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The big picture

Two graphs may appear different, but their underlying functions may be the same up to a transformation. Why do you explore transformations? Each class of functions has inherent and specific features; for example, all quadratic functions have one turning point. However, a quadratic function may be transformed so that the position of its turning point can be put anywhere. This is important if you want to use a quadratic function to model, say, projectile motion.

A projectile travelling freely through the air follows a path that is a quadratic relation between the up–down and forwards directions. However, the exact shape of the projectile's path will depend on the height it was launched from and the speed and inclination it was launched at (on the figure below the launching height is the same, but the inclination is different for the three paths). Using the underlying quadratic relation between the up–down and forwards directions and the rules of transformations, you will be able to fit a unique quadratic function to a unique projectile path.

In this section you will explore the following types of transformations:

- translations
- stretches
- reflections
- composite transformations.

Overview
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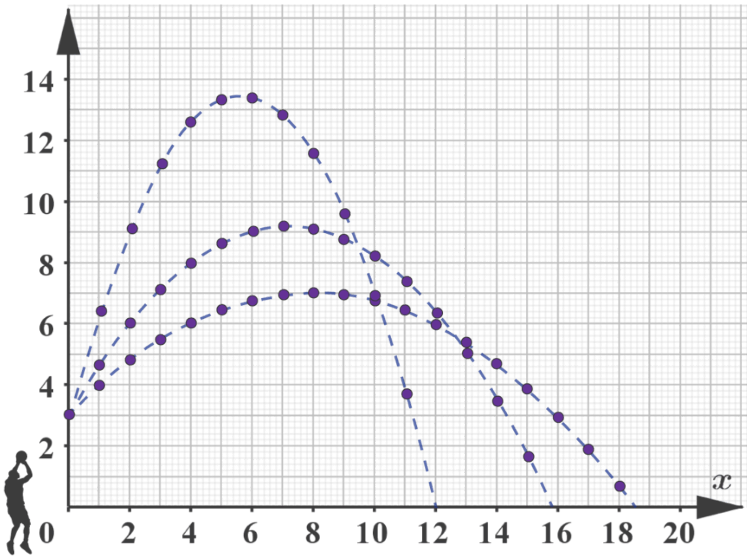

More information

The image is a graph with a grid background. It features the X-axis labeled with values from 0 to 20 increasing in increments of 2 and the Y-axis labeled with values from 0 to 16. Three dotted curves are plotted on the graph, each displaying a different trajectory. The highest curve peaks around X-axis value 8, reaching a Y-axis value slightly above 14, and then tapers off by 18. The second curve, medium in height, peaks just below the 12 Y-axis value around X 8 and then declines. The third, lowest curve reaches a Y value around 10 and shows a similar pattern of rise and fall, peaking around X 6. All curves have data points marked as dots. In the bottom left, there's a silhouette of a person poised as if preparing to throw a ball, emphasizing the context of projectile motion.

[Generated by AI]

Concept

Knowing the graphs of common functions and how the equation of a function can be systematically **changed** to transform the graph is useful when showing how transformations are applied to mathematical functions using the coordinate system. This skill is useful for sketching graphs of more complex functions by hand and for **representing relationships** using formulae and graphs.

While learning how to transform graphs, reflect on how making changes to a function affects its graphical representation.

Think about the effect of the order of transformations on the resulting graph.

Student view

2. Functions / 2.11 Transformation of graphs

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Translations

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/translations-id-26732/print/) Assign

In this section you will explore translations of the forms $y = f(x - a)$ and $y = f(x) + b$.

Activity

The following applet displays the graph of function $y = f(x)$. Adjust slider a to visualise the graph of the transformed function $y = f(x - a)$.

What effect does the constant a have?

Generalise your observations by forming a rule.

Interactive 1. Graph of a Transformed Function.

More information for interactive 1

This interactive enables users to explore the concept of horizontal transformations of functions through the relationship $y = f(x - a)$. The original function $y = f(x)$, shown in blue and the horizontally shifted version $y = f(x - a)$, shown in orange. By adjusting the slider for parameter a (ranging from −5 to 5), users can observe how the graph of $y = f(x)$ shifts left or right in real-time. The tool visually demonstrates that positive values of (a) translate the graph a units to the right, while negative values shift it |a| units to the left, reinforcing the rule that subtracting a inside the function's argument moves the graph in the direction opposite to a's sign.

Student view

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For example, when $a = 2$, the graph of $y = f(x - 2)$ shifts 2 units right from its original position, while setting $a = -3$, the graph of $y = f(x + 3)$ moves the graph 3 units left. This hands-on exploration helps users intuitively grasp how horizontal transformations affect function graphs, bridging algebraic notation with geometric interpretation.

Important

For the transformation $y = f(x - a)$, the effect of a is to translate the graph horizontally through a units.

- If $a > 0$, the transformation shifts the graph a units to the **right**.
- If $a < 0$, the transformation shifts the graph $|a|$ units to the **left**.

Notice that each point on the graph of $y = f(x)$ is mapped to a new point on the graph of $y = f(x - a)$, with a new x-coordinate $x - a$, while retaining the same value of its y-coordinate.

Activity

The following applet displays the graph of a different function $y = f(x)$. Adjust slider b to visualise the graph of the transformed function $y = f(x) + b$.

What effect does the constant b have?

Generalise your observations by forming a rule.

Student
view

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Interactive 1 . Graph of the Transformed Function.

More information for interactive 1

This interactive enables users to explore the concept of vertical transformations of functions through the relationship $y = f(x) + b$. By adjusting the slider for parameter b (ranging from −3 to 3), users can observe how the graph of $y = f(x)$ shifts up or down in real-time. The original function $y = f(x)$, shown in blue color and the horizontally shifted version $y = f(x) + b$, shown in orange color. The tool visually demonstrates that positive values of b translate the graph b units upward, while negative values shift it $|b|$ units downward, reinforcing the rule that adding b outside the function moves the graph in the same direction as b's sign.
For example, when $b = 2$, the graph of $y = f(x) + 2$ shifts 2 units up from its original position, while setting $b = -3$ moves the graph 3 units down.
This hands-on exploration helps users intuitively grasp how vertical transformations affect function graphs, connecting algebraic notation with visual representation.

Important

For the transformation $y = f(x) + b$, the effect of constant b is to translate the graph **vertically** through b units.

- If $b > 0$, the transformation shifts the graph b units **upwards.**
- If $b < 0$, the transformation shifts the graph $|b|$ units **downwards**.

Student view

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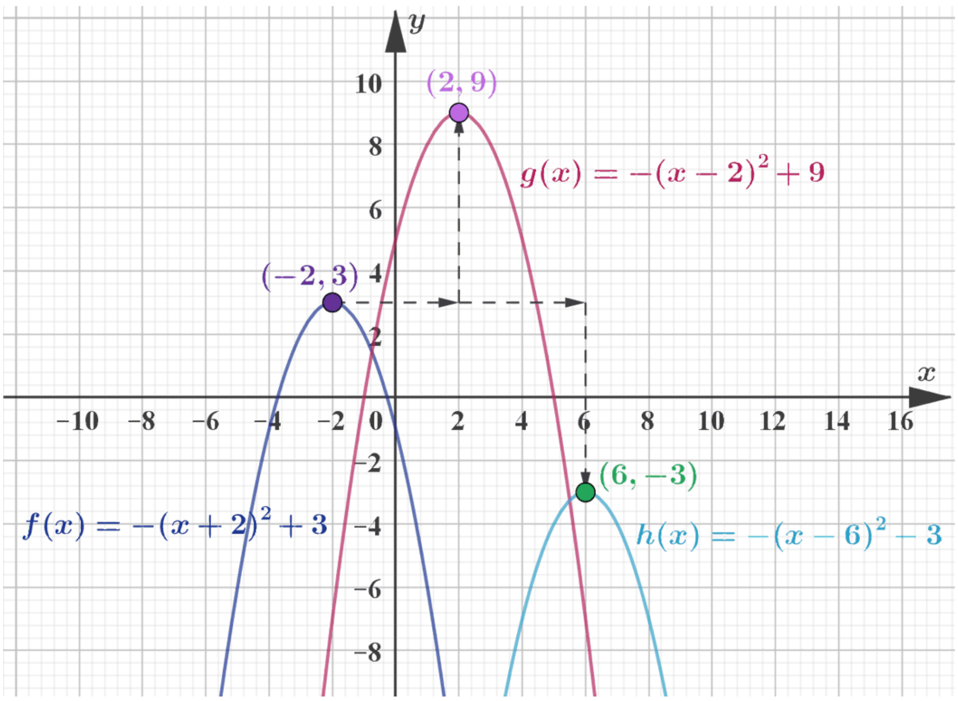
Notice that each point on the graph of $y = f(x)$ is mapped to a new point on the graph of $y = f(x) + b$, with a new y-coordinate $y + b$, while retaining the same value of its x-coordinate.

It is convenient to indicate translations in vector notation: a horizontal translation by a units and a vertical translation by b units is the transformation of the graph by vector $\begin{pmatrix} a \\ b \end{pmatrix}$. The result of a translation by $\begin{pmatrix} a \\ b \end{pmatrix}$ for a function is $f(x) \to f(x-a) + b$.

For example, consider the quadratic function $f(x) = -(x+2)^2 + 3$, which has a maximum at$(-2,\ 3)$. Under a translation of, say, $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$, the maximum point $(-2,\ 3)$ will be mapped to a new point with coordinates of $(-2+4,\ \ 3+6) = (2,\ 9)$. Function f becomes the translated function $g(x) = f(x-4) + 6 = -(x-2)^2 + 9$. Under a translation by a vector $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$, the maximum point $(-2,\ 3)$ will be mapped to a new point with coordinates of $(-2+8,\ \ 3-6) = (6,\ -3)$. Function f becomes the translated function $h(x) = f(x-8) - 6 = -(x-6)^2 - 3$. The figure below shows the graph of function f and the graphs of the transformed functions g and h.



The graph of function f (blue curve) and the graphs of the resulting functions when f is translated by a vector $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ (purple curve) and by a vector $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$ (green curve)

More information

Student view

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The image is a graph depicting three parabolic curves on a grid, each representing a quadratic function. The X-axis ranges from -10 to 10, and the Y-axis ranges from -5 to 15.

1. **Blue Curve**: Represents the original function (f(x) = -(x+2)^2 + 3). It peaks at the point ((-2, 3)) on the graph.

2. **Purple Curve**: Shows the translation of the function (g(x) = -(x-2)^2 + 9), resulting from a translation vector of (\left(

46

\right)). This curve peaks at the point ((2, 9)).

3. **Green Curve**: Represents the second translation, (h(x) = -(x-6)^2 - 3), resulting from a translation vector of (\left(

8 − 6

\right)). The peak of this curve is at the point ((6, -3)).

Each curve has its equation labeled near its vertex point. The graph illustrates the concept of translating a quadratic function by shifting its vertex along with the grid.

[Generated by AI]

Example 1

★☆☆

Describe the transformations necessary to obtain the graph of $y = (x-2)^2 - 3$ from the graph of $y = x^2$. Sketch both graphs on the same set of axes.

The graph of $y = x^2$ is translated 2 units to the right and 3 units downwards, represented by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

The parabola $y = x^2$ has vertex at $(0, 0)$ and the transformed function $y = (x-2)^2 - 3$ has vertex at $(2, -3)$.

Student
view

Overview
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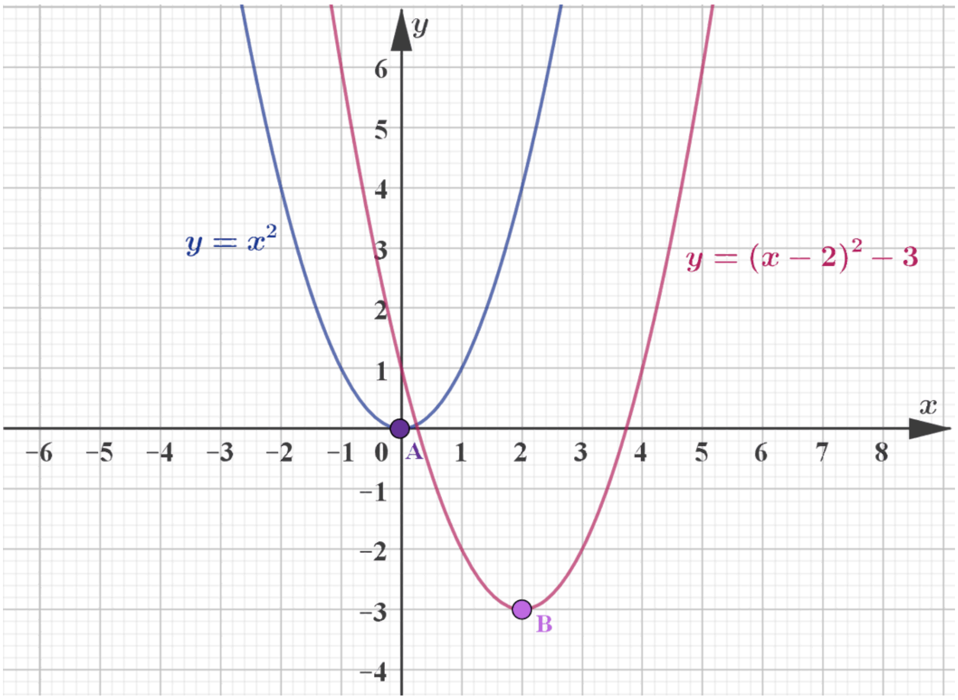

Example 2

★★☆

For the function $f(x) = \mathrm{e}^x$, sketch on the same set of axes the functions $y = f(x)$, $y = f(x-2)$, and $y = f(x+1) - 4$.

Student view

Overview
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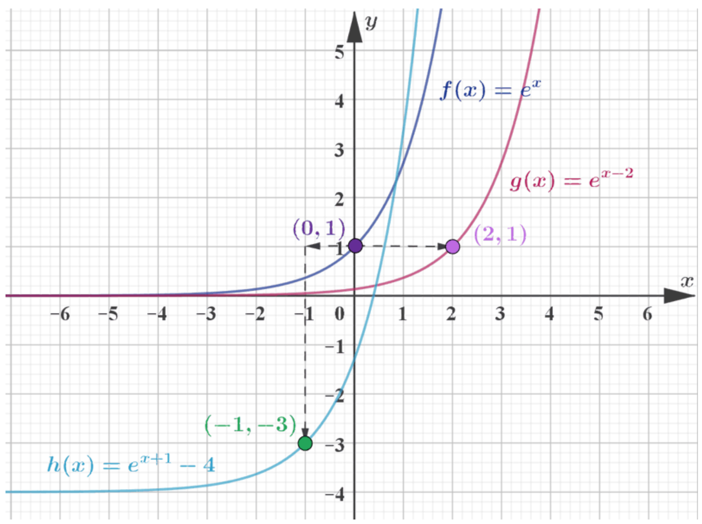
Steps	Explanation
	The graph of function $y = \mathrm{e}^x$ has y -intercept $(0, 1)$. The graph of $y = f(x-2)$ is a horizontal transformat of f by 2 units on the right. The graph of $y = f(x+1) - 4$ is a horizontal and vertica translation of f by a vector $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$.

3 section questions

2. Functions / 2.11 Transformation of graphs

Stretches

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/stretches-id-26733/print/) Assign

In this section you will explore transformations called stretches, which have the form $y = pf(x)$ and $y = f(qx)$.

Activity

The following applet displays the graph of a function $y = f(x)$. Adjust slider p to visualise the graph of the transformed function $y = pf(x)$.

What effect does the constant p have?

Generalise your observations by forming a rule.

Student view

Overview
(/study/app/math-aa-sl/sid-177-cid-761925/o

Interactive 1. Vertical Scaling and Reflection of Functions

More information for interactive 1

The interactive applet visualizes the effect of vertical scaling on the graph of a function $y = f(x)$. The applet includes a slider labeled p, which users can adjust within a range of -5 to 5. This controls the transformation of the original function $y = f(x)$ into the scaled function $y = pf(x)$.

When $p = 1$, the graph remains unchanged, displaying $y = f(x)$ in its original form as a smooth blue curve. The function retains its shape, and each point (x, y) stays the same.

If $p > 1$, the function undergoes a vertical stretch, increasing all y-values by a factor of p, making the graph appear taller. Conversely, when $0 < p < 1$, the function experiences a vertical compression, where all y-values are scaled down, making the graph appear flatter.

If $p < 0$, the function is reflected across the x-axis before being stretched or compressed. Negative values of p flip the graph upside down, while the magnitude of p determines the degree of stretching or compression.

In the provided image, the slider is initially set to $p = 1$ displaying the original function $y = f(x)$ in blue. When adjusted to $p = -2.2$, the transformed function $y = -2.2f(x)$ is shown in orange. The negative value of p causes the graph to reflect across the x-axis, while the factor of 2.2 results in a vertical stretch. This transformation is evident as each point (x, v) on the original graph is mapped to a new point $(x, -2.2v)$, where the y-coordinate is multiplied by -2.2, making peaks and troughs more pronounced and inverted.

This visualization helps users develop a deeper understanding of vertical scaling and reflection, reinforcing the relationship between the constant p and the transformation of the function.

Student view

Overview
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Important

For the transformation $y = pf(x)$, the effect of the constant p is a vertical stretch by a scale factor p.

- If $|p| > 1$, the graph is extended.
- If $|p| < 1$, the graph is compressed.
- If $p = -1$, the graph is reflected in the x-axis.

Notice that each point on the graph is mapped to a point with a new y-coordinate, $p \cdot y$, while retaining the same value of its x-coordinate.

For example, consider the function $f(x) = (x-2)^2 + 3$ with vertex at$(2,\ 3)$. Under the transformation of, say, $y = 2f(x)$ the vertex will be mapped to a point with new coordinates$(2, 2 \cdot 3) = (2,\ 6)$. The transformed function will have equation $y = 2f(x) = 2(x-2)^2 + 6$.

Activity

The following applet displays the graph of another function $y = f(x)$. Adjust slider q to visualise the graph of the transformed function $y = f(qx)$.

What effect does the constant q have?

Generalise your observations by forming a rule.

Student view

Interactive 2. Graph of a Transformed Function.

More information for interactive 2

The interactive applet allows users to explore the effects of horizontal scaling on the graph of a function by adjusting a slider for the constant q, which ranges from −2 to 2. This visualization helps users understand how the transformation from $y = f(x)$ to $y = f(qx)$ affects the function's shape.

When $q = 1$, the function remains unchanged, and the original graph of $y = f(x)$ is displayed as a smooth curve. However, when $q > 1$, the function undergoes a horizontal compression, meaning that all x-values are scaled closer together, making the graph appear narrower. Conversely, if $0 < q < 1$, the function experiences a horizontal stretch, where all x-values are spread out, making the graph appear wider.

If $q < 0$, the function is first reflected across the y-axis before being stretched or compressed. Negative values of q flip the graph horizontally, while the absolute magnitude of q determines the degree of stretching or compression.

For example, if $q = -1.5$, the function $y = f(-1.5x)$ is displayed in an orange color, illustrating both the reflection and the horizontal compression. Each point (x, y) on the original graph is mapped to $(x/q, y)$, shifting and transforming the function accordingly.

This interactive tool provides a hands-on way to understand the relationship between the scaling constant q and the transformation of the function, reinforcing key concepts of horizontal stretching, compression, and reflection.

Important

For the transformation $y = f(qx)$, the effect of the constant q is a horizontal stretch by a scale factor $\frac{1}{q}$.

- If $|q| > 1$, the graph is compressed.
- If $|q| < 1$, the graph is extended.
- If $q = -1$, the graph is reflected in the y-axis.

Notice that each point on the graph of $y = f(x)$ is mapped to a point with a new x-coordinate, $\frac{1}{q} \cdot x$, while retaining the same values of its y-coordinate.

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For example, consider the function $f(x) = (x-2)^2 + 3$ with vertex at $(2,\ 3)$. Under the transformation of, say, $y = f(2x)$ the vertex will be mapped to a point with new coordinates $\left(2 \cdot \frac{1}{2},\ 3\right) = (1,\ 3)$. The transformed function will have equation

$$y = f(2x) = (2x-2)^2 + 3 = [2(x-1)]^2 + 3 = 4(x-1)^2 + 3.$$

Example 1

★★☆

Consider the function with the graph $y = x^2$. Find the equation of the transformed function when f goes under the following transformations:

a) vertical stretch by a factor 2.

b) horizontal stretch by a factor 2.

c) vertical stretch by a factor $-\frac{1}{3}$.

d) horizontal stretch by a factor $-\frac{1}{2}$.

	Steps	Explanation
a)	A vertical stretch by a factor 2 will map function f to a new function $y = 2f(x) = 2x^2$.	Vertical stretch of a function $y = f(x)$ is the transformation $y = pf(x)$.
b)	A horizontal stretch by a factor 2 will map function f to a new function given by $y = f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$	The transformation of function $y = f(x)$ to $y = f(qx)$ is a horizontal stretch of a by a facto $\frac{1}{q}$.

Student view

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	Steps	Explanation
c)	A vertical stretch by a factor $-\frac{1}{3}$ will map function f to a new function $$y=-\frac{1}{3}f(x)=-\frac{1}{3}x^2\,.$$	Vertical stretch of a function $y=f(x)$ is the transformation $y=pf(x)$.
d)	A horizontal stretch by a factor $-\frac{1}{2}$ will map function f to a new function given by $$y=f(-2x)=(-2x)^2=4x^2.$$	The transformation of function $y=f(x)$ to $y=f(qx)$ is a horizontal stretch of a by a factc $\frac{1}{q}$.

Activity

This applet generates random stretches of a given function. Identify the transformation that the graph on the left goes through to produce the graph on the right, by choosing one of the four buttons provided.

Once the correct answer is clicked, the colour of the graph changes to green.

Interactive 3. Random Stretches of a Given Function.

More information for interactive 3

Student
view

This interactive helps users practice identifying horizontal and vertical stretches of functions. The original function is shown on the left side in blue, while its transformed version appears on the right side in red. Users must determine which transformation either a horizontal stretch or compression or a vertical stretch or compression was applied to obtain the new graph. Four answer choices are provided as options, and selecting the correct transformation turns the option green, while an incorrect choice turns red.
The applet generates multiple examples, allowing users to develop intuition for recognizing transformations by inspection.

3 section questions

Reflections

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/reflections-id-26734/print/) Assign

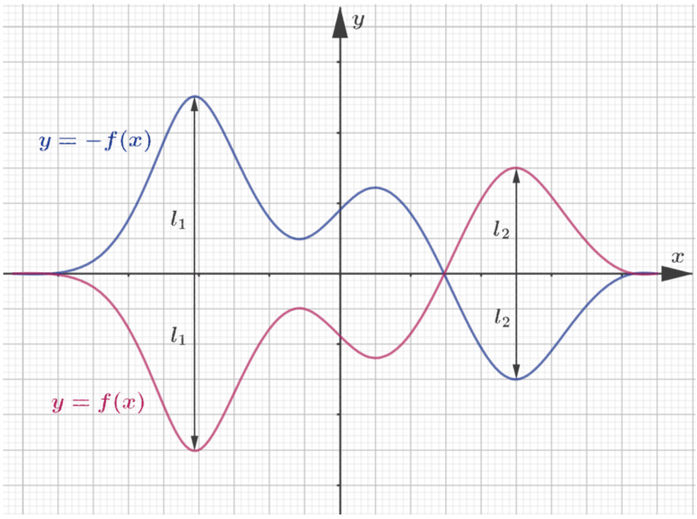
Two special cases of stretch transformations create reflections in the x-axis and y-axis.

Important

- The function $y = -f\left(x\right)$ is a reflection of $y = f\left(x\right)$ in the x-axis. This is a special case of a **vertical** stretch with scale factor -1.
- The function $y = f\left(-x\right)$ is a reflection of $y = f\left(x\right)$ in the y-axis. This is a special case of a **horizontal** stretch with scale factor -1.

The figures below show the two reflections of function $y = f\left(x\right)$ in the two axes.

Student view

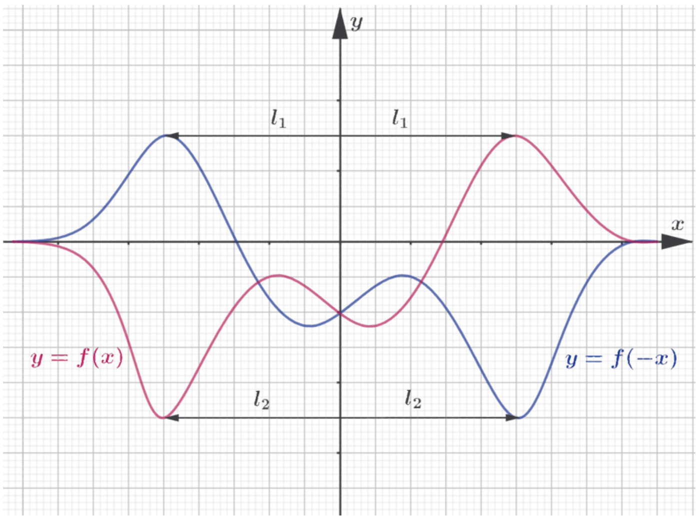


A reflection of function $y=f(x)$ in the x-axis

More information

The image is a graph depicting two functions, represented as curves, with one being the reflection of the other on the x-axis. The x-axis measures values from negative to positive, marked at regular intervals. The y-axis is labeled with values denoting the amplitude of the function. The original function curve is colored in blue and is labeled as (y = f(x)), while its reflection is shown in red. The curves display periodic waves with peaks alternating above and below the x-axis. Both curves intersect the y-axis at the origin and extend symmetrically to either side. The graph's background is a grid, helping visualize the scale and positioning of the curves along the axes.

[Generated by AI]



A reflection of function $y=f(x)$ in the y-axis

More information

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The image displays two reflections of a function on a coordinate grid. The X-axis represents variable x, with intervals marked at π/2, π, 3π/2, and 2π. The Y-axis represents the function values without specific units.

There are two wave-like curves plotted:

1. The red curve illustrates the function y = f(x) which has a peak around x = π/2 and a trough around x = 3π/2.
2. The blue curve represents the reflection y = f(-x), showing symmetry over the Y-axis.

The red curve crosses the x-axis between π and 3π/2, demonstrating typical wave behavior, and the blue curve replicates this on the negative side. Each wave exhibits oscillatory behavior with obvious symmetry over the Y-axis.

[Generated by AI]

Notice that when function $y = f(x)$ is reflected in the x-axis, the points with identical x-coordinates are equidistant from the x-axis, but on opposite sides, and when function $y = f(x)$ is reflected in the y-axis, the points with identical y-coordinates are equidistant from the y-axis, but on opposite sides.

Example 1

★☆☆

Consider a function f with a zero at $x = -3$ and a y-intercept at $y = 2$. Find the zero and y-intercept of the resulting function:

a) if f is reflected in the x-axis

b) if f is reflected in the y-axis.

a) A reflection in the x-axis will leave zero unchanged and transform the y-intercept to $y = -2$.

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view

Overview (/study/app/math-aa-sl/sid-177-cid-761925/o

b) A reflection in the y-axis will transform the zero to $x = 3$ and leave the y-intercept unchanged.

✓ **Important**

Points that are not moved under a transformation are called invariant points .

Example 2

★☆☆

Consider the function $y = 2x - 1$. Find the formula of the function:

1. when function $y = f(x)$ is reflected in the x-axis
2. when function $y = f(x)$ is reflected in the y-axis
3. graph all three functions on the same set of axes.

a) A reflection in the x -axis will map function $y = f(x)$ to the function

$$y = -f(x) = -(2x - 1) = -2x + 1\,.$$

b) A reflection in the y -axis will map function $y = f(x)$ to the function

$$y = f(-x) = 2(-x) - 1 = -2x - 1\,.$$

c) The graphs of all three functions are shown below.

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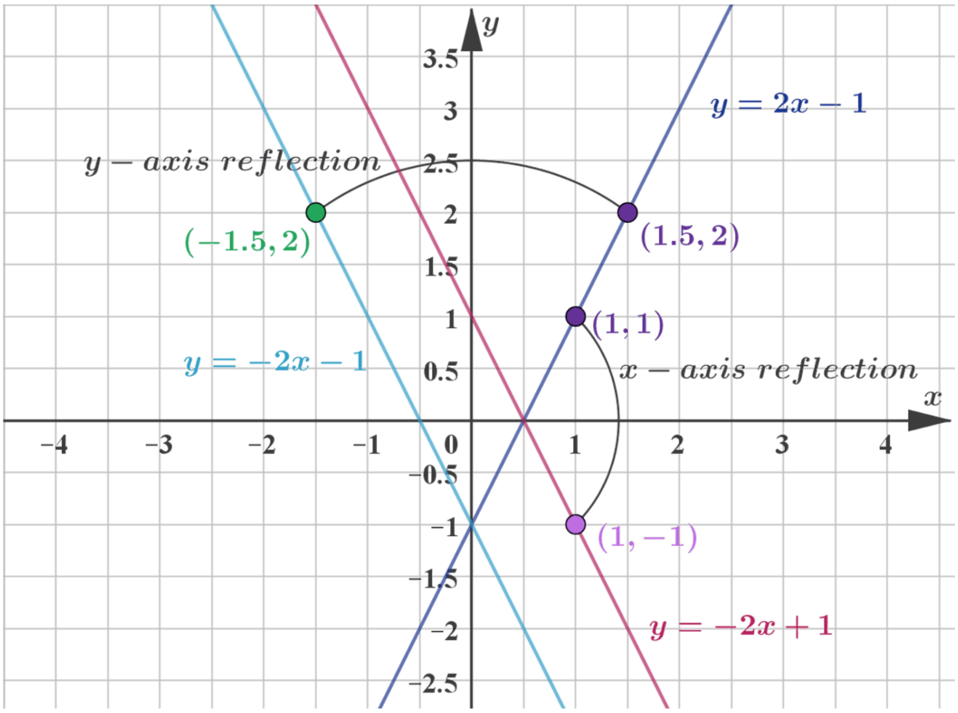

Making connections

Recall from subtopic 2.5 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26449/)that, given a function $y = f(x)$, the graph of the inverse function $y = f^{-1}(x)$ is a reflection of the graph of f in the line $y = x$.

Similarly, the graph of $y = f(x)$ is a reflection of $y = f^{-1}(x)$ in the line $y = x$.

Note that, in particular, if $y = f(x)$ intersects the line $y = x$, the graph $y = f^{-1}(x)$ intersects the line $y = x$ at the same point.

Be aware

In general, there are two types of transformations:

- **Rigid** transformations change the location of the graph in the coordinate plane but leave the size and shape unchanged.
- **Non-rigid** transformations change the size and/or shape of the graph.

Which type are translations, stretches and reflections?

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view

Overview
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International Mindedness

Dutch artist M. C. Escher (1898—1972) created mathematically inspired drawings and prints. His work was informed by his study of mathematical patterns in nature and architecture, such as this tiling at the Alhambra palace in Spain.

Mathematical depth can be found in mosaics
Credit: brytta Getty Images

Theory of Knowledge

The recognisable symmetry of the reflections in the first figure in this section can spark more TOK discussions of mathematical symmetry in other Areas of Knowledge. Mathematician Marcus du Sautoy examines in the following TED Talk not only what symmetry is but also how symmetry impacts our daily lives. He examines how mathematics is embedded in our world and shows how we may or may not recognise, measure or — as he keenly explores — move symmetrical objects.

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view

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Marcus du Sautoy: Symmetry, reality's riddle

5 section questions

2. Functions / 2.11 Transformation of graphs

Composite transformations

Section Student... (0/0) Feedback Print (/study/app/math-aa-sl/sid-177-cid-761925/book/composite-transformations-id-26735/print/) Assign

So far you have combined only two translations when transforming a function; i.e., translating a function by a units horizontally then b units vertically can be indicated by a translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$ to give $f(x) \to f(x-a)+b$. You can, of course, combine any of the four transformations in any sequence. However, be careful with the order in which you apply a composite transformation.

Do you think the order of execution of the four individual transformations generates the same resulting function? In other words, does the order of the transformations matter?

Making connections

In subtopic 2.5 (/study/app/math-aa-sl/sid-177-cid-761925/book/the-big-picture-id-26449/)you have seen that the composition of two functions f and g, where function g is applied first and function f is applied second, is the function

$$(f \circ g)(x) = f(g(x)).$$

Student view

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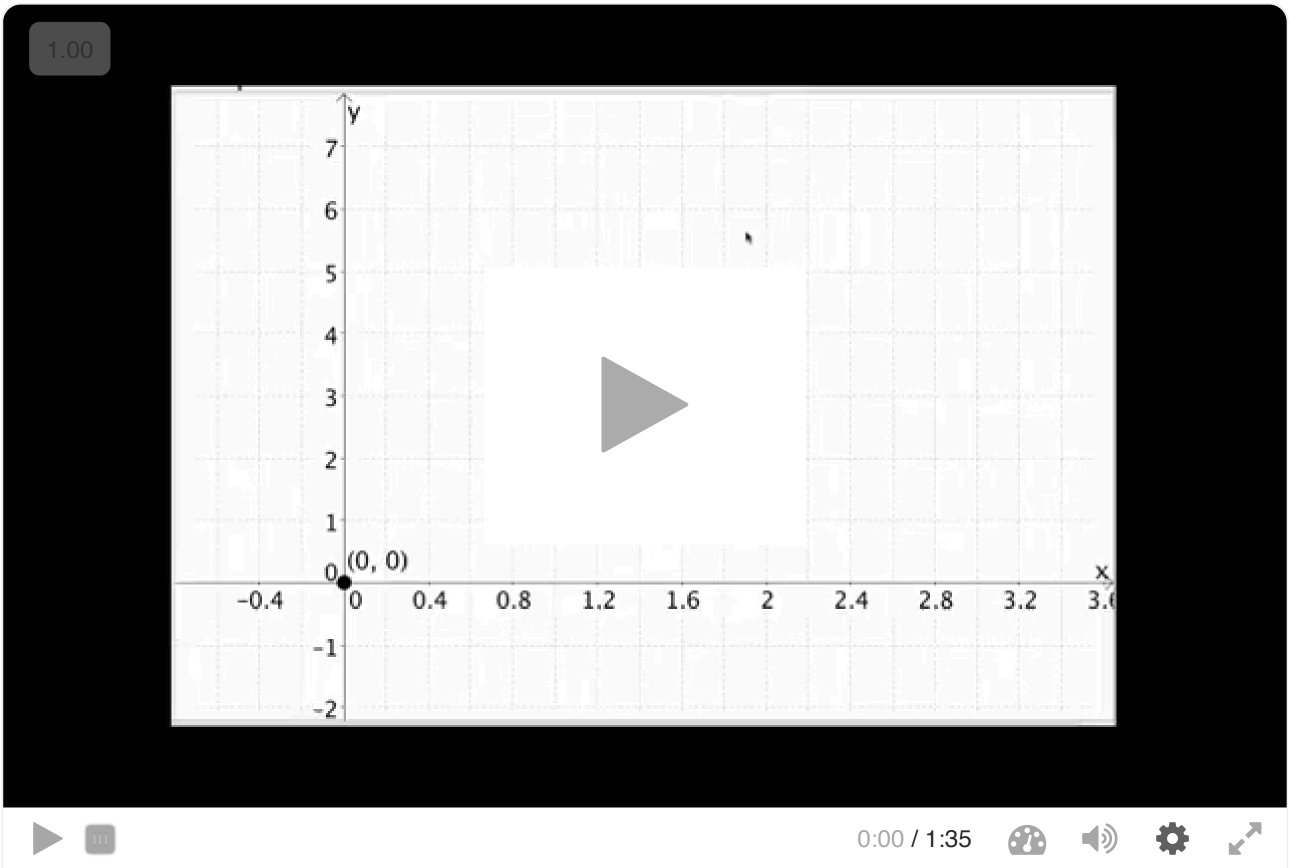
Reflect on composite functions and explain whether function transformations can be expressed as composite functions.

In the following video you will explore the transformation of a point with all four of the transformations that you have studied so far, using different orders of execution each time.



Video 1. Understanding Composite Transformations.

More information for video 1

1

00:00:00,133 --> 00:00:02,536

narrator: In this video we're going to

look at composite transformations

2

00:00:02,603 --> 00:00:04,872

where we combine the four transformation

we've been looking at.

3

00:00:04,938 --> 00:00:08,509

Student
view

Overview
(/study/app/math-aa-sl/sid-177-cid-761925/o

So we start at $(0, 0)$,

then go to $(2, 0)$,

4

00:00:08,575 --> 00:00:12,813

then go up to $(2, 3)$, $(2, 6)$,

and end up at $(1, 6)$.

5

00:00:12,880 --> 00:00:14,448

So how can we do it

with our transformations?

6

00:00:14,515 --> 00:00:17,751

Well, we can translate

horizontally by two units,

7

00:00:17,851 --> 00:00:20,454

then translate vertically by three units,

8

00:00:20,787 --> 00:00:23,357

then do a vertical

stretch by factor of two,

9

00:00:23,590 --> 00:00:26,193

and then do a horizontal

stretch by a factor of two,

10

00:00:26,260 --> 00:00:28,762

and that's how we can end up at $(1, 6)$.

11

00:00:28,996 --> 00:00:31,098

Now we may combine these

transformations differently.

12

00:00:31,164 --> 00:00:33,567

First of all, we can do

these stretch factors,

13

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00:00:33,634 --> 00:00:35,302

and multiplying zero

by anything leaves zero.

14

00:00:35,369 --> 00:00:38,672

Then we can go to $(2, 0)$

then to $(2, 3)$,

15

00:00:38,872 --> 00:00:42,509

so that we apply the p

and q stretches first.

16

00:00:42,676 --> 00:00:47,014

Then translate by two units to the right

and then three units to upwards.

17

00:00:47,181 --> 00:00:49,183

You can also see that we could have done

18

00:00:49,249 --> 00:00:52,653

after the stretch factor,

we gonna moved up by three units

19

00:00:52,786 --> 00:00:56,823

and then moved horizontally by two

and ended up at the same region.

20

00:00:56,957 --> 00:00:58,825

Now once again,

we started the origin here.

21

00:00:58,959 --> 00:01:02,396

We first go to $(2, 0)$

then to $(1, 0)$,

22

00:01:02,596 --> 00:01:03,897

and then to $(1, 3)$.

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23

00:01:03,964 --> 00:01:05,032

So how can we do this?

24

00:01:05,098 --> 00:01:07,868

Well, we can have a horizontal

translation by two units,

25

00:01:08,402 --> 00:01:11,605

then a horizonta stretch by factor of two.

26

00:01:11,672 --> 00:01:14,474

Then we can do the vertical

stretch by factor of two,

27

00:01:14,541 --> 00:01:17,411

leaving it alone,

and then translate upwards by three.

28

00:01:18,345 --> 00:01:23,150

By the four transformations,

$a = 2$, $b = 3$, $p = 2$, and $q = 2$.

29

00:01:23,283 --> 00:01:26,954

We've seen that we can end up

in three different places

30

00:01:27,287 --> 00:01:30,624

applying all those transformations.

In other words, or they may matter,

31

00:01:30,691 --> 00:01:32,559

doesn't always, as you've seen, but some.

32

00:01:32,659 --> 00:01:34,962

But it can very well matter.

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Important

When combining translations and stretches, the order in which you execute them may give different results.

Consider changing the order of a vertical translation by b and a vertical stretch by p :

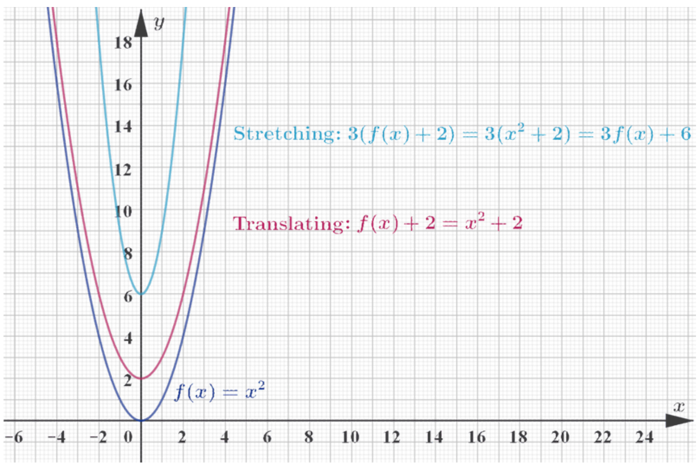
- First **translating** then **stretching** will transform:

$$f(x) \overset{translating}{\rightarrow} f(x) + b \overset{stretching}{\rightarrow} p(f(x) + b) = pf(x) + pb.$$

- First **stretching** then **translating** will transform:

$$f(x) \overset{stretching}{\rightarrow} pf(x) \overset{translating}{\rightarrow} pf(x) + b.$$

The figures below show the graph of function $f(x) = x^2$ and the graphs of the resulting functions when a translation of 2 steps upwards and a vertical stretch by a scale factor 3 are applied to the function f in different orders.



The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the vertical translation is followed by the vertical stretch.

More information

The image shows a graph illustrating the function f(x) = x^2 and the transformations it undergoes through vertical translation and stretching. The X-axis ranges from -10 to 18, labeled with integers. The Y-axis is labeled with values from 0 to 24 at intervals of 4. Three curves are visible:

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1. The original function f(x) = x^2, shown as a blue parabola symmetric about the Y-axis.
2. A translated graph representing f(x) - 2, which shifts the parabola downwards by 2 units.
3. A vertically stretched graph, which narrows the parabola compared to the original, illustrated as a pink curve.

Overall, the graphs demonstrate the effects of shifting and stretching on the basic quadratic function.

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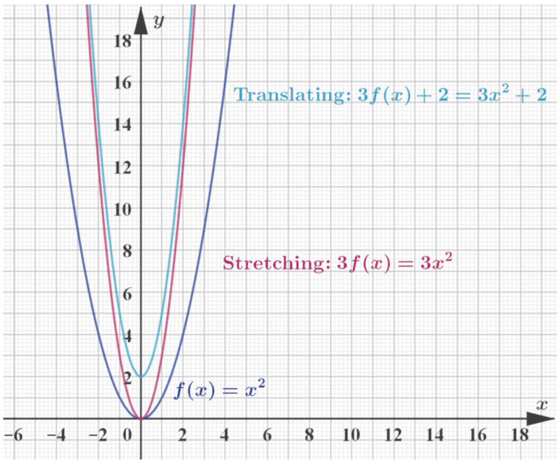

The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the vertical stretch is followed by the vertical translation.

More information

The image shows a graph depicting the quadratic function f(x) = x^2 and its transformations. The x-axis represents the input values ranging from -14 to 14, while the y-axis represents the output values of the functions from -5 to 20.

Three curves are visible: 1. A blue curve representing the original function f(x) = x^2, forming a standard parabola centered at the origin (0,0). 2. A red curve depicting a vertical stretch of the original function, expressed as g(x) = 3x^2, which is narrower than the blue curve due to the stretching factor. 3. A cyan curve representing a vertical translation, described by h(x) = 3x^2 - 9, shifted downwards, which starts from the lower section of the y-axis.

The graph visually illustrates the effect of both vertical stretching and translation on the quadratic function.

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Now consider changing the order of a horizontal translation by a and a horizontal stretch by $\frac{1}{q}$.

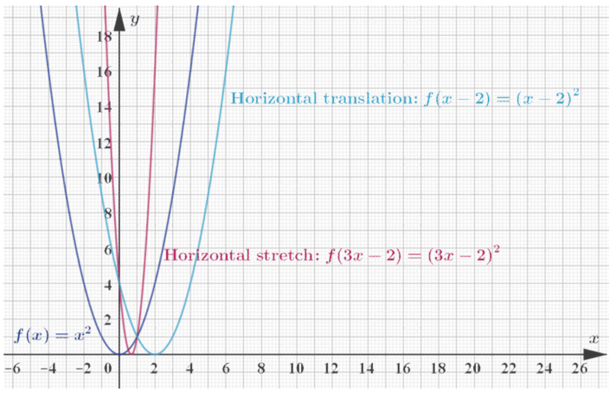
- First **translating** then **stretching** will transform:

$$f(x) \overset{translating}{\rightarrow} f(x-a) \overset{stretching}{\rightarrow} f(qx-a).$$

- First **stretching** then **translating** will transform:

$$f(x) \overset{stretching}{\rightarrow} f(qx) \overset{translating}{\rightarrow} f(q(x-a)) = f(qx-qa).$$

The following figures show the graph of function $f(x) = x^2$ and the graphs of the resulting functions when a translation of 2 steps to the right and a horizontal stretch by a scale factor $\frac{1}{3}$ are applied to the function f in different orders.



The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the horizontal translation is followed by the horizontal stretch.

More information

The image shows a graph of the quadratic function f(x) = x² and its transformations. The X-axis represents the values of x, while the Y-axis represents the values of f(x).

There are three parabolic curves, each representing a different transformation: 1. The original function f(x) = x² is centered at the origin and opens upwards. 2. The horizontally translated function f(x) = (x - 2)², shifted 2 units to the right, maintains the same shape and opens upwards. 3. The horizontally stretched function with a formula involving f(3x - 2), depicted as narrower and also opening upwards, follows the stretching transformation.

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The graph illustrates how horizontal translations and stretches affect the position and shape of the parabola on a coordinate plane.

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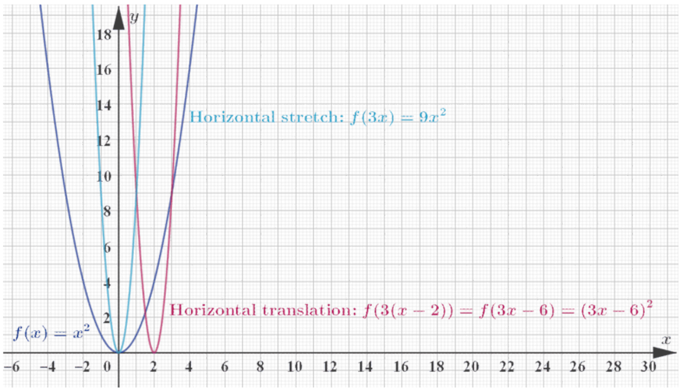

The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the horizontal stretch is followed by the horizontal translation.

More information

The image shows a graph illustrating the quadratic function f(x) = x², along with its transformations due to horizontal stretch and horizontal translation.

X-axis: Ranges from -10 to 10. Y-axis: Ranges from 0 to 100.

- The original function f(x) = x² is represented as a parabola with its vertex at the origin (0,0) and opens upwards.
- A horizontally stretched version of the function is shown, described by f(x) = (x/a)², where a represents the stretch factor. The parabola appears wider compared to the original.
- A horizontally translated version of the function, described by f(ax-b)² or similar expression, shifts the vertex of the parabola along the x-axis.

Text labels present include: "f(x) = x²", "Horizontal Stretch: f(x) = (x/a)²", and "Horizontal Translation: f(ax-b)²".

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Be aware

Different combinations of transformations may result in the same overall transformation.

Consider the line $y = 4 - 2x$. You can obtain this line by transforming the line $y = x$ in various ways:

1. A vertical stretch by a scale factor $p = -2$, followed by a vertical translation of 4 units upwards, $b = 4$: $x \to -2x \to -2x + 4$.
2. A horizontal translation of 2 units to the right, $a = 2$, followed by a vertical stretch with scale factor $p = -2$: $x \to x - 2 \to -2\left(x - 2\right) = -2x + 4$.

Now, consider the following three transformations in two different orders applied on the graph $y = x^2$:

1. A horizontal translation of 1 unit to the right, $a = 1$, a horizontal stretch by factor $q = \frac{1}{2}$ and a vertical translation of 3 units upwards, $b = 3$:

$$x^2 \to \left(x - 1\right)^2 \to \left(2x - 1\right)^2 \to \left(2x - 1\right)^2 + 3.$$

   Thus, the resulting function is $y = \left(2x - 1\right)^2 + 3 = 4x^2 - 4x + 4$.
2. A horizontal stretch by factor $q = \frac{1}{2}$, a horizontal translation of 1 unit to the right, $a = 1$, and a vertical translation of 3 units upwards, $b = 3$:

$$x^2 \to (2x)^2 \to \left(2\left(x - 1\right)\right)^2 \to 4(x - 1)^2 + 3.$$

   Thus, the resulting function is $y = 4(x - 1)^2 + 3 = 4x^2 - 8x + 7$.

Example 1

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Describe the sequence of transformations that need to be performed on the graph of $y = \cos(x)$ to obtain the graph of $y = -2\cos\left(\frac{1}{2}x + 2\right) - 1$, given that the horizontal stretch by a factor 2 occurred before the horizontal translation.

The given form $y = -2\cos\left(\frac{1}{2}x + 2\right) - 1$ of the graph implies that the horizontal translation 2 units to the left (indicated by the $+2$ in the cosine) was performed before the horizontal stretch by a scale factor 2 (indicated by the $\frac{1}{2}$ coefficient of x).

You can rewrite the function to represent that the horizontal stretch was performed before the horizontal translation, as follows:

$$y = -2\cos\left(\frac{1}{2}(x + 4)\right) - 1$$

This indicates that the horizontal stretch was performed before the horizontal translation and the series of transformations on the function $f(x) = \cos x$ is as follows:

1. Horizontal stretch by scale factor 2: $f(x) \to f\left(\frac{1}{2}x\right)$.
2. Horizontal translation by 4 units to the left: $f\left(\frac{1}{2}x\right) \to f\left(\frac{1}{2}(x+4)\right) = f\left(\frac{1}{2}x + 2\right)$.
3. Vertical stretch by scale factor -2: $f\left(\frac{1}{2}x + 2\right) \to -2f\left(\frac{1}{2}x + 2\right)$.
4. Vertical translation by 1 unit down: $-2f\left(\frac{1}{2}x + 2\right) \to -2f\left(\frac{1}{2}x + 2\right) - 1$.

Example 2

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Describe three different sequences of transformations that could be performed on the graph of $f(x) = x^2$ to obtain the graph of $g(x) = -3(2x - 1)^2 + 4$.

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	Steps	Explanation
a)	$g(x) = -3(2x-1)^2 + 4$	Start with the resulting func and describe the transforma
	$x^2 \to (x-1)^2$	Horizontal translation by 1 u the right.
	$(x-1)^2 \to (2x-1)^2$	Horizontal stretch by factor .
	$(2x-1)^2 \to -3(2x-1)^2$	Vertical stretch by factor p = Notice that this transformati also changes the function to concave down as the coeffi of x^2 becomes negative.
	$-3(2x-1)^2 \to -3(2x-1)^2 + 4$	Vertical translation by 4 unit upwards.
	In total, there are 4 transformations to obtain from $f(x) = x^2$ the graph of $$g(x) = -3(2x-1)^2 + 4\,.$$	
b)	$g(x) = -3\left(2\left(x - \frac{1}{2}\right)\right)^2 + 4$	Rewrite function f by factor out of the perfect square.
	$x^2 \to (2x)^2$	Horizontal stretch by factor .
	$(2x)^2 \to \left(2\left(x - \frac{1}{2}\right)\right)^2$	Horizontal translation by $\frac{1}{2}$ to the right.
	$\left(2\left(x - \frac{1}{2}\right)\right)^2 \to -3\left(2\left(x - \frac{1}{2}\right)\right)^2$	Vertical stretch by factor p =
	$-3\left(2\left(x - \frac{1}{2}\right)\right)^2 \to -3\left(2\left(x - \frac{1}{2}\right)\right)^2 + 4$	Vertical translation by 4 unit

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	Steps	Explanation
	In total, four transformations to obtain from $f(x) = x^2$ the graph of $$g(x) = -3(2x-1)^2 + 4\,.$$	Use that $2\left(x - \frac{1}{2}\right) = 2x$
c)	$g(x) = -12\left(x - \frac{1}{2}\right)^2 + 4$	Start with the form in the se sequence and factor -12 o the brackets.
	$x^2 \to \left(x - \frac{1}{2}\right)^2$	Horizontal translation by $\frac{1}{2}$ to the right.
	$\left(x - \frac{1}{2}\right)^2 \to -12\left(x - \frac{1}{2}\right)^2$	Vertical stretch by factor p = .
	$-12\left(x - \frac{1}{2}\right)^2 \to -12\left(x - \frac{1}{2}\right)^2 + 4$	Vertical translation by 4 unit
	In total, three transformations to obtain from $f(x) = x^2$ the graph of $$g(x) = -3(2x-1)^2 + 4\,.$$	

Exam tip

When you are asked to describe the sequence of composite transformations of the form $y = f(x) \to y = Af(Bx + C) + D$, consider the operations in the following order:

1. Horizontal translation (by $-C$ units to get the graph of $y = f(x + C)$)
2. Horizontal stretch (by a factor of $1/B$ to get the graph of $y = f(Bx + C)$)
3. Vertical stretch (by a factor of A to get the graph of $y = Af(Bx + C)$)
4. Vertical translation (by D units to get the graph of $y = Af(Bx + C) + D$)

Other order of operations can also work, but the parameters need to be adjusted according to the order used.

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For example, consider a function $y = f(x)$ that is transformed to $y = -2f\left(\frac{1}{3}x - 1\right) + 5$. Given that point $(2,\ 1)$ lies on the graph of $y = f(x)$, you can find the image of the point under the composite transformation by following the order of operations as shown in the table below.

Operation order	Transformation	Point under transformati
$y = f(x) \to f(x-1)$	Horizontal translation 1 unit to the right.	$(2, 1) \to (2+1, 1) = (3,$
$f(x-1) \to f\left(\frac{1}{3}x - 1\right)$	Horizontal stretch by a scale factor 3.	$(3, 1) \to (3 \times 3, 1) = (9,$
$f\left(\frac{1}{3}x - 1\right) \to -2f\left(\frac{1}{3}x - 1\right)$	Vertical stretch by scale factor -2 (negative sign indicates reflection)	$(9, 1) \to (9, 1 \times (-2)) = ($
$-2f\left(\frac{1}{3}x - 1\right) \to -2f\left(\frac{1}{3}x - 1\right) + 5$	Vertical translation 5 units upwards	$(9, -2) \to (9,\ -2+5) = ($

Activity

Think of your own example function and composite transformation, and explain whether the following statements are true or false.

1. Vertical or horizontal stretch order does not matter.
2. Vertical or horizontal reflection order does not matter.
3. Vertical or horizontal translation order does not matter.

3 section questions

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2. Functions / 2.11 Transformation of graphs

Checklist

What you should know

By the end of this subtopic you should be able to:

- transform graphs using horizontal and vertical translations
- transform graphs using horizontal and vertical stretches
- transform graphs using reflections about the x-axis and y-axis
- transform graphs using composite transformations.

2. Functions / 2.11 Transformation of graphs

Investigation

Below is an applet that you can use to explore all types of composite transformations.

Interactive 1. Types of Composite Transformations.

More information for interactive 1

This interactive allows users to explore composite transformations of functions by manipulating various parameters. Users begin by entering a simple parent function, such as x, x^2, x^3, e^x, in the blue input box. The applet then displays the graph of this original function, providing a visual starting point. By clicking the 'Target function' button, a randomly transformed version of the original function is generated in red color, complete with its equation and graph. This feature helps users understand how different transformations alter the parent function. The “Reset” button at the bottom-left allows users to start over.

The interactive includes sliders for adjusting parameters a, b, p, and q, which control horizontal and vertical translations, stretches, and compressions. Users can experiment with these sliders to match the transformed function's graph to the target graph. Additionally, two checkboxes allow users to toggle between the order of transformations—horizontal translation before stretch or vice versa. This flexibility enables users to observe how the sequence of transformations affects the final graph, deepening their understanding of function behavior.

As users adjust the sliders, the transformed graph updates dynamically, and the corresponding algebraic expression is displayed below. This immediate visual and algebraic representation helps users connect the graphical changes to mathematical transformations. For more precise adjustments, users can click on the sliders and use keyboard arrows for finer control, ensuring accuracy in matching the target function.

For example, if the original function is x^2 and the target function is $2(x + 2)^2 - 1$, Users can adjust the sliders to set $a = -2$, $b = -1$, $p = 2$, and $q = 1$ to achieve the desired transformation. The applet will display the transformed function as $2(x + 2)^2 - 1$, and the graph will align with the target. By toggling the order of transformations, users can explore alternative parameter combinations to reach the same result, reinforcing their grasp of composite transformations. This hands-on approach makes learning interactive and engaging.

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Start by typing a simple **original** or **parent function** in the blue input box (e.g. $x,\ x^2, x^3,\ e^x$, etc.).

Pressing the 'Target function' button generates a random transformation of the original function, displaying both its graph and formula.

The **horizontal translation before horizontal stretch** box is always ticked at the very start of the process. This order of transformations is implied because the transformed function is of the form $p\left(f\left(qx+a\right)\right)+b$.

Investigate what happens when you change the slider values. Can you remake all the transformations that are indicated in the formula of the generated function?

Investigate the effect of ticking the **horizontal stretch before horizontal translation** box. Can you find a new series of transformations to reach the same target function?

Along with the movement of the graph, the algebraic result of the transformations is shown under the sliders. You can drag the sliders, but in some cases you will probably need more accuracy, so just click on the slider and then use the arrows of your keyboard. For even finer accuracy, hold 'shift' down when moving with the arrows. You may also zoom in and out.

Rate subtopic 2.11 Transformation of graphs

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