Index

The big picture
Kepler's laws and Newton's universal law of gravitation
Activity: Solar system
Gravitational field, gravitational field lines and gravitational field strength
Gravitational potential energy and gravitational potential (HL)
Equipotential surfaces (HL)
Orbital and escape speed (HL)
Summary and key terms
Checklist
Investigation
Reflection

The big picture

? Guiding question(s)

- How are the properties of a gravitational field quantified?
- How does an understanding of gravitational fields allow for humans to explore the solar system?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

If you look up at the sky at the right time, you will see the International Space Station (ISS), which is currently orbiting the Earth at nearly 8 km s^{-1}. **Video 1** shows what astronauts on board the ISS can see.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Video 1. The view from the International Space Station.

International Mindedness

Scientists participate in international research collaborations like the ISS in order to pursue a greater understanding of science for humankind. The National Aeronautics and Space Administration (NASA) and the European Space Agency (ESA), along with Japanese, Canadian and other space agencies worked together to construct the ISS. It took over 10 years to assemble and has hosted almost 3000 experiments by researchers from 108 different countries.

When imagining the project, the NASA team (https://www.issnationallab.org/about/iss-timeline/) who initiated it asked 'what if we built a bridge, between and above all nations, to jointly discover the galaxy's great unknowns?'

The James Webb Space Telescope (JWST) was launched in 2022 and is currently orbiting the Earth at a distance of 1.5 million km. **Video 2** shows the story of the JWST.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Video 2. Exploring the Universe with the James Webb space telescope.

How do these satellites stay in such precise orbits? Why are they not drifting into space? Or falling down to the Earth over time? How far are their orbits from the surface of the Earth and what implications does this have?

What about when we do not want objects to orbit the Earth? The Voyager 1 space probe was launched in 1977, and it is still travelling away from the Earth and communicating data back to the Earth from interstellar space. What differences are there between launching a satellite and a space probe?

Creativity, activity, service

Strand: Activity
Learning outcome: Demonstrate how to initiate and plan a CAS experience

Research and create a presentation for younger students on satellites, space probes or the International space station.

Prior learning

Before you study this subtopic make sure that you understand the following:

- Vectors (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

- Circular motion (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).
    - Work done by the resultant force on a system (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)).
    - Kinetic energy, gravitational potential energy and mechanical energy (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)).

D. Fields / D.1 Gravitational fields

Kepler's laws and Newton's universal law of gravitation

D.1.1: Kepler's three laws of orbital motion D.1.2: Newton's universal law of gravitation D.1.3: Extended bodies and point masses

Learning outcomes

By the end of this section you should be able to:

- Describe Kepler's three laws of orbital motion.
- Understand Newton's universal law of gravitation and use the equation

$$F = \frac{Gm_1m_2}{r^2}.$$

- Know when extended bodies can be treated as point masses.

Planets, such as the Earth, orbit stars, such as the Sun. Stars and planets have been studied for thousands of years and are still being studied by scientists today. Satellites also orbit the Earth (**Figure 1**). What do we know about orbiting objects?

Student view

Figure 1. A satellite orbiting the Earth.

Credit: aapsky, Getty Images

Kepler's three laws of orbital motion

Johannes Kepler (1571–1630) was a German astronomer. Kepler discovered his laws of orbital motion through empirical observation of the motion of the planets across the night sky. The laws describe the orbits of planets around stars, and satellites around planets.

Kepler's first law

When you imagine a planet in orbit around a star, you might think that the planet follows a circular path. This is not always the case. Orbits can be elliptical, as shown in **Figure 2**.

A circular orbit has a single focus at the centre of the circle, but an elliptical orbit has two foci. The star is at one focus, and there is another focus with no star. No planet has an orbit that is perfectly circular.

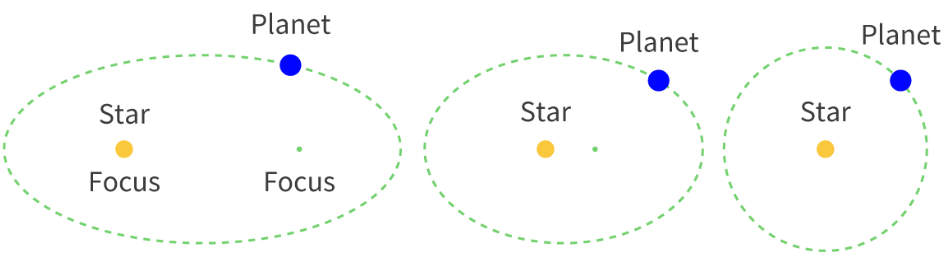


Figure 2. Elliptical and circular orbits of planets around stars.

More information for figure 2

The image depicts a diagram illustrating two types of planetary orbits around a star: elliptical and circular. It consists of three illustrations. The first shows an elliptical orbit where a planet is revolving around a star. The star is positioned at one focus point of the ellipse, and the opposite focus is unoccupied. Labels indicate 'Star' and 'Focus' at these points, with the planet labeled as 'Planet.' The second illustration is another elliptical orbit with similar labeled elements showing the planet at a different position along the orbit. Lastly, a circular orbit is shown with a single focus and a star at the center, indicating a planet moving around this circular path. All illustrations use dashed lines to represent the orbit paths. The diagram supports the concept that no planet has a perfectly circular orbit, aligning with the text that mentions a circular orbit has a single central focus, contrasting with elliptical orbits having two foci.

[Generated by AI]

Kepler's first law of orbital motion states that:

- Each planet orbits the Sun along an elliptical path. The Sun is located at a focus of the elliptical orbit.

This means that for elliptical orbits, the distance between the two objects changes constantly. (However, the sum of the distances from the planet to the two foci is constant.) For circular orbits, the distance between the two objects stays the same.

Study skills

When performing calculations in DP physics, you will only need to consider circular orbits, not elliptical ones. However, in qualitative discussions, you may need to talk about elliptical orbits.

Kepler's second law

Look at the simulation in **Interactive 1**. Can you deduce what Kepler's second law of orbital motion is?

Select the 'Show Kepler's 2nd Law Trace' box. Look at the shaded segments as the planet orbits the Sun. What do you think the segments show? Try making the orbit circular. What do the shaded segments show now?

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Interactive 1. Kepler's second law simulation.

Credit: Tom Walsh

More information for interactive 1

An interactive simulation, Kepler's second law, demonstrates Kepler's Second Law of planetary motion. The interactive allows users to adjust parameters such as the initial distance of the planet from the Sun, its initial speed, and the Sun's mass. The planet's orbit can be either elliptical or circular, depending on the parameters.

Selecting the option, Show Kepler's 2nd Law Trace, activates a series of black triangular segments, illustrating the areas swept out by the line connecting the planet to the Sun over equal time intervals. These segments change in shape depending on the planet's position in its orbit. When the planet is farther from the Sun, the segment is longer and narrower, indicating a slower orbital speed. When the planet is closer to the Sun, the segment becomes shorter and wider, showing that the planet moves faster in that region. This variation in speed follows from the conservation of angular momentum.

By adjusting the initial parameters, users can observe how different orbital shapes influence the visualization of Kepler's Second Law. If the orbit is nearly circular, the segments appear more uniform in size, indicating a relatively constant orbital speed. If the orbit is highly elliptical, the variation in segment size becomes more pronounced, with the planet moving much faster near its closest approach to the Sun.

The simulation provides an interactive way to explore the relationship between a planet's distance from the Sun and its speed, reinforcing the principle that planetary motion is governed by fundamental physical laws.

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

The simulation in **Interactive 1** shows a line connecting the planet and the star around which it orbits. In a given period of time, the line sweeps through an area of space, A:

- The further the planet is from the star, the longer the line.
- The slower the speed of the orbit, the narrower the area.

Kepler discovered through measurements that in a given time, area A is always constant (**Figure 3**).

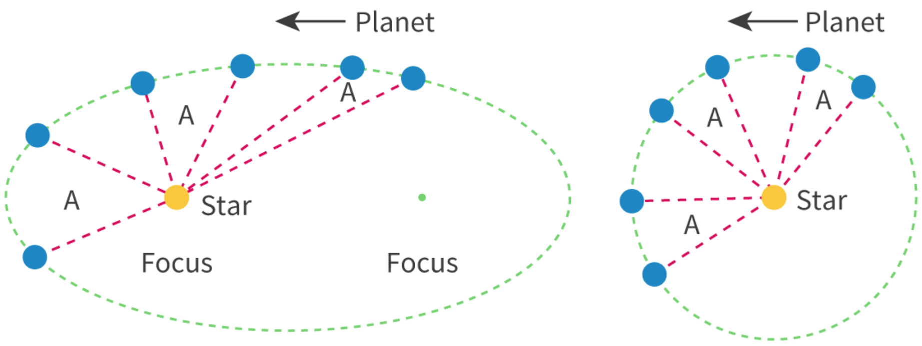


Figure 3. Kepler's second law for an elliptical orbit and a circular orbit.

More information for figure 3

The image contains two diagrams illustrating Kepler's second law, which depicts the equal-area law for planetary motion. On the left, the diagram shows an elliptical orbit with a star labeled at one focus. Dotted green lines outline the orbit, and red dashed lines connect the star to several positions of a planet along the orbit. These red lines segment the ellipse into sections, each labeled with 'A', demonstrating that the area swept by the planet in equal time intervals is the same. A dotted line labeled 'Focus' indicates the presence of another focal point of the ellipse.

On the right, a similar representation is provided for a circular orbit. The circular orbit has a star at its center, with the same configuration of red dashed lines and labeled sections 'A', illustrating the equal-area principle in a circular path.

The image highlights the concept of constant areal speed in both elliptical and circular orbits, crucial to understanding Kepler's second law.

[Generated by AI]

Kepler's second law of orbital motion states that:

Student view

- The imaginary line joining the Sun and a planet sweeps equal areas of space in equal time intervals as the planet follows its orbit.

This means that for elliptical orbits, the speed of the planet changes. The speed is fastest when the planet is the closest to the Sun and slowest when the planet is furthest from the Sun. For circular orbits, the speed of a planet at a given orbital radius is constant.

Kepler's third law

Kepler's third law of orbital motion states that:

- The square of the orbital period of a planet is directly proportional to the cube of the average distance of the planet to the Sun.

This is shown mathematically as:

$$T^2 \propto r^3$$

where T is the orbital period in seconds (s) and r is the distance of the planet to the Sun in metres (m).

Worked example 1

Planet A orbits a star with an orbital period T at an orbital radius r.

Planet B orbits the same star with an orbital period $\frac{T}{4}$.

Determine the orbital radius of planet B.

The orbital period of planet A is $T_{\mathrm{A}} = T$

The orbital radius of planet A is $r_{\mathrm{A}} = r$

The orbital period of planet B is $T_{\mathrm{B}} = \frac{T}{4}$

The orbital radius of planet B is r_{B}

By Kepler's third law:

$$T_{\mathrm{A}}^2 \propto r_{\mathrm{A}}^3$$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$T_{\mathrm{B}}^2 \propto r_{\mathrm{B}}^3$$

Since both planets orbit the same star, the proportionality constant is the same in the equations for T_{A}^2 and T_{B}^2:

$$\frac{T_{\mathrm{A}}^2}{T_{\mathrm{B}}^2} = \frac{r_{\mathrm{A}}^3}{r_{\mathrm{B}}^3}$$

$$\begin{aligned}\frac{r_{\mathrm{A}}^3}{r_{\mathrm{B}}^3} &= \frac{T_{\mathrm{A}}^2}{T_{\mathrm{B}}^2}\\ &= \frac{T^2}{\left(\frac{T}{4}\right)^2}\\ &= 16\end{aligned}$$

Hence

$$r_{\mathrm{B}}^3 = \frac{r_{\mathrm{A}}^3}{16}$$

$$\begin{aligned}r_{\mathrm{B}} &= \sqrt[3]{\frac{r^3}{16}}\\ &= \frac{r}{\sqrt[3]{16}}\end{aligned}$$

Theory of Knowledge

Laws of physics are statements that can be disproved or refined to better describe the universe. Because we call them ‘laws’, we often think of them as absolute rules that the universe obeys. Why do we call them ‘laws’? How does this terminology affect the perception of knowledge claims in the natural sciences?

Newton’s universal law of gravitation

Use the simulation in **Interactive 2** to investigate Newton’s universal law of gravitation.

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Interactive 2. Newton's universal law of gravitation simulation.

More information for interactive 2

This interactive simulation, Newton's universal law of gravitation, explores Newton's universal law of gravitation. The simulation allows users to manipulate the masses of two objects and adjust the distance between them to observe how the gravitational force changes.

The objects are represented as spheres with humanoid figures pulling them. A ruler is placed between them to measure their separation. The force acting on each mass is displayed with arrows, and the numerical values are given in scientific notation. Selecting the "Constant Size" option keeps the radius of the spheres unchanged, adjusting their density instead of their size when their mass changes.

Users can experiment by setting the masses to specific values and adjusting their separation. For example, setting both masses to 100 kilograms and placing them 10 meters apart shows a gravitational force of $F = 6.76 \times 10^{-9} N$ acting on each. Reducing the distance to 5 meters increases the magnitude of the force to $F = 2.27 \times 10^{-8} N$, while increasing the distance decreases the gravitational force, illustrating the inverse-square law. The force is proportional to the product of the masses and inversely proportional to the square of the distance between them.

By doubling one mass, users can observe how the force changes compared to the initial value. When one of the masses now sets to 200 kg, and the distance of separation between the masses sets to 10 meters. The new force will be $F = 1.34 \times 10^{-8} N$. When the distance of separation decreased to 5 meters, the force will become $F = 5.13 \times 10^{-8} N$.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

When both the masses are set to 200 kg, and the distance of separation set to 10 meters, the new force becomes $F = 2.67 \times 10^{-8} N$. If their distance is halved or set to 5 meters, the new force will become $F = 1.07 \times 10^{-7} N$. That is, when doubling both masses further increases the gravitational attraction and also when the distance between the objects is doubled, the force decreases significantly, confirming the mathematical relationship in Newton's law.

The simulation provides an interactive way to understand the effects of mass and distance on gravitational attractive force between two objects. It reinforces Newton's third law by showing that the force exerted by one object on the other is equal in magnitude and opposite in direction, regardless of their masses.

Select the 'Constant Size' box, which keeps the radius of each object constant when its mass changes (by changing the density instead of the size). In the 'Force Values' box at the bottom right, select 'Scientific Notation'.

Compare the two forces. Which of Newton's laws of motion describes their relationship?

Set the mass of each object to 100 kg, using the sliders. Move the centres of the objects to 10 m apart, by clicking and dragging them. Note the force on each object.

Now change the separation to 5 m. What do you notice about the forces? Try a separation of 2.5 m. Now experiment with doubling the mass of one object, and then doubling the masses of both. (It is important to change only one variable at a time. Why?) Compare the effects of: doubling the mass of one object; doubling the mass of both objects; doubling the distance between them.

Which of the following statements are correct for the two masses?

A. $F \propto m_1$

B. $F \propto \frac{1}{m_1}$

C. $F \propto m_1 m_2$

D. $F \propto \frac{m_1}{m_2}$

E. $F \propto r$

F. $F \propto r^2$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

G. $F \propto \frac{1}{r}$

H. $F \propto \frac{1}{r^2}$

The correct statements are A, C and H: the force is directly proportional to m_1 and to m_2, and it is inversely proportional to r^2.

Newton described these relationships in his universal law of gravitation:

- Every object in the Universe attracts every other object with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the objects.

Two variations of the equation for Newton's universal law of gravitation are shown in **Table 1** and **Table 2**.

Table 1. Equation for Newton's universal law of gravitation.

Equation	Symbols	Units
$F = G\frac{m_1 m_2}{r^2}$	F = force	newtons (N)
	G = gravitational constant (6.67×10^{-11} Nm^2 kg^{-2})	newton metres squared per kilogram squared (Nm^2 kg^{-2})
	m_1 and m_2 = masses of the two bodies	kilograms (kg)
	r = distance between the centres of the two bodies	metres (m)

Note that Newton's universal law of gravitation is also frequently written as:

Table 2. Variation of the equation for Newton's universal law of gravitation.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Equation	Symbols	Units
$F = G\frac{Mm}{r^2}$	F = force	newtons (N)
	G = gravitational constant (6.67×10^{-11} N m^2 kg^{-2})	newton metres squared per kilogram squared (N m^2 kg^{-2})
	M and m = masses of the two bodies	kilograms (kg)
	r = distance between the centres of the two bodies	

This law can be described as an 'inverse square law' because one quantity is inversely proportional to the square of another. There are many inverse square laws in physics. Another example is the relationship between the distance of an observer from a light bulb and the observed intensity (the power arriving per square metre, which we perceive as 'brightness'). The intensity is inversely proportional to the square of the distance.

Nature of Science

Aspect: Measurements

Physics uses several constants such as the gravitational constant G. The numerical values of the constants are significant because they link key concepts. Some of these constants are used to define the SI units used for measurements around the world. This means that no matter where you are in the Universe, you can use these constants to make very precise measurements.

The *r* value in Newton's universal law of gravitation refers to the distance from the centre of one body to the centre of the other body. Usually, we can approximate these bodies as point masses, meaning that their volume is negligible. We can do this if they have a uniform density or their separation is very large.

Theory of Knowledge

Simplifications and approximations are very often used in physics. For example, extended bodies are often approximated as point masses. To what extent do these approximations impact the validity of the knowledge produced?

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Worked example 2

Two objects of masses m_1 and m_2 are a distance *r* apart.

They experience a gravitational force of attraction *F*,

Determine the magnitude of the gravitational force of attraction between the two objects in terms of *F* when the distance between them is:

1. $\frac{r}{2}$
2. $2r$

The force of gravitational attraction is inversely proportional to the square of the distance between two objects:

$$F_r \propto \frac{1}{r^2}$$

1. When the distance changes from *r* to $\frac{r}{2}$, the force changes from:

$$F_r \propto \frac{1}{r^2}$$

to

$$F_{\frac{r}{2}} \propto \frac{1}{\left(\frac{r}{2}\right)^2}$$

$$F_{\frac{r}{2}} \propto \frac{4}{r^2}$$

hence

$$F_{\frac{r}{2}} \propto 4F_r$$

The force of attraction between the two objects is four times larger than the initial value when the distance between them is halved.

Student view

2. When the distance changes from *r* to 2*r*, the force changes from:

$$F_r \propto \frac{1}{r^2}$$

to

$$F_{2r} \propto \frac{1}{(2r)^2}$$

$$F_{2r} \propto \frac{1}{4r^2}$$

hence

$$F_{2r} = \frac{1}{4} F_r$$

The force of attraction between the two objects is a quarter of its initial value when the distance between them is doubled.

Study skills

Sometimes, you are given the distance between an object and the **surface** of a planet. In this case, you need to add this distance to the radius of the planet in order to get the *r* value to use in the universal law of gravitation equation.

Figure 4 shows Newton's universal law of gravitation for the Earth and the Moon. The Earth attracts the Moon with a force *F*, and the Moon attracts the Earth with the same force *F*.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

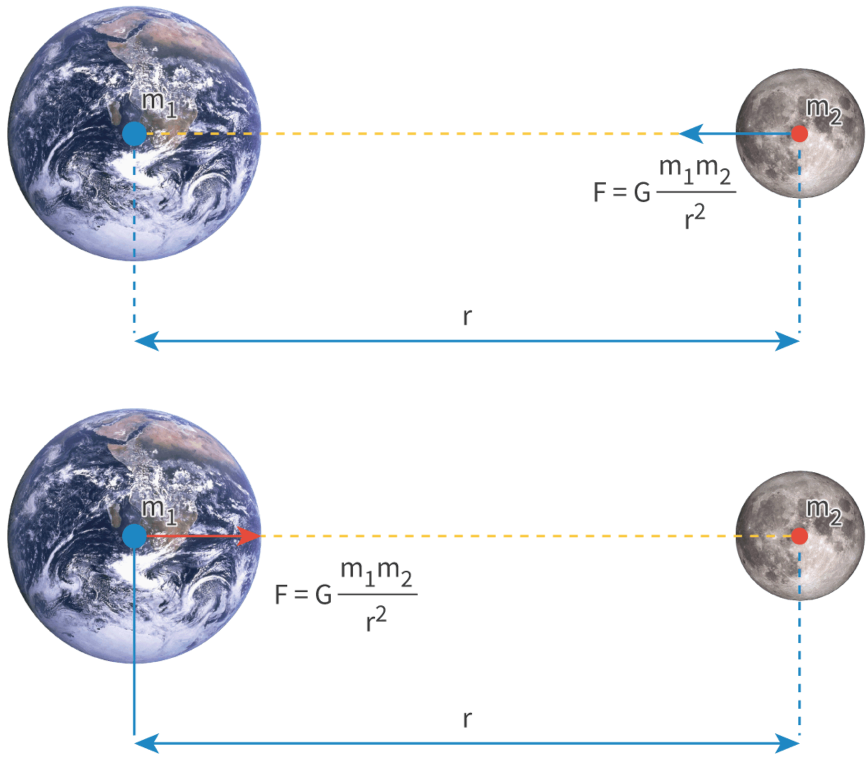


Figure 4. The Earth attracts the Moon with force F, and the Moon attracts the Earth with force F.

Credits: NASA (Earth image); Sanja Baljkas, Getty Images (Moon image)

More information for figure 4

This diagram illustrates Newton's universal law of gravitation applied to Earth and the Moon. There are two main components depicted: the Earth on the left and the Moon on the right. In both parts of the diagram, Earth is labeled with mass m1, and the Moon is labeled with mass m2. The arrows indicate the gravitational forces between them. Line segments with visible lengths represent the distance r between Earth and the Moon.

In the top part of the diagram, the Moon pulls on the Earth with a force labeled F, directed towards the Moon. The formula F = G * (m1 * m2) / r^2 is displayed next to the Moon.

In the bottom part of the diagram, the Earth exerts an equivalent force F on the Moon, directed towards Earth. The distance r is measured similarly, and the same gravitational formula is displayed next to Earth.

Each mass and force has a corresponding color-coded point or arrow representing the direction of forces and distances.

[Generated by AI]

Student view

Worked example 3

Determine the force of gravitational attraction between the Sun and the Earth.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The mass of the Sun is:

$$m_{\text{Sun}} = 1.99 \times 10^{30} \text{ kg}$$

The mass of the Earth is:

$$m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

The distance between the Sun and the Earth is:

$$r_{\text{Sun-Earth}} = 150 \times 10^{6} \text{ km}$$

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$m_{\text{Sun}} = 1.99 \times 10^{30}$ kg $m_{\text{Earth}} = 5.97 \times 10^{24}$ kg $r_{\text{Sun-Earth}} = 150 \times 10^{6}$ km $= 1.50 \times 10^{11}$ m
Step 2: Write out the equation.	$F = G\frac{Mm}{r^2}$
Step 3: Substitute the values given.	$F = 6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30} \times 5.97 \times 10^{24}}{\left(1.50 \times 10^{11}\right)^2}$ $= 3.52 \times 10^{22}$ N
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 3.5 \times 10^{22}$ N (2 s.f.)

Any orbiting object is kept in its orbit by the force of gravitational attraction (**Figure 5**). This force provides the centripetal force that keeps the orbiting object in its circular trajectory (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)).

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

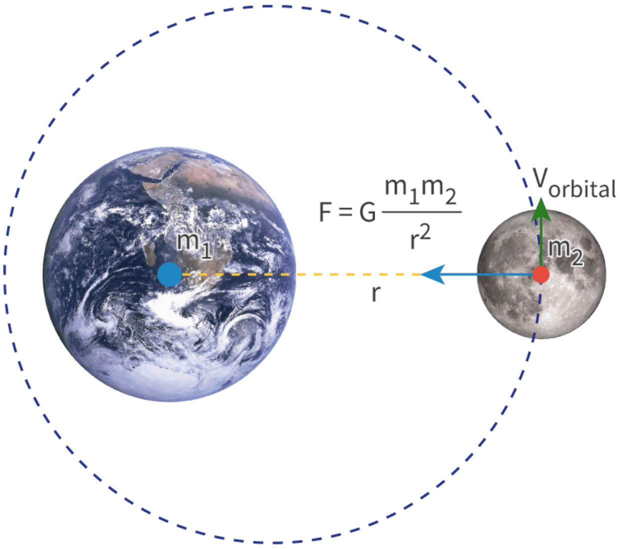


Figure 5. The Earth attracts the Moon with force F.

Credits: NASA (Earth image); Sanja Baljkas, Getty Images (Moon image)

More information for figure 5

The diagram illustrates the gravitational attraction between the Earth and the Moon. It depicts the Earth and the Moon with respective masses labeled as 'm1' and 'm2'. A force line 'F' is drawn between the two celestial bodies and is described by the equation F = G * (m1*m2) / r^2, indicating the gravitational force. The distance 'r' between them is marked, representing the radius of the Moon's orbit around Earth. An additional arrow at the Moon labeled 'Vorbit' suggests the orbital velocity direction. This diagram visually demonstrates how gravity acts as the centripetal force that keeps the Moon in orbit around the Earth.

[Generated by AI]

Making connections

The principles of circular motion are covered in subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/).

How is uniform circular motion like—and unlike—real-life orbits?

Student
view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

We can derive an equation for the orbital period, T, which is the time it takes a body of mass m to complete one orbit of another body of mass M, at a constant speed, v. The distance between the centres of the two bodies is r.

Equate the gravitational force of attraction between the two masses and the centripetal force acting on the orbiting mass:

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

The speed of mass m is given by the circumference of the circular trajectory the mass moves along per time it takes to make one full rotation around mass M (orbital period T).

This means that the speed is given by:

$$v = \frac{2\pi r}{T}$$

Substituting this value into the first equation, and cancelling values:

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}$$

Rearranging to make T^2 the subject:

$$T^2 = \frac{4\pi^2}{GM}r^3$$

This is Kepler's third law:

$$T^2 \propto r^3$$

The equation for orbital period is not included in the DP physics data booklet, but you should be able to derive it. It can also be written as:

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

Worked example 4

Student view

Mars orbits the Sun at an average distance of 228×10^6 km.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Given that the mass of the Sun is 1.99×10^{30} kg, show that the orbital period of Mars is 687 days.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$r = 228 \times 10^6 \text{ km}$ $= 2.28 \times 10^{11} \text{ m}$ $M = 1.99 \times 10^{30} \text{ kg}$
Step 2: Write out the equation.	$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$
Step 3: Substitute the values given.	$T = \sqrt{\frac{4 \times \pi^2 \times (2.28 \times 10^{11})^3}{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}}$ $= 59\,381\,203 \text{ s}$
Step 4: Calculate the period in days.	$1 \text{ day} = 24 \times 60 \times 60$ $= 86\,400 \text{ s}$ $T = \frac{59\,381\,203}{86\,400}$ $= 687.28 \text{ days}$
Step 5: State the answer with appropriate units and the number of significant figures used in rounding	$= 687 \text{ days } (3 \text{ s.f.})$

Work through the activity in the next section to check your understanding of Kepler's laws of orbital motion.

5 section questions

Question 1

SL HL Difficulty:

True or false?

Kepler's second law of orbital motion applied to a star-planet system means that the speed of a planet moving along an elliptical path is slowest when it is the closest to the star and fastest when the planet is furthest from the star.

False

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Accepted answers

False, f, false, F

Explanation

Kepler's second law states that the imaginary line joining the star and the planet sweeps out equal areas of space in equal time intervals as the planet follows its orbit. As the star is at one of the two foci of the elliptical orbit of the planet, the planet must move with a faster speed when passing closest to the star and with a slower speed when passing furthest from the star. This way, the two areas are equal.

Question 2

SL HL Difficulty:

Which statement about Newton's universal law of gravitation is correct?

1 It is applicable to all masses. ✓

2 It is equivalent to Newton's second law of motion.

3 It is only applicable to point masses.

4 It is not applicable to point masses.

Explanation

Newton's law of universal gravitation applies to all objects. It does not distinguish between point and extended masses.

Question 3

SL HL Difficulty:

Object A and object B are separated by a distance r and experience a gravitational force F.

Determine the gravitational force when their separation increases to $4r$.

1 $\frac{F}{16}$ ✓

2 $16F$

3 $4F$

4 $\frac{F}{4}$

Explanation

When the distance changes from r to $4r$ the force changes from:

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$F \propto \frac{1}{r^2}$$

to

$$F_{4r} \propto \frac{1}{(4r)^2}$$

$$F_{4r} \propto \frac{1}{16r^2}$$

$$F_{4r} = \frac{1}{16}F$$

The force of attraction between the two objects decreases 16 times when the distance between the objects increases four times.

Question 4

SL HL Difficulty:

Determine an expression for the orbital period of Mars T_M around the Sun.

The orbital radius of Earth is r_E.

The orbital period of Earth around Sun is T_E.

The orbital radius of Mars is $r_M = \frac{3}{2}r_E$

1 $T_M = T_E\sqrt{\left(\frac{3}{2}\right)^3}$ ✓

2 $T_M = \sqrt{\frac{3}{2}T_E}$

3 $T_M = T_E\left(\frac{3}{2}\right)^3$

4 $T_M = \sqrt{\left(\frac{2}{3}\right)^3 T_E}$

Explanation

$T_M^2 \propto r_M^3$ and $T_E^2 \propto r_E^3$

Since the proportionality constant is the same for T_M^2 and T_E^2 and is equal:

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

$$\frac{T_{\mathrm{M}}^2}{T_{\mathrm{E}}^2}=\frac{r_{\mathrm{M}}^3}{r_{\mathrm{E}}^3}$$

$$T_{\mathrm{M}}^2=\frac{r_{\mathrm{M}}^3}{r_{\mathrm{E}}^3}T_{\mathrm{E}}^2$$

$$T_{\mathrm{M}}=\sqrt{\frac{r_{\mathrm{M}}^3}{r_{\mathrm{E}}^3}T_{\mathrm{E}}^2}$$

$$T_{\mathrm{M}}=T_{\mathrm{E}}\sqrt{\frac{r_{\mathrm{M}}^3}{r_{\mathrm{E}}^3}}$$

Substituting $r_{\mathrm{M}}=\frac{3}{2}r_{\mathrm{E}}$ gives:

$$T_{\mathrm{M}}=T_{\mathrm{E}}\sqrt{\frac{\left(\frac{3}{2}r_{\mathrm{E}}\right)^3}{r_{\mathrm{E}}^3}}$$

$$=T_{\mathrm{E}}\sqrt{\frac{\left(\frac{3}{2}\right)^3r_{\mathrm{E}}^3}{r_{\mathrm{E}}^3}}$$

$$=T_{\mathrm{E}}\sqrt{\left(\frac{3}{2}\right)^3}$$

Question 5

SL HL Difficulty:

Determine the force of gravitational attraction acting on Mars from the Sun.

The force of gravitational attraction from Sun on Earth is:

$$F_{\mathrm{S–E}}=3.56\times10^{22}\ \mathrm{N}$$

The ratio of orbital radii of Earth and Mars is:

$$\frac{r_{\mathrm{E}}}{r_{\mathrm{M}}}=0.66$$

The ratio of mass of Earth and mass of Mars is:

$$\frac{m_{\mathrm{E}}}{m_{\mathrm{M}}}=9.35$$

1 1.7×10^{21} N (2 s.f.) ✓

2 2.5×10^{21} N (2 s.f.)

3 1.6×10^{21} N (2 s.f.)

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

4 1.4×10^{23} N (2 s.f.)

Explanation

The gravitational force between Sun and Mars is:

$$F_{\mathrm{S-M}} = G\frac{M_{\mathrm{S}}m_{\mathrm{M}}}{r_{\mathrm{M}}^2}$$

The gravitational force between Sun and Earth is:

$$F_{\mathrm{S-E}} = G\frac{M_{\mathrm{S}}m_{\mathrm{E}}}{r_{\mathrm{E}}^2}$$

The ratio of forces is:

$$\begin{aligned}\frac{F_{\mathrm{S-M}}}{F_{\mathrm{S-E}}} &= \frac{G\frac{M_{\mathrm{S}}m_{\mathrm{M}}}{r_{\mathrm{M}}^2}}{G\frac{M_{\mathrm{S}}m_{\mathrm{E}}}{r_{\mathrm{E}}^2}} \\ &= \frac{\frac{m_{\mathrm{M}}}{r_{\mathrm{M}}^2}}{\frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^2}} \\ &= \frac{m_{\mathrm{M}}r_{\mathrm{E}}^2}{m_{\mathrm{E}}r_{\mathrm{M}}^2} \\ &= \frac{m_{\mathrm{M}}}{m_{\mathrm{E}}}\left(\frac{r_{\mathrm{E}}}{r_{\mathrm{M}}}\right)^2 \\ &= \left(\frac{m_{\mathrm{E}}}{m_{\mathrm{M}}}\right)^{-1}\left(\frac{r_{\mathrm{E}}}{r_{\mathrm{M}}}\right)^2\end{aligned}$$

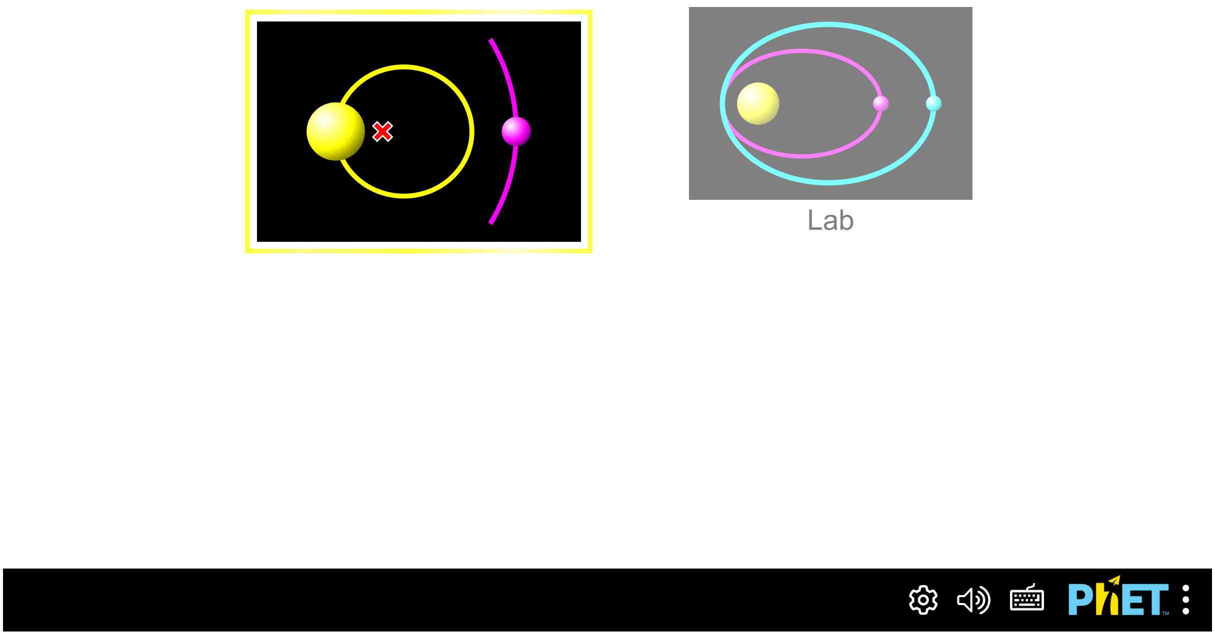
$$\begin{aligned}F_{\mathrm{S-M}} &= \left(\frac{m_{\mathrm{E}}}{m_{\mathrm{M}}}\right)^{-1}\left(\frac{r_{\mathrm{E}}}{r_{\mathrm{M}}}\right)^2 F_{\mathrm{S-E}} \\ &= 9.35^{-1} \times 0.66^2 \times 3.56 \times 10^{22} \\ &= 1.7 \times 10^{21}\ \mathrm{N}\end{aligned}$$

D. Fields / D.1 Gravitational fields

Activity: Solar system

D.1.1: Kepler's three laws of orbital motion

Student
view



Activity

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills — Asking questions and framing hypotheses based upon sensible scientific rationale
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

In the simulation, select the ‘Lab’ tab, then select the ‘Path’ and ‘Center of Mass’ boxes.

1. Choose one of these sets of orbiting objects from the drop-down menu:
    - Sun, Planet
    - Sun, Planet, Moon
    - Sun, Planet, Comet
    - Ellipses
    - Hyperbolic.
2. Observe the motion of the objects by observing their trajectories and their speeds.
3. Discuss their motion in the context of Kepler’s laws of orbital motion. Consider the following questions:

Student
view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

- Are their trajectories elliptical or circular, or do they have another shape?
- Are their speeds constant or varying?
- Is there a relationship between the distance of an object from the object it is orbiting and the speed of the first object?

4. Repeat steps 2 to 4 for all the sets of orbiting objects above.

D. Fields / D.1 Gravitational fields

Gravitational field, gravitational field lines and gravitational field strength

D.1.4: Gravitational field strength at a point D.1.5: Gravitational field lines

Learning outcomes

By the end of this section you should be able to:

- Understand the concept of gravitational fields and what gravitational field lines represent.
- Understand gravitational field strength and use the equations:

$$g = \frac{F}{m} = G\frac{M}{r^2}.$$

In 2012, Felix Baumgartner jumped from a helium balloon at around 39 000 m above sea level (**Video 1**). He fell so far and so fast that he became the first person to travel faster than the speed of sound without any form of propulsion.

Baumgartner could have made that jump from anywhere around the Earth and he would have experienced the same gravitational force (see subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)). This force is due to the presence of a gravitational field. How can we quantify and measure gravitational fields, and predict their effects?

Student view

Video 1. Felix Baumgartner experienced a gravitational force due to a gravitational field.

Gravitational fields and gravitational field lines

A gravitational field is any region of space where a mass experiences a gravitational force. Gravitational fields exist around all objects with mass. You have a gravitational field and you are exerting a gravitational force on all the objects around you (it's just too small for you to notice).

- Gravitational fields act at a distance. No contact is needed for a gravitational field to act on a mass.
- Gravitational fields have infinite range. Every gravitational field extends for an infinite distance. However, the field becomes very weak at large distances away from the mass that is the source of the field.

Making connections

There are many similarities between gravitational fields and electric fields and magnetic fields (see subtopic D.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44743/)).

We use gravitational field lines to represent gravitational fields. Gravitational field lines show the direction of the gravitational force acting on a mass at that point. **Figure 1** shows the gravitational field lines for the Earth.

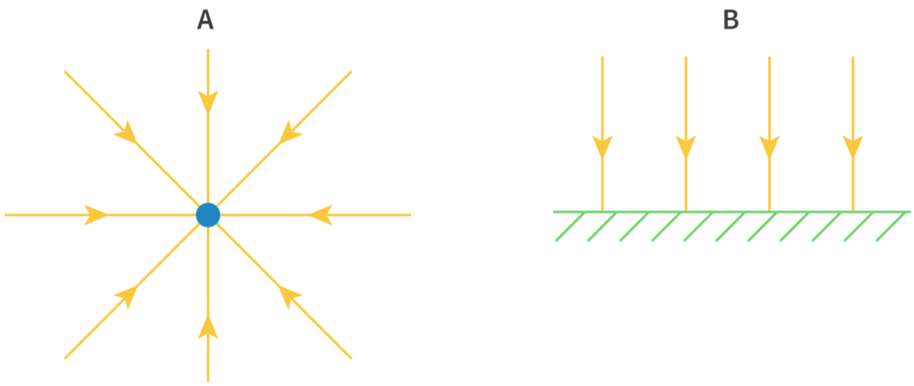


Figure 1. Gravitational field lines (A) around the Earth and (B) close to the surface of the Earth.

More information for figure 1

The image consists of two diagrams labeled A and B.

Diagram A: It shows the gravitational field lines surrounding a central point, representing Earth. The lines are depicted as arrows radiating outward in all directions from the central point, illustrating the radial nature of the gravitational field around Earth.

Diagram B: It depicts gravitational field lines close to the Earth's surface. The lines are shown as parallel arrows pointing straight downwards towards a horizontal surface, indicating that the gravitational field close to the surface is uniform.

[Generated by AI]

The gravitational field lines in **Figure 1** show that the gravitational field around the Earth is a radial field, but the gravitational field close to the surface of the Earth is a uniform field.

What do you notice about the gravitational field lines? What are the rules that they follow? Click on ‘Show or hide solution’ to see the answer.

- Gravitational field lines point towards the centre of the object.
- Gravitational field lines never cross each other.

Theory of Knowledge

Gravitational field lines are abstract mathematical constructs invented to help us understand Newton’s theory of gravity. How does the application of mathematical constructs, such as field lines, help our knowledge and understanding of real

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

phenomena?

The closer together the gravitational field lines, the stronger the gravitational field (and the stronger the force experienced by a mass). In **Figure 1A**, the lines have a greater separation at a greater distance from the Earth. This shows that the gravitational field is weaker further from the Earth.

Figure 2 shows the gravitational field lines around two equal masses. Where is the field strongest? Where is the field weakest? At what point in the field would a very small mass experience no force?

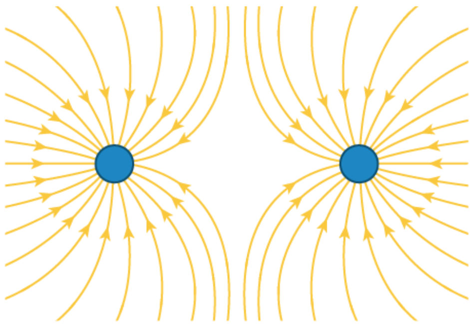


Figure 2. Gravitational field lines around two equal masses.

More information for figure 2

The image is a diagram illustrating gravitational field lines around two equal masses displayed as two blue circles. The field lines, represented by orange arrows, emanate symmetrically from each mass. They curve outwards, moving away from one mass and bending towards the other, illustrating the interaction of gravitational forces between the two masses. The concentration of arrows is denser near the masses, indicating stronger gravitational fields. In the central region between the two masses, the arrows from each mass meet and appear to fluctuate evenly, suggesting a point of equilibrium where a small mass would experience no net force. This diagram visually explains the distribution and intensity of the gravitational field in the area surrounding two equal masses.

[Generated by AI]

Study skills

When you consider the gravitational field of a planet, assume that there is a uniform field with a constant strength close to the surface of the planet. For larger distances,

Student
view

assume a radial field with varying strength.

Gravitational field strength

Gravitational field strength, g, at a point in a gravitational field is defined as the gravitational force per unit mass acting on a point mass, m.

Gravitational force, F, is given by: $F = mg$ (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)). If we rearrange this equation, we get the equation for gravitational field strength shown in **Table 1**.

Table 1. Equation for gravitational field strength.

Equation	Symbols	Units
$g = \frac{F}{m}$	g = gravitational field strength	newtons per kilogram (N kg^{-1})
	F = gravitational force	newtons (N)
	m = mass of a point mass	kilograms (kg)

The gravitational field strength (acceleration due to gravity or acceleration of free fall), g, at the surface of the Earth is 9.8 m s^{-2}. As we can see from the gravitational field lines in **Figure 1**, this value changes depending on the distance from the Earth.

The equation for gravitational force is (see section D.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/keplers-laws-and-newtons-universal-law-of-gravitation-id-46566/)):

$$F = G\frac{Mm}{r^2}$$

where M is the mass of the source of the gravitational field and m is the mass in the gravitational field.

Combining this equation with the equation for gravitational field strength gives:

$$g = \frac{GMm}{mr^2}$$

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Cancelling m, gives us an alternative equation for gravitational field strength as shown in **Table 2.**

Table 2. An alternative equation for gravitational field strength.

Equation	Symbols	Units
$g = \frac{GM}{r^2}$	g = gravitational field strength	newtons per kilogram N kg^{-1}
	G = gravitational constant (6.67×10^{-11} Nm^2 kg^{-2})	newton metres squared per kilogram squared (Nm^2 kg^{-2})
	M = mass of source of gravitational field	kilograms (kg)
	r = distance from centre of mass M	metres (m)

Worked example 1

The gravitational field strength of a planet of mass *M* is *g* at a distance of *r* from its centre.

Determine the gravitational field strength of this planet at distance:

1. $\frac{r}{2}$
2. $2r$

Gravitational field strength is inversely proportional to the square of the distance:

$$g \propto \frac{1}{r^2}$$

1. When the distance changes from *r* to $\frac{r}{2}$, the gravitational field strength changes from:

$$g_r \propto \frac{1}{r^2}$$

to

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$g_{\frac{r}{2}} \propto \frac{1}{\left(\frac{r}{2}\right)^2}$$

$$g_{\frac{r}{2}} \propto \frac{4}{r^2}$$

$$g_{\frac{r}{2}} = 4g_r$$

Gravitational field strength increases by a factor of four, when the distance from the centre of the planet is halved.

2. When the distance changes from *r* to 2*r*, the gravitational field strength changes from:

$$g_r \propto \frac{1}{r^2}$$

to

$$g_{2r} \propto \frac{1}{(2r)^2}$$

$$g_{2r} \propto \frac{1}{4r^2}$$

$$g_{2r} = \frac{g_r}{4}$$

Gravitational field strength decreases by a factor of four, when the distance from the centre of the planet is doubled.

Worked example 2

Determine the gravitational field strength at the surface of the Earth.

The mass of the Earth is:

$$M_{\mathrm{E}} = 5.97 \times 10^{24}\ \mathrm{kg}$$

The radius of the Earth is:

$$r_{\mathrm{E}} = 6400\ \mathrm{km}$$

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$M_{\mathrm{E}} = 5.97 \times 10^{24}\ \mathrm{kg}$ $r_{\mathrm{E}} = 6400\ \mathrm{km}$ $= 6.4 \times 10^{6}\ \mathrm{m}$
Step 2: Write out the equation.	
Step 3: Substitute the values given.	$g = \frac{GM}{r^2}$ $= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{\left(6.4 \times 10^{6}\right)^2}$ $= 9.7122$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 9.7\ \mathrm{ms}^{-2}$ (2 s.f.) The gravitational field strength (acceleration of free fall) at the surface of the Earth is rounded to 9.8 m s^{-2}. Because the numerical values used in the calculation are approximate, the result is different to this value, but it is in good agreement with the expected value.

We have been considering gravitational fields created by single masses, but what about a gravitational field created by two masses?

Imagine an astronaut in space in between the Earth and the Moon. The Earth exerts a gravitational force, attracting the astronaut towards the Earth, and the Moon exerts a gravitational force, attracting the astronaut towards the Moon.

Is there a point where the forces are balanced and the gravitational field strength is zero? **Figure 3** shows how to find the gravitational field strength along a straight line joining the two masses, M_1 and M_2, creating the gravitational field.

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

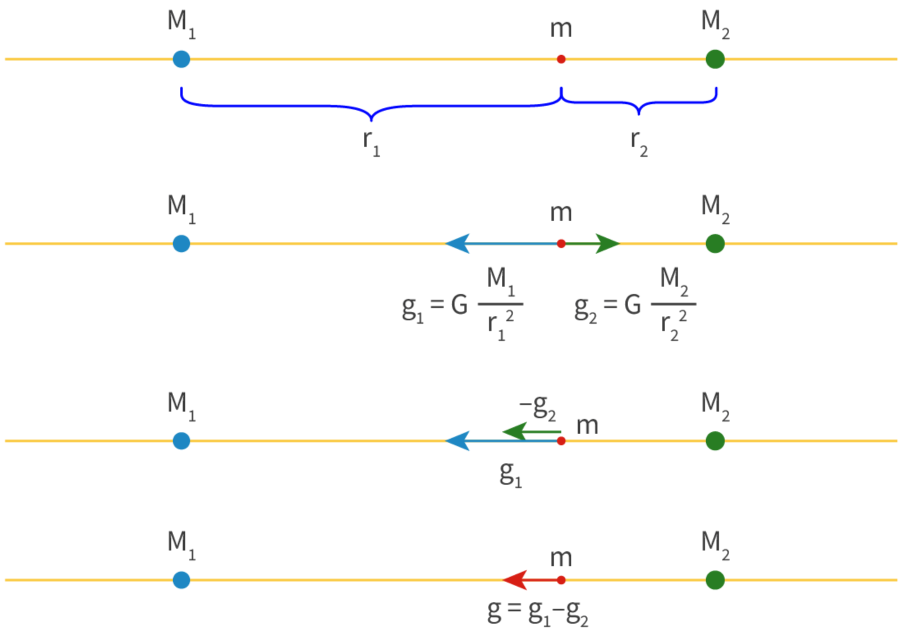


Figure 3. Determining the gravitational field strength at a point along a line joining two masses.

More information for figure 3

The diagram illustrates the gravitational field strength at a point along a line joining two masses, M1 and M2, with a smaller mass m positioned between them. The top section shows the masses M1 and M2 on a horizontal plane with the smaller mass m in the middle. Two distances, r1 and r2, are marked from m to M1 and m to M2 respectively, indicated with curved brackets.

In the middle section, the diagram details the gravitational field strengths g1 and g2, represented by arrows pointing towards M1 and M2. Formulas for g1 and g2 are shown: g1 = G M1 / r1^2 and g2 = G M2 / r2^2, where G is the gravitational constant.

The bottom section demonstrates the condition when the forces of g1 and g2 are equal, causing the resultant gravitational field strength to be zero at the position of the mass m. The arrows illustrate that g2 acts negatively, balancing the forces to achieve zero net field strength at m.

[Generated by AI]

Student view

Gravitational field strength is a vector, so the direction is important. In **Figure 3**, g_1 and g_2 are in opposite directions, so g_2 has been taken as negative. If the magnitudes of g_1 and g_2 are equal, there is no resultant gravitational field at that point.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

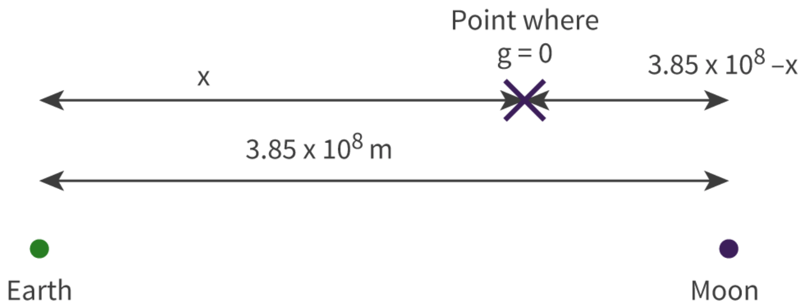
Worked example 3

Along the imaginary line connecting the Earth and the Moon, there is a point where the resultant gravitational field strength is zero.

Determine the distance of this point from the centre of the Earth.

The mass of Earth is 6.0×10^{24} kg
The mass of Moon is 7.3×10^{22} kg
The distance from Earth to Moon is 3.85×10^{8} m

Sketch a diagram to show the distances involved.



The point at which the resultant gravitational field strength is zero is the point where the gravitational field strengths from the Moon and Earth are equal in magnitude:

$$G\frac{M_{\text{Earth}}}{x^2} = G\frac{M_{\text{Moon}}}{\left(3.85 \times 10^8 - x\right)^2}$$

$$\sqrt{\frac{M_{\text{Earth}}}{M_{\text{Moon}}}} = \frac{x}{\left(3.85 \times 10^8 - x\right)}$$

$$\sqrt{\frac{6.0 \times 10^{24}}{7.3 \times 10^{22}}} = \frac{x}{\left(3.85 \times 10^8 - x\right)}$$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$\frac{x}{\left(3.85\times10^{8}-x\right)}=9.0660$$

$$3.4904\times10^{9}-9.0660x=x$$

$$3.4904\times10^{9}=x(1+9.0660)$$

$$\begin{aligned}x&=\frac{3.4904\times10^{9}}{10.0660}\\&=3.46751\times10^{8}\\&=3.5\times10^{8}\text{ m (2 s.f.)}\end{aligned}$$

This is more than half the distance between the Earth and the Moon, but less than the total distance, so this seems to be an appropriate order of magnitude.

Work through the activity to check your understanding of gravitational field strength.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

You are going to look at the change in gravitational field strength as the distance from the surface of the Earth increases.

The gravitational field strength at the surface of the Earth is $g=G\frac{M_{\text{Earth}}}{r^{2}_{\text{Earth}}}$

1. Find an equation for the gravitational field strength *g* at a height *h* above the surface of the Earth.
2. Here are some heights above the surface of the Earth:

   - Burj Khalifa, Dubai (the tallest building on Earth), *h* = 820 m
   - Mount Everest (highest mountain on the Earth), *h* = 8 849 m
   - Top of Earth's stratosphere, *h* = 50 km
   - International Space Station, *h* = 410 km

   Express the value of each height as a fraction of the radius of the Earth $(6.4\times10^{6}\text{ m})$

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

For example, the Burj Khalifa, h = 820 m:

$$\frac{820}{6.4 \times 10^6} = 1.28 \times 10^{-4}$$

$$820 = 1.28 \times 10^{-4} r_{\text{Earth}}$$

3. Use your equation from part 1. to determine the gravitational field strength for each height in 2. Click 'Show answer' to see the answers.

   (mass of the Earth = 6.0×10^{24} kg)

Burj Khalifa: 9.8 ms^{-2}

Mount Everest: 9.7 ms^{-2}

Top of Earth's stratosphere: 9.6 ms^{-2}

International Space Station: 8.6 ms^{-2}

1. Compare your results with the gravitational field strength at the surface of the Earth, 9.8 ms^{-2}. What do you notice?

5 section questions

Question 1

SL HL Difficulty:

A gravitational field is a region of space where a 1 mass ✓ experiences a gravitational 2 force ✓ .

Accepted answers and explanation

#1 mass

point mass

#2 force

General explanation

A gravitational field is a region of space where a mass experiences a gravitational force.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Question 2

SL HL Difficulty:

Gravitational field lines show that the gravitational field around the Earth is a 1 radial ✓ field, but near the surface of the Earth, it is a 2 uniform ✓ field.

The closer together the gravitational field lines, the stronger the 3 gravitational ... ✓ . The gravitational field is 4 stronger ✓ closer to the Earth.

Accepted answers and explanation

#1 radial

#2 uniform

#3 gravitational field

#4 stronger
larger
greater
bigger

General explanation

Gravitational field lines show that the gravitational field around the Earth is a radial field, but near the surface of the Earth, it is a uniform field.

The closer together the gravitational field lines, the stronger the gravitational field. The gravitational field is stronger closer to the Earth.

Question 3

SL HL Difficulty:

Object A of mass *M* has a gravitational field strength *g* at a distance *r* from its centre.

Object B has a gravitational field strength 4*g* at the same distance *r* from its centre.

What is the mass of object B?

1 $4M$ ✓

2 M

3 $2M$

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

4 $\frac{M}{4}$

Explanation

For object A, the gravitational field strength is *g* at a distance *r* from the centre of mass *M*:

$$g_{\text{A}} = G\frac{M}{r^2}$$

For object B, the gravitational field strength is 4*g* at a distance *r* from the centre of unknown mass X:

$$\begin{aligned} g_{\text{B}} &= 4g_{\text{A}} \\ &= G\frac{M}{r^2} \end{aligned}$$

$$g_{\text{B}} = 4g_{\text{A}}$$

$$\begin{aligned} \frac{g_{\text{A}}}{g_{\text{B}}} &= \frac{g_{\text{A}}}{4g_{\text{A}}} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \frac{g_{\text{A}}}{g_{\text{B}}} &= \frac{g_{\text{A}}}{4g_{\text{A}}} \\ &= \frac{G\frac{M}{r^2}}{G\frac{X}{r^2}} \\ &= \frac{M}{X} \end{aligned}$$

$$\frac{M}{X} = \frac{1}{4}$$

$$X = 4M$$

Question 4

SL HL Difficulty:

What is the change in gravitational field strength, when the distance from a mass *M* changes to a quarter of its initial value?

1 It increases by a factor of 16 ✓

2 It increases by a factor of 4

3 It decreases by a factor of 16

4 It decreases by a factor of 4

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Explanation

Gravitational field strength at a distance *r* away from mass *M* is:

$$g_{\mathrm{r}} = G\frac{M}{r^2}$$

When *r* changes to $\frac{r}{4}$, g_{r} changes to:

$$\begin{aligned} g_{\mathrm{r}} &= G\frac{M}{\left(\frac{r}{4}\right)^2} \\ &= \frac{16GM}{r^2} \\ &= 16g_{\mathrm{r}} \end{aligned}$$

Question 5

SL HL Difficulty:

Determine an expression in terms of the distance, *d*, between the centres of the Earth and the Moon for the point between the Earth and the Moon where the gravitational field strength is zero.

The mass of the Moon is:

$$M_{\mathrm{Moon}} = M$$

The mass of the Earth is:

$$M_{\mathrm{Earth}} = 81M$$

The distance between the centres of the Earth and the Moon is:

$$r_{\mathrm{Earth-Moon}} = d$$

1 $\frac{9}{10}d$ ✓

2 d

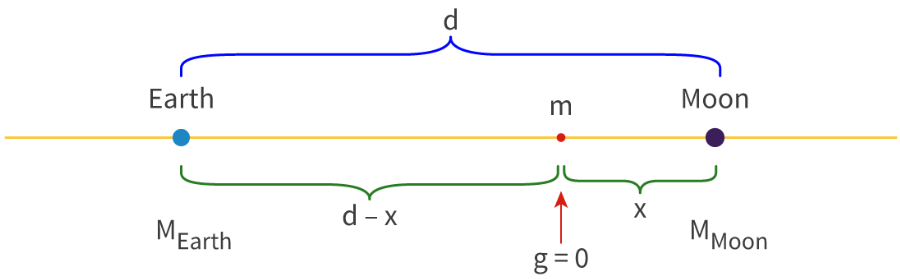
3 $\frac{80}{81}d$

4 $\frac{1}{2}d$

Explanation

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c



More information

At the point where the gravitational field strength is zero, the difference between the gravitational field strength of the Earth and the gravitational field strength of the Moon is zero:

$$g_{\text{Earth}-\text{Moon}} = 0$$

The gravitational field strength of the Earth is:

$$\begin{aligned} g_{\text{Earth}} &= G\frac{M_{\text{Earth}}}{(d-x)^2} \\ &= G\frac{81M}{(d-x)^2} \end{aligned}$$

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-field-gravitational-field-lines-and-gravitational-field-strength-id-46568/print/) Assign

The gravitational field strength of the Moon is:

$$\begin{aligned} g_{\text{Moon}} &= G\frac{M_{\text{Moon}}}{x^2} \\ &= G\frac{M}{x^2} \end{aligned}$$

When the gravitational field strength is zero:

$$g_{\text{Earth}-\text{Moon}} = 0$$

so

$$G\frac{81M}{(d-x)^2} - G\frac{M}{x^2} = 0$$

$$G\frac{81M}{(d-x)^2} = G\frac{M}{x^2}$$

$$\frac{81}{(d-x)^2} = \frac{1}{x^2}$$

$$81x^2 = (d-x)^2$$

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

$$9x = d - x$$

$$10x = d$$

$$x = \frac{d}{10}$$

$$\begin{aligned} d - x &= d - \frac{d}{10} \\ &= \frac{9}{10}d \end{aligned}$$

D. Fields / D.1 Gravitational fields

Gravitational potential energy and gravitational potential (HL)

D.1.6: Gravitational potential energy of a system (HL) D.1.7: Gravitational potential energy for a two-body system (HL)
D.1.8: Gravitational potential (HL) D.1.10: Work done in moving a mass in a gravitational field (HL)

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-potential-energy-and-gravitational-potential-hl-id-46569/print/) Assign

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Understand the concept of gravitational potential energy and use the equation:

$$E_{\mathrm{p}} = -G\frac{Mm}{r}$$

- Understand the concept of gravitational potential and use the equation:

$$V_{\mathrm{g}} = -G\frac{M}{r}$$

- Use the equation for work done moving a mass in a gravitational field:

$$W = m\Delta V_{\mathrm{g}}$$

To launch a rocket with fuel and a payload into space requires large amounts of energy and is costly (approximately $4000 per kilogram). It is so expensive that scientists and agencies have investigated building ‘space elevators’.

Student
view

Video 1. Space elevators.

How much energy is needed for a rocket to overcome the pull of the Earth's gravitational field?

Gravitational potential energy

On Earth, there is an increase in gravitational potential energy if we raise an object above ground level. This becomes more obvious if we drop the object: as it falls, gravitational potential energy transfers to kinetic energy (and thermal energy).

So is the gravitational potential energy of the object zero when it reaches the ground? At first that might seem reasonable – but what if the object falls into a hole in the ground? We would expect its gravitational potential energy to be even lower at the bottom of the hole. If it falls down a deeper hole, its gravitational potential energy will decrease further.

To make sense of this, we say that when an object is infinitely far from any other masses (so that it is not affected by any gravitational field), its gravitational potential energy is zero. When the object is in a gravitational field, its energy is less than this – so it is negative.

The gravitational potential energy of a pair of masses, such as the Earth and Moon, is the work that would be done in bringing them from infinite separation to their current positions. Since their gravitational potential energy would *decrease* during this process, positive work would not be needed. The work done is described as negative, and so the potential energy is negative. The same thinking applies to groups of more than two objects.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Theory of Knowledge

In mathematics, 'infinity' means that something is limitless. Although paradoxical, this is accepted in the ideal and abstract world of mathematics. Does infinity exist in the real world ruled by laws of physics?

We can derive a formula for the gravitational potential energy of a pair of bodies which are either points (or far enough apart to appear pointlike to each other) or spherical.

The work done when a force F moves an object a distance s is (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)):

$$W = Fs$$

Consider two small bodies of mass m_1 and m_2, a distance r apart. If m_1 is moved closer to m_2 and the change in displacement Δs is small, the work done is approximately:

$$\Delta E_{\mathrm{p}} \approx -F\Delta s$$

(It is negative because positive work would be done to increase the separation, but in this case we are decreasing the separation.)

Combining this with the equation for the gravitational force between two spherical or point masses (see section D.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-field-gravitational-field-lines-and-gravitational-field-strength-id-46568/)) gives:

$$\Delta E_{\mathrm{p}} \approx -G\frac{m_1 m_2}{r^2}\Delta s$$

An exact equation for the gravitational potential energy can be obtained using a mathematical technique called integral calculus, or integration. That method is beyond the scope of the DP Physics course, but the result is shown in **Table 1**. It is valid for bodies that are points or spheres.

Table 1. Equation for gravitational potential energy.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Equation	Symbols	Units
$E_{\mathrm{p}} = -G\frac{m_1 m_2}{r}$	E_{p} = gravitational potential energy	joules (J)
	G = gravitational constant (6.67 × 10^{-11} m^3 kg^{-1} s^{-2})	cubic metres per kilogram per second squared (m^3 kg^{-1} s^{-2})
	m_1 and m_2 = masses of the two bodies	kilograms (kg)
	r = distance between centres of mass of the two bodies	metres (m)

Worked example 1

The ISS has a mass of 4.2 × 10^3 kg.

The ISS loses altitude at a rate of around 100 m every day from its initial height of 4.0 × 10^2 km above sea level.

Deduce how much gravitational potential energy is lost by the ISS in a year.

Mass of Earth = 6.0 × 10^{24} kg
Radius of Earth = 6.4 × 10^3 km

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	mass of Earth: $m_1 = 6.0\times10^{24}\ \text{kg}$ mass of ISS: $m_2 = 4.2\times10^{3}\ \text{kg}$ radius of Earth: $r_1 = 6.4\times10^{3}\ \text{km} = 6.4\times10^{6}\ \text{m}$ height of ISS: $r_2 = 6.4\times10^{3}\ \text{km} + 4.0\times10^{2}\ \text{km} = 6.8\times10^{6}\ \text{m}$ height lost in a year: $100\times365 = 36\,500\ \text{m}$
Step 2: Write out the equation.	$E_\text{p} = -G\dfrac{m_1m_2}{r}$
Step 3: Substitute the values given.	The gravitational potential energy at the initial he ISS is: $E_\text{p1} = -6.67\times10^{-11}\times\dfrac{6.0\ \times10^{24}\times4.2}{6.8\times10^{6}} = -2.472\times10^{11}\ \text{J}$ The gravitational potential energy at the height of after a year: $E_\text{p2} = -6.67\times10^{-11}\times\dfrac{6.0\ \times10^{24}\ \times4.}{(6.8\times10^{6}-3}$ $= -2.485\times10^{11}\ \text{J}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding	The change in gravitational potential energy is: $\Delta E_p = 2.485\times10^{11} - 2.472\times10^{11} = 1.3\times10^{9}\ \text{J}\ (2\ \text{s.f.})$

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The change in gravitational potential energy of a mass as its height changes, if it is close to the Earth's surface, is $\Delta E_{\mathrm{p}} = mg\Delta h$ (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)). Why does this look so different from the equation in **Table 1**?

Remember that close to the Earth's surface, the gravitational field can be considered to be uniform. Therefore the field strength does not vary, and it equals g. The force exerted by the Earth on a mass m equals mg. When the object is moved upwards by a distance Δh, the work done is force × distance, so

$$\Delta E_p = mg\Delta h$$

Gravitational potential

Gravitational potential energy describes the energy stored by an object in a gravitational field. The gravitational potential, V_g, at a point in a gravitational field is the work done per unit mass in bringing a mass from infinity to that point.

The work done in bringing a mass m to a point in a gravitational field created by a source mass M is:

$$E_{\mathrm{p}} = -G\frac{Mm}{r}$$

The work done per unit mass is:

$$\frac{E_{\mathrm{p}}}{m} = -G\frac{M}{r}$$

This leads to the equation for gravitational potential shown in **Table 2**.

Table 2. Equation for gravitational potential.

Equation	Symbols	Units
$V_{\mathrm{g}} = -G\frac{M}{r}$	V_{g} = gravitational potential	joules per kilogram (J kg^{-1})
	G = gravitational constant (6.67×10^{-11} m^3 kg^{-1} s^{-2})	cubic metres per kilogram per second squared (m^3 kg^{-1} s^{-2})
	M = mass of source of gravitational field	kilograms (kg)
	r = distance from the centre of mass M to the point in the gravitational field	metres (m)

Student view

Like gravitational potential energy, gravitational potential is a scalar which can only take values less than or equal to zero. Remember that positive work would be required to move a body from a point in a gravitational field to a point infinitely far away – so the work done in moving a mass in the other direction is described as negative.

Worked example 2

The gravitational potential at a distance *r* away from the centre of mass *M* is V_r.

Determine the gravitational potential at a distance:

1. $\frac{r}{2}$
2. $2r$

The gravitational potential is inversely proportional to the distance *r* from the source of the field:

$$V_{\mathrm{r}} \propto \frac{1}{r}$$

1. When the distance changes from r to $\frac{r}{2}$, the force changes from:

$$V_{\mathrm{r}} \propto \frac{1}{r}$$

to

$$V_{\frac{r}{2}} \propto \frac{1}{\frac{r}{2}}$$

$$V_{\frac{r}{2}} \propto \frac{2}{r}$$

$$V_{\frac{r}{2}} = 2V_{\mathrm{r}}$$

The gravitational potential at distance $\frac{r}{2}$ is twice as large as the gravitational potential at distance *r*.

2. When the distance changes from *r* to 2*r*, the force changes from:

$$V_{\mathrm{r}} \propto \frac{1}{r}$$

to

$$V_{2r} \propto \frac{1}{2r}$$

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

$$V_{2r} \propto \frac{1}{2} \times \frac{1}{r}$$

$$V_{2r} = \frac{1}{2} V_r$$

The gravitational potential at distance 2*r* is half the gravitational potential at distance *r*.

Figure 1 shows the relationship between gravitational potential V_g and distance *r* from the centre of the Earth (mass *M*). Notice that gravitational potential is always negative.

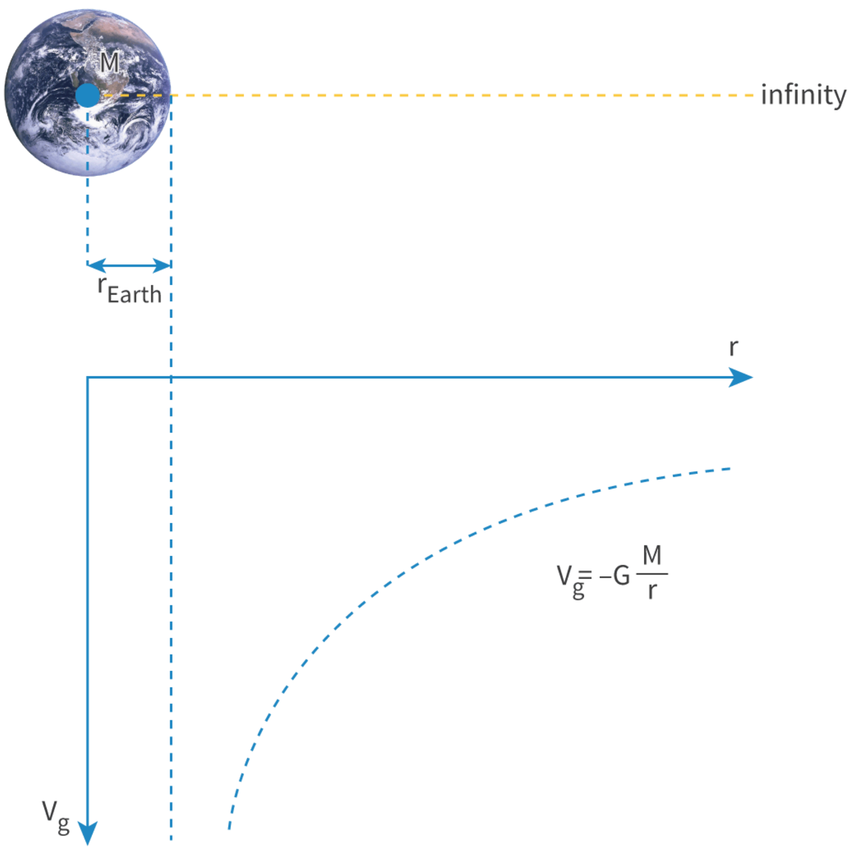


Figure 1. Gravitational potential changes as the distance from the centre of the Earth changes.

Credit: NASA (Earth image)

More information for figure 1

The image is a graph illustrating the relationship between gravitational potential (Vg) and distance (r) from the center of the Earth, denoted by mass (M). The x-axis represents the distance 'r' with an arrow pointing right, indicating distance increasing away from Earth. The y-axis represents gravitational potential 'Vg', with the direction pointing downward.

A curved line indicates the change in gravitational potential as distance increases. The graph shows that gravitational potential is always negative and decreases in magnitude as the distance from the Earth's center increases. Notably, at an infinite distance from the Earth, the gravitational potential approaches zero, while

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

close to the Earth, the potential is negative. Text on the graph includes the formula Vg = -GM/r indicating how gravitational potential varies with distance.

[Generated by AI]

Worked example 3

Determine the gravitational potential at the surface of the Earth.

The mass of the Earth is:

$$M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

The radius of the Earth is:

$$r_{\text{Earth}} = 6400 \text{ km}$$

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$M_{\text{Earth}} = 5.97 \times 10^{24}$ $r_{\text{Earth}} = 6400 \text{ km}$ $= 6.4 \times 10^{6} \text{ m}$
Step 2: Write out the equation.	$V_{\text{g}} = -G\dfrac{M}{r}$ To calculate the gravitational potential at the surface of Earth, you need to know the distance of the surface fro centre of the Earth. This distance is the radius of the Ea r_{Earth}. $V_{\text{g}} = -G\dfrac{M_{\text{Earth}}}{r_{\text{Earth}}}$
Step 3: Substitute the values given.	$V_{\text{g}} = -6.67 \times 10^{-11} \times \dfrac{5.97 \times 10^{24}}{6.4 \times 10^{6}}$ $= -6.22 \times 10^{7}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= -6.2 \times 10^{7} \text{ J kg}^{-1}$ (2 s.f.)

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Study skills

The concepts of gravitational force and gravitational potential energy rely on there being a mass in the gravitational field of another mass. The equations for gravitational force and gravitational potential energy require the magnitudes of both masses.

The concepts of gravitational field strength and gravitational potential do not rely on there being a mass in the gravitational field of another mass. The equations for gravitational field strength and gravitational potential only require the magnitude of one mass.

Worked example 4

Determine the gravitational potential halfway between Saturn and its moon, Titan.

The mass of Saturn is: 5.7×10^{26} kg
The mass of Titan is: 1.3×10^{23} kg
The distance between Saturn and Titan is: 1.2×10^{6} km

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$M_{\text{saturn}} = 5.7\times10^{26}\text{ kg}$ $M_{\text{Titan}} = 1.3\times10^{23}\text{ kg}$ $\text{distance} = 1.2\times10^{6}\text{ km} = 1.2\times10^{9}\text{ m}$ $r = 0.6\times10^{9}\text{ m from each body}$
Step 2: Write out the equation.	$V_{\text{g}} = -\dfrac{GM}{r}$

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Solution steps	Calculations
Step 3: Substitute the values given.	The gravitational potential due to Saturn: $$= -\frac{6.67 \times 10^{-11} \times 5.7 \times 10^{26}}{0.6 \times 10^{9}}$$ $$= -6.3365 \times 10^{7}\ \text{J kg}^{-1}$$ The gravitational potential due to Titan: $$= -\frac{6.67 \times 10^{-11} \times 1.3 \times 10^{23}}{0.6 \times 10^{9}}$$ $$= -1.4452 \times 10^{4}\ \text{J kg}^{-1}$$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$-6.3365 \times 10^{7} - 1.4452 \times 10^{4} = 6.338 \times 10^{7}\ \text{J kg}^{-1} = 6.3 \times 10^{7}\ \text{J k}$ This is the magnitude of the gravitational potential of Saturn. The grav potential due to Titan is negligible.

Work done moving a mass in a gravitational field

Imagine moving a mass m in the gravitational field of a mass M (**Figure 2**).

- When mass m is at a distance r_1 from the centre of mass M, it has a gravitational potential of V_{g1}.
- When mass m is at a distance r_2 from the centre of mass M, it has a gravitational potential V_{g2}.

Student
view

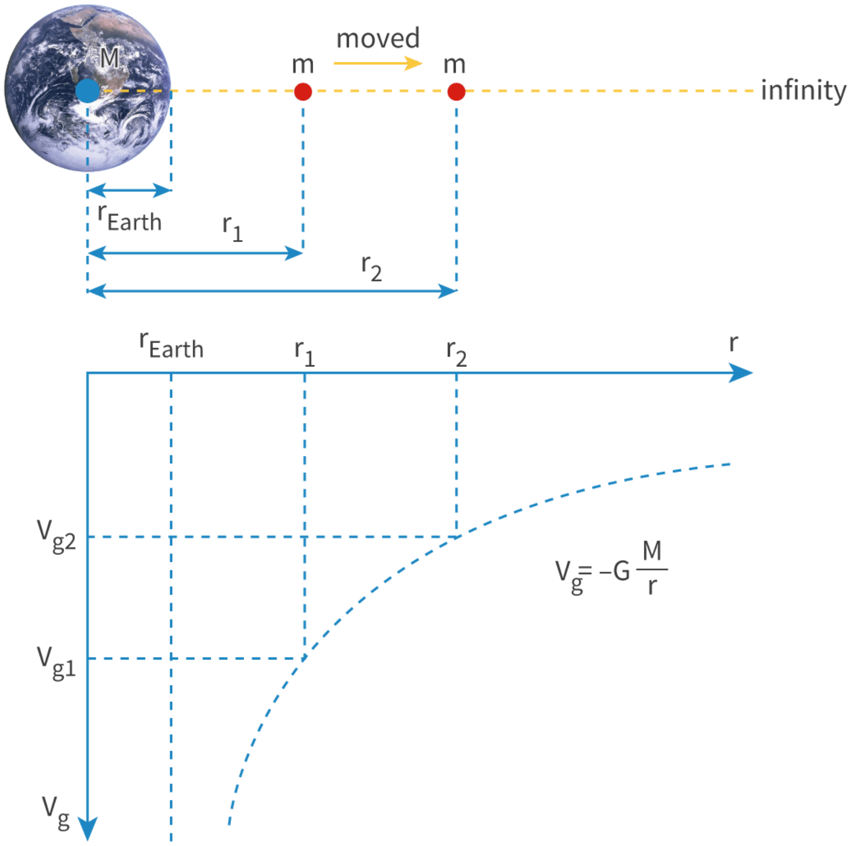


Figure 2. Moving mass m in the gravitational field of mass M.

Credit: NASA (Earth image)

More information for figure 2

The diagram shows a mass m moving in the gravitational field of a larger mass M, represented by Earth. On top of the diagram, there is a side view of Earth with an arrow indicating the distance from Earth to the starting position of the mass m. The mass m is initially placed at a distance labeled r1 and is moved to another position labeled r2. The distance between r1 and r2 is shown with a yellow arrow labeled "moved." Another yellow arrow extends further to infinity, suggesting the conceptual journey in the gravitational field.

Below this setup, a graph displays gravitational potential energy. The X-axis is labeled "r" and shows increasing distance values starting from "r_Earth," through "r1," and then "r2." The Y-axis is labeled "V_g" representing gravitational potential energy, with decreasing values from top to bottom, labeled "V_g1" and "V_g2". A downward curve represents the potential energy decreasing as the distance from Earth increases. The formula "V_g = -GM/r" appears on the curve, illustrating the mathematical relationship between gravitational potential energy, mass, and distance.

[Generated by AI]

Moving mass m between the two points in the field requires doing work against the gravitational force acting on m.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Work W is equal to the change in gravitational potential energy ΔE_p, which is the difference between the final gravitational potential energy E_{p2} and the initial gravitational potential energy E_{p1}.

$$\begin{aligned} W &= \Delta E_\mathrm{p} \\ &= E_\mathrm{p2} - E_\mathrm{p1} \end{aligned}$$

The gravitational potential energy E_p at a point in the gravitational field is given by the product of mass m and the gravitational potential Vg at that point:

$$E_\mathrm{p} = mV_\mathrm{g}$$

The gravitational potential energy E_{p2} is:

$$E_\mathrm{p2} = mV_\mathrm{g2}$$

The gravitational potential energy E_{p1} is:

$$E_\mathrm{p1} = mV_\mathrm{g1}$$

Therefore, the work done when moving mass m between the two points is:

$$\begin{aligned} W &= mV_\mathrm{g2} - mV_\mathrm{g1} \\ &= m\left(V_\mathrm{g2} - V_\mathrm{g1}\right) \end{aligned}$$

The difference in gravitational potential at different points in the field is the change in gravitational potential, ΔV_g.

The equation for the work done moving a mass in a gravitational field is shown in **Table 3**.

Table 3. The equation for work done.

Equation	Symbols	Units
$W = m\Delta V_\mathrm{g}$	W = work done moving mass m	joules (J)
	m = mass	kilograms (kg)
	ΔV_g = change in gravitational potential	joules per kilogram (J kg^{-1})

Worked example 5

Determine the work done when launching a satellite of mass 1000 kg into its orbit.

The gravitational potential at Earth’s surface is:

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

$$V_{g1} = -6.2 \times 10^7 \text{ J kg}^{-1}$$

The gravitational potential at the distance of orbit, measured from the centre of Earth is:

$$V_{g2} = -5.8 \times 10^7 \text{ J kg}^{-1}$$

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$m_{\text{satellite}} = 1000 \text{ kg}$ $V_{g1} = -6.2 \times 10^7 \text{ J kg}^{-1}$ $V_{g2} = -5.8 \times 10^7 \text{ J kg}^{-1}$
Step 2: Write out the equations.	The work done against the force of gravitational attraction between the Earth and the satellite is: $W = m_{\text{satellite}} \Delta \mathrm{V_g}$ The difference in gravitational potential at distan orbit and the surface of the Earth is: $\Delta V_g = \mathrm{V_{g2}} - \mathrm{V_{g1}}$
Step 3: Substitute the values given.	$W = m_{\text{satellite}} \Delta \mathrm{V_g}$ $W = 1000\left(-5.8 \times 10^7 - \left(-6.2 \times 10^7\right)\right) = 4.0 \times 10^9 \text{ J}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 4.0 \times 10^9 \text{ J (2 s.f.)}$

Work through the activity to check your understanding of gravitational potential energy, gravitational potential and work done moving a mass in a gravitational field.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

A satellite of mass 1 kg is launched from the surface of the Earth into an orbit 1000 km above the Earth's surface.

1. Predict the gravitational potential energy of the Earth-satellite system at the surface of the Earth (*r* = 6400 km) and when the satellite is in orbit.
2. Draw a graph of gravitational potential energy for the Earth-satellite system for the surface of the Earth up to its orbital height.
3. Using your graph, find the work done by the satellite launcher against the gravitational force to move the satellite from the surface of the Earth to a height of:

   (a) 0.1 km
   (b) 1 km
   (c) 10 km
   (d) 100 km
   (e) 1000 km

4. Repeat steps 1 to 3 (on the same axes if possible) for gravitational potential.
5. What do you notice about the results? What do you notice about the shapes of the graphs? What similarities are there? What differences?

5 section questions

Question 1

HL Difficulty:

Gravitational potential energy is defined as the 1 work ✓ done moving a mass from 2 infinity ✓ to a point in a gravitational field.

Accepted answers and explanation

#1 work

#2 infinity
infinite

General explanation

Student view

Gravitational potential energy is the work done moving a mass from infinity to a point in a gravitational field.

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Question 2

HL Difficulty:

A satellite is launched from the surface of the Earth to its orbit. The amount of energy needed to do that is equal to the work done 1 against ✓ the force of gravitational attraction between the two bodies. This work is equal to a product of the mass of the satellite and the gravitational potential 2 difference ✓ between the final and initial distance of the satellite from the centre of the Earth.

Accepted answers and explanation

#1 against

#2 difference
change

General explanation

The energy needed to move mass m in the field of mass M between two points in that field is equal to the work done against the force of gravitational attraction between the source of the gravitational field M and another mass m in that field and is expressed by $W = m\Delta V_{\mathrm{g}}$.

Question 3

HL Difficulty:

A gravitational field around a planet of mass M has gravitational potential V_r at a distance r from its centre.

Determine the gravitational potential at a distance $\frac{r}{4}$.

1 $4V_{\mathrm{r}}$ ✓

2 $16V_{\mathrm{r}}$

3 $\frac{V_{\mathrm{r}}}{16}$

4 $\frac{V_{\mathrm{r}}}{4}$

Explanation

Gravitational potential at a distance r away from the centre of the source of the field is:

Student
view

$$V_{\mathrm{r}} = -G\frac{M}{r}$$

Gravitational potential at a distance $\frac{r}{4}$ away from the centre of the field is:

$$V_{\frac{r}{4}} = -G\frac{M}{\frac{r}{4}}$$
$$= -4G\frac{M}{r}$$
$$= 4V_{\mathrm{r}}$$

Question 4

HL Difficulty:

A satellite of mass *m* is launched from the Earth (mass *M* and radius *r*) to its orbit distance *r* away from the Earth's surface. Determine the gravitational potential energy of the satellite in its orbit.

1. $-G\frac{Mm}{2r}$ ✓
2. $-G\frac{M}{r}$
3. $-G\frac{M}{2r}$
4. $-G\frac{Mm}{r}$

Explanation

The gravitational potential energy at distance *r* from centre of Earth is:

$$E_{\mathrm{p}} = -G\frac{Mm}{r}$$

The gravitational potential energy at distance 2*r* (orbit of satellite) from centre of Earth is:

$$E_{\mathrm{p}} = -G\frac{Mm}{2r}$$

Question 5

SL HL Difficulty:

A satellite is launched from the surface of the Earth to its orbit. Determine the work done in this process.

The mass of the Earth is:

$$M_{\mathrm{Earth}}$$

The mass of the satellite is:

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

$$m_{\text{satellite}} = 6.7 \times 10^{-23} \times M_{\text{Earth}}$$

The radius of the Earth is:

$$r_{\text{Earth}}$$

The distance of orbit away from the Earth's surface is:

$$d_{\text{orbit}} = 5.6 r_{\text{Earth}}$$

1 $5.7 \times 10^{-23} \times G \dfrac{M_{\text{Earth}}^2}{r_{\text{Earth}}}$ ✓

2 $7.9 \times 10^{-23} \times G \dfrac{M_{\text{Earth}}^2}{r_{\text{Earth}}}$

3 $7.7 \times 10^{-23} \times G \dfrac{M_{\text{Earth}}^2}{r_{\text{Earth}}}$

4 $5.5 \times 10^{-23} \times G \dfrac{M_{\text{Earth}}^2}{r_{\text{Earth}}}$

Explanation

The work done to move satellite from surface of Earth to its orbit is:

$$\begin{aligned}
W &= \Delta E_{\text{p}} \\
&= E_{E_{\text{p}}\text{final}} - E_{E_{\text{p}}\text{initial}} \\
&= E_{E_{\text{p}}\text{orbit}} - E_{E_{\text{p}}\text{Earth}} \\
&= -G \frac{M_{\text{Earth}} m_{\text{satellite}}}{r_{\text{Earth}} + d_{\text{orbit}}} - \left(-G \frac{M_{\text{Earth}} m_{\text{satellite}}}{r_{\text{Earth}}} \right) \\
&= G \frac{M_{\text{Earth}} m_{\text{satellite}}}{r_{\text{Earth}}} - G \frac{M_{\text{Earth}} m_{\text{satellite}}}{r_{\text{Earth}} + d_{\text{orbit}}} \\
&= G M_{\text{Earth}} m_{\text{satellite}} \left(\frac{1}{r_{\text{Earth}}} - \frac{1}{r_{\text{Earth}} + d_{\text{orbit}}} \right) \\
&= G M_{\text{Earth}} \times 6.7 \times 10^{-23} \times M_{\text{Earth}} \left(\frac{1}{r_{\text{Earth}}} - \frac{1}{r_{\text{Earth}} + 5.6 r_{\text{Earth}}} \right) \\
&= 6.7 \times 10^{-23} \times G M_{\text{Earth}}^2 \left(\frac{1}{r_{\text{Earth}}} - \frac{1}{6.6 r_{\text{Earth}}} \right) \\
&= 6.7 \times 10^{-23} \times G M_{\text{Earth}}^2 \left(\frac{6.6 r_{\text{Earth}} - r_{\text{Earth}}}{6.6 r_{\text{Earth}}^2} \right) \\
&= 6.7 \times 10^{-23} \times G M_{\text{Earth}}^2 \left(\frac{5.6 r_{\text{Earth}}}{6.6 r_{\text{Earth}}^2} \right) \\
&= 6.7 \times 10^{-23} \times G M_{\text{Earth}}^2 \left(\frac{5.6}{6.6 r_{\text{Earth}}} \right) \\
&= 5.7 \times 10^{-23} \times G \frac{M_{\text{Earth}}^2}{r_{\text{Earth}}}
\end{aligned}$$

Student view

Equipotential surfaces (HL)

Higher level (HL)

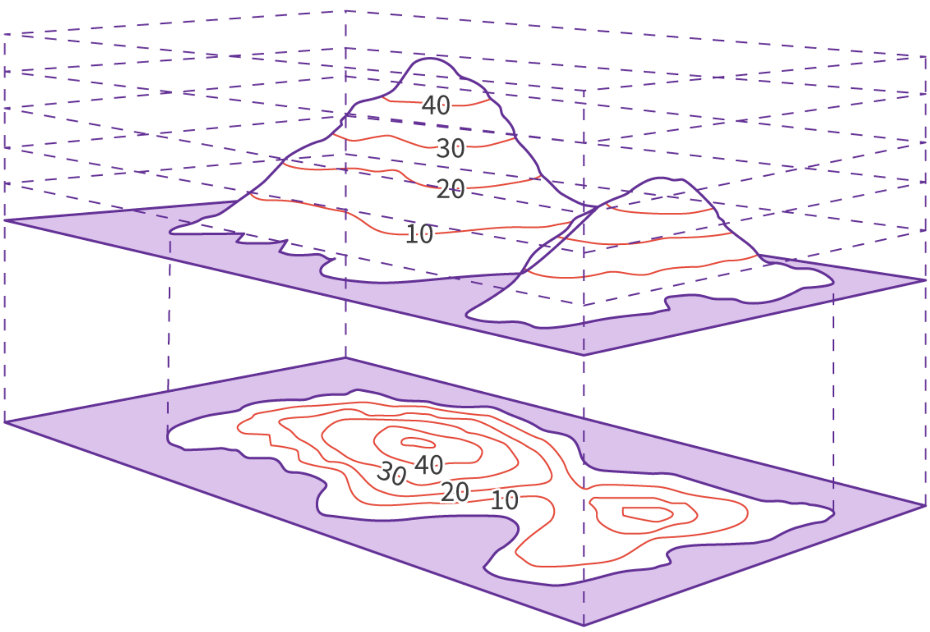
Learning outcomes

By the end of this section you should be able to:

- Understand the concept of equipotential surfaces for gravitational fields.
- Know the relationship between equipotential surfaces and gravitational field lines.
- Understand gravitational field strength as gravitational potential gradient and use the equation:

$$g = -\frac{\Delta V_{\text{g}}}{\Delta r}.$$

Have you ever been hiking with a map? If so, you might have noticed lines with numbers on the map called contour lines (**Figure 1**). Contour lines show the height of the ground above sea level.



Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Figure 1. Contour lines show height above sea level.

More information for figure 1

The image is a 3D contour plot that represents a landscape with varying heights. The plot displays two main surfaces: one in 3D and another in 2D projection at the base. The 3D surface shows a series of hills, with contour lines marked at intervals to indicate specific heights above sea level. These contour lines are labeled with numbers 10, 20, 30, and 40, indicating the height. The 2D projection below mirrors the contour lines directly as seen from above. Each contour line represents a different height, illustrating the variation in topography across the landscape, where moving from one line to another indicates a change in elevation.

[Generated by AI]

Each contour line represents a specific height above sea level and different lines represent different heights. Moving from one contour line to another on a map means going up a hill or down a hill (or, more accurately, moving closer to or further from the centre of the Earth).

In **Figure 1**, you can see that the closer the contour lines, the steeper the hill. How does this relate to gravitational fields?

Equipotential surfaces

Gravitational potential changes with distance from the mass (see section D.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-potential-energy-and-gravitational-potential-hl-id-46569/)). We can use lines, similar to contour lines, to represent areas of equal gravitational potential. These lines are called equipotential lines. When these lines are shown in 3D, they are known as equipotential surfaces. Equipotential lines and surfaces are known as equipotentials.

Making connections

The equipotential lines and surfaces we use to describe gravitational fields are conceptually equivalent to the equipotential lines and surfaces you use to describe electric fields (see subtopic D.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44743/)).

Figure 2 shows the equipotential lines around the Earth. Note that these lines are a cross-section through the equipotential surfaces that exist in three dimensions. These surfaces are spherical.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

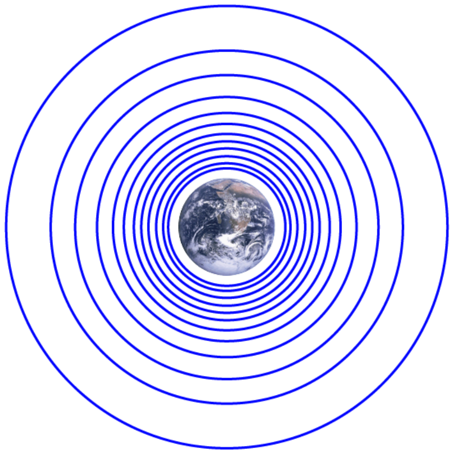


Figure 2. Equipotential lines around the Earth.

More information for figure 2

The image is a diagram showing the Earth at its center, surrounded by multiple concentric blue lines. These lines represent equipotential surfaces, which are cross-sections of a three-dimensional representation. The lines are evenly spaced and form circles around Earth, representing areas of equal gravitational potential. This visualization helps to illustrate the concept of gravitational potential around a spherical body like Earth. The diagram is simple in construction, focusing on showing the equal potential lines in two dimensions.

[Generated by AI]

Look at **Figure 2** and answer these questions:

Where are the equipotential lines the closest together?

Click on 'Show or hide solution' to see the answers.

The equipotential lines are closest near to the Earth. The lines get further apart as distance from the Earth increases.

What does this tell you about the gravitational potential?

Gravitational potential changes more with smaller changes in distance when close to the Earth. Further from the Earth, larger changes in distance are required for the same change in gravitational potential.

Concept

The difference in gravitational potential between two adjacent equipotential lines is always the same. For example, you might get equipotential lines that represent 5, 10, 15, 20, 25 J kg^{-1}, or 100, 200, 300, 400, 500 J kg^{-1}.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Equipotential lines are never drawn to show differing changes in potential such as 5, 6, 7, 9, 22, 105 J kg^{-1}.

Equipotential lines and surfaces are always perpendicular to gravitational field lines. They are more closely spaced in areas where the gravitational field is stronger (closer to the mass) and further apart when the gravitational field is weaker (further from the mass).

Figure 3 shows the equipotential lines and gravitational field lines for a radial field and a uniform field.

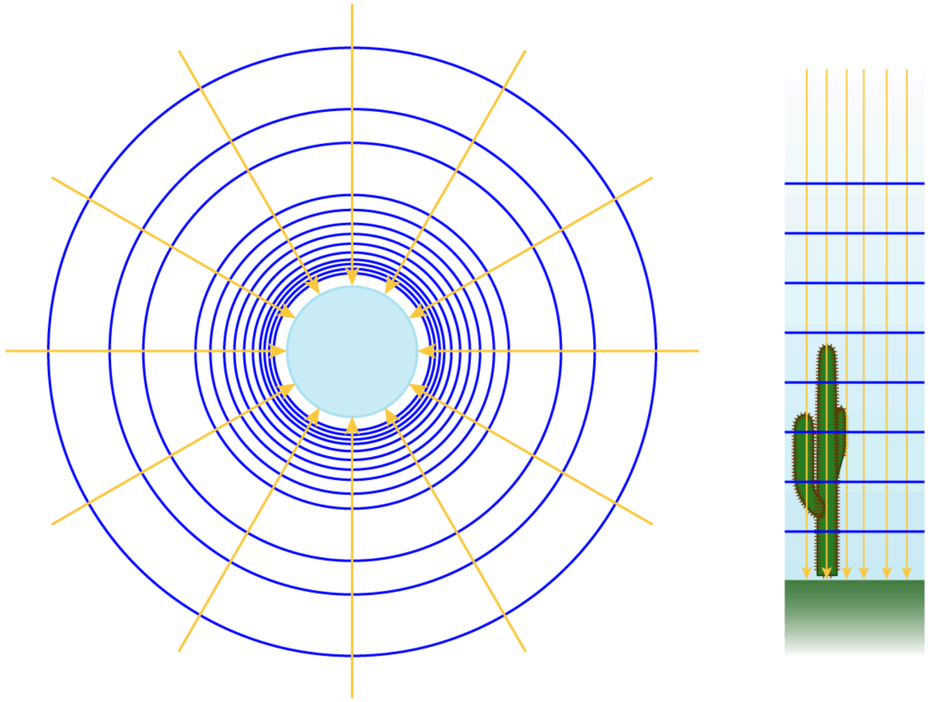


Figure 3. Equipotential lines and gravitational field lines for a radial field and a uniform field.

More information for figure 3

The image consists of two parts illustrating the equipotential lines and gravitational field lines.

On the left, there is a radial field. The field lines are depicted as yellow arrows radiating outward from a central circular shape, indicative of a gravitational source. Around these central lines, there are concentric blue circles representing equipotential lines, which become further apart as they move away from the center.

On the right, a uniform field is demonstrated. The gravitational field lines are parallel yellow lines moving vertically downwards, and the equipotential lines are horizontal and equidistant from each other. There is an illustration of a cactus, which somewhat represents the effect of the field on an object resting on the ground, to emphasize the uniform nature of the field.

[Generated by AI]

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Gravitational potential gradient

The gravitational potential gradient is how ‘steep’ the change in gravitational potential is as a function of distance.

If the equipotential lines are close together, the change in gravitational potential occurs over a short distance. Therefore, the gravitational potential gradient is steep, and more work is required to move a certain distance.

The equation for gravitational potential is section D.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-potential-energy-and-gravitational-potential-hl-id-46569/):

$$V_{\mathrm{g}} = -\frac{GM}{r}$$

The gravitational potential gradient is how much the gravitational potential changes per unit distance:

$$\frac{\Delta V_{\mathrm{g}}}{\Delta r} = -\frac{GM}{r\Delta r}$$

Using calculus (outside the scope of IBDP Physics) gives:

$$\text{gravitational potential gradient} = -\frac{GM}{r^2}$$

The equation for gravitational field strength is section D.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-field-gravitational-field-lines-and-gravitational-field-strength-id-46568/):

$$g = G\frac{M}{r^2}$$

We can conclude that gravitational field strength is equal to the negative of the magnitude of the gravitational potential gradient. The equation for gravitational field strength is shown in **Table 1**.

Table 1. The equation for gravitational field strength.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Equation	Symbols	Units
$g = -\frac{\Delta V_g}{\Delta r}$	g = gravitational field strength	newtons per kilogram (N kg^{-1})
	ΔV_g = change in gravitational potential	Joules per kilogram (J kg^{-1})
	Δr = change in distance	metres (m)

Exercise 1

Click a question to answer

Higher level (HL)

The equation for gravitational field strength can be used for objects close to the surface of the Earth, where the gravitational potential gradient is (almost) constant. How do we determine gravitational field strength when the gravitational potential gradient is not constant?

A graph of gravitational potential against distance for the Earth is a curve (**Figure 4**). By taking a tangent to this curve at any point, we can find the gravitational potential gradient and the gravitational field strength at that point.

Student view

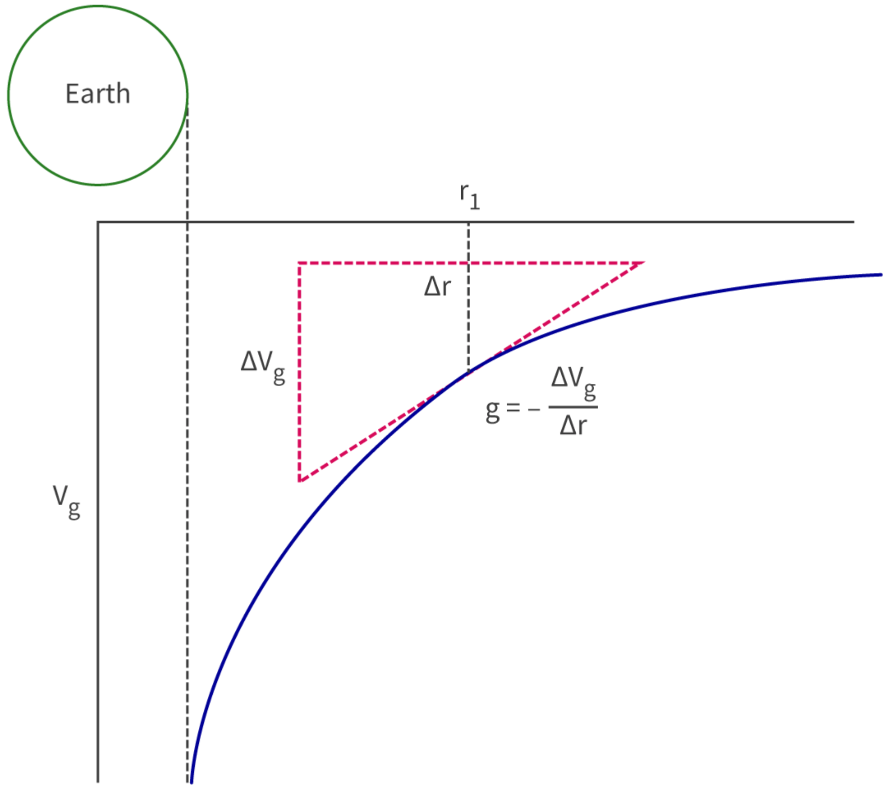


Figure 4. A graph of gravitational potential against distance for the Earth.

More information for figure 4

The graph presents gravitational potential (Vg) against distance (r) from Earth, depicted as a curve. The X-axis represents distance from Earth (r) with a specific point marked as r1, and the Y-axis represents gravitational potential (Vg). A tangent to the curve is shown, indicating the slope, which determines the gravitational potential gradient and gravitational field strength at that point. The tangent forms a right triangle with ΔVg on the Y-axis and Δr on the X-axis, demonstrating the mathematical relationship of gravitational force: g = -ΔVg/Δr. The curve shows a downward trend, indicating a decrease in potential with increased distance from Earth.

[Generated by AI]

Theory of Knowledge

Mathematical representations of physical phenomena using graphs and diagrams help us make connections between what is already intuitively understood based on our experiences from daily life and complex physical phenomena. What role do visual representations such as graphs play in communicating knowledge of scientific concepts?

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Worked example 1

The graph shows gravitational potential against distance for Venus. Determine the gravitational field strength at a distance of r from the surface of Venus.

radius of Venus, $r = 6100$ km

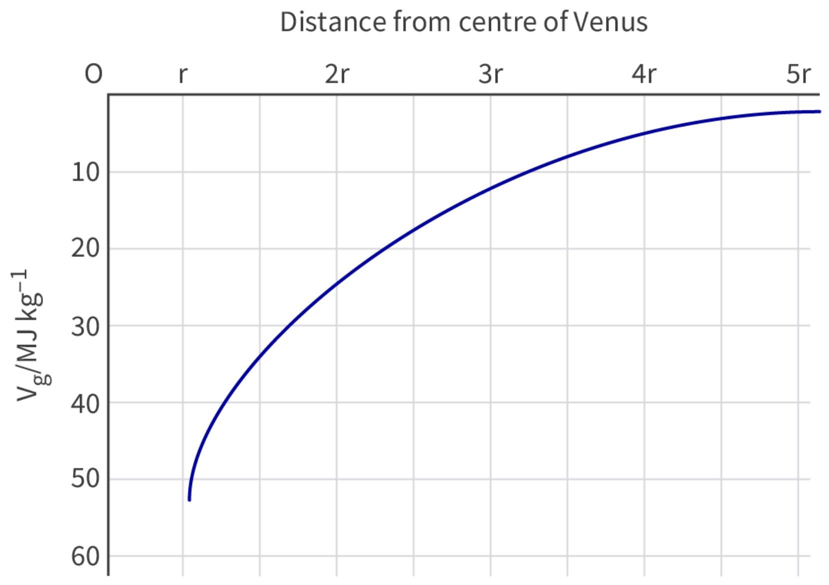


Figure 5. A graph of gravitational potential against distance for Venus.

More information for figure 5

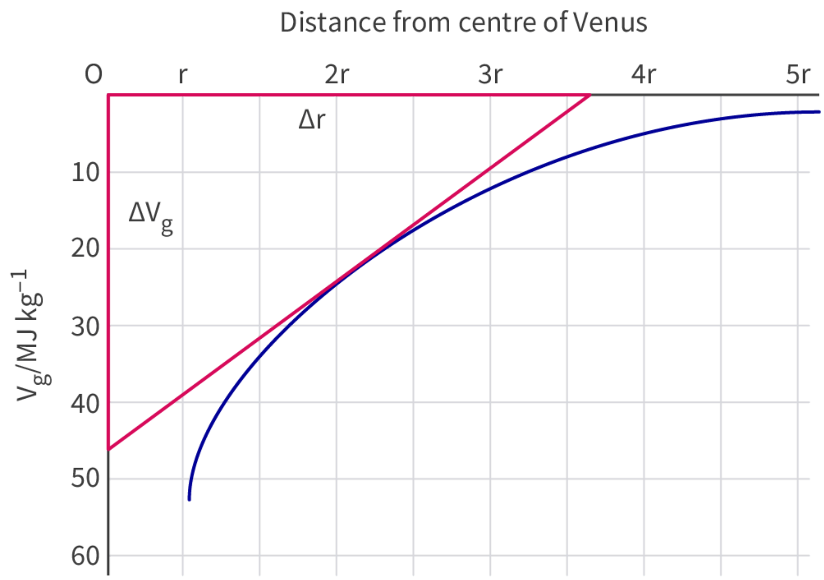
The graph illustrates the gravitational potential (Vg) in MJ kg^-1 against the distance from the center of Venus. The X-axis represents distance in multiples of the radius of Venus (r), marked as 0, r, 2r, 3r, 4r, and 5r. The Y-axis represents the gravitational potential in MJ kg^-1, ranging from 0 to 60 with a main interval of 10. A curve starts from the bottom left, representing the gravitational potential decreasing rapidly as it approaches the center of Venus and leveling off as the distance increases. The trend shows that gravitational potential decreases as distance increases.

[Generated by AI]

A distance of r from the surface is a distance of $2r$ from the centre, so find the gradient of the curve at $2r$.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c



$$
\begin{aligned}
g &= -\frac{\Delta V_g}{\Delta r} \\
&= \frac{46 \times 10^6}{3.6 \times 6100 \times 10^3} \\
&= 2.1\,\mathrm{N\,kg^{-1}}(2\ \mathrm{s.f.})
\end{aligned}
$$

Venus is a roughly similar size to the Earth and its gravitational field strength is in the same order of magnitude as Earth's gravitational field strength. So the answer seems reasonable given that 2*r* from its centre is not a long distance from Venus (compared with its size).

Study skills

When drawing a tangent on a graph, try to draw as large a triangle as possible to reduce percentage error.

Work through the activity to check your understanding of equipotential lines and gravitational field strength.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

The image shows a two-mass system of a planet (on the left) and a moon (on the right).

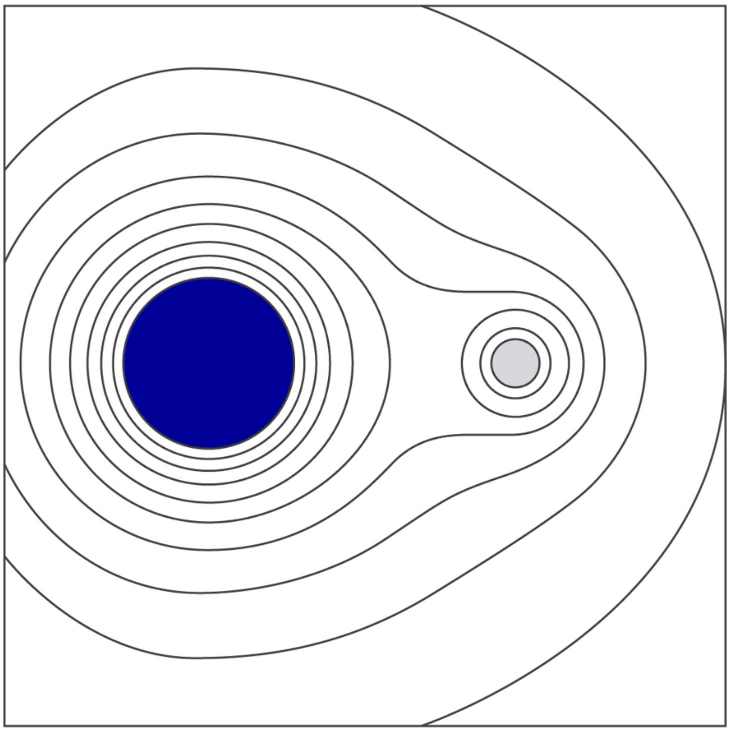


Figure 6. Two-mass system of a planet (left) and a moon (right).

More information for figure 6

The image is a contour diagram representing a two-mass system with a larger mass on the left and a smaller mass on the right. The diagram consists of a series of concentric, oval-shaped lines centered around two focal points. The lines form a series of nested loops, with the innermost loop on the left center representing the larger mass, possibly a planet. This mass is shaded in dark blue. To the right, another set of concentric loops depicts the smaller mass, possibly a moon, shaded in gray. The lines between the two sets of loops show the gravitational relationship, illustrating potential fields around the two masses. This setup visually conveys gravitational influence with contour lines demarcating equal potential zones. The structure of the diagram suggests a gravitational interaction, with elongated loops between the two masses indicating the gravitational pull they exert on each other.

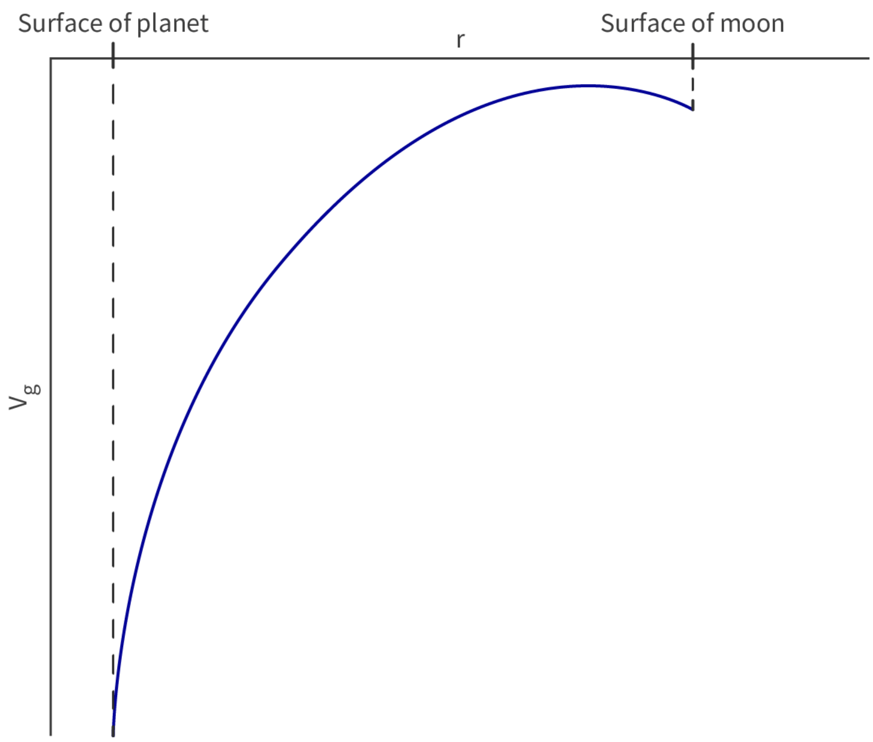
[Generated by AI]

1. Sketch a graph of gravitational potential V_g against distance r along a straight line between the surfaces of the planet and the moon.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

A graph of gravitational potential against distance for the two-mass system.



2. Look at the shape of the curve you have drawn. Which side of the curve is steeper? How can you tell this?

The equipotential lines are closer together for the Earth on the left, so the curve is steeper on the left of the graph.

3. What does the steepness of the V_g curve tell you?

The gravitational field strength at that point

4. Where would a small mass have the lowest gravitational potential energy?

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

At the surface of the planet on the left. There are more equipotential lines around the planet, and so the gravitational potential at the surface of the planet must be more negative than at the surface of the moon. Gravitational potential is the work done per unit mass moving a mass from infinity to that point, and it is always negative. So for a small mass, the lower the gravitational potential, the lower its potential energy at that point.

5. Where on your graph is the resultant gravitational field strength zero? How can you tell from the graph?

At the peak of the graph. The gravitational field strength is the potential gradient, so where the gradient is zero (the tangent to the curve is horizontal), the gravitational field strength is also zero.

5 section questions

Question 1 HL Difficulty:

1 Equipotential ✓ lines are lines of equal gravitational 2 potential ✓ . They are always 3 perpendicular ✓ to gravitational 4 field ✓ lines.

Accepted answers and explanation

#1 Equipotential

#2 potential

#3 perpendicular
at right angles

#4 field

General explanation

Equipotential lines are lines of equal gravitational potential. They are always perpendicular to gravitational field lines.

Student view

Question 2 HL Difficulty:

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Which of the following statements about the gravitational field strength of a planet is correct?

1 It is proportional to the inverse square of the distance from the planet. ✓

2 It increases as the distance from the centre of the planet increases.

3 It always has a constant value.

4 It is equal to the potential gradient.

Explanation

The gravitational field strength is given by:

$$g = G\frac{M}{r^2}$$

It is equal to the **negative** potential gradient.

Question 3

HL Difficulty:

A satellite is orbiting a planet at a distance r from the centre of the planet where the gravitational potential is $-7V_g$.

The satellite descends at a constant rate to a distance $\frac{4r}{5}$ from the centre of the planet.

The gravitational potential changes to $-8V_g$.

Determine the magnitude of the average gravitational field strength experienced by the satellite as it descends.

1 $\frac{5V_g}{r}$ ✓

2 $\frac{V_g}{r}$

3 $\frac{5V_g}{9r}$

4 $\frac{3V_g}{r}$

Explanation

Student view

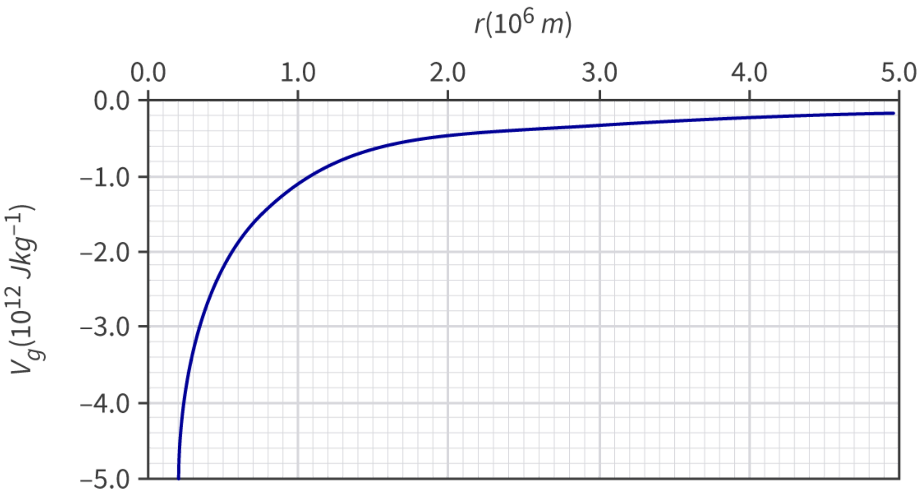
Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$
\begin{aligned}
g &= -\frac{\Delta V_g}{\Delta r} \\
&= -\frac{-8V_g - (-7V_g)}{\frac{4}{5}r - r} \\
&= -\frac{-V_g}{-\frac{1}{5}r} \\
&= -\frac{5V_g}{r}
\end{aligned}
$$

Question 4

HL Difficulty:

The graph shows gravitational potential against distance for a planet. The x-axis shows the distance from the surface of the planet.



More information

Determine the gravitational field strength at 0.5×10^6 m away from the surface of a planet.

1 -3.2×10^6 N kg^{-1} ✓

2 -2.75×10^6 N kg^{-1}

3 -3.2 N kg^{-1}

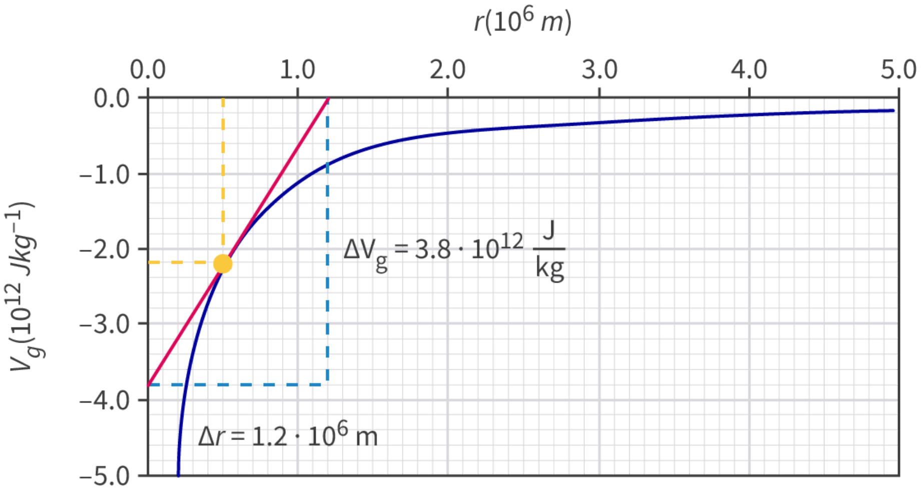
Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

4 $-2.75\ \text{N kg}^{-1}$

Explanation

Draw a tangent to the curve at $0.5 \times 10^6\ \text{m}$ and find ΔV_g and Δr.



More information

$$
\begin{aligned}
g &= -\frac{\Delta V_g}{\Delta r} \\
&= -\frac{3.8 \times 10^{12}}{1.2 \times 10^6} \\
&= -3.17 \times 10^6 \\
&= -3.2 \times 10^6\ \text{N kg}^{-1}\ (2\ \text{s.f.})
\end{aligned}
$$

Question 5

HL Difficulty:

In a single diagram, moving from any one equipotential to the adjacent equipotential means a fixed difference in 1 potential ✓ .

Accepted answers and explanation

#1 potential

Student
view

General explanation

Equipotential lines and surfaces are perpendicular to field lines at any point in the field. In a single diagram, moving from any one equipotential to the adjacent equipotential means a fixed difference in potential.

Orbital and escape speed (HL)

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Understand orbital speed v_{orbital} and use the equation:

$$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}.$$

- Understand escape speed v_{esc} and use the equation:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}.$$

- Understand how viscous drag due to the atmosphere affects the height and speed of an orbiting body.

Humans have been in space since 1961, when Yuri Gagarin entered a low Earth orbit in a human orbital spaceflight. Since then, many astronauts from many countries have been in space, and some have even stepped onto the Moon.

Figure 1 shows a SpaceX rocket being launched in 2020 to take astronauts to the International Space Station (ISS).

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Figure 1. The launch of SpaceX rocket in 2020.

Source: "SpaceX Demo-2 Launch (https://commons.wikimedia.org/wiki/File:SpaceX_Demo-2_Launch_(NHQ202005300044).jpg)" by NASA is in the public domain

How do we achieve successful launches of rockets? How do we make satellites, space probes, or observatories stay in their orbits or overcome the gravitational pull of the Earth and leave for deep space?

International Mindedness

International collaborations are needed for establishing effective rocket launch sites to benefit space programmes, and for timing re-entry into the Earth's atmosphere. What do you think might be the challenges of organising rocket launches?

Orbital speed

A satellite in a circular orbit travels at a constant orbital speed, v_{orbital}, at an orbital radius r. The force that keeps the satellite in orbit is the centripetal force. In this case, it is provided by the gravitational force (see subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)).

Figure 2 shows the ISS in orbit around the Earth.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

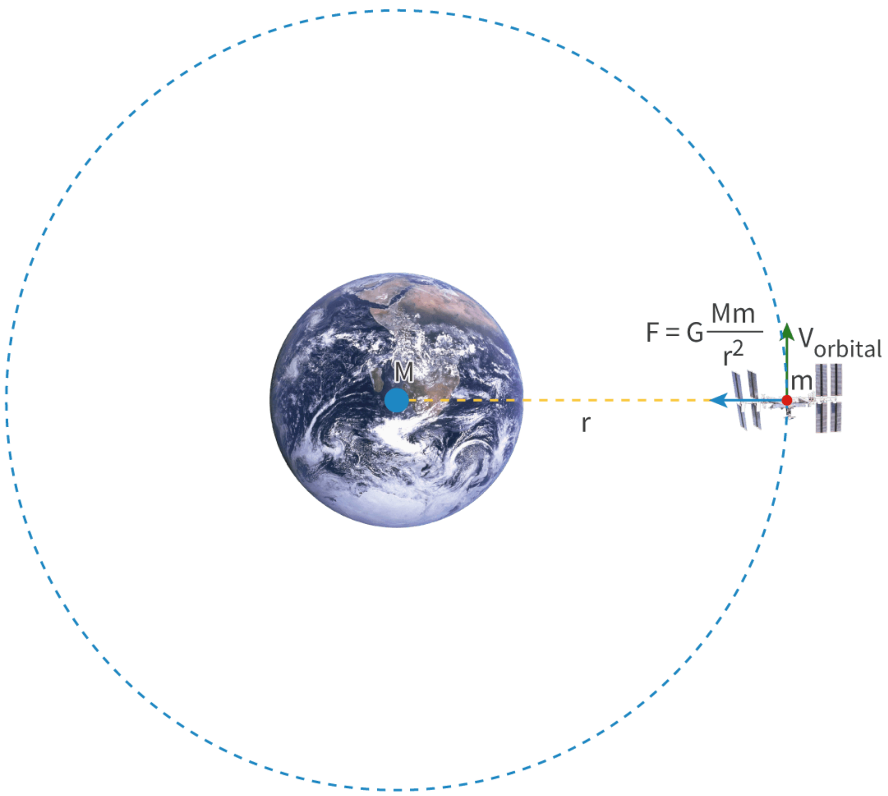


Figure 2. ISS in orbit around the Earth.

Credit: NASA (Earth and ISS images)

More information for figure 2

The image shows a diagram of the Earth with the International Space Station (ISS) in orbit around it. The Earth is depicted as a large spherical object with a noticeable cloud pattern and is labeled "M." The ISS is shown as a smaller object at a distance from the Earth's surface. A dotted line represents the ISS's orbital path around Earth.

There is a yellow dashed line indicating the distance between the center of the Earth and the ISS, labeled "r." The diagram also includes the formula for gravitational force: F = G * (Mm) / r^2, positioned near the ISS. The labels "V" and "orbital" next to an arrow on the ISS indicate its velocity vector.

Overall, the diagram provides a visual representation of the concepts involved in orbiting the Earth, including gravitational force and orbital mechanics, with relevant labels and a mathematical formula to aid understanding.

[Generated by AI]

Nature of Science

Aspect: Observations

Student view

Orbital motion is used as a model to describe motion on a very large (astronomical) scale as well as motion on a very small (atomic) scale. For example, the motion of electrons in atoms can be modelled by planetary motion, although there are some flaws in this simplified teaching tool. The Bohr model of hydrogen atom expands upon the simplified planetary model. To what extent can we apply macroscopic observations to microscopic phenomena?

We can derive an equation for the orbital speed v_{orbital} of a smaller mass m orbiting a larger mass M (the source of the gravitational field), such as a satellite orbiting the Earth.

The gravitational force of attraction between the two masses provides the centripetal force:

$$G\frac{Mm}{r^2} = \frac{mv^2_{\text{orbital}}}{r}$$

Rearranging and solving for v^2_{orbital} gives:

$$v^2_{\text{orbital}} = G\frac{M}{r}$$

The equation for orbital speed is shown in **Table 1**. Notice that as r increases, orbital speed decreases.

Table 1. The equation for orbital speed.

Equation	Symbols	Units
$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$	v_{orbital} = orbital speed	metres per second (m s^{-1})
	G = gravitational constant ($6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$)	newtons metres squared per kilogram squared ($\text{N m}^2\ \text{kg}^{-2}$)
	M = mass of source of gravitational field	kilograms (kg)
	r = orbital radius	meters (m)

Worked example 1

Calculate the orbital speed of each of the following satellites.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The distance from the surface of the Earth is given.

1. Moon, d_{Moon} = 385 700 km
2. Hubble Space Telescope (HST), d_{HST} = 535 km
3. International Space Station (ISS), d_{ISS} = 410 km

The radius of the Earth is:

$$r_{\text{Earth}} = 6400 \text{ km}$$

The mass of the Earth is:

$$m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

$$\begin{aligned} d_{\text{Moon}} &= 385\,700 + 6400 \\ &= 392\,100 \text{ km} \\ &= 3.921 \times 10^8 \text{ m} \end{aligned}$$

$$\begin{aligned} d_{\text{HST}} &= 535 + 6400 \\ &= 6935 \text{ km} \\ &= 6.935 \times 10^6 \text{ m} \end{aligned}$$

$$\begin{aligned} d_{\text{ISS}} &= 410 + 6400 \\ &= 6810 \text{ km} \\ &= 6.810 \times 10^6 \text{ m} \end{aligned}$$

$$\begin{aligned} 1.\ v_{\text{orbital}} &= \sqrt{\frac{GM}{r}} \\ &= \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{3.921 \times 10^8}} \\ &= 1.010 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} 2.\ v_{\text{orbital}} &= \sqrt{\frac{GM}{r}} \\ &= \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.935 \times 10^6}} \\ &= 7.578 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} 3.\ v_{\text{orbital}} &= \sqrt{\frac{GM}{r}} \\ &= \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.810 \times 10^6}} \\ &= 7.647 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Orbital energy

For a small mass m in orbit around a larger mass M, with orbit radius r, the kinetic energy can be expressed in terms of the orbital speed (see **Table 1**):

$$E_{\mathrm{k}} = \frac{1}{2}mv_{\mathrm{orbital}}^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2$$
$$= \frac{GMm}{2r}$$

You have already seen (in section D.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/gravitational-potential-energy-and-gravitational-potential-hl-id-46569/)) that the gravitational potential energy of two objects M and m a distance r apart is:

$$E_{\mathrm{p}} = -\frac{GMm}{r}$$

So the total energy is:

$$E_{\mathrm{orbital}} = \frac{GMm}{2r} - \frac{GMm}{r}$$
$$= -\frac{GMm}{2r}$$

which is the negative of the kinetic energy.

Therefore, as the radius of a body's orbit decreases:

- the kinetic energy increases
- the potential energy decreases (since it increases in magnitude while remaining negative)
- the total energy decreases (since the decrease in potential energy is larger than the increase in kinetic energy).

Although the Earth's atmosphere is much thinner at high altitudes, satellites still experience a small viscous drag force (air resistance). This acts in the opposite direction to a satellite's motion, reducing its speed. If the speed decreases slightly, the satellite will fall into a slightly lower altitude orbit. As it does so, its gravitational potential energy decreases and its kinetic energy increases. So the ultimate effect of drag is to make the satellite decrease its altitude and increase its speed. This can be prevented by using thrusters attached to the satellite to correct its orbit.

Escape speed

When a smaller mass m orbits a larger mass M, such as the Earth, along a circular or an elliptical path, the total mechanical energy, E, of mass m is negative.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

If mass m is supplied with enough kinetic energy that its total mechanical energy becomes zero or greater than zero, what will happen?

Mass m will escape the gravitational attraction of mass M and follow a parabolic path or a hyperbolic path, depending on the energy (**Figure 3**).

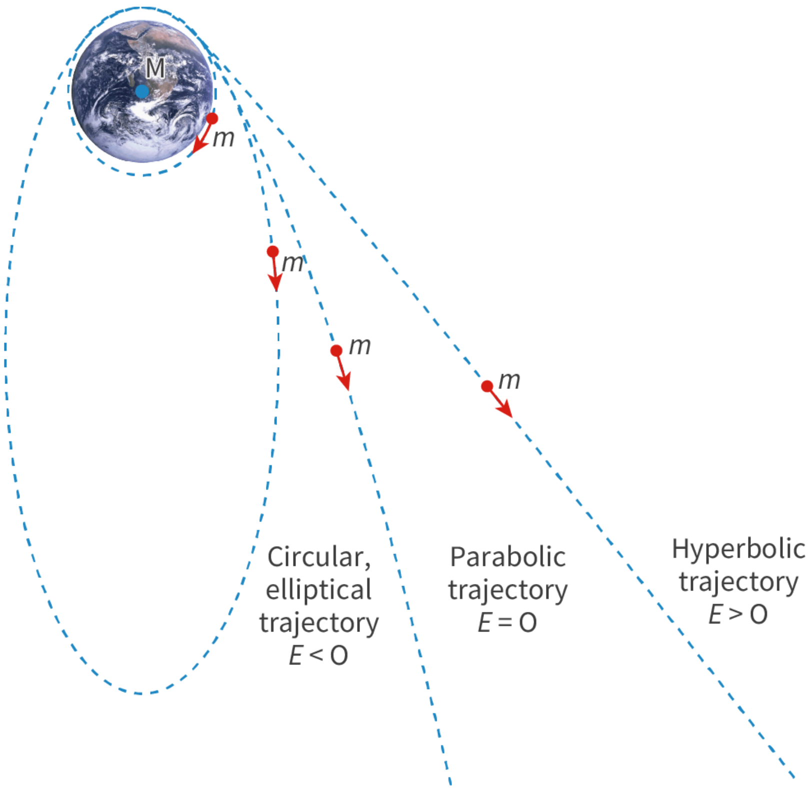


Figure 3. Mass m escaping the gravitational attraction of the Earth.

Credit: NASA (Earth image)

More information for figure 3

The image shows Earth with a large mass "M" and several potential trajectories of a smaller mass "m" as it attempts to escape Earth's gravitational pull. There are three dashed lines representing different trajectories:

1. A circular or elliptical trajectory labeled "E < 0," indicating insufficient energy to escape, resulting in the object remaining in orbit.
2. A straight line representing a parabolic trajectory labeled "E = 0," where the object just has enough energy for an escape velocity, creating a parabolic path.
3. A hyperbolic trajectory labeled "E > 0," where the object has more than enough kinetic energy to escape Earth's gravitational pull, taking a hyperbolic path.

[Generated by AI]

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

For a body with mass m, such as a Mars lander, to escape the gravitational pull of the Earth, we must supply it with enough kinetic energy so that its total mechanical energy is at least zero ($E = 0$). This means there is a minimum initial kinetic energy that will enable the body to escape instead of falling back down to Earth.

Therefore there is a minimum launch speed that will allow the body to escape the Earth. This minimum speed is known as the escape speed, v_{esc} (**Figure 4**).

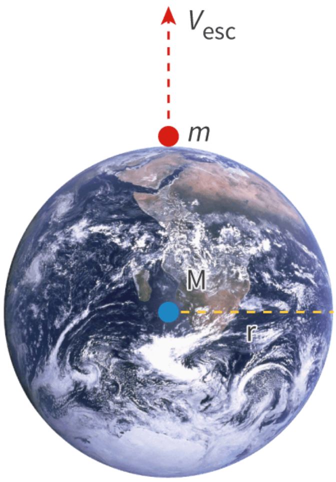


Figure 4. Escape speed.

Credit: NASA (Earth image)

More information for figure 4

The image is a diagram illustrating the concept of escape velocity from Earth. At the center of the image is a picture of Earth, shown from space. Superimposed on the Earth are elements related to physics. There is a vector labeled 'v_esc' pointing away from the Earth, representing the escape velocity. There is a small sphere labeled 'm' at the tip of the vector, indicating a mass that is projected to escape Earth's gravity. Another label 'M' indicates the mass of the Earth. A dashed yellow line is drawn horizontally, labeled 'r', which likely indicates the radius from the center of the Earth to its surface. The overall message of the diagram is to show how a mass can escape Earth's gravitational pull if it achieves the necessary escape velocity.

[Generated by AI]

We can find the escape speed by using the law of conservation of energy (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)) and considering the total mechanical energy at launch and the total mechanical energy of mass m when it reaches infinite separation from mass M.

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

When mass m is launched from mass M from rest, we need to provide enough kinetic energy that the sum of the kinetic energy and gravitational potential energy is equal to zero:

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{r} = 0$$

Rearranging and cancelling m (the mass of the launched body) gives the equation for escape speed shown in **Table 2**.

Table 2. The equation for escape speed.

Equation	Symbols	Units
$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$	v_{esc} = escape speed	metres per second (m s^{-1})
	G = gravitational constant (6.67×10^{-11} N m^2 kg^{-2})	newtons metres squared per kilogram squared (N m^2 kg^{-2})
	M = mass of source of gravitational field	kilograms (kg)
	r = distance from centre of source of gravitational field	metres (m)

Note that escape speed only applies to an object that does not use fuel to increase its own kinetic energy after its launch.

Worked example 2

Determine the escape speed from:

1. the surface of the Earth
2. Mars' moon Deimos

The mass of the Earth is:

$$m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

The radius of the Earth is:

$$r_{\text{Earth}} = 6400 \text{ km}$$

The mass of Deimos is:

$$m_{\text{Deimos}} = 1.8 \times 10^{15} \text{ kg}$$

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The radius of Deimos is:

$$r_{\text{Deimos}} = 13 \text{ km}$$

$$m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg} r_{\text{Earth}} = 6400 \text{ km} m_{\text{Deimos}} = 1.8 \times 10^{15} \text{ kg} r_{\text{Deimos}} = 13 \text{ km} v_{\text{esc}}$$

1. $$\begin{aligned} v_{\text{esc}} &= \sqrt{\frac{2GM}{r}} \\ &= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.4 \times 10^{6}}} \\ &= 1.116 \times 10^{4} \\ &= 1.1 \times 10^{4} \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

2. $$\begin{aligned} v_{\text{esc}} &= \sqrt{\frac{2GM}{r}} \\ &= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.8 \times 10^{15}}{1.3 \times 10^{4}}} \\ &= 4.297 \\ &= 4.3 \text{ m s}^{-1} \text{(2 s.f.)} \end{aligned}$$

Work through the activity to check your understanding of escape speed.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 15 minutes
- **Activity type:** Pair activity

Open the simulation (https://contrib.pbslearningmedia.org/WGBH/arct15/SimBucket/Simulations/satellite shows a satellite in orbit. In the simulation, the starting values are: satellite mass 4000 kg, satellite speed approximately 7900 m s^{-1} and orbital radius approximately 6 400 000 m.

Without changing any of the values, press play to observe the satellite's motion.

To restart the simulation at any time, reload the page.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Your task:

1. Predict and explain what will happen to the speed, orbital radius, kinetic energy and potential energy if you reduce the mass of the satellite. Press pause and change the mass to about 2000 kg, and then press play. Observe and discuss the effect of decreasing the mass on the satellite's motion and on the energy bars on the left.
2. The simulation shows that when a satellite has orbital speed $7900\,\text{m s}^{-1}$, its radius is approximately $6\,400\,000\,\text{m}$. Use a suitable equation to show that this is correct.
3. (a) What happens when a satellite in a circular orbit changes its speed, so that it now has the 'wrong' speed for its orbital radius? Reload the page to reset all the values, and then increase the speed to about 9000 m s^{-1}, to simulate the effect of using thrusters to increase the satellite's speed in the direction of motion. Before pressing play, suggest how the satellite might behave. Then check whether you were correct. If not, discuss why.
(b) Describe the satellite's new orbit. Explain how the total mechanical energy is constant even though the satellite's speed varies.
4. What will happen if the total energy is greater than zero? Reload the page and change the speed to about 5000 m s^{-1} so that the total energy is positive. Before pressing play, predict and explain how the satellite will behave. Then check your prediction.

Click on 'Show solution' to see the answers.

A faster initial speed results in a more elliptical trajectory. (Note that what looks like a circular trajectory is in fact an elliptical trajectory with a very small eccentricity (deviation from circular).)

Kinetic energy, gravitational potential energy and total energy vary at the same time as each other. The total energy stays constant, and energy is transferred between kinetic energy and potential energy.

5 section questions ^

Question 1

HL Difficulty:

A satellite is orbiting a planet of mass *M* at a distance *r* with orbital speed *v*.

Determine the orbital speed of the satellite when its distance from the planet decreases to $\frac{r}{4}$.

Student view

1 $2\sqrt{\frac{GM}{r}}$ ✓

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

2 $\sqrt{\frac{2GM}{r}}$

3 $\sqrt{\frac{GM}{2r}}$

4 $\frac{1}{2}\sqrt{\frac{GM}{r}}$

Explanation

The orbital speed of the satellite at a distance *r* from the planet is:

$$v_{\mathrm{r}} = \sqrt{\frac{GM}{r}}$$

The orbital speed of the satellite at a distance $\frac{r}{4}$ from the planet is:

$$\begin{aligned} v_{\frac{r}{4}} &= \sqrt{\frac{GM}{\frac{r}{4}}} \\ &= \sqrt{\frac{4GM}{r}} \\ &= 2\sqrt{\frac{GM}{r}} \end{aligned}$$

Question 2

HL Difficulty:

Determine the orbital speed of a satellite orbiting Mars at a height of 13 300 km above its surface.

Give your answer to two significant figures.

The mass of Mars: is 6.39 × 10^{23} kg

The radius of Mars is: 3390 km

The orbital speed is 1 1600 ✓ m s^{-1}

Accepted answers and explanation

#1 1600

General explanation

$$M = 6.39 \times 10^{23} \text{ kg}$$

$$\begin{aligned} r &= 3390 + 13\,300 \\ &= 16\,690 \text{ km} \\ &= 1.669 \times 10^{7} \text{ m} \end{aligned}$$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$v_\mathrm{r} = \sqrt{\frac{GM}{r}}$$
$$= \sqrt{\frac{6.67 \times 10^{-11} \times 6.39 \times 10^{23}}{1.669 \times 10^{7}}}$$
$$= 1598\ \mathrm{m\ s^{-1}}$$
$$= 1600\ \mathrm{m\ s^{-1}}\ (2\ \mathrm{s.f.})$$

Question 3

HL Difficulty:

Satellites in low Earth orbit experience a drag force due to the atmosphere of the Earth. This force affects their orbital radius and therefore their orbital speed. Since the presence of the drag force decreases their total mechanical energy, their orbital radius 1 decreases ✓ . This in turn 2 increases ✓ their orbital speed.

Accepted answers and explanation

#1 decreases
becomes smaller
reduces

#2 increases
gets larger
gets bigger
gets greater
boosts

General explanation

Satellites on low Earth orbit experience a drag force due to the atmosphere of the Earth. This drag force affects their orbital radius and therefore their orbital speed. Since the presence of the drag force decreases their total mechanical energy, their orbital radius decreases. This in turn increases their orbital speed. In other words, as they move closer to Earth's surface they start moving faster.

Question 4

HL Difficulty:

Determine the orbital speed of a CubeSat satellite orbiting the Earth at a distance $r_\text{Cubesat} = 0.1 r_\text{Earth}$ away from Earth's surface.

1 $\sqrt{\dfrac{GM_\text{Earth}}{1.1 \times r_\text{Earth}}}$ ✓

2 $\sqrt{\dfrac{GM_\text{Earth}}{r_\text{Cubesat}}}$

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

3 $\sqrt{\frac{GM_{\text{Earth}}}{0.9 \times r_{\text{Earth}}}}$

4 $\sqrt{\frac{GM_{\text{Earth}}}{0.1 \times r_{\text{Earth}}}}$

Explanation

$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$

M_{Earth} is the mass of the source of the gravitational field:

r is distance from the centre of mass of M_{Earth} to the centre of mass m_{Cubesat}:

$$r = r_{\text{Earth}} + r_{\text{Cubesat}}$$

The distance between centres of the two masses:

$$\begin{aligned} r &= r_{\text{Earth}} + r_{\text{Cubesat}} \\ &= r_{\text{Earth}} + 0.1r_{\text{Earth}} \\ &= 1.1r_{\text{Earth}} \end{aligned}$$

$$v_{\text{orbital}} = \sqrt{\frac{GM_{\text{Earth}}}{1.1r_{\text{Earth}}}}$$

Question 5

HL Difficulty:

Determine the escape speed of an object from Jupiter.

Give your answer correct to three significant figures.

The mass of Jupiter is 1.9 × 10^{27} kg

The radius of Jupiter is 71 500 km = 7.15 × 10^{7} m

The escape speed is 1 59500 ✓ ms^{-1}

Accepted answers and explanation

#1 59500

60000

General explanation

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$
$$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{27}}{7.15 \times 10^{7}}}$$
$$= 59\,539 \text{ m s}^{-1}$$
$$= 59\,500 \text{ m s}^{-1} \text{ (3 s.f.)}$$

D. Fields / D.1 Gravitational fields

Summary and key terms

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/summary-and-key-terms-id-46572/print/) Assign

- Kepler's first law of orbital motion states that: Each planet orbits the Sun along an elliptical path. The Sun is located at a focus of the elliptical orbit.
- Kepler's second law of orbital motion states that: The imaginary line joining the Sun and a planet sweeps equal areas of space in equal time intervals as the planet follows its orbit.
- Kepler's third law of orbital motion states that: The square of the orbital period of a planet is directly proportional to the cube of the average distance of the planet to the Sun.
- Newton's universal law of gravitation describes the force of gravitational attraction between any two masses, given by:

$$F = \frac{Gm_1m_2}{r^2}$$

- Gravitational field lines show the direction of the gravitational force acting on a mass at that point.
- Gravitational field strength at a point in a gravitational field is the force per unit mass on a point mass at that point, given by:

$$g = \frac{F}{m} \text{ and } g = G\frac{M}{r^2}$$

Higher level (HL)

- The gravitational potential energy of a system is the work done to bring together the components of the system from infinite separation, as given by:

$$E_{\text{p}} = -\frac{Gm_1m_2}{r}$$

- Gravitational potential is the work done per unit mass to bring a mass from infinity to a point in gravitational field, as given by:

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

$$V_{\mathrm{g}} = -G\frac{M}{r}$$

- The work done in moving a mass in a gravitational field is given by:

$$W = m\Delta V_{\mathrm{g}}$$

- Equipotential lines and surfaces are regions of equal gravitational potential. Equipotential lines and surfaces are perpendicular to gravitational field lines.
- Gravitational field strength is the gravitational potential gradient, as given by:

$$g = -\frac{\Delta V_{\mathrm{g}}}{\Delta r}$$

- The orbital speed of a smaller mass in orbit around a larger mass is given by:

$$v_{\mathrm{orbital}} = \sqrt{\frac{GM}{r}}$$

- The escape speed is the minimum speed needed by an object to escape the gravitational field of a mass, given by:

$$v_{\mathrm{escape}} = \sqrt{\frac{2GM}{r}}$$

- Small amounts of atmospheric drag reduce the total mechanical energy of an orbiting body, resulting in a decrease in orbital radius and an increase in orbital speed.

Student view

Key terms

**Review these key terms. Do you know them all? Fill in as many
can using the terms in this list.**

1. The is the time it takes an object to comp
orbit.
2. The is the distance between the centre o
being orbited and the orbiting object.
3. The is the speed with which an object mo
orbit.
4. A region in space where a mass experiences a gravitational force i
.
5. show the direction of the gravitational for
a mass at that point.
6. represent areas of constant gravitational
7. is the gravitational force per unit mass ex
point in a gravitational field.
8. is the work done to bring together the co
system from infinite separation.
9. is the work done per unit mass to move a
infinity to a point within the gravitational field.
10. The minimum speed needed by a mass to escape the gravitation
planet is called the .

Orbital period | Gravitational field strength | Equipotential surfa
Escape speed | Gravitational potential energy | Orbital speed
Gravitational field | Orbital radius | Gravitational field lines
Gravitational potential

Check

Interactive 1. Identify True and False Statements.

The concept diagram in **Figure 1** summarises the gravitational fields learning covered in this subtopic.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

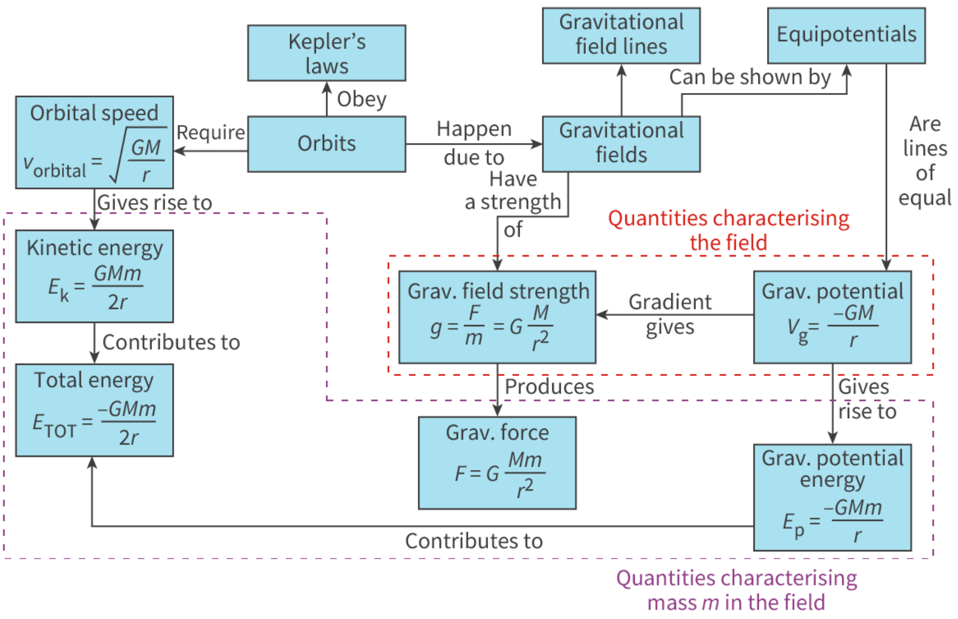


Figure 1. Concept diagram for gravitational fields.

More information for figure 1

The diagram is a detailed concept map outlining the connections between different aspects of gravitational fields. At the top, it starts with Kepler's laws, which are obeyed by orbits. Arrows indicate that orbits require orbital speed (represented by the formula (v_{orbital} = \sqrt{\frac{GM}{r}})), which gives rise to kinetic energy (E_k = \frac{GMm}{2r}). This kinetic energy contributes to total energy (E_{TOT} = -\frac{GMm}{2r}).

On another branch, gravitational fields happen due to gravitational field lines. These fields have a strength described by grav. field strength (g = \frac{F}{m} = G\frac{M}{r^2}), which produces grav. force (F = G\frac{Mm}{r^2}). The grav. field strength's gradient gives grav. potential (V_g = -\frac{GM}{r}), and this potential gives rise to grav. potential energy (E_p = -\frac{GMm}{r}).

Gravitational field lines can be shown by equipotentials, which are lines of equal gravitational potential. The red text "Quantities characterising the field" is noted near grav. field strength and potential, while "Quantities characterising mass m in the field" is associated with kinetic and potential energies.

[Generated by AI]

Student view

D. Fields / D.1 Gravitational fields

Checklist

What you should know

After studying this subtopic, you should be able to:

- Explain Kepler's three laws of orbital motion.
- Explain Newton's universal law of gravitation.
- Predict the orbital period.
- Outline conditions under which extended objects can be treated as point objects.
- Explain what is meant by gravitational field and gravitational field lines.
- State the definition of the gravitational field strength g at point in a gravitational field of an object.
- Show that $g = \frac{F}{m}$ and $g = G\frac{M}{r^2}$.

Higher level (HL)

- Explain how the gravitational potential energy E_p of a system is the work done to assemble the system from infinite separation of the components of the system.
- Show that the gravitational potential energy for a two-body system is given by $E_\text{p} = -G\frac{Mm}{r}$, where r is the separation between the centre of mass of the two bodies.
- Show that the gravitational potential V_g at a point is the work done per unit mass in bringing a mass from infinity to that point is given by $V_\text{g} = -\frac{GM}{r}$.
- Show that the work done in moving a mass m in a gravitational field is given by $W = m\Delta V_\text{g}$.
- Define equipotential surfaces for gravitational fields.
- State the relationship between equipotential surfaces and gravitational field lines.
- Show that the gravitational field strength g as the gravitational potential gradient is given by $g = \frac{\Delta V_\text{g}}{\Delta r}$.
- Show that the orbital speed v_orbital of a body orbiting a large mass as given by $v_\text{orbital} = \sqrt{\frac{GM}{r}}$.
- Show that the escape speed v_escape at any point in a gravitational field as given by $v_\text{escape} = \sqrt{\frac{2GM}{r}}$.

- Discuss the qualitative effect of a small viscous drag due to the atmosphere on the height and speed of an orbiting body.

Investigation

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills – Being curious about the natural world
- **Time required to complete activity:** 60 minutes
- **Activity type:** Pair activity

Your task

Since NASA's first mission to Mars in 1971, we have wondered whether it might be possible for humans to explore Mars.

1. Using this link on Mars exploration (https://mars.nasa.gov/all-about-mars/facts), find the following information:

   - mass of Mars
   - radius of Mars
   - distance between Sun and Mars
   - duration of a Martian year
   - gravitational field strength close to the surface of Mars

2. Using this link about the Sun (https://solarsystem.nasa.gov/solar-system/sun/by-the-numbers), find the mass of the Sun.
3. Calculate the following:

   (a) force of gravitational attraction for the Sun-Mars system
   (b) the acceleration due to gravity on the surface of Mars
   (c) orbital period of Mars
   (d) weight of a person of mass 100 kg on the surface of Mars

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Higher level (HL)

(e) escape speed of a person of mass 100 kg from Mars

1. Discuss your findings by comparing your answers with another pair, and discuss any reasons for differences.

D. Fields / D.1 Gravitational fields

Reflection

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/reflection-id-47882/print/) Assign

Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.

Reflection

Now that you've completed this subtopic, let's come back to the guiding questions introduced in The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44096/).

- How are the properties of a gravitational field quantified?
- How does an understanding of gravitational fields allow for humans to explore the solar system?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

- What main points have you learned from this subtopic?
- Is anything unclear? What questions do you still have?

Student view

- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

⚠ Once you submit your response, you won't be able to edit it.

0/2000

Submit

Rate subtopic D.1 Gravitational fields

Help us improve the content and user experience.