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The big picture

Who is ‘tall’?

Credit: simoningate GettyImages

In subtopic 4.9 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25666/), you were introduced to the normal distribution, where $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$. You learned how to find the probability that a specific value of a random variable would fall within a certain range of values. While this is very useful, it does have its limitations. Consider trying to make a comparison between values from different normal distributions. For example, if someone's height is 185 cm , would they be considered tall, short or average? We generally accept the idea that people’s heights are normally distributed, but when trying to classify people as ‘tall’ or ‘short’, we are usually thinking of them within a demographic, such as

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gender, country of origin, age group or membership in an organisation. In this way, we can classify someone relative to the other members of a certain group – an average-height 3-year-old girl, a tall Indonesian or a short basketball player. While the heights of the members of each group are normally distributed, they clearly have different means and standard deviations.

International Mindedness

Standardising the normal distribution is one essential tool researchers use when comparing data from countries with very different demographics. For example, what qualifies as 'poverty' differs depending on a country's economic development, but standardising different distributions allows us to make comparisons even though the original data sets are very different.

One way we try to compare and categorise values like these is using percentiles. However, in order to find percentiles you have to use technology to find each value independently. Another way to make comparisons is by standardising the normal distribution using transformations. In this way, we develop the standard normal distribution, where $Z \sim \mathrm{N}(0, 1)$. In this subtopic, we will derive and explore this distribution and see how it can be used in a variety of ways.

Concept

The standard normal distribution is a good example of how all normal distributions have a certain relationship that allows them to be transformed into the standard form. Recognising these relationships enables us to generalise numerous specific distributions into one familiar one that allows us to find probabilities and make comparisons more easily.

Theory of Knowledge

Normalisation of random variables allows us to consider phenomena that appear randomly distributed as normally distributed. This certainly has implications in a variety of contexts such as economics, psychology and sociology and brings up key issues in perception — if we perceive the world one way but upon further consideration through an alternative method discover the world is actually another way, what does that say about our initial perception and conclusion?

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Knowledge Question: Does mathematical knowledge provide a complete understanding of phenomena?

4. Probability and statistics / 4.12 The standard normal distribution

Understanding Z~N(0,1)

Standardising the normal distribution

The probability of a value in a normal distribution changes depending on μ and σ , but if you think in terms of how many standard deviations away from the mean that value is, you have a standard way of treating the probability. You can use basic transformations of the data to accomplish this.

First, since we want to think in terms of the distance from the mean, we will transform the data using $x-\mu$ to make the standardised mean 0 . In this way, we convert $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ into $X \sim \mathrm{N}\left(0, \sigma^2\right)$. Remember that when you transform the data with addition or subtraction, the mean changes but the standard deviation remains the same (section 4.9.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-normal-distribution-id-25667/)).

Next, we need to change the unit of measure for our normal distribution into standard deviations rather than whatever the origins unit was. When we graph the standard normal curve, we want 1 unit on the horizontal axis to represent 1σ . This means we need $\sigma\ = 1$, so we will transform the data in the distribution by dividing all the values by the standard deviation. To avoid confusion, we change the variable to z and now have the standard normal variable (or z-score), given by $z=\frac{x-\mu}{\sigma}$.

This transforms $X \sim \mathrm{N}\left(0, \sigma^2\right)$ into $Z \sim \mathrm{N}(0,1)$, the standard normal distribution.

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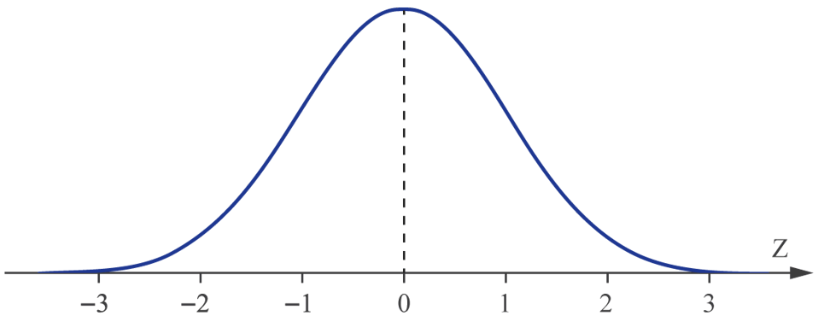

More information

The image depicts a standard normal distribution, also known as the bell curve. The X-axis represents standard units, ranging from -3 to 3. The Y-axis, labeled as Z, represents the probability density. The curve peaks at 0, indicating the mean of the distribution, and symmetrically tapers off as it approaches -3 and 3 on the X-axis. This curve is a graphical representation of the transformation of a normal distribution X~N(0, σ²) into Z~N(0, 1).

[Generated by AI]

Important

The standard normal variable z is also called the z -score.

$$z = \frac{x - \mu}{\sigma}$$

Below there is a dynamic applet, which shows that this property holds for several values of μ and σ and for several x, which are represented by the dynamic red point on the x-axis.

After setting μ and σ with the sliders, you can drag the point (which always lies between $\mu - 3\sigma$ and $\mu + 3\sigma$ for convenience) to a specific position and see that $\mathrm{P}(X < x)$ and $\mathrm{P}\left(Z < \dfrac{x - \mu}{\sigma}\right)$ are always equal.

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Interactive 1. Standard Normal Variable.

More information for interactive 1

This interactive applet allows users to change in the mean (μ) and standard deviation (σ) affect the shape and spread of a normal distribution curve. By adjusting sliders for (μ) (ranging from $-10$to 10) and σ (ranging from 0.1 to 5), users can observe real-time transformations of the blue normal distribution graph. The dynamic display demonstrates how increasing μ shifts the curve horizontally, while increasing σ flattens and widens it, reflecting greater data dispersion.

The applet reinforces the concept of standardization by showing how any normal distribution $X \sim N(\mu, \sigma^2)$ can be transformed into the standard normal distribution $Z \sim N(0, 1)$. As users modify μ and σ, the graph updates to highlight the relationship between raw values (x) and their corresponding z-scores $Z = \frac{x-\mu}{\sigma}$. This helps users understand how standardization preserves probabilities while simplifying comparisons across different distributions.

A key feature is the interactive red point, which users can drag along the x-axis (limited to $\mu \pm 3\sigma$ for practicality). The applet dynamically calculates and displays $P(X < x)$ and its equivalent $P(Z < z)$, proving their equality. This visual linkage between raw data and z-scores clarifies how standardization works, making abstract concepts tangible.

For example, setting $\mu = 4.6$ and $\sigma = 1.7$ (as in the provided image) and dragging the red point to $x = 5.5$ reveals $z = 0.53$ and $P(X < 5.5) = 0.7$. Users instantly see how the blue curve aligns with the standard normal distribution, with the shaded area under both curves matching, confirming that $P(X < 5.5) = P(Z < 0.53)$. This hands-on exploration demystifies statistical concepts like z-scores and cumulative probabilities.

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Example 1

★☆☆

If $X \sim \mathrm{N}\left(3, 0.4^2\right)$, find the z-scores for $x = 3.5$ and $x = 2.1$.

To find a z-score, use the formula $z = \frac{x - \mu}{\sigma}$, recognising that $\mu = 3$ and $\sigma = 0.4$.

For $x = 3.5$, $z = \frac{3.5 - 3}{0.4} = \frac{0.5}{0.4} = 1.25$. Since z is positive, this means that 3.5 is 1.25 standard deviations above the mean.

For $x = 2.1$, $z = \frac{2.1 - 3}{0.4} = \frac{-0.9}{0.4} = -2.25$. Since z is negative, this means that 2.1 is 2.25 standard deviations below the mean.

Example 2

★★☆

If $X \sim \mathrm{N}\left(3, 0.4^2\right)$, find the value of x that has a z-score $z = -0.8$.

Use the formula $z = \frac{x - \mu}{\sigma}$, recognising that $\mu = 3$ and $\sigma = 0.4$.

Substituting the values you know, you can solve the equation for x.

$$\begin{aligned} -0.8 &= \frac{x - 3}{0.4} \\ -0.32 &= x - 3 \\ x &= 2.68 \end{aligned}$$

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Finding probabilities with z -scores

In section 4.9.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-normal-distribution-and-calculator-functions-id-25668/), you learned how to use the normal cumulative distribution function (Normal CDF) on the calculator to calculate the probability that a random variable falls within a given range of values. You also learned to use complements and the symmetry within the normal curve to find other probabilities. Since the standard normal distribution is just a specific case of the normal distribution, you can find probabilities the same ways.

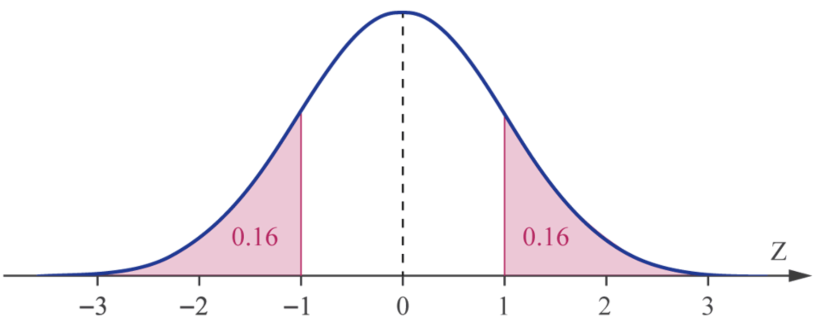
Example 3

★★☆

If $\mathrm{P}(z > 1) = 0.16$, find $\mathrm{P}(z < -1)$.

Since the normal curve is symmetrical about the mean and the mean for the standard normal distribution is 0 , you can use symmetry to find the solution (see diagram below):

$$\mathrm{P}(z < -1) = \mathrm{P}(z > 1) = 0.16 .$$



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Example 4

★★☆

For a standard normal distribution, find each probability:

a) $\mathrm{P}(-1.5 < z < 0.7)$

b) $\mathrm{P}(z < 1.1)$

c) $\mathrm{P}(z > 0.2)$

Use Normal CDF for each part, with $\mu = 0$ and $\sigma = 1$.

a) The lower bound is -1.5 and the upper bound is 0.7 , so the $\mathrm{P}(-1.5 < z < 0.7) \approx 0.691$.

b) The lower bound is negative infinity, so enter a low number like -999 into your calculator. The upper bound is 1.1 , so $\mathrm{P}(z < 1.1) \approx 0.864$.

c) The lower bound is 0.2 and the upper bound is positive infinity, so enter a large number like 999 . With this input, $\mathrm{P}(z > 0.2) \approx 0.421$.

⚠ Be aware

Make sure your answers make sense based on what you know about the normal distribution. Since half of the distribution is above $z = 0$ and half is below $z = 0$, you can easily determine that in **Example 4**, part (b) must have a probability greater than 0.5 and part (c) must be less than 0.5 .

Making connections

Previously we learned that approximately 68% of the data in a normal distribution falls within one standard deviation, 95% within two standard deviations, and 99.7% within three standard deviations. Using notation for the standard normal distribution, we have:

- $\mathrm{P}(-1 < z < 1) \approx 0.68$

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- $\mathrm{P}(-2 < z < 2) \approx 0.95$
- $\mathrm{P}(-3 < z < 3) \approx 0.997$

Activity

Draw the normal curve with divisions at one, two and three standard deviations. Use the 68% , 95% and 99.7% probabilities to label all the sections the curve is divided into.

Can you see any patterns in the probabilities?

4 section questions

Question 1

Difficulty: ★★☆

Assuming the temperatures in the month of June are normally distributed, if the Celsius temperature $T \sim \mathrm{N}\left(34, 2^2\right)$, find the value of z for a day when the temperature reaches $37°\mathrm{C}$.

Give only the number (no units) for your answer and enter it as a decimal.

1.5 ✓

Accepted answers

1.5, 1,5

Explanation

Since 37 is 3 degrees above the mean and $\sigma = 2$, $z = 3/2 = 1.5$.

You can also use the formula $z = \dfrac{x - \mu}{\sigma}$

$$z = \frac{37 - 34}{2} = \frac{3}{2} = 1.5$$

Question 2

Difficulty: ★★☆

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If $X \sim \mathrm{N}(\mathrm{e}, \ln 2)$, how many standard deviations is $X = 4.5$ from the mean?

1 2.14 ✓

2 2.57

3 1.40

4 1.35

Explanation

The number of standard deviations away from the mean is given by the z-score. For $X \sim \mathrm{N}(\mathrm{e}, \ln 2)$, i.e. normally distributed with $\mu = \mathrm{e} \approx 2.718$ and $\sigma = \sqrt{\ln 2} \approx 0.8326$, the z-score of $X = 4.5$ is given by

$z = \dfrac{4.5 - \mathrm{e}}{\sqrt{\ln 2}} = 2.14.$

Question 3

Difficulty: ★★☆

Given that $X \sim \mathrm{N}(16, 9)$, what is $\mathrm{P}(X < 10)$ in terms of the standard normal distributed variable $Z \sim \mathrm{N}(0, 1)$?

1 $\mathrm{P}(Z < -2)$ ✓

2 $\mathrm{P}\left(Z < -\dfrac{2}{3}\right)$

3 $\mathrm{P}(Z < 2)$

4 $\mathrm{P}\left(Z < \dfrac{2}{3}\right)$

Explanation

Since $Z = \dfrac{X - \mu}{\sigma}$ and if $X \sim \mathrm{N}(16, 9)$, then $\mu = 16$ and $\sigma = \sqrt{9} = 3$. Thus, for $X = 10$, we have that $Z = \dfrac{10 - 16}{3} = -2$. Hence, $\mathrm{P}(X < 10) = \mathrm{P}(Z < -2)$.

Question 4

Difficulty: ★★☆

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Consider the random variable X that represents the amount of time spent sleeping by a person. Assume that X is distributed normally with $\mu = 6.5$ hours and $\sigma = 0.4$ hours. Find the value of k, for which

$$\mathrm{P}(6.5 < X < k) = \mathrm{P}(0 < Z < 1.1),$$

where Z is the standard normal distribution.

6.94

Accepted answers

6.94, 6,94, k=6.94, k=6,94

Explanation

Use the standardization formula with the given mean and standard deviation:

$$z = \frac{x - 6.5}{0.4}.$$

When $x = 6.5$, then $z = 0$, so the lower bounds in the probabilities $\mathrm{P}(6.5 < X < k)$ and $\mathrm{P}(0 < Z < 1.1)$ correspond to each other.

Since these probabilities are equal, the upper bounds should also match. This gives an equation that you can solve for k.

$$\begin{aligned} 1.1 &= \frac{k - 6.5}{0.4} \\ 0.44 &= k - 6.5 \\ k &= 6.94 \end{aligned}$$

4. Probability and statistics / 4.12 The standard normal distribution

Using z-values to find mu and sigma

Using the Inverse Normal function with the formula for z

Student view

Another calculator function you learned to use in section 4.9.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-normal-distribution-and-calculator-functions-id-25668/) is the Inverse Normal, which gives you a value of x when you enter $\mathrm{P}(X < x)$, the mean and the

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standard deviation. Applying this to the standard normal distribution, you can find a value of z if you know $\mathrm{P}(Z < z)$, because you always know the mean and standard deviation.

For example, if $\mathrm{P}(Z < z) = 0.4$, you can use the Inverse Normal to find $z \approx -0.253$.

Example 1

★★☆

If you know $\mathrm{P}(0 < Z < z) = 0.25$, find the value of z .

First, you need $\mathrm{P}(Z < z)$ to use the Inverse Normal function on the calculator.

Since $\mu = 0$, you know that half of the normal distribution falls to the left of the mean. Therefore, $\mathrm{P}(Z < z) = 0.5 + 0.25 = 0.75$.

Now you can use the Inverse Normal on the calculator with

$\mu = 0$ and $\sigma = 1$ to find $z \approx 0.674$.

You can also use the Inverse Normal in conjunction with the formula for z to find the value of the mean or standard deviation of a normal distribution if one or both of them is unknown.

Example 2

★★☆

Suppose the mean amount of time it takes a swimmer to finish a race is 57 s. You know from the results of the race that 85% of the swimmers finish in less than 60 s. Assuming the finishing times of the swimmers are normally distributed, find the standard deviation of the finishing times.

Putting the information from the problem in mathematical form, this is what you know:

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If you let X represent the finishing time of a swimmer, $X \sim \mathrm{N}\left(57, \sigma^2\right)$.

$$\mathrm{P}(X < 60) = 0.85$$

Since you do not know the standard deviation of the finishing times, you will not be able to use your calculator to find the solution directly.

You can use the Inverse Normal function on the calculator with $\mu = 0$ and $\sigma = 1$ to find the z -score that corresponds to $x = 60$. If $\mathrm{P}(Z < z) = 0.85$, then $z = 1.03643338$.

Now you can write and solve an equation using the formula for z .

$$z = \frac{x - \mu}{\sigma}$$

$$1.03643338 = \frac{60 - 57}{\sigma}$$

$$\sigma = \frac{3}{1.03643338} \approx 2.89$$

Therefore, the standard deviation of the swimmers' finishing times is approximately 2.89 s.

Example 3

★★★

It is known that the time it takes for a train to travel from city A to city B follows a normal distribution. It is also given that 10% of the time the train arrives in 75 minutes or less, while 15% of the time the train takes 100 minutes or more to arrive at city B. Find the mean and the standard deviation of the distribution.

Since we are missing both μ and σ , we need to use the Inverse Normal function to find two values of z for the two probabilities we are given. Then we can find a system of equations to solve for both unknowns.

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Let X be the random variable of the time needed to travel from city A to city B. We know that $X \sim \mathrm{N}(\mu, \sigma^2)$. We also note that it is given that

$$\mathrm{P}(X \leq 75) = 10\% = 0.1$$

and

$$\mathrm{P}(X \geq 100) = 15\% = 0.15 \text{ or } \mathrm{P}(X < 100) = 1 - 15\% = 0.85.$$

Then use the standard normal distribution to solve

$$\begin{aligned} \mathrm{P}\left(Z \leq \frac{75-\mu}{\sigma}\right) = 0.1 \quad &\Leftrightarrow \quad \frac{75-\mu}{\sigma} = -1.28155, \quad \text{[GDC]} \\ \mathrm{P}\left(Z < \frac{100-\mu}{\sigma}\right) = 0.85 \quad &\Leftrightarrow \quad \frac{100-\mu}{\sigma} = 1.036433, \quad \text{[GDC]} \end{aligned}$$

which give in minutes

$$\begin{cases} 75-\mu = -1.28155\sigma \\ 100-\mu = 1.03643\sigma \end{cases} \Leftrightarrow \begin{cases} 75-\mu = -1.28155\sigma \\ 25 = 2.31798\sigma \end{cases} \Leftrightarrow \begin{cases} \sigma \approx 10. \\ \mu \approx 88. \end{cases}$$

Exam tip

Nearly any time you are given a problem involving the normal distribution that does not give you μ or σ , you will need to use Inverse Normal to find a z -score to use in the formula for z .

In Example 3 you have set up and solved a system of linear equations for μ and σ. This can be solved algebraically, but you can also use the calculator to solve an equation system like this. Graphic display calculators have application that can solve linear equation systems. Below you can see guidance on how to access this feature. The calculator can even solve systems with more than two unknowns, which might be useful in other problems.

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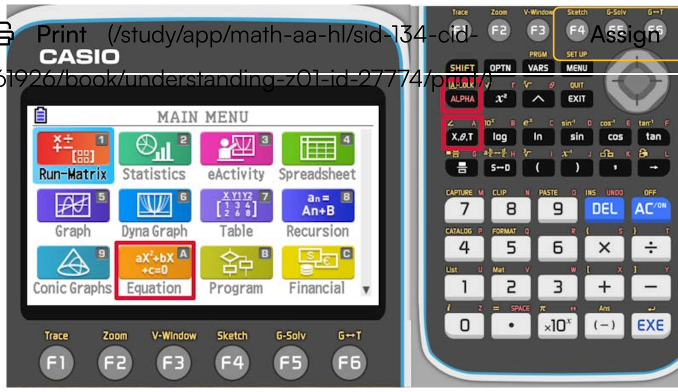
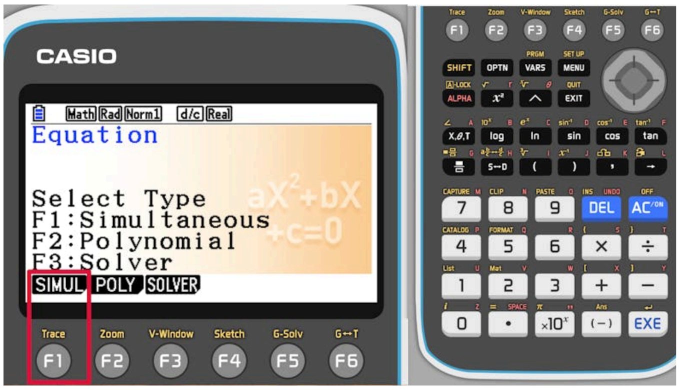
Assign

Step	Explanation
These instructions show you how to use the calculator to find the solution of the following system of equations. $$\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ 4x - 2y + 3z &= 5 \end{aligned}$$ Open the equation solving mode ...	
... and press F1 to choose the simultaneous equation solver option.	

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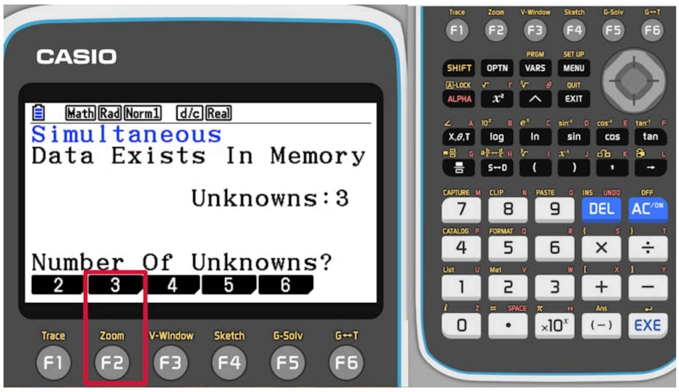
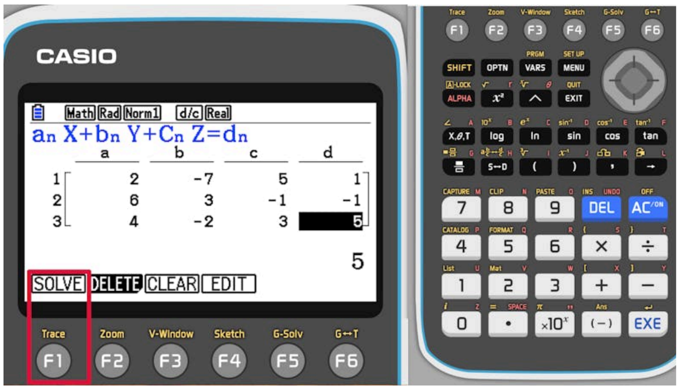
Step	Explanation
Set the number of equations (in this example, three) ...	
... and enter the coefficients. Once done, press F1 to ask the calculator to show you the solutions.	

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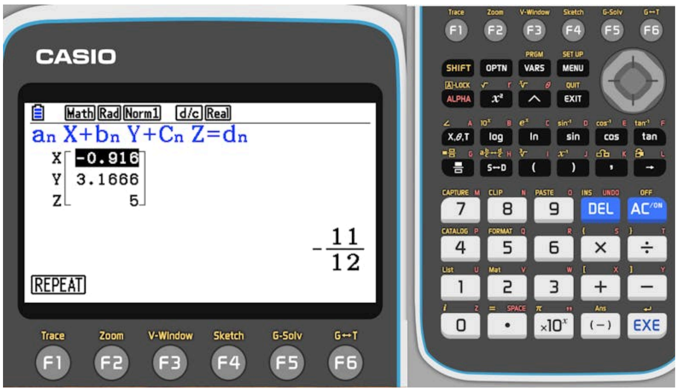
Step	Explanation
The calculator gives the approximate solution as decimals. If you move up and down, you can also view the exact value as a fractions.	

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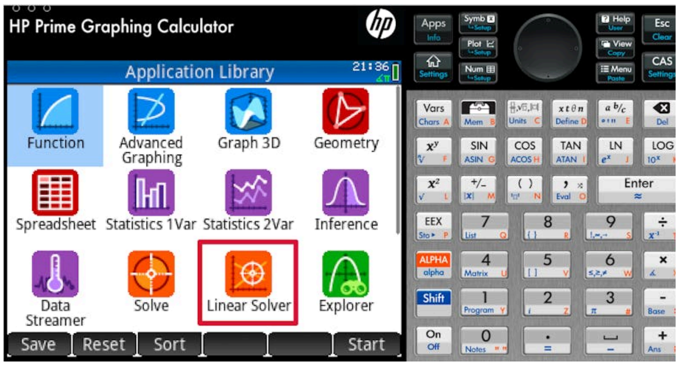
Step	Explanation
These instructions show you how to use the calculator to find the solution of the following system of equations. $$2x - 7y + 5z = 1$$ $$6x + 3y - z = -1$$ $$4x - 2y + 3z = 5$$ Open the linear solver application.	

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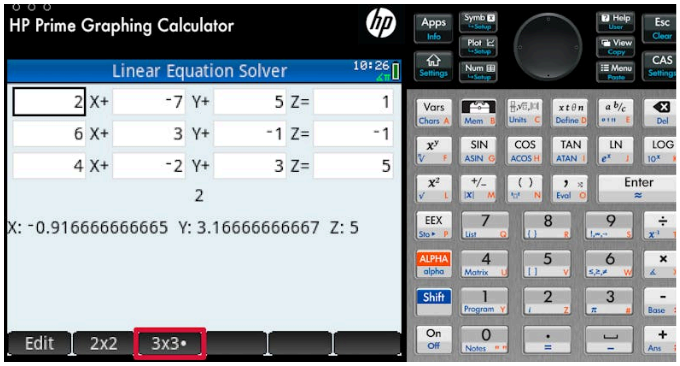
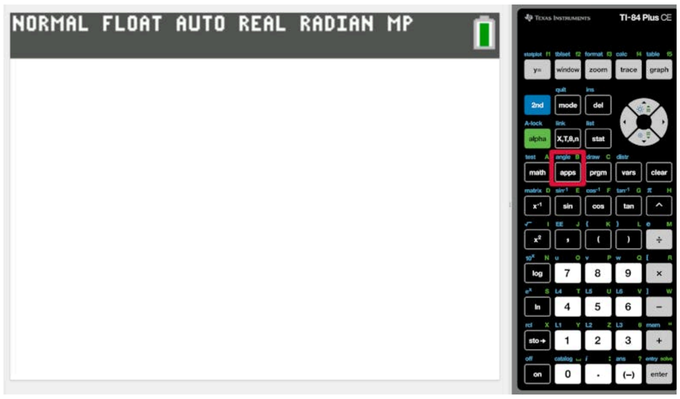
Step	Explanation
Choose the size of the system of equations (in this example three equations in three unknowns). Enter the coefficients. The calculator gives the solution in real time. It refreshes the solution set every time you change a coefficients	

Step	Explanation
These instructions show you how to use the calculator to find the solution of the following system of equations. $$\begin{aligned} 2x-7y+5z&=1\\ 6x+3y-z&=-1\\ 4x-2y+3z&=5 \end{aligned}$$ Open the application menu, ...	

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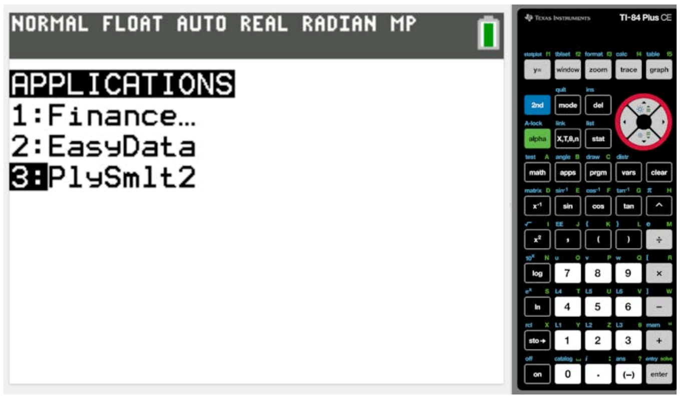
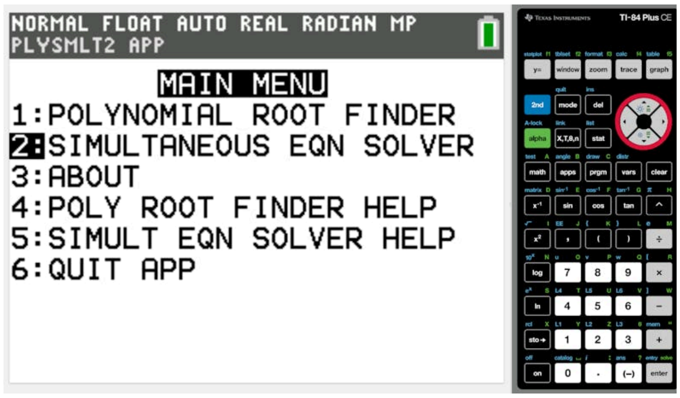
<tr><td>... choose the equation solving application (PlySmlt2) ...</td><td>
</td></tr>
<tr><td>... and choose the simultaneous equation solver option.</td><td>
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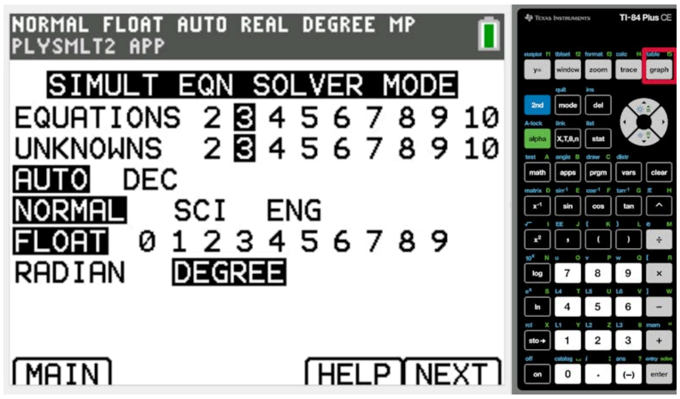
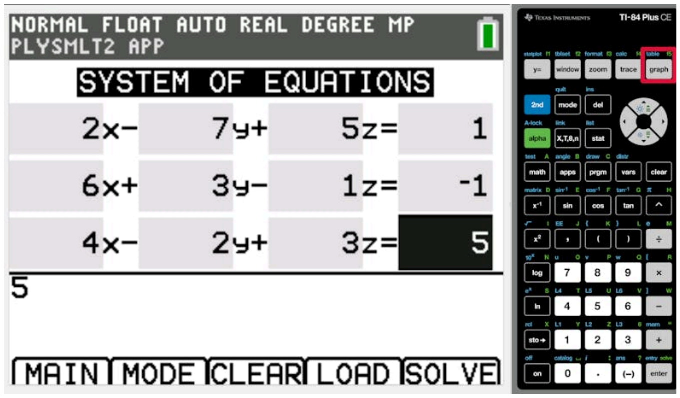
Step	Explanation
Choose the size of the system of equations (in this example three equations in three unknowns). Once done, press the graph button to move to the next screen	
Enter the coefficients. Once done, press the graph button to ask the calculator to show you the solutions.	

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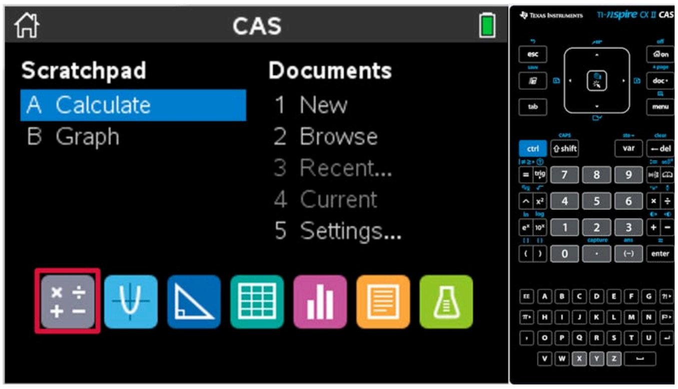
Step	Explanation
The calculator gives the solution as fractions.	

Step	Explanation
These instructions show you how to use the calculator to find the solution of the following system of equations. $2x-7y+5z=1$ $6x+3y-z=-1$ $4x-2y+3z=5$ Open a calculator page.	

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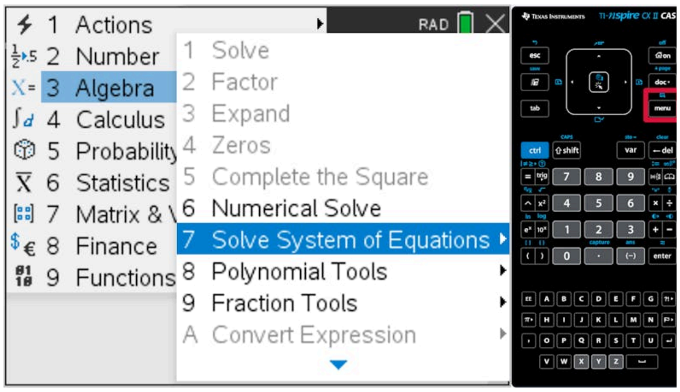
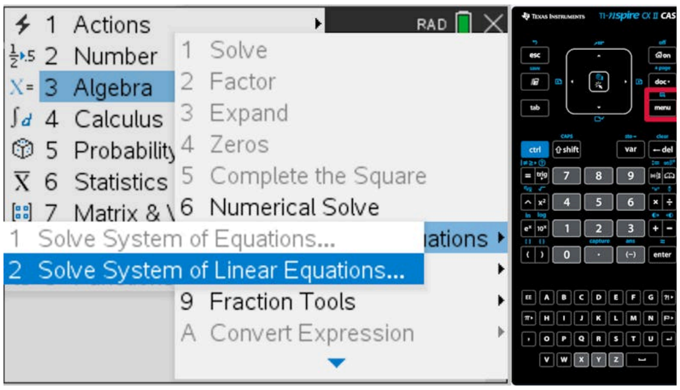
Step	Explanation
Open the menu, look for the option to solve system of equations ...	
... and choose the option to solve a system of linear equations.	

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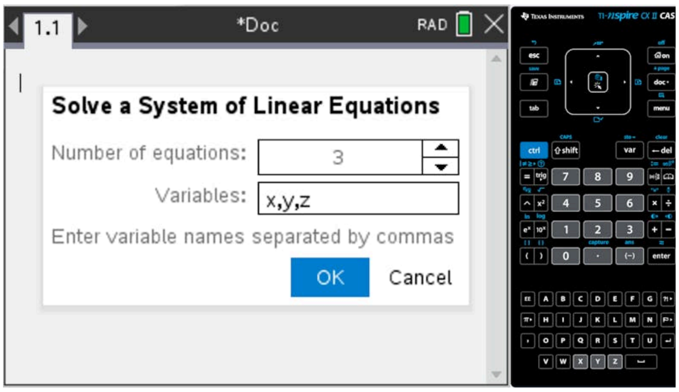
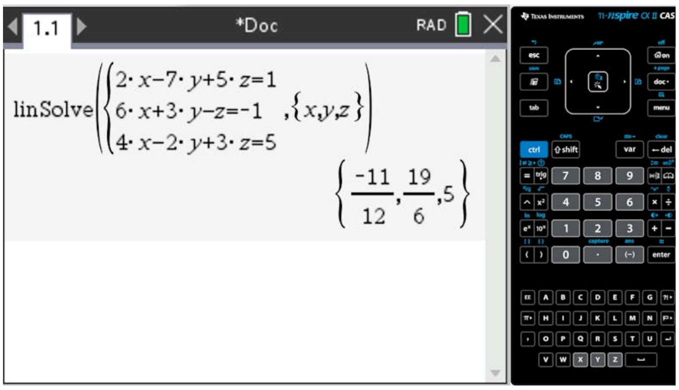
Step	Explanation
Set the number of equations (in this example three) and enter the variable names.	
Enter the equations using the variable names you specified. The calculator gives the solution as fractions.	

Theory of Knowledge

Particularly challenging aspects of mathematics, as an area of knowledge, are its scope and its applications to the real world.

Models, like the normal distribution, are very useful for representing real-life scenarios. However, they can also produce results that are mathematically correct but not realistically possible.

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How do mathematicians reconcile these facts as they use mathematical models? How are they able to discern whether or not a model is useful, even though it might produce unrealistic results?

3 section questions

Question 1

Difficulty: ★★☆

The weight in kilograms of fish caught in a day by an angler is represented by X, and $X \sim \mathrm{N}\left(\mu, 1.5^2\right)$.

If $\mathrm{P}\left(X < 38\right) = 0.2$ and $\mathrm{P}\left(Z < -0.842\right) = 0.2$, find the mean weight of fish caught in one day.

Give your answer as a decimal rounded to 3 significant figures.

39.3

Accepted answers

39.3, 39,3

Explanation

Since $\mathrm{P}\left(X < 38\right) = \mathrm{P}\left(Z < -0.842\right)$, the z-score corresponding to $x = 38$ is -0.842.

Use the formula $z = \frac{x-\mu}{\sigma}$ to write and solve an equation. $-0.842 = \frac{38-\mu}{1.5}$

$$-1.263 = 38 - \mu$$

$$\mu = 39.263 \approx 39.3$$

Therefore, the angler catches an average of 39.3 kg of fish per day.

Question 2

Difficulty: ★★★

Student view

The height of a species of cacti is normally distributed and 10% of the cacti are taller than 47.3 cm and 15% are smaller than 41.2 cm. Find the mean and standard deviation of the heights of the cacti. Round your answers to the nearest hundredth.

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1 $\mu = 43.93\,\text{cm}$, $\sigma = 2.63\,\text{cm}$ ✓

2 $\mu = 45.22\,\text{cm}$, $\sigma = 1.62\,\text{cm}$

3 $\mu = 53.68\,\text{cm}$, $\sigma = 4.98\,\text{cm}$

4 $\mu = 42.15\,\text{cm}$, $\sigma = 1.98\,\text{cm}$

Explanation

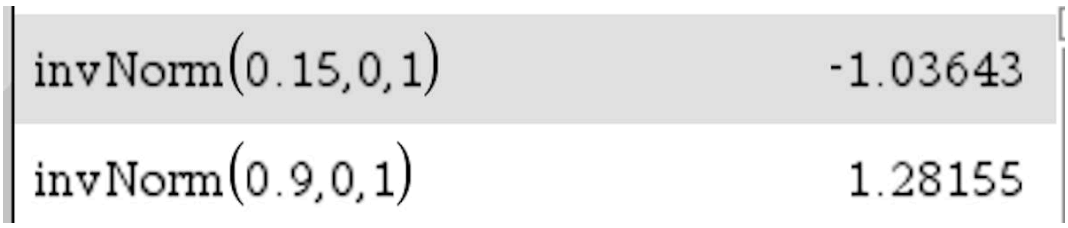
We use the standard normal distribution, and note that $\mathrm{P}(Z > a) = p \Leftrightarrow \mathrm{P}(Z < a) = 1 - p$:

$$\begin{aligned} &\mathrm{P}\left(Z < \frac{(47.3-\mu)}{\sigma}\right) = 1 - 0.1 = 0.9 && \Longleftrightarrow \frac{(47.3-\mu)}{\sigma} = 1.282 && [\text{GDC}] \\ &\mathrm{P}\left(Z < \frac{(41.2-\mu)}{\sigma}\right) = 0.15 && \Longleftrightarrow \frac{(41.2-\mu)}{\sigma} = -1.036 && [\text{GDC}] \end{aligned}$$

and

$$47.3 - \mu = 1.282\sigma \quad \text{and} \quad 41.2 - \mu = -1.036\sigma \qquad 2.318\sigma = 6.1 \Longleftrightarrow \sigma \approx 2.63,\ \mu \approx 43.93.$$

See screenshots below for GDC input and output.



More information

Question 3

Difficulty: ★★★

It is known that the lifetime of batteries in a children's toy follows a normal distribution. It is also given that 8% of times the batteries last less than 53 hours of operation, while in 12% of times the batteries last more than 61 hours of operation. Find the mean and the standard deviation of the distribution.

Give your answers rounded to 2 decimal places in the form *solution1,solution2*, i.e. separated by comma with no spaces. Use a dot as the decimal point.

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57.36,3.10

Accepted answers

57.36,3.10, 3.10,57.36

Explanation

Let X be the random variable of the lifetime of the batteries in this toy. We know that $X \sim \mathrm{N}(\mu, \sigma)$. We also note that it is given that

$$\mathrm{P}(X < 53) = 8\% = 0.08$$

and

$$\mathrm{P}(X > 61) = 12\% = 0.12 \text{ or } \mathrm{P}(X < 61) = 1 - 12\% = 0.88.$$

Then. use the standard normal distribution to solve

$$\begin{aligned} \mathrm{P}\left(Z < \frac{53-\mu}{\sigma}\right) = 0.08 \quad &\Leftrightarrow \quad \frac{53-\mu}{\sigma} = -1.4051, \quad &[\mathrm{GDC}] \\ \mathrm{P}\left(Z < \frac{61-\mu}{\sigma}\right) = 0.88 \quad &\Leftrightarrow \quad \frac{61-\mu}{\sigma} = 1.1750, \quad &[\mathrm{GDC}] \end{aligned}$$

which gives in hours of operation

$$\begin{cases} 53 - \mu = -1.4051\sigma \\ 61 - \mu = 1.1750\sigma \end{cases} \Leftrightarrow \begin{cases} 53 - \mu = -1.4051\sigma \\ 8 = 2.5801\sigma \end{cases} \Leftrightarrow \begin{cases} \sigma \approx 3.10 \\ \mu \approx 57.36 \end{cases}$$

4. Probability and statistics / 4.12 The standard normal distribution

Checklist

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What you should know

By the end of this subtopic you should be able to:

- transform values from a normal distribution into z -scores
- use the Normal CDF on the calculator to find the probability that z lies within a certain range
- use the Inverse Normal to find values of z

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- use the Inverse Normal and the formula $z = \frac{x - \mu}{\sigma}$ to find an unknown mean or standard deviation.

4. Probability and statistics / 4.12 The standard normal distribution

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-27777/print/) Assign

Before the graphic display calculator became more commonly used, functions like the Normal CDF and the Inverse Normal could only be calculated using very complex techniques.

The average student, however, would be incapable of performing those calculations, so 'standard normal tables', such as the one shown below, were used.

z	0.000	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080
1	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752

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z	0.000	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571
0.0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532

In the table, the value of z is found by adding the value in the first row to the value in the first column. Looking where the corresponding row and column intersect gives the probability that Z is less than that value.

For example, the green cell tells us that $\text{P}\left(Z < 0.82\right) = 0.794$.

Students had to get creative to find probabilities that were not included in the table. A process called linear interpolation is used to estimate probabilities for values of z that were more precise than the two decimal places the table gives, such as $z = 0.824$. Linear interpolation is a process in which you assume an approximate linear change between the values in the table and use a proportion to estimate the value.

Symmetry is used to find probabilities for negative values of z . Can you figure out how it is done?

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Use linear interpolation and symmetry to find probabilities of several values of z that are not given in the table, and check your results with your calculator.

How accurate are your estimates? Can you see a parallel between the concept of linear interpolation and another approximation tool you have learned about in this topic?