

3.13 Scalar product

Checklist

What you should know

By the end of this subtopic you should be able to:

- define the scalar product as directional multiplication of vectors:

$$\mathbf{v} \cdot \mathbf{w} = v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

- define the scalar product as the projection of one vector onto another:
 $\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}| |\mathbf{w}| \cos \theta$, where θ is the angle between the vectors \mathbf{v} and \mathbf{w}
- recall that for, two vectors \mathbf{a} and \mathbf{b} , the scalar product is:
 - commutative: $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
 - distributive for addition and subtraction: $\mathbf{a} \cdot (\mathbf{b} \pm \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} \pm \mathbf{a} \cdot \mathbf{c}$
- recall that when a scalar product is multiplied by a scalar number m ,
 $m(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (m\mathbf{b})$
- use $\mathbf{b} \cdot \mathbf{b} = |\mathbf{b}|^2$
- recall that if $\mathbf{a} \cdot \mathbf{b} = 0$, then \mathbf{a} and \mathbf{b} are perpendicular and, conversely, if \mathbf{a} and \mathbf{b} are perpendicular, then $\mathbf{a} \cdot \mathbf{b} = 0$
- recall that if \mathbf{v} and \mathbf{w} are parallel, then $\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}| |\mathbf{w}|$.

