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2. Functions / 2.5 Composite and inverse functions

The big picture

A weather balloon carries instruments up in the air to send back information on atmospheric pressure, temperature, humidity and other variables. The data is collected by small, expendable measuring devices. For example, a device called a radiosonde can measure variables such as wind speed and wind direction at high altitudes of various geographical locations.

Temperature gauge

Credit: JADEZMITH Getty Images

Data informs us that the temperature T in the atmosphere is a function of the altitude, or the height h of a weather balloon above the ground. The height h of the weather balloon is a function of the time t since the balloon was launched. A change in time will result a change

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in height, which in turn will produce a change in the air temperature. There are many real-world phenomena in which one quantity is a function of second quantity, which is in turn a function of a third quantity.

In this subtopic you will learn about the concepts of:

- composite functions
- inverse function

Concept

Functions are often used to describe relationships between quantities. There are also **relationships between functions** that can help us to model and understand more complex phenomena. A composite function is the result of combining two functions so that the output of the first function becomes the input of the second function. As you learn algebraic methods of finding composite functions in this subtopic, think about real-life applications in which composite functions can be used as **mathematical models.** An inverse function 'undoes' the effect of a given function. As you learn how to find the formula of an inverse function, think about ways in which you might verify that two functions are inverses of each other.

2. Functions / 2.5 Composite and inverse functions

Composite functions

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Combining functions

The composite function

Consider the function $f(x) = x^2$. It squares any input x, so

$$f(3) = 3^2 \quad \text{and} \quad f(x-1) = (x-1)^2.$$

Student view

In the second case, the input for f is another function of x. If we let $g(x) = x - 1$, the second result can be written as

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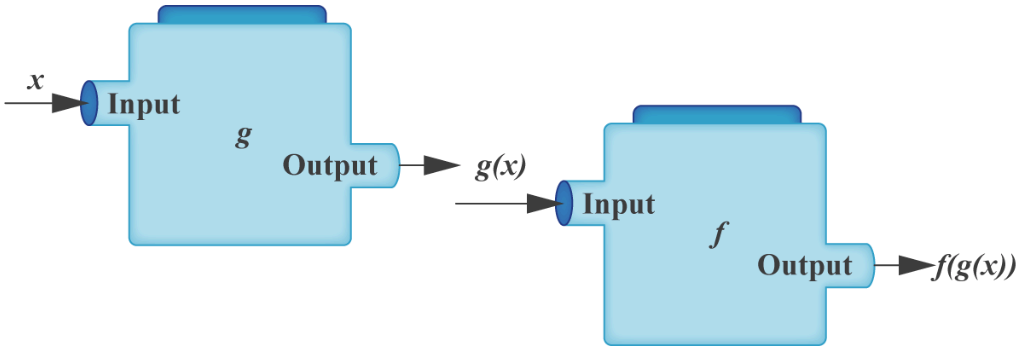
$$f(g(x)) = f(x-1) = (x-1)^2.$$

So we have combined two functions

$$f(x) = x^2 \quad \text{and} \quad g(x) = x - 1$$

to obtain the composite function $(f \circ g)(x) = f(g(x))$. This process of turning the output of one function into the input for another function is called composition . Notice that the function g is applied first and the function f is applied second. The concept of composite function is illustrated as a function machine in the below figure.



More information

The image depicts a diagram illustrating the concept of function composition. It features two separate 'function machines.' On the left side, the first machine is labeled 'g.' It has an input labeled 'x' which enters the machine, and an output labeled 'g(x).' This output then becomes the input for the second machine on the right. The second machine is labeled 'f' and takes 'g(x)' as its input, producing an output labeled 'f(g(x)).' This flow demonstrates the process of applying function g first and then function f, ultimately resulting in the composite function (f ∘ g)(x).

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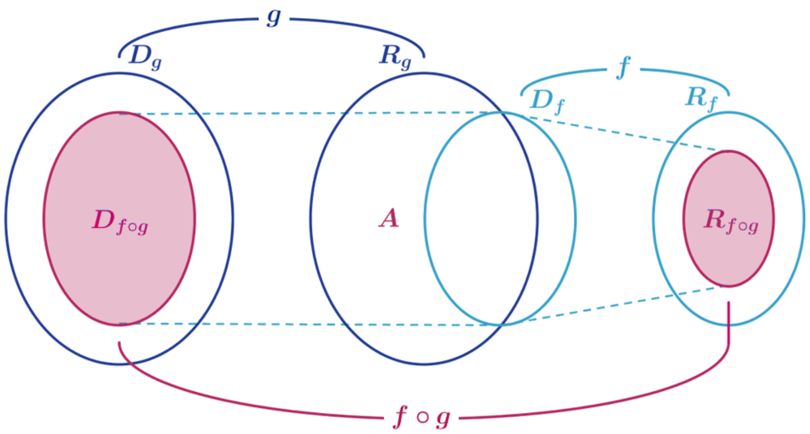
Important

A **composite function** of two functions f and g such that function g is applied first and function f is applied second is written as $f \circ g$ and maps x to $f(g(x))$. So

$$(f \circ g)(x) = f(g(x)).$$

The domain of the function $f \circ g$ is the set of all x values in the domain of g such that the corresponding $g(x)$ values belong to the domain of f.

Figure 2 illustrates the mapping for the composite function $f \circ g$. The domains and ranges of the functions are denoted by D_{function} and R_{function}, respectively. The domain $D_{f\circ g}$ of the composite function $f \circ g$ is made up of all the x values in D_g whose images $g(x)$ belong to the domain of D_f. These images are represented by the area A in the diagram. They are then mapped by the function f into its range R_f, producing the range $R_{f\circ g}$ for $f \circ g$, which is a subset of R_f.



More information

This diagram illustrates the mapping for the composite function (f \circ g). It consists of three main sections represented by overlapping ovals, showing the flow from domain to range in both functions (f) and (g).

1. On the left, the domain (D_g) of function (g) is shown with a central shaded area labeled (D_{f\circ g}). Arrows indicate mapping from (D_g) to the range (R_g).

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2. The middle section of the diagram highlights the area (A), representing the set of (x) values in (D_g) that map to (D_f) in the next function (f).

3. On the right, (D_f) maps to its range (R_f), from which the range (R_{f\circ g}) is a subset. This is visually represented with arrows indicating flow into the final range (R_{f\circ g}).

The labels and arrows illustrate how values flow from the domain of (g) through (f) to their composite range, demonstrating the hierarchical mapping process between the functions.

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You write	You say
$f \circ g$	f following g
$f(g(x))$	f of g of x

⚠ Be aware

It is important to apply the functions in the correct order:

$$(f \circ g)(x) \quad : x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$$

and

$$(g \circ f)(x) \quad : x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Example 1

★☆☆

Let $f(x) = x^2$ and $g(x) = x + 4$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. Are they the same?

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Steps	Explanation
$(f \circ g)(x) = f(g(x)) = f(x+4) = (x+4)^2$ $= x^2 + 8x + 16$	Remember the correct order (applying the two functions. For $f \circ g$ you apply g first and then feed the output of g into .
$(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2) + 4 = x^2 + 4$	For $g \circ f$ you apply f first anc then feed the output of f into .
$(f \circ g)(x) \neq (g \circ f)(x)$	This is generally the case for composite functions.

Be aware

In general, $(f \circ g)(x) \neq (g \circ f)(x)$.

Because $(f \circ g)(x) \neq (g \circ f)(x)$ in general, usually $D_{f \circ g} \neq D_{g \circ f}$. For the composite function on the left, the range of g will be the new domain of f, while for the composite function on the right it will be the opposite. **Example 3** below shows this difference.

Moreover, since in the composite function $f(g(x))$ the range of g is the new domain of f, this may affect the range of the final execution under the rule f. For example, the function $f(x) = 2x$ without any restrictions on x has range $-\infty < y < +\infty$. But if we consider $f(g(x))$ where $g(x) = \sqrt{x}$, then the range of $f(g(x)) = 2\sqrt{x}$ will be only $0 \leq y < +\infty$ because the range of $g(x) = \sqrt{x}$ is only $g(x) \geq 0$. You can see this kind of range restriction from the graph in **Example 2** (section 2.1.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/equivalent-forms-for-the-equation-of-a-line-id-24416/)).

Exam tip

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The solutions in the examples are presented in detailed steps. Although there is no need to give so much detail in an exam, you should make sure that the steps you take are clear in your mind and also easy to follow for the examiner reading your

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solution.

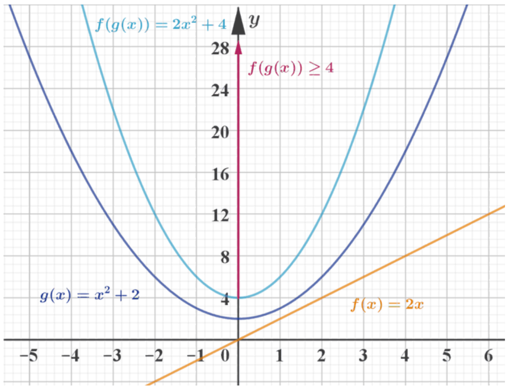
Example 2

★★☆

Consider the functions $f(x) = 2x$ and $g(x) = x^2 + 2$. Find the domain, the range and the formula of the composite function $f \circ g$.

Steps	Explanation
Let D_f, D_g and $D_{f\circ g}$ be the domains of f, g and $f \circ g$, respectively. Then $$D_f = \mathbb{R},\ D_g = \mathbb{R}$$ and $$\begin{aligned} D_{f\circ g} &= \{x \mid x \in D_g,\ g(x) \in D_f\} \\ &= \left\{x \mid x \in \mathbb{R}, x^2 + 2 \in \mathbb{R}\right\} \\ &= \mathbb{R} \end{aligned}$$	First write down the domains of the given functions. Recall the set notation you met earlier: $\{x \mid \ldots\}$ means 'the set of all x such th …'. $\mathbb{R}$ means the set of all real numbers; sc $x \in \mathbb{R}$ is the same as $-\infty < x < +\infty$
For the range of $f \circ g$: $$\begin{aligned} x &\in D_{f\circ g} \\ x &\in \mathbb{R} \\ x^2 &\geq 0 \\ x^2 + 2 &\geq 2 \\ g(x) &\geq 2 \\ 2g(x) &\geq 2 \times 2 \\ f(g(x)) &\geq 4 \\ (f \circ g)(x) &\geq 4 \end{aligned}$$ So the range of $f \circ g$ is $\{y \mid y \geq 4\}$.	To find the range of a composite function, it is helpful to go step by step. Take care with the order in which you apply the functions: For $f \circ g$, g is applied first and f is applied second..

Student view

Steps	Explanation
The formula for $f \circ g$ is $(f \circ g)(x) = f(g(x)) = 2\left(x^2+2\right) = 2x^2+4.$	The graphs of f, g and $f \circ g$, as well as the range of $f \circ g$, are shown in the following figure: 

Example 3

★★★

Consider the functions $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. Find the domain, range and formula of the functions $f \circ g$ and $g \circ f$.

Steps	Explanation
$D_f = \mathbb{R}$ and $D_g = \{x \mid x \in \mathbb{R},\ x \geq 0\}$. $D_{f\circ g} = \{x \,\vert\, x \in D_g, g(x) \in D_f\}$ $= \{x \,\vert\, x \geq 0, \sqrt{x} \in \mathbb{R}\}$ } $= \{x \mid x \in \mathbb{R}, x \geq 0\}$	Write down the domains of the given functions. Why does the range of g have to be restricted to $x \geq 0$?

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Steps	Explanation
For the range of $f \circ g$: $$\begin{aligned} x \in D_{f \circ g} x &\geq 0 \\ \sqrt{x} &\geq 0 \\ g(x) &\geq 0 \\ (g(x))^2 &\geq 0 \\ (g(x))^2 - 1 &\geq -1 \\ f(g(x)) &\geq -1 \\ (f \circ g)(x) &\geq -1. \end{aligned}$$ So the range of $f \circ g$ is $\{y \mid y \geq -1\}$.	As in **Example 2**, we write this out step by step. For $f \circ g$, g is applied first and f is applie second.
The formula for $f \circ g$ is $$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= \left(\sqrt{x}\right)^2 - 1 \\ &= x - 1 \ (as\ x \geq 0) \end{aligned}$$	
The domain of $g \circ f$ is $$\begin{aligned} D_{g \circ f} &= \{x \mid x \in D_f, f(x) \in D_g\} \\ &= \left\{x \mid x \in \mathbb{R},\ x^2 - 1 \geq 0\right\} \\ &= \{x \mid x \in \mathbb{R}, x \leq -1 \text{ or } x \geq 1\} \\ &= \ ]-\infty, -1] \cup [1, +\infty[\end{aligned}$$	Recall interval notation: $]a, b]$ means the interval $a < x \leq b$ and $[a, b[$ means the interval $a \leq x < b$.
For the range of $g \circ f$: $$\begin{gathered} x \in D_{g \circ f} \\ x \leq -1 \text{ or } x \geq 1 \\ x^2 \geq 1 \\ x^2 - 1 \geq 0 \\ f(x) \geq 0 \\ \sqrt{f(x)} \geq 0 \\ g(f(x)) \geq 0 \\ (g \circ f)(x) \geq 0 \end{gathered}$$ So the range of $g \circ f$ is $\{y \mid y \in \mathbb{R}, y \geq 0\}$.	
The formula for $g \circ f$ is $$(g \circ f)(x) = g(f(x)) = \sqrt{x^2 - 1}$$	

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Be aware

A composite function can be made up of more than two functions. The idea remains the same: the correct order of execution is from right to left. The function closest to the input variable x is the first function to be executed and the result of this is passed on to the next function to the left, and so on.

International Mindedness

Nicolas Bourbaki (the collective name of a group of mathematicians based mainly in France) first used the notation $f \circ g$ with the interpretation $(f \circ g)(x) = f(g(x))$ in 1949 (*Fonctions d'une variable réelle: Théorie élémentaire*). It is certainly conceivable that this notation for composite functions was invented by someone from the Bourbaki group, which was very concerned with good mathematical notation.

However, the symbol $\circ$ appears not only in the context of composite functions, and interpretation of the notation $f \circ g$ varies across different areas of knowledge such as category theory, group theory and computer science, as well as across different regions of the world.

Making connections

In the biological sciences there are many functional relationships where one quantity depends on another, and the second quantity is a function of a third quantity. For example, data suggests that the number of bacteria in food kept in the refrigerator can be described by a function $n(T)$, where T is the temperature of the food in degrees Celsius. After the food is removed from the refrigerator, the temperature of the food is given by a function $T(t)$, where t is the time in hours that the food has been out of the refrigerator.

Discuss with your fellow students the meaning of the composite function $n(T(t))$ in this context. Explain how this composite function could be useful for a biologist or food scientist.

Theory of Knowledge

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One of the fundamental TOK ‘takeaways’ is the ability to reconsider what you know and how you know it. This epistemological mindset works well in challenging your own ways of thinking (i.e. personal knowledge), and hopefully these changes in thinking are long lasting, affecting you as a learner through your university and professional studies. In many cases, this same mindset will help you to challenge the thinking of others as well (i.e. shared knowledge). One interesting leap, from the above Be aware box in 2.1 about the order of operations for composite functions, to TOK is the following video, which examines and challenges what was most likely the first mathematical order of operations and its acronym that you probably learned — PEMDAS, PEDMAS, BEDMAS, BIDMAS, BODMAS, etc. — depending on the region where you were taught.

This sample knowledge question (KQ) may help, as will the video: ‘To what extent does analogical thinking help link various areas of knowledge (AOKs) and/or ways of knowing (WOKs)?’ Consider specifically language and sense perception as two WOKs related to this video.

3 section questions

2. Functions / 2.5 Composite and inverse functions

The inverse function

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Finding the inverse function

Identity function

Consider a very special function that does **nothing** . What is meant by that? First, let us look at some mathematical operations we are familiar with. What is the number that I can add to any number without changing its value? Zero: $a + 0 = a$ for any a. What is the number that I can multiply by any number without changing that number's value? One: $a \times 1 = a$ for any a. We call 0 the identity under addition, and 1 the identity under multiplication. By the same token, there is an identity function that takes any value x and maps it to x itself.

Important

The **identity function** is the function defined by $I(x) = x$, which assigns any number in its domain to the number itself.

The domain D_I and the range R_I of the identity function I are the same set: $D_I = R_I$.

The inverse function f^{-1}

In subtopic 2.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25390/) we have seen that the inverse function f^{-1} 'undoes' what function f does; that is, if f maps x to y, then f^{-1} maps y back to x.

For example, consider the function $f(x) = 2x$, which doubles its input. We can 'undo' the action of f by applying the function $g(x) = \frac{1}{2}x$, which halves its input, so that $x \xrightarrow{f} 2x \xrightarrow{g} x$. Observe that the composition of functions f and g, in any order, gives the identity function:

$$g(f(x)) = \frac{1}{2}(2x) = x = I(x)$$

$$f(g(x)) = 2\left(\frac{1}{2}x\right) = x = I(x)$$

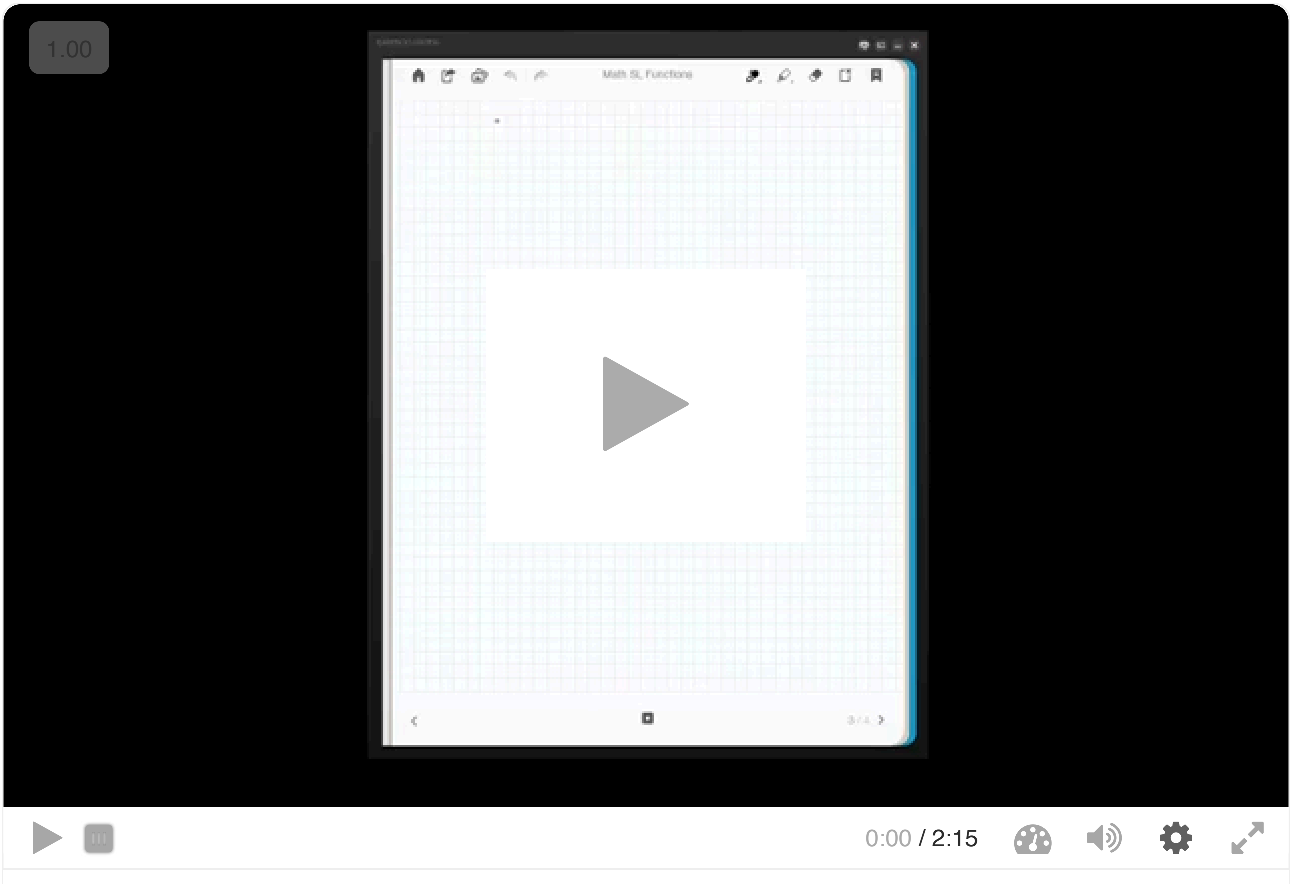
Student view

Important

Let f and g be two functions with respective domains D_f and D_g. If $(f \circ g)(x) = x$ for every $x \in D_g$ and $(g \circ f)(x) = x$ for every $x \in D_f$, then the function g is called the **inverse function of** f and is denoted by f^{-1}. Thus, we write:

$$\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$$

In subtopic 2.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25390/) you explored how to find the formula for an inverse function f^{-1} either by reversing the order of operations of the function f or by using the graphical relationship between f and f^{-1}. The video shows another method for finding the equation of the inverse function f^{-1}.



Video 1. Graphical Implications of the Inverse Function.

More information for video 1

1

00:00:00,534 --> 00:00:03,403

narrator: In this video, we're gonna

investigate inverse functions

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2

00:00:03,470 --> 00:00:06,039

and remember what inverse function are.

3

00:00:06,139 --> 00:00:07,975

They undo what another function does.

4

00:00:08,041 --> 00:00:09,643

So if x goes through f

5

00:00:10,010 --> 00:00:11,912

and the image is y equals f of x,

6

00:00:11,979 --> 00:00:14,648

if put that through f inverse,

I get x back,

7

00:00:14,715 --> 00:00:16,183

or of course the other way around.

8

00:00:16,250 --> 00:00:19,586

If x goes through f inverse

producing y equals f inverse

9

00:00:19,653 --> 00:00:22,289

of x, then bring it through f,

I get x back.

10

00:00:22,623 --> 00:00:26,627

In other words, x through f,

the result through f inverses x

11

00:00:26,693 --> 00:00:29,129

and the opposite directions through two .

12

00:00:29,296 --> 00:00:30,831

So what does it mean in terms of graphs?

13

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00:00:30,931 --> 00:00:32,733

Well let's you take a general graph,

14

00:00:33,000 --> 00:00:38,739

then x gets mapped to y,

so it goes through point x, f of x.

15

00:00:39,273 --> 00:00:41,008

Now if I take it y value,

16

00:00:41,375 --> 00:00:45,679

then if that goes

through f inverse, it should produce x.

17

00:00:45,746 --> 00:00:49,149

So the point y, f inverse of y,

18

00:00:49,449 --> 00:00:53,086

but f inverse of y

is really f inverse of f of x.

19

00:00:53,153 --> 00:00:56,356

And that should of course

produce that value x.

20

00:00:56,557 --> 00:00:59,660

Now here we have a line

f of x is 2x plus 1.

21

00:00:59,726 --> 00:01:01,428

Now let's investigate this.

22

00:01:01,795 --> 00:01:06,233

So if I take a point 1.0,

then it gets mapped

23

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00:01:06,633 --> 00:01:08,335

through f of x.

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24

00:01:08,669 --> 00:01:11,305

Now, if I take a value 3,

if I put that through

25

00:01:11,371 --> 00:01:14,641

and the inverse function,

it should produce a value 1.

26

00:01:14,775 --> 00:01:18,011

Now let's take one more

point minus 2 and 0.

27

00:01:18,078 --> 00:01:20,013

That's get mapped through f

28

00:01:20,214 --> 00:01:22,883

to the point y equals minus 3.

29

00:01:23,250 --> 00:01:25,886

So if I take minus 3

through the inverse function,

30

00:01:25,953 --> 00:01:28,288

that should get mapped to minus 2,

31

00:01:28,589 --> 00:01:30,991

'cause that's what I started

with in the blue mapping.

32

00:01:31,225 --> 00:01:32,659

Well now we've got two points

33

00:01:32,826 --> 00:01:36,797

and inverse function of a line

is another line, and here it is.

34

00:01:36,864 --> 00:01:39,766

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The inverse of 2x plus 1

is a half x minus a half.

35

00:01:40,400 --> 00:01:44,171

Now let's take one more point of interest

and I'm gonna take minus 1,

36

00:01:44,238 --> 00:01:46,139

which get mapped to minus 1

37

00:01:46,206 --> 00:01:49,543

through blue, but it also gets

mapped to minus 1 through

38

00:01:49,877 --> 00:01:50,978

the inverse function.

39

00:01:51,211 --> 00:01:53,380

It goes, it is an intersection point.

40

00:01:53,747 --> 00:01:56,817

Now very great importance is the fact that

41

00:01:56,884 --> 00:02:01,522

f and f inverse are reflections

in the y equals x axis.

42

00:02:02,122 --> 00:02:05,526

That will certainly help you to plot them

and it also helps you that

43

00:02:05,592 --> 00:02:09,696

if f intersects y equals x,

44

00:02:09,763 --> 00:02:13,800

then f inverse will intersect

y equals x at exactly the same point

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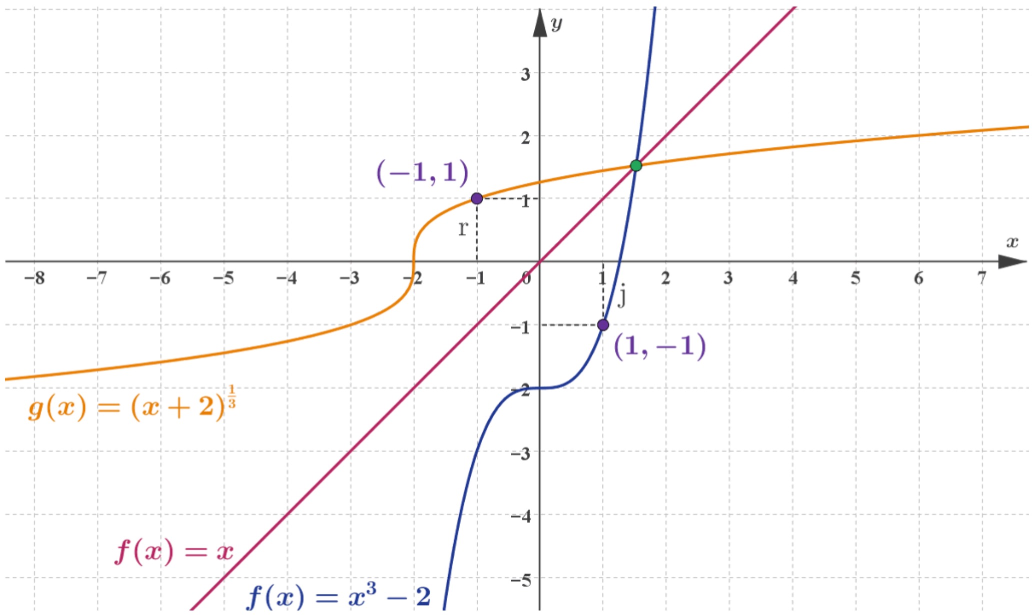
45

00:02:13,901 --> 00:02:15,269

as seen here.

Using the procedure explained in the video, we found that if $f(x) = x^3 - 2$, then $f^{-1}(x) = (x+2)^{\frac{1}{3}}$. The graphs of the two functions are shown in the figure below.



More information

The image depicts graphs of two mathematical functions and their relationship. The first function, (f(x) = x^3 - 2), is plotted in a distinct color and curves up sharply on the right side and curves downward on the left. The second function, (g(x) = (x + 2)^{1/3}), is shown in a contrasting color, appearing as the inverse of (f(x)). Additionally, there is a line (y = x) drawn in pink, serving as the reflection line for the functions. The graph shows that (f(x)) and its inverse intersect on this line. Coordinates such as ((-1, 1)) and ((1, -1)) are marked to indicate key points on the curves. The functions reflect symmetrically across the (y = x) line, highlighting their inverse relationship.

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Notice how the two functions are reflections of each other in the line $y = x$ (pink line). The point of intersection of f and f^{-1} lies on the line $y = x$, as expected.

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The procedure for finding the expression for an inverse function is summarised as follows:

Important

Procedure for finding f^{-1} :

1. Write $y = f(x)$.
2. Replace all the x's with y's and the y's by x's.
3. Solve the equation until you obtain $y = g(x)$, where $g(x)$ is an expression that does not contain y.
4. Then $g(x)$ is the formula for the inverse function; that is, $f^{-1}(x) = g(x)$.

Example 1

★★☆

Find the inverse of the function defined by $f(x) = \dfrac{1+5x}{3x-2}$.

Steps	Explanation
The domain of f is $\{x \mid x \neq \frac{2}{3}\}$.	Why does the domain of f not include all real numbers?
$y = \dfrac{1+5x}{3x-2}$	Write $y = f(x)$.
$x = \dfrac{1+5y}{3y-2}$	Swap the x's and y's and solve for y.
$(3y-2)x = 1+5y$	Multiply both sides by $(3y-2)$.
$3yx - 2x = 1+5y$	Expand brackets.
$3yx - 5y = 1+2x$	Gather all x's on LHS of the equation.
$(3x-5)y = 1+2x$	Factorise y out and solve for y.
$y = \dfrac{1+2x}{3x-5}$	This formula is valid for $x \neq \frac{5}{3}$.

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Steps	Explanation
Thus, $$f^{-1}(x)=\frac{1+2x}{3x-5},\ x\neq\frac{5}{3}$$	

Existence of an inverse function

In the Investigation of subtopic 2.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25390/), you explored conditions for a function to have an inverse.

The following activity revisits this investigation.

Activity

This applet (https://www.geogebra.org/m/xy6xawzh) shows the inverse of a given function. Follow the steps and discuss the questions with your fellow students.

Student view

Interactive 1. Investigating Inverse Functions.

Credit: GeoGebra (https://www.geogebra.org/m/jvw8kkmt) VANGELIS KARAGIANNAKIS, melbapplets

More information for interactive 1

This interactive provides an engaging way for users to explore the concept of inverse functions through a step-by-step process. Users begin by entering a function rule in the designated input box, allowing them to define the relationship they wish to analyze. To better understand how domain restrictions impact inverses, they can specify a bounded domain for the original function. By clicking the "Show inverse" button, the interactive generates and displays the graph of the inverse function, helping users visualize the reflection across the line $y = x$. To verify whether the inverse qualifies as a function, the interactive includes a "Vertical line test" feature, which checks if any vertical line intersects the inverse graph more than once—indicating whether it passes the test for functionality. $f(x) = 2x(x-1)(x-2)$ is a function, as the vertical line intersects the graph at most once.

For further exploration, users can enable the "Show points" option, which places a point on the x-axis and dynamically reveals its corresponding point on the inverse function, illustrating the one-to-one relationship between the original function and its inverse. In case the point are getting beyond the graph then you can use the Zoom function to zoom in and zoom out the graph. This interactive approach enhances understanding by combining visual and analytical elements.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

1. Enter a rule for a function in the ‘f(x) =’ box.
2. Click ‘Show points’ to display a point on the x-axis and the point(s) corresponding to $f^{-1}(x)$.
3. Drag the point to change x. What do you observe as you move x along the axis?
4. Click ‘Show inverse’ to display the graph of the inverse $f^{-1}(x)$. Is it a function? Click ‘Vertical line test’ to show a vertical line which may help you decide.
5. How could you predict from the graph of the original function whether it has an inverse function? Formulate a general rule for a function to have an inverse function.
6. Can you restrict the domain of f to make its inverse a function?

In the activity you discovered that a function has an inverse function if and only if the reflection of its graph about the line $y = x$ passes the vertical line test. This is the same as saying that the graph of the original function passes the **horizonal line test**: any horizontal line intersects the graph of the function at most once.

Making connections

Recall that a **one-to-one** function is any function such that:

- for each value of the input variable x there is only one value of the output variable y
- for each value of y there is only one value of x.

One-to-one functions satisfy both the vertical and the horizontal line tests.

Important

A function f has an inverse function if and only if f is a one-to-one function. The graph of f passes both the vertical and the horizontal line tests.

Example 2

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Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Consider the function $f(x) = x^2 + 1$.

a) Explain why this function does not have an inverse function.

b) Restrict the domain of f such that it has an inverse function.

c) Find f^{-1} if the domain of f is restricted (as in part b).

	Steps	Explanation
a)	f has domain $x \in \mathbb{R}$ and is a many-to-one function (it does not pass the horizontal line test). Therefore it does not have an inverse function.	f maps two different x values to the same y value. For example, $f(x) = 5$ when $x = -2$ and wh $x = 2$.
b)	If the domain of f is restricted to $x \geq 0$ or $x \leq 0$, the function becomes one-to-one and so has an inverse function.	The graph of f passes the horizontal line test when the domain is restricted to either $x \geq 0$ or $x \leq 0$.
c)	*Case 1:* f is defined by $y = x^2 + 1,\ x \geq 0$ To find f^{-1}: $x = y^2 + 1,\ y \geq 0$ $y^2 = x - 1,\ y \geq 0$ $y = \pm\sqrt{x-1},\ y \geq 0$ $y = \sqrt{x-1}$ So $f^{-1}(x) = \sqrt{x-1}$.	There are two cases to consider, depending on how you restrictec the domain of f.

Student view

Overview
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	Steps	Explanation
	Case 2: f is defined by $y = x^2 + 1,\ x \leq 0$ To find f^{-1}: $x = y^2 + 1,\ y \leq 0$ $y^2 = x - 1,\ y \leq 0$ $y = \pm\sqrt{x-1},\ y \leq 0$ $y = -\sqrt{x-1}$ So $f^{-1}(x) = -\sqrt{x-1}$.	

4 section questions

2. Functions / 2.5 Composite and inverse functions

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27695/print/) Assign

What you should know

By the end of this subtopic you should be able to:

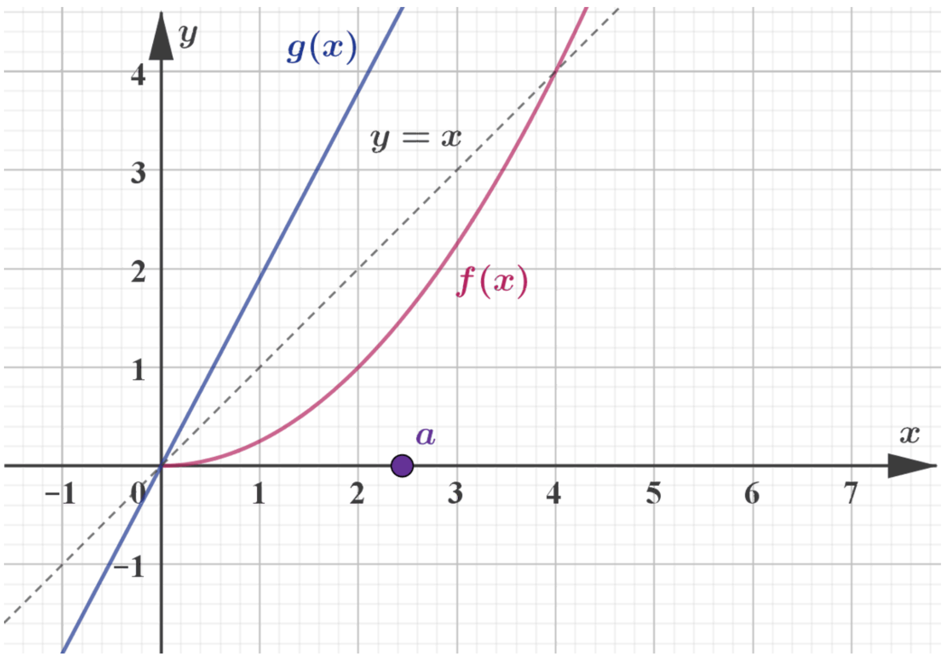
- find the composition of two functions
- find the domain and range of a composite function
- verify whether a function is the inverse of another function
- determine whether a given function has an inverse function, and appropriately modify a function so that it has an inverse function
- find the formula for the inverse function of a given function.

Student view

Investigation

In this subtopic we have explored how to find the composite function of two functions using algebra. The composition of two functions can be expressed graphically as well.

The figure shows the graphs of functions f and g, the line $y = x$ and an input value a. Think about how you could use the figure and the ideas of this subtopic to locate the point $(a,\ g(f(a)))$ that lies on the graph of the composite function $g(f(x))$. Explain your process step by step, together with the reasoning behind it.



More information

The graph displays three main lines: the function g(x) in blue, the function f(x) in red, and the line y=x in gray. The x-axis and y-axis are marked with numbers ranging from -1 to 8. The functions g(x) and f(x) curve upwards. The point labeled 'a' is marked on the x-axis and is associated with the input point for the composition of functions discussed. This graph is used to locate the point (a, g(f(a))) on the graph of the composite function g(f(x)).

[Generated by AI]

Overview
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For a hint, explore the applet below: enter some different formulas for functions f and g, choose a few different input values on the x- axis and push the slider slowly to the right.

Interactive 1. Exploring Function Compositions.

Credit: GeoGebra (https://www.geogebra.org/m/tDmXq8jz) Tim Brzezinski

More information for interactive 1

This interactive helps users explore composite functions graphically. The screen is divided into two sections: the right side shows a coordinate plane with axes labeled X and Y and a dotted line $y = x$, along with two graphs — one in red and one in blue. A draggable point $(a, 0)$ is present on the x-axis, and a “Center (0, 0)” button repositions the origin to the center. On the left side, users can edit the functions $f(x)$ (in blue) and $g(x)$ (in red), and use two buttons — “Show $f(g(a))$” and “Show $g(f(a))$” — along with a slider to animate the composition process. As the slider or point a is moved, the interactive dynamically updates to visualize the corresponding points on the composite graphs. For example, when $f(x) = 0.5x$ and $g(x) = 0.25x^2$, the blue graph is a straight line through the origin, and the red graph is a parabola opening upward. As the slider moves, a red line extends from the origin to $(a, 0)$, then rises to $g(a)$, moves horizontally to the y-axis, rotates back to the x-axis, and finally a blue line rises to $f(g(a))$. A “Reset” button in the bottom left allows users to restart the process.

This dynamic visualization helps users understand how composite functions $f(g(x))$ and $g(f(x))$ are formed and interpreted graphically, reinforcing their conceptual understanding.

Student
view

Rate subtopic 2.5 Composite and inverse functions

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