TOPIC 1
NUMBER AND ALGEBRA

SUBTOPIC 1.10
RATIONAL EXPONENTS

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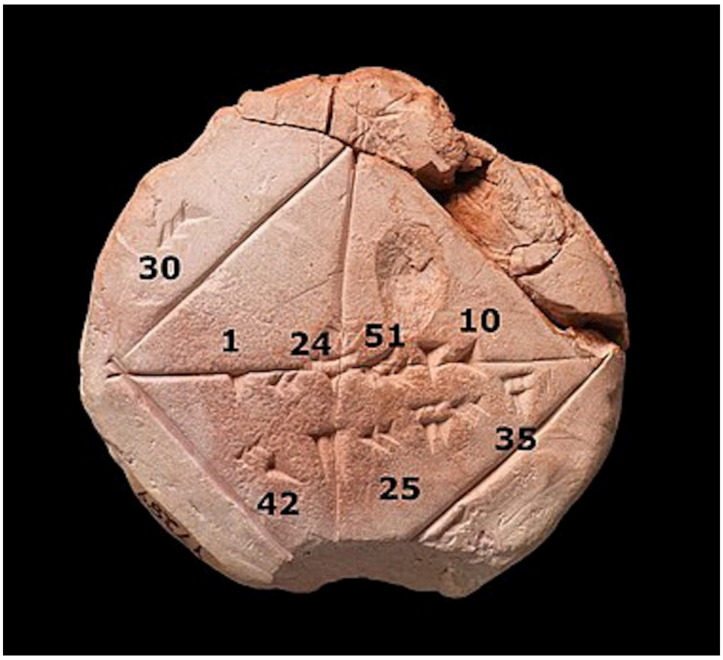
The big picture

This Babylonian clay tablet which was created almost 4000 years ago shows an approximation of $\sqrt{2}$ correct to 5 decimal places. Mathematicians have been working with rootsand refining the notation ever since then.



Babylonian clay tablet

Source: "YBC-7289-OBV-labeled" by Urcia, A., Yale Peabody Museum of Natural History is in public domain

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More information

The image depicts a Babylonian clay tablet with inscribed numbers and geometric markings. The tablet shows a series of numerical inscriptions and a star-like shape formed by intersecting lines. The numbers are prominently displayed around the shape, with some of them being: 30, 1, 24, 51, 42, 25, 10, and 35. These numbers represent an ancient approximation of the square root of 2. The tablet is circular in shape, with a rough, textured surface indicating its age and historical significance. Lines intersect across the center of the tablet, forming triangular sections with the numbers positioned in and around these sections.

[Generated by AI]

The radical symbol was introduced by Christoff Rudolff in a book published in 1526 . His radical symbol looked like this: $\surd$. It did not have the line above the number, which is called a vinculum. The modern-day radical, such as $\sqrt{a}$, was introduced by Rene Descartes in the 1600s .

Rational exponents as a way to write roots were introduced in the 1300s and acquired their modern notation in 1676 in a work by Isaac Newton.

Use of rational exponents simplifies some manipulations involving roots because it allows you to use exponent rules. For example, it is very hard to rewrite $\sqrt[3]{2} \times \sqrt[4]{2}$ as a single root in radical notation b ut this becomes much easier if you rewrite roots using rational exponent notation, where $\sqrt[3]{2} = 2^{\frac{1}{3}}$ and $\sqrt[4]{2} = 2^{\frac{1}{4}}$. Using exponent rules, you find that $2^{\frac{1}{3}} \times 2^{\frac{1}{4}} = 2^{\frac{7}{12}}$.

Concept

Roots of numbers and algebraic expressions in radical form can be written in an equivalent form using rational exponents. Writing roots as exponents allows you to apply exponent rules to your work with roots. How does converting a root to rational exponent notation reveal properties that are hidden when the same root is written in radical form? Are any of the properties that are visible in radical form hidden when a root is written in rational exponent form?

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The word 'surd' is used to describe irrational roots such as $\sqrt{2}$ or $\sqrt{3}$.

The origin of the word surd is from the Latin word, *surdus* which can be translated as deaf, mute or insensible.

Originally, such numbers were called *alogos,* which can mean both irrational and mute in Greek.

The Greek word was translated into Arabic, losing the part of the definition meaning irrational, before being translated into Latin, entering into mathematical vocabulary as the word surd, which is still used today.

1. Number and algebra / 1.10 Rational exponents

Rational exponents

Rational or fractional exponents provide another way to represent roots.

This works as follows:

Assume that the law of exponents $(a^m)^n = a^{m\times n}$ works also for fractional exponents.

So if $(a^m)^2 = a = a^1$, then $m \times 2 = 1$ and $m = \frac{1}{2}$.

But since a^m squared is equal to a , a^m must be the square root of a . Therefore

$$\sqrt{a} = a^{\frac{1}{2}}$$

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Similarly, you can show that

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$$\sqrt[3]{a} = a^{\frac{1}{3}}$$
$$\sqrt[4]{a} = a^{\frac{1}{4}}$$
$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

By convention, when n is even, $a^{\frac{1}{n}}$ refers only to the positive root of a .

Example 1

★☆☆

Evaluate $16^{\frac{1}{4}}$.

Steps	Explanation
$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	You should be able to find this root without us calculator. Remember that $a^{\frac{1}{n}}$ when n is even means th root.

Making connections

Rational exponents follow the laws of exponents that you learned in subtopic 1.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-26006/). These laws are:

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$$a^m \times a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$(a^m)^n = a^{m\times n}$$
$$(ab)^m = a^m b^m$$
$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$
$$a^{-m} = \frac{1}{a^m}$$
$$a^0 = 1$$

Important

For rational exponents:

$$a^{\frac{1}{n}} = \sqrt[n]{a},\ a > 0$$

If n is even, give only the positive root:

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

Example 2

★★☆

Evaluate $27^{\frac{2}{3}}$. Do not use a calculator for this question.

Steps	Explanation
$27^{\frac{2}{3}} = \left(27^{\dfrac{1}{3}}\right)^2 = (27^2)^{\frac{1}{3}}$	Using $(a^m)^n = a^{m\times n}$.
$\left(27^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$	It is usually easier to evaluate $\left(a^{\frac{1}{n}}\right)^m$ rath $(a^m)^{\frac{1}{n}}$.

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You can use a calculator to check your answer.

Steps	Explanation
You can use your calculator to check the result of the calculation in Example 2.	

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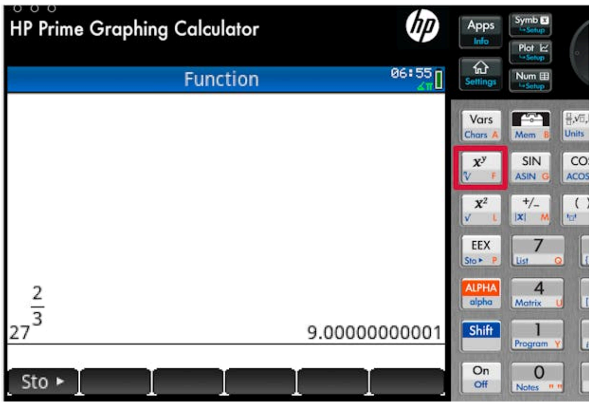
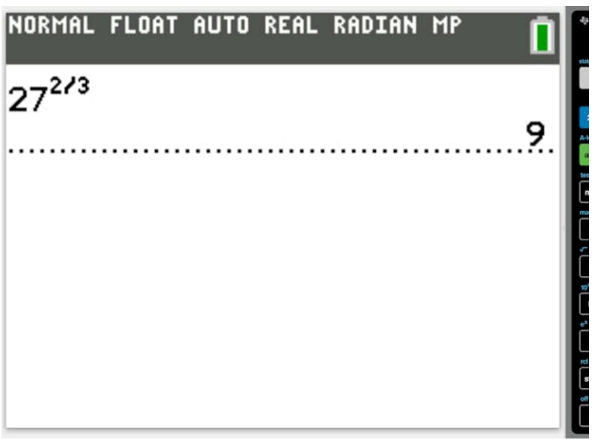
Steps	Explanation
You can use your calculator to check the result of the calculation in Example 2. Note, that the calculator uses numerical algorithms to evaluate the expression. The result is an approximate value.	

Steps	Explanation
You can use your calculator to check the result of the calculation in Example 2.	

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Steps	Explanation
You can use your calculator to check the result of the calculation in Example 2.	

Be aware

Most calculators use rational exponent notation for any root higher than $\sqrt{a}$.

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Interactive 1. Rewrite and Simplify the Fractional Exponent Expressions.

Credit: GeoGebra (https://www.geogebra.org/m/uZPZpSVh) Edward Knote

More information for interactive 1

The interactive applet allows users to practice simplifying expressions with rational exponents by rewriting them in radical form. Users are given an expression of the form $(a)^{\frac{m}{n}} = \sqrt[n]{a^m} = x$ and must determine the values of n and m. After entering their answers, the applet checks whether the values are correct. For example, $64^{\frac{4}{3}} = \sqrt[3]{64^4} = 4^4 = 256$, which means the value of $m = 4$ and $n = 3$. Users can click the “New Problem” button to generate a different question, allowing them to continue practicing and strengthening their understanding of rational exponents and how they relate to roots.

3 section questions

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Rational exponents - algebraic manipulations

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In addition to simplifying and evaluating numerical expressions with rational exponents, you can also simplify algebraic expressions with rational exponents using exponent rules.

Example 1

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Simplify $\left(x^{\frac{3}{2}} \times x^{-\frac{1}{2}}\right)^3$.

Steps	Explanatio
$\left(x^{\frac{3}{2}} \times x^{-\frac{1}{2}}\right)^3 = \left(x^{\frac{3}{2}+\left(-\frac{1}{2}\right)}\right)^3 = (x)^3 = x^3$	Simplify inside the brac

Example 2

★★☆

Simplify $\left(\frac{32x^{10}}{y^5}\right)^{\frac{2}{5}}$.

Steps	Explan
$\left(\frac{32x^{10}}{y^5}\right)^{\frac{2}{5}} = \frac{32^{\frac{2}{5}}x^{10\times\frac{2}{5}}}{y^{5\times\frac{2}{5}}} = \frac{\left(32^{\frac{1}{5}}\right)^2 x^4}{y^2} = \frac{2^2x^4}{y^2} = \frac{4x^4}{y^2}$	Use $(a^m)^n = a$ $\left(a^{\frac{1}{n}}\right)^m$.

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Example 3

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Simplify $\left(\dfrac{16x^{\frac{5}{2}}}{81y^{\frac{1}{4}}}\right)^{\frac{3}{4}}$.

Steps	Explar
$\left(\dfrac{16x^{\frac{5}{2}}}{81y^{\frac{1}{4}}}\right)^{\frac{3}{4}} = \dfrac{16^{\frac{3}{4}}x^{\frac{5}{2}\times\frac{3}{4}}}{81^{\frac{3}{4}}y^{\frac{1}{4}\times\frac{3}{4}}} = \dfrac{\left(16^{\frac{1}{4}}\right)^3 x^{\frac{15}{8}}}{\left(81^{\frac{1}{4}}\right)^3 y^{\frac{3}{16}}} = \dfrac{2^3x^{\frac{15}{8}}}{3^3y^{\frac{3}{16}}} = \dfrac{8x^{\frac{15}{8}}}{27y^{\frac{3}{16}}}$	Use $(a^m)^n = \left(a^{\frac{1}{n}}\right)^m$. $16^{\frac{1}{4}}$ $81^{\frac{1}{4}}$

You can practise more questions like this using the applet below.

Interactive 1. Practice Exercise to Simplify Algebraic Expressions with Rational Exponents.

Credit: GeoGebra (https://www.geogebra.org/m/hvr7qyxu) Audrey McLaren, Kevin Hopkins

More information for interactive 1

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This interactive allows users to practice simplifying algebraic expressions involving rational exponents, with the goal of expressing the final answer in the form $x^{p}y^{q}z^{r}$, where *p*, *q*, and *r* are simplified exponents. Users are given expressions that require applying exponent rules, such as the product rule, quotient rule, and power rule, to simplify them into this standard form. A checkbox labeled **"Show answer"** allows users to reveal the correct simplified form of the expression. This feature supports self-assessment and helps reinforce learning by allowing users to compare their solution to the correct one. Additionally, the **"New Problem"** button generates a fresh expression each time it's clicked, encouraging repeated practice and deeper understanding of rational exponent operations.

For example, simplify this expression $\left(\frac{x^{\frac{5}{6}}y^{\frac{1}{5}}}{z^{\frac{3}{5}}}\right)^{\frac{-1}{2}}\left(\frac{x^{\frac{1}{4}}z^{\frac{1}{2}}}{y^{\frac{-1}{5}}}\right)^{\frac{3}{4}}$

$$= \left(\frac{x^{\frac{5}{6}\times\left(\frac{-1}{2}\right)}y^{\frac{1}{5}\times\left(\frac{-1}{2}\right)}}{z^{\frac{3}{5}\times\left(\frac{-1}{2}\right)}}\right)\left(\frac{x^{\frac{1}{4}\times\left(\frac{3}{4}\right)}z^{\frac{1}{2}\times\left(\frac{3}{4}\right)}}{y^{\frac{-1}{5}\times\left(\frac{3}{4}\right)}}\right)$$

$$= \left(\frac{x^{-\frac{5}{12}}y^{-\frac{1}{10}}}{z^{-\frac{3}{10}}}\right)\left(\frac{x^{\frac{3}{16}}z^{\frac{3}{8}}}{y^{\frac{-3}{20}}}\right)$$

$$= \frac{\left(x^{\frac{-5}{12}}\right)\left(y^{\frac{-1}{10}}\right)\left(x^{\frac{3}{16}}\right)\left(z^{\frac{3}{8}}\right)}{\left(z^{\frac{-3}{10}}\right)\left(y^{\frac{-3}{20}}\right)}$$

$$= \left(\left(x^{\frac{-5}{12}}\right)\left(x^{\frac{3}{16}}\right)\right)\times\left(\frac{y^{\frac{-1}{10}}}{\left(y^{\frac{-3}{20}}\right)}\right)\left(\frac{z^{\frac{3}{8}}}{\left(z^{\frac{-3}{10}}\right)}\right)$$

$$= \left(x^{\frac{4\times(-5)+3\times 3}{48}}\right)\times\left(y^{\frac{-1}{10}+\frac{3}{20}}\right)\left(z^{\frac{3}{8}+\frac{3}{10}}\right)$$

$$= \left(x^{\frac{-20+9}{48}}\right)\left(y^{\frac{-2+3}{20}}\right)\left(z^{\frac{15+12}{40}}\right)$$

$$= \left(x^{\frac{-11}{48}}\right)\left(y^{\frac{-1}{20}}\right)\left(z^{\frac{27}{40}}\right)$$

Therefore, $p = \frac{-11}{48}$, $q = \frac{1}{20}$ and $r = \frac{27}{40}$.

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4 section questions

Checklist

What you should know

By the end of this subtopic you should be able to:

- interpret $a^{\frac{1}{n}}$ as $\sqrt[n]{a}$
- use $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$ to manipulate and simplify expressions containing rational exponents
- apply exponent rules to simplify expressions with rational exponents.

Investigation

Using technology, graph the equations from List 1 and List 2 on one graph with a window that is set to $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

List 1

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$y = x^8,\ y = x^7,\ y = x^6,\ y = x^5,\ y = x^4,\ y = x^3,\ y = x^2$

List 2

$y = x^{\frac{1}{8}},\ y = x^{\frac{1}{7}},\ y = x^{\frac{1}{6}},\ y = x^{\frac{1}{5}},\ y = x^{\frac{1}{4}},\ y = x^{\frac{1}{3}},\ y = x^{\frac{1}{2}}$

Comment on any patterns that you notice.

Now add $x = a$ and $y = a$ where $0.1 < a < 0.9$ to your graph.

Note the intersection points between $x = a$ and the curves from List 1 and between $y = a$ and the curves from List 2.

Do this for $2 - 3$ values of a of your own choosing.

What do you notice about the intersection points? Can you explain your observations algebraically?

Generalise your findings to $y = x^n$.

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