

Checklist

What you should know

By the end of this subtopic you should be able to:

- recall that vectors have a size (magnitude) and a direction
- represent a displacement between two points as a vector
- decompose a 2D vector into its components in the x - and y -directions and a 3D vector into its components in the x -, y - and z -directions

- write a vector in column form, e.g. $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ or base vector form

$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$, where \mathbf{i} , \mathbf{j} and \mathbf{k} are unit base vectors in the x -, y - and z -directions, respectively:

$$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- recall that two vectors are equal if and only if all their components are equal

- add vectors by adding their components: e.g. if $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}, \text{ then } \mathbf{v} + \mathbf{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_3 + w_3 \end{pmatrix}$$

- recall that addition of vectors is commutative: $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$

- recall that if $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, then $-\mathbf{v} = -\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \mathbf{v} = \begin{pmatrix} -v_1 \\ -v_2 \\ -v_3 \end{pmatrix}$

- recall that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$, where $\mathbf{0}$ is the zero vector

- recall how to subtract one vector from another by adding the negative

components of the vector being subtracted: i.e. if $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}, \text{ then } \mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w}) \begin{pmatrix} v_1 - w_1 \\ v_2 - w_2 \\ v_3 - w_3 \end{pmatrix}$$

- recall that subtraction of vectors is not commutative: i.e. $\mathbf{v} - \mathbf{w} \neq \mathbf{w} - \mathbf{v}$

- multiply a vector by a scalar, e.g. if $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is multiplied by scalar

$$k \in \mathbb{R} \text{ the result is } k\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

- recall that $k\mathbf{v}$ is a vector in the same direction as \mathbf{v} but with a different magnitude (unless $k = \pm 1$)
- recall that two vectors are parallel if they are scalar multiples of the same vector: i.e. \mathbf{v} and \mathbf{w} are parallel if there exist $m \in \mathbb{R}$ and $n \in \mathbb{R}$ such that $\mathbf{v} = m\mathbf{u}$ and $\mathbf{w} = n\mathbf{u}$

- find the magnitude of a vector using Pythagoras' theorem: if $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

, the magnitude of \mathbf{v} is denoted by $|\mathbf{v}|$ and is given by

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

- recall the properties of the magnitude of a vector:
 - $|\mathbf{v}| \geq 0$, i.e. it is never negative
 - $|k\mathbf{v}| = |k||\mathbf{v}|, k \in \mathbb{R}$
 - $|\mathbf{v}| = |-\mathbf{v}|$
 - in general, $|\mathbf{v} + \mathbf{w}| \neq |\mathbf{v}| + |\mathbf{w}|$
- recall that a unit vector in a specified direction is denoted by $\hat{\mathbf{v}}$
- calculate a unit vector in the direction of a given vector using $\hat{\mathbf{v}} = \frac{1}{|\mathbf{v}|}\mathbf{v}$
- write the position vector of point A relative to a fixed origin O: if point A has coordinates (x, y, z) then the position vector of A is denoted by

$$\overrightarrow{OA} = \mathbf{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = xi + yj + zk$$
- recall that the displacement vector between two points A and B is given in terms of their position vectors $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ by

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\mathbf{a} + \mathbf{b}$$
- calculate displacement vectors between two points: e.g. if A has coordinates (x_A, y_A, z_A) and B has coordinates (x_B, y_B, z_B) then

$$\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix}.$$