TOPIC 2
FUNCTIONS

SUBTOPIC 2.8
TRANSFORMATION OF GRAPHS

Index

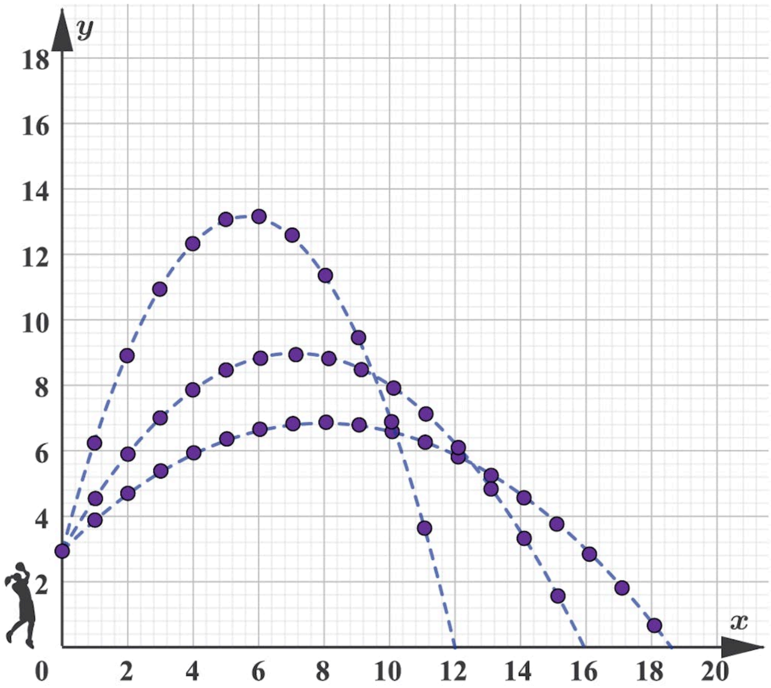
The big picture

Two graphs may appear different, but their underlying functions may be the same up to a transformation. For example, the decay of two different radioactive isotopes with different half-lives. Why do we explore transformations? Each class of functions has inherent and specific features. For example, all quadratic functions have one turning point. You can take the simplest quadratic function, $y = x^2$, and transform it using the completing the square form $y = a(x - b)^2 + c$ to produce any quadratic curve. Therefore, you can model any quadratic situation in real life, such as parabolic flight.

Neglecting air resistance, a projectile travelling freely through the air follows a quadratic path. However, the exact shape of the path will depend on the height it was launched from, its initial speed and the angle of inclination (see the figure below). Using the underlying quadratic relation between the vertical and horizontal directions, and the rules of transformations, you will be able to fit a quadratic function to the movement of any projectile.

In this section you will explore the following types of transformations:

- Horizontal and vertical translations
- Stretches
- Reflections
- Composite translations



More information

The graph illustrates three quadratic paths that represent the motion trajectories of basketballs thrown from a starting point marked at (0, 2) on the graph. The X-axis represents the horizontal distance, labeled 'x', ranging from 0 to 20. The Y-axis represents the vertical height, labeled 'y', ranging from 0 to 18.

Each trajectory is depicted as a series of purple dots forming parabolic curves. These paths start at the same vertical point but spread horizontally, demonstrating different launch angles and forces:

- The first trajectory peaks around (6, 15) and descends sharply.
- The second trajectory peaks lower, around (8, 11), and follows a similar descent.
- The third trajectory peaks around (10, 8) with the shallowest arc.

All paths intersect at the origin where they begin, emphasizing different possible paths based on varying initial conditions.

[Generated by AI]

In the figure above, the basketballs moving freely through the air follow a path that can be modelled as a quadratic relation between the vertical and horizontal directions. The exact path depends on the launch height, speed and angle of inclination.

Concept

Knowing the graphs of common functions and knowing how to translate, shift, reflect and stretch graphs of function can help you sketch a variety of more complex functions by hand. This is useful for sketching graphs of functions that **model** real-life phenomena, which can give us a better understanding of the phenomena.

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Translations

In this section, you will explore translations of the form $y = f(x - a)$ and $y = f(x) + b$.

Activity

The applet plots the graph of some function $y = f(x)$. Adjust slider a to visualise the graph of the transformed function $y = f(x - a)$.

- What effect does the constant a have when $y = f(x)$ is transformed to $y = f(x - a)$? Generalise your observations by forming a rule.

Interactive 1. Visualise the Graph of the Transformed Function and Find the Effect of Constant a.

More information for interactive 1

This interactive allows users to explore the effects of horizontal translations on the graph of a function. The display features an xy-coordinate plane, with the x-axis ranging from -6 to 6 and the y-axis from -2 to 2. Two curves are shown: the blue curve represents the original function $y = f(x)$, and the yellow curve represents the transformed function $y = f(x - a)$.

Users can adjust a horizontal slider labeled aaa, located at the top right of the graph, which ranges from -5 to 5. As the slider is moved, the yellow curve dynamically shifts to represent the transformation. The applet visually demonstrates that when $a > 0$, the graph shifts right by a units, and when $a < 0$, the graph shifts left by $|a|$ units. Each point on the original function is translated horizontally, such that the x-coordinate becomes $x + a$, while the y-coordinate remains the same.

This interactive tool helps learners generalize that in the transformation $y = f(x - a)$, the graph of the function shifts right when a is positive and left when a is negative — opposite in direction to the sign of a. It reinforces the rule that subtracting a from the input of a function results in a horizontal shift of the graph.

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

Important

For the transformation $y = f(x - a)$, the effect of a is to translate the graph horizontally through a units.

- If $a > 0$ the transformation moves the graph a units to the **right**.
- If $a < 0$ the transformation moves the graph a units to the **left**.

Notice that each point on the graph of $y = f(x)$ is mapped to a new point on the graph of $y = f(x - a)$, with a new x-coordinate $x - a$, while retaining the same y-coordinate.

Activity

The applet plots the graph of some function $y = f(x)$. Adjust slider b to visualise the graph of the transformed function $y = f(x) + b$.

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27620/print/) Assign

- What effect does the constant b have when $y = f(x)$ is transformed to $y = f(x) + b$? Generalise your observations by forming a rule.

Interactive 2. Visualise the Graph of the Transformed Function and Find the Effect of Constant b.

More information for interactive 2

The interactive applet allows users to explore the effects of vertical translations on the graph of a function by adjusting a slider for the constant b, which ranges from -5 to 3.
A graph is displayed with xy-axes, where both the x-axis and y-axis range from -4 to 4. Two curves are shown: the blue curve represents the original function $y = f(x)$, and the yellow curve represents the transformed function $y = f(x) + b$.

Student
view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

As the user moves the horizontal slider for b, located at the top left corner, the graph of y = f (x) dynamically transforms into y = f (x) + b. The applet demonstrates that when b > 0, the graph shifts **upward** by b units, and when b < 0, it shifts **downward** by |b| units. Each point on the original graph is mapped to a new point with an updated y-coordinate of y + b, while the x-coordinate remains unchanged.
This interactive tool provides a hands-on way to understand how vertical translations affect the position of a function's graph, helping users generalize the relationship between the value of b and the direction of the vertical shift.

✓ **Important**

For the transformation $y = f(x) + b$, the effect of constant b is to translate the graph **vertically** through b units.

- If $b > 0$, the transformation moves the graph b units **upwards.**
- If $b < 0$, the transformation moves the graph b units **downwards.**

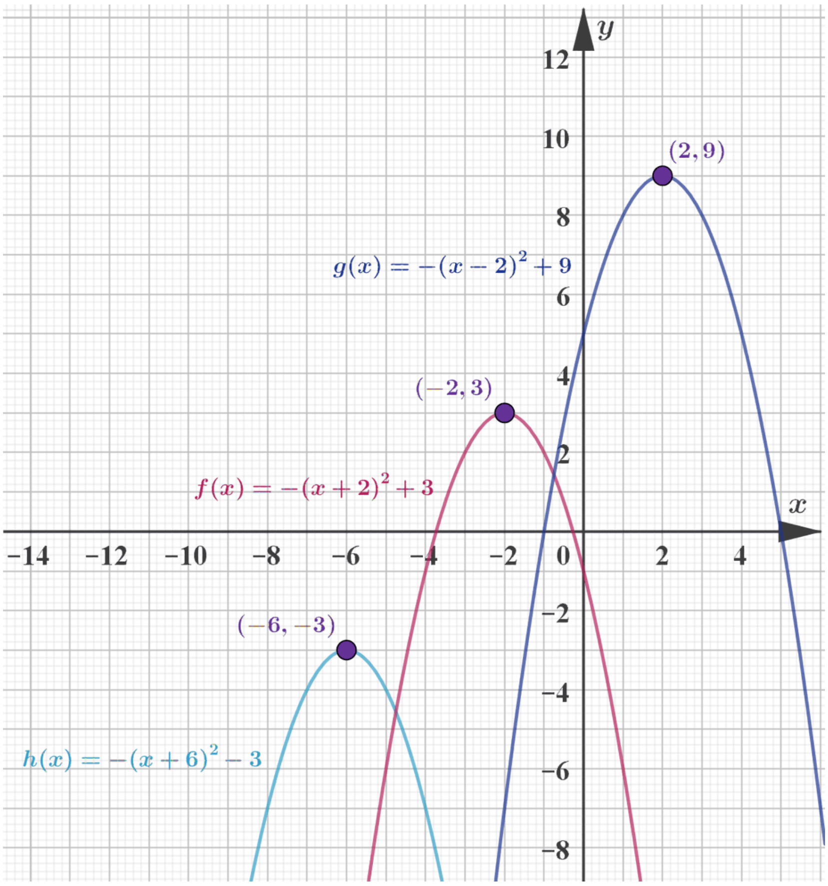
Notice that each point on the graph of $y = f(x)$ is mapped to a new point on the graph of $y = f(x) + b$, with a new y-coordinate $y + b$, while retaining the same x-coordinate.

It is convenient to use vector notation for translations: a horizontal translation by a units and a vertical translation by b units is the transformation of the graph by translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$. The result of a translation by $\begin{pmatrix} a \\ b \end{pmatrix}$ for a function is $f(x) \to f(x - a) + b$.

For example, consider the quadratic function $f(x) = -(x + 2)^2 + 3$, which has a maximum at $(-2, 3)$. Under a translation of, say, $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$, the maximum point $(-2, 3)$ will be mapped to a new point with coordinates of $(-2 + 4,\ 3 + 6) = (2, 9)$. Function f becomes the translated function $g(x) = f(x - 4) + 6 = -(x - 2)^2 + 9$.
Under a translation by the vector $\begin{pmatrix} -4 \\ -6 \end{pmatrix}$, the maximum point $(-2, 3)$ will be mapped to a new point with coordinates of $(-2 - 4,\ 3 - 6) = (-6, -3)$. Function f becomes the translated function $h(x) = f(x + 4) - 6 = -(x + 6)^2 - 3$. The figure below shows the graph of function f and the graphs of the transformed functions g and h.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov



More information

The image portrays a graph depicting three quadratic functions: f(x) = - (x + 2)^2 + 3, g(x) = - (x - 2)^2 + 9, and h(x) = - (x + 6)^2 - 3. The graph has a grid background with a horizontal (X-axis) and vertical (Y-axis), both marked with intervals. The X-axis ranges from -10 to 10, and the Y-axis ranges from -10 to 10. Three curves are shown:

1. The function f(x) appears in blue, peaking at (-2, 3).
2. The function g(x) in pink, with a maximum at (2, 9).
3. The function h(x) in cyan, with a maximum at (-6, -3).

All maximum points are marked with purple dots on the graph and are annotated next to each point. The graph visually represents transformations of the quadratic function f into g and h through translations, as described in the accompanying text, highlighting the movement of maximum points due to these transformations.

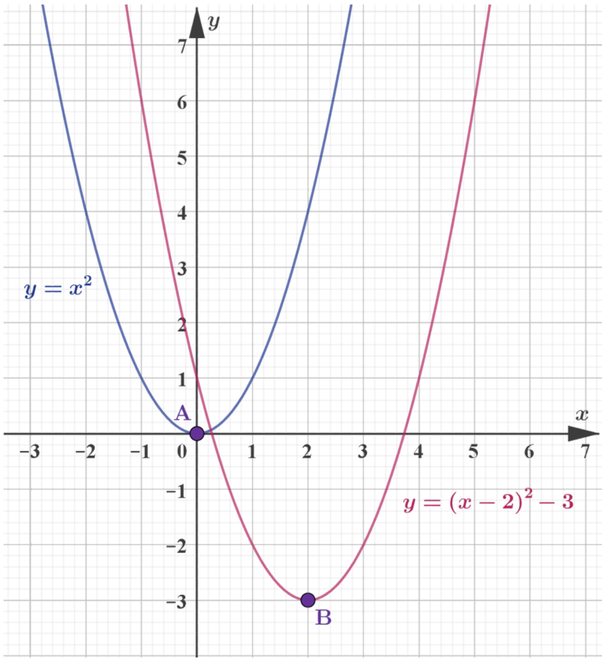
[Generated by AI]

Example 1

★☆☆

Student view

Describe the transformations necessary to obtain the graph of $y = (x-2)^2 - 3$ from the graph of $y = x^2$. Sketch both graphs on the same set of axes.

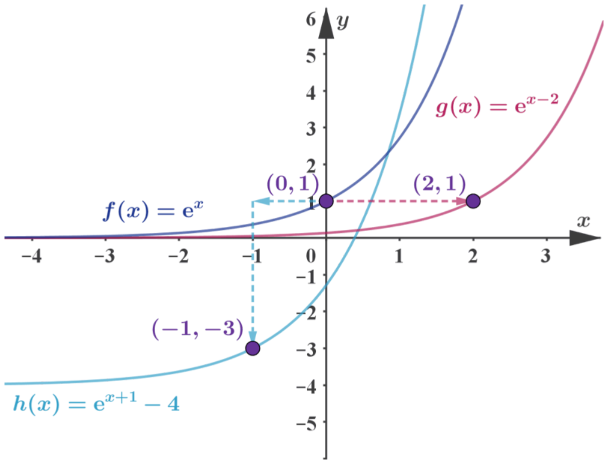
Step	Explanation
The graph of $y = x^2$ is translated 2 units to the right and 3 units downwards, represented by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$. 	The parabola $y = x^2$ has a vertex at $(0,\ 0)$ and the transformed function $y = (x-2)^2 - 3$ has a vertex at $(2, -3)$.

Example 2

★★☆

For the function $f(x) = \mathrm{e}^x$, sketch on the same set of axes the functions $y = f(x)$, $y = f(x-2)$, and $y = f(x+1) - 4$.

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

Step	Explanation
	The graph of function $y = e^x$ has y-intercept the point $(0,\ 1)$. The graph of $y = f\,(x - 2)$ is a horizontal transformation of f by 2 units to the right. The graph of $y = f\,(x + 1) - 4$ is a horizontal and vertical translation of f by a vector $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$.

3 section questions

2. Functions / 2.8 Transformation of graphs

Stretches

In this section, you will explore transformations of graphs called **stretches**, which have the form $y = pf\,(x)$ and $y = f\,(qx)$.

Activity

The following applet displays the graph of some function $y = f\,(x)$. Adjust slider p to visualise the graph of the transformed function $y = pf\,(x)$.

- What effect does the constant p have when $y = f\,(x)$ is transformed to $y = pf\,(x)$? Generalise your observations by forming a rule.

Student
view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

Interactive 1. Vertical Scaling and Reflection of Functions

More information for interactive 1

The interactive applet visualizes the effect of vertical scaling on the graph of a function $y = f(x)$. The applet includes a slider labeled p, which users can adjust within a range of -5 to 5. This controls the transformation of the original function $y = f(x)$ into the scaled function $y = pf(x)$.

When $p = 1$, the graph remains unchanged, displaying $y = f(x)$ in its original form as a smooth blue curve. The function retains its shape, and each point (x, y) stays the same.

If $p > 1$, the function undergoes a vertical stretch, increasing all y-values by a factor of p, making the graph appear taller. Conversely, when $0 < p < 1$, the function experiences a vertical compression, where all y-values are scaled down, making the graph appear flatter.

If $p < 0$, the function is reflected across the x-axis before being stretched or compressed. Negative values of p flip the graph upside down, while the magnitude of p determines the degree of stretching or compression.

In the provided image, the slider is initially set to $p = 1$ displaying the original function $y = f(x)$ in blue. When adjusted to $p = -2.2$, the transformed function $y = -2.2f(x)$ is shown in orange. The negative value of p causes the graph to reflect across the x-axis, while the factor of 2.2 results in a vertical stretch. This transformation is evident as each point (x, v) on the original graph is mapped to a new point $(x, -2.2v)$, where the y-coordinate is multiplied by -2.2, making peaks and troughs more pronounced and inverted.

This visualization helps users develop a deeper understanding of vertical scaling and reflection, reinforcing the relationship between the constant p and the transformation of the function.

Important

For the transformation $y = pf(x)$, the effect of constant p is a vertical stretch in the y-direction by a scale factor p.

- If $|p| > 1$, the transformation stretches the graph by a scale factor p.
- If $|p| < 1$, the transformation shrinks the graph by a scale factor p.

Student
view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

- If $p < 0$, the transformation reflects the graph of $y = f(x)$ in the x -axis and stretches or shrinks it as above.

Notice that each point on the graph is mapped to a point with a new y-coordinate, py, while the x-coordinate is unchanged.

For example, consider the function $f(x) = (x-2)^2 + 3$ with a vertex at $(2, 3)$. Under a transformation of, say, $y = 2f(x)$, the vertex will be mapped to a point with new coordinates $(2, 2 \times 3) = (2,\ 6)$. The transformed function will have equation $y = 2f(x) = 2(x-2)^2 + 6$.

Activity

In the following applet the graph of function $y = f(x)$ is displayed. Adjust slider q to visualise the graph of the transformed function $y = f(qx)$.

- What effect does the constant p have when $y = f(x)$ is transformed to $y = pf(x)$? Generalise your observations by forming a rule.

Interactive 2. Graph of a Transformed Function.

More information for interactive 2

The interactive applet allows users to explore the effects of horizontal scaling on the graph of a function by adjusting a slider for the constant q, which ranges from −2 to 2. This visualization helps users understand how the transformation from y = f (x) to y = f (qx) affects the function's shape.

When q = 1, the function remains unchanged, and the original graph of y = f (x) is displayed as a smooth curve. However, when q > 1, the function undergoes a horizontal compression, meaning that all x-values are scaled closer together, making the graph appear narrower. Conversely, if 0 < q < 1, the function experiences a horizontal stretch, where all x-values are spread out, making the graph appear wider.

If q < 0, the function is first reflected across the y-axis before being stretched or compressed. Negative values of q flip the graph horizontally, while the absolute magnitude of q determines the degree of stretching or compression.

For example, if q = −1.5, the function y = f (−1.5x) is displayed in an orange color, illustrating both the reflection and the horizontal compression. Each point (x, y) on the original graph is mapped to (x/q, y), shifting and transforming the function accordingly.

This interactive tool provides a hands-on way to understand the relationship between the scaling constant q and the transformation of the function, reinforcing key concepts of horizontal stretching, compression, and reflection.

Student view

Important

For the transformation $y = f(qx)$, the effect of constant q is a horizontal stretch in the x -direction by a scale factor $\frac{1}{q}$.

- If $|q| > 1$, the transformation shrinks the graph by a scale factor $\frac{1}{q}$.
- If $|q| < 1$, the transformation stretches the graph by a scale factor $\frac{1}{q}$.
- If $q < 0$, the transformation reflects the graph of $y = f(x)$ in the y -axis and stretches or shrinks it as above.

Notice that each point on the graph of $y = f(x)$ is mapped to a point with a new x-coordinate, $\frac{1}{q} \cdot x$, while the y-coordinate is unchanged.

For example, consider the function $f(x) = (x-2)^2 + 3$ with a vertex at $(2, 3)$. Under a transformation of, say, $y = f(2x)$, the vertex will be mapped to a point with new coordinates $\left(2 \times \frac{1}{2}, 3\right) = (1, 3)$. The transformed function will have equation

$$y = f(2x) = (2x-2)^2 + 3 = [2(x-1)]^2 + 3 = 4(x-1)^2 + 3.$$

Example 1

★★☆

Consider the function with a graph $y = x^2$. Find the equation of the transformed function, when f goes under the following transformations:

a) Vertical stretch by a factor of 2

b) Horizontal stretch by a factor of 2

c) Vertical stretch by a factor of $-\frac{1}{3}$

d) Horizontal stretch by a factor of $-\frac{1}{2}$

Overview (/study/ap ai-hl/sid-132-cid-761618/ov

	Step	Explanation
a)	A vertical stretch by a factor of 2 will map function f to a new function $y = 2f(x) = 2x^2$.	A vertical stretch of a function $y = f(x)$ is the transformation $y = pf(x)$.
b)	A horizontal stretch by a factor of 2 will map function f to a new function given by $y = f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$.	A horizontal stretch by a factor $\frac{1}{q}$ of a function $y = f(x)$ is the transformation $y = f(qx)$.
c)	A vertical stretch by a factor $-\frac{1}{3}$ will map function f to a new function $y = -\frac{1}{3}f(x) = -\frac{1}{3}x^2$.	A vertical stretch of a function $y = f(x)$ is the transformation $y = pf(x)$.
d)	A horizontal stretch by a factor $-\frac{1}{2}$ will map function f to a new function given by $y = f(-2x) = (-2x)^2 = 4x^2$.	A horizontal stretch by a factor $\frac{1}{q}$ of a function $y = f(x)$ is the transformation $y = f(qx)$.

Activity

Below is an applet that **generates random examples** of horizontal and vertical stretches of a function, which are shown in blue. Find the transformation that the blue function goes through to produce the graph of the green function, by choosing one of the four buttons provided.

Once the button with the correct answer is clicked, the colour changes to green, otherwise it is red. There are quite a few combinations to practise with. Make sure you can recognise each case by inspection of the given graphs.

Interactive 3. Examples of Horizontal and Vertical Stretches of a Function.

Student view

More information for interactive 3

This interactive helps users practice identifying horizontal and vertical stretches or compressions of functions.

The screen is divided into two halves: the left side displays the original function as a blue curve, while the right side shows its transformed version as a pink curve.

Users are asked to determine which transformation—horizontal stretch/compression or vertical stretch/compression—was applied to produce the new graph. Four answer choices are provided as buttons. When a user selects the correct transformation, the button turns green; an incorrect choice turns red.

A “New example” button in the top right allows users to generate different transformation scenarios, enabling repeated practice.

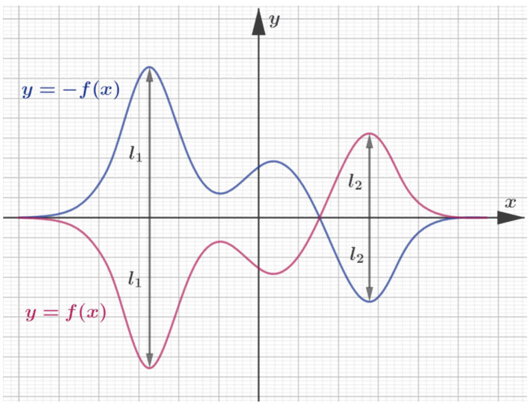
This interactive tool encourages pattern recognition and builds intuition for identifying graph transformations by visual inspection.

3 section questions

Reflections

Two special cases of stretch transformations result in reflections in the x-axis or the y-axis.

The figures below show the two reflections of function $y = f(x)$ in the two axes.

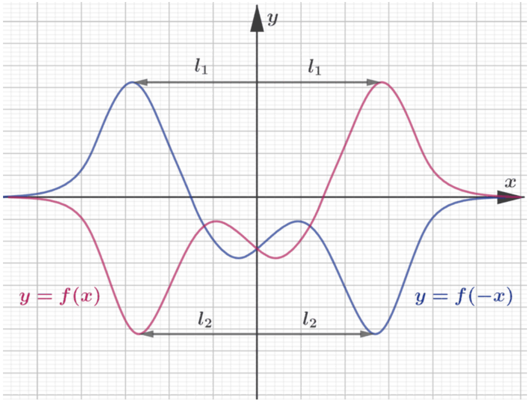


More information

The image is a graph displaying two functions, y = f(x) and y = 1/f(x), on a coordinate plane with a grid background. The x-axis is horizontal and ranges from negative to positive values, while the y-axis is vertical, typically representing y-values from negative to positive. The first curve, labeled y = f(x), is a blue curve showing periodic oscillations with peaks and troughs. The second curve, y = 1/f(x), is a red curve also showing fluctuations that inversely mirror the behavior of the first curve. Both curves intersect the y-axis, with the blue curve starting at a positive value and the red curve at a negative. The grid lines provide visual reference for plotting, with both axes marked with units and labels. The two graphs illustrate inverse functionalities, as evident by the behavior of the curves when approaching zero and their symmetry about the y-axis.

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

[Generated by AI]



More information

The image displays a graph illustrating the reflection of the function (y = f(x)) in the (x)-axis. The graph has a grid background with both x-axis and y-axis intersecting at the origin. The x-axis represents independent variable (x) with no specific units or range marked, except the position of the origin at zero. The y-axis also lacks specific units, serving to illustrate the shape of the function curves. Two curves are plotted; one is above the x-axis and dips below, labeled (y = f(x)); the other is a reflection beneath the x-axis, labeled (y = f(-x)). This reflection demonstrates the formal algebraic equality (f(x) = -f(-x)) around the origin, with peaks and valleys symmetrically distributed on either side of the y-axis, suggesting a periodic nature in appearance. Overall, the graph visually communicates an example of even-odd symmetry in mathematical functions.

[Generated by AI]

A reflection of function $y = f(x)$ in the x-axis

A reflection of function $y = f(x)$ in the y-axis

Important

- The transformation of function $y = f(x)$ to the function $y = -f(x)$ is a reflection in the x-axis. This transformation $f(x) \to -f(x)$ is a special case of a vertical stretch of f with scale factor $p = -1$.
- The transformation of function $y = f(x)$ to the function $y = f(-x)$ is a reflection in the y-axis. This transformation $f(x) \to f(-x)$ is a special case of a horizontal stretch with $q = -1$.

Student
view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Notice that when function $y = f(x)$ is reflected in the x-axis, the points with identical x-coordinates are equidistant from the x-axis, but on opposite sides of it. When function $y = f(x)$ is reflected in the y-axis, the points with identical y-coordinates are equidistant from the y-axis, but on opposite sides of it.

Example 1

★☆☆

Consider a function f with a zero at $x = -3$ and a y-intercept at $y = 2$. Find the zero and y-intercept of the resulting function:

a) When f is reflected in the x-axis.

b) When f is reflected in the y-axis.

a) A reflection in the x-axis will leave the zero in the same place and transform the y-intercept to $y = -2$.

b) A reflection in the y-axis will transform the zero to $x = 3$ and leave the y-intercept in the same place.

⚠ **Be aware**

Points that are not moved under a transformation are called **invariant points**.

Example 2

★☆☆

Consider the function $y = 2x - 1$. Find an equation for each of the functions:

a) When function $y = f(x)$ is reflected in the x-axis.

b) When function $y = f(x)$ is reflected in the y-axis.

Graph all three functions on the same set of axes.

a) A reflection in the x-axis will map function $y = f(x)$ to the function

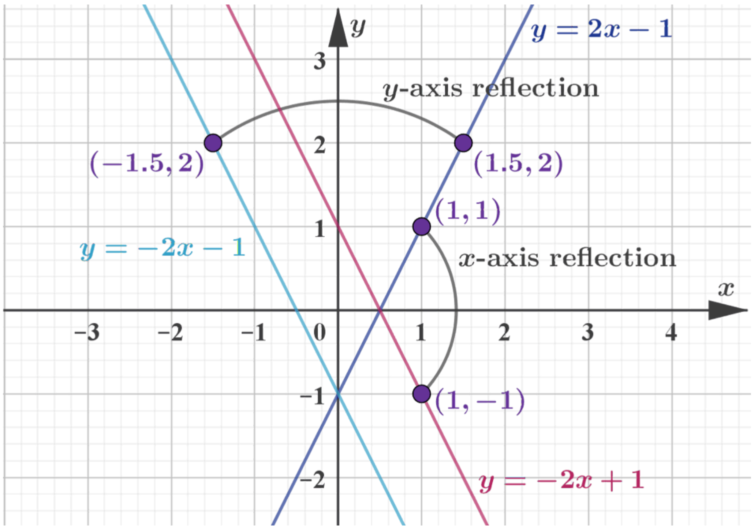
$$y = -f(x) = -(2x - 1) = -2x + 1.$$

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

b) A reflection in the y-axis will map function $y = f(x)$ to the function

$$y = f(-x) = 2(-x) - 1 = -2x - 1.$$



Making connections

Recall from subtopic 2.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27838/), that, given a function $y = f(x)$, the graph of the inverse function, $y = f^{-1}(x)$, is a reflection of the graph of $f(x)$ in the line $y = x$; similarly, the graph of $y = f(x)$ is a reflection of $y = f^{-1}(x)$ in the line $y = x$. Note that, in particular, if $y = f(x)$ intersects the line $y = x$, the graph $y = f^{-1}(x)$ intersects the line $y = x$ at the same point.

Be aware

In general, there are two types of transformation:

1. Rigid transformations that change the location of the function in the coordinate plane but leave the size and shape of its curve unchanged.
2. Non-rigid transformations that change the size or shape of the curve.

What types of transformation are translations, stretches and reflections? What types of transformation produce congruent objects? Similar objects?

Student view

Theory of Knowledge

Mathematician Marcus du Sautoy examines in the following TED Talk not only what symmetry is but also how it affects our daily lives. He examines how mathematics is embedded in our world and shows how we may or may not recognise, measure, or, as he keenly explores, move, symmetrical objects.

Marcus du Sautoy: Symmetry, reality's riddle

5 section questions

Composite transformations

So far you have seen that translating a function by a and b units horizontally and vertically is denoted by the translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$ and gives $f(x) \to f(x-a)+b$. You can, of course, use any combination of translations, stretches and reflections. However, be careful with the order in which you apply functions in a composite transformation. Does changing the order produce a different result?

Making connections

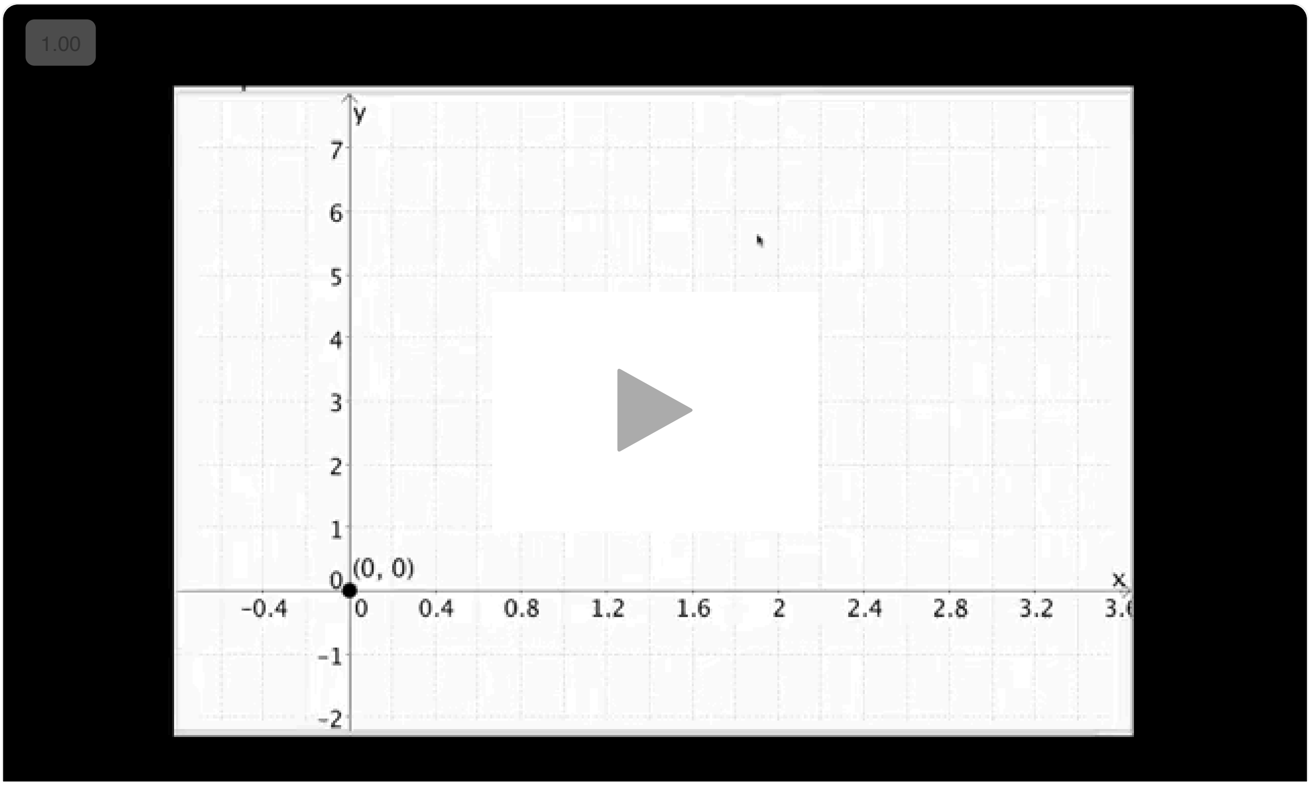
In subtopic 2.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27838/)you saw that the composition of two functions f and g , where function g is applied first and function f is applied second, is the function

$$(f \circ g)(x) = f(g(x)).$$

Reflect on composite functions and explain whether function transformations can be expressed as composite functions.

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

In the following video, you will explore the transformation of a point by the transformations you have studied so far, but doing so in a different order each time.



Video 1. Understanding Composite Transformations.

More information for video 1

1

00:00:00,133 --> 00:00:02,536

narrator: In this video we're going to

look at composite transformations

2

00:00:02,603 --> 00:00:04,872

where we combine the four transformation

we've been looking at.

3

00:00:04,938 --> 00:00:08,509

So we start at $(0, 0)$,

then go to $(2, 0)$,

4

00:00:08,575 --> 00:00:12,813

then go up to $(2, 3)$, $(2, 6)$,

and end up at $(1, 6)$.

5

00:00:12,880 --> 00:00:14,448

So how can we do it

with our transformations?

6

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

00:00:14,515 --> 00:00:17,751

Well, we can translate

horizontally by two units,

7

00:00:17,851 --> 00:00:20,454

then translate vertically by three units,

8

00:00:20,787 --> 00:00:23,357

then do a vertical

stretch by factor of two,

9

00:00:23,590 --> 00:00:26,193

and then do a horizontal

stretch by a factor of two,

10

00:00:26,260 --> 00:00:28,762

and that's how we can end up at 1.6.

11

00:00:28,996 --> 00:00:31,098

Now we may combine these

transformations differently.

12

00:00:31,164 --> 00:00:33,567

First of all, we can do

these stretch factors,

13

00:00:33,634 --> 00:00:35,302

and multiplying zero

by anything leaves zero.

14

00:00:35,369 --> 00:00:38,672

Then we can go to $(2, 0)$ then to $(2, 3)$,

15

00:00:38,872 --> 00:00:42,509

so that we apply the p

and q stretches first.

16

00:00:42,676 --> 00:00:47,014

Then translate by two units to the right

and then three units to upwards.

17

00:00:47,181 --> 00:00:49,183

You can also see that we could have done

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

18

00:00:49,249 --> 00:00:52,653

after the stretch factor,

we gonna moved up by three units

19

00:00:52,786 --> 00:00:56,823

and then moved horizontally by two

and ended up at the same region.

20

00:00:56,957 --> 00:00:58,825

Now once again,

we started the origin here.

21

00:00:58,959 --> 00:01:02,396

We first go to $(2, 0)$

then to $(1, 0)$,

22

00:01:02,596 --> 00:01:03,897

and then to $(1, 3)$.

23

00:01:03,964 --> 00:01:05,032

So how can we do this?

24

00:01:05,098 --> 00:01:07,868

Well, we can have a horizontal

translation by two units,

25

00:01:08,402 --> 00:01:11,605

then a horizontal stretch by factor of two.

26

00:01:11,672 --> 00:01:14,474

Then we can do the vertical

stretch by factor of two,

27

00:01:14,541 --> 00:01:17,411

leaving it alone,

and then translate upwards by three.

28

00:01:18,345 --> 00:01:23,150

By the four transformations,

$a = 2$, $b = 3$, $p = 2$, and $q = 2$.

29

00:01:23,283 --> 00:01:26,954

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

We've seen that we can end up

in three different places

30

00:01:27,287 --> 00:01:30,624

applying all those transformations.

In other words, or they may matter,

31

00:01:30,691 --> 00:01:32,559

doesn't always, as you've seen, but some.

32

00:01:32,659 --> 00:01:34,962

But it can very well matter.

Important

Combining translations and stretches in a different order may give a different result.

Consider a vertical translation by b and a vertical stretch by p:

- First translating then stretching will transform:

$$f(x) \overset{\text{translating}}{\rightarrow} f(x)+b \overset{\text{stretching}}{\rightarrow} p(f(x)+b) = pf(x)+pb.$$

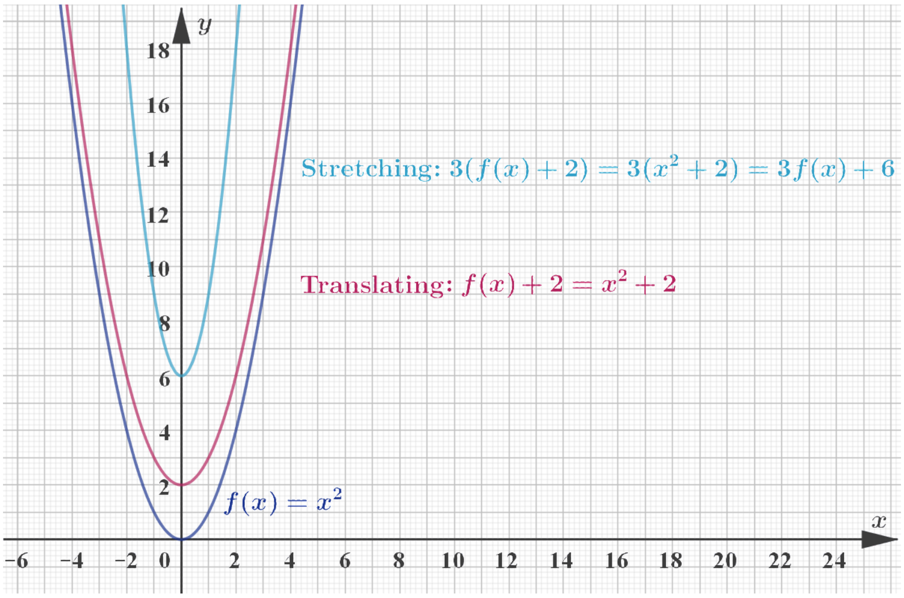
- First stretching then translating will transform:

$$f(x) \overset{\text{stretching}}{\rightarrow} pf(x) \overset{\text{translating}}{\rightarrow} pf(x)+b.$$

The figures below show the graph of function $f(x) = x^2$ and the graphs of the resulting functions when a translation of 2 steps upwards and a vertical stretch by a scale factor 3 are applied to the function f in different orders.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov



The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the vertical translation is followed by the vertical stretch

More information

The image depicts a graph with three curves illustrating the function (f(x) = x^2) and its transformations. The X-axis is labeled with numbers ranging from (-5) to (5). The Y-axis is labeled with numbers increasing by 2, specifically (-2, 0, 2, 4, 6, 8, 10), etc.

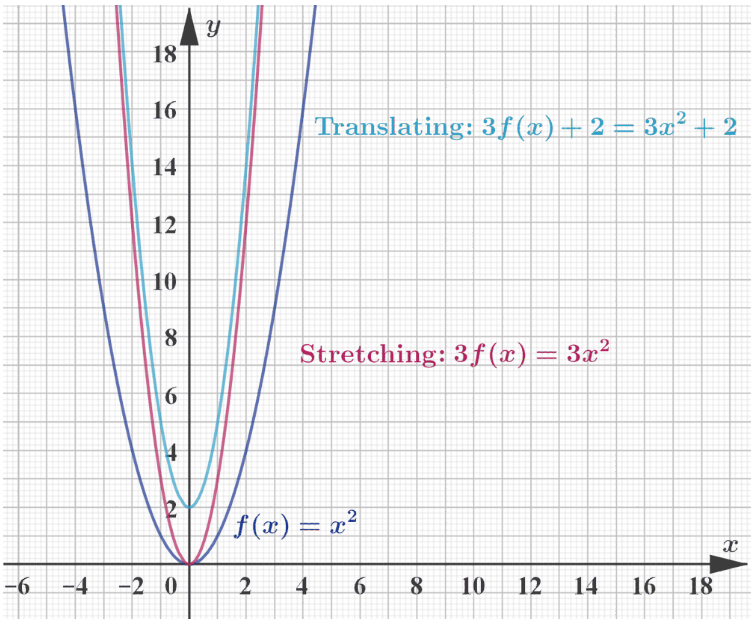
1. **Blue Curve**: Represents the original function (f(x) = x^2), a parabolic curve opening upwards with its vertex at the origin (0, 0).
2. **Red Curve**: Represents the function after a vertical translation by 2 units upwards, (f(x) + 2 = x^2 + 2). The vertex of this curve is at (0, 2).
3. **Cyan Curve**: Represents the function after a vertical stretch by a factor of 3 applied: (3 * f(x) - 6 = 3(x^2) - 6). The curve is narrower than the original, and has been shifted downward.

The text within the image indicates the equations of the transformations: "Stretching: 3/f(x) - 9 = 3(x^2 - 3) = 3/f(x) - 6" and "Translating: f(x) + 2 = x^2 + 2."

[Generated by AI]

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov



The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the vertical stretch is followed by the vertical translation

More information

The image is a graph illustrating the function f(x) = x^2 along with its transformation through vertical stretch and translation. The graph features the X-axis labeled with values ranging from -15 to 15 and the Y-axis labeled with values from -5 to 25. The original function is a parabola opening upwards centered at the origin. The graph shows two additional parabolas: one representing a vertical stretch, labeled 'Stretching 3f(x) = 3x^2,' which is narrower and steeper than the original, and the other showing a translation labeled 'Translating f(x) = x^2 + 3x^2 + 3', which shifts the graph upwards. The transformations demonstrate the effects of vertical stretching and vertical translation on the shape and position of the parabola.

[Generated by AI]

Now consider a horizontal translation by a and a horizontal stretch by $\frac{1}{q}$:

- First translating then stretching will transform:

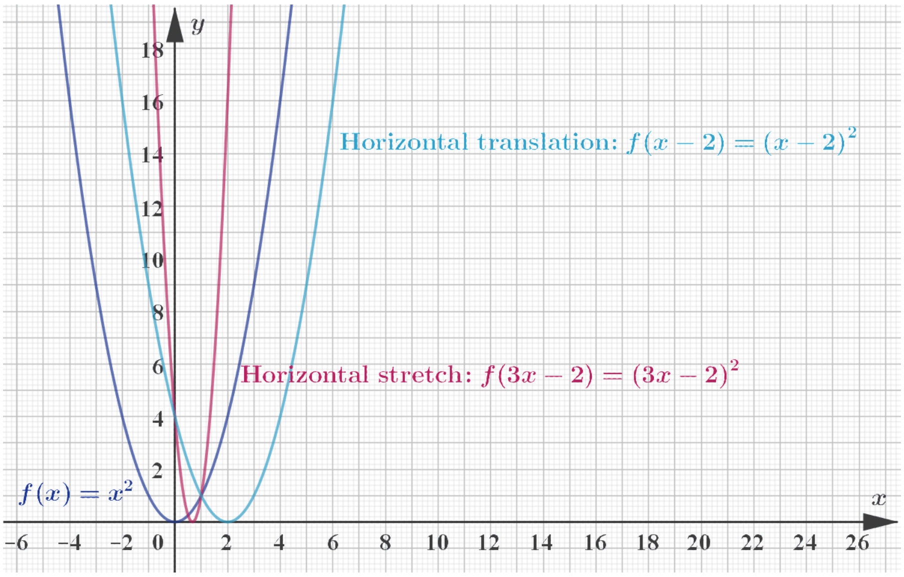
$$f(x) \overset{\text{translating}}{\to} f(x-a) \overset{\text{stretching}}{\to} f(qx-a).$$

- First stretching then translating will transform:

$$f(x) \overset{\text{stretching}}{\to} f(qx) \overset{\text{translating}}{\to} f(q(x-a)) = f(qx-qa).$$

The figures below show the graph of function $f(x) = x^2$ and the graphs of the resulting functions when a translation of 2 steps to the right and a horizontal stretch by a scale factor $\frac{1}{3}$ are applied to the function f in different orders.

Student view



The graph of function $f(x)=x^2$ and the graphs of the resulting functions when the horizontal translation is followed by the horizontal stretch

More information

The image presents a graph illustrating the function transformations of (f(x) = x^2). The X-axis ranges from -10 to 10, and the Y-axis ranges from -10 to 10, with labeled intervals at every integer point on both axes.

There are three curves visible:

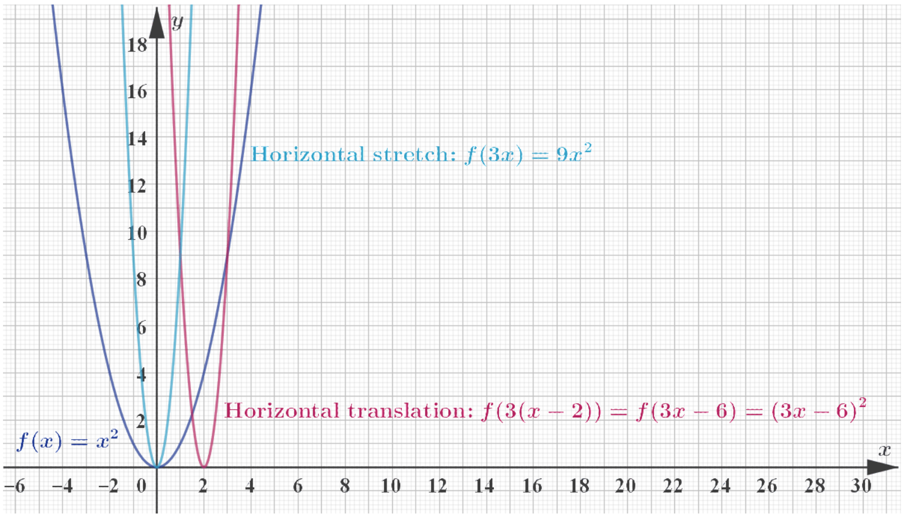
1. The original function (f(x) = x^2) is a parabola centered at the origin (0,0) and opens upwards.
2. The second curve represents a horizontal translation of the original function to the right by 2 units, described by (f(x-2) = (x-2)^2). This curve is shifted right and intersects the X-axis at (2,0).
3. The third curve shows a horizontal stretch by a scale factor of (\frac{1}{3}), applied after the translation. It represents (f(3(x-2)) = (3x-6)^2). This curve is narrower compared to the original, indicating the horizontal stretch and intersects the X-axis still at 2 (where changes mainly affect the steepness rather than position here). The vertex of this parabola remains in line with the translated curve's vertex along the x-axis.

Overall, the graph depicts the sequential impact of a translation followed by a horizontal stretch on a quadratic function.

[Generated by AI]

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov



The graph of function $f(x) = x^2$ and the graphs of the resulting functions when the horizontal stretch is followed by the horizontal translation

More information

The image is a graph displaying the function f(x) = x^2 along with two transformed versions of the function. The horizontal axis is labeled with numbers ranging from -10 to 10, and the vertical axis from 0 to 100.

1. The original function, f(x) = x^2, is a parabola centered at the origin (0,0).
2. The first transformation shown is a horizontal stretch, represented by the function f(3x) = 9x^2. This parabola appears narrower than the original.
3. The second transformation involves a horizontal translation of the function f(3(x - 2)), which becomes 9(x - 2)^2. This shifts the parabola 2 units to the right.

The image visually demonstrates how the stretching and translating affect the shape and position of the original parabola described by f(x) = x^2.

[Generated by AI]

Be aware

Note that an overall transformation can be produced from many different combinations of transformations.

To illustrate this, consider the line $y = 4 - 2x$. You can obtain this line by transforming the line $y = x$ in various ways:

1. A vertical stretch by a scale factor $p = -2$, followed by a vertical translation upwards of 4 units $b = 4$ to give: $x \to -2x \to -2x + 4$.

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

2. A horizontal translation of 2 units to the right $a = 2$, followed by a vertical stretch with scale factor $p = -2$ to give: $x \to x - 2 \to -2(x-2) = -2x + 4$.

Next, consider three transformations applied in two different orders to the curve for $y = x^2$:

1. A horizontal translation of 1 unit to the right, $a = 1$, a horizontal stretch by factor $q = \dfrac{1}{2}$ and a vertical translation upwards of 3 units, $b = 3$:

$$x^2 \to (x-1)^2 \to (2x-1)^2 \to (2x-1)^2 + 3.$$

Thus, the resulting function is $y = (2x-1)^2 + 3 = 4x^2 - 4x + 4$.

2. A horizontal stretch by factor $q = \dfrac{1}{2}$, a horizontal translation of 1 unit to the right, $a = 1$, and a vertical translation upwards of 3 units, $b = 3$:

$$x^2 \to (2x)^2 \to (2(x-1))^2 \to 4(x-1)^2 + 3.$$

Thus, the resulting function is $y = 4(x-1)^2 + 3 = 4x^2 - 8x + 7$.

Therefore, the order in which transformations are applied is important, and changing the order can change the resultant function.

Exam tip

Take care to identify the correct order of transformations. Using the wrong order would lead to a completely different resultant function.

When you are asked to describe the sequence of composite transformations of the form $y = f(x) \to y = Af(Bx + C) + D$, consider the operations in the following order:

1. Horizontal translation (C)
2. Horizontal stretch (B)
3. Vertical stretch (A)
4. Reflection in the x-axis if $A < 0$
5. Vertical translation (D)

International Mindedness

Dutch artist M. C. Escher (1898—1972) created mathematically inspired drawings and prints. His work was informed by his study of mathematical patterns in nature and architecture, such as this tiling at the Alhambra palace in Spain.

Student
view

Overview (/study/ap ai- hl/sid- 132- cid- 761618/ov

Alhambra palace tile

Credit: brytta Getty Images

Example 1

★★★

Describe the sequence of transformations that need to be performed on $y = \cos(x)$ to obtain $y = -2\cos\left(\frac{1}{2}x + 2\right) - 1$, given that the horizontal stretch by a factor of 2 occurred before the horizontal translation.

If the horizontal translation was 2 units to the left (indicated by the $+2$ in the cosine), then, in contradiction to the question, it would have been performed before the horizontal stretch by a scale factor 2 (indicated by the $\frac{1}{2}$ coefficient of x).

You can rewrite the function to show that the horizontal stretch was performed before the horizontal translation, as follows:

$$y = -2\cos\left(\frac{1}{2}(x+4)\right) - 1$$

This indicates that the horizontal stretch was performed before the horizontal translation and the series of transformations on the function $f(x) = \cos x$ is as follows:

1. Horizontal stretch by scale factor : $f(x) \to f\left(\frac{1}{2}x\right)$.
2. Horizontal translation by 4 units to the left: $f\left(\frac{1}{2}x\right) \to f\left(\frac{1}{2}(x+4)\right) = f\left(\frac{1}{2}x + 2\right)$.
3. Vertical stretch by scale factor 2 : $f\left(\frac{1}{2}x + 2\right) \to 2f\left(\frac{1}{2}x + 2\right)$.

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

4. Reflection in the x-axis: $2f\left(\frac{1}{2}x+2\right) \rightarrow -2f\left(\frac{1}{2}x+2\right)$.

5. Vertical translation by 1 unit down: $-2f\left(\frac{1}{2}x+2\right) \rightarrow -2f\left(\frac{1}{2}x+2\right)-1$.

Example 2

★★★

Describe three different sequences of transformations that could have been performed on $f(x)=x^2$ to obtain $g(x)=-3(2x-1)^2+4$.

	Step	Explanation
1.	$g(x)=-3(2x-1)^2+4$	Start with the resulting function g and describe the obvious transformations.
	$x^2 \rightarrow (x-1)^2$	Horizontal translation by 1 unit to the right.
	$(x-1)^2 \rightarrow (2x-1)^2$	Horizontal stretch by factor $q=\frac{1}{2}$.
	$(2x-1)^2 \rightarrow 3(2x-1)^2$	Vertical stretch by factor $p=3$.
	$3(2x-1)^2 \rightarrow -3(2x-1)^2$	Reflection in the x-axis because of the negative sign before the leading coefficient (-3).
	$-3(2x-1)^2 \rightarrow -3(2x-1)^2+4$	Vertical translation by 4 units upwards.
	In total, five transformations were used to obtain $g(x)=-3(2x-1)^2+4$ from $f(x)=x^2$.	
2.	$g(x)=-3\left(2\left(x-\frac{1}{2}\right)\right)^2+4$	Rewrite function f by factoring 2 out of the square.
	$x^2 \rightarrow (2x)^2$	Horizontal stretch by factor $q=\frac{1}{2}$.
	$(2x)^2 \rightarrow \left(2\left(x-\frac{1}{2}\right)\right)^2$	Horizontal translation by $\frac{1}{2}$ units to the right.
	$\left(2\left(x-\frac{1}{2}\right)\right)^2 \rightarrow 3\left(2\left(x-\frac{1}{2}\right)\right)^2$	Vertical stretch by factor $p=3$.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

	Step	Explanation
	$3\left(2\left(x-\frac{1}{2}\right)\right)^2 \rightarrow -3\left(2\left(x-\frac{1}{2}\right)\right)^2$	Reflection in the x-axis.
	$-3\left(2\left(x-\frac{1}{2}\right)\right)^2 \rightarrow -3\left(2\left(x-\frac{1}{2}\right)\right)^2+4$	Vertical translation by 4 units up.
	Again, five transformations were required.	
3.	$g(x)=-12\left(x-\frac{1}{2}\right)^2+4$	From the form of $g(x)$ used in the second sequence, take factor -12 out of the brackets.
	$x^2 \rightarrow \left(x-\frac{1}{2}\right)^2$	Horizontal translation by $\frac{1}{2}$ units to the right.
	$\left(x-\frac{1}{2}\right)^2 \rightarrow 12\left(x-\frac{1}{2}\right)^2$	Vertical stretch by factor $p=12$.
	$12\left(x-\frac{1}{2}\right)^2 \rightarrow -12\left(x-\frac{1}{2}\right)^2$	Reflection in the x-axis.
	$-12\left(x-\frac{1}{2}\right)^2 \rightarrow -12\left(x-\frac{1}{2}\right)^2+4$	Vertical translation by 4 units up.
	Here, only four transformations were required.	

3 section questions

2. Functions / 2.8 Transformation of graphs

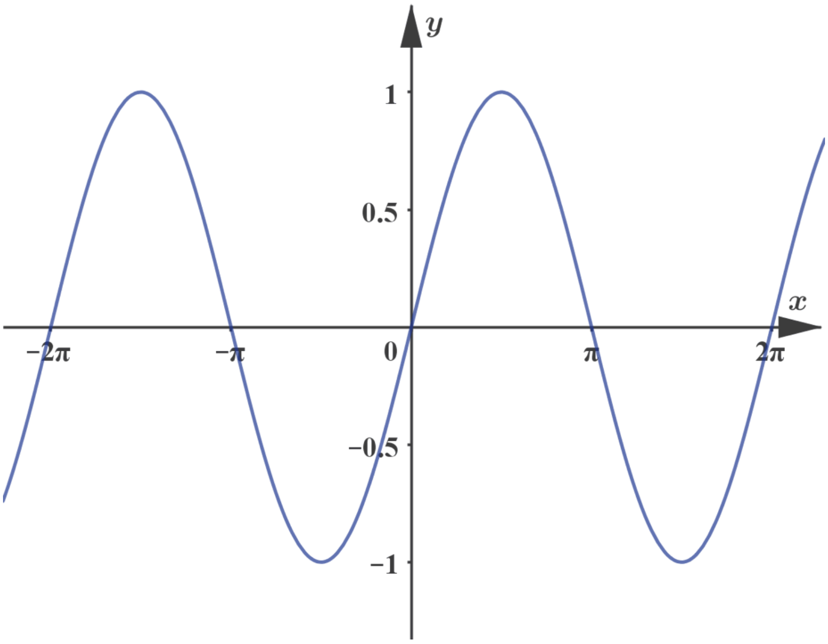
Real-life applications

Periodic functions

Many real-life problems can be solved using transformations of periodic functions.

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov



More information

The image is a graph depicting a sinusoidal wave. The X-axis is labeled with values ranging from -2π to 2π, indicating the periodic nature of the function. The Y-axis is labeled with values from -1 to 1. The wave oscillates, crossing the X-axis at -2π, -π, 0, π, and 2π. Peaks of the wave reach a Y-value of 1, occurring around -3π/2 and π/2, while valleys go down to -1 at -π/2 and 3π/2. This represents a typical sine wave, showcasing periodic motion.

[Generated by AI]

Sinusoidal models are used to represent periodic physical phenomena such as tide times, variations in sunset times throughout the year, voltages in AC circuits and so on.

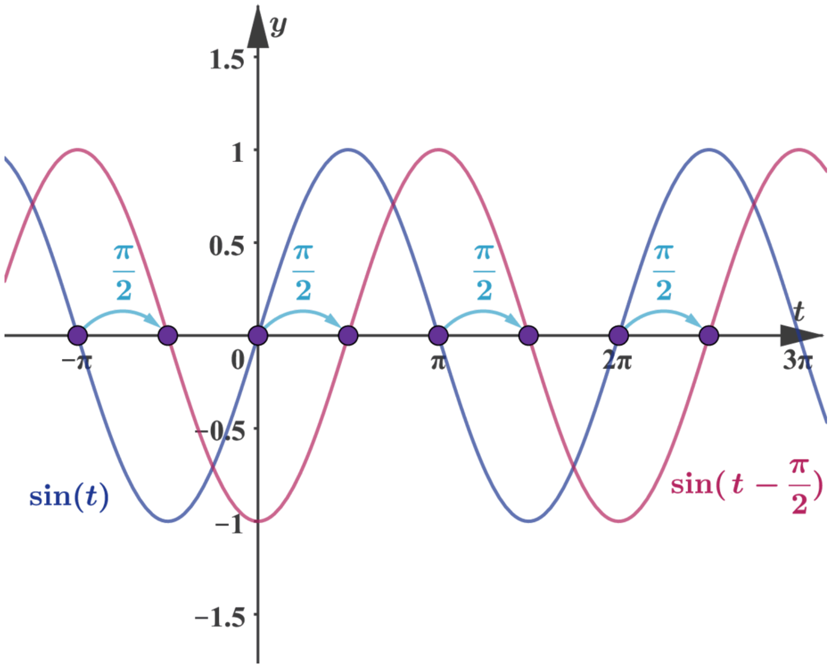
You learned about sinusoidal models of the form $f(x) = a\sin(bx) + c$ in subtopic 2.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27838/). In this section, you will add an extra transformation: $f(x) = a\sin(b(x - c)) + d$.

This transformation c is referred to as a **phase shift** in physical terms or a **horizontal shift** in mathematical terms. This shift moves the graph to the left or the right horizontally. The term phase shift is used for waves. The phase difference between two waves is the difference in their x values for corresponding minima (or maxima). In the transformation, the phase difference is c (in the same units as x). Basically, one curve has shifted c units horizontally.

For example, two waves represented by $y_1 = \sin x$ and $y_2 = \sin\left(x - \frac{\pi}{2}\right)$ have a phase difference of $\frac{\pi}{2}$ radians.

Student
view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov



More information

The image is a graph showing two sine waves represented by (y = \sin(t)) and (y = \sin(t - \frac{\pi}{2})), indicating a phase shift of (\frac{\pi}{2}) radians. The X-axis is labeled (t) and ranges from (-\pi) to (3\pi), while the Y-axis is labeled (y) and ranges from -1.5 to 1.5.

The graph displays two curves: a blue wave (\sin(t)) starting at the origin (0,0), peaking at (\frac{\pi}{2}), then dipping at (\pi) and repeating this pattern. The magenta wave (\sin(t - \frac{\pi}{2})) starts at its peak at (0,1), which indicates a right phase shift.

Both waves cross the X-axis periodically: the blue wave intersects the X-axis at 0, (\pi), and (2\pi), while the magenta wave intersects at (\frac{\pi}{2}), (\frac{3\pi}{2}), and (\frac{5\pi}{2}). The phase difference is shown with horizontal arrows between corresponding points on the curves, indicating (\frac{\pi}{2}) radians shift.

[Generated by AI]

The figure above shows two out-of-phase sound waves. Both have the same amplitude, period and axis. You can clearly see that the first sound wave intersects the axis at $t = 0$ sec and the second one at $\frac{\pi}{2}$ sec. Thus, $\sin\left(t - \frac{\pi}{2}\right)$ has been shifted $\frac{\pi}{2}$ radians to the right compared to $\sin t$.

In subtopic 2.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27838/) you learned about the amplitude, period and the equation of the axis for a sine function $f(x) = a\sin(bx) + c$. All these are the same for $f(x) = a\sin(b(x - c)) + d$ except that now we additionally have the phase shift.

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

For example, for a model $f(x) = 2\sin\left(3\left(x - \frac{\pi}{2}\right)\right) + 1$,

- the amplitude is 2
- the period is $\frac{2\pi}{b} = \frac{2\pi}{3}$
- the axis is $y = 1$
- the phase shift is $\frac{\pi}{2}$

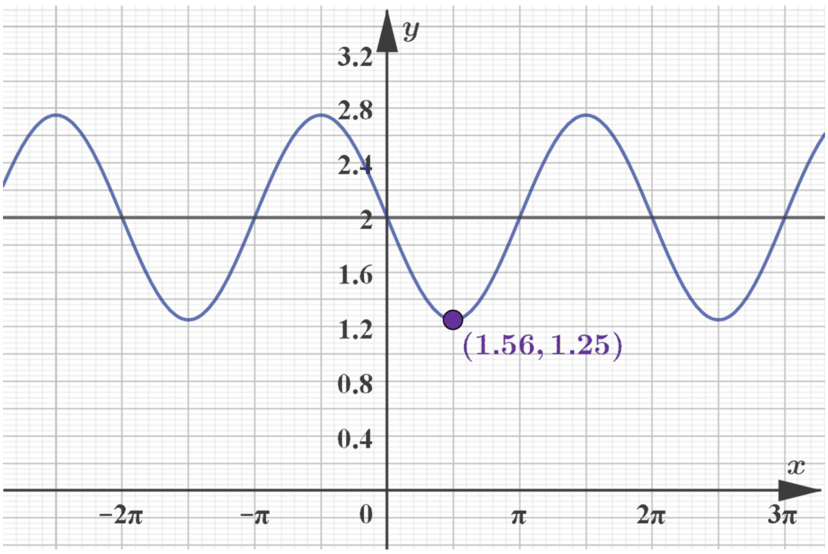
Important

Note the bracket with the period and the horizontal shift. If the model is $f(x) = a\sin(bx - c) + d$, the horizontal shift is $\frac{c}{b}$ units to the right or left and if the model is $f(x) = a\sin(b(x - c)) + d$, the horizontal shift is just c units to the right or left.

The graph moves to the right if $c > 0$ and to the left if $c < 0$.

How can you find the model from a graph?

Look at the figure below and find a model in the form $f(x) = a\sin(b(x - c)) + d$.



More information

The image is a graph of a sine wave plotted on a grid. The X-axis is marked with intervals labeled at 2π, π, 0, -π, and -2π. The Y-axis is marked from -2 to 2 in 0.5 increments. The sine curve has peaks, troughs, and crosses the Y-axis at regular intervals. A specific point on the graph is labeled with the coordinates (1.56, -1.25), indicated by a purple dot on the curve. The sine wave displays periodic behavior typical of a transformed sine function.

[Generated by AI]

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

This can be considered as a transformed form of a standard sine graph: $y = \sin x$. By considering each type of transformation, the values of a, b, c and d can be found, as explained below.

- From the figure, you can see that the vertical shift is 2 units up, so $d = 2$.
- The amplitude can be found using the minimum point given: the distance between the minimum and the axis is the amplitude. Therefore, the amplitude is $2 - 1.25 = 0.75$. Hence, $a = 0.75$
- One full cycle in the graph takes 2π radians and hence, the period is 2π. Therefore, $b = 1$.
- To find the value of c, look for the phase shift. While the graph does intersect the axis at $(0, 2)$ it is going down to a minimum not up to a maximum. Hence, there is a phase shift. It next intersects the axis at π on the right side, so we have $c = \pi$.

Putting all the values together: $f(x) = 0.75 \sin (x - \pi) + 2$.

Quadratic models

Quadratic models are used in many real-life situations. One of them is shown in the following example:

Example 1

★★☆

A tank is filled with water. A tap at its base is opened and the tank drains in such a way that the height of the water in the tank h (in meters) after time t (in hours) can be modelled by a transformed form of equation $y = x^2$.

a) Find a model in the form $h(t) = a(t - b)^2$ using the following information:

1. The initial height of the water is 65 m.
2. The tank is drained completely after 70 hours.

b) Hence find the level of water after 30 hours.

a) The conditions given in the question can be changed into mathematical statements:

$$a(0 - b)^2 = 65 \rightarrow ab^2 = 65 \qquad (1)$$

$$a(70 - b)^2 = 0 \qquad (2)$$

Student view

From (2), we get $b = 70$.

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Substituting $b = 70$ in equation (1):

$$a(70)^2 = 65 \Rightarrow a = \frac{65}{70^2} = 0.013$$

Hence, a model is:

$$h(t) = 0.013(t - 70)^2$$

b) To find the water level after 30 hours, substitute $t = 30$ into the model:

$$h = 0.013(30 - 70)^2 = 20.8 \text{ m}$$

Example 2

★★☆

The table shows the number (in thousands) of a species of fish at the start of each year over 22 years.

Year	0	2	4	6	8	10	12	14	16	18	20
Population	10.2	11.1	12.0	11.7	10.6	10.0	10.6	11.7	12.0	11.1	10.2

Why do the numbers oscillate? Find an appropriate sinusoidal model that fits the above data.

It is not possible to find the exact values of the parameters ; however, they can be estimated.

Looking at the data, the highest value is 12 and the lowest is 10, giving the vertical shift $\frac{10+12}{2} = 11$ and the amplitude $\frac{12-10}{2} = 1$.

So, $a = 1$ and $d = 11$.

Next, we will find the period and phase shift.

The minimum seems to be 10, and this occurs in year 10. There is another minimum between years 20 and 22. We can assume it occurred in year 21. Since, $21 - 10 = 11$, the period is 11 and so $b = \frac{2\pi}{11}$.

Since the vertical shift is 11, the axis is $y = 11$. The data suggests that the graph first intersects the axis somewhere between $t = 0$ and $t = 2$, closer to $t = 2$. Let's use, $c = -1.5$.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Hence, the model is

$$p(t) = \sin\left(\frac{2\pi}{11}(t - 1.5)\right) + 11$$

2 section questions

2. Functions / 2.8 Transformation of graphs

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-27626/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- graph functions using horizontal and vertical translations
- graph functions using horizontal and vertical stretches
- graph functions using reflections about the x -axis and y -axis
- graph functions using composite transformations
- use transformation of functions in real-life applications.

2. Functions / 2.8 Transformation of graphs

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-27627/print/) Assign

Use the applet to explore all the types of transformations of functions.

Student view

Interactive 1. Exploring function transformations.

More information for interactive 1

This interactive applet allows users to explore function transformations by manipulating sliders to match a randomly generated target function with a given parent function. It is a dynamic learning tool designed to enhance understanding of function transformations, including translations, stretches, and reflections.

An interactivity using a graph with four quadrants is displayed. The Original function option is at the top left, along with an input box. Users start by typing a basic function in a blue input box, such as x, x^2, x^3. After selecting a parent function, users press the “Target function” button. the target function curve is displayed as a graph alongside the parent function, along with its mathematical equation. Four sliders labeled a, b, p, and q are located at the top right. The a and b sliders range between −75 and 75, while the p and q sliders range between −50 and 50.

Users adjust sliders to modify transformation parameters and attempt to align their transformed function with the random target function.

Available transformations include:

Horizontal Translation (p) — Moves the function left or right.

Vertical Translation (q) — Moves the function up or down.

Horizontal Stretch/Compression (a) — Adjusts the width of the function.

Vertical Stretch/Compression (b) — Adjusts the height of the function.

As users move the sliders, the pink curve dynamically updates, allowing real-time visualization of the effect of each transformation. The equation of the transform function is displayed below the sliders for reference.

Horizontal translation before horizontal stretch and horizontal stretch before horizontal translation indicated by checked boxes. Users can experiment by changing the order of transformations to observe how this affects the final function.

For example,

When the Original function is Capital X^2, the Target Function changes when it is selected, and the Transformed function is Small x^2. The checkbox for horizontal translation before horizontal stretch is selected. The values of a and b are 0 and p and q are 1. The graph displays an upward parabola for the original function that starts at (−4, 12), passes through the origin, and ends at (4, 12). Another upward parabola for the target function starts at (−1, 12), passes through the origin, and ends at (1, 12). All values are estimated.

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

- Start by typing a simple **original** or **parent function** in the blue input box (e.g. x, x^2, x^3, e^x, etc.).
- When you press the 'Target function' button, the applet generates a randomly transformed function, displaying both its graph and formula.
  - You can press the 'Target function' button again if you feel that you want a new random function to be generated before you start working on it.
- As the randomly transformed function is of the form $p(f\,(qx + a) + b$, it is implied that the **horizontal translation precedes the horizontal stretch**, so the relevant box is always ticked initially.
- First, set the sliders to the values for the random function so that the two curves are the same.
  - As you move the sliders, the pink curve changes. The equation for the pink curve is shown underneath the sliders. You can drag the sliders but if you need to be more accurate, click on the slider and then use the arrows on your keyboard. For even finer accuracy, hold 'shift' down when pressing the arrows. You can also zoom in and out.
- Next, select the option that the horizontal stretch precedes the horizontal translation and find new values for the sliders for the random function.
  - Notice how changing the order of transformations, but with the same parameters, can lead to a different result.
- Finally, can you find any more combinations of transformations and the right parameter values that produces the random function?
  - For example, if your parent function is x^2 and $q \neq 1$, make q equal to 1 and try to get the same overall transformation with the remaining three transformations.

Rate subtopic 2.8 Transformation of graphs

Help us improve the content and user experience.

Student view