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SIMPLE HARMONIC MOTION

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C. Wave behaviour / C.1 Simple harmonic motion

The big picture

? Guiding question(s)

- What makes the harmonic oscillator model applicable to a wide range of physical phenomena?
- Why must the defining equation of simple harmonic motion take the form it does?
- How can the energy and motion of an oscillation be analysed both graphically and algebraically?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

Take a ruler and place it on a table so the end of the ruler hangs over the edge of the table. Hold the ruler tight against the table, push the free end of the ruler down then release it. The ruler will move quickly up and down, and eventually come to rest back in its original position.

This kind of motion, where something moves back and forth repeatedly, is called an oscillation. Oscillations are all around us (**Interactive 1**).

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Interactive 1. Real-life Oscillations.

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More information for interactive 1

An interactive with four short videos illustrates the examples of real-life examples to understand the concept of oscillation. At the bottom, a segmented grey block indicates the number of videos present. When a specific video is viewed, the respective grey block gets highlighted in blue. Below this, the slide viewed out of the four is given in numbers. At the bottom right, the zoom-out icon allows the users to view the videos in full-screen mode.

Read the following to know the examples of different oscillations in real life:

Slide 1: A wooden pendulum clock showing ten past ten is viewed. Once the video is played, the hour and minute hands move in a periodic motion in a circular path and again stop at ten past ten. Simultaneously, the pendulum follows a rhythmic motion from left to right at regular, repetitive intervals.

Slide 2: In a grass field, a girl wearing a peach-colored dress is seated on the swing hung from a tree branch. The girl swings happily back and forth by applying force on the ground with both her legs. The movement of the swing exhibits typical oscillatory motion as it follows the force applied and gravity's pull.

Slide 3: A close-up view of one of the tires in a car. As the car travels through a dry, arid region, the tire of the car moves back and forth at regular intervals in an oscillatory manner to maintain a smooth ride.

Slide 4: A close-up view of the waves on a beach. It shows the natural oscillation of waves as it rises and falls due to gravitational forces.

How is a child swinging on a swing like the tide coming in and out? How is a car's suspension like a clock's pendulum? And how are all these things related to your bouncing ruler?

Prior learning

Before you study this subtopic make sure that you understand the following:

- Displacement, velocity and acceleration (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).
- Time period and frequency (see subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/))
- Kinetic energy and potential energy (see subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/))

Simple harmonic motion (SHM)

Student view

Learning outcomes

At the end of this section you will be able to:

- Identify examples of simple harmonic motion.
- Understand and use the terms: time period, T, frequency, f, angular frequency, ω, amplitude, equilibrium position and displacement.
- Define simple harmonic motion, and use the equation $a = -\omega^2 x$.

The Nevis Swing in New Zealand is reportedly the world's biggest swing. Strapped to the enormous swing, people are dropped from a platform and swing, backwards and forwards, across a valley. The Nevis Swing is similar to a swing in a children's playground, and the Nevis Swing follows the same laws of physics.

Video 1. The Nevis Swing.

How can we describe the motion of the Nevis Swing? Saying 'I swung really far' does not give as much information as 'I was freefalling for 90 metres, then swung 300 metres across the valley and back in a few seconds'.

Being able to measure and quantify motion makes it easier to describe motion. So we need a shared language to talk about simple harmonic motion, similar to the language used for the motion of objects in subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/).

International Mindedness

In New Zealand, the Accident Compensation Corporation (https://en.wikipedia.org/wiki/Accident_Compensation_Corporation) is a government-led initiative which provides medical and other financial support to people who are injured doing sports or adventure activities like the Nevis Swing. It also means that activities providers cannot be sued if they make a mistake. Conversely, in the USA, for example, companies are regularly sued if they are at fault when someone gets hurt.

In New Zealand, companies may feel like they can provide a wider range of activities and be more innovative with the way they offer experiences. On the other hand, in the USA, companies may be more likely to be more cautious about their clients' safety.

Is there a 'better' way to do this? Should there be internationally accepted rules, so we know that no matter where we are in the world, the physics of things like the Nevis Swing is always thoroughly understood? Could the threat of legal action prevent companies from pushing the boundaries of experiential science? Can we ever know 'all' the physics needed to ensure 100% safety?

Equilibrium position

Imagine that you are pushing a friend on a swing. The friend is swinging backwards and forwards. You stop pushing. If the friend does not swing themselves, what happens?

Eventually, the friend will stop swinging, and will come to rest with their centre of mass directly underneath the bar from which the swing is hanging – position B in **Figure 1**.

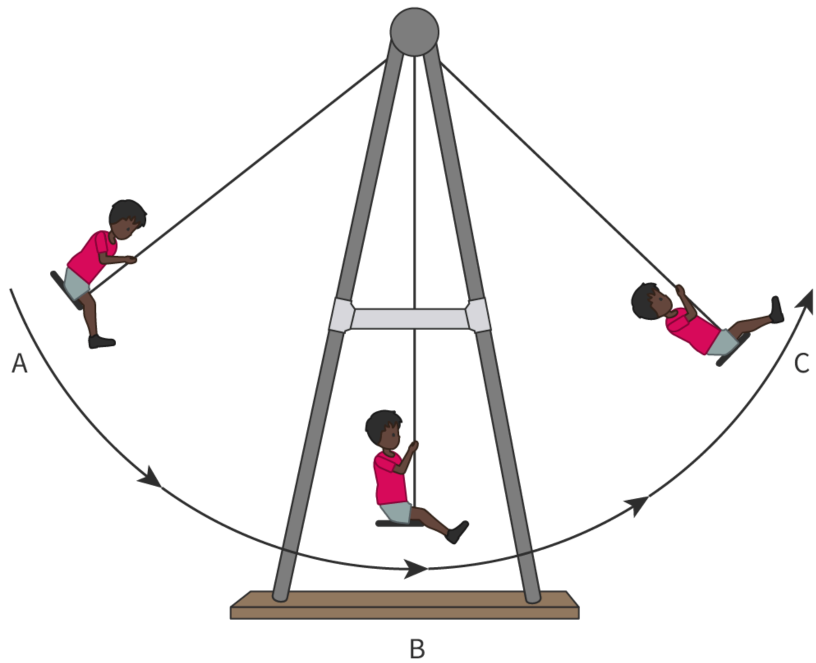


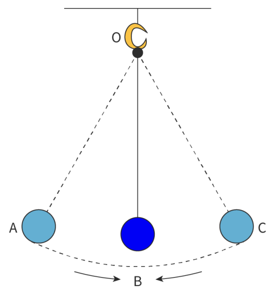
Figure 1. The motion of a swing.

More information for figure 1

The image is a diagram illustrating the motion of a swing. It shows three positions of a person on a swing. Position A is to the left, where the swing is pulled back. Position B is at the bottom, directly underneath the hinge, indicating the equilibrium position where the swing would rest if not moving. Position C is to the right, mirroring position A. The swing's path is shown as an arc with arrows indicating the direction of motion.

[Generated by AI]

Position B, where the swing would be if it was not moving, is called the equilibrium position. Identify the equilibrium position in each of the examples in **Interactive 1**.



At which point is the equlibrium position?

- A
- B
- C

Check

Interactive 1. What Is the Equilibrium Position in Each Example?

More information for interactive 1

A slider-based interactive gallery allows users to navigate through multiple images using left or/ right arrow buttons on the bottom left corner of the screen. At the bottom, there is a row of three small circular indicators showing the total number of images and highlighting the currently active ones. Users can click on these indicators to jump to any one of the three choose-the-best-option interactives in this slider. There is a “Check” box at the bottom left that users can use to check

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whether the selected answer is correct or not. There is also a plus sign present at the top right of each image. Users can use this to zoom in on the images. Once the image is zoomed in, there is a minus sign at the top right, which the users can use to zoom out the image.

Let us now learn about each slide.

Slide 1: An interactive display of a pendulum bob being hung from a central pivot point labeled "O". The black line from the pivot point represents a string with which the pendulum is hung. The pendulum swings from side to side, and the three motion positions are marked as A, B, and C. Positions A and C, highlighted in light blue, represent the outermost points of the motion. These positions are marked with dashed lines from the pivot point indicating the motion range. On the other hand, two dashed curves, one from position A and one from position C are drawn towards point B indicating the motion from A to C through B. Position B, highlighted in dark blue, is the lowest central position, and the actual position of the pendulum bob before swinging. The user is prompted to answer the question: At which point is the equilibrium position? The three choices are A, B, and C. The user has to choose their answer and check it with the "Check" option at the bottom left.

The user will now use the right arrow at the bottom left to navigate to the next page.

Slide 2: An interactive with three string-mass systems. The three strings are suspended from a fixed inverted trapezium-shaped yellow component. A rectangular yellow mass is attached to the end of all three strings and is marked A, B, and C. The three spring-mass systems differ from each other in terms of their spring length. The system with mass C has the longest spring length followed by B and A. The user is prompted to answer the question: At which point is the equilibrium position? The users have to choose from the three options: A, B, and C.

The user will now use the right arrow at the bottom left to navigate to the next page.

Slide 3: An interactive that features the bending of a flexible beam (rectangular blue strip) at different angles. The flexible beam is placed on a rigid table with orange legs and a brown surface. The portion of the beam placed on the table is fixed with a rectangle brown load. The portion of the beam outside the table is free to bend either upwards or downwards. The highest upward bend is marked A and the lowest downward bend is marked C. The horizontal position of the beam is marked B. The user is asked to answer: At which point is the equilibrium position? The three choices are A, B, and C.

The solution is B for all the three questions.

If the user has chosen the wrong answer, a progressive slider at the bottom left would indicate 0/1 along with "Show solution" and "Retry". The users can use these options to either see the correct option or retry the correct answer.

What did you notice? You may have noticed that the equilibrium position is always in the centre of these types of oscillations.

Displacement and amplitude

Let's go back to the friend on the swing. If you push them a few times, what happens? The friend starts to swing. Push the friend harder a few more times, and they start to swing higher. How does this swing compare with the Nevis Swing?

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You might say the Nevis Swing is 'bigger' or 'higher' or 'faster'. But these are not scientific terms, so we need some terminology to describe this motion.

Figure 2 shows the Nevis swing and its equilibrium position.

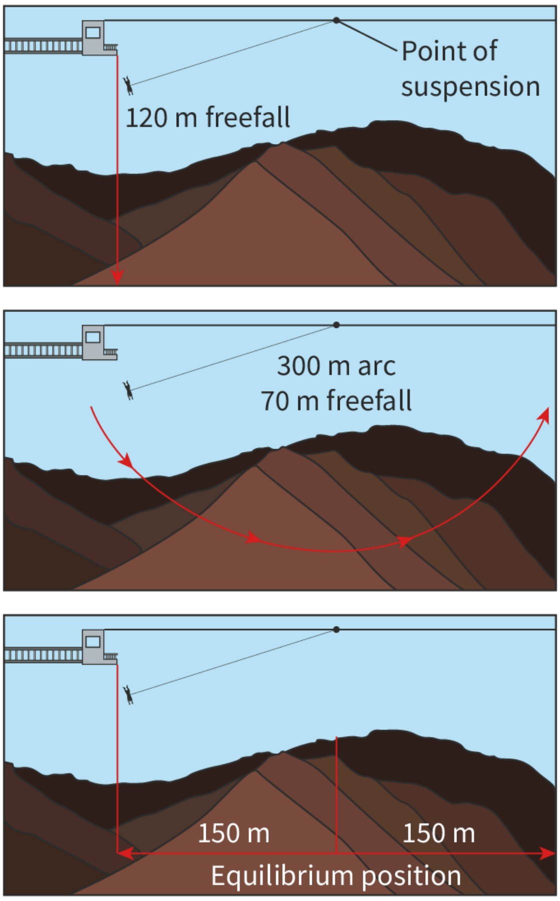

Figure 2. The Nevis Swing.

More information for figure 2

The image is a diagram illustrating the Nevis Swing in three stages. Each stage is shown in a separate panel from top to bottom.

Top panel: Depicts a point of suspension with a 120-meter freefall. A straight line descends vertically with an arrow pointing downwards.

Middle panel: Shows an arc labeled as a 300-meter arc and a 70-meter freefall. An arrow curves across the arc, indicating motion from one side to the other.

Bottom panel: Displays the swing at the equilibrium position with two 150-meter lines extending horizontally to the left and right. There is an additional vertical line marking the equilibrium position.

The diagram visually represents the freefall distances and positions in relation to the equilibrium position of the swing.

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[Generated by AI]

As the person starts to fall, they move closer and closer to the equilibrium position. How far they are from the equilibrium position is their displacement (subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)). Displacement is a vector. One side of the equilibrium position has a **positive** displacement, and the other side has a **negative** displacement.

When the person is swinging, and as far from the equilibrium position as they can be, they are about 150 m away. This maximum distance is called the amplitude of the swing. We can talk about this in terms of the linear distance, or in terms of the angle from the horizontal.

Concept

When talking about oscillations, we say that the displacement is how far the body is from the equilibrium position. The amplitude is the maximum displacement that the body has from the equilibrium position.

Time period and frequency

Amplitude can differ greatly between different oscillating objects. Use the simulation in **Interactive 2** to investigate two oscillating objects. Select 'Intro'. Add a 250 g mass to one spring and a 50 g mass to the other spring. Select the box 'Equilibrium Position'. What is different about the two oscillations?

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Interactive 2. Investigating oscillations.

More information for interactive 2

An interactive simulation, Investigating oscillations, allows users to explore the behavior of masses oscillating on springs and investigate key concepts related to simple harmonic motion. The interface features two vertical springs, each with an adjustable spring constant, and a set of masses that can be attached to them. The simulation takes place under the influence of gravity, which can be adjusted using a slider that ranges from "None" to "Lots". Additional options include toggling equilibrium position markers, displaying natural length references, and adjusting damping.

In this particular setup, a 250 gram mass is attached to one spring, while a 50 gram mass is attached to the other. The equilibrium position is marked with a green dashed line, showing that the heavier mass stretches its spring further down than the lighter mass. This difference is due to the force of gravity acting on the masses, which causes the heavier mass to exert a greater force, leading to a greater displacement from the natural length of the spring.

When the masses are displaced and released, they oscillate up and down. One visible difference between the two is the frequency of oscillation. The 50 grams mass oscillates faster, completing one full cycle in less time than the 250 grams mass. Another aspect is amplitude. The amplitude of oscillation depends on how much the mass is initially displaced before being released. By stopping and restarting the oscillations using the red stop button, users can try to match the amplitudes of both masses to observe how they behave under similar conditions.

The simulation also allows users to experiment with different spring constants by adjusting sliders that range from "Small" to "Large". A stiffer spring (higher spring constant) results in less stretch under the same force and leads to faster oscillations. Conversely, a weaker spring allows for more displacement and slower oscillations.

The simulation provides a dynamic environment to visualize and understand the relationships between mass, spring constant, equilibrium position, oscillation frequency, and amplitude in simple harmonic motion. By manipulating various parameters, users can observe real-time changes and develop a deeper understanding of oscillatory motion.

There are key differences between the two oscillating objects.

The equilibrium position for the 250 g mass is lower than the equilibrium position for the 50 g mass.

The amplitude will probably be different. Amplitude is not dependent on mass, but on the amount of initial displacement. Try to stop and restart the oscillations so they have the same amplitude using the red stop button at the top and dragging the masses down to restart them.

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The 50 g mass is oscillating faster than the 250 g mass. The 50 g mass completes one cycle in less time than the 250 g mass. You will encounter this interactive again when the mathematical relationship is explored in section C.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-pendulums-and-mass-spring-systems-id-44870/).

Making connections

The extension of a spring, x, is proportional to the amount of force exerted on the spring: $F_{\mathrm{H}} = -kx$ (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)). When the 250 g mass is hanging from the spring, there is a larger force acting than if the 50 g mass was hanging. The equilibrium position for the 250 g mass is lower than the equilibrium position of the 50 g mass.

This does **not** mean that the amplitude of the oscillations is always greater for greater masses. It depends on the magnitude of the initial displacement.

The amount of time taken for an oscillator to complete one full cycle is called the time period , *T*. Time period is measured in seconds (s).

Time period is inversely related to the frequency, *f*, which is the number of cycles completed per second. Frequency is measured in hertz (Hz) (which is equivalent to s^{-1}).

Time period and frequency are related by the equation shown in **Table 1** (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)).

Table 1. Equation for time period using frequency.

Equation	Symbols	Units
$T = \frac{1}{f}$	T = time period	seconds (s)
	f = frequency	hertz (Hz)

Worked example 1

The string of a guitar is plucked so that it completes 158 oscillations in 1.0 seconds. Calculate the time period of the vibrations.

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Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	The number of cycles in one second is the frequency: $$f = 158\ \text{Hz}$$
Step 2: Write out the equation.	$T = \frac{1}{f}$
Step 3: Substitute the values given.	$= \frac{1}{158}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding	$= 0.00633\ \text{s} = 0.0063\ \text{s}\ (2\ \text{s.f.})$

Angular frequency

Oscillations occur in cycles, and these cycles are repeated over and over. Look at the simulation in **Interactive 3**. Set the sliders for 'Spread', 'Number' and 'Seperation' to the minimum value by dragging left. Start the oscillation by pressing play. It shows an object oscillating from left to right and back. Now select 'Circle View'. Then select '3D View'. What do you notice? Go back to 'SHM View' and move all the sliders to right. Then select 'Circle View'. Then select '3D View'. What do you notice now?

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Interactive 3. Particle motion simulation.

More information for interactive 3

The interactive simulation titled, Particle motion simulation, visualizes particle motion in oscillatory and wave-like behavior, allowing users to manipulate various parameters to explore different perspectives of motion. The left panel provides control options, including toggles for animation, circles, and axes display, along with buttons to switch between different views such as Simple Harmonic Motion, Wave View, Circle View, and 3 D View. Sliders adjust parameters like spread, number of oscillators, amplitude, and separation.

When the sliders for spread, number, and separation are set to the minimum, the simulation depicts a single particle moving back and forth along a straight line in SHM (Simple Harmonic Motion) view. This motion represents a basic oscillation where the particle moves from left to right and back in a sinusoidal pattern. Selecting the Circle View transforms this linear motion into circular motion, showing that SHM can be visualized as the projection of uniform circular motion.

In one time period, an oscillating particle will complete one complete cycle 360°, or 2π radians. So, mathematically, the rate at which the oscillator completes these cycles or the angular frequency of the oscillating particle will be:

$$\omega = \frac{2\pi}{T}$$

Here, ω is the angular frequency of the oscillating particle, T is the period of oscillation.

Switching to 3D view presents a helical motion where the particle moves in a spiral-like path, reinforcing the connection between oscillation and circular motion.

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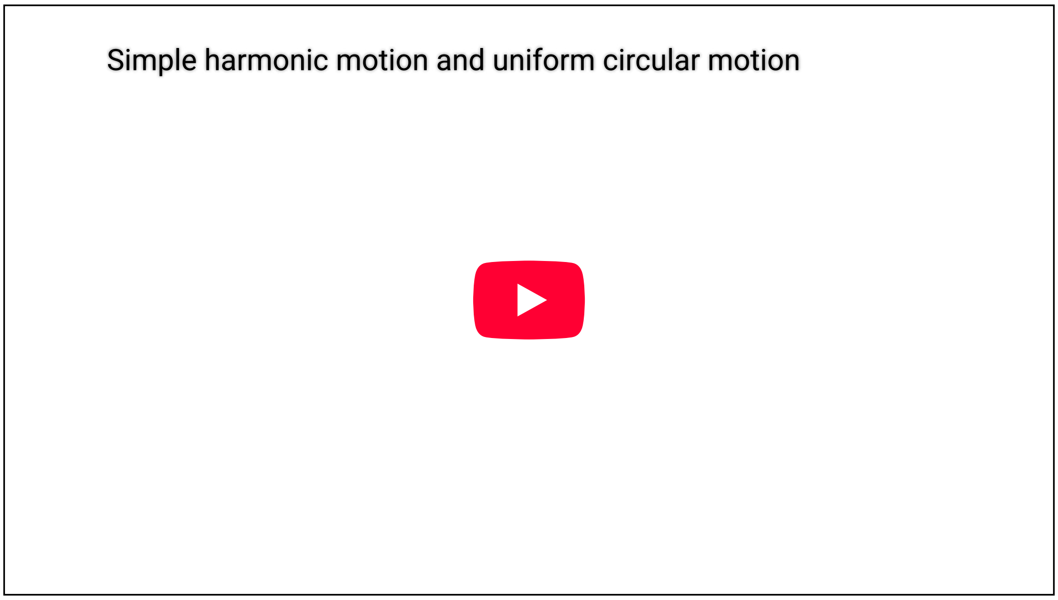
When all the sliders are set to the maximum, the SHM View now displays multiple oscillating particles spread across the screen, moving with a greater amplitude and separation. The Wave View reveals a traveling wave pattern, showing how individual oscillators contribute to wave propagation. In Circle View, the particles trace multiple circular paths, emphasizing their uniform angular motion. Finally, in 3D View, the particles create a complex helical structure, illustrating how wave motion extends into three dimensions.

Thus, the simulation effectively demonstrates the relationships between SHM, circular motion, and wave formation. It allows users to observe the transformation of simple oscillatory motion into more complex wave behavior by adjusting parameters. This simulation helps in understanding fundamental physics concepts such as angular frequency, wave propagation, and the link between periodic motion and rotational motion.

Looking at a particle oscillating around an equilibrium position is like looking at a particle moving in a circle, but from the side. **Video 2** and **Video 3** show this effect clearly.



Video 2. Simple harmonic motion and circular motion.

More information for video 2

The video presents a visual demonstration of oscillatory motion and its relationship to circular motion, illustrating how a particle oscillating around an equilibrium position can be understood in terms of a circular path observed from a particular perspective.

At the start, a stationary setup features a silver metal spring attached to a red cylinder, standing against a black background. To its right, a gold ring lies horizontally, with a green sphere resting at its edge. As the scene unfolds, the green sphere begins to move in a rhythmic, pendulum-like fashion along the edge of the gold ring. Its motion follows a smooth and predictable path, swinging back and forth in a manner characteristic of simple harmonic motion. Meanwhile, the spring and red cylinder remain static, serving as a visual anchor that contrasts with the dynamic movement beside it.

As the oscillation continues, the sphere's swing arc gradually increases. This change enhances the contrast between motion and stillness. The growing oscillation also begins to affect the gold ring itself, which was initially stable and

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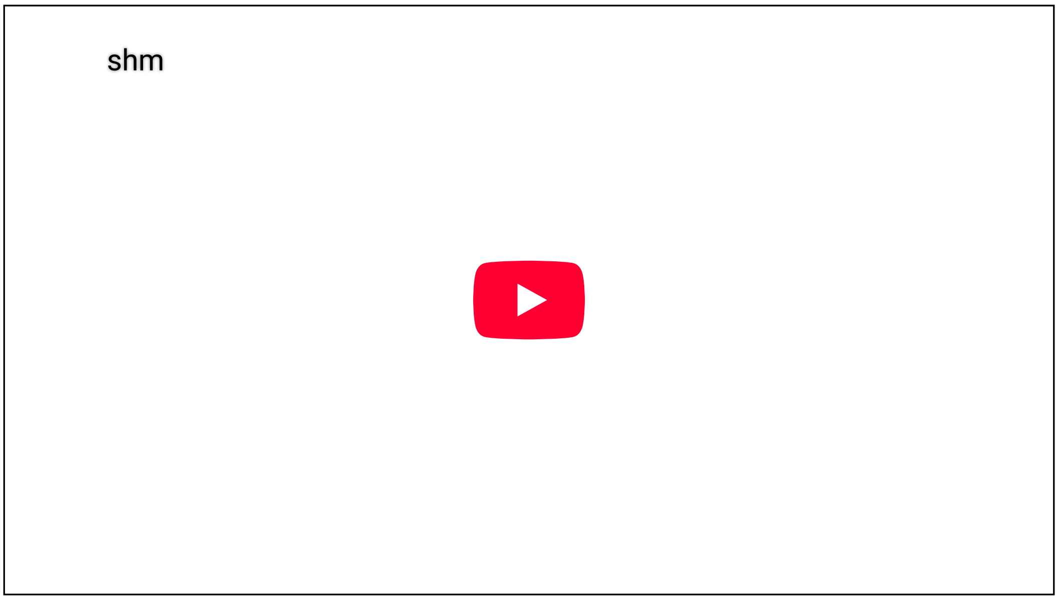
motionless. With each passing swing, the ring tilts slightly, responding dynamically to the increasing amplitude of the sphere's motion.

Over time, the tilting of the ring becomes more pronounced. The green sphere continues to follow a pendulum-like trajectory, but now, the gold ring actively responds to its motion, transitioning from a near-horizontal position to an increasingly inclined state. The effect grows more dramatic as the oscillation amplitude further increases, causing the ring to tilt through a greater range of angles. The synchronized movement of the sphere and the ring showcases how systems with interdependent components can exhibit more complex behaviors than isolated oscillatory motion alone.

As the system continues to evolve, the coordinated oscillations of the sphere and the ring persist, demonstrating a clear relationship between the forces at play. The movement of the sphere influences the motion of the ring, and the ring, in turn, subtly affects the oscillation of the sphere.



Video 3. Simple harmonic motion and circular motion using a turntable.

Theory of Knowledge

We can apply simple harmonic motion to an object moving in a circle if we consider only one of the dimensions of its movement. There are many examples in physics where the situation is simplified.

- If we ignore half of the motion of a body, does this make the equations less valid? If we simplify a situation so much that it does not exist in real life, does this make the science less valid?

Student view

In addition to talking about the position of an oscillating particle, we can also consider the angle it travels through. A full cycle is a complete circle, 360° or 2π radians.

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Study skills

Radians are a useful unit for angles when talking about circles (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)). When you are describing oscillations and cycles, you will almost always need to use radians.

Angular frequency, ω, is the angle that an oscillator covers per second. It is measured in radians per second (rad s^{-1}).

In one time period, T, an oscillator will complete one complete cycle, or 2π radians. So, mathematically, the rate at which the oscillator completes these cycles is:

$$\omega = \frac{2\pi}{T}$$

This equation can be rearranged as in **Table 2**.

Table 2. Equation for time period using angular frequency.

Equation	Symbols	Units
$T = \frac{2\pi}{\omega}$	T = time period	seconds (s)
	ω = angular frequency	radians per second (rad s^{-1})

This equation can be rearranged for the frequency of oscillation:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$f = \frac{\omega}{2\pi}$$

Worked example 2

A vibrating ruler oscillates with a frequency of 55 Hz. What is its angular frequency?

Student view

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$f = 55\ \text{Hz}$
Step 2: Write out the equation and rearrange to make ω the subject.	$T = \frac{1}{f}$ $= \frac{2\pi}{\omega}$ $\omega = 2\pi \times f$
Step 3: Substitute the values given.	$= 2\pi \times 55$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 345.58\ \text{rad s}^{-1} = 350\ \text{rad s}^{-1}$ (2 s.f.

Worked example 3 is a typical question that you might be asked – there are two situations and you have to compare them.

Worked example 3

Oscillator A has a time period of 2.5 s. Oscillator B has a frequency of 0.2 Hz. Determine the ratio:

$$\frac{\omega_A}{\omega_B}$$

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$T_A = 2.5\ \text{s}$ $f_B = 0.2\ \text{Hz}$ $T_B = \frac{1}{f_B}$ $= 5.0\ \text{s}$
Step 2: Write out the equation.	$T = \frac{2\pi}{\omega}$

Solution steps	Calculations
Step 3: Determine the equations for ω_A and ω_B.	Oscillator A: $$T_\text{A} = \frac{2\pi}{\omega_\text{A}}$$ $$\omega_\text{A} = \frac{2\pi}{T_\text{A}} = \frac{2\pi}{2.5}$$ Oscillator B: $$T_\text{B} = \frac{2\pi}{\omega_\text{B}}$$ $$\omega_\text{B} = \frac{2\pi}{T_\text{B}} = \frac{2\pi}{5.0}$$
Step 4: Compare the ratio $\frac{\omega_\text{A}}{\omega_\text{B}}$.	$$\frac{\omega_\text{A}}{\omega_\text{B}} = \frac{\left(\frac{2\pi}{2.5}\right)}{\left(\frac{2\pi}{5.0}\right)}$$ $$\frac{\omega_\text{A}}{\omega_\text{B}} = \frac{\left(\frac{1}{2.5}\right)}{\left(\frac{1}{5.0}\right)}$$ $$\frac{\omega_\text{A}}{\omega_\text{B}} = 2.0$$

Simple harmonic motion (SHM)

We have looked at different kinds of oscillators. Simple harmonic motion (SHM), is a specific type of oscillation that has certain properties. It has a very specific definition:

Simple harmonic motion is defined as repeated motion around an equilibrium position where the acceleration of the object is proportional to its displacement, but in the opposite direction.

Look at the top part of **Interactive 4**. It shows the acceleration, velocity and displacement of a mass on a spring. The bottom part shows the graph of acceleration against displacement. You can pause the video at various points to study if you wish.

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Interactive 4. A Mass on a Spring Showing Acceleration, Velocity and Displacement and an Acceleration—displacement Graph.

More information for interactive 4

A simple animation of a mass-spring system undergoing simple harmonic motion. The animation shows a horizontal spring attached to the vertical part of a L-shaped brown wooden block. A yellow mass is attached to the right end of the spring. The dashed lines passing through the yellow block indicates the equilibrium position. Three arrows positioned from top to bottom concerning the yellow block are black, green, and red. The back and green arrows represent x and v for displacement and velocity and point outwards. The red arrow represents acceleration and points inwards to the system.
An acceleration-displacement graph is provided by the spring-mass system. The x-axis represents displacement in meters (m). The y-axis represents acceleration in meters/seconds square or $\text{ms}^{(-2)}$. A straight pink line passes from the second to fourth quadrant through point O, which is marked as a yellow dot. Point O represents the equilibrium position.
As users click on the “Play” button at the bottom left, the spring-mass system moves across the equilibrium position back and forth. Simultaneously, the three arrows also change their directions concerning the position of the mass, and the acceleration-displacement graph updates below, illustrating the inverse relationship between acceleration and displacement.
The animation demonstrates the fundamental principles of simple harmonic motion, emphasizing how acceleration is always directed toward the equilibrium position and is proportional to displacement.

Most, if not all, of the examples of motion we have looked at have fulfilled this criteria and therefore would be classed as simple harmonic motion.

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Theory of Knowledge

Categorising knowledge can help scientists decide which rules to apply to certain situations. To what extent are these categories defined by nature, and to what extent are they human inventions?

If an oscillation does not fit the criteria for SHM, does that make it completely different? Will the oscillation obey any of the same rules? Is there a grey area between SHM and non-SHM?

Straight-line graphs through the origin, like the one in **Interactive 4**, always take the form: $y = mx$, where m is the gradient of the graph. What is the gradient of this graph?

Imagine your arm is a pendulum (move your arm as a pendulum, if it helps). A steeper gradient of the acceleration–displacement graph means a greater acceleration for the same displacement. A greater acceleration will mean that your hand moves back to the equilibrium position faster with each swing. Thus, the frequency is higher, and the angular frequency is higher. A less steep gradient means a smaller acceleration, which means a lower frequency and angular frequency.

The gradient of the graph is equivalent to the square of the angular frequency and, because acceleration and displacement are in opposite directions, it is negative. So the equation for acceleration becomes the equation for simple harmonic motion in **Table 3**.

Table 3. Equation for simple harmonic motion.

Equation	Symbols	Units
$a = -\omega^2 x$	a = acceleration	metres per second per second (m s^{-2})
	ω = angular frequency	radians per second (rad s^{-1})
	x = displacement	metres (m)

This is the defining equation for SHM. If an oscillator obeys this equation, it is undergoing simple harmonic motion.

Work through the activity in the next section to check your understanding of SHM oscillators.

Student
view

5 section questions

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Question 1

SL HL Difficulty:

When an object is undergoing simple harmonic motion, how far the object is from the equilibrium position at any moment is called the 1 displacement ✓ . The maximum value this has for any given oscillation is called the 2 amplitude ✓ .

Accepted answers and explanation

#1 displacement

#2 amplitude

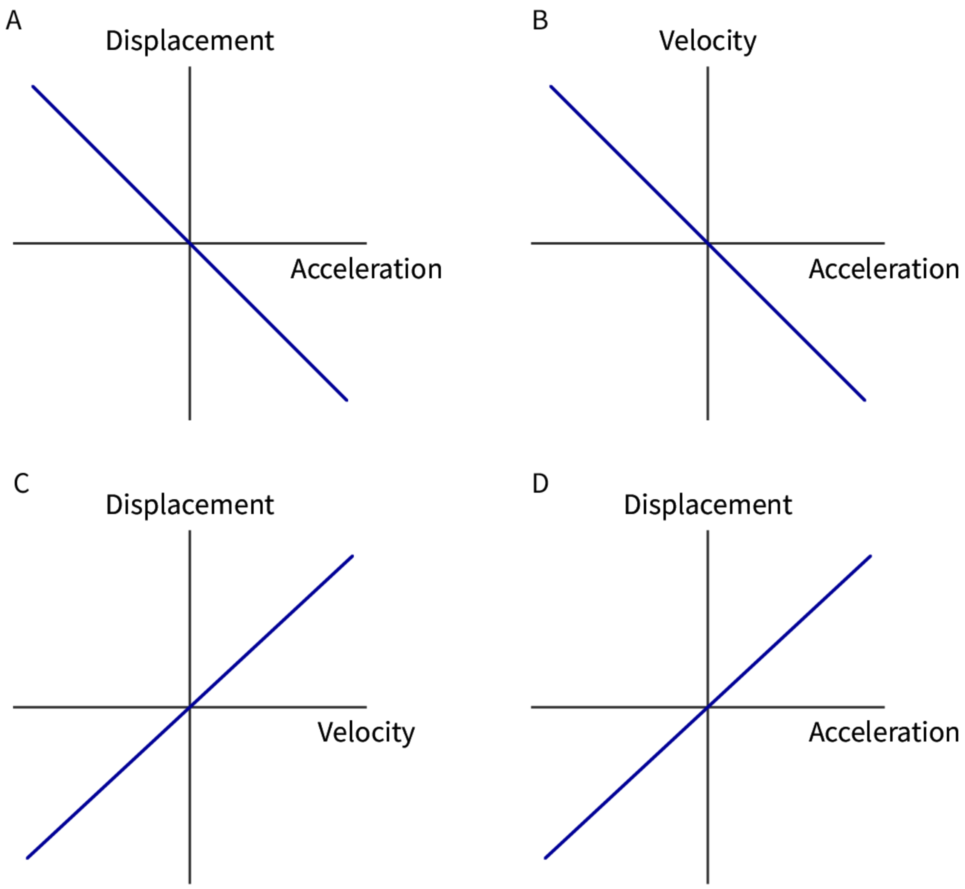
General explanation

The displacement is the vector describing how far the object is from the equilibrium position. The amplitude is the maximum value that the displacement has in any cycle.

Question 2

SL HL Difficulty:

Which of these graphs (A, B C or D) shows an object undergoing SHM?



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More information

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1 A ✓

2 B

3 C

4 D

Explanation

The defining equation of SHM states that $a = -\omega^2 x$, so acceleration, *a*, is proportional to negative displacement, *x*.

The only graph that shows this relationship is graph A.

Question 3

SL HL Difficulty:

The number of full cycles completed per second by an oscillator is the 1 frequency ✓ . The number of radians travelled through per second by an oscillator is the 2 angular frequ... ✓ .

Accepted answers and explanation

#1 frequency

#2 angular frequency

General explanation

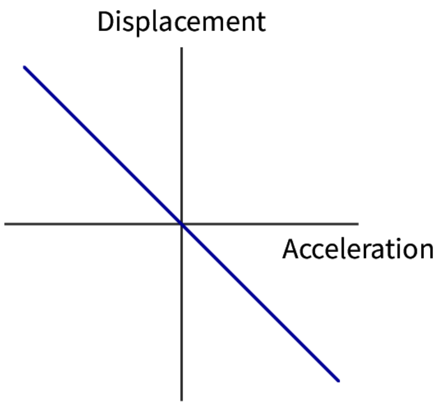
Frequency is measured in Hz and is 'cycles per second'. Angular frequency is measured in rad s^{-1} and is 'angle per second'.

Question 4

SL HL Difficulty:

The graph represents a particle oscillating with simple harmonic motion with a frequency of 4.0 Hz.

Student view



More information

What is the magnitude of the gradient of the graph?

1 -1.6×10^{-3} ✓

2 -6.3×10^{2}

3 -4.0×10^{-2}

4 -1.0×10^{-3}

Explanation

The defining equation of SHM states that:

$$a = -\omega^2 x$$

In the graph, the *y*-axis is displacement and the *x*-axis is acceleration, so rearranging the equation to fit the form $y = mx$, gives:

$$x = -\left(\frac{1}{\omega^2}\right) a$$

The gradient of the graph is:

$$-\left(\frac{1}{\omega^2}\right)$$

$$\frac{1}{f} = \frac{2\pi}{\omega} \text{ so } \omega = 2\pi f$$

Substituting this gives a gradient of:

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$$-\left(\frac{1}{4\pi^2 f^2}\right) = -\left(\frac{1}{(4\pi^2 \times 4.0^2)}\right)$$
$$= -1.583 \times 10^{-3}$$
$$= -1.6 \times 10^{-3}\ (2\ \text{s.f.})$$

Question 5

SL HL Difficulty:

A child is being pushed on a swing with a time period of 4 s. At a particular time, *t*, the child is at maximum displacement from the equilibrium position. When will the magnitude of the child's acceleration be maximum?

1 *t* ✓

2 *t* + 1 s

3 *t* + 3 s

4 *t* + 5 s

Explanation

The defining equation for SHM states that acceleration is proportional to negative displacement. So when displacement is at a maximum, so is acceleration.

The displacement is at a maximum at time *t*, then again at time *t* + 2 s (when the displacement is maximum in the opposite direction), then at *t* + 4 s (when the child returns to the same position they were in at time *t*), then at *t* + 6 s, and so on

The only suitable answer is '*t*'.

C. Wave behaviour / C.1 Simple harmonic motion

Activity: SHM oscillators

C.1.1: Conditions that lead to simple harmonic motion

Student view

Interactive 1. Pendulum simulation.

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Activity

- **IB learner profile attribute:**
  - Thinker
  - Reflective
- **Approaches to learning:**
  - Thinking skills — Applying key ideas and facts in new contexts
  - Communication skills — Reflecting on the needs of the audience when creating engaging presentations
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

In physics, we often use approximations and assumptions to simplify situations to make them more predictable. In this activity, you will compare simulations to reality and see what differences you observe.

There are two main types of SHM oscillator: simple pendulum and mass—spring system.

Interactive 2. Mass–spring simulation.

You will need: a piece of string; masses (can be made from modelling clay); a spring; a clamp stand.

Tie one mass to the end of the string. This is your pendulum. Attach the other mass to the spring. This is your mass—spring system. You can use a clamp stand and clamp to hold the string and spring in place, or you can just use your hands.

Try to set up your systems so they match those in the pendulum simulation and the mass—spring system in **Interactive 1** and **Interactive 2**.

Try to keep conditions constant between the simulation and reality.

Discuss the following questions with your partner.

1. How accurate were the simulations?
2. What did the simulations get right?
3. What happened in reality that the simulations did not take into account?
4. Do you think the simulations use acceptable approximations and assumptions?
5. What changes would you make to the simulations to enhance learning? Would you add any tick boxes, sliders or hidden variables? Would you add or remove anything?

Student
view

You could discuss the findings with your partner, make a presentation to the class, or create a video.

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Simple pendulums and mass–spring systems

C.1.5: Time period of a mass-spring system C.1.6: Time period of a simple pendulum C.1.7: Energy changes during one cycle of an oscillation

Learning outcomes

At the end of this section you will be able to:

- Determine the time period of a simple pendulum using the equation:

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$

- Determine the time period of a mass—spring system using the equation:

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

- Describe and graph the energy changes during SHM.

Imagine a father is on a swing set with two identical swings (**Figure 1**). The father is on one swing, swinging back and forth, experiencing simple harmonic motion. Their child sits on the other swing and starts swinging.

Who swings faster? In other words, who has the shorter time period (see section C.1.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/))?

Does time period depend on how heavy the person swinging is? Does time period depend on how high each person is swinging (their amplitude)?

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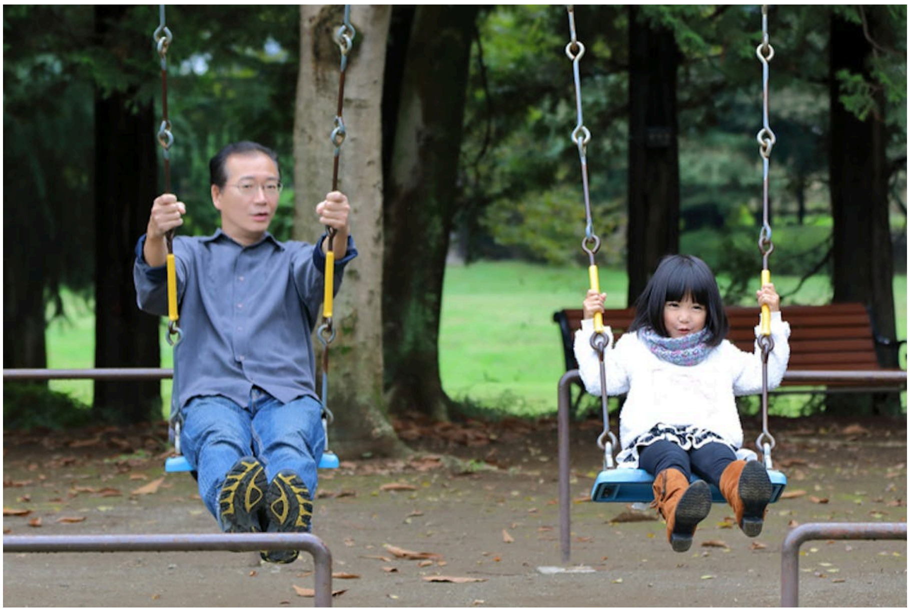

Figure 1. Who will have the shorter time period — father or child?

Credit: Copyright Crezalyn Nerona Uratsuji, Getty Images

In this section, you will see that it does not matter how high the father swings, or how heavy they are, they will always have the same time period as their child!

The simple pendulum

Use the simulation in **Interactive 1** to investigate the motion of a simple pendulum. Select the 'Intro' tab. Start the pendulum swinging, then change the variables one at a time and observe the motion.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/activity-shm-oscillators-id-44901/print/) Assign

Interactive 1. Pendulum simulation.

More information for interactive 1

Student view

The interactive simulation titled, Pendulum simulation, models the motion of a simple pendulum, allowing users to explore the factors influencing its period of oscillation. The pendulum consists of a mass attached to a string, suspended from a fixed point, and swings back and forth under the influence of gravity. The interface provides various controls to manipulate parameters such as length, mass, gravity, and friction. Additionally, tools like a ruler, stopwatch, and period trace can be toggled on to aid in measurements and observations.

The length of the pendulum can be adjusted between 0.1 meter and 1 meter, affecting the arc of motion and the time it takes to complete a full swing. The mass of the pendulum, ranging from 0.1 kilogram to 1.5 kilogram, can also be modified, though it does not influence the period of oscillation. The gravity setting allows users to select different environments, including Earth and other celestial bodies, or adjust the strength of gravity manually. Friction can be varied from none to a significant amount, demonstrating how air resistance or damping affects the pendulum's motion by gradually reducing its amplitude over time.

Users can initiate motion by pulling the pendulum to an angle and releasing it. The normal and slow-motion playback options help in closely analyzing the oscillations. The period of one oscillation can be measured by enabling the stopwatch feature displayed on the box at the bottom left. Period of oscillation is the time taken for one complete oscillation. Start the stopwatch, when the blue line starts from the equilibrium position and stop the stopwatch when the blue line ends at the equilibrium position, completing one oscillation. Observing the pendulum's movement while systematically changing variables helps reveal key principles of simple harmonic motion.

It is observed that the period of the pendulum depends on its length and the gravitational force, but remains independent of the mass. Increasing the length results in a longer period, while increasing gravitational acceleration shortens the period. These observations align with the theoretical equation for the period of a simple pendulum.

The theoretical equation for the period of a simple pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Here, l is the length of the pendulum, g is the acceleration due to gravity and T is the period of oscillation of the pendulum. As per the above equation, the period of oscillation of the pendulum is directly proportional to the length of the pendulum and inversely proportional to the acceleration due to gravity. Also, the period of oscillation does not depend on the mass of the pendulum.

The interactive provides an intuitive way to explore physics concepts, reinforcing the relationship between length, gravity, and period of oscillation. It visually demonstrates damping effects and highlights the independence of period from mass, which may initially seem counterintuitive.

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- Which variables affect the time period? What effect do these variables have?
- Which variables do not have an effect on the time period? Does anything surprise you?

You may have noticed that the only factors that affect the time period of the pendulum are the length of the pendulum and the gravity (gravitational force). Time period is independent of mass.

Deriving the time period of a pendulum

Figure 2 shows a pendulum with a displacement, x, from the equilibrium position, forming an angle θ with the vertical.

The restoring force acting towards the equilibrium position is a component of the weight of the pendulum bob, as shown in **Figure 2**. The restoring force is $mg\ \sin\theta$, while $mg\ \cos\theta$, does no work on the bob because it is perpendicular to the direction of motion.

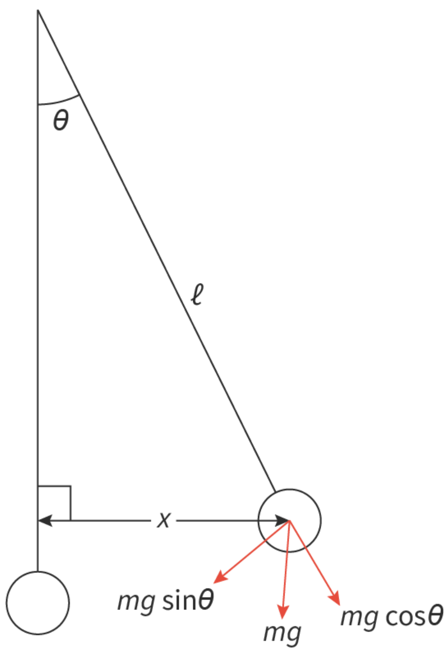

Figure 2. The motion of a pendulum.

More information for figure 2

The diagram illustrates a pendulum consisting of a weight, labeled "bob," suspended by a rod of length (\ell). The pendulum bob is displaced at an angle (\theta) from the vertical. A right triangle is formed with (\ell) being the hypotenuse and the adjacent side labeled as (x), indicating horizontal displacement. At the bob, several force vectors are depicted:

1. (mg), the gravitational force acting downward.
2. (mg \sin\theta), the restoring force acting parallel and opposite to (x) towards the equilibrium position.

Student
view

3. (mg \cos\theta), perpendicular to (mg \sin\theta), acting along the rod.

The angle (\theta) is marked near the pivot point where the rod connects to the horizontal line. The diagram is used to show the components of the gravitational force acting on the pendulum bob and is useful for understanding the pendulum's motion dynamics.

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From Newton's second law of motion (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)), the restoring force is $F = ma$, so:

$$ma = mg\sin\theta$$

The mass cancels out on each side, giving:

$$a = g\sin\theta$$

We can use small angle approximation here, for angles smaller than about 10°.

Aspect: Measurement

When measuring small angles in radians, we can use a model to simplify trigonometric equations. Change your calculator to **radians** mode, then put in sin 0.1:

sin 0.05 = 0.04998...
sin 0.10 = 0.0998...
sin 0.20 = 0.1987...

Using the sin function for these small angles barely changes the value at all. So we can say, for small angles:

$$\sin\theta \approx \theta$$

This is called small angle approximation and it only works for small angles. In the context of SHM, the equations only work when angles are small (smaller than about 10°).

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Using small angle approximation:

$$a = g\theta \quad \text{(equation 1)}$$

We can also see another right-angled triangle in **Figure 2** – a triangle formed by the length of the pendulum string when it is displaced, *l*, and the displacement, *x*. For this triangle:

$$\sin\theta = \frac{-x}{l}$$

(Notice that the displacement is negative, because we said the direction of acceleration was positive.)

Using small angle approximation:

$$\theta = \frac{-x}{l} \quad \text{(equation 2)}$$

Combine equation 1 and 2. What do you get? Click on 'Show answer' to see the answer.

Combining equation 1 and 2 gives:

$$\frac{a}{g} = \frac{-x}{l} \quad \text{(equation 3)}$$

The defining equation for SHM is:

$$a = -\omega^2 x \quad \text{(equation 4)}$$

And the equation for angular frequency is:

$$\omega = \frac{2\pi}{T} \quad \text{(equation 5)}$$

Combine equations 3, 4 and 5. What do you get? Click on 'Show or hide solution' to see the answer.

Combining equations 3, 4 and 5, gives:

Student view

$$\frac{4\pi^2 x}{T^2 g} = \frac{x}{l}$$

Cancelling x and rearranging to make T the subject gives:

$$\frac{4\pi^2 l}{g} = T^2$$

Taking the square root of each side gives us the equation for the time period of a pendulum shown in **Table 1.**

Table 1. Equation for the time period of a pendulum.

Equation	Symbols	Units
$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$	T = time period	seconds (s)
	l = length of the pendulum	metres (m)
	g = gravitational force	metres per second per second (m s^{-2})

Worked example 1

A simple pendulum completes 4 full cycles in 2 seconds. What is the length of the pendulum?

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	The pendulum completes 4 cycles in 2 s, so will complete 2 cycles in 1 s: $f = 2\text{ Hz}$ $T = \frac{1}{f} = \frac{1}{2} = 0.5\text{ s}$

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Solution steps	Calculations
Step 2: Write out the equation and rearrange to make l the subject.	$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$ $l = \frac{T^2 g}{4\pi^2}$
Step 3: Substitute the values given.	$= \frac{(0.5^2 \times 9.8)}{4\pi^2}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 0.0621 \text{ m} = 0.06 \text{ m } (1 \text{ s.f.})$

Knowing the equations for time period can also allow us to compare two different oscillations, like in the example below.

Worked example 2

Pendulum A has twice the mass and half the length of pendulum B. What is the ratio: $\frac{T_A}{T_B}$?

Solution steps	Calculations
Step 1: Write out the equation for each situation.	Remember: the time period is independent of mass. $T_A = 2\pi\sqrt{\left(\frac{l_A}{g}\right)}$ $T_B = 2\pi\sqrt{\left(\frac{l_B}{g}\right)}$ $l_B = 2l_A$ $T_B = 2\pi\sqrt{\left(\frac{2l_A}{g}\right)}$

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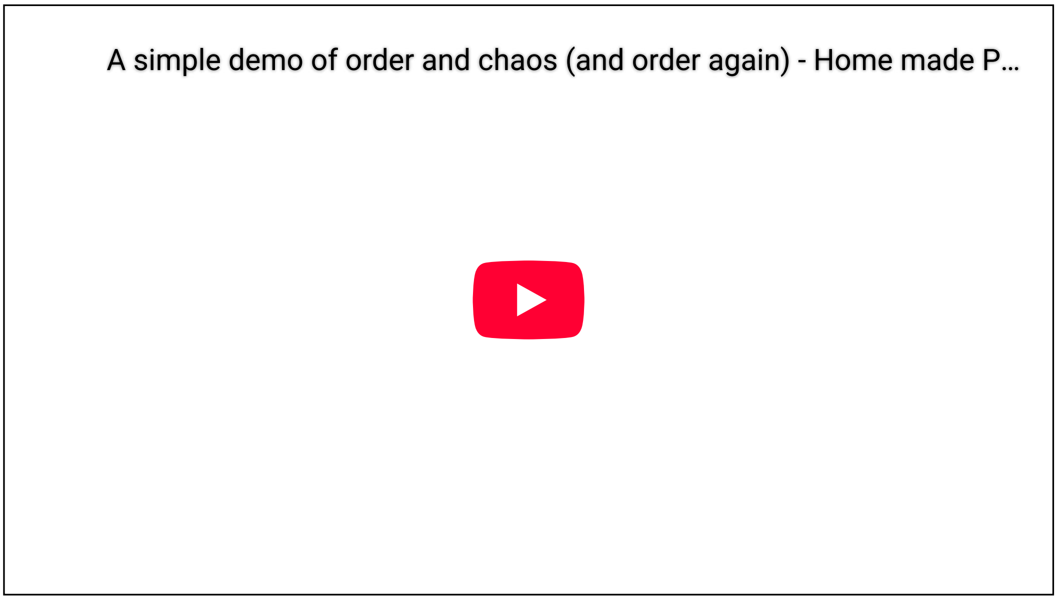
Solution steps	Calculations
Step 2: Rearrange the equations to make them equal to the same constant.	$T_{\mathrm{B}}=\sqrt{2}\times 2\pi\sqrt{\left(\frac{l_{\mathrm{A}}}{g}\right)}$ $\frac{T_{\mathrm{B}}}{\sqrt{2}}=2\pi\sqrt{\left(\frac{l_{\mathrm{A}}}{g}\right)}$ $T_{\mathrm{A}}=2\pi\sqrt{\left(\frac{l_{\mathrm{A}}}{g}\right)}$
Step 3: Substitute T_{A} and rearrange to find the answer.	$\frac{T_{\mathrm{B}}}{\sqrt{2}}=T_{\mathrm{A}}$ $\frac{T_{\mathrm{A}}}{T_{\mathrm{B}}}=\frac{1}{\sqrt{2}}$

Creativity, activity, service

Strand: Creativity
Learning outcome: Demonstrate that challenges have been undertaken, developing new skills in the process

When multiple pendulums of different, incrementally increasing lengths are set in motion together, they produce remarkable patterns.



Video 1. A pendulum wave using billiard balls.

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Overview
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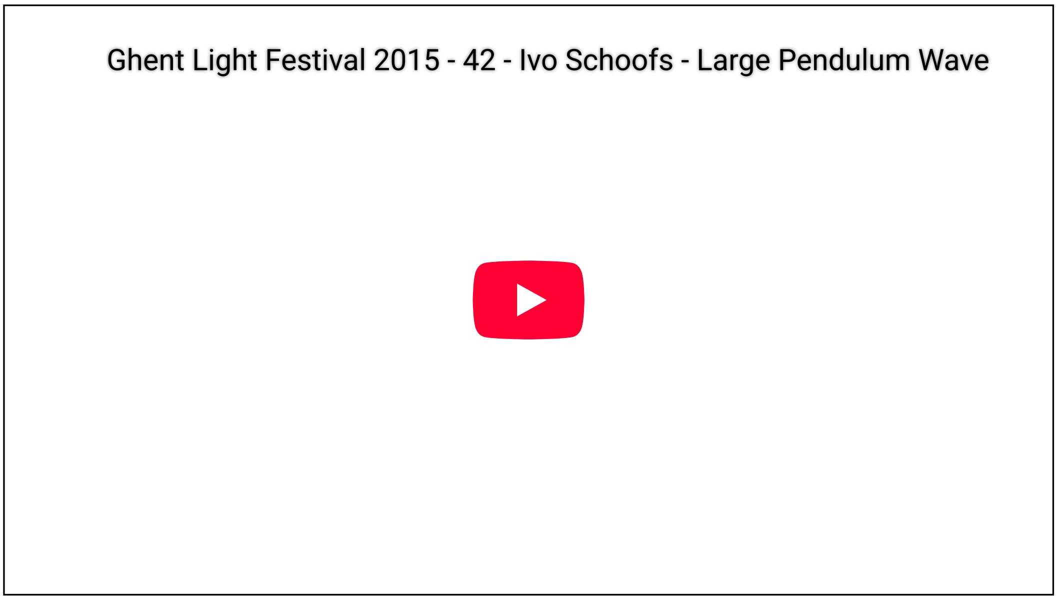
Can you create a display using pendulums in this way? What does it look like when filmed from different angles? You could recreate it using lights or even recreate it digitally! Put your own creative spin on this phenomenon!



Video 2. Ghent Light Festival 2015 - 42 - Ivo Schoofs - Large Pendulum Wave

More information for video 2

In this video, a visual demonstration of pendulum motion is shown. Set against a nighttime scene, the installation consists of a vertical array of glowing spheres suspended from a tripod structure. Each sphere functions as a pendulum, and while they are initially aligned vertically, they are set into motion simultaneously. Importantly, each pendulum has a slightly different length. The use of light, color, and atmospheric effects like fog enhances the visual impact of the demonstration.

As the pendulums begin to swing, their differing lengths result in variations in their individual periods—the time it takes each to complete a full back-and-forth swing. This difference causes the pendulums to gradually drift out of sync with one another, and then, remarkably, come back into near alignment at periodic intervals. This cyclical divergence and convergence of the pendulums create a series of shifting wave patterns. The spheres appear to form flowing arcs, spirals, and clusters. Longer pendulums swing more slowly than shorter ones. Because the spheres are arranged with incrementally increasing lengths, their oscillations gradually fall out of phase.

By observing how the pendulums interact over time, the viewer is expected to identify and describe the patterns that emerge from these interactions. It becomes evident that while each pendulum moves independently according to its own period, their collective behavior exhibits periodic alignment and misdisalignment, forming wave-like structures that visually demonstrate principles of harmonic motion and synchronization. The video demonstrates how a system of simple elements, when varied systematically, can produce complex, patterned behavior over time.

Student
view

Overview
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Videos 3 and 4. Pendulum waves form remarkable patterns.

More information

This video offers a captivating visual demonstration of pendulum wave mechanics. The video features 10 glowing pendulums, each with a slightly different length and captures how the pendulums synchronize and desynchronize to form intricate, evolving wave patterns. Set against a dark backdrop, the pendulums' neon glow highlights their rhythmic motion, transforming abstract physics principles into a mesmerizing display of order and chaos.

At the start, the pendulums are released simultaneously, swinging in near-unison. However, due to their carefully varied lengths, their oscillations gradually fall out of phase. This phase difference creates traveling waves, standing waves, and spiral-like patterns that shift and reform over time. The pendulums' glowing trails trace their paths, emphasizing the interplay between individual harmonic motion and collective wave behavior.

As the pendulums swing, their synchronized motion first aligns into smooth, undulating curves that sweep across the row. Over time, the differences in their oscillation frequencies cause the patterns to fragment into seemingly chaotic arrangements, only to reorganize into new symmetrical shapes. Moments of temporary synchronization—such as when all pendulums briefly align mid-swing—highlight the cyclical nature of periodic systems.

The experiment vividly illustrates simple harmonic motion, wave superposition, and frequency relationships. The varying pendulum lengths ensure each has a unique time period, emphasizing how small differences in oscillation rates lead to large-scale visual phenomena. The glowing design highlights concepts like amplitude, phase, and energy conservation.

By the end, the pendulums return to their initial synchronized state, completing a full cycle of motion. This seamless loop reinforces the predictability of physical laws while showcasing the beauty of mathematical patterns in nature.

Student view

Mass—spring system

Use the simulation in **Interactive 2** to investigate the motion of a mass–spring system. Select the 'Intro' tab. Add weights to the springs, then change the variables one at time and observe the motion.

Interactive 2. Investigating oscillations.

More information for interactive 2

An interactive simulation, Investigating oscillations, allows users to explore the behavior of masses oscillating on springs and investigate key concepts related to simple harmonic motion. The interface features two vertical springs, each with an adjustable spring constant, and a set of masses that can be attached to them. The simulation takes place under the influence of gravity, which can be adjusted using a slider that ranges from "None" to "Lots". Additional options include toggling equilibrium position markers, displaying natural length references, and adjusting damping.

In this particular setup, a 250 gram mass is attached to one spring, while a 50 gram mass is attached to the other. The equilibrium position is marked with a green dashed line, showing that the heavier mass stretches its spring further down than the lighter mass. This difference is due to the force of gravity acting on the masses, which causes the heavier mass to exert a greater force, leading to a greater displacement from the natural length of the spring.

When the masses are displaced and released, they oscillate up and down. One visible difference between the two is the frequency of oscillation. The 50 grams mass oscillates faster, completing one full cycle in less time than the 250 grams mass. Another aspect is amplitude. The amplitude of oscillation depends on how much the mass is initially displaced before being released. By stopping and restarting the oscillations using the red stop button, users can try to match the amplitudes of both masses to observe how they behave under similar conditions.

The simulation also allows users to experiment with different spring constants by adjusting sliders that range from "Small" to "Large". A stiffer spring (higher spring constant) results in less stretch under the same force and leads to faster oscillations. Conversely, a weaker spring allows for more displacement and slower oscillations.

The simulation provides a dynamic environment to visualize and understand the relationships between mass, spring constant, equilibrium position, oscillation frequency, and amplitude in simple harmonic motion. By manipulating various parameters, users can observe real-time changes and develop a deeper understanding of oscillatory motion.

- Which factors affect the time period? Do the factors increase or decrease the time period?
- What factors do not affect the time period? Are there any surprises?

Drag and drop the variables into the correct columns in **Interactive 3** to show whether they **do affect** or **do not affect** the time period.

Do affect *T*	Do not affect *T*

mass | amplitude | gravity | spring constant

Check

Interactive 3. Which Variables Affect the Time Period?

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Consider a mass moving horizontally on a frictionless surface attached to a spring (**Figure 3**). The spring exerts an elastic restoring force, F_{H}, on the mass (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)), towards the equilibrium position, where x is the displacement from that equilibrium.

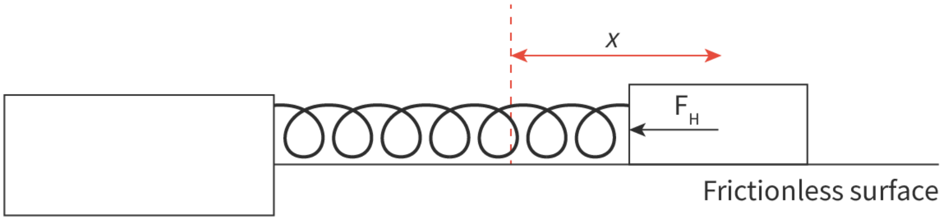

Figure 3. A mass on a spring, showing forces.

More information for figure 3

The diagram illustrates a mass attached to a spring on a horizontal frictionless surface. The spring is under tension, pulling the mass towards an equilibrium position. The displacement from the equilibrium, marked as 'x', is shown with a horizontal arrow pointing away from the equilibrium. The spring exerts an elastic restoring force labeled 'F_H', directed towards the left, opposing the displacement. The diagram clearly shows the spring, the direction of the forces, and the relevant notations for understanding the dynamics of the system.

[Generated by AI]

The restoring force provided by the spring is given by (subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)):

$$F_{\mathrm{H}} = -kx$$

where k is the spring constant of the spring.

This force provides the acceleration towards the equilibrium position, so from Newton's second law:

$$ma = -kx$$

The defining equation for SHM tells us that:

$$a = -\omega^2 x$$

Student view

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So, substituting this in the equation above gives:

$$m\omega^2 x = kx$$

The angular frequency is given by:

$$\omega = \frac{2\pi}{T}$$

Combining these equations and cancelling x gives:

$$\left(\frac{4\pi^2}{T^2}\right) m = k$$

Rearranging to make T the subject gives the equation for the time period of a mass–spring system in **Table 2**.

Table 2. Equation for the time period of a mass—spring system.

Equation	Symbols	Units
$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$	T = time period	seconds (s)
	m = mass	kilograms (kg)
	k = spring constant	newtons per metre (N m^{-1})

Worked example 3

A 230 g mass on a spring undergoes SHM with a frequency of 16 Hz. Calculate the spring constant.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Solution Steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$m = 230 \text{ g}$ $= 0.23 \text{ kg}$ $f = 16 \text{ Hz}$ $T = \frac{1}{f}$ $= \frac{1}{16}$ $= 0.0625 \text{ s}$
Step 2: Write out the equation and rearrange to make k the subject.	$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$ $k = \frac{4\pi^2 m}{T^2}$
Step 3: Substitute the values given.	$= \frac{(4\pi^2 \times 0.23)}{0.0625^2}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 2324.5 \text{ N m}^{-1}$ $= 2300 \text{ N m}^{-1}$ (2 s.f.)

Energy of SHM

As an object falls, its gravitational potential energy is transferred to kinetic energy (subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)). The same happens with a pendulum. When the pendulum is at the peak of its path (maximum displacement), all of its energy is gravitational potential energy. When the pendulum is at the equilibrium position, all of this energy has been transferred to kinetic energy.

For a horizontal mass–spring system, the potential energy is elastic potential energy, but the same transfer to kinetic energy occurs.

For a vertical mass–spring system, the potential energy is gravitational potential energy and elastic potential energy at the top, and elastic potential energy at the bottom. But the general principle still applies.

Student view

Overview
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In the absence of resistive forces, no energy is lost and therefore the total energy of the system remains constant.

Nature of Science

Aspect: Theories

Friction and drag are often ignored in physics theory. However, in the case of oscillations, friction and drag are often used extensively by engineers who need to design a product to meet certain specifications.

For example, when designing car suspension systems, automotive engineers do not want the car to continue vibrating endlessly every time the car hits a bump in the road — this would be both uncomfortable and impractical — so they design the suspension to use drag to take energy out of the oscillating system.

In a pendulum clock, air resistance has to be compensated for. Engineers need to understand exactly how much energy will be removed from the system by air resistance in order to *supply* the right amount of energy (with a battery or by winding) to keep the clock ticking.

Often, in practice, many theories and equations need to be combined in order to manufacture seemingly simple objects.

We can represent the energy transfers using the graph in **Figure 4**.

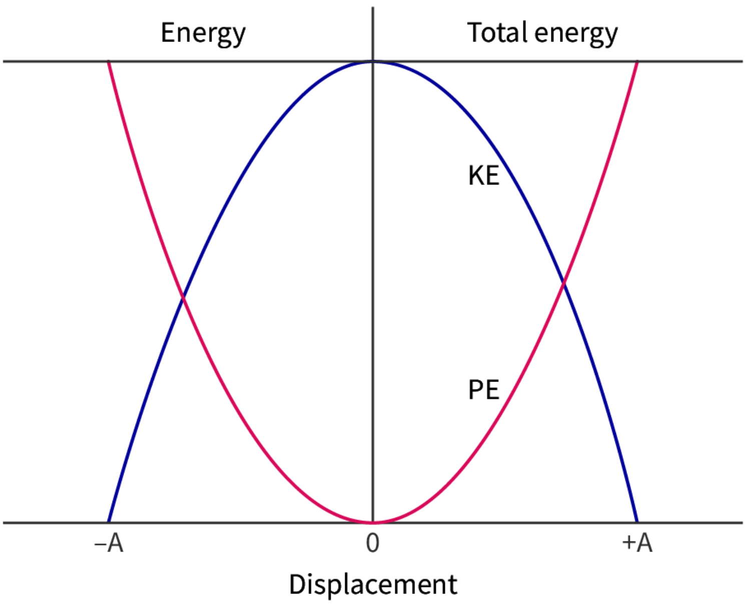

Figure 4. Energy—displacement graph.

Student view

More information for figure 4

This image is a graph illustrating the relationship between energy and displacement in a mass-spring system. The X-axis represents the displacement labeled from -A to +A, with 0 at the center. The Y-axis represents energy and total energy.

There are two main curves on the graph:

1. **Kinetic Energy (KE):** Represented by a blue curve that starts low at -A, peaks at 0, and decreases again at +A. At zero displacement, the kinetic energy is at its maximum.

2. **Potential Energy (PE):** Depicted by a red curve that does the opposite, starting high at both -A and +A and reaching a minimum at 0. This shows that the maximum potential energy occurs at maximum positive and negative displacements.

The graph effectively illustrates that as the displacement moves from one extreme to the other, kinetic energy and potential energy swap their magnitudes, maintaining a constant total energy, depicted by a horizontal line across the top.

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Work through the activity to investigate the energy in a mass–spring system.

Activity

- **IB learner profile attribute:**
    - Thinker
    - Reflective
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 15 minutes
- **Activity type:** Individual activity

What about the energy changes in a mass—spring system? Take a look at the simulation in **Interactive 4.**

Select the 'Energy' tab. Adjust the 'Mass' and 'Spring Constant 1' sliders into the middle. Then attach the block to the spring and press the red button at the top of the spring to stop it oscillating. Click on 'Slow' (bottom right) so you the spring oscillates

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slowly. Pull down the mass and let go and watch the motion and the energy changes. Kinetic energy is shown by 'KE' and potential energy is shown by 'PE_{grav}' and 'PE_{elas}' in the chart on the left.

Interactive 4. Mass—spring simulation.

Try and answer the following questions:

- When is kinetic energy greatest?
- When is potential energy greatest?
- What are the different types of potential energy?
- What simplifications are present in this model?
- What would be the differences in energy transfer if this was a horizontal mass—spring system?
- What are the differences, if any, from the simple pendulum?
- In practice, how would the amplitude of the oscillation change over time? How would this affect the energy transfers?

Record your observations and share them with a classmate.

5 section questions

Question 1

SL HL Difficulty:

A pendulum is experiencing SHM with an amplitude of oscillation of Y. It experiences an external resultant force that reduces the amplitude to 0.5Y. Which of the following has also changed?

1 Maximum kinetic energy ✓

2 Time period

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3 Frequency

4 Angular frequency

Explanation

If the amplitude is reduced, then the maximum amount of potential energy has decreased because the pendulum reaches a smaller maximum height. As all this potential energy is transferred to kinetic energy, the maximum kinetic energy must also have decreased.

Time period (and therefore frequency and angular frequency) is only dependent on length of the pendulum and gravity, neither of which has changed.

Question 2

SL HL Difficulty:

An accelerometer contains a mass of 18 g attached to a spring with a spring constant of 6.3 kN m^{-1}. Determine the frequency of its oscillations when undergoing SHM.

1 94 Hz ✓

2 11 Hz

3 0.092 Hz

4 0.011 Hz

Explanation

$$m = 18\ \mathrm{g} \\ = 0.018\ \mathrm{kg}$$

$$k = 6.3\ \mathrm{kN\ m^{-1}} \\ = 6300\ \mathrm{N\ m^{-1}}$$

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)} \\ = 2\pi\sqrt{\left(\frac{0.018}{6300}\right)} \\ = 0.01062$$

$$f = \frac{1}{T} \\ = \frac{1}{0.01062} \\ = 94.16\ \mathrm{Hz} \\ = 94\ \mathrm{Hz}\ (2\ \mathrm{s.f.})$$

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Question 3

SL HL Difficulty:

In which row (A, B, C or D) do all three quantities remain constant when a particle oscillates with SHM?

A	Time period	Displacement	Total energy
B	Speed	Kinetic energy	Time period
C	Amplitude	Total energy	Angular frequency
D	Acceleration	Angular frequency	Kinetic energy

1 C ✓

2 A

3 B

4 D

Explanation

In SHM, we assume no frictional forces, and so the amplitude of the oscillations stay constant. No energy is lost so total energy stays constant as well. The angular frequency is equal to $\frac{2\pi}{T}$, and the time period stays constant, so angular frequency must be constant too. Therefore, the answer is C.

Question 4

SL HL Difficulty:

A student measures the displacement of a simple pendulum every second. The results are shown in the table. Determine the time period of the oscillations.

Time (s)	Displacement (m)
0.0	0.0
1.0	0.38
2.0	0.71

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Time (s)	Displacement (m)
3.0	0.92
4.0	1.0
5.0	0.92
6.0	0.71
7.0	0.38
8.0	0

1 16 s ✓

2 8.0 s

3 4.0 s

4 32 s

Explanation

The amount of time taken to go from the equilibrium position (displacement = 0) back to the equilibrium position is 8 s. But this only takes into account the positive displacements, and therefore one side of the swing. This pattern is repeated on the other side of the swing until a full cycle is completed, and so the time period is twice as long: 2 × 8 s = 16 s

Question 5

SL HL Difficulty:

A student measures the displacement of a simple pendulum every second. The results are shown in the table. Calculate the length of the pendulum.

Time (s)	Displacement (cm)
0.0	0.0
1.0	5.0
2.0	8.7
3.0	10.0

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Time (s)	Displacement (cm)
4.0	8.7
5.0	5.0
6.0	0.0
7.0	−5.0
8.0	−8.7

1 36 m ✓

2 36 cm

3 19 cm

4 19 m

Explanation

The time period is twice the amount of time it takes to return to the equilibrium position (0.0):

$$\begin{aligned} T &= 2\times 6.0 \\ &= 12 \text{ s} \end{aligned}$$

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$

$$\begin{aligned} l &= \frac{T^2 g}{4\pi^2} \\ &= \frac{(12^2\times 9.8)}{4\pi^2} \\ &= 35.7 \text{ m} \\ &= 36 \text{ m (2 s.f.)} \end{aligned}$$

The unit for length is in metres. It does not matter which unit the displacement is stated in, because the time period is independent of amplitude and displacement. As long as time is in seconds, length is in metres.

C. Wave behaviour / C.1 Simple harmonic motion

Position and velocity (HL)

Student view

C.1.8: Phase angle (HL) C.1.9: Equations for simple harmonic motion (HL)

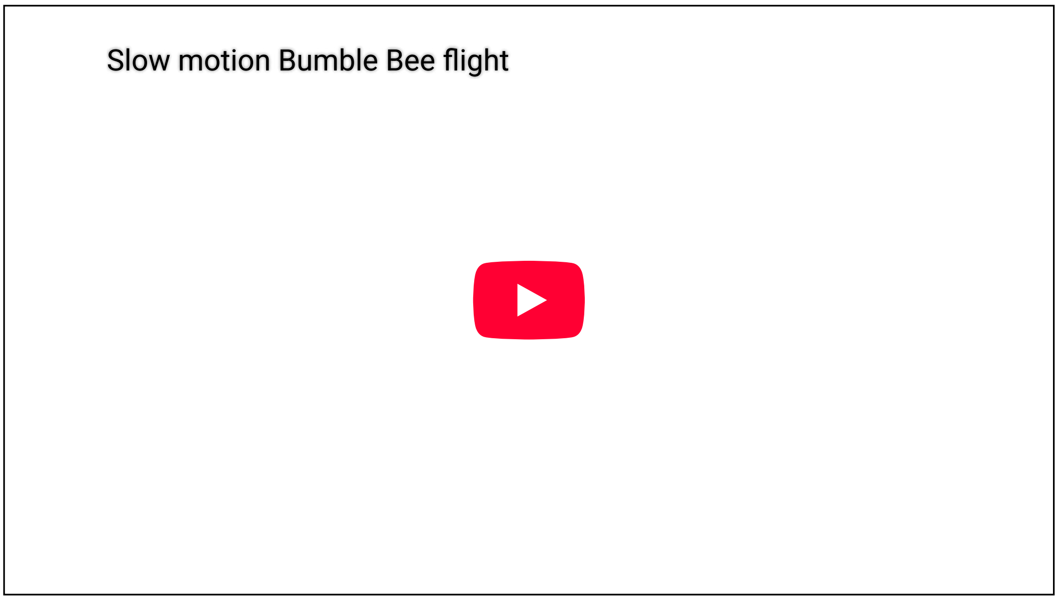
Higher level (HL)

Learning outcomes

At the end of this section you will be able to:

- Determine the velocity and displacement of a SHM oscillator at any point in its cycle.
- Describe the position of an oscillator in its cycle based on its phase angle in radians.

The wings of a bee as it is flying can be modelled using SHM. How fast do a bee’s wings move? Faster or slower than a child on a swing? What about the vibrations of a ruler that is vibrated on the edge of a table? **Video 1** shows the motion of a bee's wings in slow motion.



Video 1. A bee's wings in slow motion.

The useful thing about models like SHM is that they make things predictable and calculable. We can apply the laws of SHM to anything that fits the model, and predict velocity, position and acceleration at any point in a cycle.

Calculating displacement

A mass on a spring is oscillating up and down. What will a graph of displacement against time look like?

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The mass starts off at equilibrium. Sketch what you think the graph will look like. What will happen if you change the spring constant?

Use the simulation in **Interactive 1** to investigate the graph.

Interactive 1. Displacement—time graph for mass—spring system.

More information for interactive 1

An interactive titled, displacement—time graph for mass—spring system, models the motion of a mass-spring system, displaying a displacement-time graph that follows simple harmonic motion. Users can adjust the spring constant by using a slider which affects the oscillation frequency. The mass moves up and down, passing through an equilibrium position, with maximum displacement occurring at the peaks of the sine wave.

The interface includes a slider that ranges from 10 to 100, for modifying the spring constant and buttons for controlling the animation. As the mass oscillates, its position is plotted against time, revealing a sine curve. The graph's amplitude corresponds to the maximum displacement, while the frequency depends on the spring constant. Increasing the spring constant makes the mass oscillate more rapidly, whereas decreasing it results in slower oscillations.

The interactive helps users understand the mathematical representation of oscillatory motion. The equation $x = x_0 \sin wt$, describes how displacement varies with time, where x_0 is the amplitude and omega *w* is the angular frequency. The frequency of oscillation is directly related to the spring constant, showing that a stiffer spring leads to faster oscillations.

By exploring the interactive, users gain insight into key concepts such as equilibrium position, amplitude, and the relationship between force, displacement, and oscillation frequency. This allows learners to connect theoretical equations with real-world motion, reinforcing the principles of simple harmonic motion in mass-spring systems.

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What do you notice about the shape of the graph? It is a sine graph, but the sine of what?

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The position of the oscillating object in the cycle can be given as an angle in radians (section C.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/)). The angular frequency, ω, tells us the amount of radians covered per second by the oscillator:

ω = angular displacement per second in radians per second

So if we multiply this by the time that has elapsed, we will get the total amount of radians covered in that time:

ωt = angular displacement in radians

We know the displacement in **Interactive 1** produces a sine graph, so taking the sine of this angle will give us the correct shape of the graph:

$\sin \omega t$ = description of where the oscillator is at in its displacement cycle

Finally, to get the equation that describes the variation of the displacement of the oscillating mass with time, we must multiply the sin function above by the amplitude x_0 of the oscillation. So the displacement at time t is given by:

$$x = x_0 \sin \omega t$$

The above equation shows that, at time $t = 0$, the displacement is $x = 0$, since $\sin 0 = 0$; this is expected, since the mass started oscillating from the equilibrium position (i.e. $x = 0$). The equation also shows that, when the sine function has its maximum value (i.e. 1), the displacement is $x = x_0$, which is the amplitude of the oscillation (i.e. the maximum displacement).

Study skills

When doing SHM calculations, make sure your calculator is in radians mode. You will need it in degrees mode for other calculations, such as kinematics calculations, so make sure you can switch quickly between modes.

What is the difference between the graph in **Interactive 1** and the graph in **Interactive 2**?

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Interactive 2. Displacement—time graph for a mass—spring system.

More information for interactive 2

The interactivity is titled "Displacement—time graph for mass—spring system undergoing simple harmonic motion." It demonstrates the oscillatory motion of a mass attached to a vertical spring, showing its displacement over time. The displacement-time graph, displayed alongside the physical setup, takes the form of a sinusoidal wave. The equilibrium position is marked as the reference point, with maximum upward and downward displacements indicated.

Users can manipulate the spring constant using a slider, adjusting the stiffness of the spring. This affects the frequency of oscillation, with higher values leading to faster oscillations. The play and pause buttons allow users to observe the motion dynamically, while a reset button restores initial conditions.

The displacement-time graph helps visualize the periodic nature of the motion, reinforcing the mathematical representation of simple harmonic motion. The sinusoidal nature of the graph reflects the function:

$x = x_0 \sin \omega t$,

where x_0 is the amplitude, ω is the angular frequency and t is the time. At $t = 0$, the displacement is zero, meaning the motion starts from the equilibrium position.

The interactive encourages users to explore the relationship between displacement and time and understand how changes in system parameters influence oscillatory behavior.

The only difference is the position the mass has at time $t = 0$. This changes the equation slightly.

The graph is displaced by a quarter of a cycle. This is known as a phase angle. A simple way to visualise the phase angle of a cycle is to consider a clock face, **Figure 1**. The start and end of the cycle is the 12 o'clock position. As the hands sweep around, the phase angle increases by $\frac{\pi}{2}$ rad for each quarter of the cycle. The phase angle in this case is a quarter of a cycle, or $\frac{\pi}{2}$ rad.

Student
view

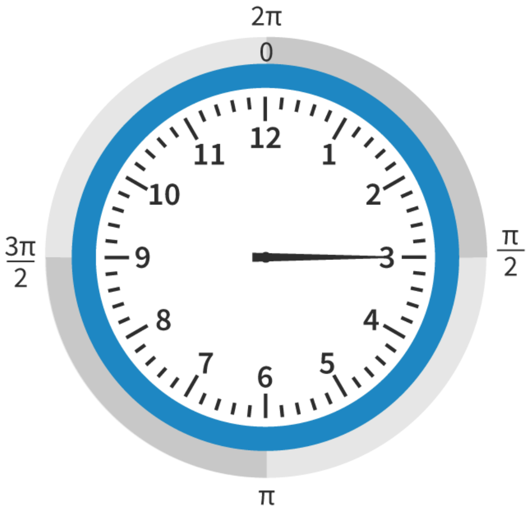

Figure 1. Phase angles of a cycle in radians.

More information for figure 1

The image depicts a clock face used to illustrate phase angles in radians. The clock has traditional hour marks from 1 to 12. Around the clock, there are additional labels for radians: 0 at the 12 o'clock position, π/2 at the 3 o'clock position, π at the 6 o'clock position, 3π/2 at the 9 o'clock position, and 2π again at the 12 o'clock position. The hour hand points to 3, representing a phase angle of π/2 radians, indicating a quarter cycle of displacement.

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We can represent this phase angle using φ, so the equation to find the displacement becomes the equation in **Table 1.**

Table 1. Equation for displacement in SHM.

Equation	Symbols	Units
$x = x_0 \sin(\omega t + \varphi)$	x = displacement at time	metres (m)
	x_0 = amplitude	metres (m)
	ω = angular frequency	radians per second (rad s^{-1})
	t = time	seconds (s)
	φ = phase angle from sine curve	radians (rad)

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You can visualise the phase angle using the simulation in **Interactive 3**. Change the phase angle and see what happens to the graph.

Interactive 3. What happens to the graph when phase angle is changed?

More information for interactive 3

An interactive simulation titled What happens to the graph when the phase angle is changed?, explores the effect of phase shift on the displacement-time graph of two oscillating mass-spring systems. The simulation helps users understand how phase differences influence oscillatory motion by adjusting the timing of two sine waves.

The interface displays two vertically hanging springs, each with an attached mass. The displacement of each mass is plotted as a function of time, creating sinusoidal graphs. Users can adjust the phase shift using a slider, which moves one wave horizontally without changing its amplitude or frequency. The slider ranges from 0 to $\frac{3\pi}{2}$, with intervals of $\frac{3\pi}{2}$.

When the phase shift is zero, both masses oscillate in sync, reaching their highest and lowest points simultaneously. As the phase shift increases, one oscillation lags behind the other. At a phase shift of π or 180°, the two masses move completely out of phase, meaning that when one is at maximum displacement, the other is at its minimum.

This simulation helps users understand the concept of phase difference in oscillatory motion. It visually demonstrates how phase shifts impact wave behavior without changing other properties like amplitude or period. By adjusting the phase shift, users can explore constructive and destructive interference in simple harmonic motion and see how oscillating systems can be synchronized or offset. By engaging with the simulation, users can relate phase differences to real-world oscillatory systems such as coupled pendulums, sound waves, or alternating current circuits.

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Worked example 1

A crate of bricks hangs from a crane on a building site and swings with SHM with an angular frequency of 0.73 rad s^{-1}. At time t = 0, the displacement is at its maximum, 14 m. Determine the displacement at time t = 15 s?

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$\omega = 0.73\ \text{rad s}^{-1}$ $x_0 = 14\ \text{m}$ $t = 15\ \text{s}$ At time $t = 0$, the displacement is at its maximum (positive direction). The phase angle from sine is a c cycle: $\varphi = \frac{\pi}{2}\ \text{rad}$
Step 2: Write out the equation.	$x = x_0 \sin(\omega t + \varphi)$
Step 3: Substitute the values given.	$= 14\times \sin\left(0.73\times 15 + \frac{\pi}{2}\right)$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= -0.6378\ \text{m} = -0.64\ \text{m}\ (2\ \text{s.f.})$ The answer is negative because it is in the opposite direction to the displacement at time $t = 0$ (which been given as a positive number)

This can apply to both pendulum motion and mass-spring systems.

Worked example 2

After the initial jump, a bungee jumper oscillates up and down with SHM like a mass on a spring. Their displacement is given by the equation:

$$x = 5.2 \sin(6.5t)$$

There is no phase angle between this motion and a sine graph. Deduce the time period of the motion.

Student
view

The equation to describe the displacement is:

$$x = x_0 \sin(\omega t + \varphi)$$

The phase angleis given as:

$$\varphi = 0$$

The equation in the question is:

$$x = 5.2 \sin(6.5t)$$

So:

$$x_0 = 5.2 \text{ m}$$

$$\omega = 6.5 \text{ rads s}^{-1}$$

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ &= \frac{2\pi}{6.5} \\ &= 0.967 \text{ s} \\ &= 0.97 \text{ s (2 s.f.)} \end{aligned}$$

Calculating velocity

We have a way to calculate the displacement of an object exhibiting SHM at a given time. But what about velocity?

Activity

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 15 minutes
- **Activity type:** Pair activity

A displacement—time graph can be drawn for an object undergoing SHM. In this activity, you are going to think about other motion graphs for SHM. Work in pairs.

1. Individually, sketch graphs of what you think the velocity—time and acceleration—time graphs will look like for SHM.
2. As a pair, look at the motion graphs in subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/).
3. Reflect on your initial graphs, comparing and contrasting them. Would you make any changes to them?
4. Discuss and decide what you think the graphs will look like.

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5. Repeat the process for a graph of velocity against displacement.

Extension

Can you derive equations to describe these graphs?

You may have identified from the Activity that velocity can be found by calculating the gradient of the displacement–time graph. If the displacement–time graph is a sine curve, the velocity–time graph will be a cosine curve.

Look at the displacement–time graph in **Figure 2**. It is a sine curve. Some tangents are drawn on it, numbered 1 to 5:

- Tangent at 1: maximum gradient, positive direction
- Tangent at 2: gradient = 0
- Tangent at 3: maximum gradient, negative direction
- Tangent at 4: gradient = 0
- Tangent at 5: maximum gradient, positive direction

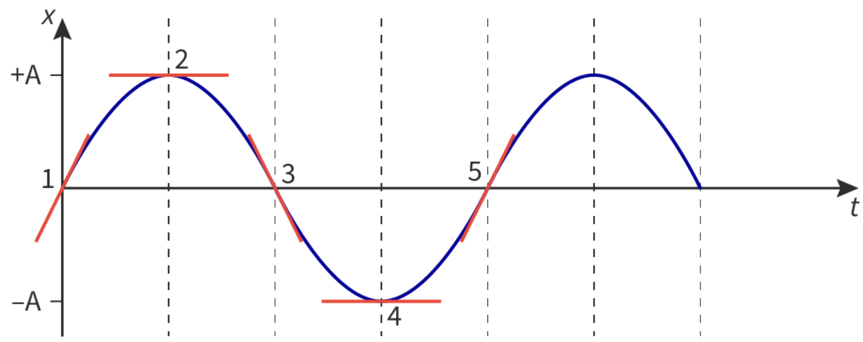

Figure 2. A sine graph with tangents.

More information for figure 2

The image displays a sine wave graph with several tangent lines. The X-axis represents time (t) with intervals marked at five equidistant points. The Y-axis represents a variable (x) ranging from -A to +A. A smooth sine wave curves above and below the horizontal axis with peaks and troughs. Tangent lines are drawn at five points: near the initial upward curve, at the first peak, approaching the mid-descent, at the trough, and during the next rise. The visual indicates how tangents lie against different segments of the sine wave, illustrating the gradient at specific points.

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Drawing a graph of these gradients will form the shape of a cosine curve. Calculus is outside of the scope of this course, but the gradient of the displacement–time graph, $\frac{dx}{dt}$ gives the equation for velocity in **Table 2**.

Table 2. Equation for velocity in SHM.

Equation	Symbols	Units
$v = \omega x_0 \cos(\omega t + \varphi)$	v = velocity at time t	metres per second (m s^{-1})
	ω = angular frequency	radians per second (rad s^{-1})
	x_0 = amplitude	metres (m)
	t = time	seconds (s)
	φ = phase angle from cosine curve	radians (rad)

Study skills

It is helpful to remember that the maximum magnitude that cos(value) or sin(value) can have is 1, and the minimum magnitude is zero.

To find the maximum velocity, v_0, we substitute $cos(\omega t + \varphi)$ with its maximum value, 1, giving:

$$v_0 = \omega x_0$$

To find the minimum velocity, v_{min}, we substitute $cos(\omega t + \varphi)$ with its minimum value, 0, giving:

$$v_{\text{min}} = \omega x_0 \times 0 = 0$$

This works with the displacement equations too.

We know that $a = -\omega^2 x$, so the acceleration–time graph will have the inverse shape of the displacement–time graph. If the displacement–time graph is a sine curve, the acceleration–time graph will be a negative sine curve.

The graphs you drew in the Activity may look, therefore, like the graphs in **Figure 3**. The starting position is when time, t = 0 s.

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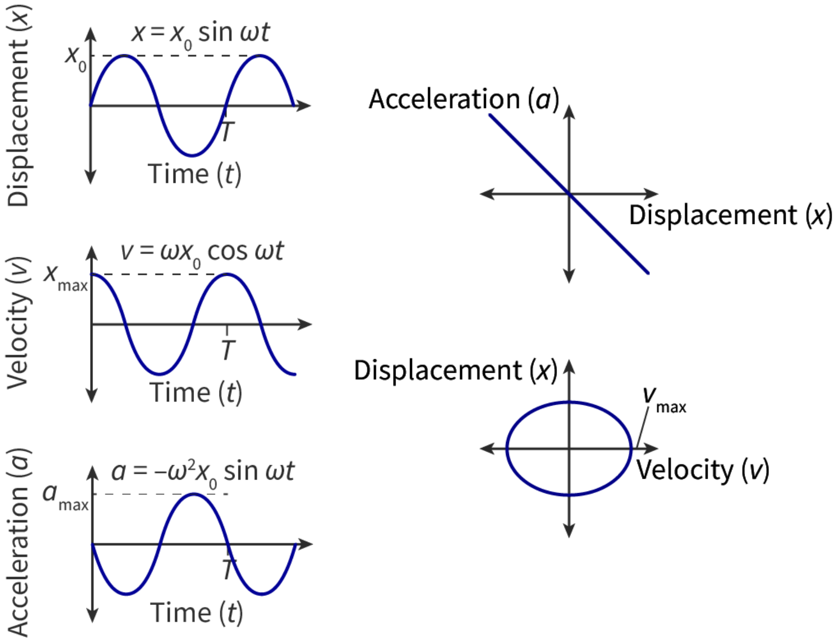

Figure 3. Displacement, velocity and acceleration graphs.

More information for figure 3

The image contains three graphs representing the characteristics of simple harmonic motion and two additional diagrams.

1. The first graph shows displacement (x) as a function of time (t). The Y-axis represents displacement labeled as 'Displacement (x)', while the X-axis represents time labeled as 'Time (t)'. The curve is a sinusoidal wave described by the equation $x = x_0 \sin(\omega t)$, with peaks at x_0 and zero crossings at regular intervals labeled T.
2. The second graph depicts velocity (v) as a function of time (t). The Y-axis is labeled 'Velocity (v)', and the X-axis is 'Time (t)'. The graph follows a cosine wave pattern described by $v = \omega x_0 \cos(\omega t)$, with maxima at x_max.
3. The third graph illustrates acceleration (a) versus time (t). The Y-axis is labeled 'Acceleration (a)', and the X-axis is 'Time (t)'. It features another sinusoidal wave described by $a = -\omega^2 x_0 \sin(\omega t)$, with maxima at a_max.

On the right, there are two additional diagrams:

1. An angled straight line graph representing acceleration (a) as a function of displacement (x).
2. An elliptical path showing the relationship between displacement (x) and velocity (v), with axes labeled 'Displacement (x)' and 'Velocity (v_max)'.

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Drag and drop 'displacement, x =', 'velocity, v =' and 'acceleration, a =' into the correct places in **Interactive 4** to show what the equations for simple harmonic motion are now.

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	$= x_0 \sin \omega t$	(1)
	$= \omega x_0 \cos \omega t$	(2)
	$= -\omega^2 x$	(3)

acceleration, a

displacement, x

velocity, v

Check

Interactive 4. Complete the Equations for Simple Harmonic Motion.

We can use equation 1 in **Interactive 4** to derive one more equation for velocity (the derivation is not required by the IB syllabus):

So our simple harmonic equations are now:

$$\text{displacement, } x = x_0 \sin \omega t \quad (1)$$

$$\text{velocity, } v = \omega x_0 \cos \omega t \quad (2)$$

$$\text{acceleration, } a = -\omega^2 x_0 \sin \omega t \quad (3)$$

Rearranging and squaring equation 1 gives:

$$\left(\frac{x}{x_0}\right)^2 = \sin^2 \omega t \quad (4)$$

You may know already that $\sin^2 \theta + \cos^2 \theta = 1$ and therefore $\cos^2 \omega t = 1 - \sin^2 \omega t$. Thus:

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$$\cos\ \omega t = \pm\sqrt{1-\sin^2 \omega t}$$

Note that there is a plus or minus sign, because either would give the same answer.

Note that these trigonometric relationships are outside of the scope of IB Physics.

Combining this with equation 2 above, gives:

$$\frac{v}{\omega x_0} = \pm\sqrt{1-\sin^2 \omega t}$$

Substituting in from equation 4 above gives:

$$\frac{v}{\omega x_0} = \pm\sqrt{1-\left(\frac{x}{x_0}\right)^2}$$

Rearranging this gives:

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

Rearranging this equation gives the equation for velocity shown in **Table 3**.

Table 3. Equation for velocity in SHM.

Equation	Symbols	Units
$v = \pm\omega\sqrt{({x_0}^2{-}x^2)}$	v = velocity at time t	metres per second (m s^{-1})
	ω = angular frequency	radians per second (rad s^{-1})
	x_0 = amplitude	metres (m)
	x = displacement at time t	metres (m)

Worked example 3

An object oscillates with a frequency of 65 Hz and amplitude 250 mm. Calculate the speed of the object when its displacement is 82 mm.

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Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$f = 65\text{ Hz}$ $x_0 = 250\text{ mm} = 0.25\text{ m}$ $x = 82\text{ mm} = 0.082\text{ m}$
Step 2: Write out the equation.	$v = \pm\omega\sqrt{(x_0{}^2 - x^2)} = \pm 2\pi f\sqrt{(x_0{}^2 - x^2)}$
Step 3: Substitute the values given.	$= \pm 2\pi \times 65 \times \sqrt{(0.25^2 - 0.082^2)}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding	$= 96\text{ m s}^{-1}\ (2\text{ s.f.})$ Note: the question asks for speed s direction is irrelevant

Work through the activity to check your understanding of displacement and velocity in SHM.

Activity

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills — Experimenting with new strategies for learning
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

Using someone's phone, download a physics lab app such as the Phyphox app (https://phyphox.org/). This uses the internal sensors in your phone to make measurements.

First, use the 'Acceleration (without g)' function. Ensure that you are in a space large enough, hold the phone tightly in your hand and swing it freely.

- Use the graph of acceleration to sketch graphs of velocity and position.

Student
view

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- Using SHM equations, calculate the maximum speed that your phone was travelling.

Next, use the 'pendulum' function. Securely place the phone inside a bag or a case. Attach the handles of the bag or case to a length of suitable string or rope. Ensure that you are in a space large enough, swing the bag/case gently like a pendulum.

- Using just the 'results' tab, can you calculate *g*? Does your value agree with the '*g*' tab?
- What errors are inherent in this experiment? What uncertainties are there? How could this experiment be improved to get a more accurate value for *g*?

5 section questions

Question 1

HL Difficulty:

A particle oscillates in simple harmonic motion with a period of 0.50 seconds and an amplitude of 2.0 metres. What is the maximum speed of the particle?

1 25 m s^{-1} ✓

2 1.0 m s^{-1}

3 8.0 m s^{-1}

4 13 m s^{-1}

Explanation

$$T = 0.50\text{ s}$$

$$x_0 = 2.0\text{ m}$$

$$v = \omega x_0 \cos(\omega t + \varphi)$$

The maximum value that $\cos(\omega t + \varphi)$ can have is 1, which gives a maximum value for v, so:

$$v = \omega x_0 \times 1$$

$$\begin{aligned}\omega &= \frac{2\pi}{T} \\ &= \frac{2\pi}{0.50} \\ &= 4\pi\end{aligned}$$

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Substituting for ω:

$$\begin{aligned} v &= \omega x_0 \times 1 \\ &= 4\pi \times x_0 \times 1 \\ &= 4\pi \times 2.0 \times 1 \\ &= 25.1 \text{ m s}^{-1} \\ &= 25 \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

Question 2

HL Difficulty:

Two particles oscillating with SHM are $\frac{\pi}{2}$ rad out of phase. For what percentage of the time are their displacements in the same direction?

1 50.0% ✓

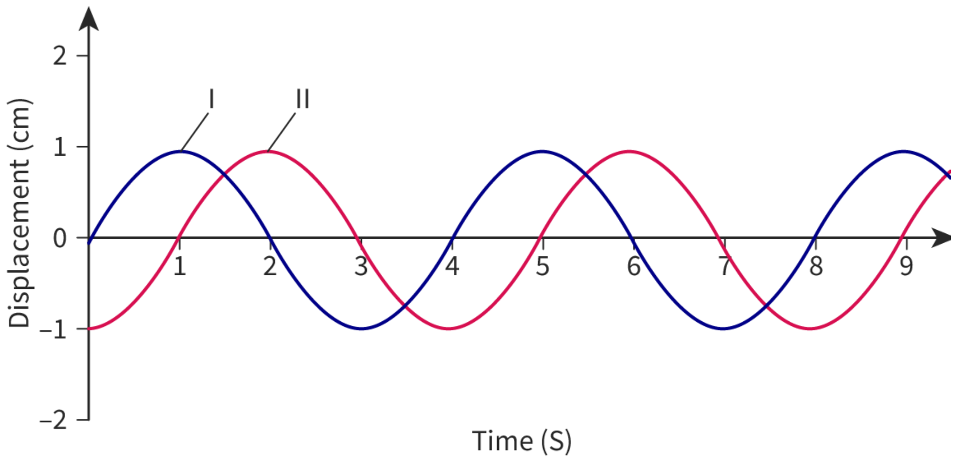
2 25.0%

3 75.0%

4 62.5%

Explanation

The graph below shows both displacement graphs. Look at where in the cycles they are on the same side of the *x*-axis. A phase angle of $\frac{\pi}{2}$ rad is a quarter cycle.



More information

Student view

The period is 4 s, so if you calculate the percentage for 4 s, this will be representative of the whole time of oscillation.

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Between 0 s and 1 s, they are on opposite sides.
Between 1 s and 2 s, they are on the same side.
Between 2 s and 3 s, they are on opposite sides.
Between 3 s and 4 s they are on the same side.

They are on the same side of the *x*-axis for 2 of the 4 seconds — 50%.

Question 3

HL Difficulty:

A mass on a spring oscillates with SHM with a velocity given by:

$v = 0.46 \cos(3.1t)$

Deduce the amplitude of the oscillations.

1 15 cm ✓

2 0.46 cm

3 3.1 cm

4 6.7 cm

Explanation

Compare the equation to one you know for velocity:

$$v = 0.46 \cos(3.1t)$$

$$v = \omega x_0 \cos(\omega t + \varphi)$$

From this, you can deduce:

$$\varphi = 0$$

$$\omega = 3.1$$

$$\omega x_0 = 0.46$$

$$3.1 x_0 = 0.46$$

Student
view

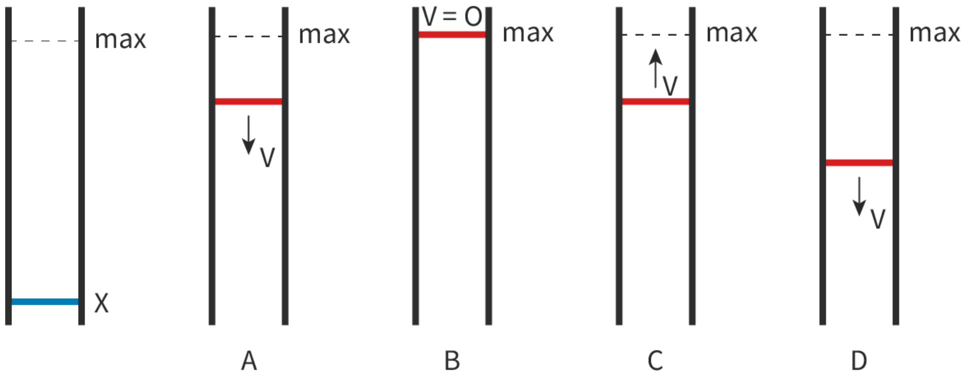
$$x_0 = \frac{0.46}{3.1}$$
$$= 0.148 \text{ m}$$
$$= 0.15 \text{ m (2 s.f.)}$$
$$= 15 \text{ cm (2 s.f.)}$$

Question 4

HL Difficulty:

A fluid oscillates up and down in a cylinder and exhibits SHM. At a given time, the fluid is at its lowest point, X, as shown in blue in the first diagram.

Which diagram corresponds to a phase angle from X of $\frac{2}{3}\pi$ radians? The velocity (if any) is shown by the red arrows.



More information

1 C ✓

2 B

3 A

4 D

Explanation

$\frac{2}{3}\pi$ radians phase angleis equal to $\frac{1}{3}$ of a cycle (2π radians is a full cycle).

X is the lowest point (maximum displacement downwards) so after $\frac{1}{4}$ of a cycle, the water level will be at the equilibrium position (the centre of the tube) moving upwards.

After $\frac{1}{2}$ of a cycle, the water level will be at maximum displacement in the other direction, at the top (shown by B).

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So after $\frac{1}{3}$ of a cycle, it will be moving up towards the 'max' line, but will not have got there yet. So the answer must be C.

A gives the situation after $\frac{2}{3}$ of a cycle $\left(\frac{4}{3}\pi \text{ radians phase angle}\right)$.

B gives the situation after $\frac{1}{2}$ of a cycle (π radians phase angle).

D gives the situation after $\frac{3}{4}$ of a cycle $\left(\frac{3}{2}\pi \text{ radians phase angle}\right)$.

Question 5

HL Difficulty:

A spring with a spring constant of 150 N m^{-1} is attached to a block with a mass of 2.0 kg. The block is pulled 12 cm to the right of its equilibrium position and released. Calculate the magnitude of the velocity of the block when it is 3.0 cm from equilibrium.

1 1.0 m s^{-1} ✓

2 0.12 m s^{-1}

3 17 m s^{-1}

4 0.37 m s^{-1}

Explanation

$m = 2.0 \text{ kg}$

$k = 150 \text{ N m}^{-1}$

$$\begin{aligned} x_0 &= 12 \text{ cm} \\ &= 0.12 \text{ m} \end{aligned}$$

$$\begin{aligned} x &= 3.0 \text{ cm} \\ &= 0.03 \text{ m} \end{aligned}$$

$v = \pm\omega\sqrt{({x_0}^2 - x^2)}$

Calculate the time period:

$$\begin{aligned} T &= 2\pi\sqrt{\left(\frac{m}{k}\right)} \\ &= 2\pi\sqrt{\left(\frac{2.0}{150}\right)} \end{aligned}$$

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Calculate the angular frequency:

$$\begin{aligned}\omega &= \frac{2\pi}{T} \\ &= \frac{2\pi}{\left[2\pi\sqrt{\left(\frac{2.0}{150}\right)}\right]} \\ &= \sqrt{75}\end{aligned}$$

Substituting for ω gives:

$$\begin{aligned}v &= \pm\sqrt{75}\times\sqrt{(0.12^2 - 0.03^2)} \\ &= 1.006\ \mathrm{m\ s^{-1}} \\ &= 1.0\ \mathrm{m\ s^{-1}}\ (2\ \mathrm{s.f.})\end{aligned}$$

C. Wave behaviour / C.1 Simple harmonic motion

Energy (HL)

C.1.9: Equations for simple harmonic motion (HL)

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/energy-hl-id-44902/print/) Assign

Higher level (HL)

Learning outcomes

At the end of this section you will be able to:

- Determine the potential energy and kinetic energy of an oscillator at a given point in a SHM cycle.
- Describe quantitatively the energy transfers during SHM.

In the The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/), a car suspension is used as an example of SHM. But this is not what you experience in a car – you do not bounce up and down constantly. That would be really annoying (**Video 1**)!

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Video 1. Not a Comfortable Way to Travel!

More information for video 1

The animation shows a car driving on a rough, uneven road, causing the wheel and suspension system to oscillate up and down. The movement demonstrates the concept of simple harmonic motion in car suspensions. However, unlike pure simple harmonic motion, real car suspensions include damping, which helps absorb shocks and prevents continuous bouncing. The animation highlights how excessive oscillations would make for an uncomfortable ride and illustrates the importance of damping in stabilizing the vehicle.

Engineers use damping (see subtopic C.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43788/)) in cars to remove energy from the system and bring you back to equilibrium as quickly as possible.

Making connections

How does damping affect periodic motion? This question is answered in subtopic C.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43788/).

Before engineers can remove energy from the system, they need to know the energy at each point in the cycle.

Student view

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Energy transfers

There is a periodic interchange between kinetic energy and potential energy in a simple harmonic motion system (see section C.1.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-pendulums-and-mass-spring-systems-id-44870/)) (**Figure 1**).

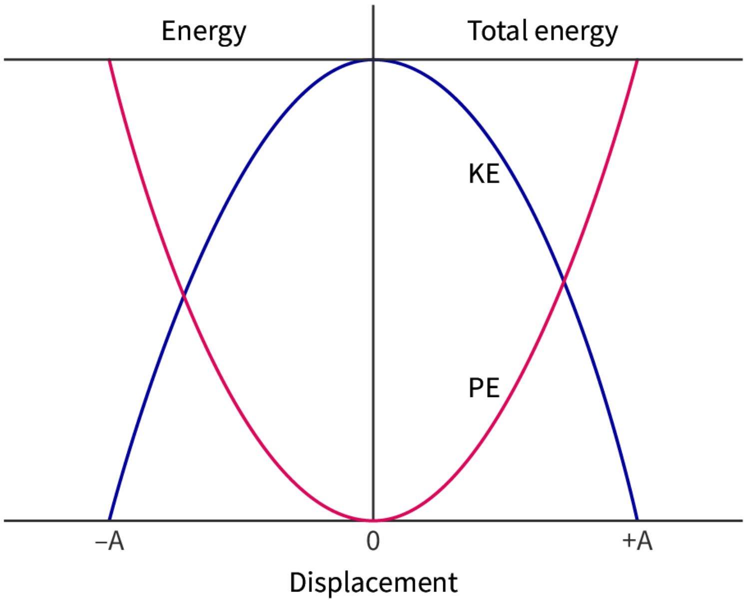

Figure 1. Potential energy and kinetic energy in SHM.

More information for figure 1

The graph displays the relationship between energy and displacement in a simple harmonic motion (SHM) system. It features two main curves on a 2D plane. The X-axis, labeled 'Displacement,' ranges from -A to +A, with 0 at the center, indicating the equilibrium position. The Y-axis, labeled as 'Energy,' is not numerically scaled but represents relative energy values.

The first curve, representing Kinetic Energy (KE), is a blue parabola opening downward and centered at the origin where displacement is zero. This indicates that kinetic energy is highest at the midpoint of the motion.

The second curve, representing Potential Energy (PE), is a red parabola opening upward, centered along the zero displacement line. Its peak is at the maximum displacements of -A and +A, showing that potential energy is highest at these points and lowest at the midpoint.

Both curves are symmetrical, and their intersection point at the origin shows where kinetic and potential energies are equal. The total energy remains constant throughout and can be imagined as a horizontal line above the highest energy points of both kinetic and potential energies.

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An adult on the Nevis Swing (see section C.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/)) will almost certainly be going faster and have more mass than a child on a playground swing, so clearly there is a difference in their energies.

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Looking at the graph in **Figure 1**, consider the total energy at 0 displacement:

$$E_{\text{total}} = E_{\text{k}}$$

To calculate the total energy, we calculate the maximum kinetic energy. We know that (subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)):

$$E_{\text{k}} = \frac{1}{2}mv^2$$

And so E_{k} has a maximum value when the velocity is maximum (assuming a constant mass).

During SHM, velocity is given by (section C.1.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/position-and-velocity-hl-id-44871/)):

$$v = \omega x_0 \cos(\omega t + \varphi)$$

The maximum value that v can have will be when $cos(\omega t + \varphi) = 1$, so:

$$v_{\text{max}} = \omega x_0$$

$$\begin{aligned} E_{\text{T}} &= E_{\text{k max}} \\ &= \frac{1}{2}m{v_{\text{max}}}^2 \end{aligned}$$

Combining this with the equation for kinetic energy gives the equation for total energy in **Table 1**.

Table 1. Equation for total energy.

Equation	Symbols	Units
$E_{\text{T}} = \frac{1}{2}m\omega^2{x_0}^2$	E_{T} = total energy	joules (J)
	m = mass	kilograms (kg)
	ω = angular frequency	radians per second (rad s^{-1})
	x_0 = amplitude	metres (m)

The potential energy in the system is equal to the total energy when $x = x_o$, so we can interchange E_{T} with E_{P}, and x_0 with x, giving the equation for potential energy in **Table 2**.

Student view

Table 2. Equation for potential energy.

Equation	Symbols	Units
$E_{\mathrm{p}} = \frac{1}{2} m\omega^2 x^2$	E_{p} = potential energy	joules (J)
	m = mass	kilograms (kg)
	ω = angular frequency	radians per second (rad s^{-1})
	x = displacement	metres (m)

Worked example 1

After their jump from a bridge, a bungee jumper is swinging from a rope of length 22 m. Their motion is that of a simple pendulum undergoing SHM. The mass of the bungee jumper is 87 kg.

(a) Calculate the frequency of the swing.

(b) Calculate their potential energy at a displacement of 2.5 m.

$l = 22 \text{ m}$

$m = 87 \text{ kg}$

$x = 2.5 \text{ m}$

(a)
The bungee jumper is acting as a pendulum:

$$\begin{aligned} T &= 2\pi\sqrt{\left(\frac{l}{g}\right)} \\ &= 2\pi \times \sqrt{\left(\frac{22}{9.8}\right)} \\ &= 9.41 \text{ s} \end{aligned}$$

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{9.41} \\ &= 0.106 \text{ Hz} \\ &= 0.11 \text{ Hz (2 s.f.)} \end{aligned}$$

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(b)

$$\begin{aligned}\omega &= 2\pi f\\ &= 2\pi \times 0.11\\ &= 0.69\ \text{rad s}^{-1}\end{aligned}$$

$$\begin{aligned}E_\text{p} &= \frac{1}{2}m\omega^2x^2\\ &= \frac{1}{2}\times 87\times 0.69^2\times 2.5^2\\ &= 129.4\ \text{J}\\ &= 130\ \text{J}\ (2\ \text{s.f.})\end{aligned}$$

Nature of Science

Aspect: Observations

We can also see SHM on an atomic scale. Increasing the energy of the oscillations of molecules increases the energy stored by the molecules.

An example of this is the greenhouse effect (see subtopic B.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43134/)). Carbon dioxide (CO_2) absorbs energy at a particular (infrared) frequency because this frequency is the same as the frequency of oscillation of a molecule of CO_2. The CO_2 in the air absorbs the radiation coming from the Earth at this frequency, the energy of the molecules' SHM increases, and this energy is stored in the atmosphere.

Graphing the energy

Look at the pendulum simulation in **Interactive 1**. Select the 'Energy' tab, then start the pendulum swinging. Look at the graph of the energies on the left-hand side.

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Interactive 1. Pendulum simulation.

More information for interactive 1

The interactive simulation titled, Pendulum simulation, models the motion of a simple pendulum, allowing users to explore the factors influencing its period of oscillation. The pendulum consists of a mass attached to a string, suspended from a fixed point, and swings back and forth under the influence of gravity. The interface provides various controls to manipulate parameters such as length, mass, gravity, and friction. Additionally, tools like a ruler, stopwatch, and period trace can be toggled on to aid in measurements and observations.

The length of the pendulum can be adjusted between 0.1 meter and 1 meter, affecting the arc of motion and the time it takes to complete a full swing. The mass of the pendulum, ranging from 0.1 kilogram to 1.5 kilogram, can also be modified, though it does not influence the period of oscillation. The gravity setting allows users to select different environments, including Earth and other celestial bodies, or adjust the strength of gravity manually. Friction can be varied from none to a significant amount, demonstrating how air resistance or damping affects the pendulum's motion by gradually reducing its amplitude over time.

Users can initiate motion by pulling the pendulum to an angle and releasing it. The normal and slow-motion playback options help in closely analyzing the oscillations. The period of one oscillation can be measured by enabling the stopwatch feature displayed on the box at the bottom left. Period of oscillation is the time taken for one complete oscillation. Start the stopwatch, when the blue line starts from the equilibrium position and stop the stopwatch when the blue line ends at the equilibrium position, completing one oscillation. Observing the pendulum's movement while systematically changing variables helps reveal key principles of simple harmonic motion.

It is observed that the period of the pendulum depends on its length and the gravitational force, but remains independent of the mass. Increasing the length results in a longer period, while increasing gravitational acceleration shortens the period. These observations align with the theoretical equation for the period of a simple pendulum.

The theoretical equation for the period of a simple pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Here, l is the length of the pendulum, g is the acceleration due to gravity and T is the period of oscillation of the pendulum. As per the above equation, the period of oscillation of the pendulum is directly proportional to the length of the pendulum and inversely proportional to the acceleration due to gravity. Also, the period of

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oscillation does not depend on the mass of the pendulum.

The interactive provides an intuitive way to explore physics concepts, reinforcing the relationship between length, gravity, and period of oscillation. It visually demonstrates damping effects and highlights the independence of period from mass, which may initially seem counterintuitive.

What do you notice about the frequency of the energy transfer? Can you work this out mathematically? Try to draw a graph of kinetic energy against time.

You may have noticed in the simulation in **Interactive 1** that the kinetic energy goes through two cycles for every one cycle of SHM. Why is this?

Qualitatively, it is because the kinetic energy is a scalar and velocity is a vector. This means that the velocity experiences two ‘peaks’ per cycle (one positive and one negative). For the kinetic energy, this corresponds to two cycles because the energy does not have a direction.

Mathematically, this is valid, because to work out the kinetic energy, we square the velocity, removing any negative values.

Worked example 2

The graph shows the energy transfers for a 2.0 kg body in SHM. Determine the frequency of the oscillations.

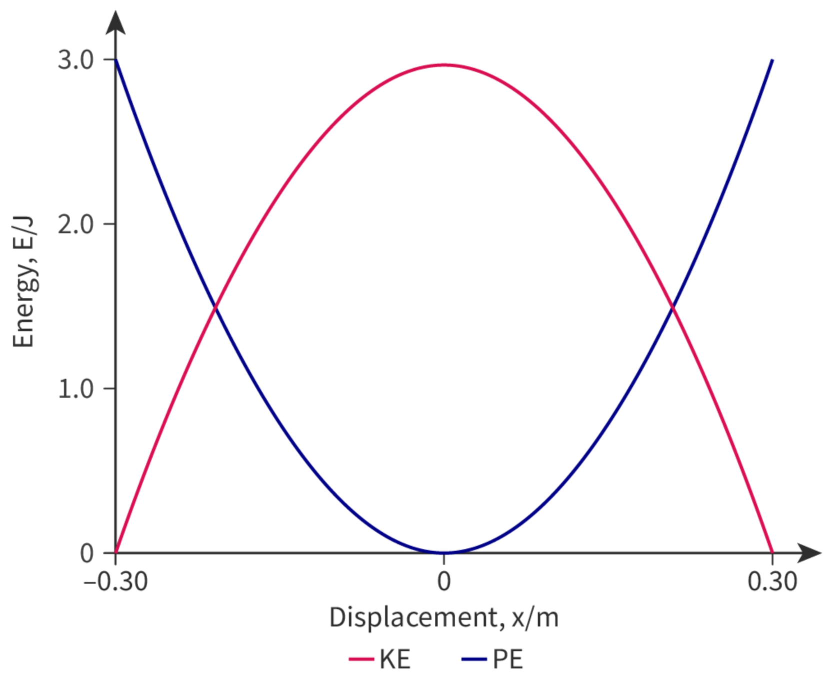

Figure 2. Energy—displacement graph.

More information for figure 2

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The graph depicts energy versus displacement, showcasing two curves labeled KE (kinetic energy) and PE (potential energy). The X-axis represents displacement in meters, ranging from -0.30 m to 0.30 m. The Y-axis represents energy in joules, ranging from 0 to 3.0 J. The red curve (KE) starts from the origin (0,0) and increases to a peak at the maximum displacement (0.3 m), then decreases symmetrically back to zero at the opposite end. The blue curve (PE) begins at 3.0 J for displacement -0.30 m, decreases to a minimum at zero displacement, and increases symmetrically back to 3.0 J at 0.30 m. This setup illustrates the energy transfer in simple harmonic motion (SHM), with kinetic and potential energy shifting as displacement changes.

[Generated by AI]

$m = 2.0\ \mathrm{kg}$

This can be solved using either of the energy equations.

Using potential energy, $E_\mathrm{p} = 3.0$, and maximum displacement, $x = 0.30\ \mathrm{m}$:

$$E_\mathrm{p} = \frac{1}{2}m\omega^2x^2$$

$$\omega^2 = \frac{Ep}{\left(\frac{1}{2}mx^2\right)}$$

$$= \frac{3.0}{\left(\frac{1}{2}\times2.0\times0.30^2\right)}$$

$$= 33.33$$

$$\omega = 5.77\ \mathrm{rad\ s^{-1}}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$= \frac{5.77}{2\pi}$$

$$= 0.918\ \mathrm{Hz}$$

$$= 0.92\ \mathrm{Hz}\ (2\ \mathrm{s.f.})$$

Work through the activity to check your understanding of energy in SHM.

Activity

- **IB learner profile attribute:**
  - Inquirer
  - Communicator
- **Approaches to learning:**

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- Thinking skills — Applying key ideas and facts in new contexts,
- Communication skills — Reflecting on the needs of the audience when creating engaging presentations

- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

Develop a resource for your classmates to explain the shape of a kinetic energy graph of SHM.

Look at the pendulum simulation in **Interactive 1** as well as the equations in section 1.6.C (/study/app/math-aa-hl/sid-423-cid-762593/book/wave-behaviour-id-45162/) of the DP *Physics data booklet*.

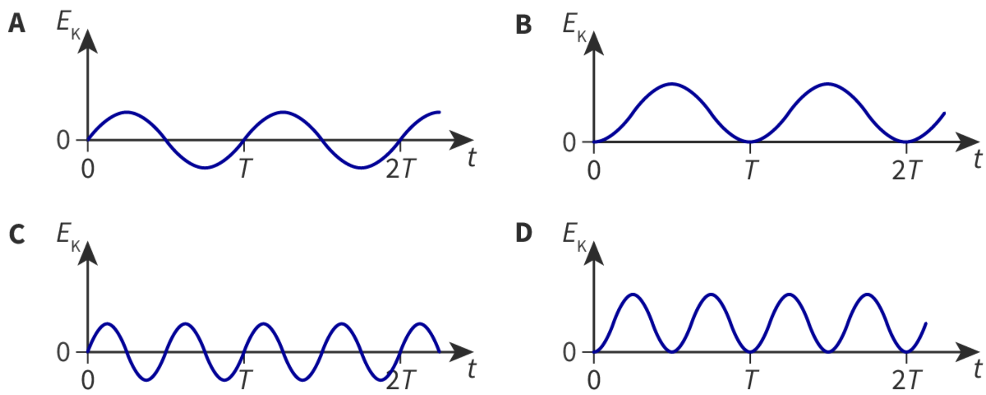
You could present your findings as a poster, with a detailed diagram and written explanation, or with a slide presentation.

5 section questions

Question 1

HL Difficulty:

Which graph (A, B, C or D) shows the variation of kinetic energy with time of a mass—spring system exhibiting SHM?



More information

1 D

2 C

3 B

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4 A

Explanation

Kinetic energy is a scalar and so cannot be negative, so A and C are incorrect.

The mass—spring experiences maximum velocity and therefore maximum kinetic energy twice per cycle, when the object passes through the equilibrium position (this happens twice in one time period — once in each direction). So the answer must be D.

Question 2

HL Difficulty:

State the phase angle between the kinetic energy and the potential energy of an object exhibiting SHM.

1 π radians ✓

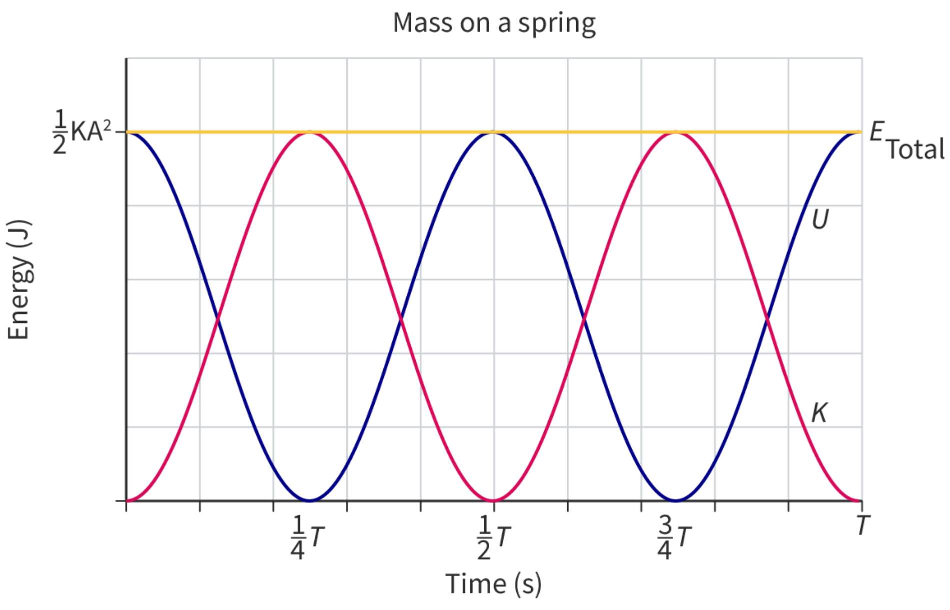
2 $\frac{\pi}{2}$ radians

3 2π radians

4 $\frac{3\pi}{2}$ radians

Explanation

The kinetic energy is at a maximum when the potential energy is zero, and vice versa.



More information

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The two energies are exactly half a cycle apart. A full cycle is 2π radians, so their phase angle is π radians.

Question 3

HL Difficulty:

A piston of mass 140 g inside an engine oscillates with SHM 420 times per second. The amplitude of the oscillation is 2.5 cm. Determine the total energy of the piston and the kinetic energy of the piston when the displacement is 2.5 cm. Give your answers to an appropriate number of significant figures.

The total energy of the piston is 1 300 ✓ J.

The kinetic energy of the piston at 2.5 cm is 2 0 ✓ J.

Accepted answers and explanation

#1 300

300J

300 J

#2 0

0.0

0J

0 J

General explanation

$$\begin{aligned} m &= 140 \text{ g} \\ &= 0.14 \text{ kg} \end{aligned}$$

$$f = 420 \text{ Hz}$$

$$\begin{aligned} x_0 &= 2.5 \text{ cm} \\ &= 0.025 \text{ m} \end{aligned}$$

$$\begin{aligned} \omega &= 2\pi f \\ &= 2\pi \times 420 \\ &= 2638.9 \text{ rad s}^{-1} \end{aligned}$$

$$\begin{aligned} E_{\text{T}} &= \frac{1}{2} m \omega^2 {x_0}^2 \\ &= \frac{1}{2} \times 0.14 \times 2638.9^2 \times 0.025^2 \\ &= 304.7 \text{ J} \\ &= 300 \text{ J (2 s.f.)} \end{aligned}$$

The kinetic energy when the displacement is maximum is zero (0), because the velocity is zero (0) at this point. All the energy is potential.

Student view

Question 4

HL Difficulty:

A mass of 75 g on a spring is pulled up from its equilibrium position by 2.5 cm and released. It oscillates with SHM with a frequency of *f*.

What is the maximum potential energy of the mass?

1 $E_p = 9.25 \times 10^{-4} f^2$ J ✓

2 $E_p = 9.25f^2$ J

3 $E_p = 1.47f^2$ J

4 $E_p = 1.47 \times 10^{-4} f^2$ J

Explanation

When potential energy is max = 2.5 cm = 0.025 m

$$m = 0.075 \text{ kg}$$

$$Ep = \frac{1}{2} m\omega^2 x^2$$

$$E_\text{p} = 0.5 \times 0.075 \times (2\pi f)^2 \times 0.025^2$$

$$E_\text{p} = 0.5 \times 0.075 \times 4\pi^2 \times f^2 \times 0.025^2$$

$$E_\text{p} = 9.25 \times 10^{-4} f^2 \text{ J}$$

Question 5

HL Difficulty:

The graph shows the potential energy of a simple harmonic oscillator of mass 0.15 kg. Determine the amplitude and frequency of the oscillations. Give your answers to an appropriate number of significant figures.

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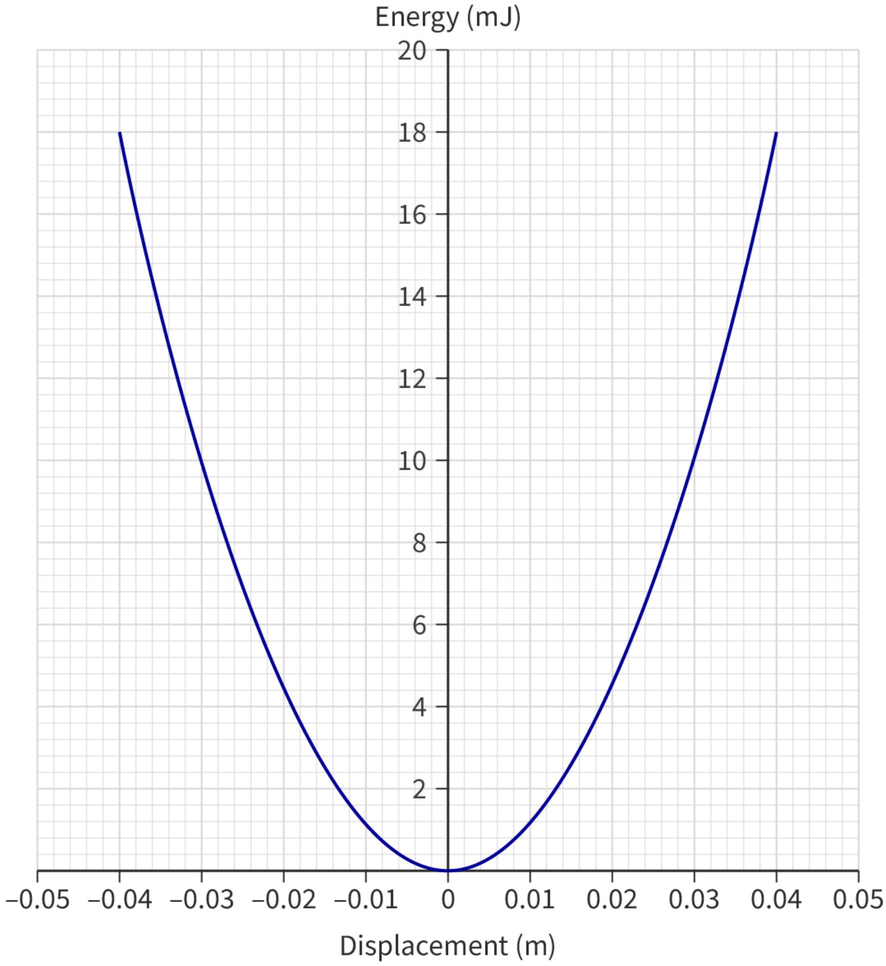

More information

The amplitude of the oscillations is 1 0.04 m.

The frequency of the oscillations is 2 1.9 Hz

Accepted answers and explanation

#1 0.04

0.040

#2 1.9

1.9Hz

1.9 Hz

2.0

2.0Hz

2.0 Hz

General explanation

$$m = 0.15 \text{ kg}$$

$$E_{\text{T}} = 18 \text{ mJ} \\ = 0.018 \text{ J}$$

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The amplitude is the maximum displacement, which can be read from the graph:

$$x_0 = 0.04 \text{ m}$$

$$E_{\mathrm{T}} = \frac{1}{2}m\omega^2{x_0}^2 \text{ and } \omega = 2\pi f$$

$$E_{\mathrm{T}} = \frac{1}{2}m(2\pi f)^2{x_0}^2$$

$$\begin{aligned} f &= \sqrt{\frac{(2E_{\mathrm{T}})}{(4m\pi^2{x_0}^2)}} \\ &= \sqrt{\frac{(2E_{\mathrm{T}})}{(4m\pi^2x_0^2)}} \\ &= \sqrt{\frac{(2\times 0.018)}{(4\times 0.15\pi^2\times 0.04^2)}} \\ &= 1.9 \text{ Hz (2 s.f.)} \end{aligned}$$

C. Wave behaviour / C.1 Simple harmonic motion

Summary and key terms

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/summary-and-key-terms-id-44872/print/) Assign

- An oscillating body can be described using displacement, velocity, acceleration, frequency, time period, angular frequency and amplitude.
- Simple harmonic motion (SHM) is defined as oscillations around an equilibrium position, where the acceleration is proportional to the displacement but in the opposite direction, which can be stated mathematically as:

$$a = -\omega^2 x$$

- In simple harmonic motion without damping, the total energy stays constant and is periodically transferred between kinetic energy and potential energy.
- The time period, T, frequency, f, and angular frequency, ω, of an oscillator are related by the equation:

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{2\pi}{\omega} \end{aligned}$$

- There are two main types of simple harmonic oscillator: simple pendulum and mass–spring system.

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- The time period of a simple pendulum can be calculated by:

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$

and the time period of a mass–spring system can be calculated by:

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Higher level (HL)

- Simple harmonic motion is a model of one-dimensional circular motion, and it can be described in terms of phase angle, or the angle through which the object has travelled, where a full cycle is 2π radians.
- The displacement and velocity of an oscillator at a given time can be calculated using the equations:
  - $x = x_0 \sin(\omega t + \varphi)$
  - $v = \omega x_0 \cos(\omega t + \varphi)$
  - $v = \pm\omega\sqrt{({x_0}^2 - x^2)}$
- The phase angle, ϕ, is the angle by which the oscillator differs from the sine curve.
- The energy of an object exhibiting SHM can be calculated using the equations:
  - $E_\mathrm{T} = \frac{1}{2}m\omega^2{x_0}^2$
  - $E_\mathrm{p} = \frac{1}{2}m\omega^2x^2$

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Key terms

Review these key terms. Do you know them all? Fill in as r can using the terms in this list.

1. An is a periodic back and forth mo
2. are oscillations around an equilibr the displacement is proportional to the negative
3. The of an oscillation is the numbe per and the to complete a full cycle.
4. The angle covered by an SHM oscillator per second is called t , whilst the amplitude is the maximun during a full cycle.
5. The position where the object wou was removed from the system.

time period | acceleration | displacement | second | a
Simple harmonic motion | oscillation | equilibrium | freq

Check

Interactive 1. Simple Harmonic Motion.

C. Wave behaviour / C.1 Simple harmonic motion

Checklist

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What you should know

After studying this subtopic, you should be able to:

- Identify examples of simple harmonic motion (SHM).

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- Understand and use the terms: time period, T, frequency, f, angular frequency, ω, amplitude, equilibrium position and displacement.
- Define simple harmonic motion, and use the equation $a = -\omega^2 x$.
- Determine the time period of a simple pendulum using the equation:

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$

- Determine the time period of a mass—spring system using the equation:

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

- Describe and graph the energy changes during SHM.

Higher level (HL)

- Determine the velocity and displacement of a SHM oscillator at any point in its cycle.
- Describe the position of an oscillator in its cycle based on its phase angle in radians.
- Determine the potential energy and kinetic energy of an oscillator at a given point in a SHM cycle.
- Describe quantitatively the energy transfers during SHM.

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Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/investigation-id-44874/print/) Assign

- **Learner profile attribute:** Inquirer
- **Approaches to learning:** Thinkings skills – Being curious about the natural world
- **Time to complete activity:** 40 minutes
- **Activity type:** Individual activity

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Your task

Simple harmonic motion can be applied to pendulums and atomic vibrations. But what about on a much larger scale?

In this investigation, you are going to extend your understanding of the link between SHM and one-dimensional circular motion. Circular motion and orbits are covered in subtopics A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/) and D.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44096/).

Table 1 shows planetary data for four planets in the solar system. Using your knowledge of SHM, you are going to test the one-dimensional circular motion model.

Table 1. Data for four planets.

Data	Venus	Earth	Jupiter	Neptune
Mass (10^{24} kg)	4.87	5.97	1898	102
Diameter (km)	12 104	12 756	142 984	49 528
Density (kg m^{-3})	5243	5514	1326	1638
Gravity (m s^{-2})	8.9	9.8	23.1	11.0
Rotation period (hours)	−5832.5	23.9	9.9	16.1
Length of day (hours)	2802.0	24.0	9.9	16.1
Distance from Sun (10^6 km)	108.2	149.6	778.5	4515.0
Orbital velocity (km s^{-1})	35.0	29.8	13.1	5.4
Orbital inclination (degrees)	3.4	0.0	1.3	1.8
Mean temperature (°C)	464	15	−110	−200
Surface pressure (bars)	91	2	Unknown*	Unknown*
Number of moons	0	1	79	14
Ring system?	No	No	Yes	Yes

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Try and answer the following questions. Click on 'Hint' to see the hint for each question.

- Calculate the orbital period of each planet (the amount of time for one complete orbit, or cycle).

You can determine the time period using the equation for maximum velocity: $v = \omega x_0 sin(\omega t + \varphi)$. When the outcome of the sine function is 1, v is max, so: $v_{\max} = \omega x_0 sin$. This maximum velocity is the orbital velocity of the planet (See **Video 2** and **Video 3** in section C.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/)). The amplitude of the oscillations is the orbital radius.

- Calculate the maximum acceleration of each planet.

Use the defining equation for SHM.

- What force provides this acceleration?

Think about what you know about how the planets stay in orbit.

- Determine the gravitational field strength from the Sun at the distance of each planet.

Gravitational field strength is equivalent to acceleration. If you imagine the planet is a pendulum, you can also calculate gravitational field strength using $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$. Is this a reasonable approximation?

- What is the total energy of each orbit?

You can calculate this using either of the energy equations.

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Theory of Knowledge

If we can approximate a planetary orbit to SHM, which form of SHM are we using? Is it like a pendulum, or a mass—spring? Are a pendulum or a mass—spring adequate models? How far can we extend the models before they lose their relevance?

C. Wave behaviour / C.1 Simple harmonic motion

Reflection

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Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.

Reflection

Now that you've completed this subtopic, let's come back to the guiding questions introduced in The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/).

- What makes the harmonic oscillator model applicable to a wide range of physical phenomena?
- Why must the defining equation of simple harmonic motion take the form it does?
- How can the energy and motion of an oscillation be analysed both graphically and algebraically?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

- What main points have you learned from this subtopic?

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- Is anything unclear? What questions do you still have?
- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

Once you submit your response, you won't be able to edit it.

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Submit

Rate subtopic C.1 Simple harmonic motion

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