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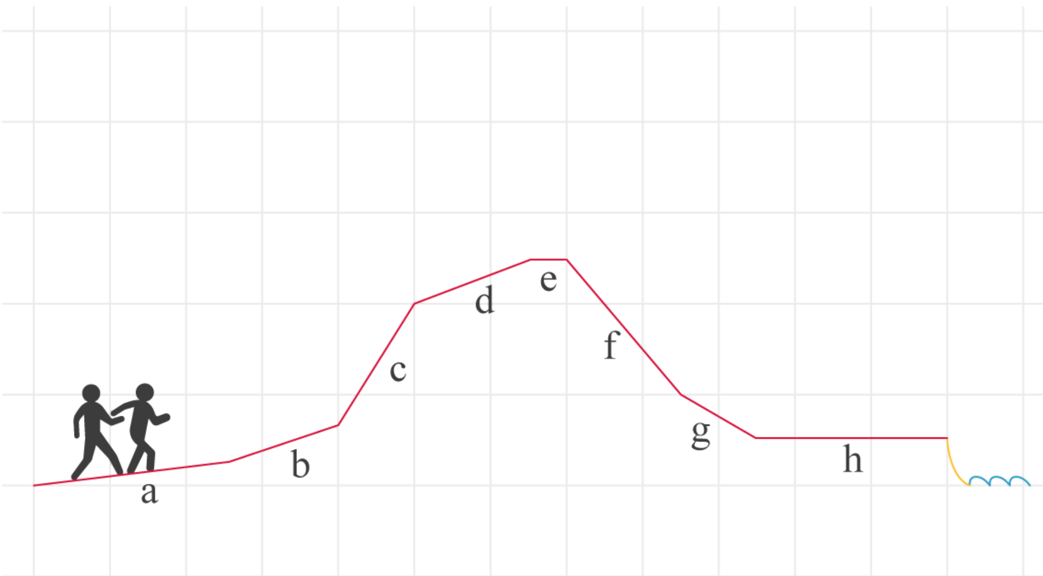
The big picture

Sarwar and John decide to go for a hike one day.

Their path takes them over a hillside as shown in the figure below.



More information

The image is a graph showing a hiking path over a hillside, divided into sections labeled from 'a' to 'h'. The graph consists of a red line tracing the elevation changes along the path. The path begins at 'a', where it is flat, then it inclines gently at 'b', becomes steeper at 'c', and levels off at the top at 'e'. After 'e', the path

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descends starting at 'f', becomes less steep at 'g', and flattens out at 'h'. This visual provides a way to compare the steepness of each segment of the hike, which can later be described more precisely using mathematical means.

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In this figure, note that the hill goes up (a), gets steeper (b then c), gets less steep (d), flattens at the top (e), goes down (f then g), and then flattens out again (h) before they reach the beach. How can the steepness of the different sections of the hike be compared? To do this you need a precise mathematical way of describing the different parts of the hike.

The field of mathematics that allows you to describe geometric figures, such as straight lines, with the use of algebraic methods is called coordinate geometry. In this subtopic, you will learn about the following concepts of coordinate geometry:

- The gradient (or slope) of lines
- Equivalent forms of equations that represent straight lines
- The algebraic relationship of parallel lines
- The algebraic relationship of perpendicular lines

Concept

Coordinate geometry provides a precise way to represent straight lines and geometric curves, using the language of algebra. Throughout this unit, while you are learning how to find equations that express straight lines, consider what information equivalent forms of lines gives you. Think about how linear models can be used to describe relationships in which quantities are related with a constant ratio.

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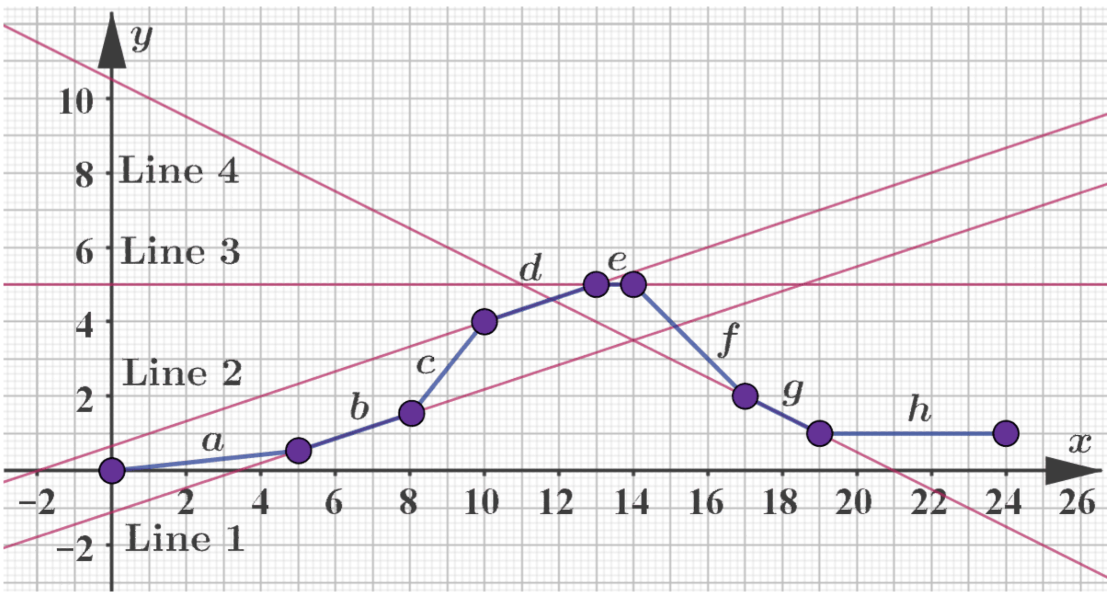
The gradient–intercept form of a line

Finding the equation of a line

The Cartesian plane

To describe the parts of the hillside we use the Cartesian coordinate system or Cartesian plane, which is named after the French mathematician René Descartes (1596–1650). The Cartesian system is the basis of coordinate geometry and allows you to determine the exact position of points and lines on the plane.

In the figure below, you can draw four straight lines to describe four different parts of the hillside. Two lines can be described as going uphill, one line is horizontal, and one line is going downhill. In mathematics, you follow a graph from left to right along the positive direction of the x -axis.



More information

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view

The image is a graph displaying a piecewise linear function divided into four segments on an X-Y axis grid. The X-axis represents the horizontal axis moving left to right, with values from 0 to 24, and the Y-axis represents the vertical axis, with values from 0 to 12.

1. **Line 1**: The line begins at the origin (X=0, Y=0), and the first segment travels uphill, with data points indicated by purple dots, reaching (X=6, Y=6).
2. **Line 2**: The second segment continues uphill, starting from (X=6, Y=6) to the peak at (X=12, Y=10).
3. **Line 3**: The next segment travels downhill, moving from (X=12, Y=10) to (X=18, Y=5).
4. **Line 4**: Finally, the line flattens, running horizontally from (X=18, Y=5) to (X=24, Y=5).

Each segment is represented with a different line style, and transition points are marked with purple circles. Red diagonal guide lines overlay the graph, intersecting various points for reference.

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International Mindedness

René Descartes, who lived and worked in many countries, is considered to be the father of coordinate geometry and his work *Discours de la Methode* (published in 1637) was one of the most important mathematical contributions of the 17th century. René Descartes commented in the book that 'Every problem that I solved became a rule which served afterwards to solve other problems'. He introduced the revolutionary idea that the position of a point on the plane can be described by two numbers — the coordinates of a point as you know them today. This notion provided new tools to mathematicians and physicists, as they could describe phenomena such as harmonic motion, alternating current, diffraction and the motion of particles with the use of equations and graphs.

The gradient of a line

The steepness of a line is called the gradient (or slope). The gradient of a line is the number that measures how much a line rises vertically compared with how much it runs horizontally.

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Important

The gradient of the line through points $\mathrm{A}\left(x_1, y_1\right)$ and $\mathrm{B}\left(x_2, y_2\right)$ is given by the formula

$$m=\frac{y_2-y_1}{x_2-x_1}$$

where y_2-y_1 is the vertical distance between points A and B and x_2-x_1 is the horizontal distance between points A and B .

The gradient measures the inclination of a line as you move along the x -axis in the positive direction, and it can be positive, negative, zero or undefined.

In the applet below, drag points A and B around to adjust the position of the line. Tick the box ‘Show gradient’ and observe how the value of the gradient changes.

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Interactive 2. The Gradient of a Line.

More information for interactive 2

This interactive tool provides an engaging way to explore the fundamental concepts of gradient (slope) and linear equations through direct manipulation and visualization. Users can drag two points on a coordinate plane to adjust the position and steepness of a line, with the tool instantly calculating and displaying the gradient using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Checkboxes allow users to show or hide the gradient calculation and the gradient—intercept form of the equation for the line, including the gradient determination and the gradient-intercept form of the equation $(y = mx + c)$, where m represents the slope and c is the y-intercept. A slider is also available to adjust the y-intercept independently, enabling users to observe how changes to c affect the line's vertical position while maintaining its slope. This dynamic approach helps build intuition about the relationship between a line's algebraic equation and its graphical representation, reinforcing key concepts like the meaning of slope as a rate of change and the y-intercept as the starting value.

For example, when points $A(1.06, 0.93)$ and $B(3.43, 1.93)$ are plotted, the tool calculates the gradient (m) by finding the ratio of vertical change to horizontal change: $m = \frac{1.93 - 0.93}{3.43 - 1.06} = \frac{1}{2.37} \approx 0.42$, Using this slope and point A, it determines the y-intercept (c) by solving $0.93 = 0.42 \times 1.06 + c$, yielding $c \approx 0.49$, resulting in the equation $y = 0.42x + 0.49$. If the user then moves point B to $(4, 2.5)$, the tool recalculates: the new gradient becomes $\frac{2.5 - 0.93}{4 - 1.06} = \frac{1.57}{2.94} \approx 0.53$, and the updated equation becomes $y = 0.53x + (0.93 - 0.53 \times 1.06) \approx y = 0.53x + 0.37$. Through these live calculations, learners can observe exactly how each adjustment affects both the line's steepness and position, transforming abstract algebraic concepts into tangible visual relationships.

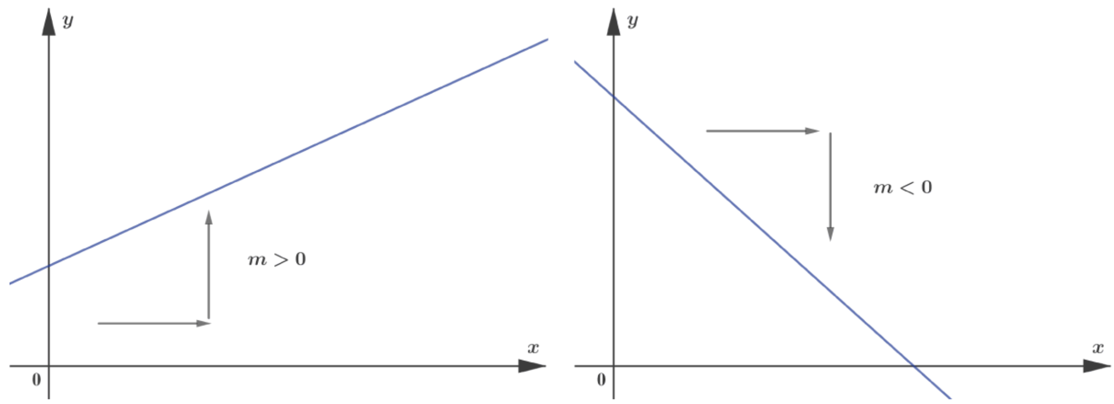
Activity

Use the applet above to discuss, with your fellow students, the following:

1. Adjust point B to obtain a positive gradient, m . As the gradient takes larger values describe the direction and steepness of the line.
2. Adjust point B to obtain a negative gradient m . As the gradient takes smaller values describe the direction of the line.
3. Adjust point B to obtain a horizontal line. Explain why the gradient of the line equals to zero.

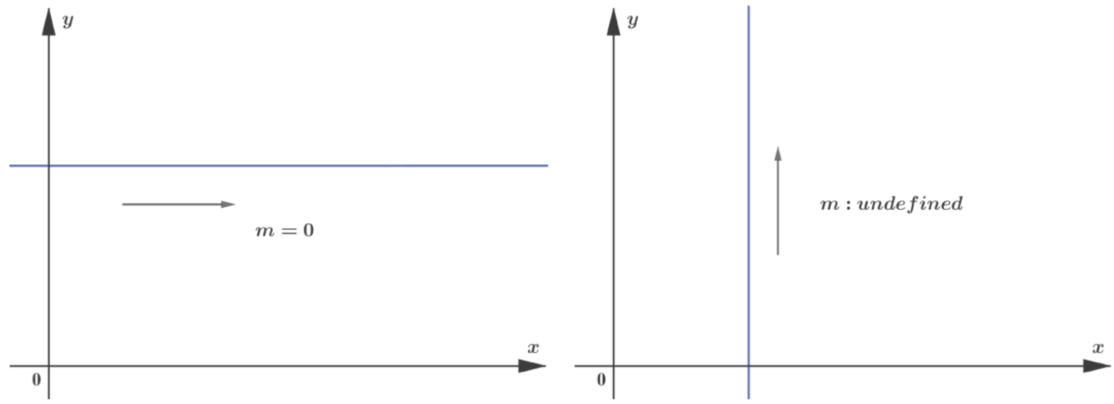
Student view

4. Drag point B to obtain a vertical line. Explain why the software does not calculate the gradient of the vertical line.



When the gradient m of a line is positive, the line is increasing in the positive direction. As the gradient takes larger values the line becomes steeper.

For negative values of gradient, the line is decreasing in the positive direction.



Horizontal lines have gradient equal to zero, as there is no vertical rise between any points on the line.

For vertical lines the gradient cannot be defined, as the horizontal distance between any two points on the line is equal to zero.

More information

The image is a diagram with four quadrant graphs illustrating different types of line gradients on the Cartesian coordinate system, labeled with X and Y axes.

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1. **Top Left Quadrant:** A line with a positive gradient (m > 0) is shown rising from left to right. An explanatory text below states, "When the gradient m of a line is positive, the line is increasing in the positive direction. As the gradient takes larger values the line becomes steeper."

2. **Top Right Quadrant:** A line with a negative gradient (m < 0) is shown descending from left to right. The description below reads, "For negative values of gradient, the line is decreasing in the positive direction."

3. **Bottom Left Quadrant:** A horizontal line with zero gradient (m = 0) is illustrated. The accompanying text explains, "Horizontal lines have gradient equal to zero, as there is no vertical rise between any points on the line."

4. **Bottom Right Quadrant:** A vertical line with undefined gradient is shown. The text reads, "For vertical lines the gradient cannot be defined, as the horizontal distance between any two points on the line is equal to zero."

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The gradient—intercept form of a line: $y = mx + c$

The gradient-intercept form of a line is

$$y = mx + c$$

where m is the gradient of the line and c is the y -intercept of the line (the point where the line intersects the y -axis). The gradient–intercept form of a line is a mathematical model that connects two quantities x and y with a constant ratio, and it is called a linear model as its graph is a straight line.

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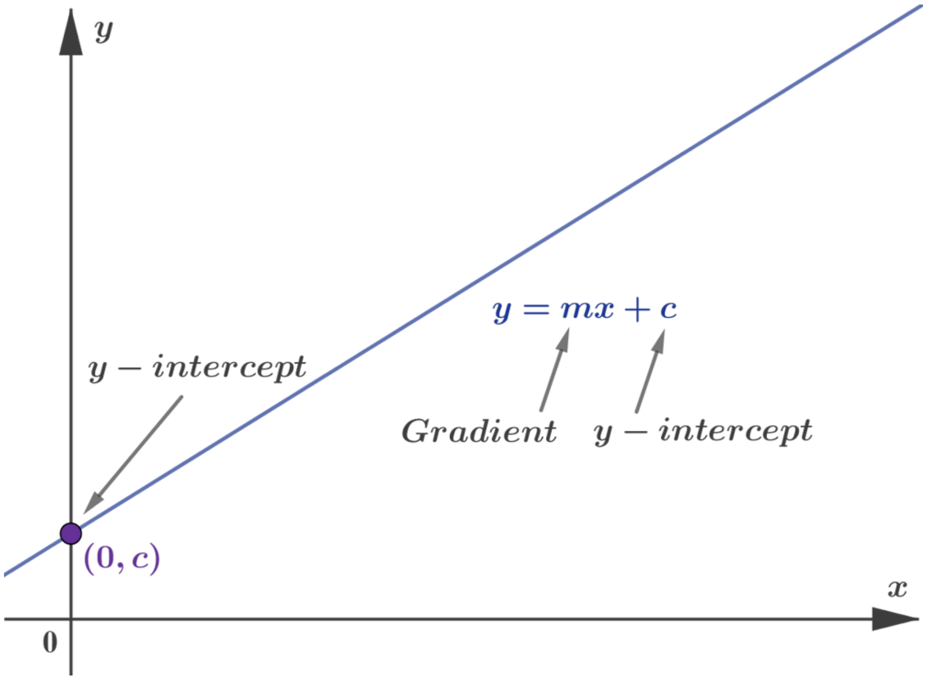

More information

The image is a graph depicting the gradient-intercept form of a linear equation, represented by the formula (y = mx + c). It features both the x-axis and y-axis, intersecting at (0,0). The x-axis is labeled 'x' and extends horizontally, while the y-axis is labeled 'y' and extends vertically. The graph includes a straight blue line illustrating a positive linear relationship. This line intersects the y-axis at point (0, c), which is the y-intercept, marked with a purple dot.

The line's equation, (y = mx + c), is indicated near the line with arrows pointing to sections of the line that represent the gradient (m) and the y-intercept (c). Labels such as 'Gradient', 'y-intercept', and the equation (y = mx + c) are shown to the right of the line. The graph serves as a visual representation of the linear model by showing how the y-intercept and gradient determine the position and steepness of the line on the graph.

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The equation of a straight line gives the relationship between the x - and the y - coordinates of the points that lie on the line. The equation $y = mx + c$ is an explicit formula for y in terms of x , as it tells us how to calculate the y -coordinate when given the x -coordinate.

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For example, the formula $y = 3x - 4$ states that the value of the y -coordinate can be found if you multiply the x -coordinate of a point by 3 , and then subtract 4 .

Activity

Look again at this applet and adjust the position of the line by dragging points A or B . This time, click both ‘Show gradient’ and ‘Show gradient—intercept form’ to observe how the gradient, y -intercept and equation of the line change.

Theory of Knowledge

‘I am thinking, therefore I exist.’

René Descartes: *Discours de la Methode, 1637*

René Descartes’ work had an impact on the modern philosophy by introducing the epistemological view of Rationalism, in which the criterion of truth is not sensory but intellectual. He believed that the system of mathematical knowledge should be constructed on a basis of self-evident propositions and then proceed mathematically to a series of deductive reasoning.

The Cartesian coordinate system, and the x - and y -axes on which the slope (as well as the unit circle) is overlaid, models and highlights the relationships between (trigonometric) ratios and angles. In particular, in design, understanding these relationships helps engineers, architects and graphic designers to consider the efficient use of space, and conversely, it helps those who use these urban designs to better understand the world in which they live.

Theory of Knowledge

Consider, with reference to the first two links below, the simultaneous simplicity and constraints of right angles, common to historic city design and to topographical depictions of these designs. How, for example, would

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the design of a subway map or railway map be affected by a geographical barrier such as a river (for instance, the Thames in London)?

200th Birthday for the Map that Made New York (https://www.nytimes.com/2011/03/21/nyregion/21grid.html)

London Tube (http://content.tfl.gov.uk/standard-tube-map.pdf)

How do those designs of New York and of London differ from, and align with, the circular designs for ancient Baghdad? Consider this article (https://www.theguardian.com/cities/2016/mar/16/story-cities-day-3-baghdad-iraq-world-civilisation) to help with this comparison.

Mathematics, indigenous knowledge systems and natural sciences can be thought-provoking lenses by which students can examine city design. Looking at the ways in which different cultures and their histories connect to the physical world around them allows for rich multi-layered analyses.

Such investigations may challenge some concepts that some cultures take for granted, such as cardinal directions. For example, if you fly over parts of the world and examine the shape of the settlements on the ground, how much of the landscape has been partitioned into grids reminiscent of a Cartesian graph, respective of cardinal directions and of corresponding rectangular grids? To what degree are the overarching geometric shapes due to the topography? To what degree could the shapes viewed below reflect cultural attitudes about land ownership and agriculture? To what degree do the shapes reflect geopolitical borders?

To examine these questions further using a real-life situation (RLS), consider this example (http://ethnomath.coe.hawaii.edu/pdf/university_thehawaiian.pdf), which explains how the Hawaiian compass and the Western unit circle correlate (and, in the broadest terms, how the unit circle connects with the directional compass).

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Also consider this RLS link (http://longnow.org/seminars/02010/jan/13/wayfinders-why-ancient-wisdom-matters-modern-world/) for an overview of some of the ideas and research of anthropologist Wade Davis, who offers interesting insights into other cultures and their means of wayfinding.

Horizontal lines

Visualise a horizontal line in the following applet (click ‘Horizontal line’). Adjust the position of the line by moving point A around and observe the y -coordinate of point A and the equation of the line. What do you notice?

Interactive 5. The Gradient of Horizontal and Vertical Lines.

More information for interactive 5

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This interactive tool helps users explore the fundamental properties of horizontal and vertical lines through direct manipulation. Horizontal line ($y = c$) is blue with a blue dot as point A, and the vertical line ($x = k$) is in red with a red dot as point B. Learners can drag two points on a coordinate plane. Real-time feedback shows the equations updating as points move, visually demonstrating how these special lines relate to their algebraic forms. Each checkbox allows users to toggle each line's visibility, helping focus on one concept at a time while seeing the connection between coordinates and equations.

For example, when Point A is placed at $(3.27, 1.68)$, the horizontal line's equation becomes $y = 1.68$, showing a flat line crossing the y-axis at 1.68. Similarly, positioning Point B at $(1.1, 0.81)$ creates the vertical line $x = 1.1$. These concrete examples make abstract concepts tangible, showing why horizontal lines have zero slope (no vertical change) while vertical lines have undefined slope (no horizontal change). The interactive's hands-on approach reinforces how constant x or y values create these special cases of linear equations.

✓ **Important**

The equation of a horizontal line is $y = c$, where c is the y -coordinate of any point of the line.

The gradient of a horizontal line is zero, as the vertical distance between any two points of the line is equal to zero.

Vertical lines

Use the applet above to visualise a vertical line (click ‘Vertical line’). Adjust the position of the line by moving point B around and observe the x -coordinate of point B and the equation of the line. What do you notice?

✓ **Important**

The equation of a vertical line has the form $x = a$, where a is the x -coordinate of any point of the line.

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For vertical lines the horizontal distance between any two points is zero and thus the gradient is not defined (the denominator cannot be zero). Hence, the gradient–intercept form $y = mx + c$ is not used for vertical lines.

Example 1

★☆☆

Determine whether the following points lie on the line $y = 5x - 2$.

a) $(3,\ 13)$

b) $(5,\ 3)$

c) $(0,\ -2)$

a)	✓	because	$13 = 5(3) - 2$
b)	×	because	$3 \neq 5(5) - 2$
c)	✓	because	$-2 = 5(0) - 2$

Example 2

★☆☆

Determine the y-intercept and calculate the x-intercept of the line $y = 2x - 10$.

According to the gradient–intercept form of a line, the y-intercept is the constant value -10.

Verification:

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The point where the line intersects the y -axis has x -coordinate equal to 0 . Thus:

$$y = 2(0) - 10 = -10$$

The x -intercept of the line has y -coordinate equal to 0.

$$\begin{aligned} 0 &= 2x - 10 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

The x -intercept of the line is $(5,\ 0)$.

Example 3

★☆☆

A line has gradient $m = \dfrac{1}{2}$ and passes through the point $\mathrm{A}\,(4,\ 8)$.

Find the equation of the line in the form $y = mx + c$.

Steps	Explanation
$y = \dfrac{1}{2}x + c$	Begin with the gradient—intercept forr $y = mx + c$ and substitute $m = \dfrac{1}{2}$.

The line passes through point $A\,(4,\ \ 8)$, so the coordinates of the point satisfy the equation of the line.

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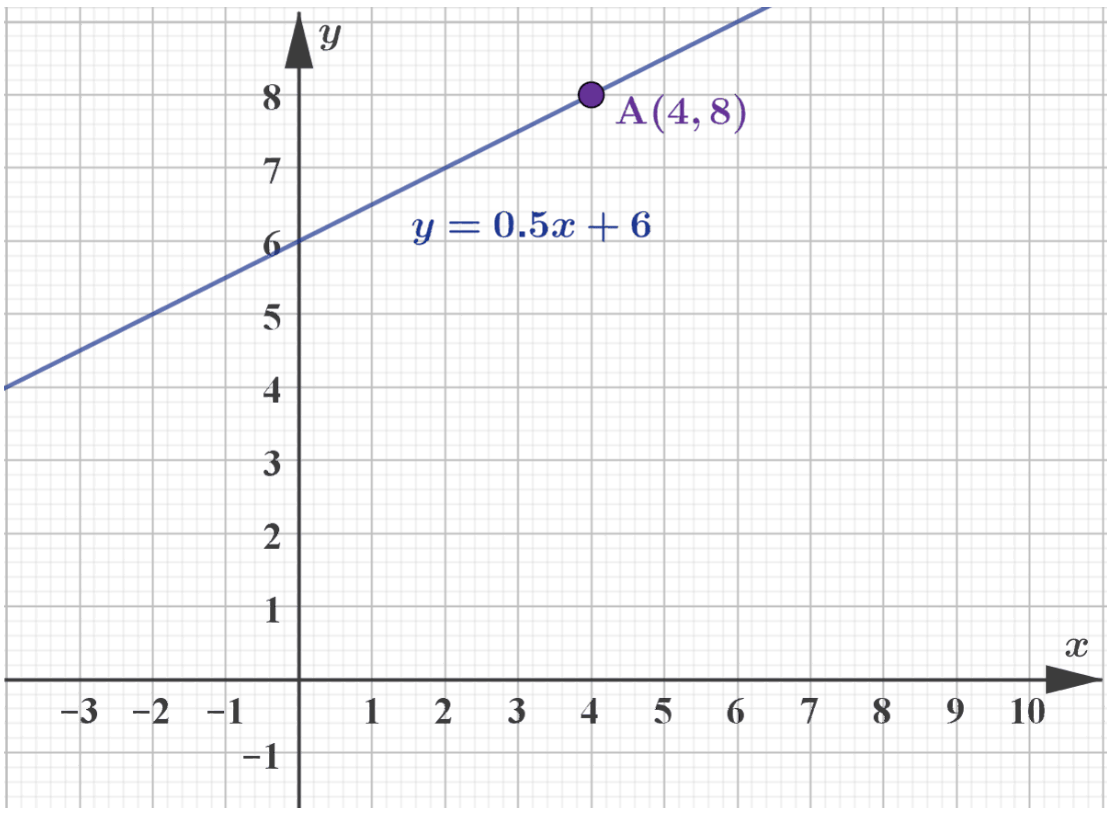
Steps	Explanation
$y = \frac{1}{2}x + c$ $8 = \frac{1}{2}(4) + c$ $8 = 2 + c$ $6 = c$ $c = 6$	Substitute x = 4 and y = 8 in the equat line and solve for c .

Therefore, the line has the equation

$$y = \frac{1}{2}x + 6\,.$$

The graph of this line is shown below.



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Example 4

★★☆

A line passes through points $\text{A}\ (-2,\ -5)$ and $\text{B}\ (1,\ 13)$.

Find the equation of the line in the form $y = mx + c$.

First, find the gradient of the line using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Steps	Explanation
$\begin{aligned} m &= \dfrac{y_2 - y_1}{x_2 - x_1} \\ &= \dfrac{13 - (-5)}{1 - (-2)} \\ &= \dfrac{18}{3} \\ &= 6 \end{aligned}$	Use the gradient formula and substitute in the coordinates of the given points. Simplify.

So, $y = 6x + c$.

To calculate the value of c, substitute the coordinates of either point A or B into the formula. Using point $\text{B}\ (1,\ 13)$, $x = 1$ and $y = 13$:

Steps	Explanation
$\begin{aligned} y &= 6x + c \\ 13 &= 6(1) + c \\ 13 &= 6 + c \\ 7 &= c \end{aligned}$	Write the equation of the line. Substitute the coordinates of point B into the e Solve for c.

Finally, the equation of the line is $y = 6x + 7$.

The graph of this line is shown below.

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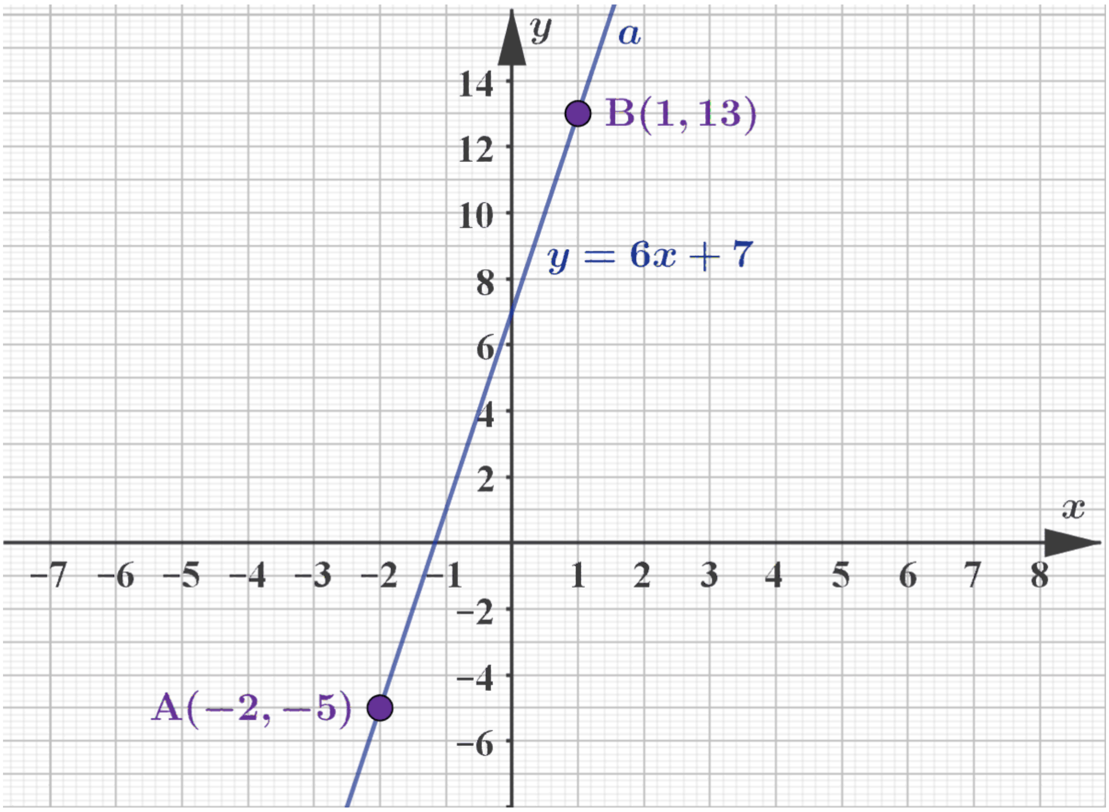

Making connections

Topic 4: In statistics, straight lines are used to model relationships between two variables. An example might be where variable 1 is the distribution of wealth across a population and variable 2 is the standard of living index experienced by the population.

Topic 5: Differential calculus describes rates of change between variables. The study of the rate of change of a variable, such as the growth of a population or the heating of a metal is important in many areas of study. The primary tool for analysing a rate of change is the straight line.

Making connections

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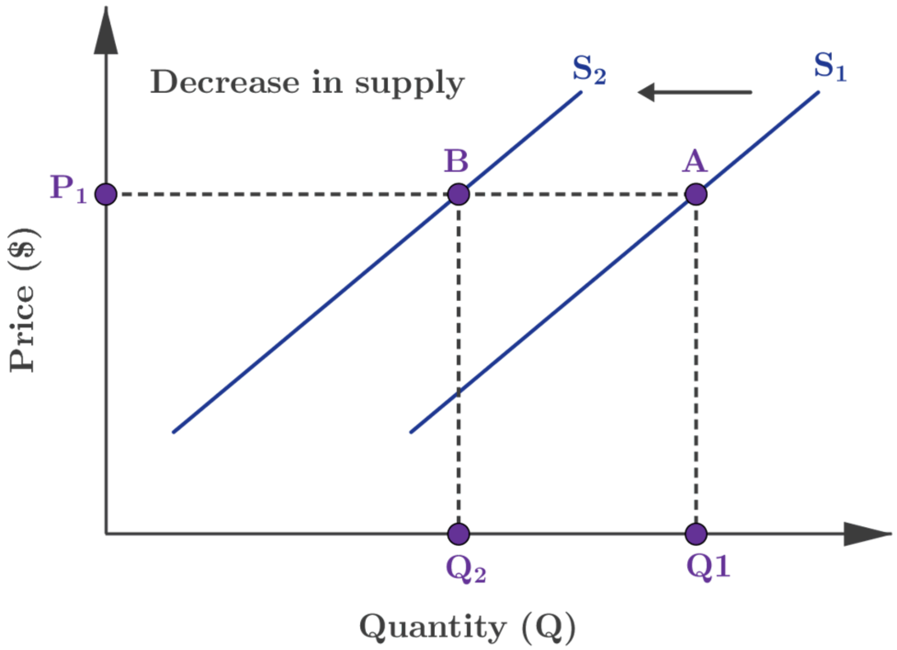
view

Linear models are used in economics to describe various relationships between quantities such as exchange rates and income elasticity, demand and supply, etc . For example, the graph below shows the relationship between the price of a product and the number of items buyers are willing to buy at a given price. Interpret the significance of the gradient in this context.



More information

The graph illustrates the relationship between price and quantity for a product. The X-axis represents Quantity (Q), and the Y-axis represents Price ($). There are two supply lines: S1 is the original supply line, and S2 is the new supply line after a decrease in supply, indicated by an arrow pointing leftwards.

The original equilibrium point A is at the intersection of line S1 and a horizontal line at Price P1, corresponding to a quantity of Q1. Following the decrease in supply, the new equilibrium point B is at the intersection of line S2 and the same horizontal line at Price P1, corresponding to a smaller quantity Q2.

The shift from point A to point B represents a reduction in the quantity available at the same price level due to the decrease in supply. This illustrates the inverse relationship between supply and price, where a decrease in supply leads to a higher price if demand remains constant.

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4 section questions

Equivalent forms for the equation of a line

The general form of a line: $ax + by + c = 0$

Lines on the plane can be described mathematically with the use of equations. Another equivalent form to describe lines is the general form.

Important

The formula $ax + by + c = 0$ is the general form of a line, where a and b are not both zero.

To transform the gradient–intercept form $y = mx + c$ to the general form $ax + by + c = 0$, start by moving all terms on to the left-hand side of the equation. It is often useful if the coefficients of x and y and the constant term are all integers (whole numbers). You can often get a form like this by multiplying the rearranged equation by an appropriate number.

For example, the gradient–intercept form for the line $y = 3x - 4$ is written in general form as $-3x + y + 4 = 0$. Note that multiplying this equation by any number gives a different equation of the same line. The equations $3x - y - 4 = 0$ and $-6x + 2y + 8 = 0$ describe the same line. In this section, you will learn to write and interpret the general form of a line.

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Example 1

★☆☆

A straight line has equation $y = \frac{2}{3}x - 7$. Rearrange this equation into the form $ax + by + c = 0$.

$$
\begin{aligned}
y &= \frac{2}{3}x - 7 && \text{Start with the given gradient–-intercept form.}\\
-\frac{2}{3}x + y + 7 &= 0 && \text{Move all terms on to the left-hand side; rememl}\\
-2x + 3y + 21 &= 0 && \text{Multiply each term in the equation by 3.}
\end{aligned}
$$

Example 2

★★☆

A straight line has equation $15x - 3y + 4 = 0$. Find:

1. the coordinates of the y-intercept
2. the coordinates of the x-intercept
3. the gradient of the line.

1. To find the coordinates of the y-intercept, use the fact that $x = 0$ anywhere on the y-axis.

$$
\begin{aligned}
15x - 3y + 4 &= 0\\
15(0) - 3y + 4 &= 0\\
-3y + 4 &= 0\\
-3y &= -4\\
y &= \frac{4}{3}
\end{aligned}
$$

Substitute $x = 0$ into the formula and solve for y.

The coordinates of the y-intercept are $\left(0, \frac{4}{3}\right)$.

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2. Similarly, to find the coordinates of the x -intercept, use the fact that $y=0$ on the x -axis.

$$\begin{aligned}15x-3y+4&=0\\15x-3(0)+4&=0\\15x+4&=0\\15x&=-4\\x&=-\frac{4}{15}\end{aligned}$$

Substitute $y=0$ into the formula and solve for x.

The coordinates of the x -intercept are $\left(-\frac{4}{15},\ 0\right)$.

3. To find the gradient of the line, rearrange to the form $y=mx+c$.

$$\begin{aligned}15x-3y+4&=0\\-3y&=-15x-4\\y&=\frac{-15}{-3}x-\frac{4}{-3}\\y&=5x+\frac{4}{3}\end{aligned}$$

Rearrange the terms of the equation to make y the subject.

The gradient of the line is 5.

Example 3

★★☆

A straight line passes through points $\mathrm{A}\ (-2,\ \ 1)$ and $\mathrm{B}\ (3,\ -4)$.

Find the equation of the line in the form $ax+by+c=0$.

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First, find the slope of the line:

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Steps	Explanation
$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4-1}{3-(-2)} \\ &= -\frac{5}{5} \\ &= -1 \end{aligned}$	Use the gradient formula and substitute the coordina given points into the formula. Simplify. Simplify.

Therefore, the gradient–intercept form of the line is $y = -x + c$.

Steps	Explanation
$\begin{aligned} y &= -x + c \\ 1 &= -(-2) + c \\ 1 &= 2 + c \\ c &= -1 \end{aligned}$	To find the value of c, use the coordinates of either given points to substitute x and y into the equation Here we use point $\text{A}\,(-2,\ \ 1)$.

So, the equation of the line in gradient–intercept form is $y = -x - 1$.

Steps	Explanation
$\begin{aligned} y &= -x - 1 \\ x + y + 1 &= 0 \end{aligned}$	Add $x + 1$ to both sides of the equation.

The equation of the line in general form is $x + y + 1 = 0$.

The point—gradient form of a line: $y - y_1 = m\,(x - x_1)$

Now consider a line with gradient m that passes through point $\text{A}\,(x_1,\ y_1)$. In this section, you will find another form for the equation of a straight line.

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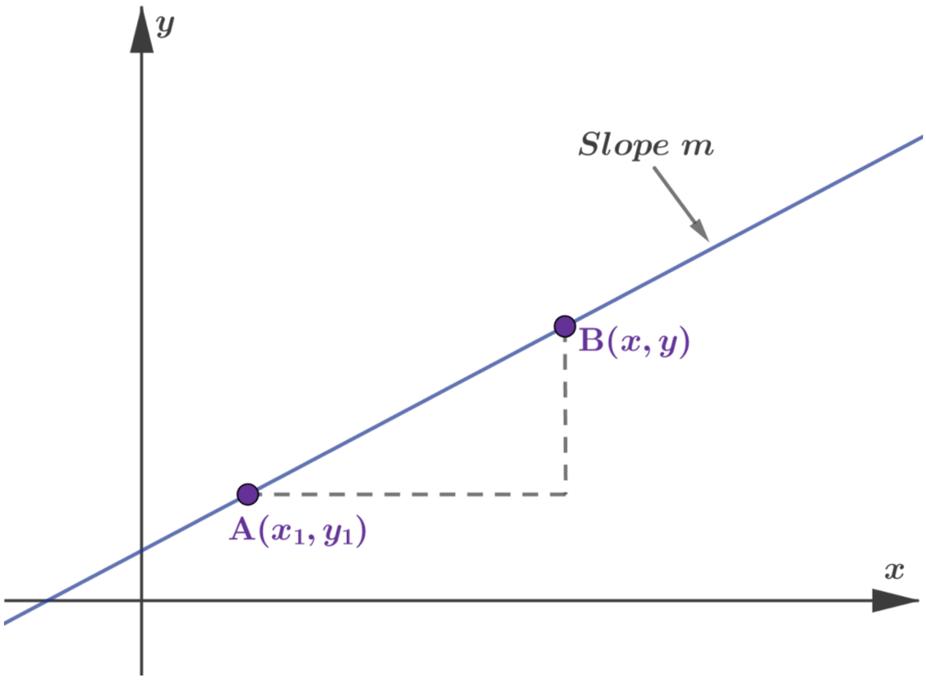
In the figure below, consider a line with gradient m that passes through point $\text{A}\ (x_1, y_1)$. Also, consider any random point on the line, B, with coordinates $(x,\ y)$. As both points A and B lie on the line, the gradient between the points must be equal to m.



More information

The graph shows a coordinate plane with a line passing through two points, A and B. Point A is located at coordinates (x1, y1) and point B at (x, y). The line has a slope denoted as 'm' and is represented by an arrow pointing along the line. The x and y axes are labeled, with the x-axis running horizontally and the y-axis vertically. A right triangle is indicated between points A and B showing the rise and run, visualizing the slope calculation.

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Applying the gradient formula for the points A and B gives

$$m = \frac{y - y_1}{x - x_1}$$

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Simplify this expression to obtain the point–gradient form of the line:

$$y - y_1 = m\left(x - x_1\right)$$

The advantage of this formula is that both substitutions (the gradient and the given point) are carried out at the same time.

In the following applet, you visualise a line and its equation in the point–gradient form. The slider controls the gradient m of the line. Drag the purple point of the line and observe how the equation of the line changes accordingly.

Interactive 2. A Line and Its Equation in Point Gradient Form.

More information for interactive 2

Student view

This interactive allows users to explore how the equation of a line changes when adjusting the slope or moving a fixed point. The interactive features a dynamic grid where users can drag the purple point (on the coordinate plane) to set its position, while a slider controls the slope m of the line, ranging from -10 to 10

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by the black point (on the slope slider). As the user adjusts these parameters, the equation of the line updates in real-time, providing an intuitive way to visualize the relationship between the line's graph and its algebraic representation.

The line follows the 'point-slope form equation': $y - y1 = m(x - x1)$, Here, $(x1, y1)$ is the fixed point on the line, and m is the slope. For example, if the fixed point is at $(2.1, 1.63)$ and the slope is set to $m = 0.3$, the equation becomes $y - 1.63 = 0.3(x - 2.1)$. Users can drag the point to new coordinates or adjust the slider to see how the equation and graph respond instantly.

This tool provides a dynamic way to explore how changes in the gradient and point position affect the line's equation, helping users understand the relationship between a line's graphical representation and its mathematical equation.

Example 4

★★☆

Find the equation of the line that has gradient $m = \frac{1}{2}$ and goes through the point $\mathrm{P}\left(4,\ 8\right)$. Give the equation of the line in the gradient–point form.

Transform the equation to the gradient–intercept form $y = mx + c$ and general form $ax + by + c = 0$.

Steps	Explanation
$y - y_1 = m\left(x - x_1\right)$ $y - 8 = \frac{1}{2}\left(x - 4\right)$	Use the gradient—point form $y - y_1 = m\left(x -\right.$ which $m = \frac{1}{2}$ is the gradient and (x_1, y_1) is th point $\mathrm{P}\left(4,\ 8\right)$.

Therefore, the point–gradient form of the line is $y - 8 = \frac{1}{2}\left(x - 4\right)$

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Steps	Explanation
$y - 8 = \frac{1}{2}(x - 4)$	Then transform the equation $y - 8 = \frac{1}{2}(x - 4$ gradient—intercept form by first expanding the
$y - 8 = \frac{1}{2}x - 2$ $y = \frac{1}{2}x + 6$	Then rearrange the terms to make y the subject

Finally, the general form of the equation of the line is

Steps	Explanation
$y = \frac{1}{2}x + 6$ $-\frac{1}{2}x + y - 6 = 0$ $-x + 2y - 12 = 0$	Multiply by 2.

Exam tip

Students are often unsure of how many lines of algebra to include in their exam solutions. Example 1 above shows 3 lines of working when transforming the equation of a line from one form to another.

In Paper 2 , full working is required to score full marks. On both Paper 1 and Paper 2, an answer that is approximately correct but not fully correct is awarded zero marks if no working is shown.

Now that you have seen all three forms of expressing the equation of a straight line, reflect on the advantages of each form and the information that each form provides. Consider some advantages and disadvantages of each.

3 section questions

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2. Functions / 2.1 Straight lines

Parallel lines and perpendicular lines

Parallel lines

From geometry you know that two lines are parallel if they lie on the same plane and remain the same distance apart over their entire length. In other words, two parallel lines on the same plane never cross each other. In this section, you will investigate a property of parallel lines.

In the following applet, the two lines are parallel. Adjust the position of the blue line by moving point A around. Moving point C around adjusts the position of the pink line. Adjust the position of both lines and observe how the gradients of the lines are related.

What do you notice?

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view

Interactive 1. Exploring Parallel Lines and Their Gradients.

More information for interactive 1

This interactive tool visually demonstrates the properties of parallel lines by allowing users to move two lines and observe how their equations change. The two lines one pink and one blue are always parallel, meaning they have the same gradient or slope.

The equation of the pink line is given as $y = 0.59x + 1.26$, and the equation of the blue line is $y = 0.59x + 0.44$. The key takeaway from this interactive is that parallel lines always share the same gradient but may have different y-intercepts, which determines their vertical position on the graph.

Users can interact with two key points: point A on the blue line and point C on the pink (red) line. Moving point C shifts the pink line vertically or parallel to the blue line without tilting, meaning that only the y-intercept in its equation changes while the gradient remains constant. This demonstrates that parallel lines can exist at different vertical positions but always maintain the same slope.

On the other hand, moving point A affects both the position and gradient of the blue line. When the blue line moves, the pink line also moves with it, maintaining parallelism. If the gradient of the blue line changes, the pink line automatically adjusts to ensure that both lines remain parallel, meaning they continue to have the same slope. However, shifting only the pink line does not alter its gradient; it simply changes its position

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while staying parallel to the blue line.

Through this interactive, users can observe how parallel lines behave when their position and gradient change, reinforcing the concept that parallel lines have identical slopes but can have different y-intercepts.

Important

For lines l_1 and l_2 with gradients m_1 and m_2 , respectively, l_1 is parallel to l_2 if and only if $m_1 = m_2$.

$$l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$$

Be aware

In mathematics, the symbol $\Leftrightarrow$ means 'if and only if' and indicates a two-way implication between two statements. For example, $A \Leftrightarrow B$ means that if A is true, then B is true and also, if B is true, then A is true.

For example, the lines $y = 3x - 4$ and $y = 3x + 7$ both have gradient $m = 3$.

The lines are parallel and are drawn in below.

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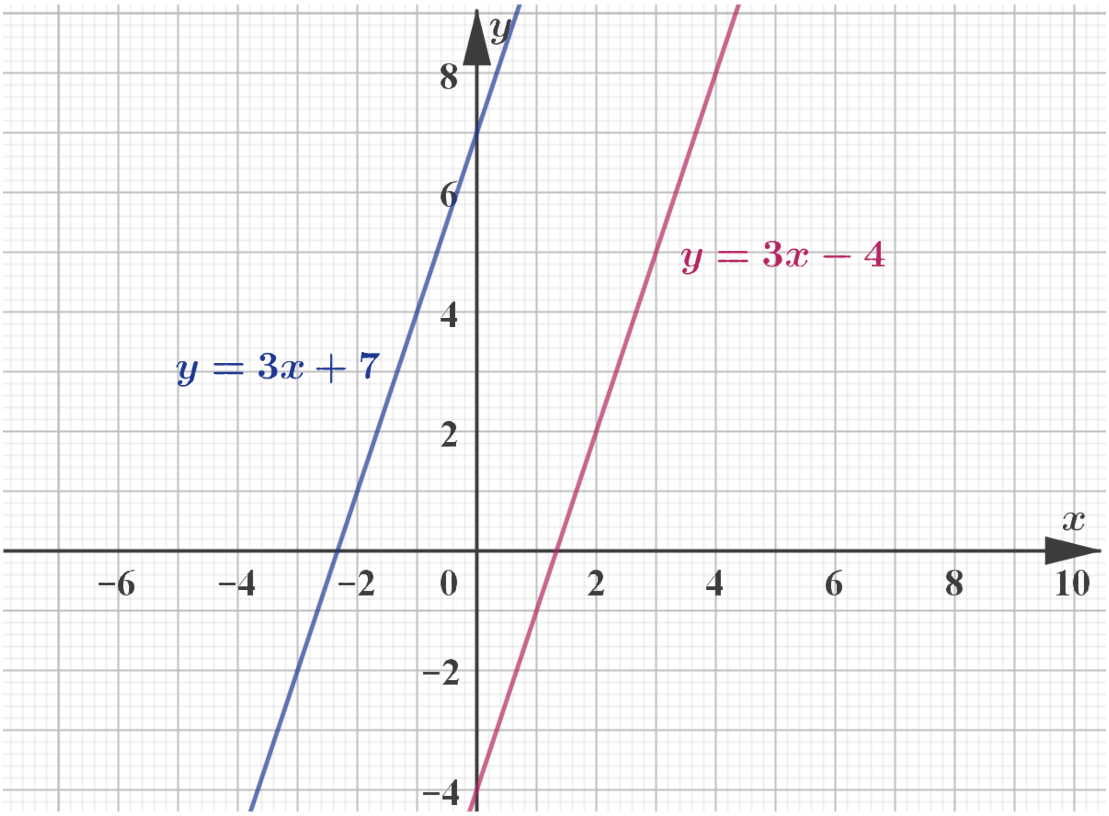

More information

The image is a graph with a grid background showing two linear equations represented as lines. The first line, labeled 'y = 3x + 7', is drawn in blue and is slanted upwards from left to right through the y-axis at +7. The second line, labeled 'y = 3x - 4', is drawn in pink and is also slanted upwards from left to right but passes through the y-axis at -4. Both lines have the same slope, indicating they are parallel and will never intersect. The x-axis is labeled with negative and positive values at equal intervals, while the y-axis includes markings at each integer value. This graph illustrates that even though both lines have the same slope, the y-intercepts are different, confirming the theory that parallel lines with different y-intercepts never meet.

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Theoretically, if you could continue flat space infinitely, these lines would never cross (intersect) each other. Using algebra and setting the equations for y equal to each other, what do you notice? What is the geometrical significance of this?

This would give

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$$\begin{aligned} 3x - 4 &= 3x + 7 \\ -4 &= 7 \end{aligned}$$ subtracting $3x$ from both sides.

But −4 does not equal 7, which demonstrates that *no* value for x exists such that the value of the expression $3x - 4$ is equal to the value of the expression $3x + 7$. Geometrically, this means that there is no point in common on both lines and that the lines do not intersect, so they must be parallel.

Perpendicular lines

Lines are perpendicular to each other when they intersect at 90° . In the following applet, you visualise lines that are perpendicular. Drag point A or C to adjust the position of the perpendicular lines and observe the relationship between their gradients. What do you notice?

Interactive 3. Property of Parallel Lines.

More information for interactive 3

This interactive allows users to explore the properties of perpendicular lines. In the interactivity, there are two perpendicular lines, one is pink and other is blue. Users can adjust the gradient of the blue line by dragging point A and move the pink line by dragging point C without changing its gradient. The interactive

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demonstrates that when two lines are perpendicular, the product of their gradients is always -1. This tool helps users understand the relationship between the gradients of perpendicular lines and how their positions affect their equations, providing a hands-on way to explore geometric concepts.

This interactive allows users to explore and verify the fundamental relationship between the gradients of perpendicular lines. By dragging points A and C, users can manipulate two lines on the coordinate plane and observe in real-time how their gradients and equations change. The key learning objective is to demonstrate that when two lines are perpendicular, the product of their gradients always equals -1 (i.e., $m1 \times m2 = -1$). The tool visually reinforces this concept by displaying both the lines' equations and their gradient product calculation, making abstract algebraic principles concrete and intuitive.

For example, Consider the two lines represented by the equations $y = -2.27x + 4.54$ and $y = 0.44x + 0.44$. When we examine these in slope-intercept form (y = mx + c), we find their slopes are $m1 = -2.27$ and $m2 = 0.44$ respectively. The key insight comes when we multiply these slopes: $-2.27 \times 0.44 = -1$ (within rounding error), perfectly demonstrating the fundamental rule that perpendicular lines have slopes whose product equals -1.

This approach bridges visual geometry with algebraic reasoning, helping learners internalize why perpendicular lines have negative reciprocal slopes.

Important

For two lines l_1 and l_2 with gradients m_1 and m_2 , respectively, l_1 is perpendicular to l_2 if and only if $m_1 \times m_2 = -1$.

$$l_1 \perp l_2 \Leftrightarrow m_1 \times m_2 = -1$$

The line $y = \frac{3}{4}x + 2$ and the line $y = -\frac{4}{3}x - 5$ are perpendicular and are drawn in below.

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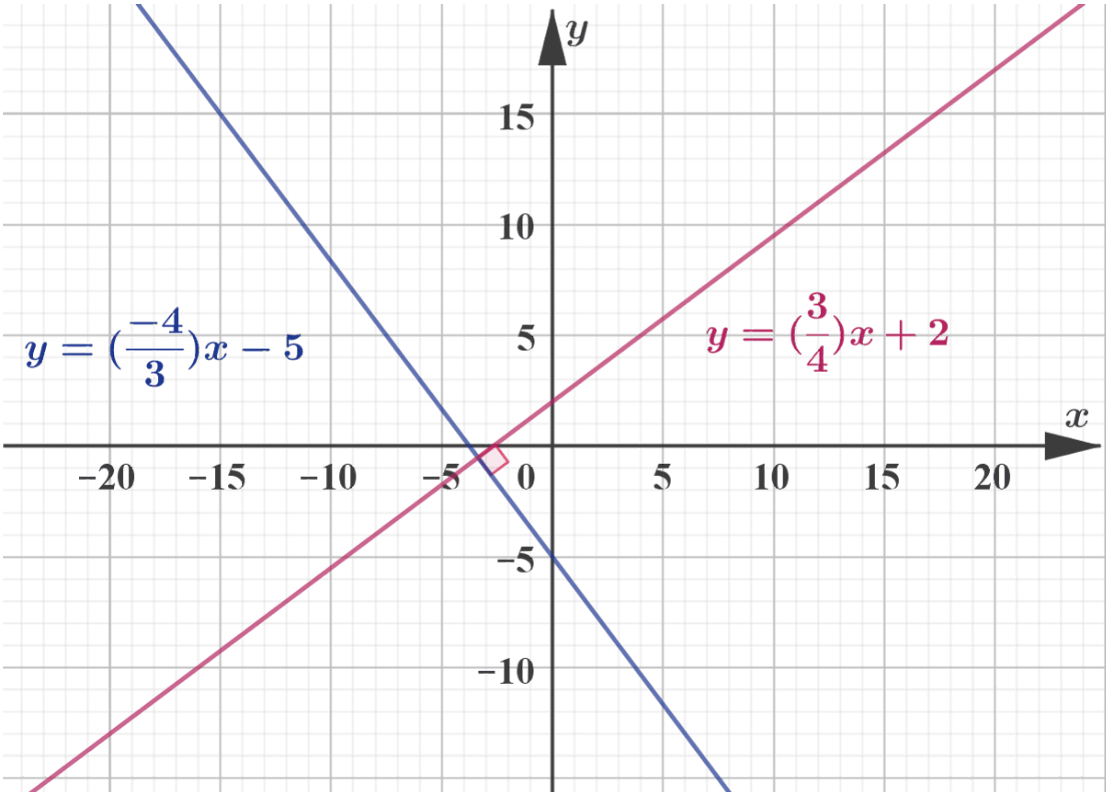

More information

This is a graph on a grid displaying two lines: one is y = (3/4)x + 2 and the other is y = -(4/3)x - 5. Both lines are drawn on an x-y plane.

- The x-axis spans from -20 to 20 with intervals marked along the axis, while the y-axis ranges from -15 to 15.
- The line y = (3/4)x + 2 is highlighted in a red or magenta color. It starts from the point (0, 2) on the y-axis and moves diagonally upward from left to right.
- The line y = -(4/3)x - 5 is depicted in blue. It begins at (0, -5) on the y-axis and also moves diagonally but downward from left to right.
- These two lines intersect at a right angle (90 degrees) at the point (0, 0), which is the origin.
- The grid provides detailed coordinates for determining the slope and intersection of these lines based on the given linear equations.

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These lines intersect at 90° . What is the product of their gradients?

The product of their gradients is

$$m_1 \times m_2 = \frac{3}{4} \times \left(-\frac{4}{3}\right) = -1$$

The gradient of the first line is the negative reciprocal of the gradient of the second line, as their product is equal to -1.

Does the relationship $m_1 \times m_2 = -1$ always hold true for perpendicular lines? What if one of the lines is horizontal?

Be aware

Consider a horizontal line l_1 with gradient $m_1 = 0$. The perpendicular line l_2 to l_1 is a vertical line with undefined gradient m_2 . Therefore, the relationship $m_1 \times m_2 = -1$ is **not** valid for horizontal and vertical lines that are perpendicular.

Example 1

★★☆

Write, in general form, the equation of the line parallel to $2x + y - 6 = 0$ that passes through the point $(2, -6)$.

Steps	Explanation
$2x + y - 6 = 0$ $y = -2x + 6$ So, gradient *m* = –2.	First, find the gradient of the given line by transformi equation $2x + y - 6 = 0$ into gradient—intercept fo
$y = -2x + c$	The lines are parallel, so they have equal gradients. Substitute *m* = –2 into *y* = *mx* + *c*.

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Steps	Explanation
$-6 = -2(2) + c$	Since the point $(2, -6)$ lies on the line, its coordinate satisfy the equation of the line.
$-6 = -4 + c$	Substitute for x and y using the coordinates of the p line passes through.
$c = -2$	Solve for c.
So, the equation of the line parallel to $2x + y - 6 = 0$ that passes through the point (2 equation $y = -2x - 2$.	
So, the equation of the line is $2x + y + 2 = 0$.	Write the equation in general form.

Example 2

★★☆

Write the equation of the line perpendicular to $3x + 2y - 1 = 0$ that passes through point $(-2,\ 5)$.

Steps	Explanation
$3x + 2y - 1 = 0$ $2y = -3x + 1$ $y = -\frac{3}{2}x + \frac{1}{2}$ The gradient of the given line is $m = -\frac{3}{2}$.	First, transform the equation of th line into gradient—intercept form the gradient of the line.

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Steps	Explanation
$$m = \frac{2}{3}$$ $$m \times \left(-\frac{3}{2}\right) = -1$$	The lines are perpendicular, so us $m_1 \times m_2 = -1$: Solve for m.
Therefore, the equation of the perpendicular line is $$y - 5 = \frac{2}{3}(x - (-2))$$ $$y - 5 = \frac{2}{3}(x + 2)$$	Write the equation of the perpenc in gradient-point form.

Example 3

★★☆

Determine the equation of the line perpendicular to $y = 4$ that passes through $(-1,\ 7)$.

The line $y = 4$ is horizontal and parallel to the x -axis. A line perpendicular to a horizontal line will be a vertical line with equation $x = a$. Since the line you are looking for passes through point $(-1,\ 7)$, its equation is $x = -1$.

Example 4

★★★

Determine the equation of the line perpendicular to the line $2x + 4y - 7 = 0$ that has the same x -intercept as the line $3x + y + 2 = 0$.

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Steps	Explan
$$3x + y + 2 = 0.$$ When $y = 0$, $$\begin{aligned} 3x &= -2 \\ x &= -\frac{2}{3} \end{aligned}$$	First, find the intercept of $3x + y + 2$ substituting the equation line.
So, the line passes through the point $$\left(-\frac{2}{3}, 0\right).$$	This is the x
$$\begin{aligned} 2x + 4y - 7 &= 0 \\ 4y &= -2x + 7 \\ y &= -\frac{2}{4}x + \frac{7}{4} \\ y &= -\frac{1}{2}x + \frac{7}{4} \end{aligned}$$	Next, transfo equation of l $2x + 4y - 7$ gradient—in form in orde its gradient.
The gradient of the perpendicular line is the negative reciprocal of $-\frac{1}{2}$, which is equal to 2.	Use $m_1 \times m$
The equation of the perpendicular line is: $$\begin{aligned} y - y_1 &= m\left(x - x_1\right) \\ y - 0 &= 2\left(x - \left(-\frac{2}{3}\right)\right) \\ y &= 2\left(x + \frac{2}{3}\right) \\ y &= 2x + \frac{4}{3} \end{aligned}$$	Use the grad point form w $m = 2$ and $\left(-\frac{2}{3}, 0\right)$ to equation of t perpendicul

Example 5

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Find the value of k if the lines $2x + 4y - 1 = 0$ and $kx - 3y + 2 = 0$ are

- parallel
- perpendicular.

Steps	Explanation
Let l_1 be the line $2x + 4y - 1 = 0$ with gradient m_1 and let l_2 be the line $kx - 3y + 2$ gradient m_2.	
$$\begin{aligned} 2x + 4y - 1 &= 0 \\ 4y &= -2x + 1 \\ y &= -\frac{1}{2}x + \frac{1}{4} \end{aligned}$$	Transform equation $2x + 4y - 1 = 0$ into gradient—intercept form to find its gradier
Therefore, the gradient of l_1 is $m_1 = -\frac{1}{2}$.	
$$\begin{aligned} kx - 3y + 2 &= 0 \\ -3y &= -kx - 2 \\ y &= \frac{-k}{-3}x - \frac{2}{-3} \\ y &= \frac{k}{3}x + \frac{2}{3} \end{aligned}$$	Transform equation $kx - 3y + 2 = 0$ int gradient—intercept form to find its gradier
Therefore, the gradient of l_2 is $m_2 = \frac{k}{3}$.	
$$\begin{aligned} m_1 &= m_2 \\ -\frac{1}{2} &= \frac{k}{3} \\ k &= -\frac{3}{2} \end{aligned}$$	The lines are parallel; therefore they have t gradient.
$$\begin{aligned} m_1 \times m_2 &= -1 \\ -\frac{1}{2} \times \frac{k}{3} &= -1 \\ -\frac{k}{6} &= -1 \\ k &= 6 \end{aligned}$$	The lines l_1 and l_2 are perpendicular, there $m_1 \times m_2 = -1$.

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Theory of Knowledge

As your breadth of knowledge in regard to both math and philosophy grows, you may be struck by the fact that some of the most famous mathematicians of all time are also some of the most famous philosophers. Rene Descartes, Gottfried Leibniz, Francis Bacon, Alfred North Whitehead, Bertrand Russell, and many other great thinkers were attracted to mathematics because of their belief in the ability of maths to explain existence.

Though their applications and goals were different, Descartes and Russell in particular believed that language could be used to 'prove' different elements of existence in the same way mathematics can be used to prove that two lines are parallel to one another.

A question of **scope**, as well as **method** and **tools**, emerges, 'Do the methods of mathematical inquiry limit the scope of mathematical application?'

4 section questions

2. Functions / 2.1 Straight lines

Checklist

What you should know

By the end of this subtopic you should be able to:

- calculate the gradient between points $\mathrm{A}\left(x_1, y_1\right)$ and $\mathrm{B}\left(x_2, y_2\right)$ using the formula $m=\frac{y_2-y_1}{x_2-x_1}$
- find the equation of a line in the gradient—intercept form $y=mx+c$
- find the equation of horizontal lines of the form $y=a$
- find the equation of vertical lines of the form $x=a$

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- find the equation of a line in the general form $ax + by + c = 0$, where $a,\ b$ and c are integers
- find the equation of a line in the gradient-point form $y - y_1 = m\,(x - x_1)$, where $\mathrm{A}\,(x_1, y_1)$ is point on the line and m is the gradient
- find the equation of a line that is parallel to a given line, using the fact that parallel lines have equal gradients
- find the equation of a line l_1 that is perpendicular to a line l_2, using the fact that the gradients of perpendicular lines are negative reciprocals of each other $(m_1 \times m_2 = -1)$.

2. Functions / 2.1 Straight lines

Investigation

Important

The tangent to a circle is a straight line that touches the circle at only one point. This point is called the point of tangency.

From any point outside a circle, there is a pair of tangents to the circle.

In the following applet, visualise the tangents to a circle from a point outside the circle. The radii of the circle to the points of tangency are shown as well.

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Interactive 1. Points of Tangency.

More information for interactive 1

This interactive tool allows users to investigate the geometric properties of circle tangents on a Cartesian plane (with an x-axis typically ranging from -0.5 to 4.5 and a y-axis ranging from -0.5 to 5.5). Users can drag 2 key points: the circle's center (A), and an external point (C), which in turn updates the two points of tangency (D and E) and observe how the tangents from this point to the circle change. The applet also displays the radii from the center of the circle (point A) to the points of tangency (points D and E). Users can form hypotheses about the relationship between the tangents and the radii at the points of tangency, particularly focusing on the gradients of these lines.

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First, when the "Show gradients" option is enabled, it displays how the tangent at any point is always perpendicular to the radius at that point, mathematically verified by the product of their slopes equaling -1

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(e.g., when radius AD has slope 1.56, its tangent CD has slope -0.64, satisfying $1.56 \times -0.64 \approx -1$).

Second, the "Show lengths" feature proves that the two tangent segments from any external point to a circle are always equal in length. As users move to point C, they can observe both tangents adjusting while maintaining equal lengths and perpendicular relationships with their corresponding radii.

This interactive provides a comprehensive tool for understanding the geometric properties of tangents to a circle, including their gradients and lengths, and offers a hands-on approach to exploring and formalizing these concepts.

- Adjust point C and observe the tangents to the circle. Form a hypothesis about the relative position between the tangents of the circle and the radii of the circle at the point of tangency.
- For your own points A and C, find the gradient of the tangent lines and the corresponding radii at the point of tangency.
- Formalise your observation by forming a rule consistent with your findings.
- Check box ‘Show gradients’ to verify your rule.
- Adjust point C and form a hypothesis about the lengths of line segments CD and CE. Click the box ‘Show lengths’ to verify your hypothesis.
- For your own points A and C, calculate the distance of line segments CD and CE.
- Check box ‘Show lengths’ to verify your calculations.

Extension:

- In this video, you can see how to construct the tangent lines from a point outside a circle using compasses and a ruler.

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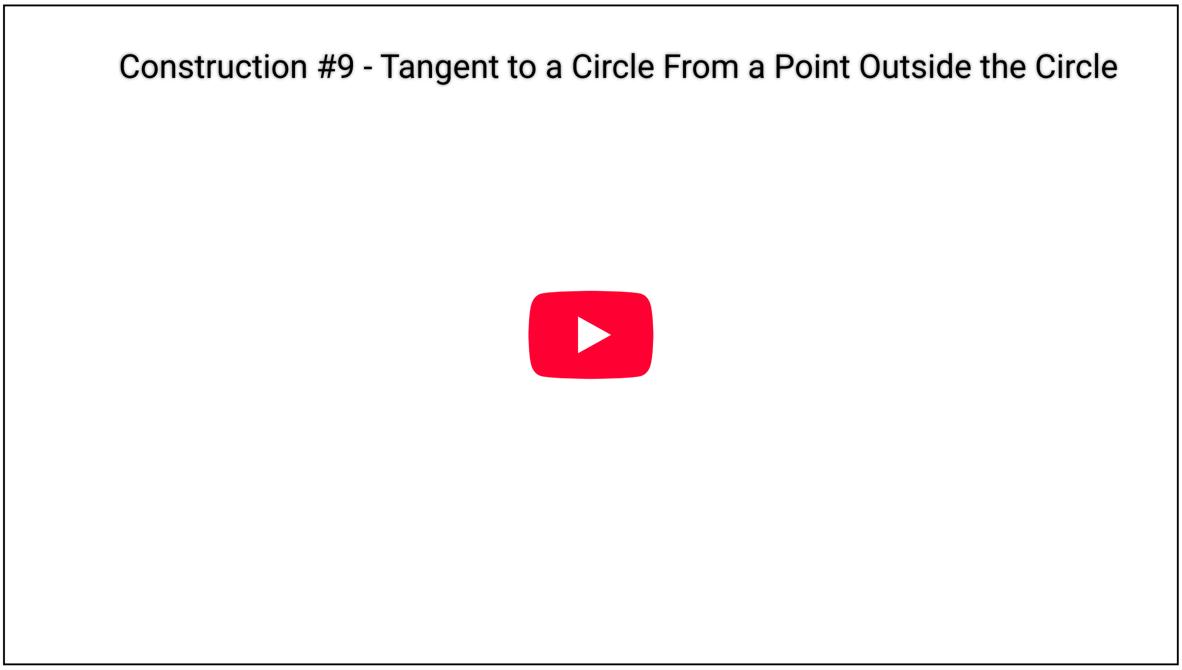

- Using compasses and a ruler, apply the method given in the applet below.

Interactive 2. Tangent Line Using Compasses and Ruler.

More information for interactive 2

This interactive allows users to construct tangent lines from an external point, referred to as point A, to a circle using compass and a ruler. Users can draw points, line segments, circles, and utilize the compass tool to perform geometric constructions. The interactive includes options to delete elements and a "undo" button for easy corrections.

To construct the tangent lines, users follow these steps: First, they draw a circle and mark point A outside it.

Next, they connect point A to the circle's center, O, with a line segment. Using the compass, they bisect this line segment to find the midpoint, M. With M as the center and MO as the radius, they draw a new circle that intersects the original circle at two points, say B and C. Finally, they draw straight lines from point A to points B and C. Now on joining the line segments AB and AC, they get two tangents to the circle from point A. This interactive provides a practical way to explore and understand the geometric principles behind tangent constructions.

Rate subtopic 2.1 Straight lines

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