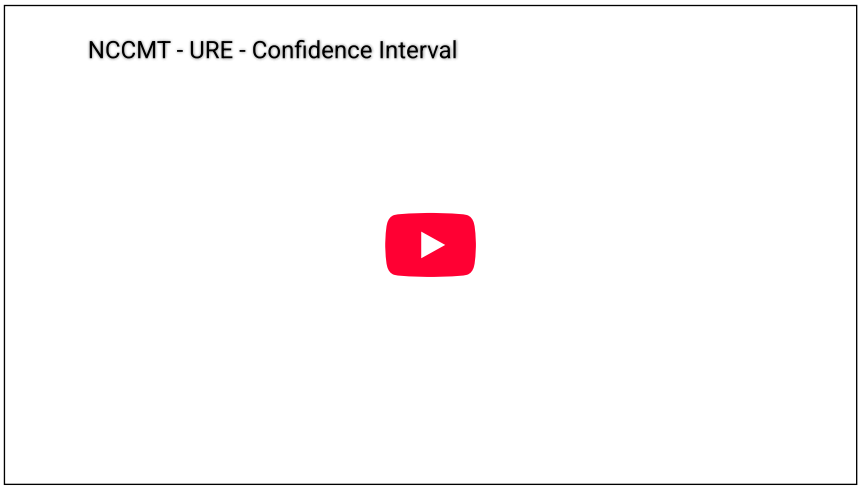
The big picture

In section 4.14 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27539/), you saw that the sample mean is an unbiased estimator for the population mean. This implies that the average of all possible sample means is the population mean. However, usually you do not have access to all possible samples. You have one sample, or a few samples, and you would like to predict the population mean. In doing this, how confident can you be about your prediction?

Making connections

Physics students are made aware of the importance of uncertainties early on in their studies. Walter Lewin, the longtime physics professor at M.I.T., once said that 'any measurement that you make without a knowledge of its uncertainty is completely meaningless.' Understanding the uncertainty of your sample mean, in physics and in other subjects, is crucial when using the sample mean to make predictions about the population.

To understand this idea in more practical terms, consider the following video.



In this subtopic, you will deal only with populations that are normally distributed, or at least approximately normally distributed.

Concept

The validity of using a sample mean to predict the population mean hinges on understanding the limitations of the sample mean.

Confidence intervals for the mean when sigma is known

Confidence intervals when σ is known

Imagine that you are a researcher trying to find the mean height for a certain population of people. We will assume that the population is normally distributed. For your research, you decide to collect the heights of 10 people. You know that it is unlikely that the mean of your sample is exactly the same as the mean of the population. However, is it possible to estimate how far off your sample mean is from the population mean? Can you use the sample mean to derive a range, an interval, within which you are confident that the true population mean lies?

Important

A confidence interval is a range calculated using the sample mean in order to estimate the unknown population mean.

In order to form a confidence interval, you first need to specify a confidence level.

Important

The confidence level is the percentage of all possible samples for which the corresponding confidence interval contains the unknown population mean. As such it describes the likelihood that the confidence interval around the sample mean contains the real mean.

The most commonly used confidence level is 95%. However, you may also see 90% or 99% used in questions. For practical reasons, the confidence level is usually stated using the significance level, α, where:

$$\text{confidence level} = 1 - \alpha$$

Example 1

★☆☆

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27987/print/) Assign

You have been instructed to carry out your research using a significance level of 0.05. State the confidence level of your results for this significance level.

$$\begin{aligned}\text{confidence level} &= 1 - \alpha \\ &= 1 - 0.05 \\ &= 0.95\end{aligned}$$

$\therefore$ there is a 95% confidence level.

Overview (/study/ap ai-hl/sid-132-cid-761618/ov

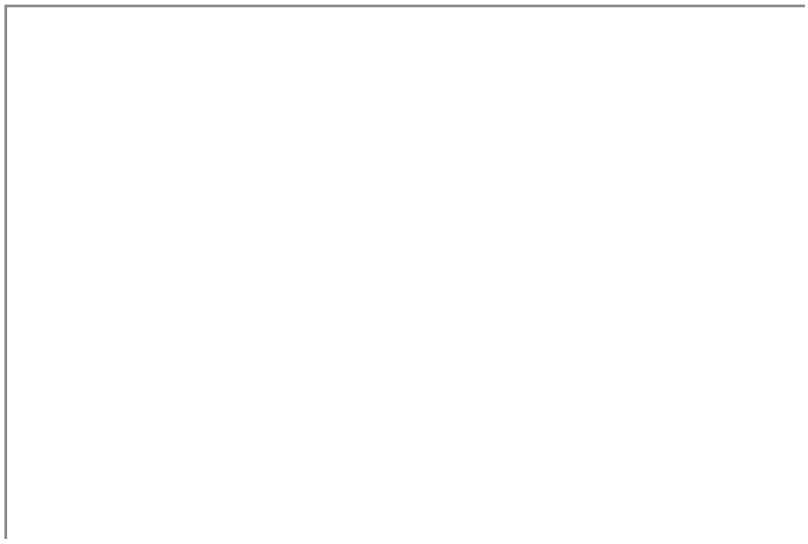
Use the formula for the confidence level, shown above.

Let us explore how the confidence level affects the confidence interval by looking at the applet shown below. Take note of the following aspects of the applet:

The points on the graph represent the sample means of 100 different possible random samples of 10 people from the population.

The yellow vertical line on the graph represents the actual population mean that you are trying to estimate using the sample means.

Remember, that for your research you are only collecting one sample of 10 people, so yours could be any one of the 100 samples shown in the graph. What happens to the data points as you drag the slider to change the confidence level?



Interactive 1. Exploring How Confidence Level Affects Confidence Interval.

More information for interactive 1

This interactive enables users to explore the concept of confidence intervals by adjusting key parameters through intuitive controls. Users can drag a slider to change the sample size ($n = 10, 20, 30, 40, 50$) and select confidence levels (15% to 99%).
Each time you generate new data, the applet creates 100 random samples, with each blue line representing a confidence interval that contains the true mean and green lines showing intervals that miss it. At higher confidence levels (e.g., 99%), intervals widen to capture the true mean more often, resulting in fewer green lines (~ 1% misses). Smaller samples ($n = 10$) produce wider, less consistent intervals, while larger samples ($n = 50$) yield precise estimates. The tool highlights the trade-off between certainty (wider intervals) and precision (narrower intervals).
By testing different combinations, users learn why researchers often use 95% confidence: it balances reliability with reasonable interval width. The dynamic visualization reinforces how proper sample sizing and confidence selection are critical for accurate statistical inference in real-world applications.

As you see, when you increase the confidence level, the data points turn into longer and longer lines. These lines represent the confidence interval for each collected sample. Notice that the intervals are blue if they contain the population mean and green if they do not.

Student view

Activity

Keep the sample size n set at 10 and generate several new sets of 100 random samples.

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1. How many of the intervals are green when you set the confidence level to 99%?
2. What about when the confidence level is set to 95%? What about 90%?
3. Comment on what happens to the intervals as you change the sample size.
4. Using your findings from the above steps, describe the effects that the sample size and confidence level has on the confidence interval.

You have now seen that the goal of creating a confidence interval is to ensure that the population mean has been included within the interval. The next step is to calculate the end points of the confidence interval. For a normally distributed population, we can use an expression including a factor, $z_{\frac{\alpha}{2}}$, that depends on the confidence level:

$$\text{end points} = \bar{x} \pm z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$$

where σ is the standard deviation of the population and n is the sample size. The table below shows some possible values for $z_{\frac{\alpha}{2}}$:

Confidence level	α	$z_{\frac{\alpha}{2}}$
99%	0.01	2.58
95%	0.05	1.96
90%	0.10	1.65

Be aware

Note that it is the standard deviation of the population, σ , that is used in the expression shown above, not the standard deviation of the sample.

Exam tip

In the IB examinations, you will not be expected to use this expression to calculate the confidence intervals. All confidence intervals will be calculated using your graphic display calculator.

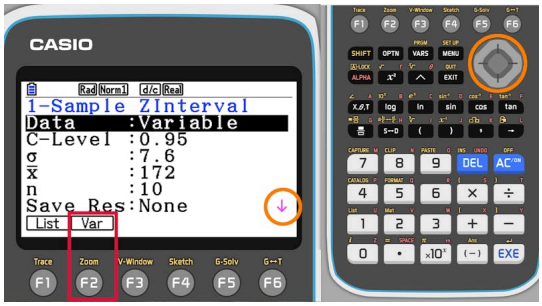
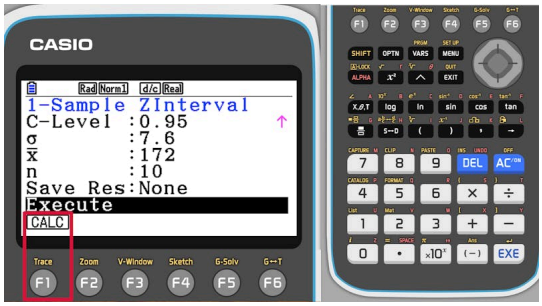
Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
In these instructions you will see how to find the 95% confidence interval for the mean of the population when you know that the standard deviation of the population is 7.6. In this example the confidence interval is based on a sample of size 10 and the information, that the mean of the sample is 172. To start, open the statistics mode.	
In this example the prediction is based on the mean of the sample and not the sample itself, so there is no data to enter here. In other questions you may know the data. In this case enter it here. Press F4 to access the interval options.	
There are two options here, Z or t. In this case the population standard deviation is known, so press F1 to choose to use the Z-distribution. The case, when the population standard deviation is not known is discussed in the next section.	

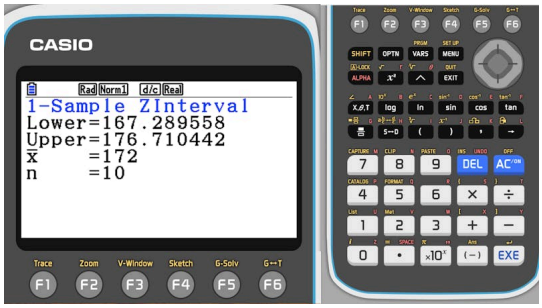
Student view

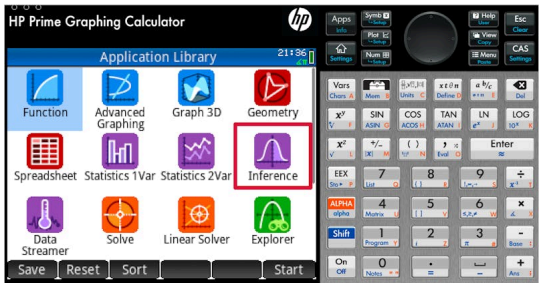
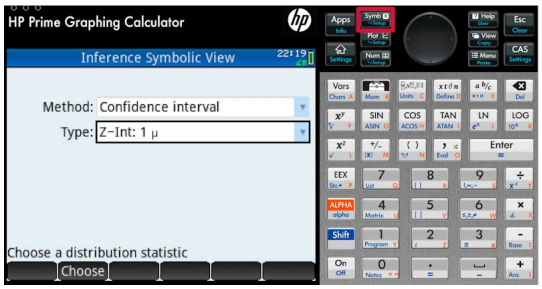
Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Press F1 to choose the option to find the confidence interval for the mean of one random variable.	
The calculator is now waiting for the information as a base of finding the confidence interval. • In the first line, press F2 to tell the calculator that you do not have the data, just some information about it (of course, if you know the data, choose the other option). You need to tell the calculator what you know. • confidence level (note, that the calculator needs this in decimal form, not as a percentage) • standard deviation of the population (σ) • mean of the sample ($\overline{x}$) • size of the sample (n) When you have entered all this, scroll down ...	
... and press F1 to calculate the confidence interval.	

Student view

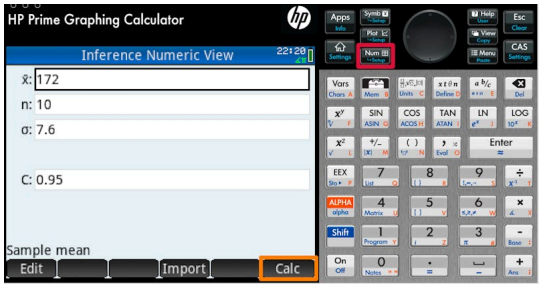
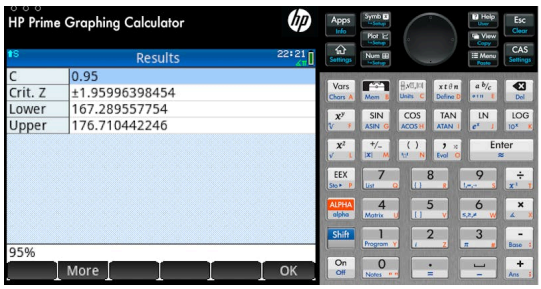
Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

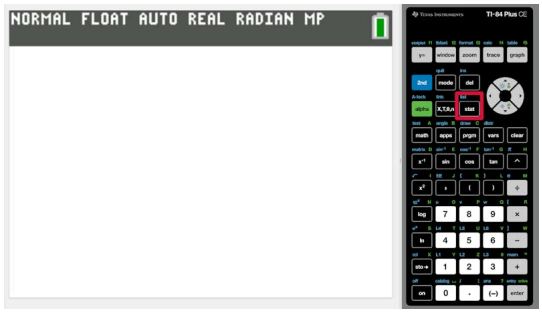
Steps	Explanation
On the result screen the lower and upper bound of the confidence interval is given. Note, that the sample mean is at the midpoint of this interval.	

Steps	Explanation
In these instructions you will see how to find the 95% confidence interval for the mean of the population when you know that the standard deviation of the population is 7.6. In this example the confidence interval is based on a sample of size 10 and the information, that the mean of the sample is 172. Since the data is not known, go straight to the inference applicatioon. In other examples the sample is given instead of just its mean. In this case you need to enter the data for example in the 1-variable statistiics application before you open the inference app.	
In symbolic view you need to select the test to run. There are several options here. Choose the option to find the confidence interval for the mean of one random variable. You will also need to decide whether to use the Z or t-distribution.. In this case the population standard deviation is known, so choose to use the Z-distribution. The case, when the population standard deviation is not known is discussed in the next section.	

Student view

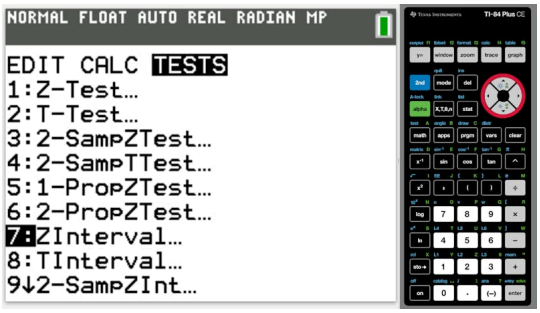
Overview (/study/app/ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Change to numeric view and enter the information as a base of finding the confidence interval. • mean of the sample ($\overline{x}$) • size of the sample (n) • standard deviation of the population (σ) • confidence level (note, that the calculator needs this in decimal form, not as a percentage) In this example the prediction is based on the mean of the sample and not the sample itself. In other questions you may know the data. In this case you can import the important information here. For details, see the instructions in the next section. When you have entered all this, tap on Calc to get the confidence interval.	
On the result screen the lower and upper bound of the confidence interval is given.	

Steps	Explanation
In these instructions you will see how to find the 95% confidence interval for the mean of the population when you know that the standard deviation of the population is 7.6. In this example the confidence interval is based on a sample of size 10 and the information, that the mean of the sample is 172. To start, open the statistics menu.	

Student view

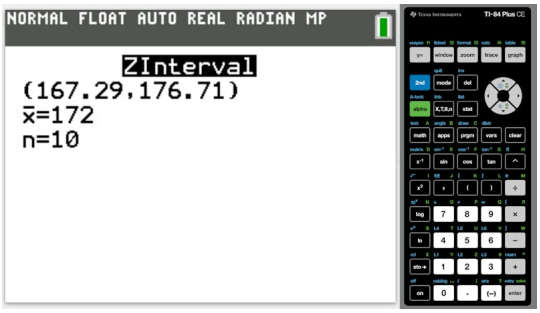
Overview (/study/ap ai-hl/sid-132-cid-761618/ov

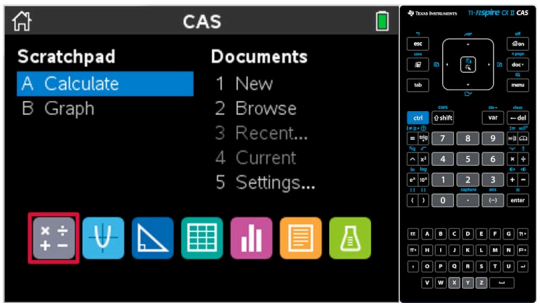
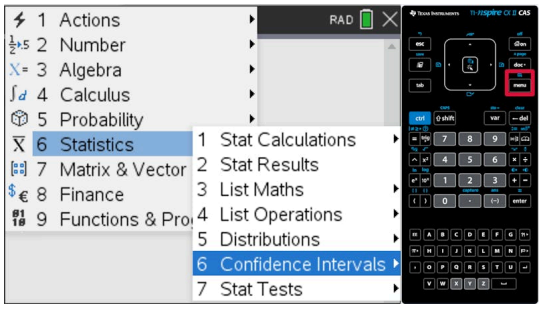
Steps	Explanation
Since the data is not known, go straight to TESTS. There are several options here. Choose the option to find the confidence interval for the mean of one random variable. You will also need to decide whether to use the Z or t-distribution.. In this case the population standard deviation is known, so choose to use the Z-distribution. The case, when the population standard deviation is not known is discussed in the next section. In other examples the sample is given instead of just its mean. In this case you need to enter the data using the EDIT menu before you choose the test.	
The calculator is now waiting for the information as a base of finding the confidence interval. • In the first line tell the calculator that you do not have the data, just some information about it (of course, if you know the data, choose the other option). You need to tell the calculator what you know. • standard deviation of the population (σ) • mean of the sample ($\overline{x}$) • size of the sample (n) • confidence level (note, that the calculator needs this in decimal form, not as a percentage) When you have entered all this, scroll down to the last line and press enter to calculate the confidence interval.	

Student view

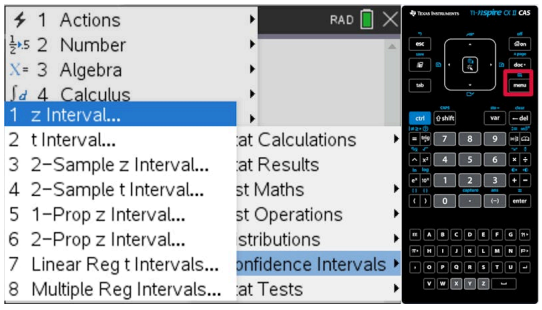
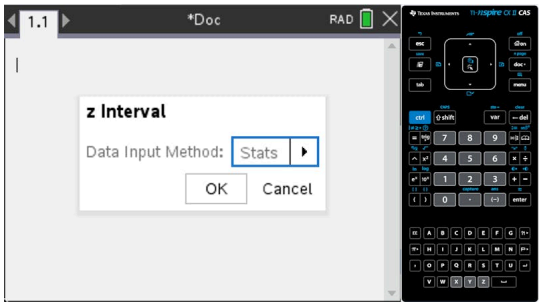
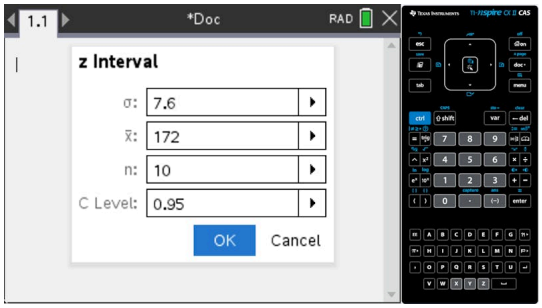
Overview (/study/ap ai-hl/sid-132-cid-761618/ov

Student view

Steps	Explanation
On the result screen you can see the confidence interval. Note, that the sample mean is at the midpoint of this interval.	

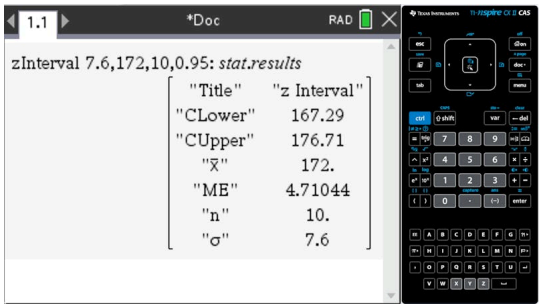
Steps	Explanation
In these instructions you will see how to find the 95% confidence interval for the mean of the population when you know that the standard deviation of the population is 7.6. In this example the confidence interval is based on a sample of size 10 and the information, that the mean of the sample is 172. Since the data is not known, open a calculator page, where you can do the calculations. In other examples the sample is given instead of just its mean. In this case you need to enter the data for example on a spreadsheet page before you open the calculator page.	
Open the menu and look for the option to work with confidence intervals.	

Overview (/study/ap ai-hl/sid-132-cid-761618/ov

Steps	Explanation
There are several options here. Choose the option to find the confidence interval for the mean of one random variable. You will also need to decide whether to use the Z or t-distribution.. In this case the population standard deviation is known, so choose to use the Z-distribution. The case, when the population standard deviation is not known is discussed in the next section.	
The calculator is now waiting for the information as a base of finding the confidence interval. On this first screen tell the calculator that you do not have the data, just some information about it (of course, if you know the data, choose the other option).	
You need to tell the calculator what you know. • standard deviation of the population (σ) • mean of the sample ($\bar{x}$) • size of the sample (n) • confidence level (note, that the calculator needs this in decimal form, not as a percentage) When you have entered all this, scroll down to OK and press enter to calculate the confidence interval	

Student view

Overview (/study/app/ai-hl/sid-132-cid-761618/ov

Steps	Explanation
On the result screen the lower and upper bound of the confidence interval is given. Note, that the sample mean is at the midpoint of this interval.	

Even though you will not need to use the expression in examinations, it is worth noting how the confidence level and the sample size affect the length of a confidence interval.

✓ Important

A confidence interval will become longer as the confidence level is increased or the sample size is decreased.

A confidence interval will become shorter as the confidence level is decreased or the sample size is increased.

Example 2

★☆☆

After collecting your random sample of the heights of 10 people, you find the mean of your sample to be 172 cm. Given that the standard deviation for the heights of humans in this population is 7.6 cm , calculate a 95% confidence interval for the mean height of the population and give an interpretation of the result.

You are given the mean of the sample, the sample size and the standard deviation for the population. These values can be input into your calculator to find the confidence interval.

The confidence interval is $[167.29, 176.71]$.

Interpret the results.

Therefore, you are 95% confident that the mean height of the total population is between 167.29 cm and 176.71 cm.

Student view

International Mindedness

Overview (/study/ap ai-hl/sid-132-cid-761618/ov

What is the average height of humans worldwide? Could you calculate a confidence interval for that height if you knew the mean height of the population of India? Would the confidence interval be different if instead you knew the mean height of the population of New Zealand?

While the average height of humans varies between countries, the overall average height for humans has been steadily increasing over the past century.

https://ourworldindata.org/human-height (https://ourworldindata.org/human-height)

The average human height has been increasing over the past century faster than any other time in history. What factors do you think are contributing to this rapid increase? Is there a limit to how much the average height can increase?

Example 3

★☆☆

A farmer needs to calculate a confidence interval for the mean mass of the plums she has harvested. The mass in grams is a random variable with a normal distribution and variance 16 g. The following is a random sample drawn from this distribution:

56.9553	49.7247	67.7432	67.4273	61.6756
64.9226	61.7774	66.1004	64.7733	67.7561
63.6987	62.2073	63.1392	59.4469	53.3063

Find the length of the 85% confidence interval for the mean of the random variable. Give your answer correct to 3 significant figures.

In this question you are given a list of values. These values can be input into your calculator to find the confidence interval.

The confidence interval is $[60.557, 63.530]$.

To find the length of the interval, you find the difference between the upper and lower bounds of the interval.

$$\begin{aligned}\text{length} &= 63.530 - 60.557 \\ &= 2.97 \text{ (3 significant figures)}\end{aligned}$$

3 section questions

4. Probability and statistics / 4.16 Confidence intervals

Confidence intervals for the mean when sigma is not known

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

In the previous section, you constructed confidence intervals to estimate the unknown mean of a normally distributed population where the standard deviation was known . For the construction, you used the expression $\bar{x} \pm z\frac{\alpha}{2} \times \frac{\sigma}{\sqrt{n}}$. This expression included the use of $z\frac{\alpha}{2}$, which is a reference to z-intervals from the standard normal distribution. While this method is helpful when the standard deviation of the population is known, it is much more likely that the standard deviation of the population will not be known and must be estimated.

Making connections

Recall from section 4.14.3 (/study/app/math-ai-hl/sid-132-cid-761618/book/unbiased-estimates-of-the-mean-and-variance-id-27542/) that ${s_{n-1}}^2$ can be used as unbiased estimator of σ^2 (the population variance), where ${s_{n-1}}^2 = \frac{n}{n-1}{s_n}^2$ and ${s_n}^2$ is the variance of the sample.

With rearrangement and substitution, the following expression can be obtained:

$$\bar{x} \pm t \times \frac{s_{n-1}}{\sqrt{n}}$$

Note that the expression now includes t, which is a reference to t-intervals from t-distribution, rather than $z\frac{\alpha}{2}$.

Important

For any sample size, confidence intervals for the population mean should be found

- by using t - intervals when σ is not known
- by using z -intervals when σ is known.

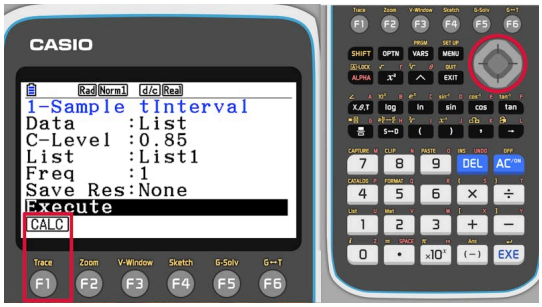
Exam tip

As noted in the previous section, in the IB examinations you will be expected to find all confidence intervals, using your graphic display calculator.

Student view

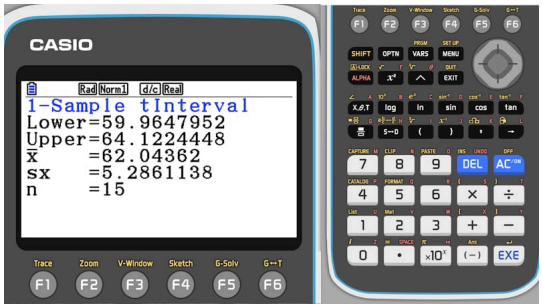
Steps	Explanation
In these instructions you will see how to find the 85% confidence interval for the mean of the population when you do not know the standard deviation of the population. In this example the confidence interval is based on a sample of data given in Example 2 below. To start, open the statistics mode.	
In this example the prediction is based on a data set, so enter it here. In other questions you may only know the mean and standard deviation of the the data. In this case you have nothing to enter here. In either case, press F4 to access the interval options.	
There are two options here, Z or t. In this case the population standard deviation is not known, so press F2 to choose to use the t-distribution. The case, when the population standard deviation is known is discussed in the previous section.	

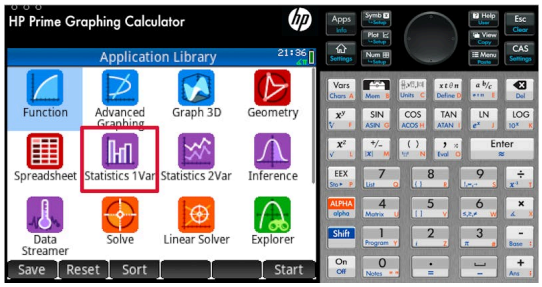
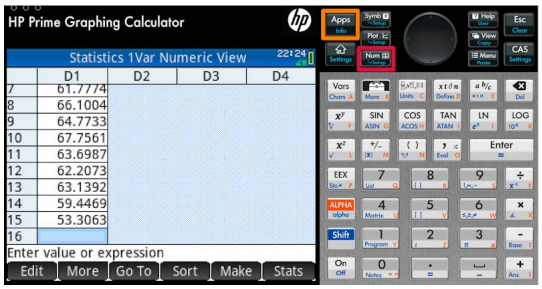
Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Press F1 to choose the option to find the confidence interval for the mean of one random variable.	
The calculator is now waiting for the information as a base of finding the confidence interval. • In the first line, press F1 to tell the calculator that you have the data, (of course, if you do not know the data, choose the other option). You need to tell the calculator what you know. • confidence level (note, that the calculator needs this in decimal form, not as a percentage) • the place where you stored the data • make sure that the frequency is set to 1.	
When you have entered all this, scroll down and press F1 to calculate the confidence interval.	

Student view

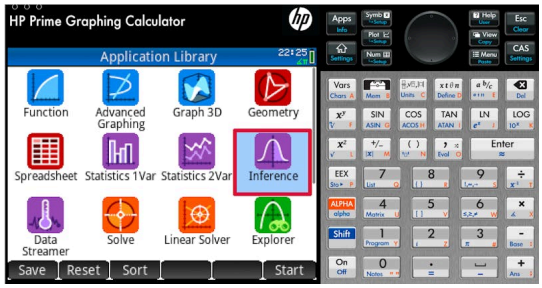
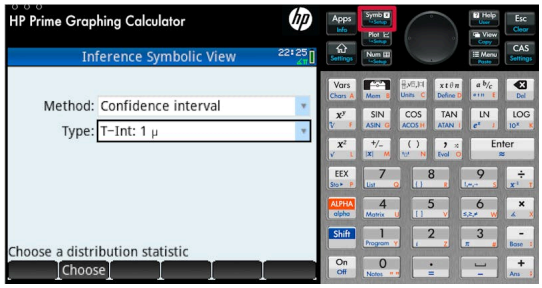
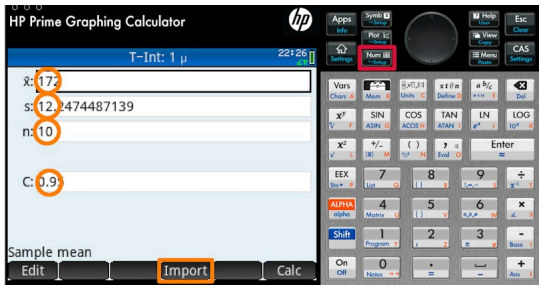
Overview (/study/ap ai-hl/sid-132-cid-761618/ov

Steps	Explanation
On the result screen the lower and upper bound of the confidence interval is given. The sample mean and the sample standard deviation is also given on this screen as a reference.	

Steps	Explanation
In these instructions you will see how to find the 85% confidence interval for the mean of the population when you do not know the standard deviation of the population. In this example the confidence interval is based on a sample of data given in Example 2 below. Since the data is known, open the 1-variable statistics application to enter the data values. In other questions you may only know the mean and standard deviation of the the data. In this case you skip this step.	
In numeric view, choose any column and enter the data. Once done, go back to the application selector screen.	

Student view

Overview (/study/app/ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Once the data is entered (or not known), open the inference application.	
In symbolic view you need to select the test to run. There are several options here. Choose the option to find the confidence interval for the mean of one random variable. You will also need to decide whether to use the Z or t-distribution.. In this case the population standard deviation is not known, so choose to use the t-distribution. The case, when the population standard deviation is known is discussed in the previous section.	
When you change to numeric view, you will see some values in the fields related to a previous problem. To replace these with the statistics of the sample, you need to import these from the 1-variable statistics application, where the data is stored.	

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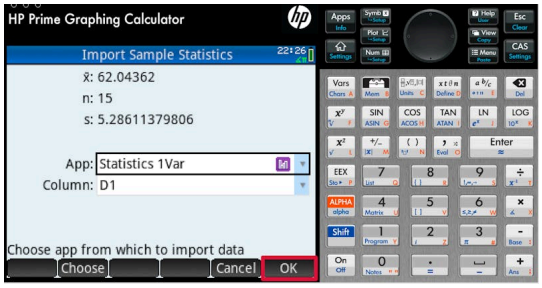
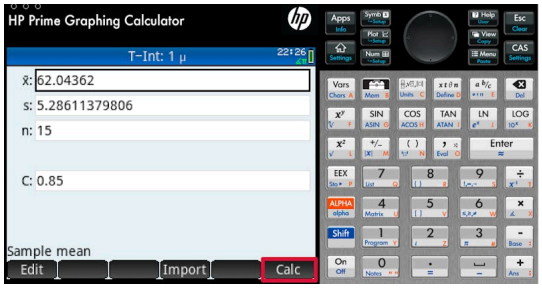
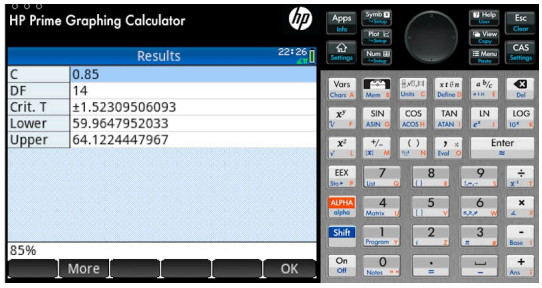
Steps	Explanation
Choose the place where the data is stored and tap OK.	
Back in numeric view you can see, that the fields are filled with the statistics of the data. • mean of the sample ($\overline{x}$) • sample standard deviation (s) • size of the sample (n) You need to also enter the confidence level (note, that the calculator needs this in decimal form, not as a percentage). In questions where the sample is not known, you will be given these values to enter manually instead of importing. When you have entered all this, tap on Calc to get the confidence interval.	
On the result screen the lower and upper bound of the confidence interval is given.	

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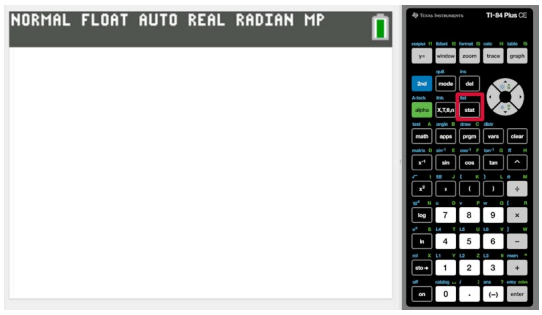
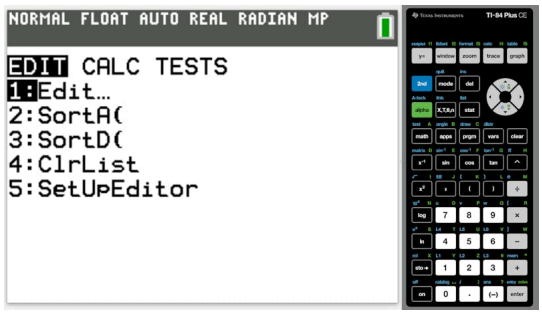
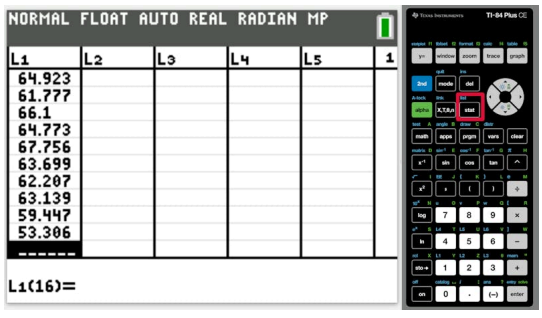
Steps	Explanation
In these instructions you will see how to find the 85% confidence interval for the mean of the population when you do not know the standard deviation of the population. In this example the confidence interval is based on a sample of data given in Example 2 below. To start, open the statistics menu.	
Since the data is known, choose to edit a list to enter the data in the memory of the calculator. In other questions you may only know the mean and standard deviation of the the data. In this case you skip this step.	
Choose any column and enter the data. Once done, open the statistics menu again.	

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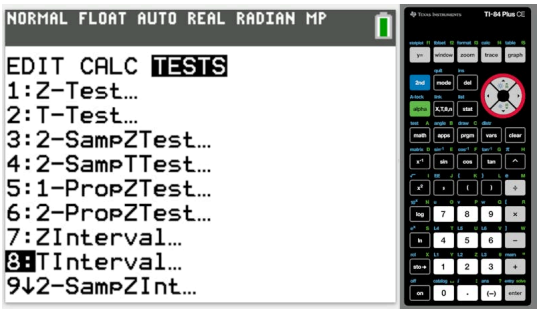
Steps	Explanation
Navigate to TESTS. There are several options here. Choose the option to find the confidence interval for the mean of one random variable. You will also need to decide whether to use the Z or t-distribution.. In this case the population standard deviation is not known, so choose to use the t-distribution. The case, when the population standard deviation is known is discussed in the previous section.	
The calculator is now waiting for the information as a base of finding the confidence interval. • In the first line tell the calculator that you have the data, (of course, if you do not know the data, choose the other option). You need to tell the calculator what you know. • the place where you stored the data • make sure that the frequency is set to 1. • confidence level (note, that the calculator needs this in decimal form, not as a percentage) When you have entered all this, scroll down to the last line and press enter to calculate the confidence interval.	
On the result screen you can see the confidence interval. The sample mean and the sample standard deviation is also given on this screen as a reference.	

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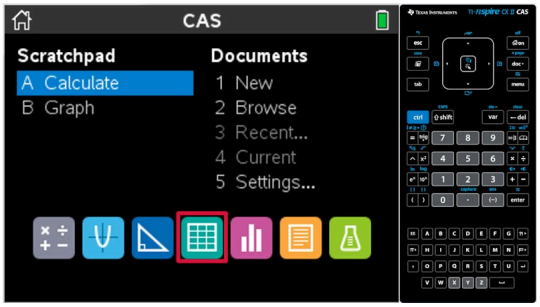
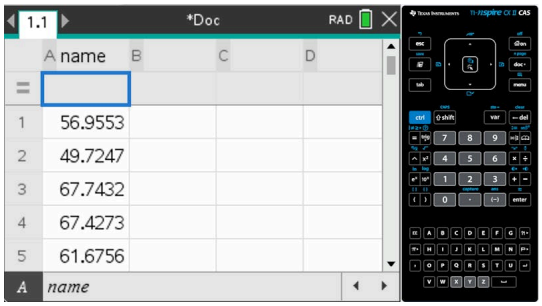
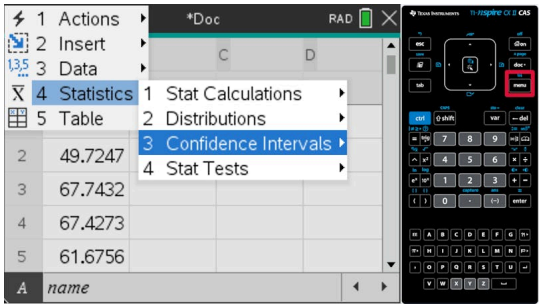
Steps	Explanation
In these instructions you will see how to find the 85% confidence interval for the mean of the population when you do not know the standard deviation of the population. In this example the confidence interval is based on a sample of data given in Example 2 below. Since the data is known, open a spreadsheet page to enter the data values. In other questions you may only know the mean and standard deviation of the the data. In this case you work in a calculator page.	
Enter the data and give a name to the column.	
Open the menu and look for the option to work with confidence intervals.	

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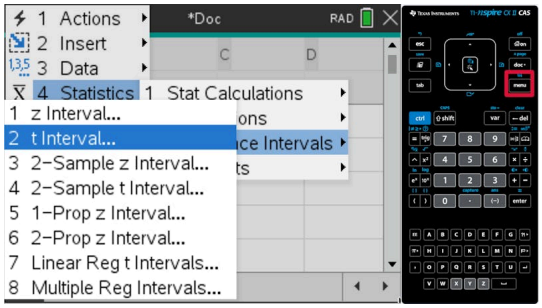
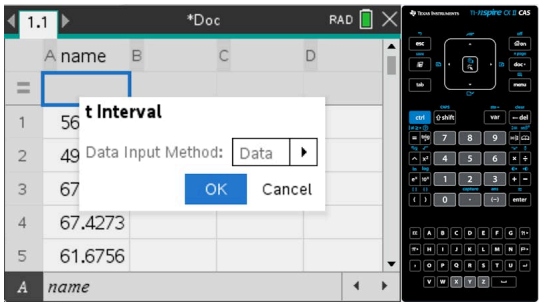
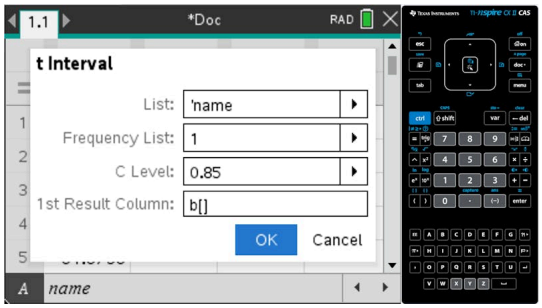
Steps	Explanation
There are several options here. Choose the option to find the confidence interval for the mean of one random variable. You will also need to decide whether to use the Z or t-distribution.. In this case the population standard deviation is not known, so choose to use the t-distribution. The case, when the population standard deviation is known is discussed in the previous section.	
The calculator is now waiting for the information as a base of finding the confidence interval. On this first screen tell the calculator that you have the data (of course, if you do not know the data, choose the other option).	
You need to tell the calculator what you know. • the place where you stored the data (use the name you gave to your column) • make sure that the frequency is set to 1. • confidence level (note, that the calculator needs this in decimal form, not as a percentage) When you have entered all this, scroll down to OK and press enter to calculate the confidence interval	

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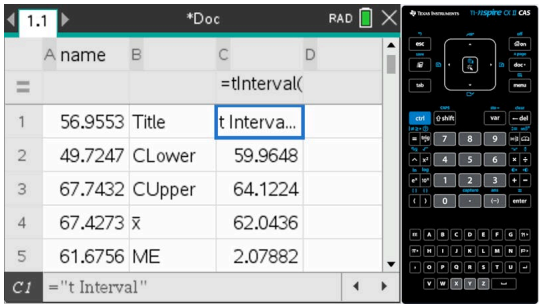
Steps	Explanation
On the result screen the lower and upper bound of the confidence interval is given.	

Example 1

★☆☆

A random variable has a normal distribution and a random sample of size 10 is taken. The mean of this sample is 17 and the sample standard deviation is 5. Find the 95% confidence interval for the mean of the random variable and interpret your result.

In this question you are given the mean of the sample, the sample standard deviation, and the sample size. These values can be input into your calculator to find the confidence interval.

The confidence interval is $[13.423, 20.577]$.

Interpret the results:

Therefore, you are 95% confident that the population mean is between 13.423 and 20.577 . In other words, since 95% of the random samples give confidence intervals that contain the population mean, there is a good chance that your sample is one of the good samples.

Example 2

★☆☆

A civil engineer takes a random sample of pebbles from a beach where construction of sea defences is planned. The mass of the pebbles is known to follow a normal distribution. The following are the masses of the pebbles in the sample:

$$56.9553 \quad 49.7247 \quad 67.7432 \quad 67.4273 \quad 61.6756$$

$$64.9226 \quad 61.7774 \quad 66.1004 \quad 64.7733 \quad 67.7561$$

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63.6987 62.2073 63.1392 59.4469 53.3063

Find the length of the 85% confidence interval for the mean of the random variable.

In this question you are given a list of values. These values can be input into your calculator to find the confidence interval.

The confidence interval is $[59.965, 64.122]$.

To find the length of the interval, you find the difference between the upper and lower bounds of the interval.

$$\begin{aligned}\text{length} &= 64.122 - 59.965 \\ &= 4.16 \,(3 \text{ significant figures})\end{aligned}$$

Activity

Compare the answer to **Example 2** with the answer to **Example 3** of the previous section, which used the same numbers. You will see that the confidence interval is wider than it was before. The reason for this is that you have less information now (only an estimate of the population standard deviation instead of the actual value), so you cannot expect the same precision at the same confidence level.

3 section questions

Checklist

What you should know

By the end of this subtopic you should be able to:

- state that a confidence interval is a range of values calculated using a sample mean, and is intended to estimate an unknown population mean
- state that a confidence level is the percentage of all possible samples for which the corresponding confidence interval contains the unknown population mean
- interpret the calculated confidence interval in terms of the situation presented in the problem
- describe how the confidence level and sample size affect the length of the confidence interval
- calculate confidence intervals with your graphic display calculator using z - intervals for situations when the population standard deviation is known and using t - intervals for when it is not known.

Investigation

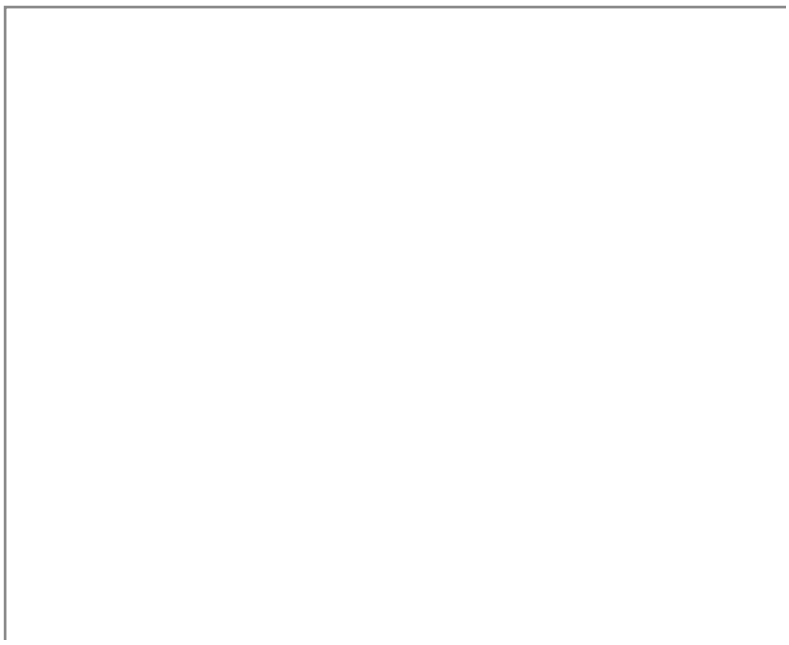
In section 4.16.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/confidence-intervals-for-mean-when-sigma-not-known-id-27989/), you saw that the t-distribution is related to the standard normal distribution. While the full explanation is not required for this course, an investigation into the connection can provide greater insights into the meaning of the two distributions.

To begin the investigation, you will first need to think about the standard normal distribution. Standardising the normal distribution involves comparing the normal distribution of the random variable you are interested in with the standard normal distribution in which

$$\mu = 0 \text{ and } \sigma = 1.$$

- What do the numbers on the horizontal axis of the graph of the standard normal distribution represent?
- What are the benefits of comparing distributions of random variables to the standard normal distribution?

Below you will find an applet that graphs a t-distribution and the standard normal distribution on the same axes. The slider in the app allows you to change the degrees of freedom associated with the t-distribution.



Interactive 1. Investigating t-Distribution and Standard Normal Distribution.

More information for interactive 1

This interactive allows you to explore the relationship between the t-distribution and standard normal distribution by adjusting the degrees of freedom. The screen is divided into two halves. On the top side of the screen, a graph is displayed with XY axis, with x-axis ranging from -4 to 4 and y-axis ranging from 0 to 0.4. Two bell-shaped curves are projected on the graph, one representing the standard normal distribution in blue, and the other representing the t-distribution in red. On the bottom of the screen, a horizontal slider is given (d) ranging from 1 to 40 which represents the degrees of freedom. As users increase the degrees of freedom using the, the red t-distribution curve gradually converges toward the blue standard normal curve. When d is small (like 1 or 2), the t-distribution appears noticeably wider with heavier tails, reflecting greater uncertainty in small sample sizes. However, as d approaches 40, the two distributions become nearly identical, demonstrating how the t-distribution approximates the normal distribution for larger samples. The horizontal axis represents standard deviations from the mean (z-scores), helping you visualize how these distributions compare in terms of

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spread and shape.

This dynamic visualization illustrates a fundamental statistical concept: while the t-distribution is essential for analyzing small sample data due to its wider confidence intervals, it becomes practically indistinguishable from the normal distribution when sample sizes grow sufficiently large (typically around $d = 30$ or more).

The interactive provides valuable insight into why statisticians use the t-distribution for small sample hypothesis testing and confidence intervals, while often switching to normal distribution approximations for larger datasets. By experimenting with different degrees of freedom, you can develop an intuitive understanding of how sample size affects the shape of these critical probability distributions.

- What are the similarities and differences between the two distributions?
- What effect does changing the degrees of freedom have on the graphs?

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