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The big picture

You first learned about the four operations of numbers – addition, subtraction, division and multiplication – when you were in primary school. But when you worked with complex numbers, in subtopic 1.13 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26989/), the meaning of these operations changed. Addition, subtraction, multiplication and division of complex numbers were described in geometrical terms by means of rotations and dilations. You learned that a complex number has real and imaginary parts which are often treated separately and have different physical meaning.

Making connections

In subtopic 1.13 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26989/), multiplication of two complex numbers

$$z_1 = r_1\mathrm{e}^{\mathrm{i}\theta_1} \text{ and } z_2 = r_2\mathrm{e}^{\mathrm{i}\theta_2} \text{ was written as } z_1 \times z_1 = r_1 r_2 \mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}$$

This was described geometrically as a rotation of z_1 by θ_2 radians and a stretch by scale factor r_2 , as seen in the diagram below.

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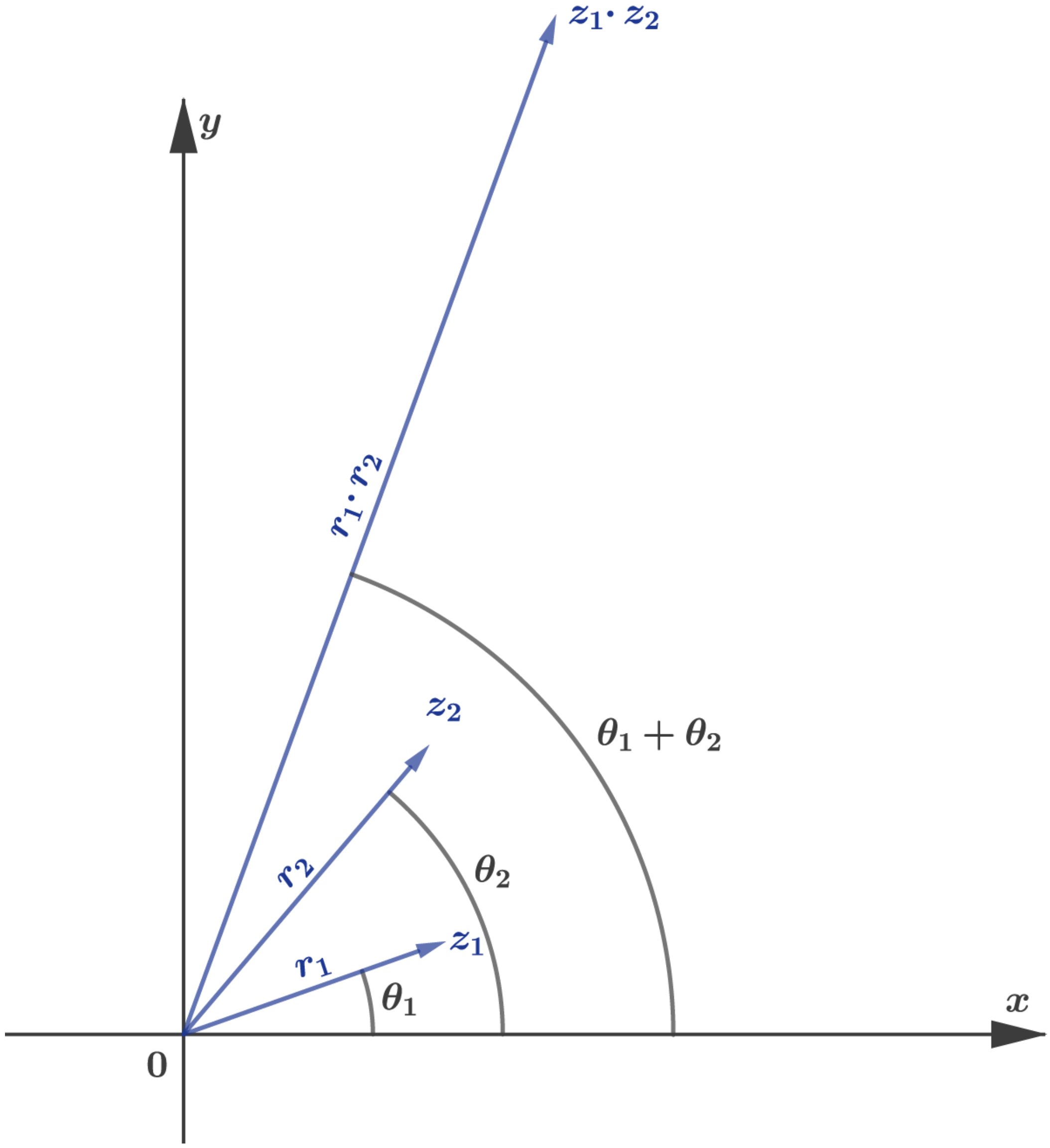

More information

The diagram shows vectors represented on a two-dimensional Cartesian plane with x and y axes. There are three vectors labeled as z1, z2, and z1·z2, originating from the origin (0,0). The vector z1 extends at an angle θ1 from the positive x-axis. The vector z2 extends at an angle θ2 from the vector z1. This creates a new resultant vector z1·z2, which extends from the origin at an angle that is the sum of θ1 and θ2. Each vector is associated with a scale factor, labeled as r1 and r2, which modifies its length. The diagram illustrates the geometric representation of complex number multiplication through rotational and scaling transformations.

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Similarly to complex numbers, operations involving vectors have a different meaning compared with operations involving numbers (scalars).

In subtopic 3.12 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-28035/) you learned how to add and subtract vectors by considering the two perpendicular components separately and you considered the

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geometrical meaning of these operations. You also learned how to multiply a vector by a scalar and the geometrical interpretation of this.

In this subtopic you will learn that multiplication of vectors has a different meaning to multiplication of scalars.

There are two types of multiplication involving vectors: scalar and vector multiplication. First you will focus on scalar multiplication and its geometrical interpretation. Later, in subtopic 3.16 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-28028/) you will learn about vector multiplication.

Concept

Points in space can be defined using vectors. Vectors can show interactions in space. If an object is acted on by electric, magnetic or gravitational fields, then vectors can be used to predict the effect on an object at a point in space. These forces may speed up or slow down a moving object depending on the direction in which they act. The strength of the interaction will depend on both the magnitudes and the direction of the forces.

Theory of Knowledge

Scalar product, sine and cosine are often used to discover unknown knowledge. When you think about it, it's pretty amazing that you can use mathematics to create new unknown knowledge that is also valid!

Consider the other areas of knowledge: do they operate in a similar way? Can you identify any other areas of knowledge that are self-perpetuating in regard to knowledge construction in the same way that mathematics is? For example, could a historian use history to create ‘new’ knowledge? What about an artist?

3. Geometry and trigonometry / 3.13 Scalar product

The scalar product

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When you learned about addition and subtraction of vectors, you saw the geometric interpretation and also an algebraic approach using components. You saw that the sum and difference can be found by adding and subtracting the corresponding components. It would be a natural approach to define the product of vectors similarly. However, as it turns out, this is not a useful definition. Instead, mathematicians agreed on two different ways of defining products of vectors. Both of these are useful in certain applications.

- In one of the definitions the product of two vectors is a number. For this reason, this is called the **scalar product**. The notation used for this product is a dot between the two vectors, $\boldsymbol{v} \cdot \boldsymbol{w}$. This product is also called dot product, this is what you will learn about in this section.
- In the second definition the product of two vectors is a vector. For this reason, this is called the **vector product**. The notation used for this product is a cross between the two vectors, $\boldsymbol{v} \times \boldsymbol{w}$. This product is also called cross product, you will learn about this later.

As with addition and subtraction, the scalar product can also be defined algebraically and geometrically. The formula booklet contains both definitions.

✓ **Important**

If $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ then the scalar or dot product of these two vectors can be defined the following two ways.

- $\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}||\boldsymbol{w}| \cos \theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$.
- $\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$

It can be shown that these two definitions are equivalent, but this proof is not presented here. You can find the connection and a motivation behind these definitions in the investigation in section 3.13.4 (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-28307/).

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To get used to the concept, let's use the second definition to find the scalar product of two given vectors.

Example 1

★☆☆

Find the scalar product of $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

Use the second definition of scalar product.

$\boldsymbol{v} \cdot \boldsymbol{w} = 1 \cdot 2 + (-1)(-1) + 0 \cdot 2 = 3$

Example 2

★☆☆

Find the scalar product of $\boldsymbol{v} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$. Hence comment on the relationship between the two vectors.

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Use the second definition of scalar product.

$$\boldsymbol{v} \cdot \boldsymbol{w} = 2 \cdot 3 + 2 \cdot (-1) + (-1) \cdot 4 = 0$$

$$\boldsymbol{v} \cdot \boldsymbol{w} = 0$$

Let's compare this result with the first definition.

As the scalar product is 0, you know that $|\boldsymbol{v}||\boldsymbol{w}| \cos \theta = 0$. This can only happen if θ, the angle between the vectors, is 90°, so the vectors are perpendicular.

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In Example 2 you discovered the following property of the scalar product.

✓ **Important**

Two vectors are perpendicular if and only if their scalar product is 0.

Example 3

★☆☆

The vectors $\boldsymbol{v} = \begin{pmatrix} a \\ 1 \\ 3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ are perpendicular.

Find the value of a.

If two vectors are perpendicular

$$\boldsymbol{v} \cdot \boldsymbol{w} = 0$$

$$\begin{aligned} \boldsymbol{v} \cdot \boldsymbol{w} &= a \cdot 1 + 1(-1) + 3 \cdot 1 = 0 \\ a + 2 &= 0 \\ a &= -2 \end{aligned}$$

Geometric property of the scalar product

You saw above that the scalar product of perpendicular vectors is 0. What happens if the scalar product is not 0? Let's rearrange the first definition.

$$\begin{aligned} \boldsymbol{v} \cdot \boldsymbol{w} &= |\boldsymbol{v}||\boldsymbol{w}| \cos\theta \\ \cos\theta &= \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{|\boldsymbol{v}||\boldsymbol{w}|} \end{aligned}$$

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If you replace the scalar product with the expression in the second definition, you get the formula that helps you find the angle between two vectors. You can find this formula in the formula booklet.

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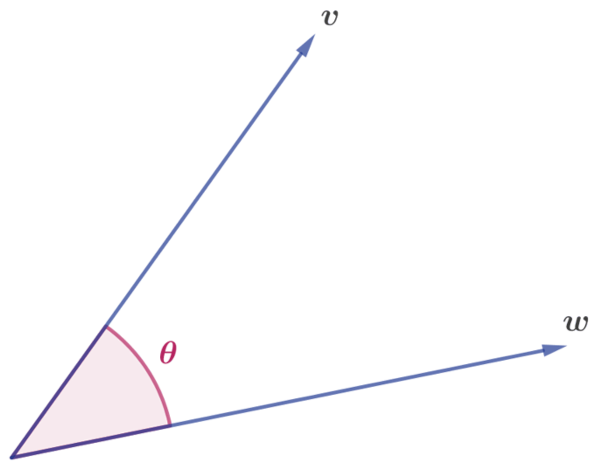
Important

If $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ then the angle between the two vectors can be found using the following formula.

$$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{|\boldsymbol{v}||\boldsymbol{w}|}$$



More information

The image is a diagram depicting two vectors, (\mathbf{v}) and (\mathbf{w}), originating from a common point and forming an angle (\theta) between them. The vector (\mathbf{v}) is shown pointing upwards to the right, and the vector (\mathbf{w}) is shown pointing downwards to the right. The angle (\theta) is highlighted in pink. The diagram illustrates the geometric representation of the dot product formula given as (\cos\theta=\dfrac{v_1w_1+v_2w_2+v_3w_3}{|\boldsymbol{v}||\boldsymbol{w}|}), emphasizing the relationship between the vectors and the cosine of the angle they form.

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Let's look at some examples.

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Example 4

★☆☆

Find the exact value of the cosine of the angle between the vectors $\boldsymbol{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

Steps	Explanation
$\boldsymbol{v} \cdot \boldsymbol{w} = 1 \cdot 1 + 1(-1) + 1 \cdot 2 = 2$	Use the scalar product $$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3$$
$\lvert\boldsymbol{v}\rvert = \sqrt{3}$ $\lvert\boldsymbol{w}\rvert = \sqrt{6}$	Find the magnitude of vectors $\boldsymbol{v}$ and $\boldsymbol{w}$. Use $\lvert\boldsymbol{u}\rvert = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$
$2 = \sqrt{3} \times \sqrt{6} \times \cos\theta$	Use $\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert \cos\theta$
$\cos\theta = \frac{2}{\sqrt{3} \times \sqrt{6}} = \frac{2}{3\sqrt{2}}$	Rearrange and simplify.
Therefore, $$\cos\theta = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$	

Example 5

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Consider the two vectors $\boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$

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Use the scalar product to show that they are parallel.

Steps	Explanation
$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert$ $$\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}3\\6\\9\end{pmatrix} = 1 \times 3 + 2 \times 6 + 3 \times 9 = 42.$$ Also $$\lvert\boldsymbol{v}\rvert = \left\lvert\begin{pmatrix}1\\2\\3\end{pmatrix}\right\rvert = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$ and $$\lvert\boldsymbol{w}\rvert = \left\lvert\begin{pmatrix}3\\6\\9\end{pmatrix}\right\rvert = \sqrt{3^2+6^2+9^2} = \sqrt{126}$$ $\lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert = \sqrt{14}\sqrt{126} = 42$ So $\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert$ and therefore $\boldsymbol{v} \| \boldsymbol{w}$.	If $\boldsymbol{v} \| \boldsymbol{w}$, then $\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert \lvert\boldsymbol{w}\rvert$ since $\cos\theta = 1$ when $\theta = 0$. However, since $3\boldsymbol{v} =$ this conclusion could been made without u the scalar product as a scalar multiple of $\boldsymbol{v}$.

Example 6

★☆☆

Find the angle between the two vectors $\boldsymbol{a} = \begin{pmatrix}-1\\3\\2\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$

Give your answer in degrees correct to 3 significant figures.

Use the expression for θ in terms of the components:

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$$\theta = \cos^{-1}\left[\frac{(-1)\times 2 + 3\times(-1) + 2\times 3}{\sqrt{(-1)^2+3^2+2^2}\sqrt{2^2+(-1)^2+3^2}}\right]$$
$$= \cos^{-1}\left[\frac{-2-3+6}{\sqrt{14}\sqrt{14}}\right]$$
$$= \cos^{-1}\left[\frac{1}{14}\right] \approx 85.9^\circ (3 \text{ significant figures})$$

6 section questions

Question 1

★☆☆

Find $\boldsymbol{a}\cdot\boldsymbol{b}$ if $\boldsymbol{a} = -\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$

1 -1 ✓

2 1

3 0

4 3

Explanation

It is a good idea to write the vectors in column format.

$$\boldsymbol{a} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\boldsymbol{a}\cdot\boldsymbol{b} = 0\times 1 + 0\times 1 + (-1)\times 1 = -1$$

Therefore, the correct answer is -1.

Question 2

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If the vectors $\boldsymbol{v} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 1 \\ a \\ 1 \end{pmatrix}$ are perpendicular, find the value of a.

1 -2 ✓

2 4

3 6

4 -5

Explanation

$$\begin{aligned} \boldsymbol{v} \cdot \boldsymbol{w} = 3 \cdot 1 + 2(a) + 1 \cdot 1 &= 0 \\ 2a + 4 &= 0 \\ 2a &= -4 \\ a &= -2 \end{aligned}$$

Therefore, the correct answer is -2.

Question 3

★☆☆

Select which of the following is the value (to one decimal place) of the angle between the vectors $-2\boldsymbol{i} + 3\boldsymbol{j}$ and $4\boldsymbol{i} + \boldsymbol{j}$.

1 $109.7°$ ✓

2 $19.7°$

3 $70.3°$

4 $80.7°$

Explanation

Let $\boldsymbol{v} = -2\boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{w} = 4\boldsymbol{i} + \boldsymbol{j}$. Then

$$\boldsymbol{v} \cdot \boldsymbol{w} = (-2) \times 4 + 3 \times 1 = -5$$

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and

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$$|\boldsymbol{v}| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

and

$$|\boldsymbol{w}| = \sqrt{4^2 + 1^2} = \sqrt{17}.$$

Hence, the angle between these vectors is

$$\begin{aligned}
\theta &= \cos^{-1}\left[\frac{\boldsymbol{v}\cdot\boldsymbol{w}}{|\boldsymbol{v}|\,|\boldsymbol{w}|}\right] \\
\Rightarrow &= \cos^{-1}\left[\frac{-5}{\sqrt{13}\sqrt{17}}\right] \\
\Rightarrow &= \cos^{-1}\left[-\frac{5}{\sqrt{221}}\right] \\
\Rightarrow &= 109.7^\circ \text{ (1 d.p.)}
\end{aligned}$$

Question 4

★★☆

Select which of the following is the value (to 3 significant figures) of the angle between the vectors $2\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$ and $-\boldsymbol{i} - \boldsymbol{j}$.

1 1.26 rad ✓

2 1.25 rad

3 1.27 rad

4 1.28 rad

Explanation

Write the vectors as column vectors:

$$\boldsymbol{u} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} \quad \boldsymbol{v} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$$\cos\theta = \frac{u\cdot v}{|u|\,|v|} = \frac{-2+4+0}{\sqrt{21}\sqrt{2}} = \frac{2}{\sqrt{21}\sqrt{2}}$$

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$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{21}\sqrt{2}}\right) \approx 1.257 \text{ rad}$$

So the answer is 1.26 radians (3 significant figures).

Question 5

★★☆

The points A and B have position vectors $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$. Find the obtuse angle between $\overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OA}}$ correct to the nearest degree.

1 $99°$ ✓

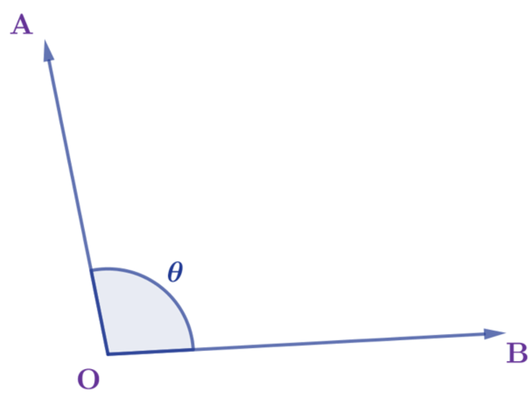
2 $81°$

3 $37°$

4 $143°$

Explanation

Draw a diagram:



More information

$$\cos\theta = \frac{\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}}}{\left|\overrightarrow{\mathrm{OA}}\right|\left|\overrightarrow{\mathrm{OB}}\right|} = \frac{2+0-3}{\sqrt{3}\sqrt{13}} = -\frac{1}{\sqrt{3}\sqrt{13}}$$

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$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{3}\sqrt{13}}\right) \approx 99.2^\circ \text{ (3 significant figures)}$$

Therefore, the correct answer is 99°.

Question 6

★☆☆

Select which of the following is the exact value of the cosine of the angle between the vectors $\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$.

1 $\frac{1}{3}$ ✓

2 $\frac{1}{\sqrt{3}}$

3 $\frac{-1}{3}$

4 $-\frac{1}{\sqrt{3}}$

Explanation

Write vectors as column vectors:

$$\boldsymbol{u} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \quad \boldsymbol{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\cos\theta = \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{|\boldsymbol{u}|\,|\boldsymbol{v}|} = \frac{1 - 1 + 1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

3. Geometry and trigonometry / 3.13 Scalar product

Properties of the scalar product

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Although the scalar product of two vectors is different from just multiplying two scalar numbers, it has some properties which are similar to multiplication. In this section, you will have a closer look at the properties of the scalar product and its geometrical implications.

Example 1

★☆☆

If $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{k}$, find $\boldsymbol{a} \cdot \boldsymbol{a}$ and $|\boldsymbol{a}|$. Hence write the relationship between $\boldsymbol{a} \cdot \boldsymbol{a}$ and $|\boldsymbol{a}|$.

Steps	Explanation
$\boldsymbol{a} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$	Write as a column vector.
$\boldsymbol{a} \cdot \boldsymbol{a} = 2 \times 2 + 0 \times 0 + (-1) \times (-1)$	Use the formula for scalar product.
$\boldsymbol{a} \cdot \boldsymbol{a} = 5$	
$\|\boldsymbol{a}\| = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{5}$	Use the formula for the magnitude of vector.
Therefore, $\boldsymbol{a} \cdot \boldsymbol{a} = \|\boldsymbol{a}\| \|\boldsymbol{a}\| = \|\boldsymbol{a}\|^{\mathbf{2}}$ You can check that this works for other vectors.	

Making connections

If an object is moving with constant acceleration its motion can be predicted using a set of equations called the $suvat$ equations, where

- s is displacement
- u is initial velocity
- v is final velocity

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- a is acceleration and
- t is time

The equations are:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

These equations are not a requirement of the syllabus and they are not included in the formula booklet. We use them here to illustrate the appropriate vector calculations.

As displacement, velocity and acceleration are vector quantities, these equations can be written in vector form:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

$$\boldsymbol{s} = \left(\frac{\boldsymbol{u}+\boldsymbol{v}}{2}\right)t \text{ and}$$

$$\boldsymbol{s} = \boldsymbol{v}t - \frac{1}{2}\boldsymbol{a}t^2$$

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The final equation $v^2 = u^2 + 2as$ needs to be written using the scalar product:

$$\boldsymbol{v} \cdot \boldsymbol{v} = \boldsymbol{u} \cdot \boldsymbol{u} + 2\boldsymbol{a} \cdot \boldsymbol{s}$$

Important

For two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$

$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta$, where θ is the angle between the two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

If $\boldsymbol{a}$ and $\boldsymbol{b}$ point in the same direction, then

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos 0 = |\boldsymbol{a}|\,|\boldsymbol{b}| \times 1 = |\boldsymbol{a}|\,|\boldsymbol{b}|$$

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If $\boldsymbol{a}$ and $\boldsymbol{b}$ point in opposite directions, then

$$\boldsymbol{a}\cdot\boldsymbol{b}=|\boldsymbol{a}|\,|\boldsymbol{b}|\cos 180^{\circ}=|\boldsymbol{a}|\,|\boldsymbol{b}|\times(-1)=-\,|\boldsymbol{a}|\,|\boldsymbol{b}|$$

Example 2

★★☆

Find a vector parallel to $\boldsymbol{v}=\boldsymbol{i}+2\boldsymbol{j}-3\boldsymbol{k}$ that has magnitude $\sqrt{126}$.

Steps	Explanation
$\boldsymbol{v}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$ Let $\boldsymbol{w}=k\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}k\\2k\\-3k\end{pmatrix}$	Write $\boldsymbol{v}$ as a column vector. Let $\boldsymbol{w}$ be a vector parallel to $\boldsymbol{w}=k\boldsymbol{v}$, where k
$\boldsymbol{v}\cdot\boldsymbol{w}=\begin{pmatrix}1\\2\\-3\end{pmatrix}\cdot\begin{pmatrix}k\\2k\\-3k\end{pmatrix}=k+4k+9k=14k$	Use the formula for scalar product $\boldsymbol{a}\cdot\boldsymbol{b}=\|\boldsymbol{a}\|\,\|\boldsymbol{b}\|\cos$
$\|\boldsymbol{v}\|=\sqrt{1^2+2^2+3^2}=\sqrt{14}$ $14k=\sqrt{14}\sqrt{126}$ $k=\dfrac{\sqrt{14}\sqrt{126}}{14}=3$	Vectors are parallel for examp when $\theta=0$ so $\cos\theta=1$ In this case $\boldsymbol{a}\cdot\boldsymbol{b}=\|\boldsymbol{a}\|\,\|\boldsymbol{b}\|$
Therefore a vector parallel to $\boldsymbol{v}=\boldsymbol{i}+2\boldsymbol{j}-3\boldsymbol{k}$ with magnitude $\sqrt{126}$ is $\boldsymbol{w}=3\boldsymbol{i}+6\boldsymbol{j}-9\boldsymbol{k}$	
The other possibility is $\boldsymbol{w}=-3\boldsymbol{i}-6\boldsymbol{j}+9\boldsymbol{k}$	Vectors are also parallel whe $\theta=180^{\circ}$ so $\cos\theta=-1$.

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Example 3

★★☆

For the vectors $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i}$ and $\boldsymbol{c} = \boldsymbol{i} - \boldsymbol{k}$, find

a) $\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c})$

b) $\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c}$

	Steps	Explanation
a)	$\boldsymbol{a} = \begin{pmatrix}1\\1\\1\end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \boldsymbol{c} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$	Write vectors as column vectors.
	$\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = \begin{pmatrix}1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}2\\0\\-1\end{pmatrix}$	$\boldsymbol{b} + \boldsymbol{c} = \begin{pmatrix}2\\0\\-1\end{pmatrix}$
	$\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = 1 \times 2 + 1 \times 0 + 1 \times (-1) = 1$ $\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = 1$	Multiply and sim
b)	$\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c} = \begin{pmatrix}1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\0\\-1\end{pmatrix}$	
	$\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c} = 1 + 1 - 1 = 1$ $\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c} = 1$	Multiply and sim

What do you notice about your answers to parts a) and b) in **Example 3** ?

The result that $\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{a} \cdot \boldsymbol{c}$ is called the distributive property of scalar multiplication. A similar rule is true for subtraction **:** $\boldsymbol{a} \cdot (\boldsymbol{b} - \boldsymbol{c}) = \boldsymbol{a} \cdot \boldsymbol{b} - \boldsymbol{a} \cdot \boldsymbol{c}$.

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Important

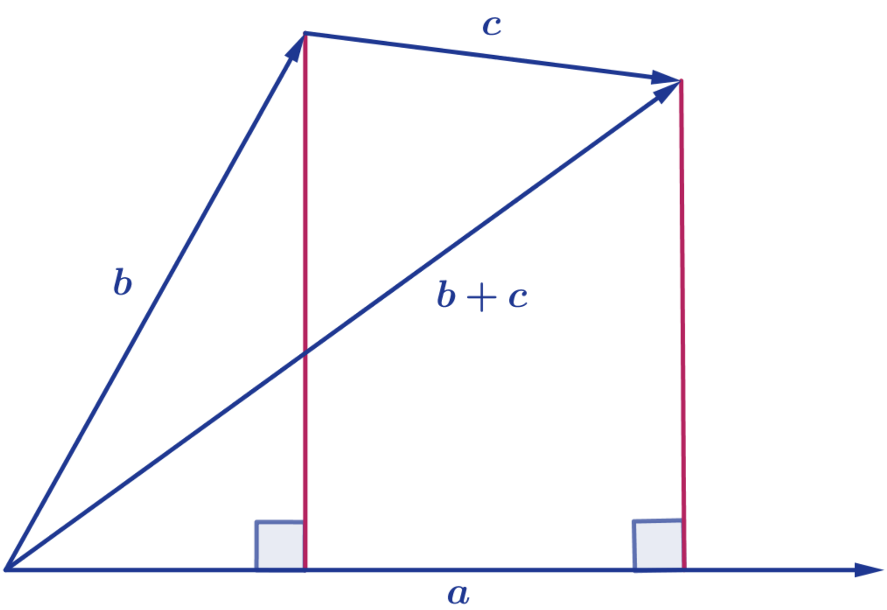
Summary of the properties of the scalar product

For three vectors $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$,

1. The scalar product is:
   - commutative $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{a}$
   - distributive for addition and subtraction $\boldsymbol{a} \cdot (\boldsymbol{b} \pm \boldsymbol{c}) = \boldsymbol{a} \cdot \boldsymbol{b} \pm \boldsymbol{a} \cdot \boldsymbol{c}$
2. When the scalar product is multiplied by a scalar m, $m(\boldsymbol{a} \cdot \boldsymbol{b}) = \boldsymbol{a} \cdot (m\boldsymbol{b})$
3. If two vectors are parallel, $\boldsymbol{b} \cdot \boldsymbol{b} = |\boldsymbol{b}|\,|\boldsymbol{b}| = |\boldsymbol{b}|^2$
4. If $\boldsymbol{a} \cdot \boldsymbol{b} = 0$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular. Conversely, if $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, then $\boldsymbol{a} \cdot \boldsymbol{b} = 0$.

Activity

In Example 3 you checked the distributive property for three given vectors. In the diagram below you can see the geometrical illustration of the proof of the distributive property of scalar product for any two vectors. Work out the details of the proof.



More information

Student view

The image is a geometric diagram that illustrates the proof of the distributive property of scalar product for vectors. The diagram consists of two main arrows representing vectors b and c extending from a common point along the horizontal line labeled 'a'. Vector b is slanted upward to the left, while vector c is slanted upward to the right. The resultant vector b + c is shown as an arrow starting from the origin of a and extending to the endpoint of vector c, forming a parallelogram.

Perpendicular lines are drawn from the tips of vectors b and c to the horizontal axis. They intersect this axis at two distinct right angles, indicating that the addition of vectors can be visualized through this parallelogram construction. The vectors themselves are labeled 'b', 'c', and 'b + c', respectively. The horizontal line representing vector 'a' is at the base of the parallelogram and extends outward to the right, indicating the direction of scalar multiplication. This visual representation aids in understanding the vector addition and scalar multiplication interaction as part of the distributive property.

[Generated by AI]

Example 4

★★☆

ABCD is a rhombus with $\overrightarrow{\mathrm{AB}} = \boldsymbol{a}$ and $\overrightarrow{\mathrm{BC}} = \boldsymbol{b}$.

Show that its diagonals are perpendicular.

Student
view

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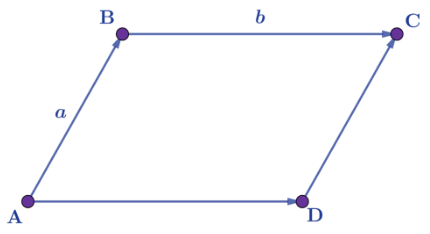
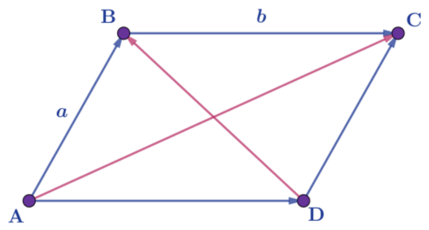
Steps	Explanation
	Sketch a diagram. Note that when you label diagrams you should continue either clockwise or anticlockwise from your starting vertex. not switch direction.
$\overrightarrow{\mathrm{AC}} = \boldsymbol{a} + \boldsymbol{b}$ $\overrightarrow{\mathrm{DB}} = \boldsymbol{a} - \boldsymbol{b}$	The diagonals are $\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{DB}}$. 
$\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{DB}} = (\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b})$	Scalar product of two vectors.
$\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{DB}} = \boldsymbol{a} \cdot \boldsymbol{a} - \boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{b} \cdot \boldsymbol{a} - \boldsymbol{b} \cdot \boldsymbol{b}$	Using the distributive property of the sc product.
$\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{DB}} = \lvert\boldsymbol{a}\rvert^2 - \lvert\boldsymbol{b}\rvert^2$	Do not write $\boldsymbol{b} \cdot \boldsymbol{b}$ as $\boldsymbol{b}^2$. Remember that correct notation is $\lvert\boldsymbol{b}\rvert^2$
$\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{DB}} = \lvert\boldsymbol{b}\rvert^2 - \lvert\boldsymbol{b}\rvert^2 = 0$	As the shape is rhombus, all the sides ha equal length. Therefore $\lvert\boldsymbol{a}\rvert = \lvert\boldsymbol{b}\rvert$

Student view

Steps	Explanation
Therefore, diagonals $\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{DB}}$ are perpendicular.	As $\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{DB}} = 0$

Activity

In a triangle, the intersection point of the medians is called the centroid of the triangle. You can investigate properties of this point using the following applet.

Prove your observations using vectors.

Interactive 1. Centroid and Vector Relationships in Triangles.

More information for interactive 1

This interactive helps users explore geometric properties of triangles using vectors, specifically focusing on the centroid—the point where all three medians of a triangle intersect. The screen shows a triangle with labeled vertices A, B, and C. Midpoints D, E, and F are marked on sides BC, AC, and AB respectively. A point G represents the centroid of the triangle, where the medians intersect.
Two vectors are highlighted: vector u points from vertex B to the centroid G, and vector v points from the centroid G to midpoint D. These vectors are shown with arrows, and their magnitudes are displayed numerically on the screen. Users can click and drag the triangle's vertices (A, B, and C), which automatically updates the triangle's shape and recalculates the

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lengths of vectors u and v.

As users move the vertices, they observe that the magnitude of vector u is always exactly twice the magnitude of vector v. This pattern is consistent regardless of the triangle's shape. This reflects a key geometric property: the centroid divides each median in a 2:1 ratio, with the longer segment extending from the vertex to the centroid and the shorter segment from the centroid to the midpoint of the opposite side.

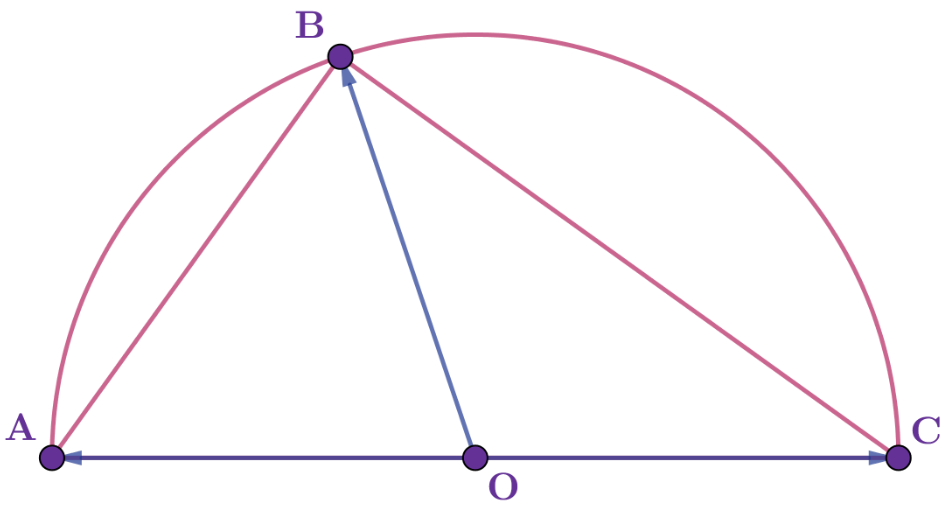
The activity also maintains the direction of both vectors u and v along the same line—the median—helping users understand the alignment and proportional relationship between them. This visual and interactive experience deepens user understanding of the geometric and vector-based properties of a triangle's centroid.

For example, if the triangle is adjusted so that vector u has a magnitude of 1.9479, vector v will always have a magnitude of approximately 0.974, confirming the 2:1 ratio. This consistency builds intuition about how medians behave and how vector representations can illustrate geometric relationships.

Example 5

★★☆

The diagram below shows a semicircle with centre at origin, O. If $\overrightarrow{\mathrm{OA}}$, $\overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OC}}$ are position vectors of points A, B and C, respectively, prove that angle ABC is $90°$.



More information

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The diagram shows a semicircle with its center at the origin, labeled as O. There are three labeled points on the semicircle, A, B, and C. Vectors are drawn from the center O to each point, referred to as (\overrightarrow{\text{OA}}), (\overrightarrow{\text{OB}}), and (\overrightarrow{\text{OC}}). Point A is on the left end of the diameter, point C on the right end, and point B is located on the arc, above the diameter, forming right triangle (\triangle ABC). The angle (\angle ABC) is supposed to be (90^\circ).

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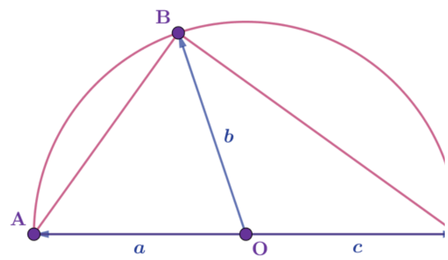
Steps	Explanation
Let $$\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$$ $$\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$$ $$\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$$	
$\overrightarrow{AB} = \boldsymbol{b} - a$ and $\overrightarrow{BC} = \boldsymbol{c} - \boldsymbol{b}$	$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$ $\overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OB}}$
$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = (\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{c} - \boldsymbol{b})$	Use the scalar product of $\overrightarrow{\mathrm{AB}}$ and .
$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = \boldsymbol{b} \cdot \boldsymbol{c} - \boldsymbol{b} \cdot \boldsymbol{b} - \boldsymbol{a} \cdot \boldsymbol{c} + \boldsymbol{b} \cdot \boldsymbol{a}$	Expanding the brackets. Do not $\boldsymbol{b} \cdot \boldsymbol{b}$ as $\boldsymbol{b}^2$ as $\boldsymbol{b}$ is a vector. the correct notation.

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Steps	Explanation
$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = \boldsymbol{b} \cdot (-\boldsymbol{a}) - \|\boldsymbol{b}\|^2 - \boldsymbol{a} \cdot (-\boldsymbol{a}) + \boldsymbol{b} \cdot \boldsymbol{a}$	$\boldsymbol{b} \cdot \boldsymbol{b} = \|\boldsymbol{b}\|^2$ $\boldsymbol{c} = -\boldsymbol{a}$
$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = -\boldsymbol{b} \cdot \boldsymbol{a} - \|\boldsymbol{b}\|^2 + \|\boldsymbol{a}\|^2 + \boldsymbol{b} \cdot \boldsymbol{a}$	
$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = -\|\boldsymbol{b}\|^2 + \|\boldsymbol{b}\|^2$	Simplify:$-\boldsymbol{b} \cdot \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{a} = 0$ and $\|\boldsymbol{a}\| = \|\boldsymbol{b}\|$.
$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{BC}} = 0$	
Therefore, the angle between vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{BC}}$ is $90°$. Therefore $\overrightarrow{\mathrm{AB}} \perp \overrightarrow{\mathrm{BC}}$.	

Example 6

★★★

For non-zero vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, show that if $|\boldsymbol{u} - \boldsymbol{v}| = |\boldsymbol{u} + \boldsymbol{v}|$, then the two vectors are perpendicular.

Steps	Explanation
$\|\boldsymbol{u} - \boldsymbol{v}\|^2 = \|\boldsymbol{u} + \boldsymbol{v}\|^2$	Since $\|\boldsymbol{u} - \boldsymbol{v}\| = \|\boldsymbol{u}$
$\|\boldsymbol{u} - \boldsymbol{v}\|^2 = (\boldsymbol{u} - \boldsymbol{v})(\boldsymbol{u} - \boldsymbol{v})$ $\|\boldsymbol{u} + \boldsymbol{v}\|^2 = (\boldsymbol{u} + \boldsymbol{v})(\boldsymbol{u} + \boldsymbol{v})$	Use $\boldsymbol{b} \cdot \boldsymbol{b} = \|\boldsymbol{b}\|^2$
$(\boldsymbol{u} - \boldsymbol{v})(\boldsymbol{u} - \boldsymbol{v}) = (\boldsymbol{u} + \boldsymbol{v})(\boldsymbol{u} + \boldsymbol{v})$	Use $\boldsymbol{b} \cdot \boldsymbol{b} = \|\boldsymbol{b}\|^2$

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Steps	Explanation
$\boldsymbol{u}\cdot\boldsymbol{u}-\boldsymbol{u}\cdot\boldsymbol{v}-\boldsymbol{v}\cdot\boldsymbol{u}+\boldsymbol{v}\cdot\boldsymbol{v}=\boldsymbol{u}\cdot\boldsymbol{u}+\boldsymbol{u}\cdot\boldsymbol{v}+\boldsymbol{v}\cdot\boldsymbol{u}+\boldsymbol{v}\cdot\boldsymbol{v}$	Use the distributive property of the scal product.
$\lvert\boldsymbol{u}\rvert^2-2\boldsymbol{v}\cdot\boldsymbol{u}+\lvert\boldsymbol{v}\rvert^2=\lvert\boldsymbol{u}\rvert^2+2\boldsymbol{u}\cdot\boldsymbol{v}+\lvert\boldsymbol{v}\rvert^2$	$\boldsymbol{u}\cdot\boldsymbol{u}=\lvert$ and $\boldsymbol{v}\cdot\boldsymbol{v}=\lvert\boldsymbol{v}$ The scalar product i commutative so $v\cdot u=\ u$
$4\boldsymbol{u}\cdot\boldsymbol{v}=0$	Simplify and rearran
$\boldsymbol{u}\cdot\boldsymbol{v}=0$	
Therefore, vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular.	

3 section questions

Question 1

★☆☆

For non-zero vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, find $\boldsymbol{u}\cdot(\boldsymbol{u}-3\boldsymbol{v})$ if $\boldsymbol{u}$ is a unit vector perpendicular to $\boldsymbol{v}$.

1 1 ✓

2 0

3 3

4 -3

Student view

Explanation

Using the distributive property

$$\boldsymbol{u}\cdot(\boldsymbol{u}-3\boldsymbol{v})=\boldsymbol{u}\cdot\boldsymbol{u}-3\boldsymbol{u}\cdot\boldsymbol{v}=|\boldsymbol{u}|^2-3\boldsymbol{u}\cdot\boldsymbol{v}$$

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As $\boldsymbol{u}$ is a unit vector, $|\boldsymbol{u}|=1$ and it is perpendicular to $\boldsymbol{v}$ so

$$\boldsymbol{u}\cdot\boldsymbol{v}=0$$

$$|\boldsymbol{u}|^2-3\boldsymbol{u}\cdot\boldsymbol{v}=1-0=1$$

Therefore, the correct answer is 1.

Question 2

★☆☆

A plane is flying at a constant speed of $150\,\mathrm{m\,s^{-1}}$ parallel to the vector $\boldsymbol{w}=3\boldsymbol{i}-4\boldsymbol{j}$.

Find the velocity of the plane.

1 $90\boldsymbol{i}-120\boldsymbol{j}\,\mathrm{m\,s^{-1}}$ ✓

2 $30\boldsymbol{i}-40\boldsymbol{j}\,\mathrm{m\,s^{-1}}$

3 $450\boldsymbol{i}-600\boldsymbol{j}\,\mathrm{m\,s^{-1}}$

4 $50\boldsymbol{i}-37.5\boldsymbol{j}\,\mathrm{m\,s^{-1}}$

Explanation

Let $\boldsymbol{v}\,\mathrm{m\,s^{-1}}$ be the velocity.

The velocity is parallel to $\begin{pmatrix}3\\-4\end{pmatrix}$ so it is a scalar multiple of this vector:

$$\boldsymbol{v}=k\begin{pmatrix}3\\-4\end{pmatrix}$$

Student
view

The magnitude of $\begin{pmatrix}3\\-4\end{pmatrix}$ is $\sqrt{3^2+(-4)^2}=5$

$|\boldsymbol{v}|=150$ (since the speed is the magnitude of the velocity vector), so

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$5k = 150$

$k = \frac{150}{5} = 30$

Therefore the velocity of the plane is $30\begin{pmatrix}3\\-4\end{pmatrix} = \begin{pmatrix}90\\-120\end{pmatrix}$ or $90\boldsymbol{i} - 120\boldsymbol{j}\ \mathrm{m\,s^{-1}}$

Question 3

★★★

For non-zero vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, $|\boldsymbol{u}| = 2$ and the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\frac{\pi}{6}$.

If $\boldsymbol{u}\cdot(\boldsymbol{u}-\boldsymbol{v}) = 1$, find the magnitude of $\boldsymbol{v}$.

1 $\sqrt{3}$ ✓

2 $\frac{1}{\sqrt{3}}$

3 $3\sqrt{3}$

4 $-\sqrt{3}$

Explanation

Using the distributive property

$$\boldsymbol{u}\cdot(\boldsymbol{u}-\boldsymbol{v}) = \boldsymbol{u}\cdot\boldsymbol{u} - \boldsymbol{u}\cdot\boldsymbol{v} = |\boldsymbol{u}|^2 - \boldsymbol{u}\cdot\boldsymbol{v}$$

$$|\boldsymbol{u}|^2 - \boldsymbol{u}\cdot\boldsymbol{v} = |\boldsymbol{u}|^2 - |\boldsymbol{u}|\,|\boldsymbol{v}|\cos\frac{\pi}{6}$$

As $|\boldsymbol{u}| = 2$

$$|\boldsymbol{u}|^2 - \boldsymbol{u}\cdot\boldsymbol{v} = 4 - 2\,|\boldsymbol{v}|\cos\frac{\pi}{6}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{and } \boldsymbol{u}\cdot(\boldsymbol{u}-\boldsymbol{v}) = 1$$

So $4 - 2\,|\boldsymbol{v}|\,\frac{\sqrt{3}}{2} = 1$

Rearrange and solve

Student view

$$\sqrt{3}\,|\boldsymbol{v}| = 3$$

$$|\boldsymbol{v}| = \frac{3}{\sqrt{3}} = \sqrt{3}$$

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3. Geometry and trigonometry / 3.13 Scalar product

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-28306/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- define the scalar product as directional multiplication of vectors:

$$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3, \text{ where } \boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

- define the scalar product as the projection of one vector onto another: $\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}|\,|\boldsymbol{w}| \cos\theta$, where θ is the angle between the vectors $\boldsymbol{v}$ and $\boldsymbol{w}$
- recall that for, two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, the scalar product is:
  - commutative: $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{a}$
  - distributive for addition and subtraction: $\boldsymbol{a} \cdot (\boldsymbol{b} \pm \boldsymbol{c}) = \boldsymbol{a} \cdot \boldsymbol{b} \pm \boldsymbol{a} \cdot \boldsymbol{c}$
- recall that when a scalar product is multiplied by a scalar number $\boldsymbol{m}$, $m(\boldsymbol{a} \cdot \boldsymbol{b}) = \boldsymbol{a} \cdot (m\boldsymbol{b})$
- use $\boldsymbol{b} \cdot \boldsymbol{b} = |\boldsymbol{b}|^2$
- recall that if $\boldsymbol{a} \cdot \boldsymbol{b} = 0$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular and, conversely, if $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, then $\boldsymbol{a} \cdot \boldsymbol{b} = 0$
- recall that if $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel, then $\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}|\,|\boldsymbol{w}|$.

Student view

3. Geometry and trigonometry / 3.13 Scalar product

Investigation

In this investigation you will explore the relationship between the two definitions of the scalar product.

Consider a triangle OVW with vertices $\text{O}(0,0)$, $\text{V}(v_1,v_2)$ and $\text{W}(w_1,w_2)$ in the coordinate plane.

- Express the lengths of the sides of the triangle in terms of the coordinates of the vertices.
- Use the cosine rule to find the angle at vertex O.
- Did you get $\cos\theta = \dfrac{v_1w_1+v_2w_2}{\sqrt{v_1^2+v_2^2}\sqrt{w_1^2+w_2^2}}$?

If you now introduce vectors $\boldsymbol{v} = \overrightarrow{\text{OV}}$ and $\boldsymbol{w} = \overrightarrow{\text{OW}}$, the equation above can be rearranged to the following form.

$$|\boldsymbol{v}||\boldsymbol{w}|\cos\theta = v_1w_1 + w_2w_2.$$

This equality justifies that both sides of the equality can be used to define the scalar product.

- In the formula booklet you have the definitions using three coordinates. Repeat the steps above for a triangle in space with vertices $\text{O}(0,0,0)$, $\text{V}(v_1,v_2,v_3)$ and $\text{W}(w_1,w_2,w_3)$. Did you get a similar equality?

Rate subtopic 3.13 Scalar product

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