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Teacher view

(https://intercom.help/kognity)

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4. Probability and statistics / 4.5 Probability and expected outcomes

The big picture

How do you choose?

Credit: esseffe GettyImages

How do you make decisions? When you are striving for a certain outcome, but life is uncertain, how do you decide? Throughout your life, you will be faced with major and minor decisions, and will need to ask ‘What are the chances this will help me to reach my goal?’

Investors analyse charts full of data trying to determine the best way to make their money grow for retirement. Doctors and patients carefully weigh the potential for improved quality of life against the risks of an operation when deciding whether to undergo a procedure. But not every decision is this significant. You have probably had to decide whether another hour of sleep or another hour of study would be more likely to help you pass the exam the next day. Or maybe you have to decide which route you should take to avoid being late to school. These decisions,

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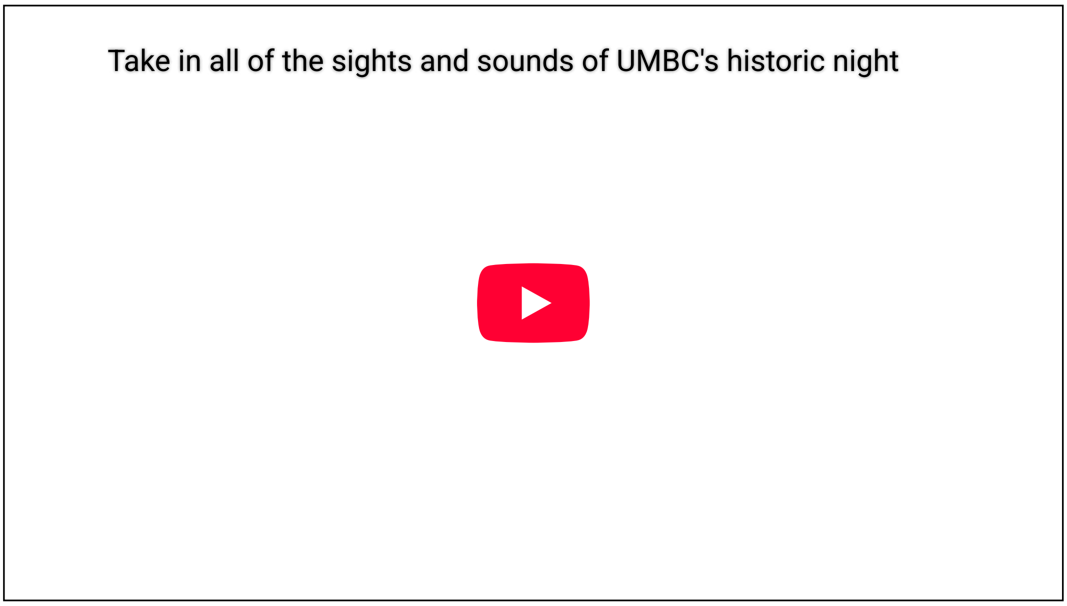
both dire and mundane, are all driven by experience, data, patterns and prediction – by probability. In its most basic sense, probability answers the question 'What are the chances that ____ will happen?' Some probabilities are determined through inductive means, by examining past events and assuming certain factors will remain consistent in the future. This is called experimental probability. Other probabilities are determined logically using a more deductive method. This is known as theoretical probability. Regardless of how you calculate probability, remember that it is simply a measure of how **likely** something is. There are no guarantees in life.

Making connections

Each March in the United States, college basketball teams compete in the March Madness Basketball Tournament. The 16th-ranked team plays the first-ranked team in the first round, and in the history of the tournament, the first-ranked team never lost — until 2018. Until that year, the odds for the first-ranked team winning were 132—0, so when the University of Virginia played the University of Maryland — Baltimore County (UMBC), the experimental probability that Virginia would win was 100%. UMBC played an incredible game, however, and made history, winning 74—54.



As you study probability, it is important to remember that both experimental and theoretical probability are uncertain. No matter how close the probability is to 100%, you still cannot be certain of the outcome.

Important

No matter how high the probability, remember that it is still just a measure of **likelihood**, not certainty. Have you ever been surprised when you were sure something

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was going to happen, but it didn't?

Concept

The study of probability is really the study of **patterns**. Through a combination of inductive and deductive reasoning, you can make predictions based on what is **likely** to happen. For example, you predict that it will be better to take a longer route to school because in your experience there is more traffic on the shorter route, making a delay more likely. There are many benefits to understanding probability, but there are also plenty of ways that you could misuse or misapply what you will learn in this subtopic. What are the limitations of making predictions based on previous events? What assumptions are you making when you use probability to make predictions? What factors prevent you from being absolutely certain of an outcome?

4. Probability and statistics / 4.5 Probability and expected outcomes

The probability of an event

Events and probability

Random events and sample space

Let us consider a random experiment, for example rolling a regular dice. Random experiments are those where, although we don't know the exact result in advance, we do know the set of all possible results. This set of all possible outcomes of a random experiment is called the sample space, U, of the experiment. For example, if the random experiment is rolling a dice, then the sample space of this experiment is the set of all possible outcomes on the upper face of the dice, i.e. $U = \{1, 2, 3, 4, 5, 6\}$.

Be aware

Assume a dice has six sides unless you are told otherwise.

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We denote the number of all possible outcomes that make up the sample space U as $n(U)$. Thus, for the random experiment of rolling a dice, $n(U) = 6$.

Any *subset* of a sample space of a random experiment is called an event . The event consists of one or more outcomes of the experiment. Each time an experiment is repeated, it is considered a trial of the experiment. For example, in the experiment of rolling a dice, the subset $\{2\}$ of the sample space is an event that can be described as 'the number on the face is 2 '. The subset $\{1, 3, 5\}$ is an event that can be described as ‘the number on the face is odd’.

Example 1

★★☆

A coin is flipped four times. Find all the possible outcomes of the four coin-flips, that is, the sample space S for this experiment.

The experiment is repeated, but this time only the number of heads is counted. Find the sample space S for the second experiment.

Heads or tails — what are the chances?

Credit: robynmac Getty Images

If we let H represent heads and T represent tails, then we can list all the possible outcomes of the four coin-flips as follows.

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- Four heads can occur one way.

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- $H-H-H-H$
- Three heads can occur four ways.
  - $H-H-H-T$
  - $H-H-T-H,$
  - $H-T-H-H$
  - $T-H-H-H$
- Two heads can occur six ways.
  - $H-H-T-T$
  - $H-T-H-T$
  - $T-H-H-T$
  - $H-T-T-H$
  - $T-H-T-H$
  - $T-T-H-H$
- One head can occur four ways.
  - $H-T-T-T$
  - $T-H-T-T$
  - $T-T-H-T$
  - $T-T-T-H$
- No heads can occur one way.
  - $T-T-T-T$

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As you can see, there is a total of 16 possible outcomes for the four coin-flips. The sample space is:

$$S=\begin{Bmatrix} HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, \\ HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT \end{Bmatrix}$$

The goal of the second experiment was simply to count the number of heads in four flips, regardless of their sequence.

Therefore, the sample space you are interested in is $S=\{4H, 3H, 2H, 1H, 0H\}$.

International Mindedness

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The name ‘heads’ refers to the fact that many countries have faces of notable people on their coin currency. The other side has another design and is referred to as ‘tails’. Some countries do not use faces on their coins, but you can assign heads to one side for an experiment.

Experimental and theoretical probability

As we stated earlier, there are two types of probability: theoretical and experimental. Experimental probability, also known as relative frequency , is found inductively by repeating an experiment a number of times and counting the number of times that particular outcome occurs.

Activity

Experimental statistics is used extensively in a wide variety of fields. In sports, managers determine which player would be the best one to play in certain situations from data on how they have performed in similar situations previously. When analysing large records of data, financial analysts can detect fraud using Benford’s Law (http://mathworld.wolfram.com/BenfordsLaw.html), a process dependent on how often the leading digit of numbers in a list tends to be a $1, 2, 3$, etc. You might be surprised to learn that they are not equally likely — 1 occurs approximately 30% of the time. Why do you think that might happen?

A truly random experiment will have a long-term regularity in the outcomes. For example, if we repeat the experiment of rolling a regular (or fair) dice many times, i.e. for a really big number of trials, say N, we will see that the number of times 2 appears on the upper face of the dice tends to stabilise to one-sixth of the N rolls. Thus, we say that when rolling a fair dice, the theoretical probability of getting a 2 on the upper face is $\frac{1}{6}$. This is also the theoretical probability of getting any other number . We would expect that in a large number of trials, one-sixth of the rolls would have resulted in a 1, one-sixth of the rolls would have resulted in a 3, and so on, as the experimental probability tends to approach the theoretical probability for a large number of trials. Therefore, rolling a dice is a random experiment.

A significant concept when considering related events is whether or not the events are mutually exclusive , meaning they do not share any outcomes – they cannot occur together. For example, rolling a 2 and rolling an odd number would be mutually exclusive because 2 is not in the set of odd numbers. Rolling a 2 and rolling a prime number would not be mutually exclusive because 2 is also a prime number.

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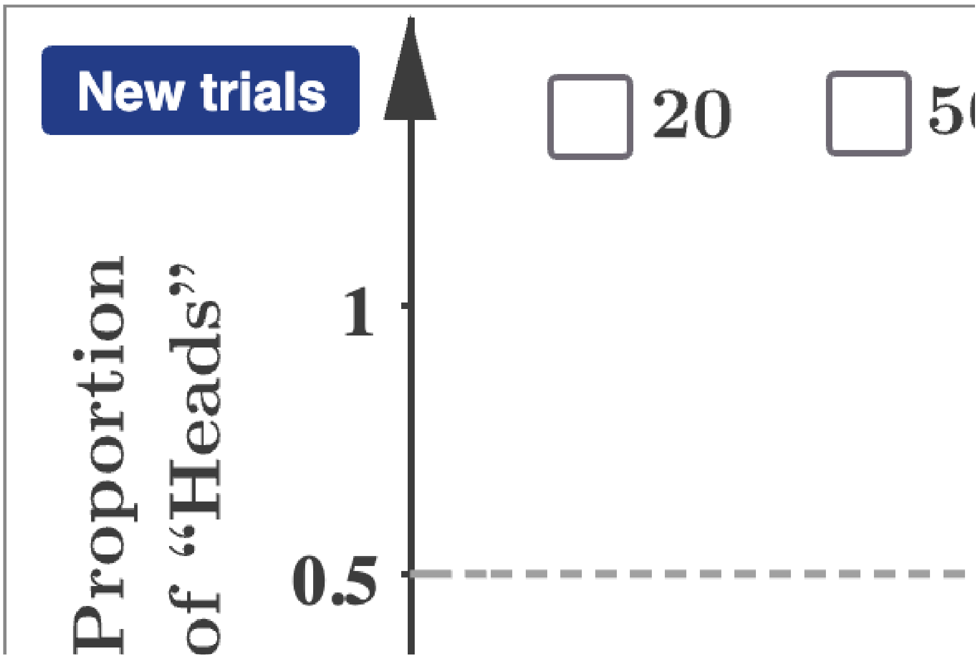
Below is an applet that simulates the random experiment of tossing a fair coin. We can see that as the number of trials increases (from 20 to 50 and then to 100) the proportion of heads in the number of trials approaches 0.5. Why do you think a greater number of trials makes such a difference?



Interactive 1. Simulating the Probability of Getting a 'Heads' Flipping a Fair Coin.

More information for interactive 1

This interactive simulates the random experiment of tossing a fair coin, helping users explore probability and long-term regularity in outcomes. Users can select the number of trials (e.g., 20, 50, or 100) and observe how the proportion of "Heads" evolves over time.

The x-axis represents the number of coin tosses, and the y-axis shows the proportion of "Heads" obtained. As the number of trials increases, users will see the proportion approach the theoretical probability of 0.5, illustrating the law of large numbers.

This hands-on tool reinforces how experimental probability converges to theoretical probability with repeated trials, deepening understanding of probability and randomness.

Thus, when running an experiment for N trials that results in an event A occurring $n(A)$ times, the probability of A happening, $\mathrm{P(A)}$, is

$$\mathrm{P(A)} = \lim_{\mathrm{N}\to\infty} \frac{\mathrm{n(A)}}{\mathrm{N}}.$$

In probability theory, if an experiment has equally likely outcomes, the probability of event A occurring is defined as

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$$\mathrm{P(A)} = \frac{n(A)}{n(U)}$$

$$\mathrm{P(A)} = \frac{\text{Number of outcomes in which A occurs}}{\text{Total number of outcomes in the sample space}}$$

Finding probabilities deductively like this is what is known as theoretical probability.

We have the following axioms for probability:

✓ **Important**

For any event A, $0 \leqslant \mathrm{P(A)} \leqslant 1$.

The probability of nothing (o) occurring is zero, i.e. $\mathrm{P(o)} = 0$, and the probability of one of all the outcomes in the sample space occurring , that is, of something occurring, is 1, i.e. $\mathrm{P(U)} = 1$.

If A and B are in U and are mutually exclusive, then the probability of either A or B happening is $\mathrm{P(A \cup B)} = \mathrm{P(A)} + \mathrm{P(B)}$.

Example 2

★☆☆

Four boys and six girls are waiting to see the doctor, but the receptionist does not remember who arrived first. If the receptionist chooses one of the patients at random, find the probability that a boy is chosen to see the doctor next.

Start with the formula for probability, letting B represent the boys waiting and U represent all the people waiting for the doctor:

$$\mathrm{P(B)} = \frac{\mathrm{n(B)}}{\mathrm{n(U)}}$$

Since there are four boys, $n(B) = 4$, while $n(U) = 10$.

Therefore, $\mathrm{P(B)} = \frac{4}{10} = \frac{2}{5} = 0.4$

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Exam tip

Unless a problem specifically asks for an answer in a certain form, probabilities can be given as a fraction or decimal. When giving answers as a decimal, remember to give them accurate to 3 significant figures.

Example 3

★★☆

For a normal 365-day year, find the theoretical probability that a person has a birthday in August, assuming that a person is equally likely to be born any day of the year.

Start with the formula for probability, letting A represent the event days in August and U represent the days in a year:

$$\mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(U)}}$$

Since August has 31 days in it, $n(A) = 31$, while $n(U) = 365$.

Therefore, $\mathrm{P(A)} = \dfrac{31}{365} \approx 0.0849$.

The complement to an event is all the other events in the sample space. We write the complement of A as A' (spoken as 'not A'). Thus, A and A' exhaust all possible outcomes, and are necessarily mutually exclusive, $\mathrm{P(A \cup A')} = 1$. Hence,

$$\mathrm{P(A')} = 1 - \mathrm{P(A)}.$$

Thus, the complement to rolling a 2 with a regular dice, which itself has a probability of $\frac{1}{6}$, is rolling a $1, 3, 4, 5$ or 6 , which has a probability of $\frac{5}{6}$. C ombined, these exhaust all possible outcomes for rolling a regular dice and thus, add as $\frac{1}{6} + \frac{5}{6} = 1$.

Example 4

Student view

★☆☆

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Given your answer from Example 3 above, find the probability that a person does **not** have their birthday in August.

Previously, we found the probability that someone is born in August to be $\mathrm{P(A)} = 0.0849$.

The complement of being born in August is **not** being born in August, so $\mathrm{P(A')} = 1 - \mathrm{P(A)} = 1 - 0.0849 = 0.9151$

Some probabilities are not difficult to find, but they require numerous, repetitive calculations. For example, suppose A represents serving an ace on any serve in a volleyball match (an ace is a serve that your opponent cannot return). If you know the probability of serving an ace on one serve and your team serves the ball 25 times in a game, you could find the probability of serving at least two aces by calculating $\mathrm{P(2A)} + \mathrm{P(3A)} + \mathrm{P(4A)} + \ldots + \mathrm{P(25A)}$. But, looking at it from a different perspective, you could more easily use the complement to find it, by calculating $1 - \mathrm{P(0A)} - \mathrm{P(1A)}$.

Be aware

Some of the more challenging probabilities to calculate are those that ask for the probability of getting 'at least 1' of event A. The easiest way to solve these problems and avoid performing many tedious calculations is to realise that $\mathrm{P}(\text{at least } 1A) = 1 - \mathrm{P}(\text{no } A) = 1 - \mathrm{P}(A')$.

In more advanced scenarios, combinations may also be used to calculate theoretical probability. Since finding combinations is not part of all syllabi, the rest of this section is only relevant for exam preparation if you read this section as part of the **higher level analysis and approaches book**.

Combinations are used to determine how many different ways you can *choose* a group of r items in any order out of a group of n items, given by the formula ${}^{n}\mathrm{C}_{r} = \frac{n!}{r!(n-r)!}$. For example, if you want to assemble a committee of students and teachers to plan a school event, you could determine how many possible committees you could have. If you want the committee to contain 3 of the 24 teachers and 12 of the 75 students, you can choose the 3 teachers out of the group of 24 using the combination ${}^{24}\mathrm{C}_{3} = 2024$. You can choose the 12 students out of the 75 using the combination ${}^{75}\mathrm{C}_{12} = 26\,123\,889\,412\,400$. Each of the 2024 different groups

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of teachers could be paired with any of the 26 123 889 412 400 groups of students to form the committee, so there are $2024 \times 26\,123\,889\,412\,400 = 52\,874\,752\,170\,697\,600$ different committees that could be formed.

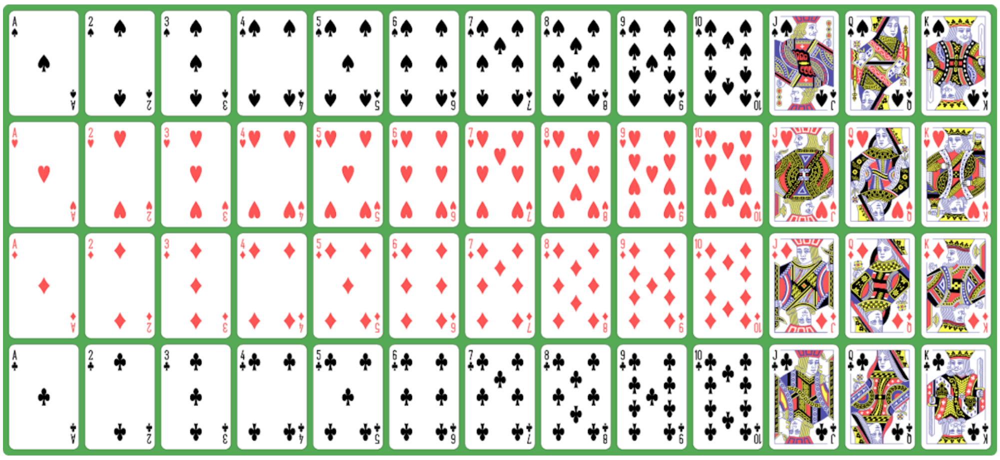
Scenarios like these are typically beyond the level of this course, but it is worth exploring briefly the fact that combinations can be applied when using the basic probability formula, as you can see in Example 5.

Example 5

★★★

A deck of playing cards contains 52 cards. Suppose you select 5 cards. Find the probability that exactly 2 of the cards are face cards (jack, queen, king).

Note that in a standard 52-card deck there are four suits (clubs, diamonds, hearts and spades). In each suit there are 13 cards (an ace, a jack, a queen, a king and nine cards with numerals from 2 to 10).



Standard 52-card deck

Source: "English pattern playing cards deck (https://commons.wikimedia.org/wiki/File:English_pattern_playing_cards_deck.svg)" by Dmitry Fomin is in public domain.

More information

The image shows a complete set of a standard 52-card deck. The cards are organized in a grid format according to the four suits: spades, hearts, diamonds, and clubs. Each row represents a different suit and contains 13 cards ranging from Ace, numbers 2 to 10, and the face cards Jack, Queen, King. The spades and clubs are in black, while the hearts and diamonds are in red. The arrangement highlights the uniform structure of a standard card deck with the identical pattern of numbering and face cards repeated in each suit.

Student view

[Generated by AI]

To use the probability formula, you need to use combinations to find the number of ways to select 2 face cards and the number of ways to select 5 cards.

If you want to select 2 face cards out of the 12 face cards in the deck, there are ${}^{12}\mathrm{C}_2 = 66$ ways to do that.

For each of those ways to get 2 face cards, there are many ways to draw the other 3 cards. They need to be something other than a face card, and there are ${}^{40}\mathrm{C}_3 = 9880$ ways to draw them.

If F is the event 'selecting two face cards', then $n(F) = 66 \times 9880 = 652\,080$.

The total number of ways you can choose 5 cards out of the 52 cards in the deck is given by ${}^{52}\mathrm{C}_5 = 2\,598\,960$.

Finally, you can use the probability formula to find the answer:

$$\mathrm{P(F)} = \frac{652\,080}{2\,598\,960} = \frac{209}{833} \approx 0.251.$$

5 section questions ^

Question 1

Difficulty: ★☆☆

Identify which of the following represents the event that has the least probability of occurring.

1 $\mathrm{P(T)} = 0.3$ ✓

2 $\mathrm{P(L)} = -2$

3 $\mathrm{P(C)} = -0.1$

4 $\mathrm{P(H)} = 0.9$

Explanation

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While -2 and -0.1 are smaller values than the others, probability cannot be negative. Therefore, 0.3 is the smallest valid probability listed, so T is the least likely event to occur.

Question 2

Difficulty: ★★☆

In rolling two fair six sided dice and adding the number appearing on each face, what is the probability that the number is a multiple of 4?

1 $\frac{1}{4}$ ✓

2 $\frac{1}{12}$

3 $\frac{2}{9}$

4 $\frac{1}{2}$

Explanation

We show this using a lattice diagram.

Roll of 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

More information

Thus, $n(\text{multiple of } 4) = 9$ in a sample of space of $n(U) = 6 \times 6 = 36$. Hence, $\mathrm{P}(\text{multiple of } 4) = \frac{9}{36} = \frac{1}{4}$.

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Question 3

Difficulty:

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★★☆

When tossing a coin four times and recording whether it was a head or tails after each toss, what is the probability that you have at least three tails?

1 $\frac{5}{16}$ ✓

2 $\frac{1}{4}$

3 $\frac{1}{16}$

4 $\frac{1}{2}$

Explanation

In four tosses of a coin, there are $2^4 = 16$ possible outcomes, i.e. $n(U) = 16$. The only ones that satisfy "at least three tails" are: $TTTH, TTHT, THTT, HTTT, TTTT$. Thus, $n(\text{at least three tails}) = 5$. Hence, $\text{P}(\text{at least three tails}) = \frac{5}{16}$.

Question 4

Difficulty:

★☆☆

A football goalkeeper has saved 78% of the shots on her goal this season. Find the experimental probability that she will *not* be able to save a shot on goal. Give your answer as a percentage.

22% ✓

Accepted answers

22%, 22, 0.22, .22, 0,22

Explanation

If the shot is on goal, then the only two possibilities are a save and a goal. This means that a goal would be the complement of a save in this situation.

Since the goalkeeper has saved 78% of the shots this season, we would say that $\text{P}(\text{Save}) = 78\%$.

Therefore, $\text{P}(\text{Goal}) = \text{P}(\text{Save'}) = 100\% - 78\% = 22\%$

Student view

Question 5

Difficulty:

★★☆

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A box contains three balls, one red, one green and one blue. You draw one ball out of the box and note its colour. Without putting that ball back, you select a second ball out of the box and note its colour. Find the probability that you did not select the green ball. Give your answer as a fraction in lowest terms, using / to separate the numerator from the denominator.

1/3 ✓

Accepted answers

1/3

Explanation

When you draw the first ball, $\text{P(not green)} = \frac{2}{3}$.

When you draw the second ball, there are only two left in the box. Now, $\text{P(not green)} = \frac{1}{2}$.

Putting these two instances together, you find the probability.

$$\text{P(not green)} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}.$$

Consider the problem from a different perspective. You could calculate the probability of 'selecting' the green ball to remain in the box, which would also be $\frac{1}{3}$.

4. Probability and statistics / 4.5 Probability and expected outcomes

Expected outcomes

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view

Determining what to expect

University graduates

Credit: skynesher Getty Images

Calculating expectation

One of the primary uses of statistics is to make predictions of certain outcomes based on the probability of an event happening. For example, if you estimate that 40% of the people in a city have a college degree, and $75\,000$ people live in the city, then your estimate of the number of people with college degree is 40% of $75\,000$, so $75\,000 \times 0.4 = 30\,000$.

You can also be interested in the number of people with college degree among 200 randomly chosen people. The mathematical concept at work here is expectation, using the probability of an event to predict the number of outcomes that will fall into that event. In the city described above, the probability of a randomly chosen person having a college degree is 0.4, so you can expect that among the randomly choosen 200 people $200 \times 0.4 = 80$ will have college degree.

Using this strategy, if you are given the probabilities of different demographics within a population, you can find the expected number of people that have those characteristics.

Important

The formula for expected number of members of group A in a sample of size n is:

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$$n\mathrm{P}(\mathrm{A})$$

If you were to choose n members of a population, this formula gives an approximation of how many of them would be members of group A.

Human populations are not the only situations in which you can apply expectation. You can use probabilities to make predictions about any event. A few examples are given below.

Activity

The theoretical probability of rolling a 5 on a dice is $\frac{1}{6}$. If you rolled a dice 20 times, how many 5s would you expect to roll? Try the experiment several times and see if what you roll matches the expectation. What principal of probability do you observe through your experiment? What do you think you could do to make the results of your experiment more consistent with the theoretical probability?

Example 1

★★☆

Imagine that you were to draw a card from a deck of 52 cards, note which card it is and replace it in the deck. The deck is divided into four colours – red, blue, green and black – each with thirteen cards. Then you repeated this until you had recorded a total of 24 cards. How many of them you would expect to be green.

Let $\mathrm{P}(\mathrm{G})$ be the probability of drawing a green card.

You recorded 24 cards, so $n = 24$.

Since you put the card back in the deck and there are 13 greens in the deck, $\mathrm{P}(\mathrm{G}) = \frac{13}{52} = \frac{1}{4}$.

Now that you know the probability that any card is green, $n\mathrm{P}(\mathrm{G}) = 24 \times \frac{1}{4} = 6$.

Therefore, you could expect that 6 of the cards you draw would be green.

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Example 2

★★☆

You are playing a game with two friends. Each turn begins by rolling two six-sided dice and adding the numbers shown on them together. If each player rolls the dice 12 times, determine how many times you would expect the dice to add up to 5.

Two six-sided dice

To determine the probability of rolling a sum of 5, a table can be useful.

Dice	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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From the table, you can see that a sum of 5 can be rolled 4 different ways, and that there are 36 different ways the dice can be rolled. Therefore, $\mathrm{P}(5) = \frac{4}{36} = \frac{1}{9}$.

Each of the three players rolls 12 times, so $n = 3 \times 12 = 36$.

Therefore, the expectation is $36 \times \frac{1}{9} = 4$.

This means you and your friends are likely to roll a sum of 5 about 4 times during the game.

Consider what we have discussed about conditional and theoretical probabilities. Why do we say 5 will be rolled **about** 4 times in the game, rather than **exactly** ?

Example 3

★★☆

In the game described in Example 2, each time someone rolls a 2, 3, 11 or 12, they automatically lose all their points. Find how many times you could expect all the players to roll during the game **without** losing their points.

From the table we constructed above, we find that 2, 3, 11 and 12 occur in a total of 6 different ways, so $\mathrm{P}(\text{Losing points}) = \frac{6}{36} = \frac{1}{6}$.

We want to know the probability of someone **not** losing their points. This is the **complement**, $\mathrm{P}(\text{Losing points'}) = 1 - \frac{1}{6} = \frac{5}{6}$.

Therefore, you can expect the players to roll the dice safely approximately $36 \times \frac{5}{6} = 30$ times in the game.

We will revisit this topic in section 4.7.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/expected-value-of-a-discrete-random-variable-id-25658/), where we will determine the expected result in scenarios with multiple outcomes.

Student view

Theory of Knowledge

What percentage of accuracy must be present to consider a statistical prediction accurate? 100%? Think about weather forecasters: they are often right, though can sometimes be wrong. Is it fair to say that weather forecasters have 'valid knowledge' or **certainty** of the weather as a result of their mathematical models?

This leads to the knowledge question: 'How do the standards of evaluation affect the knowledge we create?'

Why It's Hard to Forecast the Weather | National Geographic

2 section questions

Question 1

Difficulty:

★★☆

A group of 50 TOK students are choosing one of the six prescribed titles to write their essay on. Their choices are shown in the table below.

Title number	Number of students
A	14
B	9

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Title number	Number of students
C	1
D	8
E	7
F	11

If the teacher chooses 20 of the essays at random, find how many she should expect to discuss Title D or Title E.

6

Accepted answers

6

Explanation

There are a total of $8 + 7 = 15$ essays written on Title D or Title E, so $\mathrm{P}(\mathrm{D\ or\ E}) = \frac{15}{50} = \frac{3}{10}$.

Since the teacher chose 20 essays, the expected number of essays discussing Title D or Title E is $20 \times \frac{3}{10} = 6$.

Question 2

Difficulty: ★★☆

If you roll one six-sided dice and one four-sided dice and write down their sum 16 times, find the number of times you would expect to roll a sum of 4.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/the-probability-of-an-event-id-25646/print/) Assign

1 2

2 6

3 $4\frac{1}{2}$, so either 4 or 5.

4 $13\frac{1}{2}$, so either 13 or 14.

Explanation

Make a table to illustrate the sums of the dice.

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

From the table, $\mathrm{P}(4) = \frac{3}{24} = \frac{1}{8}$.

Thus, you can expect to roll a total of 4 approximately $16 \times \frac{1}{8} = 2$ times.

4. Probability and statistics / 4.5 Probability and expected outcomes

Checklist

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What you should know

By the end of this subtopic you should be able to:

- identify the sample space for a random experiment
- understand the difference between experimental and theoretical probability
- calculate the theoretical probability of an event A using the formula $\mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(U)}}$
- use the complement of an event to determine the probability that it does not happen
- use probability to calculate the expected number of outcomes in n trials.

Student view

4. Probability and statistics / 4.5 Probability and expected outcomes

Investigation

Think about the games you like to play that involve rolling dice. Do any of them have special rules for when a player rolls a certain way? Many games do have such rules, like getting another turn if you roll doubles (two of the same number) or performing a special task if you roll a 7. Why do you think this might be?

Choose one game that has rules like this and explore the probabilities of triggering the special rules. Do you think these special events occur too often or not often enough? How could you change the rules to make these special things happen on other rolls, and what impact do you think your rules would have on the game?

Try playing the game with a friend using your new rules and think about how your rules affected game play. Did your rules make the game more exciting? Did they ruin the game?

Whether you design a simple board game or a complex computer game, considering the probabilities of each event in the game is one essential ingredient to make it the kind of game people love to play.