3.14 Lines in two and three dimensions

Checklist

What you should know

By the end of this subtopic you should be able to:

- write the vector equation of a straight line as $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$, where $\boldsymbol{a}$ is the position vector of a point on the line, $\boldsymbol{b}$ is a vector describing the direction of the line and the parameterλ is a scalar
- write the position of a point on a straight line with coordinates (x, y, z) in terms of a vector equation:
$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$
- recall that the angle between two straight lines is given by the angle between their direction vectors:
  - if $\boldsymbol{b}$ and $\boldsymbol{d}$ are the direction vectors of two straight lines, then the angle θ between these lines is given using the scalar product as $\theta = \cos^{-1}\left(\frac{\boldsymbol{b} \cdot \boldsymbol{d}}{|\boldsymbol{b}||\boldsymbol{d}|}\right)$
  - recall that the angle between two straight lines is usually given as the acute angle not the obtuse angle
- write the equation of a straight line in vector form, parametric form and Cartesian form: given a point (x_0, y_0, z_0) and a direction vector $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$, the equation of a straight line can be written
  - in vector form as $\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$
  - in parametric form as $x = x_0 + \lambda l, y = y_0 + \lambda m, z_0 + \lambda n$
  - in Cartesian form as $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
- describe the motion of an object moving in a straight line with constant velocity by the vector equation $\boldsymbol{r} = \boldsymbol{r}_0 + vt$, where $\boldsymbol{r_0} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ is the initial position vector relative to a fixed origin, $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ is the velocity and t is the time.

- recall that speed is the magnitude of the velocity vector $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and use Pythagoras' theorem to find it: $|\boldsymbol{v}| = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$