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Teacher view



(https://intercom.help/kognity)

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- Solving polynomial equations
- Systems of linear equations
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Notebook



Glossary



Reading  
assistance

1. Number and algebra / 1.8 Equations and equation systems

# The big picture

Mathematicians have been interested in solving equations at least as far back as the Babylonian civilization which started around 2000 BC. Applications included estimation of volumes of grain storage containers and calculations of land areas.

Thousands of years of work on this topic produced a lot of different ways to solve equations and systems of equations. Solution techniques include the use of approximation, algebraic manipulations and graphing.

## International Mindedness

In order to develop the algebra needed to solve polynomial equations, mathematicians built on ideas and techniques of their predecessors, spanning centuries and all corners of the world. If you do more research into the history of polynomial equations, you will see that our current understanding rests on contributions from many civilizations and societies.

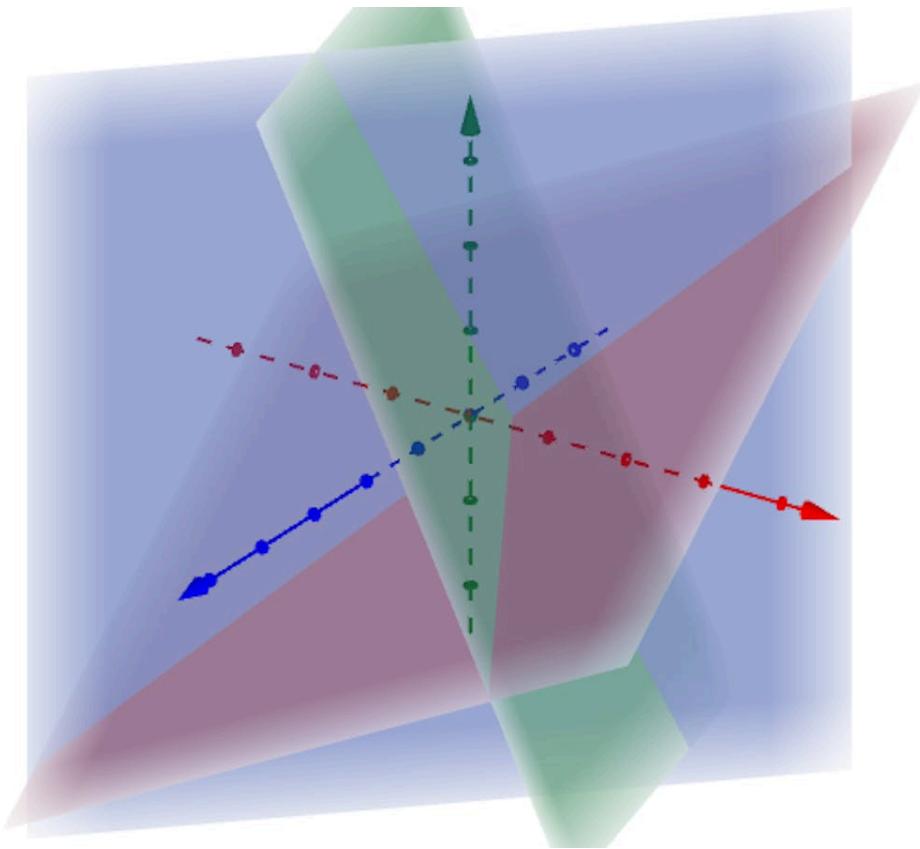
Computers and graphing calculators provide many tools for solving these types of questions. For example, a system of 3 linear equations can be solved by creating a three-dimensional graph as shown in the figure below.



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More information

The image is a three-dimensional graph depicting a system of three linear equations. It features three planes intersecting within the space. The planes are colored in translucent shades of blue, green, and red, creating a visual representation of each equation. At the center where these planes intersect, there is an axis system with arrows pointing in different directions, signifying the X, Y, and Z axes. The red, green, and blue dashed lines along these axes highlight the directionality and orientation within the 3D space. This 3D representation visually demonstrates how the three planes intersect at a point, which represents the solution to the system of equations.

[Generated by AI]

## Concept

In this subtopic you will use your graphic display calculator to generate solutions to equations. Consider the following questions:

- How will you know that the solutions are valid?
- Are there ways to check that they are correct?



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- Does the validity of the solution depend on the context of the question?

## Theory of Knowledge

Solving a linear equation can be very satisfying. When you solve for  $y$  or  $x$  you have confidence in your solution because it ‘works’. Your solution elegantly completes the puzzle so to speak. It is this certainty that creates the level of confidence many people have in the area of knowledge of mathematics.

This leads to the knowledge question, ‘Is the level of certainty within an area of knowledge positively correlated with the value of that area of knowledge?’

1. Number and algebra / 1.8 Equations and equation systems

# Solving polynomial equations

A polynomial,  $P(x)$ , is defined as follows:

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0,$$

where  $a_n \neq 0$  and  $a_n, a_{n-1}, a_{n-2}, \dots, a_1$  and  $a_0$ , which are called coefficients, are real numbers and  $n$  is a positive integer.

The degree of the polynomial is the highest power of  $x$ .

A quadratic such as  $P(x) = 2x^2 - 3x + 2$  is an example of a polynomial that you have probably seen before.



Other examples of common polynomials are shown in the table below.



## Several common polynomials with their degree, name, form and an example.

Degree	Name	General form	Example
0	Constant	$P(x) = a$	$P(x) = 3$
1	Linear	$P(x) = ax + b$	$P(x) = -2x + 5$
2	Quadratic	$P(x) = ax^2 + bx + c$	$P(x) = \frac{1}{2}x^2 - 1$
3	Cubic	$P(x) = ax^3 + bx^2 + cx + d$	$P(x) = x^3 - 2x^2 - \dots$
4	Quartic	$P(x) = ax^4 + bx^3 + cx^2 + dx + e$	$P(x) = 3x^4 + x - \frac{2}{3}$

✓ **Important**

If  $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$  is a polynomial, then the equation of the form  $P(x) = 0$  is called a polynomial equation.

The zeros or roots of a polynomial  $P(x)$  are the solutions to the polynomial equation  $P(x) = 0$ . These are also the  $x$ -intercepts of the graph of  $y = P(x)$ .

Why does it make sense to call solutions to polynomial equations zeros and  $x$ -intercepts?

**Example 1**

Determine whether any of the following values, 3, 2.5, -2, or 5, are roots of the polynomial  $P(x) = x^3 - 2x^2 - 5x + 6$ .



Steps	Explanation
<p>For <math>x = 3</math>:</p> <p><b>Section</b> <math>P(3) = (3)^3 - 2(3)^2 - 5(3) + 6 = 0</math> (<a href="#">/study/app/m/sid-122-cid-754029/book/the-big-picture-id-27394/print/</a>)</p> <p><math>x = 3</math> is a root.</p> <p>For <math>x = 2.5</math>:</p> $P(2.5) = (2.5)^3 - 2(2.5)^2 - 5(2.5) + 6 = -3.375$ <p><math>x = 2.5</math> is not a root.</p> <p>For <math>x = -2</math>:</p> $P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6 = 0$ <p><math>x = -2</math> is a root.</p> <p>For <math>x = 5</math>:</p> $P(5) = (5)^3 - 2(5)^2 - 5(5) + 6 = 56$ <p><math>x = 5</math> is not a root.</p>	<p>If <math>a</math> is a root of <math>P(a) = 0</math>.</p> <p><b>Assign</b></p> <p>You can check if <math>a</math> is a root by calculating <math>P(a)</math>. If it equals zero, then <math>a</math> is a root.</p>

## Activity

Solutions to polynomial equations are values of  $x$  for which  $P(x) = 0$ .

Explain why the solutions to  $P(x) = 0$  can be found on a graph by looking at the  $x$ -intercepts.

The applet below shows a graph of a polynomial function. You can change the degree of the polynomial and the values of the coefficients to generate new examples. Use the applet below to explore how the number of solutions to a polynomial equation is related to its degree.





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## Interactive 1. A Graph of a Polynomial Function.

More information for interactive 1

This interactive applet allows users to explore and visualize polynomial functions by adjusting both the degree of the polynomial and the values of its coefficients.

Users can set the polynomial degree anywhere from 0 to 5 and modify the coefficients to create different polynomial expressions. The graph of the polynomial function is displayed in real time, and red points appear on the graph that users can drag to observe how the shape and behavior of the curve change with different coefficient values. As users experiment, they develop an understanding of how the degree of a polynomial influences the number of turning points, the overall shape of the graph, and the possible number of real roots or x-intercepts.

For example, a degree 1 polynomial such as  $f(x) = 2x + 1$  produces a straight line; a degree 2 polynomial like  $f(x) = -6.56x^2 + 8.43x + 10$  forms a parabola; and a degree 3 polynomial such as  $f(x) = 7.65x^3 - 3.35x^2 + 2x$  creates an S-shaped curve.

This tool offers a dynamic and hands-on way to explore key properties of polynomial functions and helps users build intuition about their graphical behavior and algebraic structure.

Your findings in the activity above are related to the fundamental theorem of algebra.



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 **Important**

According to the fundamental theorem of algebra, a polynomial defined as  $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$  will have no more than  $n$  roots.

In this subtopic, you will only find the real solutions to polynomial equations and you will use your calculator for all questions.

 **Theory of Knowledge**

When mathematicians refer to ‘real’ and ‘imaginary’ solutions they are using technical terms that do not have the same meaning as the words used in everyday language. What role does language play in the sharing of knowledge in mathematics? Does your understanding of everyday words help you to understand mathematical language?

 **Making connections**

Complex (non-real) solutions to quadratic equations are considered in AIHL subtopic 1.12 .

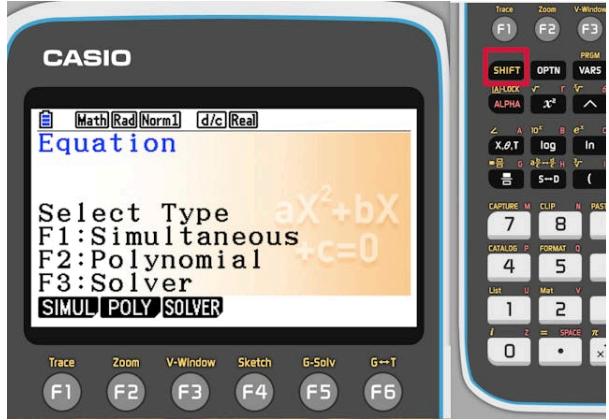
 **Exam tip**

In the exam, you will be expected to solve polynomial equations using your calculator. You will only be expected to find the real solutions unless the question specifies otherwise. You do not need to know how to find these solutions algebraically.

When solving polynomial equations using your calculator, it is useful to remember that there will be at most  $n$  real solutions to a polynomial with degree  $n$  .



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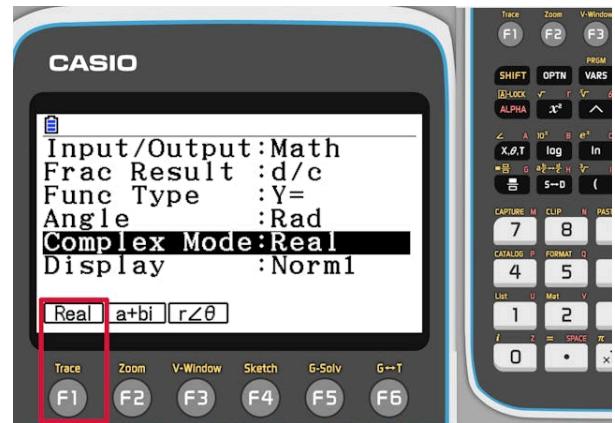
Steps	Explanation
<p>These instructions will show you how to use the calculator to find the real solutions of the equation</p> $x^3 - 2x^2 - 5x + 4 = 0.$ <p>Open the equation solver mode.</p>	
<p>To make sure the solver gives only the real roots, enter set up.</p>	



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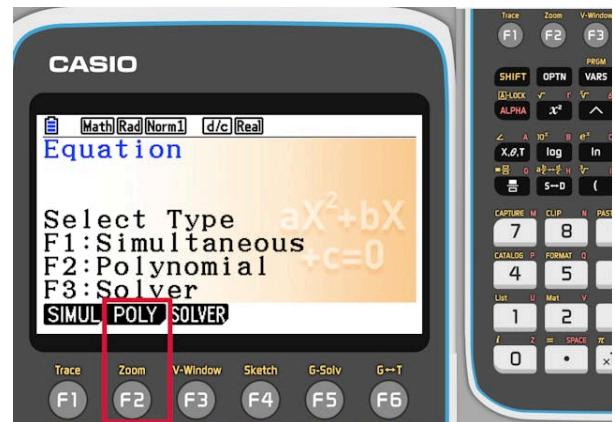


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Move down to the line of complex mode and press F1 to choose real numbers.

Once done, press EXIT.



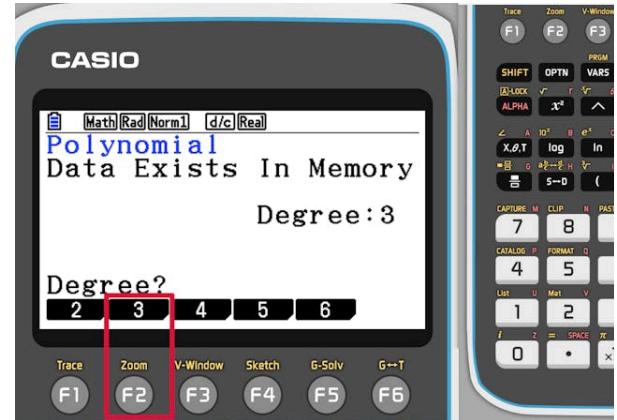
Back in the equation solver main screen press F2 to access the polynomial root finder.



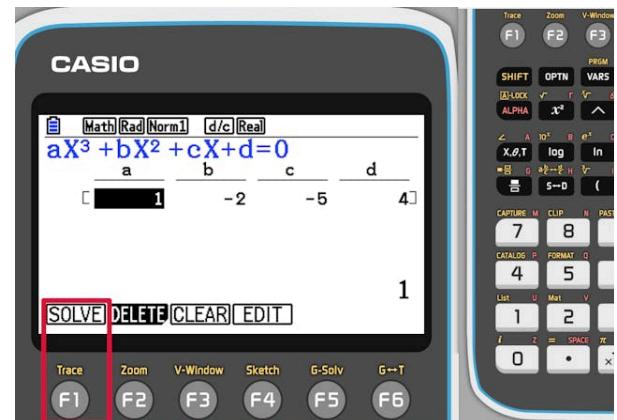
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Choose the degree of the polynomial ...



... and enter the coefficients.

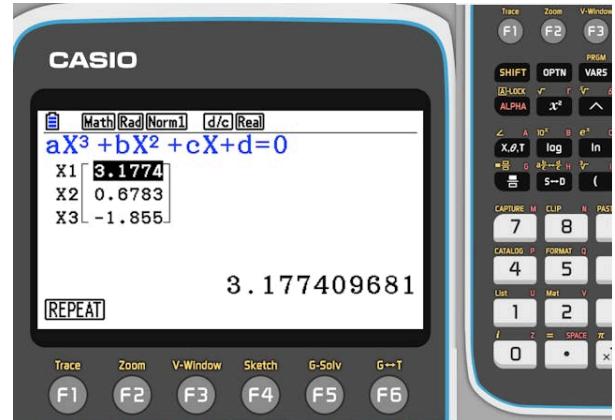
Once done, press F1 to find the roots.



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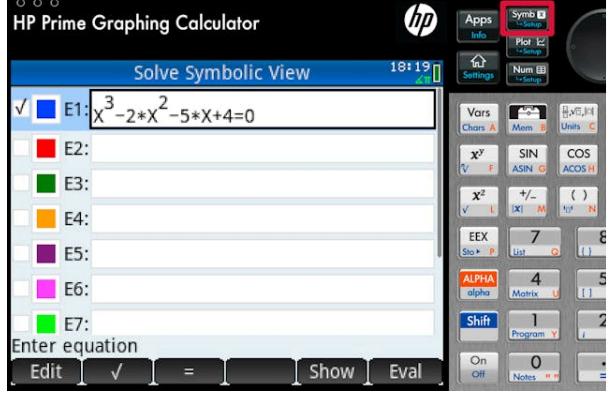
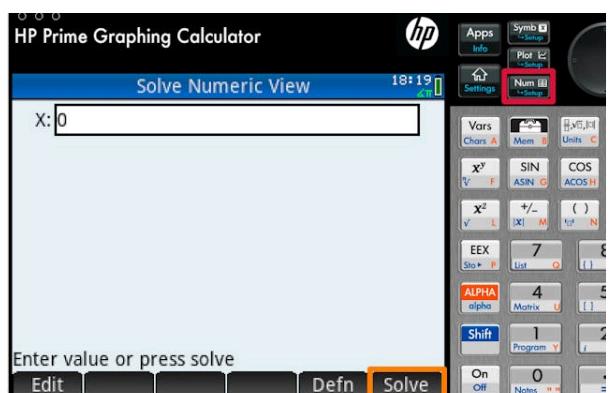
**This equation has three real solutions.**

Steps	Explanation
<p>These instructions will show you how to use the calculator to find the real solutions of the equation</p> $x^3 - 2x^2 - 5x + 4 = 0.$ <p>Open the equation solver application.</p>	



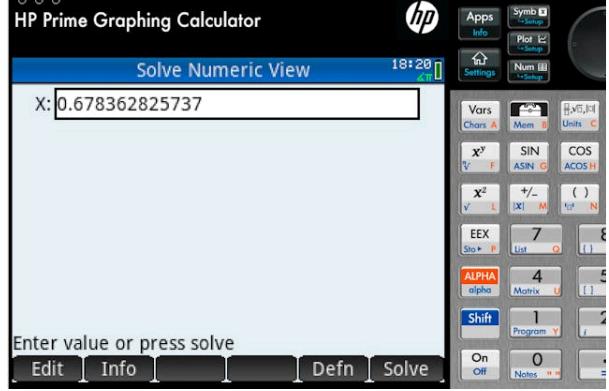
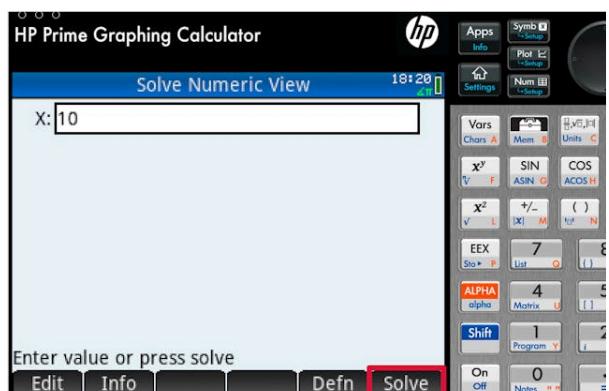
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Steps	Explanation
<p>In symbolic view, enter the equation you would like to solve.</p>	
<p>To find the solutions, change to numeric view.  This solver finds the solutions one at a time. The calculator needs a starting value for the search for the solution. Type any value for <math>x</math>.  Tap on solve, ...</p>	



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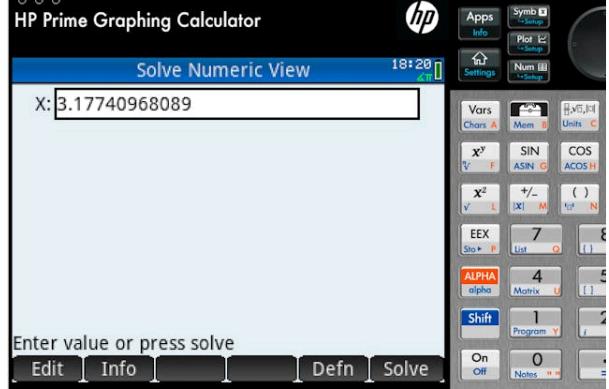
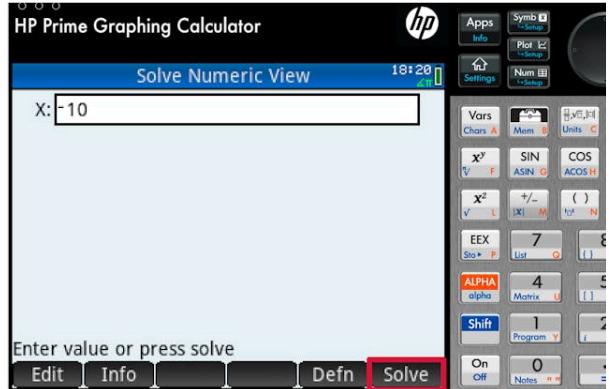
	Steps	Explanation
<p>Overview (/study/ap 122- cid- 754029/)</p> <hr/>	<p>... the calculator will find the solution closest to the value you typed for <math>x</math>.</p> <p>If you know from the context of the question, that your equation has a unique solution, you can stop here.</p> <p>The equation in this is cubic, which can have three solutions.</p>	
	<p>You can enter a different starting point for the solution search and repeat the process.</p>	



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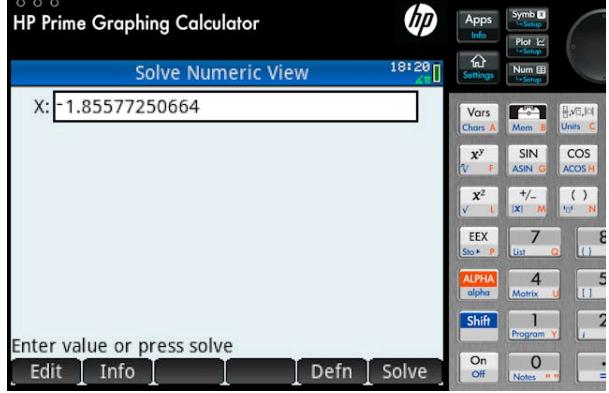
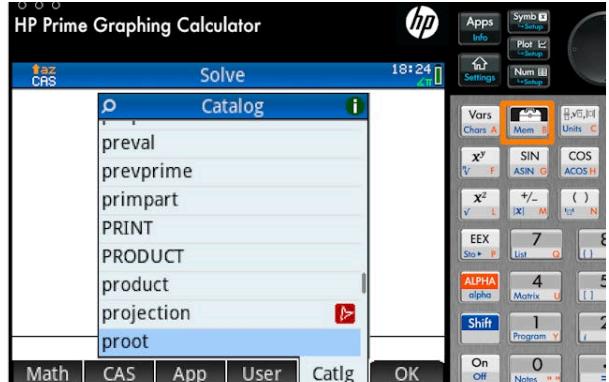
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Steps	Explanation
<p>In this example, with 10 as the starting value the calculator finds a second solution.</p> <p>It may happen, that you get the same solution as before. In this case you need to investigate different starting values.</p>	 <p>The calculator screen shows the Solve Numeric View. The input field contains X: 3.17740968089. Below the input field is the text "Enter value or press solve". At the bottom are buttons for Edit, Info, Defn, and Solve. The Solve button is highlighted with a red box.</p>
<p>You can also try a starting value smaller than the first solution.</p>	 <p>The calculator screen shows the Solve Numeric View. The input field contains X: -10. Below the input field is the text "Enter value or press solve". At the bottom are buttons for Edit, Info, Defn, and Solve. The Solve button is highlighted with a red box.</p>



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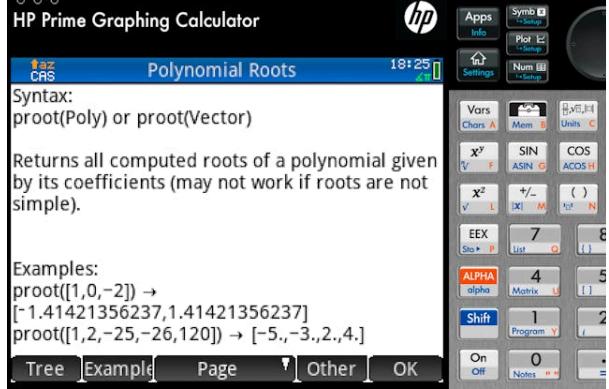
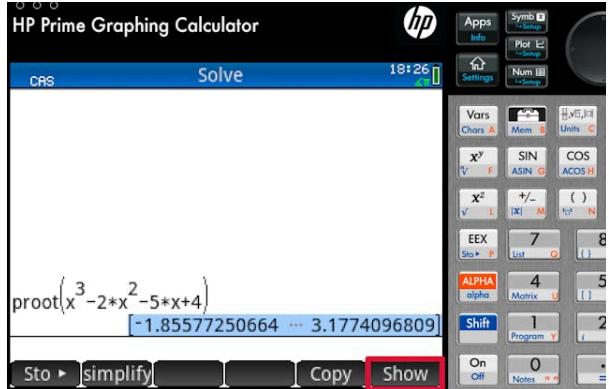
Steps	Explanation
<p>In this example, with <math>-10</math> as the starting value the calculator finds a third solution.</p> <p>Since cubic equations cannot have more than three solutions, you can stop here.</p> <p>This process of course involves trial and error. For different approaches, see the screenshots below.</p>	
<p>You can also find solutions by graphing the left hand side and finding the <math>x</math>-intercepts. This approach is not illustrated here.</p> <p>Instead, let's look at an approach, which currently is available only in CAS mode and CAS mode needs to be disabled on exam. However, it can happen, that by the time of the exam the operating system changes and this approach is available on exams.</p> <p>Open the toolbox and find the polynomial root finder (proot) option.</p>	



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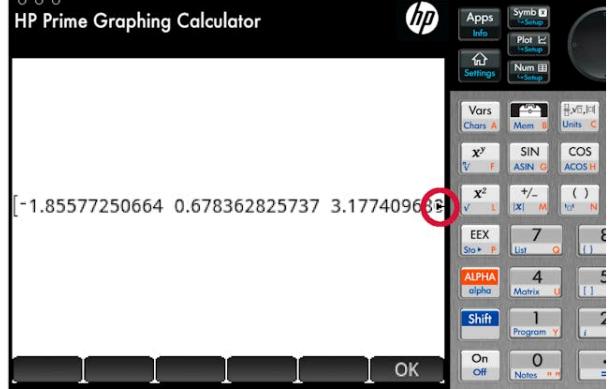
Steps	Explanation
<p>The help screen explains how this tool works.</p>	 <p>HP Prime Graphing Calculator 18:25 Polynomial Roots Syntax: proot(Poly) or proot(Vector) Returns all computed roots of a polynomial given by its coefficients (may not work if roots are not simple). Examples: proot([1,0,-2]) → [-1.41421356237,1.41421356237] proot([1,2,-25,-26,120]) → [-5.,-3.,2.,4.] Tree Example Page Other OK</p>
<p>The three dots indicate, that the calculator did not display all information. Tap on the solutions and then choose the option to show the full list.</p>	 <p>HP Prime Graphing Calculator 18:26 CAS Solve proot(x^3 -2*x^2 -5*x+4) [-1.85577250664 ... 3.1774096809] Stop Simplify Copy Show</p>

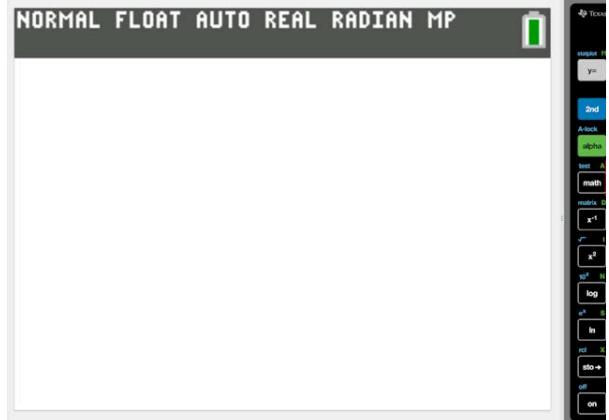


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Steps	Explanation
<p>This equation has three real solutions.</p> <p>Notice, that these solutions do not fit on one screen. This is indicated by the arrow on the right. You need to scroll the screen to see the end of the list.</p>	

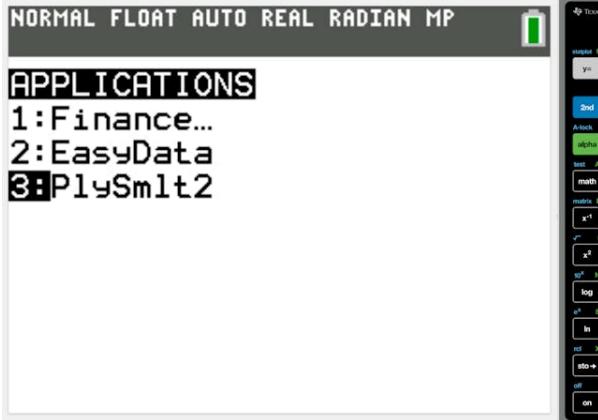
Steps	Explanation
<p>These instructions will show you how to use the calculator to find the real solutions of the equation</p> $x^3 - 2x^2 - 5x + 4 = 0.$ <p>Open the application menu, ...</p>	



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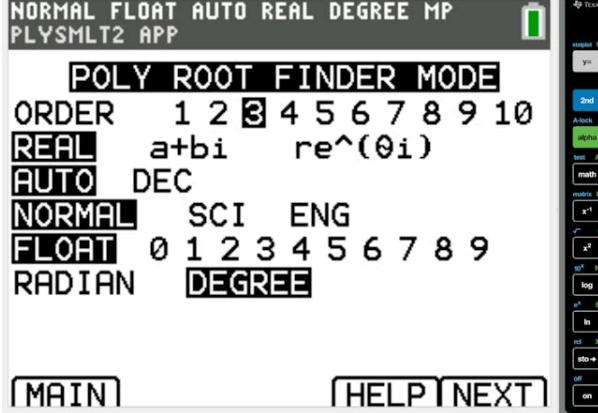
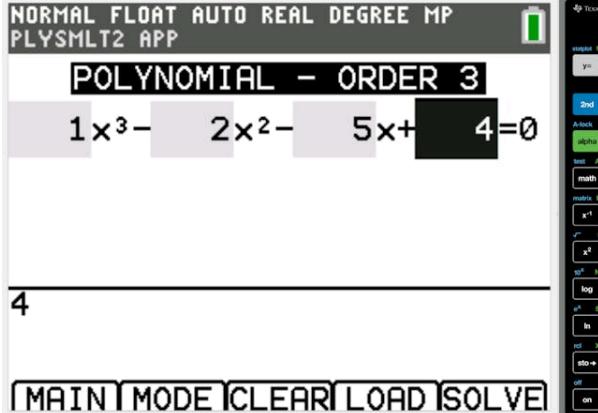
Steps	Explanation
<p>... choose the equation solver application (PlySmlt2) ...</p>	 <p>NORMAL FLOAT AUTO REAL RADIAN MP</p> <p>APPLICATIONS</p> <p>1:Finance... 2:EasyData <b>3:PlySmlt2</b></p>
<p>... and choose the polynomial root finder option.</p>	 <p>NORMAL FLOAT AUTO REAL RADIAN MP</p> <p>PLYSMLT2 APP</p> <p>MAIN MENU</p> <p>1:POLYNOMIAL ROOT FINDER 2:SIMULTANEOUS EQN SOLVER 3:ABOUT 4:POLY ROOT FINDER HELP 5:SIMULT EQN SOLVER HELP 6:QUIT APP</p>



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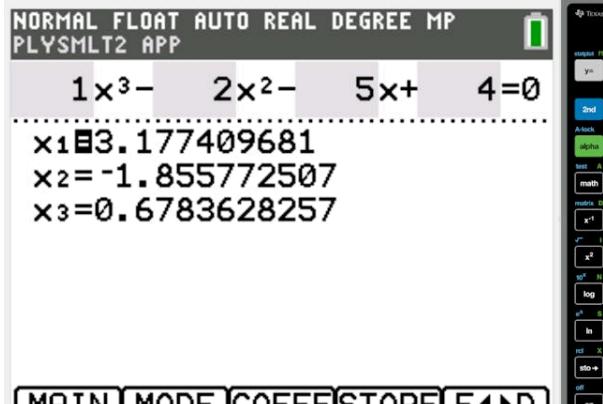
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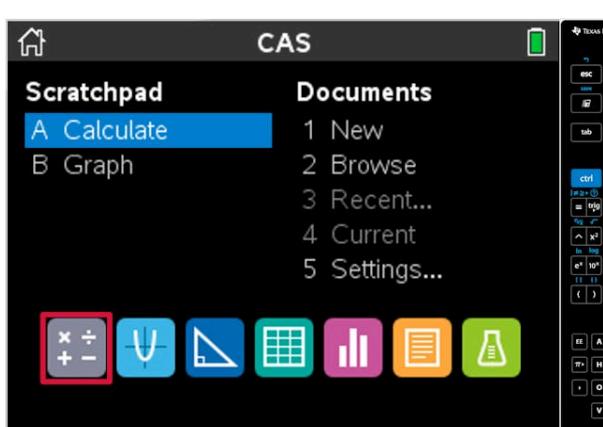
Steps	Explanation
<p>Set the order of the polynomial and choose to look for only the real roots.</p> <p>Once done, press the graph button to move to the next screen.</p>	 <p>NORMAL FLOAT AUTO REAL DEGREE MP PLYSMLT2 APP</p> <p>POLY ROOT FINDER MODE</p> <p>ORDER 1 2 3 4 5 6 7 8 9 10</p> <p>REAL a+bi re^(θi)</p> <p>AUTO DEC</p> <p>NORMAL SCI ENG</p> <p>FLOAT 0 1 2 3 4 5 6 7 8 9</p> <p>RADIANT DEGREE</p> <p>[MAIN] [HELP] [NEXT]</p>
<p>Enter the coefficients of the polynomial and press the graph button to solve the equation.</p>	 <p>NORMAL FLOAT AUTO REAL DEGREE MP PLYSMLT2 APP</p> <p>POLYNOMIAL - ORDER 3</p> <p><math>1x^3 - 2x^2 - 5x + 4 = 0</math></p> <p>4</p> <p>[MAIN] [MODE] [CLEAR] [LOAD] [SOLVE]</p>

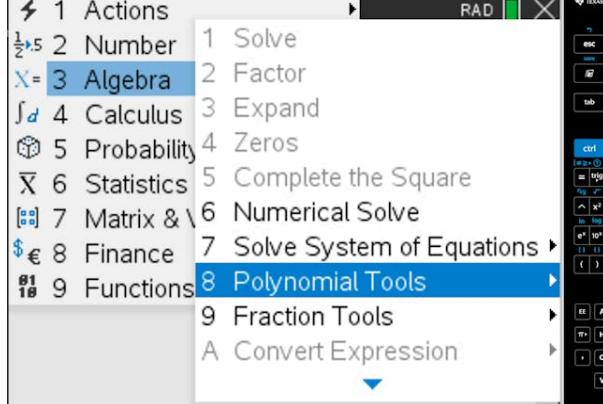
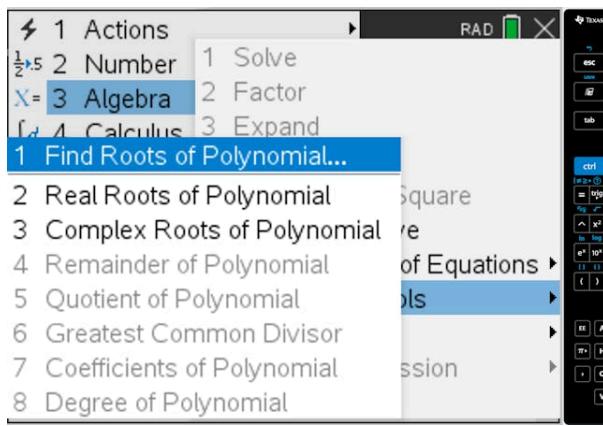


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Steps	Explanation
This equation has three real solutions.	

Steps	Explanation
<p>These instructions will show you how to use the calculator to find the real solutions of the equation</p> $x^3 - 2x^2 - 5x + 4 = 0.$ <p>Open a calculator page.</p>	

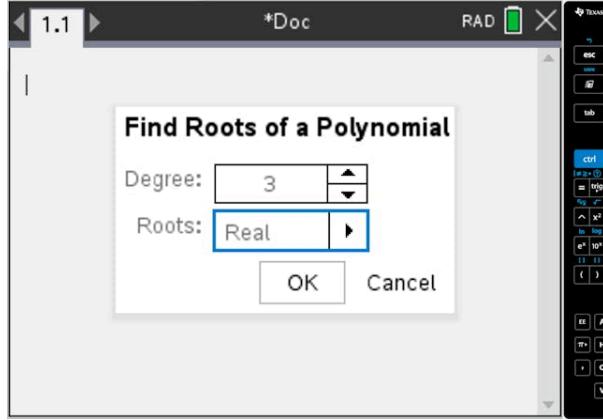
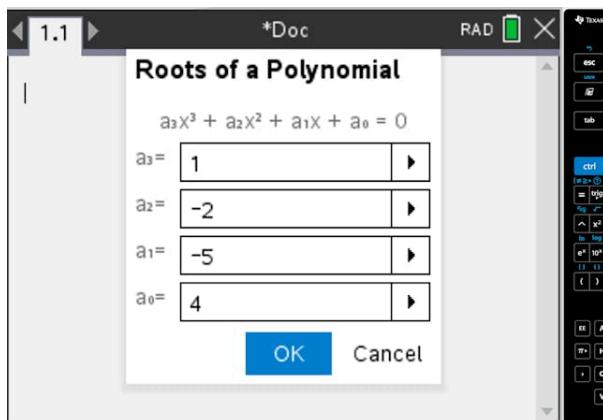
Steps	Explanation
<p>Open the menu, look for the polynomial tools ...</p>	 <p>The screenshot shows the TI-Nspire CX CAS calculator's main menu. The 'Algebra' icon (represented by a blue X=) is selected. The 'Polynomial Tools' option under 'Algebra' is also highlighted in blue. Other menu items include Actions, Number, Calculus, Probability, Statistics, Matrix &amp; Vectors, Finance, and Functions.</p>
<p>... and choose the option to find roots of polynomials.</p>	 <p>The screenshot shows the TI-Nspire CX CAS calculator's sub-menu for 'Algebra'. The 'Find Roots of Polynomial...' option is highlighted in blue. Other options in this sub-menu include Real Roots of Polynomial, Complex Roots of Polynomial, Remainder of Polynomial, Quotient of Polynomial, Greatest Common Divisor, Coefficients of Polynomial, and Degree of Polynomial.</p>



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Steps	Explanation
<p>Set the degree of the polynomial and choose to look for only the real roots.</p>	
<p>Enter the coefficients of the polynomial and press enter to solve the equation.</p>	



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Steps	Explanation
This equation has three real solutions.	A screenshot of a Texas Instruments TI-Nspire CX CAS calculator. The screen displays the command "polyRoots(x^3 - 2*x^2 - 5*x + 4, x)" and its result, which is a set of three real numbers: {-1.85577, 0.678363, 3.17741}. The calculator's interface includes a menu bar at the top with "1.1", "RAD", and a battery icon. The left side shows function keys like sin, cos, tan, and log. The right side shows mode keys like DEG, RAD, and FIX. The bottom of the screen shows a keypad with digits 0-9 and arithmetic operators.

## Example 2



Write down the solutions to  $0 = x^3 + x^2 + x + 1$ .

Steps	Explanation
$x = -1$	There is one real solution. See calculator instructions for th

## Example 3



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Find the roots of the polynomial  $P(x) = -2x^3 - 3x^2 + 2x + 1$ .

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Steps	Explanation
$x = -1.89 \text{ or } -0.355 \text{ or } 0.745 \text{ (3 significant figures)}$	<p>There are three real roots.</p> <p>See calculator instructions in this section.</p>

## Example 4



Find the  $x$  values for which  $P(x) = 2x^2 - 3x + 5$  is equal to  $G(x) = x^4 - 3x^3 - 4$ .

Steps	Explanation
$2x^2 - 3x + 5 = x^4 - 3x^3 - 4 \Leftrightarrow 0 = x^4 - 3x^3 - 2x^2 + 3x - 9$	<p>Move terms from one side to the other to create a polynomial equation that has zero on the left-hand side.</p>
$x = -1.60 \text{ or } 3.53 \text{ (3 significant figures)}$	<p>Solve the equation.</p> <p>Two real roots.</p> <p>See calculator instructions in this section.</p>



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How can you check that the solutions obtained from the calculator are correct?

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## Example 5



A diver jumps off a 10-metre high diving board into a pool. The height of the diver above the water is modelled by  $h(t) = -1.2t^2 + 1.4t + 10$  where  $h(t)$  is the height above the water measured in metres and  $t$  is the time after the diver jumps off the diving board measured in seconds.

Find how much time passes after the jump before the diver enters the water in the pool.

Give your answer in seconds correct to 2 decimal places.

Steps	Explanation
$0 = -1.2t^2 + 1.4t + 10$ $t = -2.36 \text{ or } 3.53$	The diver enters the water when $h(t) = 0$ . Use your calculator to solve the polynomial equation following the same procedure as shown for Example 4.
The amount of time that passes is 3.53 s .	There are two solutions to the polynomial equation, but the positive one makes sense in the context of the question.

## 2 section questions



1. Number and algebra / 1.8 Equations and equation systems

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# Systems of linear equations

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Linear equations can be used to model real-world situations. For example, the total monthly price of a gym membership that charges a monthly fee of €15 and an additional €5 for each instructor-led class can be represented by  $y = 5x + 15$ , where  $y$  is the total monthly cost and  $x$  is the number of instructor-led classes attended.

You could compare this gym membership with another one that charges €50 per month and only €1 for instructor-led classes.

To compare these two plans, you can start by finding the point at which both membership plans cost the same amount. To do so, you will need to solve a system of 2 linear equations,

$$\begin{cases} y = 5x + 15 \\ y = 1x + 50. \end{cases}$$

Why would it be helpful to know at which point the gym memberships cost the same amount?

## ✓ Important

A system of linear equations consists of two or more linear equations.

In this course, you will consider the following two cases:

- A system of two equations of the form:

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

- A system of three equations of the form:



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$$\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \\ ix + jy + kz = l \end{cases}$$

A solution to a system of linear equations is the set of values of the variables that make each equation true.

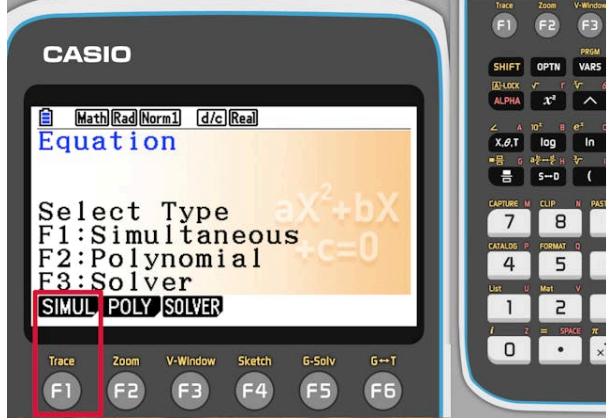
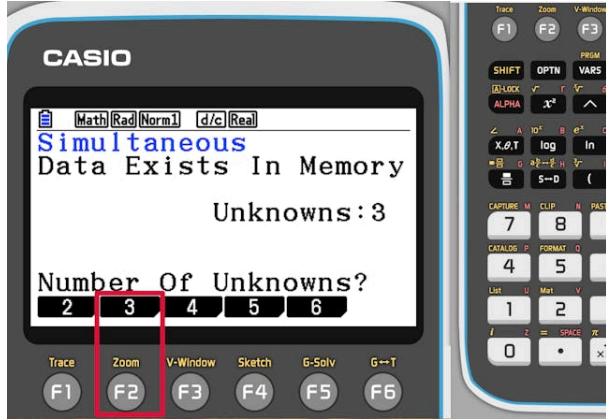
## ① Exam tip

In the IB examination, it is expected that solutions to systems of linear equations will only be found by using your calculator. You do not need to solve systems of linear equations algebraically.

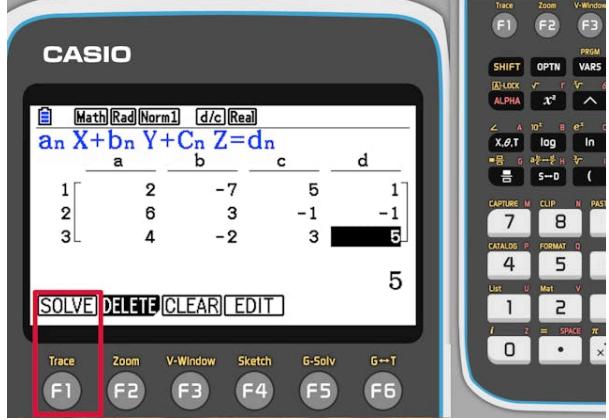
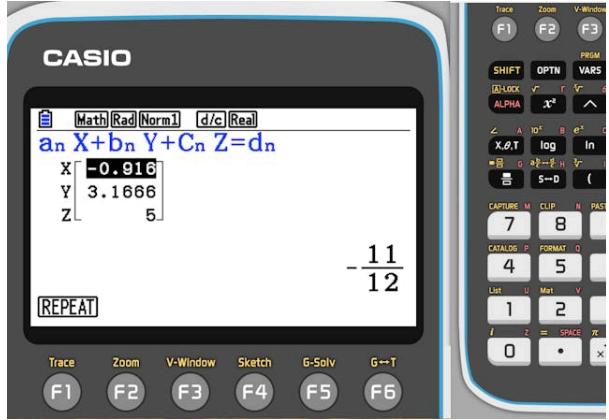
Steps	Explanation
<p>These instructions show you how to use the calculator to find the solution of the following system of equations.</p> $\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ 4x - 2y + 3z &= 5 \end{aligned}$ <p>Open the equation solving mode ...</p>	



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Steps	Explanation
<p>... and press F1 to choose the simultaneous equation solver option.</p>	
<p>Set the number of equations (in this example, three) ...</p>	



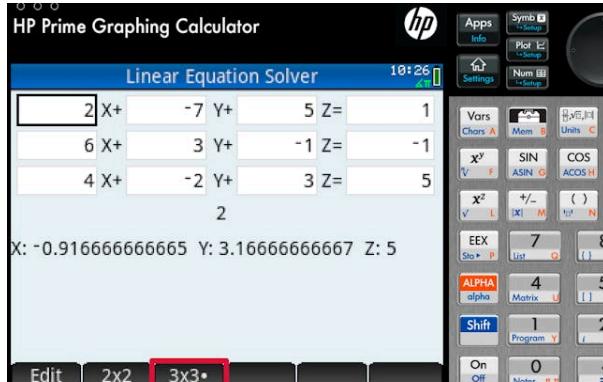
Steps	Explanation
<p>... and enter the coefficients.</p> <p>Once done, press F1 to ask the calculator to show you the solutions.</p>	
<p>The calculator gives the approximate solution as decimals. If you move up and down, you can also view the exact value as a fractions.</p>	



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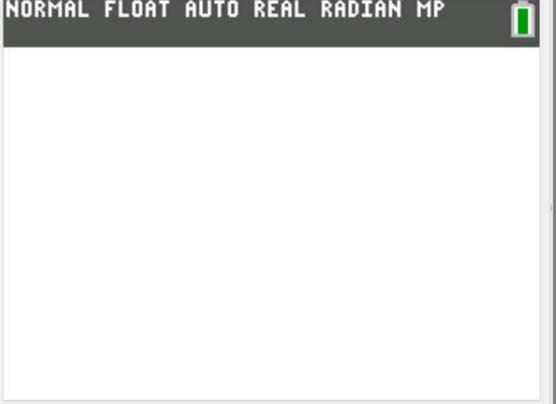
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Steps	Explanation
<p>These instructions show you how to use the calculator to find the solution of the following system of equations.</p> $\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ 4x - 2y + 3z &= 5 \end{aligned}$ <p>Open the linear solver application.</p>	
<p>Choose the size of the system of equations (in this example three equations in three unknowns).</p> <p>Enter the coefficients.</p> <p>The calculator gives the solution in real time. It refreshes the solution set every time you change a coefficient.</p>	



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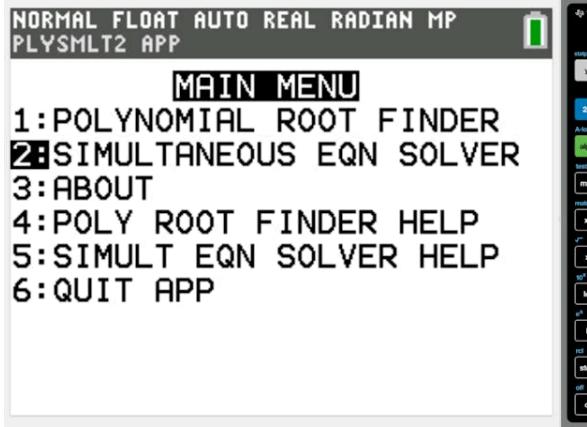
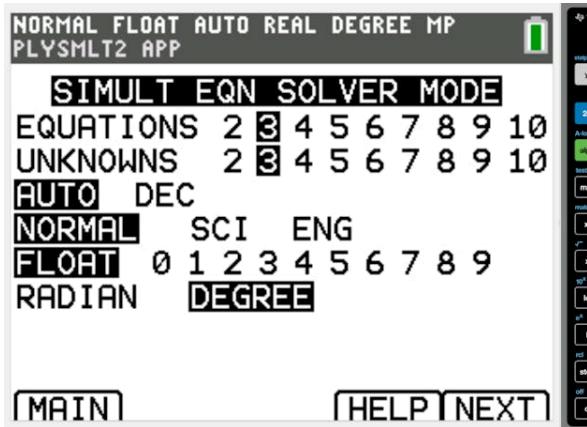
Steps	Explanation
<p>These instructions show you how to use the calculator to find the solution of the following system of equations.</p> $\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ 4x - 2y + 3z &= 5 \end{aligned}$ <p>Open the application menu, ...</p>	
<p>... choose the equation solving application (PlySmlt2) ...</p>	



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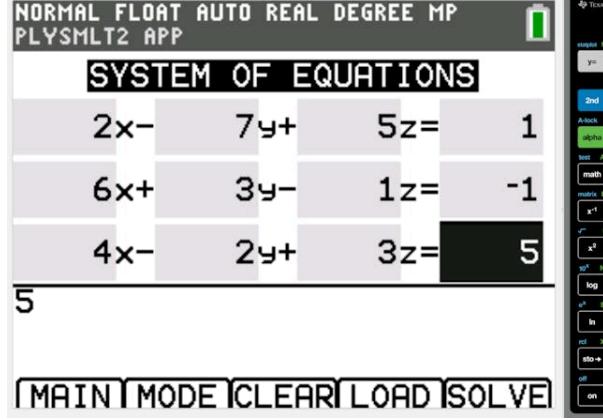
Steps	Explanation
<p>... and choose the simultaneous equation solver option.</p>	
<p>Choose the size of the system of equations (in this example three equations in three unknowns).</p> <p>Once done, press the graph button to move to the next screen</p>	



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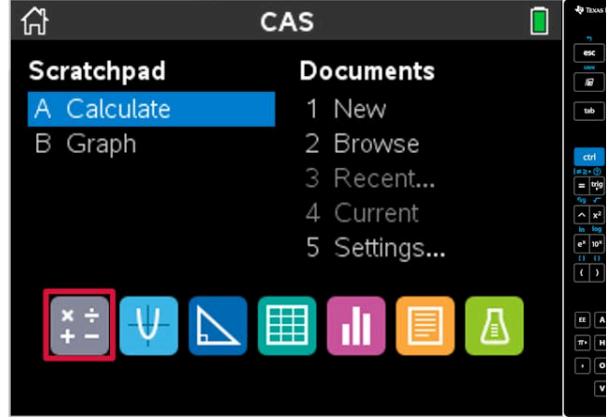
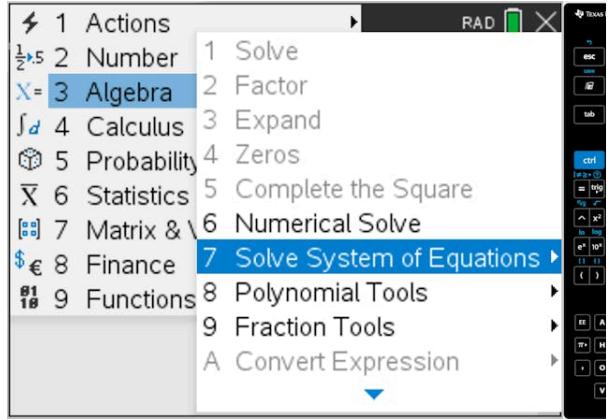
Steps	Explanation
<p>Enter the coefficients.</p> <p>Once done, press the graph button to ask the calculator to show you the solutions.</p>	 <p>NORMAL FLOAT AUTO REAL DEGREE MP PLYSMLT2 APP</p> <p>SYSTEM OF EQUATIONS</p> $\begin{array}{l} 2x - 7y + 5z = 1 \\ 6x + 3y - 1z = -1 \\ 4x - 2y + 3z = 5 \end{array}$ <p>MAIN MODE CLEAR LOAD SOLVE</p>
<p>The calculator gives the solution as fractions.</p>	 <p>NORMAL FLOAT AUTO REAL DEGREE MP PLYSMLT2 APP</p> <p>SOLUTION</p> $\begin{array}{l} x = -\frac{11}{12} \\ y = \frac{19}{6} \\ z = 5 \end{array}$ <p>MAIN MODE SYSM STORE FWD</p>



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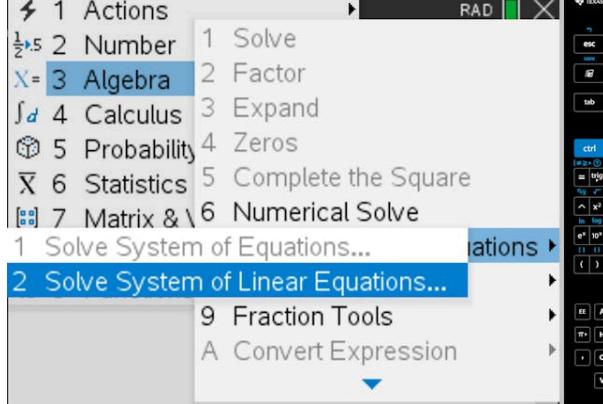
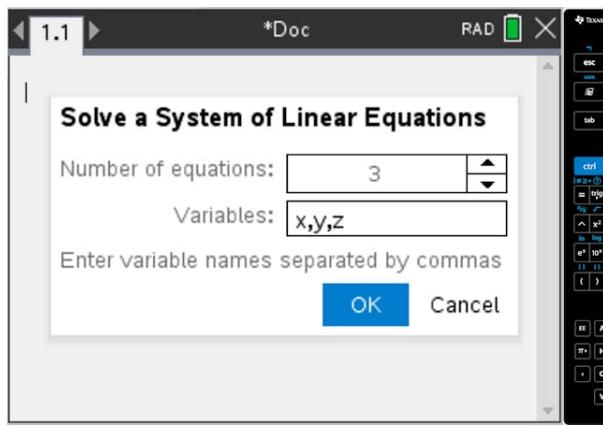


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Steps	Explanation
<p>These instructions show you how to use the calculator to find the solution of the following system of equations.</p> $\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ 4x - 2y + 3z &= 5 \end{aligned}$ <p>Open a calculator page.</p>	 <p>The screenshot shows the TI-Nspire CX CAS calculator's main menu. The 'Calculate' option is highlighted in blue. Other menu items include 'Graph', 'Recent...', 'Current', and 'Settings...'. Below the menu are several icons representing different functions: a red calculator for arithmetic, a blue circle with a square for trigonometry, a blue triangle for geometry, a green grid for matrices, a purple bar chart for statistics, an orange document for documents, and a green flask for chemistry.</p>
<p>Open the menu, look for the option to solve system of equations ...</p>	 <p>The screenshot shows the TI-Nspire CX CAS calculator's Algebra menu. The 'Algebra' option is highlighted in blue. A sub-menu is open under 'Algebra' with the following options: 'Solve', 'Factor', 'Expand', 'Zeros', 'Complete the Square', 'Numerical Solve', 'Solve System of Equations', 'Polynomial Tools', 'Fraction Tools', and 'Convert Expression'. The 'Solve System of Equations' option is also highlighted in blue.</p>



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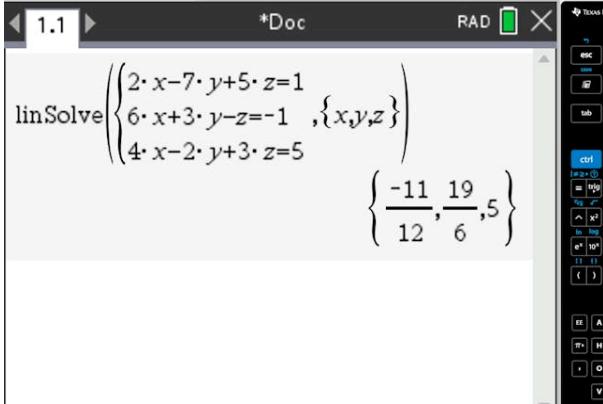
Steps	Explanation
<p>... and choose the option to solve a system of linear equations.</p>	
<p>Set the number of equations (in this example three) and enter the variable names.</p>	



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Steps	Explanation
<p>Enter the equations using the variable names you specified.</p> <p>The calculator gives the solution as fractions.</p>	 <pre>linSolve({2·x-7·y+5·z=1, 6·x+3·y-z=-1, 4·x-2·y+3·z=5}, {x,y,z}) { -11/12, 19/6, 5 }</pre>

## Example 1



The monthly costs of two types of gym membership, as given in the introduction to this section, are summarised below.

Gym A: €15 per month plus an additional €5 per instructor-led class.

Gym B: €50 per month plus an additional €1 per instructor-led class.

- a) The following equations can be used to calculate the total monthly cost of each type of membership.

$$\begin{cases} y = 5x + 15 \\ y = 1x + 50 \end{cases}$$

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## Solve these equations.

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- b) Hence, determine which of the following two gym memberships will have a lower monthly cost if you plan to take 20 instructor-led classes.

	Steps	Explar
Section	<p>a) The solution is:  <math>x = 8.75</math> and <math>y = 58.75</math></p>	Solve using y calculator. See instructions for question.
	<p>b) Let <math>y</math> be the total monthly price of each gym membership and <math>x</math> the number of instructor-led classes.</p> <p>Student... (0/0)    Feedback    Print (/study/app/m/sid-122-cid-754029/book/systems-of-linear-equations-id-27396/print/)</p> <p><b>The model for Gym A</b></p> $y = 5x + 15$ <p><b>The model for Gym B</b></p> $y = 1x + 50$ <p>According to part a, the gym memberships cost the same amount if you attend 8.75 instructor-led classes.</p> <p>For <math>x &gt; 8.75</math>, Gym B has the lower total monthly cost.</p>	

## Example 2



Solve  $\begin{cases} 2x + 3y = 10 \\ -x + 4y = -5 \end{cases}$



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Steps	Explanation
$x = 5, y = 0$	Solve using your calculator. See calculator instructions section.

## Example 3



Solve 
$$\begin{cases} -2x + y - z = 3 \\ x + 4y = -2 \\ 3x + 2y + 5z = 0 \end{cases}$$

Steps	Explanation
$x = -2.11, y = 0.0286, z = 1.26$ (3 significant figures)	Solve using your calculator. See calculator instructions section.

## Example 4



a) Solve 
$$\begin{cases} -x + y - 3z = 2 \\ 2x + 2y + z = 11 \\ x - 4y = 5 \end{cases}$$

- b) Check your solution.



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	Steps	Explanation
a)	$x = 6.48, y = 0.370, z = -2.70$ (3 significant figures)	Solve using your calculator instructions for section.
	For $-x + y - 3z = 2$ : $-6.48 + 0.370 - 3(-2.70) = 1.99 \approx 2$ . For $2x + 2y + z = 11$ : $2(-6.48) + 2(0.370) - 2.70 = 11$ . For $x - 4y = 5$ : $6.48 - 4(0.370) = 5$ . The solution works in all three equations.	The solution to the system of equations should make all three equations true. Since you are using rounding answers, do not expect the answers to match exactly.



## Activity

Solutions to systems of linear equations can be interpreted graphically as points of intersection between the lines in a system of 2 equations.

Solve each of the following using your calculator and then examine the graphs for each system.

$$\begin{cases} x + 5y = 7 \\ -2x + y = -3 \end{cases}$$

$$\begin{cases} 2x + 3y = 5 \\ -4x - 6y = 10 \end{cases}$$

$$\begin{cases} 3x - 2y = 4 \\ 6x - 4y = 8 \end{cases}$$

Explain your calculator-generated results using the graphs.



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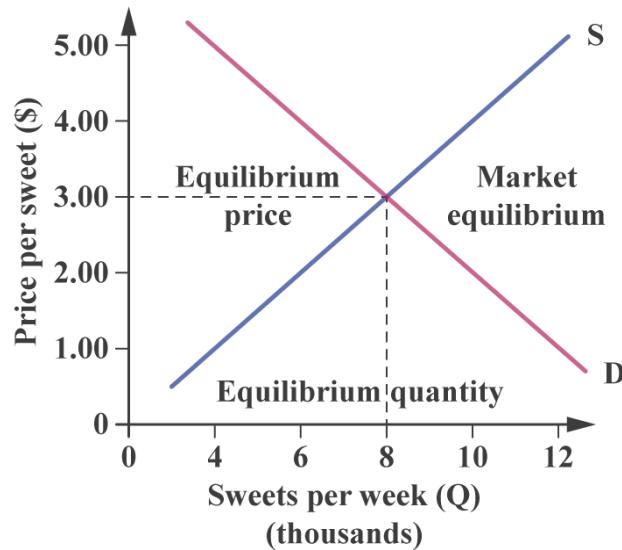


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## Making connections

If you study supply and demand curves in Economics, you will draw graphs such as the one shown in the diagram below. Finding the equilibrium price and equilibrium quantity involves solving a system of 2 linear equations.



More information

This image is a graph illustrating supply and demand curves typical in economics. The X-axis is labeled 'Sweets per week (Q)' measured in thousands, with a range from 0 to 12. The Y-axis is labeled 'Price per sweet (\$)', with a range from 0 to 5.00. A red line represents the supply (S), sloping upwards from the origin, indicating that as price per sweet increases, the quantity supplied also increases. A blue line represents demand (D), sloping downwards, showing that as price decreases, quantity demanded increases. The point where these two lines intersect is marked as 'Market equilibrium'. The intersection determines the 'Equilibrium price', which is approximately \$3.00, and 'Equilibrium quantity', around 6,000 sweets per week. The intersection indicates the balance between supply and demand, suggesting that this is the most efficient price and quantity for the market.

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## Activity

Solutions to a system of 3 linear equations can be interpreted graphically as the intersection of the three planes represented by the three equations. Sketch a situation where the three planes

- do not intersect with each other at all
- intersect in a point
- intersect in a line.

These two activities should have made you think about the possible solutions to a system of equations.

### ✓ Important

There are three possible outcomes of solving a system of equations involving two or three variables:

1. A unique solution— the lines or planes intersect at one unique point.  
There is one value for each  $x$ , and  $y$  or  $x$ ,  $y$  and  $z$ .
2. An infinite number of solutions — the lines or planes intersect in a line producing an infinite set of intersection points.
3. No solution — the lines or the planes do not have any intersection points. There are no values for  $x$  and  $y$  or  $x$ ,  $y$  and  $z$ .

### ⌚ Exam tip

Exam questions will only involve systems of equations that have a unique solution.



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## 3 section questions ▾



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1. Number and algebra / 1.8 Equations and equation systems

# Checklist

**Section**

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Feedback

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## What you should know

By the end of this subtopic you should be able to:

- solve polynomial equations using a calculator

**Section**

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- know that a polynomial equation of degree  $n$  has at most  $n$  unique solutions

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- solve systems of linear equations using a calculator.

1. Number and algebra / 1.8 Equations and equation systems

# Investigation

**Section**

Student... (0/0)

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## Part 1

The applet below allows you to manipulate the coefficients  $a$ ,  $b$  and  $c$  in a quadratic equation of the form  $y = ax^2 + bx + c$  in order to fit the curve to the parabola shown in the picture. Move the sliders around until you get a well-fitting curve. Explain how the quadratic equation that you made can be applied to the situation shown in the image.



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## Part 2



In addition to using sliders, you can create a quadratic model to fit data algebraically.

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Consider the following situation:

A basketball follows a parabolic path from the hand of the player to the basket. It is known that the ball is released by the player at a height of 1.7 m when the player is 6 m away from the basket. The ball hits the basket at a height of 3.1 m. The height of the ball when it is 3 m away from the basket is recorded as 6.2 m. Use this information and what you know about systems of linear equations to create a quadratic model for the path of the ball. Explain how this model can be used to determine how high a defence player standing 0.2 m from the basket would need to jump to block the ball from going into the basket.

Determine whether the same type of approach can be used to find a model for the path of a rollercoaster that looks like a cubic polynomial. How much information would you need in this case?



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## Interactive 1. A Graphical Representation of the Quadratic Equation (Parabola).

Credit: [GeoGebra ↗](https://www.geogebra.org/m/hcP2qAw8) (<https://www.geogebra.org/m/hcP2qAw8>) Alexandra Schmidt, jclarkrivers

More information for interactive 1

This interactive allows users to explore the behavior of quadratic functions by adjusting the coefficients a, b, and c in the standard form equation  $y = ax^2 + bx + c$ .

Users can manipulate sliders to change the value of a within a range from -2.5 to 2.5, and b and c within ranges from -20 to 20. As users adjust these values, the graph updates in real time, showing how the shape and position of the parabola change based on the selected coefficients. The interactive includes a target parabola, and the user's goal is to adjust the sliders to match the curve of their quadratic equation to the displayed parabola, providing a hands-on and visual way to understand the roles of each coefficient.

For example, if  $a = 0.45$ ,  $b = -4.4$ , and  $c = 13$ , the resulting quadratic equation is  $y = 0.45x^2 - 4.4x + 13$ , which produces a parabola that opens upward because a is positive. Changing the value of a affects how wide or narrow the parabola appears, b shifts its vertex left or right and alters the tilt, and c moves the entire graph up or down.

Through interactive experimentation, users gain an intuitive understanding of how each term in a quadratic equation influences the graph's shape and position, reinforcing the connection between algebraic expressions and their geometric representations.

### Rate subtopic 1.8 Equations and equation systems

Help us improve the content and user experience.



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