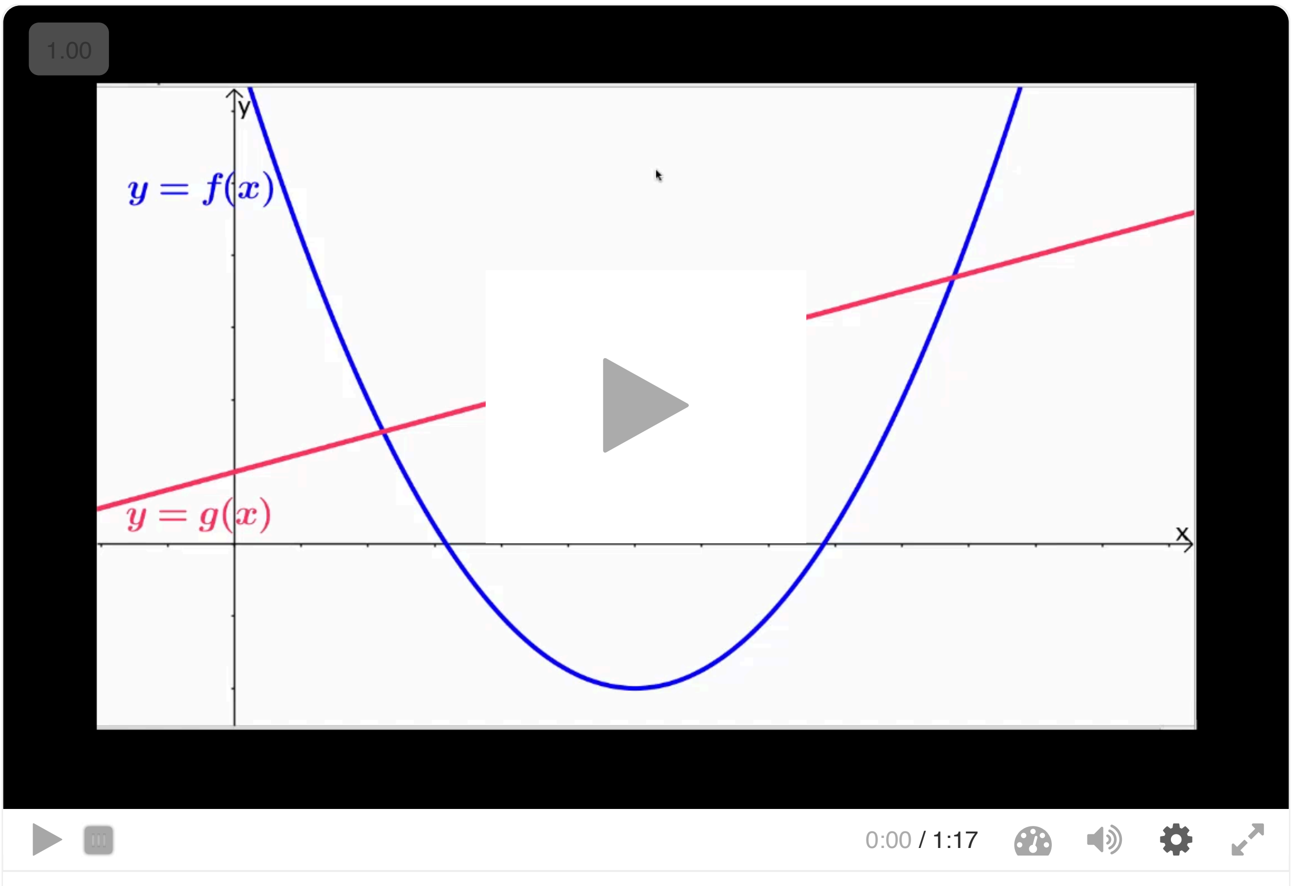
Index

2. Functions / 2.15 Solving inequalities

The big picture

You have already studied equalities and equations, solving statements of the form $f(x) = g(x)$, which corresponds graphically to finding intersection points of the curves $y = f(x)$ and $y = g(x)$. We can extend that scenario to involve inequalities, i.e. solving statements of the form $f(x) < g(x)$.



Video 1. Understanding Quadratic Inequalities: Solving and Graphing.

More information for video 1

Overview
(/study/ap
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1

00:00:00,300 --> 00:00:02,433

narrator: In this video, we're going

to look at inequalities

2

00:00:02,500 --> 00:00:03,767

when it comes to functions.

3

00:00:04,000 --> 00:00:06,633

So here we've got two functions,

y equals f of x,

4

00:00:06,700 --> 00:00:09,100

the blue one,

and y equals g of x, the reddish one.

5

00:00:09,633 --> 00:00:10,567

Now we've already seen

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00:00:10,633 --> 00:00:12,067

that we can ask ourselves the question,

7

00:00:12,133 --> 00:00:14,633

what are the points of intersection

between those two functions?

8

00:00:14,800 --> 00:00:18,433

And of course, those are the places

where for a certain x,

9

00:00:18,500 --> 00:00:20,967

they evaluate to the same y value.

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00:00:21,033 --> 00:00:23,200

In other words, f of x equals to g of x,

11

00:00:23,267 --> 00:00:25,633

and this can happen in one point

Student
view

or more than one point.

12

00:00:25,700 --> 00:00:28,467

In this case, we see two points

of intersection.

13

00:00:29,233 --> 00:00:31,367

But of course, we can also

ask ourselves a question

14

00:00:31,433 --> 00:00:34,800

instead of an equality,

what about an inequality, for example?

15

00:00:34,933 --> 00:00:37,633

Whereas f of x is small than g of x.

16

00:00:38,067 --> 00:00:41,633

Now the most important thing to realize

is that in this case,

17

00:00:41,700 --> 00:00:44,133

you are looking for a region of x.

18

00:00:44,300 --> 00:00:47,667

So don't be surprised

when you have one or more regions.

19

00:00:47,733 --> 00:00:49,133

So for example, here, the fact

20

00:00:49,200 --> 00:00:51,600

that we have a regions

indicated by the shaded area,

21

00:00:52,333 --> 00:00:56,233

among all of which that statement is true,

f of x is small than g of x.

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22

00:00:56,300 --> 00:00:58,900

Or you can do it with a number line

as indicated here,

23

00:00:58,967 --> 00:01:02,667

where the open end,

the open circle indicates

24

00:01:02,733 --> 00:01:06,000

that it is an inequality,

so not an equality.

25

00:01:06,067 --> 00:01:08,167

So that is very important to realize

26

00:01:08,233 --> 00:01:10,433

and keep in mind that

when you solve inequalities,

27

00:01:10,500 --> 00:01:15,333

you're looking for regions

where the inequality is true.

28

00:01:15,400 --> 00:01:17,000

That is where it holds.

In this section you will solve inequalities relating to two functions both graphically and analytically .

Concept

Equivalence

Two functions may be equal at one or more points. However, inequality between two functions occurs at an infinite number of points, which can be represented in interval form.

Student view

How is the inequality between two functions dependent on their conditions for equality?

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2. Functions / 2.15 Solving inequalities

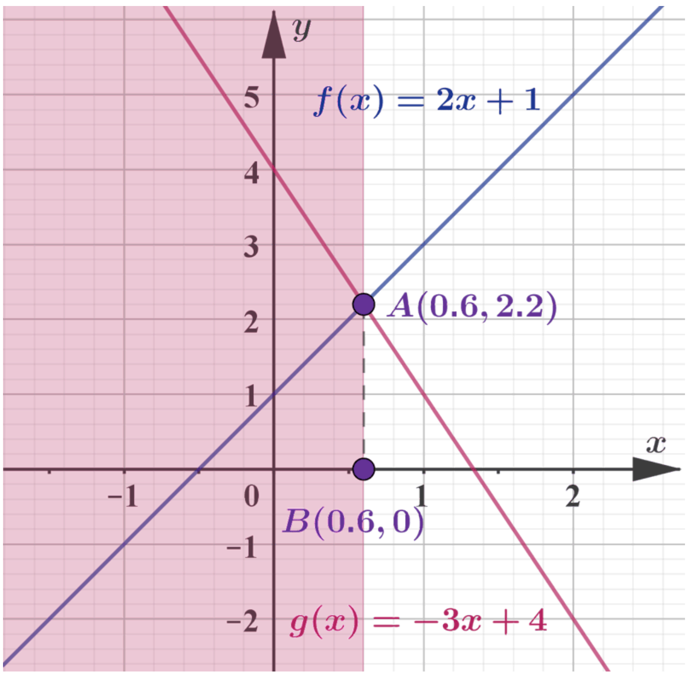
Solving graphically

The following graph shows two functions $f(x)$ and $g(x)$, where $f(x) = 2x + 1$ and $g(x) = -3x + 4$.

The coordinates of the point of intersection are $A(0.6,\ 2.2)$.

Important

The inequality condition before the point of intersection of two lines is opposite to that after the point. For example, if $f(x) < g(x)$ for $x < a$, then $f(x) > g(x)$ for $x > a$, where the point of intersection is at $x = a$.



More information

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The image depicts a graph with two linear functions, f(x) = 2x + 1 (in blue) and g(x) = -3x + 4 (in pink), plotted on a Cartesian plane with a grid background. The x-axis ranges from -2 to 3, while the y-axis ranges from -2 to 5. Point A (0.6, 2.2) and Point B (0.6, 0) are marked, highlighting where the lines intersect with x = 0.6. The region where x < 0.6 is shaded pink, indicating that, in this area, f(x) is below g(x).

[Generated by AI]

The shaded region represents the part of the x-axis where $x < 0.6$, and for these x-values note that the function $f(x)$ is below the function $g(x)$.

Mathematically you will write the above statement as:

$$f(x) < g(x) \text{ for } x < 0.6$$

On the other hand, the unshaded region gives:

$$f(x) > g(x) \text{ for } x > 0.6$$

Example 1

★☆☆

Find the values of x for which $x + 1 < 3 - 2x$.

Let:

$$f(x) = x + 1$$
$$g(x) = 3 - 2x$$

You need to find the x-values for which $f(x) < g(x)$. Follow the steps given below.

1. Graph both the functions on the same set of axes.

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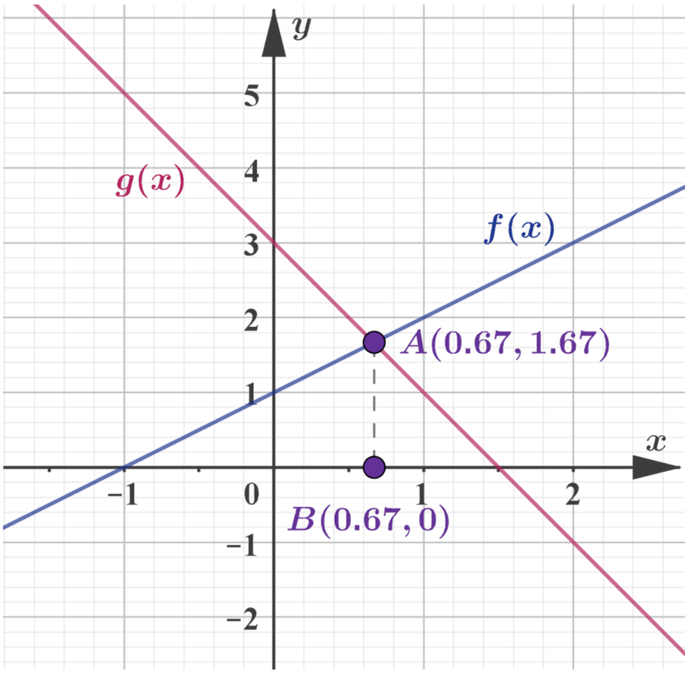

2. Find the point of intersection: $\left(\frac{2}{3}, \frac{5}{3}\right)$.
3. Check which side of $x = 0.67$ has $f(x)$ below $g(x)$.

Refer to the calculator instructions in section 2.4.3.

In this example it is on the left-hand side of $x = 0.67$, i.e. $x < 0.67$.

Hence the solution for $x + 1 < 3 - 2x$ is $x < 0.67$ or $(-\infty,\ 0.67)$ as shown by the shaded region on the graph below (note the broken line $x = 0.67$, which indicates that the points on this line are not included in the solution set).

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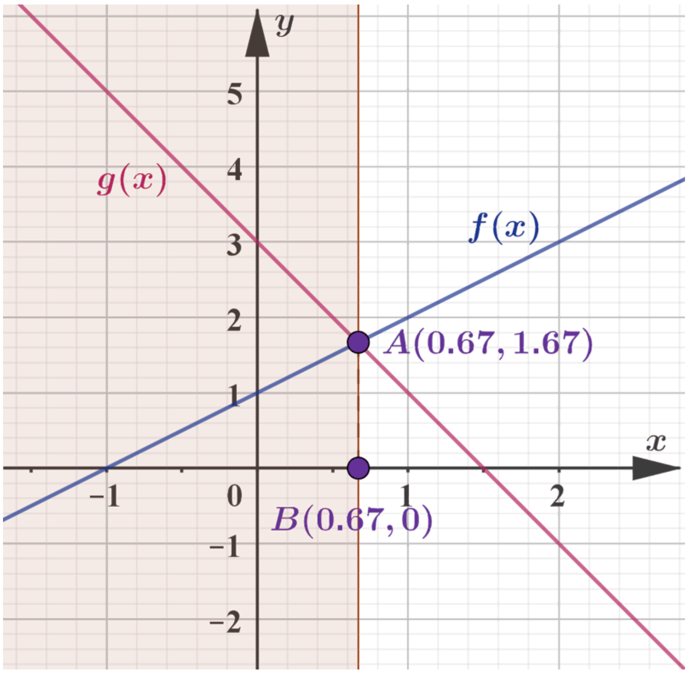
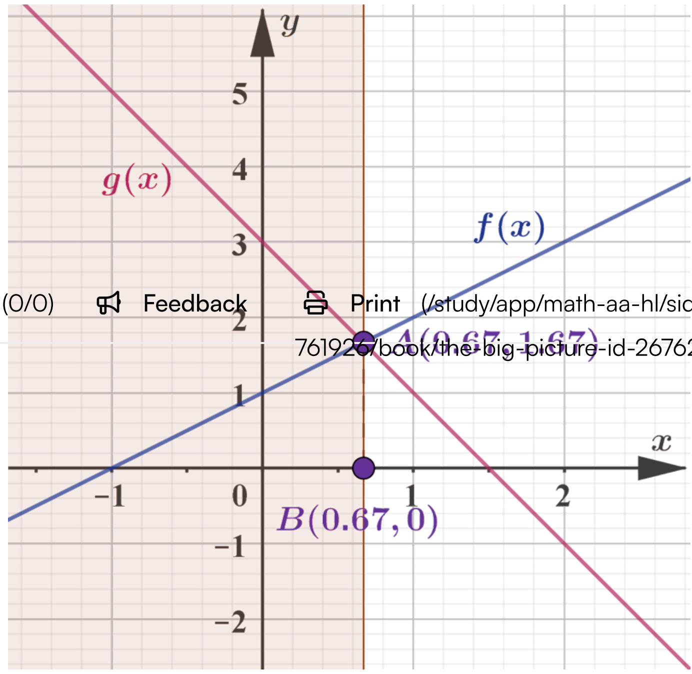


However, if the question were to find the x-values for which $x + 1 \leqslant 3 - 2x$, then your answer would be $x \leqslant 0.67$ or $(-\infty,\ 0.67]$ because equality is included in the question, and at $x = 0.67$ the functions are equal. In this case, the solution on the graph will be the shaded region including the points on the vertical line $x = 0.67$.



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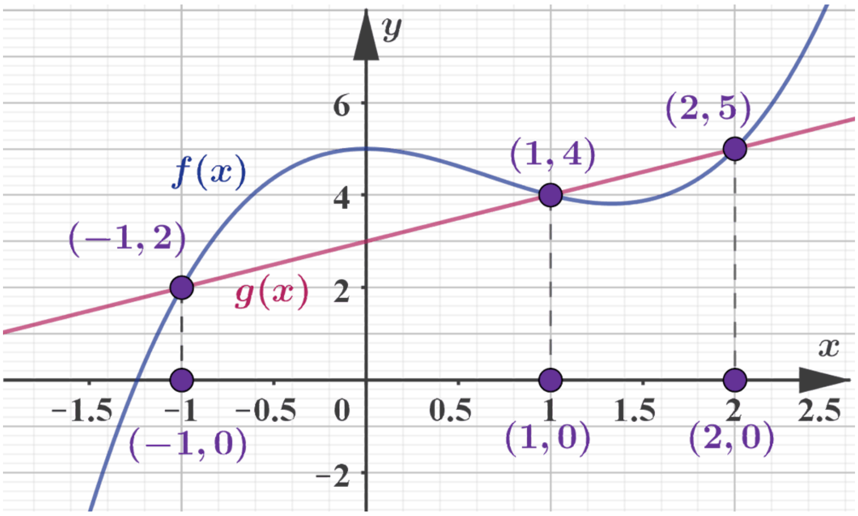
Be aware

Always keep in mind the sign of the inequality in the question and give your answer accordingly (and consider whether or not to include the end points).

In some cases, there will be more than one point of intersection. For example, consider the functions $f(x) = x^3 - 2x^2 + 5$ and $g(x) = x + 3$.



More information

The image is a graph illustrating two functions, labeled as (f(x)) and (g(x)), that intersect at three distinct points. The X-axis is labeled from approximately -1.5 to 2.5, and the Y-axis is labeled from -2 to 6, both with grid lines indicating intervals of 0.5.

The function (f(x)) is displayed as a blue curve, representing the cubic equation (f(x) = x^3 - 2x^2 + 5). It starts in the upper portion on the left side, then dips down, intersects with (g(x)), and rises steeply on the right half of the graph.

The function (g(x)) is shown as a pink straight line, representing the linear equation (g(x) = x + 3). It crosses through the center, sloping upward.

The three points of intersection, marked clearly on the graph, are labeled with their coordinates:

- ((-1, 2)) - where the functions cross as (f(x)) is on the downward curve and (g(x)) is ascending. - ((1, 4)) - another intersection point further along where both functions cross again. - ((2, 5)) - at this point, the functions intersect for

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the last time in the visible graph area.

The graph also marks additional points such as ((-1, 0)), ((1, 0)), and ((2, 0)) on the X-axis, indicating key points where (f(x)) crosses the axis.

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Looking at the graphs of the two given functions, you can see that there are three points of intersection. These are $(-1,\ 2)$, $(1,\ 4)$ and $(2,\ 5)$. This gives four sections where one of the functions is below or above the other. The table gives you the 4 intervals in which the functions can be compared with < or > .

Sections	$(-\infty, -1)$	$(-1, 1)$	$(1, 2)$	$(2, \infty)$
Graphs comparison	$f(x) < g(x)$	$g(x) < f(x)$	$f(x) < g(x)$	$g(x) < f(x)$

Needless to say, if we are allowed to use technology, i.e. our graphical display calculator (GDC), then we may explore the solutions to many more complicated inequalities than those for which we can use only pen and paper.

For example, consider $e^{-x^2+1} \geqslant x^3 - x^2 - \frac{6}{5}x + 2$. We can use technology to solve this inequality. We need to find the intersection points between the curves and find the sets of values of x for which the exponential lies above the cubic function. In the figure this is where the blue curve lies above the green curve.

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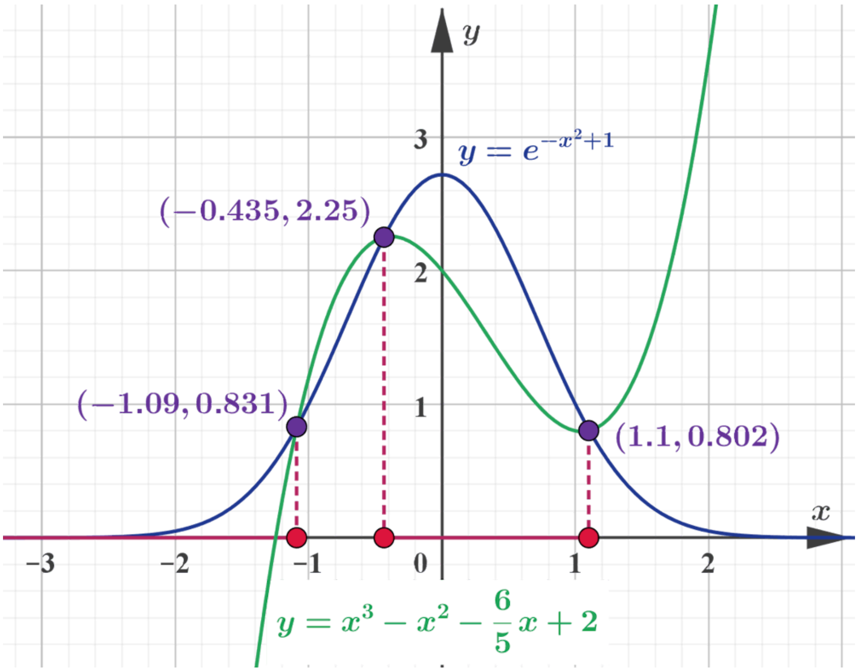

More information

The image is a graph depicting the intersection points of a blue curve, representing (y = e^{-x^2+1}), and a green curve, representing (y = x^3 - x^2 - \frac{6}{5}x + 2). The graph is set on a grid with both the x-axis and y-axis labeled and marked with numerical intervals. Three intersection points are marked with coordinates: ((-1.09, 0.831)), ((-0.435, 2.25)), and ((1.10, 0.802)). The red vertical dashed lines mark the x-values of interest that solve the inequality (e^{-x^2+1} ≥ x^3-x^2-\frac{6}{5}x+2). The area where the blue curve is above the green curve is the solution to the inequality, and these intervals are highlighted within the graph. The x-axis extends from -3 to 3, and the y-axis ranges from -1 to 3.

[Generated by AI]

To find the points of interest and thus, solve the inequality with three different GDC models to give the solution $x \leqslant -1.09, -0.435 \leqslant x \leqslant 1.10$, refer to the calculator instructions in section 2.4.3. (/study/app/math-aa-hl/sid-134-cid-761926/book/intersection-points-id-25404/)

3 section questions

Question 1

Difficulty:

★★★

Solve: $\frac{1}{x} \geqslant x - 1$

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1 $(-\infty, -0.62] \cup (0, 1.62]$ ✓

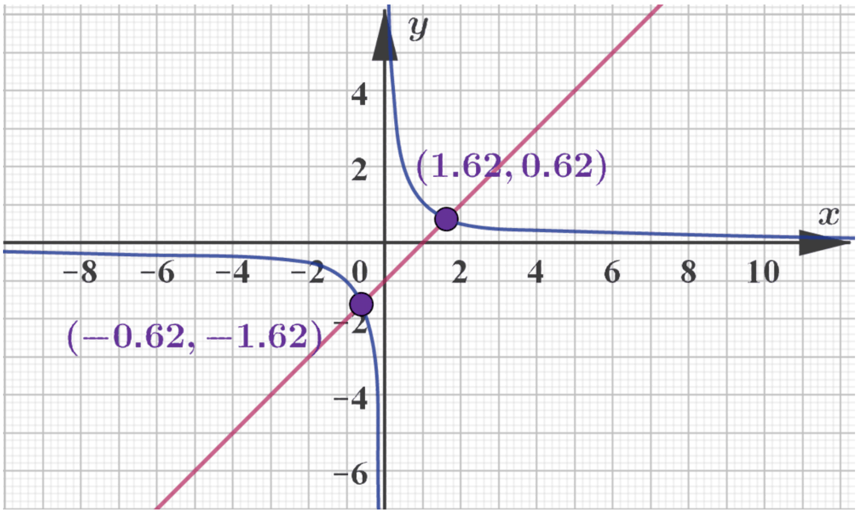
2 $[-0.62, 1.62]$

3 $[-0.62, 0) \cup [1.62, \infty)$

4 $(-\infty, 0) \cup (1.62, \infty)$

Explanation

Graph both the functions on the same set of axes and find the points of intersection. These are $(-0.62, -1.62)$ and $(1.62, 0.62)$.



More information

The solution for the inequality is where the blue curve is above the red line, which occurs in the intervals

$$(-\infty, -0.62] \cup (0, 1.62].$$

Include the points of intersection but not 0 as $x = 0$ is an asymptote.

Question 2

Difficulty:

★★★

Solve: $-x^3 + 2x^2 + 5x - 3 \leqslant x^2 - 3x - 2$

1 $[-2.44,\ 0.12] \cup [3.32, \infty)$ ✓

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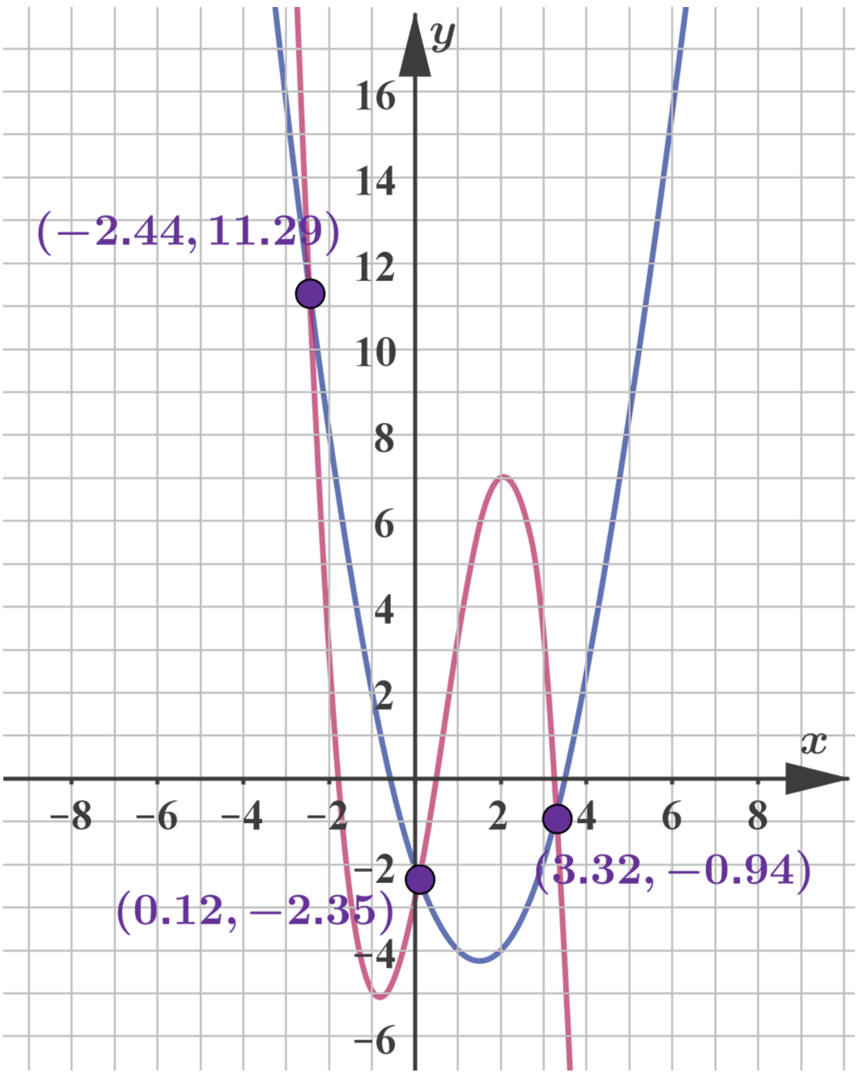
2 $(-\infty, -2.44] \cup [0.12,\ 3.32]$

3 $(-2.44, 0.12)$

4 $[-2.44,\ 3.32]$

Explanation

Graph the two functions on the same set of axes and note the points of intersection. These are at $x = -2.44, 0.12$ and 3.32 to 2 decimal places.



More information

The part where the cubic (red) is below the quadratic (blue) gives the solution to the inequality as:

$$[-2.44,\ 0.12] \cup [3.32, \infty)$$

Student view

Question 3

Difficulty:

★☆☆

Overview
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What is the solution to $\frac{1}{|x|} > \frac{1}{e}$?

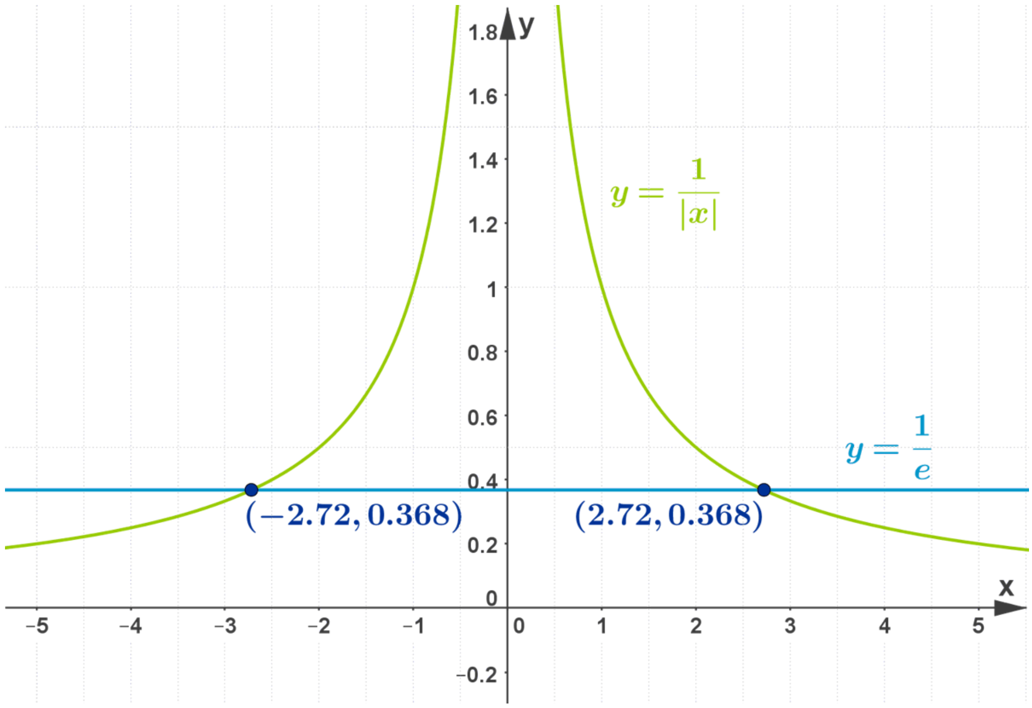
1 $x \in]-2.72, 2.72[\setminus\{0\}$ ✓

2 $x \in [-2.72, 2.72] \setminus \{0\}$

3 $x < -2.72,\ x > 2.72$

4 $x \in]-2.72, 2.72[$

Explanation



More information

2. Functions / 2.15 Solving inequalities

Solving analytically

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Student view

In order to solve an inequality $f(x) \leqslant g(x)$ analytically , you can first simplify it to the form $f(x) - g(x) \leqslant 0$.

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Example 1

★☆☆

Consider the functions $f(x) = 2x + 3$ and $g(x) = 4x - 5$.

Solve $f(x) \leqslant g(x)$

Rearranging this to the form $f(x) - g(x) \leqslant 0$ gives

$$(2x + 3) - (4x - 5) \leqslant 0$$

$$-2x + 8 \leqslant 0$$

$$-2x \leqslant -8$$

$x \geqslant 4$ (Dividing by -2 on both sides reverses the inequality)

Hence the solution is $[4, \infty)$

Example 2

★☆☆

Consider the functions $f(x) = 2x^2 + 3x - 1$ and $g(x) = 2x + 3$.

Solve $f(x) \geqslant g(x)$

Rearranging this to the form $f(x) - g(x) \geqslant 0$ gives

$$\left(2x^2 + 3x - 1\right) - (2x + 3) \geqslant 0$$

$$2x^2 + x - 4 \geqslant 0$$

Sign diagram method:

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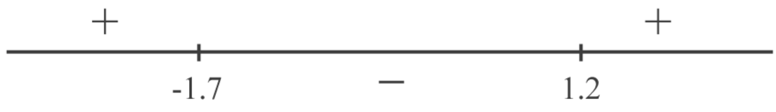
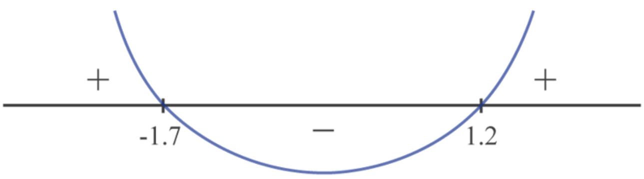
For $2x^2 + x - 4$, the leading coefficient is positive so, the function will be positive before the first root and after the second root and negative in – between the roots.



Graphical representation:

Sketch the graph of the function $2x^2 + x - 4$ corresponding to the x-intercepts as shown below:



From the sign diagram and the graph, you can conclude that the solution for $2x^2 + x - 4 \geqslant 0$ is $(-\infty, -1.7] \cup [1.2, \infty)$.

Making connections

Making connections

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Recall how you solved quadratic inequalities in subtopic 2.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27705/)

Solving for $2x^2 + x - 4 = 0$ you get, $x = -1.7$ or $x = 1.2$.

Use the sign diagram or the graph to know which interval gives the solution for

$$2x^2 + x - 4 \geqslant 0$$

Example 3

★★☆

Consider the functions $f(x) = x^3 + 3x^2 - 2x - 1$ and $g(x) = 2x^2 + 3x + 4$.

Solve $f(x) \leqslant g(x)$.

Rearranging this to the form $f(x) - g(x) \leqslant 0$ gives

$$\left(x^3 + 3x^2 - 2x - 1\right) - \left(2x^2 + 3x + 4\right) \leqslant 0$$

$$x^3 + x^2 - 5x - 5 \leqslant 0$$

Now solve $x^3 + x^2 - 5x - 5 = 0$ (see subtopic 2.12).

Analytically, you can find the first root by trial and error. In this case $x = -1$ will be a root. Then divide $x^3 + x^2 - 5x - 5$ by $x + 1$ to get a quadratic factor. Equate this to zero and solve.

$$\frac{x^3 + x^2 - 5x - 5}{x + 1} = x^2 - 5$$

Solving $x^2 - 5 = 0$, you get $x = \pm\sqrt{5} = \pm 2.24$

Hence, the three roots of the cubic equation are $x = -1,\ x = 2.24$ and $x = -2.24$.

Using a sign diagram, as shown below, you can identify the regions where the function will be positive and where it will be negative related to its x-intercepts

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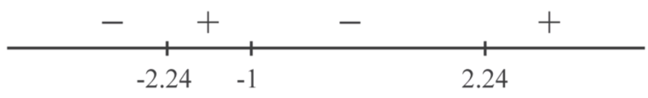
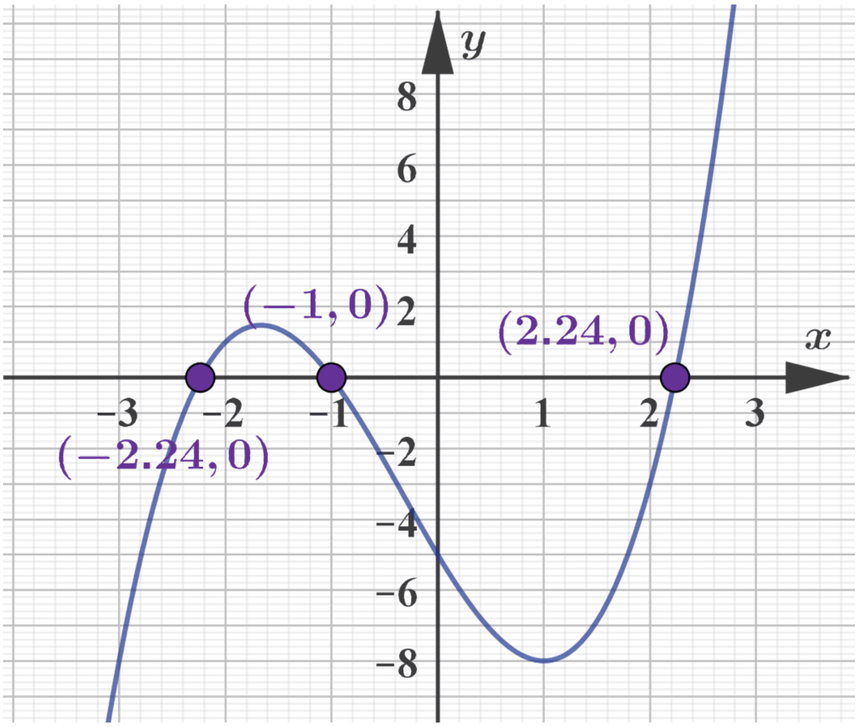


you get the solution for the inequality $x^3 + x^2 - 5x - 5 \leqslant 0$ as: (−∞, −2.24] ∪ [−1, 2.24]

Check your answer by graphing the cubic function $x^3 + x^2 - 5x - 5$:



Student view

Inequalities: analytic solutions

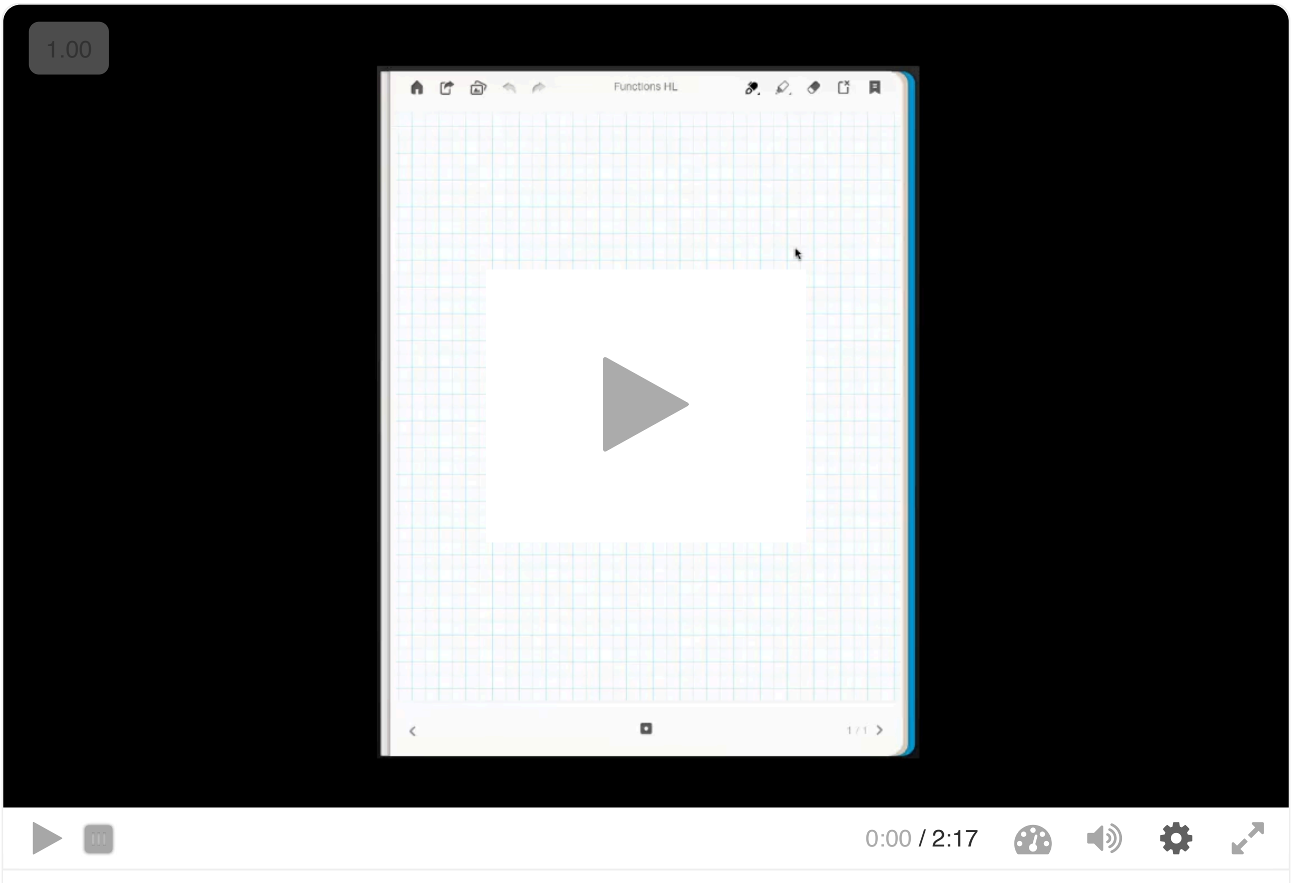
To start with, it is always helpful to remember that whether or not you are solving an inequality analytically, it is always worth quickly sketching the function(s). It is likely that those inequalities you will be asked to solve without your GDC lend themselves well for a quick sketch, i.e. they will not be overly complicated.

There are two classes of inequalities we will look at:

1. Those of the form $f(x) < 0$ or $f(x) > 0$, or forms that can be rephrased to give one of these two possibilities.
2. Those of the form $f(x) < g(x)$.

Inequalities of the form $f(x) < 0$ or $f(x) > 0$

Let us consider the inequality $(x-1)(x^2-x-5) < x-1$. We explore the analytic solution to this in the following video.



Video 1. Exploring Inequality.

More information for video 1

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00:00:00,333 --> 00:00:01,867

narrator: In this video,

we're going to take a look

2

00:00:01,933 --> 00:00:06,567

at how to solve inequalities

using an analytic method,

3

00:00:06,733 --> 00:00:10,733

but not forgetting that a sketch

is always worth you all.

4

00:00:11,033 --> 00:00:12,733

So when you look

at the following, inequality,

5

00:00:12,800 --> 00:00:19,633

$(x - 1)(x^2 - x - 5) < x - 1$.

6

00:00:20,100 --> 00:00:23,233

Now of course, you will have noticed

that there's an effect of $(x - 1)$

7

00:00:23,533 --> 00:00:24,967

on both sides with inequalities.

8

00:00:25,033 --> 00:00:27,300

So you think, well, why not cross them out

9

00:00:28,000 --> 00:00:29,533

leaving us with this inequality?

10

00:00:29,633 --> 00:00:32,133

$x^2 - x - 5 < 1$.

11

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But of course, just as with equalities,

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12

00:00:33,633 --> 00:00:36,133

that is not what we do,

because you are actually throwing out

13

00:00:36,200 --> 00:00:39,367

a solution.

So the same with inequalities

as with equalities, we do not do that.

14

00:00:39,700 --> 00:00:41,967

So let's quickly erase that.

15

00:00:43,133 --> 00:00:44,167

Now what we will do,

16

00:00:44,233 --> 00:00:48,767

we're going to bring the right

hand side factor $(x - 1)$

over to the left hand side,

17

00:00:49,300 --> 00:00:55,000

leaving $(x - 1)(x^2 - x - 5) - (x - 1) < 0$.

18

00:00:55,467 --> 00:00:58,133

And now we see

that is a factor of $(x - 1)$.

19

00:00:58,200 --> 00:01:03,500

So we can take that out leaving

$(x - 1)(x^2 - x - 5 - 1) < 0$.

20

00:01:03,867 --> 00:01:08,367

So now we have $(x - 1)(x^2 - x - 6) < 0$,

and we can now

factorize the quadratic into

Student view

21

00:01:08,667 --> 00:01:16,600

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$(x - 1)(x + 2)(x - 3) < 0.$

22

00:01:16,700 --> 00:01:19,767

So now we've got a cubic function,

which we clearly see.

23

00:01:19,833 --> 00:01:23,667

We've got the factor, so we know

where it crosses the x axis.

24

00:01:23,867 --> 00:01:33,367

So we have a cubic function

of this form because the leading

coefficient is positive

and across the x axis

at -2, 1, and 3.

25

00:01:33,500 --> 00:01:34,933

And now we look at the inequality.

26

00:01:35,100 --> 00:01:37,467

So it has to be less than zero.

27

00:01:37,533 --> 00:01:40,000

That is the solutions

that we are looking for.

28

00:01:40,300 --> 00:01:45,567

And now you can clearly

identify which portion

of this graph lies below the x axis.

29

00:01:45,633 --> 00:01:48,600

And of course, realize that on the left

hand side it continues.

30

00:01:49,133 --> 00:01:56,367

Therefore, the solutions

Student view

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(/study/app/math-aa-hl/sid-134-cid-761926/o

to this inequality are $x < -2$ and $1 < x < 3$.

31

00:01:56,633 --> 00:01:58,800

Now we can also write it as follows,

32

00:01:58,867 --> 00:02:08,600

$x \in (-\infty, -2) \cup (1, 3)$.

33

00:02:08,700 --> 00:02:11,900

So this factor over here

corresponds to there

34

00:02:12,067 --> 00:02:14,700

and the 1 to 3 corresponds

to that 1 over there.

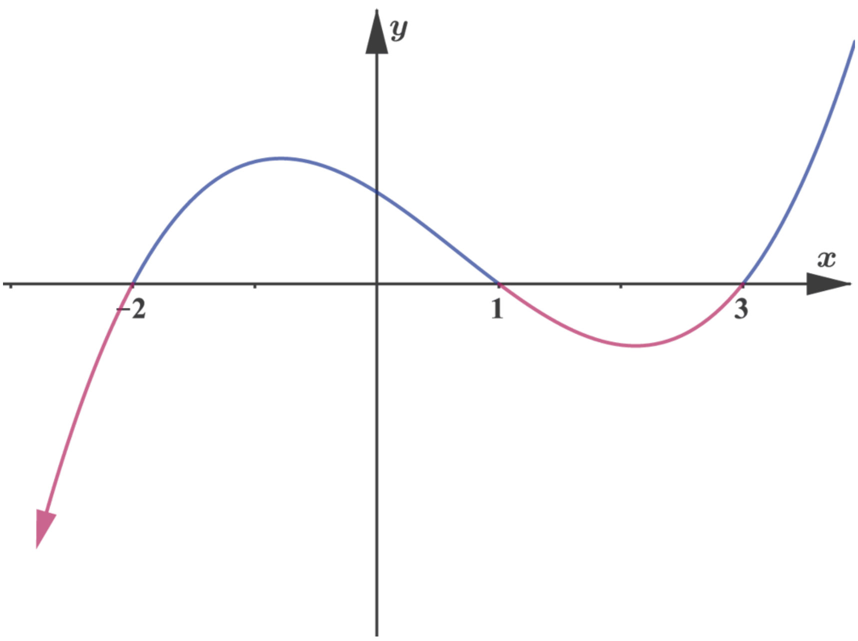
35

00:02:14,867 --> 00:02:17,533

And those are the solutions

to this inequality.

In the video, we wrote the inequality as one of the form $f(x) < 0$, then proceeded to find the x-axis intercepts and then identified the range of solutions (see figure below).



Student view

More information

The image is a graph showing a curve plotted on Cartesian coordinates. The x-axis and y-axis are labeled, with the x-axis marked at intervals including -2, 1, and 3. The curve intersects the x-axis at approximately -2 and 3, indicating the zeros of the function. It travels in a wave-like shape, moving downward from the left, reaching a low point below the x-axis before crossing upward through the -2 mark. It then peaks and dips again, crossing the x-axis at 3, before rising sharply moving to the right. This indicates a general trend of negative to positive values across the interval, with notable inflection points at the crossings.

[Generated by AI]

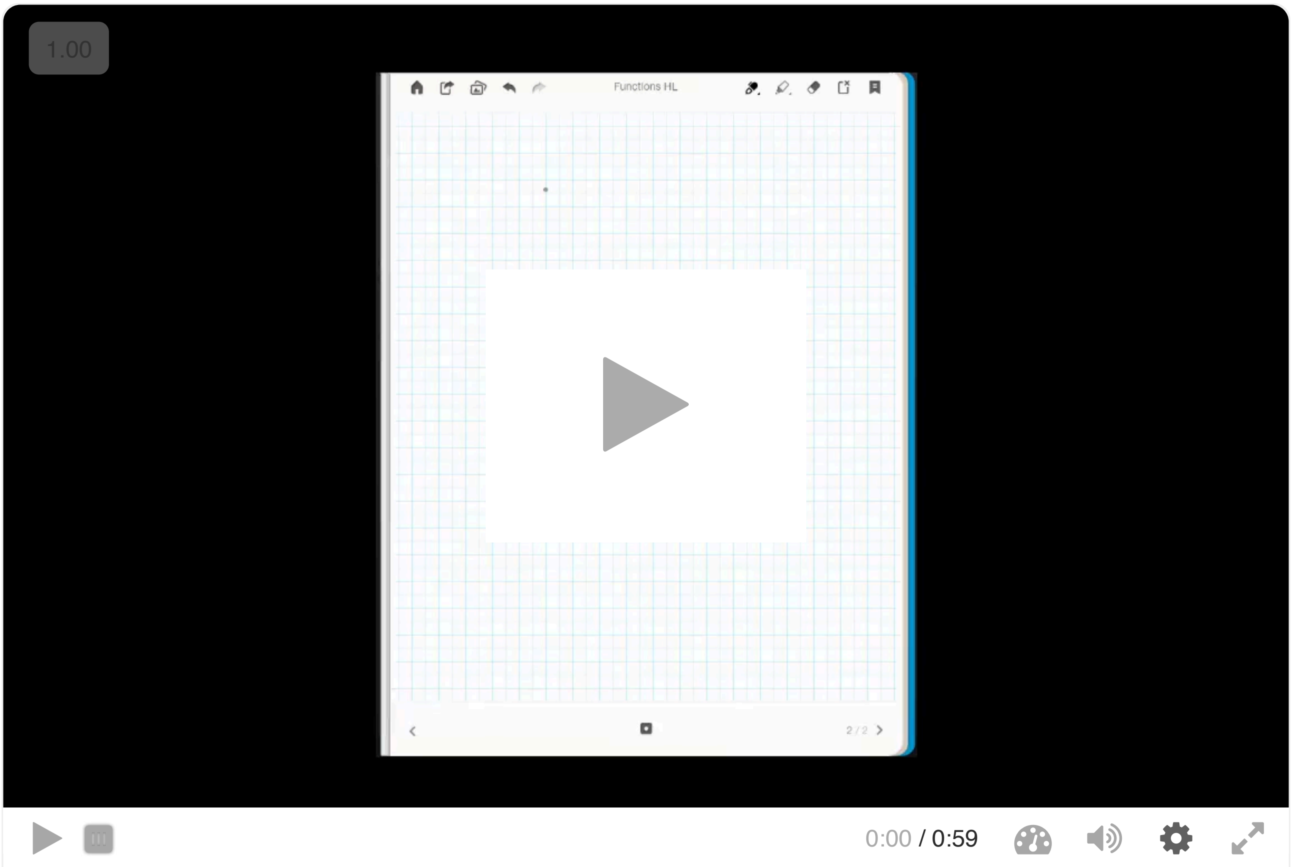
One could solve the above inequality in a totally algebraic manner (without using a graph) by creating a table like the one below in which the zeros of each factor are placed in the first row in ascending order. The factors are set in the first column. The sign of each factor in each interval is determined by using trial values (e.g. by taking -3, you can see that $x-1=-3-1=-4<0$ between $-\infty$ and -2). The sign of the product of all factors in each interval is written in the last row. Since the inequality that we want to solve for is a negative product of factors, we understand that the solution is the union of the intervals where the product is negative, i.e. $]-\infty,-2[\ \cup\ ]1,3[$.

	$-\infty$		-2		1		3		$+\infty$
$x-1$		$-$	$-$	$-$	0	$+$	$+$	$+$	
$x+2$		$-$	0	$+$	$+$	$+$	$+$	$+$	
$x-3$		$-$	$-$	$-$	$-$	$-$	0	$+$	
Product		$-$	0	$+$	0	$-$	0	$+$	

Inequalities of the form $f(x)<g(x)$

It is not always advantageous to write an inequality in the form that has a zero on one side of the inequality. For example, let us consider the inequality $1<\ln x$. We explore the analytic solution to this in the following video.

Student
view



Video 2. Video for Exploring Inequality.

More information for video 2

1

00:00:00,200 --> 00:00:02,767

narrator: Let's look at a second

example of an inequality,

2

00:00:02,833 --> 00:00:05,900

which we can solve analytically

without a graphical calculator.

3

00:00:06,100 --> 00:00:09,167

$1 < \ln(x)$,

natural logarithm of x.

4

00:00:09,700 --> 00:00:12,033

So let's quickly draw

a sketch of two functions.

5

00:00:12,100 --> 00:00:13,867

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So one, of course $y = 1$.

6

00:00:13,933 --> 00:00:17,267

So this horizontal line

and the other one is $y = \ln(x)$,

7

00:00:17,833 --> 00:00:22,500

increases like this of course,

across the x axis at 1.

8

00:00:22,800 --> 00:00:26,500

So we are looking

for the branch that is above

9

00:00:27,133 --> 00:00:28,567

the point of intersection.

10

00:00:28,933 --> 00:00:30,300

So this branch over here.

11

00:00:30,633 --> 00:00:33,267

So of course we need to find

that point of intersection,

12

00:00:33,600 --> 00:00:42,033

but we can of course write this as

$e^1 < x$ by taking the exponential

of both sides

13

00:00:42,100 --> 00:00:45,367

and therefore that point is e

and therefore the solutions is x

14

00:00:45,433 --> 00:00:48,500

is greater than e,

or again, we can rewrite that

15

00:00:48,667 --> 00:00:56,900

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761926/o

or we can write that

as follows, x lies in the set (e, ∞) ,

not including

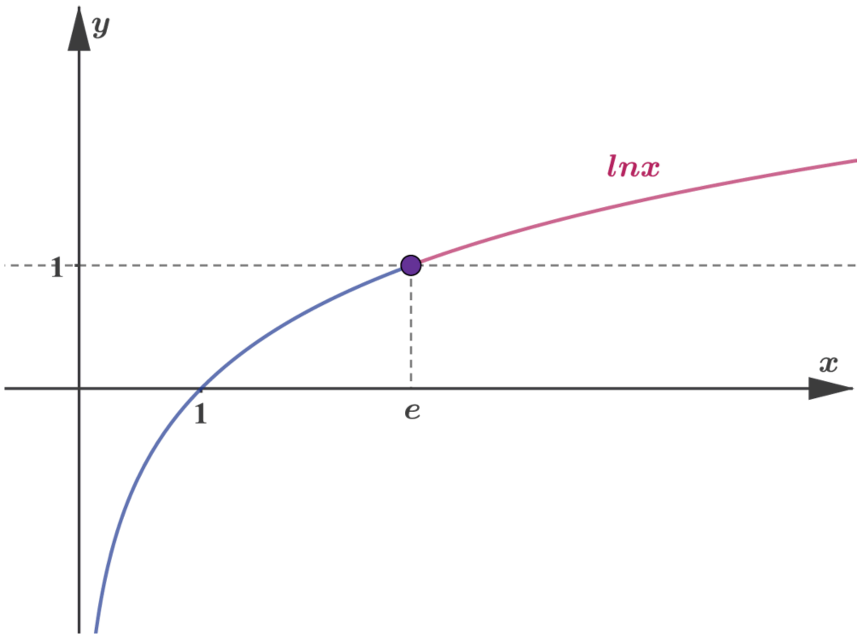
either one of the endpoints

16

00:00:56,967 --> 00:00:59,500

and that is solution to that inequality.

Again, using a sketch as a guide to decide which region to look for, we found the solution as shown in the figure below.



More information

The image is a graph illustrating the function of the natural logarithm, ln(x), drawn on a Cartesian coordinate system. The x-axis and y-axis are both labeled, with 'x' representing the horizontal axis, and 'y' the vertical. The curve of the natural logarithm begins from the y-axis, increasing gradually and intersecting the y-axis at the point where x equals 1, indicating ln(1) = 0.

The graph also highlights a point at (e, 1), represented by a purple dot, where the function ln(x) has a tangent. The curve for ln(x) passes smoothly through this point, continuing upwards along the x-axis towards infinity, representing the positive domain. The function is colored differently from the rest of the graph for emphasis, transitioning from blue to pink as it crosses the highlighted point at x = e. There are dashed lines connecting the point (e, 1) to both the x and y axes for reference.

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view

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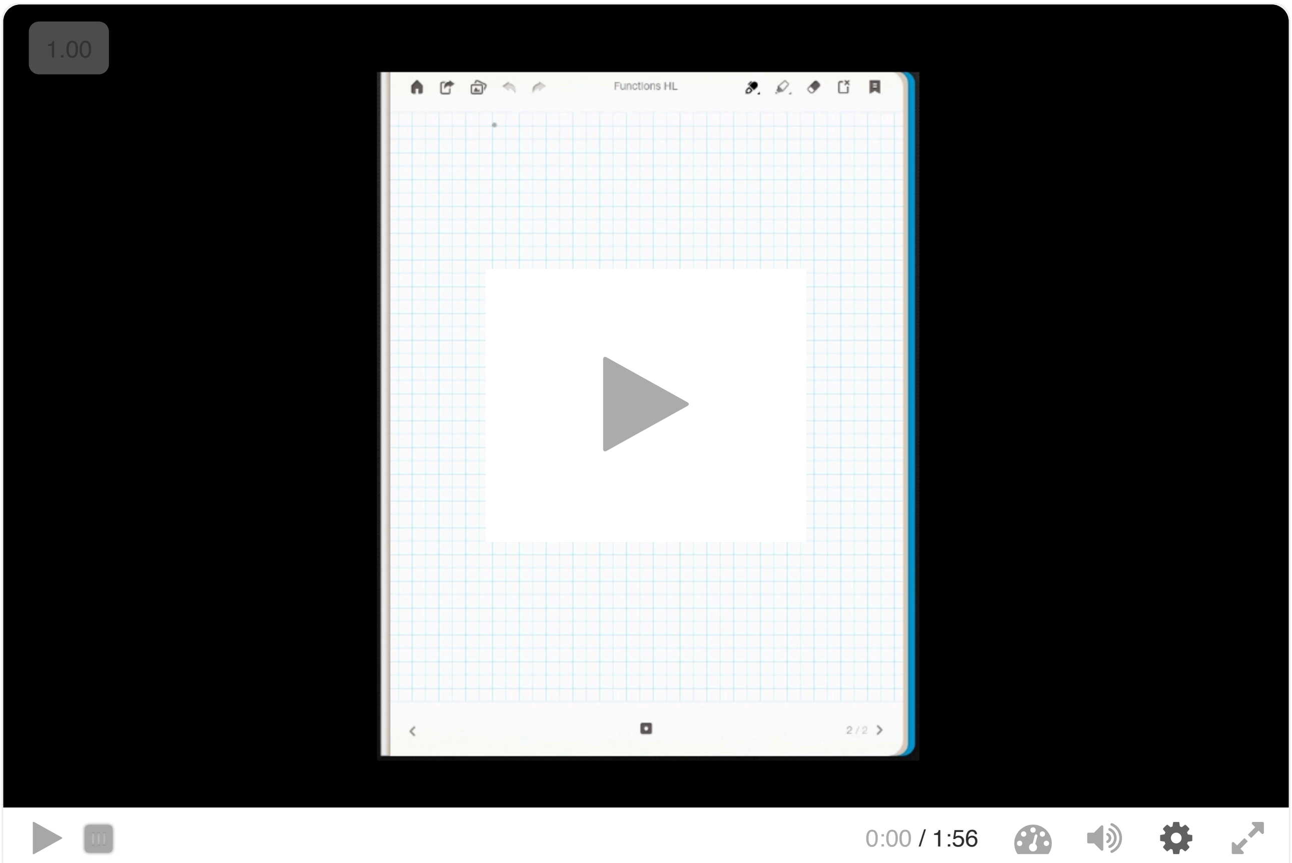
[Generated by AI]

Inequalities and the absolute value function

In the following video, we explore how to find the solution to an inequality involving the absolute value function by considering the example $|2x-1|<x$.



Video 3. Inequalities and the Absolute Value Function.

More information for video 3

1

00:00:00,467 --> 00:00:02,733

narrator: In this video, we're going to

take a look at inequalities

2

00:00:02,800 --> 00:00:04,567

using the absolute value function.

3

00:00:04,633 --> 00:00:08,167

Student view

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134-
cid-
761926/o

So here we have the absolute value

of $|2x - 1|$ is less than x,

4

00:00:08,267 --> 00:00:11,467

and we're going to start by quickly

sketching those two functions

5

00:00:11,533 --> 00:00:13,133

on the $y = x$ axis.

6

00:00:13,300 --> 00:00:16,500

So the first one is $y = x$

axis is simplest one by far.

7

00:00:16,567 --> 00:00:20,733

And of course it goes through the origin

and the point $(1, 1)$, for example.

8

00:00:21,633 --> 00:00:25,567

And then we're going to plot the function

$y = 2x - 1$.

9

00:00:26,300 --> 00:00:29,333

It cuts $y = -1$,

in the x axis and a half,

10

00:00:29,600 --> 00:00:32,967

and this is the function

$y = 2x - 1$.

11

00:00:33,600 --> 00:00:35,067

But now we have to worry about the fact

12

00:00:35,133 --> 00:00:37,767

that we need

to take the absolute value of that.

13

00:00:37,833 --> 00:00:39,333

Student
view

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And of course, wherever that function,

14

00:00:39,400 --> 00:00:41,967

$2x - 1$ is positive,

it remains the same,

15

00:00:42,033 --> 00:00:43,067

taking the absolute value,

16

00:00:43,133 --> 00:00:45,967

but wherever it's negative,

we reflect it in the x axis.

17

00:00:46,200 --> 00:00:49,133

So here the blue function

is the absolute value

18

00:00:49,200 --> 00:00:53,533

of $|2x - 1|$, and then we see that over

this orange range,

19

00:00:53,600 --> 00:00:56,333

that blue function lies

below the black function.

20

00:00:56,433 --> 00:00:58,800

But how do we find those

points of intersection?

21

00:00:58,867 --> 00:01:00,600

For this, we have to solve

a couple of equations.

22

00:01:00,667 --> 00:01:03,800

The first one is $2x - 1 = x$.

23

00:01:03,867 --> 00:01:06,433

The solution of which is $x = 1$

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and $y = 1$.

24

00:01:06,500 --> 00:01:08,567

So that's the right hand side

of the orange region.

25

00:01:08,667 --> 00:01:09,567

$(1, 1)$.

26

00:01:10,000 --> 00:01:14,200

The other one is the solution,

$-(2x - 1) = x$.

27

00:01:14,267 --> 00:01:17,100

The solution of which is $x = \frac{1}{3}$

and $y = \frac{1}{3}$,

28

00:01:18,000 --> 00:01:21,033

and that is the point on the left hand

side of the orange region.

29

00:01:22,033 --> 00:01:31,467

And therefore the solution

to the absolute value

$|2x - 1| < x$,

is the solution $\frac{1}{3} < x < 1$.

30

00:01:31,767 --> 00:01:34,967

We could of course,

have asked the question,

what is the solution to $|2x - 1| > x$,

31

00:01:35,033 --> 00:01:41,867

and that corresponds to the green region

that you see over here.

32

00:01:42,433 --> 00:01:46,633

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And so these boundaries

of the green region match up

with the boundaries of the orange region.

33

00:01:46,700 --> 00:01:48,333

So we have already solved this problem.

34

00:01:48,400 --> 00:01:56,000

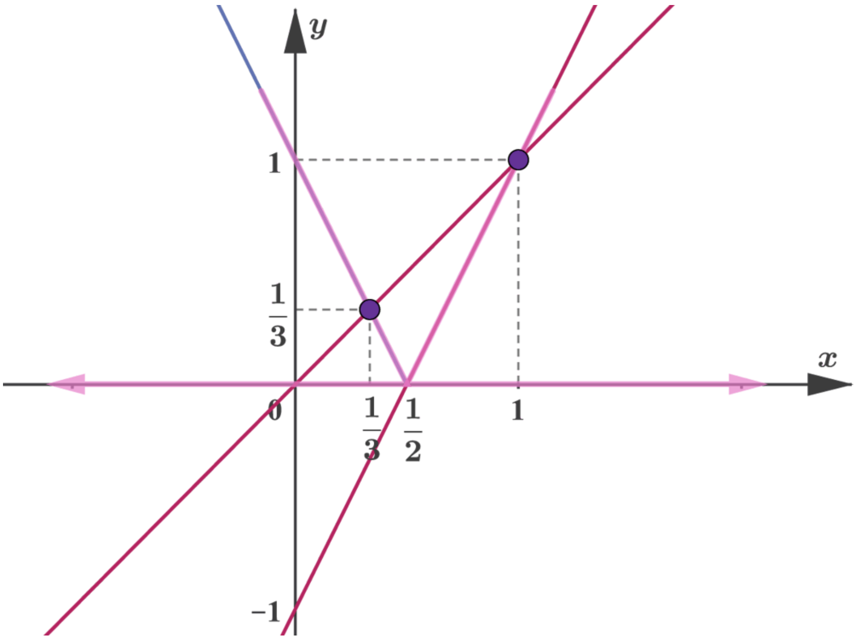
$x < \dfrac{1}{3}$ and

$x > 1$ is the solution

to the absolute value,

$|2x - 1| > x$.

The solution we found was $\dfrac{1}{3} < x < 1$ and it is illustrated in the figure below.



More information

The image is a graph illustrating the solution (\dfrac{1}{3}<x<1) on a coordinate plane. The X-axis is labeled with numbers from -1 to 1, and the Y-axis is labeled from -1 to 1. Two points are highlighted on the graph: one at (\left(\dfrac{1}{3}, \dfrac{1}{3} \right)) and another at (1,1). Multiple lines intersect at these points. The graph is used to

Student view

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represent the solution set for inequalities involving x. Each line has a distinct trajectory, showing how they relate to the solution region for the variable x within the given inequality. The dashed lines aid in visualizing the precise intersection points, and the coordinates are clearly marked.

[Generated by AI]

Be aware

Pay extra attention when you have negative signs and inequalities. Thus,

$$-x < a \Leftrightarrow x > -a$$

and

$$x > a \Leftrightarrow -x < -a.$$

Activity

Consider the following functions:

$$f(x) = 3x + 4$$

$$f(x) = x^2 + 2x - 1$$

$$f(x) = \mathrm{e}^x$$

$$f(x) = x^3 - 3x^2 + 2x - 4$$

Graph each of the above functions and find the solutions for $f(x) \leqslant \dfrac{1}{f(x)}$.

In general, for any f(x), the inequality $f(x) \leqslant \dfrac{1}{f(x)}$ can be rewritten as

$f(x) \cdot f(x) \leqslant 1 \to (f(x))^2 \leqslant 1$

Does the above statement hold good for all values of $f(x)$? Why?

How can you find analytically, the solution for $f(x) \leqslant \dfrac{1}{f(x)}$ for any $f(x)$? Relate this inequality to sine and cosine functions. What do you conclude?

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Exam tip

You can use a graphing calculator to solve the inequality directly in the exam.

International Mindedness

The concept of inequality was used by ancient mathematicians as just a support to other mathematical concepts. But it has developed into a special discipline of study in modern mathematics and has applications in computer programming. Find out about the long journey of 'inequalities' over the time.

Theory of Knowledge

Is there a part of 'equality' within 'inequality' both in mathematical and philosophical interpretations? You find the solutions to the equations before finding the solution to the inequalities. Does this apply to philosophy of life as well? Is it important to understand the commonalities between two people before analysing the differences between them?

3 section questions

2. Functions / 2.15 Solving inequalities

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-26765/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- solve inequalities algebraically for polynomials up to the 3rd degree
- sketching the graphs for which you are investigating the inequality is always a good idea, even if the solution is found algebraically
- you must be able to find intersection points of functions with your GDC and thus, solve inequalities of the form $f(x) \geqslant g(x)$

Student view

- $-x < a \Leftrightarrow x > -a$ and $-x > a \Leftrightarrow x < -a$.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

2. Functions / 2.15 Solving inequalities

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-26766/print/) Assign

Consider $f(x) = \sin x$ and $g(x) = \cos x$.

Find the intervals for which $\sin x \leqslant \cos x$ in the domain $-2\pi \leqslant x \leqslant 2\pi$

Find the general solution to the inequality $\sin x \leqslant \cos x$ in the domain $(-\infty, \infty)$.

How will the solution differ if the inequality were $\sin x \geqslant \cos x$?

Extend the above results and investigate on the solutions of $a \sin(bx) \leqslant c \cos(dx)$. Create a GeoGebra applet with sliders for a, b, c and d to identify the solution sets.

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Interactive 1. Visualizing Solutions to Inequalities.

Credit: GeoGebra (https://www.geogebra.org/m/X6u3rgNy) Gara Szabolcs

More information for interactive 1

This interactive will help the users to graphically visualise the representation of the solution of the inequality of a function.

The screen is divided into two halves. The top half displays a Cartesian coordinate graph with the *x*-axis ranging from -10 to 12 and the *y*-axis ranging from -12 to 10. A horizontal yellow line represents a specific *y*-value which rests on the *x*-axis (where $y = 0$). Users can adjust the *y*-value, by dragging the yellow line vertically along the *y*-axis. At the bottom half, Users enter an equation in the “Left side” textbox which represents the function, while the “Right side” textbox which represents the y-value updates as the user moves the yellow line on the graph. Users can also choose the inequality sign as $<, \leq, >, \geq$ using the horizontal slider. For example, the $LHS = x^2 - x - 6$ and the chosen inequality is “$<$” i.e. $x^2 - x - 6 < 0$.

In this case, the parabola intersects the x-axis at $x = -2$ and $x = 3$. The pink region between the dotted lines represents the values of x for which $x^2 - x - 6 < 0$. This means that for any value of x within this interval not including -2 and 3, the value of the quadratic expression is negative, placing the corresponding point on the

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parabola below the x-axis.

In the case of $x^2 - x - 6 \leq 0$, this means that any value of x within this interval including -2 and 3 the value of quadratic expression is negative.

In the case $x^2 - x - 6 > 0$, this means that any value of $x < -2$ and $x > 3$ will be the solution to this inequality.

The shaded part highlights the region on the x-axis where the inequality holds true.

In case of $x^2 - x - 6 \geq 0$, this means that for any value of $x \leq -2$ and $x \geq 3$ will be the solution to this inequality.

The shaded part highlights the region on the x axis where the inequality holds true.

This helps the users to graphically visualize the representation of the solution of the inequality to a function.

Rate subtopic 2.15 Solving inequalities

Help us improve the content and user experience.

Student view