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The big picture

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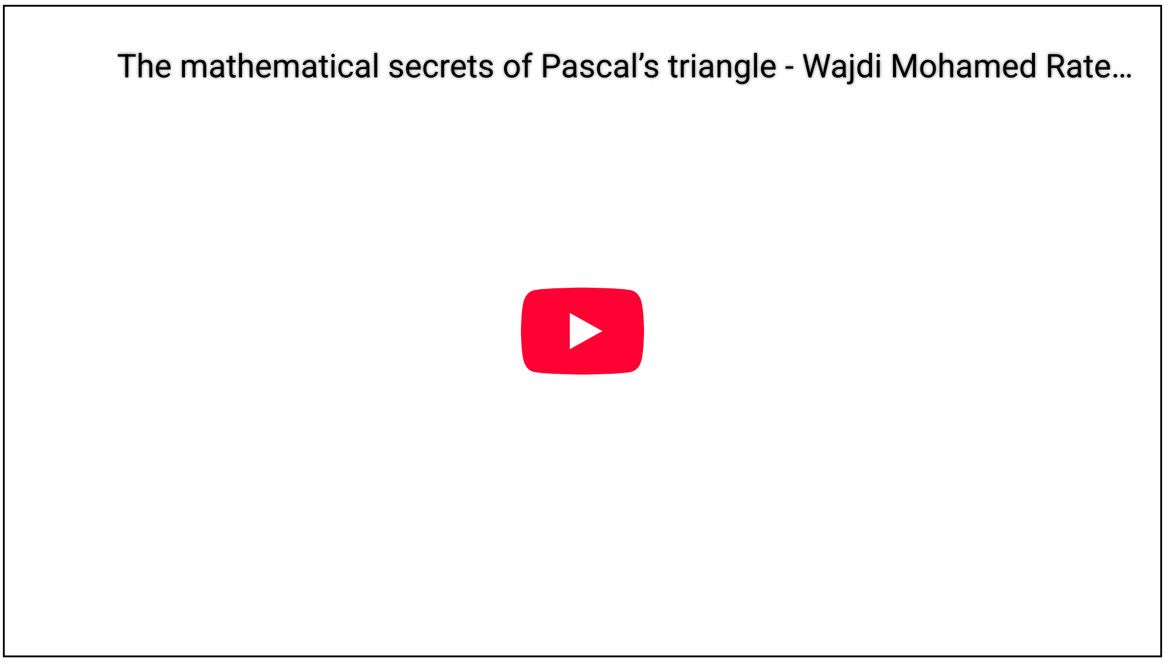
In this subtopic you will learn about a fascinating pattern of numbers called Pascal's triangle and one of its applications.

Watch the video below to see why Pascal’s triangle is so fascinating.



International Mindedness

In English, the special triangle in this subtopic is called Pascal’s triangle, attributing its discovery to the French mathematician Blaise Pascal. Keep in mind that this triangle could be called a different name in other languages

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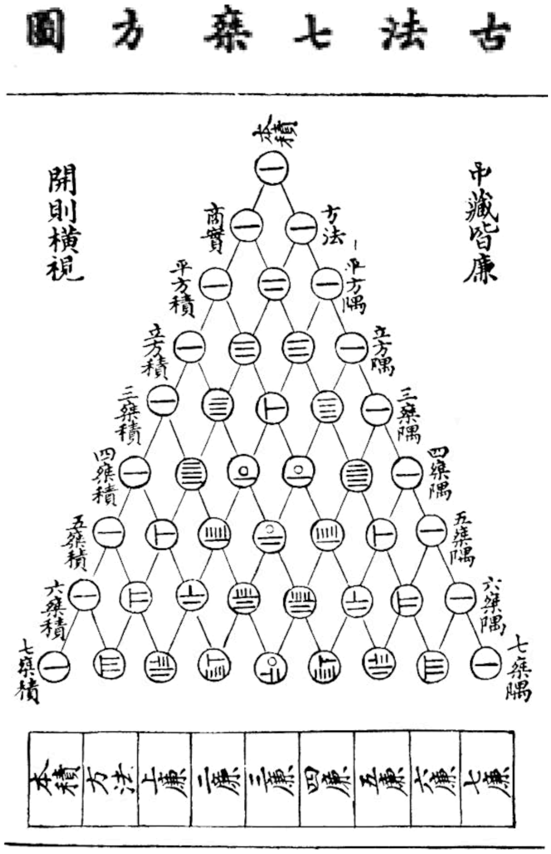
and that it was known way before Pascal's lifetime in the 17th century. The diagram below shows a version of this triangle made by Yang Hui, a 13th-century Chinese mathematician.



Source: " Yanghui triangle (https://commons.wikimedia.org/wiki/File:Yanghui_triangle.PNG) " by Yáng Huī is in public domain.

More information

The image shows a historical diagram of the Yanghui triangle, known in the West as Pascal's triangle. The triangle is composed of several rows of circular nodes interconnected by lines, forming a diamond-like grid structure. Each node contains texts represented by stylized Chinese characters, which denote numerical patterns or mathematical concepts. The triangle begins at a top single node and expands downward into multiple nodes per row, creating a symmetrical pyramid shape. To the sides and below the pyramid, additional labels and annotations in Chinese characters explain various aspects and layers of the triangle. The structure and annotations highlight mathematical relationships and computations within the triangle.

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Concept

Pascal's triangle contains many patterns, one of which you will learn to use for binomial expansions. However, binomial expansion can also be done by using combination numbers. What does this tell you about the nature of patterns in Pascal's triangle in particular and in mathematics more generally?

1. Number and algebra / 1.9 The binomial theorem

Pascal's triangle and combination numbers

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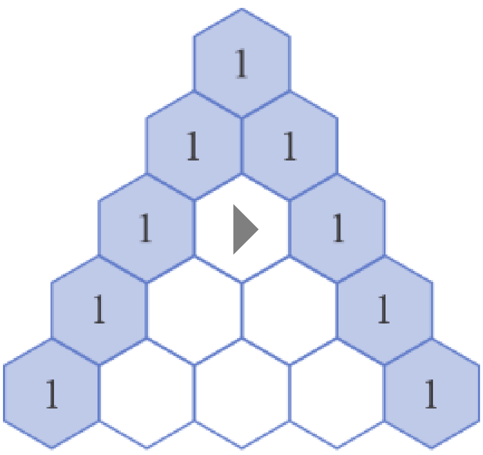
Pascal's triangle

Recognising the pattern

Pascal's triangle is constructed according to the pattern shown in the animation below.



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More information

This illustration depicts a portion of Pascal's triangle arranged in a triangular format using hexagons. The triangle is constructed with layers of hexagons, each filled with the number 1. The top row, labeled with a single hexagon, is marked with the number 1. Each subsequent row has an additional hexagon, all labeled with the number 1, forming a symmetrical pattern. The hexagons are outlined, with blue shaded backgrounds for filled hexagons. This representation visually aligns with how Pascal's triangle constructs each row based on the sum of the numbers directly above each position. The pattern demonstrates symmetry and mathematical placement, critical for understanding how Pascal's triangle is constructed.

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Example 1

★☆☆

Write down the terms for the row that follows the last one shown in the applet above.

Steps	Explanation
The last row is $1 \;\; 4 \;\; 6 \;\; 4 \;\; 1$ The next row will be: $1 \;\; 5 \;\; 10 \;\; 10 \;\; 5 \;\; 1$	Add together each pair of adjacent numbers to get the numb between them in the next row. The numbers at both ends are 1.

The rows in Pascal's triangle are numbered starting with row 0 as shown in the diagram below. The numbers in each row are called terms. The terms are numbered starting from 1 going from left to right.

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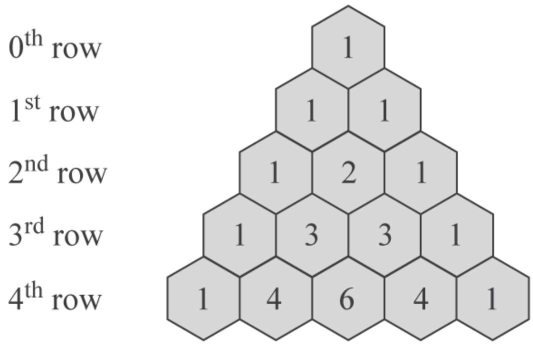

More information

The image is a diagram illustrating Pascal’s triangle from row 0 to row 4. The structure of the triangle is presented as a series of hexagonal shapes. Each number in a hexagon represents a term in Pascal's triangle. On the left, rows are labeled from the 0th to the 4th row.\n\n- Row 0: A single term, 1.\n- Row 1: Two terms, both are 1.\n- Row 2: Three terms, 1, 2, 1 from left to right.\n- Row 3: Four terms, 1, 3, 3, 1 from left to right.\n- Row 4: Five terms, 1, 4, 6, 4, 1 from left to right.

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Example 2

★☆☆

State the value of 5th term of the 6th row in Pascal’s triangle.

Steps	Explanation
The 6th row is: $$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$$ The 5th term is: $$15$$	The 5th term is the 5th number from the le

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Combination numbers

The entries in Pascal's triangle are linked to combination numbers. These numbers give the number of ways in which a combination of r objects can be chosen from n objects, if the order of arranging the objects does not matter. Combination numbers are written in the form $\binom{n}{r}$ or C_r^n or ${}^nC_r$ and read as 'n-choose-r' or 'n-c-r.'

Important

You can evaluate combination numbers by using a calculator or the following formula:

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Exam tip

The formula for combination numbers is given in the IB formula booklet. But factorial notation is not explained in the formula booklet, so you need to remember what $n!$ means.

The ! symbol in a mathematical context might be new to you. It is notation that represents a factorial.

Important

'n factorial' is defined as:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

It is the product of all positive integers between 1 and n inclusive.

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Example 3

★☆☆

Evaluate $6!$.

Steps	Explanation
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$	You should be able to write out the multiplicatic do $6!$ on your calculator.

Example 4

★☆☆

Evaluate $\binom{5}{3}$.

Steps	Explanation
$\binom{5}{3} = \frac{5!}{3!\,(5-3)!}$ $= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 10$	Use the formula $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$ with $n = 5$ $r = 3$. You can check your work by finding $\binom{5}{3}$ on the calculator.

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Steps	Explanation
In this guide you will see how to use the calculator to find the value of $6!$ and ${}^5\mathrm{C}_3 = \binom{5}{3}$. Choose the calculator mode.	
Press OPTN to display some options ...	

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Steps	Explanation
... then F6 to scroll to view more possibilities ...	
... and then F3 for the tools related to probability.	

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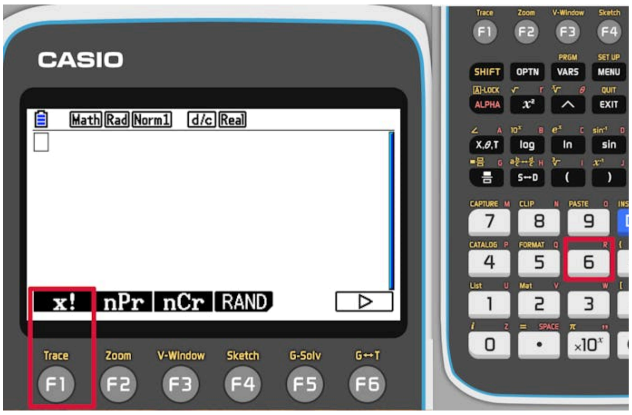
Steps	Explanation
Press the number 6 and then F1 to find $6!$.	
To find the combination number, press F3. This will bring up a template to fill.	

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Steps	Explanation
To find the value of $^5\mathrm{C}_3 = \binom{5}{3}$, the number 5 needs to go in the box in front of C, and 3 needs to go in the box after C. Press EXE to see the combination number.	

Steps	Explanation
In this guide you will see how to use the calculator to find the value of $6!$ and $^5\mathrm{C}_3 = \binom{5}{3}$. Enter the home screen of any application.	

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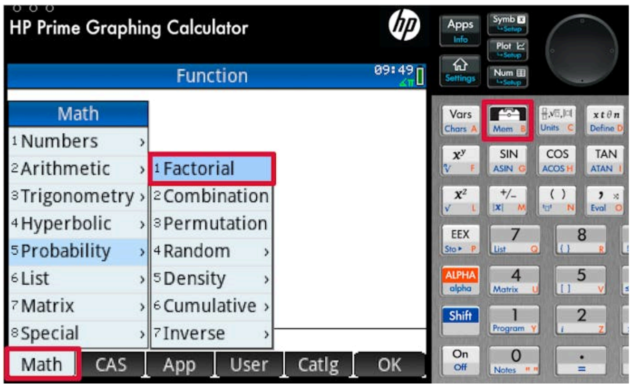
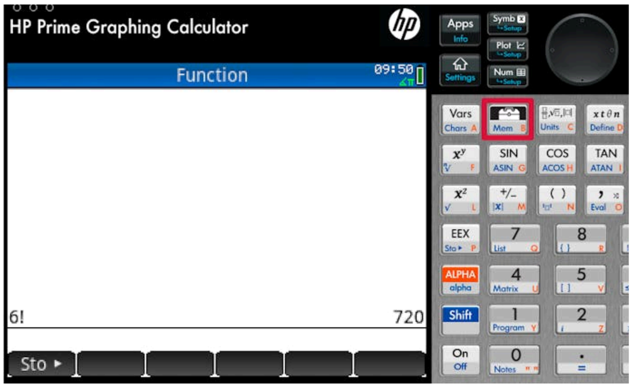
Steps	Explanation
In the toolbox you can find the tool to find factorials.	
After finding the value of $6!$, open the toolbox again ...	

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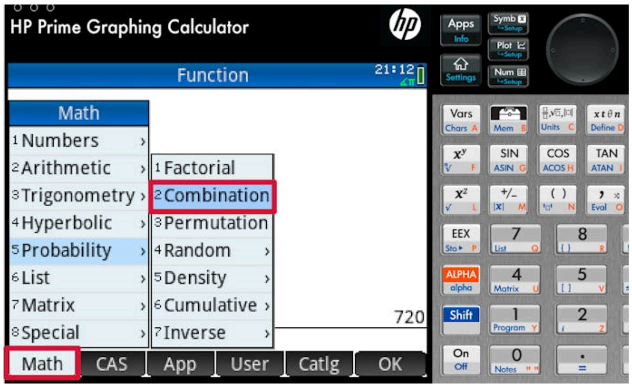
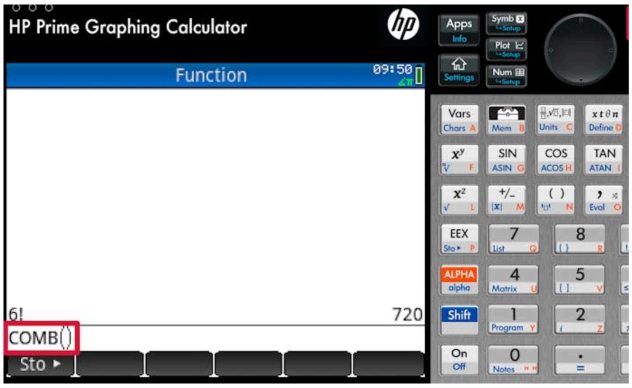
view

Steps	Explanation
... and this time choose the combination option to find the combination number ${}^5\mathrm{C}_3 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.	
This time the calculator does not give you a template to fill. Rather, it gives a function and it waits for you to enter the arguments. If you do not remember what the calculator is waiting for, open the help screen.	

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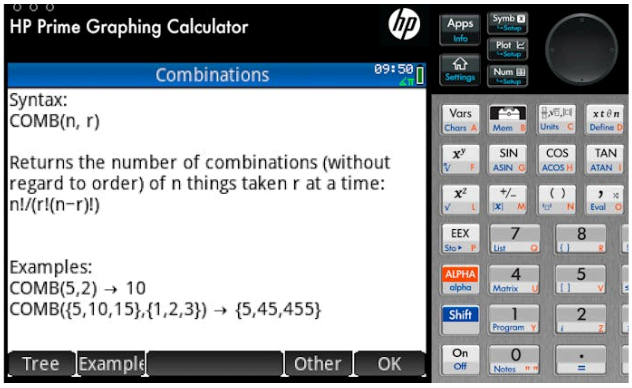
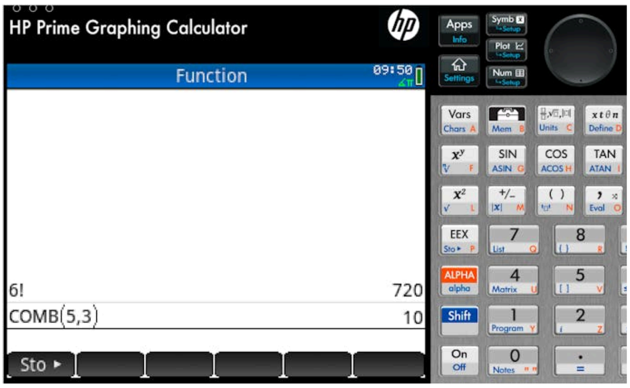
Steps	Explanation
The help screen explains the possible ways of entering arguments.	
To find ${}^5\mathrm{C}_3 = \binom{5}{3}$, you need to use the form COMB(5,3).	

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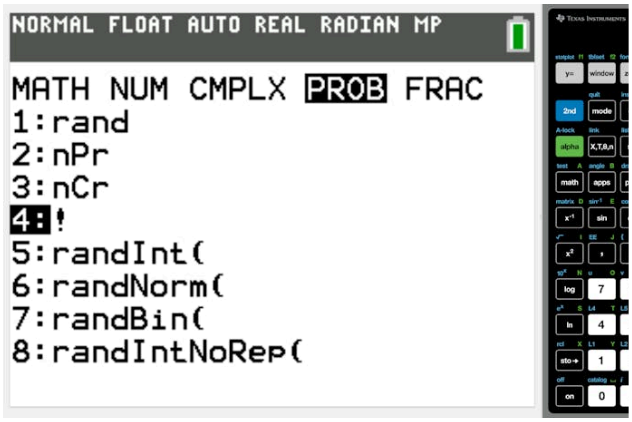
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>In this guide you will see how to use the calculator to find the value of $6!$ and ${}^5\mathrm{C}_3 = \binom{5}{3}$.

First press the number 6 and open the math options to find the symbol for factorial.</td><td>
</td></tr>
<tr><td>The factorial symbol is among the tools related to probability.</td><td>
</td></tr>
</table>

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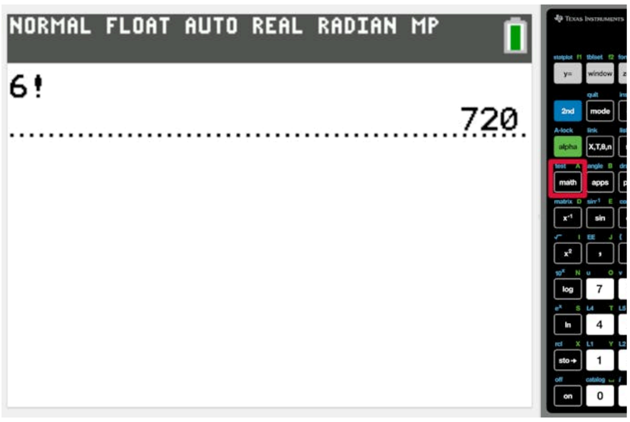
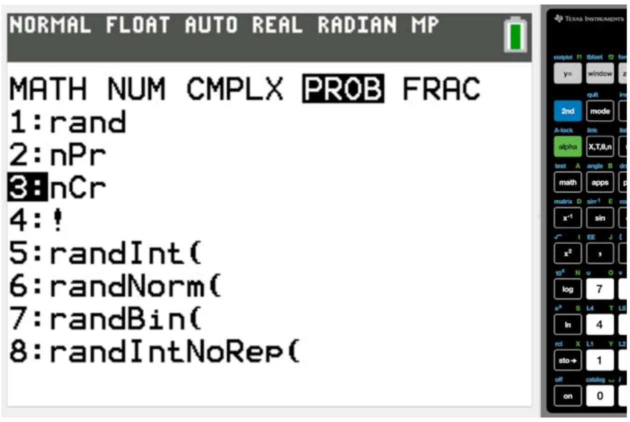
Steps	Explanation
After finding the value of $6!$, open the math options again ...	
... and this time choose the nCr option to find the combination number ${}^5\mathrm{C}_3 = \binom{5}{3}$. This will bring up a template to fill.	

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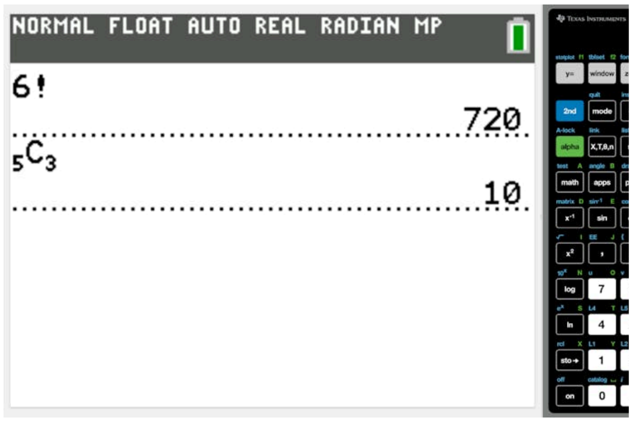
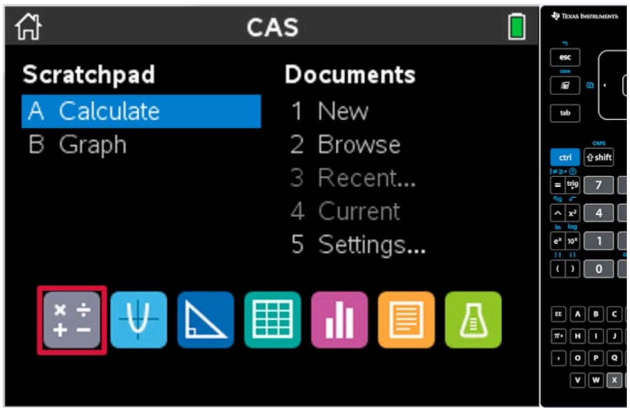
Steps	Explanation
To find the value of $^5\mathrm{C}_3 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, the number 5 needs to go in the box in front of C, and 3 needs to go in the box after C. Press enter to see the combination number.	

Steps	Explanation
In this guide you will see how to use the calculator to find the value of $6!$ and $^5\mathrm{C}_3 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$. These calculaton can be done on a calculator page.	

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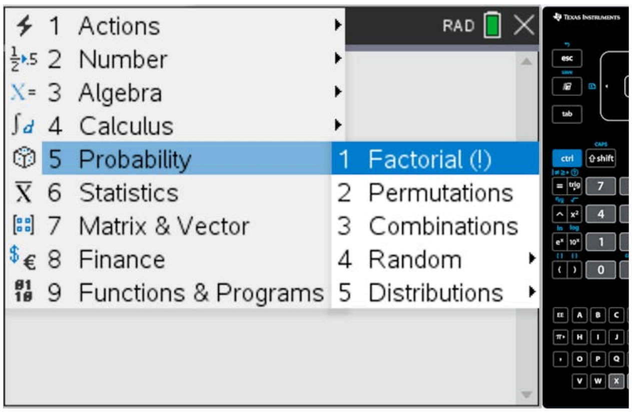
Steps	Explanation
First press the number 6 and open the menu to find the symbol for factorial.	
The factorial symbol is among the tools related to probability.	

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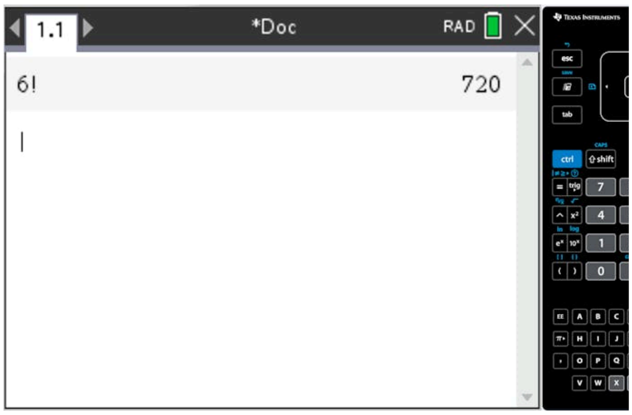
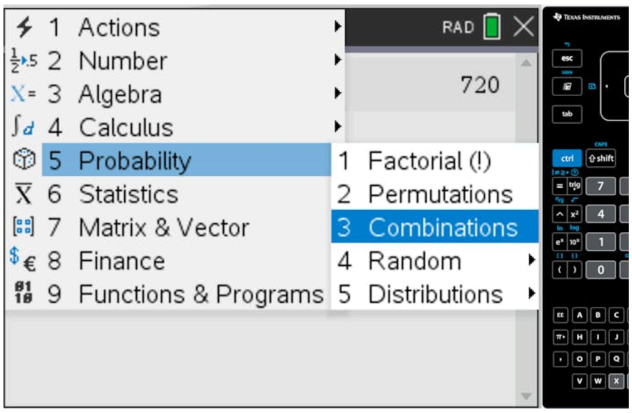
Steps	Explanation
After finding the value of $6!$, open the menu again ...	
... and this time choose the combinations option to find the combination number ${}^5\mathrm{C}_3 = \binom{5}{3}$. This time the calculator does not give you a template to fill. Rather, it gives a function and it waits for you to enter the arguments.	

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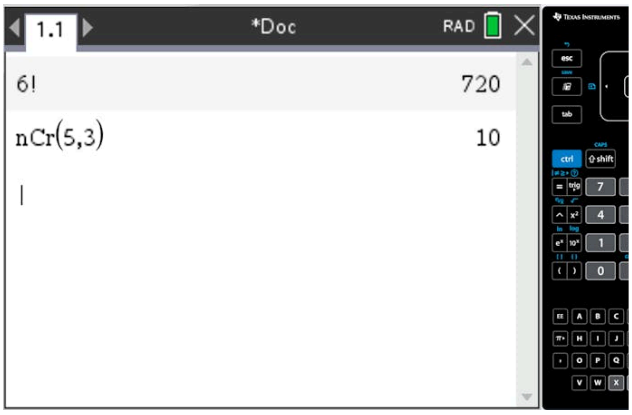
Steps	Explanation
To find ${}^5\mathrm{C}_3 = \binom{5}{3}$, you need to use the form nCr(5,3).	

Be aware

If you need to find combination numbers without a calculator, it is useful to keep in mind that factorials cancel and simplify in the following way:

$$\frac{n!}{(n-3)!} = \frac{n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \cdots \times 2 \times 1}{(n-3) \times (n-4) \times \cdots \times 2 \times 1}$$
$$= n \times (n-1) \times (n-2)$$

Activity

An interesting fact about factorials is that $0! = 1$ by definition.

Discuss with a peer what you find strange about this fact and why defining $0!$ in this way might make sense.

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Learn more about $0!$ by watching the video below.

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Zero Factorial - Numberphile

Example 5

★★☆

Write $\frac{(n!)^2}{(n-1)!\,(n-2)!}$ without using factorial notation.

Steps	E
$\frac{(n!)^2}{(n-1)!\,(n-2)!}$ $= \frac{(n\times(n-1)\times(n-2)\times\cdots)\,(n\times(n-1)\times(n-2)\times(n-3)\times\cdots)}{((n-1)\times(n-2)\times\cdots)\,((n-2)\times(n-3)\times\cdots)}$ $= n\times n\times(n-1) = n^3 - n^2$	Yo wh wi by th te fa

Activity

Evaluate each of the following:

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$$\binom{4}{0},\ \binom{4}{1},\ \binom{4}{2},\ \binom{4}{3}\ \text{and}\ \binom{4}{4}$$

Propose a relationship between these values and Pascal's triangle. Use further examples to check your idea.

Example 6

★☆☆

Evaluate $\binom{6}{3}$ by using Pascal's triangle.

Steps	Explanation
$\binom{6}{3}$ corresponds to the 4th term in the 6th row $$1\ \ 6\ \ 15\ \ 20\ \ 15\ \ 6\ \ 1$$ So $\binom{6}{3} = 20$	The following diagram show the rows and terms of Pasca triangle are related to comb numbers.

$$\binom{0}{0}$$
$$\binom{1}{0}\ \binom{1}{1}$$
$$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$$
$$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \binom{3}{3}$$
$$\binom{4}{0}\ \binom{4}{1}\ \binom{4}{2}\ \binom{4}{3}\ \binom{4}{4}$$
$$\binom{5}{0}\ \binom{5}{1}\ \binom{5}{2}\ \binom{5}{3}\ \binom{5}{4}\ \binom{5}{5}$$

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The image shows a triangular arrangement of numbers representing binomial coefficients. Starting from the top, there is a single number, (0 0). Below it, the second row contains two numbers, (1 0) and (1 1). Each subsequent row contains one more number than the row above, corresponding to the coefficients in the expansion of a binomial power.\n\nFor example, the third row has (2 0), (2 1), and (2 2). The fourth row contains (3 0), (3 1), (3 2), and (3 3). Each number at the edges of the triangle is 1, and the interior numbers follow the rule of binomial coefficients, where each number is the sum of the two numbers directly above it in the previous row. This pattern continues downwards, forming a pyramid-like shape, ending with the sixth row: (5 0), (5 1), (5 2), (5 3), (5 4), and (5 5).

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Exam tip

For exam questions where a calculator is not allowed, you can use Pascal's triangle to evaluate combination numbers.

You can also use the formula

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

This formula, unlike Pascal's triangle, is given in the IB formula booklet.

3 section questions

1. Number and algebra / 1.9 The binomial theorem

Binomial theorem

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Binomial expansion and Pascal’s triangle

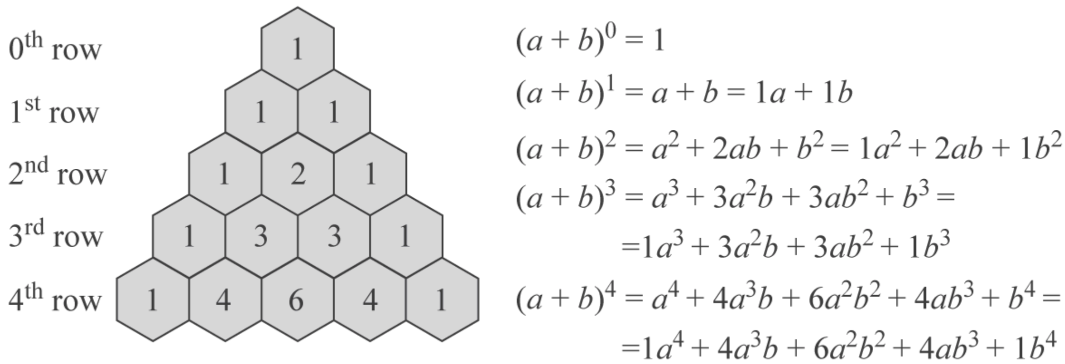
Activity

Expand each of the following powers of the binomial $a + b$. The first one is done for you.

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a+b)^3 =$
3. $(a+b)^4 =$

Compare your results to Pascal’s triangle and comment on any patterns that you notice.

A binomial is an algebraic expression consisting of two terms connected with a plus or a minus sign. A binomial expansion is the result of expanding a binomial raised to some power n. As you may have observed in the previous activity, the coefficients of a binomial expansion match the terms in Pascal’s triangle as shown in the diagram below. Also, note that the sum of the exponents for a and b is always equal to n.



More information

The image contains a diagram of Pascal's Triangle alongside binomial expansion equations. On the left, Pascal's Triangle is displayed in a triangular grid, where each row starts and ends with the number 1, and each internal number is the sum of the two numbers above it. The triangle shows the first 5 rows:

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- 0^{th} row: 1
- 1^{st} row: 1, 1
- 2^{nd} row: 1, 2, 1
- 3^{rd} row: 1, 3, 3, 1
- 4^{th} row: 1, 4, 6, 4, 1

To the right, the diagram includes binomial expansion equations corresponding to each row:

1. $(a + b)^0 = 1$
2. $(a + b)^1 = a + b = 1a + 1b$
3. $(a + b)^2 = a^2 + 2ab + b^2 = 1a^2 + 2ab + 1b^2$
4. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$
5. $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

This visual representation connects Pascal's Triangle to the coefficients in binomial expansions, illustrating the mathematical relationship described.

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Example 1

★☆☆

Write $(a+b)^5$ in expanded form.

Steps	Explanation
Using the 5th row of Pascal's triangle: $1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$	For the expansion of t power of a binomial, l the 5th row of Pascal' triangle.

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Steps	Explanation
$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	Write out the terms ir descending powers o ascending powers of coefficients are the nu in Pascal's triangle.

Binomial theorem

The pattern that you used in **Example 1** is generalised in the binomial theorem.

Important

The binomial theorem says that the expansion of $(a+b)^n$ for $(n \in \mathbb{N})$ is given by:

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b^1 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$$

where ${}^n\mathrm{C}_r$ or $\binom{n}{r}$ is called the binomial coefficient .

Making connections

Binomial expansions can also be written in sigma notation:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r.$$

Example 2

★★☆

Expand $(2x+3)^4$ using the binomial theorem.

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view

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$a = 2x$

$b = 3$

a and b can represent more complex algebraic expressions or constants.

$$(2x+3)^4 = (2x)^4 + \binom{4}{1}(2x)^3(3) + \binom{4}{2}(2x)^2(3)^2 + \binom{4}{3}(2x)(3)^3 + 3^4$$

$$= 16x^4 + 4 \times 8x^3 \times 3 + 6 \times 4x^2 \times 9 + 4 \times 2x \times 27 + 81$$

$$= 16x^4 + 96x^3 + 216x^2 + 216x + 81$$

Binomial coefficients can be found using Pascal's triangle, your calculator or the formula

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

Exam tip

The formula for binomial expansion is given in the IB formula booklet.

When you write down the terms of the expansion, check that for every term the two exponents add up to n.

Be aware

When you write out a binomial expansion it is important to use parentheses for the a and b expressions.

For example, in the expansion for $(4x-1)^3$ you should write

$$(4x)^3 + \binom{3}{1}(4x)^2(-1) + \binom{3}{2}(4x)(-1)^2 + \ldots$$

Student view

Remember that $(4x)^2 \neq 4x^2$ and $(-1)^2 \neq -1^2$.

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Binomial expansion and the general term

Exam questions will often ask for a specific term in an expansion. In this case it is more helpful to use the general term of the binomial expansion, which is

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

The general term is labelled with the $r+1$ subscript because $r = 0$ for the first term, $r = 1$ for the second term, and so on.

Example 3

★★☆

Find the 3rd term in the expansion of $\left(-xy^2 + x\right)^6$.

Steps	Explanation
$T_3 = \binom{6}{2} \times \left(-xy^2\right)^4 \times (x)^2$ $= 15 \times x^4 \times y^8 \times x^2$ $= 15x^6y^8$	Here $a = -xy^2$ and $b = x$. Using $$T_{r+1} = \binom{n}{r} a^{n-r} b^r,$$ the 3rd term is T_3 so $r = 2$.

Exam tip

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Although the general term is not given explicitly in the formula booklet as $T_{r+1} = \binom{n}{r} a^{n-r}b^r$, you can see it as one of the terms in the binomial theorem formula. Just be careful to choose the correct r value.

Example 4

★★★

Find the coefficient of x in the expansion of $\left(-2x^3 + \frac{1}{x^4}\right)^5$.

Steps	Explanation
$T_{r+1} = \binom{5}{r} \left(-2x^3\right)^{5-r} \left(\frac{1}{x^4}\right)^r$	Write an expression for the general term with w know. In this case $a = -2x^3$ and $b = \frac{1}{x^4} = x^{-4}$.
$\left(x^3\right)^{5-r} \left(x^{-4r}\right) = x^1$ $x^{15-3r}x^{-4r} = x^1$ $x^{15-7r} = x^1$ $\therefore 15 - 7r = 1$ $r = \frac{-14}{-7} = 2$	You want to find the r value for the term that co x^1, so work only with powers of x in this step (ig the constants).

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Steps	Explanation
When $r = 2$, $T_3 = \binom{5}{2}\left(-2x^3\right)^3\left(\frac{1}{x^4}\right)^2$ $= 10\left(-8x^9\right)\left(x^{-8}\right)$ $= -80x$	Find the x term.
The coefficient is -80.	Only give the coefficient in your final answer.

Exam tip

Exam questions will often ask for a specific term of an expansion or the coefficient of a specific term.

The coefficient refers to the number multiplying the variables in the term. If a question asks for the coefficient, do not write the variables in your answer.

It is useful to remember that the constant term does not contain variables; the variables are raised to the power of 0.

Example 5

★★☆

Find the constant term in the expansion of $\left(2x^3 - \frac{4}{x^6}\right)^9$.

Steps	Explanation
$T_{r+1} = \binom{9}{r}\left(2x^3\right)^{9-r}\left(-\frac{4}{x^6}\right)^r$	Write an expression for the general term wit you know.

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Steps	Explanation
$\left(x^3\right)^{9-r}\left(x^{-6r}\right)=x^0$ $x^{27-3r}x^{-6r}=x^0$ $x^{27-9r}=x^0$ $\therefore 27-9r=0$ $r=3$	You want to find the r value for the constar (the term containing x^0, so work only with p of x in this step.
When $r=3$, $T_4=\binom{9}{3}\left(2x^3\right)^6\left(-\frac{4}{x^6}\right)^3$ $=84\left(64x^{18}\right)\left(-\frac{64}{x^{18}}\right)$ $=-344\ 064$	Find the constant term.
The constant term is $-344\ 064$.	

Example 6

★★★

In the expansion of $(1+ax)^n$ the second term is $12x$ and the fourth term is $160x^3$. Find a and n.

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view

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Steps	Explanation
$T_2 = \binom{n}{1}(1)^{n-1}(ax)^1 = 12x$ $T_4 = \binom{n}{3}(1)^{n-3}(ax)^3 = 160x^3$	Write a general term equation for ea given terms.
$\binom{n}{1} \times ax = 12x$ $a \times \binom{n}{1} = 12$ $a\times \dfrac{n!}{1! \times (n-1)!} = 12$ $a \times n = 12$ $a = \dfrac{12}{n}$	Simplify the equation for T_2 by usin $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$
$\binom{n}{3} \times a^3x^3 = 160x^3$ $a^3 \times \binom{n}{3} = 160$ $a^3 \times \dfrac{n!}{3!\,(n-3)!} = 160$ $a^3 \times \dfrac{n \times (n-1) \times (n-2)}{6} = 160$	Simplify the equation for T_4 by usin $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$
$\left(\dfrac{12}{n}\right)^3 \times \dfrac{n \times (n-1) \times (n-2)}{6} = 160$ $\dfrac{(n-1)\,(n-2)}{6n^2} = \dfrac{160}{12^3}$ $\dfrac{n^2 - 3n + 2}{6n^2} = \dfrac{5}{54}$ $n^2 - 3n + 2 = \dfrac{5}{9}n^2$ $\dfrac{4}{9}n^2 - 3n + 2 = 0$	Substitute $a = \dfrac{12}{n}$ into $a^3 \times \dfrac{n \times (n-1) \times (n-2)}{6}$ =

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Steps	Explanation
$n = \dfrac{3 \pm \sqrt{(-3)^2 - 4\left(\frac{4}{9}\right)(2)}}{2\left(\frac{4}{9}\right)}$ $= \dfrac{3 \pm \sqrt{\frac{49}{9}}}{\frac{8}{9}} = \left(3 \pm \dfrac{7}{3}\right) \times \dfrac{9}{8}$ $n = 6 \text{ or } n = \dfrac{3}{4} \text{ (reject since } n \in \mathbb{N})$ So $n = 6$	Solve the quadratic equation. You c quadratic formula.
$a = \dfrac{12}{n} = \dfrac{12}{6} = 2$	Substitute $n = 6$ into $a = \dfrac{12}{n}$.

The solution to **Example 6** is also explained in the video below.

Student
view

Video 1. Solution to Example 6: Binomial Expansion.

More information for video 1

1

00:00:00,100 --> 00:00:01,500

narrator: Hello students!

Let's have a look at this

2

00:00:01,567 --> 00:00:03,500

binomial expansion question.

3

00:00:03,833 --> 00:00:08,567

It says that $(1 + ax)^n$ has a second term

4

00:00:08,633 --> 00:00:12,667

equals to $12x$

Overview
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and a fourth term equals to $160x^3$.

5

00:00:12,733 --> 00:00:15,667

So let's expand to at least a fourth term

6

00:00:15,733 --> 00:00:19,933

using the general binomial expansion

in terms of its coefficients

7

00:00:20,033 --> 00:00:24,033

and in this case the only real term is ax.

8

00:00:24,267 --> 00:00:25,800

So here we have the fourth terms

9

00:00:25,900 --> 00:00:27,867

and now we're going to

gather the information.

10

00:00:27,933 --> 00:00:32,067

It says that the second term,

which of course is na^1x^1

11

00:00:32,400 --> 00:00:36,233

is equal to $12x$

and this of course implies straight away

12

00:00:36,300 --> 00:00:38,333

that $n \times a = 12$.

13-15

00:00:38,833 --> 00:00:47,933

Now the fourth term, if we use a binomial

coefficient is $\frac{n!}{(n-3)!3!}(ax)^3$

16

00:00:48,600 --> 00:00:52,300

Student view

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Now let's look at these factorials first

and let's simplify a little bit.

17

00:00:52,667 --> 00:01:01,067

Now, $n!$ divided by $(n-3)!$

times $3!$

can be written as $n(n-1)(n-2)(n-3)!$

divided by $(n-3)! \times 3!$.

18

00:01:01,133 --> 00:01:04,267

all factorial divided by the denominator.

19

00:01:04,333 --> 00:01:06,767

So you can see that $n-3$

factorials cancel out.

20

00:01:06,833 --> 00:01:11,033

So we left with $n(n-1)(n-2)$ divided by $3!$.

21

00:01:11,100 --> 00:01:12,933

In fact, in general, $n!$

22

00:01:13,000 --> 00:01:15,667

divided by $(n-r)!r!$

23

00:01:15,733 --> 00:01:18,233

can be written as $n(n-1)$

24

00:01:18,300 --> 00:01:22,500

all the way to $n-(r-1)$

over $r!$.

25

00:01:22,567 --> 00:01:26,467

So that's a simplification

of this which is worth keeping a minus.

26

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00:01:26,533 --> 00:01:29,633

So when we left with this

$n(n-1)(n-2)$

27

00:01:29,900 --> 00:01:33,000

times a^3x^3

divided by $3!$,

28

00:01:33,067 --> 00:01:38,567

and this has to be equal $160x^3$,

therefore $n(n-1)(n-2)$

29

00:01:38,700 --> 00:01:41,367

a^3 divided by 6 is 160.

30

00:01:42,600 --> 00:01:44,833

Now, we already noted $n \times a = 12$,

31

00:01:44,900 --> 00:01:47,267

so a can be written as $\frac{12}{n}$

32

00:01:47,333 --> 00:01:51,600

and this then we can substitute

into the other equation.

33

00:01:51,667 --> 00:01:52,967

So let's do that.

34

00:01:54,467 --> 00:01:59,033

So $n(n-1)(n-2) \times 12^3$

35

00:01:59,400 --> 00:02:03,400

divided by $6 \times n^3$ is 160.

36

00:02:03,600 --> 00:02:06,333

Student
view

So 12^3 is 12×12^2

37

00:02:06,400 --> 00:02:11,967

into $n(n-1)(n-2)$

divided by $6n^3$ is 160.

38

00:02:12,200 --> 00:02:13,967

The 12 and 6 leaves a 2,

39

00:02:14,033 --> 00:02:18,967

and a 2 and a 160 leave 80,

12^2 of course is 12×12

40

00:02:19,033 --> 00:02:23,833

into $n(n-1)(n-2) = 80n^3$.

41

00:02:23,933 --> 00:02:27,700

Now 12 of course is 4×3,

and 80 is 4×20.

42

00:02:27,833 --> 00:02:31,200

So we can cancel out

the fours leaving us with 36

43

00:02:31,433 --> 00:02:35,167

n into $n - 1$ to

$n - 2 = 20n^3$.

44

00:02:35,233 --> 00:02:37,767

So now we can open the brackets $36n$

45

00:02:38,200 --> 00:02:42,033

into $n^2 - 3n + 2 = 20n^3$,

46

00:02:42,267 --> 00:02:44,600

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giving us $36n^3$

47

00:02:44,767 --> 00:02:50,167

$-108n^2 + 72n - 20n^3 = 0.$

48

00:02:50,233 --> 00:02:54,467

Leaving us with $16n^3 - 108n^2$

49

00:02:54,533 --> 00:02:55,933

$+72n.$

50

00:02:56,700 --> 00:03:00,333

And now we need to decide

how to solve this cubic function.

51

00:03:01,067 --> 00:03:02,600

Let's first solve it by hand.

52

00:03:03,200 --> 00:03:05,100

So we've got $16n^3$

53

00:03:05,567 --> 00:03:09,400

$-108n^2 + 72n = 0.$

54

00:03:09,633 --> 00:03:10,967

Well, we can take out a factor

55

00:03:11,033 --> 00:03:15,800

of $4n$ leaving $4n^2 - 27n + 18 = 0,$

56

00:03:16,233 --> 00:03:18,833

and we can solve this by hand $4n$

57

00:03:19,000 --> 00:03:22,167

and then we've got $4n$ and n,

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58

00:03:22,333 --> 00:03:25,600

and then we need at least

one integer solution.

59

00:03:25,667 --> 00:03:28,633

And you can see

that the solutions are $n = 0$,

60

00:03:28,700 --> 00:03:31,033

$n = \frac{3}{4}$, and $n = 6$.

61

00:03:31,100 --> 00:03:33,933

We reject, of course, $n = 0$

and $n = \frac{3}{4}$

62

00:03:34,000 --> 00:03:38,567

because n has to be large equal to 1,

and of course also an integer.

63

00:03:38,900 --> 00:03:42,800

In other words, our solution is $n = 6$

and a is then 2,

64

00:03:42,867 --> 00:03:48,167

such that the binomial expansions $(1 + 2x)^6$.

65

00:03:48,733 --> 00:03:50,267

However, if this were a paper two,

66

00:03:50,333 --> 00:03:53,300

you of course could also

use your GDC to solve this.

67

00:03:53,367 --> 00:03:57,133

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view

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So the cubic equation was $16n^3 - 108n^2$

68

00:03:57,200 --> 00:03:58,967

$+72n = 0$.

69

00:03:59,667 --> 00:04:02,267

You can either use a graphical

method and finding the zeros,

70

00:04:02,333 --> 00:04:04,100

or of course you can use a solver.

71

00:04:04,233 --> 00:04:06,033

I definitely prefer the graphical method.

72

00:04:06,433 --> 00:04:08,333

So let's grab this cubic function.

73

00:04:08,833 --> 00:04:10,967

In terms of x doesn't really matter.

74

00:04:11,033 --> 00:04:12,900

Here I use TI-84.

75

00:04:12,967 --> 00:04:17,600

And in the function Y1 I put in that

76

00:04:18,100 --> 00:04:22,133

cubic function and up plotting it

over a preferable window.

77

00:04:22,233 --> 00:04:24,100

And you can see the three solutions here.

78

00:04:24,433 --> 00:04:26,133

So we can calculate the zero.

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79

00:04:26,400 --> 00:04:28,967

We're gonna take one on the right first

80

00:04:29,033 --> 00:04:31,167

and giving the left and right bound

81

00:04:31,233 --> 00:04:34,467

and I guess in between

gives us one of the zeros

82

00:04:34,633 --> 00:04:36,167

for X equals six.

83

00:04:36,500 --> 00:04:37,767

Now let's see what the other one is.

84

00:04:37,833 --> 00:04:40,633

One is obviously zero

because you can see it's on the Y axis

85

00:04:41,000 --> 00:04:44,700

and the one in between the zero

and a 6 is going to be

86

00:04:45,700 --> 00:04:49,267

0.75, which of course

is this three quarters that we found.

87

00:04:49,333 --> 00:04:53,600

We reject those two relieving

we are left with $n = 6$

88

00:04:53,667 --> 00:04:56,000

and therefore $a = 2$ as a solution,

89

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00:04:56,300 --> 00:04:57,933

same as before, of course,

90

00:04:56,300 --> 00:04:57,933

same as before, of course,

91

00:04:58,000 --> 00:05:01,633

such that the binomial expansion

is $(1 + 2x)^6$.

92

00:05:01,700 --> 00:05:03,400

And so we've solved this problem.

93

00:05:05,300 --> 00:05:06,767

Now let's have a look.

94

00:05:06,833 --> 00:05:11,233

We expanded the binomial

to include at least a second

95

00:05:11,300 --> 00:05:13,867

and fourth term

because that was a question we're given

96

00:05:14,067 --> 00:05:16,400

and we also had a look at its useful

97

00:05:16,733 --> 00:05:22,533

form of the binomial coefficients,

and that was that.

5 section questions

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

1. Number and algebra / 1.9 The binomial theorem

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27690/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- write out terms in Pascal's triangle by extending the pattern from the first few rows
- use Pascal's triangle to evaluate $\binom{n}{r}$ and to find coefficients in a binomial expansion
- evaluate and simplify expressions with factorial notation
- evaluate $\binom{n}{r}$ by using $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$
- use the binomial theorem to find terms in the expansion of $(a+b)^n$.

1. Number and algebra / 1.9 The binomial theorem

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-27691/print/) Assign

Use the binomial theorem to expand $(3x+y)^4$.

Hence, by choosing appropriate values for x and y, find the exact value of 3.02^4. Compare your answer to the one given by your calculator.

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Use a similar approach to find the exact value of 2.97^4.

Explain how your findings can be used to obtain the exact value for any a^n where a is a rational number.

Rate subtopic 1.9 The binomial theorem

Help us improve the content and user experience.

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