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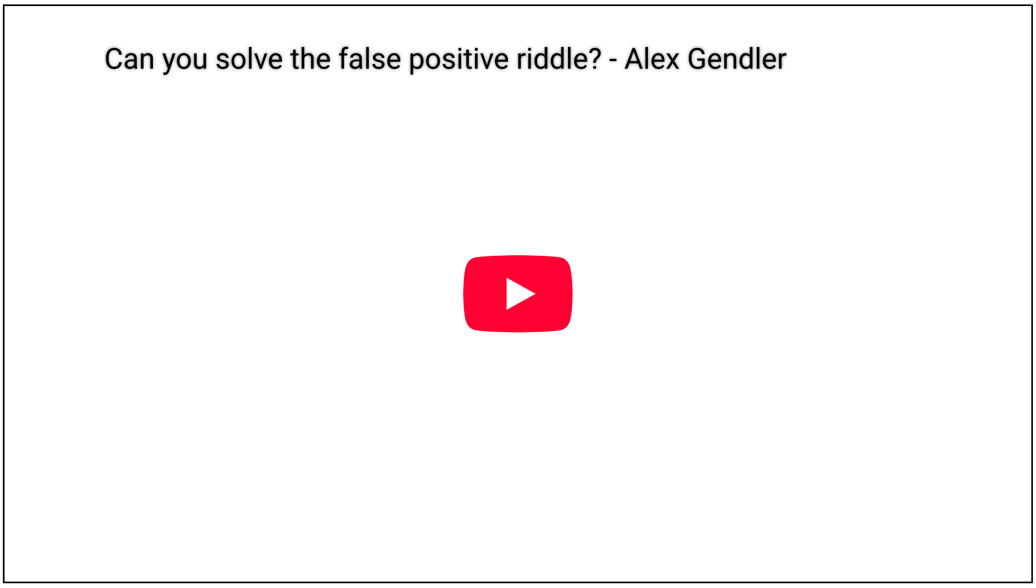
The big picture

In statistical testing, a false positive occurs when a test incorrectly states that an event has occurred. Being unaware of the possibility of these false positives could have a serious effect.

Consider a medical test result that states a patient has a certain disease or condition. Learning about this result could be very stressful for the patient. However, the initial result could very well be a false positive, meaning that the patient does not have the disease or condition. Such false positives are not uncommon when a disease is rare, even if the test is fairly reliable.

False positives can also occur within your email. The email-filtering algorithm used by your email provider could determine that a valid email is spam. If you do not know this and periodically check your spam folder, you could be missing important emails that have wrongly been sent to your spam folder. (Spam messages that the algorithm mistakes for genuine messages and allows through are the opposite, false negatives.)

The video below explores how these false positives can affect decision-making.



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Concept

In this subtopic you will study the relationship between the conditional probabilities of two events. This relationship can cause reliable statistical tests to produce false positives more frequently than we might intuitively expect. Given that these false positives occur, what steps can you take to ensure the validity of your interpretation of a test result?

4. Probability and statistics / 4.13 Bayes' theorem

Bayes' theorem for two events

Bayes' theorem

Let's explore the example of the email filtering algorithm mentioned in The big picture (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27251/).

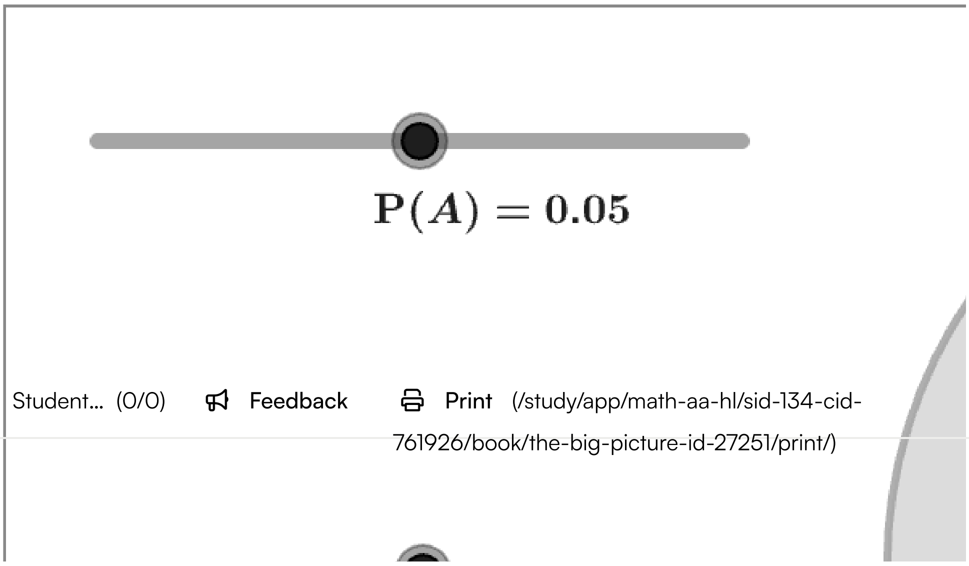
In the Venn diagram below:

- U represents all the emails you receive that contain the word 'lottery'.
- A represents the emails containing the word 'lottery' that are spam.
- B represents the emails containing the word 'lottery' that your email filter marks as spam.

This means that:

- $\mathrm{P}(A)$ is the probability that an email containing the word 'lottery' is spam.
- $\mathrm{P}(B)$ is the probability that your email filter marks an email containing the word 'lottery' as spam.
- $\mathrm{P}(B \mid A)$ is the probability that your email filter correctly identifies a spam email containing the word 'lottery' as spam.

Student view



Interactive 1. Venn Diagram for the Email Filter.

Credit: GeoGebra (https://www.geogebra.org/m/refe4uxm) Nicholas Broom

More information for interactive 1

This interactive will help the users to use Bayes theorem, with the example of the email filtering algorithm. In the given interactive there is a Venn diagram with three circles, grey, blue and red.

The grey circle U represents all the emails received that contain the word ‘lottery’.

The red circle $\mathrm{P}(\mathrm{A})$ represents the probability that an email containing the word 'lottery' is spam.

The blue circle $\mathrm{P}(\mathrm{B})$ represents the probability that the email filter marks an email containing the word 'lottery' as spam.

$\mathrm{P}(\mathrm{B}|\mathrm{A})$ represents the probability that the email filter correctly identifies a spam email containing the word 'lottery' as spam.

Users can slide the value of $\mathrm{P}(\mathrm{A})$ from 0 to 0.1 with difference of 0.01, $\mathrm{P}(\mathrm{B})$ from 0.1 to 0.2 with the difference of 0.01 and $\mathrm{P}(\mathrm{B}|\mathrm{A})$ from 0 to 1 with the difference of 0.01.

Now users will notice that as the value of $\mathrm{P}(\mathrm{A})$ increases, the red circle increases in size and at 1 it will fit exactly with the blue circle or $\mathrm{P}(\mathrm{B})$. Users can conclude that in this case $\mathrm{P}(\mathrm{A}|\mathrm{B}) = 1$ that means if A occurs B must occur. Additionally, since $\mathrm{P}(\mathrm{A})$ and $\mathrm{P}(\mathrm{B})$ are both 0.1 and B is a subset of A, this implies that A can only occur if B also occurs. Similarly users can use different values to conclude other observations giving them better insight of the concept of probability.

Activity

Explore what happens to each of the section of the applet as the probabilities are changed.

What do each of the four sections of the Venn diagram — grey (U), red (A), purple ($A \cap B$), and blue (B) — represent?

What effect do the false positives have on the overall effectiveness of the filter?

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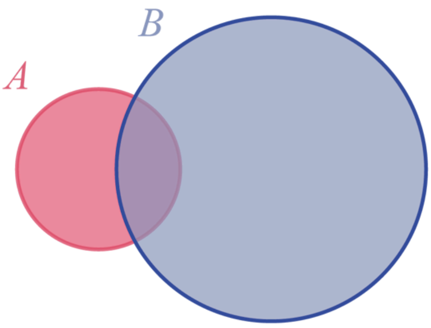
Deriving Bayes' theorem

Let us explore the Venn diagram for the email filter shown above a little more. A portion of the Venn diagram is shown below.



More information

The image is a Venn diagram displaying two circles. The left circle is labeled 'A' and is red in color, while the right circle is larger, labeled 'B', and blue in color. The circles overlap, indicating a conditional probability relationship between sets A and B. The overlap represents the intersection where conditions or elements are common to both circles. This visualization helps in understanding the probability of A occurring given that B has occurred, in line with the conditional probability notation P(A|B).

[Generated by AI]

To begin, recall that the notation for conditional probability, $\mathrm{P}\left(A \mid B\right)$, is read as 'the probability of A given B'. In other words, what is the probability that the event falls within the red circle (A) given that you know the event falls within the blue circle (B)?

Making connections

Now recall the equations for conditional probability that were covered in section 4.6.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/probabilities-of-related-events-id-25652/):

$$\mathrm{P}\left(A \mid B\right) = \frac{\mathrm{P}\left(A \cap B\right)}{\mathrm{P}\left(B\right)} \quad \rightarrow \quad \mathrm{P}\left(A \cap B\right) = \mathrm{P}\left(B\right)\mathrm{P}\left(A \mid B\right)$$

$$\mathrm{P}\left(B \mid A\right) = \frac{\mathrm{P}\left(B \cap A\right)}{\mathrm{P}\left(A\right)} \quad \rightarrow \quad \mathrm{P}\left(B \cap A\right) = \mathrm{P}\left(A\right)\mathrm{P}\left(B \mid A\right)$$

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view

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Note that each of the equations have been rearranged so that the left-hand side of each equation represents the intersection of the events A and B. According to the commutative property for intersection $\mathrm{P}(A \cap B) = \mathrm{P}(B \cap A)$.

Therefore:

$$\begin{aligned} \mathrm{P}(B)\ \mathrm{P}(A \mid B) &= \mathrm{P}(A)\ \mathrm{P}(B \mid A) \\ \mathrm{P}(A \mid B) &= \frac{\mathrm{P}(A)\ \mathrm{P}(B \mid A)}{\mathrm{P}(B)} \end{aligned}$$

✓ **Important**

$\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \dfrac{\mathrm{P}(\mathrm{A})\ \mathrm{P}(\mathrm{B} \mid \mathrm{A})}{\mathrm{P}(\mathrm{B})}$ is one version of Bayes’ theorem.

Example 1

★☆☆

Consider the email filtering situation described above. Given that $\mathrm{P}(A) = 0.01$, $\mathrm{P}(B) = 0.15$ and $\mathrm{P}(B \mid A) = 0.95$, calculate the probability that an email is spam given that your email filter has marked it as spam.

Substitute the values for the probabilities into the formula:

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A)\, P(B \mid A)}{\mathrm{P}(B)} = \frac{(0.01)\,(0.95)}{(0.15)} = 0.0633\ldots$$

Therefore, even though the filter correctly recognises 95% of the spam emails, the large number of false positives cause the filter to only be effective 6.3% of the time!

Now consider the tree diagram for the same spam email scenario:

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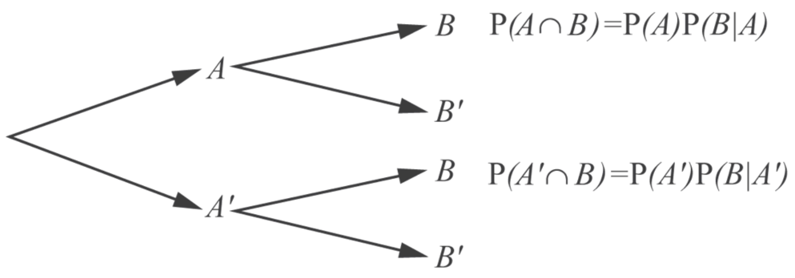

More information

The image is a tree diagram showing a probability scenario involving events A and B. It is structured as follows:

- The diagram starts with a node that splits into two branches.
- The first branch leads to event A, further splitting into two branches:
  - One leads to event B, labeled with P(A ∩ B) = P(A) P(B | A).
  - The other goes to B', indicating the event not involving B.
- The second branch from the starting point leads to event A', marking the absence of event A, and also splits into two branches:
  - One reaches B, labeled P(A' ∩ B) = P(A') P(B | A').
  - The other ends at B', similar to the first split.

The diagram visually represents the paths and probabilities of combined events, illustrating conditional probabilities and intersections of events.

[Generated by AI]

As you can see, there are two paths that include the occurrence of event B. According to the law of total probability, you can find $\mathrm{P}\left(B\right)$ by adding the probability of each path together:

$$\mathrm{P}\left(B\right)=\mathrm{P}\left(A\cap B\right)+\mathrm{P}\left(A'\cap B\right)=\mathrm{P}\left(A\right)\ \mathrm{P}\left(B\mid A\right)+\mathrm{P}\left(A'\right)\ \mathrm{P}\left(B\mid A'\right)$$

This can now be substituted into the equation used in **Example 1** to arrive at Bayes' theorem.

Important

Bayes' theorem tells us that, given any two events A and B,

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$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{\mathrm{P}(A)\ \mathrm{P}(B \mid A)}{\mathrm{P}(A)\ \mathrm{P}(B \mid A) + \mathrm{P}(A')\ \mathrm{P}(B \mid A')}.$$

Exam tip

The Bayes' theorem formula is given in the formula booklet in the following format:

$$\mathrm{P}(B \mid A) = \frac{\mathrm{P}(B)\ \mathrm{P}(A \mid B)}{\mathrm{P}(B)\ \mathrm{P}(A \mid B) + \mathrm{P}(B')\ \mathrm{P}(A \mid B')}$$

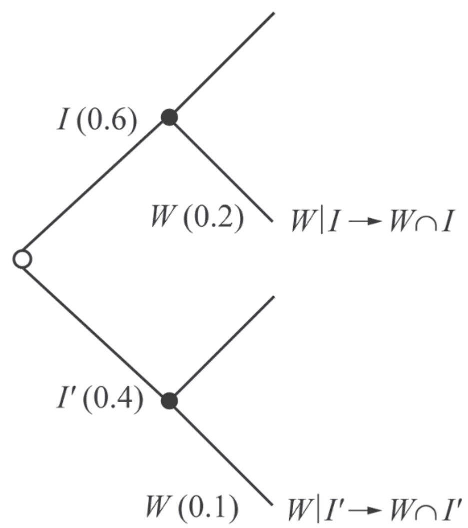
Example 2

★★☆

60% of smartphones run iOS as their operating system. 20% of the people who have an iOS smartphone use WhatsApp for messaging. However, only 10% of those who do not have an iOS smartphone use WhatsApp.

Find the probability that a smartphone user who uses WhatsApp has an iOS phone.

Let I be the event that a smartphone runs iOS and W that WhatsApp is used. The diagram shows the tree diagram that represents this information.



Student view

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The first level covers all the possible outcomes. In this case, smartphones either do or do not run iOS. Then you can use Bayes' theorem to calculate $\mathrm{P}(I \mid W)$, the probability that a given smartphone runs iOS given that the owner uses WhatsApp.

$$\begin{aligned}\mathrm{P}(I \mid W) &= \frac{\mathrm{P}(I \cap W)}{\mathrm{P}(W)} \\ &= \frac{\mathrm{P}(I)\ \mathrm{P}(W \mid I)}{\mathrm{P}(I)\ \mathrm{P}(W \mid I) + \mathrm{P}(I')\ \mathrm{P}(W \mid I')} \\ &= \frac{(0.6)(0.2)}{(0.6)(0.2) + (0.4)(0.1)} \\ &= \frac{0.12}{0.12 + 0.04} \\ &= \frac{3}{4}\end{aligned}$$

International Mindedness

Messaging apps like WhatsApp have become very popular and, by some accounts, crucial to our social networking and communication. It might seem obvious to you that almost everybody uses one particular app. However, the messaging app that you use may very well depend on where you live. While WhatsApp is one of the most popular throughout the world, Facebook messenger still dominates the US market while WeChat is the leader within China. Other apps such as LINE, imo, and Viber also dominate other smaller countries. This map (https://infogram.com/april-2022-worldwide-messaging-app-1hd12yx7dmzxw6k) gives an overview of the distribution of messaging app popularity around the world (as of April 2022).

Example 3

★★☆

A bag contains 9 white marbles and 6 black marbles. A marble is drawn from the bag, its colour is noted, and the marble is then placed to the side. Another marble is drawn from the bag and its colour is noted.

If the two marbles are different colours, find the probability that the first one drawn was white.

Let W_i be the event that a white marble is selected at time i, and B_i be the event that a black marble is selected at time i, where $i = 1$ or 2. The diagram below shows the tree diagram for two successive selections, along with the probability of each event occurring in each

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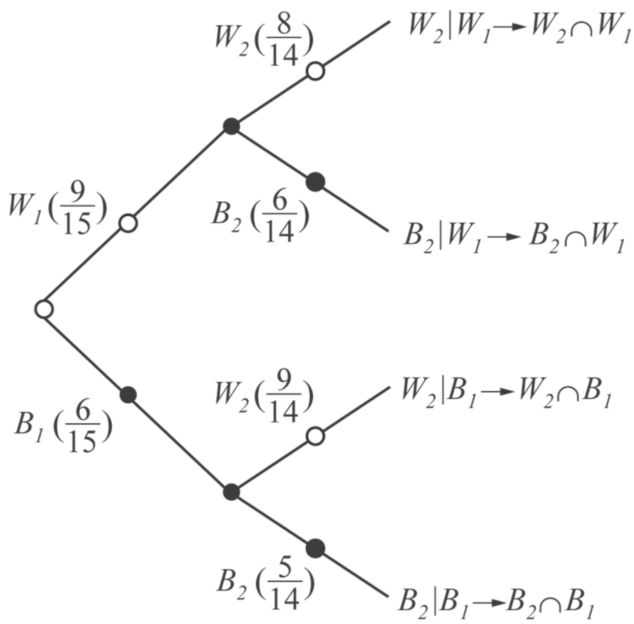
step:



Extract the needed probabilities from the tree diagram.

$$\mathrm{P}\left(\text{different}|\text{ colour } W_1\right) = \mathrm{P}\left(B_2 \mid W_1\right) = \frac{6}{14}$$

$$\mathrm{P}\left(W_1\right) = \frac{9}{15} \qquad \mathrm{P}\left(B_1\right) = \frac{6}{15}$$

$$\mathrm{P}\left(\text{different}|\text{ colour } B_1\right) = \mathrm{P}\left(W_2 \mid B_1\right) = \frac{9}{14}$$

Now substitute into Bayes' theorem.

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$$\begin{aligned}\mathrm{P}\left(W_1 \mid \text{different colour}\right) &= \frac{\mathrm{P}\left(W_1\right)\mathrm{P}\left(\text{different}\mid\ \text{colour}\ \ W_1\right)}{\mathrm{P}\left(W_1\right)\mathrm{P}\left(\text{different}\mid\ \text{colour}\ \ W_1\right)+\mathrm{P}\left(B_1\right)\mathrm{P}\left(\text{different}\mid\ \text{co}\right.}\\ &= \frac{\left(\frac{9}{15}\right)\left(\frac{6}{14}\right)}{\left(\frac{9}{15}\right)\left(\frac{6}{14}\right)+\left(\frac{6}{15}\right)\left(\frac{9}{14}\right)}\\ &= \frac{1}{2}\end{aligned}$$

3 section questions

Question 1

Difficulty:

★★☆

Two-thirds of the students in a certain school class are female. One-fifth of the girls in the class study biology and half of the boys study biology.

Calculate the probability that a randomly selected student is female given that they do not take biology.

Give your answer as a decimal correct to three significant figures.

0.762

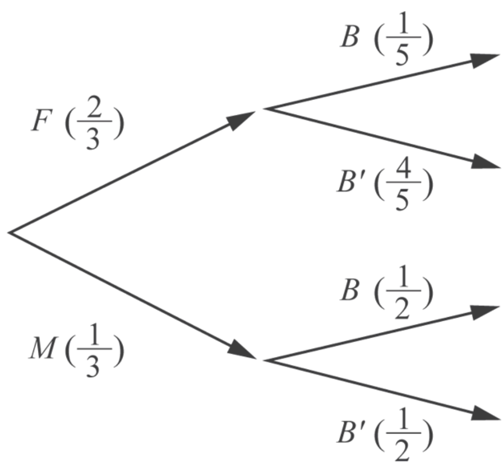
Accepted answers

0.762, 0,762, .762

Explanation

First, define event F as a student being female, event M as a student being male, and event B as a student taking biology.

Then sketch a tree diagram:



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Now use Bayes' theorem.

$$\mathrm{P}\left(F \mid B'\right) = \frac{\mathrm{P}\left(F\right)\,\mathrm{P}\left(B' \mid F\right)}{\mathrm{P}\left(F\right)\,\mathrm{P}\left(B' \mid F\right) + \mathrm{P}\left(M\right)\,\mathrm{P}\left(B' \mid M\right)}$$

$$= \frac{\frac{2}{3}\,\frac{4}{5}}{\frac{2}{3}\,\frac{4}{5} + \frac{1}{3}\,\frac{1}{2}}$$

$$= \frac{\frac{8}{15}}{\frac{8}{15} + \frac{1}{6}}$$

$$= 0.761904\ldots$$
$$= 0.762$$

Question 2

Difficulty:

★★☆

When dressing for work, Sean wears his red shirt $\frac{1}{5}$ of the time. There is a $\frac{3}{10}$ chance he will wear a pink tie when he wears his red shirt and a $\frac{7}{10}$ chance he will wear a pink tie when he does not wear his red shirt.

Given that he is currently wearing a pink tie, find the probability that he is also wearing his red shirt.

Give your answer in decimal form correct to three significant figures.

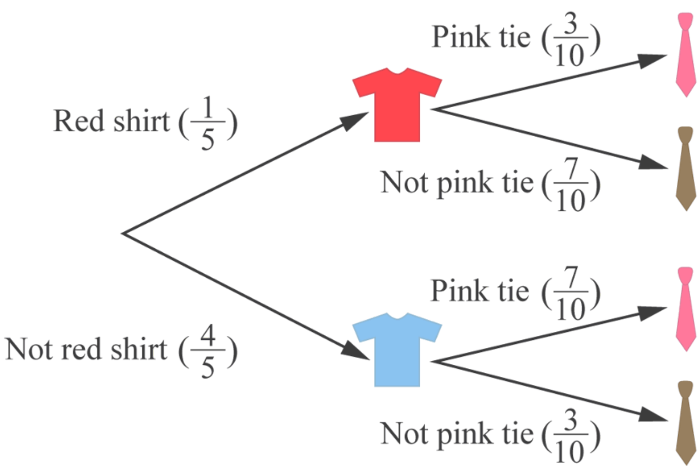
0.0968 ✓

Accepted answers

0.0968, 0,0968, .0968

Explanation

First create a tree diagram of the situation:



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Then substitute into Bayes' theorem:

$$\begin{aligned}
\mathrm{P}(R \mid P) &= \frac{\mathrm{P}(R)\,\mathrm{P}(P \mid R)}{\mathrm{P}(R)\,\mathrm{P}(\mathrm{P} \mid R) + \mathrm{P}(R')\,\mathrm{P}(\mathrm{P} \mid R')} \\
&= \frac{\left(\frac{1}{5}\right)\left(\frac{3}{10}\right)}{\left(\frac{1}{5}\right)\left(\frac{3}{10}\right) + \left(\frac{4}{5}\right)\left(\frac{7}{10}\right)} \\
&= \frac{\frac{3}{50}}{\frac{3}{50} + \frac{28}{50}} \\
&= 0.0967741\ldots \\
&\approx 0.0968
\end{aligned}$$

Question 3

Difficulty: ★☆☆

Find $\mathrm{P}(A' \mid B)$ given that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B \mid A) = 0.2$ and $\mathrm{P}(B \mid A') = 0.4$.

Give your answer correct to three significant figures.

0.824 ✓

Accepted answers

0.824, .824

Explanation

First, you realise that if $\mathrm{P}(A) = 0.3$ then $\mathrm{P}(A') = 0.7$.

You can then use Bayes' theorem:

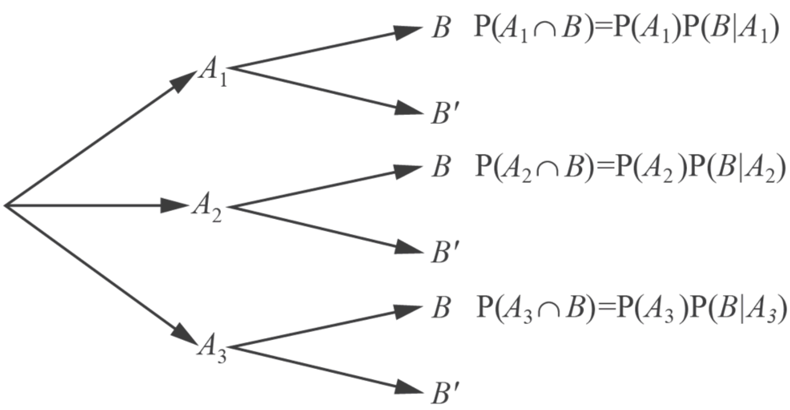
$$\begin{aligned}
\mathrm{P}\left(A' \mid B\right) &= \frac{\mathrm{P}(A')\ \mathrm{P}(B \mid A')}{\mathrm{P}(A')\ \mathrm{P}(B \mid A') + \mathrm{P}(A)\ \mathrm{P}(B \mid A)} \\
&= \frac{(0.7)(0.4)}{(0.7)(0.4) + (0.3)(0.2)} \\
&= \frac{0.28}{0.28 + 0.06} \\
&= 0.823529\ldots \\
&= 0.824 \text{ (to 3 significant figures)}
\end{aligned}$$

Student view

4. Probability and statistics / 4.13 Bayes' theorem

Bayes' theorem for three events

How can Bayes' theorem be used if there are three different outcomes for the first event? Consider the following tree diagram:



More information

The image is a tree diagram used to illustrate the application of Bayes' theorem with three different outcomes for the first event. The diagram has three main branches labeled A1, A2, and A3. Each branch splits into two sub-branches, one leading to B and the other to B'. For each branch, a probability expression is depicted:

- A1 splits into two arrows, one pointing to B with the expression P(A1 B) = P(A1)P(B|A1).
- Another points to B'.
- A2 has similar paths, with the expression P(A2 B) = P(A2)P(B|A2) on the B branch.
- A3 also splits into paths leading to B and B', with P(A3 B) = P(A3)P(B|A3).

Each expression shows the intersection and conditional probabilities for each outcome, demonstrating different paths to event B.

[Generated by AI]

You can see that $\text{P}(B)$ is now present in three different paths of the tree diagram, therefore:

$$\begin{aligned}\text{P}(B) &= \text{P}(A_1 \cap B) + \text{P}(A_2 \cap B) + \text{P}(A_3 \cap B) \\ &= \text{P}(A_1)\text{P}(B \mid A_1) + \text{P}(A_2)\text{P}(B \mid A_2) + \text{P}(A_3)\text{P}(B \mid A_3)\end{aligned}$$

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Substitute this into the equation from **Example 1** in section 4.13.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/bayes-theorem-for-two-events-id-27252/) to obtain the equation for Bayes' theorem for three events as given below.

Important

Bayes' theorem for three events:

$$\mathrm{P}(A_i \mid B) = \frac{\mathrm{P}(A_i)\mathrm{P}(B \mid A_i)}{\mathrm{P}(A_1)\mathrm{P}(B \mid A_1) + \mathrm{P}(A_2)\mathrm{P}(B \mid A_2) + \mathrm{P}(A_3)\mathrm{P}(B \mid A_3)}.$$

Exam tip

The Bayes' theorem formula is given in the formula booklet in the following format:

$$\mathrm{P}(B_i \mid A) = \frac{\mathrm{P}(B_i)\mathrm{P}(A \mid B_i)}{\mathrm{P}(B_1)\mathrm{P}(A \mid B_1) + \mathrm{P}(B_2)\mathrm{P}(A \mid B_2) + \mathrm{P}(B_3)\mathrm{P}(A \mid B_3)}$$

Example 1

★★☆

Zoe competes in a chess tournament. Each game results in a win, draw or loss. The probability that Zoe wins the tournament if she wins her first match is 70%. The probability that she wins the tournament if she draws her first match is 40%. The probability that she wins the tournament if she loses her first match is 20%. There is a 60% chance that she wins her first game and a 30% chance that she draws.

Given that Zoe wins the tournament, calculate the probability that she drew her first match.

Let the event W be a first match win, D a first match draw, L a first match loss, and T a tournament win. The tree diagram organises the information given in the question.

Notice that $\mathrm{P}(L) = 1 - (\mathrm{P}(W) + \mathrm{P}(D)) = 0.1$.

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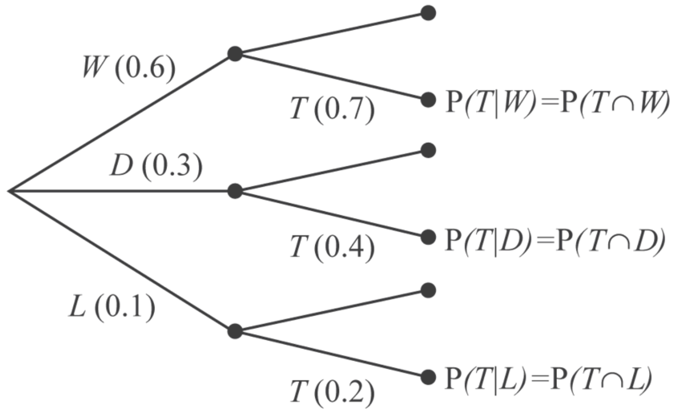

You can now substitute this information into the expanded Bayes' theorem equation:

$$\begin{aligned}
\mathrm{P}(D \mid T) &= \frac{\mathrm{P}(D)\, P(T \mid D)}{\mathrm{P}(L)\, P(T \mid L) + \mathrm{P}(D)\, \mathrm{P}(T \mid D) + \mathrm{P}(W)\, \mathrm{P}(T \mid W)} \\
&= \frac{(0.3)(0.4)}{(0.1)(0.2) + (0.3)(0.4) + (0.6)(0.7)} \\
&= \frac{0.12}{0.02 + 0.12 + 0.42} \\
&= \frac{0.12}{0.56} \\
&\approx 0.214
\end{aligned}$$

Be aware

Bayes' theorem becomes considerably easier when you represent a situation with a tree diagram. This helps you to visualise which branch needs to be selected for the numerator of the equation.

3 section questions

Question 1

Difficulty: ★☆☆

Calculate $\mathrm{P}(B_1)\,\mathrm{P}(A \mid B_1)$ where B_1 is one of three possible outcomes, given that $\mathrm{P}(B_1 \mid A) = \frac{2}{15}$, $\mathrm{P}(B_2)\,\mathrm{P}(A \mid B_2) = \frac{1}{3}$, and $\mathrm{P}(B_3)\,\mathrm{P}(A \mid B_3) = \frac{5}{24}$.

Student view

Give your answer in decimal form correct to three significant figures.

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0.0833

Accepted answers

0.0833, 0,0833, .0833, 1/12

Explanation

Insert the given information in Bayes' theorem for three events:

$$\mathrm{P}(B_1 \mid A) = \frac{\mathrm{P}(B_1)\mathrm{P}(A \mid B_1)}{\mathrm{P}(B_1)\mathrm{P}(A \mid B_1) + \mathrm{P}(B_2)\mathrm{P}(A \mid B_2) + \mathrm{P}(B_3)\mathrm{P}(A \mid B_3)}$$
$$\frac{2}{15} = \frac{\mathrm{P}(B_1)\mathrm{P}(A \mid B_1)}{\mathrm{P}(B_1)\mathrm{P}(A \mid B_1) + \frac{1}{3} + \frac{5}{24}}$$

Let $x = \mathrm{P}(B_1)\mathrm{P}(A \mid B_1)$ and solve for x:

$$\frac{2}{15} = \frac{x}{x + \frac{1}{3} + \frac{5}{24}}$$
$$\frac{2}{15} = \frac{x}{x + \frac{13}{24}}$$
$$\frac{2}{15}\left(x + \frac{13}{24}\right) = x$$
$$\frac{2}{15}x + \frac{26}{360} = x$$
$$\frac{13}{15}x = \frac{26}{360}$$
$$x = \frac{1}{12} \approx 0.0833$$

Question 2

Difficulty: ★★☆

During the week, Jaxon likes to change how he travels to work. On Mondays and Fridays, he travels by bus and is on time 90% of the time. On Tuesdays and Wednesdays, he calls a taxi and is on time 99% of the time. He uses the subway on Thursdays and is on time 95% of the time.

Given that he was late on an unknown working day of the week, calculate the probability that he travelled to work by bus. Give your answer in decimal form correct to three significant figures.

0.741

Accepted answers

0.741, .741, 0,741

Explanation

Let B be the event that Jaxon travels by bus, T be the event that he travels by taxi, S be the event that he travels by subway and L be the event that he is late. You can summarize the information given in the question.

- Since there are five working days in a week, $\mathrm{P}(B) = 2/5 = 0.4$ and $\mathrm{P}(L|B) = 1 - 0.9 = 0.1$.
- Similarly, $\mathrm{P}(T) = 2/5 = 0.4$ and $\mathrm{P}(L|T) = 1 - 0.99 = 0.01$.

Student view
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- Similarly, $\mathrm{P}(S) = 1/5 = 0.2$ and $\mathrm{P}(L|S) = 1 - 0.95 = 0.05$.

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You can use Bayes' theorem to find $\mathrm{P}(B|L)$, which is the question is asking you to find.

$$\begin{aligned}\mathrm{P}(B|L) &= \frac{\mathrm{P}(B)\,\mathrm{P}(L|B)}{\mathrm{P}(B)\,\mathrm{P}(L|B) + \mathrm{P}(T)\,\mathrm{P}(L|T) + \mathrm{P}(S)\,\mathrm{P}(L|S)} \\ &= \frac{0.4 \times 0.1}{0.4 \times 0.1 + 0.4 \times 0.01 + 0.2 \times 0.05} \\ &= \frac{0.04}{0.054} \approx 0.741\end{aligned}$$

Question 3

Difficulty: ★★★

Vroom Corporation manufactures cars at three different factories. Factory A makes 40% of the cars, factory B makes 35%, and factory C makes the rest. 10% of the cars that are produced at factory A and 8% of the cars produced at factory B have engine failure within the first 6 months of operation. It is known that 15% of all of Vroom's cars that have engine failure within the first 6 months of operation are produced at factory C.

Calculate the percentage of cars produced at factory C that have engine failure within the first 6 months of operation.

4.8 ✓

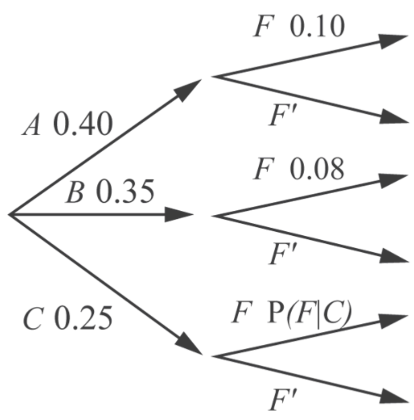
Accepted answers

4.8, 4.8%

Explanation

First define event A as a car being produced at factory A, event$B$ as a car being produced at factory B, C as a car being produced at factory C, and F as a car having engine failure within the first 6 months of operation.

You can then sketch a tree diagram:



More information

Student view

From the question, you know that $\mathrm{P}\left(C \mid F\right) = 0.15$, therefore you can use Bayes' theorem to find $\mathrm{P}\left(F \mid C\right)$:

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

$$\begin{aligned}
\mathrm{P}(C \mid F) &= \frac{\mathrm{P}(C)\,\mathrm{P}(F \mid C)}{\mathrm{P}(C)\,\mathrm{P}(F \mid C) + \mathrm{P}(B)\,\mathrm{P}(F \mid B) + \mathrm{P}(A)\,\mathrm{P}(F \mid A)} \\
0.15 &= \frac{0.25 \times \mathrm{P}(F \mid C)}{0.25 \times \mathrm{P}(F \mid C) + (0.35)(0.08) + (0.4)(0.10)} \\
0.15 &= \frac{0.25 \times \mathrm{P}(F \mid C)}{0.25 \times \mathrm{P}(F \mid C) + 0.068} \\
0.15\,(0.25 \times \mathrm{P}(F \mid C) + 0.068) &= 0.25 \times \mathrm{P}(F \mid C) \\
0.0375 \times \mathrm{P}(F \mid C) + 0.0102 &= 0.25 \times \mathrm{P}(F \mid C) \\
0.0102 &= 0.2125 \times \mathrm{P}(F \mid C) \\
0.048 &= \mathrm{P}(F \mid C)
\end{aligned}$$

Therefore, 4.8% of the cars produced at factory C have engine failure within the first 6 months of operation.

4. Probability and statistics / 4.13 Bayes' theorem

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27254/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- identify how many events are present within a probability question
- calculate probabilities using Bayes' theorem for questions with two events,

$$\mathrm{P}(B \mid A) = \frac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)} = \frac{\mathrm{P}(B)\,\mathrm{P}(A \mid B)}{\mathrm{P}(B)\,\mathrm{P}(A \mid B) + \mathrm{P}(B')\,\mathrm{P}(A \mid B')}$$

- use the extended version of Bayes' theorem for questions with three events,

$$\mathrm{P}(B_i \mid A) = \frac{\mathrm{P}(B_i)\,\mathrm{P}(A \mid B_i)}{\mathrm{P}(B_1)\,P(A \mid B_1) + \mathrm{P}(B_2)\,\mathrm{P}(A \mid B_2) + \mathrm{P}(B_3)\,\mathrm{P}(A \mid B_3)}$$

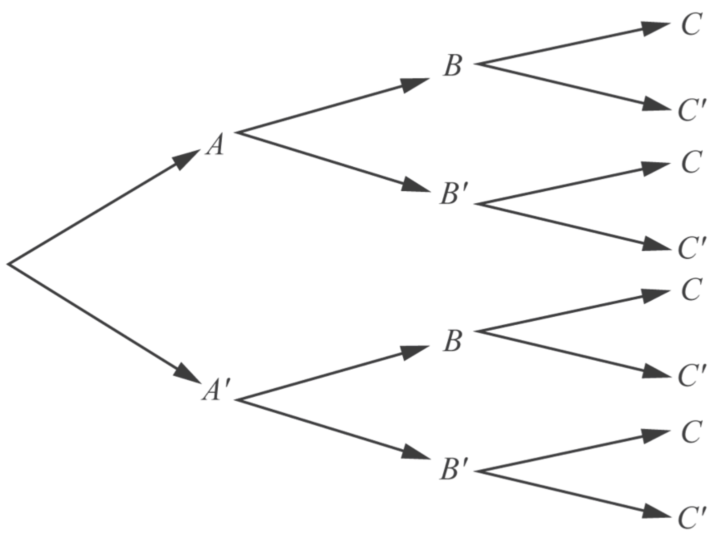
- sketch tree diagrams to represent information given with a question.

4. Probability and statistics / 4.13 Bayes' theorem

Student view

Investigation

Consider the following tree diagram.



More information

This is a tree diagram used to extend Bayes' theorem. The diagram starts with an initial node that splits into two paths labeled A and A'. From node A, there are two branches leading to nodes labeled B and B'. Node B further splits into two paths, leading to nodes C and C'. Similarly, node B' divides into two paths leading to nodes C and C'. The A' node splits into nodes B and B', each of which splits further into two paths leading to nodes C and C'. The structure represents a sequence of conditional probabilities and logical paths that aid in visualization and computation of Bayesian probabilities.

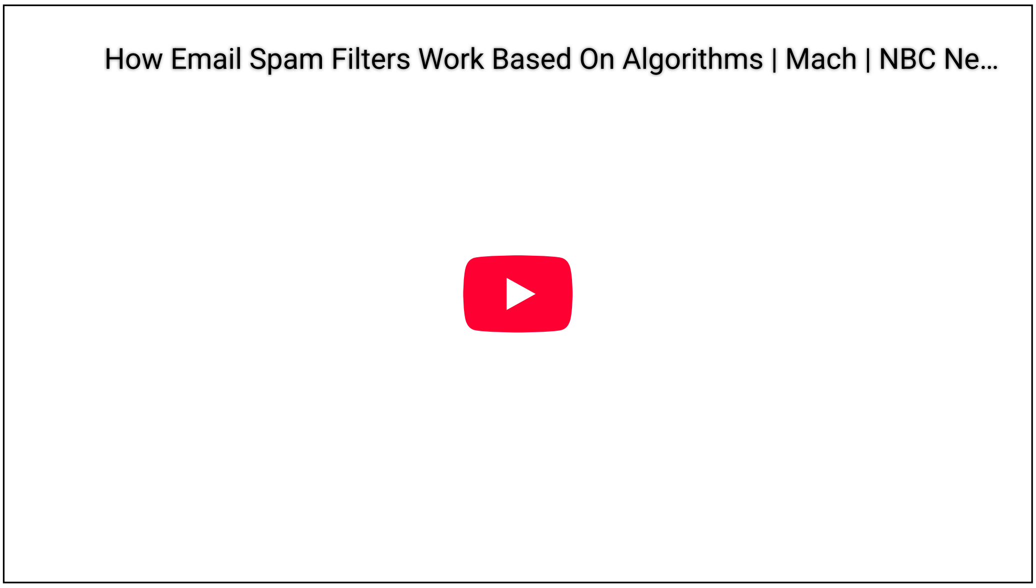
[Generated by AI]

How can you extend Bayes' theorem to find $\mathrm{P}\left(A \mid C\right)$?

We have already mentioned email filters in this subtopic. In fact, Bayes' theorem is used in filtering algorithms: each additional piece of information about the email (i.e. it contains the word 'lottery', or contains a link, or has been sent to multiple recipients, etc.) affects the calculated probability that the email is spam.

Now watch the following video about Bayesian filtering.

Student view



After watching the above video, read through this article (https://digitalcommons.morris.umn.edu/cgi/viewcontent.cgi?article=1024&context=horizons) by Jeremy J. Eberhardt from the University of Minnesota.

Now, with the use of examples, investigate the differences between the Multivariable Naïve Bayes' filtering method and the Multinomial Naïve Bayes' filtering method.

Rate subtopic 4.13 Bayes' theorem

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