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TOPIC 5
CALCULUS

SUBTOPIC 5.4
TANGENTS AND NORMALS

(https://intercom.help/kognity)

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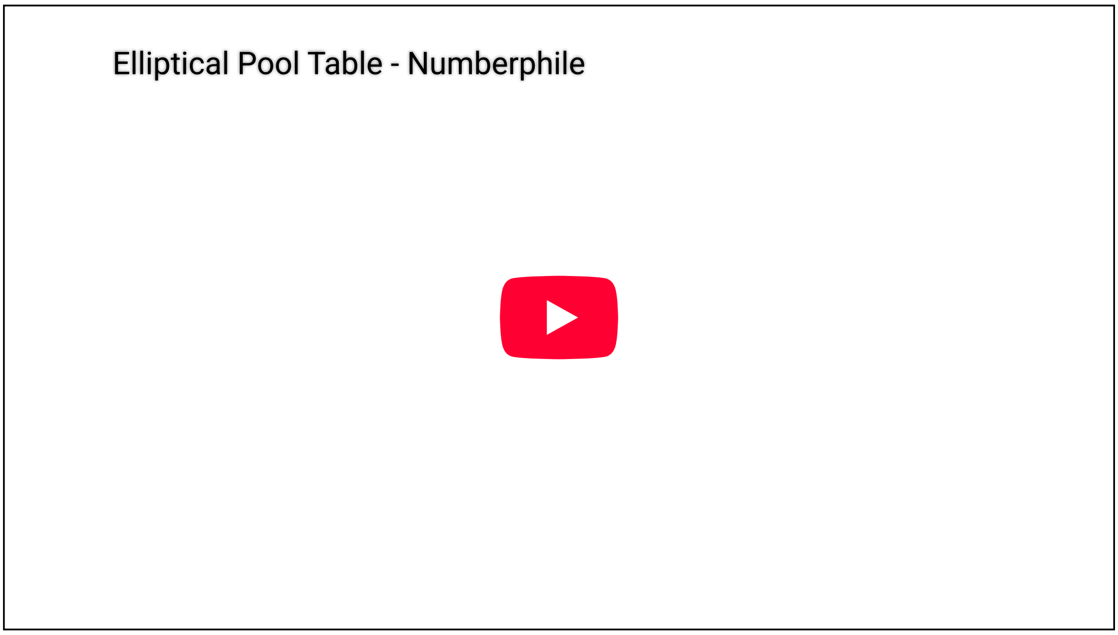
The big picture

In this subtopic, you will learn how to find the equation of the tangent to a given curve at a given point. You will also learn about a new concept; the normal to a curve.

To start, take a look at the following video.



Theory of Knowledge

As demonstrated in the video, mathematics can make predictions that are theoretically correct but turn out to not hold true in the 'real world'. For example, that the ball will go in the hole if shot in any direction from the appropriate focus spot. The mathematician in the video explains this is due to confounding factors, such as friction and human error. This relates to the knowledge question, 'Should the value of knowledge be assessed based on its application? If so, what is the 'most valuable' knowledge?'

Activity

The applet below shows an elliptical pool table. You can change the shape, the position of the target, the position of the white ball and the aim of the cue.

- Can you find the correct direction so as to hit the target without the help lines?
- On the table in the video, the target is at the focus of the ellipse. On the applet you can move the target away from the focus.
- Can you hit the target using different directions? Does the number of good directions depend on the position of the white ball and the target?

Interactive 1. Tangents and Normals Exploration.

More information for interactive 1

This interactive game uses an elliptical pool table to explore the concepts of tangents and normals in an engaging and dynamic way. Users can modify the shape and size of the table by dragging the red points, enabled by the Adjust Table, checkbox. The Adjust Target option allows repositioning of the target ball within the ellipse, while the Move Ball option enables users to place the white ball at any desired starting point. The Adjust Direction control lets users set the cue direction to aim the white ball.

In this interactivity some more checkboxes are provided: Show Focus reveals the foci of the ellipse, illustrating their geometric relationship. Show Path displays the trajectory of the white ball after being struck. Show Tangent and Show Normal draw the tangent and normal lines at the point of impact, helping users visualize their relationship to the curve's slope.

The interactive challenges users to find the correct shot direction without relying on guidelines, encouraging them to apply their understanding of tangents and normals. By experimenting with different ball positions and angles, users can observe how the number of possible shot directions changes. This hands-on activity provides a practical and visual approach to understanding geometric principles through an elliptical pool table.

Concept

While learning about tangents and normals, think about why the tangent is sometimes used as an **approximation** to a curve. In applications (for example, describing motion along a curve in physics) it is sometimes helpful to think of the tangent and normal lines as axes of a coordinate **system**. Can you think of a reason for this?

The tangent

In this section, you will learn how to find the equation of a tangent to a given curve.

Important

Recall that, in section 5.1.3 (/study/app/m/sid-122-cid-754029/book/gradient-function-id-26273/) , you saw that the gradient of the tangent to the graph of $y = f(x)$ at the point $(a, f(a))$ is $m = f'(a)$.

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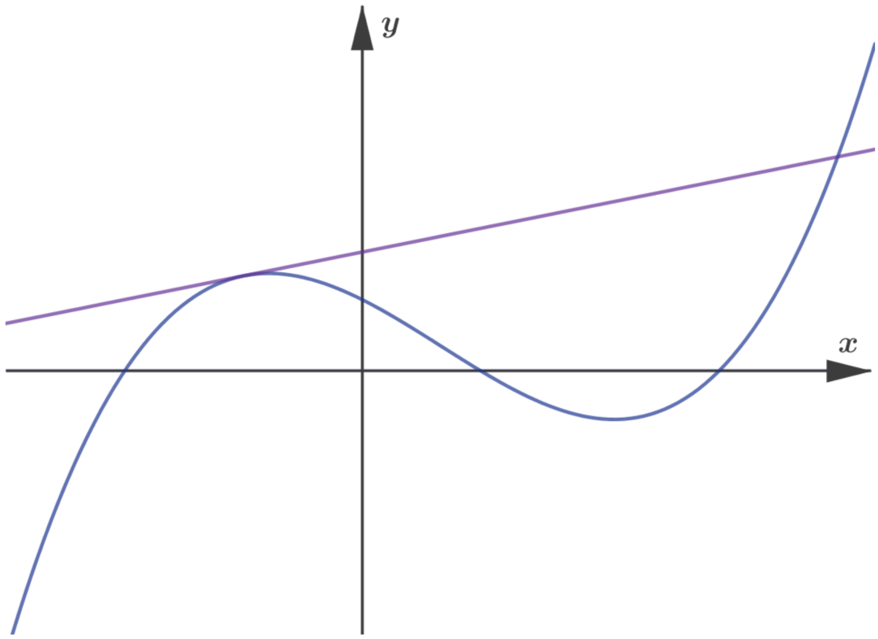
Activity

There are different ways in which information can be given about a tangent to a curve. The diagram below shows a curve and a tangent. There is no numerical information given on the diagram, not even the scale. Think about what information you would need to be able to find the equation of the tangent.



More information

The image is a diagram depicting a mathematical curve and a tangent line. The diagram includes an X-axis, labeled as 'x', and a Y-axis, labeled as 'y', forming a coordinate system. The curve (depicted in blue) intersects the X-axis at different points and passes through both positive and negative Y-values, indicating a non-linear function. The tangent line (depicted in purple) touches the curve at a single point, showing it is tangent to the curve at that point. No numerical values or scales are provided on the axes or the curve, and the purpose seems to demonstrate the geometric relationship between the curve and the tangent line.

[Generated by AI]

In the first two examples, a point is given on a graph and you are asked to find the equation of the tangent at that point.

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Example 1

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- Verify that the point $(2, 1)$ is on the graph of $y = \log(6 + x^2)$.
- Use your calculator to find the equation of the tangent to this graph at this point.

Steps	Explanation
$\log(6 + 2^2) = \log 10 = 1$	Substitute $x = 2$ and $y = 1$ in the equation of the gra and check that the equality is true.
$m = 0.1737\ldots$	Graphic display calculators have applications that car find the gradient of a curve at a given point. Do not rc the value at this point.
$y = mx + c$ $1 = 0.1737\cdots \times 2 + c$ $1 = 0.3474\cdots + c$ $c = 0.653$	Look for the equation in the form $y = mx + c$. Use th coordinates of the point and the gradient to find the v of c. You can now round to 3 significant figures.
The equation of the tangent (using rounded coefficients) is $y = 0.174x + 0.653$	Graphic display calculators also have applications tha can draw the tangent. Some models can even tell you equation of the tangent. Using these models, the calculation above is not necessary.

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26288/print/) Assign

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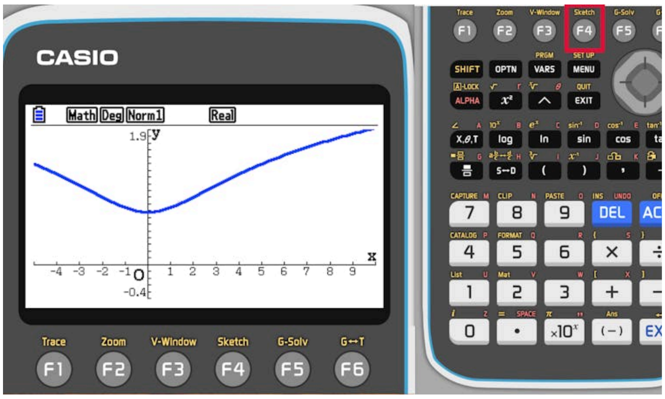
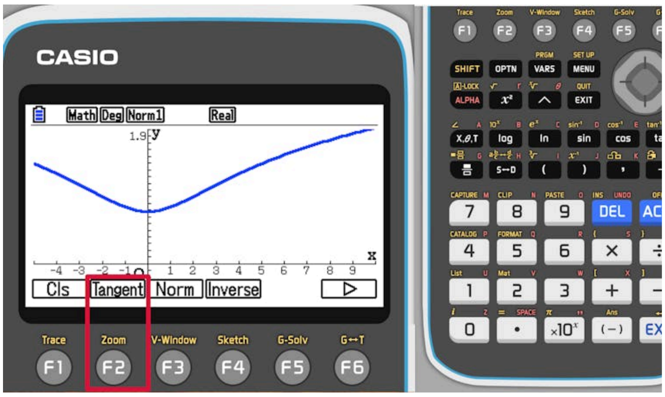
Steps	Explanation
These instructions assume that you have the graph of $y = \log(6 + x^2)$ on the screen in the viewing window $-5 \leq x \leq 10$ and $-0.5 \leq y \leq 2$. You will find guidance on how to find the equation of the tangent line to the graph at the point $(2, 1)$. Press F4 to choose the sketch option.	
Press F2 to draw a tangent line.	

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-tangent-id-26289/print/) Assign

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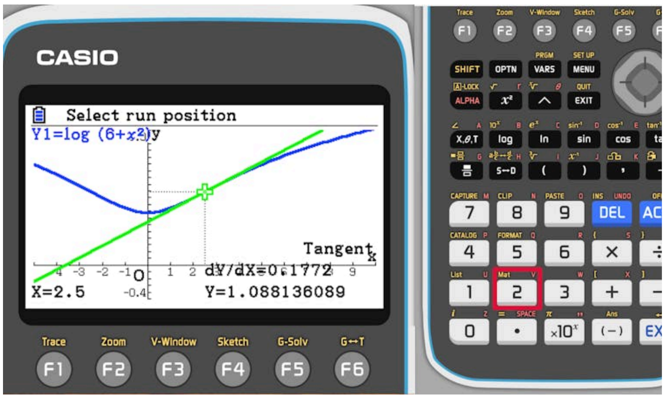
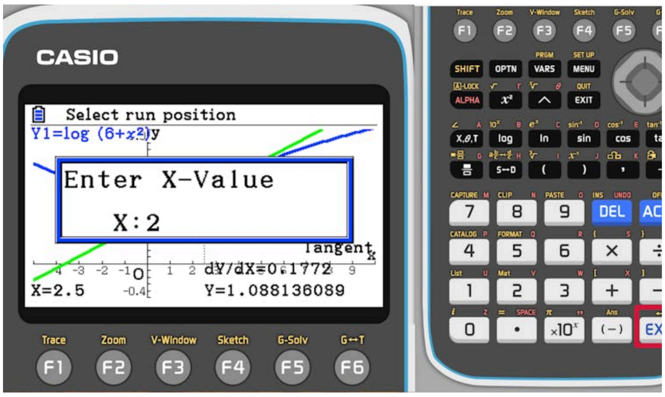
Steps	Explanation
A tangent line is drawn. To move it to the place corresponding to $x = 2$, press the number 2.	
Press EXE to confirm the x-value.	

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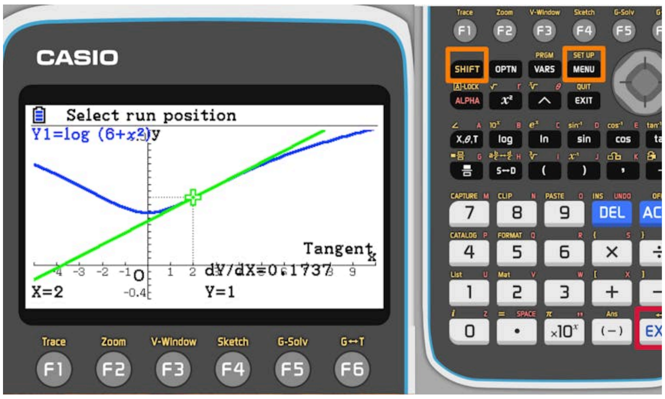
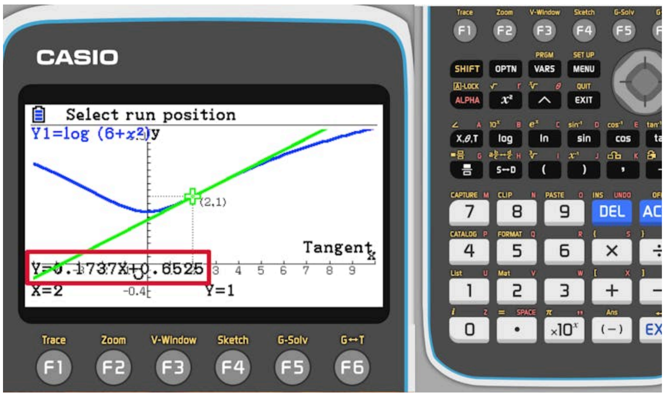
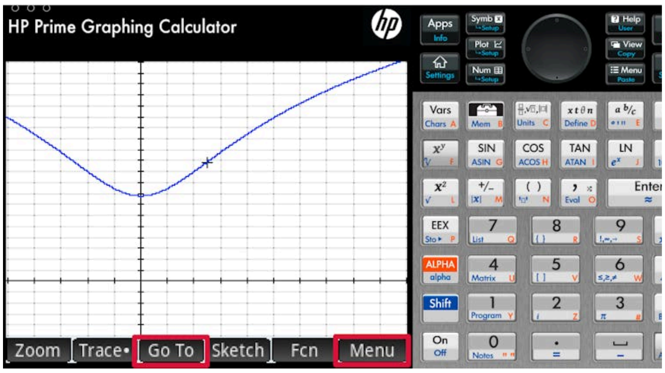
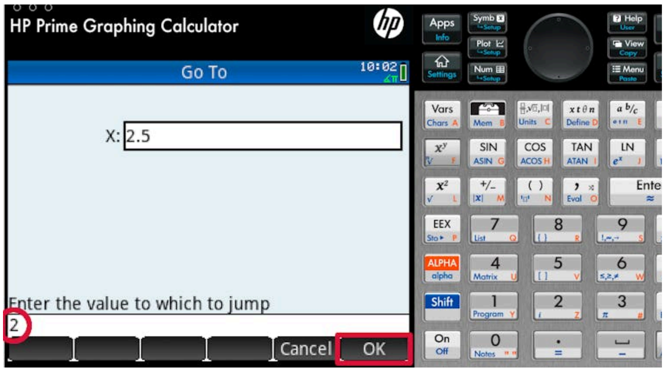
Steps	Explanation
You can see now the tangent at the right position. To see the equation of the line, press EXE. If you do not see the value of the gradient on this screen, you can turn on the derivative option in the set up menu.	
The equation of the tangent line is displayed on the screen.	

Student view

Steps	Explanation
These instructions assume that you have the graph of $y = \log(6 + x^2)$ on the screen in the viewing window $-5 \leq x \leq 10$ and $-0.5 \leq y \leq 2$. You will find guidance on how to find the equation of the tangent line to the graph at the point $(2, 1)$. The first step is to move the cursor to the point $(2, 1)$.	
Enter the value and confirm your choice with OK.	

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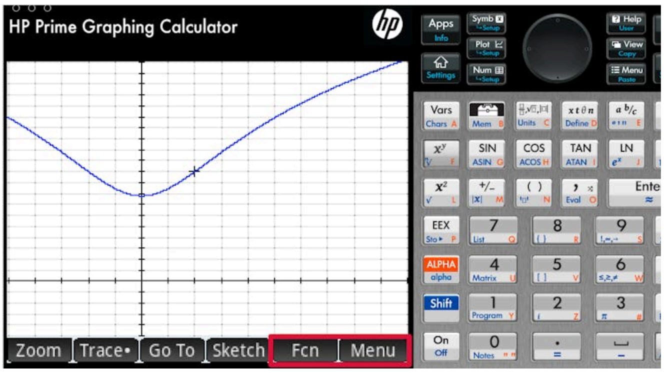
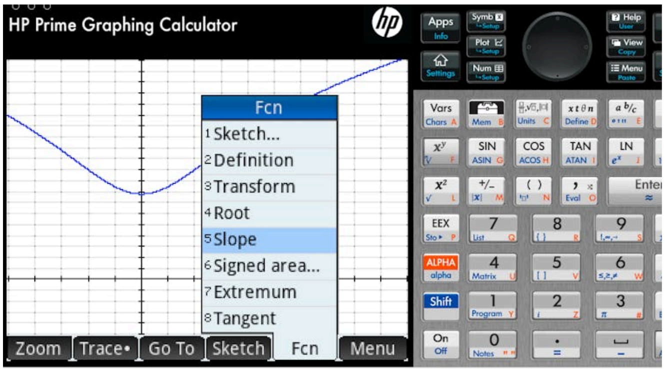
Steps	Explanation
Once the cursor is at the correct position, choose the function (Fcn) submenu.	
Choose to find the slope.	

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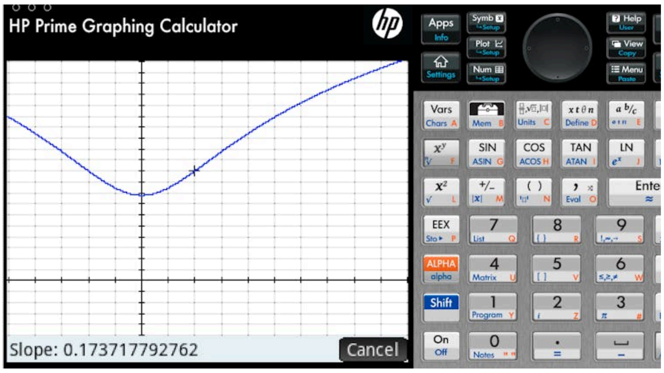
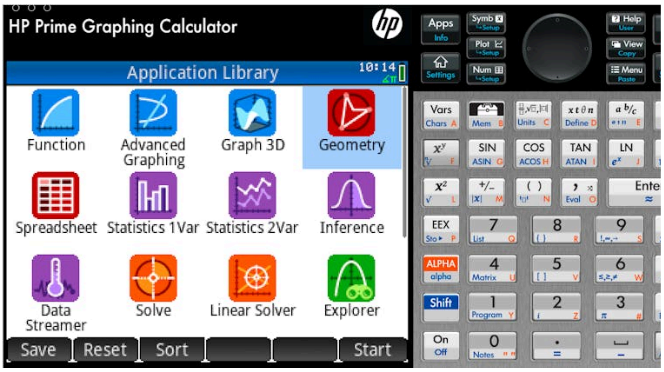
Steps	Explanation
The slope of the graph (which is the same as the slope of the tangent line) is now displayed on the screen.	
In the current operating system the option to draw the tangent line is part of the geometry application, which is currently not allowed in exams. This may change in the future, but for now the calculator does not support this shortcut. In exams, you need to follow the algebraic approach to find the equation of the tangent line.	

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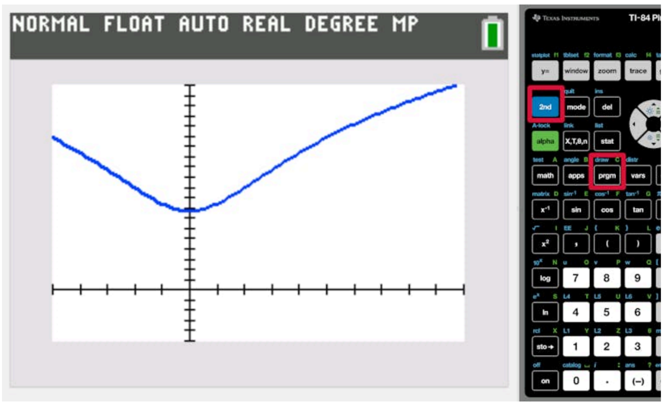
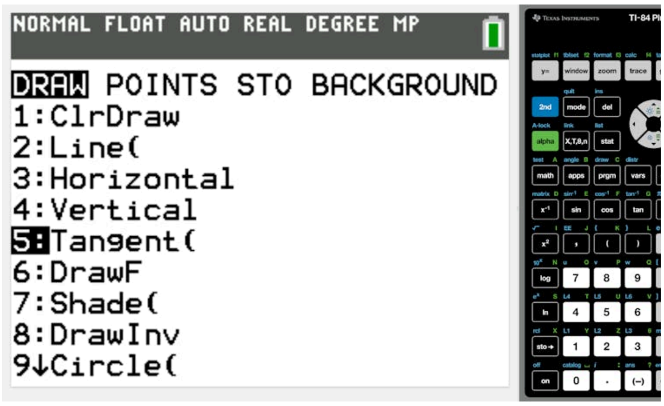
Steps	Explanation
These instructions assume that you have the graph of $y = \log(6 + x^2)$ on the screen in the viewing window $-5 \leq x \leq 10$ and $-0.5 \leq y \leq 2$. You will find guidance on how to find the equation of the tangent line to the graph at the point $(2, 1)$. Open the drawing options ...	
... and choose to draw a tangent.	

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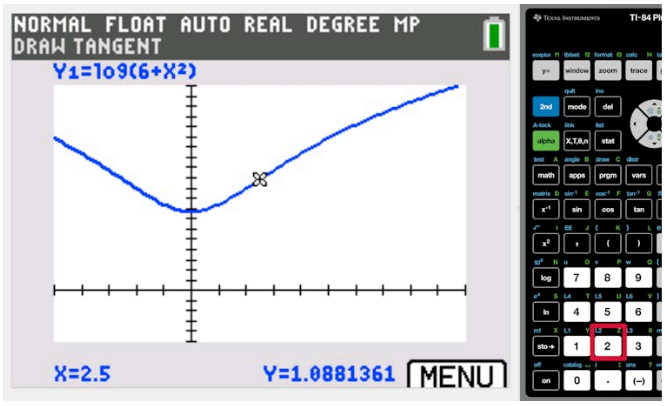
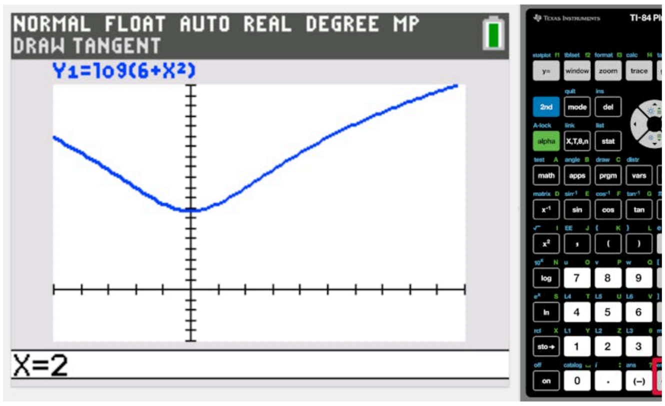
Steps	Explanation
The calculator is waiting for the position where the tangent should be drawn. Start typing in the x-coordinate of the position (in this case press 2).	
Press enter to confirm your choice of position.	

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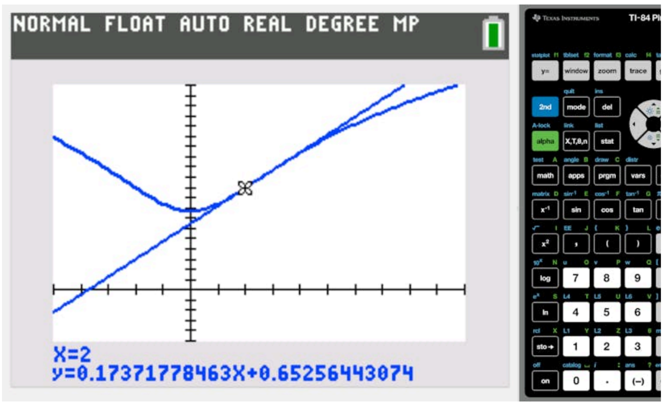
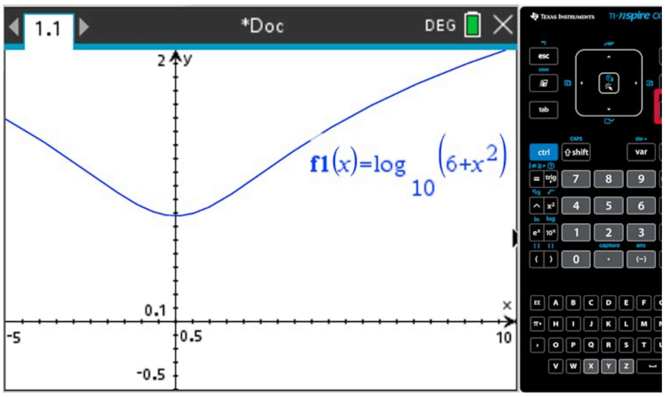
Steps	Explanation
The tangent line is drawn and the equation is displayed.	

Steps	Explanation
These instructions assume that you have the graph of $y = \log(6 + x^2)$ on the screen in the viewing window $-5 \leq x \leq 10$ and $-0.5 \leq y \leq 2$. You will find guidance on how to find the equation of the tangent line to the graph at the point $(2, 1)$. Open the menu ...	

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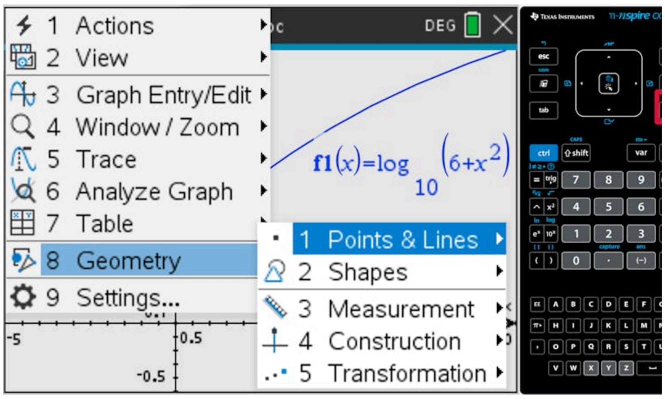
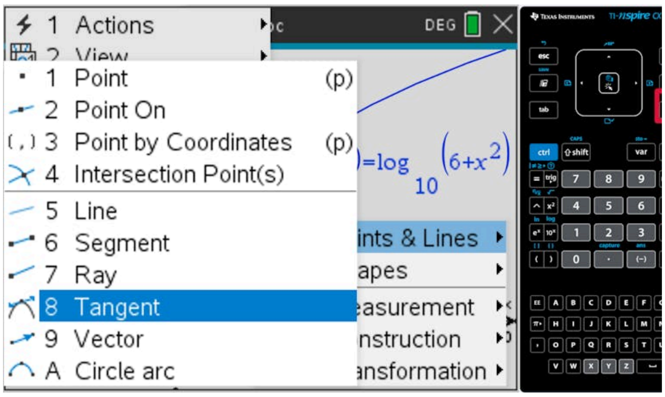
Steps	Explanation
... find points and lines among the geometry options, ...	
... and choose the option to draw a tangent.	

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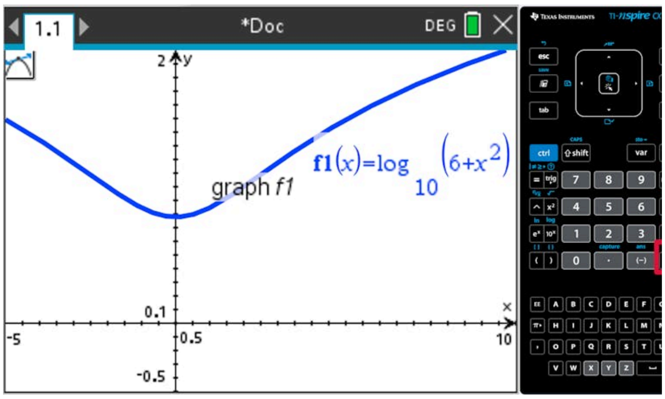
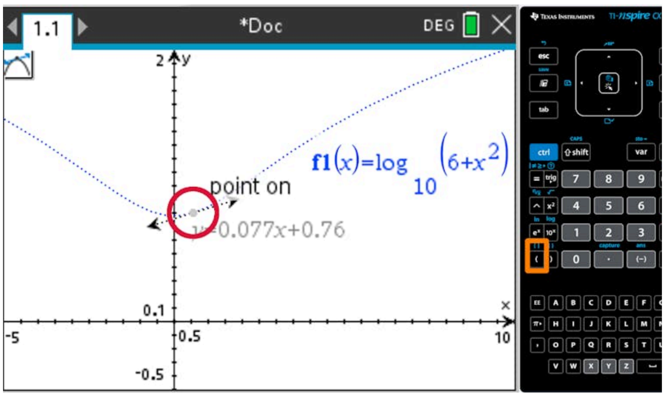
Steps	Explanation
The calculator needs some information. Select the graph and press enter.	
Now you can investigate the tangent lines by moving the point along the curve. If you want the tangent at an exact specific point, press the opening parenthesis (as if you would start typing in a pair of coordinates) ...	

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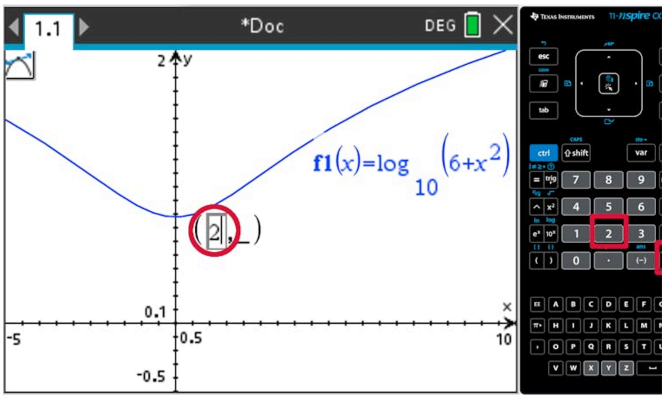
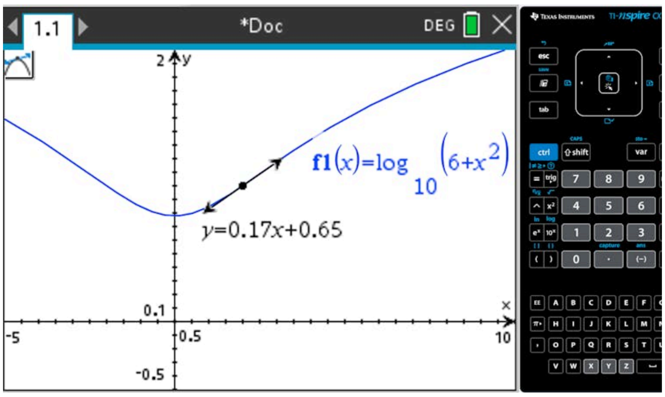
Steps	Explanation
... and type in the x-coordinate of the point (in this case 2). Press enter to confirm the value.	
The tangent line is drawn (you can extend it by dragging the arrows) and the equation is written on the screen.	

Example 2

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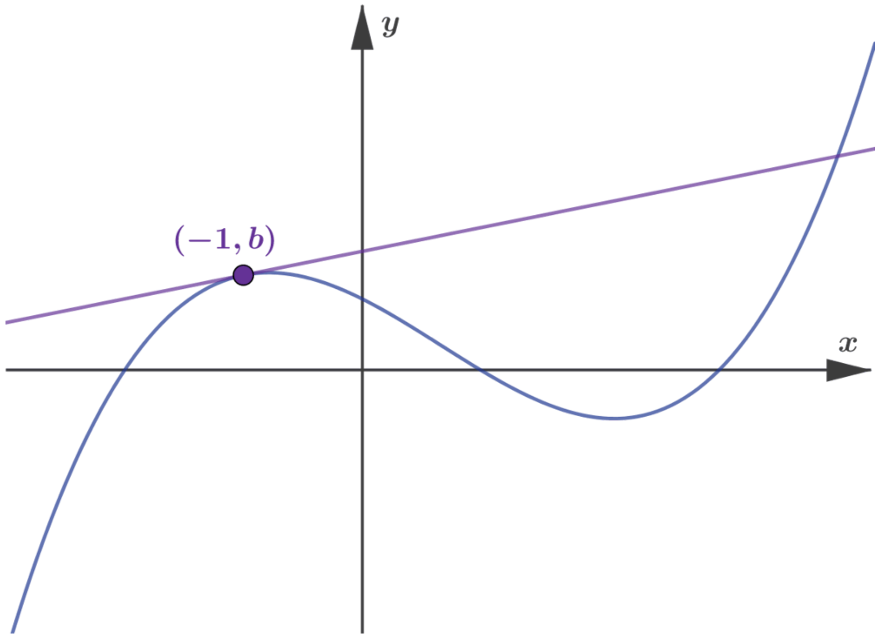
The diagram below shows part of the graph of $y = x^3 - 2x^2 - 5x + 6$. It also shows a point on the graph with x- coordinate -1 and the tangent to the graph at this point.



More information

The image shows a graph representing the curve described by the function (y = x^3−2x^2−5x+6). The X-axis and Y-axis are marked, with the X-axis depicting horizontal movement and the Y-axis depicting vertical movement. There is a noticeable curve on the graph that represents the cubic function. A tangent line intersects the curve at the point ((-1, b)). This specific point on the graph is marked by a dot and labeled with its coordinates. The tangent line, which is linear, passes through this point, indicating the slope of the curve at this location.

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Find the equation of this tangent.

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Steps	Explanation
$y = mx + c$	You can look for the equation in gradient-intercept form.
$y = x^3 - 2x^2 - 5x + 6$ $\frac{dy}{dx} = 3x^2 - 4x - 5$	You can differentiate to find the gradient function.
$m = 3 \times (-1)^2 - 4 \times (-1) - 5 = 2$	Since the x-coordinate of the given point -1, the gradient of the tangent is the valu the gradient function at $x = -1$.
$y = (-1)^3 - 2 \times (-1)^2 - 5 \times (-1) + 6 = 8$	You can substitute $x = -1$ into the equati of the curve to find the y- coordinate of th point on the curve.
$y = mx + c$ $8 = 2 \times (-1) + c$ $8 = -2 + c$ $c = 10$	You can find the value of c in the general equation of the line by substituting the coordinates of the point and the value of t gradient into $y = mx + c$.
Hence, the equation of the tangent is $y = 2x + 10$.	

In the next example, you are given the direction of the tangent line and asked to find the equation.

Example 3

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The diagram below shows part of the graph of $y = x^3 - 2x^2 - 5x + 6$, a point P on the graph, the line $y = 27x + 6$ and the tangent to the curve through point P that is parallel to this line.

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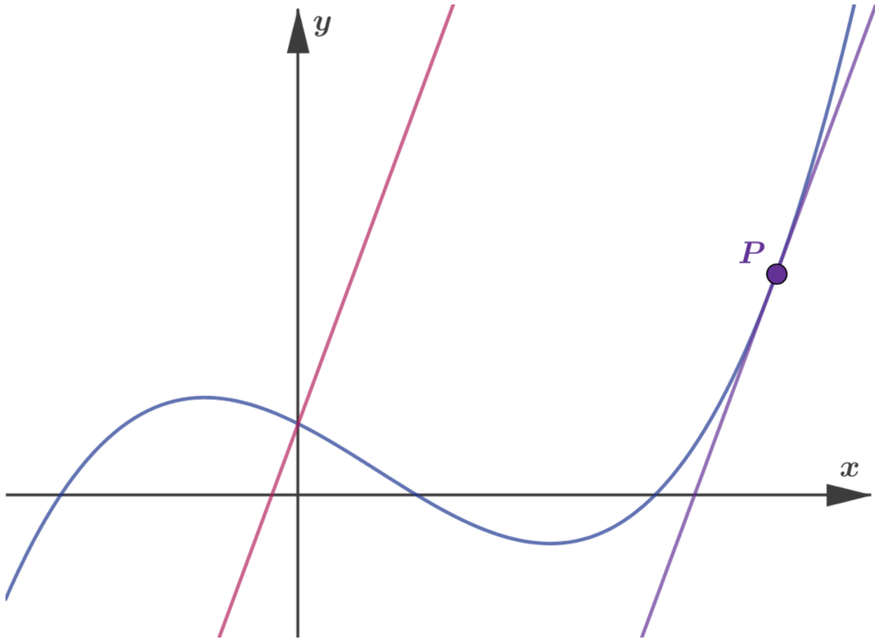

More information

The image displays a graph of the cubic function (y=x^3−2x^2−5x+6) with its characteristic curve. The x-axis is horizontal while the y-axis is vertical, each labeled "x" and "y" respectively. There are grid lines marking intervals on both axes. The curve crosses the x-axis multiple times, indicating its roots, and shows a wave-like shape with visible peaks and troughs. A point labeled as (\mathrm{P}) is marked on the curve, where it appears a tangent line is drawn. This tangent is parallel to a bright straight line representing (y=27x+6), which also appears on the graph and intersects with the curve. Highlighting such features provides insight into the tangent's slope matching that of the given line, emphasizing the geometric relationship at point (\mathrm{P}).

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- Find the gradient of the two lines.
- Find the coordinates of P.
- Find the equation of the tangent to the curve through point P.

Steps	Explanation
$m = 27$	Parallel lines have the same gradient. In tl equation $y = mx + c$, the value of m is tl gradient.

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Steps	Explanation
$y = x^3 - 2x^2 - 5x + 6$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 5$	The gradient function gives the gradient c curve at any point.
$3x^2 - 4x - 5 = 27$ $3x^2 - 4x - 32 = 0$	You know that the gradient of the curve a is 27 (*m* = 27), so the x-coordinate of P i the solution to $\frac{\mathrm{d}y}{\mathrm{d}x} = 27$.
$x = 4$ or $x = -\frac{8}{3}$	You can find the solution of this quadratic equation using various methods. One possibility is to use the appropriate application of your graphic display calculator.
$x > 0$, so $x = 4$	Since P is in the first quadrant of the coordinate system, you should choose the positive solution.
$y = 4^3 - 2 \times 4^2 - 5 \times 4 + 6 = 18$ Hence, P is $(4, 18)$.	You can substitute this value into the equa of the curve to find the y -coordinate of I
$y = mx + c$ $18 = 27 \times 4 + c$ $18 = 108 + c$ $c = -90$	You can find the value of c in the general equation of the line by substituting the coordinates of the point and the value of gradient into $y = mx + c$.
Hence, the equation of the tangent at P is $y = 27x - 90$.	

Note, that the other solution of the quadratic equation, $x = -\frac{8}{3}$, leads to another tangent parallel to the one you already found. This is shown on the diagram below. Can you find the equation of this line?

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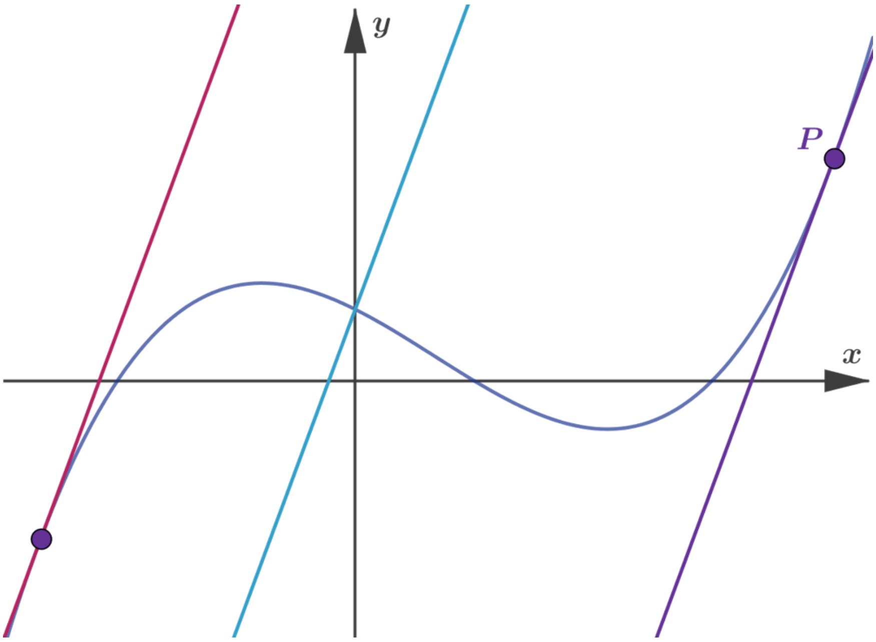

In the next example, you are asked to find the equation of a tangent through a point not on the curve. In the standard level Application and interpretation course you will not meet questions like this on the exam.

Example 4

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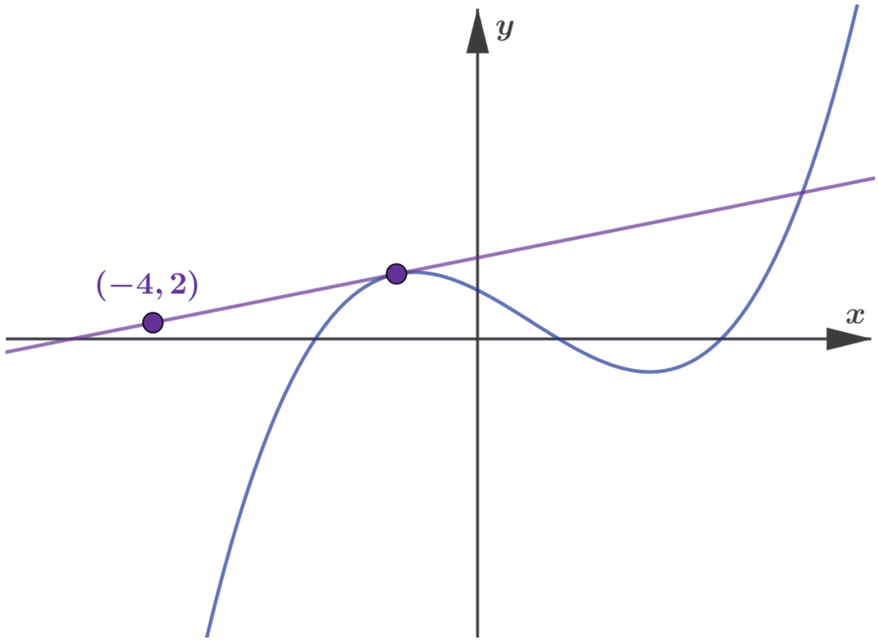
The diagram below shows part of the graph of $y = x^3 - 2x^2 - 5x + 6$, a point $\mathrm{P}(-4, 2)$ not on the graph and a tangent to the curve through point P.



More information

The image is a graph representing the curve of the function y=x^3-2x^2-5x+6. The X-axis and Y-axis are labeled. The curve is cubic with a visible inflection point. A tangent line is drawn on the graph, passing through the point P(-4,2), which is not on the curve. The tangent intersects the curve at a point along its path. The X-axis values seem to range from around -5 to 5, and the Y-axis values from about -10 to 10. The point P(-4,2) is labeled and marked on the graph, and the intersection of the tangent with the curve is also indicated with a dot, suggesting the point of tangency.

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Find the equation of this tangent.

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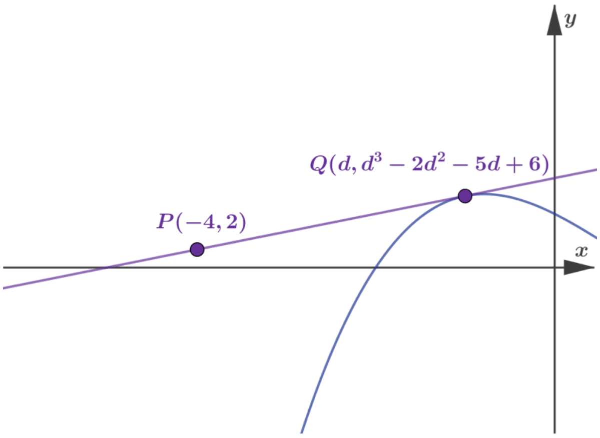
Steps	Explanation
	To answer this question, it is useful to introduce some notation. If Q is the point where the tangent meets the graph and d i the x-coordinate of this point, then the y-coordinate is the value of the expression at $x = d$.
	You can express the gradient of the line through P and Q in two different ways.
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{(d^3 - 2d^2 - 5d + 6) - 2}{d - (-4))}$ $= \frac{d^3 - 2d^2 - 5d + 4}{d + 4}$	The gradient can be expressed using the coordinates of the points.
$y'(x) = 3x^2 - 4x - 5,$ so the gradient is $m = y'(d) = 3d^2 - 4d - 5.$	Since the line is a tangent to the graph, the gradient can also be expressed as the valu the gradient function at $x = d$.
$\frac{d^3 - 2d^2 - 5d + 4}{d + 4} = 3d^2 - 4d - 5$	Since both of these expressions give the gradient of the same line, they must be equ Hence, the first coordinate of point Q is th solution of this equation.

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Steps	Explanation
The solutions are $d=-6$, $d=-1$ and $d=2$.	You can use your graphic display calculato find the solutions. For example you can fin the solutions by finding the intersection po of the graphs of the left-hand and right-ha sides. Or you can rearrange the equation t polynomial equation and use the polynomi equation solver application of your calcula
Since Q is to the right of P and to the left of the *y* -axis, $d=-1$. So the x-coordinate of Q is —1.	
Hence, the y-coordinate of Q is $(-1)^3-2\times(-1)^2-5\times(-1)+6=8.$	
$\frac{y-8}{x-(-1))}=\frac{2-8}{-4-(-1))}$ $\frac{y-8}{x+1}=\frac{-6}{-3}$ $\frac{y-8}{x+1}=2$ $y-8=2x+2$ $y=2x+10$	Using the coordinates of P and Q, you car now find the equation of the tangent.

Note, that the other solutions of the equation, $d=-6$ and $d=2$, lead to the other two tangents to the curve. These are shown on the diagram below. Can you find the equations of these line?

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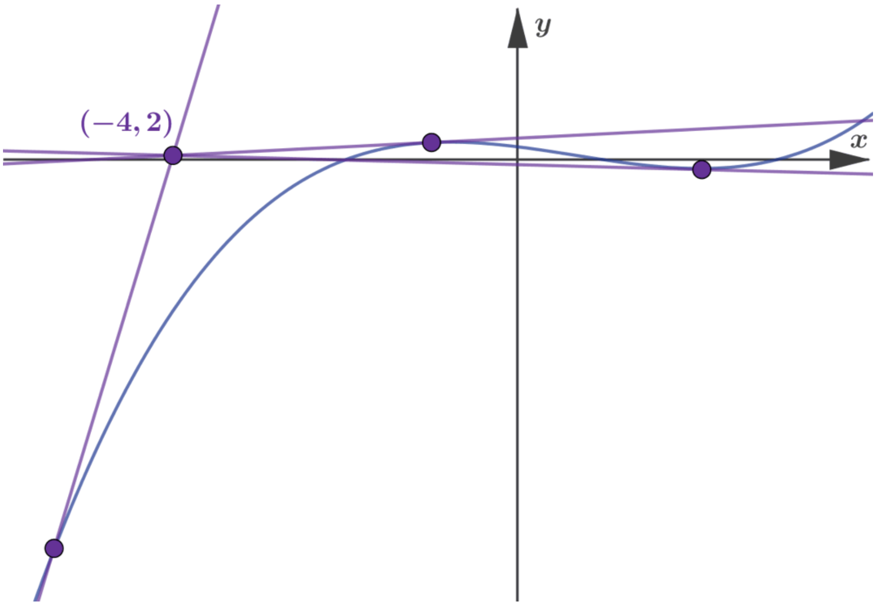
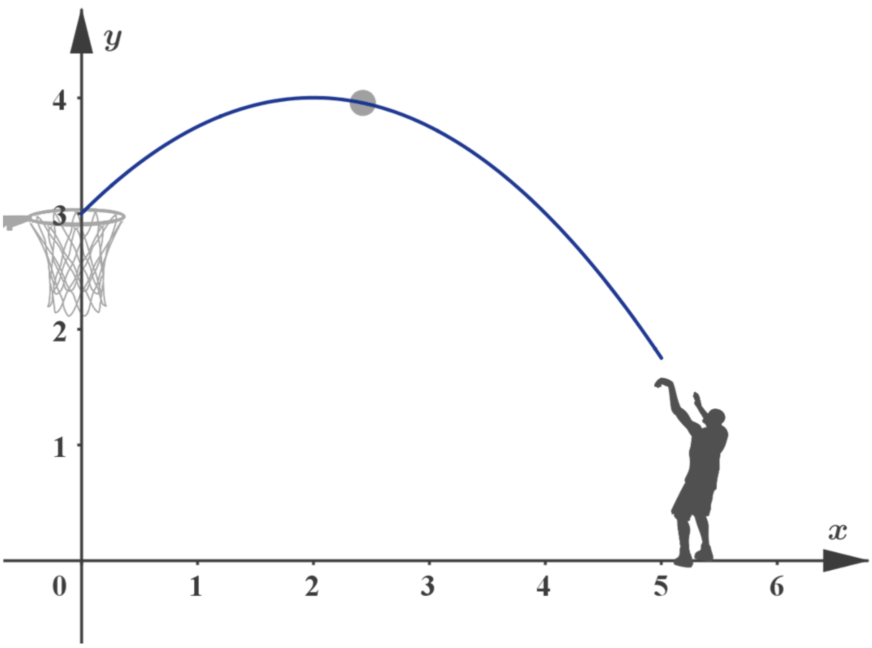


Example 5

★★☆

The diagram below shows a picture of Kyrie Irving shooting a free throw during the basketball game between the Dallas Mavericks and the Cleveland Cavaliers on 17 November, 2012.



Student
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More information

The diagram illustrates the path of a basketball being shot by Kyrie Irving during a game. The horizontal axis corresponds to the central line from the player to the hoop (x-axis), and the vertical axis represents the vertical line through the center of the hoop (y-axis). Both axes are measured in meters. Three points on the basketball's trajectory are marked: (0,3.05), (2.36,4.01), and (4.87,2.22). The path follows a parabolic curve described by the equation $y=ax^2+bx+c$, where 'a', 'b', and 'c' are specific constants. A silhouette of a basketball player is positioned near the point (5,0), indicating the release point of the ball. The ball follows a blue arc towards the net, which is located on the left at approximately (0, 3) on the grid.

[Generated by AI]

The distances on the coordinate axes are in metres. The horizontal axis is the central line from the player to the hoop and the vertical axis is the vertical line through the centre of the hoop. Three points on the path of the ball are $(0, 3.05)$, $(2.36, 4.01)$ and $(4.87, 2.22)$. The equation of the path of the ball is $y = ax^2 + bx + c$ for particular values of a, b and c.

- Find the value of a, of b and of c.
- Find the equation of the tangent to the path of the ball at the point where the ball reaches the centre of the hoop.
- At what angle does the ball reach the centre of the hoop?

Steps	Explanation
$a \times 0^2 + b \times 0 + c = 3.05$ $c = 3.05$	Since $(0, 3.05)$ is on the path, substituting $x = 0$ in $ax^2 + bx + c$ gives $y = 3.05$.
$a \times 2.36^2 + b \times 2.36 + 3.05 = 4.01$ $5.5696a + 2.36b = 0.96$ $a \times 4.87^2 + b \times 4.87 + 3.05 = 2.22$ $23.7169a + 4.87b = -0.83$	Using the other two points an the value of c found above, yo can write two equations for th remaining unknowns a and b.

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Steps	Explanation
$a = -0.230$ $b = 0.949$	Graphic display calculators ha applications that can solve lin equation systems like this.
$y = mx + c$	Look for the equation of the tangent in the gradient-interc form.
$c = 3.05$	Since $(0, 3.05)$ is on the y -axi
$y = -0.230x^2 + 0.949x + 3.05$ $y' = -0.460x + 0.949$ $m = y'(0) = 0.949$	The gradient of the tangent is value of the derivative at $x =$
Hence, the equation of the tangent to the path of the ball at the point where the ball reaches the centre of the hoop is $y = 0.949x + 3.05$.	
$\tan\theta = 0.949$ $\theta = \tan^{-1}(0.949) = 43.5°$	The gradient of a line is the tangent of the angle between line and the horizontal axis.
Hence, the ball reaches the centre of the hoop at an angle of 43.5° to the horizontal.	

3 section questions

5. Calculus / 5.4 Tangents and normals

The normal

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-normal-id-26290/print/) Assign

In this section, you will learn about the line normal to a curve at a given point.

Use the applet below to investigate the relationship between the tangent and normal to a curve.

Student view

Drag the purple dot along the curve. What do you observe?

Interactive 1. Understanding the Normal to a Curve.

More information for interactive 1

There is a graph with a curve, and users can explore the tangent and normal lines by dragging a purple dot along the curve. The tangent line touches the curve at a single point, representing its instantaneous direction at that location. The normal line is perpendicular to the tangent, showing the direction orthogonal to the curve's slope. As the user moves the dot, both lines update dynamically in distinct colors, providing a visual representation of how they change with the curve's shape. This interactive tool helps users understand the geometric relationship between a curve and its tangent and normal lines at various points.

Important

The line that is perpendicular to the tangent to a curve at the point of tangency is called the normal .

To find the equation of the normal to a curve at a particular point, you will need the gradient of the line at that point.

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Activity

The applet below shows you two perpendicular lines and gives the gradient of these two lines.

- Move the purple points to change the lines.
- Can you formulate a relationship between the two gradients?

Interactive 1. Normals in Tangents and Normals.

More information for interactive 1

In this interactive, users are presented with two perpendicular lines and provided with the gradients of these lines. By moving the purple points, users can adjust the positions and slopes of the lines. The interactive displays the gradients of both lines, allowing users to observe how changes in one line affect the other. The key relationship to explore is that the product of the gradients of two perpendicular lines is always -1. This means if one line has a gradient of m, the other line will have a gradient of $\frac{-1}{m}$. So, the gradient of the normal to the graph of $y = f(x)$ at the point $(a, f(a))$ is given by $m = \frac{-1}{f'(a)}$. It is the negative reciprocal of the gradient of the tangent

For instance, in the interactivity, the gradient of the blue line is $\frac{1}{4}$ and the gradient of the red line is $\frac{-4}{1}$. Since both lines are perpendicular to each other, the product of their gradients is -1. This demonstrates the key property that when two lines are perpendicular, the product of their slopes (gradients) is always -1.

Through this interactive exploration, users can visually and numerically verify this relationship, enhancing their understanding of the properties of perpendicular lines and their gradients.

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view

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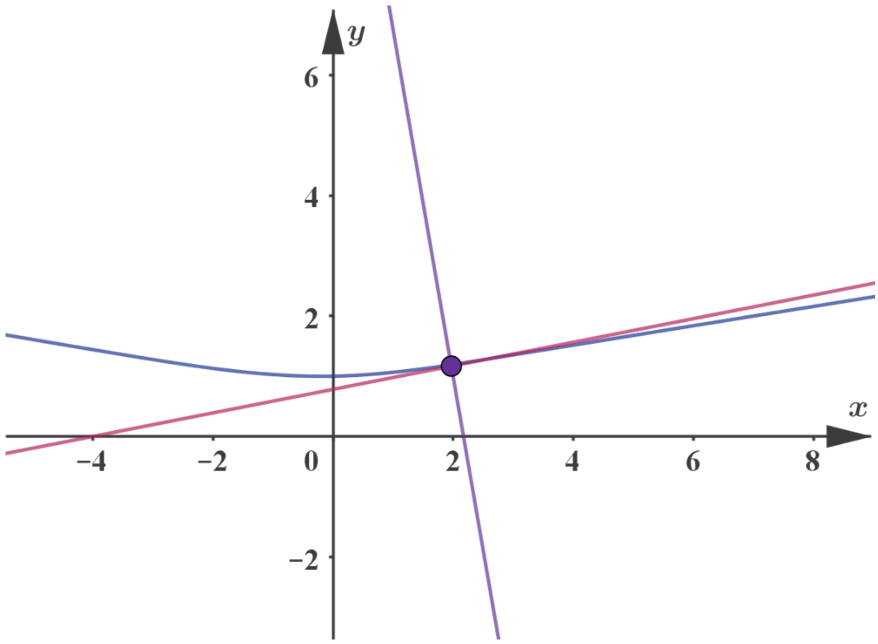
Important

- If two lines with gradients m_1 and m_2 are perpendicular, then $m_1m_2 = -1$.
- So, the gradient of the normal to the graph of $y = f(x)$ at the point $(a, f(a))$ is given by $m = \frac{-1}{f'(a)}$. It is the negative reciprocal of the gradient of the tangent.

Example 1

★☆☆

Find the equation of the normal to the graph of $y = \log(6 + x^2)$ at the point $(2, 1)$.



More information

The image is a graph depicting the function y=log(6+x²). The X-axis ranges from -8 to 8, and the Y-axis ranges from -2 to 6. The graph shows a curve that represents the function. At the point (2, 1), a dot is placed where the tangent line and normal line intersect the curve. The tangent line is represented by a blue line intersecting the

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point (2, 1) and runs slightly upwards as it moves to the right. The normal line is represented by a purple line and it's perpendicular to the tangent at the point (2,1), running steeply upwards. The axes are labeled with 'x' for the horizontal axis and 'y' for the vertical axis.

[Generated by AI]

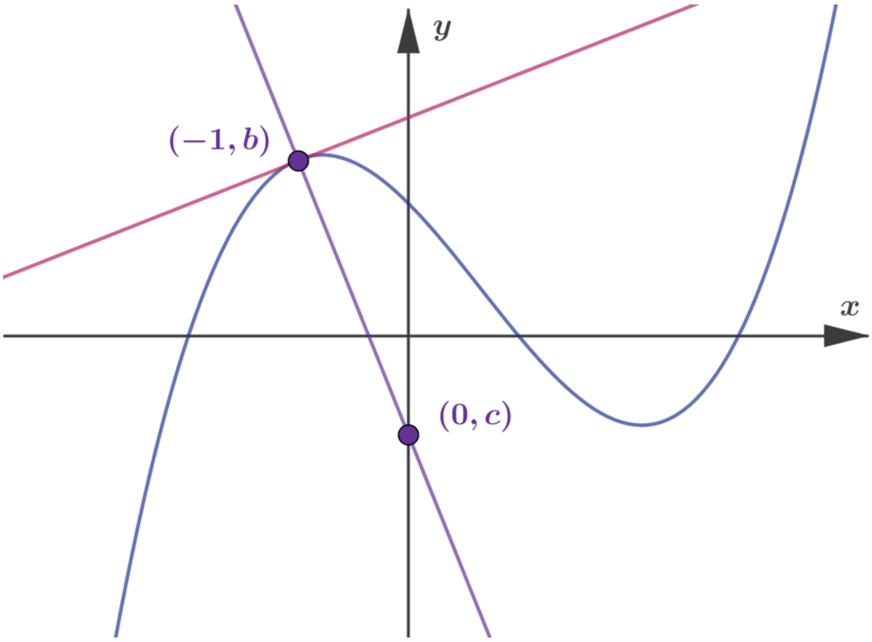
Steps	Explanation
$y'(2) = 0.1737\ldots$	Graphic display calculators have applications that can fi the gradient of a curve at a given point.
$m = \dfrac{-1}{0.1737\ldots} = -5.756\ldots$	The gradient of the normal is the negative reciprocal of gradient of the tangent.
$y = mx + c$ $1 = -5.756\cdots \times 2 + c$ $1 = -11.512\cdots + c$ $c = 12.512\ldots$	Look for the equation in the form $y = mx + c$. Use the coordinates of the point and the gradient to find the val of c.
$y = -5.76x + 12.5$	Use rounded values for the coefficients in the answer.

Example 2

★★☆

The diagram below shows part of the graph of $y = 0.2x^3 - 0.4x^2 - x + 1.2$. It also shows a point on the graph with x - coordinate -1 and the tangent and the normal to the graph at this point.

Student
view



More information

The image is a graph depicting the curve of the function y=0.2x^3−0.4x^2−x+1.2. The X-axis represents the variable 'x' and the Y-axis represents the function 'y'. The curve passes through a certain point where the x-coordinate is -1. At this specific point, a tangent and a normal line to the curve are drawn. The tangent is represented by a straight line that touches the curve at the point (-1, b), where 'b' is the corresponding y-coordinate. The normal is a line perpendicular to the tangent at the point of tangency and intersects the Y-axis at the point (0, c). The diagram emphasizes the relationships between these points and lines.

[Generated by AI]

The normal intersects the y -axis at $(0, c)$.

Find the value of c.

Steps	Explanation
$y = mx + c$	You can look for the equation in gradient-intercept form.

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Steps	Explanation
$y = 0.2x^3 - 0.4x^2 - x + 1.2$ $\frac{dy}{dx} = 0.6x^2 - 0.8x - 1$	You can use the gradient functi to find the gradient of the tange
$m_{\text{tangent}} = 0.6 \times (-1)^2 - 0.8 \times (-1) - 1 = 0.4$	Since the x - coordinate of the given point is -1, the gradient the tangent is the value of the gradient function at $x = -1$.
$m = m_{\text{normal}} = \frac{-1}{0.4} = -2.5$	The gradient of the normal is th negative reciprocal of the gradi of the tangent.
$y = 0.2 \times (-1)^3 - 0.4 \times (-1)^2 - (-1) + 1.2 = 1.6$	You can substitute $x = -1$ into equation of the curve to find the coordinate of the point on the curve.
$y = mx + c$ $1.6 = -2.5 \times (-1) + c$ $1.6 = 2.5 + c$ $c = -0.9$	You can find the value of c in th general equation of the line (wh is the y-intercept you are lookin for) by substituting the coordina of the point and the value of the gradient into $y = mx + c$.

Example 3

★★☆

The diagram below shows part of the graph of $y = 0.2x^3 - 0.4x^2 - x + 1.2$, a point P on the graph, the line $y = 5x + 7$ and the normal to the curve through point P that is parallel to this line. This normal crosses the x -axis at point $(a, 0)$.

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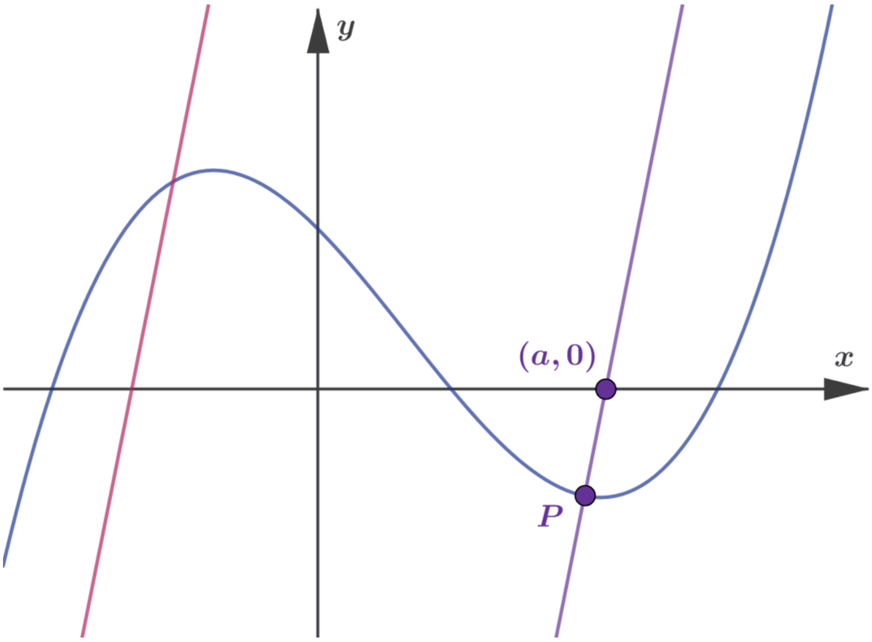

More information

The image is a graph displaying the cubic function (y = 0.2x^3 - 0.4x^2 - x + 1.2). The graph features a curve which includes a high point and a low point, indicating changes in slope direction. There is a point labeled (P) on the curve. Two straight lines intersect the curve at this point.

One line, labeled as (y = 5x + 7), is a tangent to the curve at point (P), implying it touches the curve without crossing it. The other line is a normal, parallel to the line and crossing the x-axis at a, 0. The axes are labeled as (x) and (y), with the x-axis showing the origin where the normal line meets it at point ((a,0)). The diagram illustrates the relations between these mathematical elements and the curve.

[Generated by AI]

a) Find the gradient of the normal line.

b) Find the gradient of the curve at P.

c) Find the coordinates of P.

d) Find the value of a.

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	Steps	Explanation
a)	$y = 5x + 7$ so $m = 5$. So the normal has gradient 5.	Parallel lines have the same gradient. In equation $y = mx + c$ the value of m is gradient.
b)	$\frac{-1}{5} = -0.2$ The gradient of the curve at P is -0.2.	Since the tangent is perpendicular to th normal, the gradient of the graph is the negative reciprocal of the gradient of th normal.
c)	$y = 0.2x^3 - 0.4x^2 - x + 1.2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.6x^2 - 0.8x - 1$	The gradient function gives the gradien a curve at any point.
	$0.6x^2 - 0.8x - 1 = -0.2$ $0.6x^2 - 0.8x - 0.8 = 0$	The gradient of the curve at P is —0.2, so the x-coordinate of P is the solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = -0.2$.
	$x = 2$ or $x = -\frac{2}{3}$	You can find the solution of this quadra equation using various methods. One possibility is to use the appropriate application of your graphic display calculator.
	$x > 0$, so $x = 2$	Since P is in the fourth quadrant of the coordinate system, you should choose t positive solution.
	$y = 0.2 \times 2^3 - 0.4 \times 2^2 - 2 + 1.2 = -0.8$ Hence, P is $(2, -0.8)$.	You can substitute this value into the equation of the curve to find the y-coordinate of P.
d)	$y = mx + c$ $-0.8 = 5 \times 2 + c$ $-0.8 = 10 + c$ $c = -10.8$	You can find the value of c in the gener equation of the line by substituting the coordinates of the point and the value c the gradient into $y = mx + c$.
	Hence, the equation of the normal to the curve at P is $y = 5x - 10.8$.	

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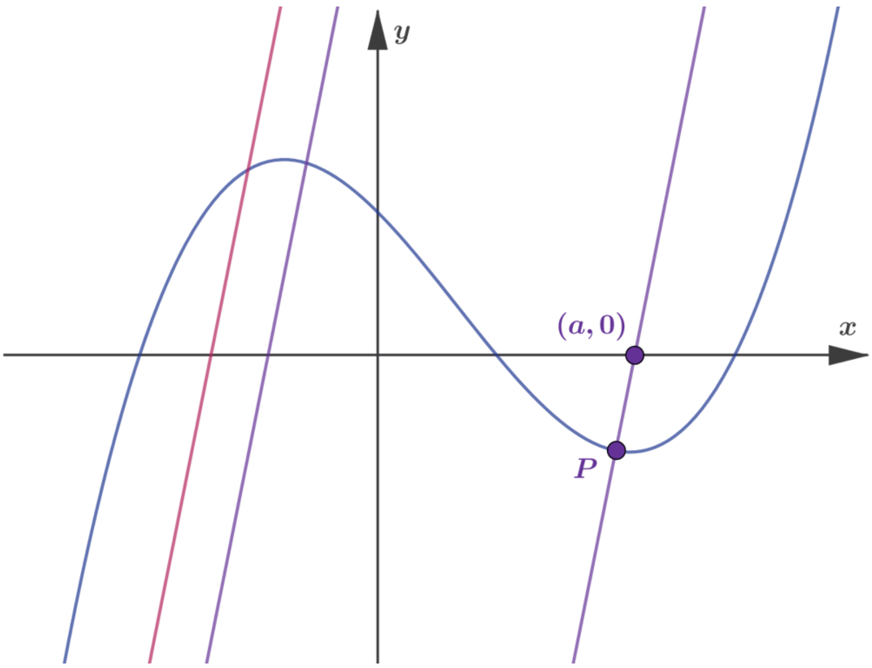
	Steps	Explanation
	$0 = 5a - 10.8$ $5a = 10.8$. $a = 2.16$ The normal to the curve at P cuts the x-axis at $(2.16, 0)$.	At the x-intercept the y-coordinate is 0

Note, that the other solution of the quadratic equation, $x = -\dfrac{2}{3}$, leads to another normal parallel to the one you already found. This is shown on the diagram below. Can you find the equation of this line?



Example 4

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There are two points on the graph of $y = 0.2x^3 - 0.4x^2 - x + 1.2$ where the normal is vertical. Find the distance between these two lines.

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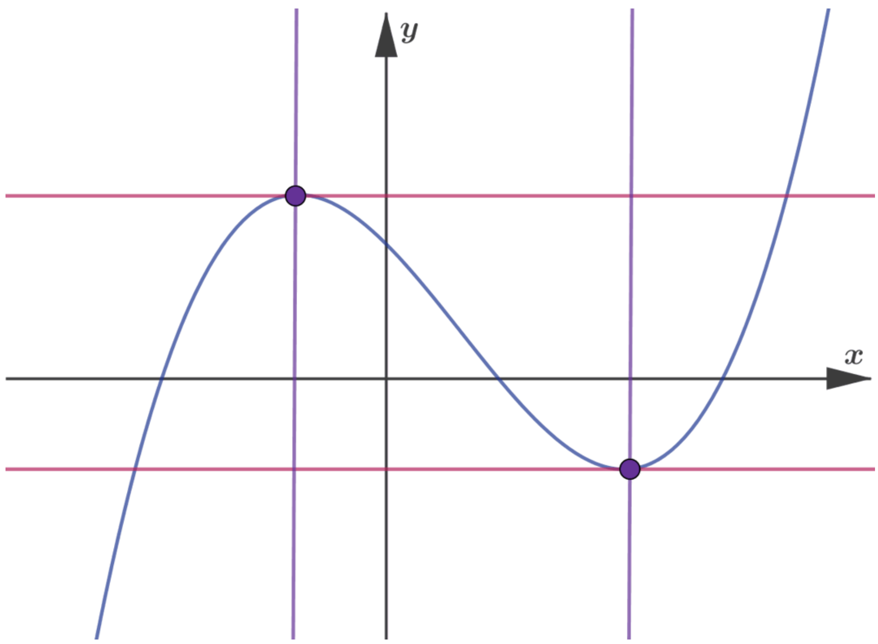
Steps	Explanation
$y = 0.2x^3 - 0.4x^2 - x + 1.2$ $\frac{dy}{dx} = 0.6x^2 - 0.8x - 1$	The gradient function gives the gradient of a curve at any point.
$0.6x^2 - 0.8x - 1 = 0$	If a normal is vertical, then the tangent is horizontal, so the gradient of the tangent is 0.
$x = 2.1196\ldots$ and $x = -0.7862\ldots$	The two solutions of this quadratic equation g the x - coordinates of the points where the tangent is horizontal and hence where the normal is vertical.
$2.1196\ldots-(-0.7862\ldots) = 2.91$ (to 3 significant figures)	The difference between these values of x give the distance between the two vertical lines.

The diagram below shows part of the graph, the two horizontal tangents and the two vertical normals.



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5. Calculus / 5.4 Tangents and normals

Checklist

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What you should know

By the end of this subtopic you should be able to:

- understand that the tangent and normal at a given point to a given curve are perpendicular to each other
- find the equation of the tangent to a given curve at a given point on the curve
- find the point on a curve if the direction of the tangent is given
- find the equation of the normal to a given curve at a given point on the curve
- find the point on a curve if the direction of the normal is given.

5. Calculus / 5.4 Tangents and normals

Investigation

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/investigation-id-26292/print/) Assign

Part 1

Graphics designers do not always draw by hand. Sometimes they use a computer to draw complicated shapes. The software they use can create curves based on a few guide points the designer specifies. Using the applet below, you can investigate how this drawing tool works.

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view

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The applet shows you a target curve (red) and lets you move four points. The applet also shows you a curve which is created using those four guiding points. This curve is created using the method that illustrator software uses to draw curves.

- Can you match the target curve?
- Can you explain how the position of the four points affect the shape of the curve?
- Can you explain why this activity is in this subtopic which covers tangents and normals?

Interactive 1. Investigating Curves and Their Tangents.

More information for interactive 1

This interactive allows users to practice creating and manipulating curves by adjusting four guiding points, similar to the tools used by graphic designers. The interactive displays a target curve (in red) and a generated curve that users can shape by moving the guiding points. Users can practice matching the target curve by experimenting with the positions of the points, observing how each adjustment influences the curve's shape. Clicking "Show Guide Lines" displays two black dashed lines that serve as visual aids, helping you align your curve with the target by showing how the points influence the slope and curvature. This hands-on approach demonstrates how control points affect a curve's shape, teaching concepts like tangents and curvature through direct manipulation.

By clicking New Curve, users can generate a fresh target curve displayed in pink. Using movable purple control points and optional hidden guidelines, they can then manipulate their own curve to match the target shape. This interactive exercise helps develop precision in curve modeling while demonstrating how control points influence a curve's path.

Student view

The tool is designed to build skills in both graphic design and mathematical modeling. Each new curve presents a fresh challenge, allowing you to develop intuition for how point positions create different curves. The guidelines

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provide helpful reference points, making it easier to understand the relationship between the control points and the resulting shape. Through repeated practice, you'll gain a deeper understanding of curve construction and improve your ability to visualize and recreate complex curves.

If you would like to draw more complicated shapes, you can use more control points and join basic curves. Think about how you would join these curves to make sure that the resulting shape is smooth at the joining points. The applet above only lets you investigate curves with four control points, but you can try this using illustrator software. You might have one type of this already on your computer. If not, you can, for example, try Inkscape (https://inkscape.org/), which is open source designer software that has this curve-drawing option.

Part 2

If you have studied charges and electric fields, then you are familiar with the terminology used in the following simulation. If you do not have the background knowledge from physics, you can still use your own words to explain the relationship that you might discover.

- Open this simulation (https://phet.colorado.edu/en/simulation/charges-and-fields) and drag and drop positive and negative charges from the bottom of the screen to the black background. The arrows you see appearing represent the electric field that these charges create. Move the charges around and see the effect on the electric field.
- Also, drag and drop a sensor. It measures the electric field. Move the sensor around to investigate the electric field.
- On the right-hand side there are two tools. The purple tool measures electrostatic potential. Drag it somewhere and use it to draw an equipotential line.
  - Move the sensor along this curve and investigate the relationship between the equipotential line and the electric field.
  - Check your understanding by trying to draw an equipotential line before you ask the tool to draw it for you.
  - If you have studied these concepts before, then you can try to explain the relationship that you have discovered.

Student view

Rate subtopic 5.4 Tangents and normals

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