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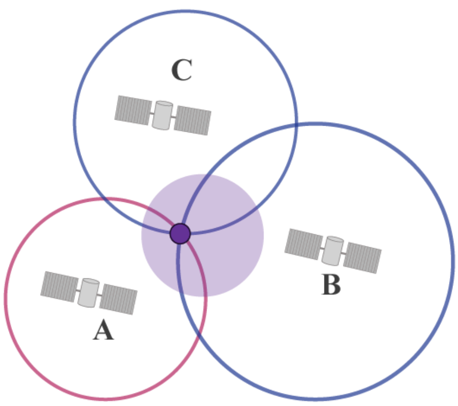
The big picture

What do you do when you are lost and want to know your location? You probably use the Global Position System (GPS) on your phone to locate your position and to guide you to your destination. But how does GPS work?

The GPS uses approximately 30 satellites orbiting the Earth. Whatever your location, at least four GPS satellites are 'visible' to your GPS device (e.g. phone) at any time. Each satellite sends out information at regular time intervals about its position. Your GPS device receives these signals and calculates how far away it is from each satellite.

Your GPS device requires information from at least three satellites to identify your location. It uses a process called trilateration. Watch the video in section 5.3.0 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25553/) if you want to learn more about how GPS works.



More information

The diagram illustrates the concept of trilateration used in GPS technology. It shows three satellites, labeled A, B, and C, each positioned at different locations in space, represented by images of satellites. Each satellite has a circle around it to indicate the area of coverage or signal range. The circles for all three satellites overlap at a central point, which is highlighted in purple, representing the position determined by the GPS device. The concept of trilateration involves the intersection of these three circles to pinpoint an exact location on Earth based on signals received from the satellites.

[Generated by AI]

GPS data can also be used plot vectors that represent the movement of the Earth's tectonic plates so as to predict the likelihood of earthquakes. You can read about this application of vectors in this article (https://spotlight.unavco.org/how-gps-works/gps-and-tectonics/gps-data.html).

In this subtopic, you will learn how to use vectors to form an alternative version of the equation of a straight line. You will see how this can be applied to kinematics problems involving motion in a straight line.

Concept

Different representations of lines help you to analyse different aspects of directional motion. Under what circumstances might a vector equation be more useful than a Cartesian equation? What extra information does it give?

Theory of Knowledge

Writing lines in various forms could be considered a type of 'model'. Models are important in knowledge production in all areas of knowledge because they serve to represent knowledge. A key knowledge issue with models, however, is that they are simplified representations of a thing — not the thing itself.

Knowledge Question: Do mathematical models hold a higher level of validity than models from other areas of knowledge?

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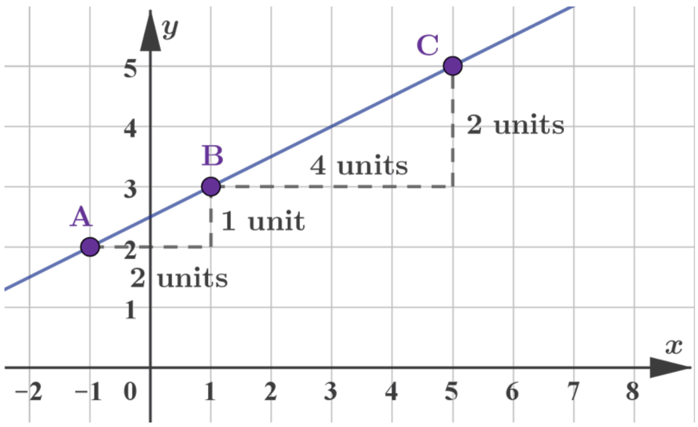
3. Geometry and trigonometry / 3.14 Lines in two and three dimensions

Vector equation of a straight line

Consider the straight line in the diagram below, which passes through the points A, B and C. The gradient is 0.5 and the y - intercept is 2.5 so, in slope–intercept form, the equation of the line is $y = 0.5x + 2.5$. The coordinates of any point on the line satisfy this equation.



More information

The image is a graph showing a straight line on a coordinate plane. The line passes through three labeled points: A, B, and C. The points are located at A(0,2), B(1,3), and C(5,5). The X-axis is labeled with values from -2 to 8 and the Y-axis from 0 to 5. The gradient of the line is 0.5, and the Y-intercept is 2.5, as indicated by the equation y = 0.5x + 2.5. The graph also visually shows the distances between points: 2 units vertically from A to B, 1 unit horizontally from A to B, and 4 units horizontally from B to C, as well as 2 units vertically from B to C.

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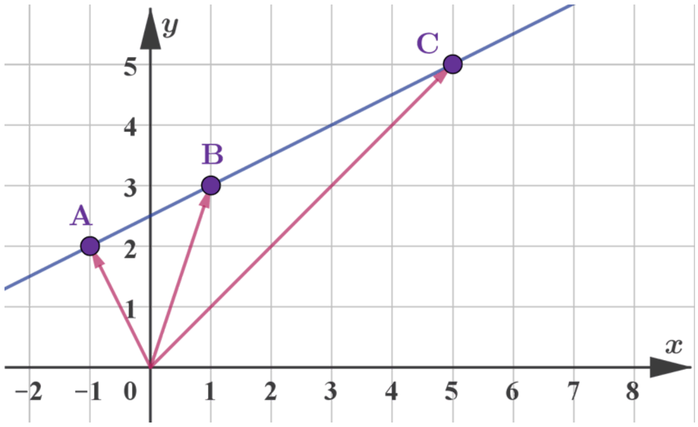
The same straight line can be represented using vectors. Instead of the gradient, you use a direction vector and instead of the y-intercept you use the position vector of a known point on the line.

Student view



More information

The diagram shows a straight line on a grid, labeled with a blue line intersecting through various points. The grid has axes labeled 'x' and 'y'. Three points, labeled A, B, and C, are visible along the line. Vectors represented by pink arrows originate from the origin (0,0) and point towards A, B, and C. Point A is located at (-1, 2), point B at (1, 3), and point C at (3, 4). All three points are collinear, demonstrating how vectors can represent a line. The vectors are labeled with letters suggesting position vectors and demonstrate direction from the origin to each point.

[Generated by AI]

Consider the line in the diagram below, where P, Q and R are collinear and $\overrightarrow{\mathrm{OP}}$, $\overrightarrow{\mathrm{OQ}}$ and $\overrightarrow{\mathrm{OR}}$ are the position vectors of these points, respectively.

As $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{QR}}$ are parallel vectors,

$$\overrightarrow{\mathrm{PQ}} = \lambda\overrightarrow{\mathrm{QR}}, \text{ where } \lambda \in \mathbb{R}$$

How could you represent the position vectors of all the points on the line l?

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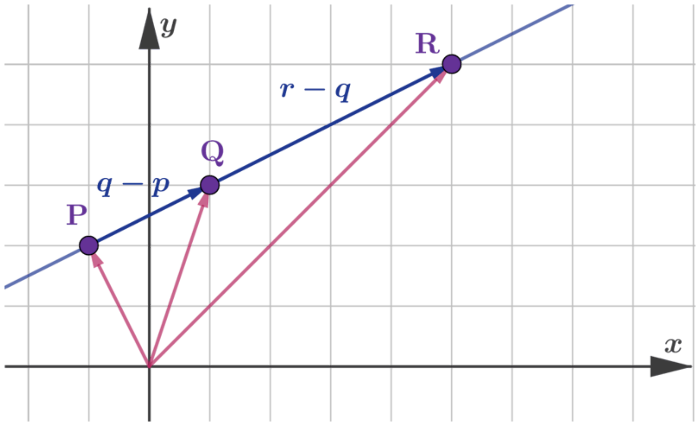

More information

The diagram illustrates a line on a grid with three distinct points labeled P, Q, and R. These points lie along a blue line trending diagonally upwards. Vectors connecting these points are depicted as arrows: (\overrightarrow{PQ}) described as (q - p) and (\overrightarrow{QR}) described as (r - q). The x and y axes are visible, with x pointing right and y upwards. The diagram focuses on the positions of these vectors and emphasizes the differences between them.

[Generated by AI]

$\overrightarrow{\mathrm{PQ}}=\overrightarrow{\mathrm{OQ}}-\overrightarrow{\mathrm{OP}}=\boldsymbol{q}-\boldsymbol{p}$ and $\overrightarrow{\mathrm{QR}}=\overrightarrow{\mathrm{OR}}-\overrightarrow{\mathrm{OQ}}=\boldsymbol{r}-\boldsymbol{q}$.

$$\overrightarrow{\mathrm{QR}}=\lambda\overrightarrow{\mathrm{PQ}}\Rightarrow\boldsymbol{r}-\boldsymbol{q}=\lambda(\boldsymbol{q}-\boldsymbol{p})$$

Rearranging gives

$$\boldsymbol{r}=\boldsymbol{q}+\lambda(\boldsymbol{q}-\boldsymbol{p})$$

Now let $\boldsymbol{q}-\boldsymbol{p}=\boldsymbol{b}$, where $\boldsymbol{b}$ is a vector in the same direction as $\overrightarrow{\mathrm{PQ}}$.

Therefore, the position vector of any point on the line should satisfy the equation.

$$\boldsymbol{r}=\boldsymbol{q}+\lambda\boldsymbol{b}$$

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view

This equation represents the vector equation of a straight line, where $\boldsymbol{q}$ is the vector representing the position vector of a point on the line and $\boldsymbol{b}$ is the direction vector.

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Example 1

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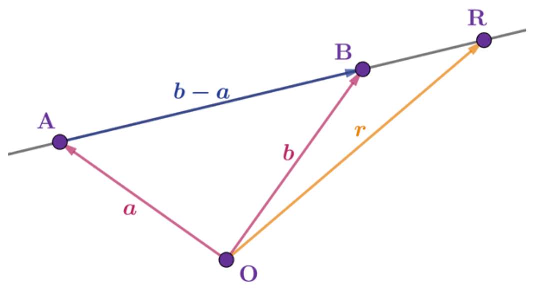
a) Write a vector equation of the line passing through points $\mathrm{A}\,(1, 1, 1)$ and $\mathrm{B}\,(-1, 2, 3)$.

b) C is the point $(2, -2, 2)$. Use your answer to part **a** to determine whether A, B and C are collinear.

	Steps	Explanation
a)		Sketch the points and position vectors.
	$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	$\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$
	$\boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ or $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	$\boldsymbol{r} = \overrightarrow{\mathrm{OA}} + \lambda\overrightarrow{\mathrm{AB}}$ or $\boldsymbol{r} = \overrightarrow{\mathrm{OB}} + \lambda\overrightarrow{\mathrm{AB}}$
b)	$\overrightarrow{OC} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$	Position vector of point C.

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	Steps	Explanation
	$\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	If point C is on the line, its positic vector will satisfy the vector equation of the line
	$2 = 1 - 2\lambda \Rightarrow \lambda = -\frac{1}{2}$ $-2 = 1 + \lambda \Rightarrow \lambda = -3$ $2 = 1 + 2\lambda \Rightarrow \lambda = \frac{1}{2}$	$\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1-2\lambda \\ 1+\lambda \\ 1+2\lambda \end{pmatrix}$
	As λ is different for each component, point C is not on line AB, so A, B and C are not collinear.	

Important

If three points A, B and C with respective position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are collinear, then there will be a unique $k \in \mathbb{R}$ which satisfies the equation

$$\boldsymbol{a} - \boldsymbol{c} = k(\boldsymbol{b} - \boldsymbol{c})$$

Be aware

The vector equation of a line is not unique.

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \text{ or } \boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \boldsymbol{t} \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix}$$

all represent the same line using different points and equivalent direction vectors.

Activity

Use the following applet to investigate how using different parameters in the equation changes the line and the points on the line.

The vector equation of the line is given by $\boldsymbol{r} = \begin{pmatrix} a \\ b \end{pmatrix} + \boldsymbol{t} \begin{pmatrix} c \\ d \end{pmatrix}$

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Use the sliders to change the values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ and $\boldsymbol{t}$ and see how this changes the position of point P on the line.

Interactive 1. Investigating How Different Parameters Affect the Line and Its Points.

More information for interactive 1

This interactive allows users to explore how different parameters affect a line's position and orientation in 2D space through its vector equation. The visualization demonstrates the relationship between the algebraic representation $r = (a, b) + t(c, d)$ and its geometric manifestation, enabling users to manipulate the equation's components and immediately observe their impact on the line's behavior in the coordinate plane.

The display presents a 2D coordinate system with x and y axes, showing a line generated by the current vector equation. Users can adjust five interactive sliders controlling parameters a and b in blue (the position vector components ranging from -3 to 3), c and d orange (the direction vector components from -3 to 3), and t in orange (a scalar parameter from -2 to 2). A movable point P marks the specific location on the line corresponding to the current t-value, with all components updating in real-time as sliders are manipulated. The current vector equation appears visibly above the graph, changing dynamically with each adjustment.

By manipulating the sliders, users can observe various line transformations. For example: setting $(a, b) = (1, 2)$ and $(c, d) = (3, 1)$ with $t = 0$ places point P at $(1, 2)$; increasing t to 1 moves P to $(4, 3)$; while changing (c, d) to $(-2, 2)$ with $t = 1.5$ reorients the line's direction and moves P to $(-2, 5)$. The t-slider specifically shows how the parameter traces different points along the infinite line.

Through this exploration, users develop a concrete understanding of how vector equations represent lines in 2D space. They learn that (a, b) determines a fixed point on the line while (c, d) controls its direction, and ‘t’ acts as a scalar that generates all points on the infinite line. The activity clarifies why changing the direction vector's components alters the line's slope and how different parameter combinations can produce identical lines through distinct equations.

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Exam tip

In IB examinations the vector equation of a line will be given as

$$\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$$

where $\boldsymbol{a}$ is the position vector of any point on the line and $\boldsymbol{b}$ is the direction vector.

Be aware

Although the vector equation is given as $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$ you will need to use the handwriting notation $\vec{r} = \vec{a} + \lambda \vec{b}$ or $\underline{r} = \underline{a} + \lambda \underline{b}$ in the exam. Otherwise you might be penalised.

Example 2

★☆☆

Consider the two points $\mathrm{A}(1, 2, -1)$ and $\mathrm{B}(11, -2, -7)$.

a) Find a vector equation of the line containing these two points.

b) Show that the point $\mathrm{S}(-14, 8, 8)$ lies on the line, while the point $\mathrm{T}\ (6, -7, 5)$ does not lie on the line.

	Steps	Explanation
a)	$\boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	Write the position vector poin .

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	Steps	Explanation
	$\boldsymbol{b} = -\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 11 \\ -2 \\ -7 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ -6 \end{pmatrix}$	$\overrightarrow{AB}$ has direction vector $\boldsymbol{b}$.
	The vector equation is $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ -4 \\ -6 \end{pmatrix}$	
b)	$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -3 \end{pmatrix}$	Given that the λ parameter is arbitrary constant and can so any number, you can replace $2 \times \lambda$ by a new parameter, wl can also be represented by λ This has no effect on the shap of the vector line.
	$\begin{pmatrix} -14 \\ 8 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ -3 \end{pmatrix}$	If point $\mathrm{S}\,(-14, 8, 8)$ lies on t line, then there is a single λ s that this equation is true.
	So $\lambda = \dfrac{-14-1}{5} = -3$	Find λ using the information the x -coordinate.
	$2 + (-3)(-2) = 8$	Substitute -3 for λ into the other two equations .
	$-1 + (-3)(-3) = 8$ So $\mathrm{S}\,(-14, 8, 8)$ lies on the line. $\mathrm{T}\,(6, -7, 5)$ $\lambda = \dfrac{6-1}{5} = 1$	Use the x-coordinate to find t value of λ .
	$-7 \neq 2 + 1 \times (-2)$	Check whether this satisfies tl y-coordinate.
	Hence, $\mathrm{T}\,(6, -7, 5)$ does not lie on the line.	

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Example 3

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Create a line that includes $\mathrm{D}(1,-4)$ and is parallel to the vector $\begin{pmatrix}2\\-3\end{pmatrix}$

This is the position vector of a point on the line.

$$\boldsymbol{d}=\begin{pmatrix}1\\-4\end{pmatrix}$$

Let $\boldsymbol{e}$ be the direction vector of the line.

$$\boldsymbol{e}=\begin{pmatrix}2\\-3\end{pmatrix}$$

So the line has a vector equation given by $\boldsymbol{r}=\begin{pmatrix}1\\-4\end{pmatrix}+\boldsymbol{t}\begin{pmatrix}2\\-3\end{pmatrix}$

or, equivalently, $\boldsymbol{r}=\boldsymbol{i}-4\boldsymbol{j}+\boldsymbol{t}(2\boldsymbol{i}-3\boldsymbol{j})$

Example 4

★★☆

Find a vector equation of a line passing through the points $\mathrm{K}(1,-1,1)$ and $\mathrm{L}(0,0,2)$.

Hence find the possible coordinates of a point R on the line which satisfies $\mathrm{KR}=3\mathrm{RL}$.

Steps	Explanation
$\overrightarrow{\mathrm{KL}}=\overrightarrow{\mathrm{OL}}-\overrightarrow{\mathrm{OK}}=\begin{pmatrix}0\\0\\2\end{pmatrix}-\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}-1\\1\\1\end{pmatrix}$	$\overrightarrow{\mathrm{OK}}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\overrightarrow{\mathrm{OL}}=$

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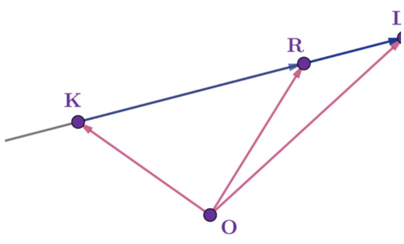
Steps	Explanation
$\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	
Position of point R is $\overrightarrow{\text{OR}} = \begin{pmatrix} 1-\lambda \\ -1+\lambda \\ 1+\lambda \end{pmatrix}$	
$\begin{pmatrix} 1-\lambda \\ -1+\lambda \\ 1+\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 3\left[\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 1-\lambda \\ -1+\lambda \\ 1+\lambda \end{pmatrix}\right]$	$\overrightarrow{\text{KR}} = 3\overrightarrow{\text{RL}} \Rightarrow \overrightarrow{\text{OR}} - \overrightarrow{\text{OK}} = 3(\overrightarrow{\text{OL}} - \overrightarrow{\text{OR}})$
$\begin{pmatrix} -\lambda \\ \lambda \\ \lambda \end{pmatrix} = \begin{pmatrix} -3+3\lambda \\ 3-3\lambda \\ 3-3\lambda \end{pmatrix}$	Simplify both sides.
$-\lambda = -3 + 3\lambda \Rightarrow \lambda = \frac{3}{4}$	Solve for λ .
$\overrightarrow{\text{OR}} = \begin{pmatrix} 1-\frac{3}{4} \\ -1+\frac{3}{4} \\ 1+\frac{3}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \\ \frac{7}{4} \end{pmatrix}$	
Therefore, the coordinates of point R are $\left(\frac{1}{4}, -\frac{1}{4}, \frac{7}{4}\right)$	

Student view

Be aware

The coordinates of a point A are given in the form $\mathrm{A}\left(x, y, z\right)$, while the position vector of point A relative to a fixed point O is given in vector format as $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Make sure you give the answer in the correct form in the exam.

4 section questions

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Question 1

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Select which of the following is a vector equation of the line with direction vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$ that passes through $\mathrm{A}(1, 1, 2)$.

1 $\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}+\lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$ ✓

2 $\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}+\lambda(\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k})$

3 $\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}+\lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$

4 $\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}+\lambda(\boldsymbol{i}-\boldsymbol{j}+2\boldsymbol{k})$

Explanation

The position vector of point A is $\begin{pmatrix}1\\1\\2\end{pmatrix}$

and the direction vector of the line is $\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\boldsymbol{r}=\begin{pmatrix}1\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\1\end{pmatrix}=\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}+\lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$$

Question 2

★☆☆

Point $\mathrm{A}(a, 2, 3)$ lies on the line with the vector equation $\boldsymbol{r}=\boldsymbol{i}+\boldsymbol{k}+\lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$.

Find the value of a.

1 3 ✓

2 2

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3 1

4 4

Explanation

The position vector of A is $\begin{pmatrix} a \\ 2 \\ 3 \end{pmatrix}$

and it lies on the line $\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

So

$$\begin{pmatrix} a \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

The x, y, and z components on the left-hand side will be equal to those on the right-hand side:

$$\begin{aligned} &a = 1 + \lambda \\ &2 = 0 + \lambda \Rightarrow \lambda = 2 \\ &3 = 1 + \lambda \Rightarrow \lambda = 2 \end{aligned}$$

Since $\lambda = 2$, $a = 3$

Question 3

★☆☆

Select which of the following points lies on the line $\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} + \boldsymbol{t} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

1 $(5, -2, -11)$ ✓

2 $(5, 2, -11)$

3 $(-5, -2, -11)$

4 $(5, -2, 11)$

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Explanation

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With the four options given, you can try any number for $\boldsymbol{t}$ as long as it gives you a vector like $\begin{pmatrix}5\\y\\z\end{pmatrix}$ or $\begin{pmatrix}-5\\y\\z\end{pmatrix}$

So for $1-2\boldsymbol{t}=5$, you must have $\boldsymbol{t}=-2$ similarly, for $1-2\boldsymbol{t}=-5, \boldsymbol{t}=3$.

However, only $\boldsymbol{t}=-2$ gives the correct y- and z-coordinates.

That is, with $\boldsymbol{t}=-2$ we obtain the point

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\-3\end{pmatrix}-2\begin{pmatrix}-2\\1\\4\end{pmatrix}=\begin{pmatrix}5\\-2\\-11\end{pmatrix}\text{, i.e. } (5,-2,-11).$$

Question 4

★☆☆

Select which of the following is **not** a vector equation of the line containing the point $(1,-2)$ and parallel to the vector $\begin{pmatrix}-2\\3\end{pmatrix}$.

1 $\boldsymbol{r}=\begin{pmatrix}1\\-2\end{pmatrix}+\boldsymbol{t}\begin{pmatrix}-3\\2\end{pmatrix}$ ✓

2 $\boldsymbol{r}=\begin{pmatrix}1\\-2\end{pmatrix}+\boldsymbol{t}\begin{pmatrix}-2\\3\end{pmatrix}$

3 $\boldsymbol{r}=\begin{pmatrix}1\\-2\end{pmatrix}+\boldsymbol{t}\begin{pmatrix}2\\-3\end{pmatrix}$

4 $\boldsymbol{r}=\begin{pmatrix}-1\\1\end{pmatrix}+\boldsymbol{t}\begin{pmatrix}-4\\6\end{pmatrix}$

Explanation

Since $\begin{pmatrix}-3\\2\end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix}-2\\3\end{pmatrix}$, it cannot be a vector equation of the line that is parallel to the vector $\begin{pmatrix}-2\\3\end{pmatrix}$.

3. Geometry and trigonometry / 3.14 Lines in two and three dimensions

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Parametric and Cartesian forms of the equation of a straight line

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Parametric form

The equation of a straight line can be written in several equivalent ways.

For a line with equation in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$, where $\boldsymbol{a} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$,

you can write

$$\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} x_0 + \lambda l \\ y_0 + \lambda m \\ z_0 + \lambda n \end{pmatrix}$$

Thus,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 + \lambda l \\ y_0 + \lambda m \\ z_0 + \lambda n \end{pmatrix},$$

where (x, y, z) are the coordinates of a general point on the line.

As the components on the left-hand side are equal to those on the right,

$$x = x_0 + \lambda l, y = y_0 + \lambda m \text{ and } z = z_0 + \lambda n$$

This is the parametric form of the equation of a straight line.

✓ **Important**

If the parametric equation of a line is

$$x = x_0 + \lambda l \text{ , } y = y_0 + \lambda m \text{ and } z = z_0 + \lambda n$$

then the vector equation of the line is

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$$\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

Example 1

★☆☆

Write the parametric equation of a line passing through $\text{A}(-1, 2, 0)$ and $\text{B}(3, 1, 1)$.

Vector form:

$$\begin{aligned} \boldsymbol{r} &= \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \lambda \left(- \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right) \\ &= \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} \end{aligned}$$

where point A is a point on the line and the vector $\overrightarrow{\text{AB}}$ is the direction vector. This leads to the parametric form

$$\begin{aligned} x &= -1 + 4\lambda \\ y &= 2 - \lambda \\ z &= \lambda \end{aligned}$$

Exam tip

In the IB formula booklet, the parametric form of the equation of a line is given as

$x = x_0 + \lambda l$, $y = y_0 + \lambda m$ and $z = z_0 + \lambda n$.

Cartesian form

In 2D, you write the Cartesian form of an equation as $y = mx + c$.

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How could you write the Cartesian form of the equation of a straight line in 3D?

As the Cartesian equation is the relationship between the x, y and z components, you can use the parametric form of the equation of a line

$$x = x_0 + \lambda l, y = y_0 + \lambda m \text{ and } z = z_0 + \lambda n$$

Rearranging each expression and solving for λ gives

$$\lambda = \frac{x - x_0}{l}$$
$$\lambda = \frac{y - y_0}{m}$$
$$\lambda = \frac{z - z_0}{n}$$

As all the equations are equal to λ, you can write

$$\lambda = \frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

This shows the relationship between the x, y and z coordinates.

Exam tip

In the IB formula booklet the Cartesian equation of a line is given as

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

Example 2

★☆☆

The vector equation of a line is $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$. Write the equation in Cartesian form.

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Steps	Explanation
$\frac{x-(-2)}{1}=\frac{y-1}{-1}=\frac{z-3}{-2}$ or $\frac{x+2}{1}=\frac{1-y}{1}=\frac{3-z}{2}$	The direction vector is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ The position vector of the point on the line is $\begin{pmatrix}-2\\1\\3\end{pmatrix}$

Example 3

★☆☆

The Cartesian equation of a line is $\frac{1+x}{3}=\frac{1-y}{2}=\frac{2+z}{1}$. Write its vector equation.

Steps	Explanation
$\frac{1+x}{3}=\frac{1-y}{2}=\frac{2+z}{1}$ $\frac{x-(-1)}{3}=\frac{y-1}{-2}=\frac{z-(-2)}{1}$	Rearrange into the form $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$
$r=\begin{pmatrix}-1\\1\\-2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}$	The direction vector is $\begin{pmatrix}3\\-2\\1\end{pmatrix}$ The position vector of the point on the line is $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$

Example 4

Student view

★☆☆

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

If the point $\mathrm{A}(2, a, b)$ is on the line $\frac{1+x}{3}=\frac{1-y}{2}=\frac{2+z}{1}$, find $a+b$.

As the point is on the line, it will satisfy the equation.

$$\frac{1+2}{3}=\frac{1-a}{2}=\frac{2+b}{1}$$

Solve for a .

$$\frac{1-a}{2}=\frac{1+2}{3}=1$$

$$a=-1$$

Solve for b .

$$\frac{2+b}{1}=1$$

$$b=-1$$

Therefore,

$$a+b=-2$$

4 section questions

Question 1

★☆☆

Select the vector form of the equation of the line

$$\frac{x-1}{2}=\frac{y+1}{2}=\frac{z-1}{3}$$

1 $\boldsymbol{r}=\boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k}+\lambda(2\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k})$ ✓

Student view

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2 $\boldsymbol{r} = 2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k} + \lambda(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$

3 $\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + \lambda(2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$

4 $\boldsymbol{r} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k} + \lambda(2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$

Explanation

The direction vector is $\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$

The position vector of the point on the line is $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k} + \lambda(2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$$

Therefore, the correct answer is

$$\boldsymbol{r} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k} + \lambda(2\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$$

Question 2

★☆☆

A line has vector equation $\boldsymbol{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

Select which of the following is the equation in Cartesian form.

1 $x = y - 1 = -0.5z$ ✓

2 $x = y = 0.5z$

3 $-x = y = -0.5z$

4 $x = -y = 0.5z$

Student
view

Explanation

Using the Cartesian equation of a line

$$\frac{x-0}{1}=\frac{y-1}{1}=\frac{z-0}{-2}$$

$$x=y-1=-\frac{z}{2}$$

$$x=y-1=-0.5z$$

Question 3

Select which of the following is a Cartesian equation of the line passing through $\mathrm{A}(0.5, 2, 1)$ and $\mathrm{B}(1, 0.5, -2)$.

1 $6x-3=-2y+4=-z+1$ ✓

2 $x-1=-2y+4=-z+1$

3 $6x-3=2y-4=-z+1$

4 $6x-3=-2y+4=z-1$

Explanation

Direction vector $\overrightarrow{\mathrm{AB}}=\begin{pmatrix}0.5\\-1.5\\-3\end{pmatrix}$ and point $\mathrm{A}(0.5, 2, 1)$:

$$\frac{x-0.5}{0.5}=\frac{y-2}{-1.5}=\frac{z-1}{-3}$$

Rearrange and simplify:

$$\frac{x-0.5}{0.5}=\frac{y-2}{-1.5}=\frac{z-1}{-3}$$

Multiply each term by 3:

$$6x-3=-2y+4=-z+1$$

Question 4

★★☆

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Select which of the following is vector equation of the line $2x - 1 = -y + 4 = 3z + 1$.

1 $\boldsymbol{r} = \begin{pmatrix} \frac{1}{2} \\ 4 \\ -\frac{1}{3} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{3} \end{pmatrix}$ ✓

2 $\boldsymbol{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ -4 \\ -\frac{1}{3} \end{pmatrix}$

3 $\boldsymbol{r} = \begin{pmatrix} \frac{1}{2} \\ -4 \\ -\frac{1}{3} \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$

4 $\boldsymbol{r} = \begin{pmatrix} \frac{1}{2} \\ 4 \\ \frac{1}{3} \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

Explanation

Rearrange and write as a Cartesian equation $\frac{x - 0.5}{\frac{1}{2}} = \frac{y - 4}{-1} = \frac{z - \left(-\frac{1}{3}\right)}{\frac{1}{3}}$

Direction vector $\begin{pmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{3} \end{pmatrix}$, position vector of a point $\begin{pmatrix} \frac{1}{2} \\ 4 \\ -\frac{1}{3} \end{pmatrix}$

So the vector equation is $\boldsymbol{r} = \begin{pmatrix} \frac{1}{2} \\ 4 \\ -\frac{1}{3} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{3} \end{pmatrix}$

3. Geometry and trigonometry / 3.14 Lines in two and three dimensions

The angle between two lines

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/the-angle-between-two-lines-id-28311/print/) Assign

Student view

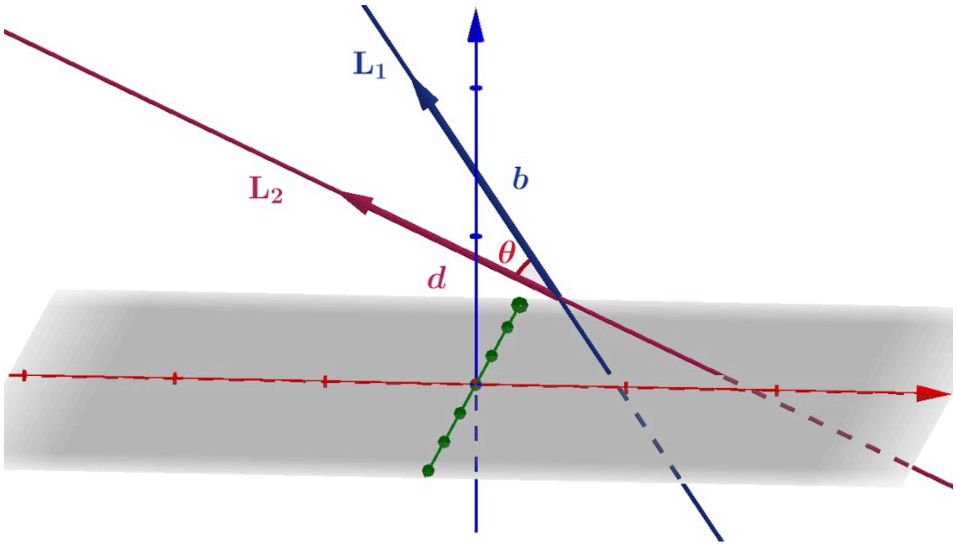
You can find the angle between two lines using their direction vectors.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Consider the lines

$$L_1 : \boldsymbol{r}_1 = \boldsymbol{a} + \lambda\boldsymbol{b} \text{ and } L_2 : \boldsymbol{r}_2 = \boldsymbol{c} + t\boldsymbol{d}$$

The angle between the two lines is the same as the angle between their direction vectors $\boldsymbol{b}$ and $\boldsymbol{d}$. How could you represent the angle between the lines L_1 and L_2?



More information

The image depicts a geometric diagram showing the intersection of two lines, L1 and L2, in a three-dimensional space. Line L1 is drawn in blue, extending upwards, with an arrow labeled 'b' representing its direction vector. Line L2 is drawn in dark red, extending diagonally across the grey plane, with an arrow labeled 'd' signifying its direction vector. The lines intersect at a point where they form an angle θ, marked in red between the direction vectors b and d. The diagram is situated within a grey plane, adding depth to the visual representation. There are also dashed projections of the lines onto the plane, enhancing the spatial perspective of the intersection.

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Writing the scalar product

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/parametric-and-cartesian-forms-of-the-equation-id-28310/print/) Assign

$$\boldsymbol{b} \cdot \boldsymbol{d} = |\boldsymbol{b}|\,|\boldsymbol{d}| \cos\theta$$

Student view

and rearranging gives

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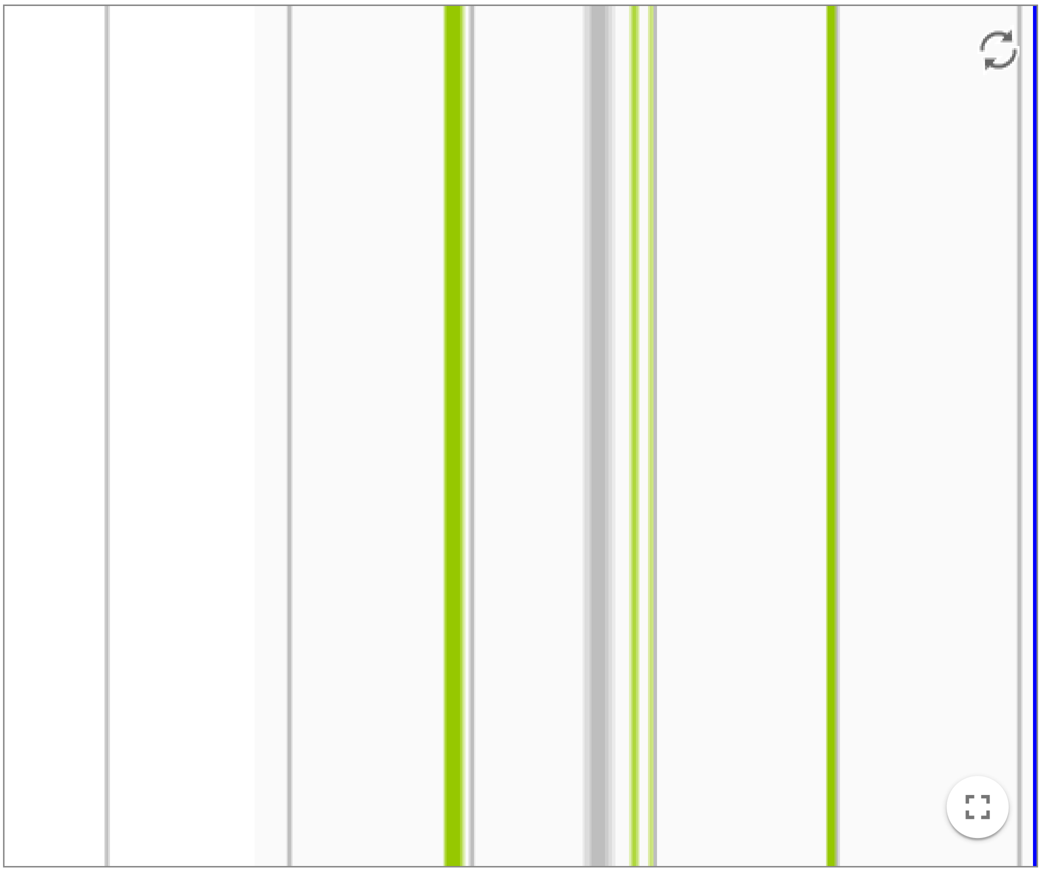
$$\cos\theta = \frac{\boldsymbol{b} \cdot \boldsymbol{d}}{|\boldsymbol{b}|\,|\boldsymbol{d}|}$$

So the angle θ between these lines is given by

$$\theta = \cos^{-1}\left[\frac{\boldsymbol{b} \cdot \boldsymbol{d}}{|\boldsymbol{b}|\,|\boldsymbol{d}|}\right]$$

You can see this relationship in the following interactive graph.



Interactive 1. Angle Between Two Lines.

More information for interactive 1

This interactive visualization helps users understand how to calculate the angle between two lines in three-dimensional space using vector mathematics.

The display features a 3D coordinate system with x-, y-, and z-axes and two colored lines labeled L1 and L2. Each line is defined by its direction vectors, represented by arrows: vectors a and b for line L1, and vectors c and d for line L2. Users can rotate and zoom the 3D system to better observe the spatial relationship between the lines. A green angle marker labeled θ highlights the angle formed between the direction vectors of L1 and L2 when extended from a common point. As users view the figure from different perspectives, they gain insight into how the

Student
view

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angle between the lines is calculated based on the cosine of the angle between their direction vectors.

This tool provides a powerful, visual explanation of a core concept in 3D vector geometry, illustrating how directional relationships in space can be analyzed using vector-based methods.

Example 1

★★☆

Find the angle between the lines L_1 and L_2 defined by

$$\mathrm{L}_1 : \boldsymbol{r}_1 = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$$

and

$$\mathrm{L}_2 : \boldsymbol{r}_2 = \begin{pmatrix} 5 \\ 1 \\ 5 \end{pmatrix} + \boldsymbol{t} \begin{pmatrix} -9 \\ 6 \\ -\frac{15}{2} \end{pmatrix}$$

Take the direction vector of these line $\boldsymbol{v} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} -9 \\ 6 \\ -\frac{15}{2} \end{pmatrix}$ and find their scalar product and their magnitudes:

$$\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -9 \\ 6 \\ -\frac{15}{2} \end{pmatrix} = 1 \times (-9) + 7 \times 6 + (-3) \times \left(-\frac{15}{2}\right) = \frac{111}{2}$$

$$|\boldsymbol{v}| = \sqrt{1^2 + 7^2 + (-3)^2} = \sqrt{59}$$

$$|\boldsymbol{w}| = \sqrt{(-9)^2 + 6^2 + \left(-\frac{15}{2}\right)^2} = \frac{\sqrt{693}}{2}$$

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Hence, the angle between these vectors is given by

$$\theta = \cos^{-1}\left[\frac{\frac{111}{2}}{\sqrt{59}\times\frac{\sqrt{693}}{2}}\right]$$
$$= \cos^{-1}\left[\frac{111}{\sqrt{59}\times\sqrt{693}}\right]$$
$$\approx 56.7^\circ$$

Thus, the angle between these lines is 56.7°.

Example 2

★★☆

Find the angle between the following lines:

$$\mathrm{T}_1 : \boldsymbol{a} = \begin{pmatrix}0\\5\end{pmatrix} + \lambda\begin{pmatrix}-1\\4\end{pmatrix}$$

and

$$\mathrm{T}_2 : \boldsymbol{b} = \begin{pmatrix}2\\-1\end{pmatrix} + \boldsymbol{t}\begin{pmatrix}1\\-2\end{pmatrix}$$

Let the direction vectors be $\boldsymbol{v} = \begin{pmatrix}-1\\4\end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix}1\\-2\end{pmatrix}$

Then evaluate:

$$\boldsymbol{v}\cdot\boldsymbol{w} = \begin{pmatrix}-1\\4\end{pmatrix}\cdot\begin{pmatrix}1\\-2\end{pmatrix} = (-1)\times 1 + 4\times(-2) = -9$$

$$|\boldsymbol{v}| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$$

$$|\boldsymbol{w}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

Student view

Thus, the angle between these vectors is

$$\begin{aligned}\theta &= \cos^{-1}\left[\frac{-9}{\sqrt{17}\times\sqrt{5}}\right]\\ &= \cos^{-1}\left[\frac{-9}{\sqrt{85}}\right]\\ &\approx 167.47^{\circ}\end{aligned}$$

As this angle is obtuse, conclude that the angle between the lines T_1 and T_2 is $180^{\circ} - 167.47^{\circ} = 12.5^{\circ}$.

Example 3

★★☆

Find the angle between the lines given by $\mathrm{L}_1 : \dfrac{x+1}{4} = 2 - y = z$ and $\mathrm{L}_2 : \dfrac{x-2}{3} = y + 1 = 2 - z$.

Clearly the angle between lines, say θ, is dependent on the direction of these lines. Thus, line L_1 is parallel to the vector $\begin{pmatrix}4\\-1\\1\end{pmatrix}$, as already seen above. Line L_2 is parallel to the vector $\begin{pmatrix}3\\1\\-1\end{pmatrix}$, where we must be careful to write $2 - z = -(z-2)$ such that it is in the form $\dfrac{z - z_0}{n}$.

Then, the angle is given through the scalar product, i.e.

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$$\theta = \cos^{-1}\left[\frac{\begin{pmatrix}4\\-1\\1\end{pmatrix}\cdot\begin{pmatrix}3\\1\\-1\end{pmatrix}}{\left|\begin{pmatrix}4\\-1\\1\end{pmatrix}\right|\left|\begin{pmatrix}3\\1\\-1\end{pmatrix}\right|}\right]$$

$$= \cos^{-1}\left[\frac{4\times 3 - 1\times 1 + 1\times(-1)}{\sqrt{4^2+(-1)^2+1^2}\sqrt{3^2+1^2+(-1)^2}}\right]$$

$$\approx 44.7^\circ$$

Be aware

Although you may find the angle between the lines to be obtuse, you must give your answer as an acute angle. So if the obtuse angle is θ then the angle between the two lines is $180^\circ - \theta$, or $\pi - \theta\,\text{radians}$.

4 section questions

Question 1

★★☆

Select which of the following is the angle between the lines $\boldsymbol{a} = \begin{pmatrix}2\\-1\end{pmatrix} + \boldsymbol{t}\begin{pmatrix}4\\3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}-2\\4\end{pmatrix} + \boldsymbol{t}\begin{pmatrix}-1\\2\end{pmatrix}$.

1 79.7° ✓

2 100°

3 63.4°

4 26.6°

Student view

Explanation

Take the direction vectors and evaluate:

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$$\begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}-1\\2\end{pmatrix}=4\times(-1)+3\times2=2$$

$$\left|\begin{pmatrix}4\\3\end{pmatrix}\right|=\sqrt{4^2+3^2}=\sqrt{25}=5$$

and

$$\left|\begin{pmatrix}-1\\2\end{pmatrix}\right|=\sqrt{(-1)^2+2^2}=\sqrt{5}$$

Then the angle between these vectors (which is the angle between these lines) is:

$$\theta=\cos^{-1}\left[\frac{2}{5\sqrt{5}}\right]\approx79.7^\circ$$

Question 2

★★☆

Select which of the following is the angle between the lines $\boldsymbol{r}=\begin{pmatrix}0\\2\\3\end{pmatrix}+\boldsymbol{s}\begin{pmatrix}3\\4\\5\end{pmatrix}$ and $\boldsymbol{a}=\begin{pmatrix}-2\\5\\3\end{pmatrix}+\boldsymbol{t}\begin{pmatrix}-1\\2\\1\end{pmatrix}$.

1 54.7° ✓

2 22.5°

3 35.3°

4 86.7°

Explanation

The direction vectors of these lines are

$$\boldsymbol{v}=\begin{pmatrix}3\\4\\5\end{pmatrix}$$

and

Student view

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$$\boldsymbol{w} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

Find the scalar product of these vectors and their magnitudes:

$$\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 3 \times (-1) + 4 \times 2 + 5 \times 1 = 10$$

$$|\boldsymbol{v}| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50}$$

and

$$|\boldsymbol{w}| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6.}$$

Hence, the angle between these vectors (which is the angle between these lines) is given by:

$$\begin{aligned} \theta &= \cos^{-1}\left[\frac{10}{\sqrt{50} \times \sqrt{6}}\right] \\ \Rightarrow &= \cos^{-1}\left[\frac{10}{\sqrt{300}}\right] \\ \Rightarrow &= \cos^{-1}\left[\frac{1}{\sqrt{3}}\right] \\ \Rightarrow &\approx 54.7^\circ \end{aligned}$$

Question 3

★★☆

Select which of the following is the angle between the lines defined by $\mathrm{L}_1 : \dfrac{x+3}{2} = y + 2 = \dfrac{z}{\sqrt{3}}$ and $\mathrm{L}_2 : \dfrac{1-x}{2} = \dfrac{y-1}{2} = \dfrac{z+1}{\sqrt{3}}$.

1 83.9° ✓

2 16.4°

3 57.8°

4 60°

Student view

Explanation

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The direction vectors are $\begin{pmatrix} 2 \\ 1 \\ \sqrt{3} \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 2 \\ \sqrt{3} \end{pmatrix}$

Then we apply the scalar product to find the angle, say θ, between the lines:

$$\theta = \cos^{-1}\left[\frac{\begin{pmatrix} 2 \\ 1 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 2 \\ \sqrt{3} \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 1 \\ \sqrt{3} \end{pmatrix}\right|\left|\begin{pmatrix} -2 \\ 2 \\ \sqrt{3} \end{pmatrix}\right|}\right]$$
$$\Rightarrow = \cos^{-1}\left[\frac{2 \times (-2) + 1 \times 2 + \sqrt{3} \times \sqrt{3}}{\sqrt{2^2 + 1^2 + (\sqrt{3})^2}\sqrt{(-2)^2 + 2^2 + (\sqrt{3})^2}}\right]$$
$$\Rightarrow \approx 83.9^\circ$$

Question 4

★★☆

Find the angle between the lines defined by

$$\text{L}_1 : \frac{x-4}{3} = y - 4 = \frac{-z-4}{2} \text{ and } \text{L}_2 : \boldsymbol{r} = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Give your answers in degrees **without** the symbol $^\circ$ or the word 'degrees', rounded to 1 decimal place.

70,9 ✓

Accepted answers

70,9, 70.9

Explanation

Write line L_1 as

$$\text{L}_1 : \frac{x-4}{3} = \frac{y-4}{1} = \frac{z+4}{-2},$$

and thus, it is clear that L_1 is parallel to the vector $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

Line L_2 is parallel to the vector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

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Then, the angle is given through the scalar product, i.e.

$$\begin{aligned}\theta &= \cos^{-1}\left[\frac{\begin{pmatrix}3\\1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\1\end{pmatrix}}{\left|\begin{pmatrix}3\\1\\-2\end{pmatrix}\right|\left|\begin{pmatrix}1\\2\\1\end{pmatrix}\right|}\right]\\ &= \cos^{-1}\left[\frac{3\times1+1\times2+(-2)\times1}{\sqrt{3^2+1^2+(-2)^2}\sqrt{1^2+2^2+1^2}}\right]\\ &\approx 70.9^\circ\end{aligned}$$

3. Geometry and trigonometry / 3.14 Lines in two and three dimensions

Kinematics

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/kinematics-id-28312/print/) Assign

Kinematics is the study of motion. All motion has both a magnitude and a direction. Therefore vectors can be used to represent the motion of an object moving with constant velocity in a straight line from an initial position.

Within this context, in the vector equation $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, vector $\boldsymbol{a}$ represents the initial position of the object and vector $\boldsymbol{b}$ represents the velocity, which takes into account the direction of motion. To make the equation dimensionally consistent, the parameter λ represents time.

Therefore the equation can be written as $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$ where $\boldsymbol{r}$ is the position vector at time t, $\boldsymbol{r}_0$ is the initial position vector and $\boldsymbol{v}$ is the velocity, which is constant.

The magnitude of the velocity vector gives the speed of the object.

Consider a ship moving north-east with a speed of $30\ \text{km h}^{-1}$ from port A, as shown in the diagrams below. How can you represent its movement using vectors?

Student view

If point A is taken as the origin, then the coordinates of A are $(0, 0)$.

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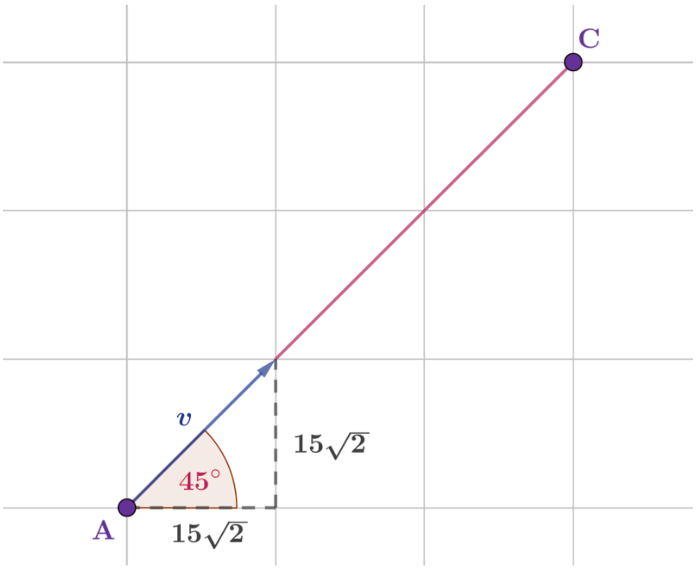
The ship is moving with a speed of 30 km h^{1} so $|\boldsymbol{v}| = 30$.

The direction is north-east so the $\boldsymbol{i}$ and $\boldsymbol{j}$ components of the velocity vector $\boldsymbol{v}$ are both $30 \cos 45$.

$$\text{Therefore } \boldsymbol{v} = \begin{pmatrix} 30\cos 45 \\ 30 \cos 45 \end{pmatrix} = \begin{pmatrix} 30 \times \frac{1}{\sqrt{2}} \\ 30 \times \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 15\sqrt{2} \\ 15\sqrt{2} \end{pmatrix}$$

The position vector of the ship at time t hours after it left A is given by

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 15\sqrt{2} \\ 15\sqrt{2} \end{pmatrix} \text{ or } \boldsymbol{r} = t \begin{pmatrix} 15\sqrt{2} \\ 15\sqrt{2} \end{pmatrix}$$



More information

The image is a diagram showing a vector on a grid with a 45-degree angle from point A to point C. The diagram includes:

- A horizontal axis and a vertical axis intersecting at the origin, labeled with grid lines spaced evenly across.
- Point A is at the origin (0,0), and point C is represented on a coordinate grid.
- The vector is drawn from point A to point C, forming a 45-degree angle with the positive x-axis.
- Both the horizontal and vertical components of the vector measure 15√2.

Student view

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- There is a right triangle formed by the vector, with the vector itself as the hypotenuse, the x-axis as the base, and the line parallel to the y-axis as the height.
- The angle within this triangle at point A is labeled as 45 degrees.
- The vector is denoted by letter 'v' and is represented as a magenta-colored line.

This diagram visually represents vector addition and direction with emphasis on the 45-degree angle and equal component sides.

[Generated by AI]

Example 1

★★☆

A drone is flying in a straight line starting from point $(2, 1, 2)$ with constant speed in $\mathrm{m\,s^{-1}}$.

After 10 s its position is $\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix}$

Find its velocity vector and its speed.

(All positions are in metres from the origin.)

The position vector is $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and let $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ be the direction vector.

Then the vector equation is

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

for which

Student view

$$\begin{pmatrix}4\\3\\3\end{pmatrix} = \begin{pmatrix}2\\1\\2\end{pmatrix} + 10\begin{pmatrix}v_x\\v_y\\v_z\end{pmatrix}$$

Thus,

$$\begin{aligned} v_x &= \frac{4-2}{10} = \frac{1}{5} \\ v_y &= \frac{3-1}{10} = \frac{1}{5} \\ v_z &= \frac{3-2}{10} = \frac{1}{10} \end{aligned}$$

So the velocity vector is

$$\boldsymbol{v} = \begin{pmatrix}\frac{1}{5}\\\frac{1}{5}\\\frac{1}{10}\end{pmatrix}$$

and the speed is

$$\left|\begin{pmatrix}\frac{1}{5}\\\frac{1}{5}\\\frac{1}{10}\end{pmatrix}\right| = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{10}\right)^2} = \sqrt{\frac{9}{100}} = \frac{3}{10}\ \mathrm{ms}^{-1}$$

Example 2

★★☆

The position vector at time t s of a moving object, P , relative to a fixed point, O, is given by

$$\overrightarrow{\mathrm{OP}} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + t(2\boldsymbol{i} - \boldsymbol{k})$$

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a) Find the coordinates of the initial position of the object.

b) Find the coordinates of the object when $t = 2$ seconds.

c) Hence, find the distance the object travelled in 2 seconds if the velocity is given in $\mathrm{m\,s^{-1}}$. Give your answer to three significant figures.

	Steps	Explanation
	$\overrightarrow{OP} = \begin{pmatrix}1\\1\\1\end{pmatrix} + t\begin{pmatrix}2\\0\\-1\end{pmatrix}$	Write the vectors as column vectors.
a)	$\overrightarrow{OP} = \begin{pmatrix}1\\1\\1\end{pmatrix} + 0\begin{pmatrix}2\\0\\-1\end{pmatrix}$ $\overrightarrow{OP} = \begin{pmatrix}1\\1\\1\end{pmatrix}$	Initial position is when $t = 0$ seconds.
	Therefore, the initial position has coordinates $(1, 1, 1)$	
b)	$\overrightarrow{OP} = \begin{pmatrix}1\\1\\1\end{pmatrix} + 2\begin{pmatrix}2\\0\\-1\end{pmatrix}$ $\overrightarrow{OP} = \begin{pmatrix}5\\1\\-1\end{pmatrix}$	Substitute $t = 2$.
	Therefore, the coordinates of the object when $t = 2$ are $(5, 1, -1)$	
c)	$d = \sqrt{(5-1)^2 + (1-1)^2 + (-1-1)^2}$	Use Pythagoras' theorem to fir the distance between the two points $(1, 1, 1)$ and $(5, 1, -$

Student view

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	Steps	Explanation
	$d = 2\sqrt{5} = 4.47\text{m}$ (3 significant figures)	
	Therefore, the distance travelled in 2 s is 4.47m (3 significant figures)	

Example 3

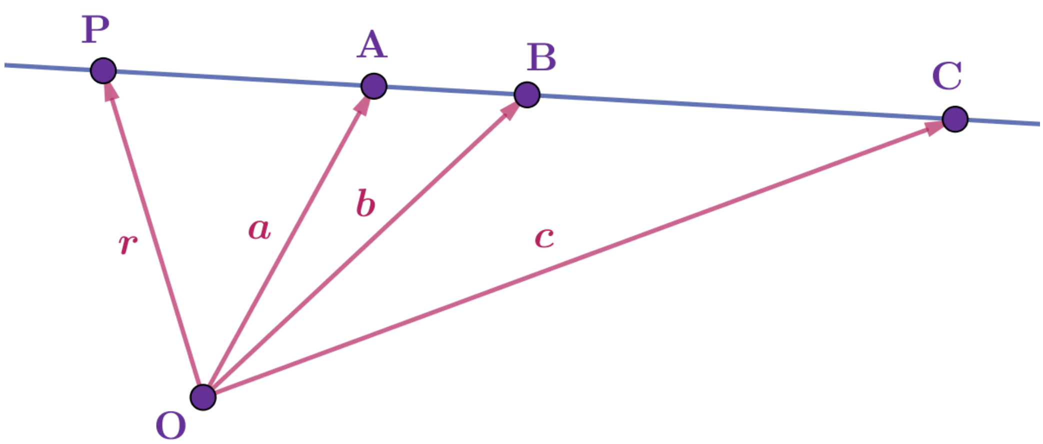
★★★

The position vector at time t seconds of a moving object it given by

$$\overrightarrow{\mathrm{OP}} = \boldsymbol{r} = \begin{pmatrix} -t^2 \\ 1 \\ 2t \end{pmatrix}$$

Show that the path of the object is not a straight line.

If the object moves in a straight line then, for any three position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$, $\boldsymbol{a} - \boldsymbol{b} = k\,(\boldsymbol{b} - \boldsymbol{c}),\ \boldsymbol{k} \in \mathbb{R}$



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Substitute some values of t to determine 3 different position vectors.

$$t = 0 \text{ then } \boldsymbol{a} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$t = 1 \text{ then } \boldsymbol{b} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$t = 2 \text{ then } \boldsymbol{c} = \begin{pmatrix} -4 \\ 1 \\ 4 \end{pmatrix}$$

$$\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$$

$$\boldsymbol{b} - \boldsymbol{c} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$$

If the object moves in a straight line, $\boldsymbol{a} - \boldsymbol{b} = k\left(\boldsymbol{b} - \boldsymbol{c}\right)$

$$\begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix} = k \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$$

Solve for k.

$$\begin{aligned} 1 &= 3k \quad \Rightarrow k = \frac{1}{3} \\ 0 &= 0k \quad \Rightarrow k = 0 \\ -2 &= -2k \Rightarrow k = 1 \end{aligned}$$

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Since there is no unique k which satisfies $\boldsymbol{a} - \boldsymbol{b} = k\left(\boldsymbol{b} - \boldsymbol{c}\right),\ \boldsymbol{k} \in \mathbb{R}$

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$\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are not collinear.

Therefore, the object does not move in a straight line.

Example 4

★★★

Two objects are moving at a constant velocity in a straight line.

Initially, object A had a position of $(1, 3, 0)$ metres and it is moving with a velocity $\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ m s^{-1}.

Object B had an initial position of $(-1, 2, -2)$ metres and it is moving with a velocity $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ m s^{-1}.

a) Show that these objects do not collide.

b) Find an equation in terms of t for the distance, d, between the objects.

c) Find their closest approach.

d) Find the velocity vector that the object B must have so that the two objects will collide after 3 s.

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a) Write the motion of the two objects in terms of two vector equations:

$$\boldsymbol{r}_A = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$$

and

$$\boldsymbol{r}_B = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}.$$

where t is the time in seconds after they depart from their initial positions.

Thus, if they collide, then there must be a time at which they are at the same positic
say, t_C , which should be the unique solution to each of the following equations:

$$\begin{aligned} 1 - 3t_C &= -1 + 2t_C \\ 3 + 4t_C &= 2 - t_C \\ 2t_C &= -2 + 3t_C \end{aligned}$$

Thus,

$$\begin{aligned} t_C &= \frac{2}{5} \\ t_C &= -\frac{1}{5} \\ t_C &= 2 \end{aligned}$$

All these solutions are different, and so there is no single time at which these object
in the same position. Therefore, they do not collide.

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b) The distance, d, between two objects A and B with coordinates $(x_\mathrm{A}, y_\mathrm{A}, z_\mathrm{A})$ and $(x_\mathrm{B}, y_\mathrm{B}, z_\mathrm{B})$, respectively, is given by

$$d = \sqrt{(x_\mathrm{A} - x_\mathrm{B})^2 + (y_\mathrm{A} - y_\mathrm{B})^2 + (z_\mathrm{A} - z_\mathrm{B})^2}.$$

Alternatively, it can be more convenient to work with

$$d^2 = (x_\mathrm{A} - x_\mathrm{B})^2 + (y_\mathrm{A} - y_\mathrm{B})^2 + (z_\mathrm{A} - z_\mathrm{B})^2.$$

For the two objects, you can write their position in terms of t explicitly:

$$\boldsymbol{r}_A = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} x_\mathrm{A} \\ y_\mathrm{A} \\ z_\mathrm{A} \end{pmatrix} = \begin{pmatrix} 1-3t \\ 3+4t \\ 2t \end{pmatrix}$$

$$\boldsymbol{r}_B = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \Leftrightarrow \begin{pmatrix} x_\mathrm{B} \\ y_\mathrm{B} \\ z_\mathrm{B} \end{pmatrix} = \begin{pmatrix} -1+2t \\ 2-t \\ -2+3t \end{pmatrix}$$

Hence,

$$\begin{aligned} d^2 &= (1-3t-(-1+2t))^2 + (3+4t-(2-t))^2 + (2t-(-2+3t))^2 \\ &= (1-3t+1-2t)^2 + (3+4t-2+t)^2 + (2t+2-3t)^2 \\ &= (2-5t)^2 + (1+5t)^2 + (2-t)^2 \\ &= 4-20t+25t^2+1+10t+25t^2+4-4t+t^2 \\ &= 9-14t+51t^2 \end{aligned}$$

giving

$$d = \sqrt{9-14t+51t^2}.$$

Since the discriminant of this quadratic is negative $(\Delta = (-14)^2 - 4 \times 51 \times 9 = -1640)$ there is no solution for $d = 0$. Hence, these objects do not collide.

c) The smallest value for d is their closest approach. As the leading coefficient of the function of d^2 is positive $(+51)$, the function is concave up and so the vertex is its minimum point.

Here you can use the graphical method, which requires finding the coordinates of the vertex of this quadratic. The vertical component of that vertex is equal to d^2. You can solve this with a graphic display calculator, as seen earlier in the course.

The solution is $d^2 = 8.04 \Rightarrow d = 2.84\ \mathrm{m}$ which occurs at $t = 0.137\ \mathrm{s}$.

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d)

Let $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be the new velocity vector for the motion of object B.

The vector equations for A and B will be

$$\boldsymbol{r}_A = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$$

and

$$\boldsymbol{r}_B = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + t \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

As you want the two objects to collide after $t = 3$ s, you have

$$\begin{aligned} 1 - 3 \times 3 &= -1 + 3 \times a \\ 3 + 4 \times 3 &= 2 + 3 \times b \\ 2 \times 3 &= -2 + 3 \times c \end{aligned}$$

from which you get

$$\begin{aligned} a &= -\frac{7}{3} \\ b &= \frac{13}{3} \\ c &= \frac{8}{3} \end{aligned}$$

Hence, the new velocity vector for object B such that the two objects collide after 3

$$\begin{pmatrix} -\frac{7}{3} \\ \frac{13}{3} \\ \frac{8}{3} \end{pmatrix} \mathrm{ms}^{-1}$$

Making connections

Have you seen ants foraging and then finding their way back to their nest? Apparently, they are aware of the magnitude and direction of their journeys. They have internal pedometer systems. You can read about the vector journeys in this article (https://plus.maths.org/content/finding-way-home).

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3 section questions

Question 1

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An object is initially at the position $(0, -5)$ and has a velocity vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Select which of the following shows its position vector in terms of time t and its speed. Units are in metres and seconds.

1 $\begin{pmatrix} 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} -4 \\ 3 \end{pmatrix}, 5 \text{ m s}^{-1}$ ✓

2 $\begin{pmatrix} 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} -4 \\ 3 \end{pmatrix}, -5 \text{ m s}^{-1}$

3 $\begin{pmatrix} 0 \\ 5 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}, 5 \text{ m s}^{-1}$

4 $\begin{pmatrix} 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}, -5 \text{ m s}^{-1}$

Explanation

The position vector is given by its initial position plus t times the velocity vector, i.e.

$$\begin{pmatrix} 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

The speed is the magnitude of the velocity vector, i.e.

$$\left|\begin{pmatrix} -4 \\ 3 \end{pmatrix}\right| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$$

Hence, the speed is 5 m s^{-1}.

Question 2

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Two objects have the following position vectors:

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$$\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

and

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ -4 \\ -8 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}.$$

Will these objects collide?

Select the correct answer.

1 Yes, the objects will collide. ✓

2 No, the paths of the objects do not meet.

3 No, the paths of the objects have a common point but the objects are not there at the same time.

4 There is not enough information given to determine.

Explanation

The two equations give the position of the two objects, where t is measuring time. The objects collide if they are at the same position at the same time, so if there is a t-value where the positions are the same. Hence they collide if there is a solution to

$$\begin{pmatrix} -1+2t \\ 2 \\ 1-t \end{pmatrix} = \begin{pmatrix} 2+t \\ -4+2t \\ -8+2t \end{pmatrix}$$

In the first component this gives the equation

$$-1+2t = 2+t$$

The solution of this is $t = 3$.

This t-value also gives the same second and third component, so the objects collide at $t = 3$ at the position $(5, 2, -2)$.

Question 3

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Object A has an initial position of $(4, 2, -1)$ and moves with a velocity $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

If an object B has an initial position of $(-1, 4, 1)$, select the correct velocity vector it must have if the two objects collide after 3 s.

1 $\begin{pmatrix} \frac{14}{3} \\ \frac{1}{3} \\ -\frac{8}{3} \end{pmatrix}$ ✓

2 $\begin{pmatrix} -\frac{14}{3} \\ \frac{1}{3} \\ -\frac{8}{3} \end{pmatrix}$

3 $\begin{pmatrix} \frac{14}{3} \\ -\frac{1}{3} \\ -\frac{8}{3} \end{pmatrix}$

4 $\begin{pmatrix} -\frac{14}{3} \\ \frac{1}{3} \\ \frac{8}{3} \end{pmatrix}$

Explanation

Let $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be the velocity vector for the motion of object B.

Then the vector equations for A and B will be

$$\boldsymbol{r}_A = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

and

$$\boldsymbol{r}_B = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

from which we have

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$$\begin{aligned} 4+3t &= -1+t\times a \\ 2+t &= 4+t\times b \\ -1-2t &= 1+t\times c \end{aligned}$$

As we want the two objects to collide after $t = 3\text{s}$, we have

$$\begin{aligned} 4+3\times 3 &= -1+3\times a \\ 2+3 &= 4+3\times b \\ -1-2\times 3 &= 1+3\times c \end{aligned}$$

So that,

$$\begin{aligned} a &= \frac{14}{3} \\ b &= \frac{1}{3} \\ c &= -\frac{8}{3} \end{aligned}$$

3. Geometry and trigonometry / 3.14 Lines in two and three dimensions

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-28313/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- write the vector equation of a straight line as $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where $\boldsymbol{a}$ is the position vector of a point on the line, $\boldsymbol{b}$ is a vector describing the direction of the line and the parameterλ is a scalar
- write the position of a point on a straight line with coordinates (x, y, z) in terms of a vector equation:
$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$
- recall that the angle between two straight lines is given by the angle between their direction vectors:
  - if $\boldsymbol{b}$ and $\boldsymbol{d}$ are the direction vectors of two straight lines, then the angle θ between these lines is given using the scalar product as
  $$\theta = \cos^{-1}\left(\frac{\boldsymbol{b}\cdot\boldsymbol{d}}{|\boldsymbol{b}||\boldsymbol{d}|}\right)$$

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- recall that the angle between two straight lines is usually given as the acute angle not the obtuse angle

- write the equation of a straight line in vector form, parametric form and Cartesian form: given a point (x_0, y_0, z_0) and a direction vector $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$, the equation of a straight line can be written
  - in vector form as $\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$
  - in parametric form as $x = x_0 + \lambda l, y = y_0 + \lambda m, z_0 + \lambda n$
  - in Cartesian form as $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
- describe the motion of an object moving in a straight line with constant velocity by the vector equation $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$, where $\boldsymbol{r_0} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ is the initial position vector relative to a fixed origin, $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ is the velocity and t is the time.
- recall that speed is the magnitude of the velocity vector $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and use Pythagoras' theorem to find it: $|\boldsymbol{v}| = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$

3. Geometry and trigonometry / 3.14 Lines in two and three dimensions

Investigation

Section Student… (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-28314/print/) Assign

The orthodromic distance is the shortest distance between two points on a sphere. It is also called the great circle distance. As the Earth is almost a sphere, the distances between points on Earth can be found using the principles of orthodromic distances.

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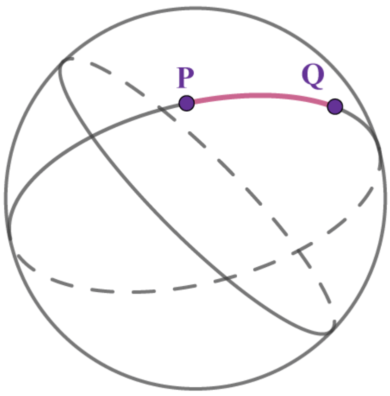
On the diagram below, the red line shows the orthodromic distance between the two points P and Q along the surface of a sphere.



More information

This diagram depicts a sphere, representing the Earth, with labeled points P and Q on its surface. The sphere is shown with several dashed lines representing different great circles and meridians. A red arc, which is part of a great circle, connects points P and Q, demonstrating the orthodromic, or shortest possible path over the Earth's surface, between these two points. The diagram illustrates the concept of orthodromic distance, which is the shortest distance between two points on a sphere, essential for navigation and cartography.

[Generated by AI]

Read this article (https://undergroundmathematics.org/trigonometry-triangles-to-functions/lost-but-lovely-the-haversine) from *Underground Mathematics* about calculating orthodromic distance.

1. Prove the formulae given in the article.
2. Find the coordinates of two places you would like to visit and calculate the orthodromic (great circle) distance between these two locations.

Rate subtopic 3.14 Lines in two and three dimensions

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