TOPIC 1
NUMBER AND ALGEBRA

SUBTOPIC 1.9
LAWS OF LOGARITHMS

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The big picture

Logarithms might seem abstract and irrelevant to anything outside mathematics when you first learn about them. It turns out that your brain thinks logarithmically in a way that you might not notice. Watch the video to learn more about this.

Weber's Law - Numberphile

After watching the video, think about the following:

In your own life have you experienced some of the logarithmic ways of thinking without noticing it?

Has the information about Weber’s law changed your perception of logarithms?

Student view

International Mindedness

Weber's law and people's perception of the ratio of $\frac{\Delta I}{I}$ can influence the response to economic and humanitarian crises. An event that has an inherently large value for the intensity I will need a larger ΔI (a larger shock to the system) to be perceived and acted upon.

Concept

Logarithms follow special rules for addition, subtraction and multiplication. These rules can be found by evaluating logarithms and noticing patterns, and they can be understood by using equivalent exponential forms. As you work through this subtopic, think about which approach appeals more to you and why.

Logarithms

You learned that $a^x = b$ (exponential form) is equivalent to $\log_a b = x$ for $a > 0$, $a \neq 1$ and $b > 0$ when you were introduced to logarithms in subtopic 1.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-26006/). You will use this relationship to further explore logarithms. In the following examples you will consider different bases to discover the properties of logarithms.

Example 1

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Rewrite $2^x = 3$ in logarithmic form and find the value of x .

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Steps	Explanation
$2^x = 3 \quad \Leftrightarrow \quad \log_2 3 = x$	The relationship, $a^x = b \Leftrightarrow \log_a b = x$ can be us for any base $a > 0, a \neq 1$.
$\log_2 3 = 1.58$ (3 significant figures)	Use your graphic display calculator. The instructic for graphic display calculator use are in subtopic

You should also be able to use the exponential form to evaluate simple logarithms without a calculator.

Example 2

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Evaluate each of the following without using a calculator.

$$\log_3 \frac{1}{9}$$

$$\log_4 64$$

$$\log_a a^m$$

	Steps	Explanation
a)	$\log_3 \frac{1}{9} = x \quad \Leftrightarrow \quad 3^x = \frac{1}{9} \quad \Leftrightarrow \quad 3^x = 3^{-2}$ $x = -2$ So $\log_3 \frac{1}{9} = -2$.	You should be able to recognise tha can be rewritten as a power of 3 .

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	Steps	Explanation
b)	$\log_4 64 = x \quad \Leftrightarrow \quad 4^x = 64 \quad \Leftrightarrow \quad 4^x = 4^3$ $x = 3$ So $\log_4 64 = 3$.	
c)	$\log_a a^m = x \quad \Leftrightarrow \quad a^x = a^m$ $x = m$ So $\log_a a^m = m.$	

Exam tip

You will be able to use your calculator to evaluate logarithms in the exam. However, you need to understand properties of logarithms when simplifying algebraic expressions such as $\log_a a^3$.

You can use the exponential form to deduce more interesting results for logarithms.

Example 3

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Evaluate each of the following without using a calculator.

$\log_a 1$

$a^{\log_a m}$

$\log_a a$

$\mathrm{e}^{\ln m}$

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	Steps	Explanation
a)	$\log_a 1 = x \quad \Leftrightarrow \quad a^x = 1$ $x = 0$ So $\log_a 1 = 0$.	
b)	Let $\log_a m = x$. Then $a^x = m$ So $a^{\log_a m} = a^x = m$.	Rewrite the logarithm in the exponent using the exponential form.
c)	$\log_a a = x \quad \Leftrightarrow \quad a^x = a$ $x = 1$ So $\log_a a = 1$.	
d)	Let $\ln m = x$. Then $\mathrm{e}^x = m$ So $\mathrm{e}^{\ln m} = \mathrm{e}^x = m$.	Rewrite the logarithm in the exponent using the exponential form. Remember that $\ln = \log_e$.

⚠ Be aware

The following properties of logarithms are useful for working with logarithms but are not included in the IB formula booklet. You should memorise them or be able to derive them from the definition of a logarithm:

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$a^{\log_a m} = m$$

$$\mathrm{e}^{\ln m} = m$$

Student view

Laws of logarithms

Activity

Evaluate each of the following:

- $\log_2 4,\ \log_2 8$ and $\log_2 32$
- $\log 100,\ \log 1000$ and $\log 100\,000$
- $\log_c c^3,\ \log_c c$ and $\log_c c^4$

Hence, deduce the relationship between $\log_a x + \log_a y$ and $\log_a xy$.

Go through a similar process to deduce the relationship between $\log_a x - \log_a y$ and $\log_a \frac{x}{y}$.

You should notice that logarithms follow a special set of rules when it comes to arithmetic. These rules are summarised as follows:

Important

Logarithms can be simplified using;

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^m = m \log_a x$$

These rules apply for $a,\ x,\ y > 0$ and $a \neq 1$.

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Exam tip

The laws of logarithms are given in the IB formula booklet and do not need to be memorised. When you use these laws it's important to remember that they only apply to logarithms with the same base.

Exam questions involving logarithm laws will only be asked for base 10 and base e.

Example 1

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Write $\log_7 xy^2 + \log_7 \frac{1}{xy}$ as a single logarithm.

Steps	Explanation
$\log_7 xy^2 + \log_7 \frac{1}{xy} = \log_7 \left(xy^2 \times \frac{1}{xy} \right) = \log_7 y$	Use $\log_a xy = \log_a x + \log_a y$.

Example 2

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Given that $\log_d 5 = 8$ and $\log_d 10 = 11$, find the value of $\log_d 2$.

Steps	Explanation
$\log_d 2 = \log_d \frac{10}{5}$	Recognise that $\frac{10}{5} = 2$.
$\log_d \frac{10}{5} = \log_d 10 - \log_d 5 = 11 - 8 = 3$	Use $\log_a \frac{x}{y} = \log_a x - \log_a y$.

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Example 3

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Given that $\log a = 2, \log b = -6$ and $\log c = 3$, find the value of $\log \dfrac{a^2c}{\sqrt[3]{b}}$.

Steps	Explanation
$\log \dfrac{a^2c}{\sqrt[3]{b}} = \log a^2 + \log c - \log \sqrt[3]{b}$ $= 2\log a + \log c - \dfrac{1}{3}\log b$	Remember that $\log a = \log_{10} a$. All the logarithms the question have the same base and the logarithn rules apply.
$= 2(2) + 3 - \dfrac{1}{3}(-6) = 9$	Replace the logarithms with their equivalent values

You can practise more questions, similar to **Example 3**, by using this applet.

Credit: GeoGebra (https://www.geogebra.org/m/eGCUSf3J) Kevin Hopkins

Making connections

In section 1.5.3 (/study/app/math-ai-hl/sid-132-cid-761618/book/natural-logarithms-id-26009/) you learned that' natural logarithms are logarithms with base e , that is, $\ln = \log_{\mathrm{e}}$.

Logarithm laws apply to natural logarithms and can be rewritten as:

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$$\ln xy = \ln x + \ln y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^m = m \ln x$$

Example 4

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Rewrite $\ln a + 3 \ln b - 12 \ln c$ as a single logarithm.

Steps	Explanation
$\ln a + 3\ln b - 12\ln c = \ln a + \ln b^3 - \ln c^{12}$ $= \ln \frac{ab^3}{c^{12}}$	Use the laws of logarithms.

You can practise more questions like **Example 4** by using this applet.

Credit: GeoGebra (https://www.geogebra.org/m/fdykgz2u) David, Kevin Hopkins

5 section questions

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Checklist

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What you should know

By the end of this subtopic you should be able to:

- rewrite exponential equations in any base in equivalent logarithmic form using $a^x = b \Leftrightarrow \log_a b = x$ for $a > 0,\ a \neq 1$ and $b > 0$
- evaluate logarithms with a calculator
- simplify algebraic expressions involving logarithms using
  - $\log_a a^m = m$
  - $\log_a 1 = 0$
  - $\log_a a = 1$
  - $a^{\log_a m} = m$
  - $e^{\ln m} = m$
- use the laws of logarithms to condense or expand logarithmic expressions.

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Investigation

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In the laws of logarithms, multiplication is related to addition, and division is related to subtraction. Where else have you seen this relationship?

Part 1

The following task will enable you to prove that $\log_a xy = \log_a x + \log_a y$.

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1. Let $\log_a x = m$ and $\log_a y = n$. Rewrite these in exponential form.

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2. Write an expression for xy and use it to show that $\log_a xy = m + n$.
3. Hence, prove that $\log_a xy = \log_a x + \log_a y$.

Deduce a similar method for proving that $\log_a \frac{x}{y} = \log_a x - \log_a y$.

Part 2

Watch the video below to learn how the laws of logarithms are derived and proved.

Rate subtopic 1.9 Laws of logarithms

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