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(https://intercom.help/kognity)

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The big picture

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In earlier subtopics you have already seen examples of moving objects and ways of modelling the movement. In this subtopic you will see further examples. The key words here are position (or displacement), velocity (which is the rate of change of position) and acceleration (the rate of change of velocity). The study of the motion of objects, specifically their displacement, velocity and acceleration, is called kinematics.

Making connections

To work out some of the problems in this subtopic, you should be familiar with the content of subtopic 5.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-26177/), subtopic 5.10 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27511/) and subtopic 5.11 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27512/).

Before you start, take a look at the following video.

At the beginning of the video, Ben Sparks asks you a question related to speed and braking distance of a car. He then explains the answer using principles you may have studied in physics. In this course, you will not learn the tools to fully understand that explanation, but you will learn other ways of finding the solution. Keep this example in mind and when you feel you have the tools, come back and work out the answer.

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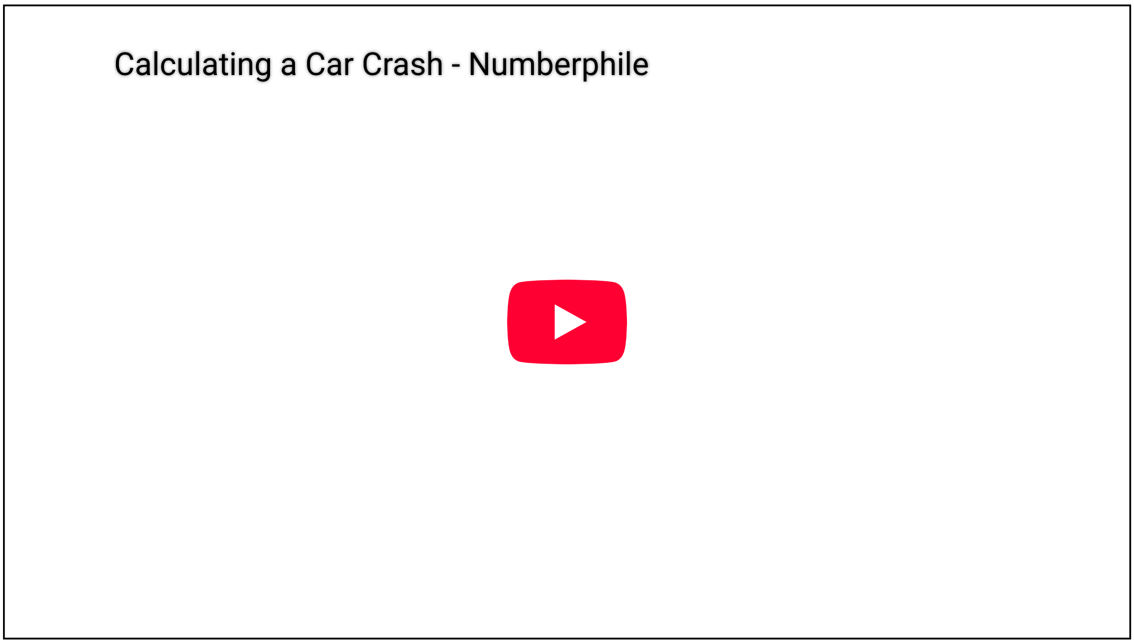

Concept

While working out the solutions of problems in this subtopic, keep in mind the **relationship** between position, velocity and acceleration, and how the information you are given **models** the motion of the object.

Theory of Knowledge

Kinematics is the application of calculus to motion. It can be considered to be both a description of something and as knowledge in and of itself, for it describes the movement of a particle or body while simultaneously providing new knowledge in the form of speed or location, for example.

Knowledge Question: Consider other areas of knowledge. Would you consider the scope of other AOKs to be both description and knowledge production?

5. Calculus / 5.13 Kinematics

Velocity

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This section concentrates on the relationship between the position and the velocity of an object moving along a straight line.

✓ Important

The displacement , $s(t)$, of an object at time t _ (https://www.codecogs.com/eqnedit.php?latex=t%251)is the difference between the position of the object at time t and the initial position. Displacement has a magnitude and also a direction. In exam questions, sometimes displacement is relative to a fixed point on the line, instead of being relative to the initial position.

The distance travelled , $d(t)$, represents the length of the path travelled by the object.

The velocity , $v(t)$ of the object is the rate of change of its displacement.

The notations $v(t) = s'(t)$, and $v = \frac{ds}{dt}$, are used to denote velocity. In addition, the notation $v = \dot{s}$ is also commonly used to indicate the first derivative of the displacement with respect to time.

The speed of the object is the magnitude of its velocity.

Several questions can be asked in relation to displacement and velocity of a moving object:

- Given the displacement at any time, can you find the velocity of the object?
  - The answer is yes, you can use differentiation. You will see examples like this in this section.
- Given the velocity at any time, can you find the position of the object?
  - The answer is no, some additional information is needed. You can think about it this way: If you are told that a car is moving at a speed of 90 _ (https://www.codecogs.com/eqnedit.php?latex=90%251)kilometres per hour, can you tell where it is after 2 hours? You can of course tell that it moved $2 \times 90 = 180$ kilometres (so the displacement relative to the starting position is 180 kilometres), but to tell where it is you need information about where it started from. These kinds of question are discussed in the next section.
- It can also happen that only a relationship between the displacement and the velocity is given. Tools that help you to find the displacement as a function of time

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in this case are covered in subtopic 5.18 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27513/).

Example 1

★★☆

A particle is moving along a straight line. The displacement (from the initial position) of the particle t seconds after the start is $s(t) = 3t - t^2$ metres.

a) What is the initial velocity of the particle?

b) When is the particle at rest?

c) How far is the particle from the initial position when it is at rest?

d) What is the speed of the particle 2 seconds after the start? Is the particle moving away from the initial position or moving towards it at this time?

e) What is the speed of the particle 4 seconds after the start? Is the particle moving away from the initial position or moving towards it at this time?

f) What is the maximum distance of the particle from the initial position for $0 \leq t \leq 4$?

	Steps	Explanation
a)	$v(t) = s'(t) = 3 - 2t$	The velocity is the rate of chang displacement.
	$v(0) = 3 - 2 \times 0 = 3$ metres per second.	The initial velocity corresponds t $t = 0$.
b)	$v(t) = 0$ $3 - 2t = 0$ $t = 1.5$	When the particle is at rest, ther $v(t) = 0$.

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	Steps	Explanation
	The particle is at rest 1.5 seconds after it starts.	
c)	$s(1.5) = 3 \times 1.5 - 1.5^2 = 2.25$	You need the displacement at th time you found above.
	The displacement of the particle from the origin when it is at rest is 2.25 metres.	
d)	$v(2) = 3 - 2 \times 2 = -1$ metre per second.	You can start with finding the velocity.
	$s(2) = 3 \times 2 - 2^2 = 2$ metres.	Negative velocity means that the particle is moving to the left on t number line. To see if it is movin towards or away from the origin position, you also need the posit
	After 2 seconds, the particle is moving towards the original position.	The displacement is positive anc velocity is negative.
	After 2 seconds, the speed is 1 metres per second.	The speed is the magnitude of tl velocity.
e)	$v(4) = 3 - 2 \times 4 = -5$ metres per second. $s(4) = 3 \times 4 - 4^2 = -4$ metres.	
	After 4 seconds, the particle is moving away from original position.	Negative velocity means that the particle is moving to the left on t number line. Negative displacement means th the particle is to the left of the or position.
	After 4 seconds, the speed is 5 metres per second.	The speed is the magnitude of tl velocity.

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	Steps	Explanation
f)	• For $0 < t < 1.5$ the particle is moving to the right, reaching a point 2.25 metres away from the starting position. • For $1.5 < t < 4$, the particle is moving to the left, passing through the initial position and reaching a point 4 metres to the left of the initial position.	You can describe the movement investigating the sign of the derivative. All the calculations needed have already been done in previous p
	Hence, the point furthest away from the initial position is the terminal point, so the maximum distance of the particle from the initial position for $0 \leq t \leq 4$ is 4 metres.	

On the applet below, you can explore the movement of the particle (represented by the boat) in relation to the initial position (shown by the purple dot). By moving the slider, you can check some of your answers.

Example 2

★★☆

A particle is moving along a straight line. The displacement (from a fixed point, P, on the line) of the particle t second after the start is $s(t) = e^{-t} + 2te^{-t}$ metres.

a) How far is the particle from P initially?

b) When is the particle changing direction?

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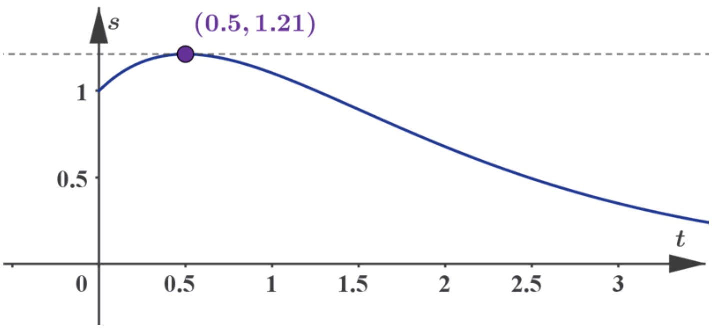
c) How far is the particle from P when it is changing direction?

d) How long does it take for the particle to get closer to P than 1 metre?

e) What is the speed of the particle when it is 0.5 metres away from P?

f) Describe the motion for $t > 0$.

	Steps	Explanation
a)	$s(0) = e^{-0} + 2 \times 0 \times e^{-0} = 1$ metre.	The term 'initially' refers tc $t = 0$.
b)	 The particle is at rest and changing direction $t = 0.5$ second after start.	At the time when the parti changing direction, the ve is changing sign, so $v(t)$ = Since $v = \dot{s}$ you can find t time by looking for the extremum point on the displacement—time graph
c)	The particle is 1.21 metres away from when it is at rest.	The second coordinate of maximum point gives the displacement.

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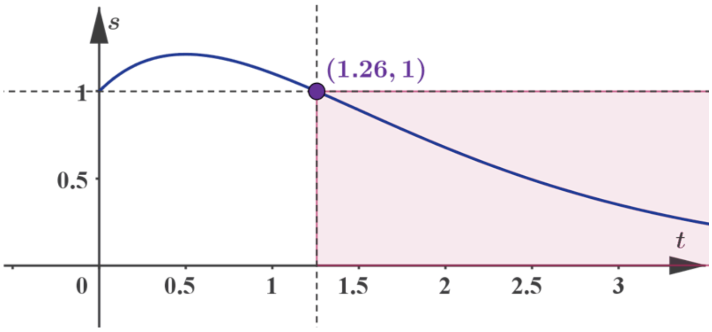
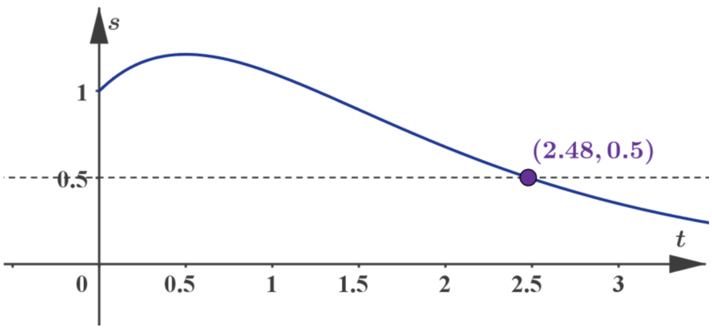
	Steps	Explanation
d)	 The particle is closer to P than 1 metre for $t > t_0$ where $t_0 \approx 1.26$.	The particle is closer to P 1 metre if $-1 < s(t) < 1$.
e)		You can use your graphic display calculator to find tl time when the particle is 0 metres away from P.
	$v(2.48) = \dot{s}(2.48) \approx -0.332$	You can also use your calc to find the gradient of a gr at any point.
	The speed of the particle is 0.332 metres per second when it is 0.5 metres away from P.	The speed is the magnituc the velocity.

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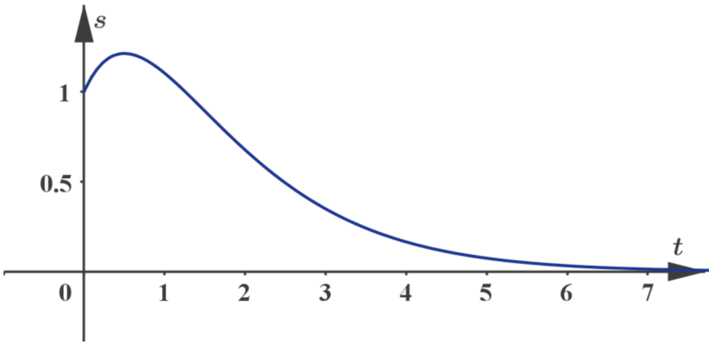
	Steps	Explanation
f)		The horizontal axis looks li horizontal asymptote to th graph.
	Initially the particle is moving away from P, then turns towards P and gets closer and closer to it.	

Note that in this example, the graphing ability of the calculator was used to answer the questions. However, in this case, the displacement function is such that you can also find the exact answers algebraically. Try this and compare your results with the approximate values found above.

Example 3

★★☆

A particle is moving along a straight line. The displacement (from a fixed point, P , (https://www.codecogs.com/eqnedit.php?latex=%5Ctext%7BP%7D%2C%251)on the line) of the particle t seconds after the start is $s(t) = e^{-t} + cte^{-t}$ metres.

For what values of $c > 0$ will there be a time when the particle is at rest?

Steps	Explanation
$\begin{aligned} s(t) &= \mathrm{e}^{-t} + ct\mathrm{e}^{-t} \\ &= (1+ct)\mathrm{e}^{-t} \\ v(t) = \dot{s}(t) &= c\mathrm{e}^{-t} + (1+ct)(-1)\mathrm{e}^{-t} \\ &= (c-1-ct)\mathrm{e}^{-t} \end{aligned}$	The velocity is the rate of ch of displacement.

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Steps	Explanation
$$\begin{aligned}(c-1-ct)\mathrm{e}^{-t}&=0\\ c-1-ct&=0\\ ct&=c-1\\ t&=\frac{c-1}{c}\end{aligned}$$	When the particle is at rest, $v(t)=0$.
For $0<c<1, \frac{c-1}{c}<0$, so for these values of c the particle is never at rest for $t>0$.	
For $c=1$, $\frac{c-1}{c}=0$, so for $c=1$ the particle is only at rest initially.	
For $c>1$, $\frac{c-1}{c}>0$ so for these values of c the particle is at rest at some point.	

Using the applet below, you can explore the movement of the particle for different values of the parameter c _(https://www.codecogs.com/eqnedit.php?latex=c%251)in the equation $s(t)=\mathrm{e}^{-t}+ct\mathrm{e}^{-t}$.

3 section questions

5. Calculus / 5.13 Kinematics

Distance travelled

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In this section, you will work on examples where the velocity is given and the questions relate to the position of a particle.

Example 1

★★☆

The velocity of a water rocket launched vertically t (https://www.codecogs.com/eqnedit.php?latex=t%251)seconds after the launch is given by $v(t) = 12 - 10t$

a) How long does it take for the rocket to get back to the ground?

b) What is the displacement (relative to the starting position) of the rocket at the time when it hits the ground?

c) What is the total distance travelled by the rocket?

	Steps	Explanation
a)	If $s(t)$ is the distance of the rocket from the ground t seconds after launch, then $$\begin{aligned} s(t) &= \int 12 - 10t \, \mathrm{d}t \\ &= 12t - 5t^2 + c \end{aligned}$$ for some value of c. Since $s(0) = 0$ (the rocket starts from the ground), $c = 0$. Hence, $s(t) = 12t - 5t^2$.	Since the velocity is the deri of displacement, $v = \frac{\mathrm{d}s}{\mathrm{d}t}$ th means that displacement is integral of the velocity, $s = \int v \, \mathrm{d}t$.

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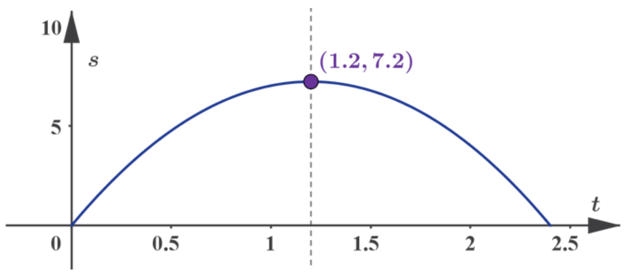
	Steps	Explanation
b)	$12t - 5t^2 = 0$ $t(12 - 5t) = 0$ The positive solution of this equation is $12 - 5t = 0$ $5t = 12$ $t = 2.4.$	When the rocket is on the g again, $s(t) = 0$.
	The rocket reaches the ground again after 2.4 seconds.	
	The displacement of the rocket at the time when it hits the ground is 0.	
c)	 $s(1.2) = 12 \times 1.2 - 5 \times 1.2^2 = 7.2$ metres.	The graph of the displaceme the rocket with respect to tir a parabola. The vertex is on the axis of symmetry, which is halfway between the two roots.
	The total distance travelled by the rocket is $2 \times 7.2 = 14.4$ metres.	The distance on the way up same as the distance on the down.

In **Example 1** you saw that the total distance travelled is not the same as the displacement. You can explore this further on the following applet.

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Activity

Using the applet you can see the velocity—time graph of the motion of a particle. By moving the point on the horizontal (time) axis, you can investigate the movement of the particle.

The applet also gives you the displacement and the total distance travelled by the particle.

- Can you see a relationship between these two values?

The applet also gives you the velocity in terms of time.

- Suggest a way of finding the displacement of the particle at any given time without finding $s(t)$ first. Use your GDC to check whether or not your method gives the value you see on the applet.

Interactive 1. Distance Travelled in Kinematics.

More information for interactive 1

This interactive is a graph that allows the user to understand the velocity—time graph of the motion of a particle. The graph ranges from 0 to 4 on the horizontal axis representing time and -1 to 1 on the vertical axis representing velocity. Below the graph, there is the equation for the velocity as a function of time: $v(t) = 0t^4 + 0.2t^3 - 1.2t^2 + 1.61t$

The blue curve represents the velocity of an object as a function of time, denoted as $v(t)$. The red dot is a movable point on the velocity-time curve. Its position indicates the velocity of the object at a specific chosen time. Using the "Adjust curve" checkbox, the curve of coefficients of the polynomial equation can be modified by the users (by dragging the red dot) which

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would in turn change the shape of the velocity-time graph. "Explore movement" Checkbox is used for animation or visual representation of the object's motion based on this velocity-time graph, which can be activated by checking this box.

For example, when users adjust the graph in such a way that it crosses the horizontal axis at 2: For the time interval, $0 < t < 2$, the velocity is mostly positive. The object would move in the positive direction. Both displacement and total distance travelled increases. At the time, $t = 2$, the velocity becomes zero, and the object momentarily stops. The "Displacement = 0.81 " and "Total distance travelled = 0.81 , is at its maximum positive value for this phase.

For the time interval, $2 < t < 4$, the velocity is negative. The object would move in the negative direction (back towards its starting point or beyond). The "Total distance traveled" continues to increase and the "Displacement" start to decrease.

At the end of the depicted time ($t = 4$), the "Displacement= 0.03" reflects the object's final position relative to its starting point, and the "Total distance travelled= 1.6" is the total length of the path it covered from $t = 0$ to $t = 4$.

The interactive will help the users to observe how these values change as the simulated movement progresses according to the velocity-time graph.

Did you notice the following relationships?

Important

If $v(t)$ is the velocity of a particle (moving on a straight line) at time t, then:

- The displacement between the position at time t_1 and the position at time t_2 is given by $\int_{t_1}^{t_2} v(t)\,\mathrm{d}t$.
- The distance travelled while getting from the position at time t_1 to the position at time t_2 is given by $\int_{t_1}^{t_2} |v(t)|\,\mathrm{d}t$.
- Since $v(t) = s'(t)$. the formula for the displacement is a consequence of the Newton—Leibniz formula (which you learn about in section 5.11.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/integrals-of-reciprocal-functions-id-27514/)),
$\int_{t_1}^{t_2} v(t)\,\mathrm{d}t = \int_{t_1}^{t_2} s'(t)\,\mathrm{d}t = s(t_2) - s(t_1)$.
- You need to take the magnitude of the velocity in the second formula because movement forwards and backwards both count positively when calculating the distance travelled.

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Exam tip

The formulae for the displacement and for the distance travelled are both in the formula booklet.

The next two examples are similar. The only difference is that in **Example 2**, algebraic methods are used to find the answers, whereas **Example 3** uses a calculator.

Example 2

★★☆

a) Find the derivative of $f(x) = \cos(\pi x^2)$.

The velocity, v, of a particle, at time t, is given by $v(t) = 2\pi t \sin(\pi t^2)$, where time is measured in seconds and distance is measured in metres.

b) Show that the particle is at rest at $t = 0$.

c) Find the first two times at which the particle changes direction.

d) Find the displacement of the particle from its initial position to the position at which it changes direction for the second time.

e) Find the distance travelled by the particle from its initial position until it changes direction for the second time.

	Steps	Explanatior
a)	$f(x) = \cos(\pi x^2)$ $f'(x) = -2\pi x \sin(\pi x^2)$	Chain rule.
b)	$v(0) = 2\pi \times 0 \times \sin(\pi \times 0^2) = 0$	

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	Steps	Explanation
c)	$2\pi t \sin(\pi t^2) = 0$ $\sin(\pi t^2) = 0$	The velocity is 0 when the particle changes direction. You need the positive solutions, so the equality can be divided by $t \neq 0$.
	• The first positive solution is $\pi t^2 = \pi$, $t = 1$. • The second positive solution is $\pi t^2 = 2\pi$, $t = \sqrt{2}$.	$\sin\theta = 0$ for $\theta = \pi, 2\pi, \ldots$
	$v(t)$ changes sign at these values, so the particle is changing direction after $t = 1$ and $t = \sqrt{2}$ second.	
d)	$\int 2\pi t \sin(\pi t^2)\,\mathrm{d}t = -\cos(\pi t^2) + c$	The first part of the question gives the indefinite integral of $v(t)$.
	$\int_0^{\sqrt{2}} 2\pi t \sin(\pi t^2)\,\mathrm{d}t = \left[-\cos(\pi t^2)\right]_0^{\sqrt{2}}$ $= -\cos\left(\pi(\sqrt{2})^2\right) - (-\cos(\pi(0)^2))$ $= -\cos(2\pi) + \cos 0 = -1 + 1 = 0$ So the displacement is 0, which means that the particle is back at its initial position when it changes direction for the second time.	The displacement can be calculated as a definite integral.
e)	$\int_0^{1} 2\pi t \sin(\pi t^2)\,\mathrm{d}t = \left[-\cos(\pi t^2)\right]_0^{1}$ $= -\cos(\pi(1)^2) - (-\cos(\pi(0)^2))$ $= -\cos\pi + \cos 0 = 1 + 1 = 2$	You can find the distance travelled in two steps. First you can find the displacement when the particle changes direction for the first time.

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	Steps	Explanation
	$\int_1^{\sqrt{2}} 2\pi t \sin(\pi t^2)\, dt = \left[-\cos(\pi t^2)\right]_1^{\sqrt{2}}$ $= -\cos\left(\pi\left(\sqrt{2}\right)^2\right) - \left(-\cos(\pi(1)^2)\right)$ $= -\cos(2\pi) + \cos\pi = -1 - 1 = -2$	Next you can find displacement bet the positions at w the particle chan direction for the and second time.
	The particle moves 2 metres forwards then 2 metres backwards, so altogether the distance travelled is 4 metres.	

Example 3

★★☆

The velocity, v, of a particle, at time t, is given by $v(t) = \sin(\pi t^2)$, where time is measured in seconds and distance is measured in metres.

a) Show that the particle is at rest at $t = 0$.

b) Find the first two times when the particle changes direction.

c) Find the displacement of the particle from its initial position to the position at which it changes direction for the second time.

d) Find the distance travelled by the particle from its initial position until it changes direction for the second time.

	Steps	Explanation
a)	$v(0) = \sin(\pi \times 0^2) = \sin 0 = 0$	

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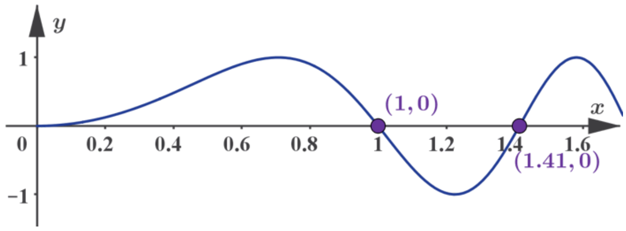
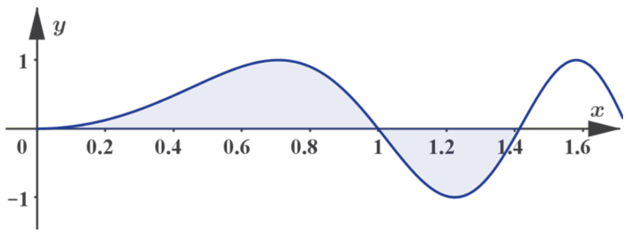
	Steps	Explanation
b)	 the particle is changing direction after $t = 1$ and $t \approx 1.41$ second.	The particle is changing direc when the velocity is changing GDCs have applications that c find the zeros of a function.
c)	The displacement is $$\int_0^{1.41} \sin(\pi t^2)\, dt \approx 0.243 \text{ metres.}$$	GDCs have applications that c find definite integrals.
d)	 The distance travelled (the area of the shaded region) is approximately 0.767 metres.	This part can be answered by finding a definite integral or b finding the area bounded by t graph and the horizontal axis.

Exam tip

In the examples above, the integral was split according to the sign of the velocity (according to the direction of the travel). On exams where calculator use is allowed, this is not necessary. Taking the integral of the absolute value of the velocity gives the total distance travelled.

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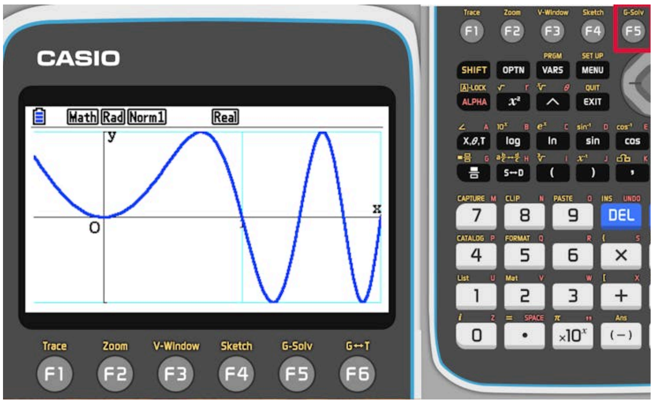
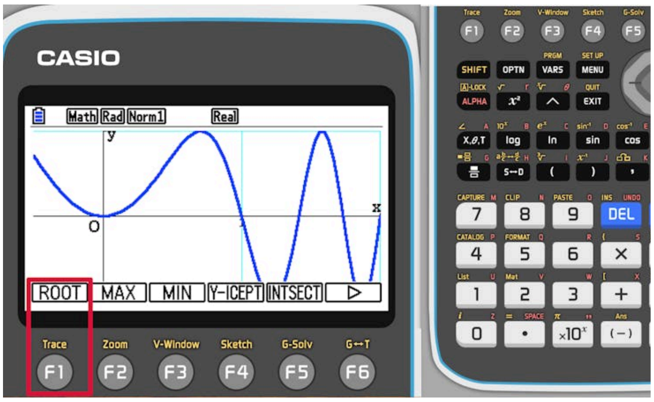
Steps	Explanation
These instructions show you a way to find the answers to the parts of Example 3. The guidance here assume that you have the graph of $y = \sin(\pi x^2)$ on the screen in a window, where the first two positive x-intercepts are visible. To find the x-intercepts, press F5 (G-Solv) to bring up the options to analyze the graph ...	
... and press F1 to look for the roots.	

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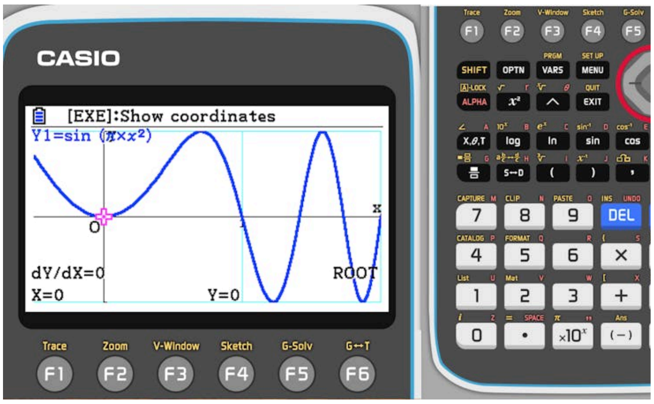
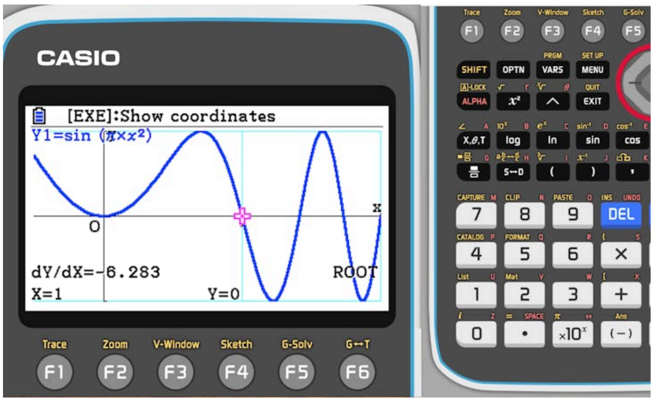
Steps	Explanation
The calculator will find one of the roots. To look for the other ones, move right (or left).	
This is one of the roots you needed to find in Example 3 ...	

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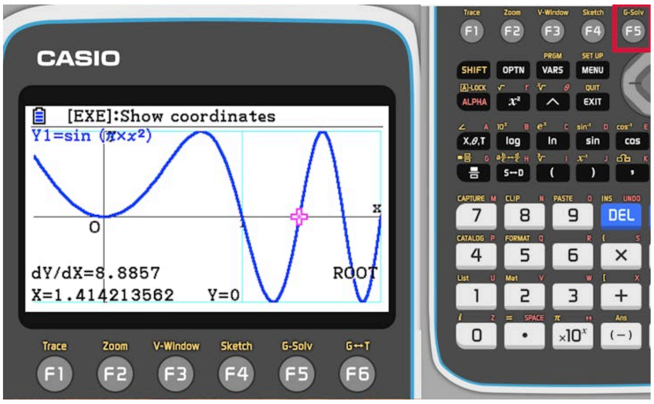
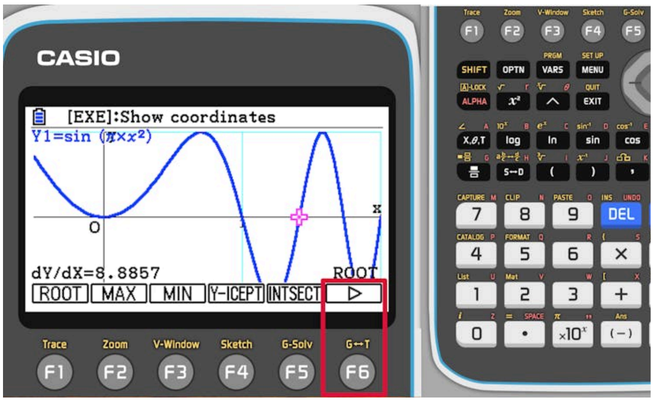
Steps	Explanation
... and this is the second one. The next step is to find the displacement and the distance traveled. Both of these can be calculated as a definite integral, so press F5 again to look for the option to find an integral.	
The definite integral is not among the first options, so press F6 to see the other possibilities ...	

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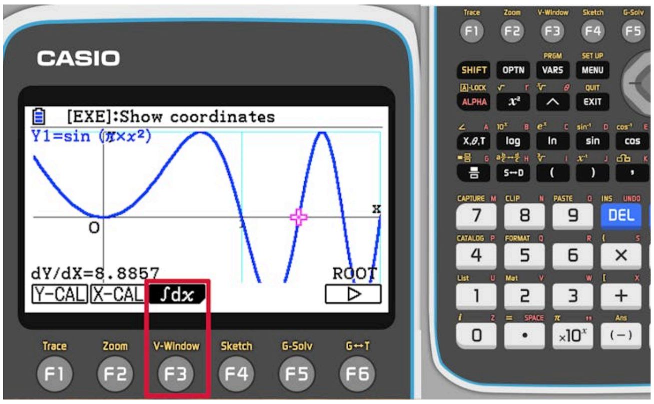
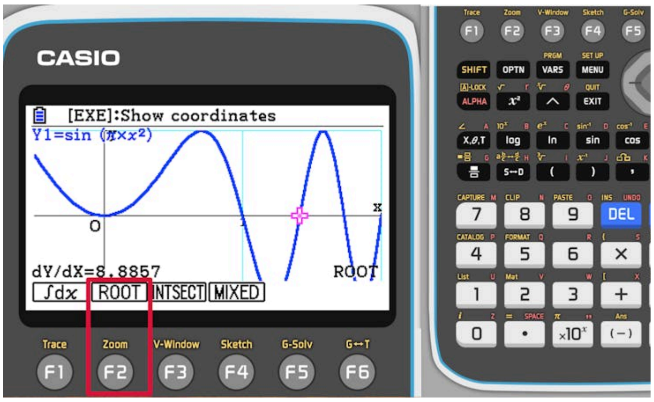
Steps	Explanation
... and press F3 to chose to find an integral.	
There are different ways you can specify the bounds of integration. Make sure you experiment with the different possibilities. In this case press F2 to use the roots as the bounds.	

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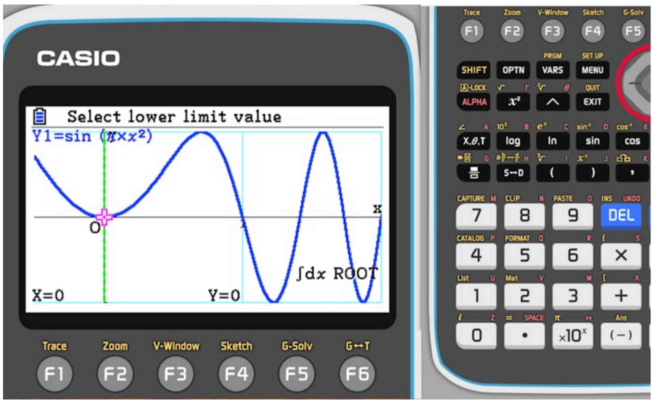
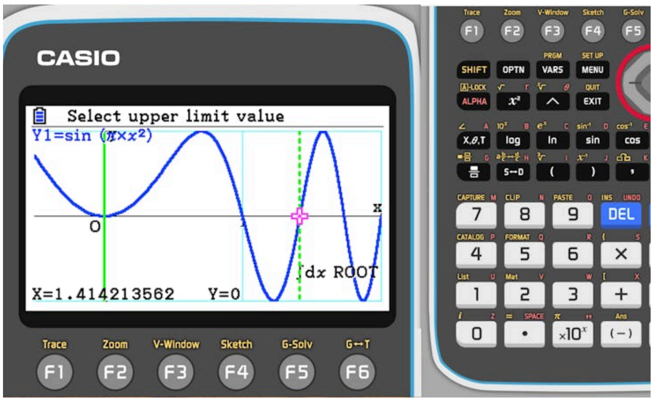
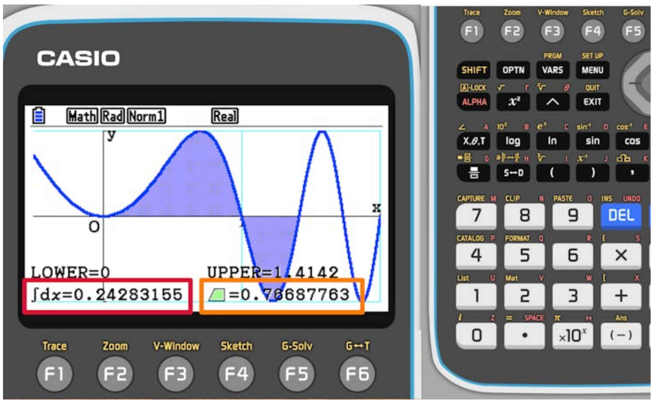
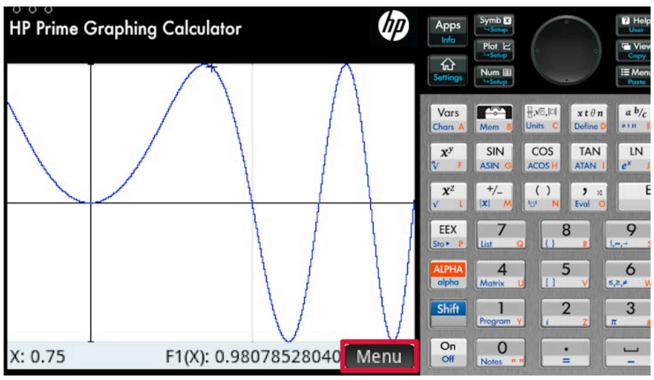
Steps	Explanation
Move left to the root and mark it as the lower bound for the integral ...	
... then move to the second positive root and select it as the upper bound.	

Student view

Steps	Explanation
The calculator gives you both $$\int_0^{1.41} \sin(\pi x^2)\mathrm{d}x$$ (which is the displacement) and $$\int_0^{1.41} \lvert \sin(\pi x^2)\rvert \mathrm{d}x$$ (which is the total distance traveled). In the integral the area of the region below the x-axis counts as negative. For the other number (indicated by the area symbol) both areas count as positive.	

Steps	Explanation
These instructions show you a way to find the answers to the parts of Example 3. The guidance here assume that you have the graph of $y = \sin(\pi x^2)$ on the screen in a window, where the first two positive x-intercepts are visible. To find the x-intercepts, open the menu to bring up the options to analyze the graph ...	

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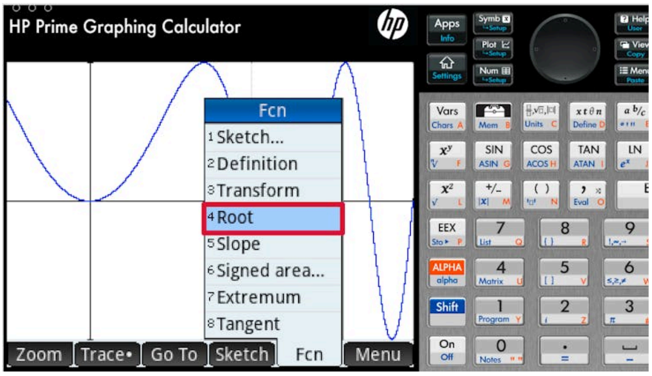
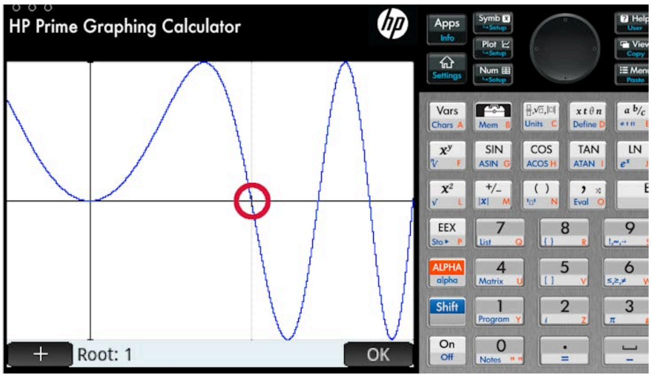
Steps	Explanation
... and choose the option to find the roots.	
The calculator moves the cursor to one of the roots and displays its x-coordinate.	

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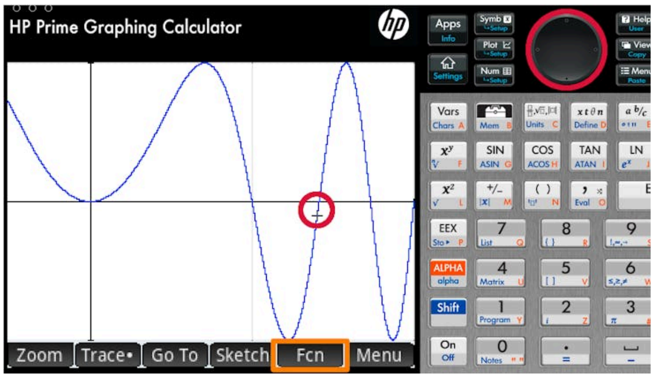
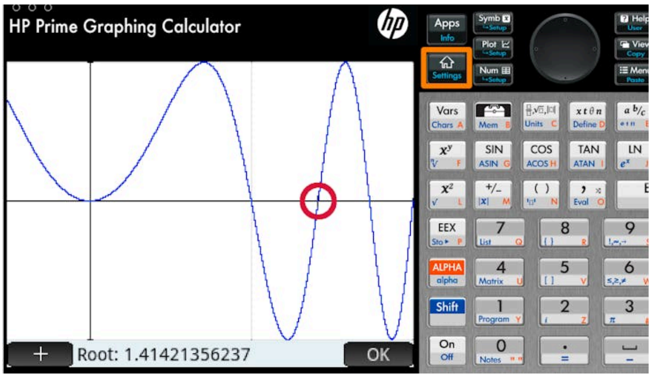
Steps	Explanation
To find the other root, move the cursor close to it and repeat the process using the function menu.	
Once again, the cursor is moved to the root and the x-coordinate is displayed. You will need this value as the limit of the integration in the next steps. To store this value, move to the home screen.	

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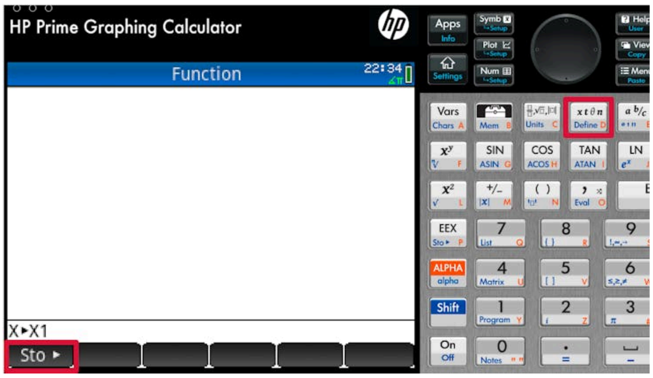
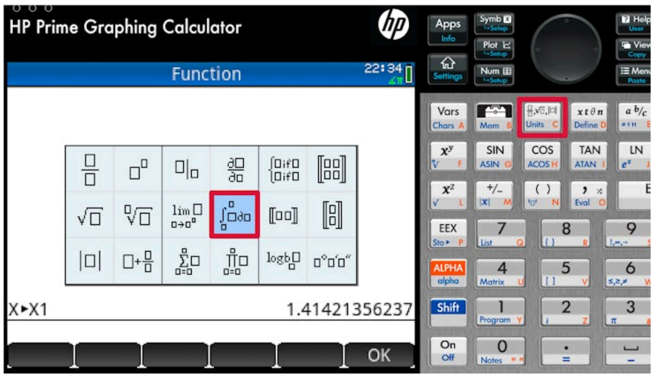
Steps	Explanation
The root is stored in the x variable, but since that variable is used all the time by the calculator, it is a good idea to store the current value to a variable of your choice. You can give any name.	
The next step is to find the displacement and the distance traveled. Both of these can be calculated as a definite integral, so open the template menu and choose the integral template.	

Overview
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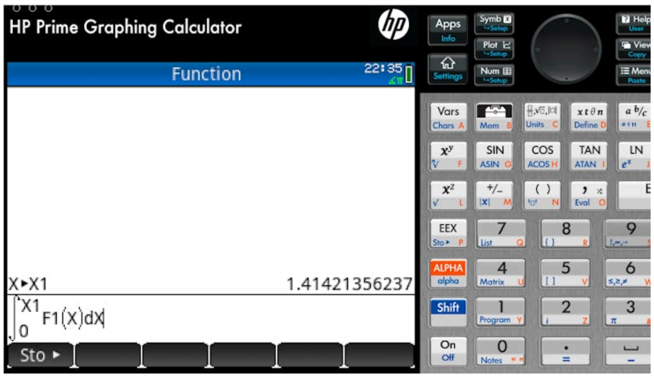
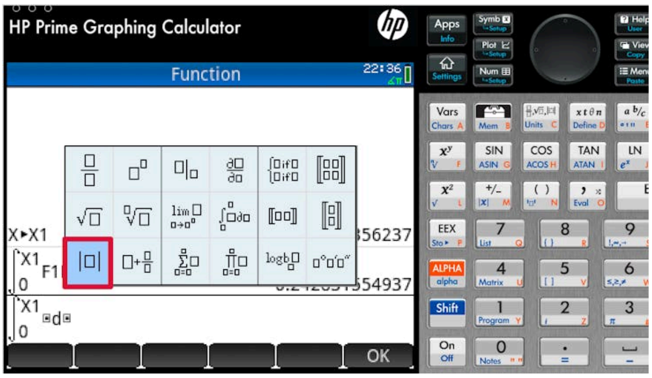
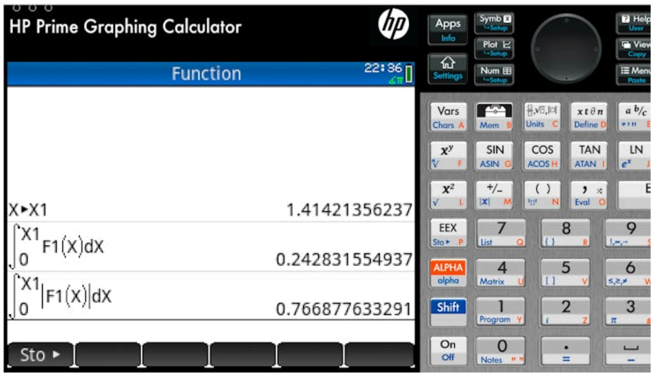
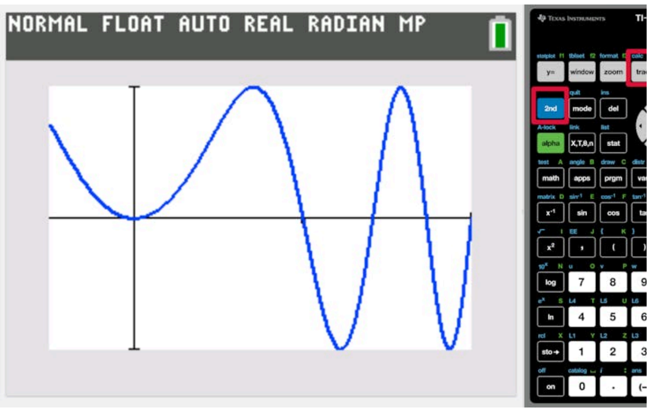
Steps	Explanation
The displacement is the integral of the velocity function. Use the variable name as the upper limit and use the function name where you stored the velocity function (for the graph) instead of typing the formula in again.	
Finding the total distance traveled is similar. The only difference is that you need to take the integral of the absolute value of the velocity function.	

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
On this screen yo can see both $$\int_0^{1.41} \sin(\pi x^2)\mathrm{d}x$$ (which is the displacement) and $$\int_0^{1.41} \lvert \sin(\pi x^2)\rvert \mathrm{d}x$$ (which is the total distance traveled).	

Steps	Explanation
These instructions show you a way to find the answers to the parts of Example 3. The guidance here assume that you have the graph of $y = \sin(\pi x^2)$ on the screen in a window, where the first two positive x-intercepts are visible. To find the x-intercepts, press 2nd/calc to bring up the options to analyze the graph ...	

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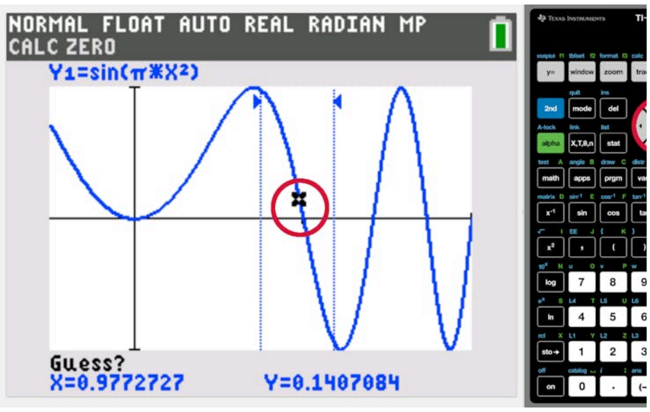
Steps	Explanation
... and choose to find the zeroes.	
After specifying a lower and upper bound for the zero, move the cursor close to the x-intercept and press enter.	

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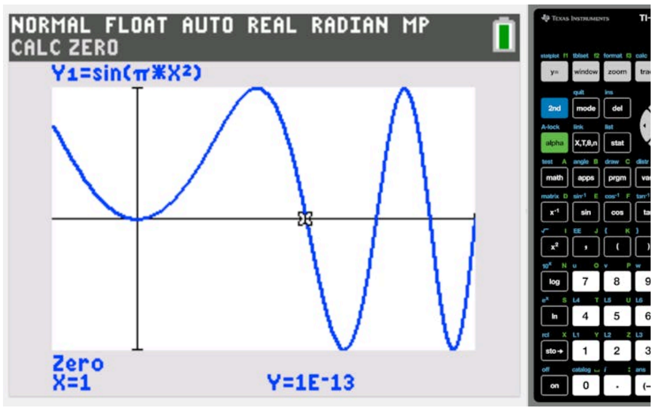
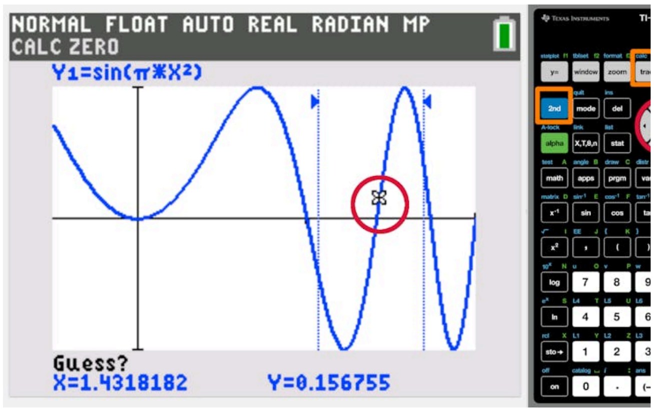
Steps	Explanation
The calculator moves the cursor to the x-intercept and displays its coordinates.	
To find the second x-intercept, repeat the process, this time with the cursor close to this point.	

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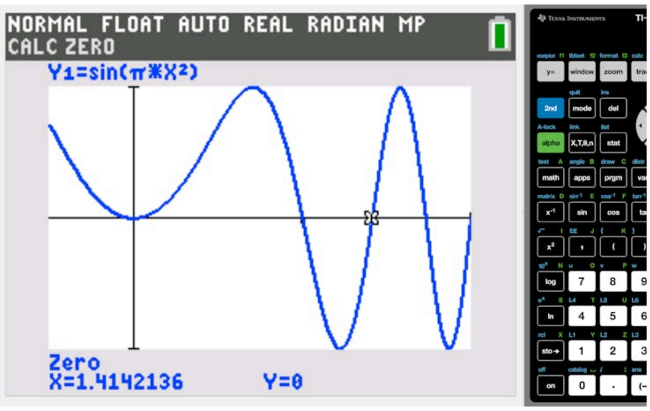
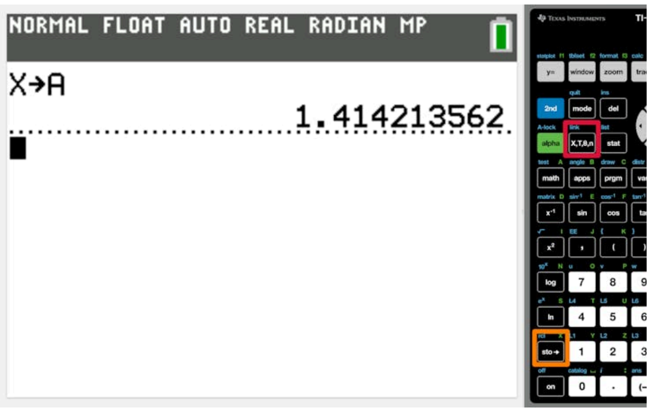
Steps	Explanation
Once again, the cursor is moved to the x-intercept and the coordinates are displayed. You will need this value as the limit of the integration in the next steps. To store this value, move to the home screen.	
The x-coordinate of the zero is stored in the x variable, but since that variable is used all the time by the calculator, it is a good idea to store the current value to a variable of your choice. You can give any name using the alpha key.	

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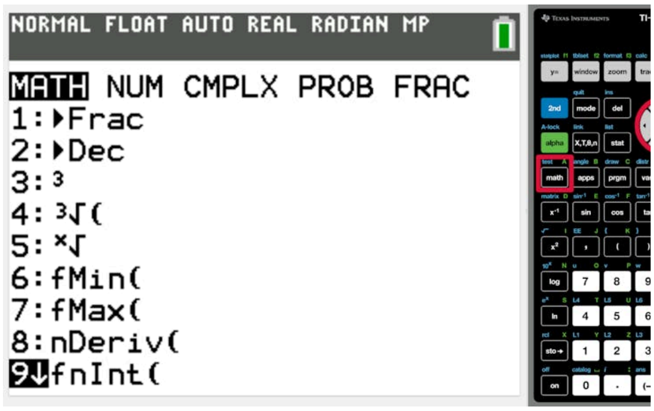
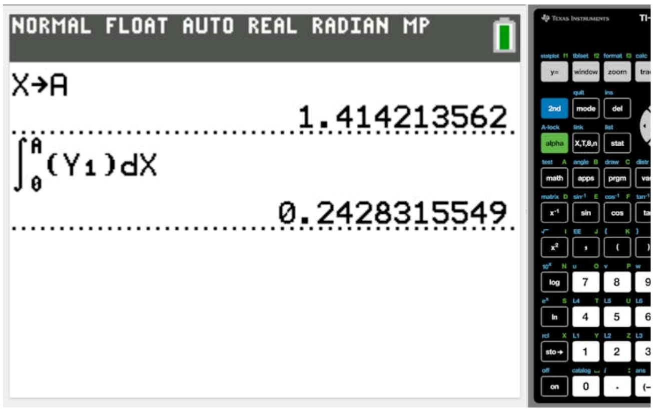
Steps	Explanation
The next step is to find the displacement and the distance traveled. Both of these can be calculated as a definite integral, so open the math menu and choose the numerical integration option (fnInt).	
The displacement is the integral of the velocity function. Use the variable name as the upper limit and use the function name where you stored the velocity function (for the graph) instead of typing the formula in again.	

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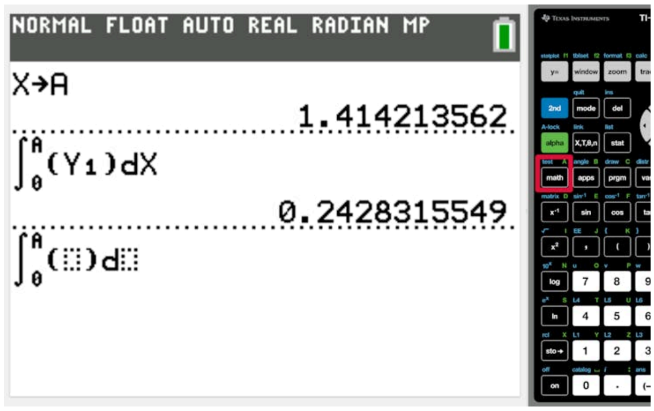
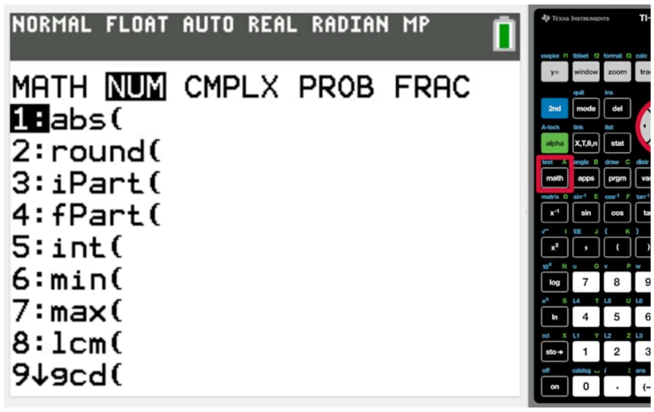
Steps	Explanation
Finding the total distance traveled is similar. The only difference is that you need to take the integral of the absolute value of the velocity function. To access the absolute value function, open the math menu again.	
The absolute value function is among the numerical options.	

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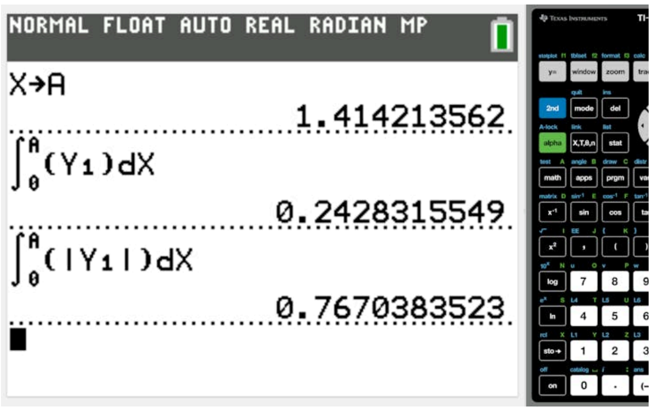
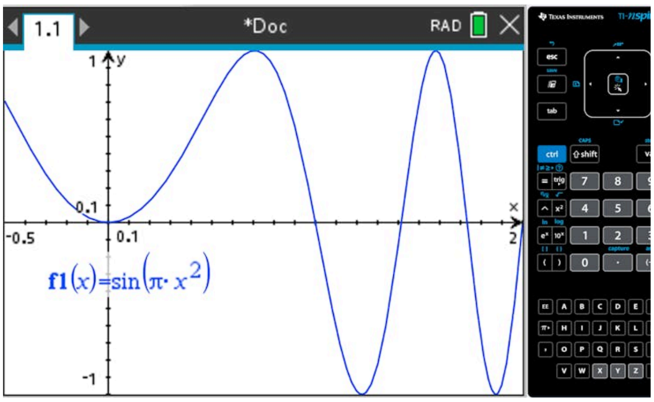
Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
On this screen yo can see both $$\int_0^{1.41} \sin(\pi x^2)\mathrm{d}x$$ (which is the displacement) and $$\int_0^{1.41} \lvert \sin(\pi x^2) \rvert \mathrm{d}x$$ (which is the total distance traveled).	

Steps	Explanation
These instructions show you a way to find the answers to the parts of Example 3. The guidance here assume that you have the graph of $y = \sin(\pi x^2)$ on the screen in a window, where the first two positive x-intercepts are visible. To find the x-intercepts, open the menu to bring up the options to analyze the graph ...	

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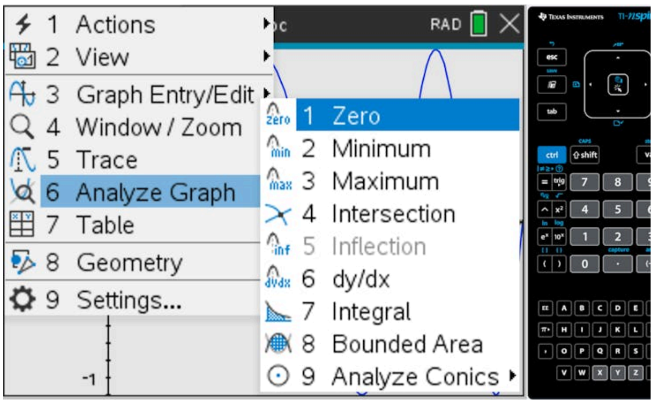
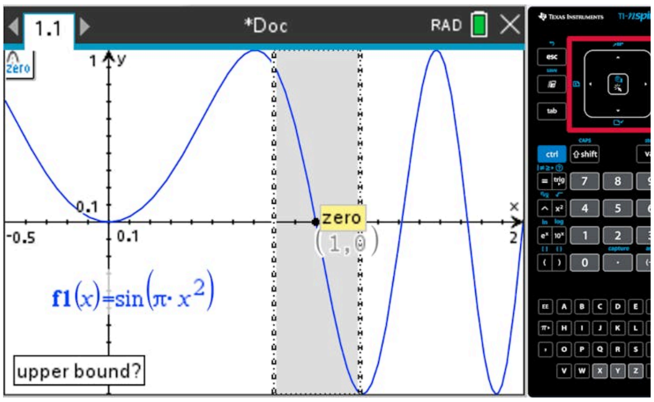
Steps	Explanation
... and choose to find the zeroes.	
After specifying a lower and upper bound for the zero, press enter.	

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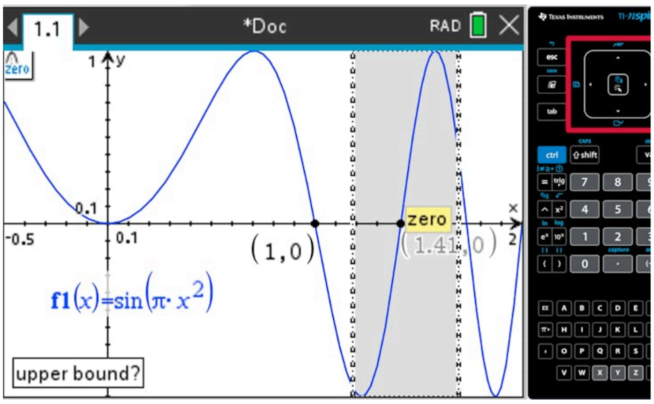
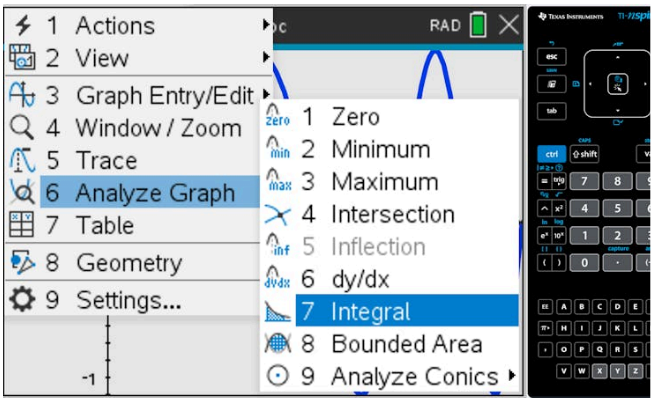
Steps	Explanation
To find the second x-intercept, repeat the process.	
By now the question in the first part of Example 3 is answered. The next step is to find the displacement and the distance traveled. Both of these can be calculated as a definite integral, so open the menu and choose the option to find an integral.	

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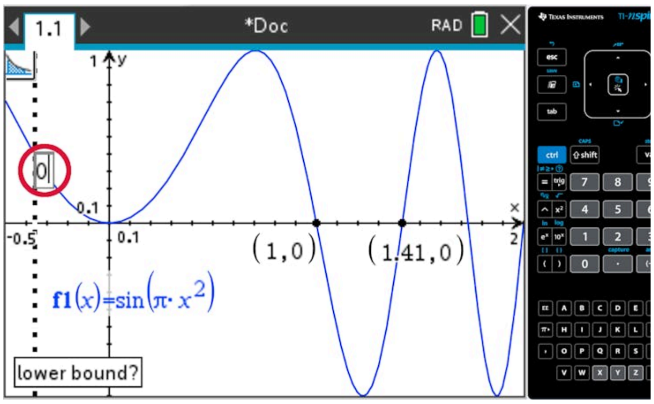
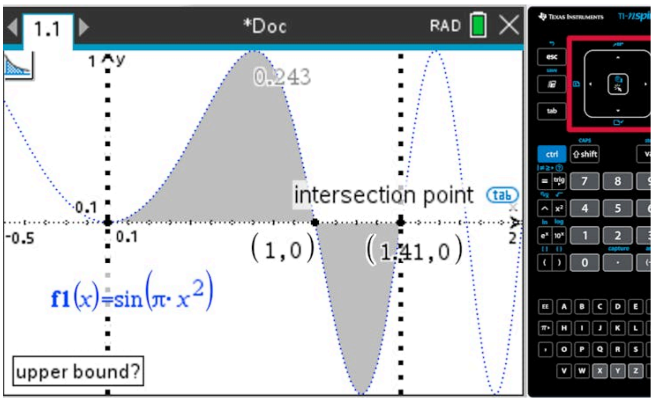
Steps	Explanation
The calculator needs to know the bounds of integration. In this example the lower bound is 0.	
Move to the second positive x-intercept to specify it as the upper bound of the intergation. You do not need to type in the value manually.	

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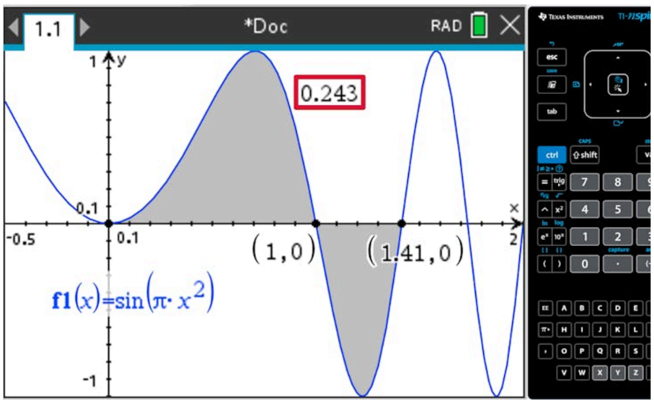
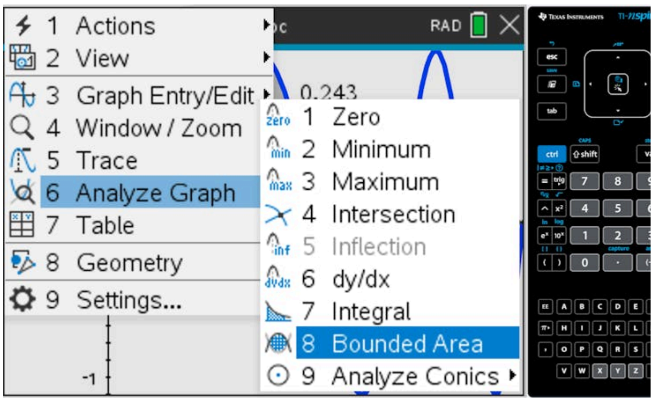
Steps	Explanation
The value of the integral $$\int_0^{1.41} \sin(\pi x^2)\mathrm{d}x$$ is displayed on the screen. This is the displacement of the particle. In finding this value the area of the region below the x-axis counts as negative.	
Finding the total distance traveled is similar. The difference is that in this case the area of both regions need to be counted as positive. This can be done by using the bounded area option of the menu.	

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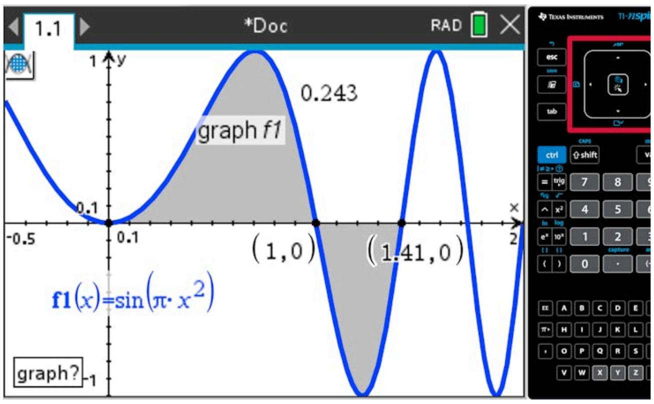
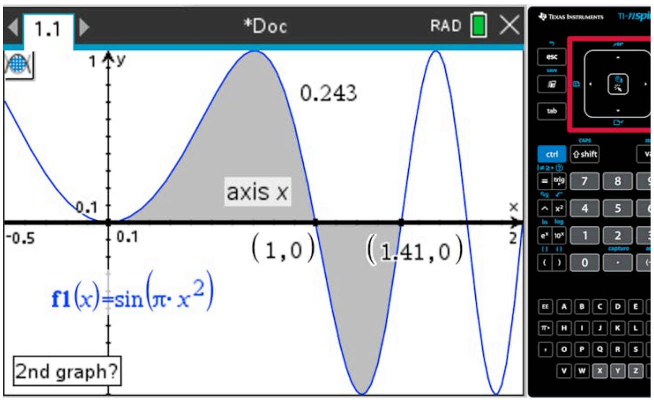
Steps	Explanation
The calculator needs some information. First, it asks for one of the bounding graphs. Move to the graph and press enter.	
The second graph is the x-axis.	

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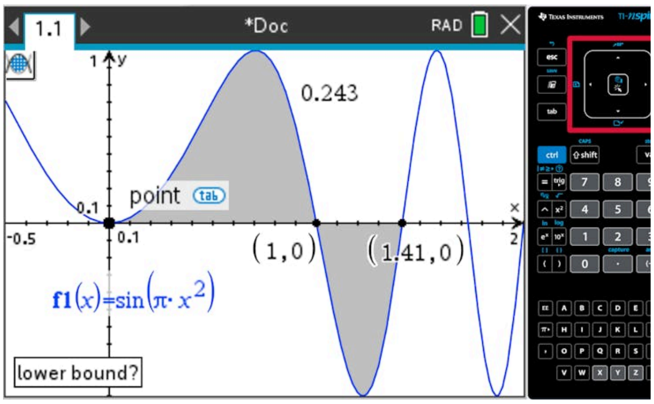
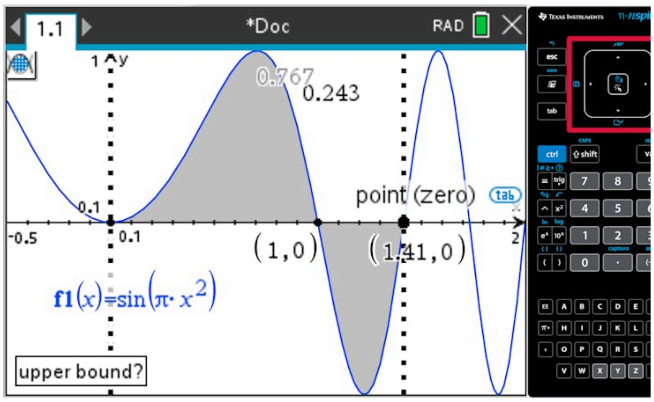
Steps	Explanation
The lower bound of the region is at the origin.	
The upper bound of the region is at the second positive x-intercept.	

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Overview
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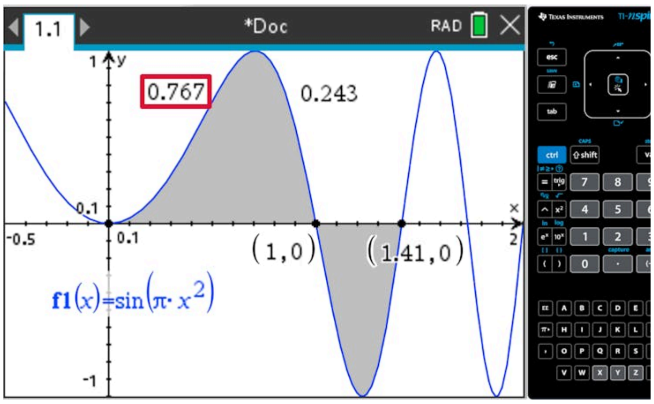
Steps	Explanation
The value of the integral $$\int_0^{1.41} \lvert\sin(\pi x^2)\rvert \mathrm{d}x$$ is displayed on the screen. This is the total distance traveled by the particle. In finding this value the area of both regions count as positive.	

4 section questions

5. Calculus / 5.13 Kinematics

Acceleration

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In this section, you will see examples related to acceleration.

Important

The acceleration , $a(t)$ of the object is the rate of change of its velocity.

As acceleration is the derivative of velocity with respect to time, acceleration can be denoted by the notation $a(t) = v'(t) = s''(t)$, and $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$, . In addition, the notation $a = \dot{v} = \ddot{s}$ is also commonly used to indicate differentiation (once and twice) with respect to time.

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Example 1

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The position of a particle moving on a straight line is given by $s(t) = e^{-t}\sin t$.

a) Find the velocity, $v(t)$.

b) Find the acceleration, $a(t)$.

	Steps	Explanation
a)	$\begin{aligned} s(t) &= \mathrm{e}^{-t}\sin t \\ v(t) = \dot{s}(t) &= -\mathrm{e}^{-t}\sin t + \mathrm{e}^{-t}\cos t \\ &= \mathrm{e}^{-t}(\cos t - \sin t) \end{aligned}$	The velocity is the ra change of position.
b)	$\begin{aligned} v(t) &= \mathrm{e}^{-t}(\cos t - \sin t) \\ a(t) = \dot{v}(t) &= -\mathrm{e}^{-t}(\cos t - \sin t) + \mathrm{e}^{-t}(-\sin t - \cos t) \\ &= -2\mathrm{e}^{-t}\cos t \end{aligned}$	The acceleration is tl rate of change of vel

Example 2

★★☆

The acceleration of a particle moving in a straight line is given by $a(t) = \cos t$, where time is measured in seconds and distance is measured in metres. The initial velocity is 2 metres per second.

a) Find the velocity of the particle in terms of t.

b) Find the displacement of the particle (from its initial position) in terms of t.

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	Steps	Explanation
a)	$v(t) = \int \cos t \, \mathrm{d}t$ $= \sin t + c$ for some value of c.	The acceleration is the rate of change of velocity, so yc use integration to find the velocity.
	$v(t) = \sin t + c$ $2 = \sin 0 + c$ $2 = 0 + c$ $c = 2$ $v(t) = 2 + \sin t$	You can use the given initial velocity to find the value o
b)	$s(t) = \int 2 + \sin t \, \mathrm{d}t$ $= 2t - \cos t + c$ for some value of c.	The velocity is the rate of change of displacement, so y can use integration to find the displacement.
	$s(t) = 2t - \cos t + c$ $0 = 2 \times 0 - \cos 0 + c$ $0 = 0 - 1 + c$ $c = 1$ $s(t) = 2t + 1 - \cos t$	Since the displacement from the initial position is askec $s(0) = 0$.

In **Example 1** of the previous section you investigated the vertical movement of a water rocket. In that example, the velocity of the water rocket was given. You might ask: How do you know that the model used in that example is realistic? This is explained using acceleration in the next example.

Example 3

★★☆

According to the laws of physics, if air resistance is ignored, the acceleration of a water rocket launched vertically is approximately constant $a(t) = -10\text{m/s}^2$.

Student view

a) What height will the rocket reach if the initial velocity is 15 metres per second?

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b) How fast does the rocket need to be launched for it to go twice as high?

	Steps	Explanation
a)	$v(t) = \int -10\,\mathrm{d}t = -10t + c$ for some value of c.	Since acceleration is the derivative of velocity, you can get the velocity by integrating the acceleration function.
	$v(t) = -10t + c$ $15 = -10 \times 0 + c$ $c = 15$ $v(t) = 15 - 10t$	The information about the initial veloc helps you to find the value of the con of integration.
	$15 - 10t = 0$ $t = 1.5$	When the rocket is at its maximum position, its velocity is changing sign, that point the velocity is 0.
	$s(t) = \int 15 - 10t\,\mathrm{d}t = 15t - 5t^2 + c$ for some value of c .	Since velocity is the derivative of displacement, you can get the displacement by integrating the veloc function.
	If $s(t)$ is the distance of the rocket above the ground, then $s(0) = 0$, so $s(t) = 15t - 5t^2 + c$ $0 = 15 \times 0 - 5 \times 0^2 + c$ $c = 0$ $s(t) = 15t - 5t^2$	You can find the value of the constan integration by using the fact that the is launched from the ground.
	The maximum height the rocket reaches is $s(1.5) = 15 \times 1.5 - 5 \times 1.5^2 = 11.25$ metres.	The rocket reaches the maximum pos 1.5 seconds after launch (see above).
b)	$v(t) = v_0 - 10t$ $s(t) = v_0 t - 5t^2$	You can use the methods of the previ part with v_0 instead of 15 as the initia velocity to get the formulae for veloci and displacement.
	The rocket reaches its maximum position $\frac{v_0}{10}$ seconds after launch	When the rocket is at its maximum position, its velocity is changing sign, that point the velocity is 0.

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	Steps	Explanation
	The displacement (compared with the initial position) at the maximum height is $$s\left(\frac{v_0}{10}\right) = v_0\frac{v_0}{10} - 5\left(\frac{v_0}{10}\right)^2 = 0.05v_0^2$$	
	$0.05v_0^2 = 22.5$ $v_0^2 = 450$ $v_0 \approx 21.2$	You can use the desired maximum position, $2 \times 11.25 = 22.5$ to set up solve an equation for the initial veloci
	If the rocket is to go twice as high, you need to improve the design to increase the initial velocity from 15 to 21.2 metres per second.	

In the solution above you found an approximate value for the answer for the second part of the question. If you explore the solution more closely, you will see that to double the height, the initial velocity needs to be increased by a factor of $\sqrt{2}$. This is related to the problem mentioned in the video in **section 5.13.0**.

Making connections

You may have noticed that the value, $10\text{m}/\text{s}^2$, that you used in the previous example is not the most accurate approximation of the standard value .of gravitational acceleration, which is $9.80665\ \text{m}/\text{s}^2$. You may also wonder why this value is the same for different objects.

Take a look at the two videos below showing experiments related to gravitational acceleration.

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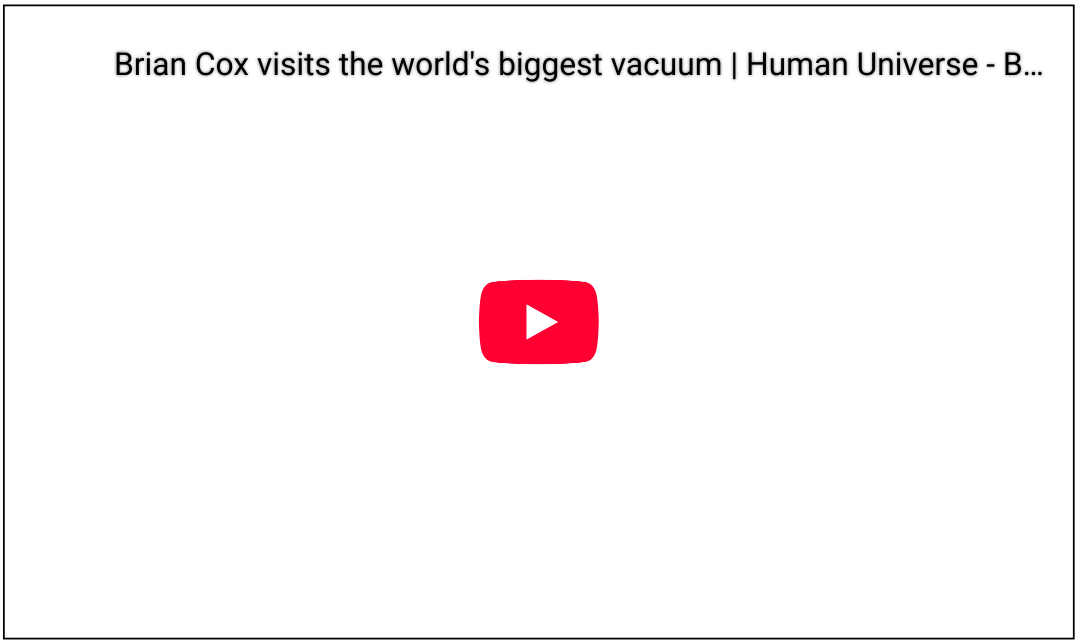

Example 4

★★★

The vertical position above ground of a parachutist, t seconds after opening their parachute is given by $s(t)$ (in metres). According to a simplified model, the acceleration of the parachutist is $a(t) = 30\mathrm{e}^{-2t}$ (measured in metres per second squared).

a) Assuming that the velocity of the parachutist is $-20\ \mathrm{m/s^2}$ when she opens the parachute, find her speed after 1 second.

b) If the parachutist opens the parachute at the altitude of 1000 metres, how long does it take her to reach the ground? At what vertical speed will she reach the ground?

c) How long after opening the parachute will her speed be $6\ \mathrm{m/s}$?

	Steps	Explanation
a)	$v(t) = \int 30\mathrm{e}^{-2t}\,\mathrm{d}t$ $= \frac{30}{-2}\mathrm{e}^{-2t} + c$ $= -15\mathrm{e}^{-2t} + c$	Since acceleration is the derivat velocity, you can get the velocit integrating the acceleration fun

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	Steps	Explanation
	$v(t) = -15e^{-2t} + c$ $-20 = -15e^{-2\times 0} + c$ $-20 = -15 \times 1 + c$ $c = -5$ $v(t) = -15e^{-2t} - 5$	The information about the initia velocity helps you to find the va the constant of integration.
	$v(1) = -15e^{-2\times 1} - 5 \approx -7.03$ Her speed 1 second after she opens the parachute is approximately 7 metres per second.	The speed is the magnitude of t velocity.
b)	$s(t) = \int -15e^{-2t} - 5t + c$ $= \frac{-15e^{-2t}}{-2} - 5t + c$ $= 7.5e^{-2t} - 5t + c$	Since velocity is the derivative c displacement, you can get the displacement by integrating the velocity function.
	$s(t) = 7.5e^{-2t} - 5t + c$ $1000 = 7.5e^{-2\times 0} - 5 \times 0 + c$ $c = 992.5$ $s(t) = 7.5e^{-2t} - 5t + 992.5$	Since $s(t)$ is the distance from t ground, $s(0) = 1000$.
	The solution of $7.5e^{-2t} - 5t + 992.5 = 0$ is approximately $t = 198.5$ so it takes around 198.5 seconds for the parachutist to reach the ground after opening the parachute.	You can use your GDC to solve $s(t) = 0$.
	$v(t) = -15e^{-2t} - 5$ $= -15e^{-2\times 198.5} - 5$ $= -5$ She will reach the ground with speed 5 metres per second.	

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	Steps	Explanation
c)	$v(t) = -15e^{-2t} - 5$ $-6 = -15e^{-2t} - 5$ $t \approx 1.35403$ The parachutist will slow down to a speed of 6 metres per second 1.35 seconds after she opens her parachute.	You can use your graphing calc to solve $v(t) = -6$. Note that y need the negative sign, since th parachutist is moving downwar the velocity is negative.

Note that the acceleration is positive all the time, but the parachutist is still slowing down from her initial speed of 20 metres per second. The reason for this is that the velocity is negative. A positive acceleration increases this negative velocity, but decreases the magnitude of the velocity (which is the speed).

Making connections

So far you have learned about velocity (the rate of change of position) and acceleration (the rate of change of velocity). The syllabus stops here, but actually in real life considering the rate of change of acceleration is also important. The technical term for this is jerk. Engineers keep this in mind when designing rollercoasters and road layouts, for example. In fact, every driver is also aware of the importance of the change in acceleration when applying the brakes to stop a car. When the brake pedal is pushed with a constant force (so the car slows down with constant deceleration), you feel a jerk when the car actually stops. To avoid this, drivers gradually decrease the pressure on the brake as the car is coming to a stop. In doing this, the change of acceleration is not as sudden when the car finally stops.

Take a look at the following video that shows equipment used in training fighter jet pilots. In the design of the experiment, the jerk (the rate of change of acceleration) is also kept in mind.

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Using vectors

Example 5

★★☆

At time t seconds, the acceleration of a particle is given by $a = (3t)\,i - (\cos t)j\ \mathrm{m/s^2}$. Its initial velocity is $v\,(0) = 3i + 2j\ \mathrm{m/s}$. Find the velocity function of the particle.

Steps	Explanation
$v(t) = \int((3t)\,i - (\cos t)j)\,\mathrm{d}t$	$v = \int a\,\mathrm{d}t$
$v = \left(\frac{3t^2}{2}\right) i - (\sin t)\,j + c$	c is the constant of integration
$\begin{pmatrix}3\\2\end{pmatrix} = (\sin 0)\,j + c$ $\begin{pmatrix}3\\2\end{pmatrix} = c$ $c = 3i + 2j$	Substitute $t = 0$ and use the initial velocity of Rearrange and solve for c.

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Steps	Explanation
$v=\left(\frac{3t^2}{2}\right)i-(\sin t)\,j+3i+2j$ $v=\left(\frac{3t^2}{2}\right)i+3i+2j-(\sin t)\,j$ $v=3\left(\frac{t^2}{2}+1\right)i+(2-\sin t)\,j$	Substitute c into the expression for the velocity vector and simplify.
Therefore the velocity function is $v=3\left(\frac{t^2}{2}+1\right)i+(2-\sin t)\,j\text{ m/s}$.	

Example 6

★★★

At time t seconds the velocity of a particle is given by $v=\begin{pmatrix}t^2-t\\t^2+2\end{pmatrix}$.

Given that the particle starts from the point with position vector $\begin{pmatrix}1\\3\end{pmatrix}$, find

a) the position vector when $t=4$ seconds

b) the acceleration of the particle.

	Steps	Explanation
a)	$r(t)=\int\left(t^2-t\right)i-(t^2+2)j\,\mathrm{d}t$	$r=\int v\,\mathrm{d}t$
	$r=\left(\frac{t^3}{3}-\frac{t^2}{2}\right)i+\left(\frac{t^3}{3}+2t\right)j+c$	c is the constant of integration

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	Steps	Explanation
	$c = \begin{pmatrix}1\\3\end{pmatrix}$	Substitute $t = 0$ and use the initia position vector $\begin{pmatrix}1\\3\end{pmatrix}$ given.
	$r = \left(\dfrac{t^3}{3} - \dfrac{t^2}{2}\right)i + \left(\dfrac{t^3}{3} + 2t\right)j + 1i + 3j$	
	Therefore $r = \begin{pmatrix}\dfrac{t^3}{3} - \dfrac{t^2}{2} + 1\\ \dfrac{t^3}{3} + 2t + 3\end{pmatrix}$	Write the position vector in colum format.
	$r = \begin{pmatrix}\dfrac{4^3}{3} - \dfrac{4^2}{2} + 1\\ \dfrac{4^3}{3} + 2 \times 4 + 3\end{pmatrix} = \begin{pmatrix}\dfrac{43}{3}\\ \dfrac{97}{3}\end{pmatrix}$	Substitute $t = 4$.
	So the position vector of the particle is $\begin{pmatrix}\dfrac{43}{3}\\ \dfrac{97}{3}\end{pmatrix}$ m	
b)	$a = \dfrac{\mathrm{d}\left[\left(t^2 - t\right)i + (t^2 + 2)j\right]}{\mathrm{dt}}$ $a = (2t - 1\ )i + 2tj$ $a = 7i + 8j$	Use $a = \dfrac{\mathrm{d}v}{\mathrm{dt}}$ Substitute $t = 4$.
	Therefore, the acceleration of the particle is $a = 7i + 8j\,\mathrm{m/s^2}$.	

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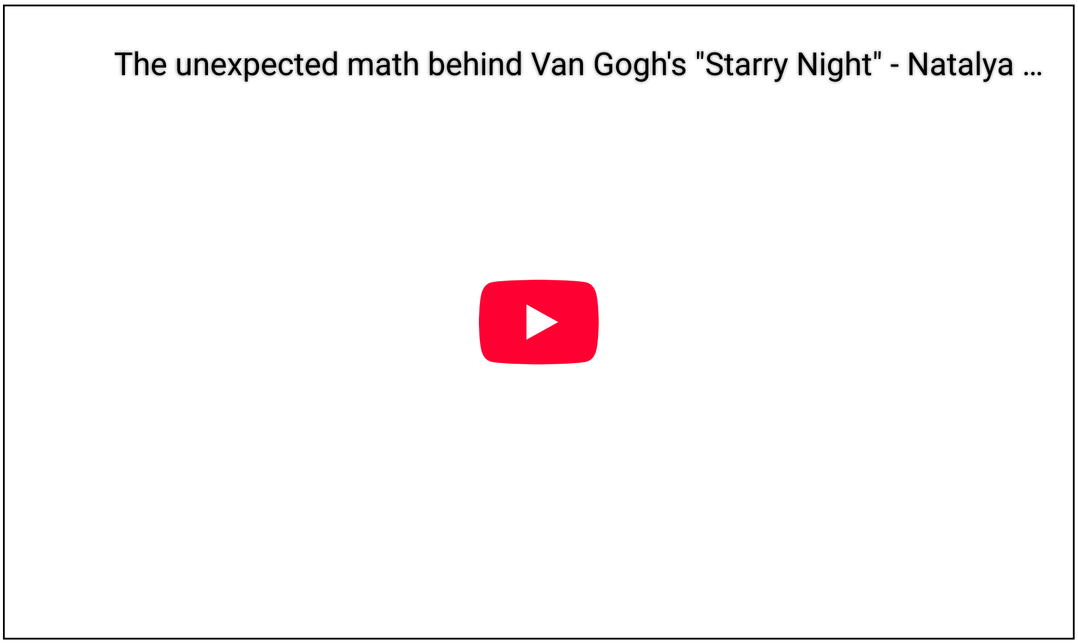
Velocity and acceleration in terms of position

In the previous sections you explored how the displacement, velocity and acceleration of a moving object are connected to each other when expressed in terms of time. In the study of fluids, scientists investigate systems where a lot of objects move simultaneously. These systems can be described by investigating the velocity of the flow as a function of position and time. For example, the flow of the water in a river can be investigated by measuring the velocity at different points. This is a difficult area of mathematics and it is beyond the scope of this syllabus. However, it gives a motivation for the examples you will see in this section.

Theory of Knowledge

The following video is related to the investigation of flows and talks about different ways of seeing and understanding the world around us.



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In the examples below, it is important to be able to think of displacement, velocity and acceleration as functions of each other, not only as functions of time. Here are some examples:

- In some cases, the velocity as a function of time can also be expressed as a composition, as a function of displacement, which itself is a function of time. This leads to the relationship
$a = \dot{v} = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}s} \times \frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{ds}} \times \dot{s} = \frac{\mathrm{d}v}{\mathrm{ds}}v\,.$
- Similarly, acceleration can be seen as a function of velocity, which itself is a function of time.
- It can also be helpful to reverse the relationship and, for example, to think of time as a function of displacement instead of displacement as a function of time. This leads to the relationship
$\frac{\mathrm{d}t}{\mathrm{d}s} = \frac{1}{\mathrm{d}s/\mathrm{d}t} = \frac{1}{v}.$

Example 1

★★★

A particle is moving along a straight line. When its displacement is s metres from a fixed point O, the velocity is given by $v(s) = \frac{1}{s}$,

a) What is the acceleration of the particle when the displacement from O is 2 metres?

b) How long does it take for the particle to move from a position that is 2 metres from O to a position that is 4 metres from O ?

	Steps	Explanation
a)	$a = \frac{\mathrm{d}v}{\mathrm{d}s}v$	You can use the expression for accelerat terms of the velocity and the change in velocity in terms of displacement.

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	Steps	Explanation
	For $v(s) = \frac{1}{s} = s^{-1}$, $$\frac{dv}{ds} = -s^{-2} = -\frac{1}{s^2}.$$ Hence, $$a = -\frac{1}{s^2} \times \frac{1}{s} = -\frac{1}{s^3}.$$	
	The acceleration when the displacement from O is 2 metres is $-\frac{1}{2^3} = -0.125$ metres per second squared.	
b)	$\frac{dt}{ds} = \frac{1}{ds/dt} = \frac{1}{v} = \frac{1}{1/s} = s$	You can think of time as a function of displacement.
	$$\frac{dt}{ds} = s$$ $$\int_2^4 s ds = t(4) - t(2)$$ $$t(4) - t(2) = \left[\frac{s^2}{2}\right]^4$$ $$= \frac{4^2}{2} - \frac{2^2}{2} = 6$$	You can integrate this relationship and us Newton—Leibniz formula.
	Hence, it takes 6 seconds for the particle to move from a position that is 2 metres from O to a position that is 4 metres from O .	$t(a)$ is the time when the displacement c object is a.

Note that in the last part of this example you could have also used your calculator to find the definite integral. This method can be useful in case the integrand is more complicated.

There are approaches other than the one used above to solve **Example 1.** For example, since $s' = v$, the relationship given in the question can be written in the form

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$$s' = \frac{1}{s}.$$

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In the higher level extension you will see methods for solving differential equations like this to find the displacement as a function of time. You can also use WolframAlpha (http://www.wolframalpha.com)to find the solution. Type:

solve s'=1/s

in the command line. Check that WolframAlpha gives the solutions you found in **Example 1**.

Example 2

★★★

A particle is moving along a straight line. The acceleration, a, of the particle as a function of its velocity, v, is given by $a = -(1+v)$. The initial velocity is 1 metre per second.

Find the displacement between the starting point and the position when the particle comes to rest.

Steps	Explanation
$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	Acceleration is the rate of change of velocity.
$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}s} \times \frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}s} \times v$	Since the question asks for a distance based on velocity change, you can thi velocity (and acceleration) as a functi displacement.
$\frac{\mathrm{d}s}{\mathrm{d}v} = -\frac{v}{1+v}$	You can reverse the direction and thir displacement as a function of velocity

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Steps	Explanation
$$\frac{\mathrm{d}s}{\mathrm{d}v} = -\frac{v}{1+v}$$ $$\int_1^0 -\frac{v}{1+v}\mathrm{d}v = s(0) - s(1)$$ $$\begin{aligned} s(0) - s(1) &= [\ln(x+1) - x]_1^0 \\ &= (\ln 1 - 0) - (\ln 2 - 1) \\ &= 1 - \ln 2 \approx 0.307 \end{aligned}$$	The question asked for the displacem between the starting point and when particle comes to rest, so this is durin time when the velocity is changing fr to 0. This means that if you think of displacement as a function of velocity need to find $s(0) - s(1)$. You can integrate the relationship you found earlier and use the Newton—Le formula.
Hence, the particle comes to rest from a speed of 1 metre per second after travelling 0.307 metres.	

Note that in the last part of this example you could have used your calculator to find the definite integral.

Again, you can use WolframAlpha (http://www.wolframalpha.com)to find the actual displacement function and check the result you got in the example above. Type

solve {s''=-(1+s'), s'(0)=1, s(0)=0}

in the command line and investigate the result you get.

Example 3

★★★

A particle is moving in a straight line. The displacement, s, (measured in metres) of the particle t seconds after the start satisfies $s^3 + s - t = 0$.

a) Find the velocity of the particle in terms of s.

b) Find the acceleration of the particle in terms of s.

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	Steps	Explanation
a)	$s^3 + s - t = 0$ $3s^2\dot{s} + \dot{s} - 1 = 0$ $\dot{s}(3s^2 + 1) = 1$ $\dot{s} = \frac{1}{3s^2 + 1}$ $v = \frac{1}{3s^2 + 1}$	You can differentiate the given relationship wi respect to time and use the chain rule.
b)	$v = \frac{1}{3s^2 + 1} = (3s^2 + 1)^{-1}$ $a = \dot{v} = -(3s^2 + 1)^{-2} \times 6s\dot{s}$ $= -\frac{6sv}{(3s^2 + 1)^2}$	You can differentiate again.
	$a = -\frac{6sv}{(3s^2 + 1)^2}$ $a = -\frac{6s\frac{1}{3s^2 + 1}}{(3s^2 + 1)^2}$ $= -\frac{6s}{(3s^2 + 1)^3}$	You can replace v with the expression you fou above.

3 section questions

5. Calculus / 5.13 Kinematics

Checklist

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What you should know

By the end of this subtopic you should be able to:

- understand the relationship between displacement, velocity and acceleration of an object moving in a straight line

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- use differentiation to find velocity from displacement and acceleration from velocity
- use integration to find velocity from acceleration and displacement from velocity
- understand the difference between displacement and distance travelled
    - know how to use the formula in the formula booklet to find displacement and distance travelled
    - find displacement and distance travelled both algebraically and using a graphing calculator
- understand that velocity and/or acceleration may be given in terms of position rather than in terms of time.

5. Calculus / 5.13 Kinematics

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-27510/print/) Assign

There are examples around us when the movement of an object follows a path approaching a stable position. For example, think about the suspension of a car or a bicycle. When the spring is compressed or stretched, it applies a force to bring the system back to its original position. Take a look at the following video showing the movement of a pendulum.

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Video 1. Critical Damping.

More information for video 1

The video features a well-equipped science laboratory, with multiple lab stations neatly arranged across the room. The camera zooms in on one station, showcasing a specific experimental setup. A wooden ruler is clamped horizontally to a metal stand, serving as the pivot point for the experiment. Two small cylindrical magnets are attached to the bottom end of the ruler, while an aluminum block is placed behind it as a conductor.

A piece of graph paper is carefully positioned beneath the ruler to serve as a reference for measurements. Labels appear on the screen, clearly identifying the "magnets" and the "aluminum block." A hand briefly pulls the ruler back, demonstrating that it can oscillate freely.

The focus shifts closer to the ruler, which is now hanging vertically. The graph paper beneath the ruler is visible, with "20.0 mm" written on it, indicating the initial distance between the magnet and the aluminum block.

A vertical green line is digitally added to the video, aligning with the center of the ruler to provide a visual guide.

The ruler is pulled back and released, causing the magnet to oscillate back and forth. The sound of the swinging ruler is audible, and the oscillations gradually decrease in amplitude due to the damping effect of eddy currents generated in the aluminum block.

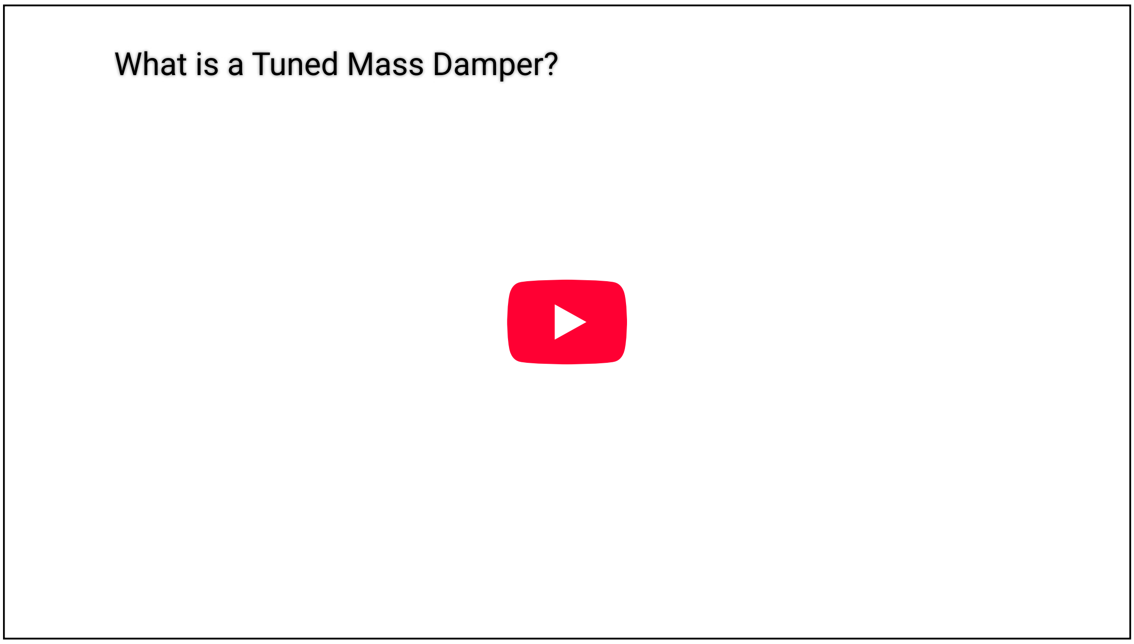
The experiment is repeated multiple times, with the distance between the magnet and the aluminum block decreasing incrementally. Each time, the graph paper displays a new measurement: "10.0 mm," "8.0 mm," "6.0 mm," "4.0 mm," "2.5 mm," "2.0 mm," "1.5 mm," "1.0 mm," "0.5 mm," and finally "0.0 mm." As the distance decreases, the damping effect becomes progressively stronger.

At 20.0 mm, the oscillations are relatively free, but as the distance shrinks, the amplitude of the oscillations diminishes more rapidly. When the distance reaches 2.0 mm, the ruler is critically damped, meaning it returns to its resting position without oscillating. At even smaller distances, such as 1.5 mm and below, the system becomes overdamped, and the ruler barely moves. At 0.0 mm, the magnet stops immediately, with no oscillation at all.

The video concludes with a montage of nine clips, each showing the ruler swinging with varying degrees of damping. The clips are labeled with the corresponding distances and the type of damping observed: 8.0 mm (underdamped), 6.0 mm (underdamped), 4.0 mm (underdamped), 2.5 mm (underdamped), 2.0 mm (critically damped), 1.5 mm (overdamped), 1.0 mm (overdamped), 0.5 mm (overdamped), and 0.0 mm (overdamped). A red dotted line appears behind the magnet in all the clips, providing a consistent visual reference for the motion. This montage effectively summarizes the experiment, illustrating the relationship between the distance of the aluminum block and the damping effect on the oscillating ruler. The demonstration clearly and engagingly highlights the principles of damping and eddy currents.

Without friction, the pendulum would go back and forth indefinitely. With more and more friction introduced, the oscillation is damped, even to a point when the pendulum does not go past the stable state. Car and bicycle manufacturers keep this in mind when they look for the optimal properties of the springs in suspensions.

Architects also use damping systems to decrease the movement of tall buildings. Take a look at the illustration of this in the following video.



Activity

On the applet below you can investigate the movement of an object at the end of a spring. By changing the damping ratio (the properties of the spring), you can investigate how it affects the movement. What do you notice?

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On the applet you can also see the equation involving the first and second derivative of the displacement function. This differential equation is based on the laws of physics. An object at the end of a spring (if no other forces are applied) will move according to a similar equation. The goal of the next part of the activity is to try to find a solution of the equations on the applet. In other words, the goal is to find the displacement function of the object satisfying an equation of the form $\ddot{s} + k\ddot{s} + s = 0$.

- First, try to guess a solution. Did you succeed?
- If you did not succeed, try functions of the form $s(t) = e^{rt}$.
  - For some values of k there will be solutions like this. Can you find these?
- Exponential graphs will not show the oscillating behaviour which you see on the applet. Try to modify these to find solutions for the values of k _(https://www.codecogs.com/eqnedit.php?latex=k%251)that did not give exponential solutions. Did you succeed?
- If you did not succeed, try functions of the form $s(t) = e^{rt}\cos(pt)$.
  - For some values of k there will be solutions like this. Can you find these?
- Did you find a solution for any value of k ?

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