3.16 Vector product

Checklist

What you should know

By the end of this subtopic you should be able to:

- recall that the vector product of vectors $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ is denoted by $\boldsymbol{v} \times \boldsymbol{w}$
- calculate the vector product from the components of $\boldsymbol{u}$ and $\boldsymbol{v}$ using the formula

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$

- recall the properties of the vector product:
  For vectors $\boldsymbol{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$:
  - $\boldsymbol{v} \times \boldsymbol{w} = -\boldsymbol{w} \times \boldsymbol{v}$ so the vector product is not commutative
  - $\boldsymbol{u}(\boldsymbol{v} + \boldsymbol{w}) = \boldsymbol{u} \times \boldsymbol{v} + \boldsymbol{u} \times \boldsymbol{w}$ so the vector product is distributive
  - $(k\boldsymbol{v}) \times \boldsymbol{w} = k(\boldsymbol{v} \times \boldsymbol{w}), k \in \mathbb{R}$
- recall that the area of a parallelogram can be calculated using $|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}| \sin\theta$
- recall that the area of a triangle can be calculated using $\frac{1}{2}|\boldsymbol{v} \times \boldsymbol{w}| = \frac{1}{2}|\boldsymbol{v}||\boldsymbol{w}| \sin\theta$.
- define the cross product as $|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}| \sin\theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
  - If vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel, then $\boldsymbol{v} \times \boldsymbol{w} = 0$
  - If vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ are perpendicular, then $|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|$.