TOPIC 3
GEOMETRY AND TRIGONOMETRY

SUBTOPIC 3.5
PERPENDICULAR BISECTOR

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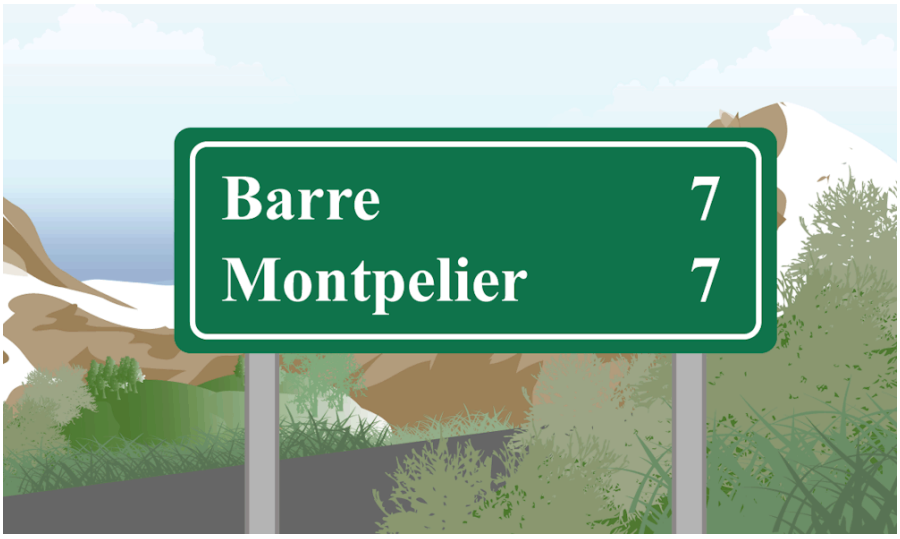
The big picture

Imagine two friends who live in different cities. They would like to meet somewhere that is the same distance from where they both live. In geometry, a set of points that satisfy the same condition or rule is called a locus of points. Examples are points on a line or a circle. In this case, the locus of points comprises the points that are equidistant from the two cities.

For two friends who would like to meet, this might be a simple problem of choosing the best restaurant within a reasonable distance for both. However, when planning a city or choosing where to site emergency services like ambulances, finding locations that are equidistant from other locations can be very important and help to save lives.



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Concept

The locus of points equidistant from two points can help to find optimum positions for siting services or meeting up. Can you see how it can be used to divide a plane into two? How is it useful for dividing a body of water between the states that surround it, such as in the allocation of fishing rights?

Theory of Knowledge

The key knowledge issue of formalism vs. Platonism in regard to mathematics is discussed in other TOK boxes throughout the course; however, it seems apropos to contemplate mathematics' rational origins in the context of rational functions.

Knower bias is a key factor in knowledge production and reception; however, at first glance, it seems that mathematics is immune to such biases because it is built on reason and has a very high level of real-world predictive validity.

Knowledge Question: To what extent can knowledge be free from bias?

Perpendicular bisector

Equidistant

Equidistant from one point

What is the geometric shape of the locus of points equidistant from a fixed point?

Student view

Say you take a piece of string and tie a piece of chalk to one end. Now fix the other end to a point C. By moving the string around the point while keeping it taut, you can draw a very familiar shape: a circle (see below). The centre of the circle is the point C and the

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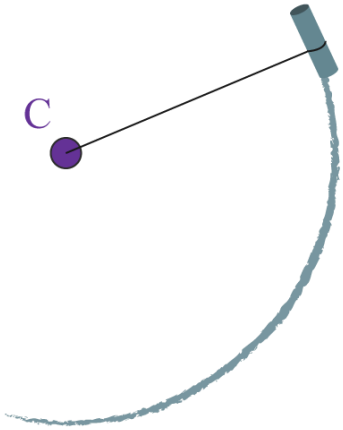
radius is the length of the string.



More information

The image depicts an example of drawing a circle using a string and a piece of chalk. The diagram shows a central point labeled 'C' around which the string is attached. The string is shown extended and taut, with a piece of chalk at one end marking out an arc on the surface as it swings around the point 'C'. The image effectively visualizes the explanation of how a circle is drawn by rotating the taut string, with 'C' being the center of the circle, and the length of the string representing the radius.

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Equidistant from two points

Say you have two fixed points. How would you find the locus of points that are equidistant from them?

Student view

Activity

You will need pieces of string of various lengths and a ruler.

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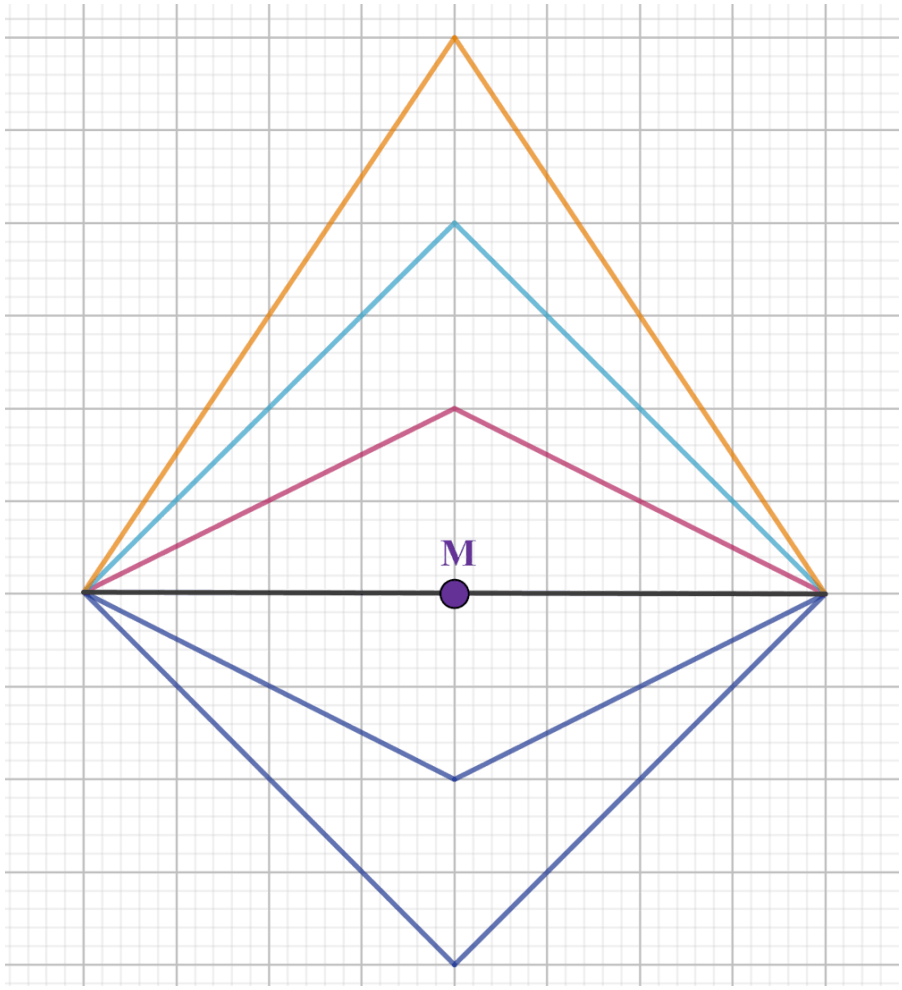
1. Draw a line and find the midpoint.
2. For each string, mark the midpoint and fix each end to the endpoints of the line.
3. Mark where the midpoint of the string is.

What do you notice? How would you describe the points?



More information

The image is a geometric diagram overlaid on a grid. It features a series of layered shapes: three triangles and a connected shape resembling a diamond, which are all aligned vertically. Each shape shares the same base point at the bottom center of the image and extends upwards to varying heights. The outermost shape is an orange triangle, the middle is a blue triangle, and the innermost is a pink triangle. In the center lies a blue shape similar to a diamond. At the heart of the diamond is a purple dot with the letter 'M' marked above it. The diagram is plotted on a grid making it easy to deduce measurements and alignments.

Student view

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International Mindedness

Some rivers and canals are the border between two or more countries, like the Akanyaru River in Africa. One bank of the river is in Rwanda and the other in Burundi.

How do countries decide which parts of a river or canal belong to them? Could this create conflict between them? How can they find the midline of a river or canal?

The locus of points equidistant from the endpoints of a line segment AB forms a line that is perpendicular to the segment and passes through its midpoint, as shown below. This line is also called the perpendicular bisector of AB , as it divides the segment into two equal parts and is perpendicular to it.

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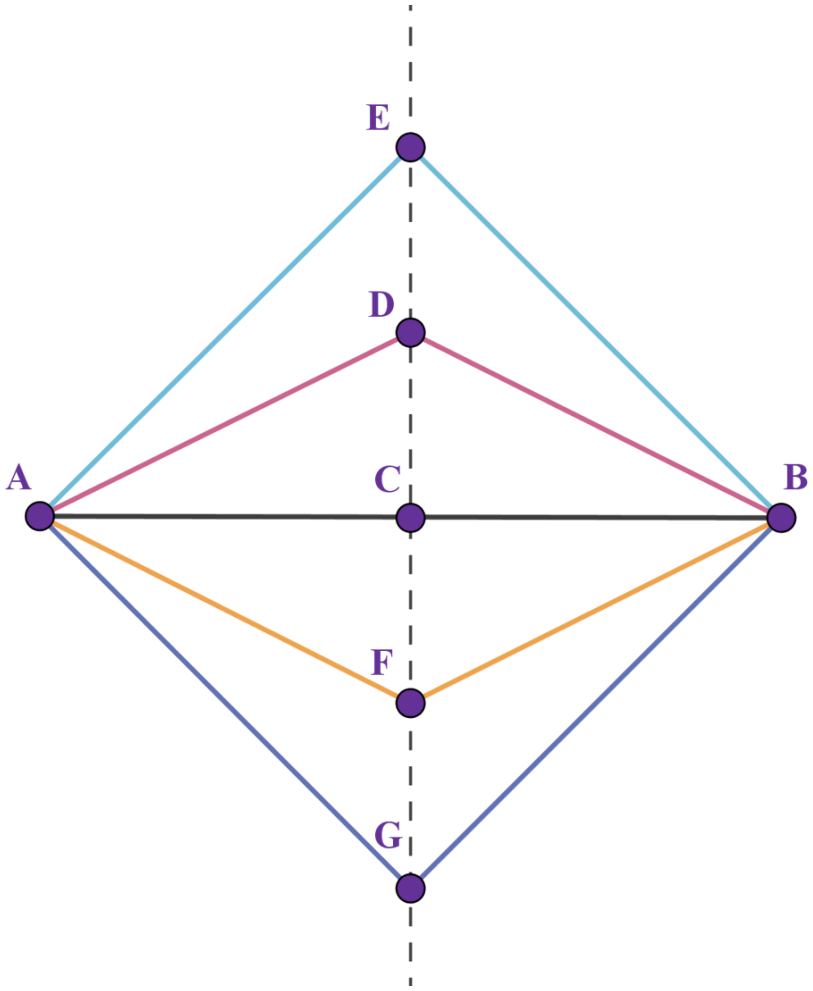

More information

The image shows a geometric diagram illustrating a perpendicular bisector of a line segment (AB). The line segment (AB) is shown horizontally with endpoints labeled (A) on the left and (B) on the right. (C) is marked at the midpoint of (AB). The diagram displays a vertical dashed line passing through (C), which represents the perpendicular bisector of (AB). The bisector is equidistant from (A) and (B) and intersects (AB) at (C).

Points (D), (E), (F), and (G) are positioned along the bisector. (E) is above (C), while (F) and (G) are below. All these points, along with (C), lie on the perpendicular bisector, maintaining equidistance from the endpoints (A) and (B). The lines (AD), (AE), (BF), and (BG) are shown in various colors, intersecting the bisector at points (D), (E), (F), and (G) respectively, illustrating the concept of equidistance.

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Student view

Now that you have defined the geometric location of these points, it is very straightforward to represent this line algebraically.

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Important

The locus of points that are equidistant from two points A and B is the perpendicular bisector of the line segment AB . The perpendicular bisector satisfies two conditions:

1. It is perpendicular to the segment AB .
2. It passes through the midpoint of AB .

Activity

You will need paper, a compass and a straight edge.

1. Draw a line.
2. Place the compass at one end, opened slightly wider than half its length.
3. Draw an arc above and below the line.
4. Keeping the same width, repeat step 3 at the other end.
5. Draw a line connecting the intersections of both arcs.

What do you notice?

Important

The intersections of two arcs drawn from the edges of a line segment AB with a radius greater than half of the length of the segment forms the perpendicular bisector. Any point on the perpendicular bisector, the red line, is equidistant from the two points A and B.

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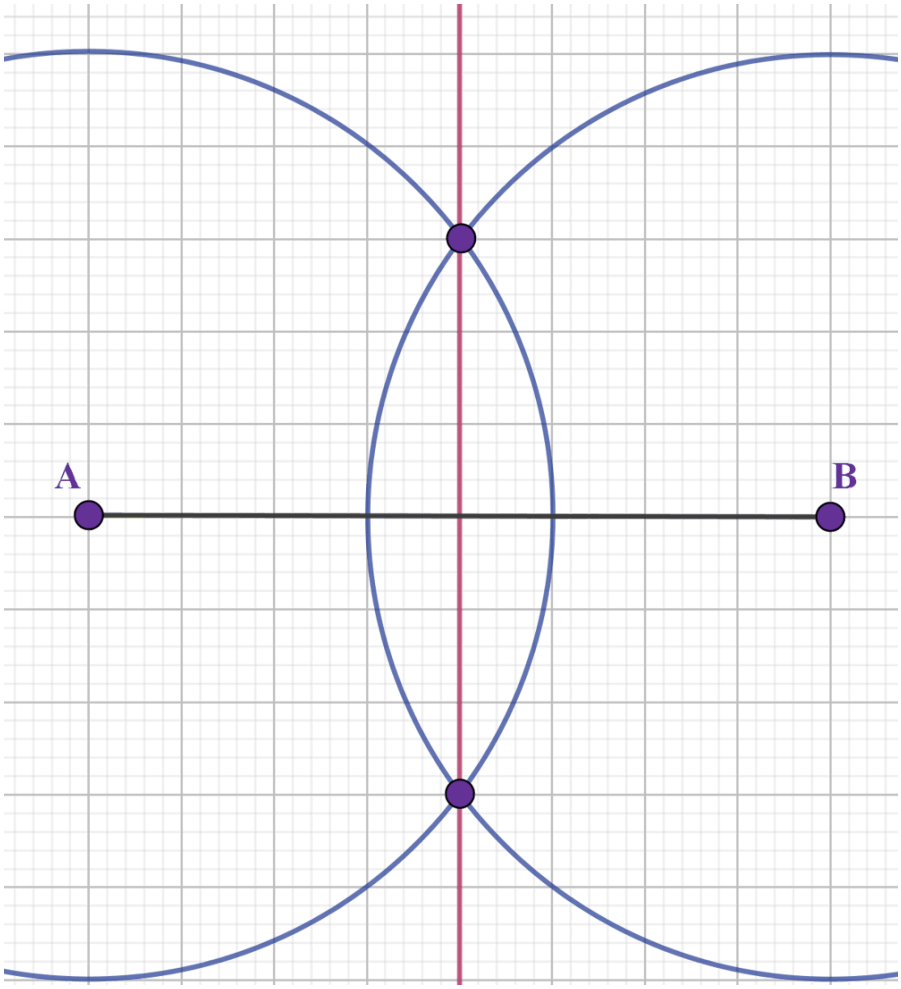

More information

The image depicts a geometric diagram on a grid. It illustrates two intersecting arcs centered at points A and B, which are positioned horizontally across from each other. These arcs intersect at two points above and below the line connecting A and B. A vertical red line passes through the intersection points, acting as the perpendicular bisector. Point A is on the left, and point B is on the right of the line connecting the two points. The perpendicular bisector divides the line segment AB into equal halves, and any point on this bisector is equidistant from A and B.

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Making connections

In subtopic 2.1 (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26160/) you studied straight lines. It might be useful to remember how to write the equation of a line and also the relationship between the gradients of two perpendicular lines.

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If two lines are perpendicular then their gradients are negative reciprocals, that is:

$$m_1 \times m_2 = -1$$

Example 1

★★☆

Find the equation of the perpendicular bisector of the line segment with endpoints $(2, 3)$ and $(4, 7)$.

Steps	Explanation
$m = \frac{7-3}{4-2} = 2$	Gradient of the line segment.
$m_P = -\frac{1}{2}$	Gradient of the perpendicular line.
$\left(\frac{2+4}{2}, \frac{3+7}{2}\right) = (3, 5)$	Midpoint of the segment.
$y - 5 = -\frac{1}{2}(x - 3)$	Use the equation of a line with a given p and gradient.
$y = -\frac{1}{2}x + 6.5$	Rearrange.
So, the equation of the perpendicular bisector is $y = -\frac{1}{2}x + 6.5$.	

Student view

Exam tip

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In IB examinations, the formula booklet gives these formulae:

- coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) :

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- gradient of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- equations of a straight line:

$$y = mx + c \text{ , } ax + by + d = 0 \text{ and } y - y_1 = m\left(x - x_1\right).$$

Example 2

★★☆

Find the equation of the points that are equidistant from the two points $\mathrm{A}(-1, 1)$ and $\mathrm{B}(3, 3)$.

Steps	Explanation
$\left(\frac{-1+3)}{2}, \frac{1+3}{2}\right) = (1, 2)$	Midpoint of the segment AB.
$m = \frac{1-3}{-1-3} = \frac{1}{2}$	Gradient of the line segment connecting A and
$m_P = -2$	Gradient of the perpendicular line.
$y - 2 = -2\left(x - 1\right)$	Use the equation of a line with a given point ar gradient.
$y = -2x + 4$	Rearrange.

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Steps	Explanation
Therefore, the equation is $$y = -2x + 4.$$	

Example 3

★★☆

In a thunderstorm, you hear the thunder after you see the lightning strike. Two friends who live near a beach were watching the same thunderstorm and heard the thunder at the same time. This means they must be equidistant from where the lightning struck.

If Anuman's house is 9 km south and 4 km east of the beach, and Boonsri's house is 8 km south and 7 km west of the beach, where did the lightning strike?

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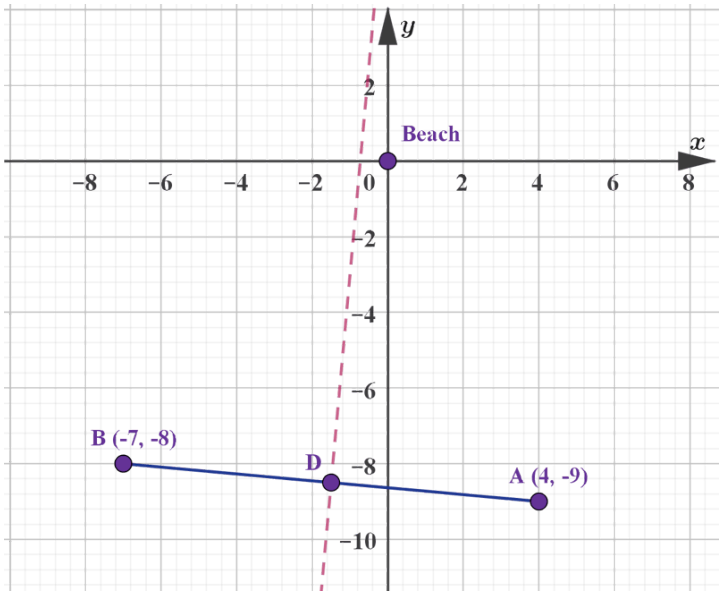
Steps	Explanation
	Since both heard the thunde same time, it must have hap at a place equidistant from b houses. Sketch the given informatior
$m = \frac{-8-(-9)}{-7-4} = -\frac{1}{11}$	Gradient of the line segment connecting A and B.
$m_P = 11$	Gradient of the perpendicul
$\left(\frac{4+(-7)}{2}, \frac{-9+(-8)}{2}\right) = (-1.5,\ -8.5)$	Midpoint of the segment.
$y-(-8.5) = 11\,(x-(-1.5))$	Use the equation of a line wi given point and gradient.
$y = 11x + 8$	Rearrange.
So, the lightning strike was somewhere on the line $y = 11x + 8.$	

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Be aware

In IB examinations, the relationship between the gradients of two perpendicular lines will not be provided. You should remember they are negative reciprocals, $m_1 \times m_2 = -1$ and you need to find the gradient using this relationship.

Thunderstorm in Thailand

Credit: Scott Brown / EyeEm

4 section questions

3. Geometry and trigonometry / 3.5 Perpendicular bisector

Checklist

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What you should know

By the end of this subtopic you should be able to:

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- find the coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) using

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- find the gradient of a line: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- use the equations of a straight line:

$$y = mx + c \text{ , } ax + by + d = 0 \text{ and } y - y_1 = m(x - x_1)$$

to find the gradient of a perpendicular bisector using

$$m_{\text{segment}} \times m_{\text{perpendicular bisector}} = -1$$

- put all this together to find the equation of a perpendicular bisector of a line segment with endpoints (x_1, y_1) and (x_2, y_2).

3. Geometry and trigonometry / 3.5 Perpendicular bisector

Investigation

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/investigation-id-27407/print/) Assign

The Republic of Kazakhstan is a large country. It is mainly in Asia but its most western areas are in Europe. As most of the land is uninhabited, it offers wonderful opportunities for sustainable tourism and a first-hand opportunity to visit areas with minimal human impact.

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Kazakhstan offers culture and nature

Credit: kiwisoul Getty Images

Imagine you are planning a trip to Kazakhstan. You are looking for a base where you can leave some of your luggage before you travel to different parts of the country.

You would like the base to be equidistant between two cities, Astana and Aktau, which is near the Caspian Sea.

Find all the possible places where you can camp. Make sure your campsite is suitable as a base, i.e. not in a lake or on a mountain top.

Paste the map of Kazakhstan into GeoGebra. You are **not** allowed to use any of the following tools:

- perpendicular bisector
- perpendicular line
- parallel line
- distance or length.

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Map of Kazakhstan

More information

A map of Kazakhstan highlighting the major cities. The map's background is colored yellow with a thick black border outlining the country's shape. Several red dots mark the locations of key cities across the map. The cities labeled are, from north to south: Petropavl, Kostanay, Kokshetau, Astana, Pavlodar, Oskemen, Semey, Karagandy, Temirtau, Satpayev, Uralsk, Aktobe, Atyrau, Aktau, Baikonur, Kyzylorda, Turkistan, Taraz, Shymkent, and Almaty. The map also shows parts of neighboring countries and water bodies surrounding Kazakhstan.

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When you arrive in Kazakhstan, you decide you should include the region around Almaty in your trip as well. Now that your base camp needs to be equidistant from three cities – Almaty, Aktau and Astana – where would you place it? Again, the location needs to be suitable for camping.

Rate subtopic 3.5 Perpendicular bisector

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