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Teacher view

0 (https://intercom.help/kognity)

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3. Geometry and trigonometry / 3.5 Trigonometric ratios beyond acute angles

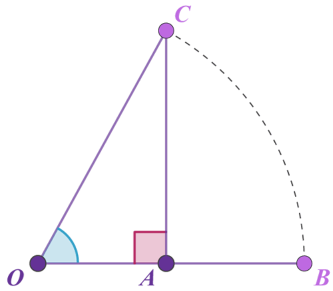
The big picture

So far you have only looked at the trigonometric ratios in a right-angled triangle. You have explored the sine and cosine of angles in any triangle. How can you extend trigonometric ratios from a right-angled triangle to any angle?

Early mathematicians were also astronomers, which meant they dealt with spheres and circles to calculate the distances between earth and moon, earth and sun, and the circumference of the earth, as well as mapping the night sky or celestial sphere. The celestial sphere is an imaginary sphere with the earth at the centre. To do these calculations they used circles and chords of circles.

International Mindedness

The word 'sine' comes from Greek sinus which means *fold* . This word is derived from Indian-Persian and Arabic words for a half a chord.



More information

The image is a geometric diagram showing a right triangle. The base of the triangle is the line segment OB, with points labeled O and B at each end. Point A is on the line segment OB, creating two segments, OA and AB. Point C is above A, and line AC is perpendicular to OB, making right angles at point A. A dotted arc from C to B represents the hypotenuse of the triangle OAC. The angle at point O is marked with a blue sector, indicating the angle of interest, while a red square at point A indicates the right angle triangle configuration.

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International Mindedness

During the 4th and 5th centuries in India, they used half chords, as shown above, and created the tables for the sine of any angle. The Jantar Mantar astronomical observatory in Jaipur, India has a structure which shows the sine of any angle.

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The structure for sine ratios for any angle in Jantar Mantar astronomical observatory in Jaipur, India
Credit: Meinzahn Getty Images

Concept

In this subtopic, you will generalise from trigonometric ratios in a right-angled triangle to trigonometric ratios for any angle.

How can circles help you to find the trigonometric ratios of any angle?

Theory of Knowledge

The concepts in this section rely on what we could call ‘a perfect circle’. Does a perfect circle exist in nature?

If you said yes, you may need to read this article from Carnegie Mellon University, *Do Perfect Circles Exist? Maybe* (https://www.cmu.edu/mcs/news-events/2019/0314_pi-day-perfect-circles.html).

If perfect circles do not exist, then how could mathematics focused on geometry of circles be said to be valid?

Knowledge Question: Must knowledge have a real-world corollary in order to be considered valid?

3. Geometry and trigonometry / 3.5 Trigonometric ratios beyond acute angles

The unit circle

Symmetry of the unit circle

A circle centred at origin $(0, 0)$ with radius 1 unit is called a unit circle. The diagram below shows a unit circle.

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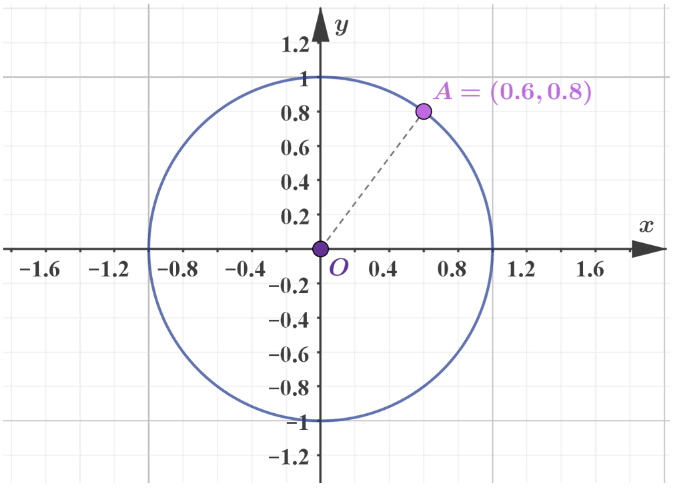

More information

The image is a diagram of a unit circle centered at the origin, point O, on a coordinate grid. The circle has a radius of 1 unit. The coordinate axes are labeled as x and y. Point A is marked on the circumference of the circle with coordinates (0.6, 0.8). A dotted line connects the origin O to point A, indicating the radius. The axes are labeled with numerical values incrementing by 0.2 both positively and negatively along their respective lines. The quadrants of the plane are not explicitly labeled, but the positioning of point A in the first quadrant assumes positive x and y values. Point O is the origin where the x and y axes intersect.

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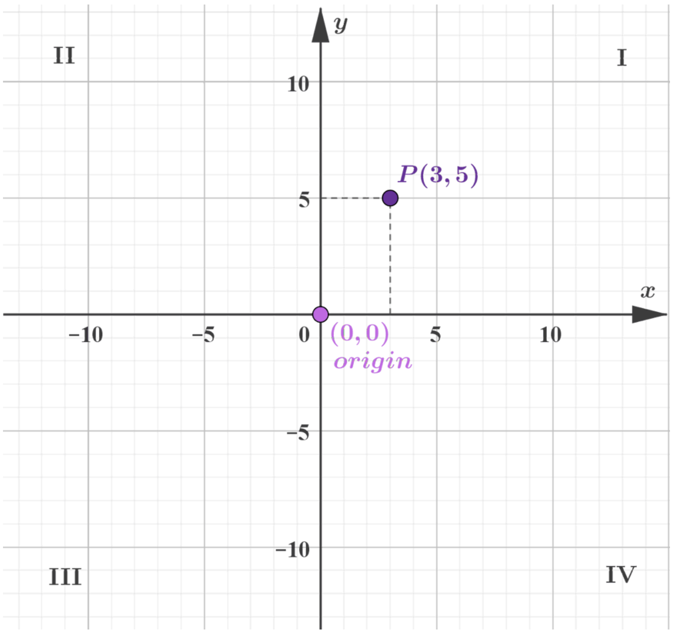
The perpendicular x -axis and y -axis divide the plane into four infinite sections. Each section is called a quadrant. The quadrants are numbered anticlockwise using roman numerals I, II, III and IV, as seen in the diagram below. In quadrant I, both x - and y - coordinates are positive; in quadrant II, x - coordinates are negative while y - coordinates are positive and so on. P oint A in the diagram above is in quadrant I, both x - and y - values are positive.



More information

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view

The image is a diagram of a Cartesian plane divided into four quadrants labeled I, II, III, and IV. The x-axis and y-axis intersect at the origin point marked as (0,0). Quadrant I is in the top right, with positive x and y coordinates. Quadrant II is top left, with negative x and positive y coordinates. Quadrant III is bottom left, with negative x and y coordinates. Quadrant IV is bottom right, with positive x and negative y coordinates. The point P (3,5) is plotted in Quadrant I, indicating that both x and y values are positive. The context added explains that the quadrants are listed in an anticlockwise manner. The grid lines help visualize the divisions, and the axes are marked with numerical values to demonstrate the scale.

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Example 1

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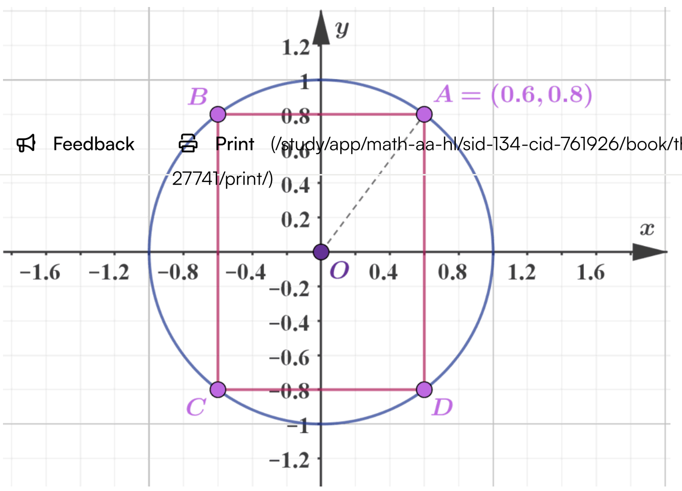
A rectangle with the vertex $\mathrm{A}(0.6,\ 0.8)$ is drawn as seen in the diagram. Find the coordinates of the other three vertices.



More information

The image is a diagram featuring a rectangle inscribed in a circle, all plotted on a grid with x and y axes. The vertices of the rectangle are marked and labeled as A, B, C, and D. The coordinates of vertex A are given as (0.6, 0.8). The rectangle and circle share a common center point labeled O. Grid lines are visible, aiding in visualizing the positions of the vertices in relation to the circle and each other. Vertex B aligns horizontally with vertex A at the same y-coordinate, while B and C align vertically at x=-0.6. Vertex D aligns vertically with A at x=0.6 and is directly opposite C in the circle's lower hemisphere. The radius of the circle appears to be 1, considering the position of point O and the grid labels.

[Generated by AI]

Steps	Explanation
	You can find the other points using symmetry.

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Steps	Explanation
$\mathrm{B}\,(-0.6, 0.8)$	As it is the reflection of A in the y -axis.
$\mathrm{C}\,(-0.6, -0.8)$	As it is the reflection of Bin the x -axis.
$\mathrm{D}\,(0.6, -0.8)$	As it is the reflection of Cin the y -axis or the reflection of A in the x -axis.

Important

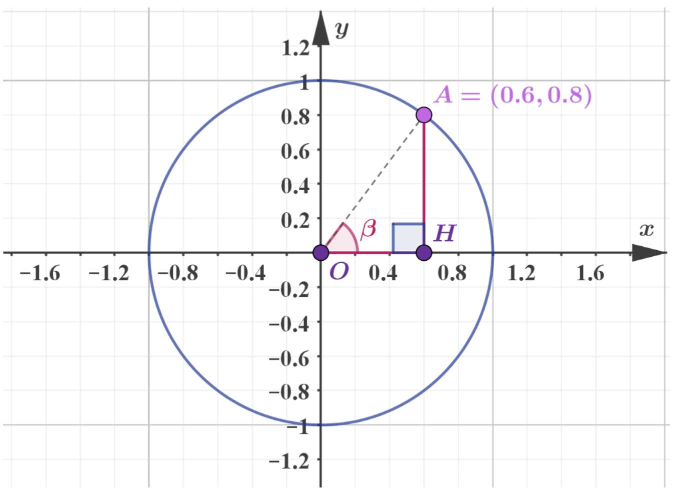
If you know one of the vertices of an inscribed rectangle with vertical and horizontal sides parallel to the x - and y -axes, you can use symmetry to find the other three vertices of this rectangle.

You can often use symmetry arguments to derive facts.

Exam tip

You can identify the sign of the x - and y - coordinates of a point by checking which quadrant the point is in.

You can also use the unit circle and the right-angled triangles you can form to write the coordinates of a point in terms of the angle formed between the x -axis and the segment OA, as in the diagram below.



More information

The image is a diagram of a unit circle superimposed on a grid. The circle is centered at the origin (0, 0) with a radius of 1. There is a right triangle present with vertices at O (origin), A (0.6, 0.8), and H (0.6, 0). The point A on the circumference of the circle is annotated with coordinates (0.6, 0.8). The angle (\beta) is formed between the positive (x)-axis and the line segment (OA). The base of the triangle runs along the (x)-axis from the origin to point H, and the height extends vertically from H to A. This triangle and angle illustrate trigonometric relationships in the context of the unit circle. The diagram uses color coding for clarity, with different colors for different lines and points. Axes are labeled (x) and (y).

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The trigonometric ratios in triangle OAH are

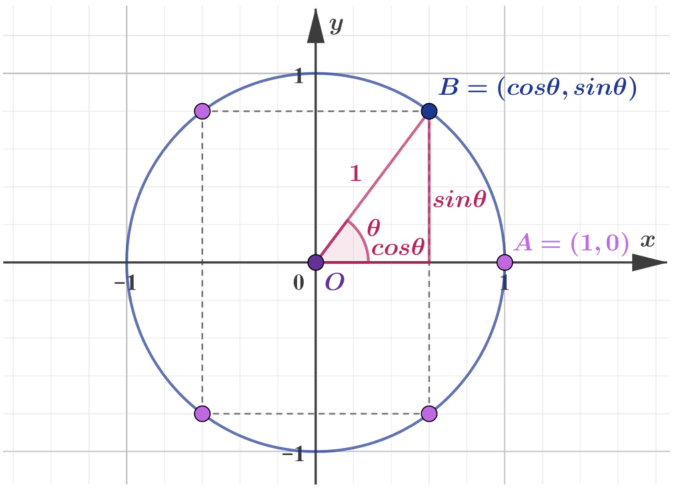
$$\sin\beta = \frac{AH}{OA} = \frac{0.8}{1} \Rightarrow \sin\beta = 0.8$$

$$\cos\beta = \frac{OH}{OA} = \frac{0.6}{1} \Rightarrow \cos\beta = 0.6$$

Therefore, for triangle OAH, you can say $A\ (0.6,\ 0.8) = A\ (\cos\beta, \sin\beta)$

Important

Angles on a unit circle are always marked from the point $(1,\ 0)$ in the anticlockwise direction. So the anticlockwise direction is the positive direction and the clockwise direction is the negative direction on the unit circle.



More information

The image depicts a unit circle on a coordinate grid with a highlighted right triangle. The circle is centered at the origin, point O (0,0), with a radius of 1. The x-axis and y-axis are labeled on the grid. Point A is labeled as (1,0) on the x-axis. Point B with coordinates (cosθ, sinθ) is marked on the circumference of the circle in the first quadrant.\n\nA right triangle is formed by the line segments OA, OB, and a perpendicular line from B to the x-axis. The angle θ is at point O. The horizontal leg of the triangle (adjacent to θ) represents cosθ, and the vertical leg (opposite to θ) represents sinθ. A 1-unit hypotenuse connects the origin O to point B. \n\nThe diagram illustrates the relationships in a unit circle where the x-coordinate of point B is the cosine of angle θ and the y-coordinate is the sine of angle θ. The grid helps show that the angle θ, sine, and cosine correspond to the lengths in the triangle on the unit circle.

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Important

Any point $B\ (x,\ y)$ on the circumference of unit circle can be written as the trigonometric ratios of the angle AOB where O is the origin and A is the point $(1, 0)$.

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As you can see in the diagram above, coordinates of B can also be written as $(\cos\theta,\ \sin\theta)$.

Example 2

★★☆

Given that $\cos 57^\circ \approx 0.545$ and $\sin 57^\circ \approx 0.839$:

a) Draw a unit circle diagram representing the angle 57° . Label the point on the circumference as A.

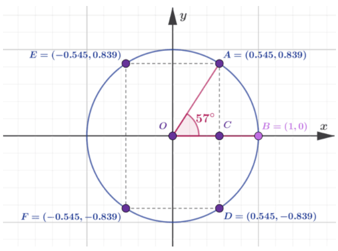
b) Draw an inscribed rectangle into the unit circle with one of the vertices A and sides parallel to the x - and y - axes.

c) Use the symmetry of the graph to label of the coordinates of the other vertices of the rectangle.

d) Use the symmetry of the graph to determine which angles are represented by the other vertices of the rectangle and label them with their coordinates using the cosine and the sine of the angle.

e) Complete the following table for the other three angles.

angle	cosine	sine
57°	0.545	0.839

	Steps	
a)		Mark point A lines to the x Mark the inte horizontal an with the circl Using symme each point.

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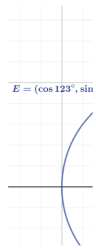
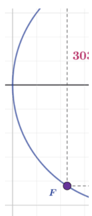
	Steps	
b)	$\mathrm{E}\left(\cos 123, \sin 123\right)$	Remember th $(1,\ 0)$ and ar 
c)	$\mathrm{F}\left(\cos 237, \sin 237\right)$	
d)	$\mathrm{D}\left(\cos 303, \sin 303\right)$	

e)

angle	cosine	sine
$57°$	0.545	0.839
$123°$	-0.545	0.839
$237°$	-0.545	-0.839
$303°$	0.545	-0.839

Using the coo

$\mathrm{E}\left(-0.545, 0\right.$

Thus

$\cos 123$

$\sin 123$

follow the sar vertices.

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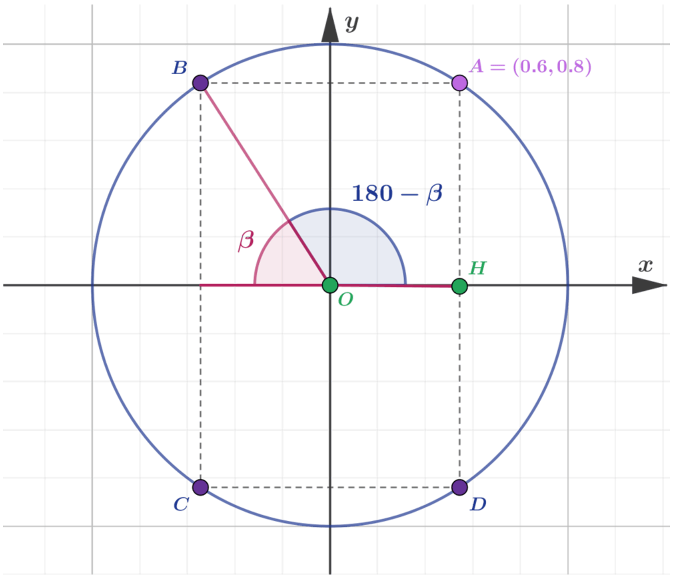
Trigonometric ratios of any angle

As you saw in **Example 2**, you can use the unit circle to find the sine and cosine of any angle. Using the points and symmetry of the unit circle, you can write the coordinates of other vertices.



More information

The image is a diagram of a unit circle drawn on a grid, which is centered at the origin 'O' of a coordinate system. The circle intersects the x and y axes. Different points on the circle include 'A' at (0.6, 0.8) and point 'B'. The diagram shows a right triangle inside the circle, with one vertex at the origin 'O', another at point 'H' on the x-axis, and the third at point 'B' on the circle. The angle ß (beta) is formed at the origin between the positive x-axis and the line OB, which represents a radius. The line extends from O to H on the x-axis. The area representing angle 180°-ß is also shown. The symmetry and trigonometric ratios are indicated by the points and the grid structure, displaying how these aspects interact to form different coordinates and angles.

[Generated by AI]

Using the symmetry and trigonometric ratios of the unit circle, in the diagram above, you can see that the coordinates of point B are

$$(\cos(180 - \beta), \sin(180 - \beta)) = (-\cos\beta, \sin\beta)$$

which also tells you that

$$\cos(180 - \beta) = -\cos\beta$$

and

$$\sin(180 - \beta) = \sin\beta$$

or, using the radian measure,

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$$\cos(\pi - \beta) = -\cos\beta$$

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and

$$\sin(\pi - \beta) = \sin \beta.$$

Example 3

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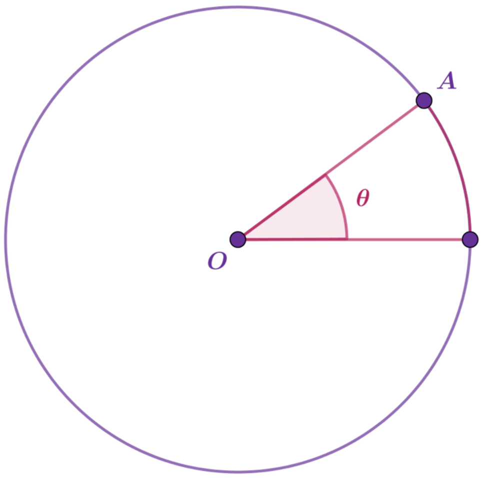

More information

The diagram illustrates a unit circle with a center labeled (O). A point (A) is marked on the circumference of the circle. The angle (\theta) is shown between the positive x-axis and the line segment (OA). The circle helps demonstrate trigonometric properties related to the angle (\theta).

[Generated by AI]

The diagram shows a point marked as A on a unit circle and the corresponding angle θ .

Copy the diagram and mark the points corresponding to the following angles.

a) $\pi - \theta$

b) $2\pi - \theta$

c) $\pi + \theta$

d) $\theta + 2\pi$

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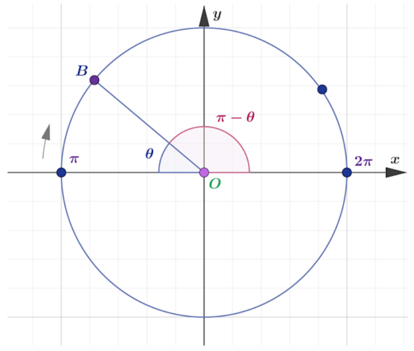
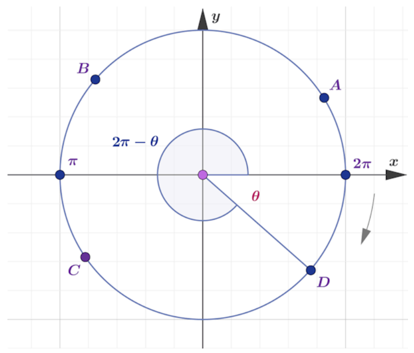
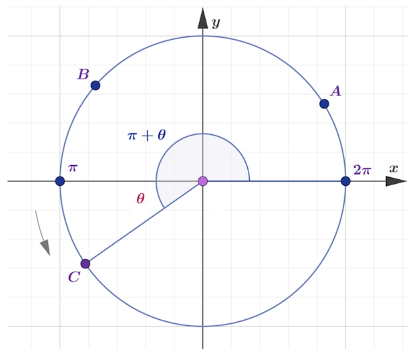
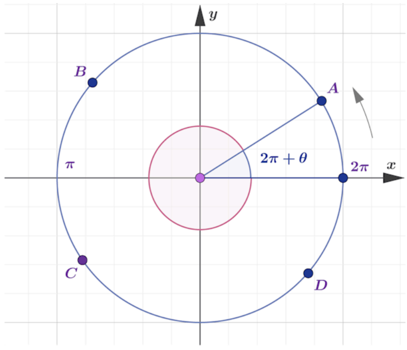
	Steps	Explanation
a)		$\pi - \ \theta$ is on the second quadrant so point B represents a turn of π minus the angles θ.
b)		$2\pi - \ \theta$ is on the fourth quadrant so point D represents a turn of 2 π minus the angle θ.
c)		$\pi + \ \theta$ is on the third quadrant so point C represents a turn of π plus the angle θ .

	Steps	Explanation
d)		$\theta + 2\pi = 2\pi + \ \theta$ is on the first quadrant so point Erepresents a full turn of 2πplus the angle θ . Note that θ and $2\pi + \ \theta$ could be represented by the same point.

Activity

Use the following applet to help you investigate the symmetry of the unit circle and four quadrants.

Interactive 1. Symmetry of the Unit Circle and Four Quadrants.

More information for interactive 1

This interactive tool helps users explore the unit circle with center O by moving point A (Red point) on the circle and observing its coordinates (cosθ, sinθ). When a rectangle ABCD is inscribed with sides parallel to the axes, the other vertices' coordinates can be systematically determined using symmetry principles. B, C and D points and their coordinates are given in purple color. Also the angle of the vector OA with x-axis is given by red color at the point O.

For example, with vertex A at $(0.6,\ 0.8)$:

- Vertex B is found by reflecting A across the y-axis, changing the x-coordinate sign: B$(-0.6,\ 0.8)$

- Vertex C is then derived by reflecting B across the x-axis, flipping the y-coordinate: C$(-0.6,\ 0.8)$

- Vertex D completes the rectangle by reflecting C back across the y-axis (or A across the x-axis): D$(-0.6,\ 0.8)$

This method demonstrates how symmetry simplifies coordinate geometry. By reflecting points across axes, users can quickly locate all vertices of axis-aligned shapes inscribed in the unit circle, reinforcing connections between algebraic coordinates and geometric

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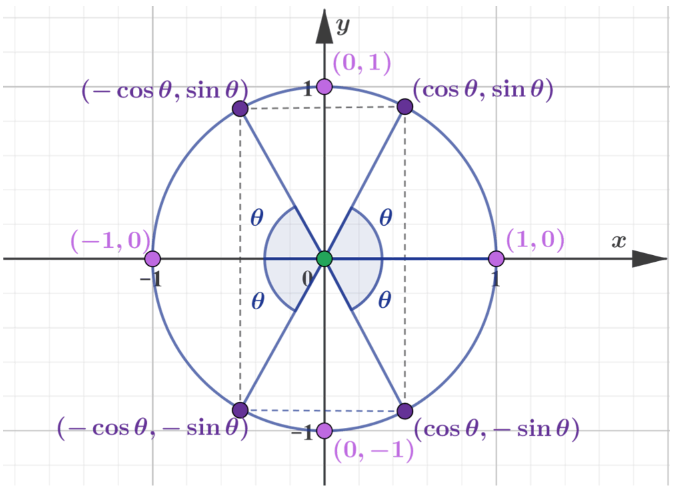
transformations. The tool visually verifies these calculations as points move, deepening understanding of trigonometric relationships and symmetry properties.

What do you notice about the coordinates of the rectangle? How could the rectangle be used to find the sine and cosine of angles in different quadrants?

Important

Points that lie on the same horizontal line have the same sine value for their angles.

Points that lie on the same vertical line have the same cosine for their angles.



More information

This image shows a unit circle on a coordinate grid. The circle is centered at the origin (0,0). Various points are marked around the circle: (−1,0), (0,1), (1,0), and (0,−1) along with specific labeled points: (cos θ, sin θ), (−cos θ, sin θ), (−cos θ, −sin θ), and (cos θ, −sin θ). The angles θ and −θ are marked within the circle, indicating positions of angles from the positive x-axis. The x-axis and y-axis are labeled, with arrows pointing in their positive directions. Intersections of dashed lines highlight square patterns and angles within the circle.

[Generated by AI]

Example 4

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If $\sin x = \dfrac{3}{5}$ find:

a) $\sin(\pi - x)$

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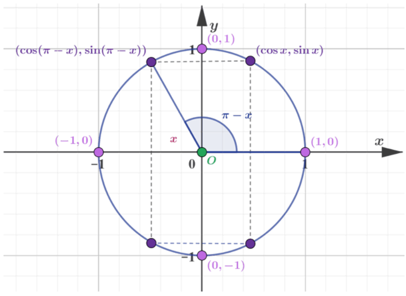
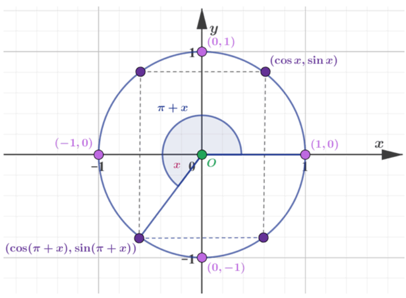
b) $\sin(\pi + x)$

c) $\sin(2\pi - x)$

	Steps	Explanation
a)		Mark angles x and $(\pi - x)$ on the unit circle.
	$\sin(\pi - x) = \sin x$ Thus $$\sin(\pi - x) = \frac{3}{5}$$	Both are on the same horizontal line. That means their sine values are equal. In the second quadrant, y values are positive and so is is the sine of the angle.
b)		Mark x and $\pi + x$ on the unit circle.
	$\sin(\pi + x) = -\sin x$ Thus $$\sin(\pi + x) = -\frac{3}{5}$$	$\sin(\pi + x)$ is in the third quadrant and it will be negative.

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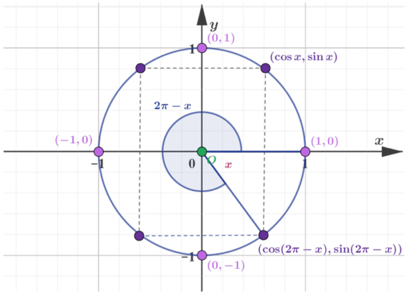
	Steps	Explanation
c)		Mark x and $2\pi - x$ on the unit circle. Note that the positive direction is anticlockwise and the negative direction is clockwise.
	$\sin(2\pi - x) = -\sin x$ Thus $$\sin(2\pi - x) = -\frac{3}{5}$$	

Important

The coordinates of any point on the unit circle are given by $(\cos\theta, \sin\theta)$, where θ is the angle measured from the x-axis in the anticlockwise direction. Hence, $\cos\theta$ is the value of the x-coordinate of the corresponding point on the unit circle, and $\sin\theta$ is the value of the y-coordinate of the corresponding point on the unit circle.

Important

The unit circle is divided into four quadrants:

1. **First quadrant** $0 < \theta < 90°$ or $0 < \theta < \frac{\pi}{2}$: Both $\sin\theta$ and $\cos\theta$ have positive values.
2. **Second quadrant** $90° < \theta < 180°$ or $\frac{\pi}{2} < \theta < \pi$: $\sin\theta$ is positive but $\cos\theta$ is negative.
3. **Third quadrant** $180° < \theta < 270°$ or $\pi < \theta < \frac{3\pi}{2}$: Both $\sin\theta$ and $\cos\theta$ have negative values.
4. **Fourth quadrant** $270° < \theta < 360°$ or $\frac{3\pi}{2} < \theta < 2\pi$: $\sin\theta$ is negative and $\cos\theta$ is positive.

Exam tip

In the IB examination, even in papers where calculators are allowed, if a question states 'exact value' it means you need to leave your answer in either surd form, as a fraction, or as a multiple of π . i.e. $\frac{1}{7}$, $\sqrt{3},\ 2\pi,\ \frac{\pi}{3}$.

Decimal answers found with a calculator are not always exact values.

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Example 5

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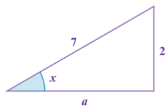
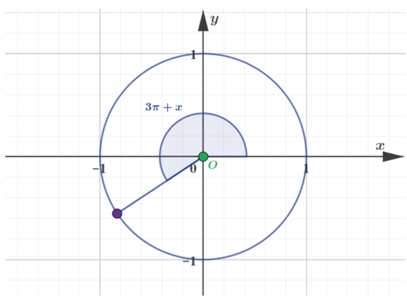
If $\sin x = \frac{2}{7}$ for an acute angle x , find the exact values of

a) $\cos x$

b) $\cos (3\pi + x)$.

	Steps	Explanation
a)		Draw and label the right-angled triangle.
	$a^2 + 2^2 = 7^2$	Using Pythagoras' theorem.
	$a = \sqrt{45} = 3\sqrt{5}$	Solve for a . Leave it in the surd form as exact answer is asked.
	$\cos x = \frac{3\sqrt{5}}{7}$	
b)	$\cos (3\pi + x) = -\cos x$ so $\cos (3\pi + x) = -\frac{3\sqrt{5}}{7}$	Use unit circle to mark the angle $3\pi + x$. This angle is bigger than 2π , so we've already completed one full cycle. 

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Example 6

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If $\cos\theta = a$, where $0 < \theta < \frac{\pi}{2}$, find $\cos(\pi - \theta)$ and $\cos(2\pi - \theta)$.

$\cos(\pi - \theta) = -\cos\theta = -a$ and

$\cos(2\pi - \theta) = \cos\theta = a.$

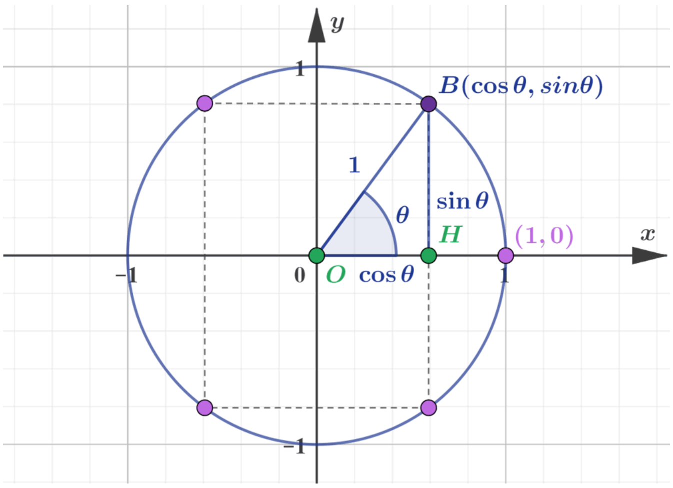
Tangent ratio using unit circle

In the diagram below,

$$\tan\theta = \frac{BH}{OH}$$

If you substitute the values of OH and BH in terms of cosine and sine of the angle θ respectively, you would get the new identity

$$\tan\theta = \frac{\sin\theta}{\cos\theta}.$$



More information

The image is a diagram of the unit circle on a Cartesian coordinate plane. The circle is centered at the origin (0,0) with a radius of 1. The positive x-axis has the point (1, 0) and intersects the circle at this point. The image shows an angle (\theta) originating from the positive x-axis, moving counter-clockwise. From the origin, a line is drawn to a point (B(\cos \theta, \sin \theta)) on the circumference.

A right-angled triangle is formed with the base on the x-axis and vertical side extending up to the point (H) on the y-axis, such that (OH) is perpendicular to the x-axis. The base of the triangle is labeled as (\cos \theta) and the vertical side is labeled as (\sin \theta). The hypotenuse of the triangle, which is the radius of the circle, has a length of 1. Text is shown in the image labeling the coordinates and functions.

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Exam tip

In the IB examination this identity for $\tan\theta$ will be in your formula booklet as

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Example 7

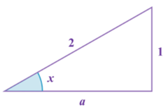
★★☆

If $\sin x = \frac{1}{2}$, $0 < x < \frac{\pi}{2}$, find the exact value of

a) $\tan\ x$

b) $\tan(\pi - x)$

c) $\tan(\pi + x)$

	Steps	Explanation
a)		Draw and label the right-angled triangle.
	$a^2 + 1^2 = 2^2$	Using Pythagoras' theorem.
	$a = \sqrt{3}$	Solve for a . Leave it in the surd form as an exact answer is asked for.
	$\tan\ x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	Using $\tan\ x = \frac{\text{opp}}{\text{adj}}$

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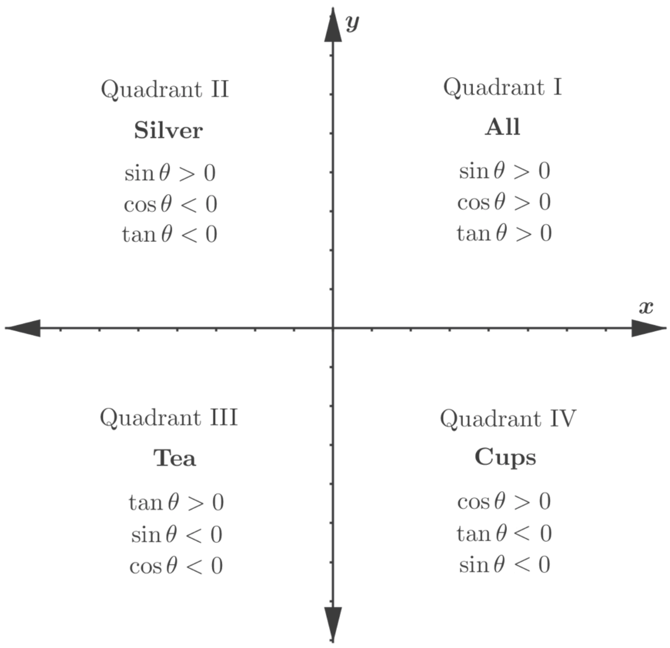
	Steps	Explanation
b)	$\tan\ (\pi - x) = -tan\ x$ therefore $$\tan\ (\pi - x) = -\frac{\sqrt{3}}{3}$$	$\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)}$ $(\pi - x\ )$ is in the 2nd quadrant as $$0 < x < \frac{\pi}{2}$$ So, $$\tan(\pi - x) = \frac{\sin(x)}{-\cos(x)} = -\frac{\sin(x)}{\cos(x)}$$
c)	$\tan\ (\pi + x) = \tan\ x$ therefore $$\tan\ (\pi + x) = \frac{\sqrt{3}}{3}$$	$$\tan(\pi + x) = \frac{\sin(\pi + x)}{\cos(\pi + x)}$$ $(\pi + x\ )$ is in the 3rd quadrant, so $$\tan(\pi + x) = \frac{-\sin(x)}{-\cos(x)} = \frac{\sin(x)}{\cos(x)}$$

Exam tip

Use the signs of x and y to determine the signs of the trigonometric ratios.

Or you could use ‘ **All S** ilver **T** ea **C** ups’ to remind you:

- Quadrant I, **all** trigonometric values are positive
- Quadrant II, **sine** is positive
- Quadrant III, **tangent** is positive
- Quadrant IV, **cosine** is positive.



More information

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This diagram illustrates the four quadrants of a Cartesian coordinate system with labeled trigonometric functions. The x-axis is the horizontal axis, and the y-axis is the vertical axis. Each of the four quadrants is detailed with a set of conditions for sine, cosine, and tangent.

- **Quadrant I (All):**
- sin θ > 0
- cos θ > 0
- tan θ > 0
- **Quadrant II (Silver):**
- sin θ > 0
- cos θ < 0
- tan θ < 0
- **Quadrant III (Tea):**
- tan θ > 0
- sin θ < 0
- cos θ < 0
- **Quadrant IV (Cups):**
- cos θ > 0
- tan θ ≷ 0
- sin θ ≷ 0

The diagram outlines the positivity or negativity of these trigonometric functions in each quadrant, providing a visual reference for understanding their mathematical properties in relation to the unit circle.

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5 section questions

Question 1

★☆☆

In which quadrant(s) is $\sin\theta$ positive?

1 I and II ✓

2 II

3 I

4 III and IV

Explanation

We may use the unit circle for this and remember that $\sin\theta$ is associated with the $y\text{-coordinate}$ of the corresponding point on the circumference, which remains positive in the first and second quadrants.

Student view

Question 2

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

★☆☆

In which quadrant(s) is $\tan\theta$ positive?

1 I and III ✓

2 I and II

3 III and IV

4 III

Explanation

We may use the unit circle for this and remember that $\tan\theta = \frac{\sin\theta}{\cos\theta}$, which is, thus, positive if $\sin\theta$ and $\cos\theta$ have the same sign. This is true in quadrants I (both positive) and III (both negative).

Question 3

★☆☆

In which quadrant of the unit circle is $\sin\theta$, $\cos\theta$ and $\tan\theta$ positive?

1 I ✓

2 II

3 III

4 IV

Explanation

We may use the unit circle for this and require that both the x- and y-coordinates are positive. This holds in the first quadrant only.

Question 4

★★☆

If $\cos x = \frac{2}{3}, 0 < x < \frac{\pi}{2}$

Which of the following is correct?

1 $\sin(\pi - x) = \frac{\sqrt{5}}{3}$ and $\tan(\pi - x) = -\frac{\sqrt{5}}{2}$ ✓

2 $\sin x = \frac{\sqrt{5}}{2}$ and $\tan x = \frac{3}{\sqrt{5}}$

3 $\sin(\pi - x) = \frac{2}{3}$ and $\tan(\pi - x) = \frac{\sqrt{5}}{3}$

4 $\sin x = \frac{\sqrt{5}}{3}$ and $\tan x = \frac{2}{\sqrt{5}}$

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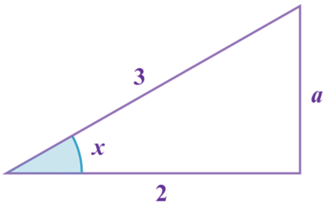
Explanation



More information

Draw and label the diagram.

Using Pythagoras $a^2 + 2^2 = 3^2$

$$a = \sqrt{5}$$

thus $\sin x = \dfrac{\sqrt{5}}{3}$ and $\tan x = \dfrac{\sqrt{5}}{2}$

$(\pi - x)$ is in the second quadrant

$\sin(\pi - x) = \sin(x) = \dfrac{\sqrt{5}}{3}$ and $\tan(\pi - x) = -\tan\ x = -\dfrac{\sqrt{5}}{2}$

so the correct answer is $\sin(\pi - x) = \dfrac{\sqrt{5}}{3}$ and $\tan(\pi - x) = -\dfrac{\sqrt{5}}{2}$

Question 5

★★☆

Evaluate $\sin(\pi - x) + \sin(2\pi - x)$

0

Accepted answers

0

Explanation

$\sin(\pi - x) = \sin x$ and $\sin(2\pi - x) = -\sin x$

Therefore

$$\sin(\pi - x) + \sin(2\pi - x) = \sin x - \sin x = 0$$

3. Geometry and trigonometry / 3.5 Trigonometric ratios beyond acute angles

Exact values

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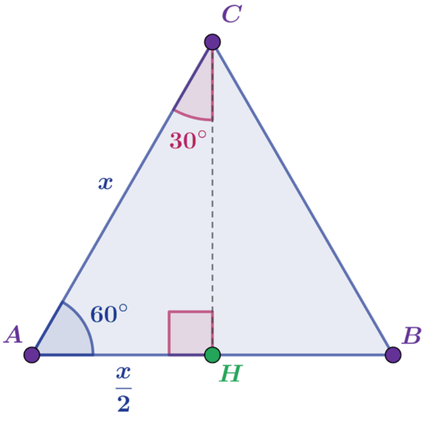
Special triangles

In the diagram below. equilateral triangle ABC has side length x. When you draw the height HC, you divide the triangle into two congruent right-angled triangles with interior angles $30°, 60°$ and $90°$.



More information

The image is a diagram of an equilateral triangle labeled ABC. The triangle has a side length labeled as x. A height line, HC, is drawn from point C to the midpoint H on side AB, dividing the triangle into two congruent right-angled triangles. The interior angles of these right-angled triangles are labeled as 30°, 60°, and 90°. The base of the right-angled triangle AHC is labeled as x/2. Points A, B, C, and H are marked with distinct colored circles.

[Generated by AI]

Using Pythagoras' theorem for triangle AHC,

$$CH^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$CH^2 = x^2 - \left(\frac{x}{2}\right)^2$$

$$CH^2 = \frac{3x^2}{4}$$

$$CH = \sqrt{\frac{3x^2}{4}} = \frac{x\sqrt{3}}{2}$$

$$CH = \frac{x\sqrt{3}}{2}$$

Now that you have all the side lengths of triangle AHC, you can write the trigonometric ratios for $60°$.

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$$\sin 60^\circ = \frac{CH}{AC} = \frac{\frac{x\sqrt{3}}{2}}{x} = \frac{\sqrt{3}}{2} \qquad \Rightarrow \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{AH}{AC} = \frac{\frac{x}{2}}{x} = \frac{1}{2} \qquad \Rightarrow \cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\text{CH}}{\text{AH}} = \frac{\frac{x\sqrt{3}}{2}}{\frac{x}{2}} = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \tan 60^\circ = \sqrt{3}$$

Following a similar approach, you can find the trigonometric ratios for 30°.

$$\sin 30^\circ = \frac{1}{2}$$

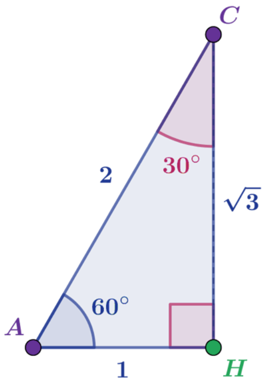
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Exam tip

In the IB examination, you will be required to know the exact values for the trigonometric ratios of the angles $\frac{\pi}{3} = 60^\circ$ and $\frac{\pi}{6} = 30^\circ$.

These ratios are not in the formula booklet, but remembering the special triangle $30^\circ, 60^\circ$ and 90° will help you to find the trigonometric ratios easily.



More information

The image is a diagram of a right triangle labeled as follows: Vertices are labeled A, C, and H. The hypotenuse AC has a length of 2. The side opposite the 30-degree angle (AH) has a length of 1, and the side opposite the 60-degree angle (CH) has a length of the square root of 3. The right angle is formed at vertex H. The triangle demonstrates a special 30-60-90 degree triangle with the side lengths in the ratio 1:√3:2. Furthermore, angle C is marked as 30 degrees, angle A as 60 degrees, and angle H is the right angle (90 degrees). The triangle is color-coded with different shades used for the angles and sides but these colors are not essential for understanding the mathematical properties of the diagram.

[Generated by AI]

Student
view

If you know these values, you can combine them to deduce other values.

Example 1

★☆☆

Find the exact values of

a) $\sin \frac{7\pi}{6}$

b) $\tan \frac{7\pi}{6}$

c) $\cos \frac{5\pi}{3}$

Steps	Explanation
$\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6} = -\frac{1}{2}$ so $\sin \frac{7\pi}{6} = -\frac{1}{2}$	$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$, it is in the 3rd quadrant where the sine of the angle will be negative.
$\tan \frac{7\pi}{6} = \tan \frac{\pi}{6}$ so $\tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$, it is in the 3rd quadrant where the tangent of the angle will be positive.
$\cos \frac{5\pi}{3} = \cos \frac{\pi}{3}$ so $\cos \frac{5\pi}{3} = \frac{1}{2}$	$\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$, it is in the 4th quadrant where the cosine of the angle will be positive.

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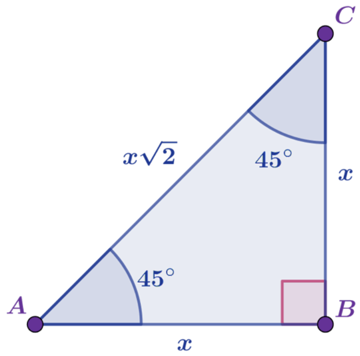


More information

The image depicts a right-angled isosceles triangle labeled ABC. The right angle is marked at B, with AB and BC labeled as 'x'. The hypotenuse AC is labeled as 'x√2'. The triangle has two 45° angles at A and C. Each vertex is marked by purple dots, and the labels are written in blue.

[Generated by AI]

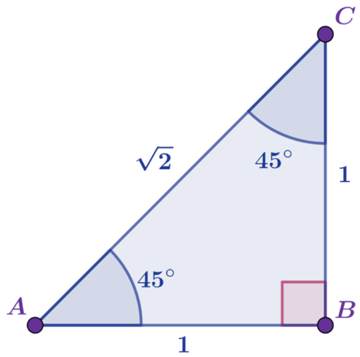
The second special triangle is the right-angled isosceles triangle.

Using the right-angled isosceles triangle ABC, you can find the trigonometric ratios of the angle 45°.

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1$$



More information

Student view

The image displays a right triangle labeled ABC with a right angle at point B. \n\n- **Sides**: The side AB measures 1 unit, BC measures 1 unit, and the hypotenuse AC measures (\sqrt{2}) units.\n\n- **Angles**: The angle at A and the angle at C are both labeled as (45^\circ). \n\n- **Special Markings**: There is a small square at point B indicating the right angle.

[Generated by AI]

Example 2

★★☆

What is $\cos\dfrac{5\pi}{4}$?

We note that $\dfrac{5\pi}{4}=\dfrac{\pi}{4}+\pi$ and so we are in the third quadrant where cos is negative. Using the unit circle, we go to $\dfrac{5\pi}{4}$ whose x value is the same as that of $\dfrac{\pi}{4}$ but with a negative sign, i.e. $-1\times\dfrac{\sqrt{2}}{2}$. Thus, $\cos\dfrac{5\pi}{4}=-\dfrac{\sqrt{2}}{2}$.

Example 3

★★☆

What is $\sin\dfrac{5\pi}{4}$?

Again we note that we are in the third quadrant. Using the unit circle, we go to $\dfrac{5\pi}{4}$ whose y value is the same as that of $\dfrac{\pi}{4}$ but negative, i.e. $-1\times\dfrac{\sqrt{2}}{2}$. Thus, $\sin\dfrac{5\pi}{4}=-\dfrac{\sqrt{2}}{2}$.

Other special angles

You have established that the coordinates of any point (x, y) on the unit circle can be written as $(\cos\theta, \sin\theta)$. You can use this to find the trigonometric ratios of the angles $0^\circ, 90^\circ, 180^\circ,\ 270^\circ$ and 360° .

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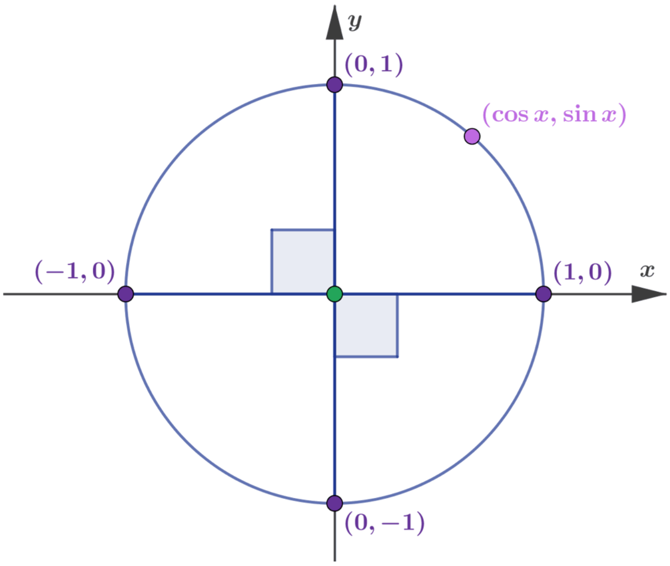

More information

The image is a diagram of the unit circle, which is placed on a coordinate system. The circle has a radius of 1 unit and is centered at the origin (0,0). There are labeled points where the circle intersects the x-axis and y-axis, specifically at (1,0), (-1,0), (0,1), and (0,-1). Additionally, a point is labeled as (cos x, sin x) on the circumference of the circle, representing a generic point on the unit circle.

The x-axis is labeled 'x', and the y-axis is labeled 'y'. The positive and negative axes indicate directions to the right, left, up, and down, respectively. At the center of the circle, the intersection of the axes is marked by a small green dot. Quadrants are visually separated by different shades of blue over certain segments, but these do not appear to correspond directly to quadrants based on trigonometric values or functions.

[Generated by AI]

If you look at the coordinates of points where the unit circle intersects the x - and y - axes:

$$(1,0):\ \text{angle } 0 \text{ radians} \Rightarrow \cos 0 = 1,\ \ \sin\ 0 = 0 \text{ and } \tan\ 0 = \frac{0}{1} = 0$$

$$(0,1):\ \text{angle } \frac{\pi}{2} \text{ radians} \Rightarrow \cos\frac{\pi}{2} = 0,\ \sin\frac{\pi}{2} = 1 \text{ and } \tan\frac{\pi}{2} = \frac{1}{0} \text{ is undefined}$$

$$(-1,0):\text{angle } \pi \text{ radians} \Rightarrow \cos\pi = -1,\ \sin\pi = 0 \text{ and } \tan\ \pi = \frac{0}{1} = 0$$

$$(0,-1):\text{angle } \frac{3\pi}{2} \text{ radians} \Rightarrow \cos\frac{3\pi}{2} = 0,\ \ \sin\frac{3\pi}{2} = -1 \text{ and } \tan\frac{3\pi}{2} = \frac{1}{0} \text{ is undefined}$$

$$(1,0):\ \text{angle } 2\pi \text{ radians} \Rightarrow \cos 2\pi = 1,\ \sin 2\pi = 0 \text{ a and } \tan\ 2\pi = \frac{0}{1} = 0$$

✓ **Important**

In the IB examination, you will be required to know the exact values for the trigonometric ratios of the angles where the unit circle intersects the x -and y - axes.

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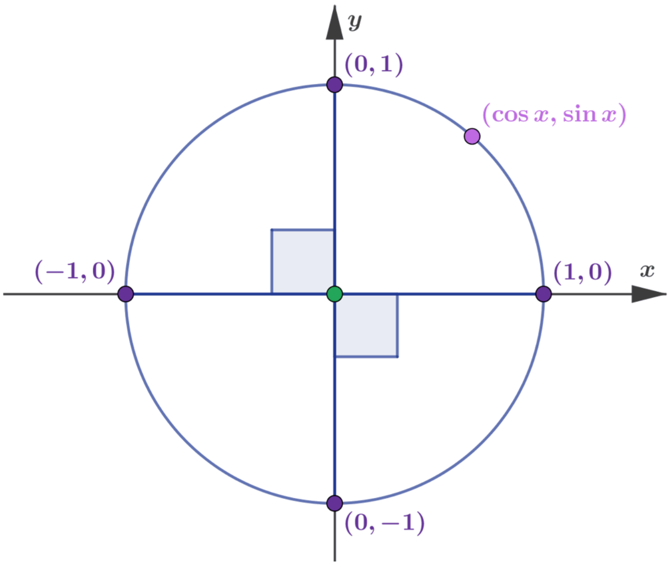
These ratios are not in the formula booklet, but remembering the unit circle will help you to find the trigonometric ratios easily.



More information

This is a diagram of the unit circle, centered at the origin of an xy-coordinate plane. The circle has a radius of 1 unit.

Key features: - The x-axis is labeled with points (1,0) and (-1,0) where the circle intersects the axis. - The y-axis is labeled with points (0,1) and (0,-1) where the circle intersects the axis. - A point on the circle is indicated by (cos x, sin x), which represents the cosine and sine of an angle x, measured from the positive x-axis. - There are small square segments inside the circle somewhere near the center, indicating specific areas or angles. The diagram illustrates the relationship between the circle, standard angle positions, and trigonometric functions.

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Activity

Draw a circle on graph paper with centre $(0,0)$ and radius 5 cm.

Using a protractor, mark points at 60° and 210° on the circumference and draw a right-angled triangle joining these points to the centre.

Calculate the values of sine, cosine and tangent using the side lengths of the triangle.

Is there a difference between your calculated values and the exact values you have learned? If yes, why?

Making connections

Forensic analysis and trigonometry

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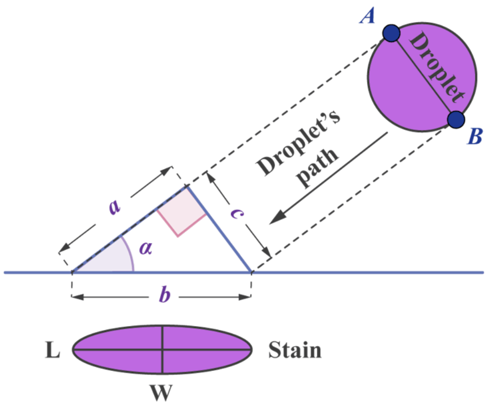
Criminal experts rely on analysis of the evidence left behind in crime scenes. Blood pattern analysts examine blood evidence left behind at the crime scene and they can recreate the sequence of events using the blood spatters. They use aspects of biology, physics, and mathematics.

The angle of impact, α , can therefore be found using the ratio between the opposite side and the hypotenuse side of the triangle, which is equivalent to $\sin \alpha$.

Experts can identify the height of the subject, angle of impact and also the trajectory of the blood droplet by using trigonometry, vectors and the projectile motion principles of physics.



More information

The diagram illustrates the trajectory of a blood droplet and the resulting stain shape. At the top right, a circular droplet labeled 'Droplet' is marked with two points A and B. A dashed line extends from the droplet, titled "Droplet's path," forming a right triangle with the base line, which represents the ground. The triangle's legs are labeled 'a' and 'b', and the hypotenuse is 'c'. The angle formed between the base and the hypotenuse is marked as 'α'. Below the triangle, a purple ellipse labeled 'Stain' is shown with its length (L) and width (W) marked, demonstrating how the droplet impacts surfaces and forms a stain. The components of the diagram demonstrate the application of trigonometry, vectors, and projectile motion in forensic analysis of blood spatter.

[Generated by AI]

If you would like to read more about how mathematics is used in forensic science click here (https://science.howstuffworks.com/bloodstain-pattern-analysis3.htm).

3 section questions

Question 1

★★☆

Which of the following is equal to $\sin \frac{7\pi}{8}$?

1 $\sin \frac{17\pi}{8}$ ✓

2 $\sin \frac{9\pi}{8}$

3 $\sin \left(-\frac{\pi}{8}\right)$

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4 $\sin\frac{15\pi}{8}$

Explanation

$\frac{7\pi}{8} = \pi - \frac{\pi}{8}$, so it is in the 2nd quadrant.

$\sin\frac{7\pi}{8} = \sin\frac{\pi}{8}$

since $\frac{17\pi}{8} = 2\,\pi + \frac{\pi}{8}$, it is in the 1st quadrant.

$\sin\frac{17\pi}{8} = \sin\frac{\pi}{8}$

Question 2

★★☆

Find the exact value of $1 - \sin^2\frac{\pi}{3}$ as a decimal.

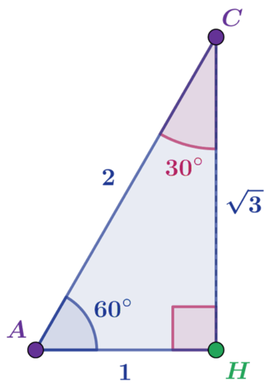
0.25

Accepted answers

0.25, 0,25, .25, 1/4

Explanation

Using the special triangle



More information

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$1 - \sin^2\frac{\pi}{3} = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = 1 - \frac{3}{4} = \frac{1}{4}$$

So

$$1 - \sin^2\frac{\pi}{3} = 0.25$$

Student
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Question 3

★★☆

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Which of the following is equal to the exact value of $\tan 210°$?

1 $\frac{1}{\sqrt{3}}$ ✓

2 $-\frac{1}{\sqrt{3}}$

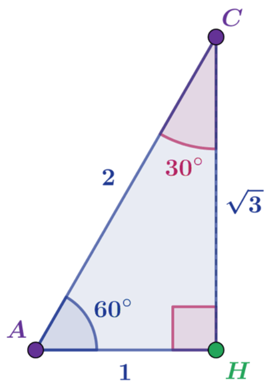
3 $\sqrt{3}$

4 $-\sqrt{3}$

Explanation

$\tan 210° = \tan(180° + 30°) = \tan 30°$

Using the special triangle



More information

$$\tan 30° = \frac{1}{\sqrt{3}}$$

Therefore,

$$\tan 210° = \frac{1}{\sqrt{3}}$$

3. Geometry and trigonometry / 3.5 Trigonometric ratios beyond acute angles

Extension of the sine rule

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/extension-of-the-sine-rule-id-27744/print/) Assign

In section 3.2.4 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-sine-rule-id-25421/), you studied the sine rule. Consider triangle ABC with $AC = 12$, $CB = 7$ and $\hat{A} = 30°$. As two sides and an angle are given, you could use the sine rule to solve this triangle. But is there a unique triangle with these measurements? Is angle B acute or obtuse? Both would give you the same sine value, as you learned in previous section.

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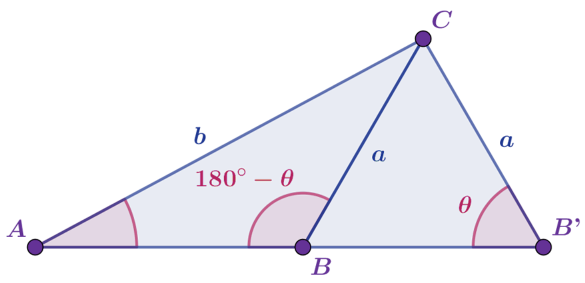
This is called the ambiguous case of the sine rule, as there is not a unique triangle with the given measurements.

Ambiguous case

Consider two given segments $[\mathrm{AC}]$, $[\mathrm{BC}]$ and a given angle $\angle\mathrm{BAC}$. As we can see in the diagram below, in certain cases there are two different triangles $\triangle\mathrm{ABC}$ and $\triangle\mathrm{AB'C}$ that can be constructed with the above two segments and the angle.



More information

The image depicts the ambiguous case of the sine rule using two triangles, (\triangle ABC) and (\triangle AB'C), which can be formed with given side lengths (a) and (b), and an angle (\angle A). The triangle (\triangle ABB') is shown as isosceles, demonstrating that two different angles, (B) and (B'), result in two valid triangles depending on the position of point (B) or (B'). The diagram includes labels for the side lengths and angles (\theta) and (180^\circ - \theta), highlighting the duality of solutions possible in this case.

[Generated by AI]

Given the sides a, b and the angle A, we may at times be able to build two triangles with different angles B and B'. We note that $\triangle\mathrm{BCB'}$ is isosceles. Thus, regardless of whether we choose the side a from $\triangle\mathrm{ABC}$ or $\triangle\mathrm{AB'C}$, the value for $\frac{a}{\sin A}$ is the same. This implies that the following ratios are also the same

$$\frac{b}{\sin B} = \frac{b}{\sin B'}.$$

However, be careful with generalising too quickly! It is not because $\frac{b}{\sin B} = \frac{b}{\sin B'}$ that $B = B'$. Instead, we have the relation $B = 180^\circ - B'$; hence one angle is acute, the other is obtuse. So, given values for a, b and A, which angle is correct, B or B'? Both are correct, and so you must give both as the solution. Depending on the triangle $\triangle\mathrm{ABC}$ or $\triangle\mathrm{AB'C}$ the angle C is also different, as can clearly be seen in the diagram above . This is tricky and not obvious straightaway; however, a must be smaller than b for the ambiguous case. Hence, always question whether you have the ambiguous case.

Let us see an example based on the triangle shown in the diagram below .

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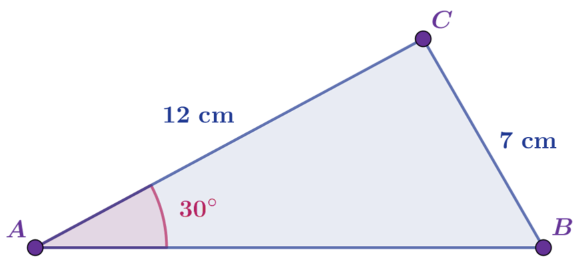

More information

The image is a diagram of a triangle labeled ABC. The side AB is labeled as 7 cm, the side AC is labeled as 12 cm, and the angle at A is labeled as 30 degrees. Vertex A is at the left, vertex B is at the bottom right, and vertex C is at the top right. This diagram is used to illustrate a potential ambiguous case in the sine rule when two sides and one angle are given.

[Generated by AI]

Example 1

★★☆

In the triangle shown above, find all possible values for angles B and C and the length of the side AB .

Steps	Explanation
$\frac{\sin B}{12}=\frac{\sin 30}{7}$ $\sin B=\frac{\frac{1}{2}}{7}\times 12$ $B=\sin^{-1}\frac{6}{7}$	Solving to find angle B , we use the sine rule.
$B\ =59.0^{\circ}$	Round your answer to 3 significant figures.
$C=180-(30+59.0)=91.0^{\circ}$	Now that we know angle B, we also know angle C.
$\frac{\sin B}{12}=\frac{\sin C}{\text{AB}}$ $\frac{\sin 59}{12}=\frac{\sin 91}{\text{AB}}$ $AB=\frac{\sin 91}{\sin 59}\times 12$	Thus, we can find the one remaining side using the sine rule.

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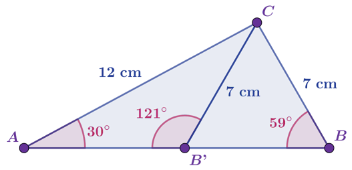
Steps	Explanation
$AB = \ 14.0\,\text{cm}.$	Round to 3 significant figures.
 	However, we are in an ambiguous situation. Since $\sin 59° = \sin 121°$, we can build two triangles given the information In the diagram. The second triangle has angle $B = 121°$, which is obtuse. This makes angle C also different, since $$\hat{\text{C}} = 180° - (30° + 121°) = 29°$$ Consequently, side $AB = \ 6.79\,\text{cm}.$

Be aware

When using the sine rule, always check if the information given results in an ambiguous case.

International Mindedness

The first mention of the sine function is found in the works of an Indian mathematician and astronomer Aryabhatiya $(476–550\text{ AD})$ of Aryabhata. He collected and expanded upon the developments of the Siddhantas (which first defined the sine as the modern relationship between half an angle and half a chord). These works contain the earliest surviving tables of sine values and versine $(1-\text{cosine})$ values, in $3.75°$ intervals from $0°$ to $90°$, to an accuracy of 4 decimal places.

Using trigonometric tables, Aryabhatiya was able to calculate a value for the size of the Earth that was only 0.2 percent smaller than the Earth's actual size. With modern technology, we know that Earth's average circumference size is $40{,}075\text{ km}$; Aryabhatiya calculates it as $39{,}968\text{ km}$.

4 section questions

Question 1

Difficulty:

★★☆

In triangle PQR, PQ is 5 units long and PR is 8 units long. If angle Q is $85°$. Find the size of angle R to 3 significant figures.

1 $= 38.5°$ ✓

2 $= 52.6°$

3 $= 1.59°$

4 $= 87.2°$

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Explanation

Sine rule:

$$\frac{\sin(85^\circ)}{8} = \frac{\sin(R)}{5}.$$

Now, using cross multiplication, we get

$$5\sin(85^\circ) = 8\sin(R)$$

$$\frac{5\sin(85^\circ)}{8} = \sin(R)$$
$$R = \arcsin\left(\frac{5}{8}\sin(85^\circ)\right) \approx 38.5^\circ.$$

When working with sine rule, we often have an ambiguous case where we have to consider the second possible solution, as $\sin x = \sin(180^\circ - x)$. However, in this case, this is not true. Note that $180^\circ - 38.5^\circ = 141.5^\circ$, but we also know that angle Q is 85°. The two angles would then add up to $141.5^\circ + 85^\circ > 180^\circ$, which is clearly not possible for a triangle! So in this case we have only one solution.

Question 2

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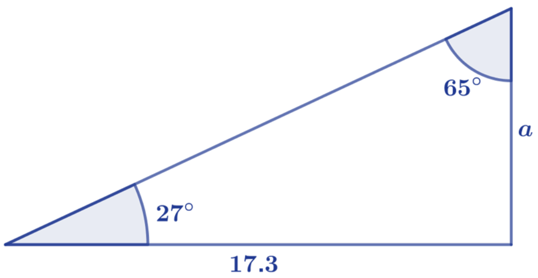
Find the length of side a given the figure below.

Give your answer rounded to 3 significant figures.



More information

8.67

Accepted answers

8.67, 8,67

Explanation

We use the sine rule:

$$\begin{aligned} & \frac{a}{\sin 27^\circ} = \frac{17.3}{\sin 65^\circ} \\ \Leftrightarrow \quad & a = \frac{17.3}{\sin 65^\circ} \times \sin 27^\circ \\ \Leftrightarrow \quad & a = 8.67. \qquad \text{[3 significant figures]} \end{aligned}$$

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Question 3

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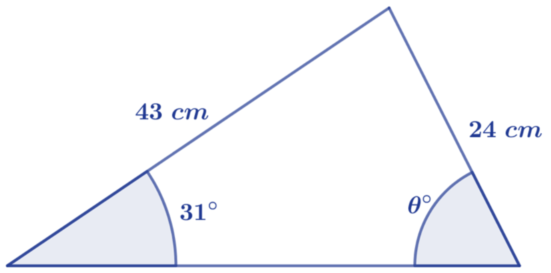
Find the angle θ given the figure below. If there is more than one solution, please give your answer in the format θ_1,θ_2 with no spaces or degree symbols. Give your answer to 3 significant figures.

The diagram below is not to scale.



More information

67.3,113

Accepted answers

67.3,113, 113,67.3

Explanation

We use the sine rule.

$$\begin{aligned} & \frac{\sin\theta}{43} = \frac{\sin 31°}{24} \\ \Leftrightarrow \quad & \sin\theta = \frac{\sin 31°}{24} \times 43 \\ \Leftrightarrow \quad & \theta = \sin^{-1}\left(\frac{\sin 31°}{24} \times 43\right) \\ \Leftrightarrow \quad & \theta = 67.3° \qquad [3 \text{ significant figures}] \end{aligned}$$

However, we are faced with the ambiguous case, since $\sin 67.3° = \sin(180° - 67.3°)$. Hence, the two solutions are $67.3°, 113°$ (to 3 significant figures).

Question 4

★★☆

In triangle $\triangle\mathrm{ABC}$, it is given that $a = 18$, $b = 27$ and $A = 30°$. To 1 decimal place, what is B?

1 $48.6°, 131.4°$

2 $46.4°, 133.6°$

3 $46.4°$

4 $48.6°$

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Explanation

We are given two sides of the triangle (a, b) and an angle that is not formed by these two sides (A), thus we are in the ambiguous case.

From the sine law we have:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 30^\circ}{18} = \frac{\sin B}{27}$$
$$\frac{\frac{1}{2}}{18} = \frac{\sin B}{27}$$
$$\sin B = \frac{27}{36}$$
$$\sin B = \frac{3}{4}$$

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Thus,

$$B = \arcsin\left(\frac{3}{4}\right) \approx 48.6^\circ$$

and as we are in the ambiguous case of the sine law, we also have

$$B \approx 180^\circ - 48.6^\circ = 131.4^\circ.$$

3. Geometry and trigonometry / 3.5 Trigonometric ratios beyond acute angles

Checklist

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What you should know

By the end of this subtopic you should be able to:

- calculate with trigonometric ratios in the unit circle
- recognise and use exact values of trigonometric ratios of special angles
- recognise the ambiguous case of the sine rule.

3. Geometry and trigonometry / 3.5 Trigonometric ratios beyond acute angles

Investigation

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Draw a unit circle on a piece of graph paper using the scale 5 cm for 1 unit.

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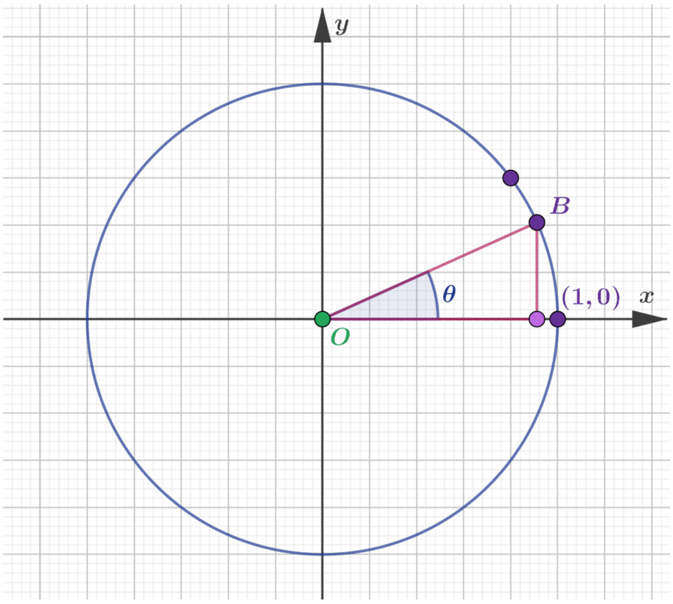

More information

The image shows a unit circle drawn on a piece of graph paper, using the scale of 5 cm for 1 unit. The circle is centered at the origin of a coordinate system plotted on the graph paper. A right triangle is inscribed inside the circle with one angle marked as θ. The triangle has its right angle on the x-axis, reaching up to a point (x, y) on the circle's circumference. The hypotenuse stretches from the origin to this point on the circle. The grid lines on the graph paper provide a reference scale for measurement, enabling the visualization of the unit circle's radius and coordinates of the point on the circumference.

[Generated by AI]

Mark a point B on the unit circle. Then:

- connect the point B to the centre and draw the right-angled triangle
- measure the angle θ using a protractor
- measure the lengths of the right sides of the triangle
- calculate the sine of the angle θ .

On a second piece of graph paper, plot the point $(\theta,\ \sin\theta)$.

Repeat this for different points on the circumference of the circle.

What do you notice? Do you recognise the shape of the graph?

Now calculate the cosine values and plot the points $(\theta,\ \cos\theta)$ in a different colour.

What are the similarities and differences between the two graphs? When do the two graphs intersect?

Rate subtopic 3.5 Trigonometric ratios beyond acute angles

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