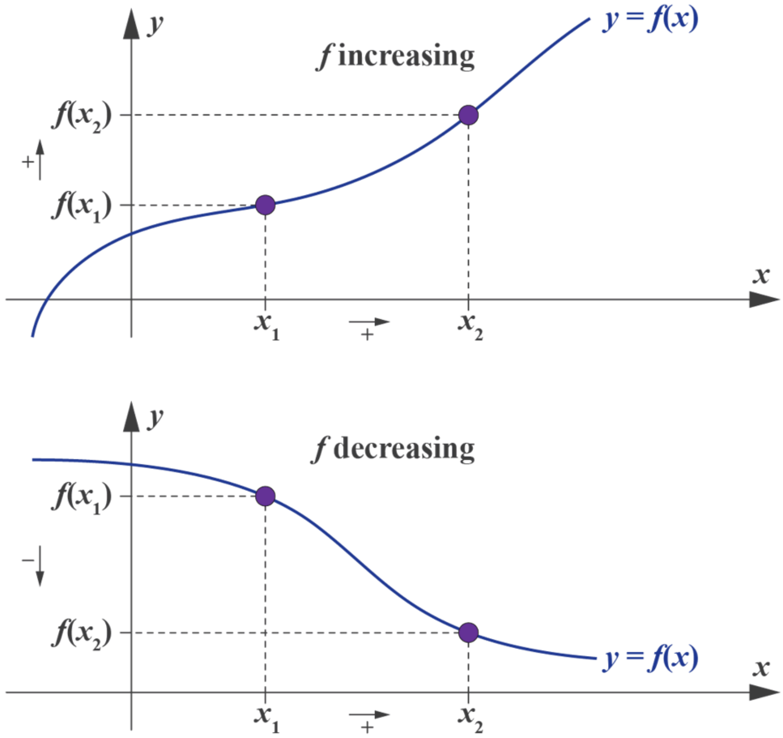
A function is decreasing when the y value decreases as the x value increases. The graph of the decreasing function goes down as you move in the positive direction of the x -axis (from left to right).



More information

The image contains two graphs showing the behavior of a function, f(x), at two different sections of its domain, demonstrating increasing and decreasing trends.

In the first graph, labeled 'f increasing', a curve shows a rising trend from left to right. The x-axis is labeled with 'x_1' and 'x_2', indicating points where x_1 < x_2. The y-axis corresponds to f(x), marked with 'f(x_1)' and 'f(x_2)', where f(x_1) < f(x_2). The curve passes through two points on the graph depicting that as x moves from x_1 to x_2, y = f(x) increases.

In the second graph, labeled 'f decreasing', the curve shows a falling trend from left to right. Similarly, the x-axis is marked with 'x_1' and 'x_2', but this time f(x_1) > f(x_2) as indicated by their positions on the y-axis. The curve passes through corresponding points demonstrating that as x moves from x_1 to x_2, y = f(x) decreases.

Both graphs have arrows on the y and x axes to indicate the directions of increase and decrease for y and x, respectively. There is a dashed line from each point on the curve to the respective axes for clarity.

Student view

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Minimum and maximum

When modelling with functions, you are often interested in points where the function takes maximum or minimum values. The points are called extrema. In your study, you will need to be able to distinguish between two types of extrema:

- Global minimum and global maximum points.
- Local minimum and local maximum (turning) points.

Important

A global minimum is the point where the function takes its smallest value on its entire domain.

A global maximum is the point where the function takes its largest value on its entire domain.

Important

A local maximum of a function is a point (x, y) on the graph of the function whose y- coordinate is larger than all other y -coordinates on the graph at points ‘close to’ (x, y) .

Similarly, a point (x, y) is local minimum if it has locally the smallest y -coordinate.

Interactive 1. Extrema of Functions.

More information for interactive 1

This interactive allows users to explore the concepts of global maximum and global minimum by moving a red point along a graph. As the red point is adjusted, users can observe how the values of the global maximum and global minimum change. Users can adjust the function's domain by dragging points along the x-axis to observe how changes affect both local and global extrema.

By checking the "Local Extrema" box, users can view local maximum and minimum points marked with green bordered circles on the graph. Similarly, checking the "Global Extrema" box displays the global maximum as a purple triangle and the global minimum as a red triangle, with their exact values (e.g., $\text{Global Maximum} = 5.3$ $\text{Global Minimum} = 1.92$) shown for reference.

This hands-on approach helps clarify the distinction between local and global extrema while reinforcing their relationship to the function's domain.

Be aware

The local maximum and local minimum points are called turning points, as the function changes direction at these points.

Student view

Given that a function has a maximum turning point, what information can you deduce regarding the shape of the function before and after the turning point? What about the shape of a function before and after a minimum turning point?

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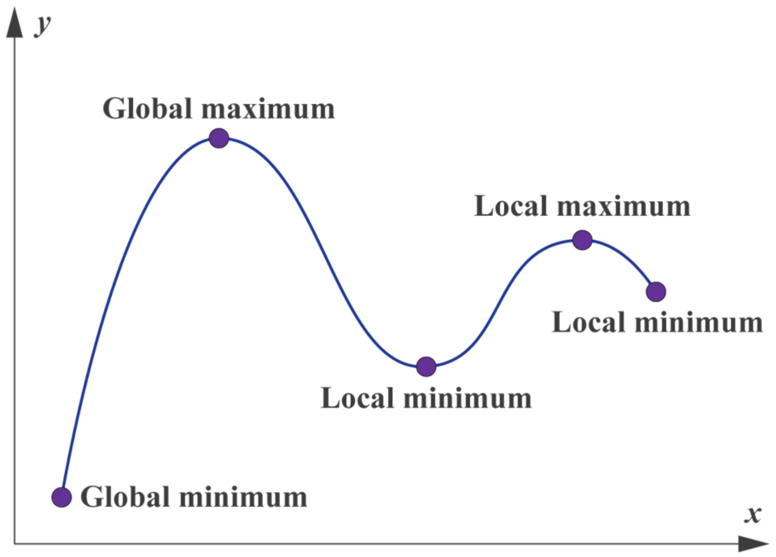

More information

The image is a graph showing the curve of a function with several key points labeled. The x-axis and y-axis represent variables typically designated as independent and dependent, respectively.

- The curve starts at a point designated as the 'Global minimum' and rises towards the 'Global maximum'.
- After reaching the global maximum, the curve descends to a 'Local minimum' before rising again to a 'Local maximum'.
- The curve then descends once more to another 'Local minimum'.

This graph specifically illustrates the concept of local and global peaks and troughs in a function, with inflection points clearly marked and labeled.

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A function is increasing on the left of a local maximum and it is decreasing on the right of the local maximum. A function is decreasing on the left of a local minimum and it becomes increasing on the right of the local minimum.

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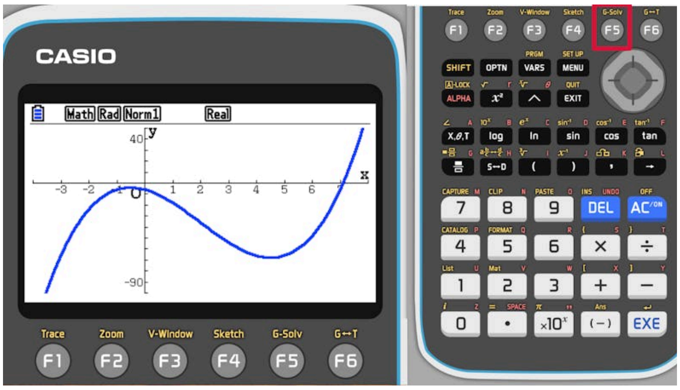
For example, consider the function

$$f(x) = x^3 - 6x^2 - 7x - 6$$

and use a GDC to determine the turning points, as shown below.

Steps	Explanation
These instructions assume that you have a graph of $y = x^3 - 6x^2 - 7x - 6$ on the screen displayed in the window $-4 \leq x \leq 8$ and $-100 \leq y \leq 50$. Press F5 (G-Solv) to bring up the options to analyse a graph.	

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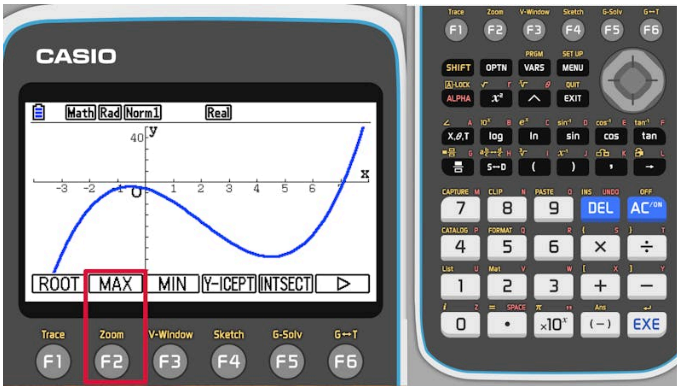
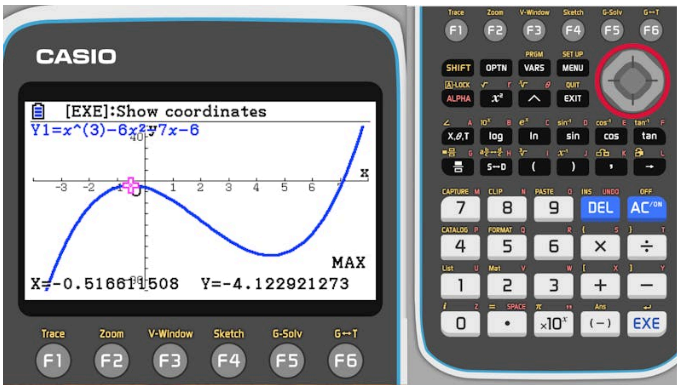
Steps	Explanation
Press F2 (MAX) to find the local maximum point. To find local minimum points, you need to press F3 at this point.	
The cursor is moved to a local maximum point and the coordinates are displayed. This graph only has one local maximum point, but in case more than one local maximum points are visible in the view, you can move between them.	

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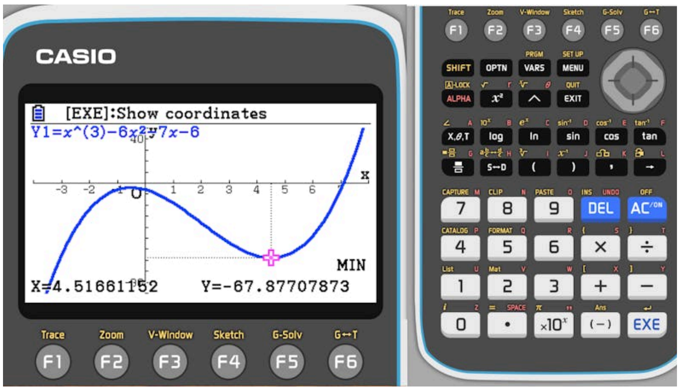
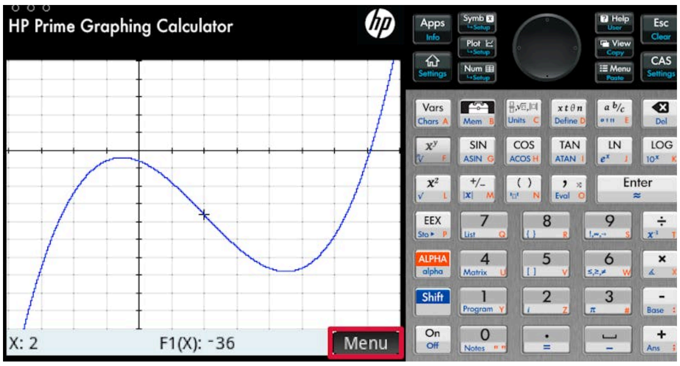
Steps	Explanation
You can find the local minimum point(s) in a similar way.	

Steps	Explanation
These instructions assume that you have a graph of $y = x^3 - 6x^2 - 7x - 6$ on the screen displayed in the window $-4 \leq x \leq 8$ and $-100 \leq y \leq 50$. Choose the menu.	

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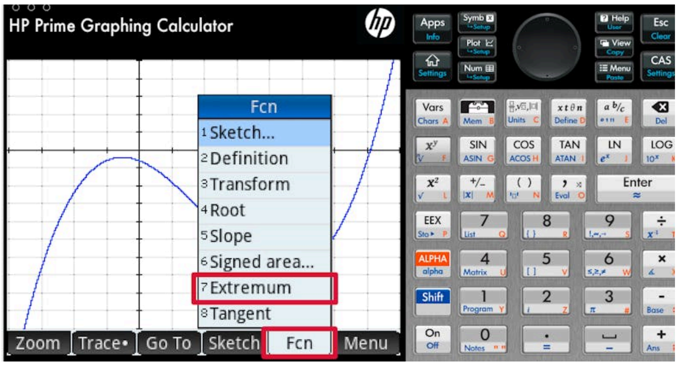
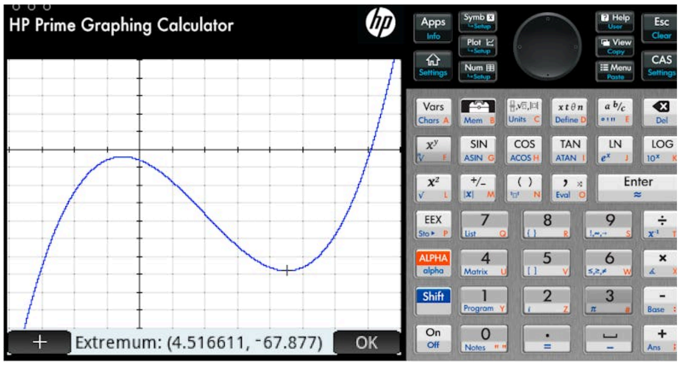
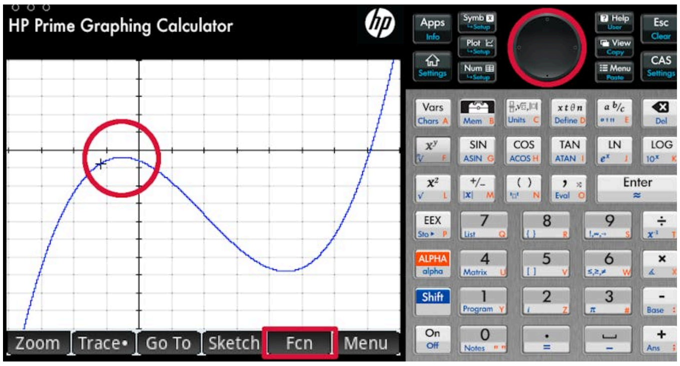
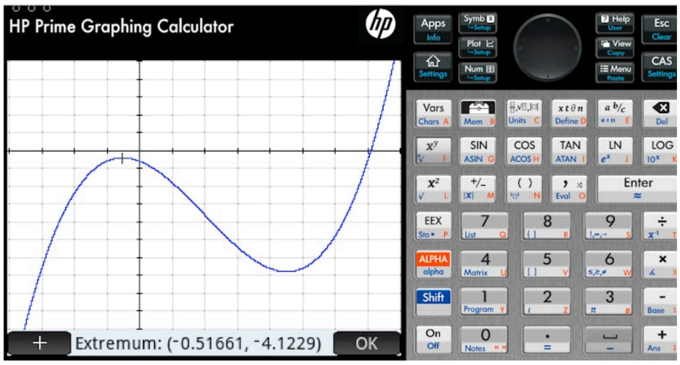
Steps	Explanation
In the function submenu choose the extremum option. This calculator finds both kind of turning points (maximum and minimum) at the same time.	
The cursor is moved to a turning point and the coordinates are displayed. In this case the cursor is at the minimum point. The calculator will find the turning point closest to the position of the cursor where you started the process.	

Student view

Steps	Explanation
To find the local maximum point, move the cursor close to it and start the process again.	
	

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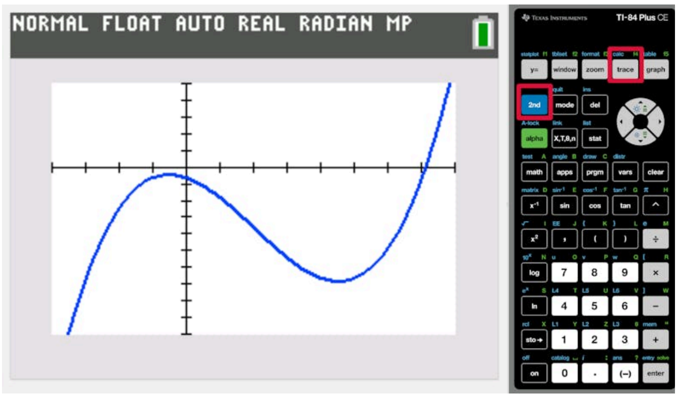
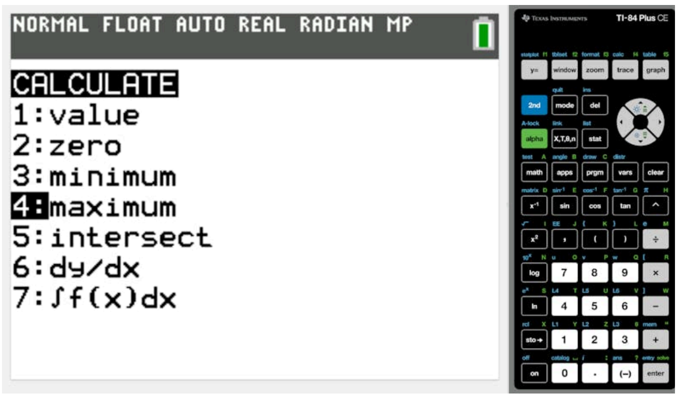
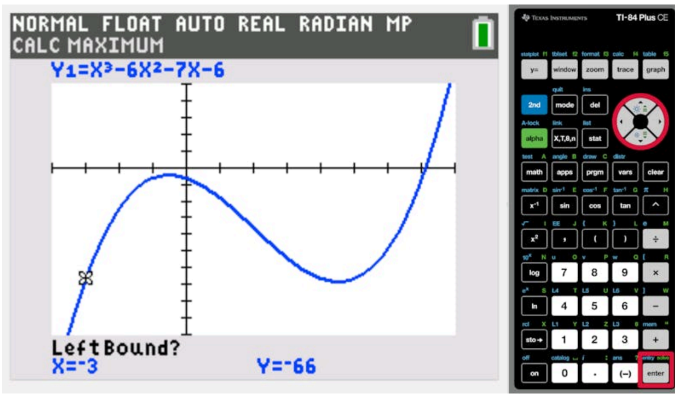
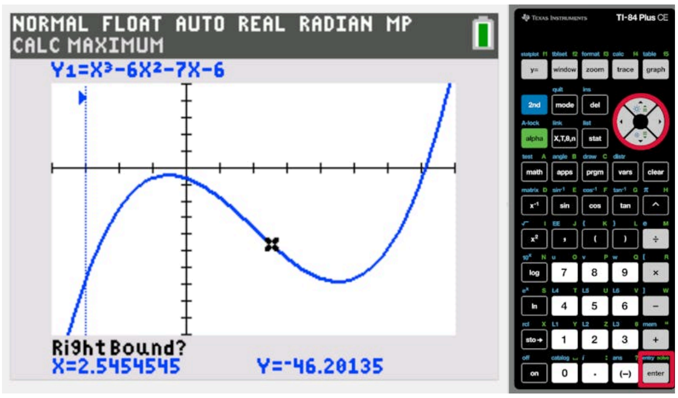
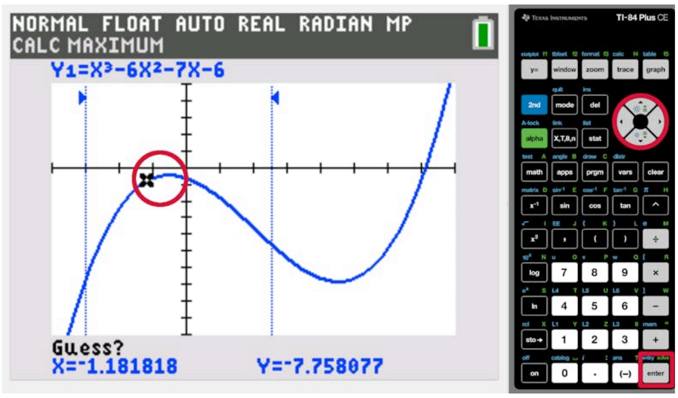
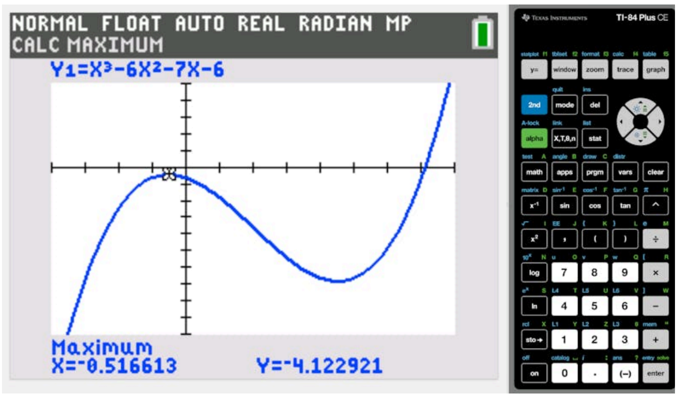
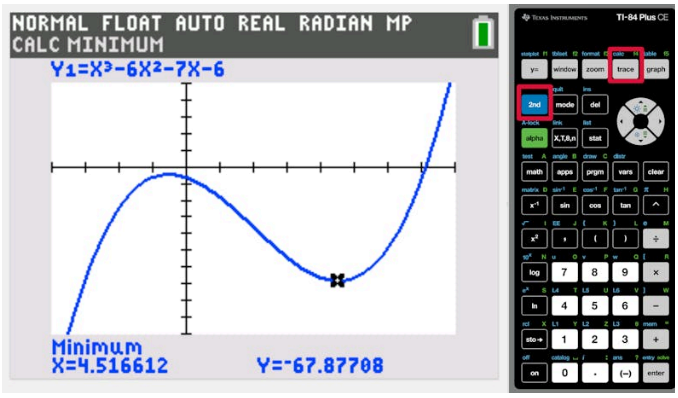
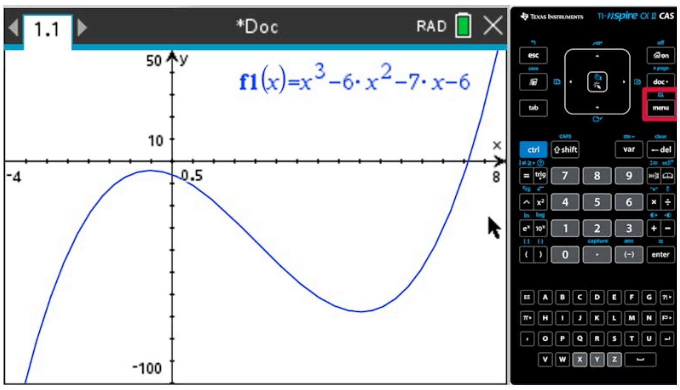
Steps	Explanation
These instructions assume that you have a graph of $y = x^3 - 6x^2 - 7x - 6$ on the screen displayed in the window $-4 \leq x \leq 8$ and $-100 \leq y \leq 50$. Choose ‘calc’ to bring up the options to analyse a graph.	
Choose the option (4) to find the local maximum point. To find local minimum points, you need to choose (3) at this point.	

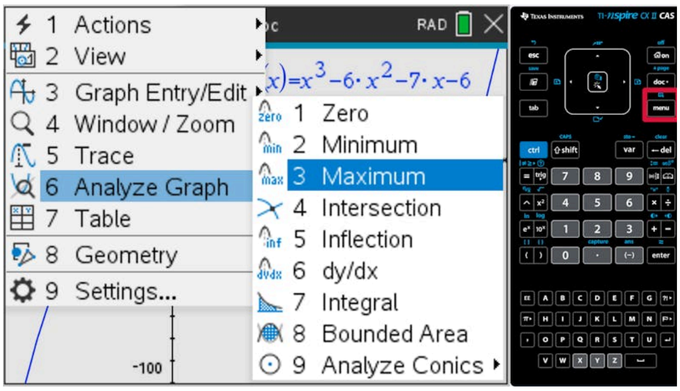
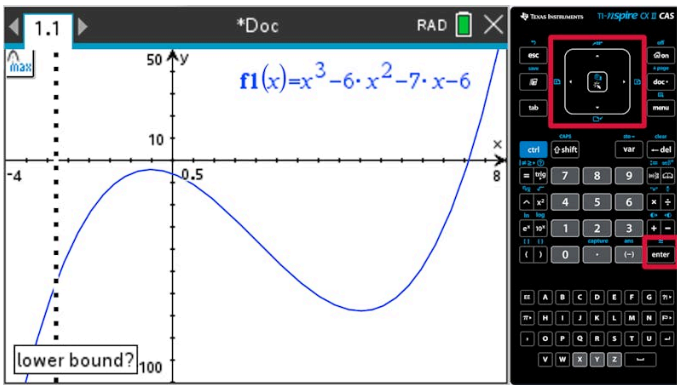
Student
view

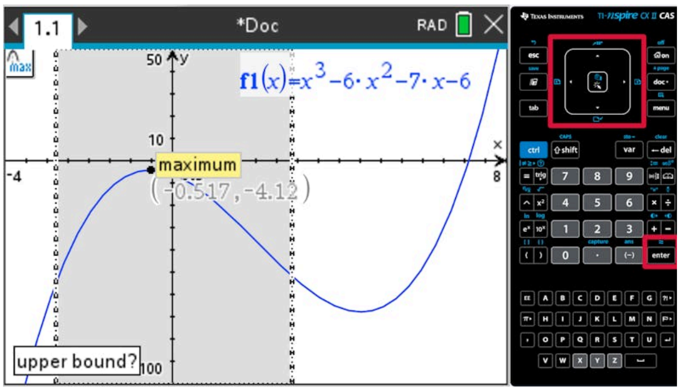
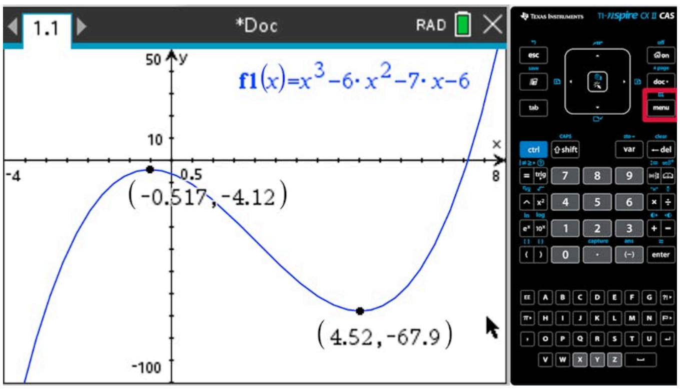
Steps	Explanation
The calculator asks for more information. First, you need to move the cursor to the left of the turning point that you are looking for.	
Next, move the cursor to the right of the turning point.	

Steps	Explanation
Move the cursor close to the turning point and press enter. Tha calculator will find the local maximum point closest to the cursor within the bounds you set.	
The cursor is moved to a local maximum point and the coordinates are displayed.	

Steps	Explanation
You can find the local minimum point(s) in a similar way.	

Steps	Explanation
These instructions assume that you have a graph of $y = x^3 - 6x^2 - 7x - 6$ on the screen displayed in the window $-4 \leq x \leq 8$ and $-100 \leq y \leq 50$. Press menu to find the option to analyse a graph.	

Steps	Explanation
Choose the option to find a maximum point. To find local minimum points, you need to choose the appropriate option at this point.	
The calculator asks for more information. First, you need to move the cursor to the left of the turning point that you are looking for.	

Steps	Explanation
Next, move the cursor to the right of the turning point.	
You can find the local minimum point(s) in a similar way.	

The function $f(x) = x^3 - 6x^2 - 7x - 6$ has two turning points: a local maximum at $(-0.517,\ -4.12)$ and a local minimum at $(4.52,\ -67.9)$.

Making connections

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In the topic of Calculus, you will learn how to determine the local and global extrema of a function analytically (without a GDC).

Example 1

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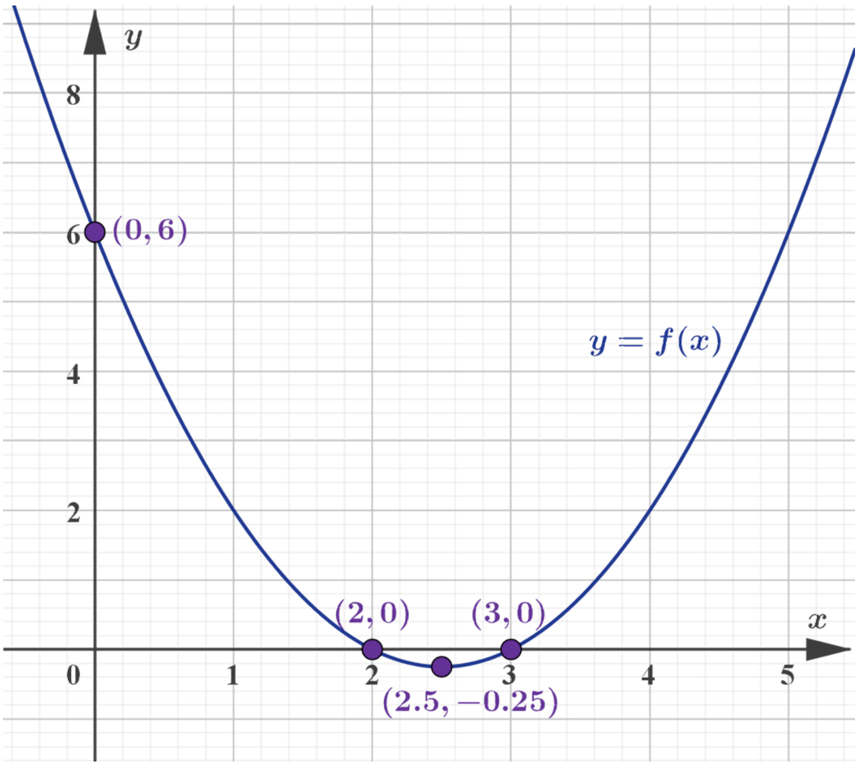
Sketch the graph of the function $f(x) = x^2 - 5x + 6$ showing all relevant features.

Use a GDC to graph the function and show the turning points and axes-intercepts of the graph in your sketch. Your sketch should look like this:



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Example 2

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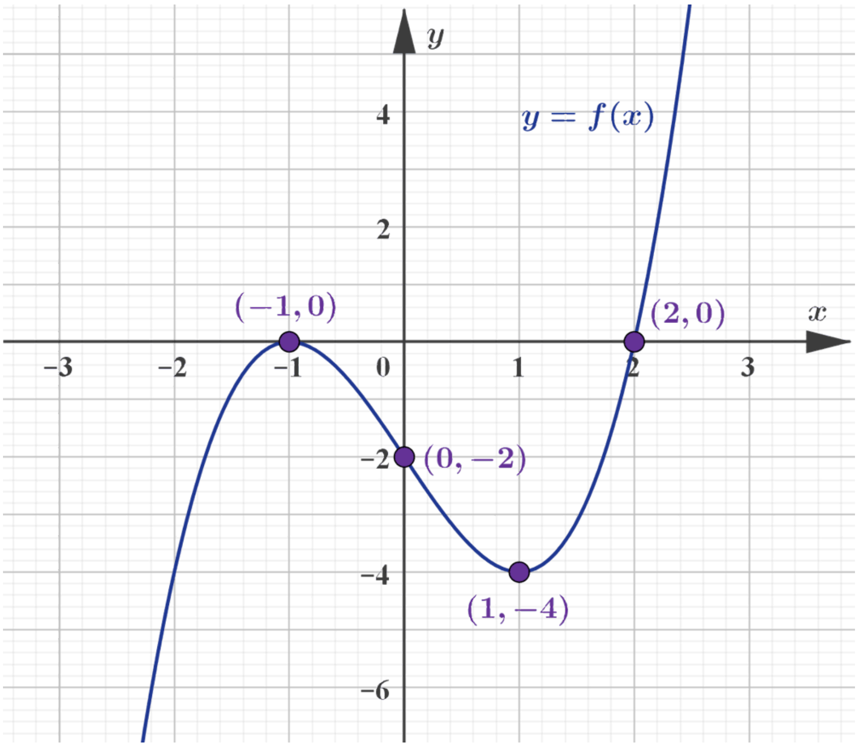
Sketch the graph of the function $f(x) = x^3 - 3x - 2$ showing all relevant features.

Use a GDC to graph the function and show the turning points and axes-intercepts of the graph. Your sketch should look like this:



Example 3

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Sketch the graph of the function $f(x) = x^3 - 6x^2 + 11x - 6$ showing all relevant features.

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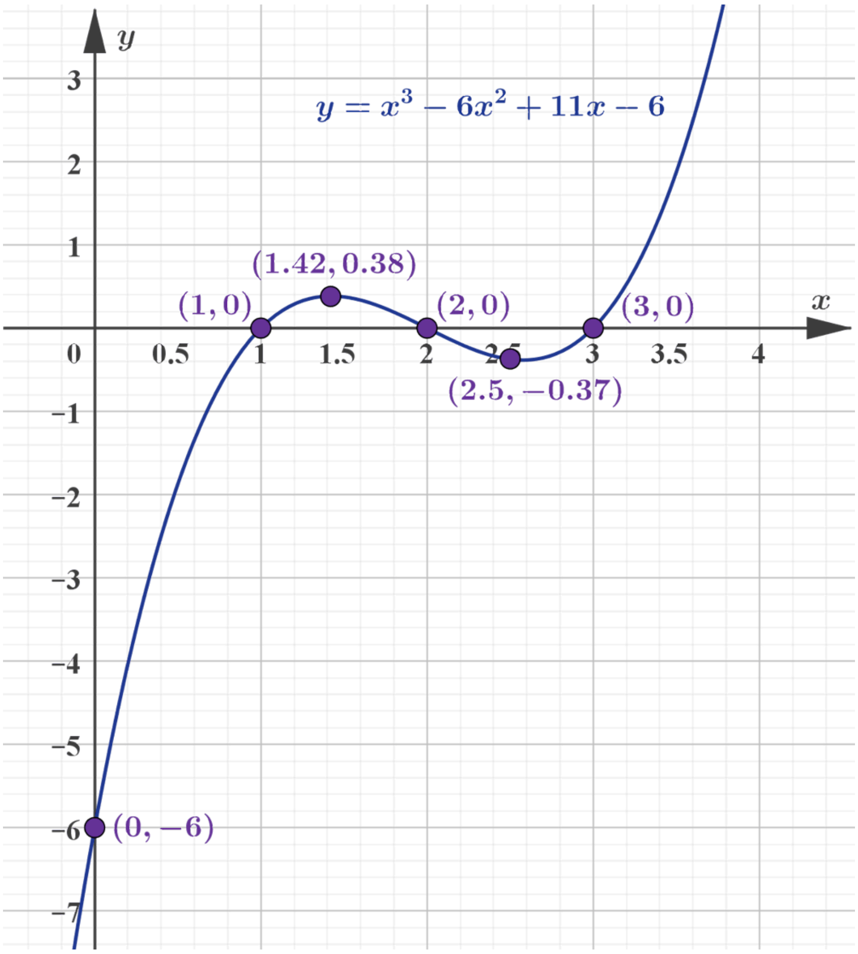
The function $f(x) = x^3 - 6x^2 + 11x - 6$ has two turning points: a local maximum at $(1.42,\ 0.38)$ and a local minimum at $(2.58,\ -0.38)$. The diagram below is the graph of the function $f(x) = x^3 - 6x^2 + 11x - 6$ showing the turning points and the axes-intercepts of the function.



3 section questions

2. Functions / 2.4 Key features of graphs

Student view

Asymptotes

Asymptotes of a graph

An asymptote of a curve is a straight line such that the distance between the graph and the line approaches zero as either the x or y coordinates tends to infinity. In this section you will see examples of graphs with horizontal and vertical asymptotes.

Horizontal asymptote

A horizontal asymptote of a graph $y = f(x)$ is a horizontal line that describes the behaviour of the value $f(x)$ as x is increasing or decreasing without bound.

You say	You write
As x is increasing in the positive direction without bound, the graph of the function tends to the horizontal asymptote $y = k$.	As $x \to +\infty$, $f(x) \to k$
As x is decreasing in the negative direction without bound, the graph of the function approaches the horizontal asymptote $y = k$.	As $x \to -\infty$, $f(x) \to k$

Important

A horizontal asymptote of a graph $y = f(x)$ is a horizontal line of the form $y = k$, where k is the value to which $f(x)$ approaches as x is increasing in the positive direction without bound, or as x is decreasing in the negative direction without bound.

For example, consider the function defined by $f(x) = \dfrac{3}{x-1} + 2.$

The graph of the function, as shown in the graph below has a horizontal asymptote given by the line $y = 2.$

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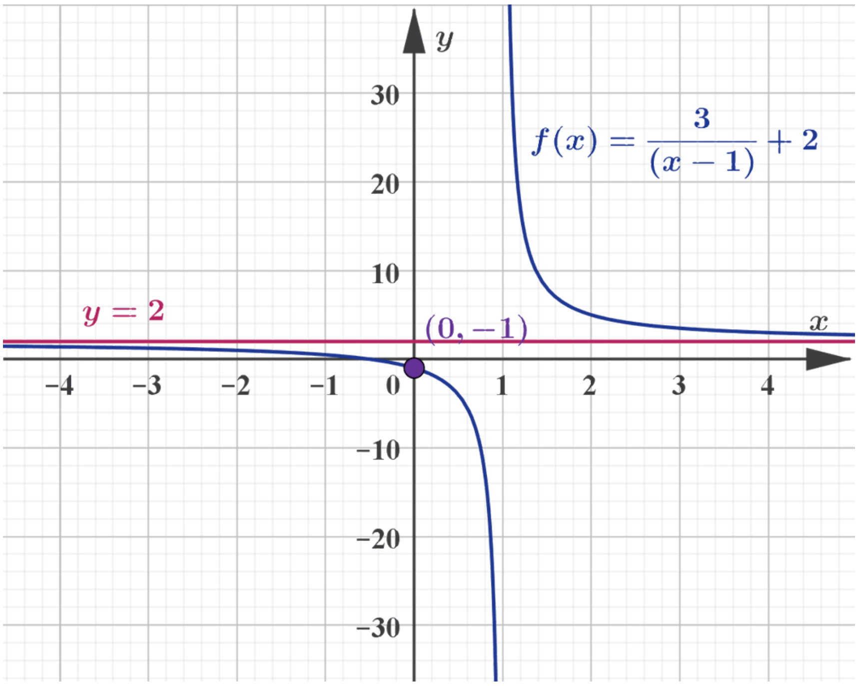

More information

The image is a graph depicting the function (f(x) = \frac{3}{x} + 2), shown with a blue curve. The graph is plotted on a grid with both the X and Y axes marked from negative to positive values. The X-axis represents the independent variable without specific units marked, while the Y-axis is similarly labeled. There's a visible key feature, a horizontal asymptote marked by a red line at (y = 2). As (x) increases positively or negatively, the curve approaches this line without touching it. One point on the graph is labeled ((0, -1)), marking a specific value on the curve. The grid lines further aid in visualizing the curve's trend as (x) tends toward infinity both positively and negatively. Overall, the graph illustrates how the function's values behave and their asymptotic nature in relation to (y = 2).

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To find, algebraically, the equation of the horizontal asymptote of the function, consider the behaviour of the values $f(x)$ as x takes larger and larger values.

As $x \to +\infty$, $\frac{3}{x-1} \to 0$. Thus, as $x \to +\infty$, $f(x) = \frac{3}{x-1} + 2 \to 2$.

Therefore, the graph of function f approaches the horizontal asymptote $y = 2$.

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Vertical asymptote

A vertical asymptote of a graph $y = f(x)$ is a vertical line that describes the behaviour of a graph $y = f(x)$ as x tends to the value where $f(x)$ is not defined.

You say	You write
As x is approaching the x-intercept of the vertical asymptote from the right, the value $\lvert f(x)\rvert$ is increasing without bound.	As $x \to h^+$, $f(x) \to \infty$ or $f(x) \to -\infty$
As x is approaching the x-intercept of the vertical asymptote from the left, the value $\lvert f(x)\rvert$ is increasing without bound.	As $x \to h^-$, $f(x) \to \infty$ or $f(x) \to -\infty$

Important

A vertical asymptote of a graph is a vertical line of the form $x = h$.

For the functions you meet in this syllabus, h is a value at which the function is not defined.

Be aware

For the functions you meet in this syllabus the graph of a function does not have point on a vertical asymptote.

Exam tip

It is important to understand that if you are asked to **sketch** a graph, the examiner expects you to give a general idea of the required shape of the graph by showing all relevant features such as:

- x -intercepts
- y -intercepts
- turning points (maximum and minimum points)
- asymptotes (which are typically represented by broken lines).

Student view

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Activity

On the applet below you can investigate the position of the horizontal and vertical asymptotes for graphs of functions of the form $f(x) = \dfrac{a}{x-h} + k$.

Use the sliders to adjust the graph of the function.

- Observe the changes in the the graph and the position of its horizontal and vertical asymptote as you change the different parameters.
- Describe the behaviour of the point $(x, f(x))$ as x is increasing in the positive direction without bound.
- Describe the behaviour of the point $(x, f(x))$ as x is decreasing in the negative direction without bound.
- Describe the behaviour of the point $(x, f(x))$ as x approaches the x-intercept of the vertical asymptote from the right.
- Describe the behaviour of the point $(x, f(x))$ as x approaches the x-intercept of the vertical asymptote from the left.

Interactive 1. Exploring Asymptotes in Rational Functions.

More information for interactive 1

This interactive tool allows users to visualize and explore rational functions of the form $f(x) = \dfrac{a}{x-h} + k$. Users can adjust three key parameters using sliders: a (vertical stretch/compression, range 1 to 10), h (horizontal shift, range -5 to 5), k (vertical shift, range -5 to 5). The graph updates in real-time as these values are modified, demonstrating

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how each parameter affects the function's shape and position. This dynamic visualization helps users understand the relationship between the algebraic equation and its graphical representation.

The tool clearly displays the function's asymptotes: a vertical asymptote at $x = h$ and a horizontal asymptote at $y = k$, both shown as dashed pink lines.

As users manipulate the sliders, they can observe how changes to h and k directly affect the asymptote positions, while changes to 'a' alter the graph's steepness and curvature. The equation is always displayed in proper mathematical formatting with the current values substituted, making the connection between parameters and graph behavior clear.

For example, when: $a = 5$, $h = 1$ and $k = 2$ the function becomes $f(x) = \frac{5}{x-1} + 2$, with asymptotes at $x = 1$ and $y = 2$. This hands-on approach enables users to develop an intuitive understanding of rational function transformations and their graphical characteristics.

Example 1

★★☆

Sketch the graph of the function $f(x) = \dfrac{1}{x-1} - 1$ showing all relevant features.

Use a GDC to graph the function $f(x) = \dfrac{1}{x-1} - 1$.

In your sketch show all relevant features such as the axes-intercepts, horizontal asymptote and vertical asymptote.

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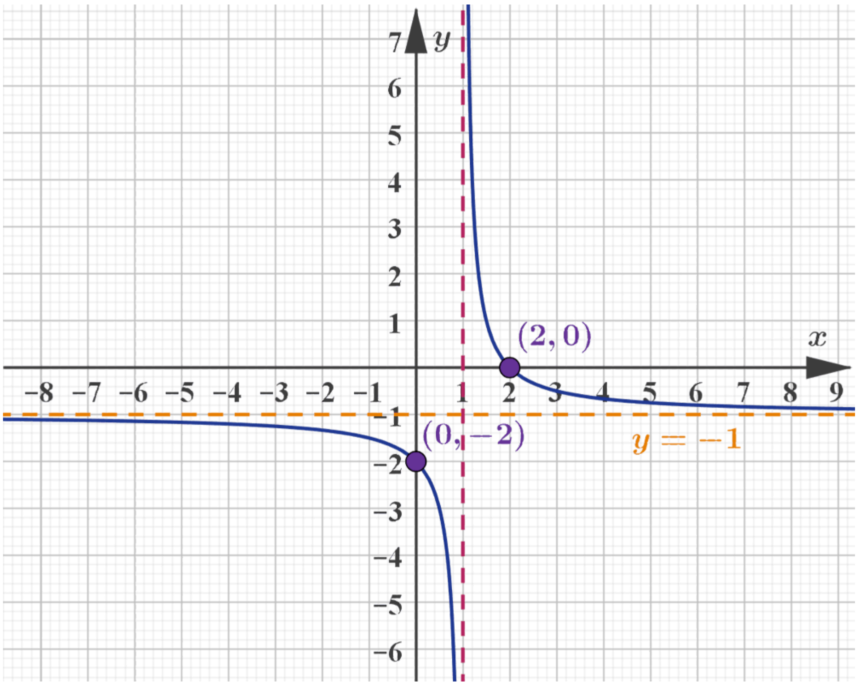

Analytically

For the horizontal asymptote, observe that as $x \to +\infty$, $\frac{1}{x-1} \to 0$ and thus $f(x) \to -1$.

For the vertical asymptote, the function $f(x) = \frac{1}{x-1} - 1$ is not defined at $x = 1$, the equation of the vertical asymptote is $x = 1$.

Theory of Knowledge

Functions and their corresponding visual representation via graphing are useful for a variety of reasons, but foremost for their ability to inform in regards to 'real world' data trends. Such graphs facilitate stock analysis, population predictions, and legislation related to human migration to name just a few. It seems then, that mathematics is ‘describing’ something; though is mathematics a ‘language’?

What elements must be present for something to be considered a language? Does mathematics share these elements? Work in pairs to debate whether or not mathematics should be considered a language.

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4 section questions

Intersection points

Intersection points using a GDC

Suppose that $f\left(x\right)$ and $g\left(x\right)$ are two functions that take any real number as input and give any real number as output. Then the intersection points of $f\left(x\right)$ and $g\left(x\right)$ appear at those values of x for which $f\left(x\right)=g\left(x\right)$. Sometimes, the exact values of x cannot be found by solving the equation $f\left(x\right)=g\left(x\right)$ analytically so a graphical method is required.

In this section, you will learn how to determine the points of intersection of two graphs using a GDC.

Consider the function $f\left(x\right)=x^3-\frac{1}{2}x^2+1$ and $g\left(x\right)=x^2+2x-1$.

Use a GDC to determine the points of intersection by following the steps below.

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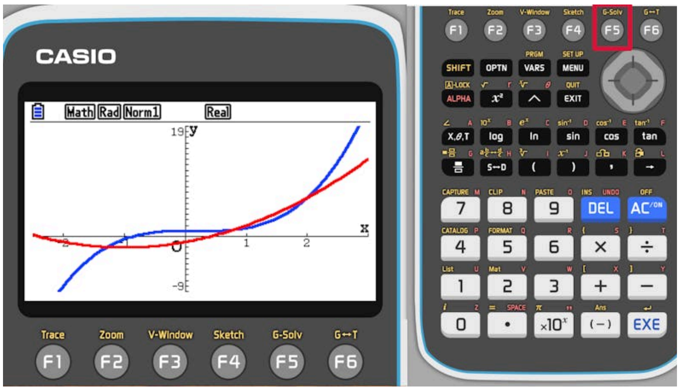
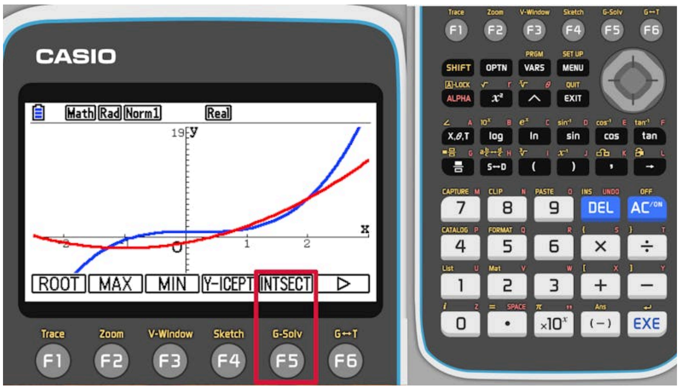
Steps	Explanation
These instructions assume that you have $y = x^3 - \frac{1}{2}x^2 + 1$ and $y = x^2 + 2x - 1$ graphed in the viewing window $-2.5 \leq x \leq 3$ and $-10 \leq y \leq 20$. Press F5 (G-Solv) to bring up the options to analyse the graphs.	
Press F5 again to find the intersection points.	

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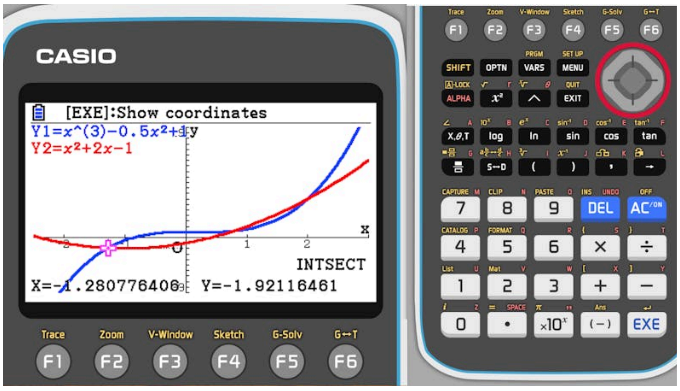
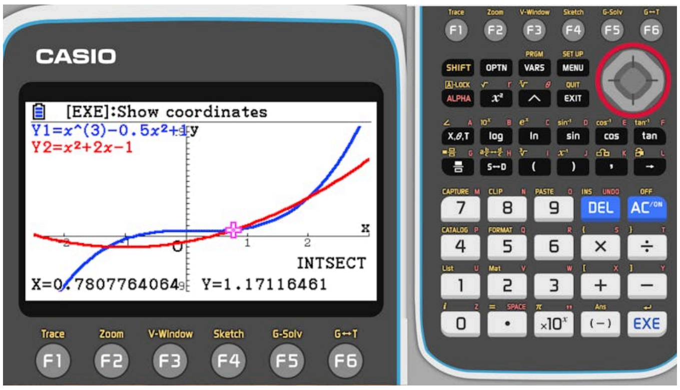
Steps	Explanation
The calculator moves the cursor to an intersection point and the coordinates are displayed. Move right to find the other intersection points.	
Move right again to find the third intersection point. You can of course move left to go back to the previous one.	

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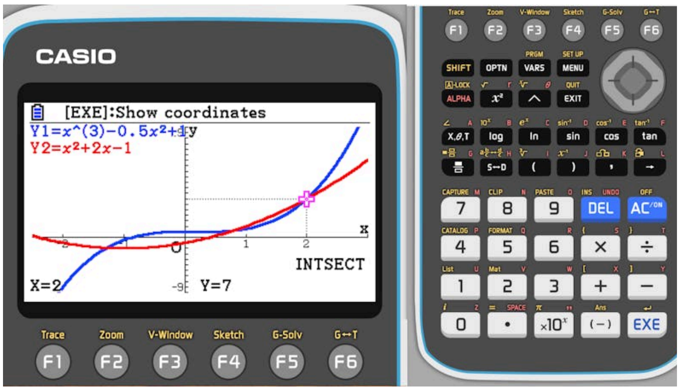
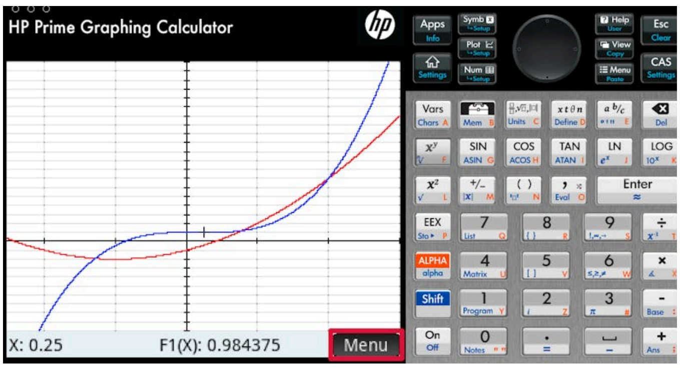
Steps	Explanation
	

Steps	Explanation
These instructions assume that you have $y = x^3 - \dfrac{1}{2}x^2 + 1$ and $y = x^2 + 2x - 1$ graphed in the viewing window $-2.5 \leq x \leq 3$ and $-10 \leq y \leq 20$. Choose the menu to access the options to analyse the graphs.	

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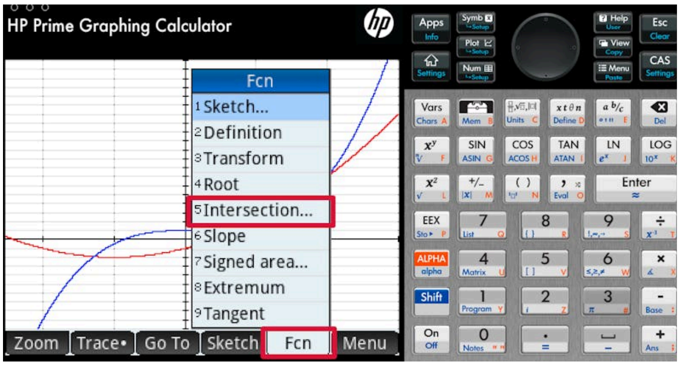
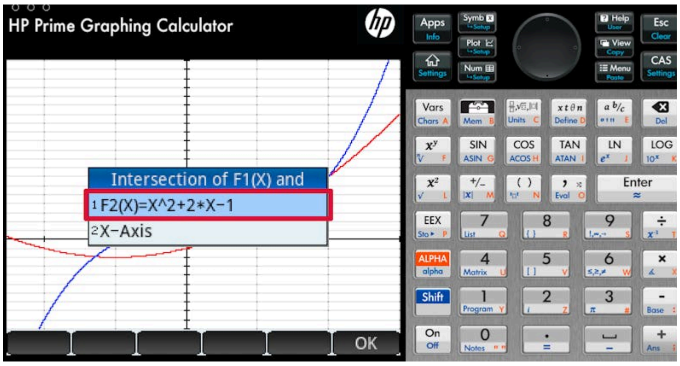
Steps	Explanation
In the function submenu choose the option that looks for the intersection points.	
You need to confirm which two functions you are interested in.	

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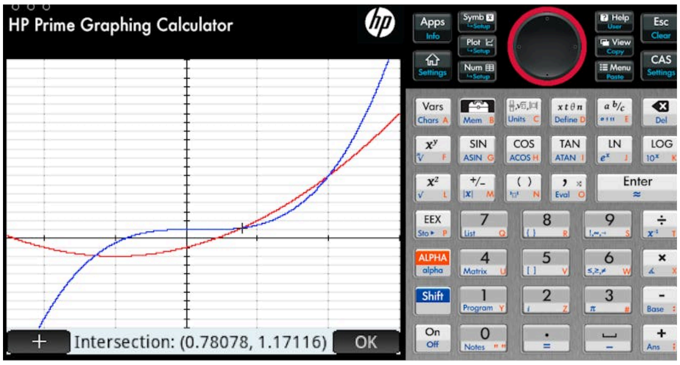
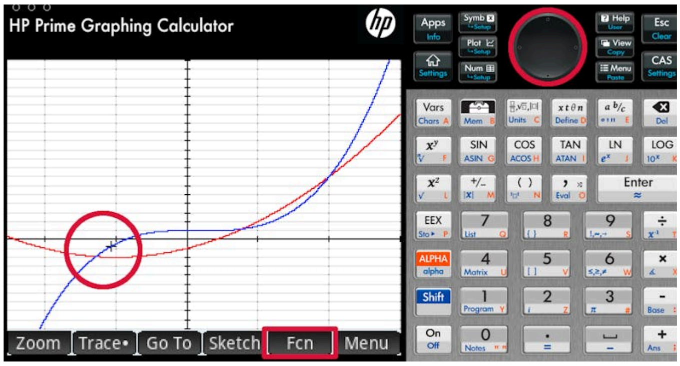
Steps	Explanation
The calculator moves the cursor to an intersection point and the coordinates are displayed. To find the other intersection points, you need to move the cursor close to the one you are interested in.	
Once the cursor is close to an intersection point, use the function submenu again to find the intersection point.	

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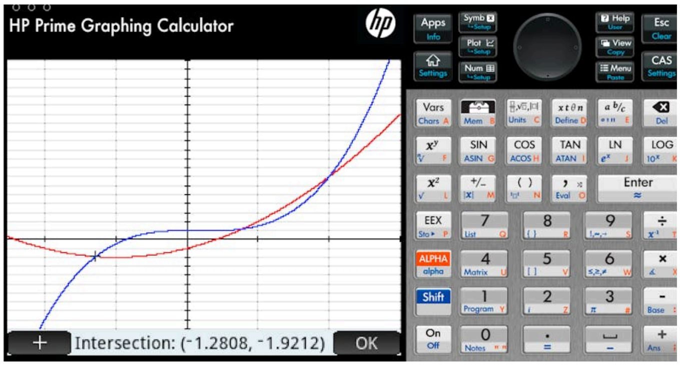
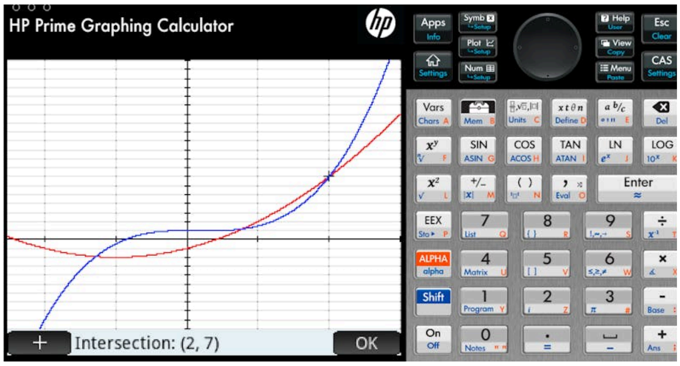
Steps	Explanation
The calculator moves the cursor to the intersection point and the coordinates are displayed.	
You can find the third intersection point in a similar way.	

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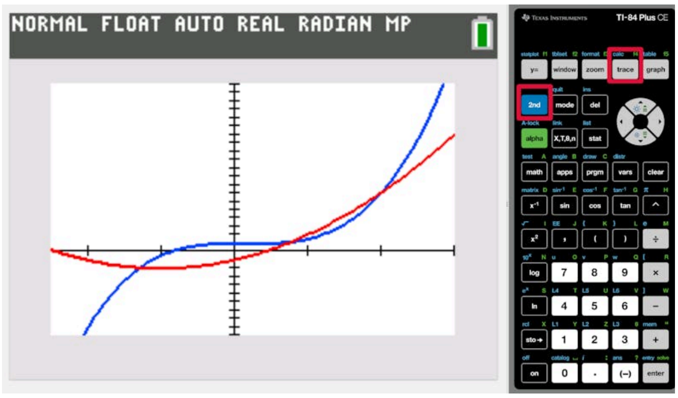
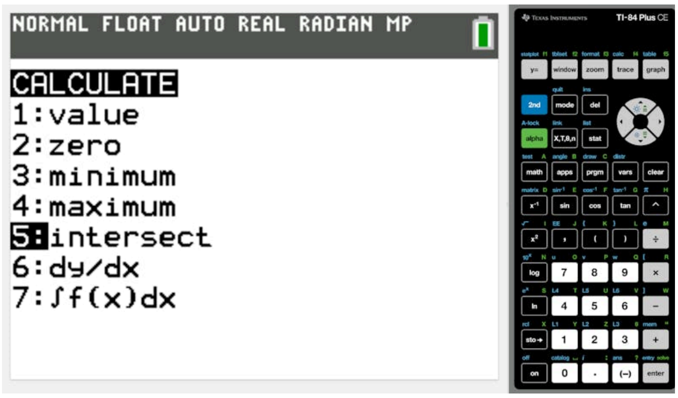
Steps	Explanation
These instructions assume that you have $y = x^3 - \frac{1}{2}x^2 + 1$ and $y = x^2 + 2x - 1$ graphed in the viewing window $-2.5 \leq x \leq 3$ and $-10 \leq y \leq 20$. Choose the calc menu to access the options to analyse the graphs.	
Choose the option to find intersection points.	

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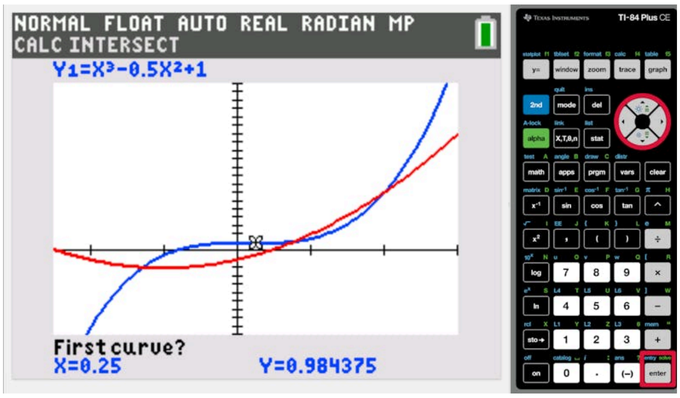
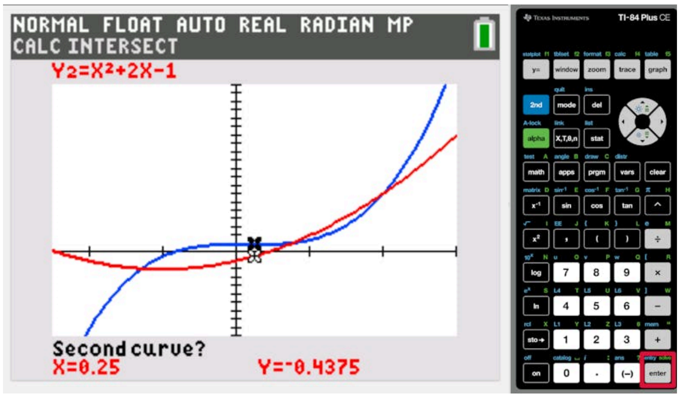
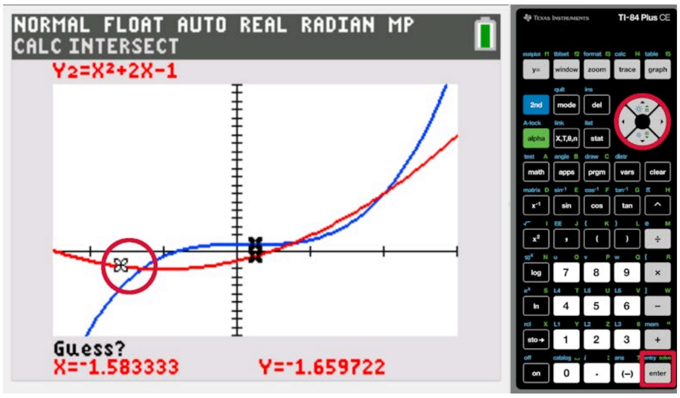
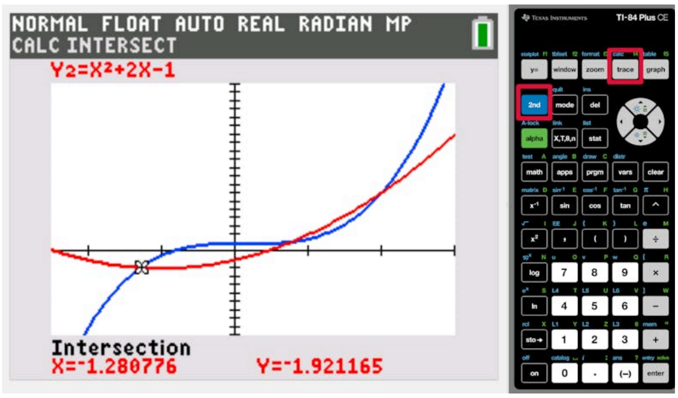
Steps	Explanation
Before it finds the intersection point, the calculator needs more information. First it asks you to identify one of the curves. You can move up/down to change between curves. Once you are satisfied with your choice, press enter.	
You also need to confirm the second curve.	

Steps	Explanation
The last thing the calculator asks is your guess. Move close to the intersection point with the cursor and press enter to confirm your guess.	
The calculator moves the cursor to the intersection point and the coordinates are displayed. To find the other two intersection points, you need to repeat the process two more times.	

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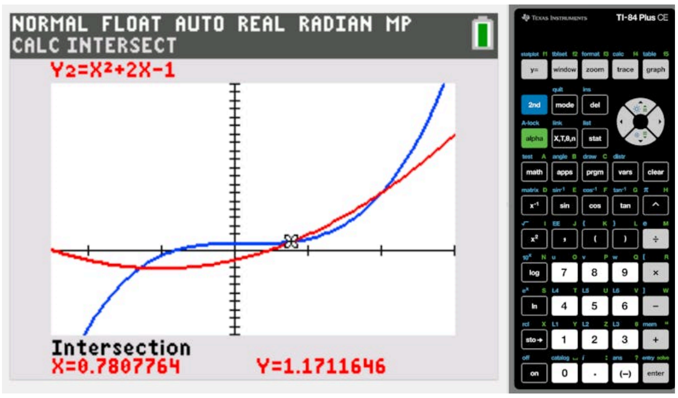
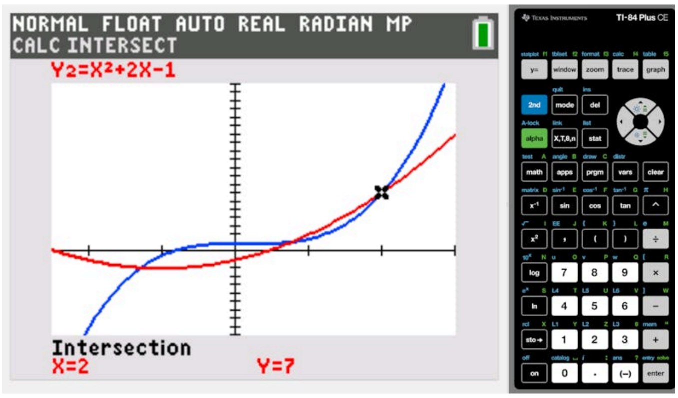
Steps	Explanation
	
	

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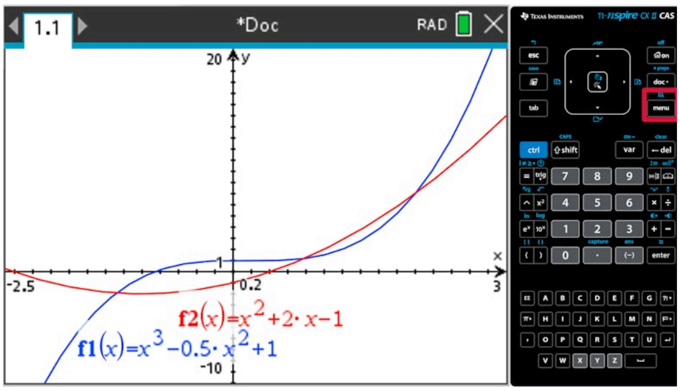
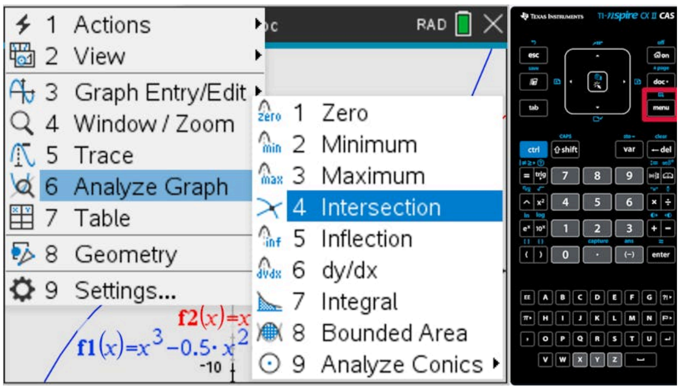
Steps	Explanation
These instructions assume that you have $y = x^3 - \frac{1}{2}x^2 + 1$ and $y = x^2 + 2x - 1$ graphed in the viewing window $-2.5 \leq x \leq 3$ and $-10 \leq y \leq 20$. Press menu to access the options to analyse the graphs.	
Choose the option to find intersection points.	

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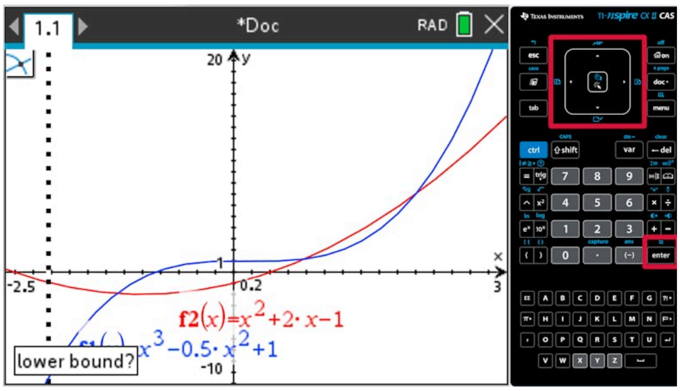
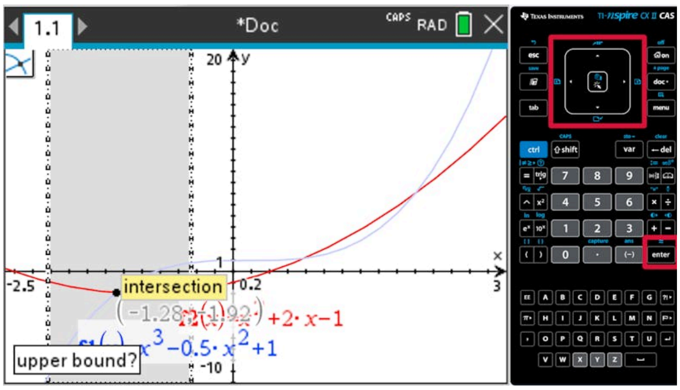
Steps	Explanation
Before it finds the intersection point, the calculator needs more information. First it asks for a lower bound for the x-coordinate of the intersection point. Move the vertical line to the left of the point and confirm your selection by pressing enter.	
Next, you need to move to the right of the intersection point.	

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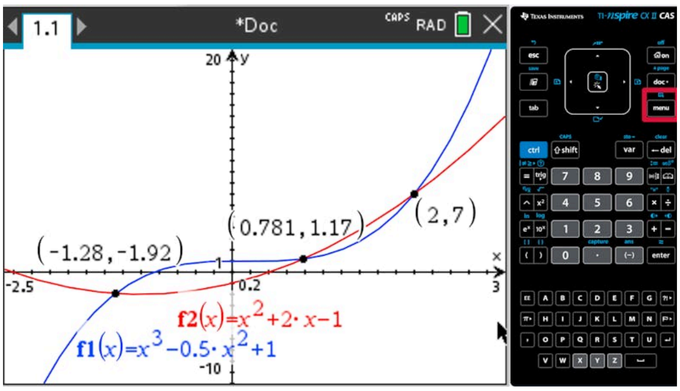
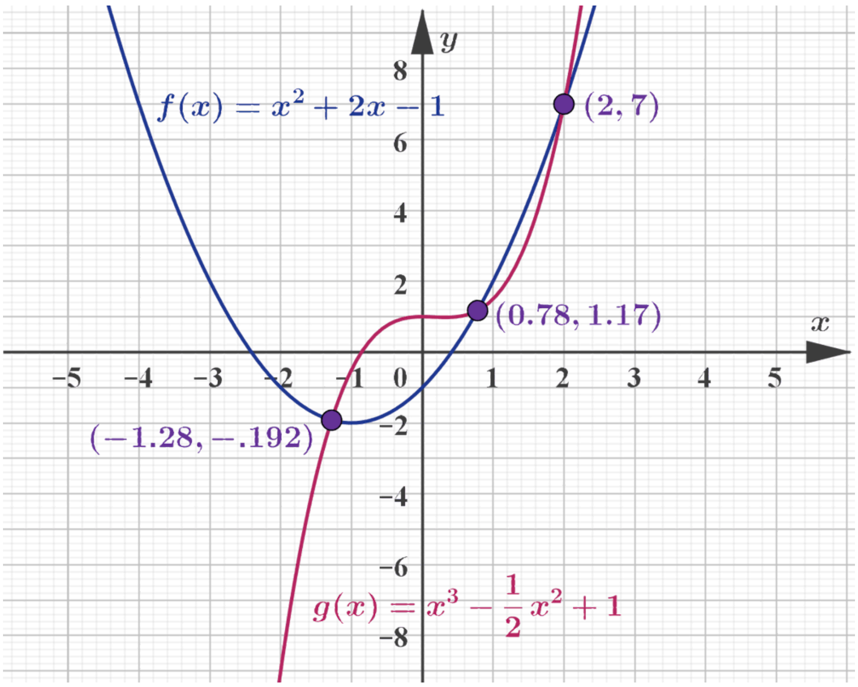
Steps	Explanation
Repeating this process two more times will give you all three intersection points.	

The graphs of $f\left(x\right)$ and $g\left(x\right)$ intersect at the points $\left(-1.28,\ -1.92\right)$, $\left(0.78,\ 1.17\right)$ and $\left(2,\ 7\right)$, as shown in the diagram below.



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view

More information

The image is a graph depicting two mathematical functions, (f(x) = -x^2 + 2x - 1) and (g(x) = 2^x - \frac{1}{2x^2} + 1). The graph is overlaid on a grid with both X and Y axes labeled with numbers ranging from -3 to 8. The points where the two functions intersect are clearly marked at ((-1.28, -1.92)), ((0.78, 1.17)), and ((2, 7)). \n\n- **X-axis:** Represents the variable (x) and ranges from around -3 to 8.\n- **Y-axis:** Represents the function values and ranges from -3 to 8.\n- **Function (f(x)):** A downward-opening parabola given by (-x^2 + 2x - 1).\n- **Function (g(x)):** A curve given by (2^x - \frac{1}{2x^2} + 1).\n- **Intersection Points:** The two curves intersect at three marked purple points where the coordinates are ((-1.28, -1.92)), ((0.78, 1.17)), and ((2, 7)).\n\nThe functions show intersection behavior typical of such mathematical equations, with the parabolic curve crossing the exponential-like curve at three locations in the visible range of the graph. This image is used to illustrate the points of intersection identified in the mathematical analysis.

[Generated by AI]

Example 1

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Identify the intersection points of the functions $f\left(x\right) = x^3 - \frac{1}{2}x + 1$ and $g\left(x\right) = 2x + 3$.

Using a GDC to find the points of intersection gives $x = 1.88679\ldots = 1.89$ and $y = 6.77359\ldots = 6.77$. Hence, the point of intersection is $(1.89,\ 6.77)$.

Activity

In the following applet, you can visualise the graph of a quadratic function f and the graph of a rational function g. Investigate the minimum and maximum number of intersection points between the two graphs. Use the black 'Drag me' points to adjust the shape and position of function $y = f\left(x\right)$. Discuss all possible cases.

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Interactive 1. Intersection of Quadratic and Rational Functions.

More information for interactive 1

This interactive allows users to visualize the graphs of a quadratic function$y = f(x)$ and a rational function $y = g(x)$. Users can adjust the shape and position of the quadratic function $y = f(x)$ by dragging the purple "Drag me" points. By manipulating these points, users can explore the minimum and maximum number of intersection points between the two graphs.

Making connections

Economics: The diagram below shows the graphs of supply and demand of a product. Interpret the meaning of the point of intersection of the two lines in this context.

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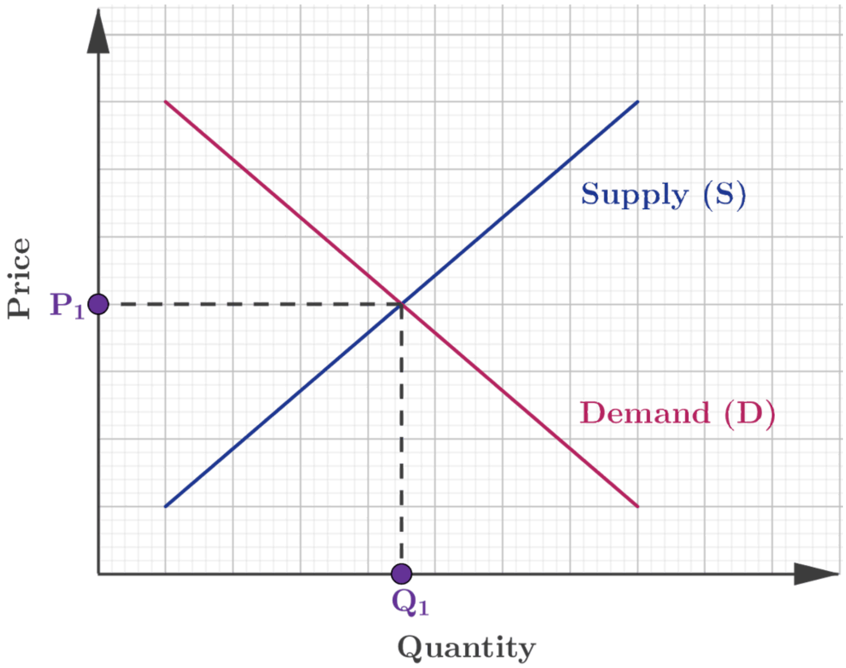

More information

The graph illustrates the relationship between supply and demand for a product, featuring a coordinate grid with labeled axes. The X-axis represents 'Quantity', while the Y-axis shows 'Price'. A positively sloped line marked 'Supply (S)' intersects with a negatively sloped line labeled 'Demand (D)'. These lines intersect at a central point, indicating the equilibrium where the quantity supplied equals the quantity demanded. In this diagram, the equilibrium point is significant as it represents the market condition under which there is neither surplus nor shortage of the product.

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Theory of Knowledge

This portion of the IB mathematics syllabus examines how using a graphic display calculator (GDC), a handheld mathematical computer that calculates, is an expectation of the curriculum itself. Since most mathematics students are still taught how to handle problems with and without computers, consider the different means by which a mathematician deals with proofs — whether entirely by pen and paper or completed using a computer.

In the following video, Conrad Wolfram talks about his book 'The Math(s) Fix' in which he wants to inspire teachers and students to make better use of computers in educating maths and promote the use of the computational thinking process. As you watch this video, evaluate to what extent computer calculations can help students and real-world maths practitioners to exhaust mathematical possibilities and to examine (in)consistencies in quantitative applications.

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Conrad Wolfram - TheMath(s)Fix

3 section questions

2. Functions / 2.4 Key features of graphs

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-25405/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- Use a GDC to determine relevant features of graphs of functions such as
  - turning points (local maxima and local minima)
  - vertical asymptotes
  - horizontal asymptotes
  - points of intersection between graphs of functions.

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2. Functions / 2.4 Key features of graphs

Investigation

The applet below allows you to visualise the graph of a function $f(x)$ in the form $f(x) = \dfrac{a}{x-h} + k$.

Use the sliders a, k and h to adjust the function.

Interactive 1. Exploring Key Features of Rational Functions.

More information for interactive 1

This interactive tool allows users to visualize and explore rational functions of the form $f(x) = \frac{a}{x-h} + k$. Users can adjust three key parameters using sliders: a (vertical stretch/compression, range 1 to 10), h (horizontal shift, range −5 to 5), k (vertical shift, range −5 to 5). The graph updates in real-time as these values are modified, demonstrating how each parameter affects the function's shape and position. This dynamic visualization helps users understand the relationship between the algebraic equation and its graphical representation.

The tool clearly displays the function's asymptotes: a vertical asymptote at $x = h$ (shown as a dashed pink line) and a horizontal asymptote at $y = k$ (shown as a dashed pink line)

As users manipulate the sliders, they can observe how changes to h and k directly affect the asymptote positions, while changes to 'a' alter the graph's steepness and curvature. The equation is always displayed in proper mathematical formatting with the current values substituted, making the connection between parameters and graph behavior clear.

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For example, when: $a = 5$, $h = 1$ and $k = 2$ the function becomes $f(x) = \frac{5}{x-1} + 2$, with asymptotes at $x = 1$ and $y = 2$. This hands-on approach enables users to develop an intuitive understanding of rational function transformations and their graphical characteristics.

How can you determine the equations of the vertical and horizontal asymptotes of the graph directly from the function?

For your own values of a, h and k, show that function f can be expressed in the form

$$f(x) = \frac{bx + c}{dx + e}$$

The following applet allows you to visualise the graph of function f in the form

$$f(x) = \frac{bx + c}{dx + e}$$

Interactive 2. Exploring Asymptotes.

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More information for interactive 2

This interactive allows users to visualize and explore rational functions of the form $f(x) = \frac{bx+c}{dx+e}$. Users can adjust four key parameters using sliders: b (numerator coefficient, range −5 to 2), c (numerator constant, range −5 to 5), d (denominator coefficient, range 1 to 5), e (denominator constant, range −5 to 5)

The tool clearly displays the function's asymptotes: A vertical asymptote at $x = -\frac{e}{d}$ (shown as a dashed pink line), A horizontal asymptote at $y = \frac{b}{d}$ (shown as a dashed pink line)

As users manipulate the sliders, they can observe how changes to b and d affect the horizontal asymptote position, while changes to d and e determine the vertical asymptote location. The equation is always displayed in proper mathematical formatting with the current values substituted, making the connection between parameters and graph behavior clear.

The graph updates in real-time as these values are modified, demonstrating how each parameter affects the function's shape and position. This dynamic visualization helps users understand the relationship between the algebraic equation and its graphical representation.

For example, when $b = 2$, $c = 3$, $d = 1$ and $e = -1$ the function becomes $f(x) = \frac{2x+3}{x-1}$, with asymptotes at $x = 1$ and $y = 2$. This hands-on approach enables users to develop an intuitive understanding of more complex rational function transformations and their graphical characteristics.

Use the sliders b, c, d and e to adjust the function and observe the position of the vertical and horizontal asymptotes.

How can you determine the equations of the vertical and horizontal asymptotes of the graph directly from the function?

Rate subtopic 2.4 Key features of graphs

Help us improve the content and user experience.

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