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Teacher view



(https://intercom.help/kognity)

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A. Space, time and motion / A.4 Rigid body mechanics (HL)



Glossary

Reading
assistance

The big picture (HL)

Higher level (HL)

? Guiding question(s)

- How can the understanding of linear motion be applied to rotational motion?
- How is the understanding of the torques acting on a system used to predict changes in rotational motion?
- How does the distribution of mass within a body affect its rotational motion?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

What do a spinning top, a ballet dancer doing fouettés and a galaxy have in common (**Figure 1**)?

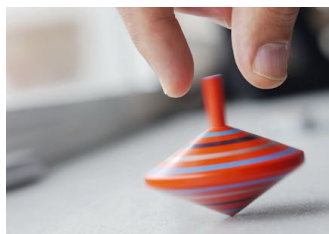


Figure 1a. Spinning top

Credits:rolfo, Getty Images

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Figure 1b. Ballet dancer doing fouettés

Credits:MaFelipe, Getty Images



Figure 1c. Galaxy

Credits:Shawn PNW / 500px, Getty Images

Figure 1. A spinning top, a ballet dancer doing fouettés and a galaxy.

Credits: rolfo, Getty Images; MaFelipe, Getty Images; Shawn PNW / 500px, Getty Images

They all spin about a central axis.

Making connections

In [subtopic A.2 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/), you studied the physics of objects moving in uniform circular motion.

In order for bodies, such as spinning tops and ballet dancers to start spinning, they must be subject to forces that have a turning effect.

For a spinning top, this turning effect is provided by the two equal and opposite forces exerted by the thumb and finger of the person who sets the spinning top in motion. As a result, the spinning top starts to rotate. Soon though, friction slows it down, and the top begins to wobble, until it eventually falls over and stops.

The fouetté is one of the most difficult moves in ballet. The ballet dancer produces a turning effect by pushing off the floor with their foot, they use leg movements to keep their body spinning. You will be able to understand how this works when you have studied this subtopic.

In a galaxy, stars and other objects rotate around an axis that passes through a point near the centre of the galaxy. Stars further away from the axis rotate slower than stars nearer to it.



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For spinning tops, ballet dancers doing fouettés and galaxies, there are big differences in the scale of the rotation and the types of forces involved. But their motion is described by the same physical variables and obeys the same physical laws, which will be discussed in this subtopic.



Concept

You can understand rotational motion by tracing back each new physical quantity, equation and law to linear motion. Spinning tops, ballet dancers and galaxies are all set in rotational motion by a resultant torque (the turning effect provided by a force). This is similar to linear motion where all moving bodies are set in motion by a resultant force.



Creativity, activity, service

Strand: Service

Learning outcome: Demonstrate how to initiate and plan a CAS experience

You could set up a CAS project to design and construct spinning tops to be given to disadvantaged children in your local community.

Activities could include:

- setting up after school sessions to design and build the toys
- visiting partner schools or organisations to show young children the designs and letting them play with the prototypes.



Prior learning

Before you study this subtopic make sure that you understand the following:

- Equations of motion (see [subtopic A.1](#) (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).
- Centripetal force, angular displacement and angular velocity (see [subtopic A.2](#) (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)).
- Newton's laws of motion, momentum and conservation of momentum (see [subtopic A.2](#) (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)).
- Kinetic energy (see [subtopic A.3](#) (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/)).

A. Space, time and motion / A.4 Rigid body mechanics (HL)



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Moment of inertia (HL)



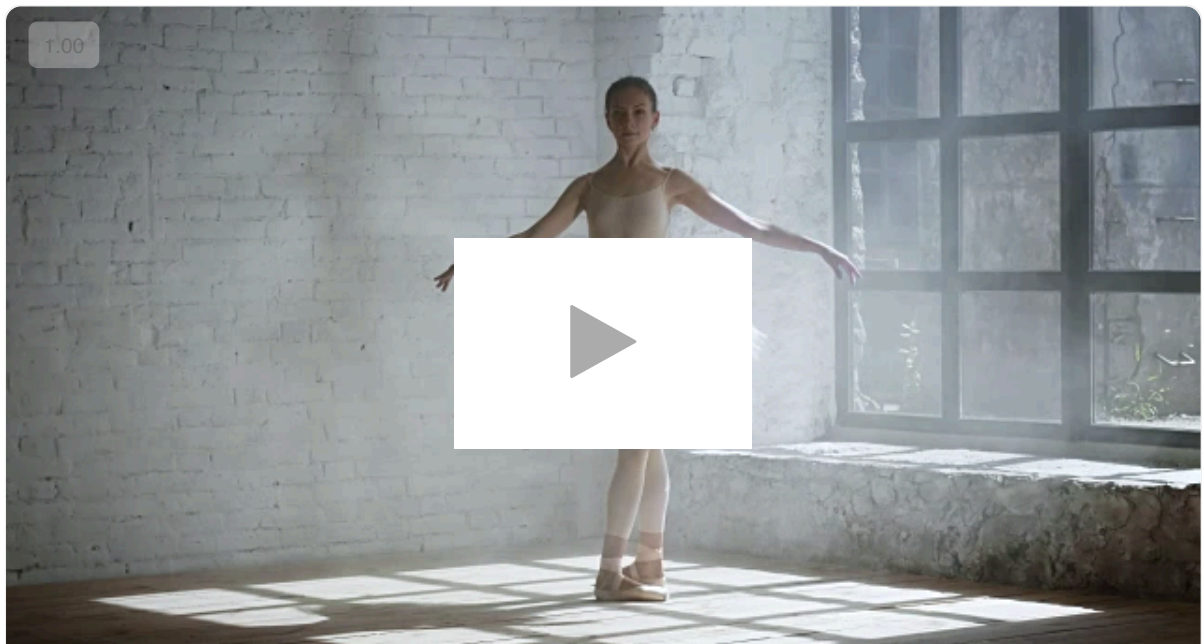
Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Define moment of inertia.
- Use the equation for the moment of inertia to solve problems.

When a ballet dancer doing fouettés moves their leg away from their body and then back towards their body, they are changing their moment of inertia (**Interactive 1**). The larger the moment of inertia, the harder it is to change the body's rotational motion (or the harder it is to make a stationary object rotate). When the ballet dancer extends their leg away from their body, their moment of inertia increases. When they pull their leg back towards their body, their moment of inertia decreases.



Interactive 1. A Ballet Dancer Extending and Pulling in Their Leg While Doing Fouettés.

 More information for interactive 1

The interactive is a video of a ballet dancer performing fouettés. The dancer is spinning while repeatedly extending and pulling in her right leg while balancing the whole body on her left leg toes. Simultaneously, the dancer also extends and pulls in her hands repeatedly at her chest level. The dancer ends her performance by landing on both her legs, and extending her right and left hands vertically and horizontally, respectively. The ballet dance explains the concept of the moment of inertia. The moment of inertia increases and decreases when the legs of the dancer are pulled towards and away from the body, respectively.





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Moment of inertia

The moment of inertia of an object is a measure of how difficult it is to start the object rotating or change its speed of rotation. The moment of inertia depends on how the mass of a body is distributed with respect to the body's axis of rotation. The further away the mass is from the axis of rotation, the larger the moment of inertia, and the greater the turning effect needed to make the object start or stop spinning, or change its angular speed (**Figure 1**).

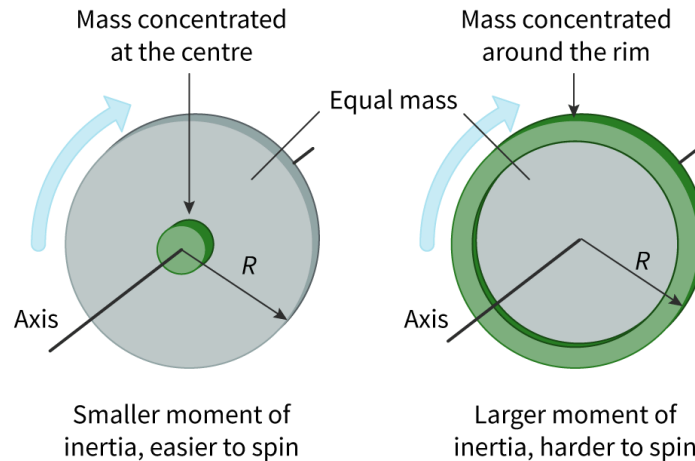


Figure 1. Moments of inertia in rotating objects.

More information for figure 1

The image presents two diagrams side by side, illustrating the concept of moments of inertia in rotating objects. On the left, there is a grey circle with a small green circle at its center. An arrow at the circumference indicates the circle is rotating clockwise. The mass is concentrated at the center, labeled as having a smaller moment of inertia, making it easier to spin. On the right, there is another grey circle, but with a large green circle at its edge, again showing clockwise rotation. Here, the mass is concentrated towards the rim, labeled as having a larger moment of inertia, meaning it is harder to spin. Both diagrams have arrows pointing inwards labeling the radius, denoted as 'R,' showing the distance from the center to the edge where the mass is distributed.

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Making connections

The role of moment of inertia in rotational motion is equivalent to the role of mass in linear motion. The inertial mass of an object is a measure of the object's opposition to a change in its linear motion.

There is only one possible value of inertial mass for an object with a given density and volume. The moment of inertia of an object depends on the mass's position relative to the axis of rotation, so the object has an infinite range of possible moments of inertia.

Like inertial mass, moment of inertia is a scalar quantity.



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A point mass has mass but is so small that it does not take up any space. The moment of inertia I of a point mass is the product of the mass of the point mass m and the square of the distance of the point mass from the axis of rotation r .

Table 1. Moment of inertia equation.

Equation	Symbols	Units
$I = mr^2$	I = moment of inertia	kilograms metre squared (kg m^2)
	m = mass of point mass	kilograms (kg)
	r = distance of point mass from axis of rotation	metres (m)

An extended body is a system of point masses. The moment of inertia of an extended body is the sum (Σ) of all the moments of inertia of the point masses that make up the extended body:

$$I = \Sigma mr^2$$

Figure 2 shows the moments of inertia of a point mass and an extended body.

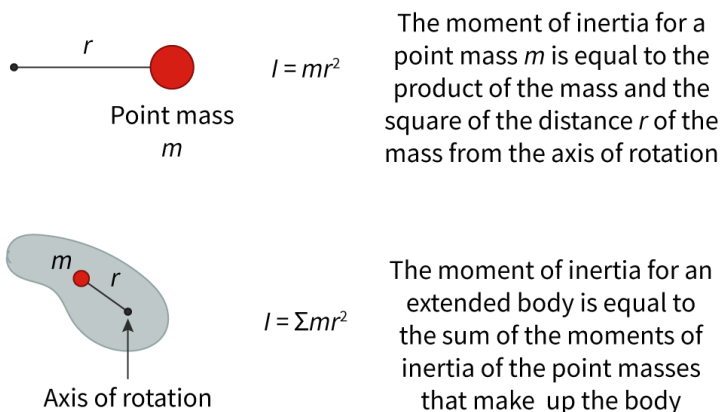


Figure 2. Moment of inertia of a point mass and an extended body.

More information for figure 2

The image consists of two diagrams illustrating the concept of moment of inertia. The top diagram shows a point mass represented by a red circle at the tip of a straight horizontal line. The equation " $I = m r^2$ " is labeled, indicating that the moment of inertia for a point mass m is the product of the mass and the square of the distance r from the axis of rotation.

The bottom diagram represents an extended body consisting of multiple point masses. A single point mass labeled " m " is shown within the extended body, with a distance " r " from the axis of rotation. The equation " $I = \Sigma m r^2$ " signifies that the moment of inertia for an extended body is the sum of the moments of inertia of the individual point masses that constitute it.

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The moment of inertia of a galaxy (which is a large collection of stars and planets, and is an extended body) is given by the sum of the moments of inertia of its stars and planets.

Nature of Science

Aspect: Models

In your study of physics so far, you have dealt with point masses. A point mass is a mass that does not occupy any space, so it cannot actually exist. A point mass is a simplification that allows us to model reality.

In rotational motion, the distribution of mass throughout an object with respect to the object's axis of rotation affects the motion. Rotating objects cannot be considered point masses and we talk about extended bodies instead.

Can you think of an example in physics where it is not appropriate to use a point mass and an example where a point mass is used successfully?

Moment of inertia of a pendulum

A simple pendulum consists of a bob (usually a metal ball) attached to a lightweight string that is suspended from a fixed point (**Figure 3**). When the pendulum is displaced from its equilibrium position, it oscillates about its axis of rotation, which passes through the point of suspension. This is a type of rotational motion.

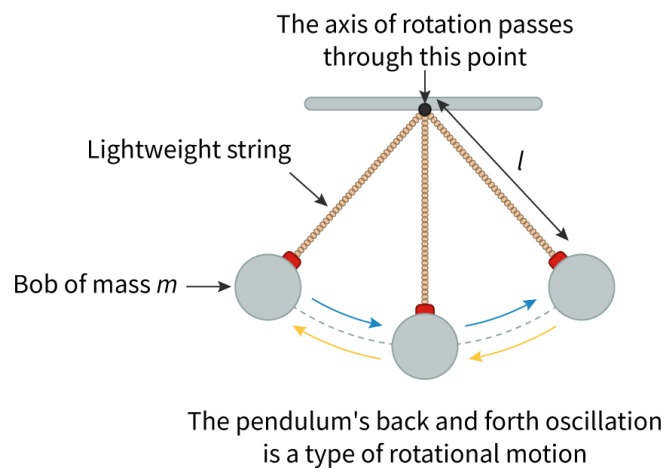


Figure 3. A simple pendulum.

More information for figure 3

The diagram shows a pendulum in three different positions to illustrate its motion. The pendulum consists of a bob, typically a metal ball, attached to a lightweight string. At the top of the pendulum, where the string is attached, there is a label indicating "The axis of rotation passes through this point." The bob of the pendulum, located at the end of the string, is labeled "Bob of mass m ." The pendulum is shown swinging to the left and right from its central position. Arrows beneath the pendulum indicate the direction of the swing: returning and advancing. Different colors are used for these arrows to



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illustrate stages of motion, such as the starting movement and return path. The string is labeled as a "lightweight string," emphasizing its role in the motion and physics of the pendulum. Arrows on either side of the pendulum's path indicate the process of oscillation.

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Table 2. Moment of inertia equation for a simple pendulum.

Equation	Symbols	Units
$I = ml^2$	I = moment of inertia of the pendulum	kilograms metre squared (kg m ²)
	m = mass of the pendulum's bob	kilograms (kg)
	l = length of the pendulum's string from pivot to mass	metres (m)

Notice that it is assumed that the mass of the pendulum's string is negligible compared to that of the bob, and that the pendulum's bob behaves like a point mass.

🔗 Making connections

The motion of a simple pendulum will be discussed in more detail in [subtopic C.1](#) ([/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/](#)).

Moment of inertia of two masses rotating around a point

Consider two point masses attached by a lightweight, rigid rod of length l . The masses rotate around a point, which is a distance $\frac{l}{2}$ from each mass (**Figure 4**).

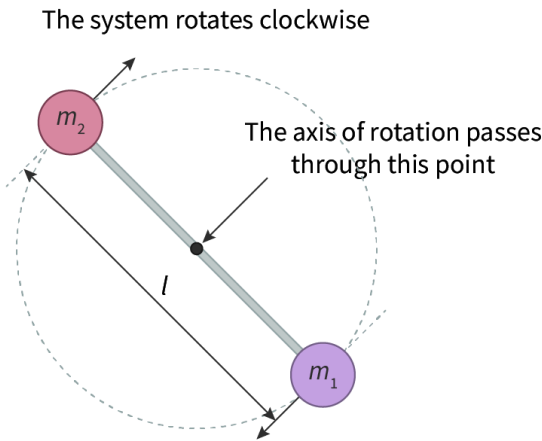


Figure 4. A system of two rotating point masses.

🔗 More information for figure 4



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The image depicts a diagram of a system with two point masses, labeled (m_1) and (m_2). These masses are connected by a grey rod of length (l), and the system rotates clockwise around a central point located in the middle of the rod. This central point is marked as the axis of rotation. The diagram includes a grey dotted circle indicating the rotational path of the masses. Arrows show the direction of rotation, and labels specify that the system rotates clockwise and that the axis of rotation passes through the center. The masses (m_1) and (m_2) are positioned at each end of the rod, and the overall concept highlights the distribution of mass relative to the axis of rotation and its effect on the moment of inertia.

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The moment of inertia of the system is given by the sum of the moments of inertia of the two point masses:

$$\begin{aligned} I &= \Sigma mr^2 \\ &= m_1 \left(\frac{l}{2} \right)^2 + m_2 \left(\frac{l}{2} \right)^2 \\ &= (m_1 + m_2) \times \left(\frac{l}{2} \right)^2 \end{aligned}$$

If the two masses are equal ($m_1 = m_2 = m$), then the equation for the moment of inertia of the system of two rotating masses is:

$$\begin{aligned} I &= 2m \times \left(\frac{l}{2} \right)^2 \\ &= \frac{1}{2} ml^2 \end{aligned}$$

Notice that it is assumed that the two masses behave as point masses and that the mass of the rod connecting them is negligible. This is similar to the pendulum.

🔧 Study skills

For more complicated systems, moment of inertia equations are derived using integral calculus. You are not expected to do this in DP physics. Equations for the moment of inertia will always be given to you in the question. You should be ready to apply these equations to unfamiliar situations when answering exam questions.

Worked example 1

Two point masses of 20 g and 35 g are attached to the ends of a lightweight rod of length 15 cm. The masses rotate about the centre of the rod. Calculate the moment of inertia of the system.



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Solution steps	Calculations
Step 1: Recall the qualitative understanding of the physics.	The moment of inertia of the system is given by the sum of the moments of inertia of the two point masses that make up the system.
Step 2: Write out the values given in the question making sure to convert to SI units.	$m_1 = 0.020 \text{ kg}$ $m_2 = 0.035 \text{ kg}$ $l = 0.15 \text{ m}$
Step 3: Write out the equation.	$I = (m_1 + m_2) \times \left(\frac{l}{2}\right)^2$
Step 4: Substitute the values given.	$= (0.020 + 0.035) \times \left(\frac{0.15}{2}\right)^2$
Step 5: State the answer with appropriate units and the number of significant figures used in rounding.	$= 3.1 \times 10^{-4} \text{ kg m}^2 \text{ (2 s.f.)}$
Step 6: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.

Try this activity about moment of inertia to check your understanding.

Activity

- **IB learner profile attribute:**
 - Inquirer
 - Thinker
 - Communicator
- **Approaches to learning:**
 - Research skills — Evaluating information sources for accuracy, bias, credibility and relevance
 - Social skills — Working collaboratively to achieve a common goal
- **Time to complete activity:** 20 minutes
- **Activity type:** Pair activity



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Instructions: With your partner, complete the following tasks.



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1. Research the equation for the moment of inertia of each of these objects and write it down in a table.

- Solid sphere
- Spherical shell
- Solid cylinder
- Cylindrical shell

Object	Moment of inertia
Solid sphere	$I = \frac{2}{5}mr^2$
Spherical shell	$I = \frac{2}{3}mr^2$
Solid cylinder	$I = \frac{1}{2}mr^2$
Cylindrical shell	$I = mr^2$

1. All the objects are made of the same number of identical point masses (the total mass of the point masses that make up each object is the same). If the objects were dropped from the top of a ramp, which object would rotate fastest? Which object would rotate slowest? Discuss this with your partner, then list the objects in order of increasing speed of rotation and justify your answer.

- Cylindrical shell: $I = mr^2$
- Spherical shell: $I = \frac{2}{3}mr^2$
- Solid cylinder: $I = \frac{1}{2}mr^2$
- Solid sphere: $I = \frac{2}{5}mr^2$

The smaller the moment of inertia, the higher the speed of rotation. The cylindrical shell has the greatest moment of inertia so it will rotate the slowest. The solid sphere has the smallest moment of inertia, so it will rotate the fastest.

1. Research the equations for the moment of inertia of more complicated shapes (for example, cone, torus). Discuss the equations with your partner.

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Watch **Video 1** to see a demonstration of how the distribution of mass of an extended body affects its rotation down a ramp.



Video 1. A practical demonstration of how the distribution of mass of an extended body affects its rotation down a ramp.

5 section questions ^

Question 1

HL Difficulty:

True or false?

Two rigid bodies, A and B, are initially at rest and are subject to an equal force applied perpendicular to the axis of rotation. The centre of mass of body A is closer to the axis of rotation than that of body B. After the force is applied, body A rotates slower.

False



Accepted answers

False, false, F, f

Explanation

For an extended body, the closer the mass is to the axis of rotation, the smaller the moment of inertia. This is because $I = \sum mr^2$, where r is the distance of each point mass making up the extended body from the axis of rotation. The smaller the moment of inertia, the faster the speed of rotation.

Question 2

HL Difficulty:

An athlete swings a hammer of 5.0 kg round their head in a horizontal circle with a radius of 1.75 m. What is the moment of inertia of the hammer? Give your answer to an appropriate number of significant figures.

The moment of inertia of the hammer is 15 ✓ kg m².



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Accepted answers and explanation

#1 15

15.3

General explanation

The moment of inertia of the hammer is given by:

$I = mr^2$, where r is the radius of the horizontal circle and m is the mass of the hammer. Substituting the numbers in:

$$\begin{aligned} I &= 5.0 \times (1.75)^2 \\ &= 15 \text{ kg m}^2 \text{ (2 s.f.)} \end{aligned}$$

Question 3

HL Difficulty:

Which of the following does **not** affect the moment of inertia of an extended body?

- 1 Angular velocity
- 2 Mass of the point masses that make up the object
- 3 Distribution of mass around the axis of rotation
- 4 Orientation of the axis of rotation



Explanation

The moment of inertia of an extended body depends on the mass of the point masses that make up the object and the distance of each point mass from the axis of rotation. Therefore, the distribution of mass around the axis of rotation and the orientation of the axis of rotation affect the moment of inertia. The angular velocity does not affect the moment of inertia.

Question 4

HL Difficulty:

A simple pendulum with mass m and length l has moment of inertia I . A second pendulum has four times the mass and half the length of the first pendulum. What is the moment of inertia of the second pendulum?

- 1 I
- 2 $2I$
- 3 $4I$
- 4 $I/2$



Explanation

The moment of inertia of the first pendulum is:

$$I = ml^2$$



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The moment of inertia of the second pendulum is:

$$4m \times \left(\frac{l}{2}\right)^2 = 4m \times \frac{l^2}{4}$$

$$= ml^2$$

Question 5

HL Difficulty:

A dumbbell consists of two masses m_1 and m_2 attached to the ends of a 15 cm rod with negligible weight. Mass m_1 is double the mass of m_2 . The moment of inertia of the dumbbell is $6.75 \times 10^{-3} \text{ kg m}^2$.

State the values of the masses m_1 and m_2 to two significant figures.

$m_1 =$ 1 0.80 ✓ kg and $m_2 =$ 2 0.40 ✓ kg

Accepted answers and explanation

#1 0.80

#2 0.40

General explanation

The moment of inertia of the dumbbell is given by the sum of the moments of inertia of the two point masses:

$$I = (m_1 + m_2) \times \left(\frac{l}{2}\right)^2$$

$$= (m_1 + m_2) \times \frac{l^2}{4}$$

Substitute $m_2 = \frac{m_1}{2}$:

$$I = \left(m_1 + \frac{m_1}{2}\right) \times \frac{l^2}{4}$$

$$= \frac{3}{2} m_1 \times \frac{l^2}{4}$$

$$= \frac{3m_1 l^2}{8}$$

Rearranging for m_1 and substituting the numbers in:

$$m_1 = \frac{8I}{3l^2}$$

$$= \frac{8 \times 6.75 \times 10^{-3}}{3 \times (0.15)^2}$$

$$= 0.80 \text{ kg (2 s.f.)}$$

$$m_2 = 0.40 \text{ kg (2 s.f.)}$$

A. Space, time and motion / A.4 Rigid body mechanics (HL)

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A.4.4: Rotation of a body described in terms of angular variables (HL)

A.4.5: The equations of motion for uniform angular acceleration (HL)

A.4.12: The kinetic energy of rotational motion (HL)

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Determine the similarities between the physical quantities of rotational motion and linear motion.
- Define angular acceleration.
- Use the equations of uniformly angularly accelerated motion to solve problems.
- Use the equation for kinetic energy in terms of angular velocity to solve problems.

In [subtopic A.1 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/), you studied the variables and equations defining linear motion. In order to determine the time it takes a falling stone to reach the ground, you need information on the stone's initial velocity, its acceleration and its vertical displacement. Similarly, to determine a car's acceleration along a flat straight road, you need to know the car's initial and final velocities and the time taken to accelerate.

What variables do you need to know in order to fully describe the motion of a spinning top? If an asteroid is spiralling into the Sun, what do you need to know in order to calculate when it will collide with the Sun?

Uniform circular motion

In [subtopic A.2 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/), you learned about circular motion. Uniform circular motion is the motion of a body in a circle at constant speed. Although the speed of the body does not change, its velocity changes continuously as it is always in a direction tangential to the circle.

For a body to move with uniform circular motion, there must be a resultant force acting on it. This resultant force is known as the centripetal force, and it is always directed towards the centre of the circle (centripetal means 'centre seeking'). For a body of mass m moving with a linear velocity v in a circle (or arc of a circle) of radius r , the centripetal force F is given in **Table 1**.

Table 1. Centripetal force equation.



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Equation	Symbols	Units
$F = \frac{mv^2}{r}$	F = centripetal force	(newtons) N
	m = mass of object	kilograms (kg)
	v = linear velocity	metres per second (m s^{-1})
	r = radius of circle (or arc)	metres (m)

According to Newton’s second law of motion, this resultant force produces an acceleration in the same direction (towards the centre of the circle).

The body’s linear velocity v is related to its angular velocity ω by the following equation:

Table 2. Angular velocity equation.

Equation	Symbols	Units
$v = \omega r$	v = linear velocity	metres per second (m s^{-1})
	ω = angular velocity	radians per second (rad s^{-1})
	r = radius of circle (or arc)	metres (m)

Figure 1 shows a point mass in uniform circular motion and the variables representing its motion.

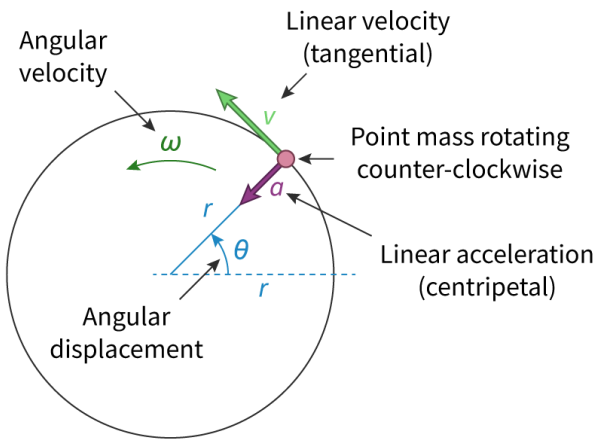


Figure 1. A point mass in uniform circular motion.

More information for figure 1

The diagram illustrates a point mass in uniform circular motion. It features a circle with several important components:

1. **Point Mass:** A small dot at the circumference of the circle represents the point mass.
2. **Linear Velocity (v):** An arrow tangent to the circle at the point mass indicates linear velocity and is labeled " v ."



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3. **Linear Acceleration (a):** An arrow pointing towards the center from the mass indicates centripetal acceleration and is labeled "a "

4. **Angular Displacement (θ):** An angle between the radius line and the horizontal center line of the circle is labeled " θ ."

5. **Angular Velocity (ω):** An inward curved arrow inside the circle pointing counterclockwise indicates angular velocity and is labeled " ω ."

6. **Radius (r):** Radii lines extend from the center of the circle to the circumference, labeled "r."

These components visually demonstrate the relationships between angular and linear measurements in circular motion, providing a clear view of how different vector quantities interact.

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Therefore, the centripetal force can also be written as:

$$F = m\omega^2 r$$

Study skills

Two of the most commonly used units for measuring angles are degrees and radians. One radian is defined as the angle formed by an arc of a circle where the length of the arc is equal to the radius. In other words:

	Degrees	Radians
Full circle	360	2π
Half circle	180	π
Quarter circle	90	$\pi/2$

We usually use radians to talk about circular motion or angular velocity, so it is important to switch your calculator to *radians* mode.

Angular velocity is how fast a rigid body rotates and is related to angular displacement by the following equation:

Table 3. Angular displacement equation.

Equation	Symbols	Units
$\omega = \frac{\Delta\theta}{\Delta t}$	ω = angular velocity	radians per second (rad s ⁻¹)
	$\Delta\theta$ = angular displacement	radians (rad)
	Δt = change in time	seconds (s)

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Angular velocity is also known as angular frequency, because it is related to the frequency of rotation f :

$$\omega = 2\pi f$$

Finally, angular velocity can also be written in terms of the rotational period T :

$$\omega = \frac{2\pi}{T}$$

Worked example 1

A student swings a mass on the end of a string in a circle with a frequency of 3.5 Hz. How long does it take for the mass to travel through an angle of $\pi/4$ radians?

Solution steps	Calculations
Step 1: Work out a strategy.	The frequency, in Hz, tells you the number of complete cycles per second. To relate this to angles, you must convert it to angular frequency.
Step 2: Determine the angular velocity.	$\begin{aligned}\omega &= 2\pi f \\ &= 2\pi \times 3.5 \\ &= 7\pi \text{ rad s}^{-1}\end{aligned}$
Step 3: Use angular velocity (ω) to find the time taken for an angular displacement(θ) of $\pi/4$ radians.	The angular frequency tells you the angle travelled through per second, so you can now determine the time taken for a particular angle to be covered.
Step 4: Write out the equation and rearrange for Δt .	$\begin{aligned}\omega &= \frac{\Delta\theta}{\Delta t} \\ \Delta t &= \frac{\Delta\theta}{\omega}\end{aligned}$
Step 5: Substitute the values given.	$= \frac{\pi/4}{7\pi}$
Step 6: State the answer with appropriate units and the number of significant figures used in rounding.	$= 0.036 \text{ s (2 s.f.)}$
Step 7: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.



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Circular motion with uniform acceleration

In uniform circular motion, the angular velocity ω is constant. However, circular motion does not have to be uniform, and an object's angular velocity can change. This will result in an angular acceleration α , which is defined as the rate of change of angular velocity with time.

Table 4. Angular acceleration.

Equation	Symbols	Units
$\alpha = \frac{\Delta\omega}{\Delta t}$	α = angular acceleration	radians per second per second (rad s^{-2})
	$\Delta\omega$ = change in angular velocity	radians per second (rad s^{-1})
	Δt = change in time	seconds (s)

The movement of a body in a circle with constant angular acceleration is known as uniformly angularly accelerated motion. This type of motion is described by angular displacement $\Delta\theta$, angular velocity ω and angular acceleration α .

Consider an object in uniformly angularly accelerated motion, with angular velocity increasing from an initial value ω_i to a final value ω_f in a time t . The equation for angular acceleration can be rewritten as:

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

This equation rearranged for ω_f is one of the **four** equations of uniformly angularly accelerated motion (**Table 5**).

Table 5. Equations of uniformly angularly accelerated motion.

Equation	Symbols	Units
$\Delta\theta = \frac{\omega_f + \omega_i}{2} t$	$\Delta\theta$ = angular displacement	radians (rad)
$\omega_f = \omega_i + \alpha t$	ω_i = initial angular velocity	radians per second (rad s^{-1})
$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$	ω_f = final angular velocity	radians per second (rad s^{-1})
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	α = angular acceleration	radians per second per second (rad s^{-2})
	t = time	seconds (s)

Making connections

The four equations of uniformly accelerated linear motion (*suvat* equations) are derived in subtopic A.1 (</study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id->



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In **Worked example 2**, the equation for final angular velocity ($\omega_f = \omega_i + \alpha t$) is used, which is the corresponding rotational equation to the equation for linear velocity ($v = u + at$).

Worked example 2

A wheel is rotated from rest. It accelerates with an angular acceleration of 9.0 rad s^{-2} for 6.0 s . Determine the wheel’s final angular velocity.

Solution steps	Calculations
Step 1: Recognise that the equation for final angular velocity will be required.	The equation for final angular velocity is available in the DP physics data booklet.
Step 2: Write out the values given in the question making sure to convert to SI units.	The wheel is rotated from rest so: $\omega_i = 0.0 \text{ rad s}^{-1}$ $\alpha = 9.0 \text{ rad s}^{-2}$ $t = 6.0 \text{ s}$
Step 3: Write out the equation.	$\omega_f = \omega_i + \alpha t$
Step 4: Substitute the values given.	$= 0.0 + (9.0 \times 6.0)$
Step 5: State the answer with appropriate units and the number of significant figures used in rounding.	$= 54 \text{ rad s}^{-1} \text{ (2 s.f.)}$
Step 6: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.

 Theory of Knowledge

Knowledge and Language

In physics, you need to use precise language to avoid ambiguity. The language of mathematics is used by scientists to allow them to communicate knowledge in an accurate and clear way.



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Each symbol in an equation has a precise meaning, and equations represent laws and theories. However, you need to understand the language of mathematics to understand this knowledge. If you are not a scientist, then this knowledge can be hard to understand.

Mathematical equations are a way of simplifying difficult concepts and a way for scientists to communicate easily. However, the language of equations makes it difficult for non-scientists to understand the knowledge.

Knowledge about the natural sciences is communicated using mathematical language that lots of people do not understand. What do you think are the negative implications and consequences of this?

Kinetic energy in uniformly angularly accelerated motion

As you learned in [subtopic A.3 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/), a body moving with a linear velocity stores kinetic energy. The amount of kinetic energy stored depends on the body's mass and the square of its velocity. For an object of mass m and linear velocity v , the kinetic energy is:

$$E_k = \frac{1}{2}mv^2$$

Since moment of inertia I and angular velocity ω in rotational motion correspond to mass m and linear velocity v in linear motion, you can write the equation as in **Table 6**.

Table 6. Equation for the kinetic energy of rotational motion.

Equation	Symbols	Units
$E_k = \frac{1}{2}I\omega^2$	E_k = kinetic energy of rotational motion	joules (J)
	I = moment of inertia	kilograms metre squared (kg m ²)
	ω = angular velocity	radians per second (rad s ⁻¹)

Often in physics, the equations used to describe different phenomena are similar to each other. Recognising and appreciating such similarities could deepen your conceptual understanding of the subject. Can you think of any similarities between two (or more) equations describing different physical phenomena?

If a body rolls down a slide, its motion is linear and rotational at the same time. In this case, the body has both translational and rotational kinetic energies, and its overall kinetic energy will be:

$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$



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Worked example 3

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The moment of inertia of a solid sphere is:

$$I = \frac{2}{5}mr^2$$

A 100 g sphere is rolling along a horizontal surface to the right, without slipping, with a linear velocity of 2.5 m s⁻¹. Calculate the sphere's total kinetic energy due to its translational and rotational motions.

Solution steps	Calculations
Step 1: Recall the qualitative understanding of the physics.	The sphere's total kinetic energy is the sum of its linear and rotational kinetic energies.
Step 2: Use a relationship to substitute known values for a quantity we do not know.	Since the sphere is rolling but not slipping, its angular velocity is related to its linear velocity according to the formula used in circular motion: $v = \omega r$ Substituting this into the equation for kinetic energy: $\begin{aligned} E_k &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \end{aligned}$
Step 3: Use information given in the question to turn the equation into a form we can solve.	Using the equation for the moment of inertia of the sphere ($I = \frac{2}{5}mr^2$): $\begin{aligned} E_k &= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}\right)mr^2\frac{v^2}{r^2} \\ &= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 \\ &= \frac{7}{10}mv^2 \end{aligned}$
Step 4: Write out the values given in the question making sure to convert to SI units.	$m = 0.1 \text{ kg}$ $v = 2.5 \text{ m s}^{-1}$
Step 5: Substitute the values given.	$= \frac{7}{10} \times 0.1 \times 2.5^2$



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Solution steps	Calculations
Step 6: State the answer with appropriate units and the number of significant figures used in rounding.	$= 0.44 \text{ J (2 s.f.)}$
Step 7: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.

The tasks in the next activity will check your understanding of the theory covered in this section.



Activity

- **IB learner profile attribute:**
 - Inquirer
 - Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 30 minutes
- **Activity type:** Individual activity

Instructions: Complete the following tasks on your own during class or at home.

1. Pair each variable of rotational motion (θ , ω_i , ω_f and α) with a variable from linear motion.

- Angular displacement $\theta \rightarrow$ linear displacement s
- Initial angular velocity $\omega_i \rightarrow$ initial linear velocity u
- Final angular velocity $\omega_f \rightarrow$ final linear velocity v
- Angular acceleration $\alpha \rightarrow$ linear acceleration a

2. Pair each equation of uniformly angularly accelerated motion with an equation from linear motion.

- $\Delta\theta = \frac{\omega_f + \omega_i}{2}t \rightarrow s = \frac{v + u}{2}t$
- $\omega_f = \omega_i + \alpha t \rightarrow v = u + at$
- $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \rightarrow s = ut + \frac{1}{2}at^2$



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$$\bullet \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \rightarrow v^2 = u^2 + 2as$$

3. Graphs of motion are often used to derive some of the variables of linear motion. The same can be done for variables of rotational motion.

(a) What variable of linear motion is represented by:

- (i) the gradient of a displacement—time graph?
- (ii) the gradient of a velocity—time graph?
- (iii) the area under the curve of a velocity—time graph?

(b) For each graph of linear motion, give the corresponding graph for rotational motion.

(c) State what variable of rotational motion is represented by the gradient, or the area under the curve, of each graph in (b).

- (a) (i) Velocity v
- (ii) Acceleration a
- (iii) Displacement s

(b)

Displacement—time graph ($s-t$) \rightarrow ($\theta-t$) angular displacement—time graph

Velocity—time graph ($v-t$) \rightarrow ($\omega-t$) angular velocity—time graph

(c)

Gradient of angular displacement—time graph \rightarrow angular velocity ω

Gradient of angular velocity—time graph \rightarrow angular acceleration α

Area under the curve of an angular velocity—time graph \rightarrow angular displacement θ

4. For a body moving in a circular path with radius r and increasing angular velocity ω , derive an equation relating angular acceleration α and linear acceleration a .

Angular acceleration is given by:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Using $v = \omega r$ and rearranging for ω :

$$\omega = \frac{v}{r}$$

Substituting this expression for ω into the equation for angular acceleration:

$$\alpha = \frac{\Delta\left(\frac{v}{r}\right)}{\Delta t}$$

Since the radius of the circle is constant and does not change with time:



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$$\alpha = \frac{\Delta v}{r \Delta t}$$

This is the equation for linear acceleration:

$$a = \frac{\Delta v}{\Delta t}$$

Therefore, the equation relating angular acceleration α and linear acceleration a is:

$$\alpha = \frac{a}{r}$$

5. Verify that the units in the equation $E_k = \frac{1}{2} I \omega^2$ are consistent (the units on the left-hand side of the equation are equal to the units on the right-hand side of the equation).

Kinetic energy is measured in joules (J):

$$\begin{aligned} 1 \text{ J} &= 1 \text{ N m} \\ &= 1 \text{ kg m s}^{-2} \text{ m} \\ &= 1 \text{ kg m}^2 \text{ s}^{-2} \end{aligned}$$

Moment of inertia is measured in kg m^2 and angular velocity is measured in rad s^{-1} (remember that this is squared in the equation for the kinetic energy). The radian is a dimensionless unit and should not be included in dimensional analysis. Therefore, on the right-hand side of the equation, you have $1 \text{ kg m}^2 \text{ s}^{-2}$. Hence, the units in the equation for rotational kinetic energy are consistent.

6. A disc initially at rest starts rotating. Its angular acceleration is constant and equal to 12 rad s^{-2} . The disc accelerates for 6.0 s . Determine:

- (a) the disc's final angular velocity
(b) the number of revolutions that the disc has rotated through

- (a) Since the disc starts rotating from rest, its initial angular velocity, $\omega_i = 0$.

The final angular velocity is given by:

$$\begin{aligned} \omega_f &= \alpha t \\ &= 12 \times 6.0 \\ &= 72 \text{ rad s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

- (b) Before you can calculate the number of revolutions that the disc has rotated through, you must calculate the disc's angular displacement $\Delta\theta$. This is given by:

$$\begin{aligned} \Delta\theta &= \frac{\omega_f t}{2} \\ &= \frac{72 \times 6.0}{2} \\ &= 216 \text{ rad} \end{aligned}$$

A complete revolution covers an angular displacement of $2\pi \text{ rad}$. Hence the number of revolutions N is given by:



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$$N = \frac{216}{2\pi}$$

$$= 34 \text{ (2 s.f.)}$$

5 section questions ^

Question 1

HL Difficulty:

What is the difference between uniform circular motion and uniformly angularly accelerated motion?

- 1 In uniform circular motion, the angular velocity is constant.
In uniformly angularly accelerated motion, the angular acceleration is constant. ✓
- 2 In uniform circular motion, the angular acceleration is constant.
In uniformly angularly accelerated motion, the angular velocity is constant.
- 3 In uniform circular motion, the angular displacement is constant.
In uniformly angularly accelerated motion, the angular acceleration is constant.
- 4 In uniform circular motion, the angular velocity is constant.
In uniformly angularly accelerated motion, the angular velocity changes at a non-constant rate.

Explanation

In uniform circular motion, the angular velocity is constant, and there is no angular acceleration. The magnitude of the linear velocity is also constant, only its direction changes, so there is linear acceleration. In uniformly angularly accelerated motion, the angular acceleration is constant. This means that the angular velocity changes at a constant rate.

Question 2

HL Difficulty:

A wheel accelerates from 2.0 rad s^{-1} with a constant angular acceleration of 1.5 rad s^{-2} . Calculate the wheel's angular displacement after 5.0 s. Give your answer in radians to an appropriate number of significant figures.

29 rad ✓

Accepted answers

29 rad, 29 radians

Also accepted

29, 28.75, 29rad, 29radians, 28.8

Explanation

Since you are given the initial angular velocity ω_i , the acceleration α and the time t , you can calculate the angular displacement of the wheel as follows:

$$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$= (2.0 \times 5.0) + \left(\frac{1}{2} \times 1.5 \times 5.0^2 \right)$$

$$= 28.75 \text{ rad}$$

$$= 29 \text{ rad (2 s.f.)}$$



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Question 3

HL Difficulty:

A solid object initially at rest starts rotating horizontally about its centre. The object's angular acceleration is constant and is 0.22 rad s^{-2} . The object reaches its maximum angular velocity in four complete rotations. What is the object's final angular velocity?

Give your answer to an appropriate number of significant figures.

The object's final angular velocity is ☒ rad s^{-1} .

Accepted answers and explanation

#1 3.3

General explanation

Since the object is initially at rest, the initial angular velocity is:

$$\omega_i = 0$$

After four complete rotations, the object's angular displacement is:

$$\begin{aligned}\Delta\theta &= 4 \times 2\pi \\ &= 8\pi\end{aligned}$$

Since you have the object's angular acceleration and angular displacement and you need to calculate its final angular velocity, you can use the equation below:

$$\begin{aligned}\omega_f^2 &= \omega_i^2 + 2\alpha\Delta\theta \\ &= 2\alpha\Delta\theta \text{ (because } \omega_i = 0\text{)}\end{aligned}$$

So the object's final angular velocity is:

$$\begin{aligned}\omega_f &= \sqrt{2\alpha\Delta\theta} \\ &= \sqrt{2 \times 0.22 \times 8\pi} \\ &= 3.3 \text{ rad s}^{-1} \text{ (2 s.f.)}\end{aligned}$$

Question 4

HL

A disc is rotating about its centre. The disc slows down uniformly until it stops in 3.5 s, having performed six complete rotations. What is the angular deceleration of the disc? Give your answer to an appropriate number of significant figures.

The angular deceleration of the disc is ☒ rad s^{-2} .

Accepted answers and explanation

#1 6.2

-6.2

General explanation

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Since the disc stops, its final angular velocity is:

$$\omega_f = 0$$

The disc's angular displacement after six complete revolutions is:

$$\begin{aligned}\Delta\theta &= 6 \times 2\pi \\ &= 12\pi\end{aligned}$$

You now need to calculate the disc's initial angular velocity. You can use the equation below:

$$\begin{aligned}\Delta\theta &= \frac{\omega_i + \omega_f}{2} t \\ &= \frac{\omega_i}{2} t \text{ (because } \omega_f = 0\text{)}\end{aligned}$$

Rearranging for the initial angular velocity:

$$\begin{aligned}\omega_i &= \frac{2\Delta\theta}{t} \\ &= \frac{2 \times 12\pi}{3.5} \\ &= 21.54 \text{ rad s}^{-1}\end{aligned}$$

Now you can use the equation:

$$\omega_f = \omega_i + \alpha t$$

Rearranging for α and remembering that $\omega_f = 0$:

$$\begin{aligned}\alpha &= -\frac{\omega_i}{t} \\ &= -\frac{21.54}{3.5} \\ &= -6.2 \text{ rad s}^{-2} \text{ (2 s.f.)}\end{aligned}$$

The negative sign is due to the fact that this is an angular deceleration.

Question 5

HL Difficulty:

The moment of inertia of a solid cylinder is:

$$I = \frac{1}{2}mr^2$$

A cylinder is located at the top of a ramp of vertical height 20 cm. It starts rolling down the ramp. Calculate the cylinder's linear velocity at the bottom of the ramp. Give your answer to an appropriate number of significant figures.

The cylinder's linear velocity at the bottom of the ramp is 1.6 ✓ m s⁻¹.

Accepted answers and explanation

#1 1.6

General explanation

Friction is negligible. Therefore, the cylinder's total mechanical energy is conserved.

When it is at rest at the top of the ramp, the cylinder has gravitational potential energy:

$$E_p = mgh$$

As it starts rolling down the ramp, the gravitational potential energy is transferred to translational and rotational kinetic energies:



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$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

At the bottom of the ramp, all of the initial gravitational potential energy is transferred to translational and rotational kinetic energies. So:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

You can use the relationship between angular velocity and linear velocity to express ω in terms of v :

$$\omega = \frac{v}{r}$$

Substituting this into the conservation of mechanical energy equation:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$

Substituting the expression for the cylinder's moment of inertia I :

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \\ &= \frac{3}{4}mv^2 \end{aligned}$$

Simplifying for the mass m and rearranging for the linear velocity v :

$$\begin{aligned} v &= \sqrt{\frac{4gh}{3}} \\ &= \sqrt{\frac{4 \times 9.8 \times 0.2}{3}} \\ &= 1.6 \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

A. Space, time and motion / A.4 Rigid body mechanics (HL)

Torques and rotational equilibrium (HL)

A.4.1: Torque of a force (HL) A.4.2: Bodies in rotational equilibrium have a resultant torque of zero (HL) A.4.3: Unbalanced torque (HL)

A.4.8: Newton's second law for rotation (HL)

Extended



Learning outcomes

By the end of this section you should be able to:

- Define torques and couples.
- Use the equation for torques to solve problems.
- Define rotational equilibrium.
- Apply Newton's second law for angular momentum to solve problems.



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How can you set an object in rotational motion? [This article](#)

(<https://www.bbc.co.uk/newsround/61081302>) talks about a cleaner way of launching projectiles into space using rotational motion and without rocket fuel. Would you be able to explain the physics behind this launch mechanism?

Torque

Imagine you are using a spanner to turn a nut (**Figure 1**). You apply a force F at the end of the spanner's handle, at a distance r from the point P at which the nut is located. The force's orientation is such that the line of action of the force makes an angle θ with the handle. Your force will cause a rotation of the spanner about an axis of rotation that passes through point P (known as the pivot). Only $F \sin \theta$ is responsible for the turning effect, while $F \cos \theta$ does not produce a rotation.

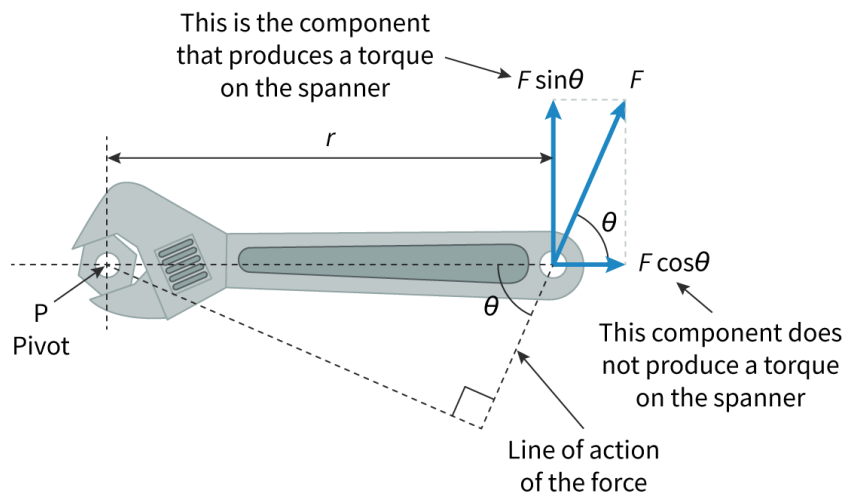


Figure 1. The turning effect of a force on the handle of a spanner.

More information for figure 1

The diagram shows a spanner with labeled components to illustrate the turning effect of a force. An arrow labeled $F \sin \theta$ points upwards from the end of the spanner, indicating that this component does not produce torque. Another arrow, labeled $F \cos \theta$, points outwards from the end. Below the spanner, an angled dotted line is labeled as the 'Line of action of the force.' The head of the spanner is marked with a point labeled 'P Pivot,' indicating the axis around which rotation occurs. Arrows and labels explain how different components of force act on the spanner.

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To understand whether or not a force (or one of its components) will have a turning effect on an object, you need to consider the pivot about which the object would rotate and the line of action of the applied force:

- Any force (or component of a force) whose line of action **passes through** the pivot, has **no turning effect**.
- Any force (or component of a force) whose line of action does **not pass through** the pivot, has a turning effect.



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Torque is a measure of the turning effect of a force. If a force F acts on an object and causes it to rotate, the torque is defined as the product of the magnitude of the force and the perpendicular distance between the line of action of the force and the pivot.

For the spanner, the torque of the force F applied to the end of the spanner's handle can be calculated as follows:

Table 1. Calculation of torque.

Equation	Symbols	Units
$\tau = Fr \sin \theta$	τ = torque	newton metres (N m)
	F = force	newtons (N)
	r = displacement between the pivot and the point at which the force is applied	metres (m)
	θ = angle between the line of action of the force and r	degrees ($^{\circ}$)



Exercise 1



Click a question to answer

Extended

You should notice from the above equation that the **maximum torque** is obtained when the line of action of the force F and the distance r are perpendicular:

$$\theta = 90^{\circ}$$

If the line of action of the force F and the distance r are parallel ($\theta = 0^{\circ}$), the **torque is zero**.

Study skills

Torque is measured in newton metres (N m). This is because it is the product of force and distance. Since work is also the product of force and distance, you might think that torque is equivalent to work (or energy). However, torque and work are not equivalent. When calculating work, the force and the distance must be parallel. When calculating torque, the force and the distance must be perpendicular. **Torque is a vector quantity**, while work is a scalar quantity.



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The direction of the torque can be expressed as **clockwise** or **counter-clockwise**. When multiple forces each have their own turning effect on an object, you need to calculate the torque of each force separately, assigning it a **positive** or **negative** sign depending on its direction. The overall torque will be given by the sum of the torques of the forces acting on it.

Couples

Rotation can often be caused by a pair of equal and opposite forces. Such a pair of forces is known as a couple.

When a couple is applied to an object, it produces a rotation, without translational motion.

Consider a couple acting on a rod of length $2r$, as shown in **Figure 2**.

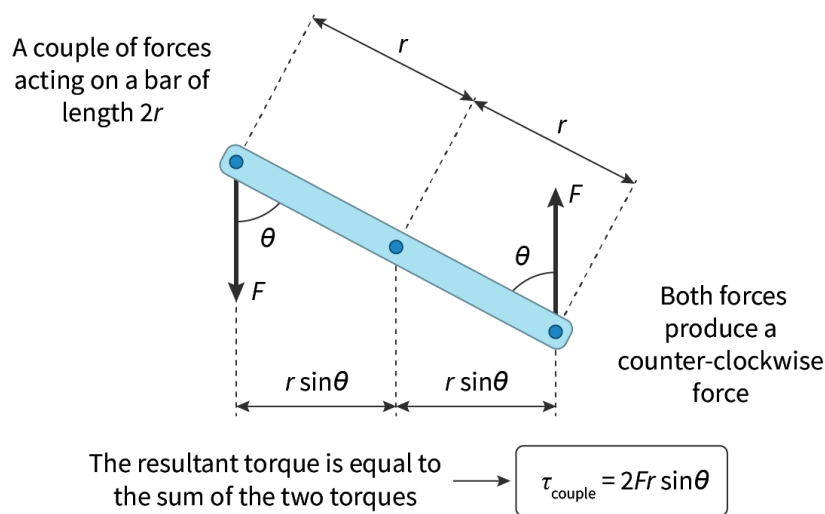


Figure 2. The torque of a couple.

More information for figure 2

The image is a diagram illustrating a couple of forces acting on a bar of length $2r$. This is depicted with a light blue rectangular bar diagonally placed. There are two force vectors, labeled F , one acting downwards on the left side of the bar and the other acting upwards on the right side. Each vector spans a distance labeled r from the center of the bar, making an angle θ with the perpendicular drawn from the point of application to the center.

The diagram shows two dashed lines extending vertically from the ends of the bar, indicating the direction of the forces. Additional dashed lines run horizontally from where each force is applied to the center line of the bar, labeled $r \sin \theta$, representing the perpendicular distance from the pivot point to the line of action of each force.

There is a notation box on the diagram with the equation for the resultant torque, labeled as $\tau_{\text{couple}} = 2F * r * \sin \theta$, indicating that the torque is the sum of the two individual torques caused by these forces, both contributing to a counter-clockwise torque around the pivot point at the center of the bar.

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Each force in the couple exerts its own counter-clockwise (anticlockwise) torque on the bar, which will rotate counter-clockwise about the central fixed point (the pivot). Since the torques are both in the same direction, they have the same sign. The overall torque is given by the sum of the two torques of the forces in the couple. This is equal to:

$$\tau_{\text{couple}} = 2Fr \sin \theta$$

Rotational equilibrium

In subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/) you learned that, according to Newton's first law of motion, a body remains at rest or continues to move with constant velocity unless it is acted upon by a resultant force. When the resultant force on a body is zero, the body is said to be in translational equilibrium. In this case, as stated by Newton's second law, the body's linear acceleration is zero. So, translational equilibrium is characterised by zero linear acceleration.

A body is said to be in rotational equilibrium when the resultant torque on it is zero, or in other words, when its angular acceleration is zero. Such a body will remain stationary or continue to rotate with constant angular velocity.

To understand whether or not a body is in rotational equilibrium, you need to calculate the individual torques acting on it, and then add them together, considering the respective signs. If the resultant torque is zero, the body is in rotational equilibrium.

Worked example 1

Consider a bar resting on a pivot located at its centre. A 100 g mass hangs 50 cm to the left of the pivot, and a 200 g mass hangs 25 cm to the right of the pivot (**Figure 3**). Determine whether the bar is in rotational equilibrium.

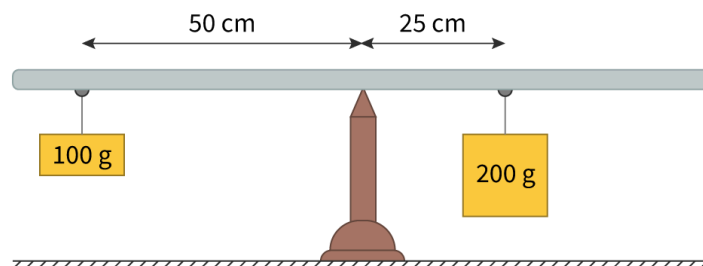


Figure 3. Bar on a pivot.

More information for figure 3



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The image shows a bar positioned on a pivot at its center. On the left side of the pivot, at a distance of 50 cm, a 100 g weight is suspended. On the right side, at a distance of 25 cm, there is a 200 g weight hanging. The image illustrates the concept of rotational equilibrium, where the moments about the pivot are calculated by multiplying the weight with the distance from the pivot. Given the values, both sides exert equal moments (torques) about the pivot, balancing the bar.

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Solution steps	Calculations
Step 1: Recall the qualitative understanding of the physics.	The bar is in rotational equilibrium if the resultant torque on it is zero.
Step 2: Write out the values given in the question making sure to convert to SI units. The value for g is in the DP physics data booklet.	$m_1 = 0.1 \text{ kg}$ $m_2 = 0.2 \text{ kg}$ $r_1 = 0.5 \text{ m}$ $r_2 = 0.25 \text{ m}$ $g = 9.8 \text{ N kg}^{-1}$
Step 3: Choose and simplify the relevant equation for this question.	The load located to the left of the pivot causes a counter-clockwise torque: $\tau_1 = F_1 r_1 \sin \theta$ The force and the distance from the pivot are perpendicular, $\theta = 90^\circ$, so $\sin \theta = 1$: $\tau_1 = F_1 r_1$
Step 4: Add values to find the required quantity.	The force applied is the weight of the object, mg : $\begin{aligned}\tau_1 &= m_1 g r_1 \\ &= 0.1 \times 9.8 \times 0.5 \\ &= 0.49 \text{ N m}\end{aligned}$
Step 5: Calculate the remaining unknown quantity.	The load located to the right of the pivot causes a clockwise torque: $\begin{aligned}\tau_2 &= F_2 r_2 \\ &= m_2 g r_2 \\ &= -0.2 \times 9.8 \times 0.25 \\ &= -0.49 \text{ N m}\end{aligned}$ The negative sign indicates the opposite direction of this torque with respect to the first torque.



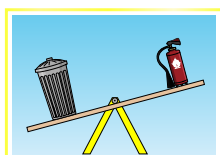
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Solution steps	Calculations
Step 6: State the answer.	<p>Since the torques have the same magnitude and opposite directions, they add to zero to give a resultant torque τ of zero:</p> $\begin{aligned}\tau &= \tau_1 + \tau_2 \\ &= 0.49 + -0.49 \\ &= 0\end{aligned}$ <p>Therefore the bar is in rotational equilibrium.</p>
Step 7: Does your answer make sense?	✓ Yes, as the torques are equal and opposite, we would expect no net torque.

Using **Interactive 1**, try to apply your understanding of torque to answer the questions:

- Click 'Balance Lab' and drag and drop the items onto the beam. Can you balance the torques to create a system in equilibrium?
- Using the 'Balance Lab' tab, can you find the masses of all eight mystery objects? (Click all the way to the right in the object list to find them.)

Using the 'Game' tab, can you get 5 stars on each level?



Interactive 1. Balance torques to create a system in equilibrium.

More information for interactive 1

An interactive simulation titled, Balance torques to create a system in equilibrium, allows users to explore the concept of torque and equilibrium by manipulating objects on a balance beam. In the Balance Lab, users can place various objects of known and unknown masses at different positions along the beam. The goal is to adjust the placements so that the beam remains level, demonstrating a system in equilibrium. The interface provides options to display mass labels, forces from objects, and level indicators. It also allows users to switch between mystery objects and standard weights such as bricks of different masses.

To achieve equilibrium, users must apply the principle of torque, which states that the sum of the torques on both sides of the pivot must be equal. Torque is calculated as the product of force (weight) and the perpendicular distance from the pivot point. By experimenting with different placements, users can determine unknown masses by balancing the system.



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For example, consider a 5 kilogram mass placed 2 meters to the left of the pivot. If an unknown mass is placed 1 meter to the right, the equation for equilibrium is:

$$(5 \text{ kilogram} \times 2 \text{ meters}) = (\text{mass} \times 1 \text{ meter})$$

Solving for mass,

$$10 = \text{mass} \times 1$$

$$\text{mass} = 10 \text{ kilogram}$$

This method can be used to determine the masses of the mystery objects by adjusting their positions until equilibrium is achieved.

The simulation also includes a Game mode where users can test their understanding by attempting to balance different objects to earn stars. By completing challenges in this mode, users can reinforce their grasp of torque and equilibrium. The interactive provides an engaging way to develop an intuitive and mathematical understanding of rotational balance.

Study skills

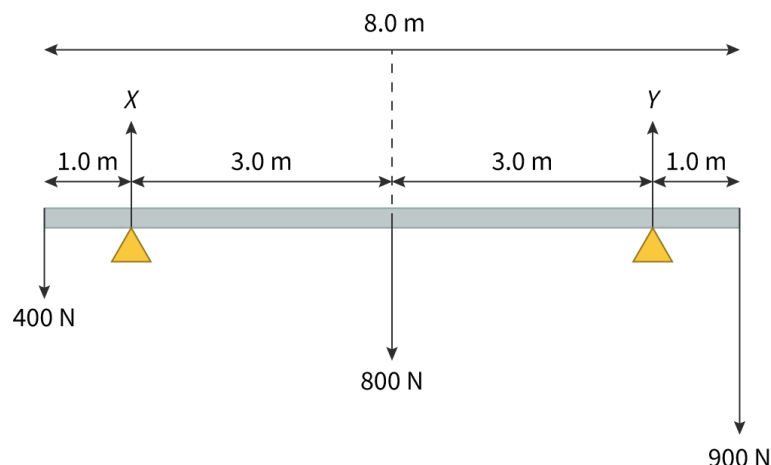
When you solve problems involving rotational equilibrium, you have to calculate the resultant torque with respect to a certain point. This point is usually the pivot (the point through which the axis of rotation passes), but it does not have to be, and it could be any other point instead.

To simplify calculations, it is often useful to choose a point through which many of the lines of action of the forces pass. Such forces will not exert a torque, and you will end up with fewer terms in the calculation of the resultant torque.

If the weight of an object produces a resultant torque on it, you must remember that the weight acts at the centre of mass of the object (in a uniform gravitational field), which for a regular shape is located at the centre. For example, a uniform bar of length l has its centre of mass at a distance $\frac{l}{2}$.


Worked example 2

An 8.0 m plank rests on two pyramidal supports. The supports exert the upwards forces X and Y on the plank. The weight of the plank is 800 N. A downwards 400 N force acts on the left-hand side of the plank and a downwards 900 N force acts on the right-hand side of the plank (**Figure 4**).



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Figure 4. Plank on two supports.

 More information for figure 4

The diagram shows an 8-meter wide plank resting horizontally on two pyramidal supports. These supports are positioned 1 meter in from each end of the plank. The plank is subjected to various forces: an upward force labeled (X) acting on the left pyramidal support and an upward force labeled (Y) on the right support. The weight of the plank is given as 800 N. Additionally, there is a downward force of 400 N acting on the left-hand side and a downward force of 900 N acting on the right-hand side of the plank. The forces and distances are proportionally indicated.

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Determine the magnitudes of the forces X and Y .

Solution steps	Calculations
Step 1: Recall the qualitative understanding of the physics.	Since the plank is at rest, it is in both translational and rotational equilibrium. The condition for linear equilibrium tells you that the resultant force on the plank is zero and the condition for rotational equilibrium tells you that the resultant torque is zero.
Step 2: Using linear equilibrium, write an expression for X and Y .	$X + Y = 400 + 800 + 900$ $= 2100 \text{ N}$
Step 3: Using rotational equilibrium, decide the point you will take torques around (remember, torques will be equal around every point, so choose one where you can remove an unknown).	Around the left side support: clockwise torque $= (800 \times 3.0) + (900 \times 7.0)$ $= 8700 \text{ N m}$ counter-clockwise torque $= (400 \times 1.0) + (Y \times 6.0)$ These must be equal for the net torque to be zero: $8700 = 400 + (Y \times 6.0)$
Step 4: Solve for Y .	$Y = \frac{8700 - 400}{6.0}$ $= 1400 \text{ N (2 s.f.)}$
Step 5: Substitute Y into the equation from step 2.	$X = 2100 - 1400$ $= 700 \text{ N}$
Step 6: State the answer with appropriate units and the number of significant figures used in rounding.	$X = 700 \text{ N}$ $Y = 1400 \text{ N (2 s.f.)}$



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Solution steps**Calculations**

Step 7: Does your answer make sense?

✓ Yes. You should notice that you would get the exact same results if you chose to calculate the torques about the right-hand side support instead. You should do this as an exercise.

Worked example 3

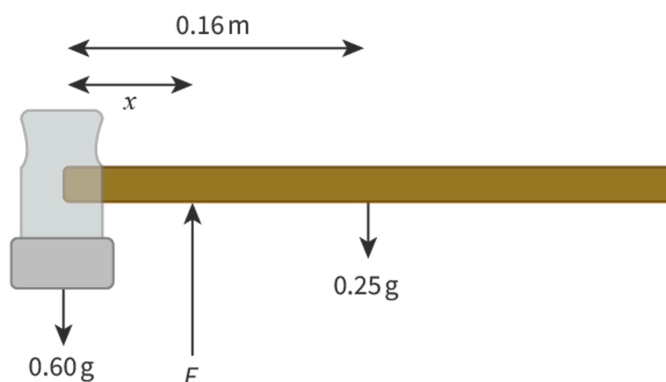
A hammer is made of a small metal head of mass 0.60 kg attached to the end of a uniform wooden rod of mass 0.25 kg and length 0.32 m .

A person balances the hammer on the end of their finger so that the hammer is horizontal and stationary. Determine the distance from the head at which the hammer balances.

Solution steps**Calculations**

Step 1: Consider the forces acting on the rod. The point at which the hammer balances on the finger must be between the head and the centre of the rod. The weights of the head and the rod then produce clockwise and anticlockwise moments about this point.

The person's finger exerts an upwards force F at a distance x from the head, as shown.



Step 2: Write expressions for the torques around a point — the position of the finger is convenient because the force F does not produce a torque about that point.

Around the position of the finger (at a distance x from the head of the hammer):

$$\text{Clockwise torque} = 0.25 \times 9.8 \times (0.16 - x)$$

$$\text{Anticlockwise torque} = 0.60 \times 9.8 \times x$$

Step 3: Use the fact that net torque around any point is zero for an object in rotational equilibrium.

$$0.25 \times 9.8 \times (0.16 - x) = 0.60 \times 9.8 \times x$$

Section

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Feedback



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Solution steps	Calculations
Step 4: Solve for x . (It is helpful to start by dividing both sides by 9.8.)	$0.25 \times 9.8 \times (0.16 - x) = 0.60 \times 9.8 \times x$ $0.25 \times (0.16 - x) = 0.60x$ $0.04 - 0.25x = 0.60x$ $0.85x = 0.04$ $x = \frac{0.04}{0.85}$ $= 0.047 \text{ m (2 s.f.)}$ $= 4.7 \text{ cm}$

Worked example 4

Outside a shop, a sign hangs from a rod. The rod is attached to the wall by a hinge, so that the right-hand end of the rod can swing freely up and down. The other end of the rod is attached to the wall by a cable. The rod is in equilibrium (**Figure 5**).

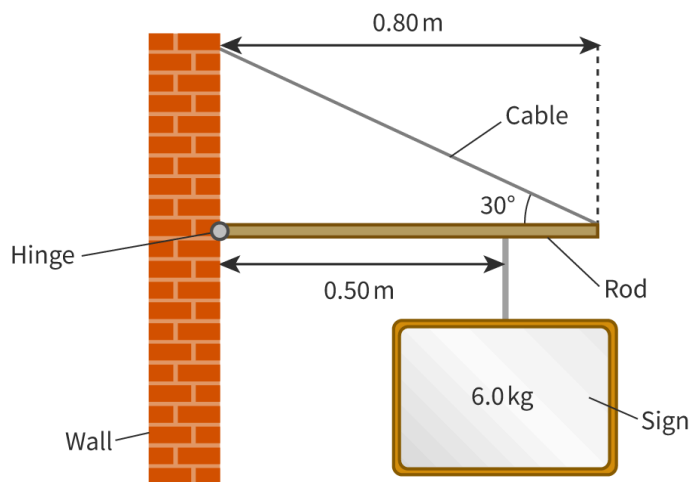


Figure 5. A sign hanging from a rod that is attached to a wall by a hinge and a cable.

More information for figure 5

The image is a diagram depicting a sign hanging system. A rod is attached to a brick wall by a hinge, allowing the rod to swing up and down. The rod is supported by a cable that runs diagonally upwards to the wall at a 30-degree angle. The cable is 0.80 meters long. The right side of the rod, which is 0.50 meters in length, supports a sign weighing 6.0 kg. Labels indicate parts of the system: hinge, cable, rod, wall, and sign.

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Determine:

- the tension in the cable;
- the horizontal and vertical components of the force exerted by the hinge on the left-hand end of the rod.

Assume that the masses of the rod and the cable are negligible.



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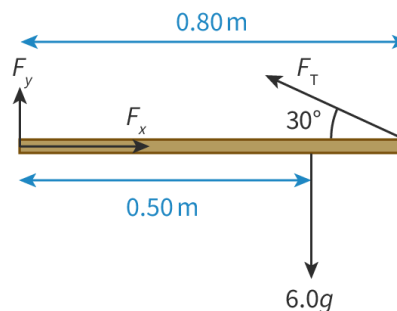
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Solution steps

Step 1: Consider the forces acting on the rod, and the conditions for equilibrium.

Calculations

This free-body diagram of the rod shows the tension F_T exerted on it by the cable, the weight of the sign (weight of sign = $6.0 \text{ kg} \times g = 6.0g$), and the horizontal and vertical forces exerted on the rod by the hinge, labelled F_x and F_y respectively.



Since the rod is at rest, it is in both translational and rotational equilibrium.

The condition for translational equilibrium is that the resultant force on the rod is zero. The condition for rotational equilibrium is that the resultant torque is zero.

Step 2: Write equations for the horizontal and vertical forces acting on the rod.

The resultant horizontal and vertical forces must both be zero, so:

$$F_x = F_T \cos 30^\circ$$

$$F_y + F_T \sin 30^\circ = 6.0g$$

There are three unknowns in these two equations, so they cannot be solved without further information.



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Solution steps	Calculations
<p>Step 3: Choose a point to take torques around (since the net torque is zero about any point for an object in equilibrium) — the left-hand end is convenient because both</p> <p>F_x and F_y cause no torque around this point.</p>	<p>Around the left-hand end:</p> <p>Clockwise torque = $6.0 \times 9.8 \times 0.50$ = 29.4 N m</p> <p>Anticlockwise torque = $F_T \times 0.80 \sin 30^\circ$</p> <p>(since the perpendicular distance in metres between the line of action of F_T and the left-hand end is $0.80 \sin 30^\circ$).</p> <p>These must be equal since the net torque is zero, so:</p> <p>$F_T \times 0.80 \sin 30^\circ = 29.4$</p> $F_T = \frac{29.4}{0.80 \sin 30^\circ}$ <p>= 73.5 N = 74 N (2 s.f.)</p>
<p>Step 4: Substitute the result from step 3 into the equations from step 2.</p>	$F_x = 73.5 \cos 30^\circ$ <p>= 64 N (2 s.f.)</p> $F_y + 73.5 \sin 30^\circ = 6.0 \times 9.8$ $F_y = 22 \text{ N (2 s.f.)}$

Newton's second law for rotational motion

In [subtopic A.2 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/), you learned that whenever a resultant force acts on a body, it produces a linear acceleration, which is directly proportional to the force. This is stated in Newton's second law, and its mathematical expression is:

$$F = ma$$

There is an equivalent to Newton's second law in rotational motion. It states that if a resultant torque acts on a body, the body will rotate with an angular acceleration that is directly proportional to the torque. This is shown mathematically in **Table 2**.

Table 2. Relationship between torque and angular acceleration.

Equation	Symbols	Units
$\tau = I\alpha$	τ = torque	newton metres (N m)
	I = moment of inertia	kilograms metre squared (kg m ²)
	α = angular acceleration	radians per second per second (rad s ⁻²)



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Worked example 5

A couple of 3.0 N forces acts tangentially (at 90 degrees to the radius) on a wheel that is initially at rest and has a 15 cm radius. Under the action of the two forces, the wheel starts rotating, making one complete rotation in 6.0 s. Determine the moment of inertia of the wheel.

Solution steps	Calculations
Step 1: Work out a strategy.	First, determine the net torque on the wheel produced by the couple. Then determine the wheel's angular acceleration, Finally, calculate its moment of inertia.
Step 2: Write out the values given in the question making sure to convert to SI units.	$F = 3.0 \text{ N}$ $r = 0.15 \text{ m}$ $t = 6.0 \text{ s}$ Since the wheel is initially at rest, the wheel's initial angular velocity is: $\omega_i = 0 \text{ rad s}^{-1}$
Step 3: Determine the net torque produced by the couple.	$\tau = 2Fr$ $= 2 \times 3.0 \times 0.15$ $= 0.90 \text{ N m}$
Step 4: Determine the angular acceleration of the wheel.	The angular displacement $\Delta\theta$ for one complete revolution is equal to 2π radians. $\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$ $= \frac{1}{2} \alpha t^2$ Rearranging for α : $\alpha = \frac{2\Delta\theta}{t^2}$ $= \frac{2 \times 2\pi}{6.0^2}$ $= 0.349 \text{ rad s}^{-2}$
Step 5: Determine the moment of inertia of the wheel by rearranging Newton's second law of rotational motion.	$I = \frac{\tau}{\alpha}$ $= \frac{0.90}{0.349}$ $= 2.6 \text{ kg m}^2 \text{ (2 s.f.)}$



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Solution steps	Calculations
Step 6: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.



Activity

- **IB learner profile attribute:** Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 30 minutes
- **Activity type:** Individual activity

Instructions: Complete the tasks on your own during class or at home. The tasks make sure that you can work with the new learning in this section.

Question 1

A force F acts on a body a distance r from the pivot.

- State the condition for which the torque of the force is a maximum.
- State the condition for which the torque of the force is zero.

(a) The torque of the force is a maximum if the line of action of the force and the distance r are perpendicular to each other (the angle between them is 90°).

(b) The torque is zero if the line of action of the force and the distance r are parallel. Or, in other words, if the line of action of the force passes through the pivot.

Question 2

State the condition for the rotational equilibrium of a body.

The resultant torque on the body must be zero.

Question 3

A spanner is used to turn a nut. The force applied is 22 N and the angle between the line of action of the force and the spanner's handle is 50° (**Figure 6**). The torque on the spanner is 5.0 N m. Determine the length of the spanner. Give your answer in centimetres.



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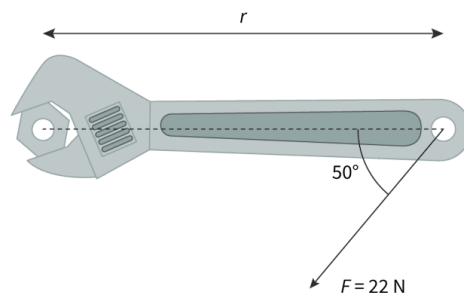


Figure 6. Spanner turning a nut.

More information for figure 6

The image is a diagram showing a spanner turning a nut. The spanner is depicted horizontally, with a dashed line extending along its length, ending at the nut, indicated at the left side. There is an arrow pointing from left to right showing the length of the spanner. Another arrow shows the direction of force applied, with the label 'Force applied 22 N' along the handle. The angle of force application, marked as 50° , is shown between the line of applied force and the spanner handle. Additionally, there are arrowheads indicating measurements related to length and force exertion on the spanner. No color information is given, focusing purely on the mechanical function depicted.

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The torque produced by the force is:

$$\tau = Fr \sin \theta$$

Rearranging this for the length r of the spanner and substituting the numbers in:

$$\begin{aligned} r &= \frac{\tau}{F \sin \theta} \\ &= \frac{5.0}{22 \times \sin(50)} \\ &= 0.30 \text{ m (2 s.f.)} \end{aligned}$$

The spanner is 30 cm long.

Question 4

A couple of 5.0 N forces acts on a wheel with a 30 cm diameter. Initially, the forces are perpendicular to the wheel's diameter. The wheel then rotates through a 20° angle (Figure 7). What is the torque produced by the couple after the 20° rotation?



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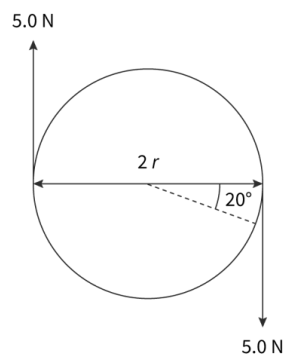


Figure 7. Turning wheel.

More information for figure 7

The diagram depicts a circle representing a wheel. There is a horizontal line passing through the center of the circle, labeled as '2 r', indicating the diameter. At two opposite ends of this diameter, arrows point outward, labeled with '5.0 N', representing forces applied perpendicularly on the wheel. Below the horizontal line, there is an angled line depicting a 20-degree rotation from the initial position of the horizontal line. This angle is marked '20°'. The diagram illustrates the change in position as the wheel rotates due to the forces applied, indicating a scenario to calculate torque after a rotation.

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The torque produced by the couple is given by:

$$\tau = 2Fr \sin \theta$$

In this case, the radius of the wheel $r = 15 \text{ cm}$.

The angle θ between F and r is 90° initially, but after the 20° rotation, it changes to:

$$\begin{aligned}\theta &= 90^\circ - 20^\circ \\ &= 70^\circ\end{aligned}$$

So, the torque produced by the couple on the wheel after it has rotated by 20° is:

$$\begin{aligned}\tau &= 2Fr \sin \theta \\ &= 2 \times 5.0 \times 0.15 \times \sin(70) \\ &= 1.4 \text{ N m (2 s.f.)}\end{aligned}$$

Question 5

A 50 cm uniform rod rests horizontally on a solid support. The mass of the rod is 300 g. The left end of the rod is pivoted and the rod is stationary. The support is 6.0 cm from the pivot (**Figure 8**). Calculate the force exerted by the support on the rod.



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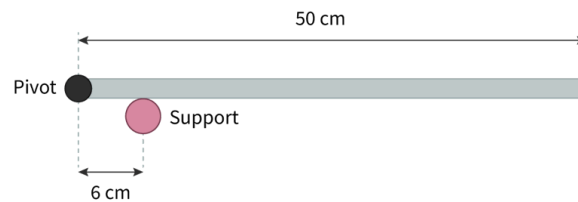


Figure 8. Rod, pivot and support.

More information for figure 8

The image depicts a horizontal rod that is 50 cm in length. On the left end of the rod, there is a circular pivot point. Six centimeters from the pivot, a support is illustrated as a smaller circle beneath the rod. The distance from the pivot to the support is marked with a double-headed arrow labeled '6 cm.' The entire length of the rod is also annotated with a double-headed arrow labeled '50 cm' at the top, indicating the rod's total length. The rod itself is shaded, showing its uniform nature. This diagram visually represents the setup described for a physics problem involving forces acting on the rod.

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Since the rod is at rest, it must be in rotational equilibrium. This means that the resultant torque about the pivot must be zero.

Let us call the force of the support S . This produces a counter-clockwise torque equal to:

$$\tau_1 = Sr_1 (r_1 = 6.0 \text{ cm} = 0.06 \text{ m})$$

The weight W of the bar causes a clockwise torque equal to:

$$\begin{aligned}\tau_2 &= Wr_2 \\ &= mgr_2\end{aligned}$$

($r_2 = 25 \text{ cm} = 0.25 \text{ m}$, since the weight acts at the centre of the bar.)

For the resultant torque to be zero, these two torques must be equal in magnitude:

$$Sr_1 = mgr_2$$

Rearranging for the force S of the support and substituting the numbers in:

$$\begin{aligned}S &= \frac{mgr_2}{r_1} \\ &= \frac{0.3 \times 9.8 \times 0.25}{0.06} \\ &= 12.25 \text{ N} \\ &= 12 \text{ N (2 s.f.)}\end{aligned}$$

Question 6



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A 20 g bar that is 12 cm in rotates horizontally about its centre with an angular acceleration of 0.33 rad s^{-2} . Calculate the torque on the bar.

A bar's moment of inertia is $I = \frac{1}{12}ml^2$.

Calculate the bar's moment of inertia:

$$\begin{aligned} I &= \frac{1}{12}ml^2 \\ &= \frac{1}{12} \times 0.02 \times 0.12^2 \\ &= 2.4 \times 10^{-5} \text{ kg m}^2 \end{aligned}$$

The torque on the bar is given by the product of the moment of inertia and the angular acceleration:

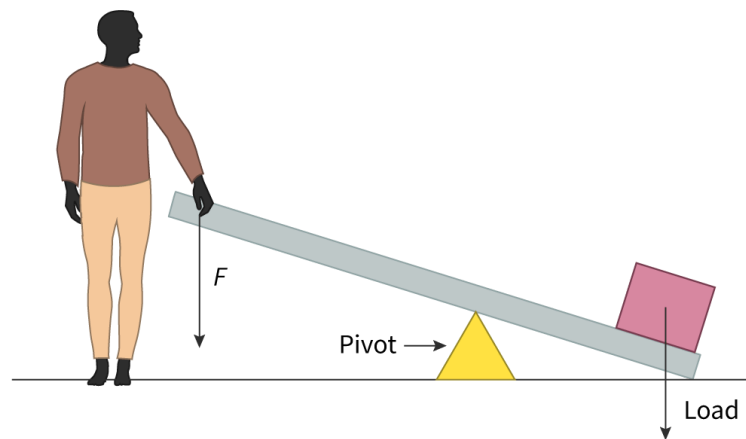
$$\begin{aligned} \tau &= I\alpha \\ &= 2.4 \times 10^{-5} \times 0.33 \\ &= 7.9 \times 10^{-6} \text{ N m (2 s.f.)} \end{aligned}$$

5 section questions ^

Question 1

HL Difficulty:

A person uses a lever to lift a heavy load, as shown in the diagram.



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Which change would allow the person to use a smaller force?

1 Move the heavy load to the left.



2 Move themselves closer to the pivot.



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3 Move the pivot to the left.

4 Move the pivot and the heavy load to the left by the same amount.

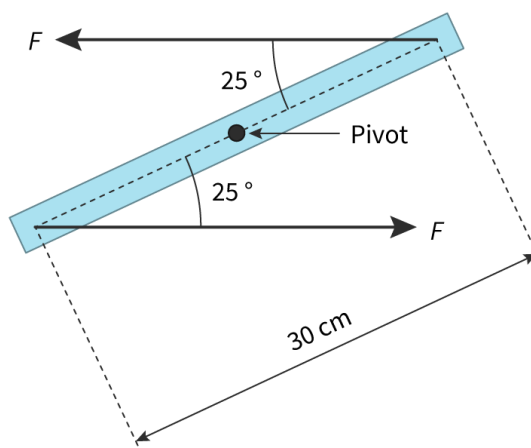
Explanation

For the person to be able to lift the heavy load with a smaller force, the torque of the heavy load must be reduced. Since torque is the product of force (the weight of the load) and perpendicular distance between the line of action of the force and the pivot (and since the weight of the load is constant), the only way of reducing the torque of the heavy load is to reduce the perpendicular distance from the pivot. This is achieved by moving the load to the left. The person can apply a reduced torque in the opposite direction in order to lift the load. Since the person is not moving further away from the pivot (this is not an option in the answers), the force applied by the person can be smaller.

Question 2

HL Difficulty:

A couple of 6 N forces acts on the two ends of a ruler as shown in the diagram.



📄 More information

The points at which the forces act are 30 cm apart. The ruler is free to rotate about the pivot, located at its centre. What is the torque generated by the couple?

1 0.8 N m

2 0.4 N m

3 0.7 N m

4 2 N m



Explanation

The torque generated by the couple is given by:



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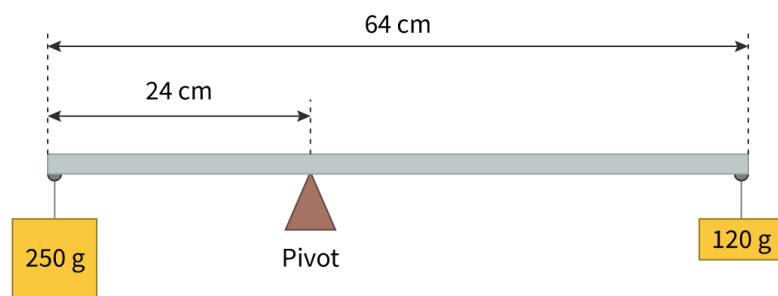
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$$\begin{aligned}\tau &= 2Fr \sin \theta \\ &= 2 \times 6 \times 0.15 \times \sin(25) \\ &= 0.76 \text{ N m (2 s.f.)} \\ &= 0.8 \text{ N m (1 s.f.)}\end{aligned}$$

Question 3

HL Difficulty:

A 64 cm uniform bar is at rest on a pyramidal support. A 250 g mass and a 120 g mass are attached to the two ends of the bar as shown in the diagram.



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Calculate the mass of the bar. Give your answer to two significant figures.

0.15 kg

**Accepted answers**

0.15 kg, 150 g

Explanation

The condition for rotational equilibrium states that the resultant torque on the bar must be zero. This means that the counter-clockwise torques must be equal to the clockwise torques.

The counter-clockwise torque is exerted by the weight of the 250 g mass:

$$W_1 = m_1 g$$

There are two clockwise torques:

- one from the weight of the 120 g mass:

$$W_2 = m_2 g$$

- one from the bar's weight:

$$W = mg$$

The condition for rotational equilibrium is:

$$W_1 r_1 = W_2 r_2 + W r$$



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Rearranging for the bar's weight W :

$$W = \frac{W_1 r_1 - W_2 r_2}{r}$$

The distances at which the forces act are:

- $r_1 = 24 \text{ cm} = 0.24 \text{ m}$
- $r_2 = 64 - 24 = 40 \text{ cm} = 0.40 \text{ m}$
- $r = \left(\frac{64}{2} - 24 \right) = 8 \text{ cm} = 0.08 \text{ m}$

So the weight of the bar is:

$$W = \frac{(0.25 \times 9.8) \times 0.24 - (0.12 \times 9.8) \times 0.40}{0.08}$$

$$= 1.5 \text{ N (2 s.f.)}$$

So the bar's mass m is:

$$m = \frac{W}{g}$$

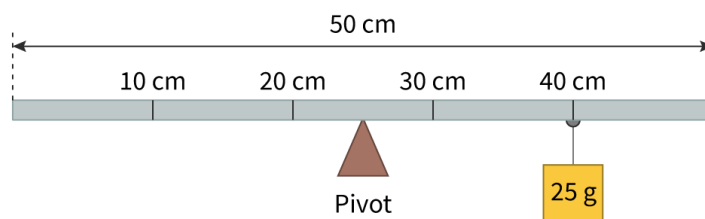
$$= \frac{1.5}{9.8}$$

$$= 0.15 \text{ kg (2 s.f.)}$$

Question 4

HL Difficulty:

A 50 cm uniform ruler is pivoted at its centre. The ruler's mass is 150 g. A 25 g mass is hung at the 40 cm mark, making the ruler rotate.



More information

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What is the ruler's angular acceleration? Give your answer to an appropriate number of significant figures.



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The moment of inertia of the ruler is $I = \frac{1}{12}MR^2$, where M is the ruler's mass and R is its length. You can assume the moment of inertia of the 25 g mass to be negligible compared to that of the ruler.

The ruler's angular acceleration is 12 rad s^{-2} . ✓

Accepted answers and explanation

#1 12

General explanation

The only torque on the ruler is caused by the weight of the 25 g mass, $W = mg$. The ruler's weight (Mg) does not produce a torque, since its line of action passes through the axis of rotation (the ruler's weight acts at the centre, and the ruler is pivoted at its centre).

The weight of the 25 g mass acts at a perpendicular distance from the pivot r equal to:

$$\begin{aligned} r &= 40 - 25 \\ &= 15 \text{ cm} \\ &= 0.15 \text{ m} \end{aligned}$$

Therefore, the resultant torque is:

$$\begin{aligned} \tau &= Wr \\ &= mgr \\ &= 0.025 \times 9.8 \times 0.15 \\ &= 0.03675 \text{ N m} \end{aligned}$$

The relationship between the resultant torque on the ruler and the ruler's angular acceleration α is:

$$\tau = I\alpha$$

Rearranging for the angular acceleration:

$$\begin{aligned} \alpha &= \frac{\tau}{I} \\ &= \frac{12\tau}{MR^2} \\ &= \frac{12 \times 0.03675}{0.15 \times 0.5^2} \\ &= 11.76 \text{ rad s}^{-2} \\ &= 12 \text{ rad s}^{-2} \text{ (2 s.f.)} \end{aligned}$$

Question 5

HL Difficulty:

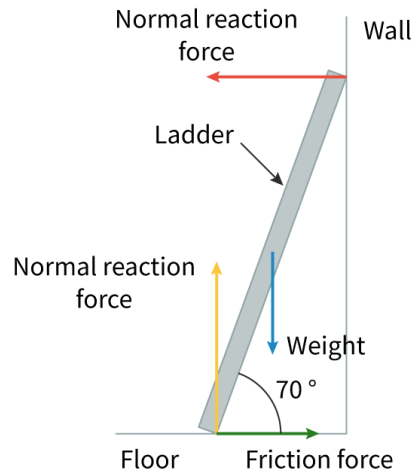
A uniform ladder rests against a frictionless wall. The bottom of the ladder makes a 70° angle with the surface of the floor. The forces acting on the ladder are shown in the diagram.



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More information

The ladder's mass is 5.0 kg and its length is 2.5 m. Calculate the magnitude of the friction from the floor. Give your answer to an appropriate number of significant figures.

8.9 N



Accepted answers

8.9 N, 8.9 newtons

Also accepted

8.9

Explanation

Since the ladder is at rest, it must be in both translational and rotational equilibrium.

The condition for translational equilibrium states that the resultant force on the ladder must be zero. This means that the magnitude of the ladder's weight must be equal to the magnitude of the normal reaction from the floor, and that the magnitude of the friction from the floor must be equal to the magnitude of the normal reaction from the wall.

To calculate the magnitude of the friction force F_f from the floor, you can use the condition for rotational equilibrium, which states that the resultant torque on the ladder must be zero.

You can take the torques about the point of contact with the ground. This way, the torques of the friction force and the normal reaction force from the floor will be equal to zero (because their lines of action pass through this point). So you will only be dealing with the counter-clockwise torque produced by the normal reaction force F_N from the wall and the clockwise torque from the ladder's weight W . These must be equal in magnitude, so:

$$Wr_W \sin(20) = F_N r_N \sin(70)$$

Rearranging for the normal reaction force and substituting the values in:

$$\begin{aligned} F_N &= \frac{Wr_W \sin(20)}{r_N \sin(70)} \\ &= \frac{(5.0 \times 9.8) \times 1.25 \times \sin(20)}{2.5 \times \sin(70)} \\ &= 8.9 \text{ N} \end{aligned}$$



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Since, according to the condition from translational equilibrium, the magnitude of the friction force from the floor is equal to the magnitude of the normal reaction force from the wall, the friction force is:



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$$F_N = 8.9 \text{ N (2 s.f.)}$$

A. Space, time and motion / A.4 Rigid body mechanics (HL)

Angular momentum (HL)

A.4.9: Extended body rotation and angular momentum (HL) A.4.10: Conservation of angular momentum (HL)

A.4.11: The action of a resultant torque constitutes an angular impulse (HL) A.4.12: The kinetic energy of rotational motion (HL)

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Define angular momentum and use its equation.
- Apply the conservation of angular momentum to solve problems.
- Explain the relationship between resultant torque and angular impulse, and between change in angular momentum and angular impulse.
- Use the equation for kinetic energy in terms of angular momentum to solve problems.

Just as ballet dancers doing fouettés extend and pull in their legs, ice skaters change the position of their arms as they spin. Why do they do it? It allows them to control their angular velocity. When an ice skater extends their arms away from their body, their moment of inertia increases, and their angular velocity decreases. When the ice skater pulls their arms back towards their body, their moment of inertia decreases, and they are able to rotate faster. This inverse relationship between the moment of inertia and the angular velocity of the rotation exists because of a fundamental law of physics known as the conservation of angular momentum.

Angular momentum

In subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/), you learned that a body of mass m moving with a linear velocity v has a linear momentum p , given by:

$$p = mv$$

Similarly, in rotational motion, the angular momentum L of a rigid body can be defined as the product of its moment of inertia and angular velocity (**Table 1**).

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Table 1. Angular momentum equation.



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Equation	Symbols	Units
$L = I\omega$	$L = \text{angular momentum}$	kilogram metres squared per second ($\text{kg m}^2 \text{s}^{-1}$)
	$I = \text{moment of inertia}$	kilogram metres squared (kg m^2)
	$\omega = \text{angular velocity}$	radians per second (rad s^{-1})

Worked example 1

Earth has a mass of $5.972 \times 10^{24} \text{ kg}$ and a radius of 6371 km. Determine the Earth's angular momentum as it rotates on its axis. You can approximate the shape of the Earth to that of a solid sphere with moment of inertia:

$$I = \frac{2}{5}mr^2$$

Solution steps	Calculations
Step 1: Work out a strategy.	First calculate the Earth's moment of inertia. Then determine the Earth's angular velocity. Finally, determine its angular momentum.
Step 2: Write out the values given in the question making sure to convert to SI units.	$m = 5.972 \times 10^{24} \text{ kg}$ $r = 6371 \text{ km}$ $= 6371 \times 10^3 \text{ m}$ $= 6.371 \times 10^6 \text{ m}$
Step 3: Calculate the Earth's moment of inertia.	$I = \frac{2}{5}mr^2$ $= \frac{2}{5} \times (5.972 \times 10^{24}) \times (6.371 \times 10^6)^2$ $= 9.696 \times 10^{37} \text{ kg m}^2$
Step 4: Determine the Earth's angular velocity.	<p>The Earth completes one rotation on its axis ($\Delta\theta = 2\pi$) in one day ($\Delta t = 1 \text{ day} = 24 \times 60 \times 60 \text{ seconds}$), therefore:</p> $\omega = \frac{\Delta\theta}{\Delta t}$ $= \frac{2\pi}{(24 \times 3600)}$ $= 7.272 \times 10^{-5} \text{ rad s}^{-1}$
Step 5: Determine the Earth's angular momentum.	$L = I\omega$ $= 9.696 \times 10^{37} \times 7.272 \times 10^{-5}$ $= 7.051 \times 10^{33} \text{ kg m}^2 \text{s}^{-1} \text{ (4 s.f.)}$



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Solution steps	Calculations
Step 6: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.

For a point mass m that is rotating with angular velocity ω at a distance r from the axis of rotation, the angular momentum is:

$$L = (mr^2)\omega$$

Worked example 2

The Earth has a mass of 5.972×10^{24} kg, and it orbits the Sun in 365 days. The average Earth–Sun distance is 148.22 million kilometres. Determine the Earth’s angular momentum when it orbits the Sun. You can assume the orbit to be perfectly circular and the Earth to be a point mass.

Solution steps	Calculations
Step 1: Work out a strategy.	First, calculate the Earth’s moment of inertia assuming it is a point mass. Then determine the Earth’s angular velocity. Finally, determine its angular momentum.
Step 2: Write out the values given in the question making sure to convert to SI units.	$m = 5.972 \times 10^{24}$ kg $r = 148.22$ million kilometres $= 1.4822 \times 10^{11}$ m
Step 3: Calculate the Earth’s moment of inertia.	$I = mr^2$ $= (5.972 \times 10^{24}) \times (1.4822 \times 10^{11})^2$ $= 1.312 \times 10^{47}$ kg m ²
Step 4: Determine the Earth’s angular velocity.	<p>The Earth completes one revolution around the Sun ($\Delta\theta = 2\pi$) in one year ($\Delta t = 365$ days), therefore:</p> $\omega = \frac{\Delta\theta}{\Delta t}$ $= \frac{2\pi}{365 \times 24 \times 3600}$ $= 1.992 \times 10^{-7} \text{ rad s}^{-1}$
Step 5: Determine the Earth’s angular momentum.	$L = I\omega$ $= 1.312 \times 10^{47} \times 1.992 \times 10^{-7}$ $= 2.614 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$ (4 s.f.)



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Solution steps	Calculations
Step 6: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.

Just like linear momentum, angular momentum is also a vector quantity.

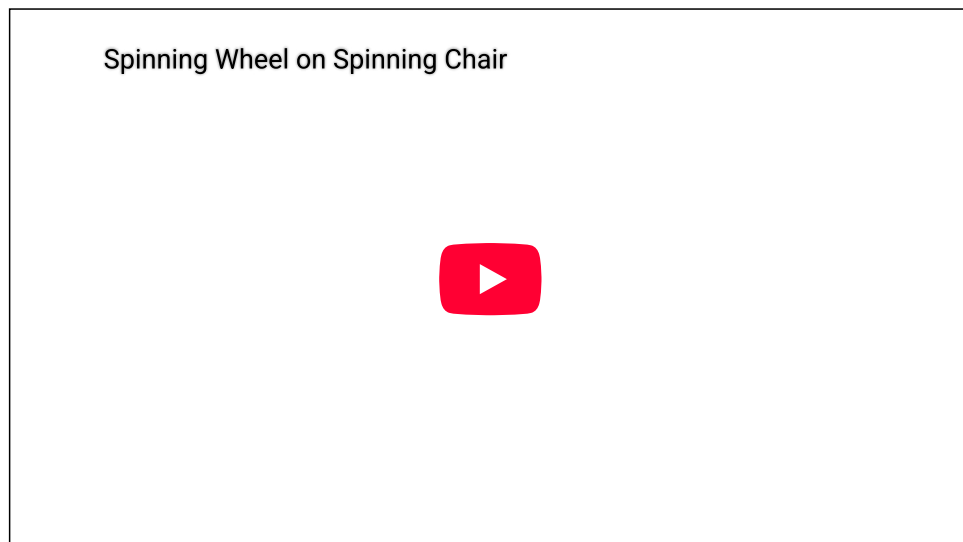
Making connections

Angular momentum is quantised. This means that, at the subatomic level, angular momentum can only assume a set of discrete values. The quantisation of angular momentum will be discussed in [subtopic E.1 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43191/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43191/).

Conservation of angular momentum

In **Video 1**, a man sits, initially stationary, on a rotating chair. Then he holds a rotating bike wheel horizontally with respect to his body. Watch the video to find out what happens.

Could you explain why this happens?



Video 1. Demonstration of angular momentum.

In linear motion, the momentum p of a body is conserved if the resultant force on the body is zero. This is known as the conservation of linear momentum.

Angular momentum is conserved in rotational motion, provided that no resultant torque acts on the rotating body.

Consider a spinning ice skater (**Figure 1**). Assume no friction between their skates and the ice, so that the resultant torque is zero, and the skater's angular momentum is conserved.



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- If the skater wants to **decrease** their angular velocity, they can spread their arms. This **increases** their moment of inertia (since the mass is distributed further away from the axis of rotation).
- If the skater wants to **increase** their angular velocity, they can move their arms closer to their body. This **decreases** their moment of inertia (since the mass is closer to the axis of rotation).

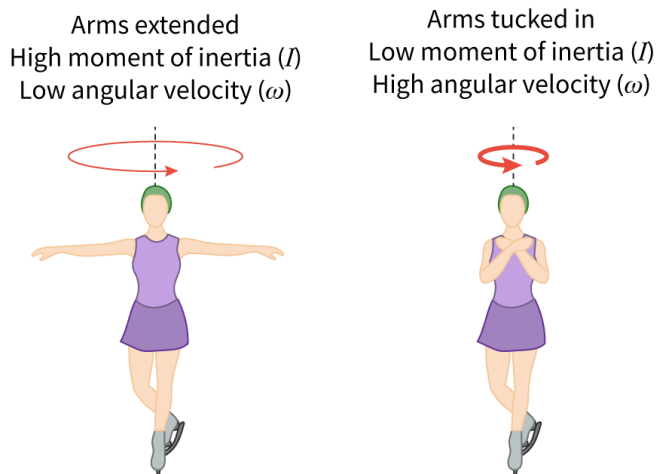


Figure 1. An ice skater using conservation of angular momentum to decrease or increase their angular velocity.

More information for figure 1

The image depicts two diagrams of an ice skater illustrating the conservation of angular momentum. The left diagram shows a skater with arms extended, labeled as having a high moment of inertia and low angular velocity. The right diagram shows the same skater with arms tucked in, labeled as having a low moment of inertia and high angular velocity. Arrows indicate the spinning direction above the skater's head, demonstrating changes in angular velocity with arm positioning.

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Angular Momentum Demo: Platform and Dumbbells



Video 2. Demonstration of the conservation of angular momentum discussed above.



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Worked example 3

An ice skater spins with his legs straight and his arms against his body. In this position his moment of inertia is 1.2 kg m^2 and he completes six complete rotations per second.

While spinning, he extends both arms. His moment of inertia is now 2.2 kg m^2 .

Calculate the time taken for the skater to make one complete rotation with his arms extended.

Solution steps	Calculations
Step 1: Determine the initial angular speed, ω_1 , of the skater using the initial time for one rotation, T_1 .	$T_1 = \frac{1}{6} \text{ s} = 0.167 \text{ s (3 s.f.)}$ $\omega_1 = \frac{2\pi}{T_1}$ $= \frac{2\pi}{0.167}$ $= 37.6 \text{ rad s}^{-1} \text{ (3 s.f.)}$
Step 2: Determine the initial angular momentum of the skater using the initial angular speed and the initial moment of inertia, I_1 .	$L = I_1 \omega_1$ $= 1.2 \times 37.6$ $= 45.1 \text{ kg m}^2 \text{ s}^{-1} \text{ (3 s.f.)}$
Step 3: Use conservation of angular momentum to calculate the final angular speed, ω_2 .	$L_1 = L_2 = L$ $\omega_2 = \frac{L}{I_2}$ $= \frac{45.1}{2.2}$ $= 20.5 \text{ rad s}^{-1} \text{ (3 s.f.)}$
Step 4: Calculate the time taken for one rotation, T_2 .	$T_2 = \frac{2\pi}{\omega_2}$ $= \frac{2\pi}{20.5}$ $= 0.31 \text{ s (2 s.f.)}$



Concept

The conservation of angular momentum is yet another conservation law in physics.

A conservation law states that, within an isolated system, a certain measurable physical quantity does not change with time.

Conservation laws enhance our understanding of the physical world, and often allow you to solve problems using a few simple equations. Many of these problems would otherwise have very complex solutions, while some may not be solvable at all.



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All conservation laws are considered to be fundamental laws of nature. Most of them are said to be absolute, since they apply to all possible physical processes. Others are partial, as they only apply to some processes.

As well as the conservation of angular momentum, you have also come across two other conservation laws: conservation of energy ([subtopic A.3 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/\)](#)) and conservation of linear momentum ([subtopic A.2 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/\)](#)). As you progress through DP physics, you will encounter many more conservation laws.

Do you know of any other conservation laws?

Angular impulse

You have learned about conservation of angular momentum. But what if angular momentum is not conserved?

Whenever a resultant torque acts on a rotating body, angular momentum is **not** conserved.

This is also true in linear motion, where the momentum p of a body is not conserved if a resultant force F acts on the body.

In this case, the rate of change of linear momentum $\frac{\Delta p}{\Delta t}$ is equal to the resultant force:

$$F = \frac{\Delta p}{\Delta t}. \text{ This is equivalent to } F = ma.$$

You should recall that the product of the force F and the time interval Δt is known as impulse ($F\Delta t$), and that this is equal to the change in momentum (Δp) of the body.

Similarly, in rotational motion, the action of a resultant torque for a given period of time produces an angular impulse ΔL (**Table 2**).

Table 2. Calculation of angular impulse.

Equation	Symbols	Units
$\Delta L = \tau \Delta t$	$\Delta L =$ angular impulse	Newton meter second (N m s)
	$\tau =$ resultant torque	newton metres (N m)
	$\Delta t =$ change in time	seconds (s)

As in linear motion, the angular impulse is equal to the change in angular momentum (**Table 3**).

Table 3. Relationship between angular impulse and angular momentum.



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Equation	Symbols	Units
$\Delta L = \Delta(I\omega)$	ΔL = angular impulse (or change in angular momentum)	kilogram metres squared per second (N m s)
	I = moment of inertia	kilogram metres squared (kg m ²)
	ω = angular velocity	radians per second (rad s ⁻¹)

Just like linear impulse, angular impulse is also a vector quantity.

Torque—time graphs

In subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/), you learned that in linear motion the change in momentum Δp (or linear impulse) is sometimes calculated as the area under the curve of a force—time graph.

In rotational motion, the angular impulse ΔL of a body (equivalent to the body’s change in angular momentum) can be calculated as the area under the curve of a torque—time graph.

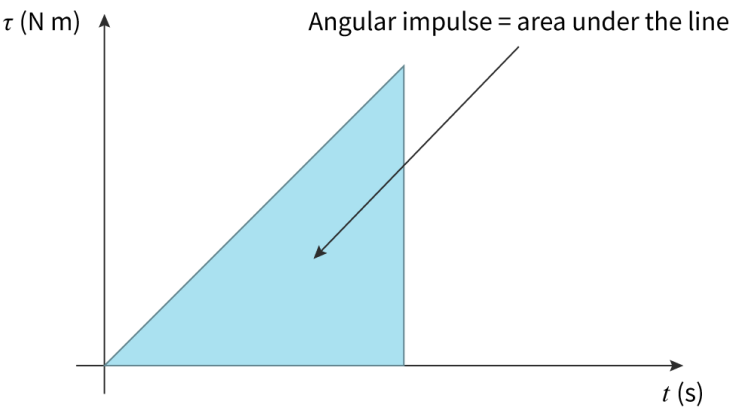


Figure 2. Angular impulse as the area under the line on a torque—time graph.

More information for figure 2

This image is a graph depicting a torque-time relationship. The x-axis represents time (t) in seconds, and the y-axis represents torque (in Newton-meters, N m). There is a line plotted at a 45-degree angle, indicating constant increase. The area under this line is shaded and labeled as "Angular impulse = area under the line," indicating that angular impulse is calculated by the shaded area beneath the line on the graph.

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Exercise 1

Click a question to answer

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Extended

Kinetic energy and angular momentum

In a previous section, you learned the equation for the kinetic energy of a body in uniformly angularly accelerated motion. For a body of moment of inertia I and angular velocity ω , this is $E_k = \frac{1}{2} I \omega^2$. Since the product $I\omega$ is equal to the body's angular momentum, the kinetic energy equation can be written as in **Table 4**.

Table 4. Kinetic energy and angular momentum.

Equation	Symbols	Units
$E_k = \frac{L^2}{2I}$	E_k = kinetic energy	joules (J)
	L = angular momentum	kilogram metres squared per second ($\text{kg m}^2 \text{s}^{-1}$)
	I = moment of inertia	kilogram metres squared (kg m^2)

Worked example 4

The Earth has a rotational kinetic energy of $2.14 \times 10^{29} \text{ J}$ as it spins on its axis. The Earth's moment of inertia is $8.04 \times 10^{37} \text{ kg m}^2$. What is the Earth's angular momentum?

Solution steps	Calculations
Step 1: Recognise that the equation for rotational kinetic energy will be required.	The equation for the rotational kinetic energy of an object is available in the DP physics data booklet.
Step 2: Write out the values given in the question making sure to convert to SI units.	$E_k = 2.14 \times 10^{29} \text{ J}$ $I = 8.04 \times 10^{37} \text{ kg m}^2$
Step 3: Write out the equation and rearrange for angular momentum.	$E_k = \frac{L^2}{2I}$ $L = \sqrt{2E_k I}$
Step 4: Substitute the values given.	$= \sqrt{2 \times 2.14 \times 10^{29} \times 8.04 \times 10^{37}}$



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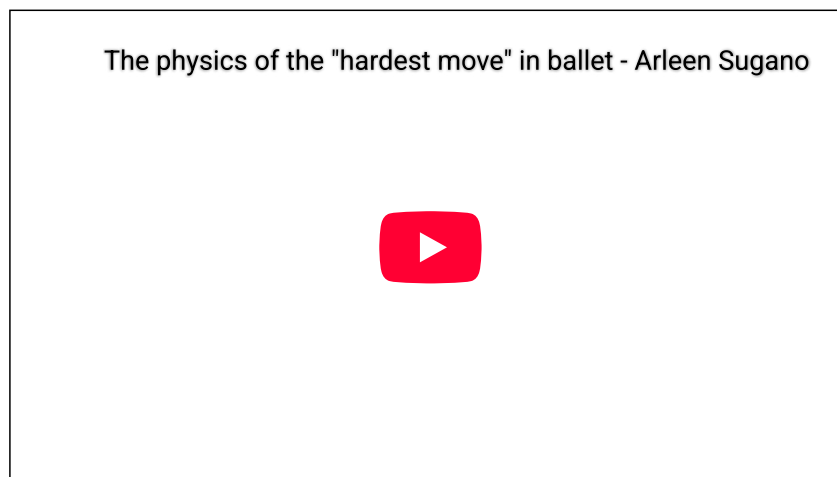
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Solution steps	Calculations
Step 5: State the answer with appropriate units and the number of significant figures used in rounding.	$= 5.87 \times 10^{33} \text{ kg m}^2 \text{ s}^{-1} \text{ (3 s.f.)}$
Step 6: Does your answer make sense?	✓ Yes. The number of significant figures is the same as the smallest number of significant figures in the question data. The units are dimensionally consistent with the equation.



Concept

In [section A.4.0 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-43153/\)](#), a spinning ballet move called the fouetté was introduced. Now that you have learned about torque, angular momentum and centre of mass, watch this video which explains the physics behind the fouetté.



Video 3. The physics behind the fouetté.



Nature of Science

Aspect: Patterns

You should have noticed the parallels between linear motion and rotational motion. The four *suvat* equations and the equation for the kinetic energy of a body moving in a straight line all have corresponding equations in rotational motion.

Equation for...	Linear motion	Rotational motion	Equation for...
Final velocity	$v = u + at$	$\omega_f = \omega_i + \alpha t$	Final angular velocity



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Equation for...	Linear motion	Rotational motion	Equation for...
Displacement	$s = \left(\frac{u+v}{2} \right) t$	$\Delta\theta = \frac{\omega_f + \omega_i}{2} t$	Angular displacement
Displacement	$s = ut + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	Angular displacement
Squared final velocity	$v^2 = u^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	Squared final angular velocity
Kinetic energy of a body in linear motion	$E_k = \frac{1}{2}mv^2$	$E_k = \frac{1}{2}I\omega^2$	Kinetic energy of a body in rotational motion
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Newton's second law of linear motion	$F = ma$	$\tau = I\alpha$	Newton's second law of rotational motion
Work done	$W = Fs \cos \theta$	$\tau = Fr \sin \theta$	Torque

Try this activity to strengthen your understanding of this section.



Activity

- **IB learner profile attribute:**
 - Inquirer
 - Thinker
 - Communicator
- **Approaches to learning:**
 - Thinking skills — Applying key ideas and facts in new contexts
 - Social skills — Working collaboratively to achieve a common goal
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

Instructions

With your partner, complete the following tasks.

Part 1

Calculate the angular momentum of Mercury, Jupiter and Neptune in the Solar System for two types of rotational motion:

- The planet's rotation on its own axis.
- The planet's orbit around the Sun.



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Before you start, discuss with your partner:

- What data do you need to research?
- What assumptions do you have to make?

Part 2

Calculate the kinetic energies of each planet as a result of the two types of rotational motion.

Part 3

Look at your results and discuss with your partner:

- Which planet has the greatest angular momentum when rotating on its own axis?
- Which planet has the smallest angular momentum when rotating on its own axis?
- Which planet has the greatest angular momentum when orbiting the Sun?
- Which planet has the smallest angular momentum when orbiting the Sun?
- Which planet has the largest kinetic energy due to its rotational motion?
- Which planet has the smallest kinetic energy due to its rotational motion?

Are there any unexpected answers?

5 section questions ^

Question 1

HL Difficulty:

Which of the given statements are true about angular impulse?

1. It is proportional to the change in angular momentum
2. It is equal to the product of the resultant torque and the time the torque is applied for
3. It is equal to the gradient of a torque-time graph
4. It acts when a body is in rotational equilibrium

1 1 and 2



2 1 only

3 2 only

4 1 and 4

Explanation

Angular impulse is equal to change in angular momentum. This comes from Newton's second law in rotational motion: whenever there is a resultant torque τ on a body with moment of inertia I , the body accelerates with an angular acceleration α , so that $\tau = I\alpha$. Angular acceleration is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$



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$$\begin{aligned}\tau &= I \frac{\Delta\omega}{\Delta t} \\ &= \frac{\Delta(I\omega)}{\Delta t} \\ &= \frac{\Delta L}{\Delta t}\end{aligned}$$

When torque is constant we can also express Angular Impulse as:

$$\text{Angular Impulse} = \tau \Delta t$$

Thus both 1 and 2 are correct

Question 2

HL Difficulty:

A small solid cylinder of mass 15 g and radius 0.50 cm, rotates about its axis with angular velocity of 3.14 rad s^{-1} . What is its angular momentum?

The moment of inertia of a solid cylinder is:

$$I = \frac{1}{2}mr^2$$

1 $5.9 \times 10^{-7} \text{ kg m}^2 \text{ s}^{-1}$



2 $1.2 \times 10^{-7} \text{ kg m}^2 \text{ s}^{-1}$

3 $1.9 \times 10^{-7} \text{ kg m}^2 \text{ s}^{-1}$

4 $1.2 \times 10^{-4} \text{ kg m}^2 \text{ s}^{-1}$

Explanation

The cylinder's moment of inertia is:

$$\begin{aligned}I &= \frac{1}{2}mr^2 \\ &= \frac{1}{2} \times (15 \times 10^{-3}) \times (0.50 \times 10^{-2})^2 \\ &= 1.875 \times 10^{-7} \text{ kg m}^2\end{aligned}$$

The cylinder's angular momentum is:

$$\begin{aligned}L &= I\omega \\ &= 1.875 \times 10^{-7} \times 3.14 \\ &= 5.9 \times 10^{-7} \text{ kg m}^2 \text{ s}^{-1} \text{ (2 s.f.)}\end{aligned}$$



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Question 3

HL Difficulty:



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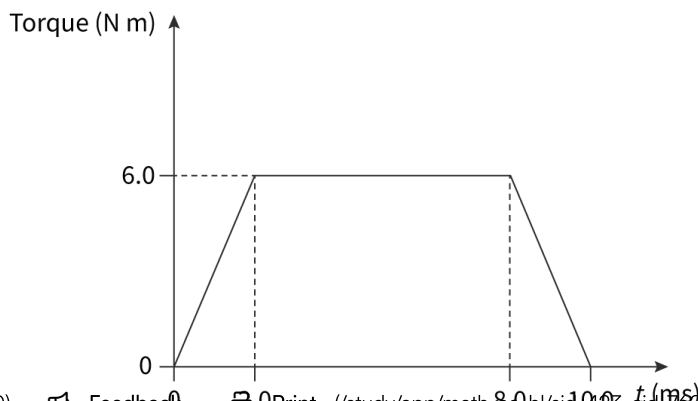
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This is a torque—time graph for a rotating object.



Section

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Feedback



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Assign

More information

What is the object's change in angular momentum? Give your answer to an appropriate number of significant figures.

The change in angular momentum is 1 4.8 ✓ $\times 10^{-2} \text{ kg m}^2 \text{ s}^{-1}$.

Accepted answers and explanation

#1 4.8

General explanation

The object's change in angular momentum ΔL is equal to the impulse $\tau \Delta t$, and can be calculated as the area under the line on the torque—time graph. This is given by the area of the two triangles plus that of the central rectangle:

$$\begin{aligned}\Delta L &= \text{area of triangle} \times 2 + \text{area of rectangle} \\ &= (\tfrac{1}{2} \times 2.0 \times 10^{-3} \times 6.0 \times 2) + (6.0 \times 10^{-3} \times 6.0) \\ &= 4.8 \times 10^{-2} \text{ kg m}^2 \text{ s}^{-1} \text{ (2 s.f.)}\end{aligned}$$

Question 4

HL Difficulty:

Body A has moment of inertia I and angular momentum L . Body B has double the moment of inertia and half the angular momentum. What is the ratio of kinetic energy of body A to kinetic energy of body B?

1 8



2 4

3 1/4

4 1/8



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Explanation

The kinetic energy of body A is:

$$E_{kA} = \frac{L^2}{2I}$$

Since body B has double the moment of inertia ($2I$) and half the angular momentum ($\frac{L}{2}$), its kinetic energy is:

$$\begin{aligned} E_{kB} &= \frac{L^2}{2 \times 4 \times 2I} \\ &= \frac{L^2}{2 \times 8I} \\ &= \frac{E_{kA}}{8} \end{aligned}$$

So the ratio of kinetic energy of body A to kinetic energy of body B is:

$$\frac{E_{kA}}{\frac{E_{kB}}{8}} = 8$$

Question 5

HL Difficulty:

A solid disc of mass 32 g and radius 2.2 cm is rotating about its centre at a constant angular velocity of 1.3 rad s^{-1} . A small 4.0 g mass is dropped onto the disc, landing 1.8 cm from the centre. What is the new angular velocity of the disc? Give your answer to an appropriate number of significant figures.

The moment of inertia of a solid disc of mass M and radius R is:

$$I = \frac{1}{2}MR^2$$

The new angular velocity is ☒ rad s^{-1} .

Accepted answers and explanation

#1 1.1

1,2

General explanation

Angular momentum $L = I\omega$ is conserved.

The final angular momentum of the disc (after the small mass is dropped) is equal to the initial angular momentum.

The disc's moment of inertia is:

$$\begin{aligned} I &= \frac{1}{2}MR^2 \\ &= \frac{1}{2} \times (32 \times 10^{-3}) \times (2.2 \times 10^{-2})^2 \\ &= 7.744 \times 10^{-6} \text{ kg m}^2 \end{aligned}$$

The initial angular momentum of the disc is:

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$$\begin{aligned} L_{\text{initial}} &= I\omega \\ &= 7.744 \times 10^{-6} \times 1.3 \\ &= 1.007 \times 10^{-5} \text{ kg m}^2 \text{ s}^{-1} \end{aligned}$$

This is equal to the final angular momentum.

The moment of inertia after the mass is dropped increases by:

$$\begin{aligned} mr^2 &= (4.0 \times 10^{-3}) \times (1.8 \times 10^{-2})^2 \\ &= 1.296 \times 10^{-6} \text{ kg m}^2 \end{aligned}$$

So the final angular momentum is:

$$L_{\text{final}} = (I + 1.296 \times 10^{-6}) \times \omega_{\text{new}}$$

Equating this to the initial angular momentum, and rearranging for the new angular velocity ω_{new} :

$$\begin{aligned} \omega_{\text{new}} &= \frac{1.007 \times 10^{-5}}{7.744 \times 10^{-6} + 1.296 \times 10^{-6}} \\ &= 1.1 \text{ rad s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

A lower new angular velocity makes physical sense. For the angular momentum to be conserved, if the moment of inertia increases, the angular velocity must decrease.

A. Space, time and motion / A.4 Rigid body mechanics (HL)

Summary and key terms (HL)

Section

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Feedback



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Higher level (HL)

- Rotating bodies have a moment of inertia, which is a measure of their ability to resist changes in their rotational motion. The further away the mass is from the axis of rotation, the larger the moment of inertia, and the slower the speed of rotation.
- A body is said to be in uniformly angularly accelerated motion when its angular acceleration is constant (and non-zero). Angular acceleration is defined as the rate of change of angular velocity with time. Uniformly angularly accelerated motion is characterised by four variables (angular displacement, angular velocity, angular acceleration and time) and by four equations (equivalent to the *suvat* equations in linear motion).
- A body is said to be in rotational equilibrium if no resultant torque acts on it. The torque of a force quantifies the force's turning effect. A net torque can also be the result of the combined action of a couple of equal and opposite forces. If a resultant torque acts on a body, the body rotates with angular acceleration directly proportional to the applied net torque.
- A rotating body has angular momentum. This is conserved if no resultant torque acts on the body. If there is a resultant torque, there is a corresponding angular impulse on the body, equal to the body's change in angular momentum. Angular impulse can be calculated as the area under the line on a torque—time graph.
- The kinetic energy of a body in rotational motion can be expressed in terms of the body's angular velocity or angular momentum. If a body rolls along a surface, its motion is both translational and rotational, therefore its overall kinetic energy is equal to the sum of its translational and rotational kinetic energies.



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Key terms

Review these key terms. Do you know them all? Fill in as many gaps as you can using the terms in this list.

1. The _____ of a body is a measure of the body's ability to resist changes in its rotational motion.
2. A body is said to be in uniform circular motion when its _____ is constant (and non-zero).
3. A body is said to be in uniform angularly accelerated motion when its _____ is constant (and non-zero).
4. If a body rolls along a surface, its kinetic energy is equal to the sum of its _____ and rotational kinetic energies.
5. _____ quantifies the turning effect of a force.
6. Rotation often results from the action of a pair of equal and opposite forces acting as a _____.
7. A body is in translational equilibrium if the resultant _____ is zero.
8. A body is in _____ equilibrium if the resultant torque on it is zero.
9. A body's angular acceleration is directly proportional to the _____ on the body.
10. _____ is conserved if the resultant torque is zero.
11. The _____ on a body is equal to the body's change in angular momentum, resulting from the action of a resultant torque.

couple

rotational

moment of inertia

angular velocity

angular impulse

force

translational

resultant torque

angular acceleration

Torque

Angular momentum



Check

Interactive 1. Key Concepts in Rotational Motion and Dynamics.



A. Space, time and motion / A.4 Rigid body mechanics (HL)

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Checklist (HL)



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Assign

Higher level (HL)**What you should know**

After studying this subtopic you should be able to:

- Define moment of inertia.
- Use the equation for the moment of inertia to solve problems.
- Determine the similarities between the physical quantities of rotational motion and linear motion.
- Define angular acceleration.
- Use the equations of uniformly angularly accelerated motion to solve problems.
- Use the equation for kinetic energy in terms of angular velocity to solve problems.
- Define torques and couples.
- Use the equation for torques to solve problems.
- Define rotational equilibrium.
- Apply Newton's second law for angular momentum to solve problems.
- Define angular momentum and use its equation.
- Apply the conservation of angular momentum to solve problems.
- Explain the relationship between resultant torque and angular impulse, and between change in angular momentum and angular impulse.
- Use the equation for kinetic energy in terms of angular momentum to solve problems.

A. Space, time and motion / A.4 Rigid body mechanics (HL)

Investigation (HL)**Section**

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Assign

Higher level (HL)

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:**
 - Research skills — Comparing, contrasting and validating information
 - Thinking skills — Asking questions and framing hypotheses based upon sensible scientific rationale
 - Self-management skills — Breaking down major tasks into a sequence of steps



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- **Time to complete activity:** 45 minutes
- **Activity type:** Pair activity

Your task

Imagine you have an inclined ramp and objects of different shapes (for example, solid sphere, hollow sphere, solid cylinder, hollow cylinder, hoop).

1. Design an investigation to answer the following research question: How does the shape of an object affect the object's final speed at the bottom of the ramp?

Your investigation should include:

- independent variable and how it will be changed
 - dependent variable and how it will be measured
 - control variables and why and how they are controlled (including any difficulties in controlling them)
 - list of apparatus (ramp, objects and other apparatus)
 - diagram of experimental set-up
 - step-by-step method
 - table of results with correct headings
 - graph with labelled axes (physical quantities and units of measure).
2. Now come up with a hypothesis and back it up with your knowledge of rotational motion from this section.
 - What variables do you need to know in order to calculate each object's linear speed at the bottom of the ramp?
 - What equations and laws should you use?
 3. Finally, discuss the following linking question: How are the laws of conservation and equations of motion in the context of rotational motion analogous to those governing linear motion?

Making connections

You may wish to revise [subtopic A.1 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/), [subtopic A.2 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/) and [subtopic A.3 \(/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/\)](/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/), where the laws of conservation and equations of motion are discussed.

Extension

- **IB learner profile attribute:**
 - Thinker
 - Inquirer
- **Approaches to learning:** Thinking skills — Being curious about the natural world
- **Time to complete activity:** 20 minutes



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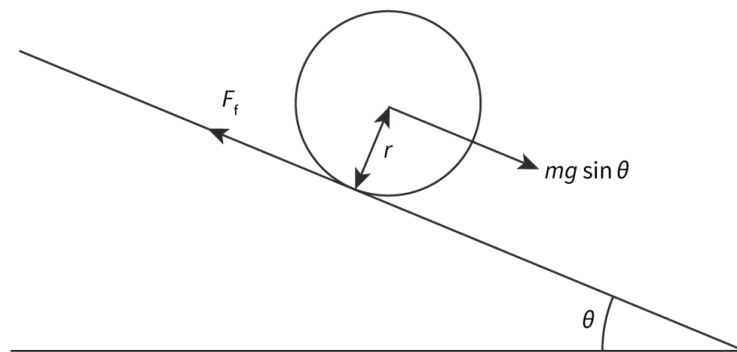
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• **Activity type:** Individual activity

1. The moment of inertia of a solid cylinder with base radius r and mass m is $\frac{1}{2}mr^2$. Show that the acceleration of a cylinder rolling (without slipping) down a slope is independent of its mass.

The forces acting on the cylinder parallel to the slope are a component of its weight, acting at the centre of the cylinder, and friction acting where the cylinder is in contact with the slope. One of these forces produces a torque and the other does not.

The diagram shows the forces acting on the cylinder parallel to the slope. These are the forces that affect the cylinder's acceleration down the slope.



The resultant force acting on the cylinder parallel to the slope is $mg \sin \theta - F_f$ where θ is the angle of the slope and F_f is the friction acting on the cylinder. So according to Newton's second law:

$$mg \sin \theta - F_f = ma$$

The resultant torque acting about the centre of the cylinder is simply the torque caused by friction, $F_f r$. So according to the rotational equivalent of Newton's second law:

$$F_f r = I\alpha$$

Now consider the relationship between the linear acceleration a and the angular acceleration α of the rolling cylinder. The cylinder rolls without slipping, so each time it makes one complete rotation, it travels a distance equal to its own circumference. The relationship between linear speed and angular speed is the same as for circular motion: $v = \omega r$. Dividing both sides of this equation by a time interval gives $a = r\alpha$, or

$$\alpha = \frac{a}{r}$$

Substituting this equation into $F_f r = I\alpha$ gives:

$$F_f r = \frac{Ia}{r} \text{ or } F_f = \frac{Ia}{r^2}$$

and substituting this into $mg \sin \theta - F_f = ma$ gives:



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$$mg \sin \theta - \frac{Ia}{r^2} = ma$$

Rearranging to make a the subject:

$$a = \frac{g \sin \theta}{1 + \frac{I}{mr^2}}$$

For a solid cylinder, the moment of inertia is $I = \frac{1}{2}mr^2$. Substituting this into the equation above gives:

$$a = \frac{g \sin \theta}{1 + \frac{\frac{1}{2}mr^2}{mr^2}} = \frac{2g \sin \theta}{3}$$

which is independent of the mass m . Therefore the acceleration of a cylinder rolling down a slope does not depend on its mass (or its radius).

1. Show that, for an object rolling (without slipping) down a slope, $v_{\text{final}} = \sqrt{\frac{2gh}{1+k}}$ where h is the change in height during the motion and k is a constant that depends on the shape of the object.

For motion with constant acceleration:

$$v_{\text{final}}^2 = u^2 + 2as$$

where s is the distance travelled (along the slope). If the initial velocity u is zero, this simplifies to:

$$v_{\text{final}} = \sqrt{2as}$$

We have already seen in the solution to Extension 1 that the acceleration of an object that rolls down a slope without slipping is given by:

$$a = \frac{g \sin \theta}{1 + \frac{I}{mr^2}}$$

The distance travelled along the slope is related to the vertical height h fallen:

$$S = \frac{h}{\sin \theta}$$

Substituting the expressions for a and s into the first equation gives:

$$v_{\text{final}} = \sqrt{2 \times \frac{g \sin \theta}{1 + \frac{I}{mr^2}} \times \frac{h}{\sin \theta}}$$

and simplifying gives:

$$v_{\text{final}} = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}$$

Comparing with the equation given in the question, $v_{\text{final}} = \sqrt{\frac{2gh}{1+k}}$, the constant k equals $\frac{I}{mr^2}$, which depends on the shape of the object.



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1. Try the following activity at home. Fill two bottles of the same shape and material with water. Place one of the bottles in the freezer. When the water is completely frozen, take the bottle out. Let the two bottles roll down an inclined ramp, making sure they start from the same point. Which bottle will roll faster? Why? Discuss your answers within your group.

Creativity, activity, service

Strand: Service

Learning outcome: Demonstrate engagement with issues of global significance

Consider the following linking question from the DP physics guide: How can rotation lead to generation of an electric current?

You could set up a CAS project to research the ways in which rotation could be used to generate an electric current. This would allow for the production of 'clean' electrical energy, with no emissions of pollutant gases.

Activities could include:

- liaising with universities to learn more about the topic
- liaising with external organisations to get the necessary funds and materials
- setting up after-school sessions to explore the physics behind the project and then construct a prototype device.

A. Space, time and motion / A.4 Rigid body mechanics (HL)

Reflection (HL)

Section

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Feedback



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Assign



Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.



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HL Extension



Reflection

Now that you've completed this subtopic, let's come back to the guiding questions introduced in The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-43153/).

- How can the understanding of linear motion be applied to rotational motion?
- How is the understanding of the torques acting on a system used to predict changes in rotational motion?
- How does the distribution of mass within a body affect its rotational motion?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

- What main points have you learned from this subtopic?
- Is anything unclear? What questions do you still have?
- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

⚠ Once you submit your response, you won't be able to edit it.

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Submit

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Help us improve the content and user experience.



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