5. Calculus / 5.15 Graphical approximations to differential equations

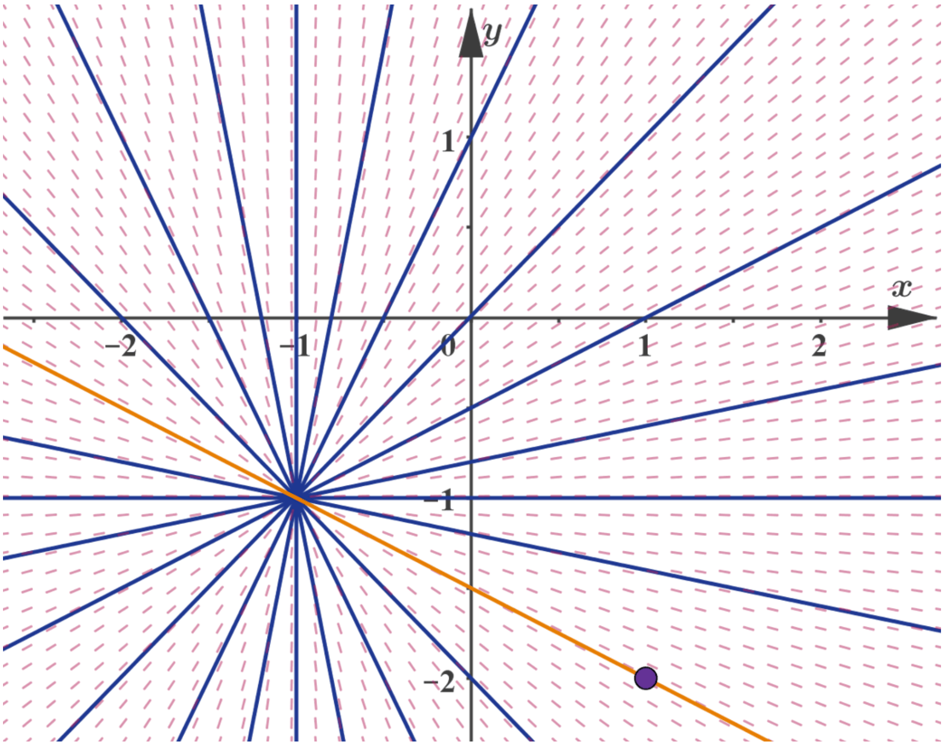
The big picture

In the study of differential equations, you analysed equations that consisted of several variables: x, y, $\frac{\mathrm{d}y}{\mathrm{d}x}$. Prior to this, you probably thought of graphs as a tool to compare dependent and independent variables, y versus x. Graphing a derivative, or the change in y with respect to x, was just another graph of y versus x. Now that you have seen that the derivative, y', can be a function of both x and y, what changes? Is it still just a single curve, where a given x-value results in a specific y'-value? The answer is 'no'. To find y', or the change in y with respect to x, you need to know both the x- and y- values.

Rather than try to represent this information in a three-dimensional graph, a slope field is a special type of graph that represents the slope at several $x - y$ combinations. That way, you can see more than just a few values of the slope. You can see patterns across the entire $x - y$ plane.

In subtopic 5.14.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/exact-solution-separable-equations-id-27924/), **Examples 5** and **6** found the general and particular solutions for $xy' - y = 1 - y'$ with $y(1) = -2$ to be $y = c(x+1) - 1$ and $y = -\frac{1}{2}x - \frac{3}{2}$, respectively. The following diagram illustrates the slope field and some solution curves of this differential equation, with the initial condition and the particular solution highlighted.

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov



More information

The image is a diagram showing a slope field and solution curves for the differential equation (xy' - y = 1 - y'). The X-axis and Y-axis are marked with small intervals, and the equation's slope field is depicted with a series of short, pink dashed lines oriented in various directions. These lines visualize the slope, or derivative, of the function at each point in the field.

Several solution curves are overlaid on the slope field. One particular curve is highlighted in orange, representing the specific solution to the equation given an initial condition (y(1) = -2). This curve passes through the point (1, -2) and follows the general direction indicated by the slope field lines.

Another set of curves, shown in blue, represent the general solutions to the differential equation. These curves are radiating outward, following the directional flow implied by the slope lines.

This diagram helps illustrate the behavior of the differential equation's solutions and how they align with the slope direction at various points.

[Generated by AI]

Student view

Overview
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Concept

As you are investigating these graphical methods, think about how slope fields and isoclines can be used in science. For example, in physics, students study a variety of forces, most notably gravity and electromagnetism. Can you **model** the strength and direction of these forces with slope fields? With a given starting point, can you predict or **approximate** where an object will travel based on isoclines? When studying weather, winds can also be **modelled** using slope fields and isoclines. Can you predict the flight of a weather balloon based on these pictures? What other examples can you think of?

5. Calculus / 5.15 Graphical approximations to differential equations

Slope fields

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/slope-fields-id-27929/print/) Assign

You will now investigate graphical methods that help sketch solution curves even if there is no known solution to the differential equation . The idea behind slope fields and isoclines is that for a differential equation of the form $y' = F(x, y)$, if a solution curve goes through a point (x_0, y_0), then the equation gives the slope of the tangent line to the solution curve at this point, $m = y'(x_0) = F(x_0, y_0)$.

A slope field, sometimes called a direction field, is a graphical representation of the rate of change across all values of x and y over a given domain and range.

To build a slope field, draw little line segments at points of a rectangular grid to illustrate the tangent line to a solution curve passing through the points of the grid. With a dense enough grid, these line segments will allow you to sketch solution curves to the differential equation.

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = xy$. When both variables are positive or negative, the slope is positive. If both values are large in magnitude, then the slope is steep. When either variable approaches zero, the slope approaches zero. Graphically, this could be portrayed as:

Student view

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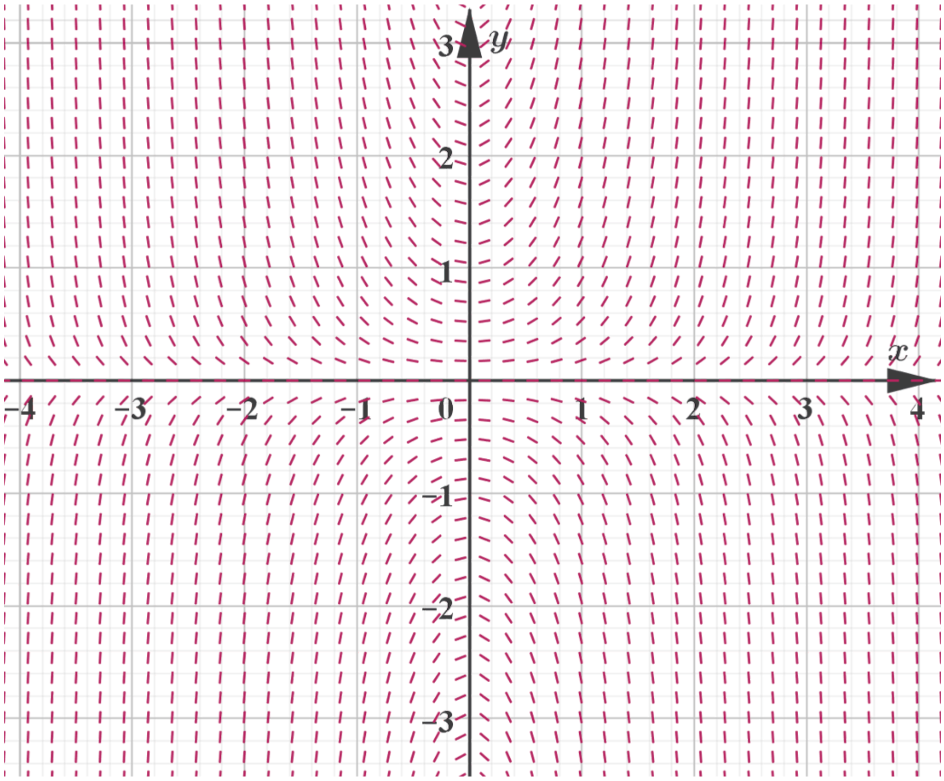


More information

The image shows a direction field for the differential equation (\frac{{\text{d}y}}{{\text{d}x}} = xy). The background consists of a grid with horizontal and vertical lines forming small squares. The x-axis is horizontal with ticks labeled from -3 to 3, and the y-axis is vertical with similar markings. Short line segments at each grid point depict the slope (xy) at that point. The line segments vary in direction and steepness, representing the positive slope when both (x) and (y) have the same sign, with increased steepness for larger magnitudes of (x) and (y). Near the origin, where either (x) or (y) is near zero, the slope approaches horizontal, with line segments nearly flat. This pattern forms a visual representation of how solutions to the differential equation behave based on initial conditions.

[Generated by AI]

Important

The slope field corresponding to the differential equation $y' = F(x, y)$ is the collection of line segments with slope $F(x, y)$ drawn at all points (x, y) of the plane. If $F(x, y)$ is not defined at a particular point, draw a vertical line segment.

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Of course, you cannot draw the full slope field because it has infinitely many line segments, so you must choose a grid of points and draw only the corresponding segments. Note, in some resources, slope fields are vector fields (they contain vectors instead of line segments).

Exam tip

Some calculator models have applications to draw slope fields. However, these options need to be disabled on IB exams. You can use these as learning tools, but you cannot rely on these applications on your exams.

You can also use online tools to plot slope fields to help your learning, but of course these are not available on exams either.

Drawing and using slope fields can help to find approximate solutions to differential equations such as $\frac{\mathrm{d}y}{\mathrm{d}x} = -y\frac{1+xy}{1-xy}$.

In this course, you will not learn how to find exact solutions to this equation but will learn a technique for attempting to sketch solution curves.

Example 1

★★☆

Draw the slope field for the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = -y\frac{1+xy}{1-xy}$ using the grid of points with integer coefficients such that $-4 \leq x \leq 4$ and $-2 \leq y \leq 2$.

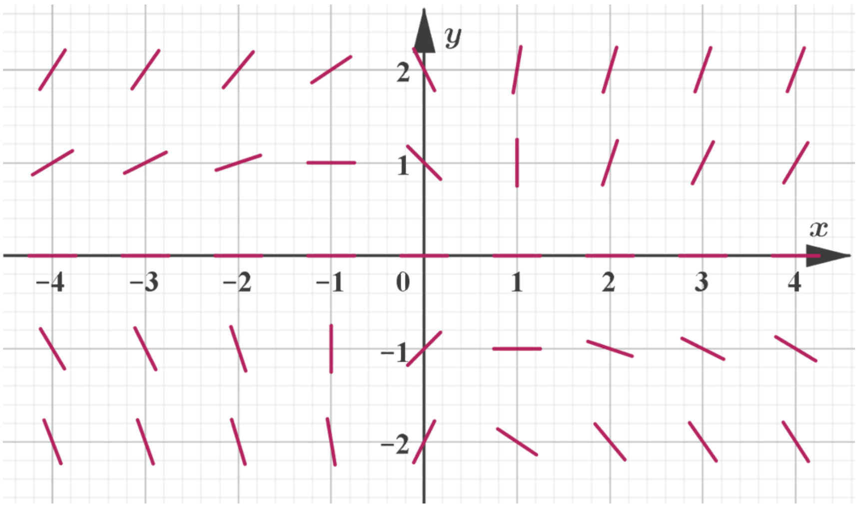
In an exam, it is unlikely that you will get a question like this. The method is not difficult but it is very repetitive, as it required repeating the same process over and over again. You need to calculate the value of the slope for any possible (x, y) pair.

For example, for $(x, y) = (-4, -2)$,
$m = -(-2)\frac{1+(-4)(-2)}{1-(-4)(-2)} = -\frac{18}{7} = -2.57$. Hence, you need to draw a line segment through the point $(-4, -2)$ with approximate slope -2.57. This segment,

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/overview)

and others, are illustrated in the diagram below.



Note, that at $(-1, -1)$ and at $(1, 1)$, the lines are vertical. At these points the denominator of the quotient is 0, so the gradient is not defined.

Note: In the example above, you were asked to draw the slopes at integer points, effectively building a grid with increments in both directions of 1.0. There is no limitation on finding the slope only at integer points. You could find the slope at any point in the x–y plane. This could have been done in increments of 0.5, or any other value.

With the applet below, you can move the point around and draw the line segment at any point.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Interactive 1. Interactive Slope Field Visualization.

More information for interactive 1

This interactive tool helps users understand slope fields for differential equations by drawing tangent line segments at various points on a graph. The tool demonstrates how these segments indicate the direction of potential solution curves, providing a graphical approach to understanding differential equations without requiring exact solutions. Users can explore how changing positions affect the slope values and corresponding tangent lines.

The display shows the x and y axes, with the x-axis ranging from -4 to 4 and the y-axis ranging from -2 to 3. A movable purple point that users can drag across the graph. At each position, the coordinates and slope value are displayed. Two buttons are available: "Draw segment at the point" at the top left corner, adds a pink tangent line segment based on the current slope, while "Clear all" on the top right, removes all drawn segments. This allows users to build a custom slope field by placing segments at selected points.

For example, if the user moves the point to $(2, 2)$ where the slope is 3.33, clicking "Draw segment at the point" will place a steep upward-sloping line there. Moving the point to $(-2, -1)$ with a negative slope and drawing another segment shows how the direction changes. By repeating this process, users can approximate solution curves by connecting the tangent segments.

Through this exploration, users learn how slope fields represent differential equations graphically, helping them predict solution behavior even without solving analytically. They gain an intuition for how initial conditions influence curves and practice sketching approximate solutions by following tangent directions.

Student view

Overview
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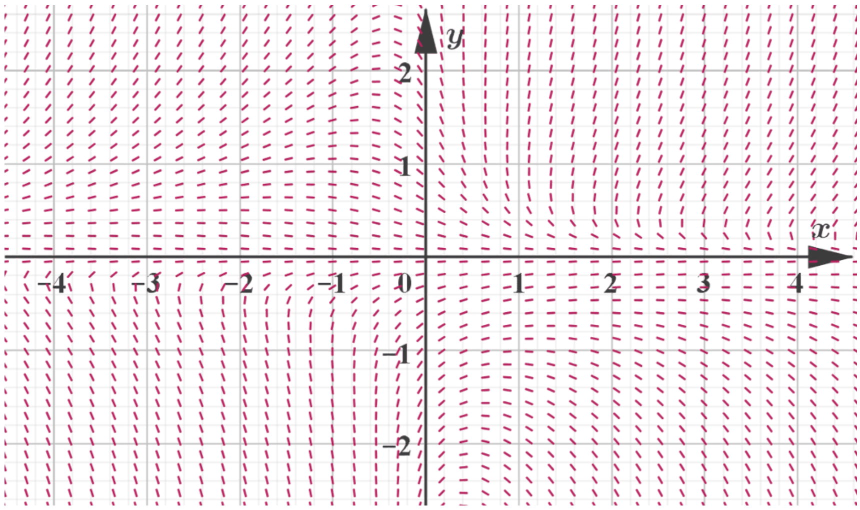
Of course, drawing a slope field by hand is time-consuming, and the one in the previous example is not really dense enough to use it to sketch solution curves. Technology, for example GeoGebra, can be used to draw slope fields on a denser grid. This will be covered further in the investigation (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-27932/).



More information

The image depicts a slope field on a dense grid, likely created using software like GeoGebra. The field consists of numerous short arrows, forming a detailed lattice across the entire grid. These arrows indicate the slope of the solution curves at various points, thereby visually demonstrating the differential equation's behavior across the plane. The X-axis is marked with numbers ranging from -4 to 4, while the Y-axis spans a similar range. The arrows change direction and density throughout the grid, suggesting areas of differing gradient which might reflect changes in the underlying differential equations.

[Generated by AI]

Although it is interesting to know this, it is not necessary to use slope fields to see how the function acts at every point. You can think of this like a river with currents. If you were to drop a leaf in at a specific starting point, where would it go? By mapping the currents, you could get a pretty good idea of where they would lead. The same is true with slope fields. If you start at a given point with predicted slopes or directions, you can determine where the solution will take you. From a given a starting point you can follow the direction of the slopes in both directions, adjusting the solution curve along with the change in the slope.

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

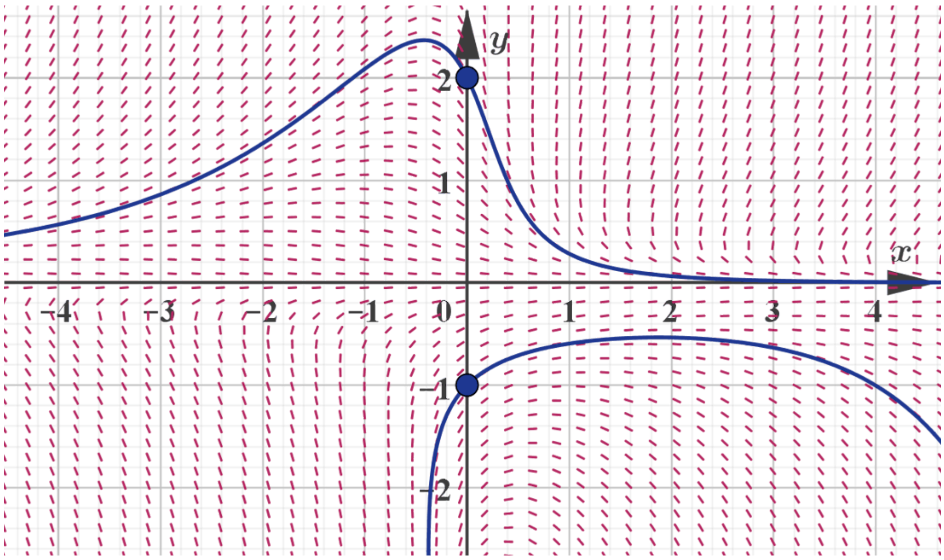
Example 2

★☆☆

Using the slope field above, sketch a solution curve to the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = -y\frac{1+xy}{1-xy}$ with initial condition $y(0) = 2$ and sketch another curve with initial condition $y(0) = -1$.

The first curve should go through the point $(0, 2)$ and the second curve should go through the point $(0, -1)$. Both curves should follow the slope field. The line segments of the slope field should be close to the tangents of the solution curves at any point on the curve.

The diagram below shows the curves. Did you get a curve close to these?



Example 3

★★☆

Student view

Draw some more solution curves for the differential equation above.

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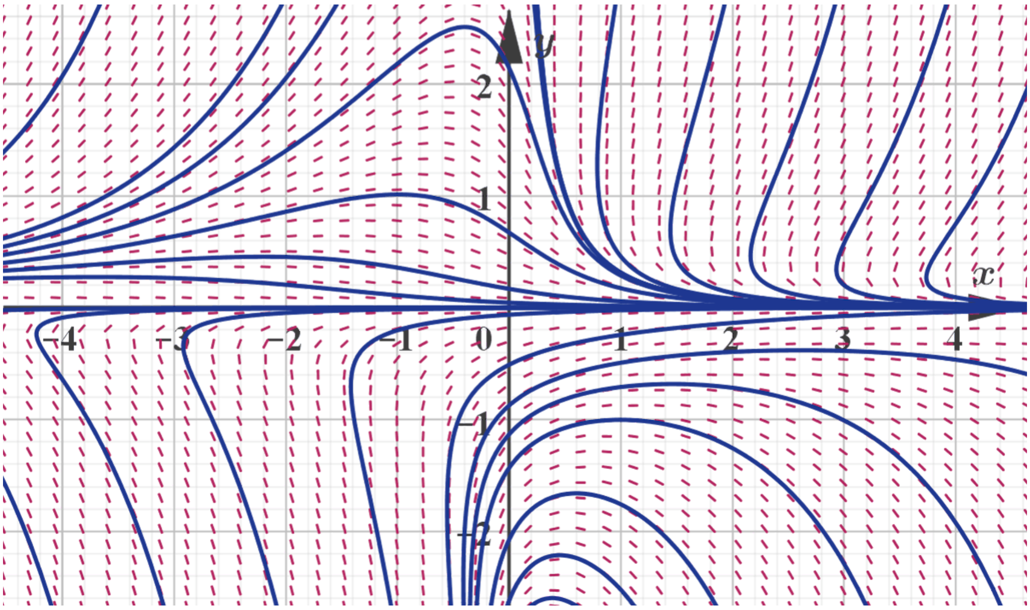

Example 4

★★☆

Sketch the solution curve of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy$ that passes through the point $(1, 1)$.

Use the slope field below:

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view

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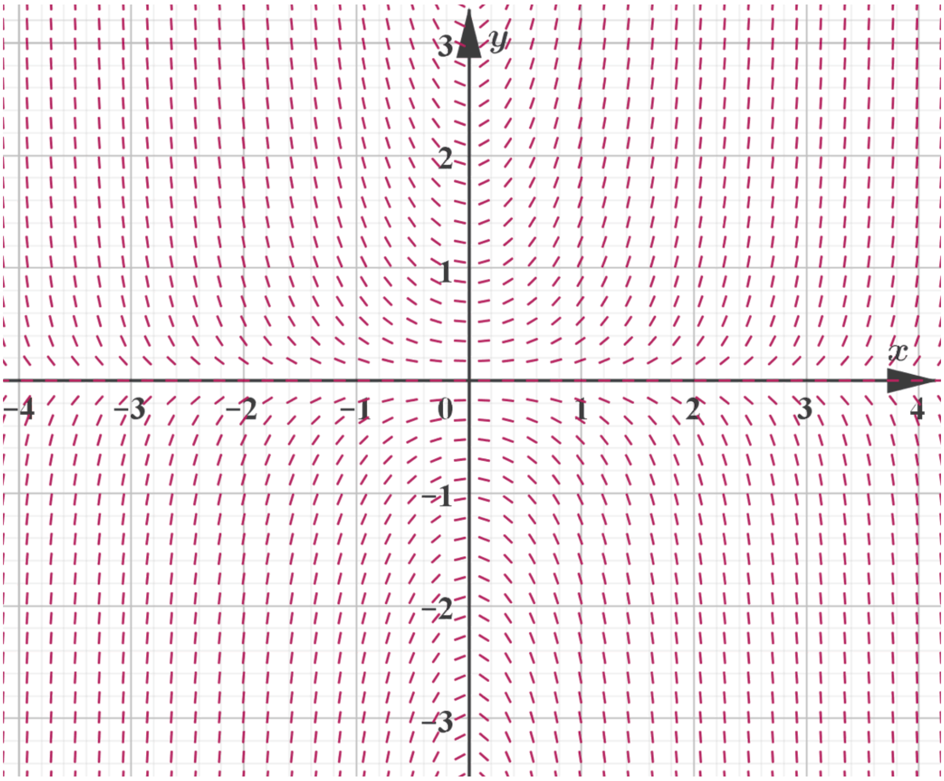

More information

The image is a slope field, which is a grid overlayed with directional arrows or lines indicating the slope of a function at various points. The horizontal axis is marked with units ranging from negative to positive values, and the vertical axis is labeled similarly. Each intersection on the grid has small lines or arrows that illustrate the direction of the slope at that point, with a consistent pattern forming curves across the grid. This type of representation is used to visualize solutions to differential equations by showing the tangent slope at each grid point. The background is neutral to emphasize the clarity of the lines and arrows in depicting these slopes. No specific labels or text are visible on the grid, reinforcing the focus on the patterns formed by the directional indicators.

[Generated by AI]

The solution curve would look something like:

Student view

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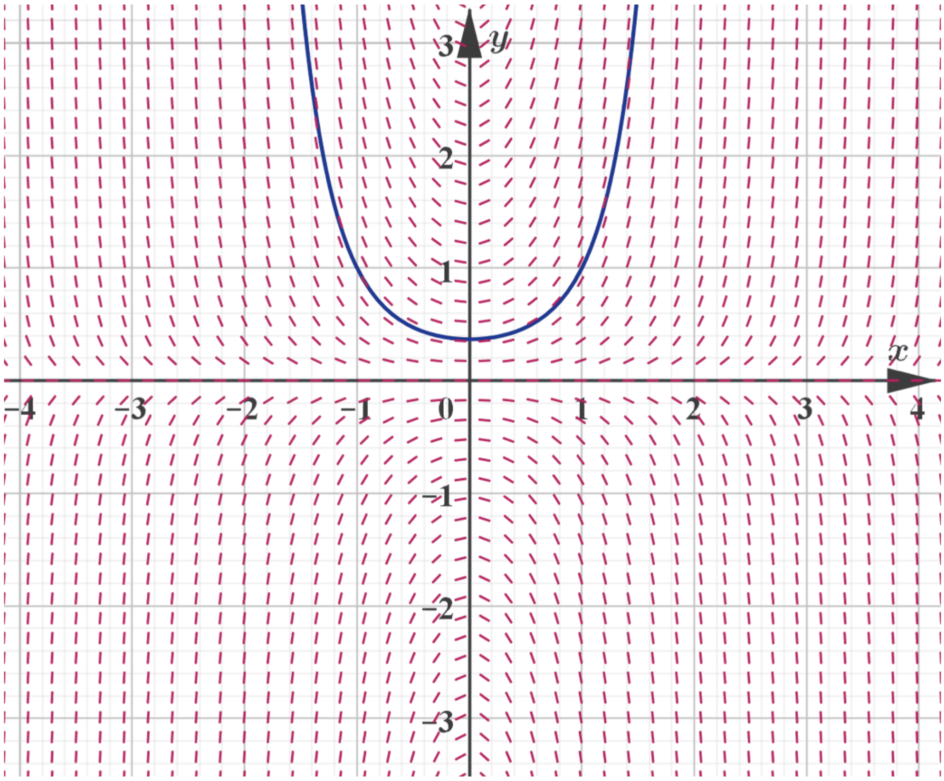

3 section questions

5. Calculus / 5.15 Graphical approximations to differential equations

Isoclines

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/isoclines-id-27930/print/) Assign

Exam tip

Isoclines will not be included in Applications and Interpretation examinations. They have been included here as additional illustrations of differential equations.

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Building entire slope fields by hand, especially with any meaningful density, is extremely time-consuming. Although you do have technology that can help, it is useful to explore other graphical techniques that are more efficient to help you study the slopes. Another such technique is the development of **isoclines**.

Important

An isocline corresponding to the differential equation $y' = F(x, y)$ is a curve with the (implicit) equation $F(x, y) = c$, for some real number c.

Alternatively, an **isocline** is a curve on a graph connecting points of equal slope c.

The name isocline comes from the Greek words 'isos' and 'klinein' and, roughly, means 'same slope'. Can you see how the definition reflects this meaning? If a solution curve of the differential equation crosses the isocline corresponding to c, then the tangent line to the solution curve at the intersection point (x_0, y_0) has slope $y'(x_0) = F(x_0, y_0) = c$.

To see how to use isoclines, study a few isoclines for the differential equation $y' = x - 2y + y^2$.

For $c = 0$, the equation is $x - 2y + y^2 = 0$, or after rearrangement $x = -y^2 + 2y$. Notice that, since the slope $c = 0$, this equation is related to the horizontal line segments from the slope field. The graph of this equation is a parabola with a horizontal axis of symmetry and a vertex at $(1, 1)$.

For $c = 1$, the equation is $x - 2y + y^2 = 1$, or after rearrangement, $x = -y^2 + 2y + 1$. The graph of this equation is a parabola with a horizontal axis of symmetry and a vertex at $(2, 1)$.

The following diagram shows these two isoclines and also indicates the directions in which a solution curve crosses these isoclines.

Student view

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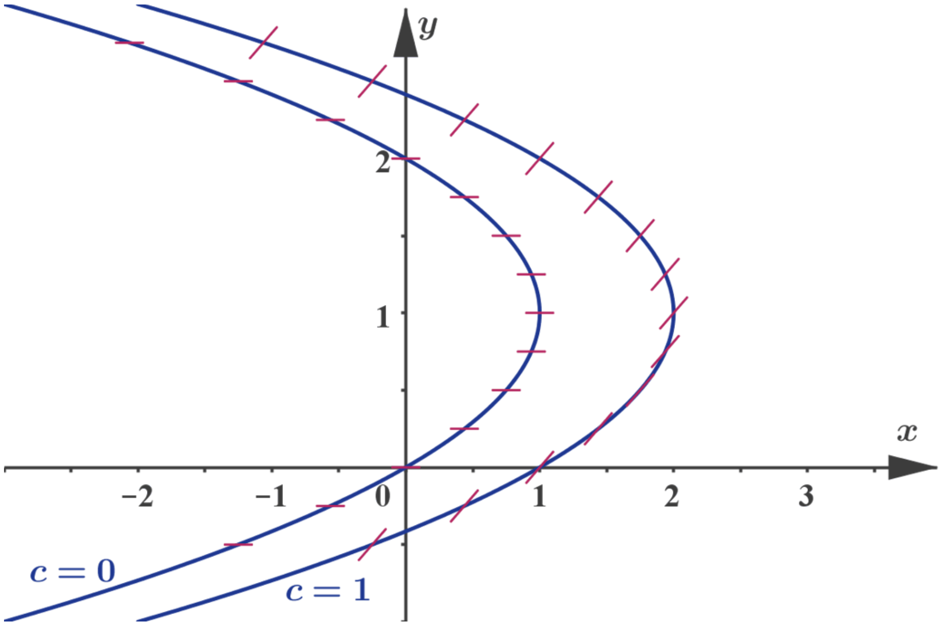

More information

The image is a diagram illustrating two isoclines on a Cartesian plane, with the x-axis labeled as 'x' and the y-axis labeled as 'y'. The diagram shows two curved lines representing isoclines, labeled 'c = 0' and 'c = 1'. These lines curve upward and are intersected by small perpendicular line segments indicating the directions in which a solution curve would cross these isoclines. The isoclines are plotted within a range of x from -3 to 3 and y from 0 to 3.

[Generated by AI]

Once you have your isoclines and a starting point, you can estimate the solution curve by sketching a curve that crosses the isoclines at a slope equal to the constant of each isocline.

Example 1

★★☆

Sketch a solution curve for the differential equation $y' = x - 2y + y^2$ that starts at the point $(0, 1)$, crosses the isocline corresponding to $c = 0$ and reaches the isocline corresponding to $c = 1$.

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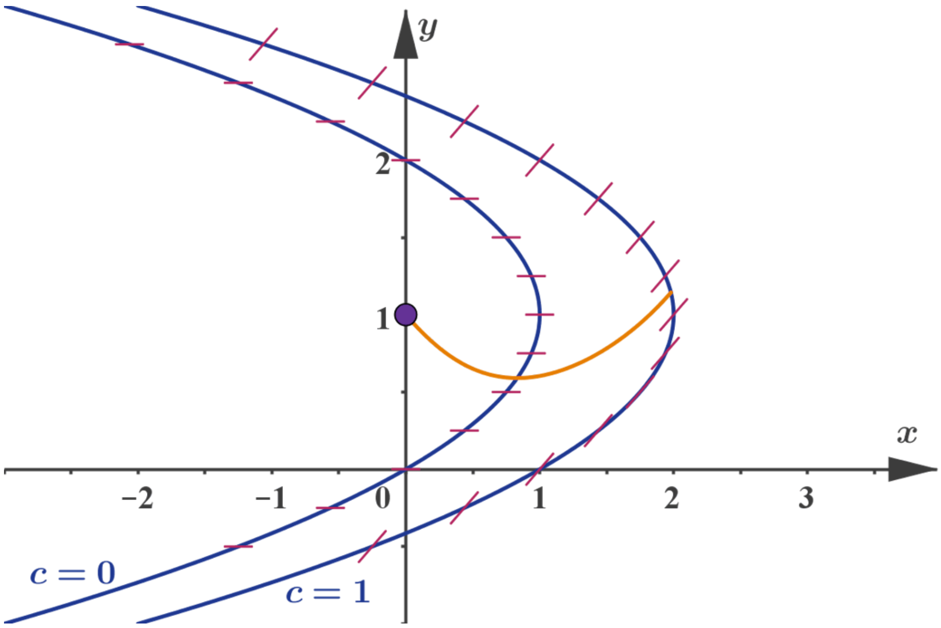
At $(0, 1)$, $y'(0) = F(0, 1) = 0 - 2 \times 1 + 1^2 = -1$. Hence, the tangent line to the solution curve at $(0, 1)$ has gradient -1. This shows that the solution curve is decreasing at this point. When it crosses the isocline corresponding to $c = 0$, it has a horizontal tangent line, and when it reaches the isocline corresponding to $c = 1$, it is increasing with gradient 1. The diagram below illustrates this part of the solution curve.



To extend the solution curve from the previous example, you can draw more isoclines.

Example 2

★★☆

Use the isoclines from the previous example to sketch a solution curve defined on $[-3, 3]$ for the differential equation $y' = x - 2y + y^2$ that passes through the point $(0, 1)$.

Student
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First, you need more isoclines. Like the calculations above, the isoclines are parabolas with equations $x = -y^2 + 2y + c$. The labelled isoclines without the slope field look something like this:

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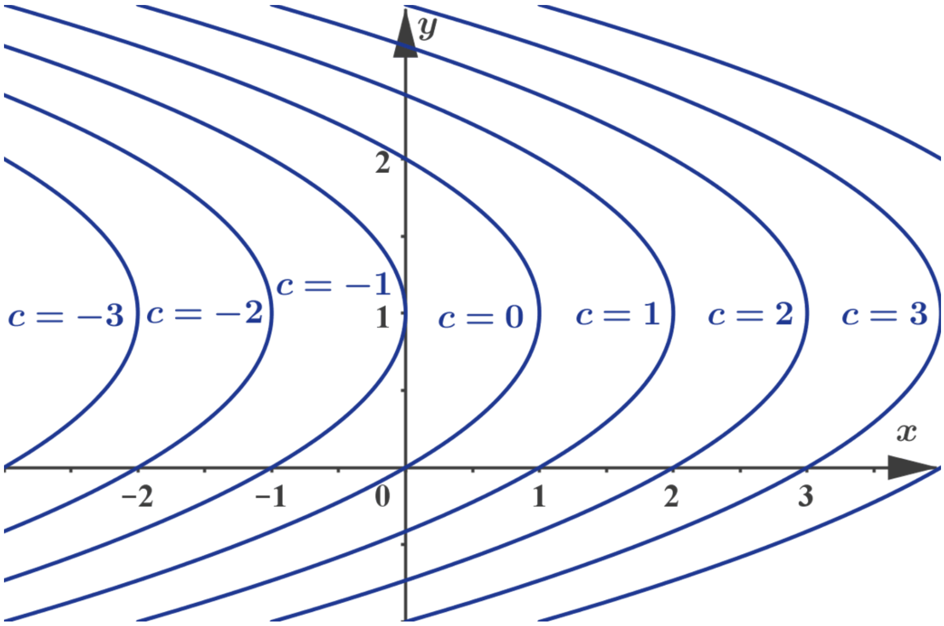


You already have part of this sketch from **Example 4** above.

To extend it to the right, notice that the gradient is increasing (since the isoclines to the right correspond to gradients greater than 1).

Extending the curve to the left is more tricky. The explanation below gives you the reasons in an informal way. In an exam, if a question like this is asked, then a correct sketch of the curve is expected, and a few words of explanation.

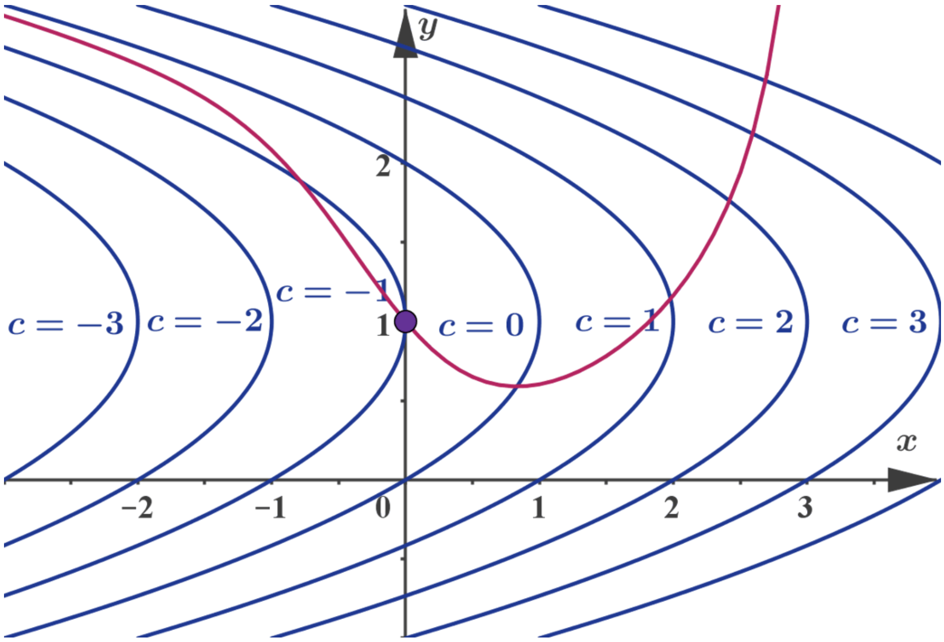
First, notice that the curve will not reach the isocline corresponding to $c = -2$. This is, of course, not a precise observation but, informally, the isocline for $c = -2$ is far from the point $(0,\ 1)$ on the isocline corresponding to $c = -1$. A solution curve needs to be quite flat to reach it. On the other hand, the gradient of the solution curve is less than –1 to the left of the isocline $c = -1$, so it cannot be too flat. As mentioned before, this is not a precise argument, but for an approximate sketch, you will not need any more details.

In addition, similar arguments (considering slopes) will also show that the curve does not stay below the isocline $c = -1$ forever, so it crosses it again somewhere to the left of the y-axis.

Student view

To the left of this second crossing point, it will now always stay between the isoclines $c = -1$ and $c = 0$. Again this will not be proved precisely, but informal arguments using slopes can be used to justify this claim. For example, it does not cross the isocline corresponding to $c = 0$, because at the crossing point the slope should be 0, but the shape of the isocline contradicts this.

The diagram below shows the actual solution curve.



The following video explores the differential equation $y' = x - 2y + y^2$ and the application of isoclines as laid out in **Examples 1** and **2** above.

Student view

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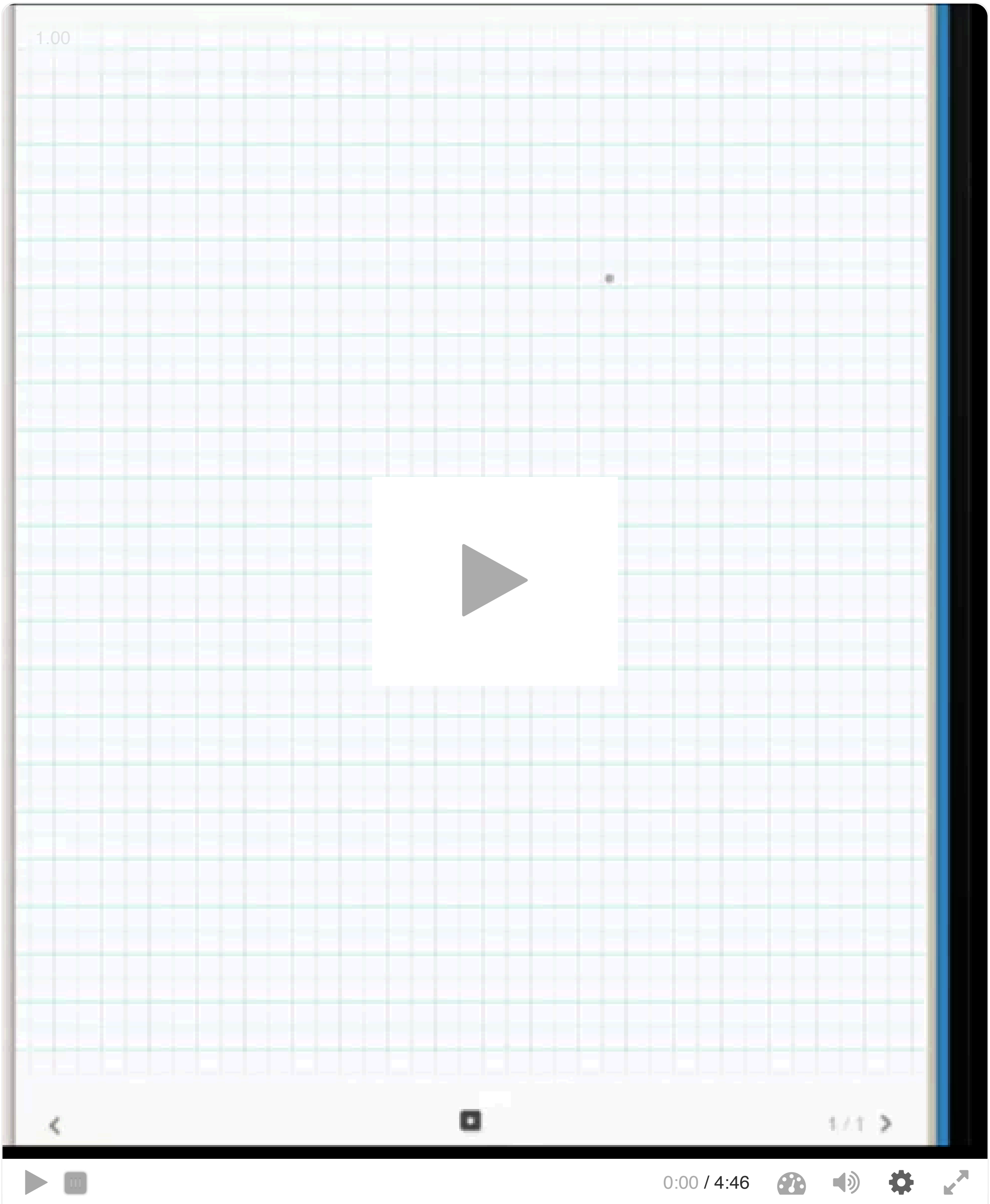

Video 1. Isoclines.

More information for video 1

1

00:00:00,033 --> 00:00:01,933

narrator: In this video,

we're going to study isoclines

2

00:00:02,000 --> 00:00:04,533

Student
view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

and how they can help

us giving the solution

3

00:00:04,633 --> 00:00:07,733

to an equation $y' = F(x, y)$.

4

00:00:08,000 --> 00:00:11,400

That is a differential equation.

5

00:00:12,267 --> 00:00:14,900

Now let us choose

the differential equation y'

6

00:00:14,967 --> 00:00:18,000

is $x - 2y + y^2$.

7

00:00:18,233 --> 00:00:22,533

Then the isoclines

are solutions to the $y' = C$,

8

00:00:22,600 --> 00:00:25,000

where C is a constant.

9

00:00:25,933 --> 00:00:28,933

For our case,

that then means of course, that

10

00:00:29,433 --> 00:00:33,567

$x - 2y + y^2 = C$, where C is a constant,

11

00:00:33,633 --> 00:00:36,467

which in fact is the gradient

12

00:00:36,533 --> 00:00:40,667

at certain points on the xy plane,

which of course gives you slope field.

13

00:00:42,467 --> 00:00:47,700

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Now this means that $x = -y^2 + 2y + C$.

14

00:00:48,333 --> 00:00:51,633

Now we need to analyze

these to give us the isocline curves.

15

00:00:51,700 --> 00:00:53,500

But let's first consider a simpler one,

16

00:00:53,567 --> 00:00:57,100

and that is $y = -x^2 + 2x + C$,

17

00:00:57,167 --> 00:00:59,933

which of course we can complete

a square quite easily.

18

00:01:00,133 --> 00:01:03,100

Make sure that you be careful

with the minus signs,

19

00:01:03,167 --> 00:01:07,333

which gives you $y = -(x - 1)^2 + 1 + C$.

20

00:01:07,433 --> 00:01:09,800

So it is a parabola upside down,

21

00:01:09,900 --> 00:01:13,933

and the vertex is given

by $(1, 1 + C)$,

22

00:01:14,233 --> 00:01:17,167

and you have drawn

this quadratic curve $y = -(x - 1)^2 + 1 + C$.

23

00:01:17,233 --> 00:01:18,867

minus 1 square plus 1 plus C.

24

00:01:19,167 --> 00:01:20,633

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Now I'm going to manipulate C,

25

00:01:20,700 --> 00:01:23,800

which of course amounts to nothing

more than a vertical translation.

26

00:01:23,967 --> 00:01:28,700

So you can see here that the green point

of vertex remains at the same x position,

27

00:01:28,767 --> 00:01:30,467

but simply change this y.

28

00:01:30,533 --> 00:01:32,933

as I change the C.

29

00:01:33,067 --> 00:01:36,600

Of course,

I can move the vertex up by increasing C,

30

00:01:36,700 --> 00:01:39,467

but of course I can also

move the vertex down

31

00:01:39,533 --> 00:01:40,800

by decreasing C.

32

00:01:41,167 --> 00:01:43,200

Now, let's create a family of curves

33

00:01:43,267 --> 00:01:46,733

by tracing all those

quadratic functions as I step

34

00:01:46,800 --> 00:01:49,167

through C in steps of one,

35

00:01:49,233 --> 00:01:51,800

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

and this creates

this family of quadratic curves.

36

00:01:52,200 --> 00:01:53,233

Now here I've shown it again.

37

00:01:53,333 --> 00:01:57,133

Now of course, this was not

the original function we had there.

38

00:01:57,200 --> 00:01:58,867

The y's and the x's were interchanged,

39

00:01:58,933 --> 00:02:02,667

but that amounts nothing more

than rotating the axis system

40

00:02:02,933 --> 00:02:05,833

y to x, which I'm simply gonna do by

41

00:02:05,900 --> 00:02:07,700

rotating physically this picture.

42

00:02:07,800 --> 00:02:11,067

So here I've done

that just rotated clockwise,

43

00:02:11,233 --> 00:02:15,400

and all I need to do now

is rename the axis, y vertical,

44

00:02:15,467 --> 00:02:17,800

x horizontal, and of course equation.

45

00:02:18,100 --> 00:02:20,833

Then I need to also

interchange the x and y,

46

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

00:02:20,900 --> 00:02:24,900

and there is our quadratic

with the isoclines.

47

00:02:24,967 --> 00:02:28,567

So here I've redrawn

the isoclines $x = -y^2 + 2y + C$ for various values

48

00:02:28,667 --> 00:02:34,567

of C ranging from minus 3 to 3.

49

00:02:34,667 --> 00:02:37,267

Now, let's make our solution

go through point $(0, 1)$.

50

00:02:37,333 --> 00:02:39,933

So I'm gonna create

that point right there.

51

00:02:40,000 --> 00:02:42,167

And now let's remember

what those values of C means.

52

00:02:42,233 --> 00:02:44,700

That means that the gradient,

53

00:02:44,767 --> 00:02:47,300

anywhere along those

isocline is that value.

54

00:02:47,400 --> 00:02:51,167

So minus 1 at a point $(0, 1)$,

55

00:02:51,233 --> 00:02:53,833

then 0 for $C = 0$, 1 at $C = 1$,

56

00:02:53,900 --> 00:02:56,433

greater 2 at $C = 2$ and 3 at $C = 3$.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

57

00:02:56,700 --> 00:02:58,633

Now I can approximate

58

00:02:58,733 --> 00:03:01,100

the solution for this

curve going through the point

59

00:03:01,533 --> 00:03:05,367

by connecting the gradient slopes,

60

00:03:05,567 --> 00:03:08,667

the gradient segments at those isoclines.

61

00:03:08,867 --> 00:03:10,033

So you have shown a few of them

62

00:03:10,100 --> 00:03:13,133

and then a smooth curve

should connect them,

63

00:03:13,667 --> 00:03:16,733

which looks roughly like so.

64

00:03:17,400 --> 00:03:19,733

Now the one to the right

of the point is rather easy.

65

00:03:19,800 --> 00:03:21,633

The one to the left is a

little bit more tricky,

66

00:03:21,967 --> 00:03:25,967

so it has a gradient of minus

1 at the point $(0, 1)$,

67

00:03:26,233 --> 00:03:27,567

and then a little bit to the left of that.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

68

00:03:27,633 --> 00:03:30,500

It's a little bit steeper,

but negative of course.

69

00:03:30,733 --> 00:03:35,000

So you can see that it reaches

the C minus line again,

70

00:03:35,100 --> 00:03:36,567

roughly where I've drawn it.

71

00:03:36,833 --> 00:03:39,800

And of course, the next

isocline up is the C equal 0.

72

00:03:39,867 --> 00:03:40,767

So it's horizontal.

73

00:03:40,833 --> 00:03:44,933

So you see that the red line never

becomes horizontal

74

00:03:45,000 --> 00:03:47,033

before it can hit the blue line,

75

00:03:47,100 --> 00:03:49,400

and so therefore it never quite

crosses it.

76

00:03:49,467 --> 00:03:52,833

Now let's trace this with a bigger

font size, and there you've got it.

77

00:03:53,267 --> 00:03:56,367

We can, of course,

verify this using technology.

78

Student view

Overview
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00:03:56,500 --> 00:03:58,567

So we have drawn the isocurves,

79

00:03:58,700 --> 00:04:01,133

and now I'm going to draw the slope field

80

00:04:01,200 --> 00:04:03,033

using technology over here.

81

00:04:03,367 --> 00:04:06,233

Of course, remember

that the little segments

82

00:04:06,333 --> 00:04:09,367

that make up a slope

field take on the values

83

00:04:09,467 --> 00:04:11,867

of C when crosses those isoclines.

84

00:04:12,033 --> 00:04:13,600

Now the point was $(0, 1)$.

85

00:04:13,667 --> 00:04:16,367

So the orange point is

what the solution needs to go through.

86

00:04:16,500 --> 00:04:18,933

And let's draw

the right hand branch first.

87

00:04:19,033 --> 00:04:22,000

And there you can see

this is the right hand branch across

88

00:04:22,067 --> 00:04:24,800

the isoclines at the

appropriate gradients.

Student
view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

89

00:04:25,100 --> 00:04:28,100

And on the left hand side,

the solution is here roughly

90

00:04:28,167 --> 00:04:29,667

what we saw.

91

00:04:30,233 --> 00:04:32,067

So this is using technology

92

00:04:32,133 --> 00:04:34,867

and let's just compare

to what we had by hand.

93

00:04:34,933 --> 00:04:39,433

And this is our solution by hand,

which agrees quite well,

94

00:04:40,200 --> 00:04:43,333

and that is how to use isoclines

to find approximate solutions,

95

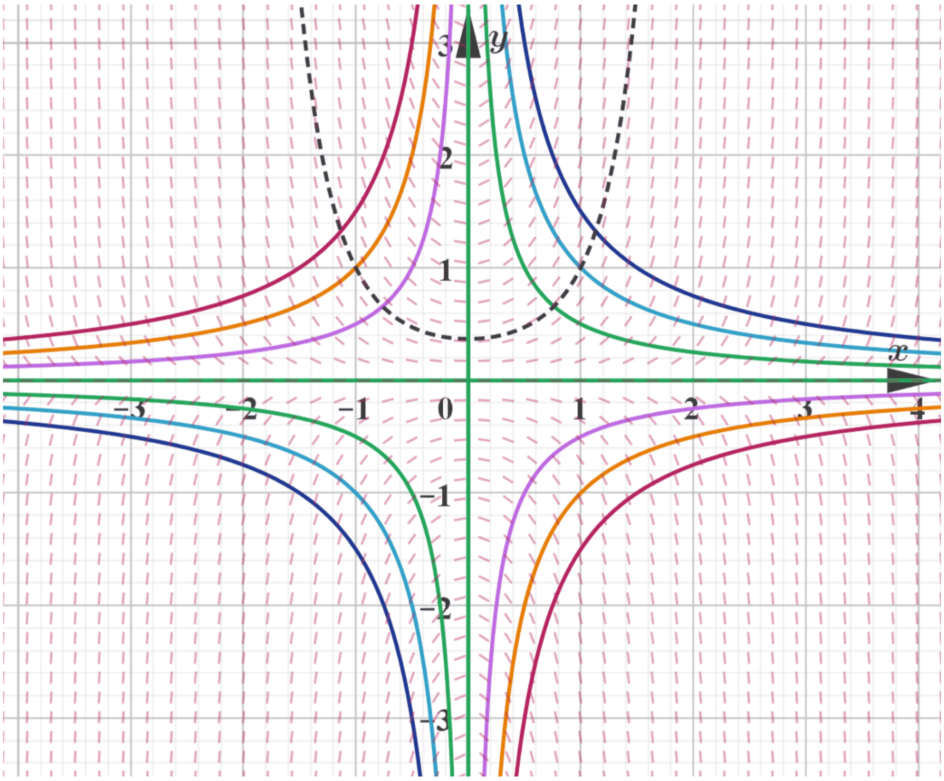
00:04:43,467 --> 00:04:46,333

curves to a differential equation.

For the function $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy$, a handful of isoclines with the solution curve would look something like this:

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov



More information

The image shows a graph depicting isoclines and a solution curve for the differential equation (\frac{{\rm{d}}y}{{\rm{d}}x} = 2xy). The x-axis and y-axis are marked with a grid, but specific labels or scales are not provided, implying a general illustrative purpose. Several curves, representing isoclines, traverse through the grid, crossing each other to form distinct patterns and inflection points. The curves are colored differently for visual distinction, with noticeable turning points and intersections suggesting changes in direction correlating to the differential function's solutions.

[Generated by AI]

As mentioned earlier, you typically would not take the time to develop both the slope field and the isoclines, but this does help show the relationship between the slope field, isoclines and solution curve. They are all related.

3 section questions

5. Calculus / 5.15 Graphical approximations to differential equations

Checklist

Student view

What you should know

By the end of this subtopic you should be able to:

- estimate the solution of a first-order differential equation of the form $y' = F(x, y)$ through the use of slope fields and isoclines
- sketch the slope field and isoclines
- sketch solution curves guided by the geometry of the slope field or isoclines.

Investigation

In this section, you learned about using slope fields and the related isoclines to approximate solutions to differential equations.

Computing and drawing slope fields by hand is tedious and time-consuming. These techniques are supposed to save you time, so there has to be a better way. Software packages, such as GeoGebra, offers a quick way to create a slope field.

Open up GeoGebra. In the space for entering equations, type in ‘=slopefield’. The program should prompt you for the required information. There is a total of seven fields, but only the first is necessary. The entire function with all fields would look like:

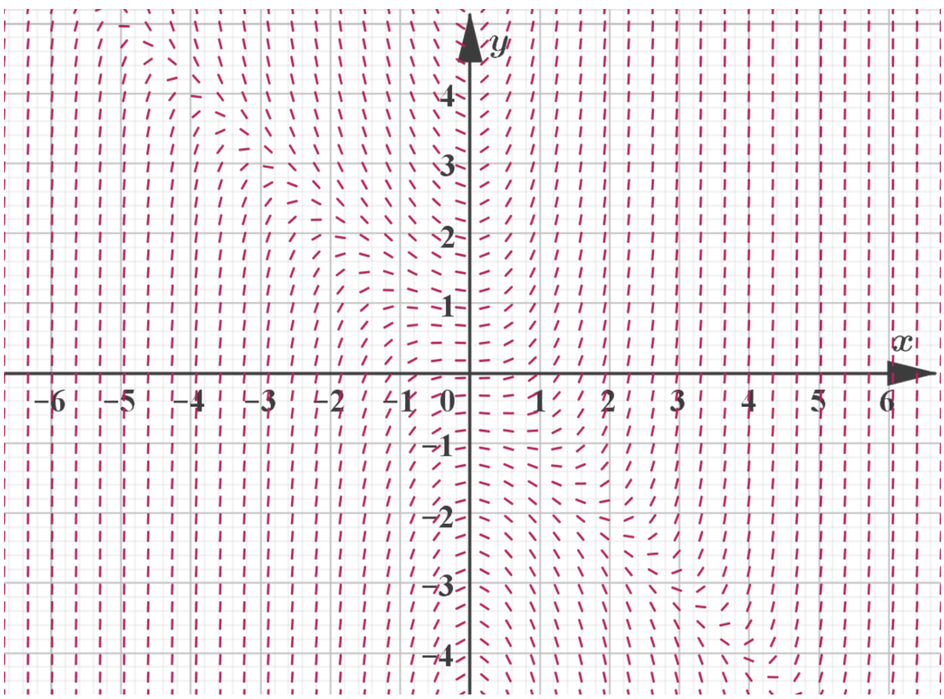
Slopefield(<f(x,y)>,<Number n>,<Length Multiplier a>,<Min x>,<Min y>,<Max x>,<Max y>)

Steps	Explanation
<f(x,y)>	This is the only required field, as it is the differential equation you are modelling.

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
<Number n>	This field sets the number of lines in each direction. For example, slopefield(xy,20) plots a slopefield with 400 lines, a 20×20 grid. GeoGebra adjusts the spacing based on the graphics view. The default is 40.
<Length Multiplier a>	With a range of $0 < a < 1$, this field determines the length of the lines. If you want longer lines, enter a number closer to 1. If you want shorter lines, enter a value closer to 0. The default is 0.5.
<Min x>, . . . ,<Max y>	If used, the last four fields work as a group. They establish the limits of the rectangular grid for the slope field. The default is to fill the entire graphics view.

To model the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = xy + x^2$ using all of the default settings, the screen would look like:



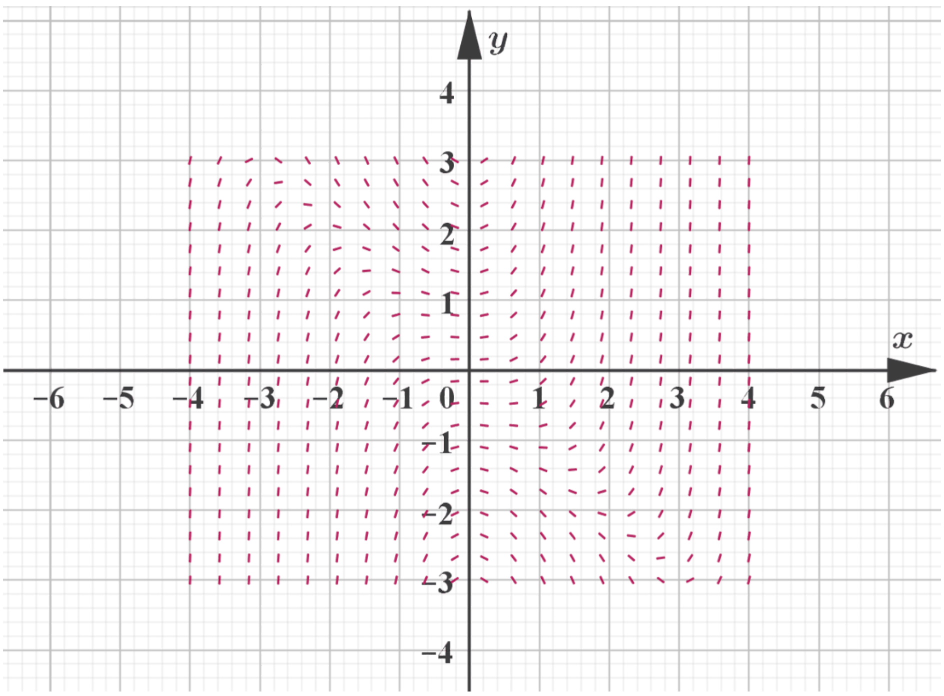
Student view

More information

The image is a graph displaying the differential equation (\frac{{\rm{d}}y}{{\rm{d}}x} = xy + x^2) on a grid. The grid is densely populated with small squares, creating a background for the vector field. Arrows within the grid show the slope direction and magnitude across different points. The axes are present with numerical labels visible along the X-axis and Y-axis, which extend to include negative and positive integer values. The arrows form a pattern indicating the direction and rate of change of (y) with respect to (x). The pattern is notably denser in the central region, indicating greater variations.

[Generated by AI]

Using the optional fields, you could reduce the number of lines in the grid, shorten the lines, and limit the grid to a specific region. The screen might look something like:



More information

The image shows a grid overlaid on a 2D graph, representing a vector field. It features intersecting lines that form squares, creating a coordinate grid on the plane. Along the grid lines, small vectors (arrows) are drawn, pointing in various directions and suggesting the flow or direction of the field within this space. The X and Y axes are labeled, with the X-axis ranging from approximately -6 to 7 and the Y-axis from approximately -6 to 7. The arrows are contained mostly within a central area, forming a pattern that suggests a systematic field orientation.

[Generated by AI]

Enter in your own differential equations and play with the fields. What settings provide you with the best balance between information and clutter?

Rate subtopic 5.15 Graphical approximations to differential equations

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