3.12 Vectors

Checklist

What you should know

By the end of this subtopic you should be able to:

- recall that vectors have a size (magnitude) and a direction
- represent a displacement between two points as a vector
- decompose a 2D vector into its components in the x- and y-directions and a 3D vector into its components in the x-, y- and z-directions
- write a vector in column form, e.g. $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ or base vector form $\boldsymbol{v} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$ where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit base vectors in the x-, y- and z-directions, respectively: $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
- recall that two vectors are equal if and only if all their components are equal
- add vectors by adding their components: e.g. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\boldsymbol{v} + \boldsymbol{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_3 + w_3 \end{pmatrix}$
- recall that addition of vectors is commutative: $\boldsymbol{v} + \boldsymbol{w} = \boldsymbol{w} + \boldsymbol{v}$
- recall that if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, then $-\boldsymbol{v} = -\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -v_1 \\ -v_2 \\ -v_3 \end{pmatrix}$
- recall that $\boldsymbol{v} + (-\boldsymbol{v}) = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector
- know how to subtract one vector from another by the adding the negative components of the vector being subtracted: i.e. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w}) = \begin{pmatrix} v_1 - w_1 \\ v_2 - w_2 \\ v_3 - w_3 \end{pmatrix}$
- recall that subtraction of vectors is not commutative: i.e. $\boldsymbol{v} - \boldsymbol{w} \neq \boldsymbol{w} - \boldsymbol{v}$

- be able to multiply a vector by a scalar, for example if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is multiplied by scalar $k \in \mathbb{R}$ the result is $k\boldsymbol{v} = \begin{pmatrix} kv_1 \\ kv_2 \\ kv_3 \end{pmatrix}$
- recall that $k\boldsymbol{v}$ is a vector in the same direction as $\boldsymbol{v}$ but with a different magnitude (unless $k = \pm 1$)
- recall that two vectors are parallel if they are scalar multiples of the same vector: i.e. $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel if there exist m, n and $\boldsymbol{u}$ such that $\boldsymbol{v} = m\boldsymbol{u}$ and $\boldsymbol{w} = n\boldsymbol{u}$
- find the magnitude of a vector using Pythagoras' theorem: if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, the magnitude of $\boldsymbol{v}$ is denoted by $|\boldsymbol{v}|$ and is given by $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- know the properties of the magnitude of a vector:
  - $|\boldsymbol{v}| \geqslant 0$, i.e. it is never negative
  - $|\boldsymbol{v}| = |-\boldsymbol{v}|$
  - in general, $|\boldsymbol{v} + \boldsymbol{w}| \neq |\boldsymbol{v}| + |\boldsymbol{w}|$
- Recall that a unit vector in a specified direction is denoted by $\widehat{\boldsymbol{v}}$
- Calculate a unit vector in the direction of a given vector using $\widehat{\boldsymbol{v}} = \frac{1}{|\boldsymbol{v}|}\boldsymbol{v}$
- write the position vector of point A relative to a fixed origin O: if point A has coordinates (x, y, z) then the position vector of A is denoted by $\overrightarrow{\mathrm{OA}} = \boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$
- recall that the displacement vector between two points A and B is given in terms of their position vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$ and $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ by $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = -\boldsymbol{a} + \boldsymbol{b}$
- be able to calculate displacement vectors between two points: for example if A has coordinates $(x_A, y_{A,} z_A)$ and B has coordinates $(x_B, y_{B,} z_B)$ then $\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix}$

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