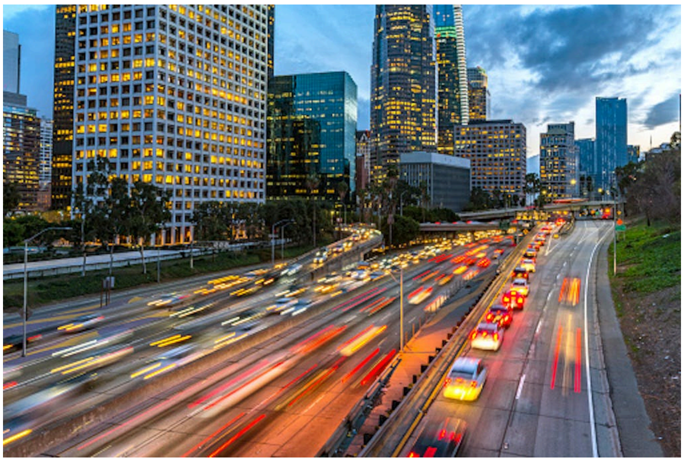
The big picture

Traffic planning is a major concern in many of the larger cities of the world. As the number of vehicles continues to increase, a city must continue to monitor traffic and determine where new traffic lights, new traffic lanes, and other traffic controls are needed.

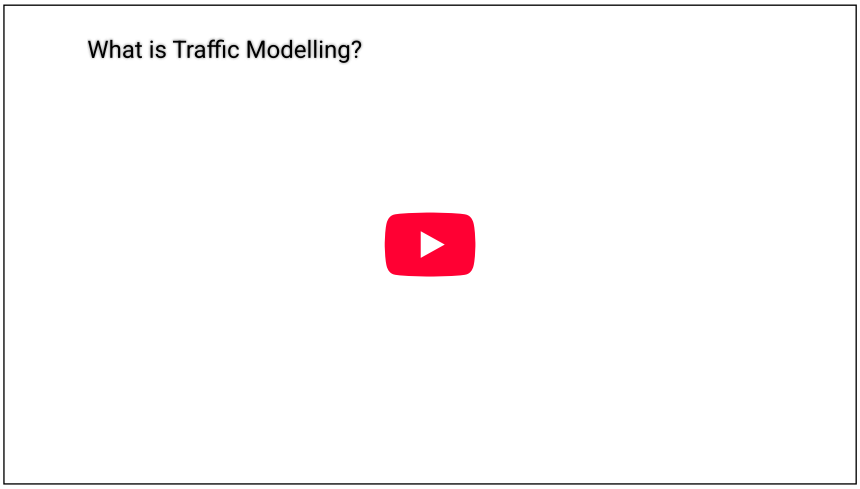


Traffic planning is a major concern for cities

Credit: 4kodiak Getty Images

How can a city determine whether a particular intersection is in need of a traffic light? How can it determine whether a road needs to be widened to add a new lane? These are questions that require traffic engineers to consider the number of cars that travel through an intersection or on a road in a given amount of time. By modelling the traffic in terms of statistical distributions, it is possible to calculate the probability that the level of traffic through the intersection or along the road is greater than a predetermined limit.

Watch this video to learn about the modelling approach taken by a traffic-planning project in Melbourne, Austrailia.



Concept

Probability distributions can provide a representation of the relationship between the theory and reality. These representations can help us make predictions about what might happen.

The Poisson distribution

The Poisson distribution for number of occurrences of an event

As you saw in subtopic 4.8 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-26114/), binomial distributions can be very helpful when modelling an event that has only two possible results and is repeated a number of times. Let us look at an example as a quick review.

Example 1

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Coraline owns a red racing bicycle and a blue road bicycle. She only rides one of the bicycles each day. On any given day there is a 70% chance that she will ride the blue bicycle.

Calculate the expected number of days in which Coraline will ride the red bicycle in one year.

Begin by identifying the important parameters:

$$n = 365 \text{ and } p = 0.3 \therefore X \sim B(365, 0.3)$$

Then substitute into the formula:

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

$$\begin{aligned}\mathrm{E}(X) &= np \\ &= 365 \times 0.3 \\ &= 109.5\end{aligned}$$

Making connections

Recall from section 4.8.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-binomial-distribution-id-26115/) that the expectation of a binomial distribution is found using the formula $\mathrm{E}(X) = np$, where n is the number of trials and p is the probability of success.

A key aspect of the binomial distribution is that it is applied to situations that have a finite number of possible outcomes. In **Example 1**, there were a total of 365 days on which Coraline could have ridden the red bicycle. However, what if you want to consider situations that have an infinite number of possible outcomes? For those types of situations, you can use the Poisson distribution.

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If there is a random variable that describes the occurrence of the number of events in a random process and we know the average occurrence of that random variable, then we have a Poisson distributed random variable.

Important

For a Poisson distribution to be used, an event of the variable must be equally likely to occur in any interval (which may be space or time) and the occurrences of the event must be independent of each other.

Two assumptions are made when determining that a random variable follows a Poisson distribution. First, each event must be independent of the others. For example, in the example above it is assumed that the bike Coraline rides on any given day is independent of the bike she rode on the previous day (or any other day). Second, the mean value of the Poisson distribution is a number based on data gathered over a period of time: it is based on historical data.

In the applet below, you can explore the distributions of random variables that follow these two assumptions. The applet draws a histogram of a random sample of size n drawn from a Poisson distribution with mean μ.

- Change the mean, μ, to see how the shape of the histogram changes.
- Experiment with different sample size, n.

When moving the sliders for the mean or the sample size, a new random sample is generated.

- You can investigate experimental probabilities by changing the values of a and b. Dividing the number given on the applet by the sample size, n, you can estimate the probability $\mathrm{P}(a \leq x \leq b)$.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Interactive 1. Investigating Experimental Probabilities.

Credit: GeoGebra (https://www.geogebra.org/m/n7d9qwtt) Heather Pierce

More information for interactive 1

This interactive provides a hands-on way to explore the Poisson distribution, which models the occurrence of rare events over a fixed interval of time or space. The interface includes a histogram and four adjustable sliders that control the mean (μ), sample size (n), and a custom interval defined by values a and b. The histogram represents the distribution of simulated data, with red bars highlighting the number of outcomes that fall within the user-defined range from a to b, and blue bars indicating values that fall outside this range. As users adjust the mean μ (ranging from 1 to 50), they observe how the distribution's shape evolves—from a steep, right-skewed form for low μ values to a more symmetric, bell-shaped curve as μ increases, illustrating how the Poisson distribution begins to resemble a normal distribution. Increasing the sample size n (from 1 to 1000) leads to smoother histograms that more closely align with theoretical expectations. The sliders for a and b (where $0 \leq a \leq b < 52$) allow users to specify a range of interest, and the applet displays the count of data points within that interval. This enables users to estimate the probability of an event falling between a and b by dividing the count by the sample size n. For example, if $\mu = 23$, $a = 19$, $b = 46$, and $n = 1000$, the tool might show that 798 outcomes fall in this range, giving an empirical probability of $798/1000 = 0.798$. Each change in the sliders generates a new random sample, providing an engaging, real-time simulation that helps users visualize statistical variability, understand how parameters affect the distribution, and intuitively grasp the meaning of probabilities like $P(a \leq X \leq b)$.

Given the limitations placed on the distribution by the assumptions mentioned above, how well can the Poisson distribution model the real world?

Consider the formula used to calculate probabilities for a Poisson distributed variable:

$$\text{If } X \sim Po(m) \text{ then } \mathrm{P}(X = x) = \frac{m^x \mathrm{e}^{-m}}{x!} \text{ where } m \text{ is the average rate of occurrence.}$$

Be aware

As the Poisson distribution is based on the number of occurrences of an event, the value of x will always be a whole number greater than or equal to zero.

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Exam tip

You will not be expected to use this equation on assessments as all Poisson calculations can be carried out using your calculator.

The average occurrence rate, m, is the only parameter needed for the Poisson distribution. Take note of the similarity between the value of m and the expectation E of a binomial distribution. This relationship can be explored further in the investigation for this subtopic.

✓ **Important**

If a random variable X is distributed such that $X \sim Po\,(m)$ then $\mathrm{E}\,(X) = \mathrm{Var}\,(X) = m$. In other words, the expected value and variance of a Poisson-distributed variable are both equal to the value of m .

As m is a rate, when determining whether to use the Poisson distribution, it is important to identify the average occurrence of the random variable and what the change in the variable depends on. The table below gives some examples.

Random process	Event
Visitors to a museum in a time interval	Number of visitors per hour time-dependent
Arrival of migratory birds	Number of birds touching down per hour time-dependent
Flaws in laying a railway line	Number of flaws per kilometre length-dependent
Red blood cell count	Number of red blood cells per 100 ml volume-dependent

Example 2

★★☆

Akinyi is investigating whether a certain intersection needs to have a traffic light installed. She has found that the average number of cars going through the intersection every minute during a work day is 33.2 cars. A traffic light should be installed if the probability of there being more than 40 cars per minute in the intersection is greater than 10%.

Determine if a new traffic light should be installed at the intersection that Akinyi is investigating.

First, identify the parameter:

$$m = 33.2 \therefore X \sim \mathrm{Po}\,(33.2).$$

Overview
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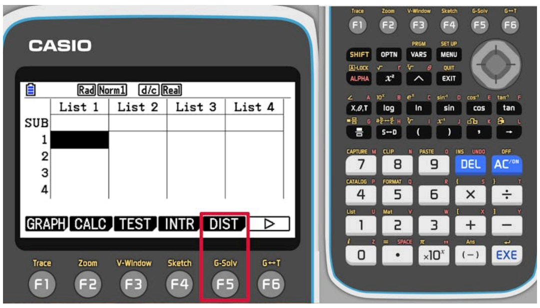
Then calculate the probability using your calculator using PoissonCDF since you are looking for the probability of a range.

$$\begin{aligned} \mathrm{P}\left(X>40\right) &= 1-\mathrm{P}\left(X\leqslant 40\right) \\ &= 1-0.894777\ldots \\ &= 0.105222\ldots \\ &\approx 10.5\% \end{aligned}$$

$\therefore$ a traffic light should be installed.

Steps	Explanation
In these instructions you will see how to use the calculator to find $\mathrm{P}(X \leq 40)$, where the random variable X follows a Poisson distribution with mean $m = 33.2$. To start, open the statistics mode ...	
... presss F5 to choose to work with distributions ...	

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Steps	Explanation
... press F6 to scroll to see more options ...	
... and press F1 to access the tools related to Poisson distribution.	
To calculate cumulative probabilities, press F2.	

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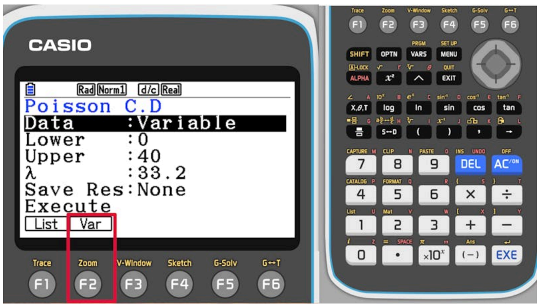
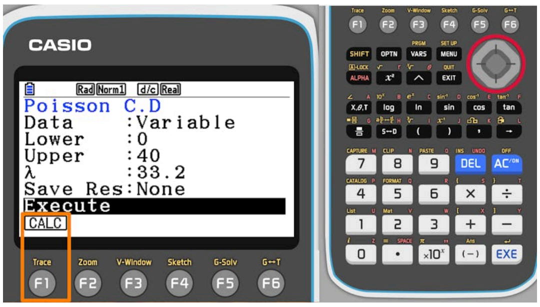
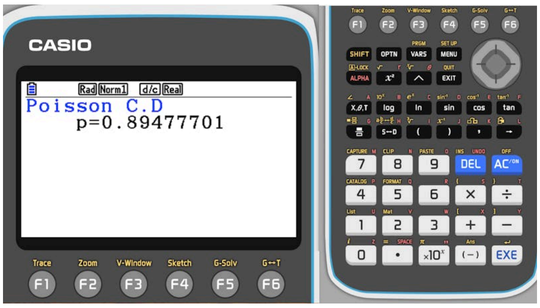
Steps	Explanation
The calculator is now waiting for information about the specific question. In the first line press F2 to indicate, that you work with random variables.	
This calculator can find probabilities of the form $\mathrm{P}(a \leq X \leq b)$. • a is the lower bound. In cases, when there is no lower bound (like in this example), enter 0. • b is the upper bound • λ is the mean of the distribution. When you entered all information, scroll to the last line and press F1 to calculate the probability.	
On the result screen you can see the probability. $$\mathrm{P}(X \leq 40) = 0.8947770\ldots$$	

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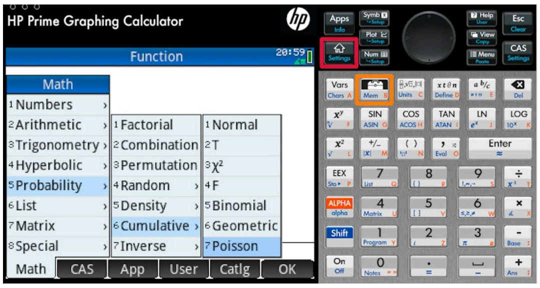
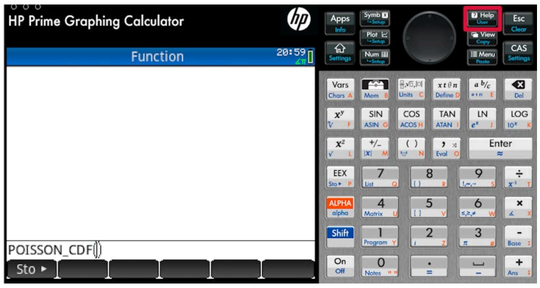
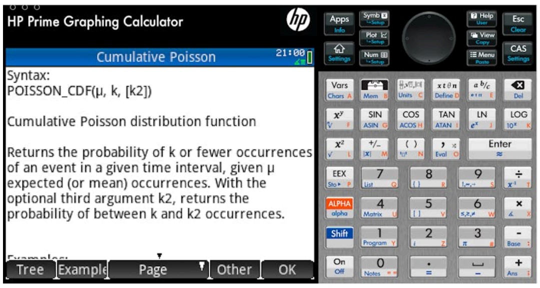
Steps	Explanation
In these instructions you will see how to use the calculator to find $$\mathrm{P}(X \leq 40),$$ where the random variable X follows a Poisson distribution with mean $m = 33.2$. On the home screen of any application open the toolbox and look for the cumulative Poisson distribution option.	
The calculator is now waiting for information about the specific question. If you do not remember how to enter the parameters, open the help screen.	
This calculator can find probabilities either of the form $\mathrm{P}(X \leq k)$ or of the form $\mathrm{P}(k \leq X \leq k2)$. In both cases, the mean of the distribution is the first argument you need to enter.	

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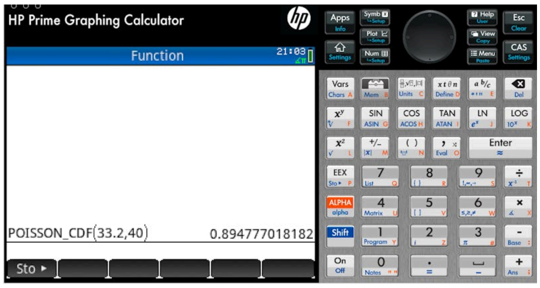
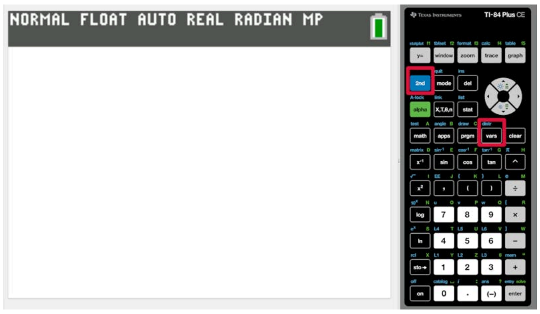
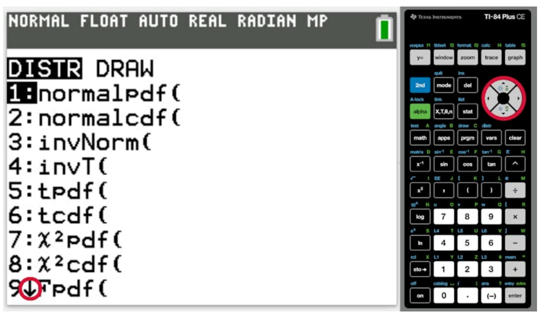
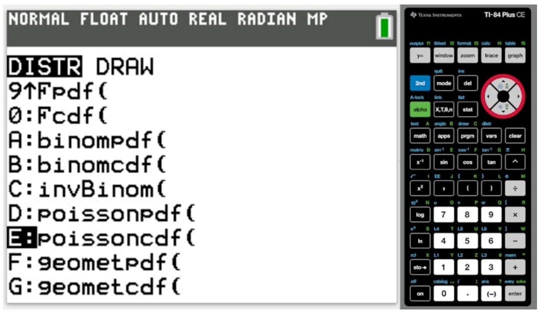
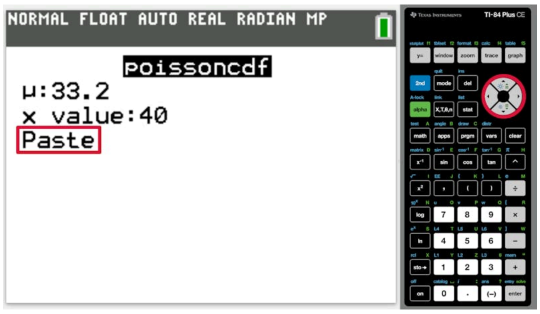
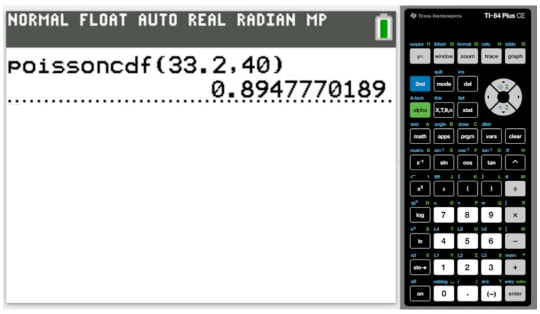
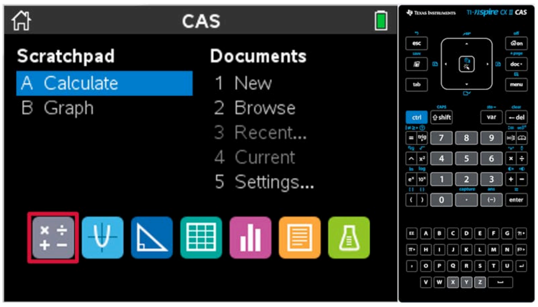
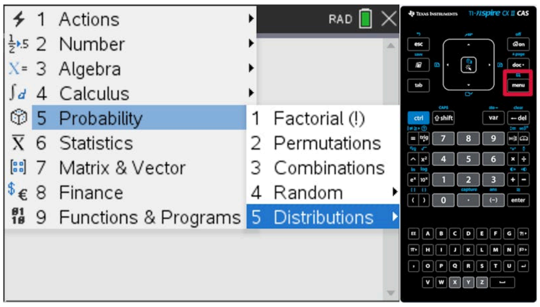
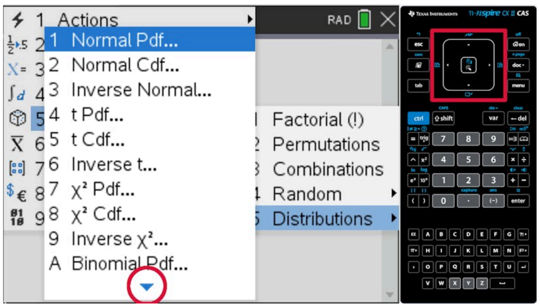
Steps	Explanation
On the result screen you can see the probability. $$\mathrm{P}(X \leq 40) = 0.8947770\ldots$$	

Steps	Explanation
In these instructions you will see how to use the calculator to find $$\mathrm{P}(X \leq 40),$$ where the random variable X follows a Poisson distribution with mean $m = 33.2$. To start, open the distribution menu ...	
... and look for options related to Poisson distribution. These are not on the first screen, so you need to scroll down.	

Student view

Steps	Explanation
Choose the option to work with cumulative Poisson distributions (poissoncdf).	
The calculator is now waiting for information about the specific question. This calculator can find probabilities of the form $\mathrm{P}(X \leq x)$. • μ is the mean of the distribution. • x is the upper bound When you entered all information, scroll to the last line and press enter to calculate the probability.	
On the result screen you can see the probability. $$\mathrm{P}(X \leq 40) = 0.8947770\ldots$$	

Steps	Explanation
In these instructions you will see how to use the calculator to find $$\mathrm{P}(X \leq 40),$$ where the random variable X follows a Poisson distribution with mean $m = 33.2$. To start, open a calculator page.	
Open the menu and choose to work with probability distribution ...	
... and look for options related to Poisson distribution. These are not on the first screen, so you need to scroll down.	

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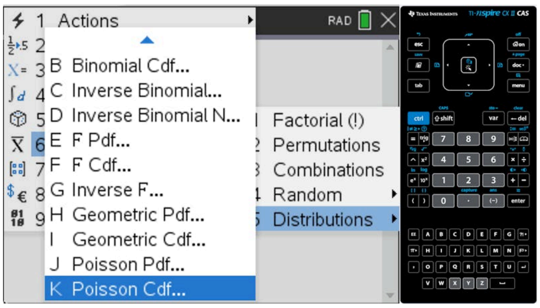
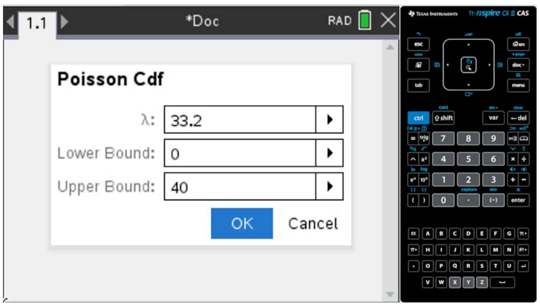
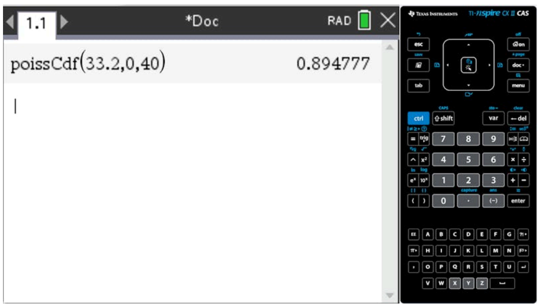
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>Choose the option to work with cumulative Poisson distributions.</td><td>
</td></tr>
<tr><td>The calculator is now waiting for information about the specific question.

This calculator can find probabilities of the form $\mathrm{P}(a \leq X \leq b)$.
λ is the mean of the distribution.
a is the lower bound. In cases, when there is no lower bound (like in this example), enter 0.
b is the upper bound
When you entered all information, scroll down and press OK to calculate the probability.</td><td>
</td></tr>
<tr><td>On the result screen you can see the probability.

$$\mathrm{P}(X \leq 40) = 0.89477\ldots$$</td><td>
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When planning new roads, cities and counties often need to consider the capacity of their existing roads. As discussed in this article (https://www.australasiantransportresearchforum.org.au/sites/default/files/2007_Laufer.pdf) from the 30th Australasian Transport Research Forum, one factor that the capacity of a road depends on is the speed at which cars are allowed to travel. The maximum speed allowed on roads is determined by each country individually and varies between countries.

What other factors do you think influence a country's decision on the maximum speed it will allow on its roads?

✓ **Important**

As the events in a Poisson distributed variable occur uniformly, they are proportional to their interval and, therefore, the value of m can be scaled to fit the new interval.

Example 3

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Visitors to a museum arrive on average at a rate of 2 per minute and follow a Poisson distribution. Find the probability that 12 visitors arrive in the first 10 minutes after noon.

First, identify the parameter:

Since the average number of visitors is 2 per minute, then the average for 10 minutes would be $2 \times 10 = 20 \therefore X \sim \text{Po}\,(20)$.

Then calculate the probability using PoissonPDF on your calculator, because you are looking for the probability of a single outcome:

$$\begin{aligned} \text{P}\,(X = 12) &= 0.0176251 \\ &\approx 0.0176. \end{aligned}$$

Adding two Poisson distributions

✓ **Important**

The sum of two independent Poisson distributions will also have a Poisson distribution. This is similar to what you saw in section 4.15.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/linear-combinations-of-normally-distributed-random-id-27546/) with normal distributions.

The sum of the means of the two independent Poisson distributions will be equal to the mean of the summed distribution.

Example 4

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Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

For a certain railway it is found that the flaws on the tracks from installation occur following a Poisson distribution. Due to the method used to install the track, there are an average of five flaws per kilometre on the right rail of the track and only two flaws per kilometre on the left rail of the track. In planning for a new 20-kilometre long track, the company wants to budget enough money for repairs so there is at least a 95% probability they will be able to repair all of the flaws on the new track.

Determine how many flaws the company should budget for.

First, identify the parameter:

Since both rails follow a Poisson distribution then their averages can be added together for the total mean of both sides of the track

$$\therefore m = 5 + 2 = 7\,.$$

Since m is the flaws per kilometre, you must multiply by 20 to get the average number of occurrences for the entire track:

$$7 \times 20 = 140 \therefore X \sim \text{Po}\,(140).$$

Next, use your calculator to find the number of flaws for which the company should budget.

The company should budget for the repair of 160 track flaws since

$$\text{P}\,(X \leqslant 159) = 0.948 \text{ and } \text{P}\,(X \leqslant 160) = 0.956\,.$$

Steps	Explanation
In these instructions you will see how to find the smallest k such that $\text{P}(X \leq k) \geq 0.95,$ where the random variable X follows a Poisson distribution with mean 140. • There are two ways this can be done. Either create a sequence of cumulative probabillities and see when this sequence gets above 0.95. • Or use the inverse Poisson option on the calculator. You will see here how to use the second method. Open the statistics mode ...	

Student view

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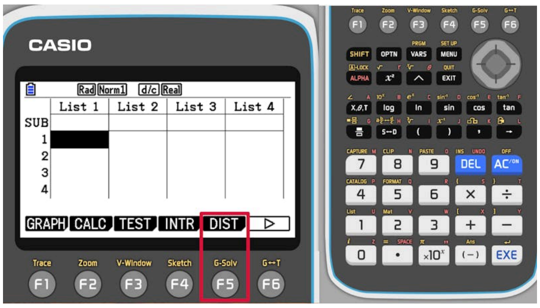
Steps	Explanation
... presss F5 to choose to work with distributions ...	
... press F6 to scroll to see more options ...	
... and press F1 to access the tools related to Poisson distribution.	

Student
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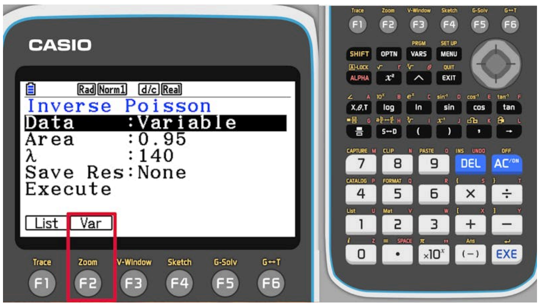
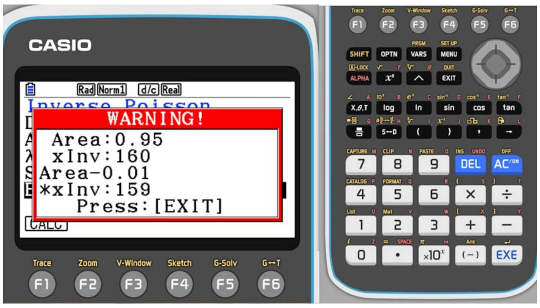
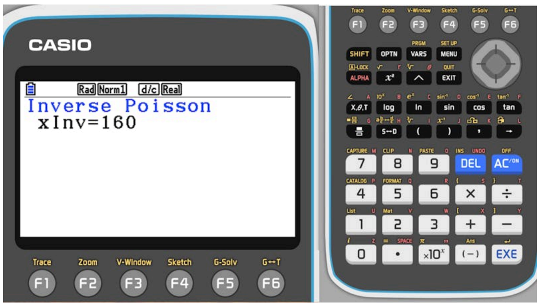
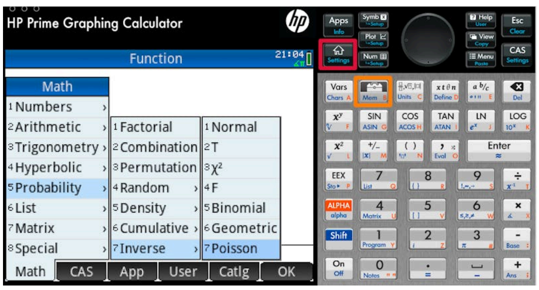
Overview (/study/ap ai-hl/sid-132-cid-761618/ov

Steps	Explanation
To work with the inverse distribution, press F3.	
The calculator is now waiting for information about the specific question. In the first line press F2 to indicate, that you work with random variables.	
You also need to tell the target probability (area) and the mean (λ) of the random variable. When you entered all information, scroll to the last line and press F1 to calculate the bound.	

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Before you can see the result, you might get a warning message if there is very little difference between neighbouring probabilities.	
On the answer screen you can see the smallest value of k such that $\mathrm{P}(X \leq k) \geq 0.95.$ In this example the result means, that $\mathrm{P}(X \leq 160) \geq 0.95$, but $\mathrm{P}(X \leq 159) < 0.95.$	

Steps	Explanation
In these instructions you will see how to find the smallest k such that $\mathrm{P}(X \leq k) \geq 0.95,$ where the random variable X follows a Poisson distribution with mean 140. • There are two ways this can be done. Either create a sequence of cumulative probabillities and see when this sequence gets above 0.95. • Or use the inverse Poisson option on the calculator. You will see here how to use the second method. On the home screen of any application open the toolbox and look for the inverse Poisson distribution option.	

Student view

Overview
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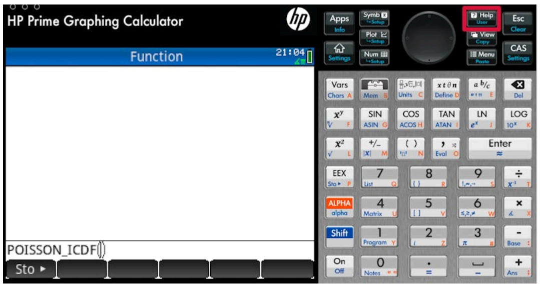
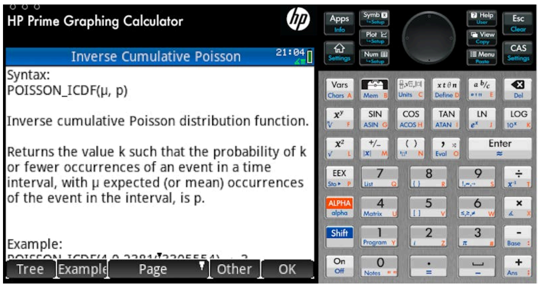
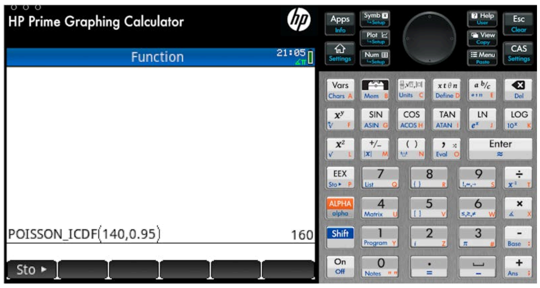
Steps	Explanation
The calculator is now waiting for information about the specific question. If you do not remember how to enter the parameters, open the help screen.	
You need to tell the mean of the distribution (μ) and the target probability (p) in this order.	
On the answer screen you can see the smallest value of k such that $\mathrm{P}(X \leq k) \geq 0.95.$ In this example the result means, that $\mathrm{P}(X \leq 160) \geq 0.95$, but $\mathrm{P}(X \leq 159) < 0.95.$	

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<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>

In these instructions you will see how to find the smallest k such that

$$\mathrm{P}(X \leq k) \geq 0.95,$$

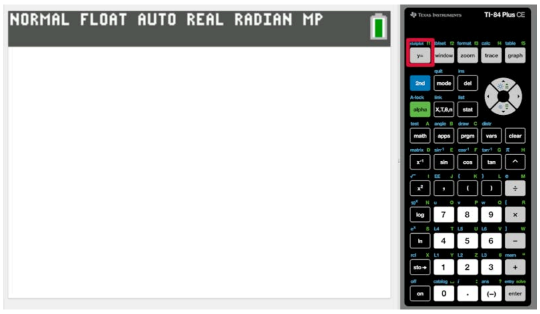
where the random variable X follows a Poisson distribution with mean 140.

- There are two ways this can be done. Either create a sequence of cumulative probabillities and see when this sequence gets above 0.95.
- Or use the inverse Poisson option on the calculator.

At the time of writing this instruction this calculator does not have an inverse Poisson option, so you will see the work using the first option.

Open the function editing mode.

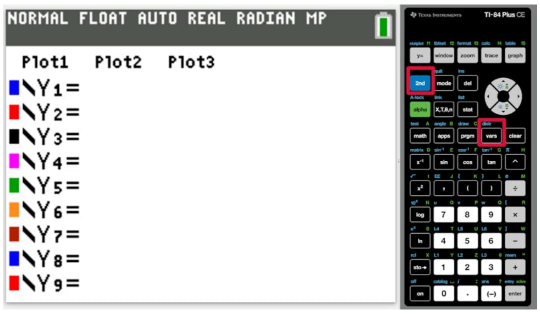
</td><td>



</td></tr>
<tr><td>

You will need to define a function that calculates cumulative Poisson probabilities, so open the distribution menu ...

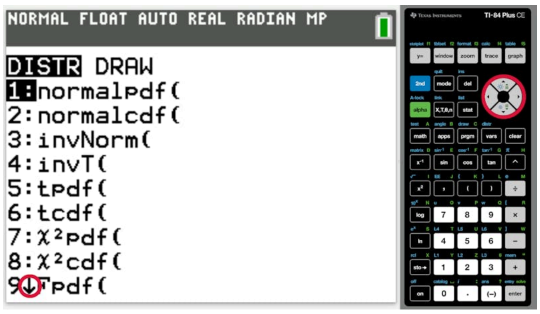
</td><td>



</td></tr>
<tr><td>

... and look for options related to Poisson distribution. These are not on the first screen, so you need to scroll down.

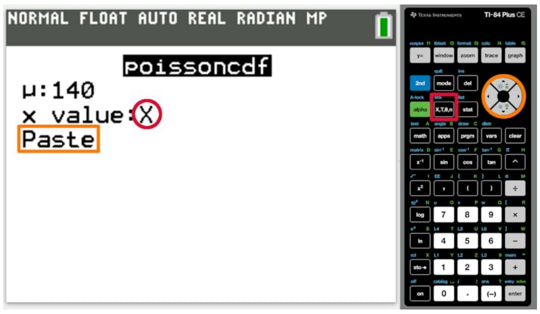
</td><td>



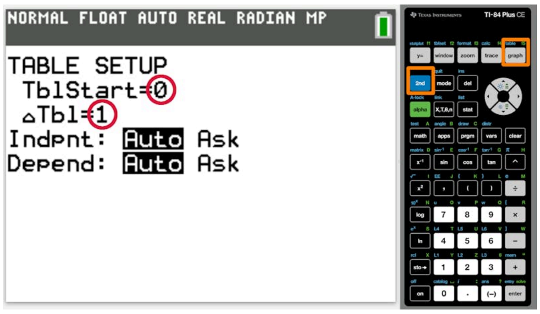
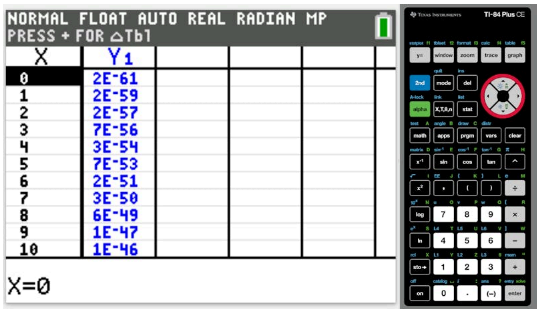
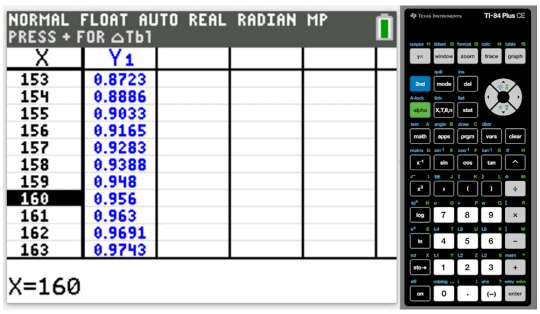
</td></tr>
</table>

Student
view

Overview (/study/ap ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Choose the option to work with cumulative Poisson distributions (poissoncdf).	
The calculator is now waiting for information about the specific question. This calculator can find probabilities of the form $\mathrm{P}(X \leq x)$. • μ is the mean of the distribution. • x is the upper bound. In this case you are defining the probability as a function of the upper bound, so enter the variable x in this line. When you entered all information, scroll to the last line and press enter.	
The function is now defined. You will need the cumulative probabilities for the different values of x so that you can choose the smallest x where the probability gets above 0.95. Instead of graphing, you can do this by investigating the table of values. First let's check the table properties.	

Student view

Steps	Explanation
You can set the starting x-value and the step size in the table. By setting the start value to 0 and the increment to 1 you will see the probabilities $$\mathrm{P}(X \le 0)$$ $$\mathrm{P}(X \le 1)$$ $$\mathrm{P}(X \le 2)$$ $$\ldots$$ Once you set the values, open the table.	
The first ten probabilities are very small. To look for probabilities above 0.95 you need to scroll down. If you find the increase in values too slow, you can adjust the table parameters any time.	
Eventually, you will see the smallest value of k such that $$\mathrm{P}(X \le k) \ge 0.95.$$ In this exmaple, $k = 160$.	

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>

In these instructions you will see how to find the smallest k such that

$$\mathrm{P}(X \leq k) \geq 0.95,$$

where the random variable X follows a Poisson distribution with mean 140.

- There are two ways this can be done. Either create a sequence of cumulative probabillities and see when this sequence gets above 0.95.
- Or use the inverse Poisson option on the calculator.

At the time of writing this instruction this calculator does not have an inverse Poisson option, so you will see the work using the first option.

Open a calculator page.

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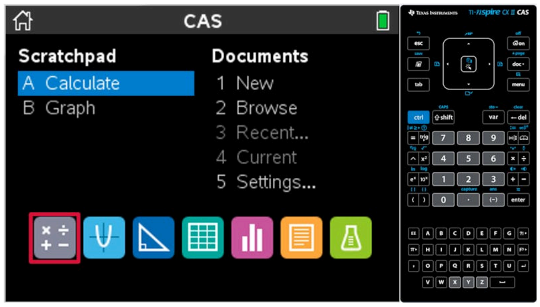

</td></tr>
<tr><td>

You will need to define a function that calculates cumulative Poisson probabilities. Give any name and use the colon equal sign for the definition.

</td><td>

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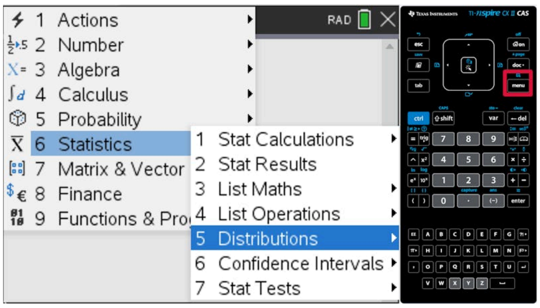
<tr><td>

To define the function, open the menu and choose to work with probability distribution ...

</td><td>



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Student
view

Overview
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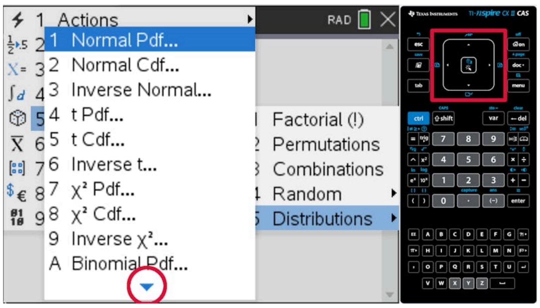
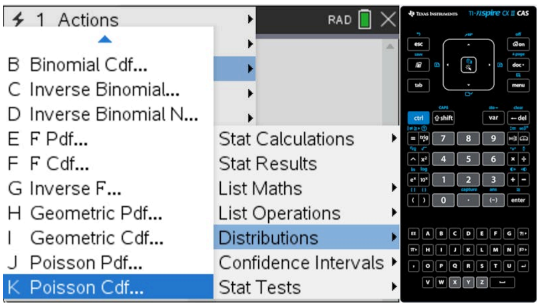
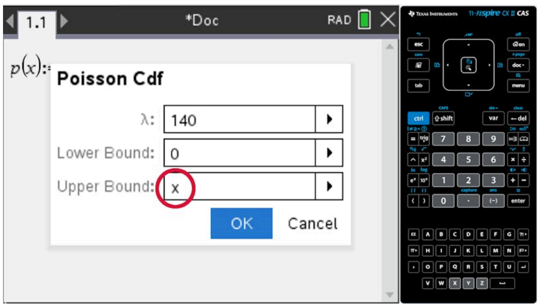
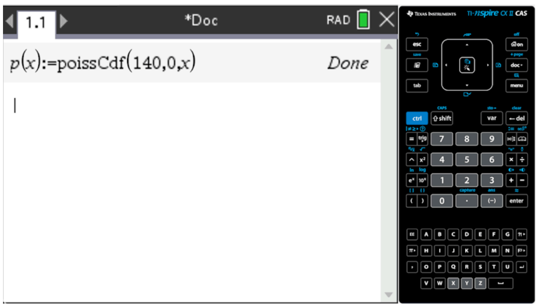
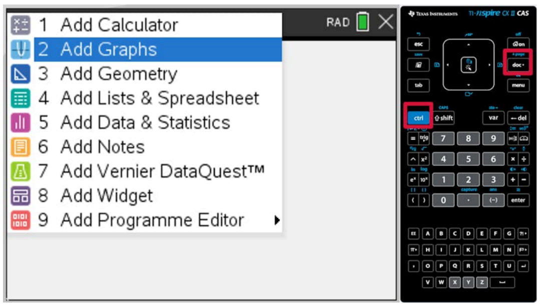
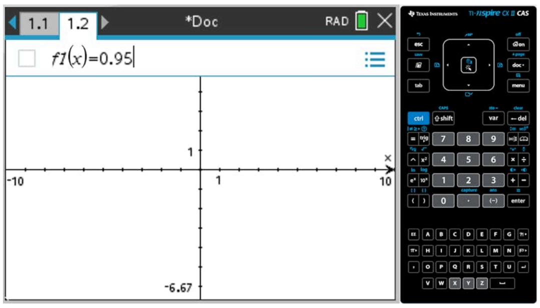
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>... and look for options related to Poisson distribution. These are not on the first screen, so you need to scroll down.</td><td>
</td></tr>
<tr><td>Choose the option to work with cumulative Poisson distributions.</td><td>
</td></tr>
<tr><td>The calculator is now waiting for information about the specific question.

This calculator can find probabilities of the form $\mathrm{P}(a \leq X \leq b)$.

• λ is the mean of the distribution.
• Use 0 as the lower bound.
• In this case you are defining the probability as a function of the upper bound, so enter the variable x in this line.

When you entered all information, scroll to OK and press enter.</td><td>
</td></tr>
</table>

Student
view

Steps	Explanation
The function is now defined. You will need the cumulative probabilities for the different values of x so that you can choose the smallest x where the probability gets above 0.95.	
You can access these values by plotting the graph, so open a graph page.	
You are looking for the a probability that is above 0.95, so define a function as a guide.	

Overview
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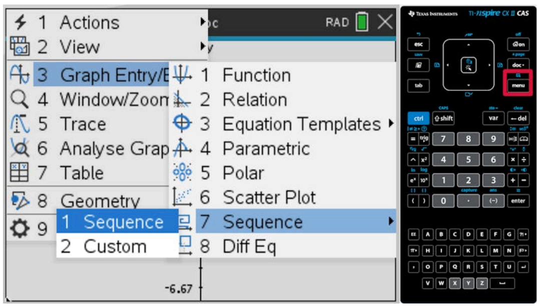
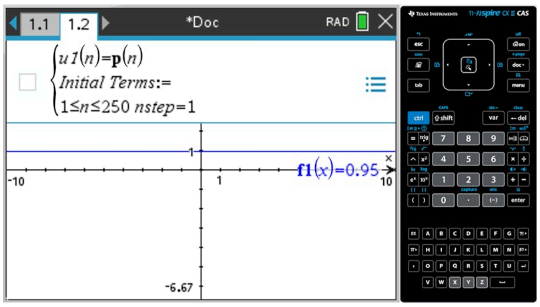
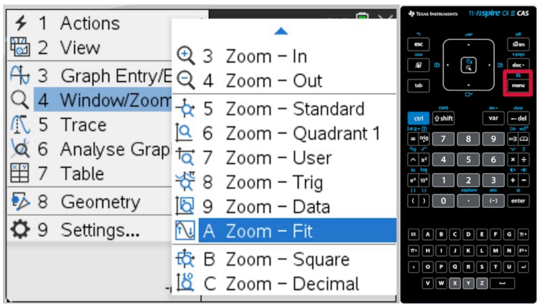
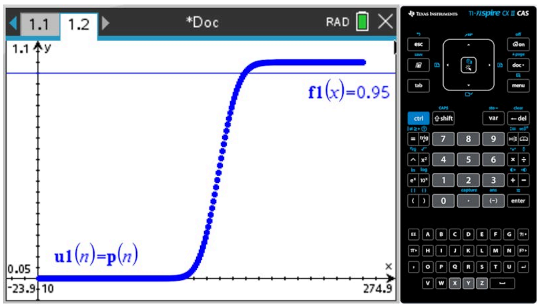
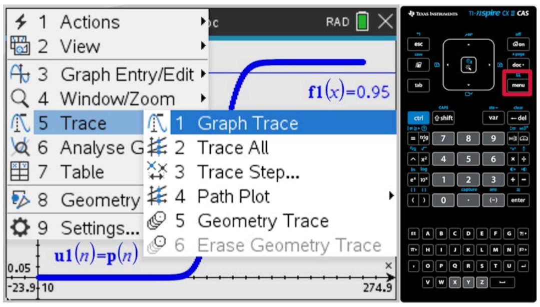
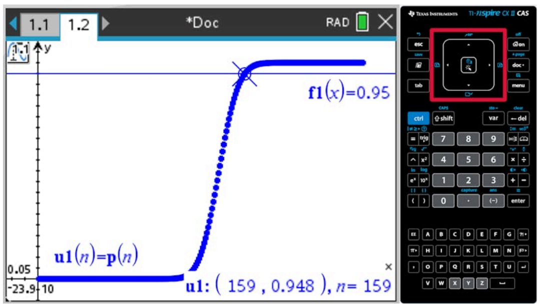
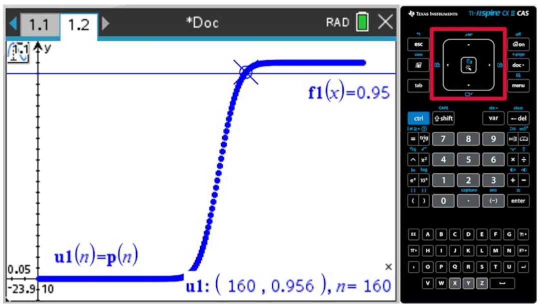
Steps	Explanation
You can add the probabilities to the graph by plotting a sequence.	
Use the name of the probability function you defined earlier. Set the bounds for the index. By setting the start value to 1 and the step size to 1 you will see the probabilities $\mathrm{P}(X \leq 1)$ $\mathrm{P}(X \leq 2)$ $\mathrm{P}(X \leq 3)$ $\ldots$ You may need to experiment with the upper bound.	
To see the plot, adjust the window from the menu.	

Student
view

Steps	Explanation
You can see, that the cumulative probabilities get above 0.95 at some point.	
To find which is the first term that gets above 0.95, open the menu and choose the option to trace the graph.	
If you move around, you will find the relevant points. On this screen you see, that $$\mathrm{P}(X \leq 159) = 0.948 < 0.95.$$	

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
On this screen you see, that $$\mathrm{P}(X \leq 160) = 0.956 \geq 0.95.$$ The combination of the last two screens gives you the smallest value of k such that $$\mathrm{P}(X \leq k) \geq 0.95.$$ In this exmaple, $k = 160$.	

1 section question

4. Probability and statistics / 4.17 The Poisson distribution

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-27994/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- state that a random variable follows a Poisson distribution when an event of the variable is equally likely to occur in any interval and the occurrences of the event are independent of each other
- calculate probabilities for a certain number of occurrences for a Poisson distributed random variable
- calculate the probabilities for a range of occurrences for a Poisson distributed random variable
- state that the value of m can be scaled to fit the interval needed
- state that the expectation and variance of a Poisson distributed variable are both equal to m
- find the probabilities of a combination of Poisson distributed variables by adding their respective values of m together to create a new Poisson-distributed variable.

4. Probability and statistics / 4.17 The Poisson distribution

Investigation

Student view

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-27995/print/) Assign

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

In this subtopic, you learned how the Poisson distribution is similar to the binomial distribution. Let us explore this relationship some more by considering the applet shown below.

Interactive 1. Relationship Between Poisson Distribution and Binomial Distribution.

More information for interactive 1

This interactive allows users to explore the relationship between the Poisson and Binomial distributions by adjusting key parameters and observing real-time changes in their graphical representations.

The screen is divided into two halves. The top half displays a histogram with an XY axis, where the x-axis ranges from 0 to 70 and the y-axis ranges from 0 to 0.4. In the bottom half of the screen, there are two checkboxes labeled "Poisson distribution" and "Binomial distribution," which enable the respective histograms on the graph. When the Poisson distribution is enabled, a horizontal slider appears on the right, allowing the user to adjust the mean (m) of the Poisson distribution, ranging from 0 to 50. Similarly, when the Binomial distribution is enabled, two horizontal sliders appear on the right: the first for the number of trials (n), ranging from 0 to 75, and the second for the success probability (p), ranging from 0 to 1. Below these sliders, there are two additional checkboxes labeled "Normal approximation of Poisson distribution" and "Normal approximation of Binomial distribution," which, when checked, display red and blue curves over their respective histograms.

Users can modify the mean (m) for the Poisson distribution and the number of trials (n) and success probability (p) for the Binomial distribution. As these values change, the tool visually demonstrates how the shapes of the two distributions converge when n is large and p is small, illustrating the key approximation condition $m = np$.

Additionally, users can compare these distributions with their Normal approximations, observing how increasing m (for Poisson) or np (for Binomial) leads to better alignment with the Normal curve. The activity reinforces the mathematical connection between the distributions while highlighting practical scenarios where the Poisson distribution serves as an efficient estimator for the Binomial distribution—particularly in cases of rare events (small p) and large sample sizes (n).

For example, when users sets the Poisson mean (m) to 12 and enables the Binomial distribution with parameters $n = 59$ and $p = 0.1$, they can observe how the two distributions nearly overlap, demonstrating the Poisson approximation to Binomial (since $m \approx np = 5.9$). When they enable the normal approximations, both the red (Poisson) and blue (Binomial) curves closely follow their respective histograms, showing how increasing m and np improve the fit. If the user then adjusts p to 0.5 while keeping $n = 59$, the Binomial distribution becomes symmetric, and its normal approximation remains accurate, while the Poisson distribution (still at $m = 12$) diverges, reinforcing that the Poisson approximation works best for rare events (small p). This hands-on comparison helps users visualize the conditions under which these distributions converge or diverge.

By experimenting with different parameter values, users gain an intuitive understanding of these statistical relationships and the role of approximations in simplifying real-world probability problems.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Activity

1. Explore how you can make the curves of the two distributions similar in shape by changing the parameters for each distribution.
2. Consider the values of the parameters once you have made the curves to be a similar shape. Is there a connection or pattern that can be found?

The connection between the parameters n and p for the binomial distribution and the parameter m for the Poisson distribution can be further developed by analysing the equation for both distributions.

$$\text{If } X \sim \mathrm{B}\left(n, p\right) \text{ then } \mathrm{P}\left(X = x\right) = \binom{n}{x} p^{n-x}(1-p)^{x}$$

$$\text{If } X \sim \mathrm{Po}\left(m\right) \text{ then } \mathrm{P}\left(X = x\right) = \frac{m^{x}e^{-x}}{x!}$$

By letting $m = np$, it can be shown algebraically that the formula for the binomial distribution can be transformed into that of the Poisson distribution. How does this connection of the parameters of the two distributions compare with your findings from the activity above?

Finally, the Poisson distribution can be used to estimate a binomial distribution when n is very large and p is small. Do those limitations of n and p match what you found in the above activity? Explore the benefits of using the Poisson distribution as an estimate of the binomial distribution.

Student view