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The big picture

So far you have learned trigonometric identities for one angle. In this subtopic, you will be extending your knowledge to compound angles. You may want to revisit subtopic 3.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27747/) to refresh your memory before you start this subtopic.

Consider a right-angled triangle with hypotenuse of length 1 unit, circumscribed in a rectangle, as seen in the diagram below.

Can you find the lengths of the line segments in terms of trigonometric ratios of angles α and β?

Can you find the side lengths of the rectangle as trigonometric ratios of angles α and β?

Can you write angle EAB (or $\alpha + \beta$) in terms of trigonometric ratios?

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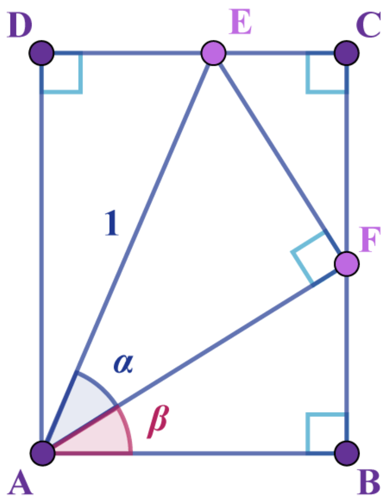

More information

The image depicts a geometric diagram involving a rectangle ABCD with four right angles at corners D, C, B, and A. Inside this rectangle is a triangle AEF with a right angle at F, where line segment EF is perpendicular to side AB of the rectangle. The rectangle is vertical, with points A and B at the bottom, and points D and C at the top. Point A is located at the bottom left corner, B at the bottom right, D at the top left, and C at the top right.

EF is a diagonal line extending from point E on the top horizontal line DC to point F on the right vertical line AB. Angle EAB at point A is labeled with two angles, α and β, indicated in a way that α + β is the angle EAB. The side AE of the triangle is labeled as 1.

The diagram's purpose is to express angle EAB in terms of trigonometric ratios involving angles α and β, as suggested by the text, "Can you write angle EAB (or α + β) in terms of trigonometric ratios?"

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Concept

In this subtopic, you will look at the relationships between trigonometric ratios and identify equivalent expressions to form identities for compound angles.

How do identities help you to prove mathematical results?

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3. Geometry and trigonometry / 3.10 Trigonometric identities revisited

Compound angle identities

Overview
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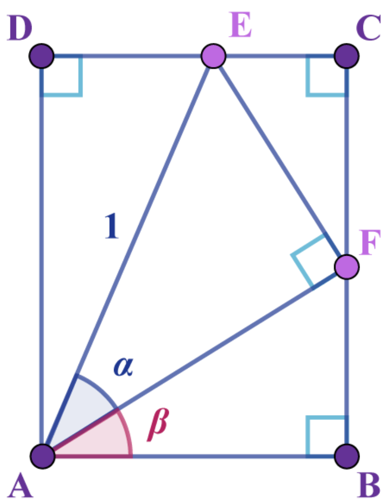
Compound angle identities for sine and cosine

In this section, you will learn about the trigonometric ratios of compound angles, that is, the sum or difference of two or more angles.

You will start with the sum of two acute angles α and β positioned in a rectangle, as shown below, where triangle EFA is a right-angled triangle with $AE = 1$ unit and angle $\mathrm{EFA} = 90°$.



More information

The image shows a rectangle labeled ABCD, with corners marked A, B, C, and D. Inside this rectangle, there is a right-angled triangle labeled EFA. Point E is on the top side of the rectangle, and point F is on the right side, creating a diagonal line from point A to point F.\n\nTriangle EFA is right-angled at F, with AE labeled as 1 unit. Angles (\alpha) and (\beta) are marked at corner A, with (\alpha) adjacent to AE and (\beta) near AB. The line segments AE, EF, and FA form the triangle, with the right angle marked at F. The rectangle has right angles at each corner, and the segment EF is perpendicular to side AB. The image captures the geometric configuration needed to analyze the sum of the angles (\alpha) and (\beta), given the properties of the triangle and rectangle.

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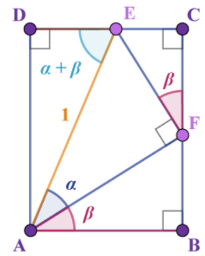
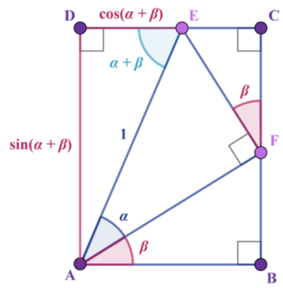
Example 1

★★☆

Write all the line segments, from the figure above, in terms of trigonometric ratios of angles α, β and $\alpha + \beta$.

Steps	Explanation
$D\hat{E}A = B\hat{A}E = \alpha + \beta$ as they are alternate angles	
In triangle DAE, write the lengths of the sides using trigonometric ratios of angle $\alpha + \beta$. As the length $AE = 1 \text{ unit}$, $AD = \sin(\alpha + \beta)$ and $DE = \cos(\alpha + \beta)$	

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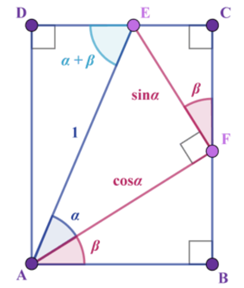
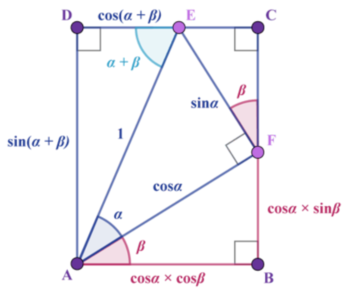
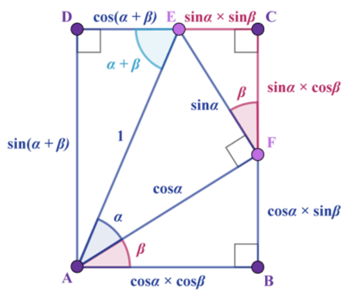
Steps	Explanation
I n triangle AFE, as angle EFA is 90° $AF = \cos\alpha$ $EF = \sin\alpha$	
In triangle ABF, $\sin\,\beta {=} \dfrac{BF}{AF} = \dfrac{BF}{\cos\,\alpha} \Rightarrow BF = \cos\,\alpha\,\sin\,\beta$ $\cos\,\beta = \dfrac{AB}{AF} = \dfrac{AB}{\cos\,\alpha} \Rightarrow AB = \cos\,\alpha\,\cos\,\beta$	
In triangle ECF, $\sin\,\beta {=} \dfrac{EC}{EF} = \dfrac{EC}{\sin\,\alpha} \Rightarrow EC = \sin\,\alpha\,\sin\,\beta$ $\cos\,\beta = \dfrac{FC}{EF} = \dfrac{FC}{\sin\,\alpha} \Rightarrow FC = \sin\,\alpha\,\cos\,\beta$	

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Now that you have found the lengths of all the line segments in terms of trigonometric ratios of angles you can use these to write equivalence relationships between these lines segments. In the diagram below, $AD = BC$ and $DC = AB$ as they are opposites sides of the rectangle ABCD.

$$AD = BC$$

$$AD = BF + FC$$

$$\sin(\alpha + \beta) = \cos\alpha\sin\beta + \sin\alpha\cos\beta$$

or

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Similarly,

$$DC = AB$$

$$DC = DE + EC$$

$$\cos\alpha\cos\beta = \cos(\alpha + \beta) + \sin\alpha\sin\beta$$

Rearranging gives

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

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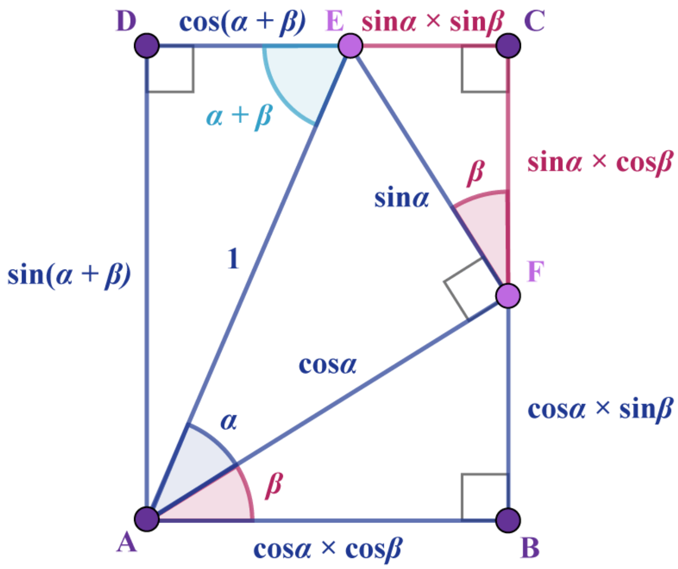

More information

The image is a geometric diagram demonstrating compound angle identities, specifically focusing on the angles α and β. The diagram consists of several labeled points: A, B, C, D, E, and F, forming various right triangles. The triangles are labeled with components pertaining to trigonometric identities.

- The top edge, labeled DE, is marked as 'cos(α + β)'.
- The left edge, labeled AD, is marked as 'sin(α + β)'.
- Right triangles are drawn in the diagram with right angles indicated.
- Angles α and β are shown in different colors within the triangles.
- Trigonometric expressions like 'sin α', 'cos α', 'sin β', and 'cos β' are used to illustrate the relationships between these angles and the triangle sides.
- The intersecting lines within the triangles illustrate various trigonometric products and sums.

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These two relationships are known as compound angle identities. You have proved them for acute angles and the sum of acute angles, but can you show that they hold true for the sum of any two angles?

How would these identities change for the difference of any two angles, for example $\alpha - \beta$?

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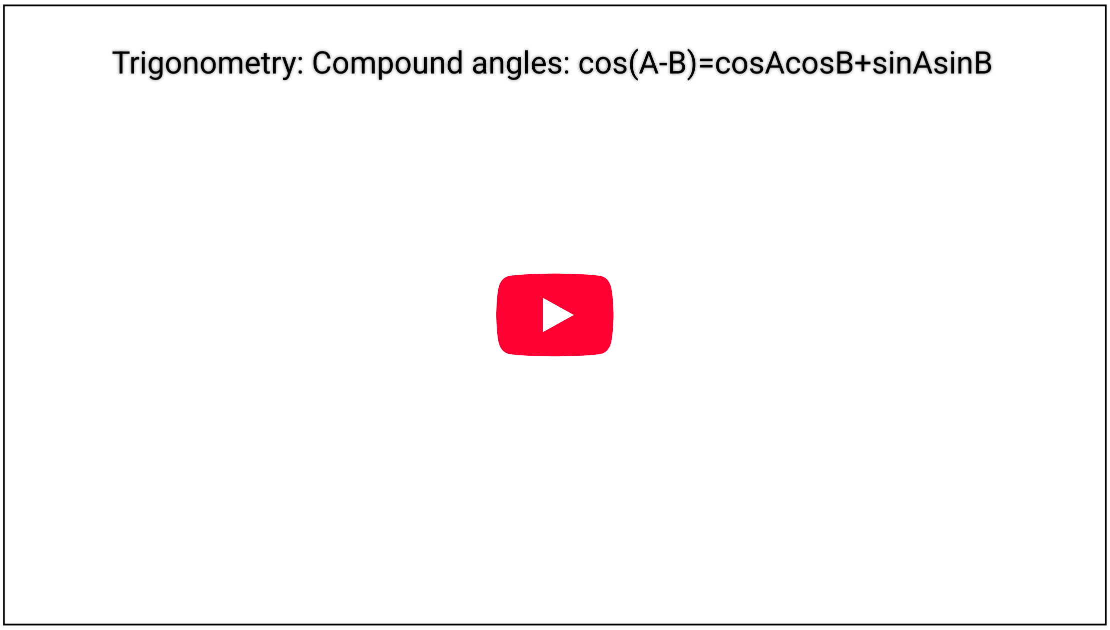
There are many ways of proving the identities for the sine and cosine of the difference between two angles. For example, you could write $\alpha - \beta$ as $\alpha + (\beta)$ and use the identities for the sum of two angles and the properties of the trigonometric functions sine and cosine to simplify the expressions with $(-\beta)$. Or you could use geometry and properties of the unit circle, as seen in this video.

Exam tip

Although, it is unlikely that you will be asked to prove these identities in the IB examination, it is a good idea to learn these proofs as you might be asked to prove other identities.



You can write the identities for the sine and cosine of the difference between two angles as

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

and

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Be aware

The signs in the identities for $\cos(\alpha + \beta)$ and $\cos(\alpha - \beta)$ are not what you might expect:

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$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$, and
$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

Exam tip

In IB examinations, the compound angle formulae will be given as

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$

Be aware

Although you will be given the compound angle identities in the examination, the sum and difference formulae are not written separately. You need to pay attention to the signs.

For example,

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

means that

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

Important

In subtopic 3.6 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27747/), you proved the Pythagorean identity $\sin^2\theta+\cos^2\theta=1$

and the double angle identities for sine and cosine

$$\sin 2\theta=2\sin\theta\cos\theta$$

$$\cos 2\theta=\cos^2\theta-\sin^2\theta=2\cos^2\theta-1=1-2\sin^2\theta$$

using graphical methods.

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These will be useful, in addition to the compound angle identities, for proving other trigonometric relationships.

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Example 2

★★☆

Prove the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

$$\begin{aligned}\cos 3\theta &= \cos(2\theta + \theta) \\ &= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta && \text{[using compound angle} \\ &= \left(2\cos^2\theta - 1\right)\cos\theta - 2\sin\theta\cos\theta\sin\theta && \text{[using double angle ider} \\ &= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta \\ &= 2\cos^3\theta - \cos\theta - 2\left(1 - \cos^2\theta\right)\cos\theta && \text{[using the Pythagorean} \\ &= 2\cos^3\theta - \cos\theta + 2\cos^3\theta - 2\cos\theta \\ &= 4\cos^3\theta - 3\cos\theta\end{aligned}$$

The above will be useful when you are working with complex numbers.

Note: Try to prove the same identity by using the $\cos 2\theta = 1 - 2\sin^2\theta$ double angle identity instead of the $\cos 2\theta = 2\cos^2\theta - 1$ identity that was used in the example. You should reach the same result.

Example 3

★★☆

Factorise $\sin 2x + \sin 6x$. Hence write it as a product of trigonometric functions.

You can write $2x = 4x - 2x$ and $6x = 4x + 2x$, so start by making these substitutions, and then use compound angle identities during rearrangement.

$$\begin{aligned}\sin 2x + \sin 6x &= \sin(4x - 2x) + \sin(4x + 2x) \\ &= \sin 4x\cos 2x - \cancel{\cos 4x\sin 2x} + \sin 4x\cos 2x + \cancel{\cos 4x\sin} \\ &= 2\sin 4x\cos 2x\end{aligned}$$

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which is a product of trigonometric functions.

Example 4

★★☆

Simplify the expression $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta}$.

Start by writing as a single fraction.

$$\begin{aligned}\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} &= \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\sin \theta \cos \theta} \\ &= \frac{\sin (3\theta - \theta)}{\sin \theta \cos \theta} && \text{[using compound an} \\ &= \frac{\sin 2\theta}{\sin \theta \cos \theta} \\ &= \frac{\sin 2\theta}{\frac{1}{2}\sin 2\theta} && \text{[using double angle i} \\ &= 2\end{aligned}$$

which is a rather pleasing simplification.

Note that, as this is an identity, this result is true for any angle θ .

Example 5

★★☆

Express $6\sin x + 8\cos x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and α is in radians.

Using the compound angle identity for cosine,

$$R\cos(x - \alpha) = R\cos x \cos \alpha + R\sin x \sin \alpha$$

So you can write

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$$6\sin x + 8\cos x = R\cos(x-\alpha)$$

as

$$6\sin x + 8\cos x = R\cos x\cos\alpha + R\sin x\sin\alpha$$

Therefore

$$R\cos\alpha = 8 \text{ and } R\sin\alpha = 6.$$

Dividing gives

$$\frac{R\sin\alpha}{R\cos\alpha} = \tan\alpha = \frac{6}{8},$$

so

$$\alpha = \tan^{-1}\left(\frac{6}{8}\right) = 0.644$$

and

$$\begin{aligned}(R\sin\alpha)^2 + (R\cos\alpha)^2 &= 6^2 + 8^2\\ R^2\left(\sin^2\alpha + \cos^2\alpha\right) &= 100\\ R^2 &= 100\\ R &= 10\end{aligned}$$

Hence,

$$6\sin x + 8\cos x = 10\cos(x - 0.644)$$

International Mindedness

Modern electricity networks rely on alternating current (AC) for the transmission of electricity. Although no single person invented the AC system, it was Nicolas Tesla who invented the first induction motor for producing AC which was licensed by Westinghouse Electric in 1888 to produce electricity in the United States.

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Tesla had previously worked for Thomas Edison, who owned a company that supplied direct current (DC) electricity to household consumers. Westinghouse and Edison engaged in what became known as the 'War of the Currents' due to their rivalry to win contracts to produce electricity by their different methods.

In the 1890s it was shown that AC was the more efficient method and the current war ended. One important fact about AC is that voltage and current vary over time according to sine and cosine waves. If you would like to know more about this, read this article (https://plus.maths.org/content/energetic-maths).

Compound angle identities for the tangent of an angle

You can use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and compound angle identities for sine and cosine to define compound angle identities for the tangent of an angle.

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\cos\ \alpha\ \sin\ \beta + \ \sin\ \alpha\ \cos\ \beta \cos\ \alpha\ \cos\ \beta - \sin}{\alpha\ \sin\ \beta}$$

Divide each term in the numerator and denominator by $\cos\ \alpha\ \cos\ \beta$ and simplify

$$\tan(\alpha+\beta) = \frac{\dfrac{\cos\ \alpha\ \sin\ \beta}{\cos\ \alpha\ \cos\ \beta} + \dfrac{\sin\ \alpha\ \cos\ \beta}{\cos\ \alpha\ \cos\ \beta}}{\dfrac{\cos\ \alpha\ \cos\ \beta}{\cos\ \alpha\ \cos\ \beta} - \dfrac{\sin\ \alpha\ \sin\ \beta}{\cos\ \alpha\ \cos\ \beta}}$$

$$\tan(\alpha+\beta) = \frac{\dfrac{\sin\ \beta}{\cos\ \beta} + \dfrac{\sin\ \alpha}{\cos\ \alpha}}{1 - \dfrac{\sin\ \alpha\ \sin\ \beta}{\cos\ \alpha\ \cos\ \beta}} = \frac{\dfrac{\sin\ \alpha}{\cos\ \alpha} + \dfrac{\sin\ \beta}{\cos\ \beta}}{1 - \dfrac{\sin\ \alpha}{\cos\ \alpha} \times \dfrac{\sin\ \beta}{\cos\ \beta}}$$

Therefore, $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

See if you can use the same process to obtain the identity for $\tan\ (\alpha-\beta)$.

You should get $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

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Example 6

★★☆

Find the exact value of $\frac{\tan 40 + \tan 20}{1 - \tan 40 \tan 20}$.

Steps	Explanation
$\frac{\tan 40 + \tan 20}{1 - \tan 40 \tan 20} = \tan(40 + 20)$	Use the compound angle identity for tanger
$\tan(40 + 20) = \tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$	Use the trigonometric ratio for 60° .
Therefore, $\frac{\tan 40 + \tan 20}{1 - \tan 40 \tan 20} = \sqrt{3}$	

Exam tip

In the IB formula booklet, the compound angle identity for the tangent is given as

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

Make sure you pay attention to the $+$ and $-$ signs as well as the order of the angles.

5 section questions ^

Question 1

★☆☆

What is the factorised form of $\cos(x + y) + \cos(x - y)$?

1 $2\cos x \cos y$ ✓

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2 $-2\cos x\cos y$

3 $2\sin x\sin y$

4 $-2\sin x\sin y$

Explanation

$$\begin{aligned} & \cos(x+y)+\cos(x-y) = \cos x\cos y - \cancel{\sin x\sin y} + \cos x\cos y + \cancel{\sin x\sin y} \\ \Leftrightarrow \quad & = 2\cos x\cos y \end{aligned}$$

Question 2

★★★

If $3\cos x - 4\sin x$ is expressed in the form $R\cos(x-\alpha)$, what are R and α?

1 $R=5,\ \alpha=-0.927$ ✓

2 $R=5,\ \alpha=0.927$

3 $R=-5,\ \alpha=-0.927$

4 $R=5,\ \alpha=53.1°$

Explanation

$$\begin{aligned} & R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha = 3\cos x - 4\sin x \\ \Leftrightarrow \quad & R\cos\alpha = 3 \quad \text{and} \quad R\sin\alpha = -4 \\ \Leftrightarrow \quad & \frac{R\sin\alpha}{R\cos\alpha} = \tan\alpha = -\frac{4}{3} \quad \Rightarrow \text{(using GDC)} \quad \alpha \approx -0.927 \end{aligned}$$

and,

$$\begin{aligned} & (R\sin\alpha)^2 + (R\cos\alpha)^2 = 3^2 + (-4)^2 \\ \Leftrightarrow \quad & R^2\left(\sin^2\alpha + \cos^2\alpha\right) = R^2 = 25 \\ \Leftrightarrow \quad & R = 5 \end{aligned}$$

Question 3

★☆☆

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Find the exact value of $\sin 50^\circ \cos 40^\circ + \cos 50^\circ \sin 40^\circ$.

1 1 ✓

2 -1

3 0

4 $\sqrt{3}$

Explanation

Recognise the compound angle identity for sine:

$$\sin 50^\circ \cos 40^\circ + \cos 50^\circ \sin 40^\circ = \sin\left(40^\circ + 50^\circ\right) = \sin 90^\circ$$

$$\sin 90^\circ = 1$$

Therefore, the correct answer is 1.

Question 4

★☆☆

Find the exact value of $\frac{\tan 40 - \tan 70}{1 + \tan 40 \tan 70}$.

1 $-\frac{\sqrt{3}}{3}$ ✓

2 $\frac{\sqrt{3}}{3}$

3 $\sqrt{3}$

4 $-\sqrt{3}$

Explanation

Recognise the compound angle formula for tangent:

$$\frac{\tan 40 - \tan 70}{1 + \tan 40 \tan 70} = \tan\left(40 - 70\right) = \tan\left(-30\right)$$

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$$\tan(-30) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$ as angle $-30°$ is in the fourth quadrant

Therefore the correct answer is $-\frac{\sqrt{3}}{3}$.

Question 5

★★☆

Simplify $\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}$.

1 $\tan\left(x+\frac{\pi}{6}\right)$ ✓

2 $\tan\left(x+\frac{\pi}{3}\right)$

3 $\tan\left(x-\frac{\pi}{6}\right)$

4 $\tan\left(x-\frac{\pi}{3}\right)$

Explanation

Divide the numerator and denominator by $\sqrt{3}$.

$$\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x} = \frac{\frac{1}{\sqrt{3}}+\tan x}{1-\frac{1}{\sqrt{3}}\tan x}$$

Recognise that $\frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

so $\frac{\frac{1}{\sqrt{3}}+\tan x}{1-\frac{1}{\sqrt{3}}\tan x} = \frac{\tan\frac{\pi}{6}+\tan x}{1-\tan\frac{\pi}{6}\tan x}$.

Using the compound angle identity for tangent:

$$\frac{\tan\frac{\pi}{6}+\tan x}{1-\tan\frac{\pi}{6}\tan x} = \tan\left(\frac{\pi}{6}+x\right)$$

Therefore the correct answer is $\tan\left(x+\frac{\pi}{6}\right)$.

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3. Geometry and trigonometry / 3.10 Trigonometric identities revisited

Double angle identities revisited

In section 3.6.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/double-angle-identities-id-27749/) you used the double angle identities for the sine and cosine ratios. Although you proved them geometrically, they can also be derived as a special case of the compound angle identities by substituting $\beta = \alpha$.

In this section, you will extend the double angle identities to the tangent ratio.

Making connections

In section 3.6.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/double-angle-identities-id-27749/), you used the following double angle identities which are given in formula booklet.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Consider $\tan 2\theta = \dfrac{\sin 2\theta}{\cos 2\theta}$.

Substitute $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$,

This gives $\tan 2\theta = \dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Divide both the numerator and denominator by $\cos^2\theta$ and simplify.

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$$\tan 2\theta = \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}} = \frac{2\frac{\sin\theta}{\cos\theta}}{1 - \frac{\sin^2\theta}{\cos^2\theta}} = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Therefore, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$.

See if you can arrive at the same result by using the compound angle formula

$$\tan\ (\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

and substituting $\beta = \alpha$.

Exam tip

In the IB formula booklet the double angle identity for tangent is given as

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Example 1

★★☆

Find the exact value of $\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$.

Steps	Explanation
$\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}} = \tan\left(2\times\frac{\pi}{12}\right) = \tan\frac{\pi}{6}$	Use the double angle identity for tangent.
$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$	Use the trigonometric ratios for $\frac{\pi}{6}$

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Steps	Explanation
Therefore, the answer is $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$.	

Example 2

★★★

Show that $\frac{\tan 2x \tan x}{\tan 2x - \tan x} \equiv \sin 2x$.

Steps	Explanation
$\text{LHS} = \frac{\frac{2\tan x}{1-\tan^2 x}\tan x}{\frac{2\tan x}{1-\tan^2 x} - \tan x}$	Use the double angle identity for tangent.
$= \frac{\frac{2\tan^2 x}{1-\tan^2 x}}{\frac{2\tan x - \tan x\left(1-\tan^2 x\right)}{1-\tan^2 x}}$ $= \frac{2\tan^2 x}{2\tan x - \tan x\left(1-\tan^2 x\right)}$	Rearrange and simplify.
$= \frac{2\tan^2 x}{\tan x\left(2-\left(1-\tan^2 x\right)\right)}$	Factorise $\tan x$.
$= \frac{2\tan^2 x}{\tan x\left(2-\left(1-\tan^2 x\right)\right)}$ $= \frac{2\tan x}{1+\tan^2 x}$	Simplify.

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Steps	Explanation
$= \dfrac{2\dfrac{\sin x}{\cos x}}{1+\left(\dfrac{\sin x}{\cos x}\right)^2}$ $= \dfrac{2\dfrac{\sin x}{\cos x}}{\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}}$	Use $\tan x = \dfrac{\sin x}{\cos x}$.
$= 2\dfrac{\sin x}{\cos x} \times \dfrac{\cos^2 x}{(\cos^2 x+\sin^2 x)}$	Rearrange and simplify.
$= 2\dfrac{\sin x}{1} \times \dfrac{\cos x}{(\cos^2 x+\sin^2 x)} = 2\sin x\cos x$	Use $\cos^2 x+\sin^2 x = 1$ and simplif
$= \sin 2x = \text{RHS}$	Use the double angle identity for sine.
$\text{LHS} = \text{RHS}$ Therefore, the proof is complete.	

3 section questions

Question 1

★☆☆

Find the exact value of $\dfrac{2\tan 67.5°}{1-\tan^2 67.5°}$.

1 -1 ✓

2 1

3 $\sqrt{2}$

Student view

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4 $-\sqrt{2}$

Explanation

Recognise the double angle formula for tangent

$$\frac{2\tan 67.5^\circ}{1-\tan^2 67.5^\circ} = \tan\left(2\times 67.5^\circ\right) = \tan 135^\circ$$

Angle $135^\circ$is in the second quadrant

$$\tan 135^\circ = -1$$

Therefore, the correct answer is -1.

Question 2

★★★

Simplify $\dfrac{\sin x}{\sin x+\cos x} - \dfrac{\cos x}{\sin x-\cos x}$.

1 $1+\tan 2x$ ✓

2 $1-\tan 2x$

3 $1+\cos 2x$

4 $1-\sin 2x$

Explanation

Write as a single fraction with a common denominator.

$$\frac{\sin x}{\sin x+\cos x} - \frac{\cos x}{\sin x-\cos x} = \frac{\sin x\left(\sin x-\cos x\right)-\cos x\left(\sin x+\cos x\right)}{\left(\sin x+\cos x\right)\left(\sin x-\cos x\right)}$$

Expand the brackets.

$$\frac{\sin^2 x-\sin x\cos x-\cos x\sin x-\cos^2 x}{\sin^2 x-\cos^2 x}$$

Rearrange.

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$$\frac{-(\cos^2 x-\sin^2 x)-2\sin x\cos x}{-(\cos^2 x-\sin^2 x)}$$

Using the double angle identities for sine and cosine.

$$\frac{-(\cos^2 x-\sin^2 x)-2\sin x\cos x}{-(\cos^2 x-\sin^2 x)}=\frac{-\cos 2x-\sin 2x}{-\cos 2x}$$

Simplify

$$\frac{-\cos 2x-\sin 2x}{-\cos 2x}=\frac{-(\cos 2x+\sin 2x)}{-\cos 2x}=1+\frac{\sin 2x}{\cos 2x}$$

Substitute $\tan 2x=\dfrac{\sin 2x}{\cos 2x}$

$$1+\frac{\sin 2x}{\cos 2x}=1+\tan 2x$$

Therefore, the correct answer is $1+\tan 2x$.

Question 3

★★☆

Simplify $\dfrac{\tan\frac{\pi}{3}+\tan\frac{\pi}{3}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{3}}$.

1 $-\sqrt{3}$ ✓

2 $\sqrt{3}$

3 $\dfrac{-\sqrt{3}}{2}$

4 $\dfrac{1}{\sqrt{3}}$

Explanation

Recognise the double angle identity for tangent

$$\frac{\tan\frac{\pi}{3}+\tan\frac{\pi}{3}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{3}}=\tan\left(2\times\frac{\pi}{3}\right)=\tan\frac{2\pi}{3}$$

Student view

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Angle $\frac{2\pi}{3}$ is in the second quadrant so $\tan\frac{2\pi}{3} = -\sqrt{3}$

Therefore, the correct answer is $-\sqrt{3}$.

3. Geometry and trigonometry / 3.10 Trigonometric identities revisited

Exact values revisited

You used exact values of trigonometric ratios in subtopic 3.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27741/) for particular angles. Here you will be extending your knowledge to compound angles.

Remember that when a question asks for an exact value it should be given as a fraction, a multiple of π or in surd form and not as a decimal value.

Exam tip

In papers 2 and 3 of the IB examinations, you will be allowed to use a GDC.

If you are asked to give an exact answer don't convert it into a decimal. You need to be able to work out exact values without a calculator.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/double-angle-identities-revisited-id-27564/print/) Assign

Example 1

★★☆

Find the exact value of $\cos 75°$.

First, note that $75 = 45 + 30$ so you can use the compound angle identity for cosine.

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$$\begin{aligned}\cos 75 &= \cos(45+30)\\ &= \cos 45 \cos 30 - \sin 45 \sin 30 && \text{[using compound angle identity]}\\ &= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\frac{1}{2} && \text{[using exact results for special triangle}\\ &= \frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

Example 2

★★★

Find the exact value of $\cos \frac{7\pi}{12}$.

First, note that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, so you can use the compound angle identity for cosine.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/compound-angle-identities-id-27563/print/) Assign

$$\begin{aligned}\cos\frac{7\pi}{12} &= \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)\\ &= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4} && \text{[using compound angle ident}\\ &= \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} && \text{[using exact results for speci}\\ &= \frac{\sqrt{2}(1-\sqrt{3})}{4}\end{aligned}$$

Example 3

★★★

If $\sin\alpha = \frac{3}{5},\ 0 \leqslant \alpha \leqslant \frac{\pi}{2}$ and $\cos\beta = -\frac{12}{13},\ \pi \leqslant \beta \leqslant \frac{3\pi}{2}$, find $\sin(\alpha+\beta)$.

Using the compound angle identity for sine

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

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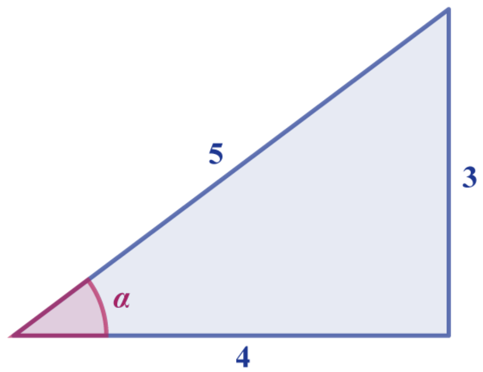
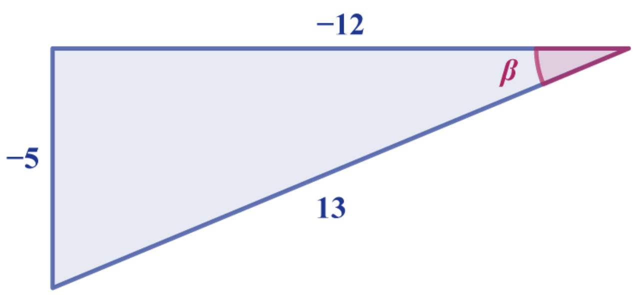
You are given values for $\sin\alpha$ and $\cos\beta$ so you need to find $\sin\beta$ and $\cos\alpha$.

Since $\sin\alpha = \frac{3}{5}$ and $\cos\beta = -\frac{12}{13}$, you need to use the 3, 4, 5 and 5, 12, 13 triangles which have integer sides: α is in the 3, 4, 5 triangle, while β is in the 5, 12, 13 triangle. As you need to consider the quadrant, it is best to sketch the triangles; see diagrams below.





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From the first diagram: $\cos\alpha = \frac{4}{5}$ and from the second diagram: $\sin\beta = -\frac{5}{13}$.

Substituting these values into the compound angle identity gives

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ &= \frac{3}{5}\times\left(-\frac{12}{13}\right)+\frac{4}{5}\times\left(-\frac{5}{13}\right)\\ &= -\frac{56}{65}\end{aligned}$$

Making connections

In section 3.9.3 (/study/app/math-aa-hl/sid-134-cid-761926/book/inverse-trigonometric-functions-id-27601/), you studied inverse trigonometric functions where the inverse trigonometric functions were defined as $\arcsin x$, $\arccos x$ and $\arctan x$.

For example, if $\sin x = a$, then angle $x = \arcsin a$.

Example 4

★★★

Find the exact value of $\sin\left(\arccos\frac{2}{3}+\arcsin\frac{1}{4}\right)$.

Both inverse functions exist. First, use the compound angle formula

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

to obtain

$$\begin{aligned}\sin\left(\arccos\frac{2}{3}+\arcsin\frac{1}{4}\right) &= \sin\left(\arccos\frac{2}{3}\right)\cos\left(\arcsin\frac{1}{4}\right)+\cos\left(\arccos\frac{2}{3}\right.\\ &= \sin\left(\arccos\frac{2}{3}\right)\cos\left(\arcsin\frac{1}{4}\right)+\frac{2}{3}\times\frac{1}{4}\\ &= \sin\left(\arccos\frac{2}{3}\right)\cos\left(\arcsin\frac{1}{4}\right)+\frac{2}{12}\end{aligned}$$

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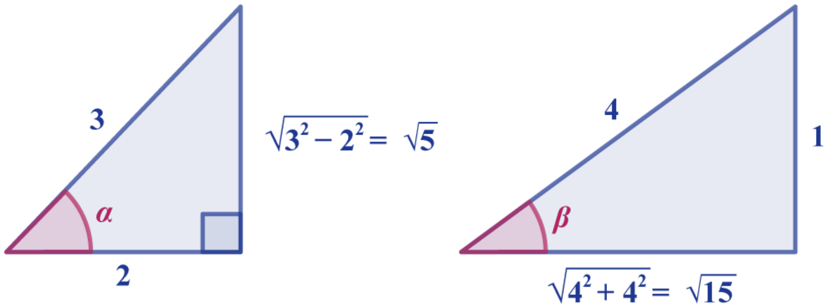
Now you need to construct two right-angled triangles to evaluate the first two terms.

Let $\alpha = \arccos \frac{2}{3}$ and $\beta = \arcsin \frac{1}{4}$.



From the diagram on the LHS, $\sin \alpha = \frac{\sqrt{5}}{3}$.

From the diagram on the RHS, $\cos \beta = \frac{\sqrt{15}}{4}$.

so

$$\sin\left(\arccos \frac{2}{3}\right) = \frac{\sqrt{5}}{3}$$

and

$$\cos\left(\arcsin \frac{1}{4}\right) = \frac{\sqrt{15}}{4}$$

which gives

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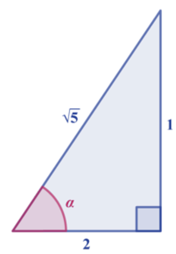
$$\sin\left(\arccos\frac{2}{3}+\arcsin\frac{1}{4}\right)=\frac{\sqrt{5}}{3}\times\frac{\sqrt{15}}{4}+\frac{2}{12}$$
$$=\frac{2+\sqrt{75}}{12}=\frac{2+5\sqrt{3}}{12}$$

Example 5

★★★

If $A=\arctan\frac{1}{2}+\arcsin\frac{2}{3}$, find the exact value of $\sin A$.

Steps	Explanation
Let $\alpha=\arctan\frac{1}{2}$ and $\beta=\arcsin\frac{2}{3}$	
$\sin A=\sin(\alpha+\beta)$	As $A=\alpha+\beta$
$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$	Using the compound angle identity for sine
$\sin\alpha=\frac{1}{\sqrt{5}}$ and $\cos\alpha=\frac{2}{\sqrt{5}}$	So, you need to find the sine and cosine of both α and β using right-angled triangles. $$\alpha=\arctan\frac{1}{2}$$  Find the missing sides using Pythagorean Theore

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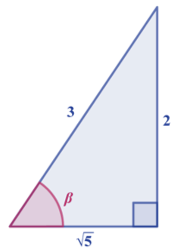
Steps	Explanation
$\sin\beta = \frac{2}{3}$ and $\cos\beta = \frac{\sqrt{5}}{3}$	$\beta = \arcsin\frac{2}{3}$ 
$\sin(\alpha+\beta) = \frac{1}{\sqrt{5}}\frac{\sqrt{5}}{3} + \frac{2}{\sqrt{5}}\frac{2}{3}$	As $$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$\sin(\alpha+\beta) = \frac{\sqrt{5}+4}{3\sqrt{5}}$	Simplify.
Therefore, the answer is $\frac{\sqrt{5}+4}{3\sqrt{5}}$.	Give an exact value.

5 section questions

Question 1

★★☆

What is the **exact** value of $\sin 105°$?

1 $\frac{\sqrt{3}+1}{2\sqrt{2}}$ ✓

2 $\frac{\sqrt{3}-1}{2\sqrt{2}}$

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3 $\dfrac{1-\sqrt{3}}{2\sqrt{2}}$

4 $\dfrac{\sqrt{2}+1}{\sqrt{6}}$

Explanation

$$\begin{aligned} & \sin 105 = \sin(60+45) \\ \Leftrightarrow \quad & = \sin 60 \cos 45 + \cos 60 \sin 45 \\ \Leftrightarrow \quad & = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}} \\ \Leftrightarrow \quad & = \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

Question 2

★★★

If $\sin\alpha = \dfrac{3}{5},\ \dfrac{\pi}{2} \leqslant \alpha \leqslant \pi$, and $\cos\beta = -\dfrac{12}{13},\ \pi \leqslant \beta \leqslant \dfrac{3\pi}{2}$, what is $\cos(\alpha+\beta)$?

1 $\dfrac{63}{65}$ ✓

2 $-\dfrac{63}{65}$

3 $-\dfrac{33}{65}$

4 $\dfrac{33}{65}$

Explanation

You have the special triangles with integer sides $3, 4, 5$ and $5, 12, 13$, with α in the second quadrant ($\sin$ positive and $\cos$ negative) and β in the third quadrant (both $\sin$ and $\cos$ negative). Thus, if $\sin\alpha = \dfrac{3}{5}$, then $\cos\alpha = -\dfrac{4}{5}$; similarly, if $\cos\beta = -\dfrac{12}{13}$, then $\sin\beta = -\dfrac{5}{13}$.

Now using the compound angle identities, you get

$$\begin{aligned} & \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \Leftrightarrow \quad & = -\frac{4}{5} \times -\frac{12}{13} - \frac{3}{5} \times -\frac{5}{13} \\ \Leftrightarrow \quad & = \frac{63}{65} \end{aligned}$$

Student view

Question 3

★★★

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What is the exact value of $\cos 165°$?

1 $-\frac{\sqrt{2}(1+\sqrt{3})}{4}$ ✓

2 $\frac{-\sqrt{2}+\sqrt{6}}{4}$

3 $-\frac{\sqrt{2}-\sqrt{6}}{4}$

4 $-\frac{\sqrt{2}+\sqrt{3}}{4}$

Explanation

$$\begin{aligned} \cos 165 &= \cos(120+45) \\ &= \cos 120 \cos 45 - \sin 120 \sin 45 && \text{[using compound angle identity]} \\ &= \cos 120 \frac{\sqrt{2}}{2} - \sin 120 \frac{\sqrt{2}}{2} \qquad (1) && \text{[using exact results for special triangles]} \end{aligned}$$

Moreover,

$$\sin 120 = \sin(180-60) = \sin 60 = \frac{\sqrt{3}}{2}$$

and

$$\cos 120 = \cos(180-60) = -\cos 60 = -\frac{1}{2}.$$

Thus,

$$(1) \Rightarrow$$

$$\begin{aligned} \cos 165 &= \cos 120 \frac{\sqrt{2}}{2} - \sin 120 \frac{\sqrt{2}}{2} \\ &= -\frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} \\ &= -\frac{\sqrt{2}(1+\sqrt{3})}{4} \end{aligned}$$

Question 4

★★★

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view

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What is the **exact** value of $\cos\left(\arccos\frac{1}{7}+\arcsin\frac{1}{5}\right)$?

1 $\frac{2\sqrt{6}}{35}-\frac{4\sqrt{3}}{35}$ ✓

2 $\frac{2\sqrt{6}}{35}+\frac{4\sqrt{3}}{35}$

3 $\frac{2\sqrt{3}}{35}$

4 $-\frac{6\sqrt{3}}{35}$

Explanation

$$\cos\left(\arccos\frac{1}{7}+\arcsin\frac{1}{5}\right)=\cos\left(\arccos\frac{1}{7}\right)\cos\left(\arcsin\frac{1}{5}\right)-\sin\left(\arccos\frac{1}{7}\right)\sin\left(\arcsin\frac{1}{5}\right)$$

$$\Leftrightarrow \quad =\frac{1}{7}\cos\left(\arcsin\frac{1}{5}\right)-\frac{1}{5}\sin\left(\arccos\frac{1}{7}\right)$$

[using two right-angled triangles for the remaining terms]

$$\Leftrightarrow \quad =\frac{1}{7}\times\frac{2\sqrt{6}}{5}-\frac{1}{5}\times\frac{4\sqrt{3}}{7}$$

$$\Leftrightarrow \quad =\frac{2\sqrt{6}}{35}-\frac{4\sqrt{3}}{35}$$

Question 5

★★★

If $\tan x=\frac{1}{2}$, find the exact value of $\tan 4x$.

1 $-\frac{24}{7}$ ✓

2 $\frac{24}{7}$

3 $-\frac{54}{27}$

4 $\frac{56}{54}$

Explanation

Using the double angle formula for tangent

$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$$

Student view

Substitute using $\tan x=\frac{1}{2}$:

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$$\tan 2x = \frac{2 \times \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} = \frac{4}{3}$$

Using the double angle formula for $\tan 4x$:

$$\tan 4x = \frac{2\tan 2x}{1 - \tan^2 2x} = \frac{2 \times \frac{4}{3}}{1 - \left(\frac{4}{3}\right)^2}$$

Simplify:

$$\tan 4x = -\frac{24}{7}$$

Therefore, the correct answer is $-\frac{24}{7}$

3. Geometry and trigonometry / 3.10 Trigonometric identities revisited

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27566/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- recognise and use
  - the compound angle identities:

$$\sin(\alpha \pm \beta) = \cos\alpha \sin\beta \pm \sin\alpha\cos\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

  - the double angle identity for tangent $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$
- find exact values of trigonometric ratios of compound angles.

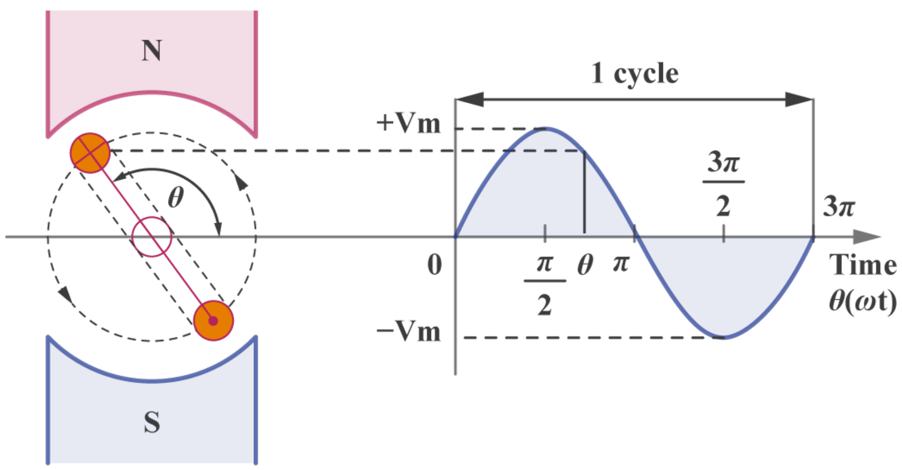
Student view

Investigation

In Europe, and many other countries in the world, mains electricity uses a voltage of 220–240 volts while most of America and Japan uses a voltage of 120–127 volts.

The difference originated from the initial use of AC and DC. Tesla, the inventor of AC generators, through his calculations, found that 60 hertz (cycles per second) was the best frequency for alternating currents which created 240 V. But Edison, the inventor of DC generators, was already producing and selling electricity at 110 V. As the AC creates more power and could be delivered over longer distances, Europe and other countries adopted the AC system. So, when you travel next time, check the voltage used in the country you are visiting before you damage your electrical appliances.

In a basic AC generator, a coil rotates anticlockwise around a central axis which is placed between magnetic poles. In the diagram below, θ is the angle of rotation, and the frequency of rotation is measured in hertz which is the number of cycles per second. This rotation creates an alternating voltage and current with varies with time, as illustrated.



Overview
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The image shows a diagram of an AC generator on the left and a corresponding sine wave graph on the right. In the generator diagram, there's a rotating coil between the poles of a magnet, labeled 'N' for north and 'S' for south. The coil rotates anticlockwise, and an angle (\theta) is marked showing its rotation. The graph shows the alternating current generated as the coil rotates, depicted as a sine wave. The x-axis, labeled as 'Time (\theta(\omega t))', is annotated with values (\frac{\pi}{2}), (\pi), and (\frac{3\pi}{2}). The y-axis indicates voltage, with values ranging from +Vm to -Vm. The sine wave completes one full cycle, and the amplitude is described by Vm.

[Generated by AI]

The instantaneous voltage at a given time, in seconds, can be found using the formula

$$V\left(t\right) = V_m \sin\left(ax + b\right)$$

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Where

- $V\left(t\right)$ is the voltage at time t seconds, in volts
- V_m is the maximum voltage, or the amplitude of the sine curve.
- ax is the angle of rotation at time t seconds, measured in radians
- b is the phase difference, measured in radians

Use the app below to investigate the effect of changing the values of V_m, a and b on the magnitude and frequency of the voltage.

Student view

Interactive 1. Exploring Trigonometric Functions in AC Voltage.

More information for interactive 1

This interactive is a graph that allows the user to investigate the effect of changing the values of V_m, a, and b on the magnitude and frequency of the voltage.

A blue sinusoidal voltage signal curve is projected on a graph, whose parameters, amplitude, angular frequency, and phase constant, can be interactively adjusted using the horizontal sliders at the bottom of the graph.

The graph plots voltage versus time. The voltage is represented by a sinusoidal curve, demonstrating the behavior of an alternating current over time. The x-axis represents time, and the scale ranges from 0 to 14. The y-axis represents the voltage of the signal; the scale ranges from -80 to 80.

On the top, the function is defined as $V(t) = V_m \sin(at + b)$. The slider V_m controls the amplitude of the sine wave ranging from 0 to 100, slider a controls the angular frequency ranging from 0π to 6π and slider b controls the phase shift of the sine wave. The blue curve shows how the voltage of the signal changes over time.

For example, on setting $V_m = 50$, $a = \frac{3}{4}\pi$ and $b = 2\pi$ the equation becomes $V(t) = 50\sin\left(\frac{3}{4}\pi t + 2\pi\right)$, it is observed that The peak voltage is $50V$, and the minimum voltage is $-50V$.

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The period is the time it takes for one complete cycle of the waveform, calculated from the angular frequency: $a = 2\pi f = 8/3$ So, the period of the waveform is approximately 2.67 time units. The frequency is the number of cycles per unit time: $f = 1/T = 3/8$ cycles per unit time. The phase constant $b = 2\pi$ means the wave is shifted by one full cycle. This means the starting point of the wave at $t = 0$ is the same as it would be if there were no phase shift ($b = 0$). The sine function starts at 0 at an angle of 0 radians, and $\sin(0 + 2\pi) = \sin(2\pi) = 0$.

The equations provide the mathematical description of this waveform, allowing users to understand and predict the voltage at any given time.

Investigate the formulae used in alternating current and write a short report showing how these formulae rely on trigonometry.

Rate subtopic 3.10 Trigonometric identities revisited

Help us improve the content and user experience.

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