4.18 Teacher view

Population tests

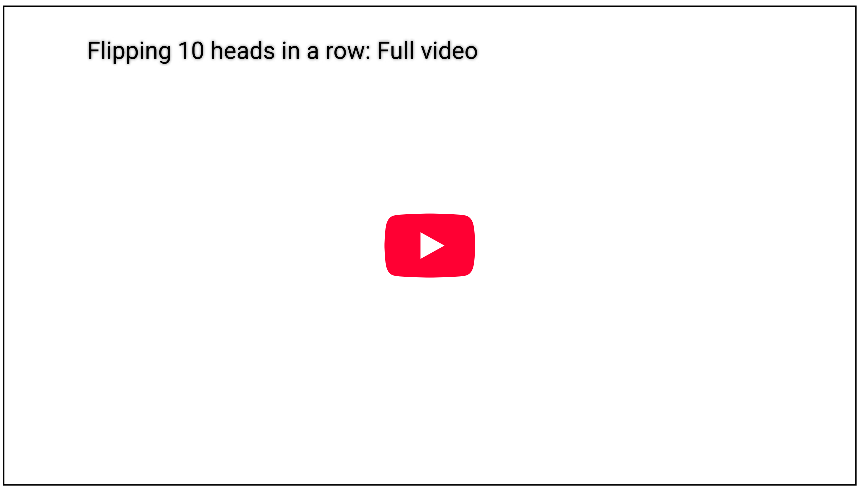
Index

The big picture

When collecting sample data for a statistical investigation it is better to collect multiple samples and to make your samples as large as possible. However, sometimes unavoidable limitations prevent you from collecting more than one small sample. When this happens, how much can you infer about the population you are sampling?

Watch this video of James Grime flipping a coin that comes up heads ten times in a row.

Now watch this video in which he reveals how he spent hours videotaping himself flipping a coin until he finally achieved ten heads in a row.



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Imagine that the trial you saw in the first video was the only sample you had with which to draw a conclusion. How much could rely on your conclusion? How could you account for errors in your conclusion that you may not know about?

Concept

Approximation in data can help you approach the truth but unavoidable errors may keep you from being able to achieve the truth.

4. Probability and statistics / 4.18 Population tests

Testing for the population mean

Testing when σ is known

Making connections

In this section, you will be using the concepts of null and alternative hypotheses, significance levels, and p-values. It may helpful to review these concepts from section 4.11.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/hypothesis-testing-and-the-chi-squared-id-27880/).

A peanut farmer from Jiangsu, China

Credit: Alexandre Morin-Laprise Getty Images

Li Wei is a peanut farmer in southern China. After harvesting his peanuts, he packages them in 25 kg bags before loading them into a truck and sending them to the processing plant. Li Wei has received a complaint that his bags do not contain 25 kg as required. He wants to investigate the complaint, but weighing every bag of peanuts would take too much time.

Therefore, Li Wei decides to take a sample of six bags. The sample contains bags weighing 24.7, 25.2, 24.2, 24.9, 23.8, and 25.6 kg. Li Wei notices that four of the bags weigh less than 25.0 kg and the mean of the sample is $24.7\bar{3}$ kg. Does this sample give enough evidence to show that the mean weight of all of Li Wei's bags of peanuts is less than 25.0 kg?

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To answer this question, let us begin by stating our null and alternative hypotheses. When testing the mean of a population, the null hypothesis should be a single value, not a range. The alternative hypothesis can either be one-tailed or two-tailed.

- $H_0 : \mu = 25.0$ (the mean weight of Li Wei's bags of peanuts is 25.0 kg, or more)
- $H_1 : \mu < 25.0$ (the mean weight of Li Wei's bags of peanuts is less than 25.0 kg)

Notice that a one-tailed test will be carried out since Li Wei is only interested in whether the mean weight of his bags is less than 25 kg.

Exam tip

In exams, you will be expected to decide from the context of the problem whether you need to carry out a two-tailed test :

$$(H_1 : \mu \neq \mu_0)$$

or a one-tailed test:

$$(H_1 : \mu < \mu_0, \text{ or } H_1 : \mu > \mu_0)\ .$$

The method used to test the above hypotheses depends on whether σ, the standard deviation of the population, is known.

Important

For any sample size, the normal distribution is used to test for the mean of a population when σ is known and the t-distribution is used when σ is not known.

Example 1

★☆☆

If the weights of Li Wei's bags of peanuts are normally distributed with a standard deviation of 1.5 kg, calculate the p-value for his sample data of 24.7, 25.2, 24.2, 24.9, 23.8, and 25.6 kg.

Use your p-value to determine whether, at a 5% level of significance, the mean weight in Li Wei's bags is likely to be less than the required weight of 25.0 kg $(H_1 : \mu < 25.0)$.

Steps	Explanation
The p-value of this sample data is $0.331612\ldots \approx 0.332$, $\therefore$ there is a 33.2% chance of Li Wei collecting a random sample of six bags from his truck that have a mean weight of $24.7\bar{3}\text{ kg}$ or less.	Use your graphic display calculator to find the p-value.

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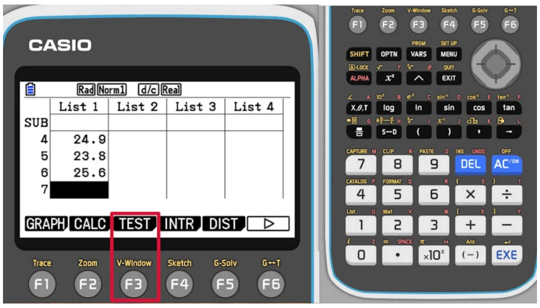
Steps	Explanation
Since $0.332 > 0.05$, Li Wei's sample does not give enough evidence to reject the null hypothesis. This sample does not give enough evidence at a 5% level of significance to conclude that the weight of the bags is less than the required 25 kg.	Use the p-value to decide whether to reject the null hypothesis or not.

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is known. You will see how to test the null hypothesis $\mu = 25$ against the one sided alternative hypothesis $\mu < 25$ when the standard deviation is $\sigma = 1.5$. The test will use the random sample $24.7, 25.2, 24.2, 24.9, 23.8, 25.6$ and gives back the p-value to use in deciding your conlcusion. To start, open the statistics mode.	
Enter the data and press F3 to open the test options.	

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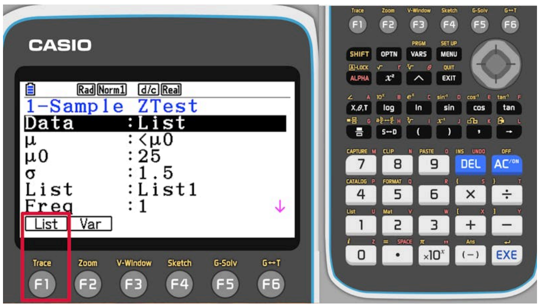
Steps	Explanation
Since the standard deviation is known, press F1 to choose to work with the Z-distribution …	
… and press F1 again to choose the 1-sample option.	
The calculator is now waiting for the information about the specific question. In the first line press F1 to select, that you want to use your data as the basis of your conclusion.	

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view

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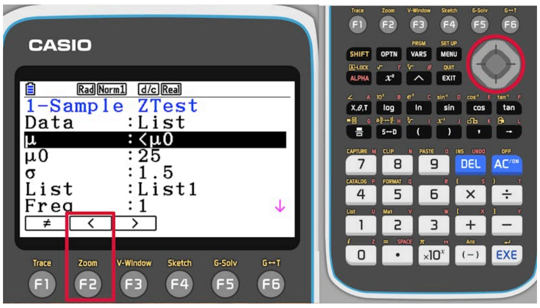
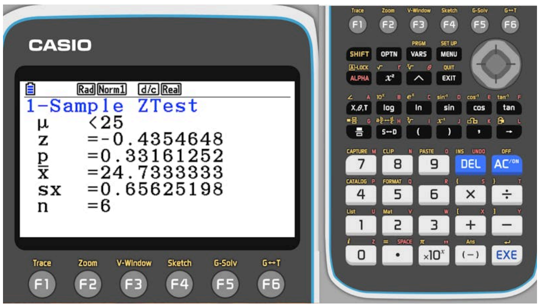
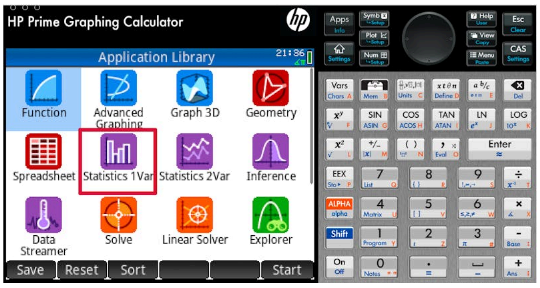
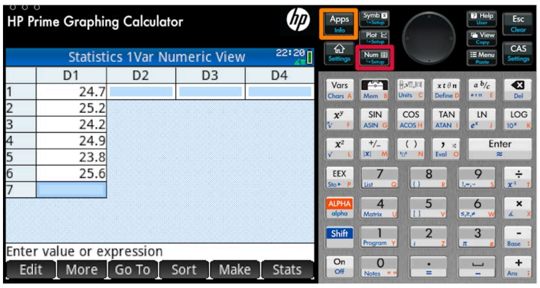
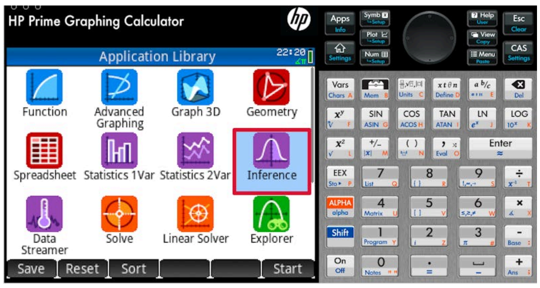
Steps	Explanation
Scroll down to the second line and choose the direction of the alternative hypothesis. Fill in also the other information. • μ_0 is the mean in the null-hypothesis. • σ is the known standard deviation • enter the list name where you stored the data and make sure the frequencies are set to 1.	
When you entered all information, scroll down to the last line and press F1 to ask the calculator to find the p-value of the test.	
Among other information, you will find the p-value on the result screen.	

Student view

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is known. You will see how to test the null hypothesis $\mu = 25$ against the one sided alternative hypothesis $\mu < 25$ when the standard deviation is $\sigma = 1.5$. The test will use the random sample $24.7, 25.2, 24.2, 24.9, 23.8, 25.6$ and gives back the p-value to use in deciding your conlcusion. You can enter the data for example in the one variable statistics mode.	
In numeric view enter the data (and make a note of the name of the column). Once done, go back to the application menu.	
This time, choose the inference application.	

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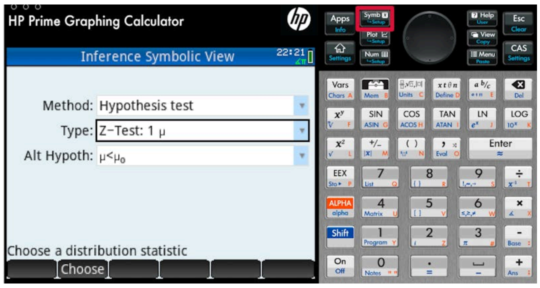
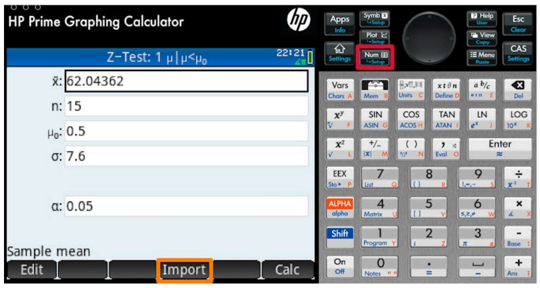
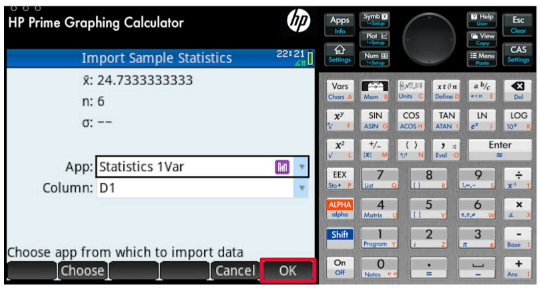
Steps	Explanation
In symbolic view set the type of test. • You would like to test a hypothesis. • Since the standard deviation is known, choose the Z-distribution. Choose also to test for one mean. • Choose the appropriate (in this example one-sided) alternative hypothesis.	
In numeric view, the fields are filled with data from earlier work. You need to replace these. Since yu want to use your data, you need to import it.	
Select the place where you stored the data and tap on OK when done.	

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view

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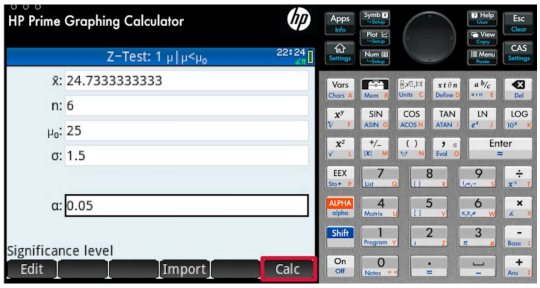
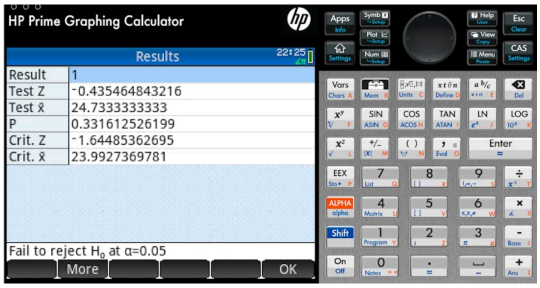
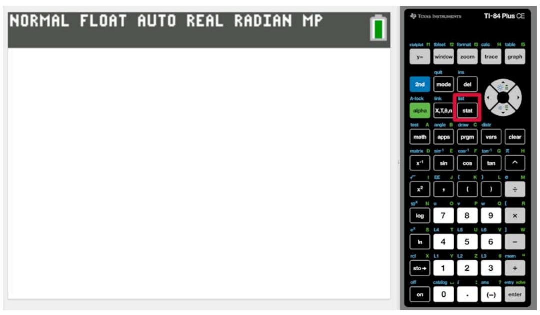
Steps	Explanation
Now the mean and size of the sample is filled in. You still need to tell the calculator • the mean value in the null-hypothesis (μ_0) and • the known standard deviation (σ). You can also tell the significance level. When done, tap on calc to run the test.	
Among other information, you will find the p -value on the result screen.	

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is known. You will see how to test the null hypothesis $\mu = 25$ against the one sided alternative hypothesis $\mu < 25$ when the standard deviation is $\sigma = 1.5$. The test will use the random sample $24.7, 25.2, 24.2, 24.9, 23.8, 25.6$ and gives back the p-value to use in deciding your conlcusion. To start, open the statistics menu.	

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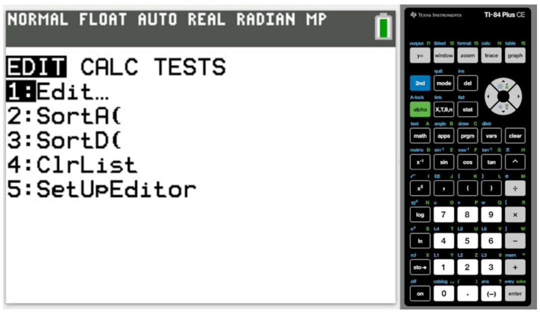
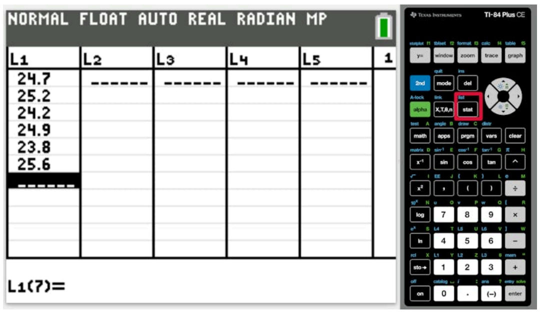
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>To enter the data, choose to edit a list.</td><td>



</td></tr>
<tr><td>Enter the data (and make a note of the name of the column).

Once done, go back to the statistics menu.</td><td>



</td></tr>
<tr><td>Scroll to see the options for tests.

Since the standard deviation is known, choose the Z-distribution and choose to test for one mean.</td><td>

</td></tr>
</table>

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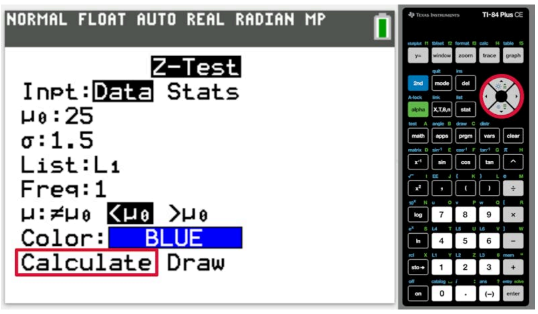
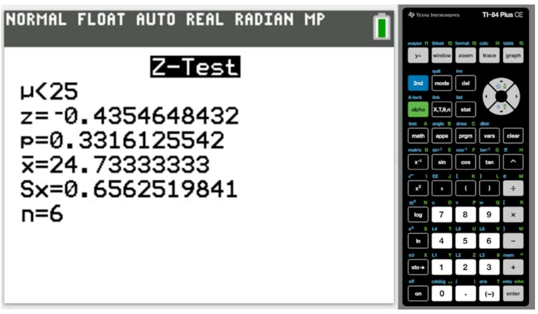
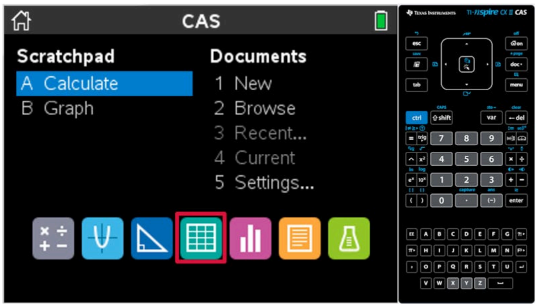
Steps	Explanation
The calculator is now waiting for the information about the specific question. • In the first line select, that you want to use your data as the basis of your conclusion. • μ_0 is the mean in the null-hypothesis. • σ is the known standard deviation. • Enter the list name where you stored the data and make sure the frequencies are set to 1. • Choose the direction of the alternative hypothesis. When you entered all information, scroll down to the last line and press enter to ask the calculator to find the p-value of the test.	
Among other information, you will find the p-value on the result screen.	

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is known. You will see how to test the null hypothesis $\mu = 25$ against the one sided alternative hypothesis $\mu < 25$ when the standard deviation is $\sigma = 1.5$. The test will use the random sample $24.7, 25.2, 24.2, 24.9, 23.8, 25.6$ and gives back the p-value to use in deciding your conlcusion. To start, open a spreadsheet page.	

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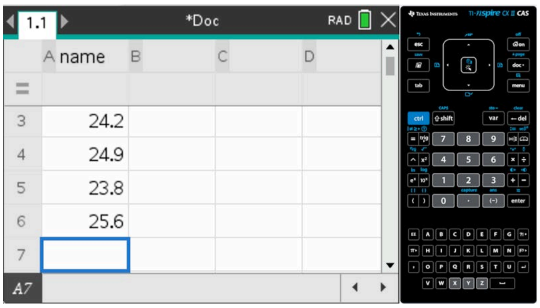
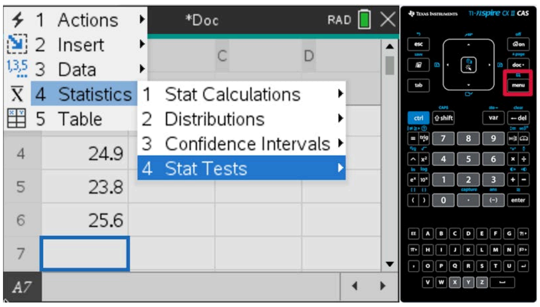
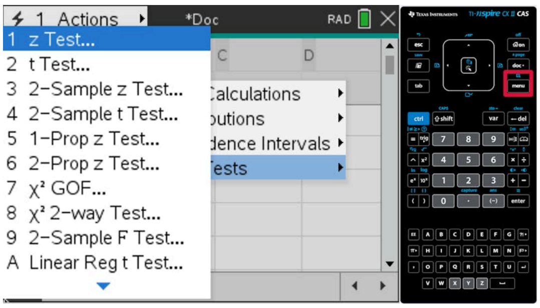
Steps	Explanation
Enter the data and give a name you can remember.	
Open the menu and look for the statistical tests.	
Since the standard deviation is known, choose the Z-distribution and choose to test for one mean.	

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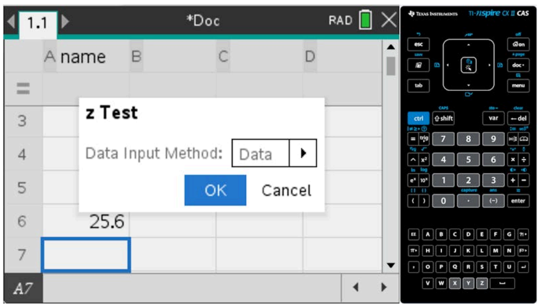
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>

The calculator is now waiting for the information about the specific question.

On this first screen select that you want to use your data as the basis of your conclusion.

</td><td>



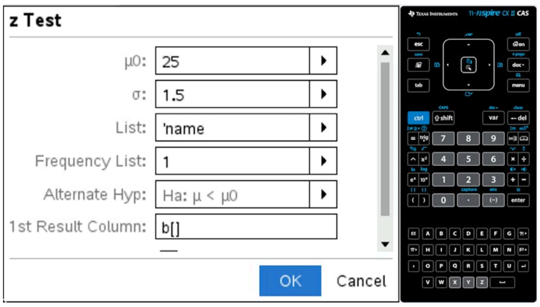
</td></tr>
<tr><td>

On the next screen fill in the other information.

- μ_0 is the mean in the null-hypothesis.
- σ is the known standard deviation.
- Enter the list name where you stored the data and make sure the frequencies are set to 1.
- Choose the direction of the alternative hypothesis.

When you entered all information, scroll down to OK and press enter to ask the calculator to find the p-value of the test.

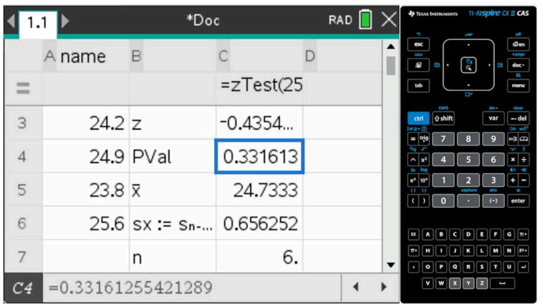
</td><td>



</td></tr>
<tr><td>

Among other information, you will find the p-value on the result screen.

</td><td>



</td></tr>
</table>

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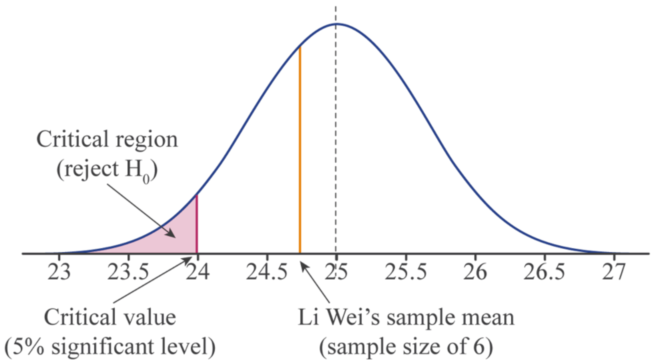
Note that the conclusion was Li Wei's sample failed to reject the null hypothesis. When testing hypotheses, you should not say that you accept the null hypothesis as you have only taken a small sample of data.

Let us explore the relationship between the p-value and the significance level by considering the diagram shown below. Recall that in **section 4.15.2** you saw that the distribution of the possible means of a sample of size 6 that could be drawn from a population with a mean of 25 and a standard deviation of 1.5 is a normal distribution with mean 25 and standard deviation $\frac{1.5}{\sqrt{6}} \approx 0.612$.



More information

The image shows a normal distribution curve with a mean of 25. The X-axis represents sample means and is labeled from 23 to 27. A vertical dashed line at 25 indicates the population mean. An orange line marks Li Wei's sample mean at approximately 24.5 with a sample size of 6. The Y-axis shows probability density without specific labeling.

A shaded red area on the left, below the curve, represents the critical region where the null hypothesis (H0) is rejected at a 5% significance level. A label points to the shaded area indicating it as the critical region (reject H0) and the critical value at the 5% significance level. The critical value aligns with a sample mean of approximately 23.5 on the X-axis. This region indicates the lowest possible sample means whose total probabilities add to 5%.

[Generated by AI]

The critical region under the curve is determined by the significance level. As the significance level for Li Wei's test was 5%, the red shaded area represents the lowest possible sample means whose total probabilities add to 5%.

The critical value is the value that defines the upper limit of the critical region. If Li Wei's sample mean was less than the critical value , then the null hypothesis would have been rejected.

Be aware

A significance level should be chosen *before* a test is carried out. It is often the case that significance levels are chosen to meet government standards or other criteria and, thus, the level is predetermined. It is inappropriate to carry out a test and then choose a significance level that would allow you to gain a desired result. For example,

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imagine that the people testing a new drug before selling it were able to set the significance level after they had received the results of the safety tests. They would be able to select a level that enabled them to declare the drug safe.

Making connections

Recall from section 4.15.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/sampling-a-normally-distributed-random-variable-id-27547/) that when sampling from a normally distributed population, the distribution of the sample mean is defined as $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ where μ is the population mean, σ is the population standard deviation, and n is the size of the sample.

Example 2

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Given that the standard deviation of the weight of Li Wei's bags of peanuts is 1.5 kg, calculate the critical region for a test using a 5% significance level on his sample data of 24.7, 25.2, 24.2, 24.9, 23.8 and 25.6 kg.

Steps	Explanation
$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ $\bar{X} \sim N\left(25, \frac{1.5^2}{6}\right)$	To begin, you need to define the distribution of a sample of size 6.
Refer to the calculator instructions in section 4.9.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-normal-distribution-and-calculator-functions-id-26127/). $\therefore$ the critical value is $23.9927\ldots$ Therefore, the probability of Li Wei's sample mean being $23.9927\ldots$ kg or less is 5% (when the sample size is 6).	Use the InvNorm function of your graphic display calculator to find the critical value.
Since this is a one-tailed test (on the left) the critical region is $\bar{X} < 23.99$.	Use the critical value and the type of test to form the critical region.

Important

The null hypothesis is rejected if either the p-value is less than the significance level or the sample mean is in the critical region.

Example 3

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A candy company has set up its packaging machine to put 60 candies in each package. The company believes that the machine accomplishes this with a standard deviation of 5 candies. The company wants to know if the mean number of candies in each package is actually 60. An employee for the company randomly selects 20 packages, counts the number of candies in each package, and finds the mean of the sample to be 57.1.

a) State the null and alternative hypothesis for this test.

b) Calculate the p-value and determine whether there is sufficient evidence to reject the null hypothesis at a 1% significance level.

	Steps	Explanation
a)	$H_0 : \mu = 60$ (the mean number of candies in each package made by the machine is 60). $H_0 : \mu \neq 60$ (the mean number of candies in each package made by the machine is not 60).	As the company is interested in whether the mean number is different from 60 (and not just above or below 60) this will be a two-tailed test.
b)	$p = 0.00949116\ldots \approx 0.00949\ldots$ $\therefore$ there is a 0.949% chance that a random sample of 20 packages would have a sample mean of 57.1 or less.	Use your graphic display calculator to find the p-value.
	Since $0.00949 < 0.01$ the sample data provides enough evidence to accept the alternative hypothesis at a 1% significance level. Therefore, the employee can report to the company the machine is not filling packages with a mean of 60 candies.	Use the p-value to decide whether to reject the null hypothesis or not.

Testing when σ is not known

There are situations in which the population standard deviation is not known. When this happens, you will use the standard deviation of the sample and perform a t -distribution test, as shown in **Example 4**.

Example 4

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The mean length of time it takes for a certain roller coaster to go completely around the track is 54 s. An employee feels that the roller coaster has started going faster around the track and decides to take some sample data. She measured the time 14 times. She does not know the overall standard deviation of the roller coaster's time. However, she calculates that the mean of her sample is 52.79 s and the sample standard deviation is 4.106 s.

a) State the null and alternative hypothesis for this test.

b) Calculate the p-value and determine whether there is sufficient evidence to reject the null hypothesis at a 10% significance level.

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	Steps	Explanation
a)	$H_0 : \mu = 54$ (the mean time it takes for the roller coaster to complete the track is 54 seconds) $H_1 : \mu < 54$ (the mean time it takes for the roller coaster to complete the track is less than 54 seconds)	As the employee is interested only in whether the roller coaster is going faster, this will be a one-tailed test.
b)	The p-value for the test is $0.145090\ldots \approx 0.145$.	Use your graphic display calculator to find the p-value.
	Therefore, the employee fails to reject the null hypothesis. (Note that this does not mean that the employee should accept the null hypothesis that the roller coaster takes 54 seconds to complete the track.)	Use the p-value to decide whether to reject the null hypothesis or not.

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is not known. You will see how to test the null hypothesis $\mu = 54$ against the one sided alternative hypothesis $\mu < 54$. The test will be based on a random sample of size 14 with mean 52.79 and standard deviation 4.106. The test will give back the p-value to use in deciding your conclusion. To start, open the statistics mode.	
There is no data to enter, so press F3 to open the test options.	

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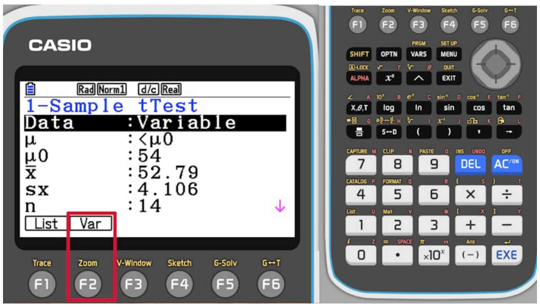
Steps	Explanation
Since the standard deviation is not known, press F2 to choose to work with the t-distribution ...	
.. and press F1 to choose the 1-sample option.	
The calculator is now waiting for the information about the specific question. In the first line press F2 to select, that you do not have the data (but you know information about it).	

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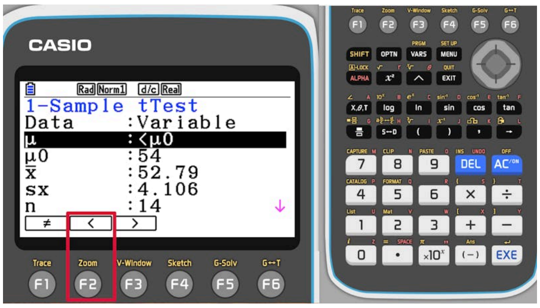
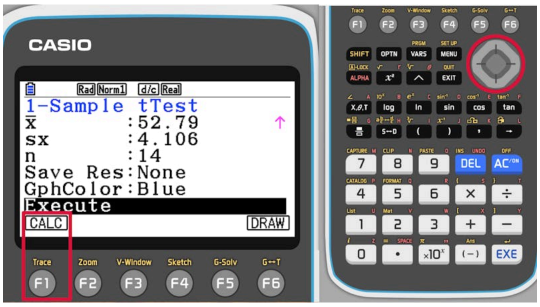
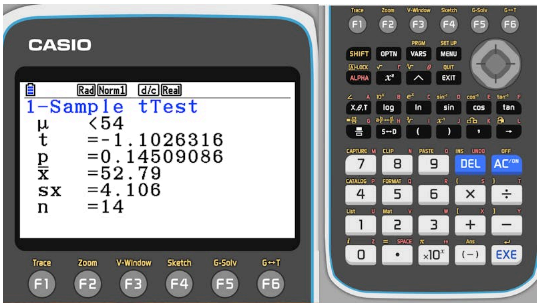
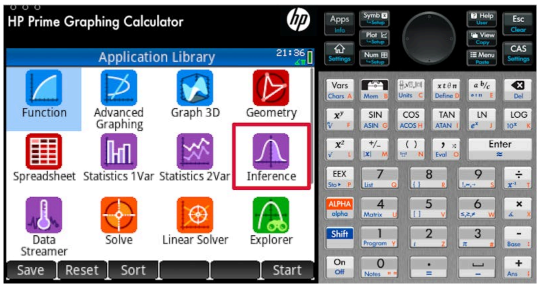
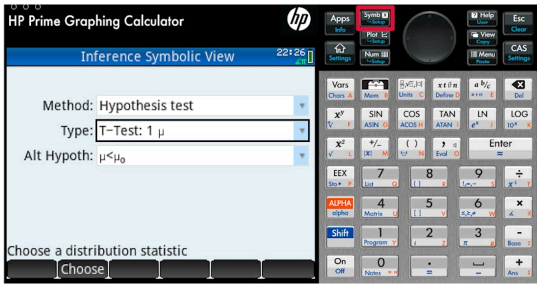
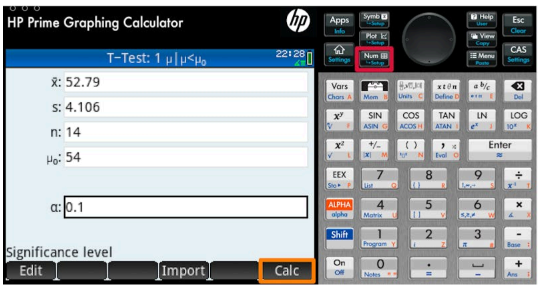
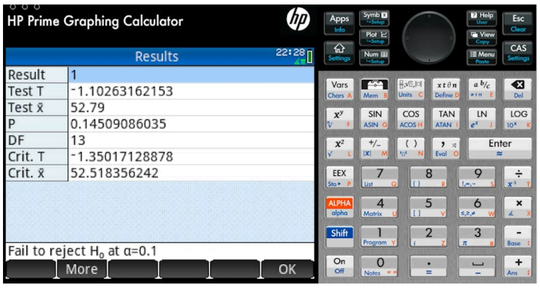
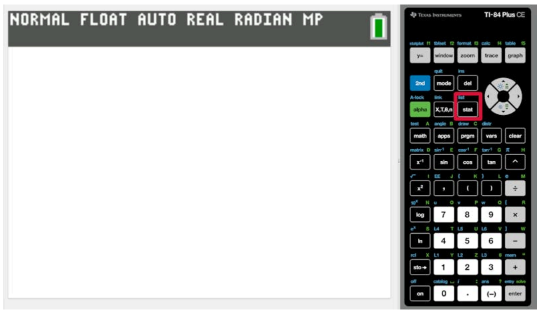
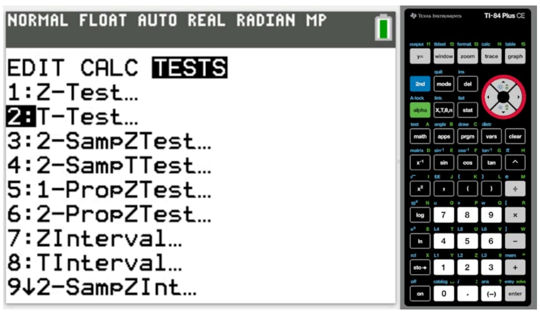
Steps	Explanation
Scroll down to the second line and choose the direction of the alternative hypothesis. Fill in also the other information. • μ_0 is the mean in the null-hypothesis. • $\overline{x}$ is the mean of the sample. • sx is the sample standard deviation • n is the size of the sample.	
When you entered all information, scroll down to the last line and press F1 to ask the calculator to find the p-value of the test.	
Among other information, you will find the p-value on the result screen.	

Student view

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is not known. You will see how to test the null hypothesis $\mu = 54$ against the one sided alternative hypothesis $\mu < 54$. The test will be based on a random sample of size 14 with mean 52.79 and standard deviation 4.106. The test will give back the p-value to use in deciding your conlcusion. To start, choose the inference application.	
In symbolic view set the type of test. • You would like to test a hypothesis. • Since the standard deviation is not known, choose the t-distribution. Choose also to test for one mean. • Choose the appropriate (in this example one-sided) alternative hypothesis.	
Change to numeric view and enter the information about the specific question. • $\overline{x}$ is the mean of the sample. • s is the sample standard deviation. • n is the size of the sample. • μ_0 is the mean in the null-hypothesis. You can also tell the significance level. When done, tap on calc to run the test.	

Steps	Explanation
Among other information, you will find the p-value on the result screen.	

Steps	Explanation
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is not known. You will see how to test the null hypothesis $\mu = 54$ against the one sided alternative hypothesis $\mu < 54$. The test will be based on a random sample of size 14 with mean 52.79 and standard deviation 4.106. The test will give back the p-value to use in deciding your conlcusion. To start, open the statistics menu ...	
... and scroll to see the options for tests. Since the standard deviation is not known, choose the t-distribution and choose to test for one mean.	

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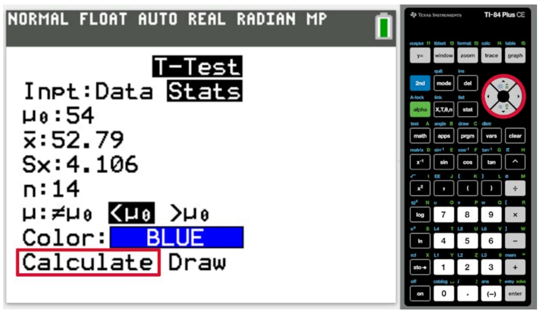
Steps	Explanation

The calculator is now waiting for the information about the specific question.

- In the first line select, that you want to use your statistics that you know about the sample.
- μ_0 is the mean in the null-hypothesis.
- $\overline{x}$ is the mean of the sample.
- Sx is the sample standard deviation
- n is the size of the sample.
- Choose the appropriate (in this example one-sided) alternative hypothesis.

When you entered all information, scroll down to the last line and press enter to ask the calculator to find the p-value of the test.



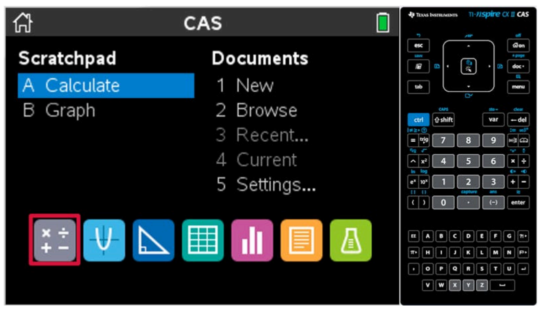
Among other information, you will find the p-value on the result screen.

Steps	Explanation

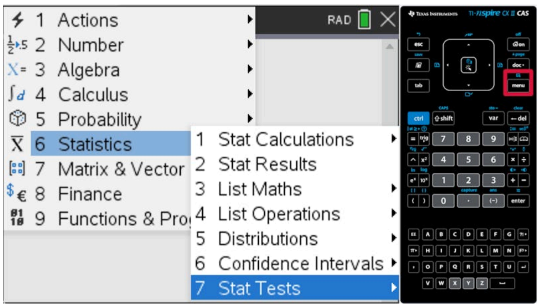
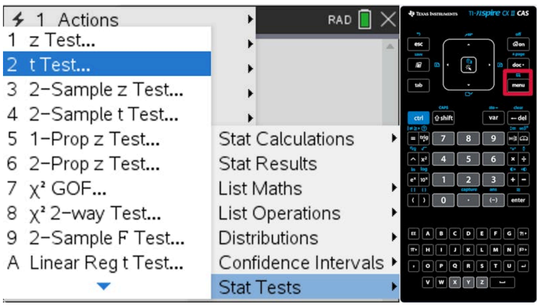
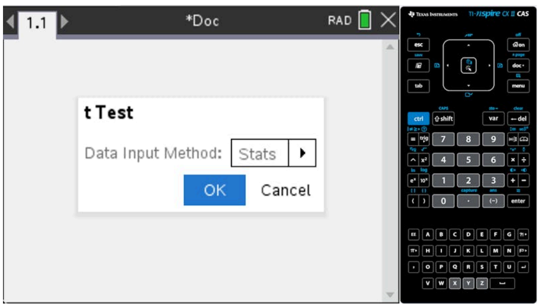
In this instruction you will see how to run a hypothesis test for the mean of a distribution when the standard deviation is not known.

You will see how to test the null hypothesis $\mu = 54$ against the one sided alternative hypothesis $\mu < 54$. The test will be based on a random sample of size 14 with mean 52.79 and standard deviation 4.106. The test will give back the p-value to use in deciding your conlcusion.

Since there is no data to enter, the calculations can be done on a calculator page.



Student view

Steps	Explanation
Open the menu and look for the statistical tests.	
Since the standard deviation is not known, choose the t-distribution and choose to test for one mean.	
The calculator is now waiting for the information about the specific question. On this first screen select that you want to use your your statistics that you know about the sample.	

Overview
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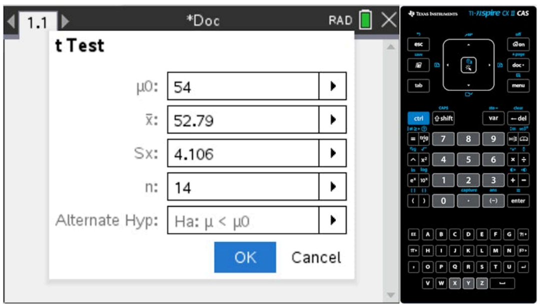
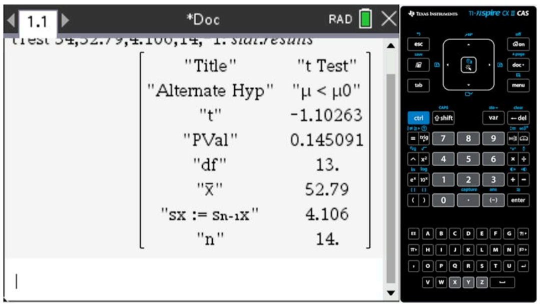
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>
On the next screen fill in the other information.

μ_0 is the mean in the null-hypothesis.
$\overline{x}$ is the mean of the sample.
Sx is the sample standard deviation
n is the size of the sample.
Choose the appropriate (in this example one-sided) alternative hypothesis.

When you entered all information, scroll down to OK and press enter to ask the calculator to find the p-value of the test.
</td><td>


</td></tr>
<tr><td>
Among other information, you will find the p -value on the result screen.
</td><td>


</td></tr>
</table>

Exam tip

In the IB examinations, you will not be expected to calculate critical regions or critical values for t-tests.

3 section questions

4. Probability and statistics / 4.18 Population tests

Matched pairs

Student view

Testing population mean for matched pairs

A fitness centre

Credit: Rob Melnychuk Getty Images

Two fitness centres have developed exercise programmes that they claim help people to lose weight. In order to prove their claim, they recruited ten people to take part in a trial of the programme. They recorded the weights of the ten people before and after using the exercise programme. Their data, in kg, is shown in the table below.

Before	72.9	74.2	76.8	65.2	81.2	82.0	70.7	84.2	72.6	75.4
After	73.2	73.1	76.4	63.8	80.3	81.7	69.9	82.4	71.4	75.4

Before	52.6	57.2	73.9	56.1	81.7	68.2	54.3	75.8	61.2	81.7
After	54.1	55.0	69.1	51.2	77.2	68.1	58.4	69.4	58.8	84.3

Activity

Discuss the following question with another classmate.

- Looking at the data, does it appear that the exercise programme helped people lose weight?

Let us consider a response to the last question in the activity. Using your graphic display calculator, you can find the mean for each of the two samples.

For the first company, these are $\bar{x}_{\text{before}} = 75.52$ kg and $\bar{x}_{\text{after}} = 74.76$ kg.

For the second company, these are $\bar{x}_{\text{before}} = 66.27$ kg and $\bar{x}_{\text{after}} = 64.56$ kg.

Does the decrease in the mean really indicate that the exercise programme is effective? To help alleviate this issue, it is often helpful to link the data for each participant together rather than finding the mean of each sample. You can evaluate them as matched pairs.

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Important

When two samples are considered to be matched pairs, a hypothesis test can be carried out on the differences between the sets of data using a null hypothesis of $H_0 : \mu = 0$.

Be aware

Note that the null hypothesis will always be $\mu = 0$ as you want to test whether there is a difference in the two samples. You will not use the population mean when using matched pairs.

In the examples below you can see how to evaluate the data sets presented earlier for the activity.

Example 1

★☆☆

The weights of the following participants in an exercise programme can be considered to have come from a normally distributed population.

Before	72.9	74.2	76.8	65.2	81.2	82.0	70.7	84.2	72.6	75.4
After	73.2	73.1	76.4	63.8	80.3	81.7	69.9	82.4	71.4	75.4

a) Find the difference between each participant's weight after the programme and their weight before the programme.

b) By choosing an appropriate alternative hypothesis, determine whether there is sufficient evidence at a 5% significance level to show that the exercise programme effectively helped the participants lose weight.

	Steps	Explanation
a)	0.3 -1.1 -0.4 -1.4 -0.9 -0.3 -0.8 -1.8 -1.2 0.0	Begin by finding the differences between the weights.

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

	Steps	Explanation
b)	Since you are investigating whether the participants lost weight, you would like to know if the mean of the differences is less than zero, therefore $H_0 : \mu = 0$ and $H_1 : \mu < 0.$	Choose an appropriate alternative hypothesis.
	$p = 0.002585878\ldots$ $= 0.00259$ (3 significant figures)	Since the standard deviation of the population's weight is not known, use the t-test to calculate a p-value.
	Since $0.00259 < 0.05$ you reject the null hypothesis. There is significant evidence to conclude that the exercise programme helped the participants lose weight.	Draw a conclusion based on the p-value.

Example 2

★☆☆

The weights of the following participants in an exercise programme can be considered to have come from a normally distributed population.

Before	52.6	57.2	73.9	56.1	81.7	68.2	54.3	75.8	61.2	81.7
After	54.1	55.0	69.1	51.2	77.2	68.1	58.4	69.4	58.8	84.3

a) Find the difference between each participant's weight after the programme and their weight before the programme.

b) By choosing an appropriate alternative hypothesis, determine whether there is sufficient evidence at a 5% significance level to show that the exercise programme effectively helped the participants lose weight.

Student view

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	Steps	Explanation
a)	1.5 -2.2 -4.8 -4.9 -4.5 -0.1 4.1 -6.4 -2.4 2.6	Begin by finding the differences between the weights.
b)	Since you are investigating whether the participants lost weight, you would like to know if the mean of the differences is less than zero, therefore $H_0 : \mu = 0$ and $H_1 : \mu < 0$.	Choose an appropriate alternative hypothesis.
	$p = 0.082686115\ldots$ $= 0.0827$ (3 significant figures)	Since the standard deviation of the population's weight is not known, use the t-test to calculate a p-value.
	Since $0.0827 > 0.05$ you do not reject the null hypothesis. There is not significant evidence to conclude that the exercise programme helped the participants lose weight.	Draw a conclusion based on the p-value.

International Mindedness

Exercise is an important part of a healthy lifestyle. However, the form of exercise that you prefer may very well depend on where you live in the world. The article found here (https://www.insider.com/popular-exercise-methods-around-the-world-2018-11)discusses some of the differences that can be found around the world.

3 section questions

Student view

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Other population tests

Testing proportion using binomial probability

Making connections

Recall from section 4.8.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/calculating-bionomial-probabilities-id-26116/) that the binomial CDF function of your calculator can be used to find the cumulative probability of a range of outcomes for the random variable X that is binomially distributed.

A basketball player shooting a free throw

Credit: simonkr Getty Images

Jamin is an IB Diploma student who loves to play basketball and claims that he succeeds in 85% of the free throws that he attempts. His friend, Aaron, does not believe Jamin and asks him to prove it by shooting 20 free throws in the gym. Jamin makes 14 of the 20 free throw shots. Based on this sample of shots, what conclusion should Aaron draw about Jamin's claim?

By this point you should be able to recognise when a question is asking you to perform a hypothesis test. You begin this test by stating your null and alternative hypotheses and determining your level of significance.

- $H_0 : \mu = 0.85$ (on average, Jamin makes 85% of his free throws)
- $H_1 : \mu < 0.85$ (on average, Jamin makes less than 85% of his free throws)
- The test will be carried out using a 5% significance level.

Exam tip

From your work in section 4.18.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/testing-for-the-population-mean-id-27997/) you should recognise this as a one-tailed test. In exams, you will only be expected to carry out one-tailed binomial tests.

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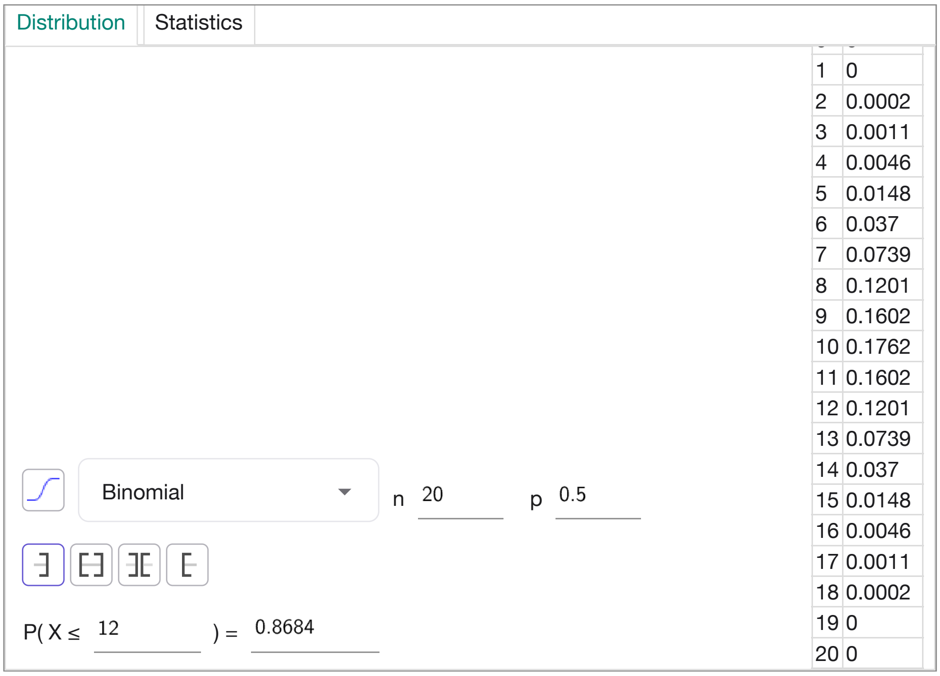
In this situation, there are two outcomes: either Jamin succeeds in the free throw or he misses it. Therefore, you can model the possible outcomes of this situation using the binomial distribution $X \sim B(20, 0.85)$. What assumption have you made in applying the binomial distribution model to Jamin's free throw shooting?

⚠ Be aware

Remember that successive events must be independent of each other for the binomial distribution to be a valid model.

You may remember the applet below from section 4.8.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/calculating-bionomial-probabilities-id-26116/) on binomial probabilities. Enter the appropriate information to find the probability that Jamin succeeds in 14 or fewer free throws, assuming that he usually succeeds in 85% of his free throw shots.



Credit: GeoGebra (https://www.geogebra.org/m/mx4qqb7s) Vivax Solutions

By setting $n = 20$ and $p = 0.85$ you can see that $\mathrm{P}(X \leqslant 14) = 0.0673$. This tells you that if, on average, Jamin succeeds in 85% of his free throw shots then there is a 6.73% chance of him making 14 or fewer shots in a sample of 20 shots. As this is greater than the level of significance chosen for the test, you fail to reject H_0 using this sample of 20 shots.

✓ Important

The binomial distribution can be used to test the proportion of a population. If the p-value falls within the critical region then the test suggests that the population's proportion is different from that originally stated.

Example 1

★☆☆

Student view

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A drug company claims that their medicine reduces back pain in 76% of patients within one hour of taking the medicine. To test this claim, 20 randomly selected patients are asked about their results. Twelve of these patients reported a reduction of back pain within one hour of taking the medicine. By choosing an appropriate alternative hypothesis, determine whether there is sufficient evidence at a 10% significance level to reject the drug company's claim.

Steps	Explanation
$H_0 : p = 0.76$ $H_1 : p < 0.76$	Begin by choosing the appropriate alternative hypothesis.
$\mathrm{P}(X \leqslant 12) = 0.0834918\ldots$ ≈ 0.0835	Assuming the distribution $X \sim B(20, 0.76)$, use the binomial CDF function on your graphic display calculator.
Since $0.0835 < 0.10$ you reject the null hypothesis and accept the alternative hypothesis that the actual percentage of patients that have a reduction in back pain within one hour of taking the medicine is less than 76%.	Draw your conclusion.

✓ **Important**

The critical value of a binomial distribution test is the first value to satisfy the condition that the probability of achieving that value or a more extreme value is less than the significance level.

Example 2

★★★

A light bulb manufacturer states that only 2% of its light bulbs are defective. However, one of its machines was recently repaired and the manufacturer wants to check whether this has increased the percentage of defective bulbs. A sample of 75 bulbs from the machine is selected at random to test the hypothesis.

Find the critical region for the test at a 5% significance level.

Steps	Explanation
$H_0 : p = 0.02$ $H_1 : p > 0.02$	Begin by choosing the appropriate alternative hypothesis.

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(/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Consider the case of having four defective bulbs in the sample: $$\begin{aligned}\mathrm{P}(X \geqslant 4) &= 1-\mathrm{P}(X \leqslant 3)\\ &= 1-0.936266\ldots\\ &= 0.0637339\ldots\\ &= 0.0637 \text{ (3 significant figures)}\end{aligned}$$ Consider the case of having five defective bulbs in the sample: $$\begin{aligned}\mathrm{P}(X \geqslant 5) &= 1-\mathrm{P}(X \leqslant 4)\\ &= 1-0.982601\ldots\\ &= 0.0173989\ldots\\ &= 0.0173 \text{ (3 significant figures)}\end{aligned}$$ Note that this method relies on a little trial and error to find the number of bulbs at which the probability falls below 0.05. There is no specific reason to begin by considering four defective bulbs.	Assuming the distribution $X \sim B(75, 0.02)$, use the binomial CDF function on your graphic display calculator.
You are carrying out the test at a 5% significance level, therefore you are looking for the first instance in which the probability of an outcome falls below 0.05. This happens when there are five defective bulbs. Therefore, the critical value is $X = 5$ and the critical region is $X \geqslant 5$. This means that if the sample of 75 bulbs contains five or more defective bulbs then the null hypothesis is rejected, and the manufacturer should conclude that the proportion of defective bulbs manufactured by the machine is now greater than 2% .	Determine the critical value and critical region.

Be aware

As the binomial distribution measures the counts of an outcome, the critical value and critical region will always be composed of integer values.

Testing mean using the Poisson distribution

Making connections

Recall from section 4.17.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-poisson-distribution-id-27993/) that the Poisson distribution is related to the binomial distribution but applies to situations in which there are an infinite number of possible outcomes.

Student view

People waiting for a subway train to arrive

Credit: choochart choochaikupt Getty Images

Example 3

★★☆

On weekday afternoons a northbound train passes a subway station in Bangkok every 10 minutes. The subway authority has decided to keep the trains at this rate unless the number of people waiting for a train becomes greater than 50. On a certain afternoon, a subway employee counts the number of people waiting for each train. Over the course of one hour, the employee counts a total of 324 people boarding the trains. The employee then carries out a hypothesis test on the data to determine whether the number of trains should be increased.

By choosing an appropriate alternative hypothesis, determine whether there is sufficient evidence at a 5% significance level to reject the null hypothesis $H_0 : \mu = 50$.

Steps	Explanation
$H_0 : \mu = 50$ $H_1 : \mu > 50$	Begin by choosing the appropriate alternative hypothesis.
As a train passes the station every 10 minutes, six trains would pass in one hour. Since the assumed mean number of passengers waiting for one train is 50, the mean number waiting for six trains is 300. Therefore, the distribution to use is $X \sim Po\,(300)$.	Set up the Poisson distribution to be used.
$\begin{aligned} \mathrm{P}\,(X \geqslant 324) &= 1 - \mathrm{P}\,(X \leqslant 323) \\ &= 1 - 0.911350\ldots \\ &= 0.0886494\ldots \\ &= 0.0886 \text{ (3 significant figures)} \end{aligned}$	Use the Poisson CDF function of your calculator to find the probability that 324 or more passengers will board a train in 1 hour.
Since $0.0886 > 0.05$ there is not sufficient evidence to reject the null hypothesis.	Draw your conclusion using the 5% significance level.

Student view

Exam tip

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Exam questions will often include the scaling of the Poisson mean.

Example 4

★★☆

Find the critical value for the hypothesis test carried out in **Example 3**.

Steps	Explanation
$H_0 : \mu = 50$ $H_1 : \mu > 50$	The hypotheses from **Example 3** will be used.
By creating a list of probabilities in your graphic display calculator, you find the following: $\mathrm{P}(X \leqslant 327) = 0.942235$ $\mathrm{P}(X \leqslant 328) = 0.948433$ $\mathrm{P}(X \leqslant 329) = 0.954085$ Choosing the last one as it is the first with a value of less than 0.05: $\begin{aligned}\mathrm{P}(X \geqslant 330) &= 1 - \mathrm{P}(X \leqslant 329)\\ &= 1 - 0.954085\ldots\\ &= 0.045915\ldots\\ &= 0.0459 \text{ (3 significant figures)}\end{aligned}$	With the distribution $X \sim Po\,(300)$, use the Poisson CDF function on your graphic display calculator.
Therefore the critical value for this hypothesis test is 330 people.	Determine the critical value and critical region.

Be aware

When using the method shown in the solution for **Example 4** it is easy to incorrectly give 329 as the critical value. It is important to remember the last step to obtain the correct answer for the critical value.

3 section questions

4. Probability and statistics / 4.18 Population tests

Testing population correlation

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Testing whether $\rho = 0$

Be aware

The discussion within this section assumes that the two variables being investigated produce a bivariate normal population.

Interactive 1. Testing Population Correlation.

More information for interactive 1

This interactive tool helps users explore how sample size affects the reliability of correlation estimates and regression lines.

The screen is divided into two halves. On the right side of the screen, a graph is displayed with an XY axis, with both axes ranging from 0 to 4. The graph shows a cloud of purple dots and some larger green circles. These represent the individual data points in the current sample (n) drawn from the population. The green circles highlight specific data points. A red line represents the sample regression line, which shows the trend observed in the current sample. This line represents the population regression line, which shows the overall trend in the entire population.

On the left side of the screen correlation is mentioned , and the equation of regression y in blue. Below that is a slider of sample size (n) which ranges from 10 to 200. The "New Sample" button below the slider generates a new set of sample data, displaying green points at random positions on the graph, changing the equation of the sample regression. A "New data" on the top left corner allows users to generate a new underlying population dataset.

Users can generate a population dataset of 500 points with varying correlation strengths (ρ), then draw samples of different sizes (from 10 to 200 points) to compare against the true population values. The visualization shows the population regression line (representing the actual relationship) alongside sample regression lines (calculated from randomly drawn subsets shown as green points). Through experimentation, users discover that smaller samples (like $n = 10$) produce correlation coefficients and regression equations that vary widely from the population values, while larger samples $(n = 200)$ yield more stable, accurate estimates. The tool allows comparison across different population correlation scenarios (strong $\rho > 0.8$ vs weak $\rho < 0.2$), demonstrating how sampling variability depends on both sample size and underlying correlation strength. By repeatedly generating new samples, users gain intuitive understanding of key statistical concepts: the precision of correlation coefficients improves with larger samples, regression lines become more consistent, and small samples can sometimes give misleading results. This hands-on experience effectively illustrates why researchers must consider sample size when interpreting correlations and regression analyses in real-world studies.

For example, if a user selects a population with a strong negative correlation $(\rho = -0.86)$ and generates a small sample of $n = 30$ points, they might obtain a sample regression equation of $y = -0.93x + 3.88$ with a correlation of $\rho = -0.53$ — noticeably weaker than the true relationship. However,

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when they increase the sample size to n=150 using the same population, the sample regression line ($y = -0.89x + 3.75$) and correlation ($\rho = -0.82$) become much closer to the population parameters ($y = -0.86x + 3.7$, $\rho = -0.86$). This demonstrates how small samples can produce unstable estimates that fluctuate around the true values, while larger samples yield more reliable results that better reflect the underlying population relationship, particularly for stronger correlations.

Activity

The applet above generates a population of 500 data points. It also generates a sample with a size determined by you. You can use the buttons in the applet to generate new population data and new samples.

1. Click the button to create new population data until the applet has generated a population with a correlation greater than 0.8.
2. Set the sample size to 10.
3. Click the button to generate new samples and take note of how the correlations of the samples vary from the correlation of the population.
4. Set the sample size to 200.
5. Again, generate new samples and take note of how the correlations of the samples vary from the correlation of the population. Was the variation different from what you saw in Step 3?
6. Repeat Steps 1—5 with a population data that has a correlation smaller than 0.2.
7. Explore what happens with other population correlations.

Now consider these questions.

- What does this tell you about the reliability of correlation coefficients calculated from smaller sample sizes?
- Imagine that you did not have the population data or the population correlation. Would the sample correlation always be a good prediction of the population correlation?

A corner store

Credit: Image Source Getty Images

Yourui works at a corner store and wants to know whether the weather affects the number of soda drinks sold at the store. He randomly selects a sample of 10 days. For each day he writes down the highest temperature for the day and the number of sodas sold on the day. His record is shown in the table below.

Student view

Temperature (°C)	32	28	34	31	34	28	30	25	35	27

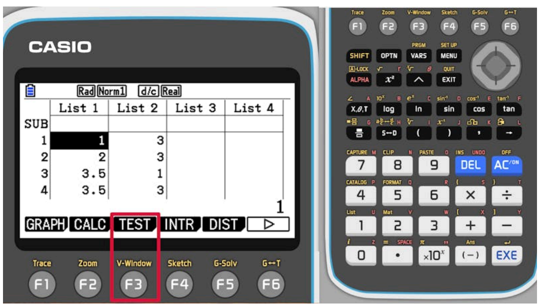
Number of sodas sold	59	60	82	64	71	65	64	45	77	60

Yourui calculates the product moment correlation coefficient for the above data and finds that $r \approx 0.848$. While this number indicates a positive correlation for the sample data between the high temperature for the day and the number of sodas sold, Yourui wants to further test whether this relationship holds true for the population of data. To do this, he decides to carry out a hypothesis test.

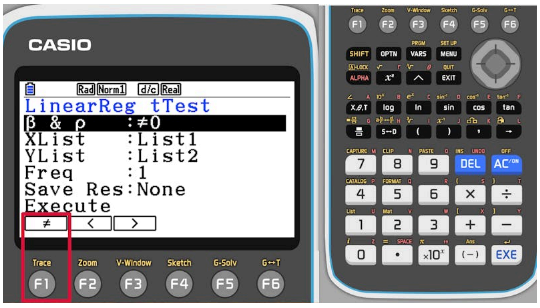
Important

The null hypothesis of a population product moment correlation coefficient test is always $\rho = 0$, where ρ represents the correlation coefficient of the population data. If $\rho = 0$, then there is no correlation between the two variables at the population level. The alternative hypothesis is either $\rho \neq 0$ (when testing whether there is any correlation), $\rho > 0$ (when testing for a positive correlation), and $\rho < 0$ (when testing for a negative correlation).

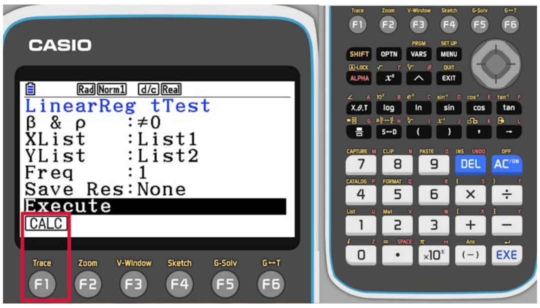
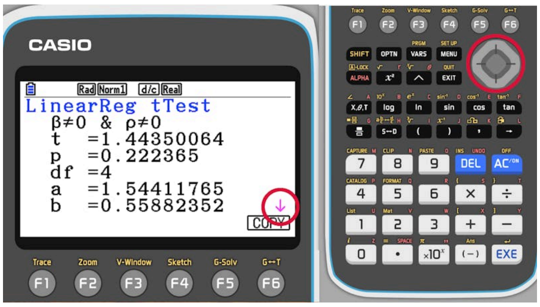
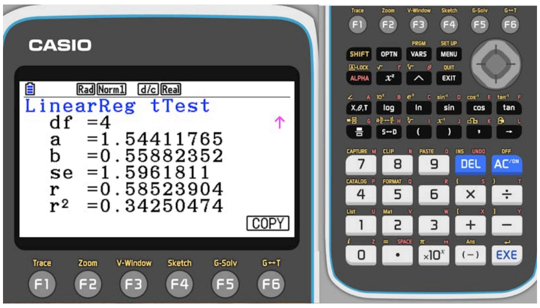
In the following instructions you can find help on how to run a linear regression t-test on your calculator.

Steps	**Explanation**
In these instructions you will see how to run a linear regression t-test. To start, open the statistics mode.	
Enter the data. Make sure you remember the lists where you stored your data. Note, that on this screenshot you only see the first four entries of the lists. Once you are done, press F3 to bring up the test options, ...	

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
... then press F2 to choose a t-test, ...	
... and finally F3 to choose the test for regression.	
You need to set some parameters for the test. In the first line you select whether you want to run a one-sided or two-sided test.	

Student view

Steps	Explanation
Make sure, that the list names are set to the lists where the data is stored and that the frequencies are set to 1. Once you checked everything, scroll down to the last line and press F1 to run the test.	
The results are displayed on two screens, you need to scroll down to see the rest. The p-value, which you need to draw conclusion, is on the first screen.	
	

Overview
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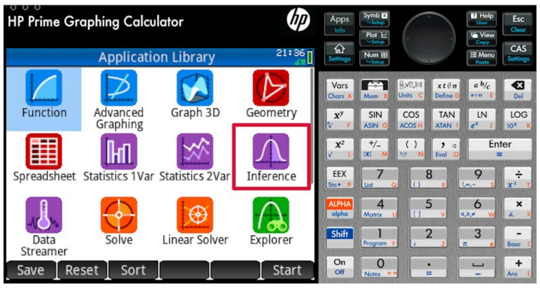
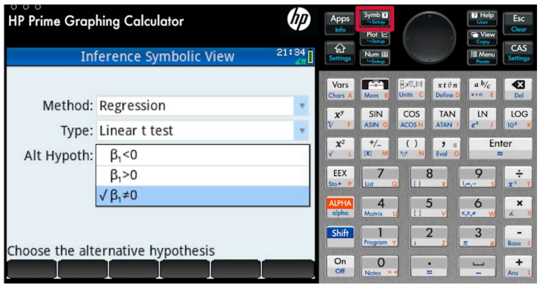
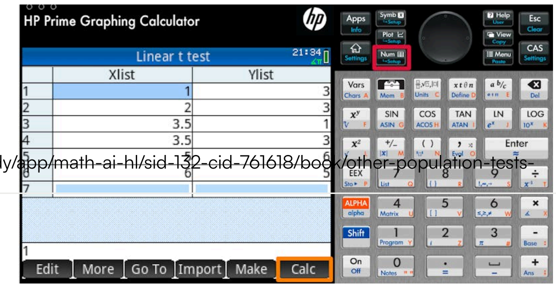
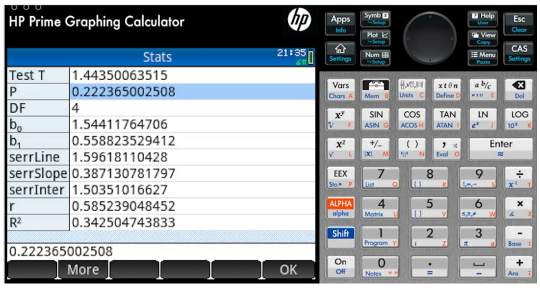
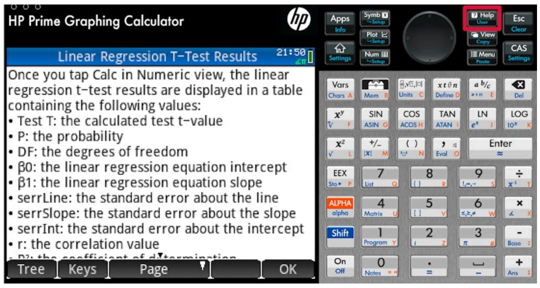
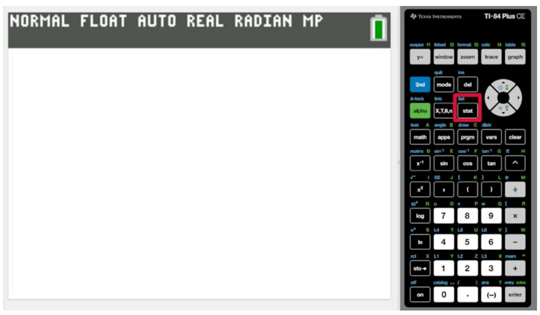
132-
cid-
761618/ov

Steps	Explanation
In these instructions you will see how to run a linear regression t-test. To start, open the inference application.	
In symbolic view, choose the regression method and the linear t-test type. In the third line you select whether you want to run a one-sided or two-sided test.	
Change to numeric view and enter the data. Once done, tap on calc to run the test.	

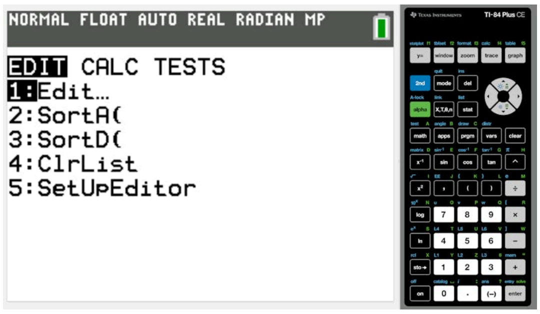
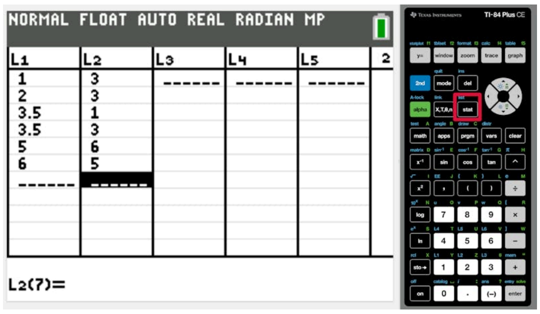
Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/other-population-tests-id-27999/print/) Assign

Student view

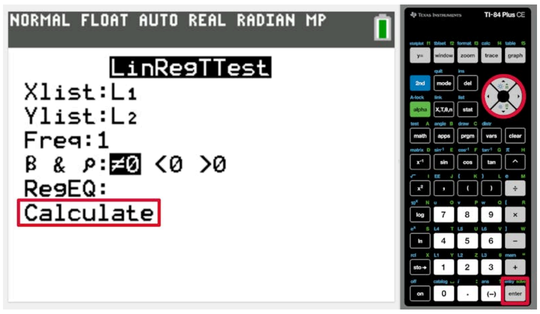
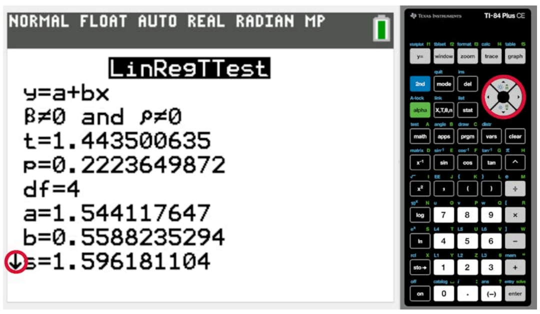
Steps	Explanation
This is the information you get as the result of the test. You will need the p-value to draw conclusion.	
Pressing the help button will bring up some explanation about the result.	

Steps	Explanation
In these instructions you will see how to run a linear regression t-test. To start, open the statistics mode ...	

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
... and choose to edit the lists.	
Enter the data. Make sure you remember the lists where you stored your data. Once you are done, bring up again the statistics options, ...	
... move to the right to see the test options and scroll down ...	

Student view

Steps	Explanation
... until you find the linear regression t-test (LinRegTTest).	
You need to set some parameters for the test. Make sure, that the list names are set to the lists where the data is stored and that the frequencies are set to 1. In the next line you select whether you want to run a one-sided or two-sided test. Once you checked everything, scroll down to the last line and press enter to run the test.	
The results are displayed on two screens, you need to scroll down to see the rest. The p-value, which you need to draw conclusion, is on the first screen.	

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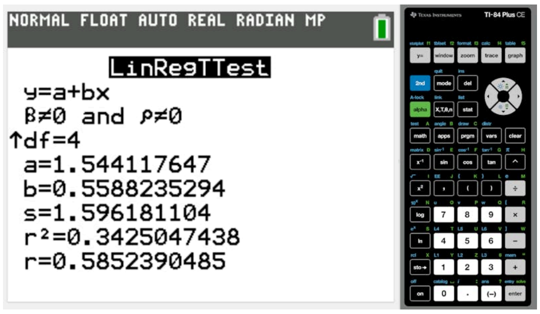
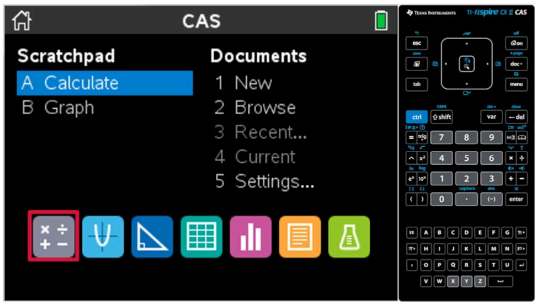
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td></td><td>
</td></tr>
</table>

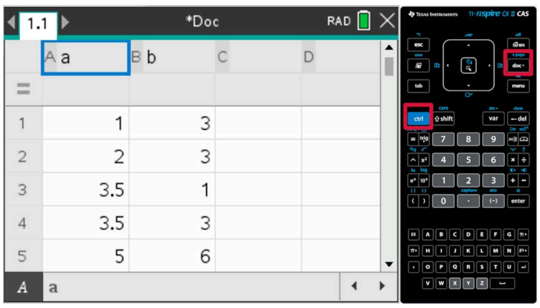
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>In these instructions you will see how to run a linear regression t-test.

To start, open a spreadsheet page.</td><td>
</td></tr>
</table>

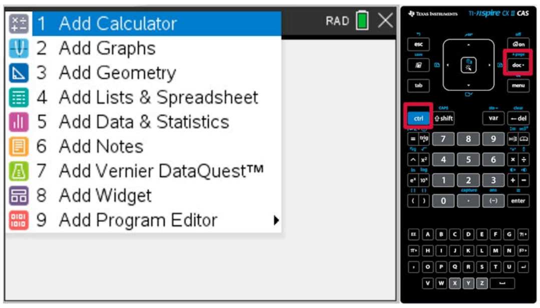
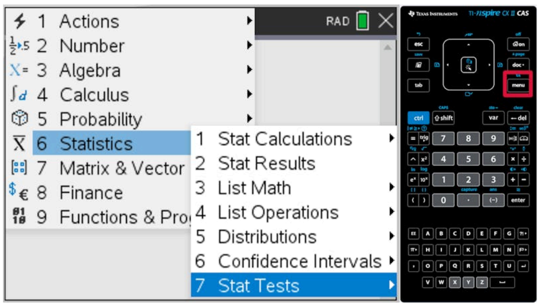
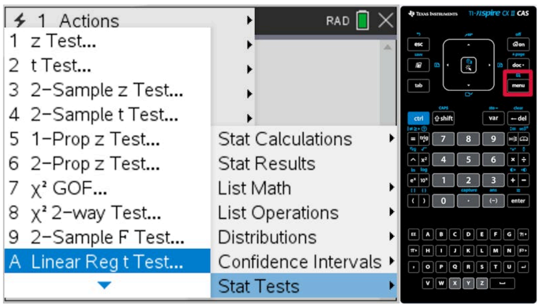
Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/testing-for-the-population-mean-id-27997/print/) Assign

<table>
<tr><td>Enter the data. You can also give a name to your lists.

Note, that on this screenshot you only see the first five entries of the lists.

Once you are done, open the option to add a new page ...</td><td>
</td></tr>
</table>

Student
view

Steps	Explanation
... and add a calculator page to your document.	
On the calculator page open the menu and look for the statistical tests ...	
... and choose the linear regression t-test.	

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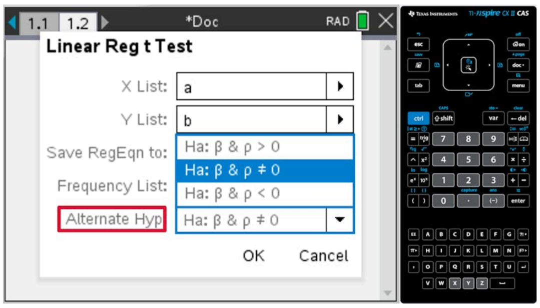
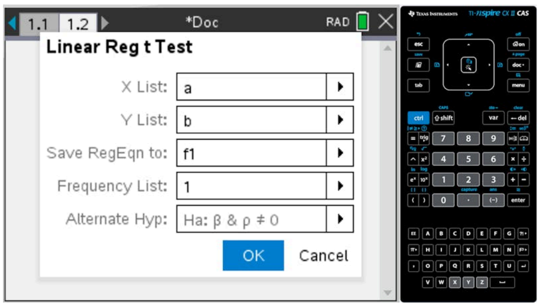
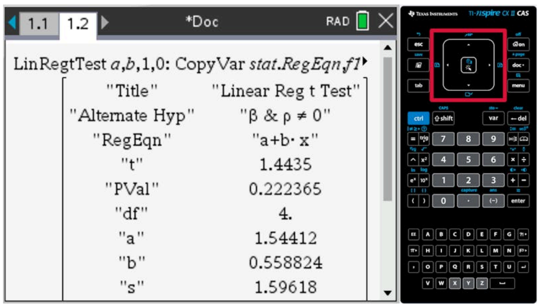
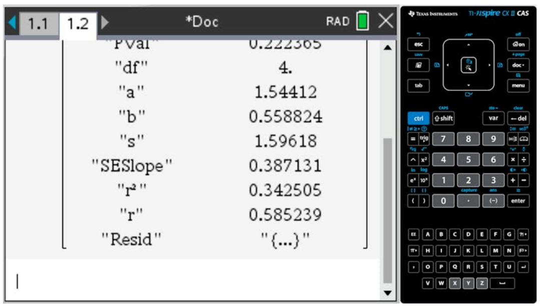
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>You need to set some parameters for the test.

In the last line you select whether you want to run a one-sided or two-sided test.</td><td>
</td></tr>
<tr><td>Make sure, that the list names are set to the lists where the data is stored and that the frequencies are set to 1.

Once you checked everything, scroll down to OK and press enter.</td><td>
</td></tr>
<tr><td>The results are displayed on two screens, you need to scroll down to see the rest.

The p-value, which you need to draw conclusion, is on the first screen.</td><td>
</td></tr>
</table>

Student
view

Steps	Explanation
	

Example 1

★★☆

Yourui has calculated a product moment correlation coefficient of $r \approx 0.848$ for his data.

Temperature (°C)	32	28	34	31	34	28	30	25	35	27
Number of sodas sold	59	60	82	64	71	65	64	45	77	60

Determine whether there is sufficient evidence at a 5% significance level to reject the null hypothesis $\rho = 0$ for Yourui's data.

Steps	Explanation
$H_0 : \rho = 0$ $H_1 : \rho \neq 0$ $p = 0.00195705\ldots$ $= 0.00196$ (3 significant figures)	Begin by choosing the appropriate alternative hypothesis.
Since $0.00196 < 0.05$ there is sufficient evidence to reject the null hypothesis that there is no correlation between the two variables within the population data.	Draw your conclusion.

Important

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(/study/app/math-ai-hl/sid-132-cid-761618/ov

When there is sufficient evidence to reject the null hypothesis that there is no correlation between the two variables, then it is appropriate to calculate the least-squares regression line for the data.

Example 2

★★☆

In an experiment, Matt and Evelyn are simultaneously measuring their reaction times. They both respond to the same input as fast as they can and a computer records their response times. The table below summarises the result of these trials. The times are given in milliseconds.

Matt (x)	224	220	242	219	204	239	243	201
Evelyn (y)	229	219	244	242	222	250	239	201

Test, at a 5% significance level, whether there is a positive correlation between their two sets of data and, if appropriate, state the least-squares regression line for the data.

Steps	Explanation
$H_0 : \rho = 0$ $H_1 : \rho > 0$ (since you are testing for a positive correlation.)	Begin by choosing the appropriate alternative hypothesis.
$p = 0.00594727\ldots$ $= 0.00595$ (3 significant figures)	Use your graphic display calculator to carry out a linear regression t-test.
Since $0.00595 < 0.05$ there is sufficient evidence to reject the null hypothesis that there is no correlation between the two variables within the population data.	Draw your conclusion.
$y = a + bx$ $a = 47.9755\ldots \approx 48.0$ $b = 0.815957\ldots \approx 0.816$ $\therefore y = 48.0 + 0.816x$	Since the result of the test was significant, it is appropriate to write down the least-squares regression line for the data.

⚠ Be aware

Double-check the form in which your calculator gives the least-squares regression line. Don't just assume that a is the slope and b is the y -intercept.

3 section questions ⌄

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4. Probability and statistics / 4.18 Population tests

Type I and II errors

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/type-i-and-ii-errors-id-28001/print/) Assign

Possible errors in hypothesis tests

In the applet below, you can adjust the null and alternative hypotheses, the population mean, the sample size and the significance level.

- The blue curve represents the values when the null hypothesis is correct and the area of the blue region below the blue curve is the significance level α .
- This significance level is used to determine the critical value separating the rejection region (illustrated by the orange part of the horizontal axis above the critical value) from the non-rejection region (illustrated by the pale blue part of the horizontal axis below the critical value).
- The red curve represents the values when the alternative hypothesis is correct.

Interactive 1. Possible Errors in Hypothesis Tests.

More information for interactive 1

This interactive helps users visualize and experiment with key concepts in hypothesis testing through dynamic adjustments.

The screen is divided into two halves, the bottom half has a x axis ranging from 80 to 130. Two bell-shaped curves are projected. The blue curve represents the distribution of the sample mean under the null hypothesis (H_0) and the red curve represents the distribution of the sample mean under the

Student view

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alternative hypothesis (H_1). On the top half of the screen, there are several horizontal sliders for the following parameters: the null hypothesis mean ($H_0: \mu$) ranging from 50 to 150, alternative hypothesis mean ($H_1: \mu$), ranging $[(H_0: \mu) + 5, (H_0: \mu) + 10]$, population standard deviation (σ) ranging from 15 to 25, sample size (n) ranging from 5 to 50, and significance level (α) ranging from 0.01 to 0.1.
Users can modify these parameters, with all changes instantly reflected in the visualization. The blue curve displays sampling distribution under H_0, with its shaded blue area showing the significance level (α), while the red curve displays the distribution under H_1. The x-axis is divided into an orange rejection region and pale blue non-rejection region, determined by the critical value.
As users adjust parameters, they can observe how these changes affect the overlap between distributions, the position of the critical value, and the sizes of the rejection regions. The tool demonstrates important statistical relationships - how increasing sample size tightens distributions, how changing α alters the critical threshold, and how the distance between H_0 and H_1 means impacts test power and error probabilities. This real-time visual feedback helps build intuition about the trade-offs between Type I and Type II errors, power analysis, and the sensitivity of hypothesis tests to different parameter choices, making abstract statistical concepts more concrete and understandable.
For example, if a user sets $H_0 = 88.6$ and $H_1 = 93$ with $\sigma = 20$ and $n = 20$, the tool shows two overlapping curves. The blue curve (H_0) marks a rejection threshold at 92.5 for $\alpha = 0.07$. The red curve (H_1) shows that many values still fall below this threshold, meaning the test might miss true effects. If the user increases n to 40, the curves narrow, the threshold drops to 90.8, and fewer H_1 values fall below it, making the test better at detecting differences while keeping the same error rate. This shows how sample size affects hypothesis testing accuracy.

Activity

Consider the following questions while using the applet.

1. You learned in section 4.18.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/testing-for-the-population-mean-id-27997/) that the critical value determined the boundary between rejecting the null hypothesis and not rejecting the null hypothesis. With this in mind, explain the possible error that occurs when the test value falls within the blue shaded region seen in the applet.
2. The red shaded region represents a possible crossover of the two hypotheses below the critical value. Describe the error that could occur in this region.

As you have seen throughout this subtopic, all of the hypothesis tests are based on the probability of certain events occurring. If the probability (p -value) is small enough to be below a predetermined significance level, you make the decision to reject the null hypothesis. However, the inherent risk in that method is that it ignores the fact that sometimes extreme values occur naturally.

For example, consider the blue shaded area under the curve seen in the applet. The values represented in that region are true for the null hypothesis but are also the values for which you reject the null hypothesis since the probability of them occurring is small. This is a type I error.

Important

A type I error is made if the null hypothesis is rejected when it is true.

For tests on normally distributed data, the probability of a type I error being made is based on the critical region and is the same as the significance level. However, this is not the case for discrete random variables. Let us have another look at **Example 2** from section 4.18.3 (/study/app/math-ai-hl/sid-132-cid-761618/book/other-population-tests-id-27999/).

Student view

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Example 1

★★☆

A light bulb manufacturer states that only 2% of its light bulbs are defective. However, one of its machines was recently repaired and the manufacturer wants to check whether this has increased the percentage of defective bulbs. A sample of 75 bulbs from the machine is selected at random to test the hypothesis.

Find the probability of making a type I error when a hypothesis test is carried out at a 5% significance level.

Steps	Explanation
$H_0: p = 0.02$ $H_1: p > 0.02$	First, remember the null and alternative hypothesis used for this test.
Consider the case of having 5 defective bulbs in the sample: $\begin{aligned} \mathrm{P}(X \geqslant 5) &= 1 - \mathrm{P}(X \leqslant 4) \\ &= 1 - 0.982601\ldots \\ &= 0.0173989\ldots \\ &= 0.0173 \text{ (3 significant figures)} \end{aligned}$ $\therefore$ the probability of making a type I error is 1.73%, which is not the same as the significance level.	Using the distribution $X \sim B(75, 0.02)$ you previously found the critical value to be 5, as it was the first occurrence of the cumulative probability being less than 0.05.

✓ **Important**

For discrete random variables, the critical region is chosen to represent the maximum probability of a type I error occurring, but the probability will always be less than the significance level.

Now consider the red shaded area in the applet. This area is to the left of the critical value, which means it falls in the region where you would not reject the null hypothesis. However, as you can see from the applet, it is also a part of the region where the alternative hypothesis is correct. This is a type II error.

✓ **Important**

A type II error is made if the null hypothesis is not rejected when the alternative hypothesis is true (and hence the null hypothesis is not true).

Let us have another look at **Example 3** from section 4.18.3 (/study/app/math-ai-hl/sid-132-cid-761618/book/other-population-tests-id-27999/).

Example 2

★★☆

Student view

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On weekday afternoons a northbound train passes a subway station in Bangkok every 10 minutes. The subway authority has decided to keep the trains at this rate unless the number of people waiting for a train becomes on average greater than 50 between two trains. On a certain afternoon, a subway employee has been asked to count the number of people waiting for a train over the course of one hour. The total number of people will then be used to carry out a hypothesis test at a 5% significance level. The subway authority then carries out a hypothesis test on the data to determine whether the number of trains should be increased.

By considering the critical value, calculate the probability of making a type II error given that the actual mean number of people waiting over the course of one hour is 335.

Steps	Explanation
$H_0 : \mu = 300$ $H_1 : \mu > 300$	Remember the null and alternative hypotheses. Since the employee counts the waiting people over an hour, if the average number of people waiting is not greater than $6 \times 50 = 300$, the rate does not have to be changed.
$\mathrm{P}(X \geqslant 329) = 1 - \mathrm{P}(X \leqslant 328) = 1 - 0.948433\ldots = 0.051567\ldots = 0.0516$ (3 significant figures) $\mathrm{P}(X \geqslant 330) = 1 - \mathrm{P}(X \leqslant 329) = 1 - 0.954085\ldots = 0.045914\ldots = 0.0459$ (3 significant figures) This means that the null hypothesis will not be rejected for values less than or equal to 329, but it will be rejected if the observed number of people waiting is 330 or more.	Use the Poisson CDF function of your calculator to find the critical value for the distribution $X \sim \mathrm{Po}(300)$.
For the distribution $X \sim \mathrm{Po}(335)$, use your graphic display calculator to find that $\mathrm{P}(X \leqslant 329) = 0.385070\ldots \approx 0.385$. Therefore, there is a 38.5% chance of making a type II error with this hypothesis test. In other words, if the mean number of people waiting over the course of one hour is 335, then there is a 38.5% chance that the sample will not give the subway authority reason to increase the number of trains.	To calculate the probability of making a type II error, you find the probability (using the actual mean and distribution) that the claim was not rejected.

Making connections

In section 4.18.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/testing-for-the-population-mean-id-27997/) you learned that it is inappropriate to conclude that you accept the null hypothesis. How does the concept of type II errors help explain the reasoning behind this?

3 section questions

Student view

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4. Probability and statistics / 4.18 Population tests

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-28002/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- carry out a hypothesis test for the mean of a normally distributed random variable
- calculate critical values and critical regions for random variables that follow a normal distribution, a binomial distribution, or a Poisson distribution
- carry out a hypothesis test for population proportion using the binomial distribution
- carry out a hypothesis test for the mean of a random variable that follows a Poisson distribution
- carry out a product moment correlation coefficient test to determine whether two bivariate normally distributed variables are correlated with each other
- use the product moment correlation coefficient test to determine whether producing a least-squares regression equation is appropriate
- describe type I and type II errors in hypothesis tests
- calculate the probability of a type I or a type II error occurring for any one-tailed hypothesis test.

4. Probability and statistics / 4.18 Population tests

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-28003/print/) Assign

Testing the reliability of packaging promises

Credit: Image Source Getty Images

Student view

Activity

1. Go to a grocery store and choose an inexpensive product that has a label showing the weight of the product. Purchase five packages of the product. Alternatively, you could seek a packaged item stored in multiples in your home store cupboard. Carry out a hypothesis test at a 5% significance level to determine whether the mean weight of the packages is accurately labelled.
2. Choose a different brand of the same product from Step 1. Carry out another hypothesis test at a 5% significance level to determine whether the mean weight of these items is accurately labelled on the packages.
3. Consider the results of the two tests you have carried out. What conclusions can you draw about the packaging of the two companies you considered?
4. Suggest some ways that you could improve the reliability of your data and the validity of your tests.

Rate subtopic 4.18 Population tests

Help us improve the content and user experience.