5.19 Maclaurin series expansions

Checklist

What you should know

By the end of this subtopic you should be able to:

- represent a function using Maclaurin series

$$f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\ldots=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k$$

- find Maclaurin series and use these for approximation for:
  - $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots=\sum_{n=0}^{\infty}\frac{x^n}{n!}$ for all x
  - $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$ for $|x|<1$
  - $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$ for all x
  - $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$ for all x
  - $\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$ for $|x|\leqslant 1$
  - $(1+x)^p=1+px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3+\cdots$ for $-1<x\leqslant 1$

- use Maclaurin series of the special functions to find other expansions using substitution, multiplication, addition, subtraction, division, composition, differentiation and integration
- use a Maclaurin series to obtain:
  - approximations of definite integrals
  - limits of the indeterminate form $\frac{0}{0}$
  - approximate solutions to differential equations.