Index

The big picture

In biology, scientists study populations using exponential models. These populations could be something as simple as bacteria growing in a food supply or as advanced as tracking the initial stages of an epidemic. In physics, radioactivity also follows exponential distributions. In financial markets, investments follow exponential distributions through the use of compound interest.

Since logarithmic functions are inverse functions of exponential functions, these are also used to model many systems. Additionally, logarithmic scales are used in areas with vastly different values, such as measuring the intensity of earthquakes or the mass and luminosity of stars.

By now, you have learned a great deal about e^x and $\ln x$. Unfortunately, most of these examples do not revolve around the natural base e. Similarly, all trigonometry problems do not necessarily rely on $\sin x$ and $\cos x$.

In subtopic 5.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27778/), you learned to use the chain, product and quotient rules. When dealing with higher-order polynomials, these rules save time by allowing you to find derivatives without expanding brackets. You have also used them to evaluate trigonometric functions, natural logarithmic functions, and natural exponential functions (exponentials with base e). The same rules can be used to find derivatives of more complex functions.

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Making connections

The concept of the derivative was first introduced in subtopic 5.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25542/), and the first derivative formulae using the power rule were introduced in subtopic 5.3 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/). In subtopic 5.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27778/), you expanded your knowledge of differentiation to simple trigonometric, exponential and logarithmic functions.

In this subtopic, you are going to build on the basic derivatives that you used at standard level (available in the formula booklet) to widen the range of functions that you are able to differentiate.

Table 1. Basic derivatives.

Steps	Explanation
$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
e^x	e^x
$\ln x$	$\frac{1}{x}$

You will need to use the notation for powers of functions, for example, $\sin^2 x = (\sin x)^2$, $\cos^2 x = (\cos x)^2$, and $\ln^2 x = (\ln x)^2$.

Exam tip

Although all these derivatives are provided in the formula booklet, it is useful to memorise them as they are commonly needed and may save you valuable time.

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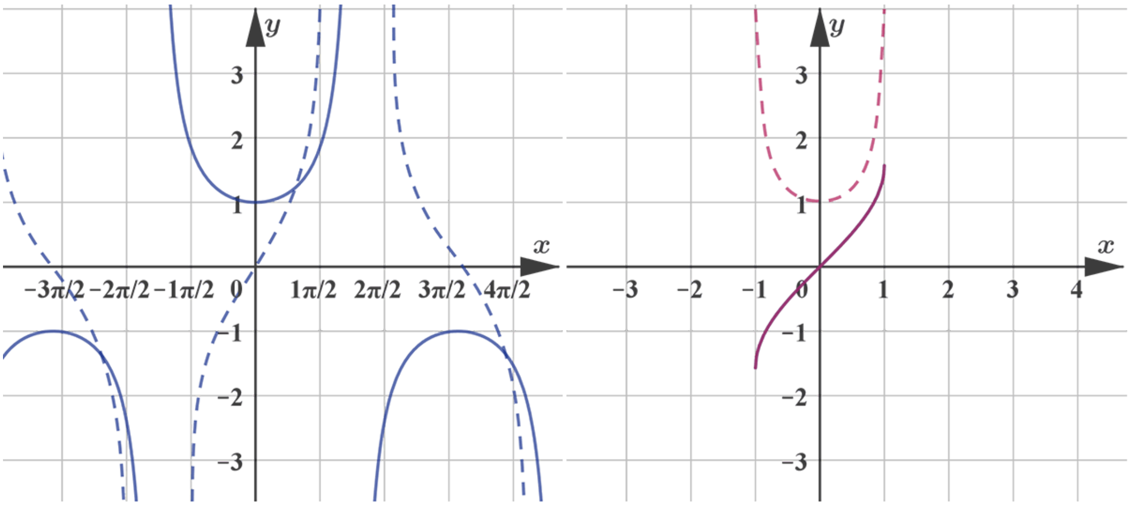
Once you cover derivatives of more advanced functions, you will also explore the indefinite integrals leading to these functions.

The graph on the left below shows the function $f(x) = \sec(x)$ in a solid line with its derivative in a dashed line, both in blue. On the right is the function $f(x) = \arcsin(x)$ with its derivative in pink.



More information

This image contains two graphs. The left graph shows the function f(x) = sec(x) and its derivative. The solid curve represents sec(x), while the dashed line indicates its derivative, both in blue. The function exhibits oscillatory behavior with vertical asymptotes around the values where x is an odd multiple of π/2, where the function is undefined. The derivative follows the same oscillatory pattern but with variations in amplitude.

The right graph displays the function f(x) = arcsin(x) in pink and its derivative. The solid line is arcsin(x) with a smooth increasing curve from -1 to 1, and the dashed line is its derivative, showing a slope that decreases from a maximum at x=0. The arcsine function is only defined between -1 and 1 along the x-axis. The graph highlights the inverse relationship and the derivative's behavior as it approaches the endpoint values.

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Student
view

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Concept

Throughout this subtopic, you will be applying what you already know about differentiation to find derivatives of more complex functions. What functions have you not learned how to differentiate yet? Consider the rules you know. Are there any derivatives you do not know how to find? What about integration? Keep in mind the **relationship** between differentiation and integration.

5. Calculus / 5.15 Further differentiation

Derivatives of tan(x), sec(x), csc(x) and cot(x)

In subtopic 5.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27778/), you learned how to find the derivatives of $\sin x$ and $\cos x$. Using these with the quotient rule will enable you to find the derivatives of some other trigonometric functions.

Making connections

Trigonometry was introduced in subtopic 3.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25417/), and reciprocal trigonometric identities were introduced at HL. Derivatives of sine and cosine were introduced in subtopic 5.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27778/).

Derivative of $y = \tan(x)$

In subtopic 3.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27741/), you learned that the tangent function is defined as $\tan x = \frac{\sin x}{\cos x}$. With that knowledge and the quotient rule (subtopic 5.8 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27795/)), you can find the derivative:

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$

Quotient rule:

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$$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

$$u = \sin x \qquad v = \cos x$$
$$\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = -\sin x$$

$$f'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Example 1

★☆☆

Find the derivative of $\tan \mathrm{e}^x$

The formula booklet gives $f(x) = \tan x \Leftrightarrow f'(x) = \sec^2 x$.

Applying the chain rule and the exponent rule, you get,

$$f(x) = \tan \mathrm{e}^x$$

Chain rule:

$$y = g(u),\ u = f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$u = f(x) = \mathrm{e}^x \qquad y = \tan u$$
$$\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x \qquad \frac{\mathrm{d}y}{\mathrm{d}u} = \sec^2 u$$

$$f'(x) = \sec^2(u) \times (\mathrm{e}^x)$$
$$f'(x) = \mathrm{e}^x \sec^2(\mathrm{e}^x)$$

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Derivative of $y = \sec(x)$

In subtopic 3.9 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26855/), you learned that the secant function is defined as $\sec x = \frac{1}{\cos x}$. You can find the derivative of this function using the quotient rule:

$$f(x) = \sec x = \frac{1}{\cos x}$$
$$f'(x) = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2}$$
$$f'(x) = \frac{\sin x}{\cos x^2} = \sec x \tan x$$

Example 2

★☆☆

Find the derivative of $\sec^2 x$

The formula booklet gives $f(x) = \sec x \Leftrightarrow f'(x) = \sec x \tan x$.

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Applying the chain rule and the power rule, you get,

$$f(x) = \sec^2 x$$
$$f'(x) = 2\sec x \times \sec x \tan x$$
$$f'(x) = 2\sec^2 x \tan x$$

Derivative of $y = \csc(x)$

In subtopic 3.9 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26855/), you learned that the cosecant function is defined as $\csc x = \frac{1}{\sin x}$. You can find the derivative of this function using the quotient rule

$$f(x) = \csc x = \frac{1}{\sin x}$$

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$$u = 1 \qquad \frac{\mathrm{d}u}{\mathrm{d}x} = 0$$
$$v = \sin \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \cos x$$

$$f(x) = \csc x = \frac{1}{\sin x}$$
$$f'(x) = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2}$$
$$f'(x) = \frac{-\cos x}{\sin x^2} = -\csc x \cot x$$

Example 3

★☆☆

Find the derivative of $\ln(\csc(2x))$

The formula booklet gives $f(x) = \csc x \Leftrightarrow f'(x) = -\csc x \cot x$.

Applying the chain rule twice, you get,

$$f(x) = \ln(\csc(2x))$$
$$f'(x) = \frac{1}{\csc(2x)} \times (-\csc(2x)\cot(2x)) \times (2)$$
$$f'(x) = \frac{-2\csc(2x)\cot(2x)}{\csc(2x)} = -2\cot(2x)$$

Derivative of $y = \cot(x)$

In subtopic 3.9 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26855/), you learned that the cotangent function is defined as $\cot x = \frac{\cos x}{\sin x}$. You can find the derivative of this function using the quotient rule:

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$
$$f'(x) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$
$$f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

Student view

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Example 4

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Find the derivative of $\cot 3x^2$

The formula booklet gives $f(x) = \cot x \Leftrightarrow f'(x) = -\csc^2 x$.

Applying the chain rule, you get,

$$\begin{aligned} f(x) &= \cot 3x^2 \\ f'(x) &= -\csc^2\left(3x^2\right) \times (6x) \\ f'(x) &= -6x\csc^2\left(3x^2\right) \end{aligned}$$

Important

The trigonometric derivatives you are now able to work with include:

- $f(x) = \sin x \Leftrightarrow f'(x) = \cos x$
- $f(x) = \cos x \Leftrightarrow f'(x) = -\sin x$
- $f(x) = \tan x \Leftrightarrow f'(x) = \sec^2 x$
- $f(x) = \sec x \Leftrightarrow f'(x) = \sec x \tan x$
- $f(x) = \csc x \Leftrightarrow f'(x) = -\csc x \cot x$
- $f(x) = \cot x \Leftrightarrow f'(x) = -\csc^2 x$

Be aware

These formulae for derivatives of trigonometric functions only work when angles are measured in radians. If angles are given in any other measuring system, they must be converted to radians before evaluation.

3 section questions

Student view

Question 1

Difficulty:

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Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

What is the derivative of sec $(2x)$?

1 $2\sec(2x)\tan(2x)$ ✓

2 $-2\sec(2x)\tan(2x)$

3 $2\tan(2x)$

4 $2\csc(2x)\tan(2x)$

Explanation

Using the standard derivative and the (trivial) chain rule:

$$f(x)=\sec(2x) \ \Rightarrow\ f'(x)=\sec(2x)\tan(2x)\times 2=2\sec(2x)\tan(2x).$$

Question 2

Difficulty:

★☆☆

What is the derivative of cot $\left(\frac{\pi}{4}-x\right)$?

1 $\csc^2\left(\frac{\pi}{4}-x\right)$ ✓

2 $-\csc^2\left(\frac{\pi}{4}-x\right)$

3 $\sec^2\left(\frac{\pi}{4}-x\right)$

4 $-\sec^2\left(\frac{\pi}{4}+x\right)$

Explanation

Using the standard derivative and the chain rule:

$$f(x)=\cot\left(\frac{\pi}{4}-x\right)\Rightarrow f'(x)=-\csc^2\left(\frac{\pi}{4}-x\right)\times -1=\csc^2\left(\frac{\pi}{4}-x\right).$$

Student view

Question 3

Difficulty:

★☆☆

Overview
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What is the derivative of sec $(2x - \pi)$?

1 $-2\sec(2x)\tan(2x)$ ✓

2 $2\sec(2x)\tan(2x)$

3 $-2\sec(2x-\pi)\tan(2x-\pi)$

4 $-2\sec(2x)\cot(2x)$

Explanation

Using standard derivative and the (trivial) chain rule:

$$f(x) = \sec(2x-\pi) \Rightarrow f'(x) = 2\sec(2x-\pi)\tan(2x-\pi),$$

and then using the properties $\sec(x-\pi) = -\sec x$ and $\tan(x-\pi) = \tan x$, we get the result:

$$f(x) = \sec(2x-\pi) \Rightarrow f'(x) = 2\sec(2x-\pi)\tan(2x-\pi) = -2\sec(2x)\tan(2x).$$

5. Calculus / 5.15 Further differentiation

Derivatives of exponential and logarithmic functions

In subtopic 5.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27778/), you found the derivatives of e^x and $\ln x$, but you did not use any other bases. In this section, you will expand your knowledge to bases other than e.

Making connections

Logarithms and exponents were introduced in subtopic 1.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25055/). Derivatives of logarithmic and exponential functions with base e were introduced in subtopic 5.6

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Derivative of $y = a^x$

You will use properties of logarithms that you learned in subtopic 1.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25055/).

First, you rewrite the function:

$$y = a^x$$

$$y = \mathrm{e}^{\ln a^x} \qquad \mathrm{e}^x \text{ and } \ln x \text{ are inverse functions}$$

$$y = \mathrm{e}^{x \ln a} \qquad \text{Power rule for logarithms}$$

Next, find the derivative using the chain rule:

$$y' = \mathrm{e}^{x \ln a} \times \ln a \qquad \text{Chain rule}$$

$$y' = \mathrm{e}^{\ln a^x} \ln a$$

As $\ln a$ is just a constant, there is no need for an imbedded product rule. Now you can go back and substitute the original function of $a^x = \mathrm{e}^{\ln a^x}$ into this equation:

$$y' = e^{\ln a^x} \ln a = a^x \ln a$$

This works for any constant a. This formula can be found in the formula booklet.

Example 1

★☆☆

Find the derivative of $y = 5^{3x-1}$

Student view

The formula booklet gives $f(x) = a^x \Leftrightarrow f'(x) = a^x \ln a$.

Applying the quotient rule and chain rule, you get,

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$$f(x) = 5^{3x-1}$$
$$f'(x) = 5^{3x-1} \ln 5 \times 3$$
$$f'(x) = 3(\ln 5)\, 5^{3x-1} = (\ln 5^3)\, 5^{3x-1} = (\ln 125)\, 5^{3x-1}$$

Example 2

★★☆

Find the derivative of $y = \dfrac{x}{1+3^x}$

The formula booklet gives $f(x) = a^x \Leftrightarrow f'(x) = a^x \ln a$.

Applying the quotient rule, you get,

$$f(x) = \frac{x}{1+3^x}$$
$$f'(x) = \frac{(1+3^x)(1) - x(3^x \ln 3)}{(1+3^x)^2}$$
$$f'(x) = \frac{1 + 3^x - x3^x \ln 3}{(1+3^x)^2} = \frac{3^x(1 - x\ln 3) + 1}{(1+3^x)^2}$$

Derivative of $y = log_a x$

Again, you will use properties of logarithms that you learned in subtopic 1.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25055/).

Rewrite the function:

$$y = \log_a x \Leftrightarrow a^y = a^{\log_a x} = x$$

This changes the function from logarithmic form to exponential form, staying in some given base.

Next, find the derivative using the chain rule and implicit differentiation. Simplify and substitute $a^y = x$:

Student view

$$a^y = x$$
$$a^y \times \ln a \times y' = 1$$
$$y' = \frac{1}{a^y \ln a} = \frac{1}{x \ln a}$$

Example 3

★★☆

Find the derivative of $y = \log_3 \left(2x^4 - 3\right)$

The formula booklet gives $f(x) = \log_a x \Leftrightarrow f'(x) = \frac{1}{x \ln a}$.

Applying the chain rule and power rule, you get,

$$f(x) = \log_3 \left(2x^4 - 3\right)$$
$$f'(x) = \frac{1}{\left(2x^4 - 3\right) \ln 3} \times \left(8x^3\right)$$
$$f'(x) = \frac{8x^3}{\ln 3 \left(2x^4 - 3\right)}$$

Example 4

★★☆

Find the derivative of $y = \log_4 \tan 3x$

The formula booklet gives $f(x) = \log_a x \Leftrightarrow f'(x) = \frac{1}{x \ln a}$.

Applying the chain rule and product rule, you get,

$$f(x) = \log_4 \tan 3x$$
$$f'(x) = \frac{1}{\tan 3x \ln 4} \times 3 \sec^2 3x$$
$$f'(x) = \frac{3}{\ln 2^2} \frac{\cos 3x}{\sin 3x} \frac{1}{\cos^2 3x} = \frac{3}{\ln 2 \left(2 \sin 3x \cos 3x\right)}$$

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From subtopic 3.2, recall that $2\sin x\cos x = \sin 2x$ or, in this case, $2\sin 3x\cos 3x = \sin 6x$, so

$$f'(x) = \frac{3}{\ln 2\sin 6x}$$

Important

The exponential and logarithmic derivatives you are now able to work with include:

- $f(x) = \mathrm{e}^x \Leftrightarrow f'(x) = \mathrm{e}^x$
- $f(x) = a^x \Leftrightarrow f'(x) = a^x \ln a$
- $f(x) = \ln x \Leftrightarrow f'(x) = \dfrac{1}{x}$
- $f(x) = \log_a x \Leftrightarrow f'(x) = \dfrac{1}{x\ln a}$

4 section questions

Question 1

Difficulty:

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What is the derivative of 4^{3x}?

1 $3\ln 4 \times 4^{3x}$ ✓

2 $\ln 4 \times 4^{3x}$

3 3×4^{3x}

4 $4\ln 3 \times 4^{3x}$

Explanation

Using the standard derivative and the, simple, chain rule:

$$f(x) = 4^{3x} \ \Rightarrow\ f'(x) = \ln 4 \times 4^{3x} \times 3 = 3\ln 4 \times 4^{3x}.$$

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Question 2

Difficulty:

★☆☆

What is the derivative of $y = x^3\, 3^x$?

1 $x^2\, 3^x\, (3 + x \ln 3)$ ✓

2 $x^3 \ln 3\, 3^x$

3 $3x^2\, 3^x\, (x \ln 3 + 1)$

4 $x^2\, 3^x\, (\ln x + 3)$

Explanation

Using the standard derivatives and the product rule, we get

$$\begin{aligned} & \frac{dy}{dx} = \frac{d[x^3\, 3^x]}{dx} \\ \Rightarrow \quad & = \frac{d[x^3]}{dx} 3^x + x^3 \frac{d[3^x]}{dx} \\ \Rightarrow \quad & = 3x^2\, 3^x + x^3 \ln 3\; 3^x \\ \Rightarrow \quad & = x^2\, 3^x\, (3 + x \ln 3) \end{aligned}$$

Question 3

Difficulty:

★☆☆

What is the derivative of $\log_{10} 25x$?

1 $\dfrac{1}{x \ln 10}$ ✓

2 $\dfrac{25}{x \ln 10}$

3 $\dfrac{1}{25x \ln 10}$

4 $\dfrac{10}{x \ln 25}$

Explanation

Firstly, we need to get this into the form of an equation we know how to differentiate. We cannot differentiate log with base 10, but we can do the natural log. So let's change the base!

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$$\log_{10} 25x = \frac{\ln 25\,x}{\ln 10}$$

Then, using the standard derivative, and remembering that if $f(x) = \ln(kx) \Rightarrow f'(x) = \frac{1}{x}$, $\forall k$, we get the following ($\ln 10$ is just a constant, so we don't need to worry about differentiating it; it just stays in the equation!)

$$y = \log_{10} 25x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{x \ln 10}$$

Question 4

Difficulty:

★☆☆

What is the derivative of $y = a^x \ \log_a x$?

1 $a^x \left(\ln x + \frac{1}{x \ln a}\right)$ ✓

2 $a^x \left(\log_a x + \frac{1}{x \ln a}\right)$

3 $\frac{a^x}{\ln a} \left(\ln x + \frac{1}{x}\right)$

4 $a^x \left(\ln a - \frac{1}{x \ln a}\right)$

Explanation

Using the standard derivatives and the product rule, and recalling the change of base for logarithms, such that $\log_a x = \frac{\ln x}{\ln a}$, we get

$$\begin{aligned} & \frac{dy}{dx} = \frac{d[a^x \ \log_a x]}{dx} \\ \Rightarrow \quad & = \frac{d[a^x]}{dx} \log_a x + a^x \frac{d[\log_a x]}{dx} \\ \Rightarrow \quad & = a^x \ln a \log_a x + a^x \frac{1}{x \ln a} \\ \Rightarrow \quad & = a^x \left(\ln x + \frac{1}{x \ln a}\right) \end{aligned}$$

5. Calculus / 5.15 Further differentiation

Derivatives of inverse trigonometric functions

Student view

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In section 3.9.3 (/study/app/math-aa-hl/sid-134-cid-761926/book/inverse-trigonometric-functions-id-27601/), you learned about inverse trigonometric functions. In this section, you will explore finding derivatives of inverse trigonometric functions.

Making connections

Trigonometric functions, including inverse trigonometric functions, were introduced in subtopic 3.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25417/). Now that you are able to carry out implicit differentiation you will be able to find their derivatives.

Be aware

Notation for inverse functions can vary, for example, $\arcsin x = \sin^{-1}x$, $\arccos x = \cos^{-1}x$ and $\arctan x = \tan^{-1}x$. If you see the latter, remember that $\sin^{-1}x \neq (\sin x)^{-1}$ because $(\sin x)^{-1} = \dfrac{1}{\sin x} = \csc x$.

Derivative of $y = \arcsin(x)$

It is necessary to use implicit differentiation.

$$\begin{aligned} y &= \arcsin x \\ \sin y &= x \\ \cos y \times y' &= 1 \\ y' &= \frac{1}{\cos y} \end{aligned}$$

At this point, you need to recall some right-triangle trigonometry from section 3.2.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/rightangled-triangles-id-25419/). First, define a right-angled triangle with angle y, a hypotenuse of length 1, and an opposite of length x.

Student view

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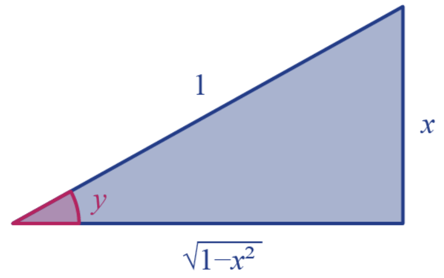

More information

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The image is a diagram of a right-angled triangle. It is oriented such that the right angle is at the bottom left. The triangle is labeled with an angle denoted as (y) at the bottom left corner. The hypotenuse is labeled with a length of (1), extending from the bottom left to the top right. The side opposite the angle (y) is labeled (x), forming the vertical side of the triangle. The base of the triangle has a length labeled as (\sqrt{1-x^2}), extending horizontally from the bottom left to the bottom right.

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Notice that by choosing the opposite side to be x and the hypotenuse to be 1, you can use the definition $\sin y = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{1} = x$, relating to the original function. Pythagoras' theorem allows you to find an expression for the adjacent.

Then $\cos y = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

So the required derivative $y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

An alternative to developing this geometrically is through trigonometric properties. You start in the same way:

$$\begin{aligned} y &= \arcsin x \\ \sin y &= x \\ \cos y \times y' &= 1 \\ y' &= \frac{1}{\cos y} \end{aligned}$$

Student view

Using a Pythagorean identity, you can solve:

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$$\begin{aligned} \sin^2 y + \cos^2 y &= 1 \\ \cos^2 y &= 1 - \sin^2 y \\ \cos y &= \sqrt{1 - \sin^2 y} \end{aligned}$$

Remember that $\sin y = x$, so substitution results in:

$$\cos y = \sqrt{1 - x^2}, \text{ and } y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - x^2}}$$

⚠ **Be aware**

The expression under the radical sign in the denominator will force the domain to be limited to $-1 < x < 1$.

Example 1

★☆☆

Find the derivative of $\arcsin{(3x)}$.

The formula booklet gives $f(x) = \arcsin x \Leftrightarrow f'(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

Applying the chain rule, you get,

$$\begin{aligned} f(x) &= \arcsin{(3x)} \\ f'(x) &= \frac{1}{\sqrt{1 - (3x)^2}} \times 3, -1 < 3x < 1 \\ f'(x) &= \frac{3}{\sqrt{1 - 9x^2}} = \frac{1}{\sqrt{\frac{1}{9} - x^2}}, -\frac{1}{3} < x < \frac{1}{3} \end{aligned}$$

Derivative of $y = \arccos(x)$

Similarly to $\arcsin x$, you use implicit differentiation.

Student view

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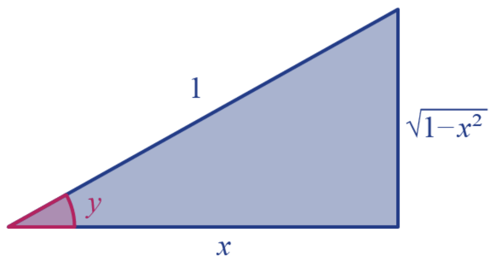
$$
\begin{aligned}
y &= \arccos x \\
\cos y &= x \\
-\sin y \times y' &= 1 \\
y' &= -\frac{1}{\sin y}
\end{aligned}
$$

Using right-triangle trigonometry, define a right-angled triangle with angle y, a hypotenuse of length 1, and an adjacent of length x.



More information

The image depicts a right-angled triangle. The triangle has:

1. A hypotenuse with a length labeled as 1.
2. An adjacent side, relative to the angle (y), labeled as (x).
3. An angle (y) located between the adjacent and the hypotenuse.
4. An opposite side to angle (y) with a length labeled as (\sqrt{1-x^2}).

The triangle is oriented with the hypotenuse slanting upward from left to right, the adjacent side horizontal along the bottom, and the opposite side vertical.

[Generated by AI]

Notice that choosing the adjacent side to be x this time allows you to use the definition $\cos y = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{1} = x$, relating to the original function. Pythagoras' theorem enables you to solve for the adjacent:

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view

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$$\sin y = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

Using this substitution, you can now find the derivative, $y' = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}}$

The trigonometric alternative is:

$$\begin{aligned} y &= \arccos x \\ \cos y &= x \\ -\sin y \times y' &= 1 \\ y' &= -\frac{1}{\sin y} \end{aligned}$$

Using a Pythagorean identity, you can solve:

$$\begin{aligned} \sin^2 y + \cos^2 y &= 1 \\ \sin^2 y &= 1 - \cos^2 y \\ \sin y &= \sqrt{1 - \cos^2 y} \end{aligned}$$

Substitution results in:

$$\sin y = \sqrt{1-x^2}, \text{and} y' = \frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}}$$

Example 2

★☆☆

Find the derivative of $\arccos\left(x^2-4\right)$

The formula booklet gives $f(x) = \arccos x \Leftrightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$.

Applying the chain rule, you get,

Student view

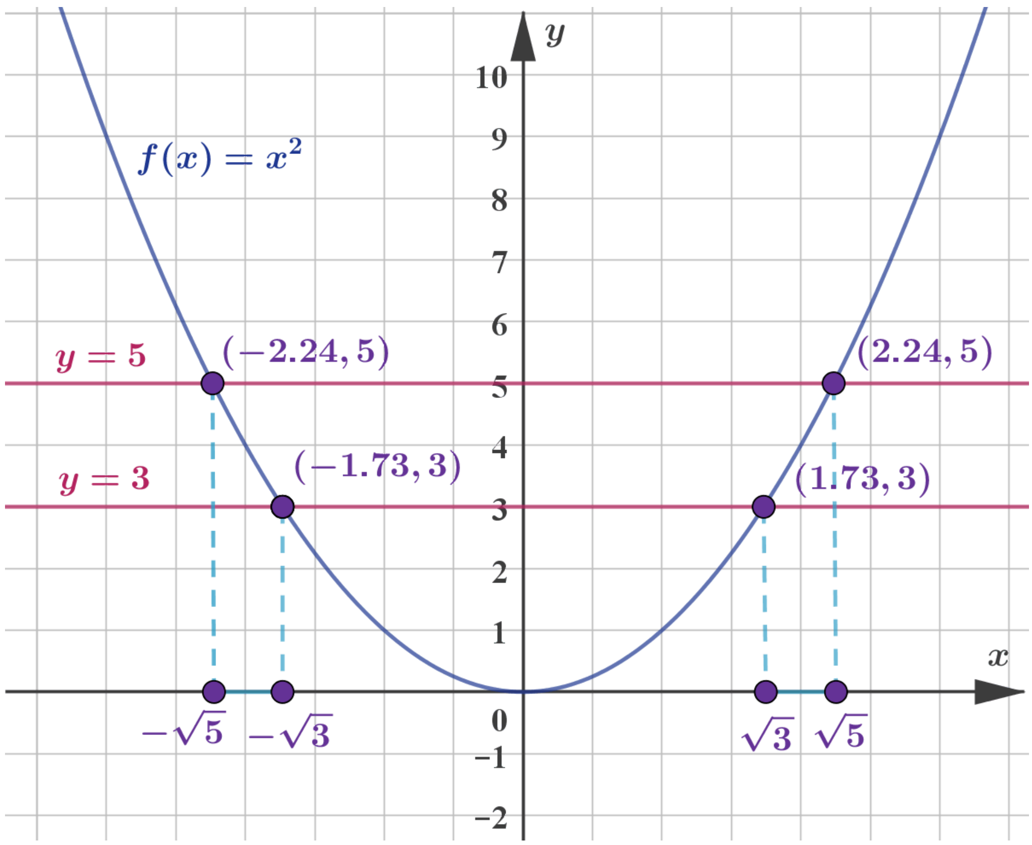
$$f(x) = \arccos\left(x^2 - 4\right)$$

$$f'(x) = -\frac{1}{\sqrt{1-\left(x^2-4\right)^2}} \times 2x, -1 < x^2 - 4 < 1$$

$$f'(x) = -\frac{2x}{\sqrt{1-\left(x^2-4\right)^2}}, 3 < x^2 < 5$$

$$f'(x) = -\frac{2x}{\sqrt{1-\left(x^2-4\right)^2}}, x \in \left(-\sqrt{5}, -\sqrt{3}\right) \cup \left(\sqrt{3}, \sqrt{5}\right)$$

The domain of this function is also of interest. A quick sketch would be one way of obtaining it.



The graphical method of solving the inequality $-1 < (x-4) < 3$ which leads to the inequality $3 < x^2 < 5$, the solution of which is shown in the figure above, i.e. $x \in \left(-\sqrt{5}, -\sqrt{3}\right) \cup \left(\sqrt{3}, \sqrt{5}\right)$.

Student view

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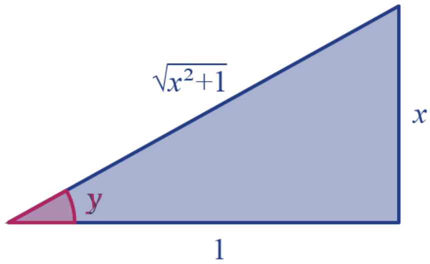
Derivative of $y = \arctan(x)$

Here is a third inverse function.

$$\begin{aligned} y &= \arctan x \\ \tan y &= x \\ \sec^2 y \times y' &= 1 \\ y' &= \cos^2 y \end{aligned}$$

This time, the right-angled triangle with angle y has an adjacent of length 1 and an opposite of length x.



More information

The image shows a right-angled triangle. One of the angles is labeled as y. The side adjacent to angle y is labeled as 1, and the side opposite angle y is labeled as x. The hypotenuse is labeled with the expression (\sqrt{x^2+1}). The triangle demonstrates a trigonometric relationship useful in understanding the expression (\tan y = x).

[Generated by AI]

Notice that choosing the opposite and adjacent sides this time allows you to use the definition $\tan y = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{1} = x$, relating to the original function. Pythagoras' theorem enables you to solve for the hypotenuse:

Student view

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$\cos y = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{1}{\sqrt{x^2+1}}$, or $\cos^2 y = \dfrac{1}{x^2+1}$. Using this substitution, you can now find the derivative $y' = \cos^2 y = \dfrac{1}{x^2+1}$.

The trigonometric alternative is:

$$
\begin{aligned}
y &= \arctan x \\
\tan y &= x \\
\sec^2 y \times y' &= 1 \\
y' &= \cos^2 y
\end{aligned}
$$

Using a Pythagorean identity, you can solve:

$$\sec^2 y = 1 + \tan^2 y$$

Substitution results in:

$$\sec^2 y = 1 + x^2, \text{ or } \cos^2 y = \frac{1}{1+x^2}, \text{ and} y' = \frac{1}{1+x^2}$$

Example 3

★★☆

Find the derivative of $\dfrac{\arctan(2x)}{x^2}$

From the formula booklet, you have that $f(x) = \arctan x \Leftrightarrow f'(x) = \dfrac{1}{1+x^2}$, and applying the quotient rule and chain rule, you get

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$$f(x) = \frac{\arctan(2x)}{x^2}$$

$$f'(x) = \frac{2x^2\left(\frac{1}{1+(2x)^2}\right) - 2x\arctan(2x)}{x^4}$$

$$f'(x) = \frac{2}{x^2}\left(\frac{1}{1+4x^2}\right) - \frac{2\arctan(2x)}{x^3}$$

$$f'(x) = \frac{2}{x^2}\left(\frac{1}{1+4x^2} - \frac{\arctan(2x)}{x}\right)$$

Be aware

The derivatives of the inverse trigonometric functions are of interest in their own right. However, they will be of particular use when you study integration. It is important that you are able to recognise the general form of these derivatives.

Important

The inverse trigonometric derivatives you are now able to work with include:

- $f(x) = \arcsin x \Leftrightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
- $f(x) = \arccos x \Leftrightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$
- $f(x) = \arctan x \Leftrightarrow f'(x) = \frac{1}{1+x^2}$

Other functions such as $\mathrm{arcsec}x$, $\mathrm{arccsc}x$ and $\mathrm{arccot}x$ do have derivative formulas, but you will not need to use (undefined)them in this course.

3 section questions

Question 1

Difficulty:

★★☆

What is the derivative of $y = \arcsin\left(\frac{1}{\sqrt{x}}\right)$?

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1 $\frac{dy}{dx}=-\frac{1}{2x^{\frac{3}{2}}\sqrt{\frac{x-1}{x}}}$ ✓

2 $\frac{dy}{dx}=\frac{1}{2x^{\frac{3}{2}}\sqrt{\frac{x-1}{x}}}$

3 $\frac{dy}{dx}=-\frac{1}{2x^{\frac{1}{2}}\sqrt{\frac{x-1}{x}}}$

4 $\frac{dy}{dx}=\frac{1}{2x^{\frac{1}{2}}\sqrt{\frac{x-1}{x}}}$

Explanation

We make a substitution such that $y=\arcsin u$ where $u=x^{-\frac{1}{2}}$. Then we use the standard derivatives and the chain rule:

$$\begin{aligned}
& \frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}\\
\Rightarrow\quad & =\frac{1}{\sqrt{1-u^2}}\times-\frac{1}{2}x^{-\frac{3}{2}}\\
\Rightarrow\quad & =-\frac{1}{2x^{\frac{3}{2}}\sqrt{1-x^{-1}}}\\
\Rightarrow\quad & =-\frac{1}{2x^{\frac{3}{2}}\sqrt{\frac{x-1}{x}}}
\end{aligned}$$

Question 2

Difficulty: ★★☆

What is the derivative of $y=\arccos(\sin x),\ -\pi<x<\pi$?

1 $\frac{dy}{dx}=\begin{cases}1\,, & -\pi<x<-\frac{\pi}{2}\cup\frac{\pi}{2}<x<\pi\\ -1\,, & -\frac{\pi}{2}<x<\frac{\pi}{2}\end{cases}$ ✓

2 $\frac{dy}{dx}=\begin{cases}1\,, & -\frac{\pi}{2}<x<\frac{\pi}{2}\\ -1\,, & -\pi<x<-\frac{\pi}{2}\cup\frac{\pi}{2}<x<\pi\end{cases}$

3 $\frac{dy}{dx}=\begin{cases}\tan x\,, & -\pi<x<-\frac{\pi}{2}\cup\frac{\pi}{2}<x<\pi\\ -\tan x\,, & -\frac{\pi}{2}<x<\frac{\pi}{2}\end{cases}$

4 $\frac{dy}{dx}=\tan x\,,\ -\pi<x<\pi$

Student view

Explanation

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

We make a substitution such that $y = \arccos u$, where $u = \sin x$. Then we apply the standard derivatives and the chain rule:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ \Rightarrow \quad &= -\frac{1}{\sqrt{1-u^2}} \times \cos x \\ \Rightarrow \quad &= -\frac{\cos x}{\sqrt{1-\sin^2 x}} \\ &\qquad \text{(using the Pythagorean trigonometric identity)} \\ \Rightarrow \quad &= -\frac{\cos x}{|\cos x|} \end{aligned}$$

Thus, when $\cos x$ is positive, the derivative is $\frac{dy}{dx} = -1$, which is for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. And,similarly, we obtain the result for when $\cos x$ is negative, leading to $\frac{dy}{dx} = 1$ for the remainder of the domain of the given equation.

Question 3

Difficulty:

★★★

What is the derivative of $y = \ln\left(\arctan\left(\frac{1}{2}x\right)\right)$?

1 $\frac{dy}{dx} = \frac{2}{\arctan\left(\frac{1}{2}x\right)(4+x^2)}$ ✓

2 $\frac{dy}{dx} = \frac{4}{\arctan\left(\frac{1}{2}x\right)(4+x^2)}$

3 $\frac{dy}{dx} = \frac{1}{\arctan\left(\frac{1}{2}x\right)(2+x)}$

4 $\frac{dy}{dx} = \frac{1}{\arctan\left(\frac{1}{2}x\right)\left(\sqrt{4+x^2}\right)}$

Explanation

We make a substitution such that $y = \ln u$, where $u = \arctan\left(\frac{1}{2}x\right)$. Then we use standard derivatives and the chain rule:

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$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\Rightarrow \quad = \frac{1}{u} \times \frac{1}{1+\left(\frac{1}{2}x\right)^2} \times \frac{1}{2}$$

$$\Rightarrow \quad = \frac{1}{\arctan\left(\frac{1}{2}x\right)} \times \frac{1}{1+\frac{1}{4}x^2} \times \frac{1}{2}$$

$$\Rightarrow \quad = \frac{2}{\arctan\left(\frac{1}{2}x\right)(4+x^2)}$$

5. Calculus / 5.15 Further differentiation

Further indefinite integrals

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/further-indefinite-integrals-id-26511/print/) Assign

In subtopic 5.10 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27807/) you used integration to find the area under the curve for basic functions such as x^n , $\frac{1}{x}$, $\sin x$, $\cos x$ and e^x . You are now able to apply your understanding of derivatives of the more advanced functions to integration.

Indefinite integrals of trigonometric functions

You now have six trigonometric functions that you can differentiate.

Important

The trigonometric derivatives and integrals you are now able to apply include:

derivative	integral
$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$	$\int \cos x \, \mathrm{d}x = \sin x + C$
$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$	$\int \sin x \, \mathrm{d}x = -\cos x + C$
$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x$	$\int \sec^2 x \, \mathrm{d}x = \tan x + C$
$\frac{\mathrm{d}}{\mathrm{d}x}\sec x = \sec x \tan x$	$\int \sec x \tan x \, \mathrm{d}x = \sec x + C$
$\frac{\mathrm{d}}{\mathrm{d}x}\csc x = -\csc x \cot x$	$\int \csc x \cot x \, \mathrm{d}x = -\csc x + C$
$\frac{\mathrm{d}}{\mathrm{d}x}\cot x = -\csc^2 x$	$\int \csc^2 x \, \mathrm{d}x = -\cot x + C$

Student view

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The first two integral formulae are found in the formula booklet. If needed, the rest can be identified from the derivative formulae.

Be aware

For the cosecant function, many textbooks use the notation $\csc x$, but some use $\operatorname{cosec} x$. There is no mathematical difference between the two notations. Both are equally correct.

Why do you need the '$+\ C$' on the end of every integral?

As you learned in subtopic 5.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25533/), indefinite integrals require a constant added to the end, typically written as '$+\ C$'. This goes back to the original idea of an integral being an anti-derivative, so there may have been a constant in the original expression that was differentiated, which would not have had a derivative with respect to any variable. An entire family of functions translated vertically have the same gradient for any x-value.

Finding integrals can be more challenging than finding derivatives. It is more of a puzzle. You are looking for patterns that relate the integral to function families that you know.

To review a simple integral function, consider $\int \cos(5x+3)\mathrm{d}x$. You know that the derivative of $\sin x$ is $\cos x$, so $\int (\cos x)\, \mathrm{d}x = \sin x + C$. You can hypothesise that $\int \cos(5x+3)dx = a\sin(5x+3)+C$ where a is some constant.

To check this, take the derivative of the right-hand side: you get $\frac{\mathrm{d}}{\mathrm{d}x}(a\sin(5x+3)+C) = 5a\cos(5x+3)$. Since the coefficient of the original function is 1, you can surmise that $a = \frac{1}{5}$, so $\int \cos(5x+3)\mathrm{d}x = \frac{1}{5}\sin(5x+3)+C$.

Example 1

★☆☆

Find the indefinite integral of $f(x) = \sec^2(3x+2)$

Student view

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Using the basic integral $\int \left(\sec^2 x\right) \mathrm{d}x = \tan x + C$, you can think backwards to produce $\int \sec^2(3x+2)\mathrm{d}x = \dfrac{1}{3}\tan(3x+2) + C$. The '$\dfrac{1}{3}$' results from the chain rule when taking the derivative.

Example 2

★☆☆

Find the indefinite integral of $f(x) = \sec(x+5)\tan(x+5)$

Using the basic integral $\int (\sec x \tan x)\, dx = \sec x + C$, you can think backwards to produce $\int \sec(x+5)\tan(x+5) dx = \sec(x+5) + C$. The '$+\ 5$' has no impact on the integral.

Example 3

★☆☆

Find the indefinite integral of $f(x) = \csc\left(\dfrac{x}{4}\right)\cot\left(\dfrac{x}{4}\right)$

Using the basic integral $\int (\csc x \cot x)\, dx = -\csc x + C$, you can think backwards to produce $\int \csc\left(\dfrac{x}{4}\right)\cot\left(\dfrac{x}{4}\right) dx = -4\csc\left(\dfrac{x}{4}\right) + C$. The '–4' results from the chain rule when taking the derivative.

Example 4

★★☆

Find the indefinite integral of $f(x) = \csc^2\left(\dfrac{x+7}{3}\right)$

Student view

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Using the basic integral $\int \left(\csc^2 x\right) dx = -\cot x + C$, you can think backwards to produce $\int \csc^2\left(\frac{x+7}{3}\right) dx = -3\cot\left(\frac{x+7}{3}\right) + C$. The '−3' results from the chain rule when taking the derivative. If it helps, you can rewrite $\frac{x+7}{3} = \frac{x}{3} + \frac{7}{3}$.

Indefinite integrals of exponential and logarithmic functions

You have four logarithmic and exponential functions that you can find the derivatives of.

✓ **Important**

The exponential and logarithmic derivatives and integrals you are now able to work with include:

- $f(x) = \mathrm{e}^x \Leftrightarrow f'(x) = \mathrm{e}^x \quad \Leftrightarrow \quad \int \mathrm{e}^x \mathrm{d}x = \mathrm{e}^x + C$
- $f(x) = a^x \Leftrightarrow f'(x) = a^x \ln a \quad \Leftrightarrow \quad \int a^x \mathrm{d}x = \frac{1}{\ln a}a^x + C$
- $f(x) = \ln x \Leftrightarrow f'(x) = \frac{1}{x} \quad \Leftrightarrow \quad \int \frac{1}{x}\mathrm{d}x = \ln x + C$
- $f(x) = \log_a x \Leftrightarrow f'(x) = \frac{1}{x\ln a} \quad \Leftrightarrow \quad \text{Not needed}$

These three integral formulae are found in the formula booklet.

Example 5

★☆☆

Find the indefinite integral of $f(x) = 5^x$

Using the basic integral (found in the formula booklet)

$$\int a^x dx = \frac{1}{\ln a}a^x + C$$

gives $\int 5^x dx = \frac{5^x}{\ln 5} + C$

Student view

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The 'ln 5' results from the chain rule when taking the derivative.

Example 6

★★☆

Find the indefinite integral of $f(x) = 4 \times 3^{2x}$

Use the basic integral $\int a^x dx = \frac{1}{\ln a} a^x + C$ (found in the formula booklet) and the chain rule:

$$\begin{aligned} \int 4 \times 3^{2x} \mathrm{d}x &= \frac{1}{2} \times \frac{4 \times 3^{2x}}{\ln 3} && \text{By inspection. This can be checked by} \\ &= \frac{2 \times 3^{2x}}{\ln 3} \\ &= \frac{2 \times (3^2)^x}{\ln 3} && \text{Using properties of exponents} = \frac{2 \times 9}{\ln 3} \end{aligned}$$

When you have completed subtopic 5.16, you will also be able to find the integral using integration by substitution. The method is shown here. Can you follow the steps?

$$\begin{aligned} \text{Let} \quad u &= 2x \\ \mathrm{d}u &= 2\mathrm{d}x && \text{Differentiate} \\ \int 4 \times 3^{2x} \mathrm{d}x &= \int 2 \times 3^{2x} \times 2\mathrm{d}x \\ &= 2 \int 3^u \mathrm{d}u && \text{Substitute } u = 2x \text{ and } \mathrm{d}u = 2\mathrm{d}x \\ &= 2 \frac{1}{\ln 3} 3^u && \text{Use the integral from the formula booklet} \\ &= \frac{2 \times 3^{2x}}{\ln 3} && \text{Substitute } u = 2x \\ &= \frac{2 \times (3^2)^x}{\ln 3} && \text{Use properties of exponents} \\ &= \frac{2 \times 9^x}{\ln 3} + C && \text{Simplify} \end{aligned}$$

Student view

Example 7

★☆☆

Find the indefinite integral of $f(x) = \dfrac{3}{x \ln 10}$

Working backwards, you could hypothesise that $\int \dfrac{1}{x \ln a} \mathrm{d}x = \log_a x + C$, and you would be correct. Using this, you can produce $\int \dfrac{3}{x \ln 10} \mathrm{d}x = 3 \log x + C$. This can, however, be carried out by treating the logarithm as a constant, $\int \dfrac{3}{x \ln 10} \mathrm{d}x = \dfrac{3}{\ln 10} \int \dfrac{1}{x} \mathrm{d}x = \dfrac{3}{\ln 10} \ln |x| + C$. Notice that these answers are equivalent (using the change of base rule found in subtopic 1.7). Since both work, it is much easier to just remember one integral formula $\int \dfrac{1}{x} dx = \ln x + C$.

Indefinite integrals of inverse trigonometric functions

You now have three inverse trigonometric functions that you can find derivatives of.

✓ Important

The derivatives and integrals you can now work with include:

- $f(x) = \arcsin x \Leftrightarrow f'(x) = \dfrac{1}{\sqrt{1-x^2}} \quad \Leftrightarrow \int \dfrac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C$
- $f(x) = \arctan x \Leftrightarrow f'(x) = \dfrac{1}{1+x^2} \quad \Leftrightarrow \int \dfrac{1}{1+x^2} \mathrm{d}x = \arctan x + C$
- $f(x) = \arccos x \Leftrightarrow f'(x) = -\dfrac{1}{\sqrt{1-x^2}} \quad \Leftrightarrow$ (Not needed)

Now consider:

$$f(x) = \arcsin \frac{x}{a}$$

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Using the chain rule, you would get:

$$\frac{\mathrm{d}}{\mathrm{d}x}\arcsin\frac{x}{a}=\frac{1}{\sqrt{1-(x/a)^2}}\frac{1}{a}=\frac{1}{a\sqrt{1-x^2/a^2}}=\frac{1}{\sqrt{a^2-x^2}}$$

This implies that $\int\frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x=\arcsin\left(\frac{x}{a}\right)+C$.

Similarly, you can also consider:

$$f(x)=\frac{1}{a}\arctan\frac{x}{a}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{a}\arctan\frac{x}{a}=\frac{1}{a}\frac{1}{\left(1+(x/a)^2\right)}\frac{1}{a}=\frac{1}{a^2+x^2}$$
$$\int\frac{1}{a^2+x^2}\mathrm{d}x=\frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$$

✓ **Important**

The integrals that you can now work with include:

- $\int\frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x=\arcsin\left(\frac{x}{a}\right)+C$
- $\int\frac{1}{a^2+x^2}\mathrm{d}x=\frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$

These integral formulae are found in the formula booklet.

Notice that you do not have an integral resulting in arccos. If you are interested in learning why, check out the investigation at the end of this subtopic.

Example 8

★★☆

Find the indefinite integral of $f(x)=\frac{1}{\sqrt{16-x^2}}$.

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Using the basic integral $\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \arcsin\left(\frac{x}{a}\right) + C$, you can think backwards to produce $\int \frac{1}{\sqrt{16-x^2}}\mathrm{d}x = \int \frac{1}{\sqrt{4^2-x^2}}\mathrm{d}x = \arcsin\left(\frac{x}{4}\right) + C$.

Example 9

★★★

Find the indefinite integral of $f(x) = \frac{1}{x^2+10x+34}$.

This uses the basic integral $\int \frac{1}{a^2+x^2}\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$.

First, complete the square to rewrite the denominator:

$$x^2+10x+34 = x^2+10x+5^2+9 = (x+5)^2+3^2$$

So now $\int \frac{1}{x^2+10x+34}\mathrm{d}x = \int \frac{1}{(x+5)^2+3^2}\mathrm{d}x = \frac{1}{3}\arctan\left(\frac{x+5}{3}\right) + C$.

Notice the '$x+5$' and '3' take the place of the 'x' and 'a' in the formula.

You will explore these in more detail in subtopic 5.16 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26514/) when you cover integration by substitution and integration by parts.

7 section questions ^

Question 1

Difficulty:

★★☆

Find the indefinite integral of $f(x) = 3\sec^2\left(\frac{2x+5}{9}\right)$

1 $\frac{27}{2}\tan\left(\frac{2x+5}{9}\right) + C$ ✓

Student view

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(/study/app/math-aa-hl/sid-134-cid-761926/o

2 $3\tan\left(\frac{2x+5}{9}\right)+C$

3 $\frac{3}{2}\tan\left(\frac{2x+5}{9}\right)+C$

4 $27\tan\left(\frac{2x+5}{9}\right)+C$

Explanation

$\int 3\sec^2\left(\frac{2x+5}{9}\right)\mathrm{d}x$

$=3\left(\frac{9}{2}\right)\tan\left(\frac{2x+5}{9}\right)+C$

$=\frac{27}{2}\tan\left(\frac{2x+5}{9}\right)+C$

Question 2

Difficulty:

★★☆

Find the indefinite integral of $f(x)=\sec\left(\frac{3x}{5}\right)\tan\left(\frac{3x}{5}\right)$

1 $\frac{5}{3}\sec\left(\frac{3x}{5}\right)+C$ ✓

2 $\sec\left(\frac{3x}{5}\right)+C$

3 $5\sec\left(\frac{3x}{5}\right)+C$

4 $\frac{1}{3}\sec\left(\frac{3x}{5}\right)+C$

Explanation

$\int\sec\left(\frac{3x}{5}\right)\tan\left(\frac{3x}{5}\right)\mathrm{d}x$

$=\frac{5}{3}\sec\left(\frac{3x}{5}\right)+C$

Question 3

Difficulty:

★★☆

Student view

Find the indefinite integral of $f(x)=\csc(2x+7)\cot(2x+7)$

Overview
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1 $-\frac{1}{2}\csc(2x+7)+C$ ✓

2 $\frac{1}{2}\csc(2x+7)+C$

3 $-\csc(2x+7)+C$

4 $\csc(2x+7)+C$

Explanation

$\int \csc(2x+7)\cot(2x+7)\,\mathrm{d}x$

$= -\frac{1}{2}\csc(2x+7)+C$

Question 4

Difficulty: ★★☆

Find the indefinite integral of $f(x)=8\csc^2(4x+3)$

1 $-2\cot(4x+3)+C$ ✓

2 $-8\cot(4x+3)+C$

3 $2\cot(4x+3)+C$

4 $-\frac{1}{4}\cot(4x+3)+C$

Explanation

$\int 8\csc^2(4x+3)\,\mathrm{d}x$

$= -8\left(\frac{1}{4}\right)\cot(4x+3)+C$

$= -2\cot(4x+3)+C$

Question 5

Difficulty: ★★★

Find the indefinite integral of $f(x)=7\times 4^{x+2}$

Student view

1 $\frac{56\times 4^x}{\ln 2}+C$ ✓

Overview
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2 $\dfrac{7\times 4^{x+2}}{\ln 2}+C$

3 $\dfrac{112\times 4^{x}}{\ln 2}+C$

4 $\dfrac{4^{x}}{\ln 4}+C$

Explanation

$$\int 7\times 4^{x+2}\mathrm{d}x=\int 7\times 4^2\times 4^x\mathrm{d}x=\int 112\times 4^x\mathrm{d}x$$
$$=112\frac{4^x}{\ln 4}+C=\frac{112\times 4^x}{\ln 2^2}+C=\frac{112\times 4^x}{2\ln 2}+C$$
$$=\frac{56\times 4^x}{\ln 2}+C$$

Alternatively:

$$\int 7\times 4^{x+2}\mathrm{d}x$$
$$=7\frac{4^{x+2}}{\ln 4}+C=\frac{7\times 4^2\times 4^x}{\ln 2^2}+C=\frac{7\times 16\times 4^x}{2\ln 2}+C=\frac{7\times 8\times 4^x}{\ln 2}+C$$
$$=\frac{56\times 4^x}{\ln 2}+C$$

Question 6

Difficulty:

★★★

Find the indefinite integral of $f(x)=\dfrac{1}{\sqrt{-x^2-6x+16}}$

1 $\arcsin\left(\dfrac{x+3}{5}\right)+C$ ✓

2 $\arcsin\left(\dfrac{x}{2}\right)+C$

3 $\arcsin\left(\dfrac{x+3}{25}\right)+C$

4 $5\arcsin(x+3)+C$

Explanation

$$\int\frac{1}{\sqrt{-x^2-6x+16}}\mathrm{d}x$$

Complete the square on denominator:

$$-x^2-6x+16=-\left(x^2+6x-16\right)=-\left(x^2+6x+9-25\right)=-\left((x+3)^2-25\right)=25-(x+3)^2$$

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Integrate:

$$\int \frac{1}{\sqrt{-x^2-6x+16}}\mathrm{d}x = \int \frac{1}{\sqrt{25-(x+3)^2}}\mathrm{d}x = \arcsin\left(\frac{x+3}{5}\right) + C$$

Question 7

Difficulty:

★★★

Find the indefinite integral of $f(x) = \dfrac{4}{x^2+2x+5}$

1 $2\arctan\left(\frac{x+1}{2}\right) + C$ ✓

2 $\frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C$

3 $4\arctan\left(\frac{x+1}{\sqrt{5}}\right) + C$

4 $\arctan\left(\frac{x+1}{2}\right) + C$

Explanation

$$\int \frac{4}{x^2+2x+5}\mathrm{d}x$$

Complete the square on denominator:

$$x^2+2x+5 = x^2+2x+1+4 = (x+1)^2+2^2$$

Integrate:

$$\int \frac{4}{x^2+2x+5}\mathrm{d}x = \int \frac{4}{(x+1)^2+2^2}\mathrm{d}x$$
$$= 4\left(\frac{1}{2}\right)\arctan\left(\frac{x+1}{2}\right) + C$$
$$= 2\arctan\left(\frac{x+1}{2}\right) + C$$

Student view

5. Calculus / 5.15 Further differentiation

Indefinite integrals of rational functions

In subtopic 1.11 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26980/), you covered partial fraction decomposition. This was a technique developed to separate complex rational functions into multiple simple rational components. Paired with your knowledge of finding the integrals of simple rational functions presented in subtopic 5.10 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27807/), you can now find indefinite integrals of complex rational functions.

Exam tip

In your exam, the complexity of the denominator will be limited to two distinct linear terms and the degree of the numerator will be less than that of the denominator.

Consider the integral $\int \frac{1}{x^2+3x+2} \mathrm{d}x$. First, rearrange the integrand through partial fraction decomposition.

Making connections

For review of partial fraction decomposition, see section 1.11.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/partial-fractions-id-26981/).

Factorise the denominator

$$x^2+3x+2=(x+2)(x+1)$$

Construct partial fraction

$$\frac{1}{x^2+3x+2}=\frac{A}{x+2}+\frac{B}{x+1}$$

Clear fractions

$$\left(x^2+3x+2\right)\left(\frac{1}{x^2+3x+2}\right)=\left(\frac{A}{x+2}+\frac{B}{x+1}\right)(x+2)(x+1)$$

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Simplify

$$1 = A(x+1) + B(x+2)$$

$$1 = Ax + A + Bx + 2B$$

Set up a system of equations

0th order: $1 = A + 2B$

1st order: $0 = A + B$

Solve the system of equations

$$\begin{cases} 1 = A + 2B \\ 0 = A + B \end{cases} \rightarrow \begin{cases} 1 = A + 2B \\ A = -B \end{cases} \rightarrow \begin{cases} \begin{cases} 1 = -B + 2B & B = 1 \\ A = -B & A = -1 \end{cases} \end{cases}$$

Substitute

$$\frac{1}{x^2+3x+2} = \frac{-1}{x+2} + \frac{1}{x+1}$$

Simplify

$$\frac{1}{x^2+3x+2} = \frac{1}{x+1} - \frac{1}{x+2}$$

Substitute

$$\int \frac{1}{x^2+3x+2} \mathrm{d}x = \int \left(\frac{1}{x+1} - \frac{1}{x+2} \right) \mathrm{d}x = \int \frac{1}{x+1} \mathrm{d}x - \int \frac{1}{x+2} \mathrm{d}x$$

Integrate

$$\ln|x+1| - \ln|x+2| + C = \ln\left|\frac{x+1}{x+2}\right| + C$$

Student view

Notice the use of the rules of logarithms from subtopic 1.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27674/).

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Be aware

Throughout this section, laws of logarithms from subtopic 1.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27674/) will be important. Applied to natural logarithms, they are:

- $\ln xy = \ln x + \ln y$
- $\ln \frac{x}{y} = \ln x - \ln y$
- $\ln x^m = m \ln x$

It is a matter of personal preference whether you use these rules to make the final step.

Example 1

★★★

Find the indefinite integral of $f(x) = \frac{7x+10}{x^2+5x}$

Factorise the denominator

$$x^2 + 5x = x(x+5)$$

Construct partial fraction

$$\frac{7x+10}{x^2+5x} = \frac{A}{x} + \frac{B}{x+5}$$

Clear fractions

$$\left(x^2+5x\right)\left(\frac{7x+10}{x^2+5x}\right) = \left(\frac{A}{x} + \frac{B}{x+5}\right)x(x+5)$$

Simplify

$$7x + 10 = A(x+5) + Bx$$

Set up a system of equations

Student view

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0th order: $10 = 5A$

1st order: $7 = A + B$

Solve the system of equations

$$\begin{cases} 10 = 5A \\ 7 = A + B \end{cases} \rightarrow \begin{cases} A = 2 \\ B = 7 - A \end{cases} \rightarrow \begin{cases} A = 2 \\ B = 5 \end{cases}$$

Substitute

$$\frac{7x + 10}{x^2 + 5x} = \frac{2}{x} + \frac{5}{x + 5}$$

Substitute

$$\int \frac{7x + 10}{x^2 + 5x} \mathrm{d}x = \int \frac{2}{x} \mathrm{d}x + \int \frac{5}{x + 5} \mathrm{d}x$$

Integrate

$$= 2 \ln |x| + 5 \ln |x + 5| + C$$

Simplify

$$\int \frac{7x + 10}{x^2 + 5x} \mathrm{d}x = \ln \left(x^2 |x + 5|^5 \right) + C$$

Example 2

★★★

Find the indefinite integral of $f(x) = \dfrac{1}{x^2 + 7x + 12}$

Factorise the denominator

$$x^2 + 7x + 12 = (x + 4)(x + 3)$$

Student view

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onstruct partial fraction

$$\frac{1}{x^2+7x+12}=\frac{A}{x+4}+\frac{B}{x+3}$$

Clear fractions

$$\left(x^2+7x+12\right)\left(\frac{1}{x^2+7x+12}\right)=\left(\frac{A}{x+4}+\frac{B}{x+3}\right)(x+4)(x+3)$$

Simplify

$$1=A\left(x+3\right)+B\left(x+4\right)$$

Set up a system of equations

0th order: $1=3A+4B$

1st order: $0=A+B$

Solve the system of equations

$$\begin{cases}1=3A+4B\\0=A+B\end{cases}\rightarrow\begin{cases}1=3A+4B\\A=-B\end{cases}\rightarrow\begin{cases}1=3(-B)+4B\\A=-B\end{cases}\rightarrow\begin{cases}B=1\\A=-1\end{cases}$$

Substitute

$$\frac{1}{x^2+7x+12}=\frac{-1}{x+4}+\frac{1}{x+3}$$

Substitute

$$\int\frac{1}{x^2+7x+12}\mathrm{d}x=\int\frac{1}{x+3}\mathrm{d}x-\int\frac{1}{x+4}\mathrm{d}x$$

Integrate

$$=\ln|x+3|-\ln|x+4|+C=\ln\left|\frac{x+3}{x+4}\right|+C$$

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Example 3

★★★

Find the indefinite integral of $\frac{11x+3}{x^2+3x-18}$

Factorise the denominator

$$x^2+3x-18=(x+6)(x-3)$$

Construct partial fraction

$$\frac{11x+3}{x^2+3x-18}=\frac{A}{x+6}+\frac{B}{x-3}$$

Clear fractions

$$\left(x^2+3x-18\right)\left(\frac{11x+3}{x^2+3x-18}\right)=\left(\frac{A}{x+6}+\frac{B}{x-3}\right)(x+6)(x-3)$$

Simplify

$$11x+3=A(x-3)+B(x+6)$$

Set up a system of equations

0th order: $3=-3A+6B$

1st order: $11=A+B$

Solve system of equation

$$\begin{cases}3=-3A+6B\\11=A+B\end{cases}\rightarrow\begin{cases}1=-A+2B\\11=A+B\end{cases}\rightarrow\begin{cases}A=-1+2B\\12=3B\end{cases}\rightarrow\begin{cases}A=7\\B=4\end{cases}$$

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Substitute

$$\frac{11x+3}{x^2+3x-18}=\frac{7}{x+6}+\frac{4}{x-3}$$

Substitute

$$\int\frac{11x+3}{x^2+7x+12}\mathrm{d}x=\int\frac{7}{x+6}\mathrm{d}x+\int\frac{4}{x-3}\mathrm{d}x$$

Integrate

$$=7\ln|x+6|+4\ln|x-3|+C=\ln\left|(x+6)^7(x-3)^4\right|+C$$

Example 4

★★★

Find the indefinite integral of $\frac{36x-55}{12x^2-23x-24}$

Factorise the denominator

$$12x^2-23x-24=(3x-8)(4x+3)$$

Construct partial fraction

$$\frac{36x-55}{12x^2-23x-24}=\frac{A}{3x-8}+\frac{B}{4x+3}$$

Clear fractions

$$\left(12x^2-23x-24\right)\left(\frac{36x-55}{12x^2-23x-24}\right)=\left(\frac{A}{3x-8}+\frac{B}{4x+3}\right)(3x-8)(4x+$$

Simplify

$$36x-55=A(4x+3)+B(3x-8)$$

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Set up system of equations

0th order: $-55 = 3A - 8B$

1st order: $36 = 4A + 3B$

Solve system

$$\begin{cases} -55 = 3A - 8B \\ 36 = 4A + 3B \end{cases} \rightarrow \begin{cases} 220 = -12A + 32B \\ 108 = 12A + 9B \end{cases} \rightarrow \begin{cases} 328 = 41B \\ A = 9 - \left(\frac{3}{4}\right) B \end{cases} \rightarrow \{$$

Substitute

$$\frac{36x - 55}{12x^2 - 23x - 24} = \frac{3}{3x - 8} + \frac{8}{4x + 3}$$

Substitute

$$\int \frac{36x - 55}{12x^2 - 23x - 24} \mathrm{d}x = \int \frac{3}{3x - 8} \mathrm{d}x + \int \frac{8}{4x + 3} \mathrm{d}x$$

Integrate

$$= \ln|3x - 8| + 2\ln|4x + 3| + C = \ln\left|(3x - 8)(4x + 3)^2\right| + C$$

2 section questions ^

Question 1

Difficulty:

★★☆

Find the indefinite integral of $f(x) = \dfrac{2}{3x^2 + 2x}$

1 $\ln\left|\dfrac{x}{3x + 2}\right| + C$ ✓

2 $\ln\left|\dfrac{x}{(3x + 2)^3}\right| + C$

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3 $\ln\left|\frac{2}{3x^2+2x}\right|+C$

4 $\ln\left|\frac{x}{x^3+x^2}\right|+C$

Explanation

Use partial fraction decomposition to rearrange

$$\int \frac{2}{3x^2+2x}\mathrm{d}x = \int \frac{1}{x}\mathrm{d}x - \int \frac{3}{3x+2}\mathrm{d}x$$

Integrate:

$$\int \frac{2}{3x^2+2x}\mathrm{d}x = \int \frac{1}{x}\mathrm{d}x - \int \frac{3}{3x+2}\mathrm{d}x$$
$$= \ln|x| - \ln|3x+2| + C = \ln\left|\frac{x}{3x+2}\right| + C$$

Question 2

Difficulty:

★★★

Find the indefinite integral of $f(x) = \dfrac{3x+7}{6x^2+13x+6}$

1 $\ln|3x+2| - \frac{1}{2}\ln|2x+3| + C$ ✓

2 $\ln\left|\frac{3x+2}{2x+3}\right|+C$

3 $\frac{1}{2}\ln|2x+3| - \ln|3x+2| + C$

4 $\ln\left|\frac{2x+3}{3x+2}\right|+C$

Explanation

Use partial fraction decomposition to rearrange

$$\int \frac{3x+7}{6x^2+13x+6}\mathrm{d}x = \int \frac{3}{3x+2}\mathrm{d}x - \int \frac{1}{2x+3}\mathrm{d}x$$

Integrate:

$$\int \frac{3x+7}{6x^2+13x+6}\mathrm{d}x = \int \frac{3}{3x+2}\mathrm{d}x - \int \frac{1}{2x+3}\mathrm{d}x$$
$$= \ln|3x+2| - \frac{1}{2}\ln|2x+3| + C$$

Student view

5. Calculus / 5.15 Further differentiation

Checklist

What you should know

By the end of this subtopic you should be able to:

- find derivatives of
  - $f(x) = \tan(x)$
  - $f(x) = \sec(x)$
  - $f(x) = \csc(x)$
  - $f(x) = \cot(x)$
  - $f(x) = a^x$
  - $f(x) = \log_a x$
  - $f(x) = \arcsin(x)$
  - $f(x) = \arccos(x)$
  - $f(x) = \arctan(x)$
- find indefinite integrals using the above functions
- find indefinite integrals of complex rational functions using partial fraction decomposition.

5. Calculus / 5.15 Further differentiation

Investigation

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

As you were developing more integrals in section 5.15.4 (/study/app/math-aa-hl/sid-134-cid-761926/book/further-indefinite-integrals-id-26511/), you intentionally left out an integral that resulted in $\arccos x$ or a similar function. Let's look at why.

Just like the other cases, integrals are nothing more than anti-derivatives. Consider the functions

$$f(x) = \arccos \frac{x}{a}$$

$$g(x) = \arcsin \frac{x}{a}$$

How would you find the derivative of $f(x)$?

Using the chain rule, you would get:

$$\frac{\mathrm{d}}{\mathrm{d}x} \arccos \frac{x}{a} = -\frac{1}{\sqrt{1-(x/a)^2}} \frac{1}{a} = \frac{-1}{a\sqrt{1-x^2/a^2}} = \frac{-1}{\sqrt{a^2-x^2}}$$

This would imply that

$$\int \frac{-1}{\sqrt{a^2-x^2}} \mathrm{d}x = \arccos\left(\frac{x}{a}\right) + C.$$

Now you can try an example.

$$h(x) = \int \frac{-1}{\sqrt{9-x^2}} \mathrm{d}x$$

Using our new formula, you get:

$$\int \frac{-1}{\sqrt{9-x^2}} \mathrm{d}x = \arccos \frac{x}{3} + C$$

Alternatively, you could have found the derivative of $g(x)$.

$$\frac{\mathrm{d}}{\mathrm{d}x} \arcsin \frac{x}{a} = \frac{1}{\sqrt{1-(x/a)^2}} \frac{1}{a} = \frac{1}{a\sqrt{1-x^2/a^2}} = \frac{1}{\sqrt{a^2-x^2}}$$

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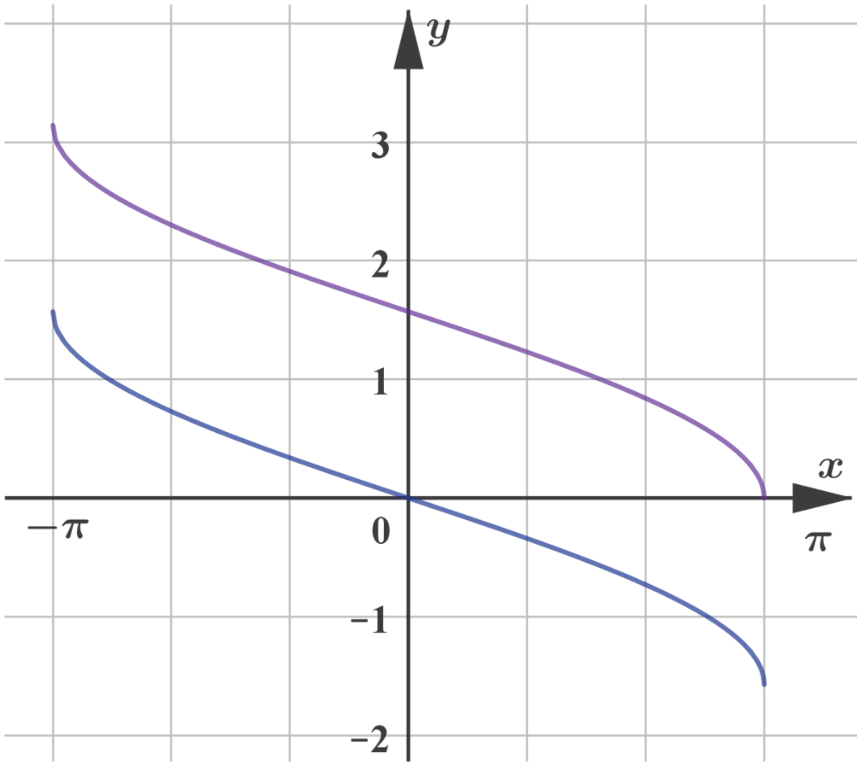
Going back to the example, can you leverage your results from $g(x)$ to find $h(x)$?

The trick is to pull out the –1 from inside the integral.

$$\int \frac{-1}{\sqrt{9-x^2}}dx = -\int \frac{1}{\sqrt{9-x^2}}dx = -\arcsin\frac{x}{3}+C$$

So, which is it, $\arccos\frac{x}{3}+C$ or $-\arcsin\frac{x}{3}+C$?

If you were to look at the graph of each answer without the constant of integration, you get:



The graphs of $f(x) = \arccos\frac{x}{3}$ (top line) and $g(x) = -\arcsin\frac{x}{3}+C$ (bottom line).

Notice that the functions look very similar. It looks like the one function is just a transformation of the other. As a matter of fact, $\arccos\frac{x}{3} = -\arcsin\frac{x}{3}+\frac{\pi}{2}$.

Does it really make much difference which form you use?

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Once you consider that both indefinite integrals have a constant of integration, it is no longer important which one you use as the difference will be accounted for in the '$+C$'.

The same argument holds true for not finding integrals that result in $\log_a x$. If you were to evaluate $\int \frac{1}{x \ln a} \mathrm{d}x$, you can just pull the $\frac{1}{\ln a}$ outside of the integral and use the integral you already know.

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