3.18 Intersections and angles between lines and planes

Checklist

What you should know

By the end of this subtopic you would be able to:

- understand that two planes in 3D can be parallel, intersecting or coincident
- recall that there are five possible relative positions for three planes in 3D
- recall that:
  - If three planes intersect at a point, then there is a single solution when the equations of the planes are solved simultaneously
  - If at least two of three planes are all parallel, then the simultaneous equations have no solutions
  - If three planes intersect along a line, then there are an infinite number of solutions when the equations of the planes are solved simultaneously
- recall that:
  - if two planes are parallel, then their normal vectors will be parallel
  - if two planes intersect, then the angle between two planes will be equal to the acute angle between the lines parallel to their normal vectors
  - if two planes intersect, then the equation of the line along which they intersect can be found by solving the equations of the planes simultaneously
- find θ, the angle between two planes using the scalar product of the normal vectors, $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$, of the planes: $\cos\theta = \frac{|\boldsymbol{n}_1 \cdot \boldsymbol{n}_2|}{|\boldsymbol{n}_1|\,|\boldsymbol{n}_2|}$
- recognise the relationship between a plane and a line:
  - if a line is parallel to the plane, then the normal vector of the plane will be perpendicular to the direction vector of the line
  - if a line is contained in the plane, then the normal vector will be perpendicular to the direction vector of the line
- find θ, the angle between a line and a plane using $\sin\theta = \frac{|\boldsymbol{n} \cdot \boldsymbol{b}|}{|\boldsymbol{n}|\,|\boldsymbol{b}|}$, where $\boldsymbol{b}$ is the direction vector of the line and $\boldsymbol{n}$ is a normal vector to the plane.