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The big picture

Kinematics is a branch of mechanics which describes the motion of objects without the consideration of the forces that create the motion. Motion involves distance travelled, the direction of travel and how fast an object moving all, of which can be described using vectors. Vectors are used to represent forces, acceleration, velocity and momentum and enable the motion of an object to be predicted and described.

Possibly the most amazing goal in the history of football was the one in 1997 when Brazilian football player Roberto Carlos scored a goal from a distance of 35 metres with no clear line to the goal. The ball flew up and changed course mid-air so that it surprisingly reached the net. Watch the following video to understand the maths and physics behind this kick. What would happen if the ball was spinning vertically?

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In this subtopic, you will learn how different aspects of motion can be modelled using vector concepts.

Concept

Vectors help you to quantify positions and movements. They provide the tools for enhancing your spatial awareness in two and three dimensions. This topic provides you with the tools for analysis, measurement and transformation of quantities, movements and relationships.

3. Geometry and trigonometry / 3.12 Vector kinematics

Constant velocity

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/constant-velocity-id-28427/print/) Assign

Recall from subtopic 3.11 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-28347/) that the equation of a straight line can be written in vector form as $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$.

In this context, vector $\boldsymbol{a}$ represents the initial position of the object and vector $\boldsymbol{b}$ represents the velocity, which takes into account the direction of motion. To make the equation dimensionally consistent, the parameter λ represents time.

Therefore, the equation can be written as

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$$

where $\boldsymbol{r}$ is the position vector at time t, $\boldsymbol{r}_0$ is the initial position vector and $\boldsymbol{v}$ is the velocity, which is constant.

Important

Student view

Velocity describes how quickly and in which direction an object is moving.

Consider a ship moving north-east with a speed of 30 km h^{-1} from port A, as shown below. How can you represent its movement using vectors?

If point A is taken as the origin, then the coordinates of A are $(0, 0)$.

The ship is moving with a speed of 30 km h^{-1} so $|\boldsymbol{v}| = 30$.

The direction is north-east, so the $\boldsymbol{i}$ and $\boldsymbol{j}$ components of the velocity vector $\boldsymbol{v}$ are

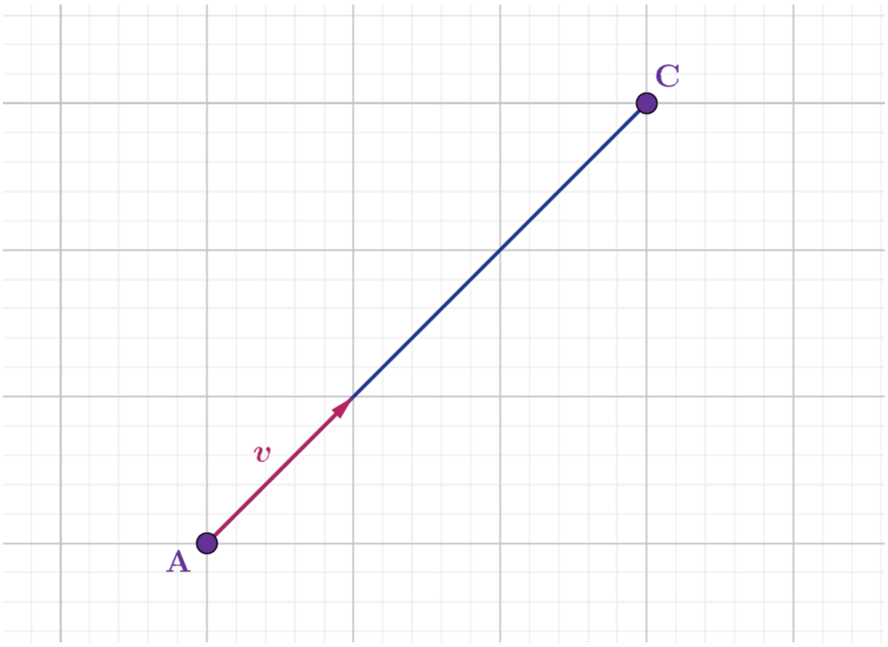
$$30 \cos 45° \text{ and } 30 \sin 45°.$$

Therefore the velocity can be written in column vector form as

$$\boldsymbol{v} = \begin{pmatrix} 30 \cos 45° \\ 30 \sin 45° \end{pmatrix} = \begin{pmatrix} 30 \times \frac{1}{\sqrt{2}} \\ 30 \times \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 15\sqrt{2} \\ 15\sqrt{2} \end{pmatrix}$$

The position vector of the ship at time t hours after it left A is given by

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 15\sqrt{2} \\ 15\sqrt{2} \end{pmatrix} \text{ or } \boldsymbol{r} = t \begin{pmatrix} 15\sqrt{2} \\ 15\sqrt{2} \end{pmatrix}$$



More information

The image is a graph on a grid showing a vector originating from point A and terminating at point C. The vector, indicated by a directional arrow, is labeled with a 'v', representing its magnitude and direction. Point A and point C are marked clearly, each with a dot and a label next to it on the graph. The background consists of a square grid that provides a reference for positioning and direction. This visual represents the mathematical equation given in the text, illustrating a vector in a two-dimensional space.

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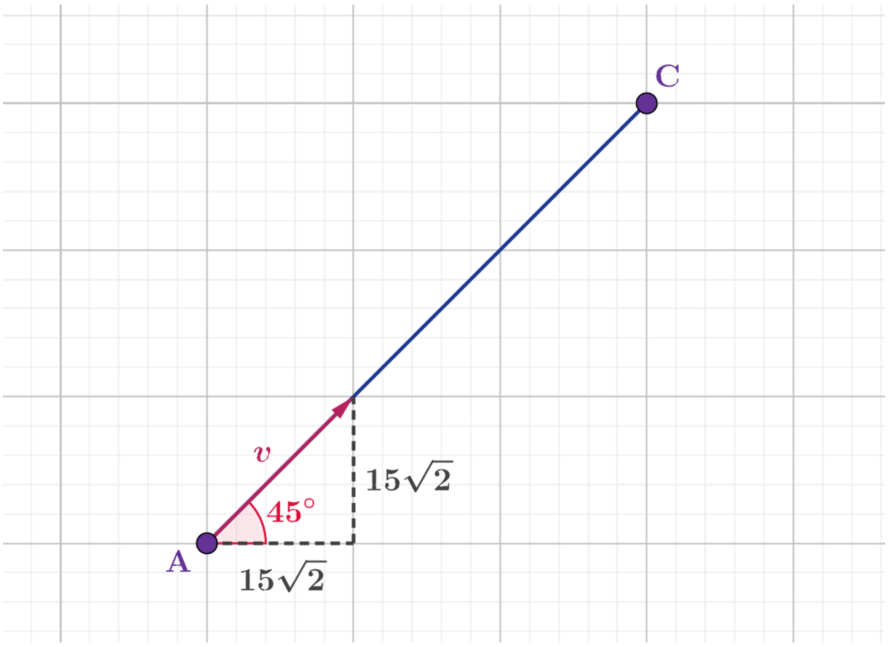

More information

The image is a vector diagram displayed on a grid background. It features a vector originating from point A and extending to point C. The vector forms a 45-degree angle with the horizontal axis. The horizontal component is labeled as 15√2, and the vertical component is also labeled as 15√2, reflecting the direction and magnitude of the vector components with respect to the axis. The vector itself is marked with an arrow and the symbol 'u' indicating its direction. The point where the vector changes direction is labeled as a vertex, creating a right-angled triangle along the gridlines.

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Important

Velocity is a vector quantity which describes both the magnitude and direction of the motion of an object.

Velocity is the rate of change of displacement.

Displacement is the difference between the final and initial positions vectors of an object.

The magnitude of the velocity vector gives the speed.

Speed, distance and time are scalar quantities so have magnitude only.

The relationship between speed, distance and time is

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$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Distance is the magnitude of the displacement vector.

Example 1

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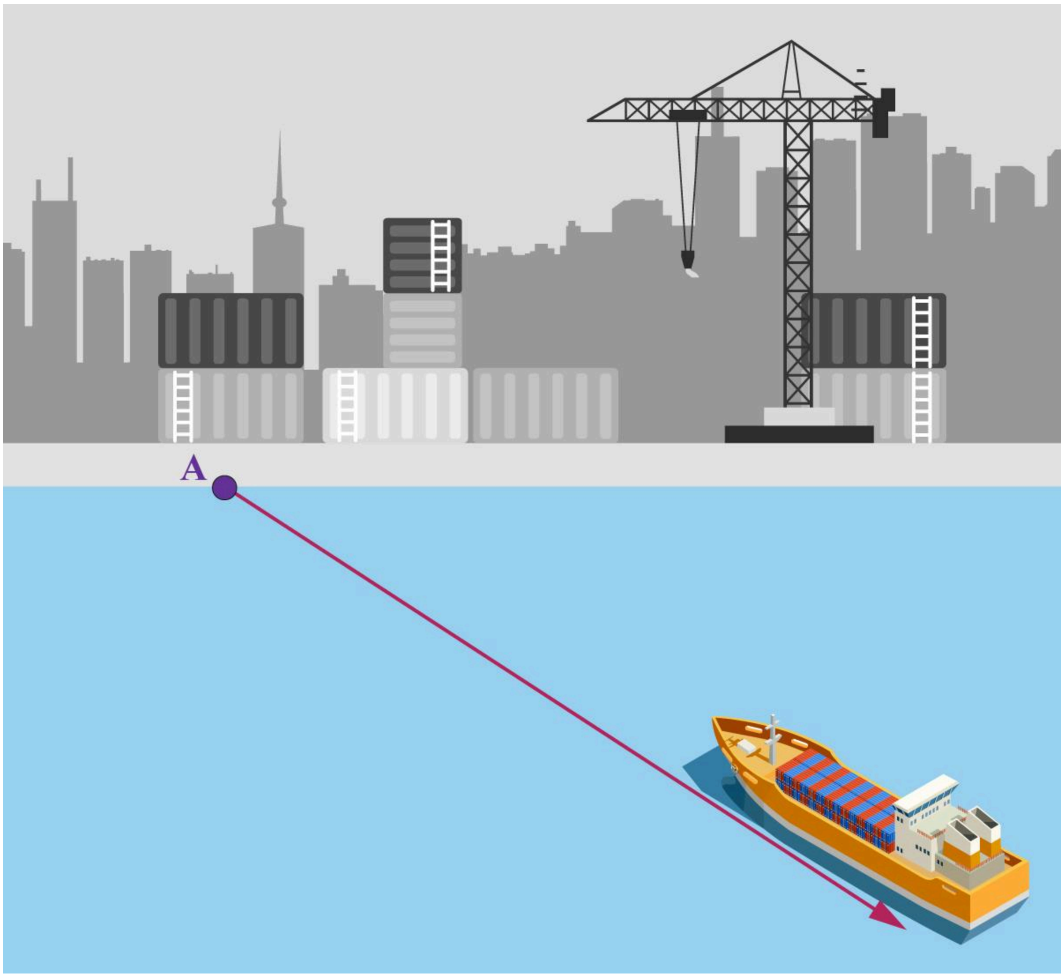
A ship travelling from port A, the origin, travels with velocity $\begin{pmatrix}30\\40\end{pmatrix}$ kilometres per hour.

a) How far will the ship be from the port after 10 hours?

b) Write the coordinates of the ship relative to the port.



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	Steps	Explanation
a)	$\boldsymbol{v} = \begin{pmatrix} 30 \\ 40 \end{pmatrix} \Rightarrow \lvert v\rvert = \sqrt{30^2+40^2} = 50\ \text{kmh}^{-1}$	The magnitude of the veloc is the speed of the ship.
	$\text{distance} = 10 \times 50 = 500\ \text{km}$	The speed is constant so $\text{distance} = \text{speed} \times \text{ti}$
b)	$\boldsymbol{p} = 10 \times \begin{pmatrix} 30 \\ 40 \end{pmatrix}$	Let $\boldsymbol{p}\ \text{km h}^{-1}$ be the positic after t hours. This will be eq the displacement relative to starting point. $\text{displacement} = \text{velocity}$
	$\boldsymbol{p} = \begin{pmatrix} 300 \\ 400 \end{pmatrix}$	
	So the coordinates are $(300, 400)$.	$300\,\text{km}$ east and $400\,\text{km}$ nc the port.

Example 2

★★☆

A drone is flying in a straight line starting from point $(2, 1, 2)$ with constant speed in m s^{-1}. After $10\,\text{s}$ its position is $\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix}$. Find its velocity vector and its speed. (All positions are in metres relative to a fixed origin.)

The initial position vector is $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and let $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ be the velocity which gives the direction vector.

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Then the vector equation is

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

As you know the new position vector after 10 seconds

$$\begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + 10 \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

So

$$v_x = \frac{4-2}{10} = \frac{1}{5}$$

$$v_y = \frac{3-1}{10} = \frac{1}{5}$$

$$v_z = \frac{3-2}{10} = \frac{1}{10}$$

Therefore the velocity vector is

$$\boldsymbol{v} = \begin{pmatrix} \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{10} \end{pmatrix}$$

and the speed is

$$\left| \begin{pmatrix} \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{10} \end{pmatrix} \right| = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{10}\right)^2} = \sqrt{\frac{9}{100}} = \frac{3}{10} \text{ ms}^{-1}$$

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Example 3

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The position vector at time t seconds of a moving object, P, relative to a fixed point, O, is given by

$$\overrightarrow{\mathrm{OP}} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + t(2\boldsymbol{i} - \boldsymbol{k})$$

a) Find the coordinates of the initial position of the object.

b) Find the coordinates of the object when $t = 2$ seconds.

c) Hence, find the distance the object travelled in 2 seconds if the velocity is given in $\mathrm{m/s}$. Give your answer to 3 significant figures.

	Steps	Explanation
	$\overrightarrow{\mathrm{OP}} = \begin{pmatrix}1\\1\\1\end{pmatrix} + t\begin{pmatrix}2\\0\\-1\end{pmatrix}$	Write the vectors as column vectors.
a)	$\overrightarrow{\mathrm{OP}} = \begin{pmatrix}1\\1\\1\end{pmatrix} + 0\begin{pmatrix}2\\0\\-1\end{pmatrix}$ $= \begin{pmatrix}1\\1\\1\end{pmatrix}$	Initial position is when $t = 0$ seconds.
	Therefore, the initial position has coordinates $(1, 1, 1)$.	
b)	$\overrightarrow{\mathrm{OP}} = \begin{pmatrix}1\\1\\1\end{pmatrix} + 2\begin{pmatrix}2\\0\\-1\end{pmatrix}$ $= \begin{pmatrix}5\\1\\-1\end{pmatrix}$	Substitute $t = 2$. This gives the displacement re to the fixed point O.

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	Steps	Explanation
	Therefore, the coordinates of the object when $t = 2$ are $(5, 1, -1)$.	
c)	$\boldsymbol{d} = \sqrt{(5-1)^2 + (1-1)^2 + (-1-1)^2}$	Distance travelled is the magn of the displacement vector. Use Pythagoras' theorem to fi the distance between the two points $(5, 1, -1)$ and $(1, 1, 1)$. $(1, 1, 1)$ and $(5, 1, -1)$.
	$\boldsymbol{d} = 2\sqrt{5} = 4.47\text{m}$ (3 significant figures)	
	Therefore, the distance travelled in 2 seconds is 4.47m (3 significant figures)	

Example 4

★★★

The position vector at time *t* seconds of a moving object it given by

$$\overrightarrow{\mathrm{OP}} = \boldsymbol{r} = \begin{pmatrix} -t^2 \\ 1 \\ 2t \end{pmatrix}$$

Show that the path of the object is not a straight line.

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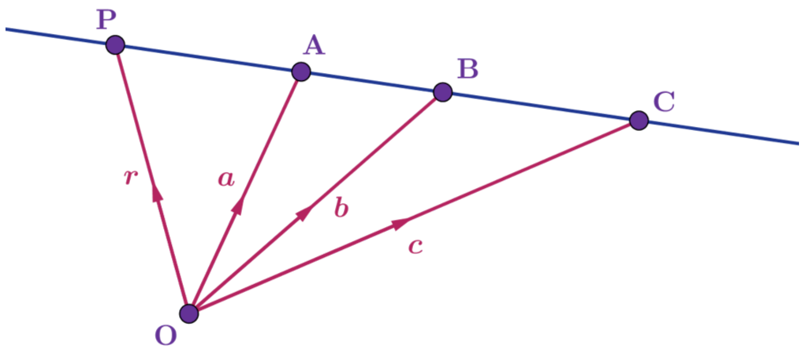
Steps	Explanat
If the object moves in a straight line then, for any three position vectors $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$, $$\boldsymbol{a}-\boldsymbol{b}=k\,(\boldsymbol{b}-\boldsymbol{c}),\ \ \boldsymbol{k}\in\mathbb{R}$$  	
$t=0$ then $\boldsymbol{a}=\begin{pmatrix}0\\1\\0\end{pmatrix}$ $t=1$ then $\boldsymbol{b}=\begin{pmatrix}-1\\1\\2\end{pmatrix}$ $t=2$ then $\boldsymbol{c}=\begin{pmatrix}-4\\1\\4\end{pmatrix}$	Substitute s values of t t determine tl different po: vectors.
$\boldsymbol{a}-\boldsymbol{b}=\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{pmatrix}-1\\1\\2\end{pmatrix}=\begin{pmatrix}-1\\0\\-2\end{pmatrix}$ $\boldsymbol{b}-\boldsymbol{c}=\begin{pmatrix}-1\\1\\2\end{pmatrix}-\begin{pmatrix}-4\\1\\4\end{pmatrix}$	

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Steps	Explanat
$\boldsymbol{b}-\boldsymbol{c}=\begin{pmatrix}3\\0\\-2\end{pmatrix}$	
If the object moves in a straight line, $\boldsymbol{a}-\boldsymbol{b}=k\left(\boldsymbol{b}-\boldsymbol{c}\right)$	
$\begin{pmatrix}-1\\0\\-2\end{pmatrix}=k\begin{pmatrix}3\\0\\-2\end{pmatrix}$	
$l1=3k\Rightarrow k=\frac{1}{3}$ $0=0k\Rightarrow k=0$ $-2=-2k\Rightarrow k=1$	Solve for k.
Since there is no unique k which satisfies $\boldsymbol{a}-\boldsymbol{b}=k\left(\boldsymbol{b}-\boldsymbol{c}\right),\ \boldsymbol{k}\in\mathbb{R}$. $\boldsymbol{a},\boldsymbol{b}$ and $\boldsymbol{c}$ are not collinear.	
Therefore, the object does not move in a straight line.	

Colliding objects

If two objects are moving for them collide (meet), they need to be at the same point at the same time. Therefore, to find out whether two objects collide, you need to find the intersection of the two vector equations and ensure that they give the same position at the same time.

Example 5

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Aircraft A has initial position vector at 0900 given by $\boldsymbol{r}=\begin{pmatrix}0\\2\\3\end{pmatrix}$ kmrelative to an airport and is moving with velocity $\begin{pmatrix}3\\4\\5\end{pmatrix}$ $\mathrm{km\,h^{-1}}$.

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A second aircraft, B, has initial position vector at 0900 given by $\begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix}$ kmand is moving with velocity $\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ $\mathrm{km\,h^{-1}}$.

Show that the two aircraft will collide at a point P and write down the coordinates of P.

Steps	Explanation
For aircraft A , $$\boldsymbol{r}_{\mathbf{A}} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$ For aircraft B , $$\boldsymbol{r}_{\mathbf{B}} = \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$$	Use $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$ to write expressions for the position vec of A and B at time t hours afte 0900.
$$\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$$	Equate the vector equations.
$0 + 3t = 4 - t \Rightarrow t = 1$ $2 + 4t = 5 + t \Rightarrow t = 1$ $3 + 5t = 5 + 3t \Rightarrow t = 1$	Solve for the x, y and z-compo and check see whether all thre times are equal.
Therefore, the two aircraft will collide after $1\mathrm{hr}$ at 1000 .	
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 8 \end{pmatrix}$$	Use either of the vector equatic find the position vector of the aircraft when they collide.
So the coordinates of the collision point are $(3, 6, 8)$.	Make sure you write the coordi of P.

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Example 6

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Two objects are moving with a constant velocity in a straight line.

Initially, object A is at a point with coordinates $(1, 3, 0)$ m, relative to a fixed origin, and is moving with velocity $\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ m s^{-1}.

Object B is initially at a point with coordinates $(-1, 2, -2)$ m, relative to the fixed origin, and is moving with velocity $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ m s^{-1}.

a) Show that these objects do not collide.

b) Find an equation in terms of t for the distance, d, between the objects.

c) Find their closest approach.

d) Find the velocity vector that the object B must have so that the two objects will collide after 3 s.

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	Steps	Explanation
a)	$\boldsymbol{r_A} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{r_B} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$	Write the motion of the two objects in te two vector equations. t is the time in seconds after they depar their initial positions.
	So $\begin{aligned} 1 - 3t_C &= -1 + 2t_C \\ 3 + 4t_C &= 2 - t_C \\ 2t_C &= -2 + 3t_C \end{aligned}$ which give $t_C = \frac{2}{5}$, $t_C = -\frac{1}{5}$, $t_C = 2$. All these solutions are different, so there is no single time at which these objects are in the same position. Therefore they do not collide.	If the objects collide, then there must be at which they are at the same position, s which should be the unique solution to these equations.

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	Steps	Explanation
b)	The position vectors of A and B in terms of t are $$\boldsymbol{r_A} = \begin{pmatrix}1\\3\\0\end{pmatrix} + t\begin{pmatrix}-3\\4\\2\end{pmatrix} \Leftrightarrow \begin{pmatrix}x_A\\y_A\\z_A\end{pmatrix} = \begin{pmatrix}1-3t\\3+4t\\2t\end{pmatrix}$$ $$\boldsymbol{r_B} = \begin{pmatrix}-1\\2\\-2\end{pmatrix} + t\begin{pmatrix}2\\-1\\3\end{pmatrix} \Leftrightarrow \begin{pmatrix}x_B\\y_B\\z_B\end{pmatrix} = \begin{pmatrix}-1+2t\\2-t\\-2+3t\end{pmatrix}$$ Hence $$\begin{aligned} d^2 &= (1-3t-(-1+2t))^2 + (3+4t-(2-t))^2 + (2t-(-2+3t))^2 \\ &= (1-3t+1-2t)^2 + (3+4t-2+t)^2 + (2t+2-3t)^2 \\ &= (2-5t)^2 + (1+5t)^2 + (2-t)^2 \\ &= 4-20t+25_t{}^2+1+10t+25_t{}^2+4-4t+t^2 \\ &= 9-14t+51t^2 \end{aligned}$$ giving $$d = \sqrt{9-14t+51t^2}$$ Since the discriminant of this quadratic is negative there is no solution for $d=0$. This confirms that these objects do not collide.	The distance, d between two objects A with coordinates (x_A, y_A, z_A) and $(x_B$, respectively, is given by $$d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z$$ Alternatively, it is sometimes more conv work with $$d^2 = (x_A - x_B)^2 + (y_A - y_B)^2 + (z_A$$

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	Steps	Explanation
c)	$d^2 = 9 - 14t + 51t^2$ The minimum of this function occurs at $t = 0.137$ seconds. So the minimum value of $d^2 = 8.04 \Rightarrow d = 2.84 \text{ m}$	The smallest value for d is their closest approach. As the leading coefficient of function of d^2 is positive $(+51)$, the fun concave up and so the vertex is its mini point. You can use a graphical method to find coordinates of the vertex of this quadra Substitute $t = 0.137$ second into $d^2 = 9 - 14t + 51t^2$. To find the minimum value of d.
d)	Let $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be the new velocity vector for the motion of object B. The vector equations for A and B will be $\boldsymbol{r_A} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{r_B} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + t\begin{pmatrix} a \\ b \\ c \end{pmatrix}$	

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	Steps	Explanation
	As you want the two objects to collide after $t = 3$ s: $$\begin{aligned}1-3\times 3 &= -1+3\times a\\ 3+4\times 3 &= 2+3\times b\\ 2\times 3 &= -2+3\times c\end{aligned}$$ which gives $$\begin{aligned}a &= -\frac{7}{3}\\ b &= \frac{13}{3}\\ c &= \frac{8}{3}\end{aligned}$$ So the new velocity vector for object B such that the two objects collide after 3 s is $$\begin{pmatrix}-\frac{7}{3}\\ \frac{13}{3}\\ \frac{8}{3}\end{pmatrix}\mathrm{ms}^{-1}$$	

International Mindedness

Have you seen ants foraging and then finding their way back to their nest? Apparently, they are aware of the magnitude and direction of their journeys. They have internal pedometer systems. You can read about the vector journeys in this article (https://plus.maths.org/content/finding-way-home).

4 section questions

3. Geometry and trigonometry / 3.12 Vector kinematics

Variable velocity: uniform acceleration

Student view

So far, you have only considered motion where velocity is constant. But in most real situations, this is not the case and an object's velocity will change.

Acceleration is the rate of change of velocity. If this is non-uniform, then you need to differentiate the velocity vector to calculate it.

If the acceleration is uniform, then you can use the equation

$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

or

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

where a is the acceleration, u is the initial velocity, v is the final velocity and t is time.

Rearranging this gives the formula for the velocity of an object at a given time as

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

In this section, you will be working with uniform acceleration only.

If the acceleration is uniform, the average velocity is given by $\left(\frac{\boldsymbol{u}+\boldsymbol{v}}{2}\right)$.

So in time t the displacement is given by $s = \left(\frac{\boldsymbol{u}+\boldsymbol{v}}{2}\right)t$.

Making connections

If an object is moving with constant acceleration its motion can be predicted using a set of equations called the **suvat** equations, where:

s is displacement
u is initial velocity
v is final velocity

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a is acceleration and
t is time

The equations are:

$$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad s = \left(\frac{u+v}{2}\right)t, \quad s = vt - \frac{1}{2}at^2$$

and

$$v^2 = u^2 + 2as$$

These equations are not a requirement of the syllabus and they are not included in the formula booklet. We use them here to illustrate the appropriate vector calculations.

As displacement, velocity and acceleration are vector quantities, these equations can be written in vector form:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t, \boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2, s = \left(\frac{\boldsymbol{u}+\boldsymbol{v}}{2}\right)t \text{ and } \boldsymbol{s} = \boldsymbol{v}t - \frac{1}{2}\boldsymbol{a}t^2$$

The final equation $v^2 = u^2 + 2as$ needs to be written using the scalar product as $\boldsymbol{v} \cdot \boldsymbol{v} = \boldsymbol{u} \cdot \boldsymbol{u} + 2\boldsymbol{a} \cdot \boldsymbol{s}$.

Example 1

★★☆

A particle moves with a constant acceleration $(i - 4j)\ \mathrm{m\,s^{-2}}$.

At time $t = 3s$, P has velocity $-3i + 2j; \mathrm{m\,s^{-1}}$, find

a) its initial velocity

b) its initial speed.

Steps	Explanation
$\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} + 3\begin{pmatrix} 1 \\ -4 \end{pmatrix}$	Using $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$.

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Steps	Explanation
$\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} + \begin{pmatrix} 3 \\ -12 \end{pmatrix}$ $\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \end{pmatrix}$ $= \begin{pmatrix} -6 \\ 14 \end{pmatrix}$	Solve for $\boldsymbol{u}$.
So the initial velocity is $\begin{pmatrix} -6 \\ 14 \end{pmatrix}$	
$\sqrt{(-6)^2 + (14)^2} = 15.23\ldots$	The speed is the magnitude of the velocity vector.
Therefore the initial speed is $15.2 \mathrm{~m\,s}^{-1}$ (3 significant figures)	

Example 2

★★☆

A particle moves with a constant acceleration.

At time $t = 1$, P has velocity $(-3\boldsymbol{i} + 2\boldsymbol{j}) \mathrm{~m\,s}^{-1}$ and at time $t = 3$, P has velocity $(-\boldsymbol{i} + \boldsymbol{j}) \mathrm{~m\,s}^{-1}$, find

a) its acceleration

b) its velocity when $t = 5$ s.

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Steps	Explanation
$\boldsymbol{a} = \frac{(-\boldsymbol{i}+\boldsymbol{j})-(-3\boldsymbol{i}+2\boldsymbol{j})}{3-1}$	Use $$a = \frac{v-u}{t}$$ The time taken is $3-1=2$ seconds
$\boldsymbol{a} = \boldsymbol{i} - \frac{1}{2}\boldsymbol{j}$	
Therefore the acceleration is $\boldsymbol{i} - \frac{1}{2}\boldsymbol{j}\ \mathrm{m\,s^{-2}}$	
$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$	Use $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$.
$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
The velocity at $t=5$ is $\boldsymbol{i}\ \mathrm{m\,s^{-1}}$	

Example 3

★★☆

The acceleration of an object given by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\ \mathrm{m\,s^{-2}}$.

If the particle has velocity $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}\ \mathrm{m\ s^{-1}}$ at time $t=0$ seconds, find:

a) the velocity of the particle after 3 seconds

b) the displacement of the particle over this 3 seconds.

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	Steps	Explanation
a)	$\boldsymbol{v} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \\ 8 \end{pmatrix}$	Use $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$ to find the final velocity.
b)	$\boldsymbol{s} = \frac{1}{2}\left(\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 9 \\ 8 \end{pmatrix}\right)3 = 3\begin{pmatrix} 3.5 \\ 6 \\ 5 \end{pmatrix}$	Use $\boldsymbol{s} = \left(\frac{\boldsymbol{u}+\boldsymbol{v}}{2}\right)t$ to find the displacement using the average velocit
	So the displacement is $\begin{pmatrix} 10.5 \\ 18 \\ 15 \end{pmatrix}$ m.	

Making connections

If an object moves in a circle, then there must be a force maintaining this circular motion. This force is called the centripetal force. It will cause the object to accelerate.

But if the object travels with uniform circular motion, its speed is constant. Therefore, the acceleration can have no component in the same direction as the velocity; otherwise, the magnitude of the velocity would change.

The acceleration caused by the centripetal force is directed toward the centre of the circle and is always at right angles to the object's velocity. It changes the direction of the object's velocity while keeping the magnitude of the velocity (i.e. the speed) constant.

So if a body is moving in a circle at constant speed, it is actually accelerating. This can happen because it is constantly changing direction so the velocity is constantly changing.

3 section questions

Student view

3. Geometry and trigonometry / 3.12 Vector kinematics

Variable velocity in two dimensions

Consider an object moving in two dimensions. Its velocity can be separated into two perpendicular components which may vary with time, i.e. $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$, where v_x and v_y depend on time t according to the relationship $v = u + at$.

If the object is moving in a vertical plane, the only force acting on it will be gravity. This will affect the vertical component of the velocity only – the horizontal component will be constant. A particle moving like this is called a projectile.

For a projectile moving freely under the action of gravity, the horizontal component of the velocity, v_x, is constant and the vertical component, v_y, at time t is found using $v = u + at$, where a is the acceleration due to gravity which is usually denoted by g.

On Earth, the value of g is approximately 9.8 m s^{-2}.

✓ **Important**

For a projectile moving freely under the action of gravity alone, the acceleration is given by

$\boldsymbol{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$, where g is the acceleration due to gravity.

Example 1

★★☆

A particle has velocity given by $\boldsymbol{v} = \begin{pmatrix} 15 \\ 3 + 4t \end{pmatrix} \text{ m s}^{-1}$.

Find the speed at time $t = 10$ seconds.

Give your answer correct to 3 significant figures.

Overview
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Steps	Explanation
$\boldsymbol{v} = \begin{pmatrix} 15 \\ 3 + 4 \times 10 \end{pmatrix} = \begin{pmatrix} 15 \\ 43 \end{pmatrix}$	Substitute $t = 10$ into the expression for
Speed = $\sqrt{15^2 + 43^2} = \sqrt{2074} = 45.54\ldots$	The speed is the magnitude of the velocit vector. Use Pythagoras' theorem.
The speed is 45.5 m s^{-1} (3 significant figures)	

Example 2

★★☆

A projectile moves freely under the action of gravity and has an initial velocity of $\begin{pmatrix} 7 \\ 10 \end{pmatrix}\text{ m s}^{-1}$.

Find the velocity of the projectile after 4 seconds.

Take the acceleration due to gravity to be 9.8 m s^{-2}.

Student
view

Overview
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Steps	Explanation
$\boldsymbol{u} = \begin{pmatrix} 7 \\ 10 \end{pmatrix}$	The initial velocity is $\begin{pmatrix} 7 \\ 10 \end{pmatrix} \text{ m s}^{-1}$
$v_x = 7$ $v_y = u + at$ where $a = -9.8$ $v_y = 10 - 4 \times 9.8 = -29.2$	The horizontal component of the velocity is constant $v_x = 7$ The vertical component of the velocity depends on t acceleration in the vertical direction, which is g, the acceleration due to gravity. This will be -9.8 because upwards is taken to be the positive direction for the vertical component of the velocity and the force of g acts downwards.
$\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 7 \\ -29.2 \end{pmatrix} \text{ m s}^{-1}$	Use $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

Making connections

When a particle is modelled as projectile moving freely under the action of gravity, certain assumptions have been made about its motion to simplify the calculations.

What forces other than gravity might be acting on an object such as a football or a tennis ball as it moves through the air? How would these affect its motion? What refinements would you need to make to the projectile model to make it more realistic?

3 section questions

3. Geometry and trigonometry / 3.12 Vector kinematics

Checklist

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What you should know

By the end of this subtopic you should be able to:

- describe the motion of an object moving in a straight line with constant velocity by the vector equation $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$, where $\boldsymbol{r}_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ is the initial position vector relative to a fixed origin, $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ is the velocity and t is the time
- recall that speed is the magnitude of the velocity vector $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and use Pythagoras' theorem to find it: $|\boldsymbol{v}| = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$
- recall that if an object is moving with constant acceleration given by $\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$, then the formula for the velocity of an object at a given time is $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$, where $\boldsymbol{a}$ is the acceleration, $\boldsymbol{u}$ is the initial velocity, $\boldsymbol{v}$ is the final velocity and t is time.
- recall that if an object is moving in two dimensions, its velocity can be separated into two perpendicular components which may vary with time, i.e. $\boldsymbol{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$, where v_x and v_y depend on t according to the relationship $v = u + at$
- recall that for a projectile moving freely under gravity, the horizontal component of the acceleration is constant and the vertical component will be given by $-g$ where g is the acceleration due to gravity, i.e. $\boldsymbol{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$.

Investigation

Student view

How can you model the trajectory of a ball when you kick it? Can you estimate the location of the ball after it is kicked by a player so that you can intercept it? If you are the goalkeeper trying to prevent a goal being score from a penalty kick, how can you predict where the ball will go?

You can answer these questions using vectors, kinematics and maybe some calculus.

Read this article (https://plus.maths.org/content/os/issue40/features/bray/index) about the maths behind kicking a ball.

To carry out this investigation you might need help from your friends; one person to be a goalkeeper, one to kick the ball and one person to record the result.

Ask a friend to kick the ball and get the goalkeeper to catch it. Record the motion of the ball and the goalkeeper. Work out an estimate of the speed of the ball and the angle at which it starts to move. Model the motion of the ball and the goalkeeper using vector equations and try to predict whether he/she will be able to intercept the ball.

Compare your model with the actual result. Why might there be differences between the model and the actual result? Will the ball move in a straight line?

What assumptions do you have to make when setting up your model? How could you make it more realistic?

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Will the goalkeeper make the save?

Credit: Klaus Vedfelt Getty Images

Find out how the electronic system Hawk-Eye is used in sports such as tennis, cricket and football to predict the motion of a ball and plot its most-likely path.

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