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The big picture

Logarithms might seem abstract and irrelevant to anything outside mathematics when you first learn about them. But in fact, your brain thinks logarithmically in a way that you might not notice. Watch the video below to learn more about this.

After watching the video, think about the following:

- In your own life have you experienced some of the logarithmic ways of thinking without noticing it?
- Has the information about Weber’s law changed your perception of logarithms?

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Concept

Logarithms follow special rules for addition, subtraction and multiplication. These rules can be understood by evaluating logarithms and noticing patterns, and they can be proved by using equivalent exponential forms. As you work through this subtopic, think about which approach appeals more to you and why.

Theory of Knowledge

As you've learned in this section and throughout your maths education, mathematics functions according to rules, procedures and laws. All of these prescribe a way of operating within mathematics that assures accuracy, cohesion and order. Other areas of knowledge (aside from perhaps the natural sciences to a certain degree) do not operate according to such strict limitations, but they do have paradigms — established ways of doing — that govern how individuals construct knowledge within a given AOK.

Knowledge Question: Do paradigms within AOKs serve to increase or limit knowledge production?

1. Number and algebra / 1.7 Further exponents and logarithms

Rational exponents

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/rational-exponents-id-27675/print/) Assign

Rational or fractional exponents provide another way to represent roots.

For example, supposing that the law of exponents $(a^m)^n = a^{m\times n}$ works also for fractional exponents, if $(a^m)^2 = a = a^1$, then $m \times 2 = 1$ so $m = \frac{1}{2}$. But since a^m squared is equal to a, a^m must be the square root of a. Therefore

$$\sqrt{a} = a^{\frac{1}{2}}$$

Similarly, you can show that

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$$\sqrt[3]{a} = a^{\frac{1}{3}}$$

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$$\sqrt[4]{a} = a^{\frac{1}{4}}$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Example 1

★☆☆

Evaluate $16^{\frac{1}{4}}$.

Steps	Explanation
$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	You should be able to find this root without using a calculator. Remember that $a^{\frac{1}{n}}$ when n is even means the positiv root.

Making connections

Rational exponents follow these laws of exponents, that you learned in subtopic 1.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25055/):

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m\times n}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^0 = 1$$

Student view

Example 2

★★☆

Evaluate $27^{\frac{2}{3}}$. Do not use a calculator for this question.

Steps	Explanation
$27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2 = \left(27^2\right)^{\frac{1}{3}}$	Using $(a^m)^n = a^{m \times n}$.
$\left(27^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$	It is usually easier to evaluate $\left(a^{\frac{1}{n}}\right)^m$ rather than $(a^n$

You can practise more of these types of questions by using the applet below.

Interactive 1. Practice Exercise on Fractional Exponents.

Credit: GeoGebra (https://www.geogebra.org/m/uZPZpSVh) Edward Knote

More information for interactive 1

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

This interactive allows users to practice simplifying expressions with rational exponents by rewriting them in radical form. Users are given an expression of the form $(a)^{\frac{m}{n}} = (\sqrt[n]{a})^m = x$ and must determine the values of n and m. After entering their answers, the interactive checks whether the values are correct. Users can generate new questions to continue practicing and thus improving their understanding of rational exponents and their relationship to roots.

For example, when presented with $(1)^{\frac{4}{3}} = (\sqrt[3]{1})^m = x$, the correct solution requires identifying $n = 3$ (the denominator of the exponent), $m = 4$ (the numerator), and $x = 1$ since $\sqrt[3]{1} = 1$ and $1^4 = 1$). The interactive helps users to check their answers, providing immediate feedback to reinforce understanding. This approach enables learners to build confidence in manipulating fractional exponents and connecting them to their equivalent radical forms through active problem-solving.

Important

For rational exponents:

$$a^{\frac{1}{n}} = \sqrt[n]{a}\,, \quad a > 0$$

If n is even, give only the positive root.

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

Exam tip

If you need to simplify or evaluate $a^{\frac{m}{n}}$ on the non-calculator part of the exam, $\left(a^{\frac{1}{n}}\right)^m$ is usually the easier form to work with.

Example 3

★★☆

Simplify $\left(\dfrac{32x^{10}}{y^5}\right)^{\frac{2}{5}}$.

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Steps	Explanation
$\left(\frac{32x^{10}}{y^5}\right)^{\frac{2}{5}} = \frac{32^{\frac{2}{5}}x^{10\times\frac{2}{5}}}{y^{5\times\frac{2}{5}}} = \frac{\left(32^{\frac{1}{5}}\right)^2 x^4}{y^2} = \frac{2^2x^4}{y^2} = \frac{4x^4}{y^2}$	Use $(a^m)^n = a^{m\times n}$ and $\left(a^{\frac{1}{n}}\right)^m$.

Be aware

Most calculators use rational exponent notation for any root higher than $\sqrt{a}$.

4 section questions

1. Number and algebra / 1.7 Further exponents and logarithms

Logarithms

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/logarithms-id-27676/print/) Assign

You learned that $a^x = b$ is equivalent to $\log_a b = x$ for $a > 0$, $a \neq 1$ and $b > 0$ when you were introduced to logarithms. You will use this relationship to further explore logarithms, starting with bases other than 10 and e.

Example 1

★☆☆

Rewrite $2^x = 3$ in logarithmic form and find the value of x.

Steps	Explanation
$2^x = 3 \quad \Leftrightarrow \quad \log_2 3 = x$	The relationship, $a^x = b \quad \Leftrightarrow \quad \log_a b = x$ can be used f any base $a > 0, a \neq 1$.

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Steps	Explanation
$\log_2 3 = 1.58$ (3 significant figures)	Use your graphic display calculator.

You should also be able to use the exponential form to evaluate simple logarithms without a calculator.

Example 2

★★☆

Evaluate each of the following without using a calculator.

a) $\log_3 \frac{1}{9}$

b) $\log_4 64$

c) $\log_{25} 5$

d) $\log_a a^m$

	Steps	Explanation
a)	$\log_3 \frac{1}{9} = x \quad \Leftrightarrow \quad 3^x = \frac{1}{9}$ $\Leftrightarrow \quad 3^x = 3^{-2}$ $x = -2$ So $\log_3 \frac{1}{9} = -2$.	You should be able to recognise that $\frac{1}{9}$ can be rewritten in base 3.

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	Steps	Explanation
b)	$\log_4 64 = x \quad \Leftrightarrow \quad 4^x = 64 \quad \Leftrightarrow \quad 4^x = 4^3$ $x = 3$ So $\log_4 64 = 3$.	
c)	$\log_{25} 5 = x \quad \Leftrightarrow \quad 25^x = 5$ $\Leftrightarrow \quad 25^x = 25^{\frac{1}{2}}$ $x = \frac{1}{2}$ So $\log_{25} 5 = \frac{1}{2}$.	
d)	$\log_a a^m = x \quad \Leftrightarrow \quad a^x = a^m$ $x = m$ So $\log_a a^m = m$.	

⚠ Be aware

The observation that $\log_a a^m = m$ is useful for evaluating logarithms without a calculator. It is not given in the IB formula booklet, but you can easily derive it straight from the definition of a logarithm.

You can use the exponential form to deduce more interesting results for logarithms.

Example 3

★★☆

Evaluate each of the following without using a calculator.

a) $\log_a 1$

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b) $a^{\log_a m}$

c) $\log_a a$

d) $\mathrm{e}^{\ln m}$

	Steps	Explanation
a)	$\log_a 1 = x \quad \Leftrightarrow \quad a^x = 1$ $x = 0$ So $\log_a 1 = 0$.	
b)	Let $\log_a m = x$. Then $a^x = m$. So $a^{\log_a m} = a^x = m$.	Rewrite the logarithm in the exponent using the expone form.
c)	$\log_a a = x \quad \Leftrightarrow \quad a^x = a$ $x = 1$ So $\log_a a = 1$.	
d)	Let $\ln m = x$. Then $\mathrm{e}^x = m$ So $\mathrm{e}^{\ln m} = \mathrm{e}^x = m$.	Rewrite the logarithm in the exponent using the expone form. Remember that $\ln = \log_\mathrm{e}$.

Be aware

The following properties of logarithms are useful for working with logarithms, but are not included in the IB formula booklet. You should memorise them or be able to derive them from the definition of a logarithm:

$$\log_a 1 = 0$$

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$$\log_a a = 1$$

$$a^{\log_a m} = m$$

$$e^{\ln m} = m$$

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1. Number and algebra / 1.7 Further exponents and logarithms

Laws of logarithms

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Activity

Evaluate each of the following.

a) $\log_2 4, \log_2 8$ and $\log_2 32$

b) $\log 100,\ \ \log 1000 \text{and} \log 100\ 000$

c) $\log_c c^3,\ \log_c c \text{ and} \log_c c^4$

Hence, deduce the relationship between $\log_a x + \log_a y$ and $\log_a xy$.

Go through a similar process to deduce the relationship between $\log_a x - \log_a y$ and $\log_a \frac{x}{y}$.

You should notice that logarithms follow a special set of rules when it comes to arithmetic. These rules are summarised as follows:

Important

Logarithms can be simplified using

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

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$$\log_a x^m = m\log_a x.$$

These rules apply for $a,\ x,\ y > 0$ and $a \neq 1$.

Exam tip

The laws of logarithms are given in the IB formula booklet and do not need to be memorised. When you use these laws it is important to remember that they only apply to logarithms with the same base.

Example 1

★★☆

Write $\log_7 xy^2 + \log_7 \frac{1}{xy}$ as a single logarithm.

Steps	Explanation
$\log_7 xy^2 + \log_7 \frac{1}{xy} = \log_7 \left(xy^2 \times \frac{1}{xy}\right) = \log_7 y$	Use $\log_a xy = \log_a x + \log_a y$.

Example 2

★★☆

Given that $\log_d 5 = 8$ and $\log_d 10 = 11$, find the value of $\log_d 2$.

Steps	Explanation
$\log_d 2 = \log_d \frac{10}{5}$	Recognise that $\frac{10}{5} = 2$.

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Steps	Explanation
$\log_d \dfrac{10}{5} = \log_d 10 - \log_d 5 = 11 - 8 = 3$	Use $\log_a \dfrac{x}{y} = \log_a x - \log_a y$.

Example 3

★★☆

Given that $\log a = 2, \log b = -6$ and $\log c = 3$, find the value of $\log \dfrac{a^2 c}{\sqrt[3]{b}}$.

Steps	Explanation
$\log \dfrac{a^2 c}{\sqrt[3]{b}} = \log a^2 + \log c - \log \sqrt[3]{b}$ $= 2 \log a + \log c - \dfrac{1}{3} \log b$	Remember that $\log a = \log_{10} a$. All the logarithms i the question have the same base and the logarithm rules apply.
$= 2(2) + 3 - \dfrac{1}{3}(-6) = 9$	Replace the logarithms with their equivalent values.

You can practise more questions like **Example 3** by using the applet below.

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Interactive 1. Logarithm Expansion Practice.

Credit: GeoGebra (https://www.geogebra.org/m/eGCUSf3J) Kevin Hopkins

More information for interactive 1

This interactive tool allows users to practice expanding logarithmic expressions by applying fundamental logarithm rules, such as the product rule, quotient rule, and power rule.

Using the logarithmic properties:

- **Product Rule**: $\ln(a.\ b) = \ln a + \ln b$
- **Quotient Rule**: $\ln(\frac{a}{b}) = \ln a - \ln b$
- **Power Rule**: $\ln(a^b) = b.\ln a$

Users can generate new questions by clicking the "New Question" button, which presents a different logarithmic expression to expand. After attempting the problem, they can click "Show Answer" to reveal the correctly expanded final expression.

For example, when a user clicks New Question, they might see the expression: $\ln(x^3.\ \frac{y^3}{z^4})$

After solving, clicking "Show Answer" will display: $3\ln x + 3\ln y - 4\ln z$

If the user clicks New Question again, another example might appear: $\ln(y^4.\ \frac{x}{z^4})$

The final expanded form shown upon revealing the answer would be: $4\ln y + \ln x - 4\ln z$.

This tool helps reinforce understanding of logarithmic expansion by allowing users to attempt multiple problems and verify their answers instantly. By engaging with different expressions, users gain familiarity with logarithmic properties and improve their problem-solving skills.

Making connections

Natural logarithms are logarithms with base e, that is, $\ln = \log_{\mathrm{e}}$.

Logarithm laws apply to natural logarithms and can be rewritten as:

$$\ln xy = \ln x + \ln y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^m = m \ln x$$

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Example 4

★☆☆

Rewrite $\ln a + 3\ln b - 12\ln c$ as a single logarithm.

Steps	Explanation
$\ln a + 3\ln b - 12\ln c = \ln a + \ln b^3 - \ln c^{12}$ $= \ln \frac{ab^3}{c^{12}}$	Use the laws of logarithms.

You can practise more questions like **Example 4** by using the applet below.

Interactive 2. Logarithm Compression Interactive

Credit: GeoGebra (https://www.geogebra.org/m/fdykgz2u) Kognity

More information for interactive 2

This interactive tool allows users to practice simplifying logarithmic expressions by applying fundamental logarithm rules. Each question presents a mathematical expression that users must condense into a single logarithm using the product rule, quotient rule, and power rule.

Users can generate new questions to test their skills and reveal answers to verify their solutions. This hands-on approach reinforces key logarithmic concepts and improves problem-solving skills in algebra.

For example, an expression like: 2ln(x) + 4ln(y) + 3ln(z)

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can be simplified to: $\ln(y^4 z^3 x^2)$

By engaging with this interactive, users gain a deeper understanding of logarithmic manipulation, making it a valuable tool for students learning exponent and logarithm properties.

In the laws of logarithms, multiplication is related to addition, and division is related to subtraction. Where else have you seen this relationship?

Watch the video below to learn how the laws of logarithms are derived and proved.

5 section questions

1. Number and algebra / 1.7 Further exponents and logarithms

Change of base of a logarithm

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/change-of-base-of-a-logarithm-id-27678/print/) Assign

The laws of logarithms that you learned in the previous section (/study/app/math-aa-hl/sid-134-cid-761926/book/laws-of-logarithms-id-27677/) work only for logarithms with the same base. If you encounter a question where the bases are different, you can use a formula to change the base.

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Important

The change of base formula is:

$$\log_a x = \frac{\log_b x}{\log_b a} \text{ for } a, b, \ x > 0 \text{ and } a \neq 1, \ b \neq 1.$$

Activity

Can you prove this law using a similar approach to the derivation of the other laws of logarithms? Carry out research to check your thinking.

Example 1

★☆☆

Given that $\log_5 a = 2$, evaluate $\log_a 25$.

Steps	Explanation
$\log_a 25 = \frac{\log_5 25}{\log_5 a} = \frac{2}{2} = 1$	Change the base to 5. Note that $25 = 5^2$, so $\log_5 25 = \log_5 5^2 = 2$.

Example 2

★☆☆

Using a scientific calculator that only performs logarithmic calculations in base 10 and base e, evaluate $\log_2 7$ to 3 significant figures.

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Steps	Explanation
$\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2} = 2.81$ (3 significant figures)	Change the base to 10 or e, then use the 'log' or 'ln' button' on the calculator. You can check that changing to base e $\left(\log_2 7 = \frac{\ln}{\ln}\right.$ gives the same answer

Example 3

★★☆

Write $\log_x y \times \log_y x$ as a single logarithm.

Steps	Explanation
$\log_y x = \frac{\log_x x}{\log_x y} = \frac{1}{\log_x y}$	Change the base of one of the logarithms. Here the logarithm with base y is changed to bas but you could also change the one with base x to base y.
$\log_x y \times \log_y x = \log_x y \times \frac{1}{\log_x y} = 1$	

Example 4

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Show that $\log_a b = \frac{1}{\log_b a}$.

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Steps	Explanation
LHS $=\log_a b = \dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a}$ LHS = RHS	Change both logarithms to the same base Use the method of LHS to RHS proof that y learned in subtopic 1.6.

4 section questions

1. Number and algebra / 1.7 Further exponents and logarithms

Solving exponential equations

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/solving-exponential-equations-id-27679/print/) Assign

Equating bases

You can solve some exponential equations by writing all exponential terms of the equation in the same base.

Example 1

★★☆

Solve $27^x = \dfrac{1}{3}$.

Steps	Explanation
$27^x = \dfrac{1}{3} \Leftrightarrow \left(3^3\right)^x = 3^{-1}$ $\Leftrightarrow 3^{3x} = 3^{-1}$	Rewrite both sides using the same base.

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Steps	Explanation
If $3^{3x} = 3^{-1}$, then $3x = -1 \quad \text{so} \quad x = -\frac{1}{3}$	Since the bases are equal, the exponents must l equal.

Example 2

★★★

Solve $2^{2-x} \times \frac{1}{16^x} = 8^{-2-x}$.

Steps	Explanation
$2^{2-x} \times \frac{1}{16^x} = 8^{-2-x}$ $\Leftrightarrow \quad 2^{2-x} \times \left(2^4\right)^{-x} = \left(2^3\right)^{-2-x}$ $\Leftrightarrow \quad 2^{2-x+(-4x)} = 2^{-6-3x}$	Rewrite all terms in base 2.
$2^{2-5x} = 2^{-6-3x}$ $\therefore 2 - 5x = -6 - 3x$	Equate the exponents.
$\Leftrightarrow \quad 8 = 2x$ so $x = 4$	Solve for x.

Using logarithms

You can use logarithms to solve exponential equations. You do this by rewriting an exponential equation in logarithmic form or by taking logarithms of both sides.

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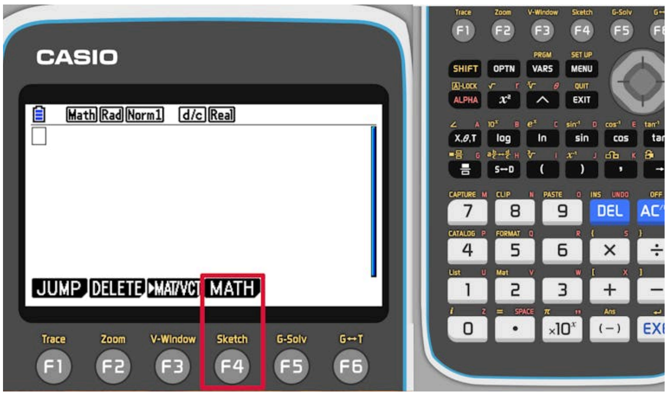
Steps	Explanation
This guide helps you use the calculator to find the solution of $2^x = 7$. The logarithmic form of this equation is $$x = \log_2 7.$$ Here you will see a way of calculating the value of this logarithmic expression. Choose the option to enter the calculator screen.	
Press F4 to see the math options ...	

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Steps	Explanation
... and press F2 to bring up the template for the logarithmic expressions.	
Fill in the base and the argument and press EXE. The displayed value is the approximate solution of the original equation.	

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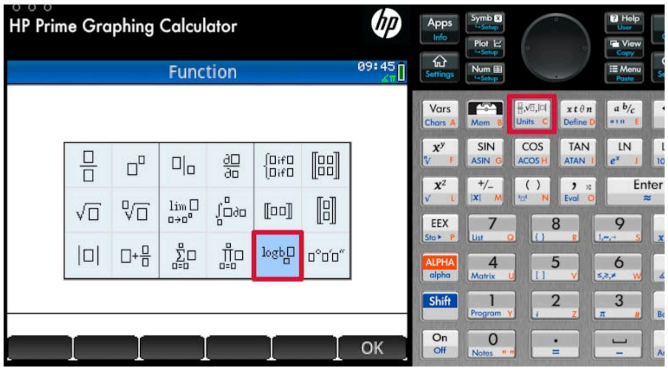
view

Steps	Explanation
This guide helps you use the calculator to find the solution of $2^x = 7$. The logarithmic form of this equation is $$x = \log_2 7.$$ Here you will see a way of calculating the value of this logarithmic expression. Enter the home screen of any application.	
Open the templates and choose the template for the logarithmic expression.	

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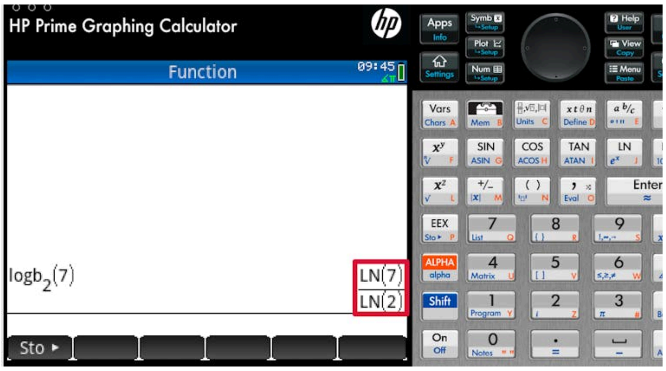
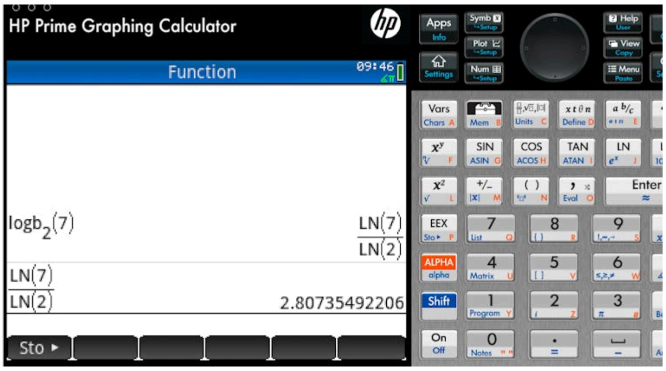
Steps	Explanation
Fill in the base and the argument and press enter. In case you see an expression instead of an approximate value, tap on it and press enter again.	
The displayed value is the approximate solution of the original equation.	

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<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>
This guide helps you use the calculator to find the solution of $2^x = 7$. The logarithmic form of this equation is

$$x = \log_2 7.$$

Here you will see a way of calculating the value of this logarithmic expression.

You can access the feature you need using the math button ...
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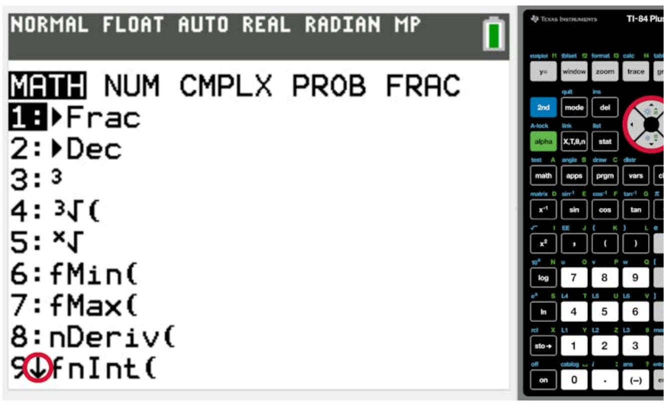

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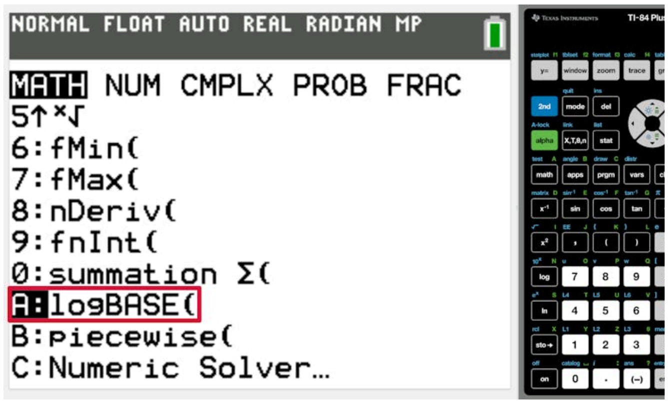
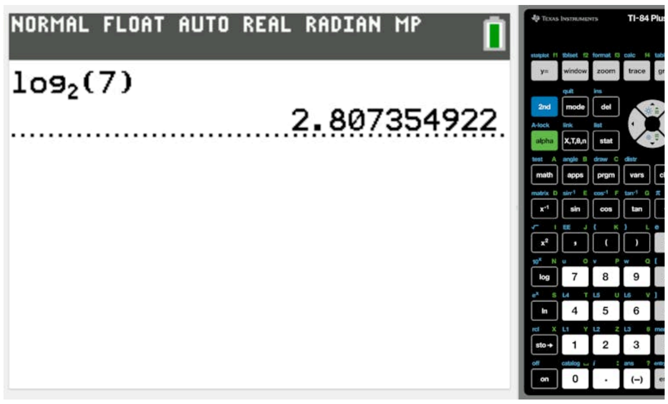
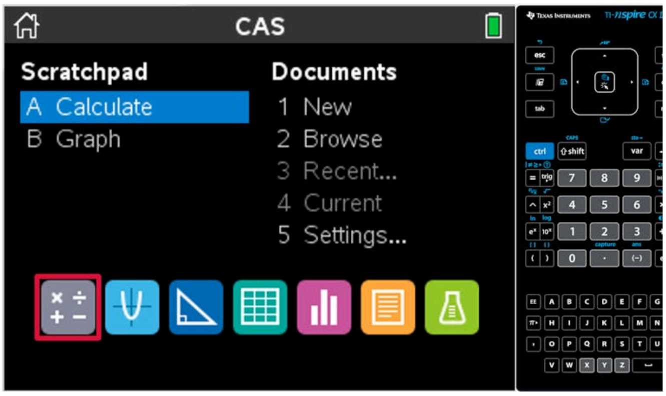
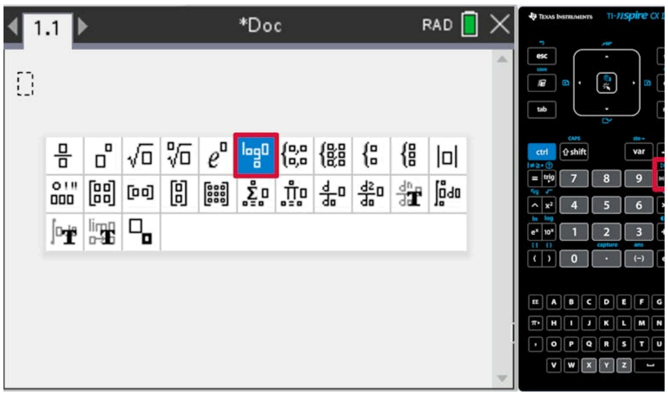
Steps	Explanation
... and choose the option to evaluate logarithmic expression.	
Fill in the base and the argument in the template and press enter. The displayed value is the approximate solution of the original equation.	

Steps	Explanation
This guide helps you use the calculator to find the solution of $2^x = 7$. The logarithmic form of this equation is $$x = \log_2 7.$$ Here you will see a way of calculating the value of this logarithmic expression. Open a calculator page.	
Open the templates and choose the template for the logarithmic expression.	

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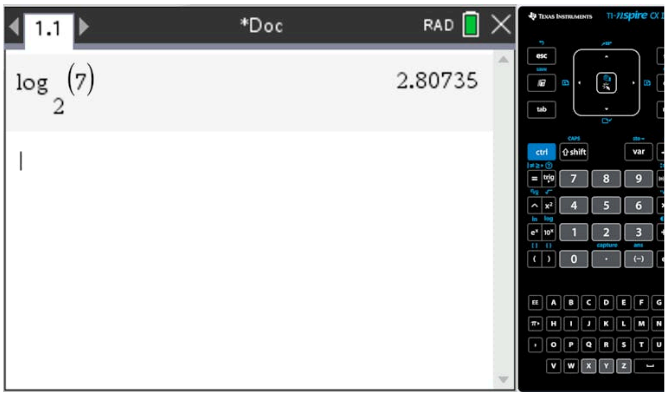
Steps	Explanation
Fill in the base and the argument and press enter. The displayed value is the approximate solution of the original equation.	

Example 3

★☆☆

Solve $3^x = 5$.

Method 1

Steps	Explanation
$3^x = 5 \quad \Leftrightarrow \quad \log_3 5 = x$ $x = 1.46$ (3 significant figures}	Rewrite in logarithmic form and evaluate on your graphic display calculator.

Method 2

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Steps	Explanation
$\log 3^x = \log 5$	Take the logarithm of both sides of the equation. Y can choose any base (as long as it's the same base both sides).
$\log 3^x = \log 5 \quad \Leftrightarrow \quad x \log 3 = \log 5$ $\Leftrightarrow \; x = \frac{\log 5}{\log 3}$ $= 1.46$ (3 significant figures)	Solve for x and evaluate on your graphic display calculator.

Example 4

★★☆

Solve $4 \times 9^{2x} = 10$.

Method 1

Steps	Explanation
$4 \times 9^{2x} = 10 \quad \Leftrightarrow \quad 9^{2x} = \frac{10}{4}$ $\Leftrightarrow \quad \log_9 2.5 = 2x$ $\Leftrightarrow \quad x = \frac{\log_9 2.5}{2}$ $x = 0.209$ (3 significant figures)	Rearrange to get into $a^b = c$ form and then rewrite in logarithmic form. Evaluate using your graphic display calculator.

Method 2

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Steps	Explanation
$\log 4 \times 9^{2x} = \log 10$ $\Leftrightarrow \quad \log 4 + 2x \log 9 = 1$ $\Leftrightarrow \quad 2x \log 9 = 1 - \log 4$ $\Leftrightarrow \quad x = \frac{1 - \log 4}{2 \log 9}$ $= 0.209$ (3 significant figures)	Take logs of both sides. You can pick any base you wan Use the laws of logarithms to rearrange the equation. Solve for x. Then evaluate on your graphic display calculator.

Example 5

★★★

Solve $1.27 \times 7^{2x-1} = 5^x$. Give your answer correct to three significant figures.

Steps	Explanation
$\log\left(1.27 \times 7^{2x-1}\right) = \log 5^x$	Take logarithms of both sides. You can choose any base you want.
$\log 1.27 + (2x - 1)\log 7 = x \log 5$ $\Leftrightarrow \log 1.27 + 2x \log 7 - \log 7 = x \log 5$ $\Leftrightarrow \log 1.27 - \log 7 = x \log 5 - 2x \log 7$ $\Leftrightarrow \log 1.27 - \log 7 = x(\log 5 - 2\log 7)$ $x = \frac{\log 1.27 - \log 7}{\log 5 - 2\log 7} = 0.748$ (3 significant figures)	Use the laws of logarithms to simplify, rearrange the equation and solve for x.

Example 6

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Solve $4^x - 2^x - 6 = 0$.

Steps	Explanation
$\left(2^2\right)^x - 2^x - 6 = 0$	Rewrite the exponentials in base 2.
$\left(2^x\right)^2 - \left(2^x\right) - 6 = 0$ Let $a = 2^x$. Then $a^2 - a - 6 = 0$ $(a-3)(a+2) = a$ $a = 3 \text{ or } a = -2$	Note that this equation looks like a quadra
$2^x = 3 \quad \Leftrightarrow \quad \log_2 3 = x$ $x = 1.58$ (3 significant figures) $2^x = -2$ has no solution.	Solve for x.
$x = 1.58$ (3 significant figures)	Write down the final answer.

Application questions

Application questions where exponential models are used can often be solved using logarithms.

Example 7

★★☆

The number of people in a school who have heard a particular rumour increases by 2% every minute. Calculate how long it will take for the number of people who have heard the rumour to triple (i.e. increase to three times the original number).

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Steps	Explanation
Let u_n be the number of people who have heard the rumour after n minutes. Then $$u_n = u_0(1.02)^n$$ and we want to find n such that $$u_n = 3u_0$$ $$3u_0 = u_0(1.02)^n \quad \Leftrightarrow \quad \frac{3u_0}{u_0} = 1.02^n$$ $$\Leftrightarrow \quad 3 = 1.02^n$$	This is an application of geome sequences. Use $u_n = u_0 r^n$.
$$n = \log_{1.02} 3 = 55.4781\ldots$$	Solve the exponential equation rewriting it in logarithmic form.
It will take 56 minutes for the number of people who have heard the rumour to triple.	

Making connections

Before you were introduced to logarithms, you learned how to solve these types of questions with the use of a graphic display calculator.

Example 8

★★☆

The population of a city grew by an average of 2.5% per year from the year 2000 to the year 2015. Given that the population was 1.89×10^7 at the end of 2000, determine the year in which the population reached 2.03×10^7.

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Steps	Explanation
Let u_n be the population of the city n years after 2000. Then $$u_n = 1.89 \times 10^7 \times (1.025)^n$$ and we want to find n for which $2.03 \times 10^7 = 1.89 \times 10^7 \times (1.025)^n$	This is an application of geometric sequences. Use $u_n = u_0 r^n$.
$$\frac{2.03 \times 10^7}{1.89 \times 10^7} = 1.025^n$$ $$\Leftrightarrow\ 1.07407 = 1.025^n$$ $n = \log_{1.025} 1.07407 = 2.89$	Solve for n.
The population of $2.03 \times 10^7$was reached in 2003.	Since the question asks for t year, round up the value of r and add to 2000.

International Mindedness

A geometric sequence model ($u_n = u_0 r^n$) is often used for population growth. The diagram below shows the population of Brazil, which follows this exponential trend.

What does this model tell you about human consumption of resources throughout the world? Do you think that the population of the world will continue to follow such a trend?

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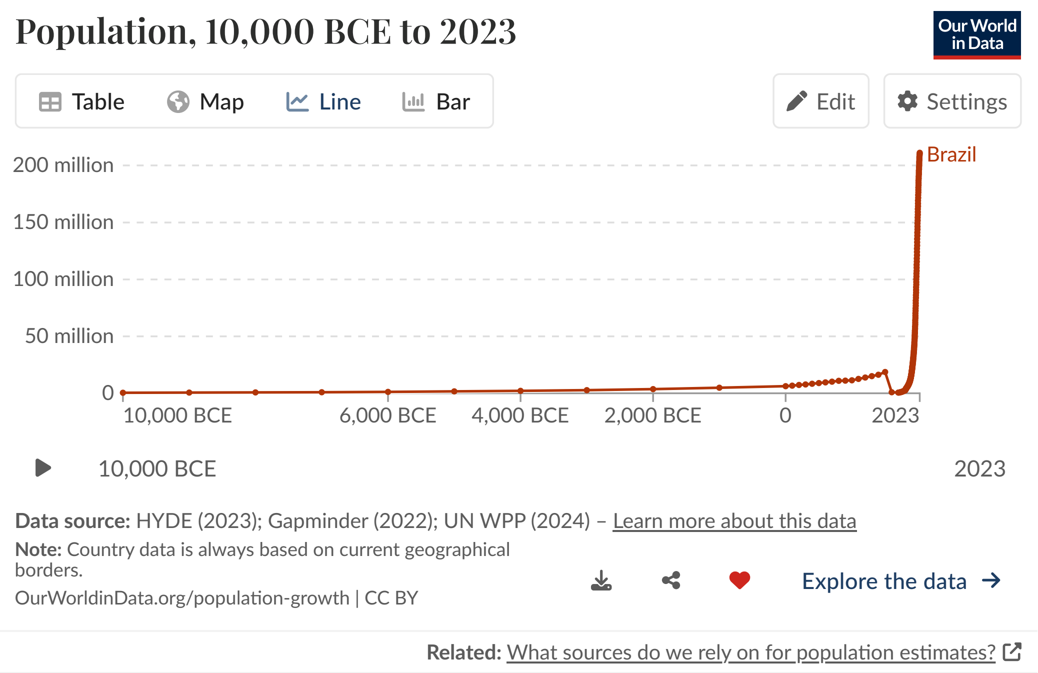

Interactive 1. A Geometric Sequence Model of Brazil's Population.

More information for interactive 1

The interactive visualization presents population data for Brazil from 10,000 BCE to 2023, modeled using a geometric sequence ($U_n = U_0 r^n$) to illustrate exponential growth trends. Users can explore this data through tables, maps, or charts, with options to adjust timeframes and compare other countries. The graph highlights Brazil's rapid population surge, particularly in recent centuries, reflecting broader global patterns of demographic expansion. This model suggests that resource consumption likely follows a similar exponential trajectory, raising questions about sustainability as populations grow.

The tool allows users to select different countries, revealing how population trends vary geographically and historically. For instance, comparing Brazil's growth to other nations shows disparities in timing and scale of demographic shifts, influenced by factors like industrialization, healthcare, and migration. The time-lapse feature emphasizes how modern advancements have accelerated population increases, with global totals rising sharply post-1800. Such trends imply that resource demands—energy, food, water—have grown disproportionately, straining planetary boundaries.

While the geometric model captures past growth, its future applicability is debated. Population growth rates are slowing in many regions due to declining birth rates and societal changes, suggesting a potential shift toward logistic rather than exponential patterns. The data underscores the need for sustainable resource management, as unchecked exponential growth could outstrip Earth's capacity. By visualizing these trends, the tool invites reflection on balancing demographic dynamics with environmental limits.

5 section questions

Checklist

What you should know

By the end of this subtopic you should be able to:

- interpret $a^{\frac{1}{n}}$ as $\sqrt[n]{a}$
- use $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$ to manipulate and simplify expressions containing rational exponents
- evaluate simple $a^{\frac{m}{n}}$ expressions without a calculator
- rewrite exponential equations in any base in equivalent logarithmic form using $a^x = b \Leftrightarrow \log_a b = x$ for $a > 0,\ a \neq 1$ and $b > 0$
- evaluate logarithms with and without a calculator
- understand that

$$\log_a a^m = m$$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$a^{\log_a m} = m$$

$$\mathrm{e}^{\ln m} = m$$

- use the laws of logarithms to condense or expand logarithmic expressions
- use the change of base formula to rewrite logarithms in a different base
- solve exponential equations by equating bases
- solve exponential equations by using logarithms.

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Investigation

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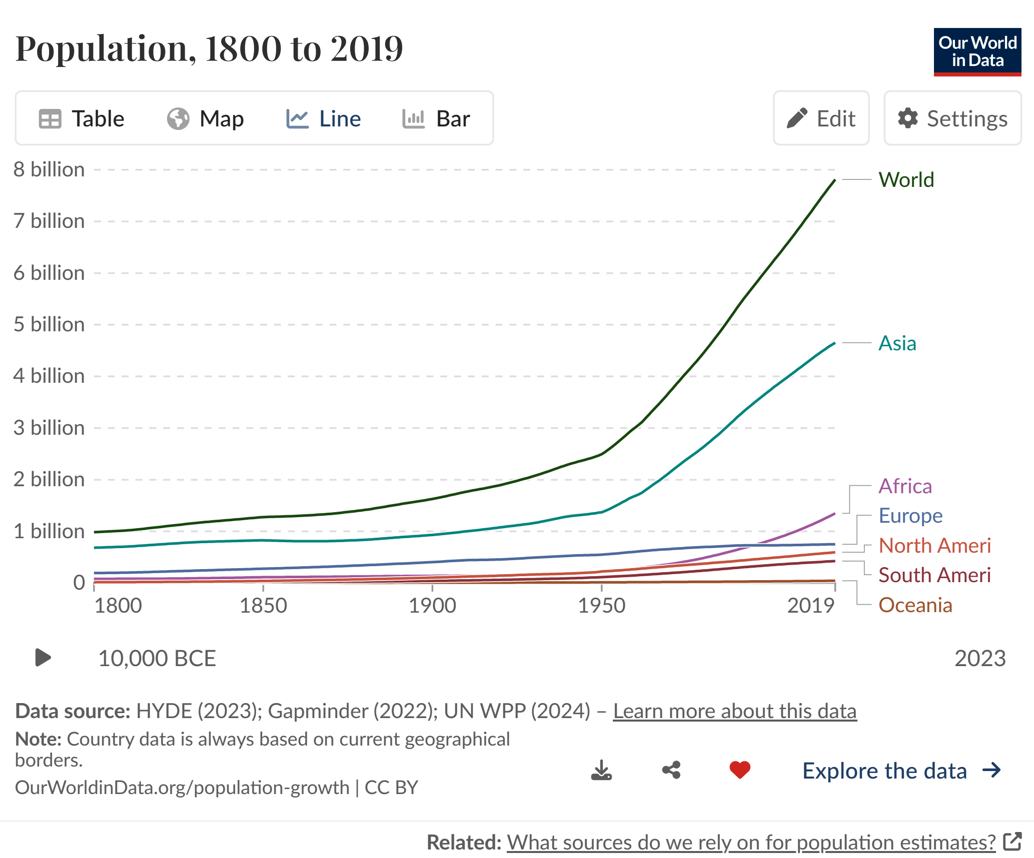
Part 1

Use the applet below to explore population trends in a variety of countries between the years 1800 and 2019.

Identify which of the countries that you explored have population trends that can be modelled using exponential functions. Justify your answer with your observations from the graphs.



Interactive 1. Population Trends in a Variety of Countries Between the Years 1800and 2019.

More information for interactive 1

This interactive allows users to analyze global population trends from 1800 to 2019, comparing data across continents and regions. By exploring the visualizations—including tables, maps, and charts—users can observe how populations have grown exponentially in some areas while following different patterns in others. In the year 1850, world population was around 1.29 billion and it reached 1.67 billion by the year 1900. The data reveals striking disparities, with Asia experiencing dramatic increases compared to slower growth in regions like Oceania.

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Users are encouraged to investigate which countries or continents exhibit trends that align with exponential growth models, considering factors like industrialization, healthcare advancements, and migration patterns that may explain these trends. The users can try to figure out which characteristics distinguish exponentially growing populations from those with linear or stagnant growth and how these trends might evolve in the future. The included time-lapse feature further enhances this exploration by illustrating the pace of change over time.

The data for the population of China is shown in the graph of the applet below.

Interactive 2. Exponential Function Model for the Population of China.

More information for interactive 2

This interactive graph lets the user experiment with exponential population growth modeling through hands-on adjustment of key parameters. This is a graph with X- axis showing 400 years from 1700 in interval of 50 years. Y axis shows population in millions from 0 to 2400. The actual change in pollution of China is denoted in orange line. The pink curve represents your model defined by the equation $y = Pr^t$, where P is the initial population and r is the annual growth factor. The user's challenge is to drag the sliders for P and r until the pink model curve closely matches the yellow curve showing actual historical population data. A good fit occurs when the pink curve follows the general trajectory of the yellow data points - users will know that they have found reasonable values when most of the historical data points either lie on or closely follow the modeled curve. For instance, in modeling China's population growth, users might find optimal values around $P = 112$ million (representing the baseline population) and $r = 1.0083$ (indicating 0.83% annual growth) to best fit the historical trend from $1700 - 1900$. This exercise demonstrates both the usefulness and limitations of exponential modeling - while such models can effectively

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describe certain historical growth periods, real-world populations often experience changing growth rates due to social, economic and environmental factors. The tool helps visualize how sensitive population projections are to small changes in growth rates, and why demographers often use more complex models for long-term forecasting.

Part 2

Use the applet above to manipulate the values of P and r to create a good model for the population of China.

Explain what the variables y, P, r and t mean in the context of this model.

Decide if this model is valid to use to predict the population of China in 2025, 2040 or 2150.

Explain how you can use this model to find the number of years it took for the population to double from 1800 and from 1900.

Decide whether you can use the same method to predict how long it will take for the population to double from 2019. Justify your decision.