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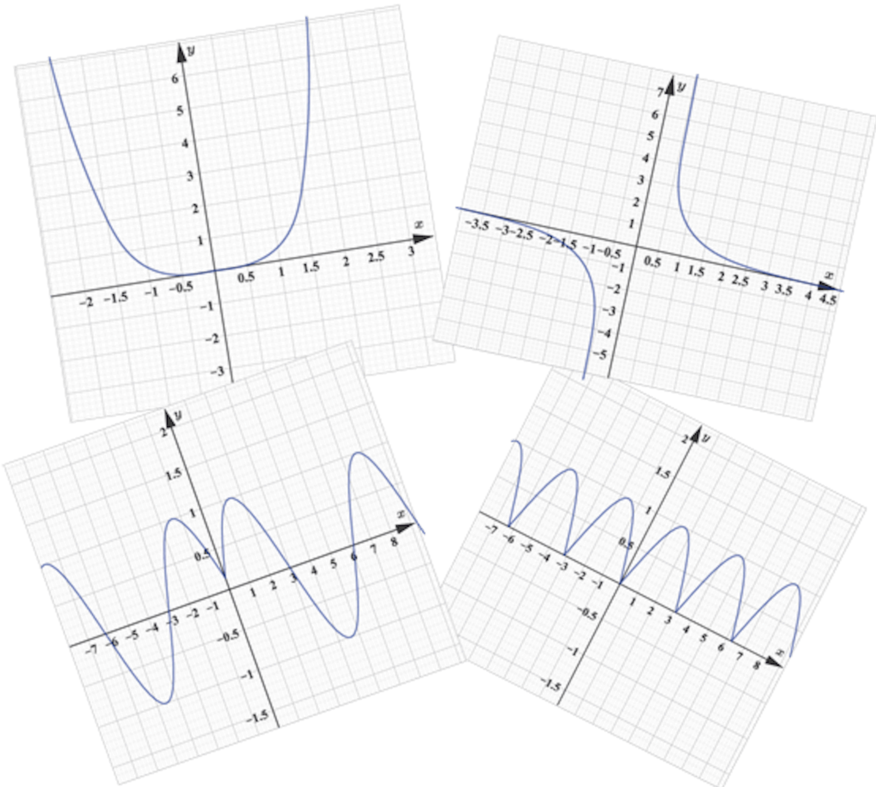
The big picture

Would you believe it if someone tells you that the graphs shown below are some form of a cubic function?



More information

The image shows four mathematical graphs arranged in a 2x2 layout. Each graph features a different curve plotted on a Cartesian coordinate system with x and y axes.

1. **Top Left Graph:** This graph shows a parabolic curve that opens upwards. The y-axis ranges from -3 to 9, and the x-axis has markings from -3 to 3 at intervals of 0.5.

2. **Top Right Graph**: This graph presents a curve resembling a vertical asymptote. The y-axis ranges from -5 to 3, while the x-axis spans -3.5 to 4.5, both in intervals of 1.

3. **Bottom Left Graph**: A sinusoidal wave is depicted, showing oscillations symmetrical about the x-axis. The x-axis ranges from -7 to 7, and the y-axis is labeled from -1.5 to 1.5 in intervals of 0.5.

4. **Bottom Right Graph**: Another sinusoidal wave with smaller amplitudes, covering a wider range on the x-axis from -9 to 9. The y-axis marks intervals from -1.5 to 1.5.

The graphs illustrate different mathematical behaviors such as polynomial and sinusoidal functions based on their shapes and characteristics.

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Or that the graphs are in fact sine functions?

In all the above graphs, some form of transformation has been done to a function that gives a completely different graph. However, there is a relationship between the original graph and the transformed graph, which you will be learning about in this section.

In this subsection, you will be learning

- how to graph some of the transformations of functions such as:

$|f(x)|$, $f(|x|)$, $\dfrac{1}{f(x)}$, $f(ax+b)$ and $(f(x))^2$ from a given function $f(x)$

- how to solve equations and inequalities with modulus functions.

Concept

Representation

Representing a function in the form of a graph helps in understanding the various transformations related to each of the parameters in the function. The transformed function could also be **represented** through a completely different function.

Which is easier — being given a transformed function and finding the original function or being given an original function and finding the transformed function? Why?

Student view

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2. Functions / 2.16 Further graph transformations

Graphs of y=|f(x)| and y=f(|x|)

Graphs of $y = |f(x)|$

Recall the meaning of absolute value of a number:

$|x|$ is always positive and it is defined as:

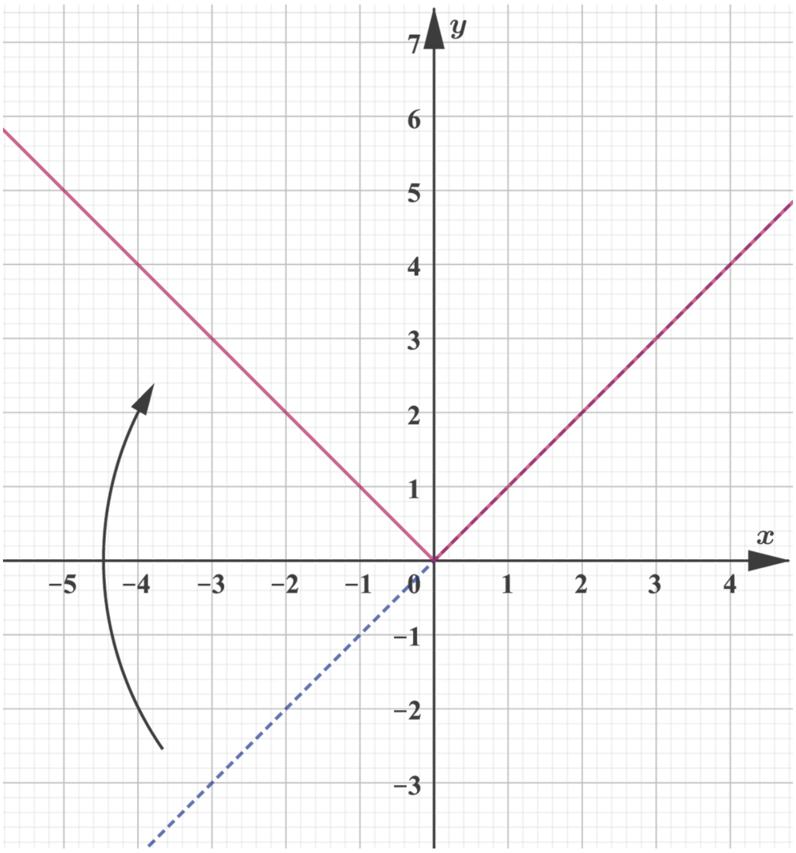
$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

For example, $|3| = 3$ and $|-3| = -(-3) = 3$.

The absolute value function or modulus function is $f(x) = |x|$.

The graph of $f(x) = |x|$ (red line) is compared with the graph of $f(x) = x$ (blue dotted line) below:

Student view



More information

The image is a graph comparing the functions (f(x) = |x|) and (f(x) = x). The graph is plotted on a coordinate plane with both x and y-axes labeled. The x-axis ranges from -6 to 6, and the y-axis ranges from -6 to 6.

The red line represents (f(x) = |x|). It forms a V shape, intersecting the origin (0,0). The line ascends at a 45-degree angle in both directions.

The blue dotted line represents (f(x) = x). It is a straight line intersecting the origin and extending with a 45-degree angle to the top right and bottom left of the graph, indicating linearity.

This comparison highlights the difference between the absolute value function, which reflects a V shape, and the linear function which is a diagonal line.

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Important

Note that the negative part of the line $f(x) = x$ is shifted to the positive y—values. This is because, when x is negative, $f\left(x\right) = |x| = -\left(-x\right) = x$. What is this transformation

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called?

So, for $x = -3$, $f(-3) = |-3| = 3$, which will be the same value as $f(3)$.

Hence, for all x values, $f(x) = f(-x)$. What type of function is $|x|$? (see subtopic 2.14 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26756/)).

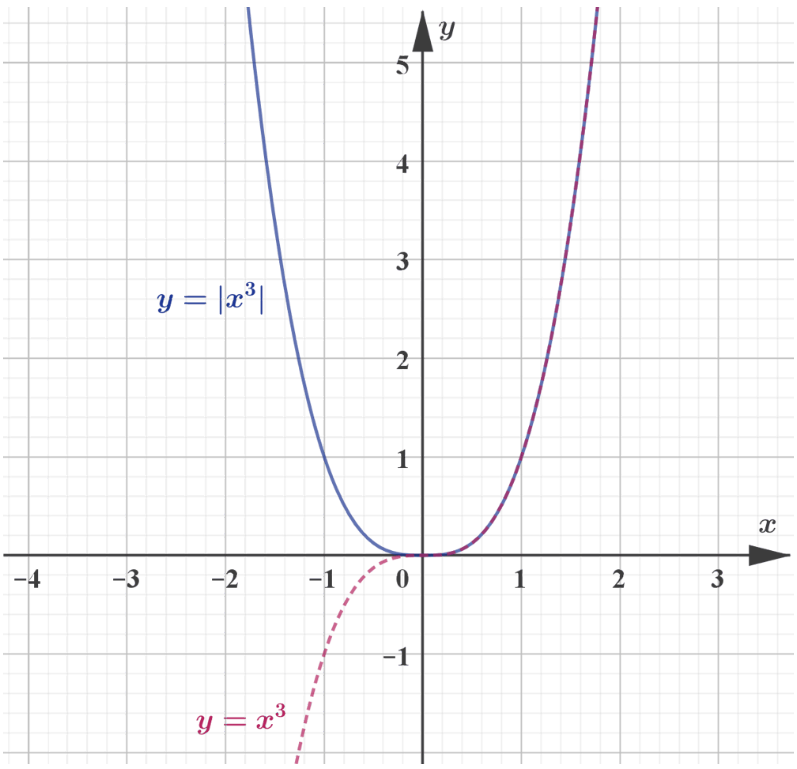
For any function $f(x)$,

$$|f(x)| = \begin{cases} f(x), & f(x) > 0 \\ -f(x), & f(x) < 0 \end{cases}$$

For example, $|x^3| = \begin{cases} x^3: & x^3 > 0 \\ -x^3: & x^3 < 0 \end{cases}$

What would be $|x^2|$? Do you have two options for x^2 as in x^3?

The graphs of $y = |x^3|$ and $y = |x^2|$ are shown below:



More information

Student view

The image is a graph showing the functions (y = |x^3|) and (y = |x^2|). The graph is plotted on a Cartesian coordinate system with grid lines for better reference.

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- The X-axis represents the value of (x) and ranges from -3 to 3.
- The Y-axis represents the absolute value of the function outputs and ranges from 0 to 5.

The curve for (y = |x^2|) forms a symmetric V-shape, touching the origin at (0,0) and rising upwards on both sides as x increases or decreases, indicating that the function value is always non-negative.

The curve for (y = |x^3|) also passes through the origin but with a softer curve, bending upwards more gradually compared to (y = |x^2|) as it moves away from the origin. This curve is steeper around the origin and flattens out as (x) approaches ±3.

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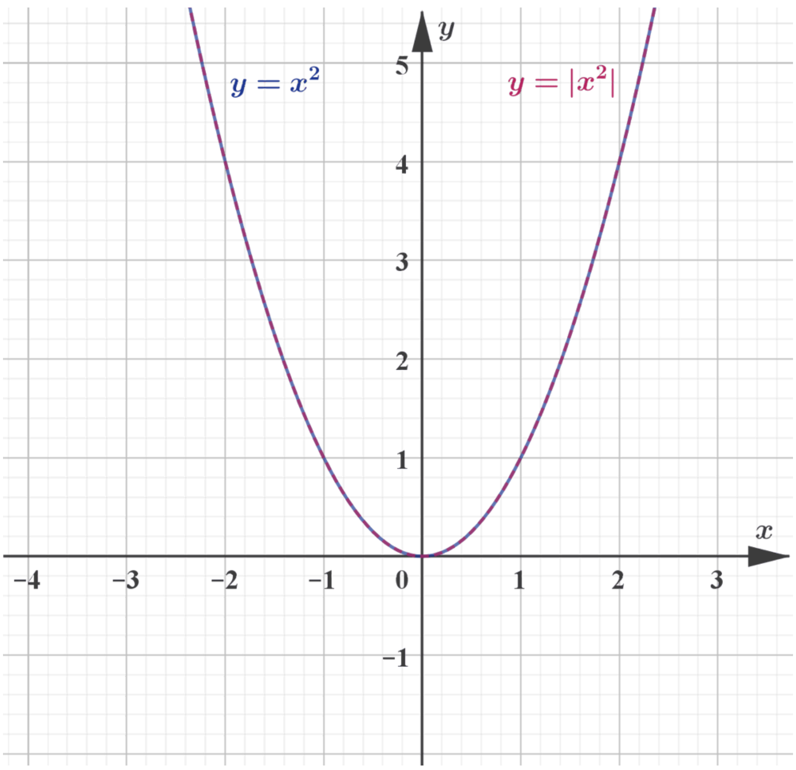

More information

The image is a graph displaying two functions. The X-axis represents the (x) values and ranges from -6 to 6. The Y-axis represents the (y) values and ranges from -7 to 7.

The first function is (y = -x^2), a parabola opening downward. The vertex is at the origin (0, 0), and it intersects the Y-axis at (0, 0). The parabola symmetrically stretches on both sides of the Y-axis, intersecting the X-axis at points (-3, -9) and (3, -9).

Student view

The second function is (y = |x^3|), starting at the origin (0, 0) and moving in both positive and negative directions of the X-axis. This curve has a steeper slope due to the cubic function and reflects across the X-axis due to the absolute value.

Both functions are superimposed on a grid with one-unit intervals, helping to highlight their symmetry and intersection points. The text notes the reflection of negative part of (x^3) in the X-axis and highlights that (x^2) does not have any negative values, explaining that in this case (f(x) = |f(x)|) for all values of (x).

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Note the reflection of negative part of x^3 in the x-axis. But x^2 does not have any negative values. Hence, in this case, $f(x) = |f(x)|$ for all values of x.

Example 1

★★☆

Sketch the graphs of $y = f(x)$ and $y = |f(x)|$ on the same set of axes for the following functions.

$$f(x) = x^2 + 2x - 1$$

$$f(x) = x^3 - 3x^2 + 2x + 4$$

$$f(x) = \sin(x)$$

What did you observe?

Compare and check your sketches with the graphs shown below: (red curves represent $y = |f(x)|$ and green dotted curves represent $y = f(x)$ on the graphs)

1. $f(x) = x^2 + 2x - 1$

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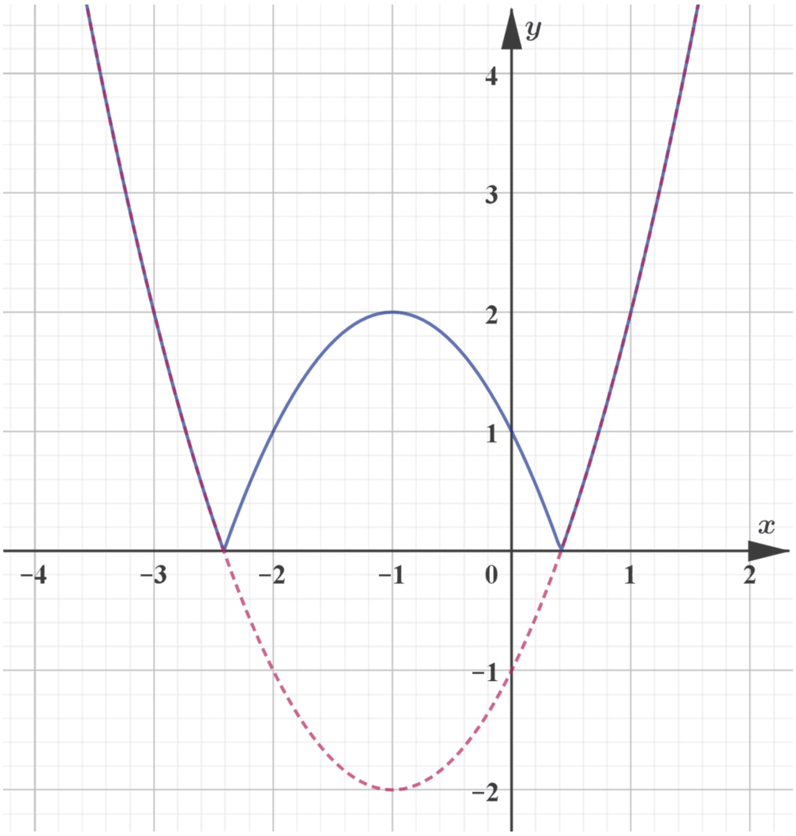

2. $f(x) = x^3 - 3x^2 + 2x + 4$

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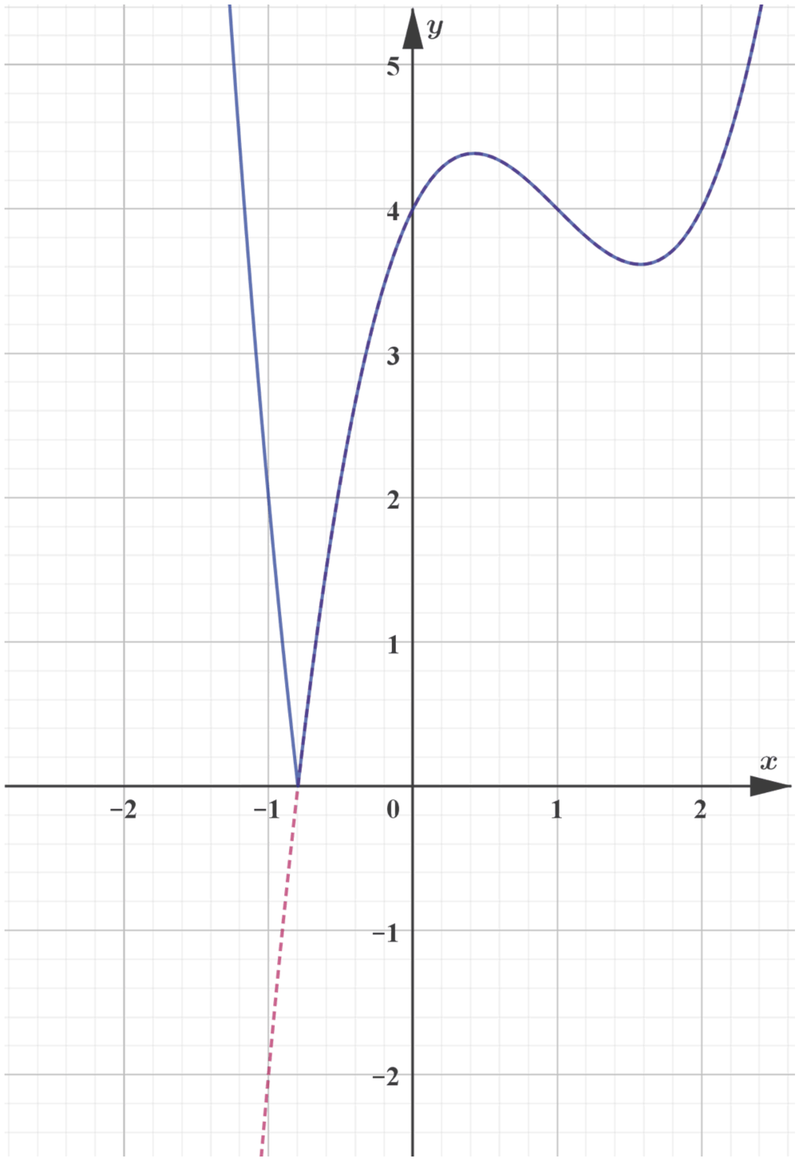

3. $f(x) = \sin(x)$

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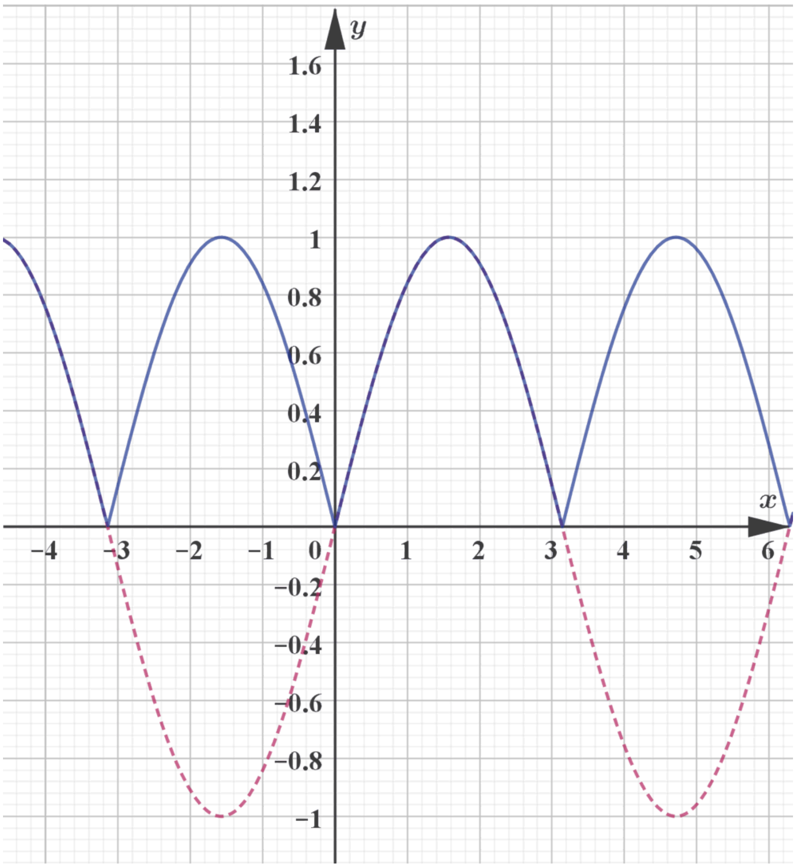

Fill in the blanks based on your observations, choosing the right answer from the options given in the brackets:

1. The graph of $y = |f(x)|$ is always above the ________. (y-axis, x-axis)
2. The negative part of $y = f(x)$ is _______________ on the x-axis. (translated , reflected)
3. Which intercept remains the same for $y = f(x)$ and $y = |f(x)|$? (x-intercept, y-intercept)
4. Which intercept changes its sign only when it is negative? (x-intercept, y-intercept)

1. The graph of $y = |f(x)|$ is always above the x-axis.
2. The negative part of $y = f(x)$ is reflected on the x-axis.
3. Which intercept remains the same for $y = f(x)$ and $y = |f(x)|$? x-intercept
4. Which intercept changes its sign only when it is negative? y-intercept

Graphs of $y = f(|x|)$

Consider the function $f(x) = 2x + 5$.

Student view

$$f(|x|) = 2|x| + 5$$

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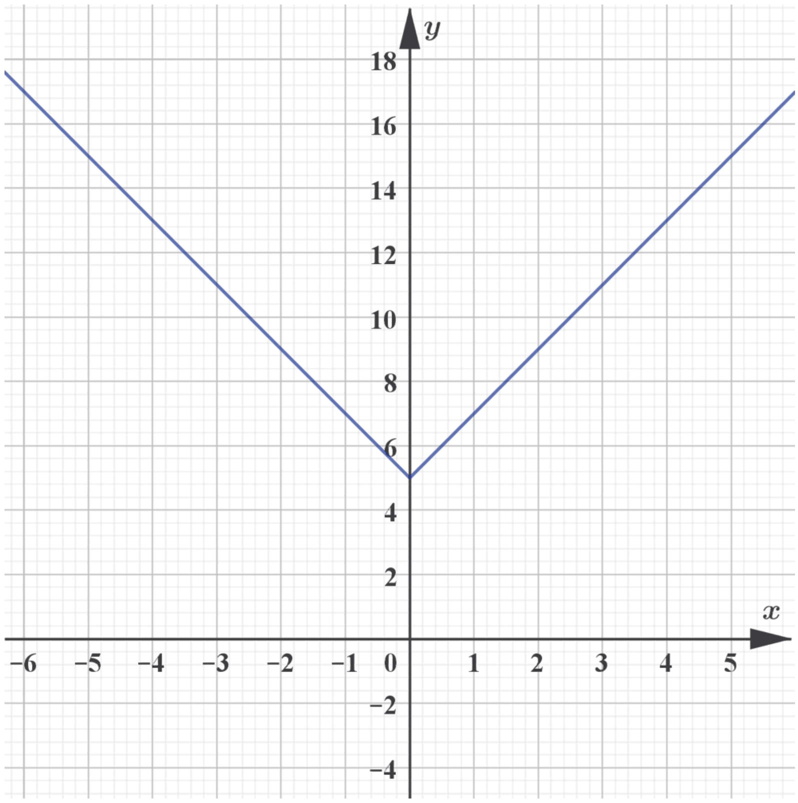
This means that both the negative and positive values of x give the same result for y.

Hence the graph for positive values of x would be reflected in the y-axis.

The following graph shows = $f\left(|x|\right) = 2\left|x\right| + 5$.



More information

The graph represents the function f(|x|) = 2|x| + 5, which is a V-shaped graph due to the absolute value. The X-axis is marked with intervals ranging from -10 to 10. The Y-axis has intervals labeled from 0 to 15. The line intersects the Y-axis at point (0, 5). It forms an upward sloping line beginning from the Y-axis extending to the right and an identical downward slope going to the left, reflecting the symmetry of the absolute value function. The overall shape indicates that for every positive and negative value of x, the y value will be the same, confirming the property of even functions.

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Hence, all functions of the form $f\left(|x|\right)$ are even functions since $f\left(-x\right) = f\left(x\right)$ for all x.

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Example 2

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Sketch the graphs of $y = f(x)$ and $y = f(|x|)$ on the same set of axes and compare them:

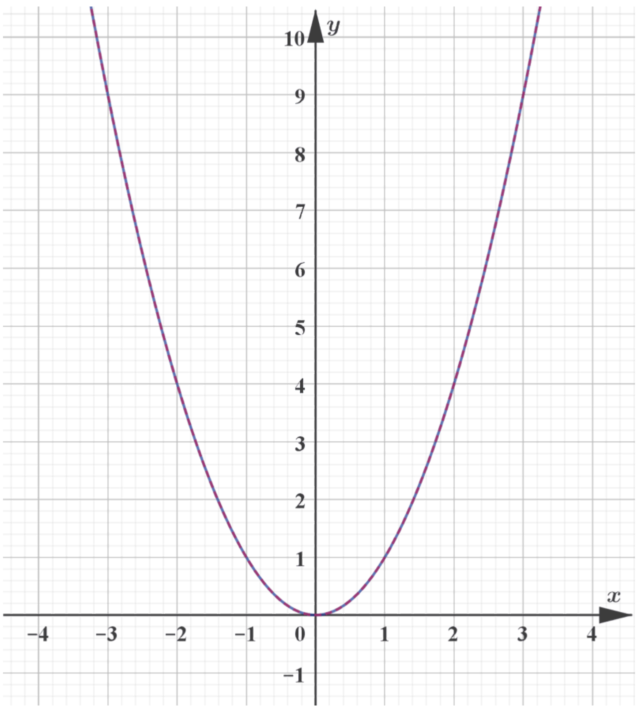
$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^2 + 2x - 1$$

$$f(x) = x^3 - 3x^2 + 2x + 4$$

$$f(x) = \sin(x)$$

	Function $f(x)$ (red dotted)	Graph of $y = f(\lvert x \rvert)$ (blue)
1.	$f(x) = x^2$	

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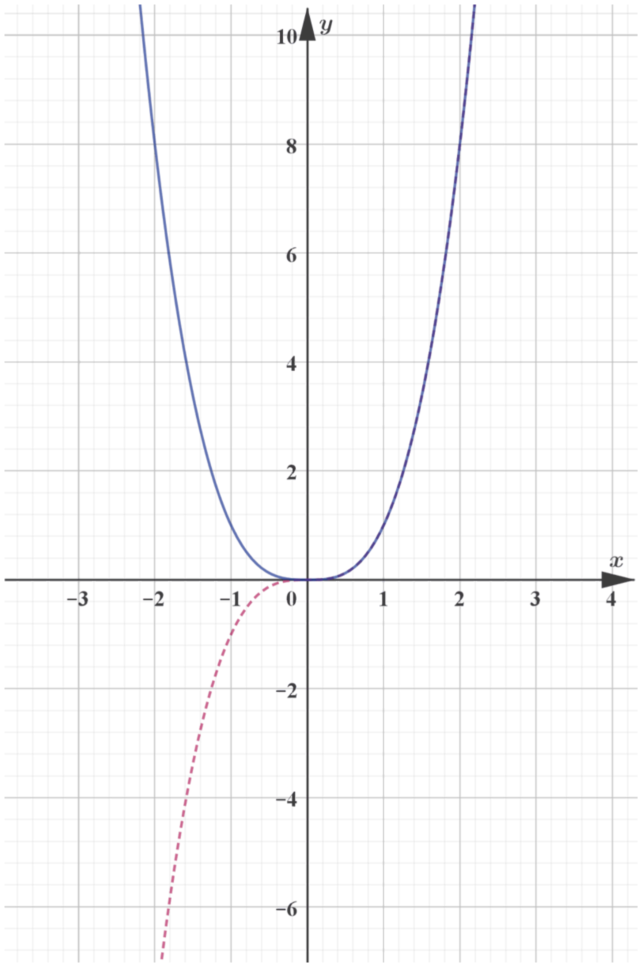
	Function $f(x)$ (red dotted)	Graph of $y = f(\|x\|)$ (blue)
2.	$f(x) = x^3$	

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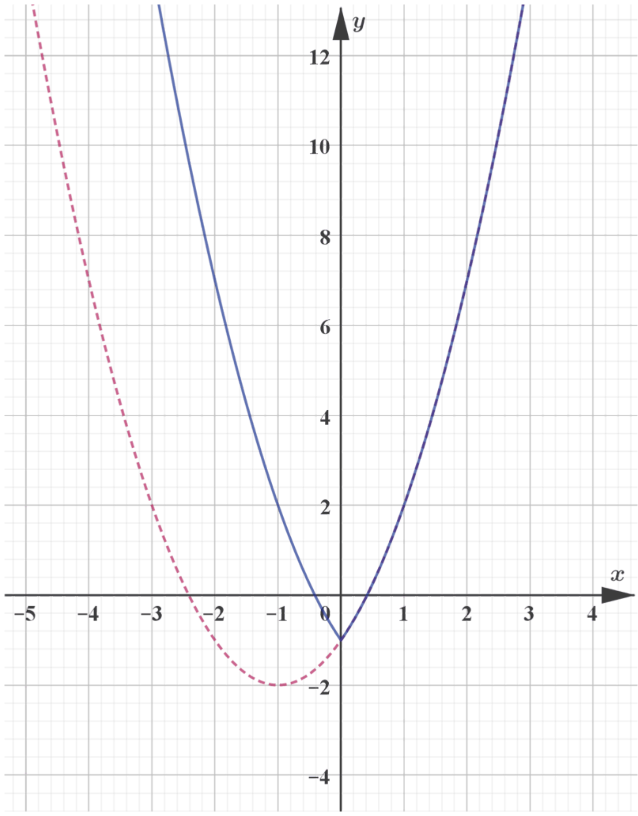
	Function $f(x)$ (red dotted)	Graph of $y = f(\|x\|)$ (blue)
3.	$f\left(x\right) = x^2 + 2x - 1$	

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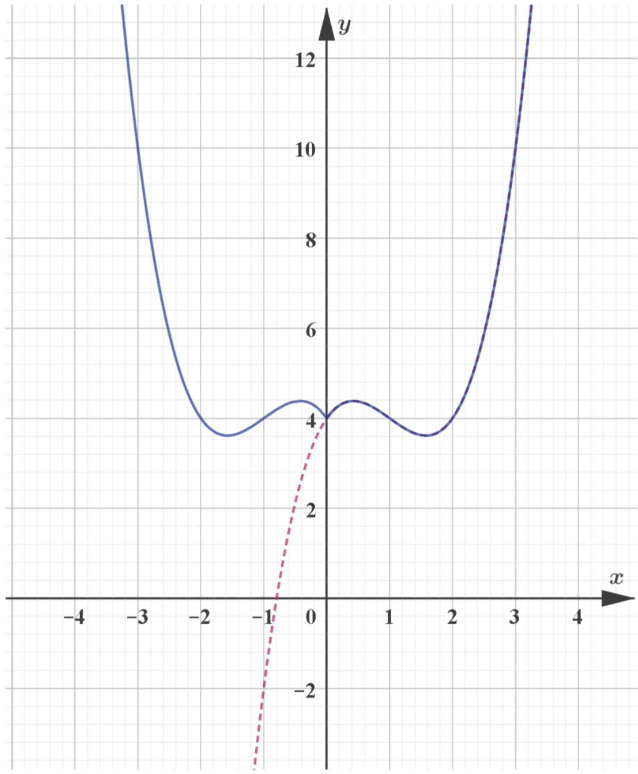
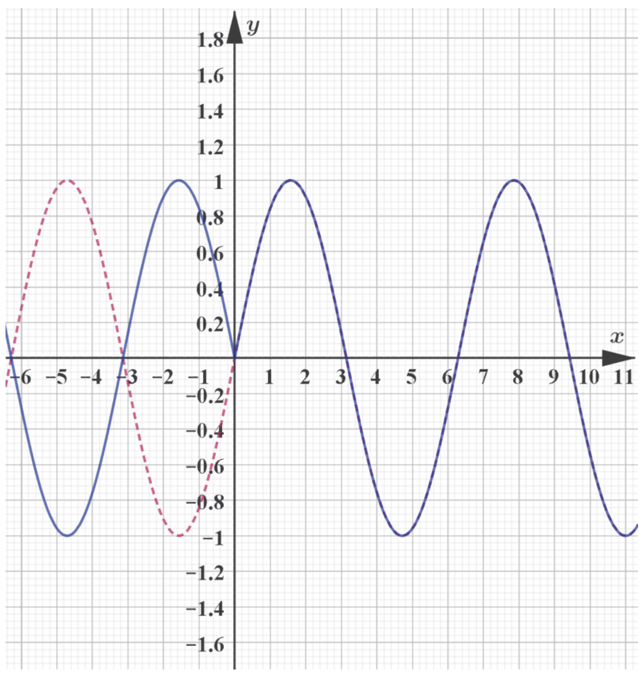
	Function $f(x)$ (red dotted)	Graph of $y = f(\|x\|)$ (blue)
4.	$f(x) = x^3 - 3x^2 + 2x + 4$	
5.	$f(x) = \sin(x)$	

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Making connections

What did you observe for the first two functions x^2 and x^3? How do the graphs for $|f(x)|$ and $f(|x|)$ compare? What are the similarities and differences? Do you have such similarities and differences for the rest of the functions? Why or why not?

Fill in the blanks based on your observation above:

In general, the graph of $y = f(|x|)$:

1. is symmetrical about the ____________. (x-axis, y-axis)
2. is always an ______________ function. (odd, even)
3. has the same ______________ as $f(x)$. (x-intercept, y-intercept)
4. The part of the graph for positive values of x is _____________ in the y-axis. (translated, reflected)

1. is symmetrical about the y-axis.
2. is always an even function.
3. has the same y-intercept as $f(x)$.
4. The part of the graph for positive values of x is reflected in the y-axis.

International Mindedness

Absolute value functions are largely used in vector space, complex numbers, calculus. And so on, or simply to indicate the distance between two objects. How do different countries use different units and terminology to indicate distance between two objects? How is this useful in space science, for example, to find the distance between two stars?

3 section questions

Question 1

Difficulty:

★☆☆

The graph of $y = |f(x)|$, where $f(x) = \cos x$, is given by which of the following?

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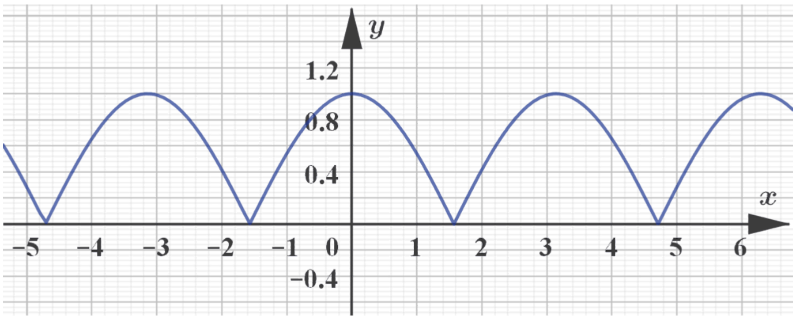
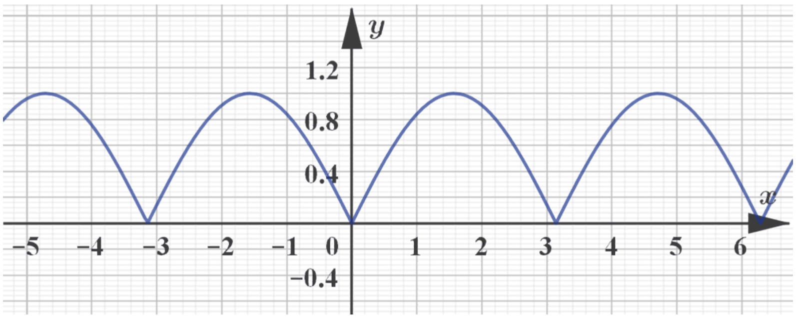
1



More information

2



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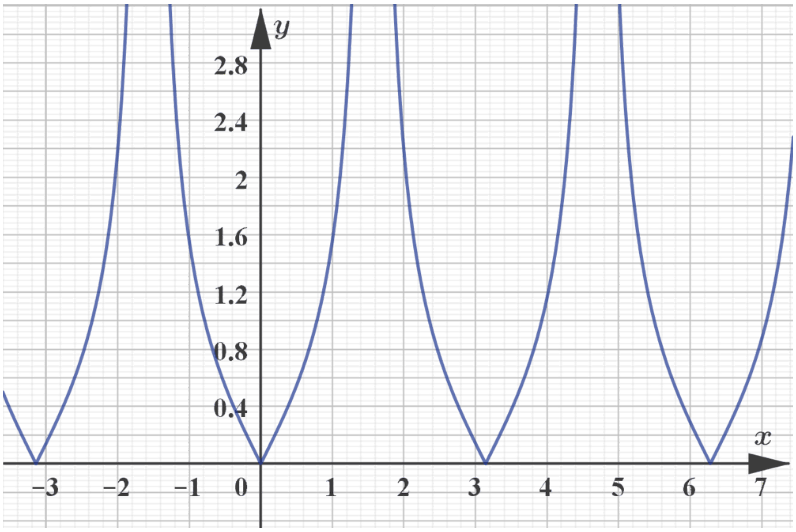

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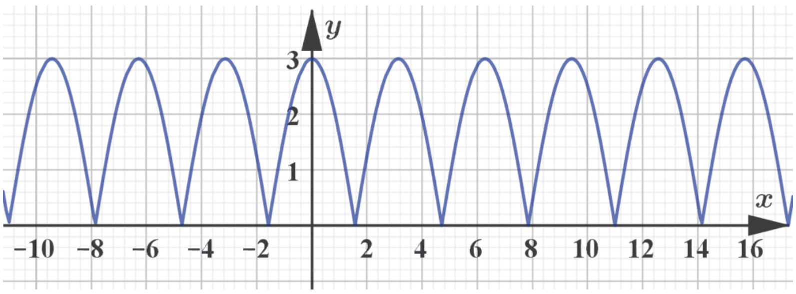

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Explanation

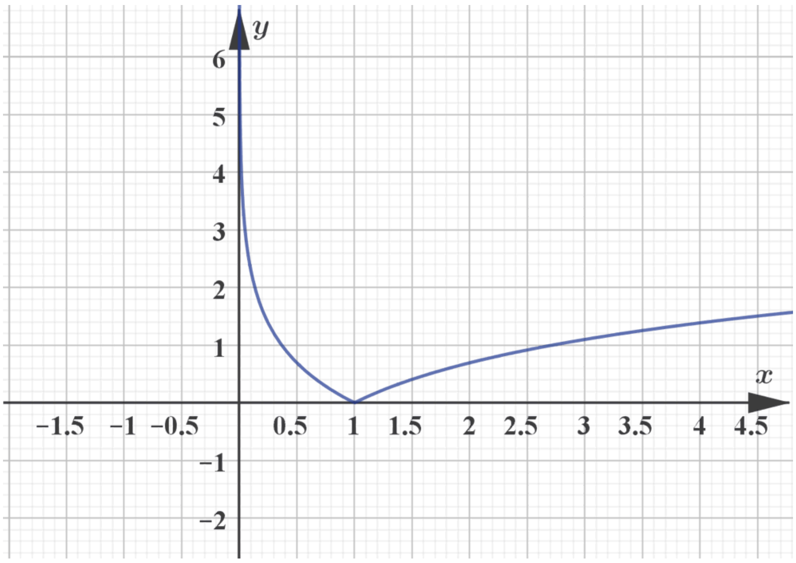
Graph $y = |\cos x|$ using graphing software and identify the correct option.

Question 2

Difficulty:

★☆☆

Which of the following functions does the graph shown below represent?



More information

1 $|\ln x|$ ✓

2 $\ln |x|$

3 $e^{|x|}$

4 $|e^x|$

Explanation

The part of the $|\ln x|$ function below the x-axis is reflected on the x-axis, which gives $|\ln x|$. Graph and compare all the given options with the graph given above.

Question 3

Difficulty:

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Which of the following graphs represents $y = \tan |x|$?

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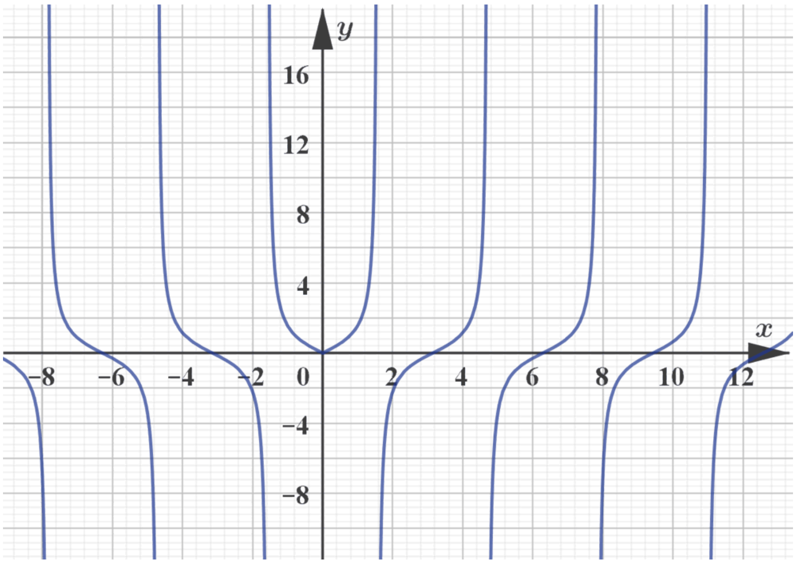
1



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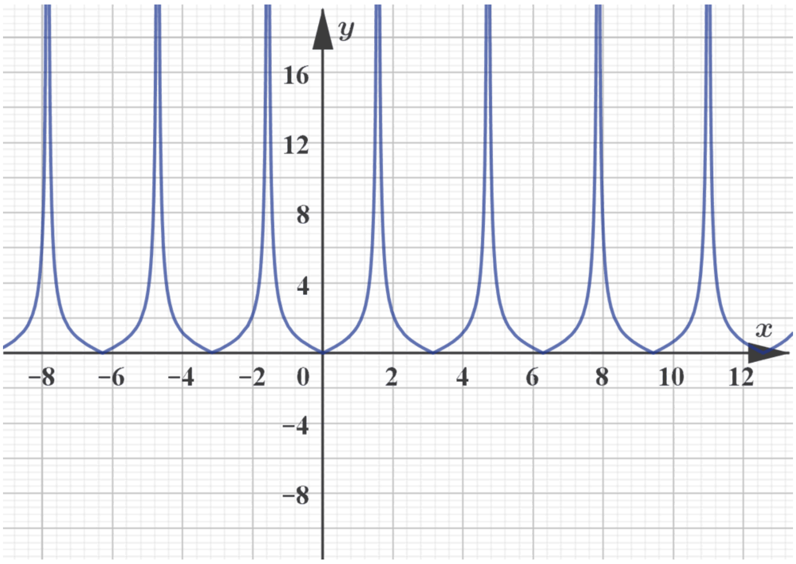
2



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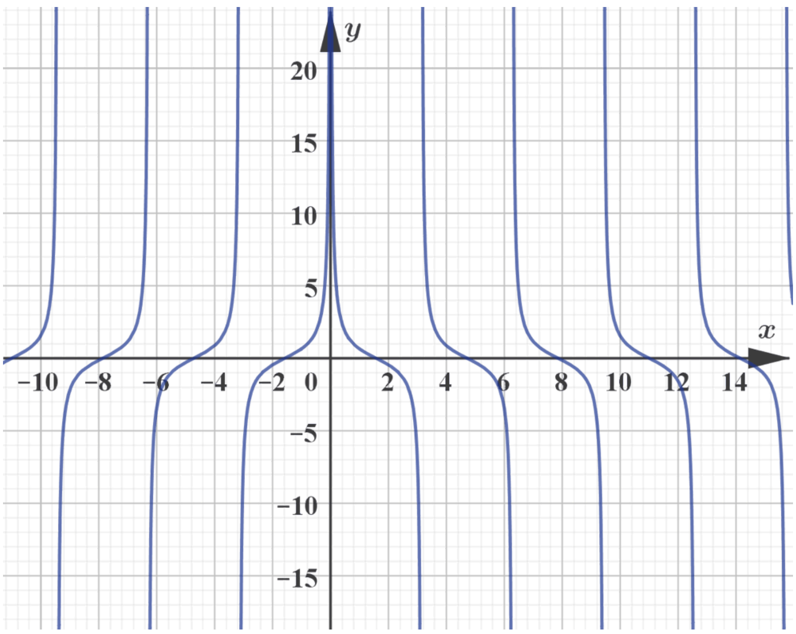
3



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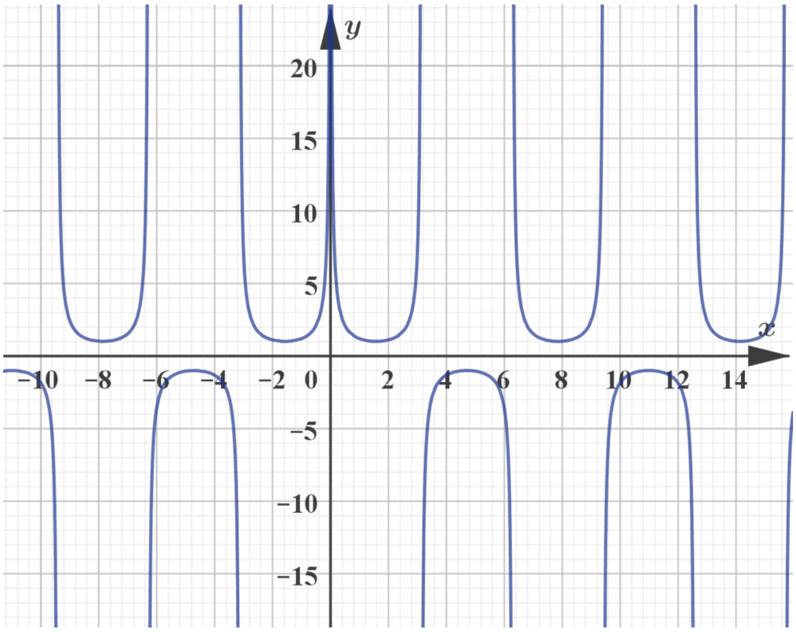

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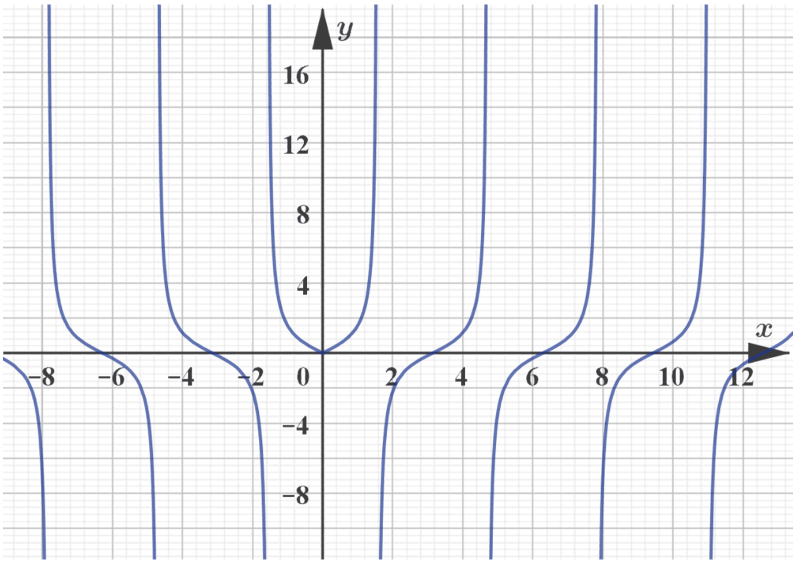
Explanation

Graphing $y = \tan|x|$ using a GDC shows that the correct answer is



More information

Answer #2 is $y = |\tan x|$

Answer #3 is $y = \dfrac{1}{\tan|x|}$

Answer #4 is $y = \dfrac{1}{\sin|x|}$

Student
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Graphs of y=1/f(x)

In order to graph $y = \dfrac{1}{f(x)}$ for any given $f(x)$, you should apply some of the rules you have studied in subtopic 2.8 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27718/).

1. For large x values, $\dfrac{1}{x}$ will be small. In other words, $\dfrac{1}{x} \to 0$ as $x \to \infty$
2. For small x values, $\dfrac{1}{x}$ will be large. In other words, $\dfrac{1}{|x|} \to \infty$ as $x \to 0$

Using the applet below, you can analyse the graph of $y = \dfrac{1}{f(x)}$, for a cubic function.

Activity

Interactive 1. Analyse the Graph of $y = 1/f(x)$ for a Cubic Function.

More information for interactive 1

This interactive graph helps the users analyze the graph of y = $\dfrac{1}{f(x)}$ for a cubic function.

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In the interactive the cubic function is in the form $f(x) = (x - a)(x - b)(x - c)$. The cubic function curve intersects the x-axis at three different points and is marked in red. The users can move the red points on the x-axis to change the roots of the function which will change the shape and position of the cubic function $f(x)$. The graph of $y = f(x)$ is given in blue color while the graph of $y = \frac{1}{f(x)}$ is given in yellow color.

Case 1: The minimum of $y = f(x)$ becomes the maximum for $y = \frac{1}{f(x)}$ and the maximum of $y = f(x)$ becomes the minimum for $y = \frac{1}{f(x)}$. That means the x-coordinates of these points remain the same while the y-coordinates become the reciprocals of each other.

Case 2: The users will also observe that when $x \to \pm\infty, f(x) \to \pm\infty$ and hence $\frac{1}{x} \to 0$, which will give a horizontal asymptote at $y = 0$.

Case 3: At x-intercepts, $f(x) = 0$. Hence, by the rule, as $f(x) \to 0 \Rightarrow \frac{1}{f(x)} \to \pm\infty$, users will observe a vertical asymptote for $y = \frac{1}{f(x)}$ at each x-intercept of $y = f(x)$.

Case 4: The y-intercept of $y = \frac{1}{f(x)}$ will be the reciprocal of the y-intercept of the original function.

Case 5: Users will find that the points of intersection occur at all points where $f(x) = \frac{1}{f(x)}$, which happens at $f(x) = \pm 1$. Therefore, the intersections of both the curves are at various x-coordinates but the y-coordinates will always be either $+1$ or -1.

Case 6: Lastly, the users will also analyze that when $f(x)$ is positive, $\frac{1}{f(x)}$ is also positive, and when $f(x)$ is negative, $\frac{1}{f(x)}$ is also negative and when $f(x)$ is increasing, $\frac{1}{f(x)}$ is decreasing and vice versa.

The interactive will give users a better visualization of the graph of $y = \frac{1}{f(x)}$ for a cubic function of the form $f(x) = (x - a)(x - b)(x - c)$.

The cubic function in the activity is given in the form $f(x) = (x-a)(x-b)(x-c)$. Move the points on the x-axis to change the roots and hence the shape and position of the cubic function $f(x)$. The graph of $y = f(x)$ is given in blue colour. The graph of $y = \frac{1}{f(x)}$ is given in yellow colour. Observe the various points shown on the graphs and analyse them using the following guidelines:

- Observe the minimum and maximum points of $y = f(x)$. How are they transformed for $\frac{1}{f(x)}$?
- Observe the graph of $f(x)$ when $x \to \pm\infty$ and hence deduce the behaviour of $y = \frac{1}{f(x)}$ when $x \to \pm\infty$.
- Observe the x-intercepts on the graph of $y = f(x)$ and explain why asymptotes are seen at these points for $y = \frac{1}{f(x)}$.
- Observe the y-intercepts of $y = f(x)$ and $y = \frac{1}{f(x)}$. How are they transformed?

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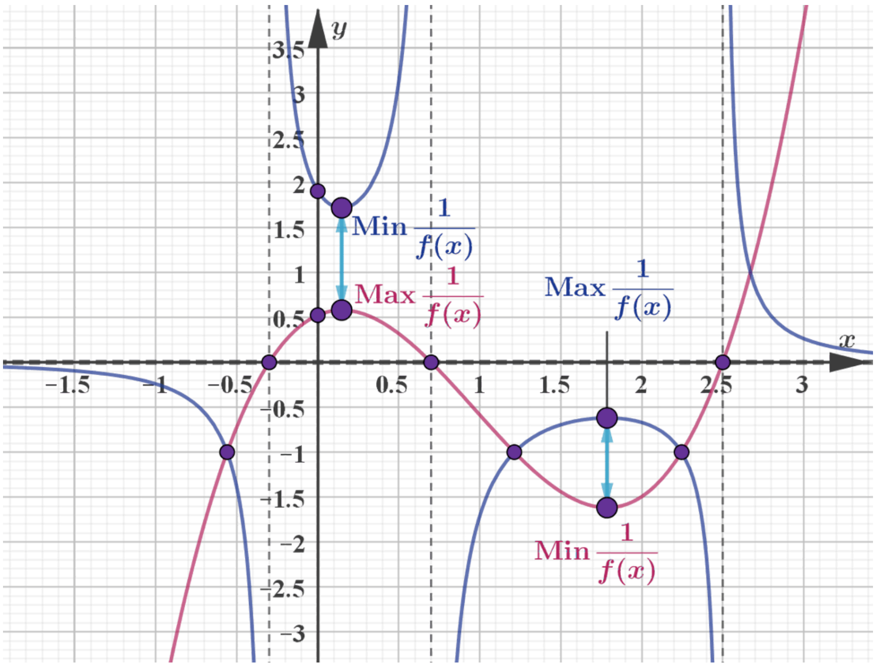
- Observe the points of intersection of $y = f(x)$ and $y = \frac{1}{f(x)}$. What is the significance of the coordinates of these points? Why?
- Record any other observations.

- Observe the minimum and maximum points of $y = f(x)$. How are they transformed for $y = \frac{1}{f(x)}$?

The minimum of $y = f(x)$ becomes the maximum for $y = \frac{1}{f(x)}$ and vice versa. The x-coordinates of these points remain the same while the y-coordinates become the reciprocals of each other. This is shown in the graph below:



- Observe the graph of $y = f(x)$ when $x \to \pm\infty$ and hence deduce the behaviour of $y = \frac{1}{f(x)}$ when $x \to \pm\infty$

When $x \to \pm\infty,\ f(x) \to \pm\infty$ and hence $\frac{1}{f(x)} \to 0$, which gives a horizontal asymptote at $y = 0$

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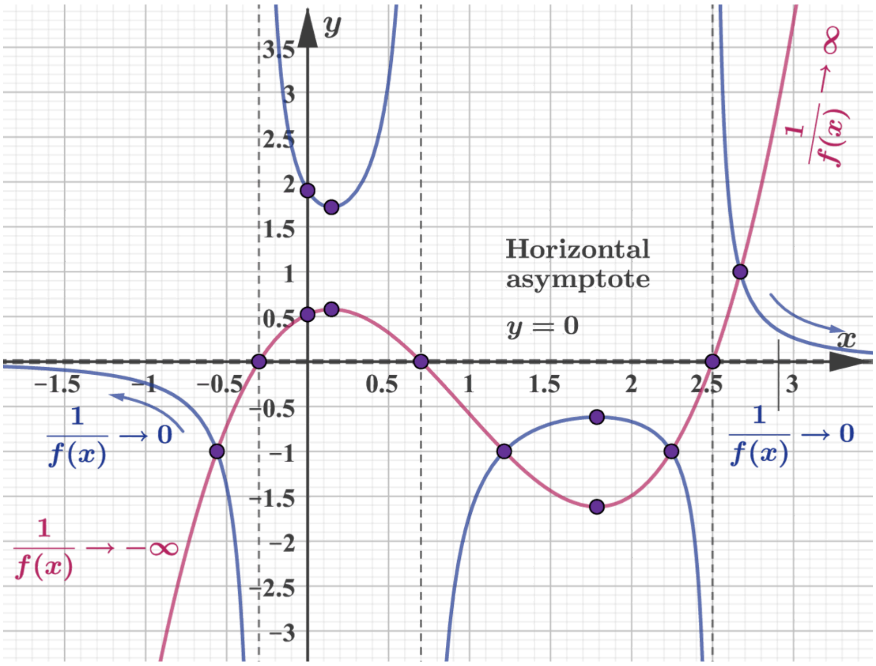
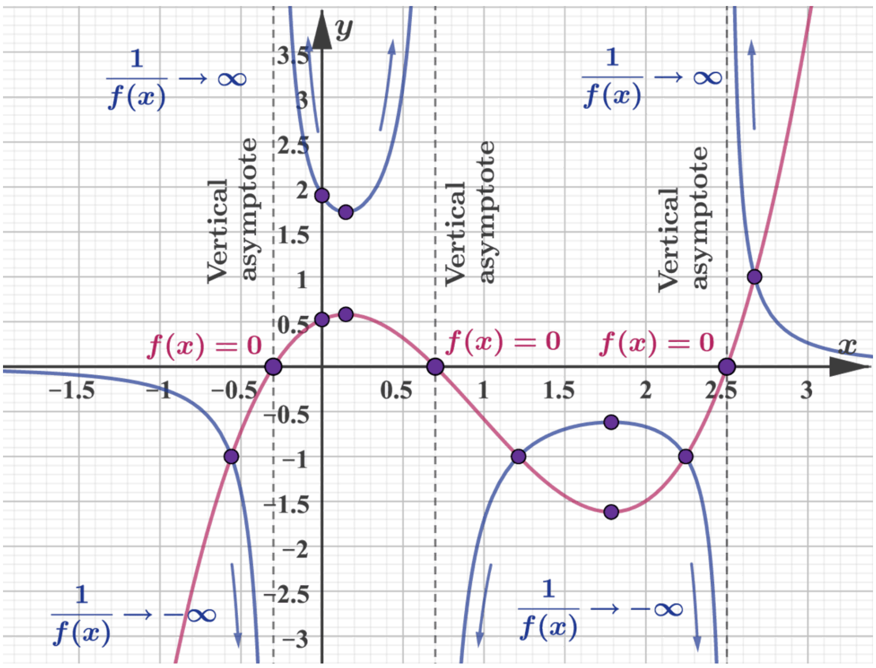


- Observe the x-intercepts on the graph of $y = f(x)$ and explain why asymptotes are seen at these points for $y = \frac{1}{f(x)}$.

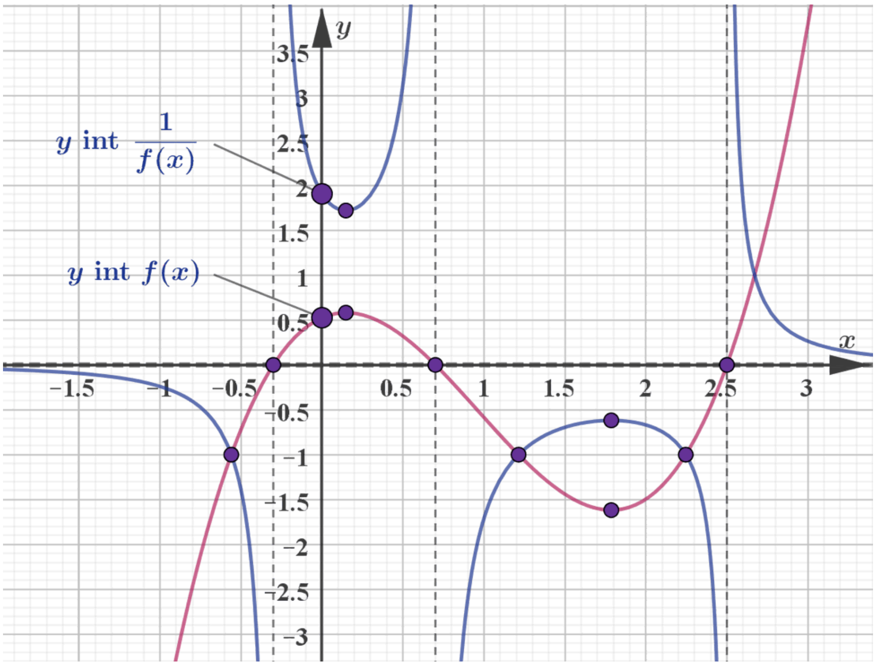
At x-intercepts, $f(x) = 0$. Hence, by the rule, $f(x) \to 0 \Rightarrow \frac{1}{f(x)} \to \pm\infty$, you will get a vertical asymptote for $y = \frac{1}{f(x)}$ at each x-intercept of $= f(x)$, as shown below:



Student view

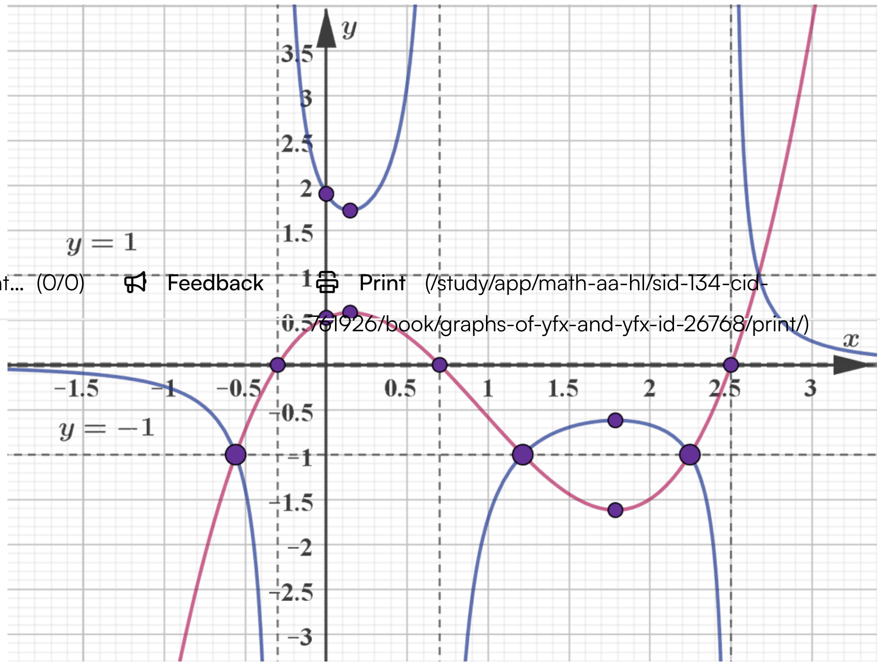
- Observe the y-intercepts of $y = f(x)$ and $y = \frac{1}{f(x)}$. How are they transformed?

The y-intercept of $y = \frac{1}{f(x)}$ is the reciprocal of the y-intercept of the original function.



- Observe the points of intersection of $y = f(x)$ and $y = \frac{1}{f(x)}$. What is the significance of the coordinates of these points? Why?

The points of intersection occur at all points where $f(x) = \frac{1}{f(x)}$, which happens at $f(x) = \pm 1$. Hence there are intersections at various x-coordinates but the y-coordinates will always be either $+1$ or -1.



- Any other observation.

  When $f(x)$ is positive, $\frac{1}{f\left(x\right)}$ is also positive, and when $f(x)$ is negative, $\frac{1}{f\left(x\right)}$ is also negative.

  When $f(x)$ is increasing, $\frac{1}{f\left(x\right)}$ is decreasing and vice versa.

Be aware

It is important to use an appropriate window in your graphing software in order to see the whole graph with all the x-intercepts and maximum/minimum points.

Example 1

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Using the above results, sketch the graphs of $y = \frac{1}{f\left(x\right)}$ for the following $f\left(x\right)$ showing all the asymptotes.

$$f\left(x\right) = x + 1$$

$$f\left(x\right) = x^2 - 3x - 1$$

$$f\left(x\right) = \sin x$$

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$f(x) = x + 1$

The x-intercept of $y = x + 1$ is $x = -1$. Hence $y = \dfrac{1}{x+1}$ will have a vertical asymptote at $x = -1$

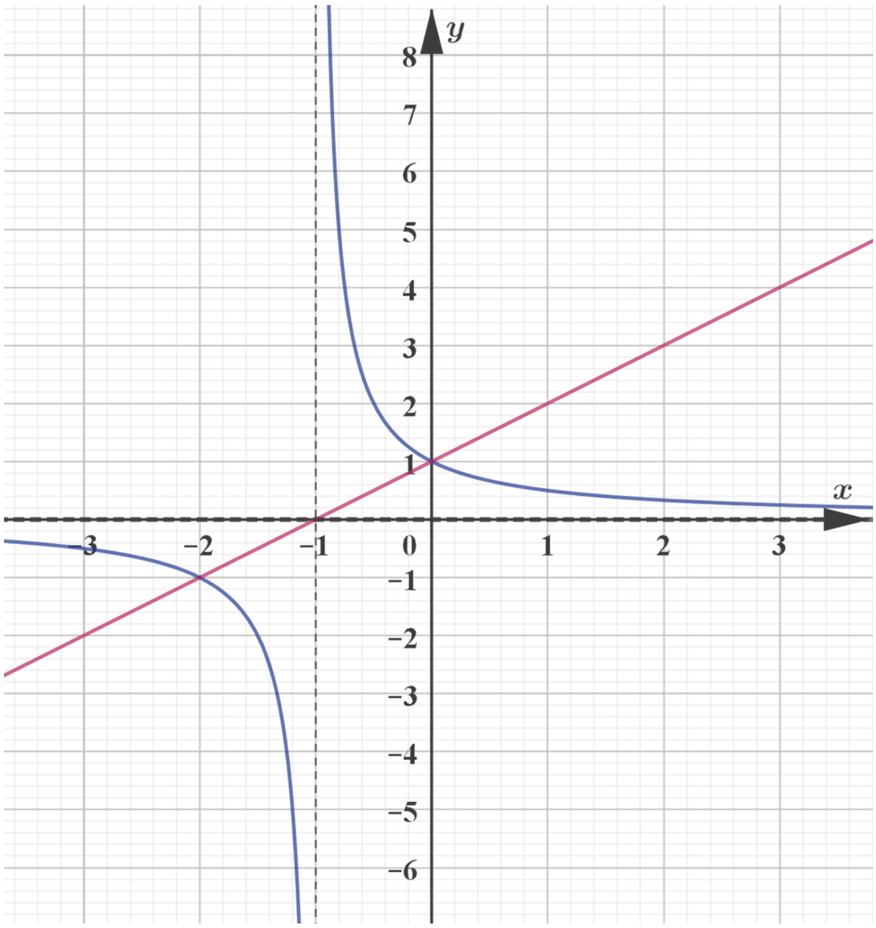
The y-intercept of $y = x + 1$ is $(0,\ 1)$. Hence $y = \dfrac{1}{x+1}$ will have y-intercept at $(0,\ 1)$

$x + 1 \to \pm\infty$ as $x \to \pm\infty$, which means that $\dfrac{1}{x+1} \to 0$ as $x \to \pm\infty$ and so it has a horizontal asymptote $y = 0$

There is no maximum or minimum for $y = f(x) = x + 1$. Hence there is none for $y = \dfrac{1}{x+1}$

Where the two graphs meet: $y = 1 \to x = 0;\ \ y = -1 \to x = -2$. Hence the points of intersection are: $(0,\ 1)$ and $(-2,\ -1)$

Using the above information, you get the following graph of $y = \dfrac{1}{x+1}$ (in red):



Student view

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$f\left(x\right)=x^2-3x-1$

Intercepts: $x:-0.3, 3.3$ which gives vertical asymptotes at $x=-0.3$ and $x=3.3$

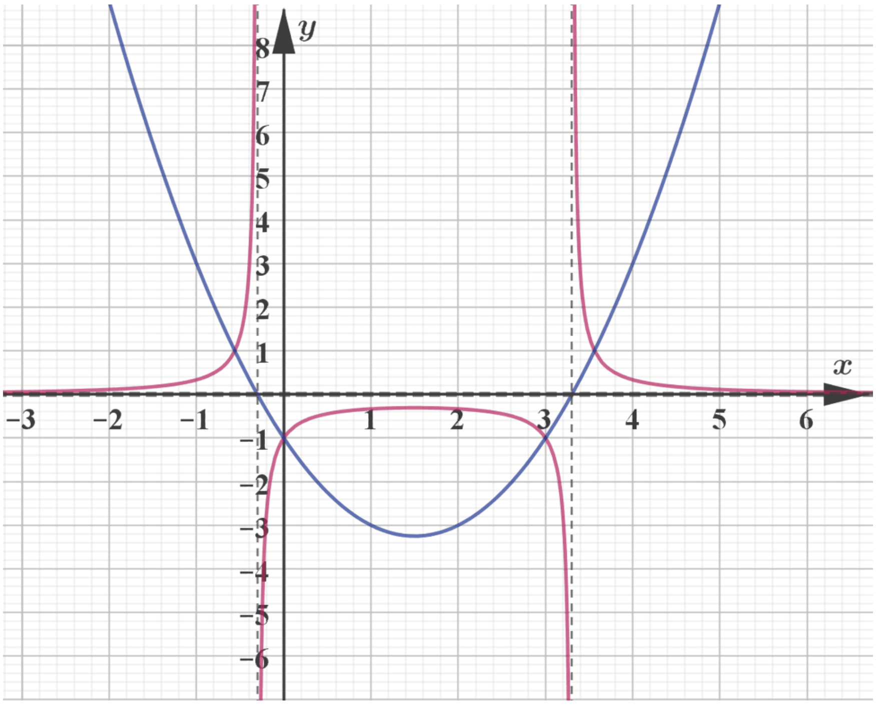
(see subtopic 2.7)

$y:-1$ which gives y-intercept of $y=\dfrac{1}{x^2-3x-1}$ to be $(0,\ -1)$

Minimum: $\left(\dfrac{3}{2},-\dfrac{13}{4}\right)$ giving maximum for $y=\dfrac{1}{x^2-3x-1}$ as $\left(\dfrac{3}{2},-\dfrac{4}{13}\right)$

Points of intersection: $y=1\rightarrow x=3.6,-0.6$; $y=-1\rightarrow x=0, x=3$; hence there are 4 points of intersection: $(3.6, 1)$, $(–0.6, 1)$, $(0, –1)$, $(3, –1)$

Using the above information, you get the following graph of $y=\dfrac{1}{x^2-3x-1}$ (in red):



Student
view

$f\left(x\right)=\sin x$

Intercepts: $x : n\pi$, which gives vertical asymptotes at each $n\pi$, $n = 0, 1, 2, 3$.

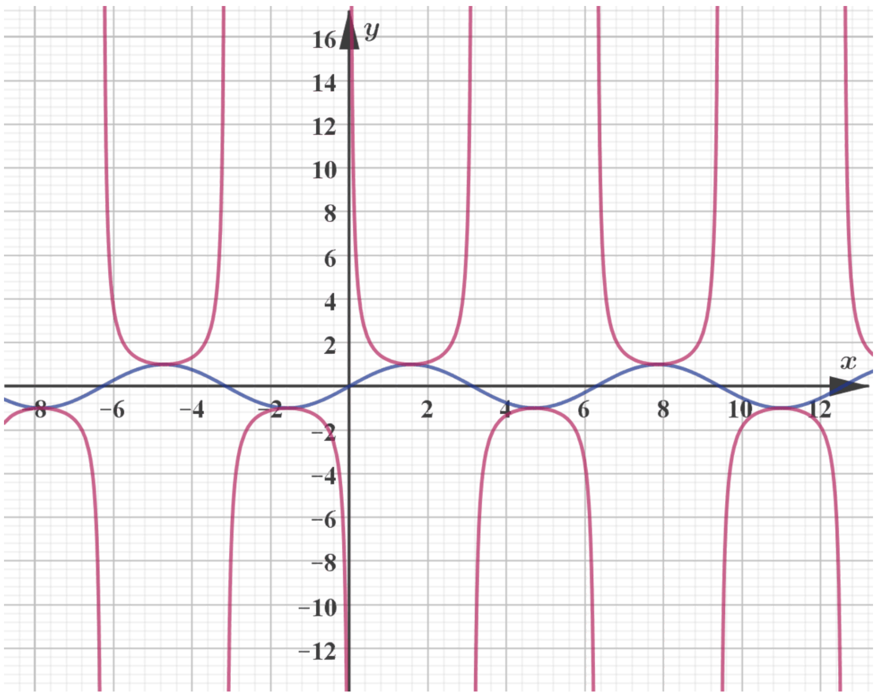
$y = 0$ hence there is no y-intercept for $y = \dfrac{1}{\sin x}$

Minimum: at each $\pm(4n-1)\dfrac{\pi}{2}$, giving maximum for $y = \dfrac{1}{\sin}x$ at each $\pm(4n-1)\dfrac{\pi}{2}$, for $n = 1, 2, 3$.

Maximum: at each $\pm(2n-1)\dfrac{\pi}{2}$, giving minimum for $y = \dfrac{1}{\sin x}$ at each $\pm(2n-1)\dfrac{\pi}{2}$, for $n = 1, 2, 3$.

Points of intersection: at each maximum and minimum points which occurs at $y = 1$ and $y = -1$

Using the above information, you get the following graph of $y = \dfrac{1}{\sin x}$ (in red):



3 section questions

Graphs of y=f(ax+b)

In subtopic 2.11 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27728/), you learnt about transformations such as horizontal translations and horizontal stretches for a function.

Translation occurs for functions of the form $f\left(x-b\right)$, in which the function is translated b units to the right if $b>0$ and to the left if $b<0$.

Horizontal stretches occur for functions of the type $f\left(ax\right)$, in which the function is stretched horizontally by $\frac{1}{a}$ units.

Now, what happens if both transformations are carried out together on the same function, that is, what type of transformation occurs for functions of the type $f\left(ax+b\right)$?

Activity

Use the GeoGebra link given below and analyse the transformation $f(ax+b)$.

Interactive 1. Analyse the Transformation $\mathrm{f}\left(\mathrm{ax}+\mathrm{b}\right)$.

More information for interactive 1

This interactive will help the users to analyze the **translations and stretches** of the function $f(ax + b)$.
A graph on a cartesian plane of the xy-axis, where the x-axis ranges from -5 to 5 and the y-axis ranges from -6 to 10.
A parabola in red is projected on the graph which is static, passing through five fixed points, $(-3.65, 3)$, $(-3, 0)$, $(-1, -4)$, $(1, 0)$, and $(1.65, 3)$. A parabola in blue is projected on the graph which the users can modify using two horizontal sliders on the top left corner that changes the value of 'a' and 'b' from -5 to 5.
For $a = 1$ and $b = 0$, the graph of the original function with this condition both the parabolas (red and blue) overlap each other, meeting the x-axis at $(-3, 0)$ and $(1, 0)$ and the vertex is at $(-1, -4)$.
Now, keeping $b = 0$ and moving $a = -5$ to 5. As the value of 'a' increases from -5 to -1, the graph of the function becomes wider, and at $a = 0$, the parabola transforms into a straight line that runs parallel to the x-axis, intersecting at the point $(0, -3)$. Now, when the value of 'a' increases further from 0 to 5, the parabola narrows, with the vertex shifts from $(0.2, -4)$ at $a = -5$ to $(-0.2, -4)$ at $a = 5$. Users will notice a horizontal stretch in the graph due to the changes in the x-coordinate, while the y-coordinate remains constant.
For $a = 1$, and changing the value of b from -5 to 5. As the value of 'b' increases from -5 to 5 the parabola shifts from right to left. Here also the user notices the coordinates of the y-axis are constant and the x-axis changes.

This helps the users get a better insight into the change in the graph of the given function.

Instruction to use the GeoGebra activity:

1. Use the sliders for a and b as shown below:
   a) First keep $a = 1$ and $b = 0$ to view the original function.
   Then change the values of a from -5 to $+5$ keeping $b = 0$ and observe the horizontal stretch.
   Observe the special points given on the graphs and note that the y-coordinates remain the same but their x-coordinates change. How do these x-coordinates change? Is the change connected to the value of a?
   Using your observation, find the x-coordinate of the point on the transformed graph corresponding to any point (x_1, y_1) of the original graph.

Points on original function	A	J	E	F	C	(
x-coordinates						
Corresponding points on $f(2x)$	B	I	G	H	D	
x-coordinates						

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Corresponding points on $f(3x)$	B	I	G	H	D	
x**-coordinates**						
Corresponding points on $f(4x)$	B	I	G	H	D	
x**-coordinates**						

b) Next, keep $a = 1$ and change the values of b from -5 to 5 and observe the horizontal translation by b units left and right.
Observe the special points given on the graphs and note that the y-coordinates remain the same but their x-coordinates change. How do these x-coordinates change? Is there any connection between the change and the value of b?
Using your observation, find the x-coordinate of the point on the transformed graph corresponding to any point (x_1, y_1) of the original graph.

Points on original function	A	J	E	F	C	(
x**-coordinates**						
Corresponding points on $f(x-2)$	B	I	G	H	D	
x**-coordinates**						
Corresponding points on $f(x+1)$	B	I	G	H	D	
x**-coordinates**						
Corresponding points on $f(x+3)$	B	I	G	H	D	
x**-coordinates**						

Student view

c) Next, keep $a = 2$ and change the b values as above. Observe the graph of $y = f(ax + b)$. Change the a value to 3 and then to 4 and repeat for different values of b. Observe what effect this has. How can you find the value of the x-coordinate of

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any point on the transformed graph $y = f(ax + b)$ using the x-coordinate of the corresponding point on the original graph $f(x)$?
For example, if $f(2x–3) = f(1)$ what is the value of x?

Answer: $x = 2$

$$2x - 3 = 1$$
$$x = \frac{1+3}{2}$$
$$x = 2$$

Example 1

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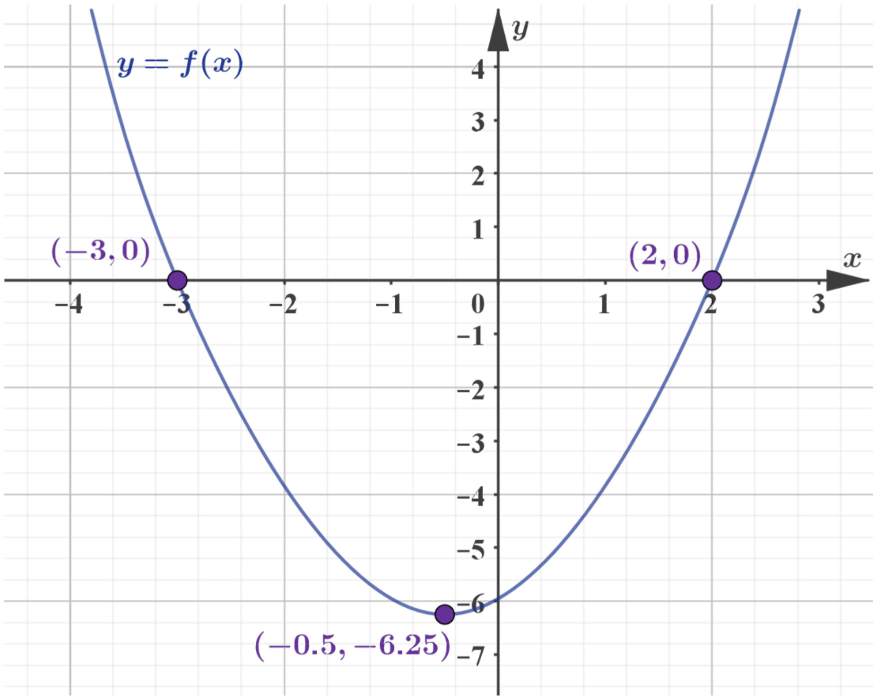
Sketch the graph of $y = f(2x + 1)$ from the graph of $y = f\,(x)$ given below:



More information

The image is a graph depicting the function y = f(x), which is a parabolic curve. The graph is plotted on a Cartesian coordinate system with both the x-axis and y-axis labeled and marked with grid lines for reference. The x-axis ranges from approximately -5 to 5, while the y-axis ranges from 0 to 7.

Student
view

There are three key labeled points on the graph: 1. (-3, 0), which is a point on the x-axis. 2. (0, -6.25), the vertex of the parabola, representing the minimum point of the graph. 3. (2, 0), another point on the x-axis.

The curve is symmetric about the vertical line x = 0, which passes through the vertex. The graph appears to represent a downward-opening parabola centered around its vertex at (0, -6.25), indicating it is likely an upside-down quadratic equation. The title inside the image denotes the function as y = f(x).

[Generated by AI]

Transform the x-intercepts $(-3 \text{ and } 2)$ and the minimum point $(-0.5, -6.25)$ as shown in the table below:

	From x=−3	From x=2	Minimum point (−0.5,−6.25)
Equation	$2x+1=-3$	$2x+1=2$	$2x+1=-\frac{1}{2}$
Solve for x	$x=\frac{-3-1}{2}$	$x=\frac{2-1}{2}$	$x=\frac{-\frac{1}{2}-1}{2}$
Simplified x	$x=-2$	$x=\frac{1}{2}$	$x=-\frac{3}{4}=-0.75$
New Coordinates	New x-intercept: $(-2,0)$	New x-intercept: $\left(\frac{1}{2},0\right)$	New turning point: $(-0.75,-\ 6.25)$

Connecting the above three new points you get the transformed graph as shown below (in brown):

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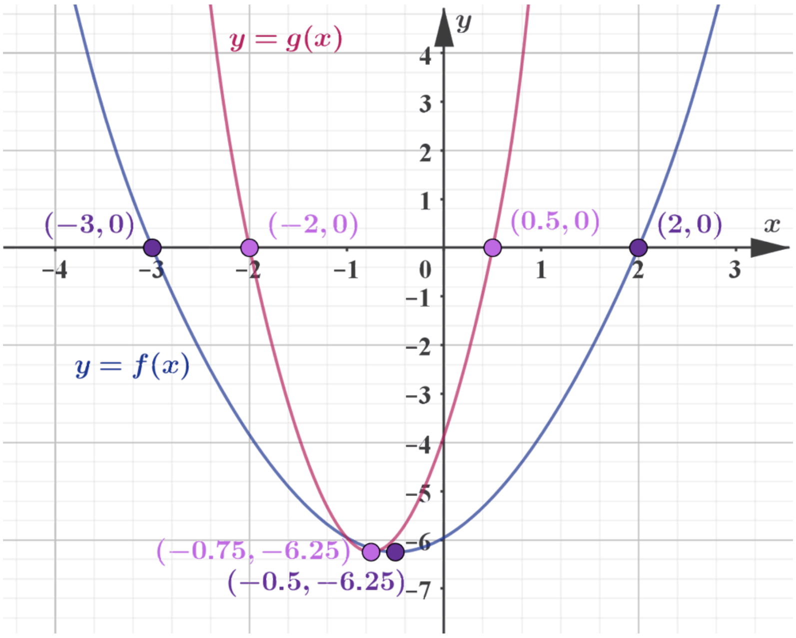

Theory of Knowledge

How does mathematical modelling using transformation of functions affect the accuracy of the data? How can you measure the degree of accuracy when modelling a best-fit function to a given set of data?

2 section questions

2. Functions / 2.16 Further graph transformations

Graphs of y=[f(x)]^2

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/graphs-of-yfx2-id-26771/print/) Assign

In order to sketch the graph of $y = [f(x)]^2$ from a given graph of $y = f(x)$, the following properties can be applied:

1. The values of $[f(x)]^2$ are always positive or zero. That is, the graph of $y = [f(x)]^2$ is always on or above the x-axis.

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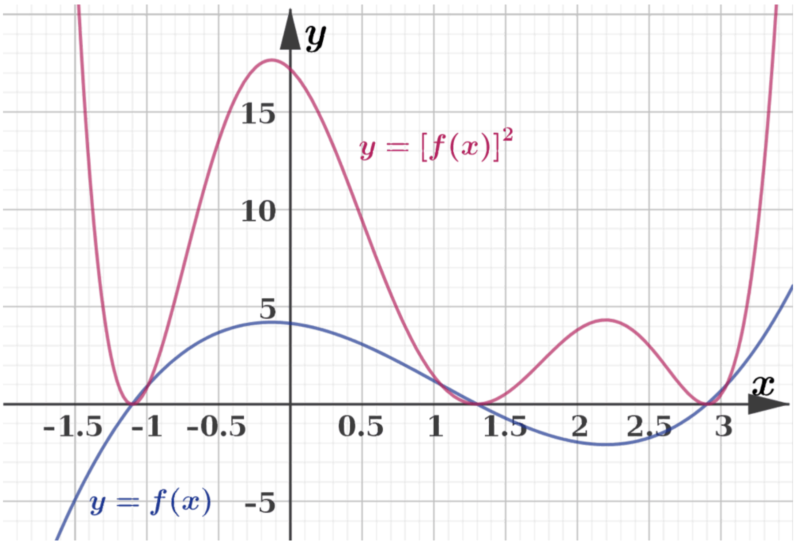


More information

The image is a graph that contains two curves plotted on a grid. The x-axis is marked at regular intervals and covers a range from approximately -2 to 3.5. The y-axis is similarly marked, ranging from -5 to about 15.

The first curve, labeled as 'y = f(x)', is drawn in blue. It undulates above and below the x-axis, and its values range from negative to positive. It intersects the x-axis at several points, including x = 0 and other points close to x = -1 and x = 1.5.

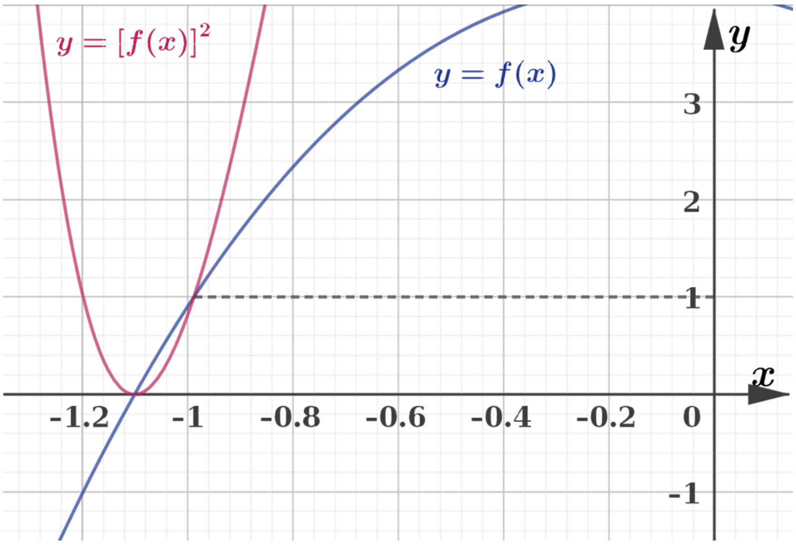
The second curve, labeled 'y = [f(x)]²', is drawn in pink. This curve remains entirely above the x-axis, with no intersections below it, as discussed in the text - since squares are always non-negative. It peaks higher above the x-axis than the blue curve, and its minimum points touch the x-axis where the blue curve intersects it.

The visual representation demonstrates that squaring a function ensures its graph is always at or above the x-axis, corresponding to its values being always positive or zero.

[Generated by AI]

2. If $y > 1$, then $y^2 > y$ and if $0 < y < 1$, then $y^2 < y$. This means that the graph of $y = [f(x)]^2$ is always below the graph of $y = f(x)$ for $0 < f(x) < 1$ and above it for $f(x) > 1$

Student
view



More information

The image is a graph with two curves plotted on a Cartesian coordinate system. The X-axis is labeled from -1.2 to 0, and the Y-axis is labeled from -1 to 4. The blue curve represents the function y = f(x), which shows a steadily increasing trend as x increases, starting from just above y = -1 and moving upwards towards y = 3. The red curve represents y = [f(x)]^2, and it is a parabola opening upwards, intersecting the blue curve and crossing the X-axis at around x = -1. The red curve lies above the blue curve when f(x) > 1 and below it when 0 < f(x) < 1, illustrating the properties mentioned before the image.

[Generated by AI]

3. Consider now the sign and the direction of the original curve. There are four possibilities: positive and decreasing, negative and decreasing, positive and increasing, negative and increasing.

a) Consider first when $f(x)$ is decreasing.

i) If $a, b > 0$, then $a < b$ implies $a^2 < b^2$. This means that when $f(x)$ is positive and decreasing, then $g(x) = [f(x)]^2$ is also decreasing.

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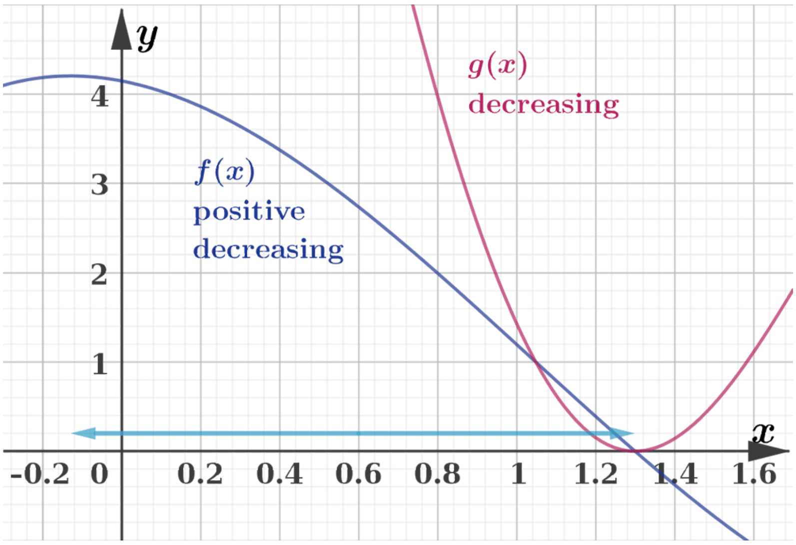
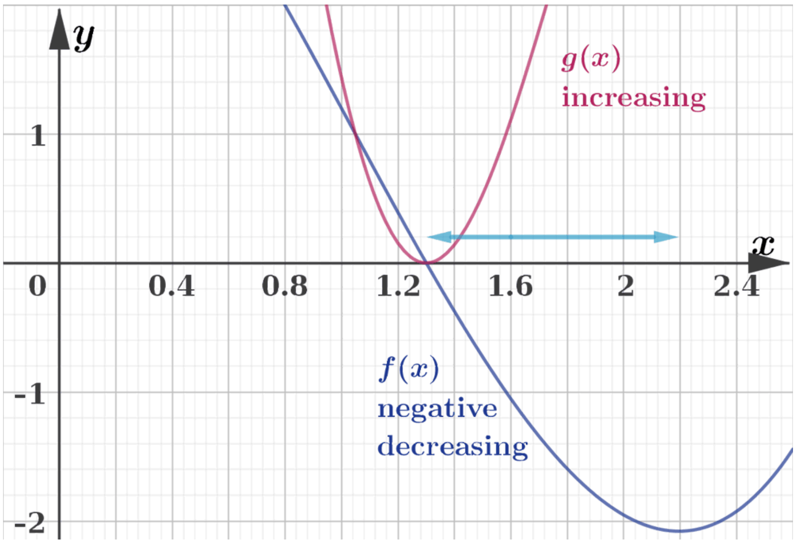


More information

ii) If $a, b < 0$, then $a < b$ implies $a^2 > b^2$. This means that when $f(x)$ is negative and decreasing, then $g(x) = [f(x)]^2$ is increasing.



More information

The image is a graph depicting two functions, f(x) and g(x). The X-axis represents numeric values from 0 to approximately 2.4, and the Y-axis depicts numeric values from around -2 to 2. There are two curves on the graph:

a. The blue curve represents f(x), labeled as 'negative decreasing.' It starts above 0 on the Y-axis, decreases, crosses the X-axis, and continues downward.
b. The red curve represents g(x), labeled as 'increasing.' This curve starts at a lower point below 0 on the Y-axis, curves upwards, intersecting with the blue curve around the X-axis, and continues rising.

An arrow is drawn from right to left above the curves, implying directionality or trend. The graph visually demonstrates the mathematical relationships described: when f(x) is negative and decreasing, g(x) is increasing. The grid lines create a clear numeric reference for observing the mathematical trends represented by the function curves.

Student view

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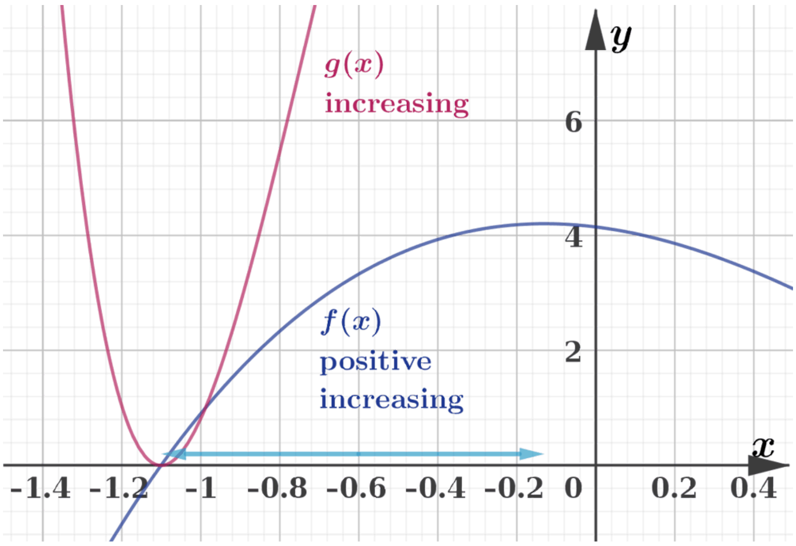
b) Next, consider the case when $f(x)$ is increasing

i) If $a, b > 0$, then $a < b$ implies $a^2 < b^2$. This means that when $f(x)$ is positive and increasing, then $g(x) = [f(x)]^2$ is also increasing.



More information

The image is a graph plotted on a grid with two functions: (f(x)) and (g(x)). The X-axis is labeled numerically from approximately -1.5 to 0.6. The Y-axis has labels at intervals of 2, up to 6. The function (f(x)) is shown as a blue curve that starts at the left with a value of about -4, rises to reach a peak around Y=6, and then decreases slightly. It is labeled as "f(x) positive increasing." The function (g(x)) is drawn as a red curve, increasing steadily and labeled "g(x) increasing." The graph illustrates the behavior of these functions as stated in the accompanying descriptions: if (f(x)) is positive and increasing, then (g(x) = [f(x)]^2) is also increasing.

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ii) If $a, b < 0$, then $a < b$ implies $a^2 > b^2$. This means that when $f(x)$ is negative and increasing, then $g(x) = [f(x)]^2$ is decreasing.

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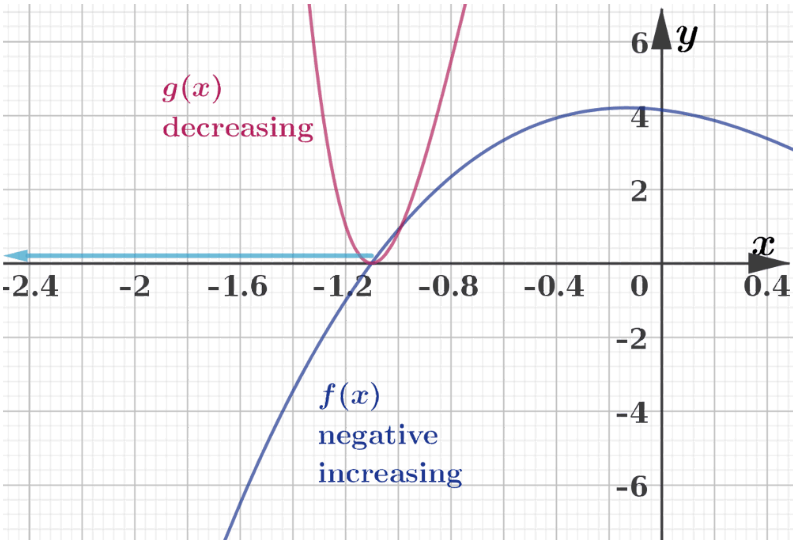

More information

The image is a graph with two functions plotted. The x-axis ranges from approximately -2.4 to 0.6, and the y-axis ranges from -6 to 6, both marked at intervals of 0.2 and 2, respectively. A grid is overlaid on the graph.

The red curve, labeled "g(x) decreasing," is a parabola with its vertex at about (-1.2, 0), opening upwards, indicating a decreasing nature in the context of the description.

The blue curve labeled "f(x) negative increasing" starts below the x-axis and rises as it moves from left to right, staying below the x-axis.

This illustration visually represents the relationship of these functions as described in the text before and after the image, exemplifying that when f(x) is negative and increasing, g(x) = [f(x)]^2 is decreasing.

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c) The previous investigation of directions also shows that for any turning point of the graph of $y = f(x)$, the corresponding point is a turning point on the graph of $y = [f(x)]^2$ at the same x-coordinate. However, the y-coordinate of the turning point will be the square of the original y-value and hence will always be positive.
For example, if $(3, -4)$ is a turning point for $f(x)$, $(3, 16)$ would be the corresponding turning point on $(f(x))^2$.

d) At the roots of $f(x)$, you know that $f(x) = 0$, so $g(x) = [f(x)]^2 = 0$. Using derivatives, which you learn in chapter 5, you can show that at these points the x-axis is a horizontal tangent line to the graph of $y = [f(x)]^2$.

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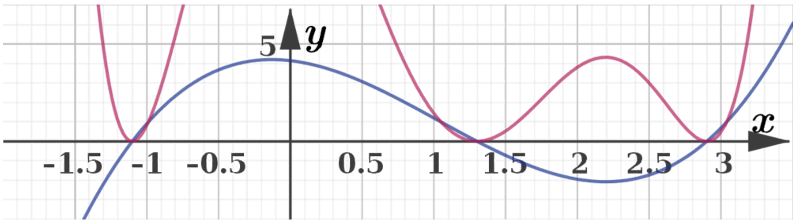

More information

This image depicts a graph with grid lines, illustrating the curves of two functions. The X-axis is marked at intervals (-1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3). The Y-axis is up to 5 and labeled at certain intervals. There are two curves shown: one in blue and one in red. The blue curve undulates gently, suggesting a smoother, perhaps sinusoidal function. The red curve has sharper, rapid oscillations. Both curves intersect at several points along the axis. Turning points and roots are visible, but specific values cannot be determined from the image alone.

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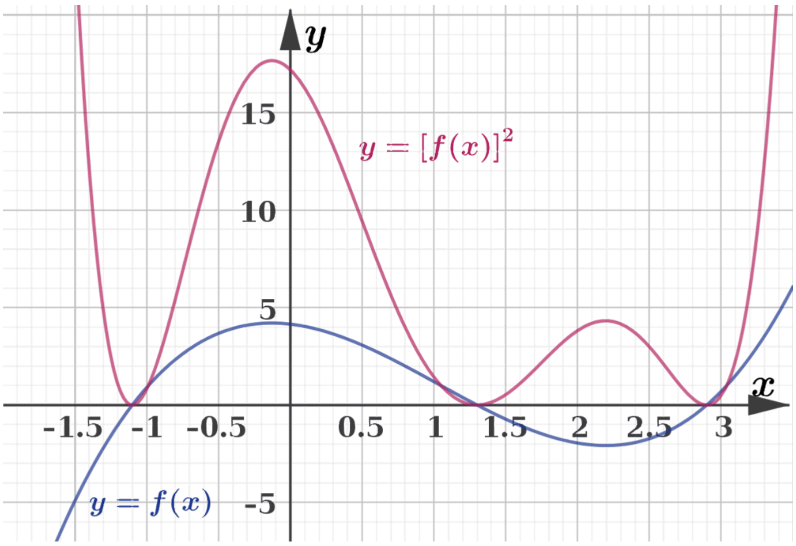
4. $(f(x))^2 = 1$ when $f(x) = \pm 1$

Now look at the fully transformed graph of $f(x) = x^3 - 3x^2 - x + 4$ (in blue) into $(f(x))^2$ (in red).



More information

This image is a graph comparing the function (f(x) = x^3 - 3x^2 - x + 4) (in blue) with its transformation (f(x)^2) (in red). The X-axis represents values ranging approximately from -2 to 3.5, while the Y-axis ranges from -5 to 20.

The blue graph represents the original cubic function (f(x)), which has a local minimum at around x = -1 and a local maximum near x = 0.5. The graph dips below the X-axis between these points.

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The red graph represents (f(x)^2) and has a noticeably different shape due to the squaring transformation. It starts high on the left side, dips to a minimum between x = -1 and 0, and reaches a peak shortly after x = 0 before continuing to fluctuate less dramatically and maintaining positive values.

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Exam tip

Make sure you fully understand the difference between the graphs of $y = |f(x)|$ and $y = [f(x)]^2$. For $y = |f(x)|$ the y-values remain the same except for the sign change, while for $y = [f(x)]^2$ each y-value is squared and hence the graph will be stretched vertically.

3 section questions

2. Functions / 2.16 Further graph transformations

Absolute equations and inequalities

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/absolute-equations-and-inequalities-id-26772/print/) Assign

Solving modulus equations

In order to solve an algebraic equation analytically, you can rearrange the equation to isolate the variable on one side and find then its value using the values you are given. If you solve it graphically, you will graph the functions and look for the common points. How accurate are the solutions when you find them graphically?

For example, if you want to solve $2x - 5 = x - 3$, you can make x the subject and solve to get $x = 2$. Or, you can graph $y = 2x - 5$ and $y = x - 3$ and find the x-coordinate of the point of intersection.

What will be the solution if there was a modulus in the equation? For example, what is the solution for $|x - 3| = 5$?

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view

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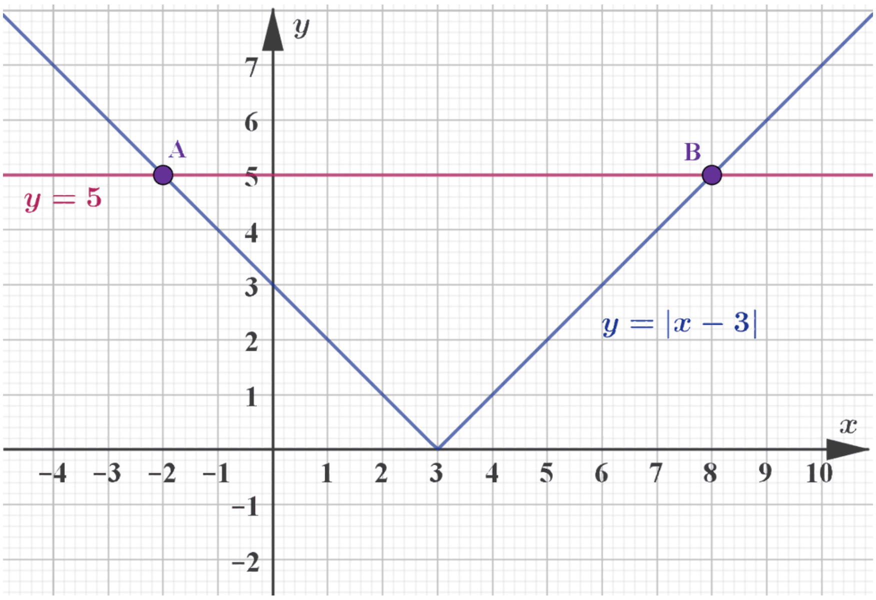
As an algebraic approach, you can start with the definition of the modulus sign. If $|x-3|=5$, then $(x-3)$ could be either 5 or −5. So, to solve $|x-3|=5$, equate $x-3=5$ or $x-3=-5$ and solve both the equations. This will give $x=8$ or $x=-2$.

Graphically, you would graph $y=|x-3|$ and $y=5$ and look for the point(s) of intersection. The graph below shows that they meet at $x=-2$ and $x=8$, which are the solutions to the equation $|x-3|=5$.



Graphs of $y_1=|x-3|$ and $y_2=5$

More information

The image displays a graph with two functions: (y_1 = |x - 3|) and (y_2 = 5).

Axes Details: - The X-axis is a number line ranging from -4 to 10, marked with intervals of 1 unit. - The Y-axis ranges from -2 to 8, in intervals of 1 unit.

Function Details: - The function (y_1 = |x - 3|) is a V-shaped line with a vertex at point (3, 0). The line decreases from right to left until it reaches point A at (2, 1) and then continues upward from left to right through point B at (4, 1). - The function (y_2 = 5) is a horizontal line intersecting the Y-axis at 5, and it spans across the graph.

Key Points: - Point A is labeled on the V-shaped graph (y = |x - 3|) at (2, 1). - Point B is labeled on the same V-shaped graph at (4, 1).

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Example 1

★☆☆

Solve $|2x-3|=8$

$2x-3=8$ or $2x-3=-8$

$x=5.5$ or $x=-2.5$

Example 2

★★☆

Solve $\left|\frac{x+1}{x}\right|=2$

$\frac{x+1}{x}=2$ or $\frac{x+1}{x}=-2$

$x+1=2x$ or $x+1=-2x$

$x=1$ or $x=-\frac{1}{3}$

Example 3

★★★

Solve $|2x-1|=|x+3|$

Let's see both the algebraic and the graphing approach to solve this equation.

1. Algebraic solution:
   The concept used in previous examples can be generalised a bit. You can use that $|a|=|b|$ is true if either $a=b$ or $a=-b$.
   $2x-1=x+3$ or $2x-1=-(x+3)$
   Giving solutions as $x=4$ and $x=-\frac{2}{3}$

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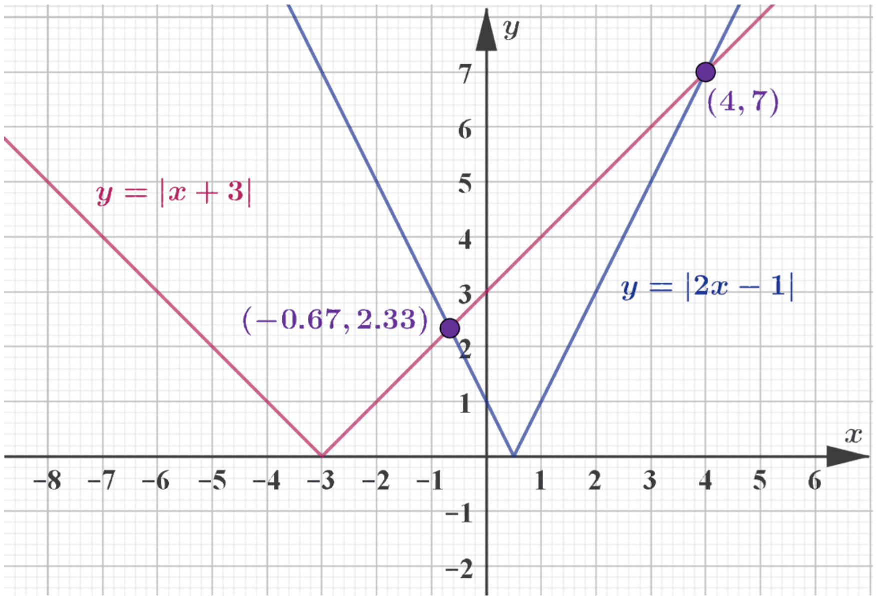
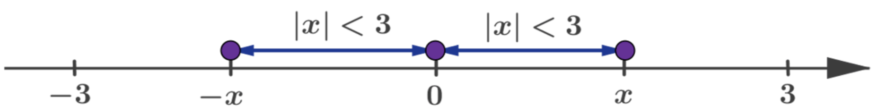
2. Graphical method:
   Sketching the graph of $y = |2x-1|$ and $y = |x+3|$ and finding the x-coordinates of the points of intersection. The solution is $x = -0.67$ or $x = 4$.
   This of course only gives an approximate solution, but if you look at the more accurate values of the first solution, you can see that the solution is in fact $x = -\frac{2}{3}$.



Modulus inequalities

Modulus inequalities can be solved in a similar way to equations.

For example, $|x| < 3$ would mean that the distance from 0 to the point is less than 3 units. On a number line, this would be as shown below:



More information

The image is a diagram of a number line illustrating the inequality (|x| < 3). The number line is horizontal with an arrow pointing right, representing positive values. The main points marked on the line are (-3), (0), and (3), with ticks extending from negative to positive infinity. There are purple dots placed at (-3), (0), and (3). The line between (-3) and (3) is shaded in purple, indicating the range of values (x) can take for the inequality (|x| < 3). This means any value of (x) between (-3) and (3), excluding (-3) and (3) themselves, satisfies the inequality.

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This means that any x between -3 and $+3$ would satisfy the inequality $|x| < 3$

Let's look at some examples.

Example 4

★★☆

Solve $|x - 4| < 3$.

Method 1 (algebraic approach)

The inequality $|x - 4| < 3$ means that $(x - 4)$ is between -3 and $+3$. This can be written as a double inequality:

$$-3 < x - 4 < 3$$

If x is a solution, it has to satisfy both inequalities. You can solve these separately

$-3 < x - 4$ and $x - 4 < 3$

$-3 + 4 < x$ and $x < 3 + 4$

$x > 1$ and $x < 7$

Together, $1 < x < 7$ is the solution set for the given inequality.

Method 2 (using technology)

The inequality can be solved using a graph by drawing the graphs of $y = |x - 4|$ and $y = 3$ as shown below:

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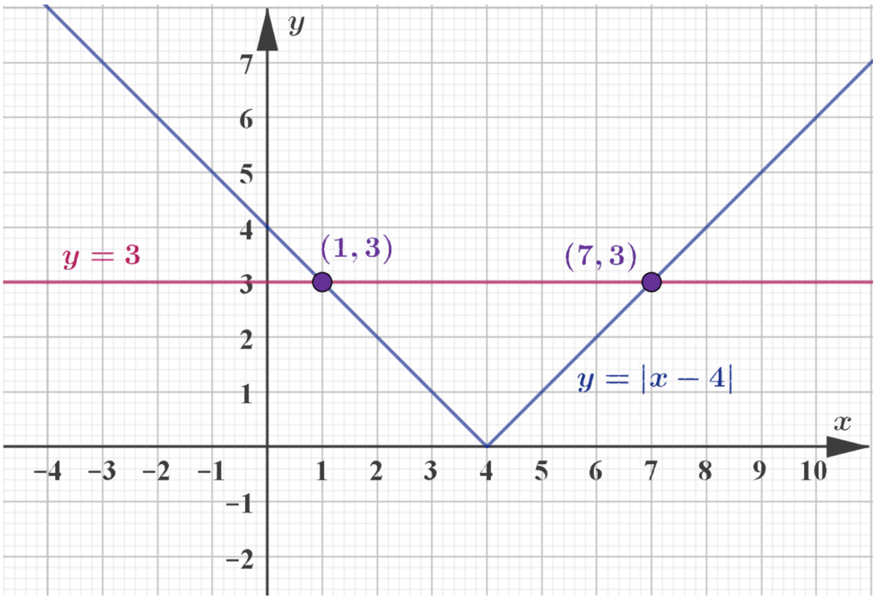

Look for the part of the graph of $y = |x - 4|$ which is below the graph of $y = 3$. This is in the interval $(1, 7)$ and this is the solution to $|x - 4| < 3$ (see subtopic 2.15 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26762/)).

In the example above the graphical solution used the general idea of solving inequalites using technology.

Exam tip

In order to solve a modulus inequality using a graphic display calculator, you can graph both the sides of the inequality and find the point(s) of intersection. Look for the part of the graphs for which one is below or above the other according to the question. The solution will be the x-values that satisfy the condition.

Example 5

★★★

Solve $\left|\frac{1}{x} + x\right| \leqslant 2$

Method 1 (using technology)

Graphically, you can graph $y_1 = \left|\frac{1}{x} + x\right|$ and $y_2 = 2$ and find the x values for which $y_1 \leqslant y_2$

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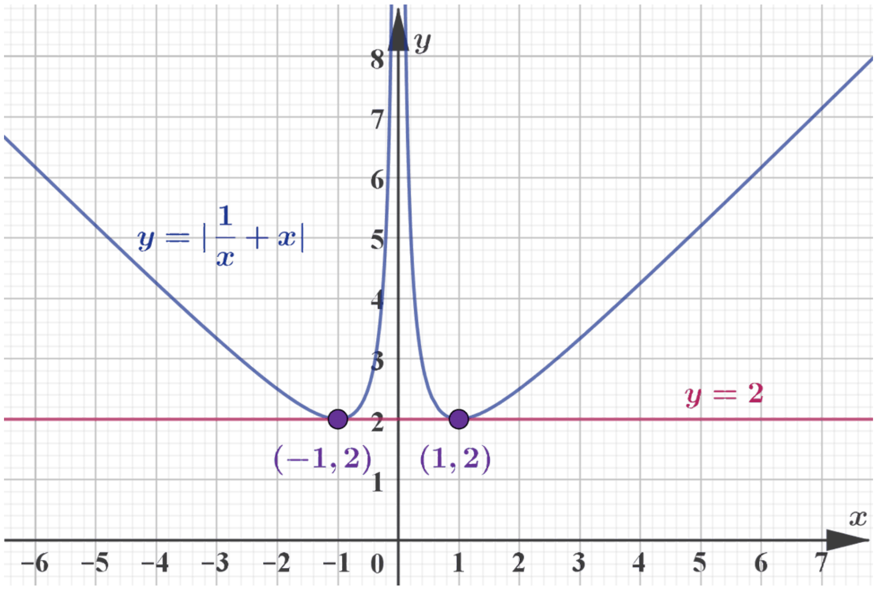

It is clearly seen that there is no region on the graph where y_1 is below y_2.

This means that the only solutions for $\left|\frac{1}{x} + x\right| \leqslant 2$ are given by the points where the line touches the curve.

$$x = -1,\ x = 1$$

Method 2 (algebraic approach)

The inequality $\left|\frac{1}{x} + x\right| \leqslant 2$ is true when $\frac{1}{x} + x$ is between -2 and 2.

$$-2 \leqslant \frac{1}{x} + x \leqslant 2$$

When multiplying the inequality with x you need to consider two cases, because when x is negative, then the inequalities are reversed after the multiplication.

- Case 1, $x > 0$
  Since in this case $\frac{1}{x} + x$ is positive, you only need to solve the inequality on the right.
  $$\begin{aligned} \frac{1}{x} + x &\leqslant 2 \\ 1 + x^2 &\leqslant 2x \\ x^2 - 2x + 1 &\leqslant 0 \\ (x-1)^2 &\leqslant 0 \end{aligned}$$
  Since a square is never negative, the only solution to this is $x = 1$, which is positive, so it also satisfies the starting condition of the case.

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- Case 2, $x < 0$

  Since in this case $\frac{1}{x} + x$ is negative, you only need to solve the inequality on the left.

$$-2 \leqslant \frac{1}{x} + x$$
$$-2x \geqslant 1 + x^2$$
$$0 \geqslant x^2 + 2x + 1$$
$$0 \geqslant (x+1)^2$$

  Since a square is never negative, the only solution to this is $x = -1$, which is negative, so it also satisfies the starting condition of the case.

Combining the two cases you get the solutions, $x = -1$ and $x = 1$.

The example above illustrates that solving an inequality using an algebraic approach may involve careful work with the direction of the inequalities.

Be aware

While multiplying or dividing by a negative number in an inequality, the direction of the inequality needs to be reversed.

3 section questions

2. Functions / 2.16 Further graph transformations

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-26773/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- sketch the graphs of $y = |f(x)|$ and $y = f(|x|)$ and know the difference between them, including that the graph of $y = |f(x)|$ is always above the x-axis and that $y = f(|x|)$ is always an even function
- sketch the graphs of $y = \frac{1}{f(x)}$ from a given function $f(x)$
- find all the asymptotes of $y = \frac{1}{f(x)}$ from a given $f(x)$
- find the shape of $y = \frac{1}{f(x)}$ on both the sides of the asymptotes

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- sketch and analyse the transformation $y = f(ax + b)$ understanding that both a and b give horizontal transformations only, i.e. a horizontal stretch and a horizontal translation respectively.
- sketch $y = f(ax + b)$ from a given graph of $y = f(x)$ and vice versa
- sketch and analyse the graph of $y = (f(x))^2$ and know the difference between them
- use the properties of the graph of $y = (f(x))^2$
- solve equations and inequalities involving modulus functions, giving solutions on a number line and in interval form and using a graph to solve modulus inequalities between two functions.

2. Functions / 2.16 Further graph transformations

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-26774/print/) Assign

In this investigation you are going to model the archway shown in the background of the applet.

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Interactive 1. Model the archway.

Credit: Getty Images Xantana

More information for interactive 1

This interactive will help the users to model the archway shown in the background of the applet.

The screen displays a cartesian graph of the xy axes, with the x-axis ranging from -5 to 5 and the y-axis ranging from -4 to 14. There is a model of an archway behind the graph. A curve line is projected on the graph representing a basic logarithmic function $f(x) = \ln x$.

At the top, there are four horizontal sliders a, b, c, and d. Here the value of a, b, and c varies from 0 to 5 and the value of d varies from 0 to 6. The parameters altering the curve line are as follows:

Parameter ‘a’ allows vertical stretching or compression, making users understand how the steepness of the curve changes, becoming less steep $(0 < a < 1)$ and steeper $(1 < a \leq 5)$.

Parameter ‘b’ allows horizontal scaling of the curve, making users understand how the rate of the curve’s growth or decline is affected.

Parameter ‘c’ allows horizontal shift from left to right, affecting the vertical asymptote and domain.

Parameter ‘d’ allows vertical shift. Users can shift the curve from 0 to 6 units changing the y-values.

Users will use the sliders to transform the function into $f(x) = a\ln(b(x + c)) + d$ so that the curve fits the outermost arch on the left-hand side.

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The picture above has been pasted into the axes and a basic logarithmic function $f(x) = \ln(x)$ is given. Four sliders a, b, c and d are provided. Use these sliders to transform the function into $f(x) = a\ln(b(x+c)) + d$ so that the curve fits the outermost arch on the left-hand side.

- What transformation would you use to get the right-hand side of the arch from the above function?
- What is the advantage of using a logarithmic function rather than a quadratic for this arch?
- W hat are its limitations when compared with reality?
- What modifications would you use to overcome these limitations?

Rate subtopic 2.16 Further graph transformations

Help us improve the content and user experience.

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