Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Table of contents

Notebook

Glossary

Reading assistance

0 (https://intercom.help/kognity)

TOPIC A
SPACE, TIME AND MOTION

SUBTOPIC A.5
GALILEAN AND SPECIAL RELATIVITY (HL)

Student view

Show all topics

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Teacher view

Index

A. Space, time and motion / A.5 Galilean and special relativity (HL)

The big picture (HL)

Higher level (HL)

? Guiding question(s)

- How do observers in different reference frames describe events in terms of space and time?
- How does special relativity change our understanding of motion compared to Galilean relativity?
- How are space—time diagrams used to represent relativistic motion?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

Light travels at an incredibly high speed – 300 000 000 metres per second. The fastest any human made object has ever travelled is, according to most estimates, NASA's Parker Solar Probe. The fastest speed this probe has reached is around 150 000 metres per second. Travelling this fast, it would take you about 4.5 minutes to travel all the way around the Earth. This is fast, but very slow compared to light.

Student view

What would you see if you could travel at the speed of light? At the age of sixteen, Albert Einstein imagined chasing after a beam of light, and tried to imagine what he would see. Would the beam seem to be frozen in space? How fast would it seem to be

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

travelling?

Science fiction is full of light-speed travel and time-warps, but is it possible to travel at the speed of light? And what would the light look like if you did (**Figure 1**)?

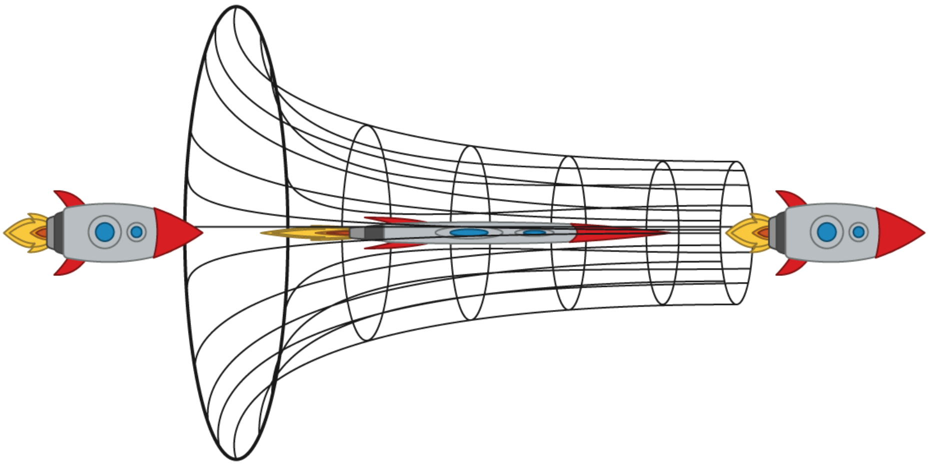


Figure 1. An artist's interpretation of 'warp speed' or 'light-speed' travel.

Creativity, activity, service

Strand: Creativity
Learning outcome: Demonstrate that challenges have been undertaken, developing new skills in the process

Writers, artists and movie makers have been imagining, visualising and imaging the most distant frontiers of science and technology for years.

What do you think near-light-speed travel would look like? What would you see if you were on board a spaceship travelling close to the speed of light?

Try creating a painting, a sketch, a digital render, an animation, a model or even a game that explores this idea.

Prior learning

Before you study this subtopic make sure that you understand the following:

- The fundamentals of kinematics (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)).

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

A. Space, time and motion / A.5 Galilean and special relativity (HL)

Reference frames and Galilean relativity (HL)

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Outline reference frames.
- Explain Galilean relativity and use the Galilean transformation equations:

$$x' = x - vt \text{ and } t' = t$$

- Know and use the velocity addition equation:

$$u' = u - v$$

Imagine you are sitting on a train, which is moving at 20 m s^{-1} through a station. You look out of the window and see someone standing on the platform. From your perspective, you are not moving and they are moving at 20 m s^{-1}. From their perspective, you are moving at 20 m s^{-1}. Who is right? How can you tell?

Figure 1 shows the motion from your perspective, sitting on the train, and from the perspective of the person watching the train from the platform.

Student view

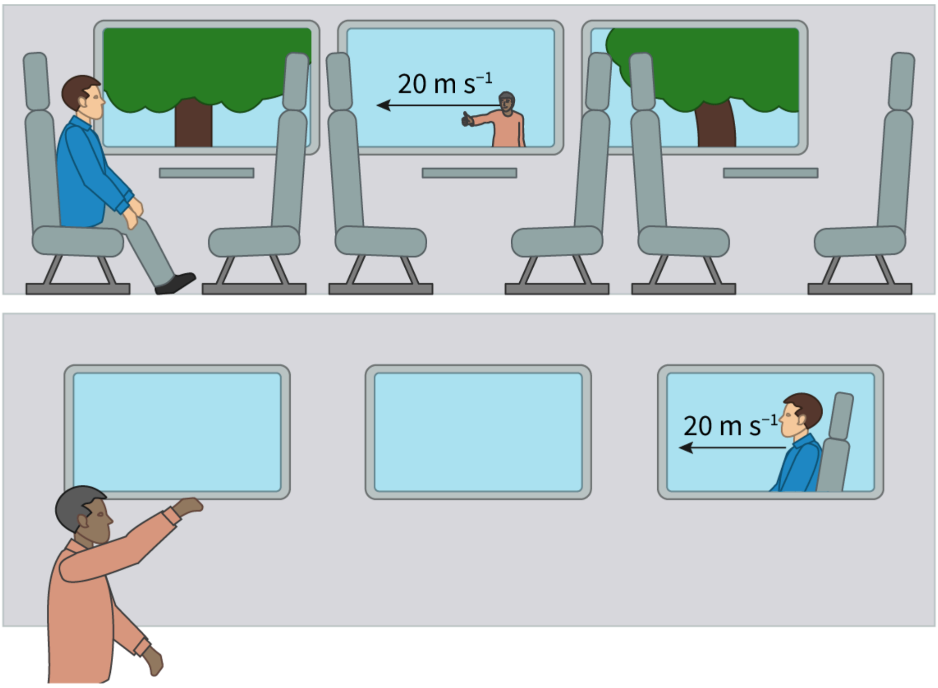


Figure 1. Are you moving or is the person on the platform moving?

More information for figure 1

The image consists of two parts. The top part shows an interior view of a train. Inside the train, a person is sitting facing forward, with trees visible through the windows. A figure standing outside the train on the platform is pointing towards the train. An arrow labeled '20 m s^-1' points left, indicating the perceived motion of the train from the inside perspective.

The bottom part of the image illustrates the exterior view of the train from the platform. Here, a person is standing on the platform, and through the train windows, another person inside the train is seen facing forward. An arrow labeled '20 m s^-1' points left again, representing the motion from the platform perspective. The diagram compares perspectives of motion from a person sitting inside a moving train and a person standing outside the train on the platform.

[Generated by AI]

Reference frames

Look at the skydiver in **Video 1** and describe the motion of the person after they pull the cord of the parachute.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Video 1. What Is the Motion of the Skydiver?

More information for video 1

The animation video features a skydiver in free fall, captured from a perspective that allows viewers to observe their motion relative to the ground. The skydiver is descending rapidly toward the earth when they pull the parachute cord. At this moment, their motion changes significantly.
From the perspective of a camera filming the skydiver, the parachute deployment causes them to accelerate upwards, giving the appearance of sudden upward motion. However, from an earth-based reference frame, the skydiver is still falling but experiences a rapid decrease in speed, or deceleration, due to the parachute's increasing air resistance. This results in a much slower descent.
From the skydivers' own perspective, the earth appears to be moving up toward them quickly. Once the parachute opens, the relative motion of the earth slows down as its falling speed decreases.
The animation visually demonstrates the concept of reference frames, showing how the same motion can appear differently depending on the observer's position.

Your answer will depend on which perspective you are looking at the situation from. These different perspectives are called reference frames. A reference frame is a coordinate system that we use to describe the motion of a body in space and time. There are three obvious reference frames we could use to describe the motion of the skydiver, and these are described in **Table 1**.

Table 1. Possible reference frames for the motion of a skydiver.

Reference frame	Motion
The camera filming the skydiver	After the skydiver pulls the cord, they accelerate **upwards** and this causes them to move **upwards**.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/o

Reference frame	Motion
The Earth	After the skydiver pulls the cord, they **decelerate** rapidly. They are still falling downwards but at a **lower speed**.
The skydiver	The Earth is moving up towards them very quickly, and then when they pull the cord, the Earth slows down.

When talking about relativity, we are only going to consider inertial reference frames. An inertial reference frame is one that is not accelerating (or decelerating). So we are not going to consider accelerating reference frames, such as the reference frame of the skydiver in **Table 1**.

Concept

An inertial reference frame is a way of looking at a situation from a non-accelerating perspective.

Imagine you are driving a car along a road at $10\,\text{m}\,\text{s}^{-1}$ and there is another car driving towards you at $10\,\text{m}\,\text{s}^{-1}$.

- In your reference frame, the other car is coming towards you at $20\,\text{m}\,\text{s}^{-1}$.
- In the reference frame of the other car, you are moving towards them at $20\,\text{m}\,\text{s}^{-1}$.
- In the reference frame of a person standing on the side of the road, you and the other car are moving in opposite directions at $10\,\text{m}\,\text{s}^{-1}$.

Each of these inertial reference frames is valid — there is not one that is 'absolutely' true.

Galilean relativity

Imagine you are sitting at a table in a train which is waiting at a station (so it is in an inertial reference frame). The windows are covered by blinds so you cannot see outside. If you place a ball on the table, it will not move as there is no resultant external force acting on it. The ball obeys Newton's first law of motion (see subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/)).

If the train is moving at a steady speed in a straight line (another inertial reference frame), and you put the ball on the table, the ball still obeys Newton's first law of motion and does not move.

The ball behaves in the same way each time, so how can you detect whether the train is stationary or moving at constant speed?

Student view

Galileo Galilei (1564–1642) was an Italian physicist who devised a thought experiment. He imagined a ship sailing on completely calm waters at a constant speed. He asked the question: ‘Could a sailor below deck with no windows tell if the ship was moving?’. What do you think? Watch **Video 2** for some clues.

Video 2. Can someone tell if they are moving?

Galileo’s response was that there was no experiment that the sailor could do that would be able to identify if the ship was moving.

We can conclude that:

Newton’s laws of motion are the same in all inertial reference frames.

Thinking back to the train, if it were accelerating (a non-inertial reference frame), the ball on the table would move without any external force acting on it. It would not obey Newton’s first law of motion.

The idea that Newton’s laws of motion are **invariant** (remain unchanged) in all inertial reference frames is known as **Galilean relativity**.

Nature of Science

Aspect: Theories

In addition to his work on relativity, Galileo also promoted the idea that the Earth orbited the Sun, not the other way around. At the time, this was an extremely rebellious idea and the Catholic Church found Galileo ‘vehemently

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

suspect of heresy' and imprisoned him in his home for the last ten years of his life as punishment. Sometimes, scientific ideas that turn out to be correct take a long time to be accepted.

Galilean transformation equations

A Galilean transformation is a way to describe the position or velocity of a moving body relative to a stationary body. Galilean transformations give correct results if the relative velocity is much less than the speed of light.

Imagine that two people at a train station synchronise their watches, and then one person gets on a train while the other person stays behind. During the train journey, the traveller sees a shooting star in the sky. The other person also sees it. If their watches are working correctly, you expect the two observers to agree on the time at which the shooting star occurred.

In Galilean relativity, there exists a definition of time that is the same regardless of the reference frame. This is known as **absolute time**. Mathematically, this is shown as the equation in **Table 2**.

Table 2. Equation for the time of an event.

Equation	Symbols	Units
$t' = t$	t' = time of event in second reference frame	seconds (s)
	t = time of event in first reference frame	seconds (s)

Using the idea of absolute time, it is possible to define the position of an object or event with respect to a particular reference frame. This is known as the relative position.

The concept of absolute time is valid as long as the relative speed of the reference frames is much less than the speed of light.

Work through the activity to help you visualise motion from different reference frames.

Activity

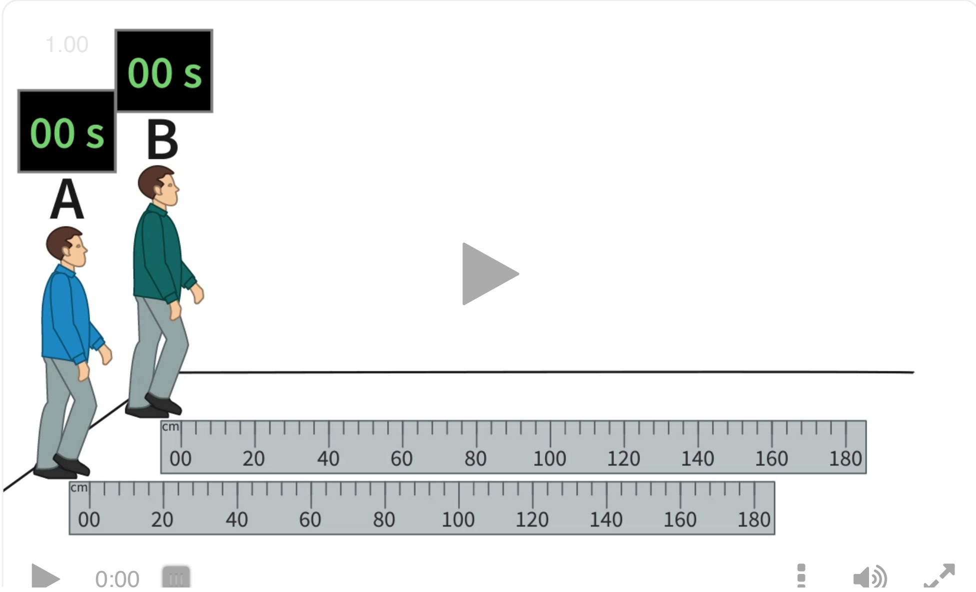
- **IB learner profile attribute:**
  - Open-minded
  - Inquirer

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

- **Approaches to learning:** Thinking skills — Asking questions and framing hypotheses based upon sensible scientific rationale
- **Time required to complete activity:** 20 minutes
- **Activity type:** Pair activity

Look at **Interactive 1.** It shows two people who are moving relative to each other. At a particular time, a ball is dropped. The event that you are going to describe is the ball hitting the floor.



Interactive 1. A Ball Hitting a Floor.

More information for interactive 1

A video depicts two individuals, labeled A and B, who are moving relative to each other. At a specific moment, a ball is dropped, and the focus is on observing its motion until it hits the floor.
There are two horizontal measuring scales marked in centimeters at the bottom. The bottom scale is fixed and ranges from 0 cm to 180 cm. It is used to measure the motion of Observer A. Another scale is placed above the bottom scale starting from the value of 20 on the bottom scale. This scale also ranges from 0 cm to 180 cm and is used to measure the motion of Observer B. Both scales increase from left to right. Two digital timers are shown, one above Observer A and the other above Observer B.
At 00 seconds (s), both Observer A and Observer B are standing next to each other at the left end of their respective scales. Observer A is aligned with the 0 cm mark on the lower scale, and Observer B is aligned with the 0 cm mark on the upper scale. Two digital timers are shown above each observer, both displaying “00 s”.
At 1 second, Observer A stands at 0 cm and Observer B has moved 56 cm. These readings are obtained from the lower and upper scales, respectively.
At 2 seconds, Observer B has walked some distance to the right, reaching a distance past 80 cm relative to the lower scale, while Observer A remains at the 0 cm mark on the lower scale. The red ball in mid-air has started falling vertically. The timers above A and B both display “02 s”.
At 3 seconds, the Observe A remains at 0 cm. On the other hand, Observer B has moved a distance past 104 cm relative to the lower scale.

Student
view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

At 4 seconds, Observer B has moved further right, reaching past 112 cm on the upper scale, while Observer A is still at the 0 cm mark. The red ball has now reached the ground at the 60 cm and 180 cm mark on the upper and lower scale, respectively, showing it has fallen vertically. The timers above both observers now read “04 s”.
A play button along with a slider at the bottom allows the users to view the video with convenience. The “Playback Rate” icon next to the slider allows the video to be played at different rates including 0.25, 0.5, 1, 1.25, 1.5, and 2. At the bottom right is the “Fullscreen” icon that enables the screen to view in full size.

1. Allow the ball to drop and watch what happens.
2. Determine:

   (a) the velocity of B as measured by A
   (b) the velocity of A as measured by B
   (c) the position of the ball when it hits the floor as measured by A
   (d) the position of the ball when it hits the floor as measured by B
   (e) the time at which the ball hits the floor as measured by A
   (f) the time at which the ball hits the floor as measured by B

3. Describe:

   (a) the path of the ball as it falls as seen by A
   (b) the path of the ball as it falls as seen by B

4. Discuss:

   (a) are either of these reference frames wrong?
   (b) what would be the difference in motion, if any, if you changed the speed of B?
   (c) what would be the difference in motion, if any, if A was moving in:

   (i) the same direction?
   (ii) the opposite direction?

In the activity, you have seen that the event (a ball falling) is different depending on the motion of the observer. The reference frames of person A and person B are different and so they observe the event differently.

Think back to Galileo’s thought experiment about ships. Imagine you are on a small boat floating on a lake in the middle of the night. There is no way for you to tell if you are moving or stationary.

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Another small boat moves past you in the darkness (**Figure 2**). In your reference frame, the other boat is travelling at a constant speed, v, and the person on it is directly opposite you at time $t = 0$.

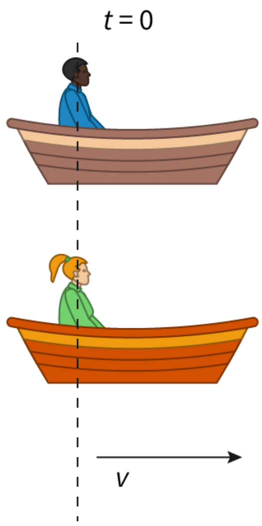


Figure 2. A boat moving past a stationary boat.

More information for figure 2

The image is a diagram illustrating the scenario where two boats are shown in a vertical arrangement. At the top, there is an individual in a stationary boat. Below, another person is in a different boat, which is moving according to an arrow labeled "v" pointing to the right. This represents the velocity of the moving boat. Both boats are aligned such that the individuals are directly opposite each other at time "t=0," which is marked near the top of the image. The setup likely visualizes a physics concept involving relative motion.

[Generated by AI]

At a time t, the person on the other boat turns on a light at the front of the boat (**Figure 3**).

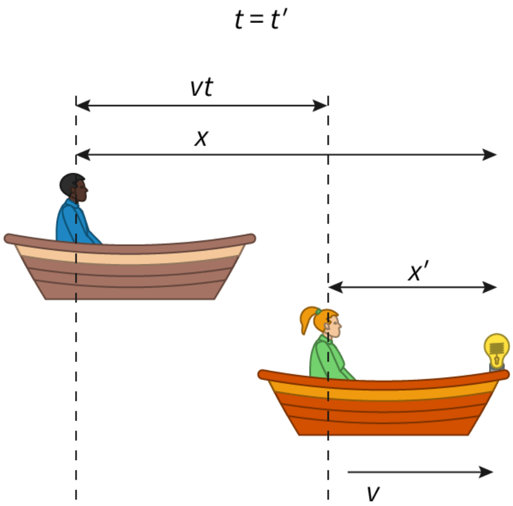


Figure 3. Position from different reference frames.

Student view

More information for figure 3

The diagram illustrates two boats on water, each with a person in it. The upper boat, occupied by a person in blue, is stationary in one reference frame. There is an arrow labeled 'vt' indicating distance from the boat to the origin, and a horizontal arrow labeled 'x' representing its position relative to a reference frame. The lower boat, occupied by a person in green, has a light bulb at the front. This boat is moving at a velocity 'v' as shown by an arrow under it. An arrow labeled 'x'' indicates the position relative to its own reference frame. The moment depicted is when time 't' equals 't''.

[Generated by AI]

In your reference frame, the light is moving and it is turned on at position x. In the other person's reference frame, the light is not moving and is turned on at position x'. The time that the light is turned on is the same in both reference frames: $t' = t$.

In your reference frame, the light is moving at speed v, and has covered a distance of vt in time t. How do we determine the position of the light, x', in the other person's reference frame?

We can use vector addition to give us the equation shown in **Table 3**.

Table 3. Equation for the position of an event.

Equation	Symbols	Units
$x' = x - vt$	x' = position of event in second reference frame	metres (m)
	x = position of event in first reference frame	metres (m)
	v = relative velocity of second reference frame, as viewed from first reference frame	metres per second (m s^{-1})
	t = time of event in first reference frame (absolute time)	seconds (s)

Study skills

When we are talking about multiple reference frames, it helps to distinguish between the variables we are using. For example, x is the position of an event in one reference frame, and x' is the position of the same event in another reference frame.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

In Galilean relativity, the time t is the same for each reference frame.

Worked example 1

A spaceship is travelling at a velocity of 3.0 km s^{-1} directly away from a planet.

At time t = 0, the displacement between the spaceship and planet is 0.0 km. After time t, two asteroids collide at a position 8800 km from the planet and 420 km in front of the spaceship.

Determine the time, t, at which the asteroids collide.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$v = 3.0 \text{ km s}^{-1}$ $x = 8800 \text{ km}$ $x' = 420 \text{ km}$ As all the values are given in km, there is need to convert the units as the units ca
Step 2: Write out the equation and rearrange to find t.	$x' = x - vt$ $t = \dfrac{x - x'}{v}$
Step 3: Substitute the values given.	$= \dfrac{8800 - 420}{3.0}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 2793 \text{ s} = 2800 \text{ s (2 s.f.)}$

We can determine the time and position of an event in a different reference frame. How do we determine the velocity of an event in a different reference frame?

Imagine a person is standing on the side of a road and a bus is driving past them at a constant velocity v. On the bus, there is a person walking in the same direction as the bus is moving.

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

In the reference frame of the bus, the person on the bus is walking at a constant velocity u' (**Figure 4**).

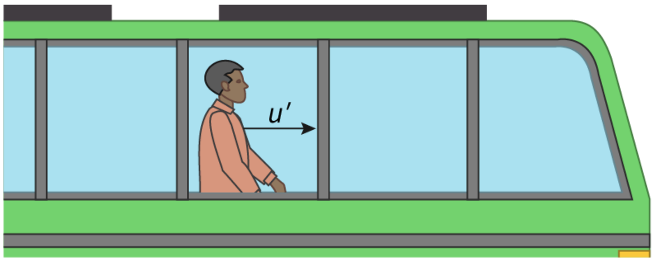


Figure 4. Passenger walking at velocity *u'* relative to the bus.

In the reference frame of the person at the side of the road, the person on the bus is moving at a higher constant velocity u (**Figure 5**).

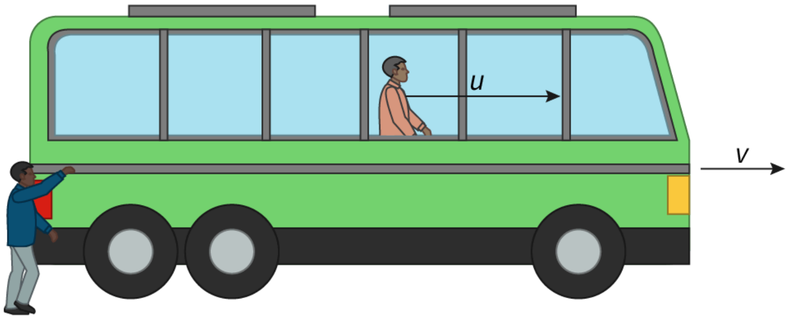


Figure 5. Velocities of the passenger and the bus as seen by the person standing by the road.

More information for figure 5

The diagram depicts a green bus moving to the right, with a person standing outside and a passenger seated inside. The bus is drawn with two large wheels and several windows, illustrating a side view. There are two arrows; one labeled 'u' within the bus, indicating the velocity of the passenger inside, and another labeled 'v' outside, indicating the velocity of the bus itself. The diagram is used to illustrate the concept of relative velocities between the bus and the passenger from the viewpoint of a stationary observer standing by the road.

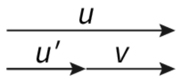
[Generated by AI]

How can we determine u' using information from the reference frame of the person on the side of the road? Click 'Show or hide solution' to see the answer.

From the reference frame of the person on the side of the road, the velocity of the person walking on the bus, u, is equal to the sum of the velocity of the bus, v, and the velocity of the person from the reference frame of the bus, u'.

Student
view

Relationships between the velocities:



This gives us the velocity addition equation shown in **Table 4**.

Table 4. Equation for the velocity of an event.

Equation	Symbols	Units
$u' = u - v$	u' = velocity in second reference frame	metres per second (m s^{-1})
	u = velocity in first reference frame	metres per second (m s^{-1})
	v = relative velocity of second reference frame to first reference frame	metres per second (m s^{-1})

International Mindedness

The concept of reference frames is a useful one to apply to lots of areas of life, not just motion. Is there such a thing as absolute truth? Or can two people with opposing views both be right? How can this idea support constructive dialogue around the world?

Worked example 2

Person A is travelling in a car along a road at a velocity of 12 m s^{-1}. Person B is standing by the side of the road. Person A leans out of the window of the car and throws a ball forwards, in the same direction as the direction of travel of the car. Person B measures the velocity of the ball to be 15.5 m s^{-1}.

What is the velocity of the ball as measured by person A?

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	Taking person B to be the first referenc and person A to be the second referen frame: $v = 12 \text{ m s}^{-1}$ $u = 15.5 \text{ m s}^{-1}$
Step 2: Write out the equation.	$u' = u - v$
Step 3: Substitute the values given.	$= 15.5 - 12$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 3.5 \text{ m s}^{-1} \text{ (2 s. f.)}$

In **Worked example 2**, you can take person A to be the first reference frame and person B to be the second reference frame. Person A is not moving (in their own reference frame) and person B (and the rest of the landscape) is moving backwards at 12 m s^{-1}. In this case:

$$v = -12 \text{ m s}^{-1}$$

$$u' = 15.5 \text{ m s}^{-1}$$

$$u' = u - v$$

$$\begin{aligned} u &= u' + v \\ &= 15.5 + (-12) \\ &= 3.5 \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

The answer is the same, whichever reference frame you label as the ‘first’. (Note that it can be helpful to draw a diagram to help you identify which velocity is which.)

Worked example 3

A train is moving at a velocity of 22 m s^{-1} relative to the ground. A second train, following the first train and moving relative to the ground at a velocity of 11 m s^{-1}, has a radar speed gun. A projectile is fired from the first train directly at the second train. The velocity of the projectile is recorded by the radar speed gun as 33 m s^{-1}.

Student
view

Overview (/study/ap aa- hl/sid- 423- cid- 762593/c

In the reference frame of the first train, what speed is the projectile fired at?

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	There is more information about the second train the radar speed gun so take it as the first referenc frame and the first train as the second reference f From the radar speed gun's perspective, the radai gun is not moving (it is the ground that is moving) the reference frame of the second train: $v = 22 - 11 = 11 \text{ m s}^{-1}$ (relative velocity of the first train as viewed from tl radar speed gun) $u = -33 \text{ m s}^{-1}$ (speed of the projectile measured by the radar sp gun — note the minus sign to indicate direction)
Step 2: Write out the equation.	$u' = u - v$
Step 3: Substitute the values given.	$= -33 - 11$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding,	$= -44 \text{ m s}^{-1}$ (2 s.f.)

Work through the activity to check your understanding of Galilean transformations.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

You are standing on a frozen, straight river in winter. An ice skater travels along the river at constant velocity *v* relative to you. The skater passes you at a time which you both agree is *t* = 0. There is a tree ahead of the skater, at the side of the river.

Interactive 2 shows your frame of reference, F, using axes, with you always at the origin of these axes. It shows the skater's frame of reference, F', with the skater at the origin of those axes. (The *x*-direction is along the river and the *y*-direction is across the river. The real frames of reference are three-dimensional, but we can ignore the vertical dimension in this simple example.)

The tree is shown as a black circle. The interactive begins at time *t* = 0.

77 / 77 2 s

Interactive 2. Galilean relativity: position of an object in two reference frames that have relative velocity *v*.

More information for interactive 2

The "Galilean Relativity" interactive illustrates how the position of an object transforms between two reference frames moving at a relative velocity v. It visually demonstrates the principles of Galilean relativity by depicting a tree as observed from both a stationary and a moving reference frame.

In this simulation, reference frame F represents a stationary observer standing on a frozen river, while reference frame F dash represents an ice skater moving at a constant velocity v. At t = 0, both observers agree on their positions. However, as time progresses, the skater moves relative to the stationary observer. The tree remains fixed in the stationary frame but appears to shift in the moving skater's frame. The displacement between the two frames, represented by the arrow labeled vt, highlights the transformation of coordinates over time.

By interacting with this simulation, users can visualize how an object's position changes when viewed from different reference frames in relative motion. This helps them grasp the fundamental idea of Galilean relativity, reinforcing the concept that while spatial relationships may appear different to moving and stationary observers, the transformation between frames remains time-independent.

Student
view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Answer the following questions:

- What does *x* represent? What does *y* ′ represent?
- Why is the distance between the origins of the frames labelled *vt*?
- At what time(s), if any, do you and the skater agree on the *x*-coordinate of the tree in your coordinate systems?
- At what time(s), if any, do you and the skater agree on the *y*-coordinate of the tree in your coordinate systems?
- Using **Interactive 2** to help you, write the relationship between *x* and *x*′.

Just after the skater passes the tree, you notice a sledge sliding along the ice, in the same direction as the skater, at velocity *u* relative to you.

Interactive 3 shows the reference frames of you F and the skater F', with the sledge represented by a black square.

Interactive 3. Galilean relativity: speed of an object in two reference frames that have relative velocity *v*.

More information for interactive 3

The "Galilean Relativity: Speed in Two Reference Frames" interactive demonstrates how the velocity of an object changes when observed from different reference frames moving at a relative velocity v. It visualizes Galilean relativity by illustrating how the speed of a sledge, represented by a black square, is perceived in both a stationary and a moving reference frame.

The stationary observer's frame F and the moving skater's frame F dash are color-coded for clarity. In the stationary frame, the sledge moves with velocity u. However, since the skater is also moving relative to the stationary observer, the velocity of the sledge appears different when measured in the skater's frame. This relationship is captured by the Galilean velocity transformation equation, u' = u - v, which shows how velocities transform between reference frames in classical mechanics.

Student
view

Overview
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The interactive represents this concept using coordinate axes and velocity vectors. The arrow labeled v t indicates the displacement of the moving frame over time, while the motion of the sledge is depicted in both frames as the skater's reference frame moves along the x-axis.

By engaging with this interactive, users can observe how the perception of motion changes depending on the observer's frame of reference. The concept of relative velocity is reinforced by demonstrating how an object's speed differs when measured in different frames, emphasizing the fundamental idea of Galilean relativity—that velocities simply add or subtract depending on the direction of relative motion.

- Write a formula for the velocity of the sledge relative to the skater.

5 section questions

Question 1

HL Difficulty:

A situation can be viewed from different perspectives. In relativity, these different perspectives are called 1 reference fra... ✓ . An 2 inertial refere... ✓ is one that is not accelerating.

Accepted answers and explanation

#1 reference frames
frames of reference

#2 inertial reference frame
inertial frame of reference

General explanation

A situation can be viewed from different perspectives. In relativity, these different perspectives are called reference frames. An inertial reference frame is one that is not accelerating.

Question 2

HL Difficulty:

According to Galilean relativity, the concept of 1 time ✓ is absolute. This means that it is the same no matter the 2 reference fra... ✓ .

Student
view

Accepted answers and explanation

Overview
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#1 time

#2 reference frame
frame of reference

General explanation

According to Galilean relativity, the concept of time is absolute. This means that it is the same no matter the reference frame: t' = t.

Question 3

HL Difficulty:

Parachutist A is falling at a constant speed of 45 m s^{-1} vertically downwards towards the Earth. Parachutist B is falling nearby at a constant speed of 38 m s^{-1}. Parachutist A throws a ball vertically upwards at an initial speed of 3.5 m s^{-1} as viewed by parachutist A.

What is the initial velocity of the ball as viewed by parachutist B?

1 3.5 m s^{-1} ✓

2 −3.5 m s^{-1}

3 10.5 m s^{-1}

4 −10.5 m s^{-1}

Explanation

From the reference frame of parachutist B (parachutist B is not moving and all movement is relative to their reference frame):

$$\begin{aligned} v &= 45 - 38 \\ &= 7.0 \text{ m s}^{-1} \end{aligned}$$

$$u' = -3.5 \text{ m s}^{-1}$$

$$u' = u - v$$

$$\begin{aligned} u &= u' + v \\ &= 7.0 + (-3.5) \\ &= 3.5 \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

(i.e. 3.5 m s^{-1} downwards)

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Question 4

HL Difficulty:

Student A and student B are running a race. Student A is running at speed v_A and student B is running at a slower speed, v_B. At time t = 0, they are the same distance away from the finish line. At time t = 5.0 s, someone at the finish line waves a flag.

As viewed by student A, the position of the flag is 65 m away.

As viewed by student B, the position of the flag is 71 m away.

What is the speed of student A as viewed by student B?

1 1.2 m s^{-1} ✓

2 6.0 m s^{-1}

3 2.8 m s^{-1}

4 0.83 m s^{-1}

Explanation

Using the reference frame of student B:

$$x = 71 \text{ m}$$

$$t = 5.0 \text{ s}$$

$$x' = 65 \text{ m}$$

$$x\prime = x - vt$$

$$\begin{aligned} v &= \frac{x - x'}{t} \\ &= \frac{71 - 65}{5} \\ &= 1.2 \text{ m s}^{-1} \text{ (2 s.f.)} \end{aligned}$$

Question 5

HL Difficulty:

Student A is riding a rollercoaster and student B is watching from the side. At a particular instant, the rollercoaster passes right next to student B, who measures the constant speed of the rollercoaster to be 16 m s^{-1}.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The rollercoaster moves in a straight line at this same constant speed for 1.2 s, at which point, a camera on board the rollercoaster takes a photo with a flash. Student B measures the position of the camera flash to be 22.1 m away.

What is the position of the camera as measured by student A?

1 2.9 m ✓

2 6.1 m

3 41 m

4 38 m

Explanation

Using the reference frame of student B:

$$v = 16\ \text{m s}^{-1}$$

$$t = 1.2\ \text{s}$$

$$x = 22.1\ \text{m}$$

$$\begin{aligned} x\prime &= x - vt \\ &= 22.1 - (16\times1.2) \\ &= 2.9\ \text{m}\ (2\ \text{s.f.}) \end{aligned}$$

A. Space, time and motion / A.5 Galilean and special relativity (HL)

Special relativity and time dilation and length contraction (HL)

A.5.5: The two postulates of special relativity (HL) A.5.6: Lorentz transformation equations (HL)

A.5.9: Proper time interval and proper length (HL) A.5.10: Time dilation (HL) A.5.11: Length contraction (HL) A.5.15: Muon decay (HL)

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Know the two postulates of special relativity.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

- Describe the concepts of proper time interval and time dilation and use the equation:

$$t = \gamma t_0$$

- Describe the concepts of proper length and length contraction and use the equation:

$$L = \frac{L_0}{\gamma}$$

- Recognise that muon decay provides evidence for time dilation and length contraction.

The two postulates of special relativity

Around 1865, James Clerk Maxwell defined four equations that describe how electromagnetic fields propagate through the Universe. Later, Albert Einstein and others noted that the equations predict that electromagnetic waves (see subtopic C.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43778/)) move through empty space at a value very close to the experimental value for the speed of light.

Einstein wondered what would happen if light were an electromagnetic wave that always had this speed, regardless of the reference frame.

He imagined running at the speed of light alongside a beam of light. Would the beam of light just look like a beam of light? What if, even if he were moving very fast, he still measured the speed of light to be the same?

In 1905, Einstein came up with his theory of special relativity, which is based on two postulates.

Theory of Knowledge

A postulate is an idea, not fully supported by scientific evidence, that allows us to consider new perspectives and predict new patterns. Does it have less value than a theory derived from empirical evidence? If so, why can postulates be helpful in the pursuit of knowledge?

Student view

The first postulate of special relativity is:

The laws of physics are the same in all inertial reference frames.

We have seen that there is no way of telling if you are moving very fast forwards, very slowly backwards, or not moving at all, since any experiment you do (for example, dropping a ball or jumping up and down) will yield the same result (see section A.5.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/reference-frames-and-galilean-relativity-hl-id-46604/)). This led Einstein to consider that absolute uniform motion cannot be detected.

Concept

Einstein's first postulate of special relativity states that the laws of physics apply in the same way in all inertial reference frames.

Imagine you are in a spaceship moving in deep space towards a star. The star is shining light towards you. If you were able to measure the speed of the light beam approaching you, you might expect to record a faster speed than if your spaceship were stationary. But this violates Einstein's first postulate – all measurements should be the same in all inertial reference frames.

Einstein suggested that the speed of the light, as recorded by you on the moving spaceship, should be the same as it would be if the spaceship was not moving towards the star.

According to Maxwell's equations, the speed that electromagnetic waves propagate through a vacuum (empty space) is constant. Einstein suspected that light is an electromagnetic wave (we now know this to be true), which led to his second postulate.

The second postulate of special relativity is:

The speed of light in a vacuum is the same for all observers, regardless of their relative motion.

Concept

Einstein's second postulate of special relativity states that the speed of light in a vacuum is the same for all observers, regardless of their relative motion.

This means that no matter how fast you travel in any direction, the speed of light you observe will be the same as for a stationary observer.

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Nature of Science

Aspect: Models

Einstein said that: 'Imagination is more important than knowledge'. Do you think this is true? Would you feel confident if your airline pilot said that? Or your surgeon? Are there any situations in science where knowledge is more important than imagination?

Time dilation

Imagine a spaceship travelling through deep space. A person inside the spaceship carries out a simple experiment. They shine a light from one side of the spaceship to the other and time how long the light takes to reach the other side. Someone watching the spaceship from a planet nearby also measures the time taken. Will they measure the same time? Work through the activity to explore this experiment.

Activity

- **IB learner profile attribute:** Inquirer
- **Approaches to learning:** Thinking skills — Asking questions and framing hypotheses based upon sensible scientific rationale
- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

Look at the simulation in **Interactive 1.** Use the 'Spaceship speed' slider to change the speed of the spaceship. Click 'Run'. What time is measured by the person on the spaceship (proper time)? What time is measured by the person on the nearby planet (clock on the bottom right)? How does the speed of the spaceship affect the observations? Can you explain why?

Student
view

Javalab

Science Simulations

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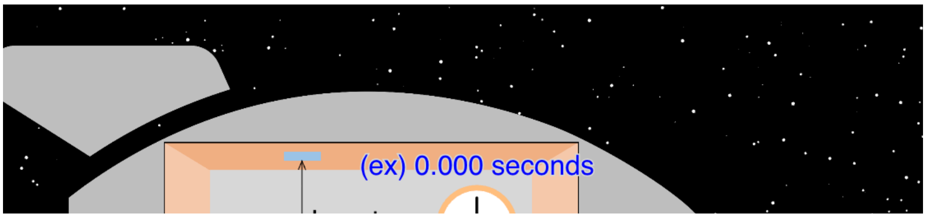
Special Relativity: Time Dilation



Interactive 1. Time dilation simulation.

Source: Javalab, Special Relativity: Time dilation (https://javalab.org/en/special_relativity_en/)

More information for interactive 1

The interactive webpage, captioned "Time dilation simulation," offers an engaging way to explore the concept of time dilation from Einstein's special relativity. It features a visual simulation that illustrates how time behaves differently for observers inside a moving spaceship compared to those on a nearby planet. The interface includes a slider labeled "Spaceship speed," allowing users to adjust the velocity of the spaceship relative to the speed of light, denoted as c (approximately 299,792,458 m/s). This slider likely ranges from a low value (perhaps near zero) up to a significant fraction of c (that is, 0.99 times c), enabling users to see the effects of relativistic speeds.

Once the speed is set, a "Run" button activates the simulation. The animation likely depicts a light beam traveling within the spaceship, perhaps from the floor to the ceiling as described in the accompanying document. The time experienced by an observer onboard the spaceship is the proper time. For the person inside the spaceship, this light travels a straight vertical path, and the proper time, t_0, is measured and displayed. This represents the transit time in the spaceship's stationary frame. Meanwhile, an observer on the nearby planet sees the spaceship moving at speed v. From this external perspective, the light follows a diagonal path (the hypotenuse of a right triangle), due to the spaceship's motion. The simulation shows this longer path and measures the time t as observed from the planet, typically displayed on a clock in the bottom right corner.

As users increase the spaceship's speed using the slider, the difference between the two times becomes more pronounced. The proper time, t_0 remains constant, as the vertical distance the light travels inside the spaceship doesn't change. However, the external time t increases. This demonstrates time dilation: time appears to slow down inside the moving spaceship from the planet's perspective. At low speeds, the difference is negligible, but as v approaches c, the effect becomes dramatic, with t growing significantly larger than t_0.

The time dilation effect follows from special relativity, where time is measured differently for observers moving relative to one another. The faster the spaceship moves, the slower its onboard clock ticks relative to the stationary observer on the planet. This is mathematically described by the Lorentz factor:

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Here:

- t is the dilated time (experienced on the spaceship),
- t_0 is the time measured on the planet,
- v is the velocity of the spaceship,
- c is the speed of light.

As v approaches c, the denominator approaches zero, causing the time dilation effect to become extremely pronounced. This explains why astronauts on a fast-moving spaceship age slower relative to those who remain on a planet.
The simulation visually explains why the motion stretches the light's path in the external frame, requiring more time to complete the journey, a core insight of special relativity.

For each spaceship speed in **Table 1**, record the time measured by the person on the planet.

Table 1. Results table.

Spaceship speed (c)	Time recorded by person on plan
0.0	
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	

Overview
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Spaceship speed (c)	Time recorded by person on plan
0.9	
0.98	

Plot a graph of *t* (*y*-axis) against speed (*x*-axis). What do you notice? When is the difference in time most noticeable? Why do we not notice this effect in our everyday lives?

Imagine that the spaceship is now travelling past the Earth. The person on the spaceship wants to make a simple clock. They do so by shining a beam of light from an emitter to a detector. They record how much time this light beam takes to reach the detector.

An observer on the Earth is watching the spaceship travel past. From their reference frame, the beam of light follows a different path. **Figure 1** shows what each observer sees.

Diagram (a) shows the light clock from the point of view of an observer (who is holding the detector) on the spaceship. They are therefore at rest relative to the light emitter.

Diagram (b) shows the light clock from the point of view of an observer on Earth (again, holding the detector) who sees the spaceship travelling from left to right.

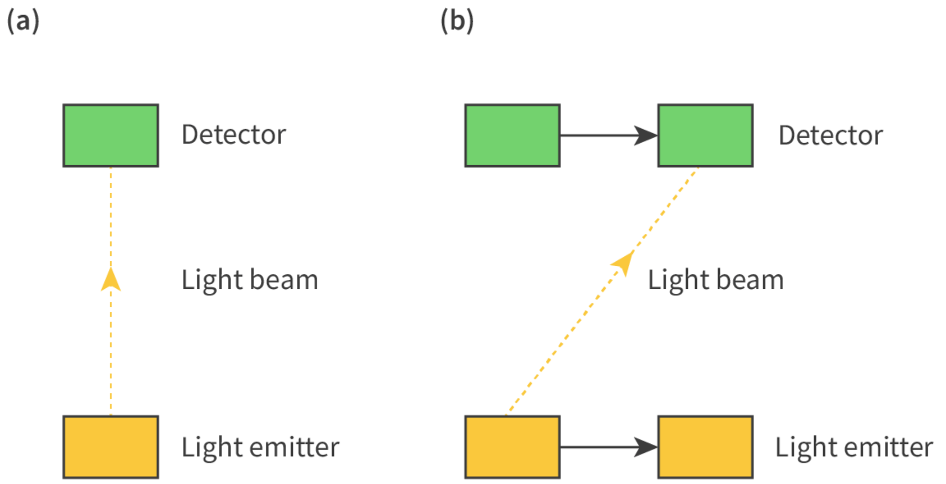

Figure 1. A light clock from the reference frame of a person on the spaceship and a person on the Earth.

More information for figure 1

Student
view

The image is a diagram with two parts labeled (a) and (b). Both parts illustrate a light clock setup. In both sections, there are two main components: a light emitter, shown as a yellow rectangle, and a detector, shown as a green rectangle.

In part (a), the light emitter is at the bottom and the detector is directly above it. A yellow dashed line labeled "Light beam" travels vertically from the light emitter to the detector.

In part (b), both the light emitter and detector are aligned horizontally at the bottom and top, respectively. The yellow dashed line representing the light beam is diagonal, moving from the light emitter towards the detector. This setup suggests the observer's perspective from a moving reference frame, possibly illustrating the relativistic effects according to Einstein's theory.

[Generated by AI]

Einstein said that the speed of light is the same in all inertial reference frames. The light beam is moving a larger distance in the Earth's inertial reference frame. How can the light beam move farther if it is travelling at the same speed in both reference frames?

The equation for constant speed is (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)):

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The speed of light measured by both observers will be the same. The person on the Earth measures the distance to be greater than the distance measured by the person on the spaceship.

So it follows that the person on the Earth must measure the amount of time that has passed as greater than the time measured by the person on the spaceship. From the reference frame of the person on the spaceship, the event happens in less time. Time for the spaceship has slowed down. We call this effect time dilation.

Concept

The idea that time can slow down is one of the strangest ideas in physics. This really is how the Universe works, and time dilation has been supported by data from experiments involving very precise atomic clocks being flown around the Earth.

We can derive the equation for time dilation as follows. This derivation is not required in the IB Physics course.

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The time interval in the reference frame of the spaceship is Δt_0. This the proper time interval, and there is no relative motion between the observer and the timed event. The time interval in the Earth's frame of reference is Δt. This time is not the proper time interval in the spaceship, as the clock is moving relative to the Earth.

The person on the spaceship shines a light from one side of the spaceship to a sensor on the other side. **Figure 2** shows the event from the reference frame of the spaceship.

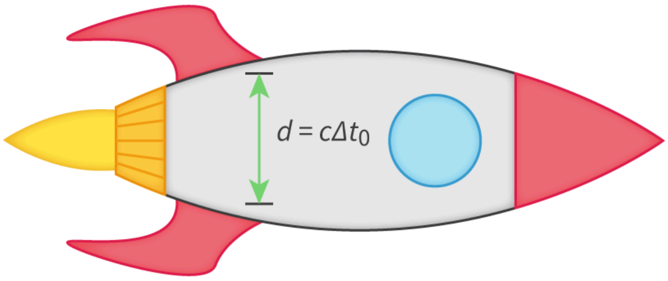

Figure 2. The path of a beam of light in a spaceship, as viewed from the spaceship.

More information for figure 2

The diagram shows the interior of a spaceship with a path of a light beam illustrated inside. The spaceship is horizontally oriented, with two fins or wings on each side. In the center, an arrow labeled with the formula "d = cΔt_0" represents the path and distance traveled by the light beam from one side of the spaceship to the other. The front of the spaceship is pointed to the right, while the back is on the left, where a yellow flame indicates propulsion. A blue circle is depicted on the right side, possibly representing a sensor or some internal feature. The arrows indicate the light's vertical travel from one side of the interior to the opposite.

[Generated by AI]

Applying the equation speed $= \dfrac{\text{distance}}{\text{time}}$ for the reference frame of the spaceship, we get:

$$d = c\Delta t_0$$

where d is the distance travelled by the beam from one side to the other.

Figure 3 shows the event from the reference frame of the Earth.

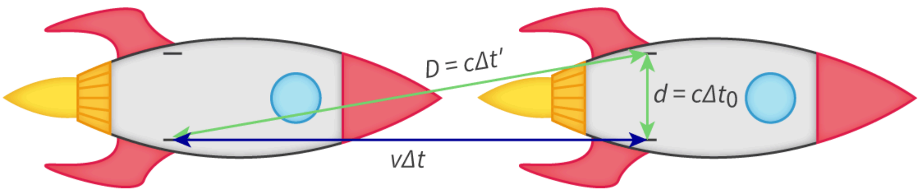

Figure 3. The path of a beam of light in a spaceship, as viewed from the Earth.

Student view

More information for figure 3

The image is a diagram illustrating the paths of a light beam within two spaceship diagrams, viewed from the Earth. The spaceship on the left has a green line inside labeled "D = cΔt'", representing the distance traveled by the light beam in Earth's reference frame. The spaceship on the right shows a similar line labeled "d = cΔt₀", representing the distance traveled by the light beam in the spaceship's reference frame. Between the spacecraft, a horizontal blue line labeled "vΔt" indicates the distance traveled by the spaceship in Earth's reference frame. Both spacecraft have a circular cabin area marked on the side and are oriented horizontally with red nose cones and tail fins on both ends.

[Generated by AI]

In **Figure 3**, D is the distance travelled by the light beam in the reference frame of the Earth, and d is the distance travelled by the light beam in the reference frame of the spaceship. The bottom of the triangle is the distance travelled by the spaceship in this time. In time Δt (measured by the observer on the Earth), the spaceship will have moved a distance $v\Delta t$.

We can now use Pythagoras' theorem:

$$a^2 + b^2 = c^2$$

Applying this formula to the triangle in **Figure 3** gives:

$$(ct_0)^2 + (vt)^2 = (ct)^2$$

Multiplying out the brackets, we get:

$$c^2{t_0}^2 + v^2t^2 = c^2t^2$$

Subtracting the term v^2t^2 from both sides gives:

$$c^2{t_0}^2 = c^2t^2 - v^2t^2$$

Factoring out the t^2 term on the right-hand side of the equation gives:

$$c^2{t_0}^2 = t^2\left(c^2 - v^2\right)$$

Dividing by the $(c^2 - v^2)$ term gives:

$$t^2 = \frac{c^2{t_0}^2}{(c^2 - v^2)}$$

Dividing top and bottom of the right-hand side of the equation by c^2 gives:

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

762593/book/reference-frames-and-galilean-relativity-hl-id-46604/print/)

$$t^2 = \frac{{t_0}^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

Taking the square root of both sides gives:

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

The **gamma factor**, *γ*, is given by the equation in **Table 2**.

Table 2. Equation for gamma factor.

Equation	Symbols	Units
$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$	γ = gamma factor	unitless
	v = velocity relative to stationary observer	metres per second (m s^{-1})
	c = speed of light (3.00 × 10^8 m s^{-1})	given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet

It is important to note that the gamma factor is always greater than 1. This is because *v* (speed of the object) cannot be greater than *c* (speed of light in a vacuum), so the equation will always have a larger numerator than denominator.

Worked example 1

A spaceship travels past the Earth with a relative velocity of 180 000 000 m s^{-1}. Determine the gamma factor as applied to time intervals on the spaceship, as observed by someone standing on the Earth. Give your answer to 3 significant figures.

Student view

Overview
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Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$v = 180\,000\,000 \text{ m s}^{-1}$ $= \frac{180\,000\,000}{3.00 \times 10^8}$ $= 0.6c$
Step 2: Write out the equation.	$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$
Step 3: Substitute the values given.	$= \frac{1}{\sqrt{\left(1 - \frac{(0.6c)^2}{c^2}\right)}} = \frac{}{\sqrt{(}}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 1.25 \text{ (3 s.f.)}$

As the spaceship speed approaches c (speed of light), the gamma factor gets larger and larger. This means that the time interval on the spaceship will get longer and longer, as observed by the person standing on the Earth.

The simplified equation for time dilation is shown in **Table 3**.

Table 3. Equation for time dilation.

Equation	Symbols	Units
$\Delta t = \gamma \Delta t_0$	Δt = time interval (time dilation)	seconds (s)
	γ = gamma factor $\frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$	unitless
	Δt_0 = proper time interval	seconds (s)

Student
view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The time dilation formula is valid when the time interval applies to two events that occur at the same location in one frame – the frame in which Δt_0 is measured. Δt_0 is the proper time interval, which is the shortest time that can be measured between the two events. The time interval in a different inertial frame is Δt, where $\Delta t > \Delta t_0$.

Which graph in **Interactive 2** shows the correct relationship between gamma factor, γ, and velocity relative to stationary observer, v?

Which graph shows the gamma factor against relative velocity

Check

Interactive 2. Graph of Gamma Factor Against Relative Velocity.

Graphs 2 and 3 can be ruled out. In both cases the value for v/c is greater than 1. However, as we have seen, it is not possible to travel faster than the speed of light, so ‘v’ cannot be greater than ‘c’, the speed of light in a vacuum. Graph 4 shows the gamma factor decreasing below 1 as ‘v’ increases. This means that the measured time would be less than the proper time. This is not supported by experiment.

Worked example 2

A spaceship with artificial gravity travels past the Earth with a velocity of 120 000 000 m s^{-1}. An observer on the Earth sees that the spaceship fires its rockets for 2.0 s. Determine how long this event takes in the reference frame of the spaceship.

Student view

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Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$v = 120\,000\,00$ $= \frac{120\ 000\ 0}{3.00\ \times\ 1}$ $= 0.4c$ $t = 2.0\text{ s}$ $\gamma = \frac{1}{\sqrt{\left(1 - \frac{v}{c}\right.}}$ $= \frac{1}{\sqrt{\left(1 - \frac{(}{}\right.}}$ $= \frac{1}{\sqrt{(1 - 0.}}$ $= 1.1$
Step 2: Write out the equation and rearrange to find t_0.	$\Delta t = \gamma \Delta t_0 t_0 =$
Step 3: Substitute the values given.	$= \frac{2.0}{1.1}$
Step 4: State the answer with appropriate units and the number of significant figures used in rounding.	$= 1.833\text{ s} = 1.$

Study skills

A helpful question to ask when solving questions of this type is: 'Who is observing the proper time interval?' This will always be the observer who is in the same inertial reference frame as the event being timed. This means the observer who is not moving relative to the event being timed. For observers who are moving relative to the event, the timed duration will be longer.

What about the observer on the spaceship? When they observe events happening on the Earth, what do they measure when the duration of events is compared?

Student
view

Overview
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hl/sid-
423-
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The observer on the spaceship sees exactly the same effect happening on the Earth. That is, the observer on the spaceship also measures time on the Earth running slower than the observer on the Earth would measure. Remember, we cannot say for certain that the spaceship is moving and the Earth is stationary, or vice versa. Therefore, we treat the reverse scenario in exactly the same way.

Length contraction

Consider a train travelling extremely fast. A scientist on the train is trying to measure the length of the carriage by shining a laser light from one end of the carriage to the other end, where the light reflects and travels back again. They calculate the length of the path by multiplying the speed of the laser, c, by the time taken, t. What do you think the scientist will measure? What will someone watching from outside the train measure?

Use the simulation in **Interactive 3** to investigate this experiment. Select the speed of the train using the slider. Select ‘Time’, then select ‘Inside perspective’ and watch what happens. Then select ‘Outside perspective’ and watch what happens. Who measures the greater length? Can you explain why the lengths measured are different?

Javalab

Science Simulations

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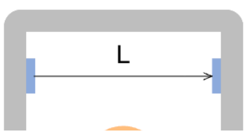
Menu

Special Relativity: Length Contraction



Interactive 3. Length contraction simulation.

Source: Javalab, Special Relativity 2 (https://javalab.org/en/special_relativity_2_en/)

More information for interactive 3

The interactive webpage, captioned "Length contraction simulation," provides a hands-on exploration of length contraction, a key phenomenon of Einstein’s special relativity. The simulation centers on a train moving at relativistic speeds, allowing users to visualize how the length of an object changes depending on the

Student
view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

observer's perspective. The user can adjust the speed of the train, ranging from stationary to near-light speed (0.91 times the speed of light). The speed of light (c) is approximately $299, 792, 458$ m/ s. This adjustment lets users observe the effects of increasing velocity on length measurements.

The simulation offers two main controls: a "Time" option and a perspective toggle between "Inside perspective" and "Outside perspective." After setting the train's speed, selecting "Time" and "Inside perspective" likely triggers an animation showing a light beam emitted from one end of the train, reflecting off the opposite end, and returning. From inside the train, the observer sees the light travel a fixed distance, twice the train's stationary length $2L_0$, at speed c. The time measured here, $t_0 = \left(\frac{2L_0}{c}\right)$, represents the proper time in the train's inertial frame, where the train appears stationary and its length is L_0).

Switching to "Outside perspective" shifts the view to an observer watching the moving train. Here, the simulation illustrates the light's path as affected by the train's motion at speed v. The light travels to the right while the train moves forward, then back after reflection, creating two segments: $t_1 = \frac{L}{c-v}$ and, $t_2 = \frac{L}{c+v}$

The total round-trip time becomes:

$$t = t_1 + t_2$$

$$= \frac{L}{c-v} + \frac{L}{c+v}$$

$$= \frac{2Lc}{\sqrt{c^2-v^2}}$$

Thus, the total round trip is longer due to the train's motion.
This phenomenon is described by the length contraction formula:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

Here:

- L is the contracted length observed by an external viewer,
- L_0 is the proper length measured in the train's frame,
- v is the speed of the train,
- c is the speed of light.

As the speed increases, the denominator decreases, leading to a shorter measured length for the moving train. This contraction becomes significant at speeds approaching the speed of light.

The inside observer measures the greater length L_0 because the train is stationary in their frame, unaffected by motion. The outside observer sees a contracted length L because the train's high speed compresses its dimension along the direction of travel, a relativistic effect rooted in the constant speed of light across all inertial frames. As v increases, the difference grows, visually explaining why measured lengths differ.

A spaceship is travelling from the Earth to a distant star, which is at rest relative to the Earth. A person on the spaceship measures the time elapsed on a clock in the spaceship. This is the proper time interval, Δt_0, as the person is moving with the

Student view

Overview
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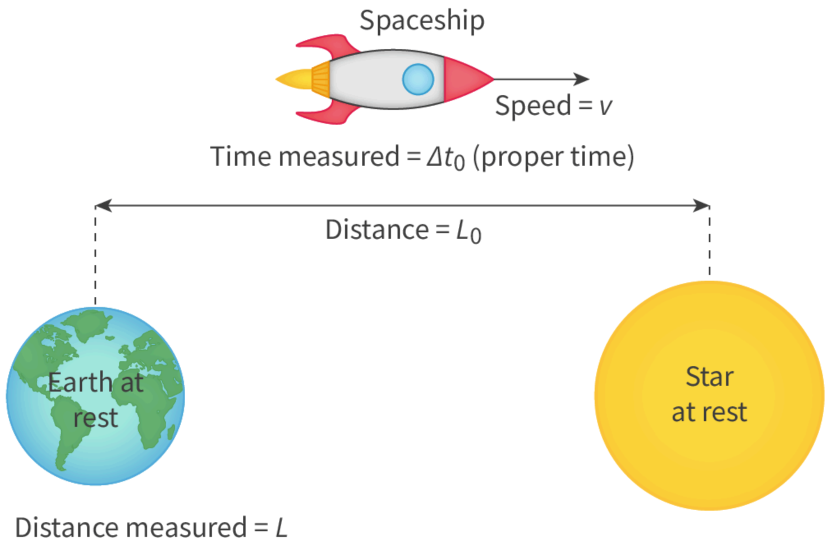
spaceship (and the clock). The duration of the journey measured by a person on the Earth is Δt, as the person on the Earth is moving relative to the clock doing the timing – Δt will be greater than Δt_0 because of time dilation.

For the person on the Earth, the distance between the Earth and the star is the proper length, L_0. This is because the person on the Earth is not moving relative to the Earth or the star. The distance between the Earth and the distant star as measured by the person on the spaceship is a different length, L. This is because there is relative motion between the person in the spaceship and the distance being measured.

Figure 4 shows the spaceship moving relative to the stationary Earth and the star, and the Earth and the star moving relative to the stationary spaceship.



More information

The diagram illustrates the concept of relative motion between a spaceship and two celestial bodies: Earth and a star. At the top, a cartoon depiction of a spaceship is shown moving horizontally to the right, labeled "Spaceship" with an arrow indicating its direction and labeled "Speed = v." Below the spaceship, a text states "Time measured = Δt_0 (proper time)". Earth is shown on the bottom left side of the diagram with the label "Earth at rest," and on the right, a yellow circle labeled "Star at rest." A dashed line connects Earth and the star, labeled as "Distance = L_0." Beneath Earth, a text states "Distance measured = L."

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Student
view

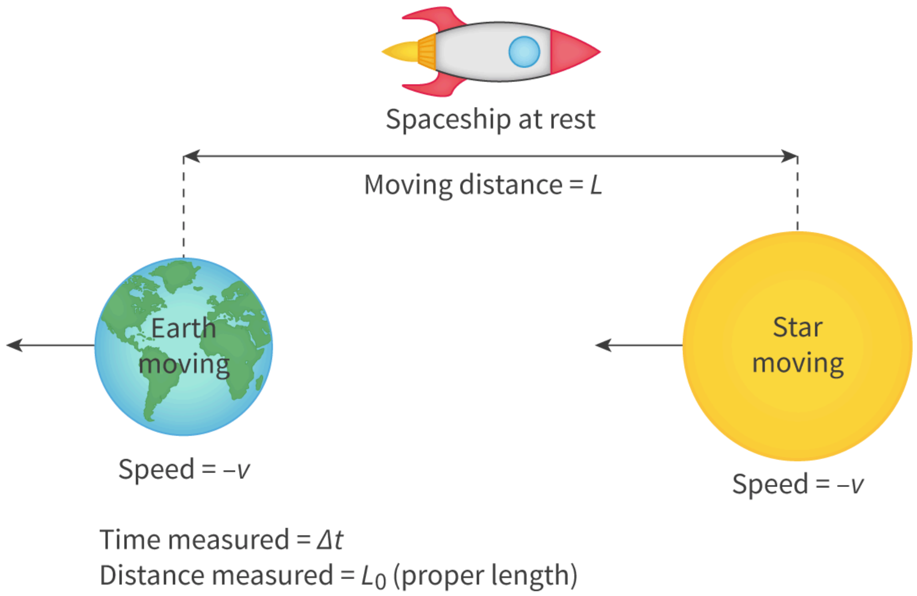

Figure 4. A spaceship moving relative to stationary Earth and star (upper image), and the Earth and star moving relative to stationary spaceship (lower image).

More information for figure 4

The diagram illustrates the concept of relative motion with a spaceship, Earth, and a star. The upper part shows a stationary spaceship with Earth and a star moving in opposite directions, both labeled with speed = -v. The distance between the spaceship and the two celestial bodies is labeled as "Moving distance = L". The lower part of the diagram explains that the time measured is Δt and the distance measured is L_0 (proper length). The Earth and star are clearly labeled in the diagram, and arrows indicate their directions of motion. The text provides key variables related to the spaceship's rest condition and the movement of the Earth and star.

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We can apply the equation speed $= \frac{\text{distance}}{\text{time}}$ to this motion to derive the equation for length contraction. The derivation of the equation is not required in the DP Physics course.

From the reference frame of the person on the spaceship:

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{L}{\Delta t_0}\end{aligned}$$

From the reference frame of the person on the Earth:

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{L_0}{\Delta t}\end{aligned}$$

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

As the speed of the Earth relative to the spaceship is the same as the speed of the spaceship relative to the Earth, we can combine the equations to give:

$$\frac{L}{\Delta t_0} = \frac{L_0}{\Delta t}$$

The proper time interval, Δt_0, as measured by the person on the spaceship, and the time interval measured by the person on the Earth, Δt, are linked by the relationship:

$$\Delta t = \gamma \Delta t_0$$

Applying this relationship to the equation above, gives:

$$\frac{L}{\Delta t_0} = \frac{L_0}{\gamma \Delta t_0}$$

Cancelling Δt_0 gives the equation for length contraction given in **Table 4**.

Table 4. Equation for length contraction.

Equation	Symbols	Units
$L = \frac{L_0}{\gamma}$	L = length (length contraction)	metres (m)
	L_0 = proper length	metres (m)
	γ = gamma factor $\left(\frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}\right)$	unitless

The distance travelled by the spaceship has, from the reference frame of the person on the spaceship, been shortened. In other words, if you are moving relative to the distance you are measuring, you will measure a shorter distance than someone in the same reference frame as the measurement being made. This is known as length contraction.

The length contraction formula is valid when the measured length is not changing with time, as for the distance between the Earth and the star in the derivation above. So the formula can be applied to the length of an object or to a fixed distance between two objects. It cannot be applied to the distance between two objects that are moving relative to each other, or to two events.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Concept

Length contraction is where the length of an object shortens, as measured in the reference frame of an observer moving relative to that object.

If the observer is moving along with the object (there is no relative motion between observer and object), there will be no length contraction for the observer. The length measured by the observer will then be the proper length (which is the maximum distance between two events — in this case, the measurements of the two ends of the object).

It is important to realise that length contraction is restricted to measurements along the direction of motion. A cubical box moving at **relativistic speeds** (speeds approaching the speed of light, $c = 3.00 \times 10^8 \text{ m s}^{-1}$) along an x-axis contracts only along the x-axis (**Figure 5**). The height (y-axis) and the depth (z-axis) measurements remain unchanged. Note that, according to the equations of relativity, the cube can approach but never reach the speed of light.

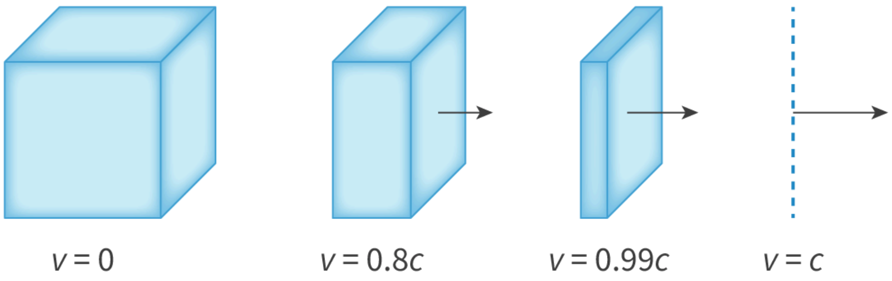

Figure 5. A cube, moving along the x-axis at different speeds, experiencing length contraction.

More information for figure 5

The diagram illustrates a cube undergoing length contraction as its velocity increases along the x-axis. There are four stages depicted:

1. **v = 0**: The cube is shown at rest with standard dimensions, appearing as a perfect cube.
2. **v = 0.8c**: The cube is moving at 80% of the speed of light (where 'c' is the speed of light). The cube is visibly contracted along the x-axis, appearing elongated in the direction perpendicular to motion.
3. **v = 0.99c**: At 99% of the speed of light, the cube appears further contracted along the x-axis, becoming even thinner.
4. **v = c**: The cube is represented as a dotted line to signify that it can never reach the speed of light according to the laws of relativity. Any further contraction to this extent is theoretical as the cube cannot physically attain this speed.

Arrows to the right indicate the direction of motion for each scenario. This illustration represents the concept of length contraction in the theory of relativity, where the length of an object contracts along the direction of motion as its speed approaches the speed of light.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

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Exercise 1

Click a question to answer

Higher level (HL)

Let’s try an example to illustrate the learning required here. Whilst the ideas discussed are very strange, the calculations required are straightforward, as the next example shows.

Worked example 3

An observer on the Earth sees a spaceship flying past at 0.8c. The observer measures an object on the spaceship. The object has a proper length of 1.0 m. What length does the observer measure the object as having?

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$v = 0.8c$ $L_0 = 1.0\ \mathrm{m}$ $= \frac{1}{\sqrt{\left(1 -\right.}}$ $= \frac{}{\sqrt{\left(1 -\right.}}$ $= 1.67$
Step 2: Write out the equation.	$L = \frac{L_0}{\gamma}$
Step 3: Substitute the values given.	$x = \frac{1.0}{1.67}$

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Solution steps	Calculations
Step 4: State the answer with appropriate units and the number of significant figures used in rounding	$0.599\ \text{m} = 0.6$

Look at the simulation in **Interactive 4**. Try to move the bus into the garage. What happens when the speed of the bus is large? Can you work out the minimum speed to park the bus in a particular length of garage (‘garage: bus ratio’)?

Interactive 4. Length contraction simulation.

More information for interactive 4

The interactive simulation, "Length Contraction," demonstrates the relativistic effect of length contraction a phenomenon where a moving object's length appears shortened from the perspective of a stationary observer. This concept, rooted in Einstein’s theory of special relativity, is illustrated through a scenario in which a bus moves toward a garage at varying speeds.

A slider labeled "V" allows users to adjust the bus’s velocity from 0 to 10, controlling how fast it moves relative to the garage. Another slider adjusts the bus-to-garage length ratio (0 to 0.999), showing how the bus’s apparent length changes as its speed increases. When moving at low speeds, the bus retains its original length. However, as the velocity approaches a significant fraction of the speed of light or relativistic speeds, the bus appears shortened due to length contraction.

The goal of the simulation is to determine the minimum speed at which the bus appears short enough to fit inside the garage, reinforcing the counterintuitive nature of relativistic motion. Users can start, pause, and reset the simulation to observe how different speeds impact length contraction.

It is important to note that the simulation presents a simplified model of relativity. Length contraction occurs only in an inertial reference frame it is not observed when the bus and garage are stationary relative to each other. Additionally, the simulation does not account for non-inertial effects introduced by deceleration when the bus slows down.

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

By experimenting with different velocities and observing the corresponding contraction, users develop an intuitive understanding of relativistic effects. This interactive allows learners to explore how increasing speed leads to significant length contraction and encourages reflection on real-world implications, such as high-energy particle physics and relativistic space travel.

Example: If the garage ratio is **3** and the bus's velocity is **0.5**, the contracted length of the bus is still too long to fit inside the garage. However, when the garage ratio is reduced to **1.62**, the bus's contracted length becomes short enough to fit within the garage.

The simulation in **Interactive 4** is a simplified simulation. The length contraction effect would not exist when the bus and the garage became stationary relative to each other. Also, special relativity only applies to inertial reference frames. When the bus is decelerating, this is not an inertial reference frame.

Muon decay

Muons are subatomic particles that are created when high energy cosmic rays from space interact with the upper atmosphere of the Earth. The muons stream towards the surface of the Earth, a distance of about 10 km (**Figure 6**).

Making connections

For the purposes of muon decay, at this stage it is enough to know that decay means the change from one particle into another. We will learn more about particle decay in subtopic E.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44319/).

Muons are unstable particles that have a very short mean lifetime of about 2.2 μs before they decay into other particles. In this lifetime, they should not be able to reach the Earth's surface, without going faster than the speed of light. Yet scientists are able to record muons reaching detectors on the surface of the Earth. How is this possible?

Student view

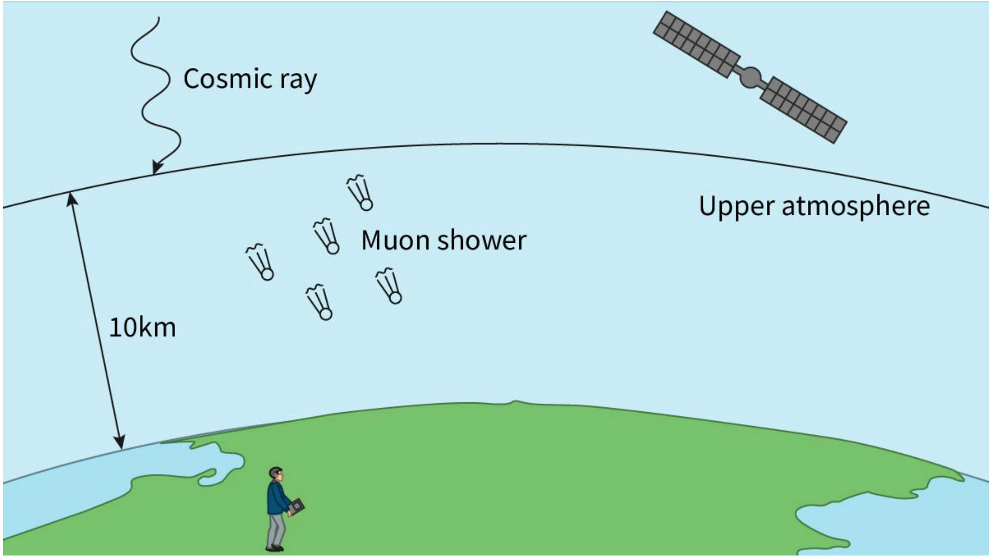

Figure 6. How can muons reach the Earth's surface without going faster than the speed of light?

More information for figure 6

The image illustrates the process by which muons reach the Earth's surface. At the top, a cosmic ray is shown entering the upper atmosphere, labeled as such. Upon impact, it creates a 'Muon shower' with several muons depicted as streaking downward. The upper atmosphere is marked as 10 km above the Earth's surface. A human figure stands on the ground holding a detector. The background shows a satellite in orbit above the atmosphere, indicating the high altitude of this process.

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The phenomenon can be explained either from the point of view of an observer on Earth, or from the point of view of a muon.

First, consider a scientist on Earth who observes a muon. The scientist is stationary relative to the atmosphere. In the reference frame of the scientist, the length of the muon's journey from the top of the atmosphere to the Earth's surface is the proper length (about 10 km).

From the point of view of the scientist, the muon travels at an extremely high speed. Since the scientist's reference frame and the muon's reference frame are moving relative to each other, the scientist does not measure the proper time interval (2.2 μs) for the muon's existence. As a result of time dilation, the scientist observes the muon's lifetime to be longer than 2.2μs.

So according to the scientist, the muon reaches the Earth before decaying because its lifetime is long enough for it to complete the 10 km journey.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Now consider the situation from the point of view of an observer who is travelling with the muon. The observer is in the muon's frame of reference, so they measure the muon's lifetime as the proper time interval, 2.2μs. However, they are travelling at high speed relative to the atmosphere, so they do not not measure the proper length (10 km) of the muon's journey. As a result of length contraction, in the muon's reference frame the journey is much shorter than its proper length of 10 km.

So according to an observer in the muon's reference frame, the muon reaches Earth because the journey is short enough to complete within 2.2μs.

These theoretical arguments agree with observations, which show that on average, one muon lands on each square centimetre of the Earth per minute.

Table 5 shows the difference between reference frames when observing muons.

Table 5. Difference between reference frames when observing muons.

	Muons' reference frame	Scientists' reference frame
Distance travelled	The distance between the upper atmosphere and the surface of the Earth undergoes length contraction. This is because the muons are moving (very quickly) relative to the atmosphere. This shorter distance allows the muons to reach the Earth's surface.	The distance between the upper atmosphere and the surface of the Earth is measured as the proper length. This is because there is no relative motion between the scientists and the atmosphere.
Time taken	The lifetime of the muons is measured as the proper time, 2.2 μs.	The lifetime of the muons undergoes time dilation. This longer time allows the muons to reach the Earth's surface.

Making connections

To more fully understand muon decay, we need to think about half-life (see subtopic E.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44319/)). The half-life of the muons is about 1.5 μs. After one half-life, half the muons will remain. Using half-life, we can more accurately determine the proportion of muons reaching the Earth's surface, and the results are close to the experimentally observed values.

Student view

Work through the activity to check your understanding of time dilation and length contraction.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 25 minutes
- **Activity type:** Individual activity

1. Muons have a mean lifetime of 2.2 μs. If the distance between the upper atmosphere and the Earth's surface is 10 km, how fast should these muons travel to **just** reach the Earth's surface before they decay into other particles? What do you notice about your answer?
2. If muons travel at about 0.999*c* in the upper atmosphere, determine their gamma factor. Click on 'Show or hide tip' to see a hint.

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

1. Muons are observed on the Earth by particle physicists. In the reference frame of the physicists, what is the lifetime of the muons, once it has undergone time dilation by the gamma factor? How fast do the muons need to travel to reach the Earth's surface in this time?
2. In the reference frame of the muons, the atmosphere is moving, and so the distance between the upper atmosphere and the Earth's surface undergoes length contraction. What will the distance be in the reference frame of the muons? How fast will the muons need to travel to reach the surface of the Earth for this distance? Remember, in the reference frame of the muons, their lifetime (2.2 μs) is the proper time.
3. Imagine that Einstein never came up with his theory of special relativity. Some scientists are mystified as to why muons keep reaching the surface of the Earth, because this implies that the muons are travelling faster than the speed of light, and the scientists know that nothing can travel faster than the speed of light. Create a presentation to explain to the scientists how it is possible for the muons to reach the surface of the Earth. You might find some of the following terms helpful:

   Postulate, Time dilation, Length contraction. Proper time interval, Proper length, Inertial reference frame, Speed of light

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

1. $$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1.0\times10^4}{2.2\times10^{-6}} = 4.5\times10^9\ \text{m s}^{-1}\ (2\ \text{s.f.})$$

This is much faster than the speed of light.

2. $$\gamma = \frac{1}{\sqrt{\left(1-\frac{(0.999c)^2}{c^2}\right)}} = 22.4$$

3. $$\Delta t = \gamma t_0 = 22.4\times2.2\times10^{-6} = 4.9\times10^{-5}\ \text{s}$$

So, the muons would need to travel at:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1.0\times10^4}{4.9\times10^{-5}} = 2.0\times10^8\ \text{m s}^{-1}$$

4. $$L = \frac{L_0}{\gamma} = \frac{1.0\times10^4}{22.4} = 447\ \text{m}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{447}{2.2\times10^{-6}} = 2.0\times10^8\ \text{m s}^{-1}$$

Notice that this is exactly the same speed as was calculated in the reference frame of the Earth — there is no absolute correct reference frame.

5 section questions

Student view

Question 1

HL Difficulty:

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

True or false?

The gamma factor can only take a value between 0 and 1:

$$0 < \gamma < 1$$

False ✓

Accepted answers

False, F, false, f

Explanation

Because of the equation for the gamma factor, the value has to be equal to 1 (when travelling at the speed of light) or greater than 1.

Question 2

HL Difficulty:

When an object travels at speeds approaching the speed of light, a stationary observer will observe that the object has undergone 1 length contra... ✓ along its axis of motion. The observer will also observe 2 time dilation ✓ for the object, meaning that time for the object as measured by the stationary observer runs more 3 slowly ✓ .

Accepted answers and explanation

#1 length contraction

#2 time dilation

#3 slowly

General explanation

When an object travels at speeds approaching the speed of light, a stationary observer will observe that the object has undergone length contraction along its axis of motion. The observer will also observe time dilation for the object, meaning that time for the object as measured by the stationary observer runs more slowly.

Question 3

HL Difficulty:

An astronaut on a spaceship is travelling at 0.70c relative to the Earth. The astronaut watches a video, which lasts for one hour in their reference frame. An observer on Earth uses a telescope to watch the same video on the spaceship. How long does the video last for the observer on the Earth?

Student view

1 1.4 hours ✓

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

2 0.71 hours

3 1.0 hours

4 2.8 hours

Explanation

As the spaceship is moving relative to the observer on the Earth, the video will be longer for the observer on the Earth, due to time dilation.

$$v = 0.70c$$

$$\begin{aligned} t_0 &= 1 \text{ hour} \\ &= \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \\ &= \frac{1}{\sqrt{\left(1 - \frac{(0.7c)^2}{c^2}\right)}} \\ &= 1.4 \end{aligned}$$

$$\begin{aligned} \Delta t &= \gamma t_0 \\ &= 1.4 \times 1 \\ &= 1.4 \text{ hours} \end{aligned}$$

Question 4

HL Difficulty:

A metre rule of proper length 1.0 m is viewed on a spaceship travelling very quickly past the Earth. An observer on the Earth measures the metre ruler to be 0.50 m long. How fast is the spaceship travelling?

1 0.87c ✓

2 0.95c

3 2.0c

4 0.50c

Explanation

$L_0 = 1.0 \text{ m}$

Student view

$L = 0.50 \text{ m}$

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

$$L = \frac{L_0}{\gamma}$$

$$\gamma = \frac{L_0}{L} = \frac{1.0}{0.50} = 2.0$$

$$= \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \sqrt{\left(1 - \frac{v^2}{c^2}\right)} = \frac{1}{2} \left(1 - \frac{v^2}{c^2}\right) = \frac{1}{4}$$

$$\frac{v^2}{c^2} = 0.75$$

$$v = \sqrt{0.75}c = 0.87c \text{ (2 s.f.)}$$

Question 5

HL Difficulty:

Which of the following is an explanation for why muons produced in the upper atmosphere are detected by scientists on the surface of the Earth, despite the muons' extremely short lifetime? The distance between the upper atmosphere and the Earth's surface is about 10 km.

1. Time dilation means that time for the muons runs slower than time for the scientists ✓
2. Time dilation means that time for the scientists runs slower than time for the muons
3. Length contraction means that the speed of the muons is faster than the speed of light
4. Length contraction means that the scientists are measuring the distance between the upper atmosphere and the Earth's atmosphere to be much smaller than it would be from the reference frame of the muons

Explanation

When measuring the muons' lifetime, scientists are effectively reading the 'clock' of the muons. In the reference frame of the scientists, the muons' clock runs slow because they are moving.

Proper distance is measured by the scientists because they are not moving relative to the atmosphere. Therefore, the scientists do not observe length contraction.

Nothing will make the muons travel faster than the speed of light, because that would be against the laws of physics, and these laws hold in all inertial reference frames.

If time ran slower for the scientists, then it would run comparatively quicker for the muons, and so they would decay in less than their mean lifetime, which would make it even less likely that they would reach the Earth.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

A. Space, time and motion / A.5 Galilean and special relativity (HL)

Lorentz transformations and the space–time interval (HL)

A.5.6: Lorentz transformation equations (HL) A.5.7: The relativistic velocity addition equation (HL)
A.5.8: The space-time interval between two events (HL) A.5.9: Proper time interval and proper length (HL)

Higher level (HL)

Learning outcomes

By the end of this section you should be able to:

- Determine that the postulates of special relativity lead to the Lorentz transformation equations:

$$x' = \gamma (x - vt) \text{ and } t' = \gamma \left(t - \frac{vx}{c^2}\right)$$

- Explain that the Lorentz transformation equations lead to the relativistic velocity addition equation:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

- Demonstrate that the space—time interval is an invariant quantity and use the equation:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

Galilean transformations work fine when we consider time to be absolute. But the only absolute is the speed of light, which takes the same value for all observers, irrespective of their relative velocities (see section A.5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/special-relativity-and-time-dilation-and-length-contraction-hl-id-46605/)).

As time can differ for observers in different frames of reference, how can we adapt the transformations?

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Lorentz transformations

Lorentz transformations are a way of finding the position and time of an event in the reference frame of a moving observer, based on the position and time of the event in the reference frame of a stationary observer. The two reference frames are inertial (see section A.5.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/reference-frames-and-galilean-relativity-hl-id-46604/)).

Look at the boats in **Figure 1**. The boats are moving relative to each other. The position of the torch as measured by the observer in the boat on the left is x and the position measured by the observer in the boat on the right is x'.

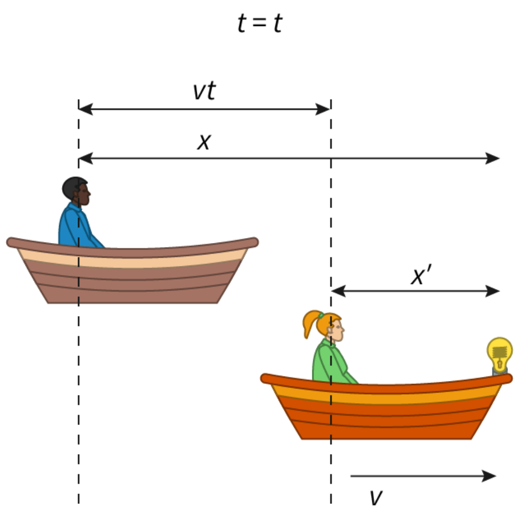

Figure 1. A moving boat and a stationary boat.

More information for figure 1

The image is a diagram illustrating the concept of relative motion between two boats. There are two boats depicted: one with an observer and the other with an observer holding a torch. The top row features the observer seated on the left, while the observer with the torch is on the right in the bottom row.

Both boats have dotted vertical lines extending upwards, labeled with short perpendicular lines indicating positions. The distance from the observer on the moving boat on the bottom is designated as x' and points towards the left of the observer, while the distance on the stationary boat on the top is labeled as x.

There is a series of arrows labeled vt and v which indicate velocity and distances in the diagram. The arrow labeled vt shows the travel of the moving observer over time, while v indicates the velocity of the moving boat in the direction of x, pointing to the right. This imagery corresponds to the study of motion in special relativity, where measurements of distance are dependent on the observer's frame of reference.

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Student view

The two postulates of special relativity are (see section A.5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/special-relativity-and-time-dilation-and-length-contraction-hl-id-46605/)):

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

- The laws of physics apply in the same way in all inertial reference frames.
- The speed of light is the same for all observers, regardless of their relative motion.

These postulates lead to the Lorentz transformation equations, where relativistic effects are taken into account. Unlike the Galilean transformations, the Lorentz transformations apply even when the two inertial frames have a large relative velocity.

The Lorentz transformation equation for the relationship between x and x' is shown in **Table 1**.

Table 1. Lorentz transformation equation for the position of an event.

Equation	Symbols	Units
$x' = \gamma\left(x - vt\right)$	x' = position of event in second reference frame	metres (m)
	γ = gamma factor $\left(\frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}\right)$	unitless
	x = position of event in first reference frame	metres (m)
	v = relative velocity of second reference frame, as viewed from first reference frame	metres per second (m s^{-1})
	t = time measured in first reference frame	seconds (s)

In Galilean transformations, we consider time to be absolute, and $t' = t$. The Lorentz transformation equation for the relationship between t and t' is shown in **Table 2**.

Table 2. Lorentz transformation equation for the time of an event.

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Equation	Symbols	Units
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$	t' = time measured in the second reference frame	seconds (s)
	γ = gamma factor $\left(\frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}\right)$	unitless
	t = time measured in first reference frame	seconds (s)
	v = relative velocity of second reference frame, as viewed from first reference frame	metres per second (m s^{-1})
	x = position of event in first reference frame	metres (m)
	c = speed of light (3.00 × 10^8 m s^{-1})	given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet

Note that the Lorentz transformations as shown in the tables above can only be used if the two reference frames are initially synchronised so that $x = x' = 0$ when $t = t' = 0$.

The derivation of the Lorentz transformation equations is not required in the DP physics course.

Concept

Differences between measurements made by observers in different frames are not caused by the time it takes for light to reach the observers.

When we say that the times and locations of events are measured in a frame of reference, we imagine that there is always an observer next to any event as it occurs. Another way to think of this is to imagine that in any inertial frame, there is a network of closely-spaced sensors that measure the times and positions of events in that frame.

Student view

Worked example 1

A spaceship is flying past the Earth at a speed of $0.60c$ towards a gamma ray burst event. In the reference frame of the Earth, the event is recorded at a time of 2.0 s and a position of 4.0×10^8 m. Determine the position and time measurements recorded by an observer on the spaceship. Both spaceship and Earth stopwatches begin timing when the spaceship is level with the Earth.

Try to rewrite the position and time equations with x and t as the subjects.

If the relative velocity of frame F' from the point of view of frame F is v, what is the relative velocity of frame F from the point of view of frame F'?

Click on 'Show or hide solution' to see the answer.

Consider the Lorentz equation for transforming a position x in frame F to a position x' in frame F':

$$x' = \gamma\left(x - vt\right)$$

The quantity on the left-hand side of the equation is measured in frame F, and the quantities on the right-hand side are measured in frame F'. Since the laws of physics must be the same in both frames, we can also write

$$x = \gamma\left(x' - v't'\right)$$

where v' is the velocity of frame F from the point of view of frame F'.

The time in frame F' therefore is given by:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

The two equations above are sometimes called the inverse Lorentz transformations. They are used when an observer does not know, and wants to find, the position or the time of an event in their own frame, but does know both of these as measured in a different inertial frame. In other words, these equations can be used to undo the effects of the Lorentz transformations.

To solve the problem:

Solve for γ first:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(1.8 \times 10^8)^2}{(3 \times 10^8)^2}}} = 1.25$$

Relativity of Simultaneity term:

Overview
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$$\frac{vx}{c^2} = \frac{(1.8 \times 10^8)(4.0 \times 10^8)}{(3.0 \times 10^8)^2} = 0.8s$$

Thus:

$$t' = 1.25\,(2.0 - 0.8) = 1.5$$

Worked example 2

A spaceship is stationary relative to a star. The spaceship launches a small robotic probe. The probe travels towards the star in a straight line, at speed 0.40 c relative to the spaceship. At the moment of launch, both the spaceship and the probe measure the time as 0 and their position as 0.

The star releases a sudden burst of radiation, called a flare, at time 0.50 s and position 1.2×10^9 m away, as measured by the probe.

What are the position and time of the flare as measured by the spaceship?

In this question, we need to find the space and time coordinates of an event in the spaceship's reference frame, given its space and time coordinates in another reference frame.

Solution steps	Calculations
Step 1: Consider the frames and quantities involved.	Let x and t be the position and time of the flare in the fram the spaceship (which is the same as the reference frame of star). Let x' and t' be the position and time of the flare in the frar of the probe. $v = 0.40c$ $\quad = 1.2 \times 10^8 \text{ m s}^{-1}$ $x' = 1.2 \times 10^9 \text{ m}$ $t' = 0.50 \text{ s}$
Step 2: Calculate γ for the two frames.	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ $= \frac{1}{\sqrt{1 - \frac{(1.2 \times 10^8)^2}{c^2}}}$ $= 1.09$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Solution steps	Calculations
Step 3: Apply an inverse Lorentz transformation to find x.	$x = \gamma\left(x' + vt'\right)$ $= 1.09\left(1.2 \times 10^{9} + 1.2 \times 10^{8} \times 0.50\right)$ $= 1.4 \times 10^{9}\ \text{m}\ \ (2\ \text{s.f.})$
Step 4: Apply an inverse Lorentz transformation to find t.	$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$ $= 1.09\left(0.50 + \frac{1.2 \times 10^{8} \times 1.2 \times 10^{9}}{\left(3.0 \times 10^{8}\right)^{2}}\right)$ $= 2.3\ \text{s}\ \ (2\ \text{s.f.})$

The Lorentz transformation equations can be used to find intervals of position, Δx, and time, $x\Delta t$:

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right)$$

where:

- $\Delta x'$ is the separation of events in the second reference frame, F'
- Δx is the separation of events in the first reference frame, F
- v is the relative velocity of frame F', as viewed from frame F
- t is the time interval measured in frame F.

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

where:

- $\Delta t'$ is the time interval measured in the second reference frame, F'
- Δt is the time interval measured in the first reference frame F
- v is the relative velocity of frame F', as viewed from frame F
- Δx is the separation of events in the frame F.

The derivation of these equations is not required in the DP physics course.

Concept

Why do the Lorentz transformation equations for intervals look different from the length contraction and time dilation equations introduced in section 5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/special-relativity-and-time-dilation-and-length-contraction-hl-id-46605/)?

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

The length contraction formula only applies to the length of an object or to the distance between two objects that are in fixed positions relative to each other. The Lorentz equation for position intervals can also be used to calculate the distance between objects that are moving relative to each other, or the distance between two separate events.

The time dilation formula only applies to two events that occur at the same place in one of the two frames being considered. The Lorentz equation for time intervals can also be applied to time intervals between events that occur in different locations in both frames.

Worked example 3

Two spaceships travel in the same direction along the same line, both with constant speed $0.60c$. According to observers who are at rest in each spaceship, the separation of the spaceships is 2.0 km.

One spaceship sends a radio signal to the other spaceship. (Note that radio is a form of electromagnetic radiation, like light.)

1. According to an observer on Earth, what is the distance between the two spaceships?
2. How long does it take, in the spaceships' reference frame and in Earth's reference frame, for the signal to travel from one spaceship to the other if it is:

   (a) sent by the spaceship that is behind and received by the spaceship that is ahead?
   (b) sent by the spaceship that is ahead and received by the spaceship that is behind?

Solution steps	Calculations
Step 1: Consider the frames and quantities involved.	Let Δx be the separation of the spaceships in the spaceships reference frame, F. Let Δt be the time taken for the signal to from one spaceship to the other, in the spaceships' reference Let $\Delta x'$ be the separation of the spaceships in the Earth's ref frame, F'. $\Delta x = 2.0\ \mathrm{km} = 2000\ \mathrm{m}$ $v = 0.60c = 1.8 \times 10^{8}\ \mathrm{m\ s^{-1}}$

Student view

Overview
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Solution steps	Calculations
Step 2: Calculate the time, in the spaceship frame, between sending and receiving the signal.	In the spaceships' reference frame, the distance between the spaceships is always $2000\ \mathrm{m}$. Light travels at speed c relative observer in an inertial frame, so: $$\begin{aligned}\Delta t &= \frac{\Delta x}{c} \\ &= \frac{2000}{3.0 \times 10^8} \\ &= 6.67 \times 10^{-6}\ \mathrm{s}\ \ (3\ \mathrm{s.f.})\end{aligned}$$ (This is the same whichever spaceship sends the signal.)
Step 3: Calculate γ for the two frames.	$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-\frac{(1.8\times 10^8)^2}{(3.0\times 10^8)^2}}} \\ &= 1.25\end{aligned}$$
Step 4: Use the length contraction equation to answer question 1, finding the spaceship separation in Earth's frame.	$$\begin{aligned}\Delta x' &= \frac{\Delta x}{\gamma} \\ &= \frac{2000}{1.25} \\ &= 1600\ \mathrm{m}\end{aligned}$$ (The length contraction equation is applicable here because t spaceships are in fixed positions relative to each other — like two ends of an object.)

Student
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Overview
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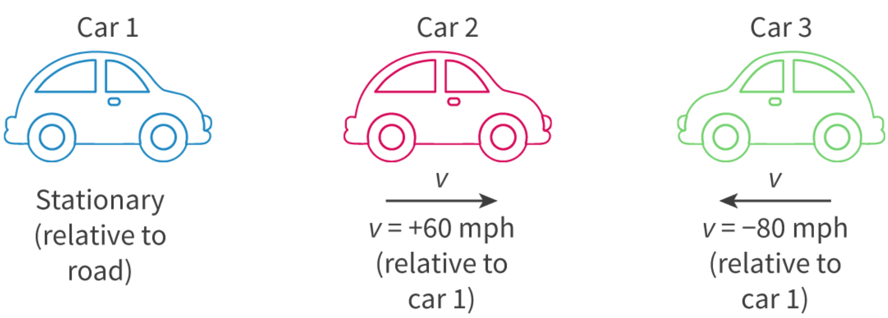
Solution steps	Calculations
Step 5: Use Galilean relativity in Earth's frame to find the travel time of the signal in 2a.	In Earth's frame, the speed of the radio signal is c and the dis between the spaceships is $1600\ \text{m}$. However, the receiving spaceship continues moving while the radio signal travels tow so the light has an additional distance to travel. We can apply Galilean relativity to find the signal's travel time we are now only considering quantities in Earth's frame). Take displacement as the location where the signal is emitted. The is received when the signal's displacement, $c\Delta t$, equals the displacement of the receiving spaceship, $1600 + 0.60c\Delta t$, sc meet when: $c\Delta t = 1600 + 0.60c\Delta t$ $\Delta t = \frac{1600}{(1-0.60)\times 3.0\times 10^{8}}$ $= 1.33\times 10^{-5}\ \text{s}\ \ (3\ \text{s.f.})$ (The time dilation equation cannot be used here, because the events — sending and receiving the signal — occur at differe locations in both frames.)
Step 6: Use Galilean relativity in Earth's frame to find the travel time of the signal in 2b.	In this scenario, the motion of the receiving spaceship is towa transmitting ship, so from Earth's point of view the light has le distance to travel. $c\Delta t = 1600 - 0.60c\Delta t$ $\Delta t = \frac{1600}{(1+0.60)\times 3.0\times 10^{8}}$ $= 3.33\times 10^{-6}\ \text{s}\ \ (3\ \text{s.f.})$

The relativistic velocity addition equation

Figure 2 shows three cars, moving relative to each other.



Student view

Figure 2. Three cars, moving relative to each other.

More information for figure 2

The image is a diagram illustrating three cars, labeled Car 1, Car 2, and Car 3, with their relative movements. Car 1 is depicted as stationary relative to the road. Car 2 is moving at +60 mph relative to Car 1, indicated by a rightward arrow, showing the direction of motion. Car 3 is moving at -80 mph relative to Car 1, indicated by a leftward arrow. The diagram shows each car with respective directional arrows and velocity notations to highlight their speeds and direction relative to Car 1.

[Generated by AI]

We can give the velocity of car 2 relative to car 3 as:

$$\begin{aligned} v_{32} &= v_2 - v_3 = 60 - (-80) \\ &= 140 \text{ mph} \end{aligned}$$

Let us now think in relativistic terms. **Figure 3** shows three objects, X, Y and Z. (If you find it helpful, you can imagine that Z is Earth and Y and X are two spaceships.) Objects X and Y are moving in the same direction at different, known velocities u and v relative to object Z.

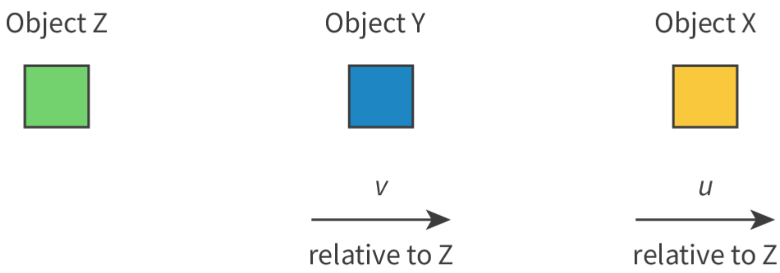


Figure 3. Velocities of objects X and Y moving in the same direction relative to a third object, Z.

More information for figure 3

The diagram illustrates three objects labeled as X, Y, and Z. Each object is represented as a colored square with Object Z in green, Object Y in blue, and Object X in yellow.

Object Z is positioned on the left side of the diagram. Object Y is in the middle, and Object X is on the right. Two arrows represent the velocity of Objects Y and X, with labels 'v relative to Z' and 'u relative to Z,' respectively. These arrows indicate the direction of motion towards the right, implying that both Y and X move away from Z. The diagram visually represents the relative velocities of objects Y and X with respect to object Z.

[Generated by AI]

What is the velocity of Y as measured in the reference frame of X?

Overview (/study/ap aa-hl/sid-423-cid-762593/c

Using u' to represent this velocity, and applying Galilean relativity:

$$u' = u - v$$

Figure 4 shows the same three objects, but this time X is moving in the opposite direction to Y.

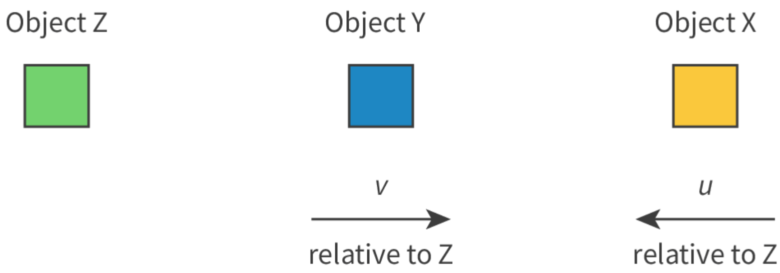


Figure 4. Velocities of objects X and Y moving in the opposite direction relative to a third object, Z.

More information for figure 4

The diagram illustrates three objects labeled Z, Y, and X, each represented by colored squares: green for Z, blue for Y, and yellow for X. The objects are arranged horizontally with object Z on the left, object Y in the middle, and object X on the right. Arrows below objects Y and X indicate their velocities relative to Z. Object Y's velocity is represented by an arrow labeled 'v' pointing to the right, indicating its movement direction. Object X's velocity has an arrow labeled 'u' pointing to the left, indicating movement in the opposite direction of Y relative to Z.

[Generated by AI]

If we apply Galilean relativity (section A.5.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/reference-frames-and-galilean-relativity-hl-id-46604/)), the velocity of Y relative to X is still $u' = u - v$, but the result will be different because this time the vectors v and u have opposite signs.

What if X and Y are travelling towards Z from opposite sides, both with speed $0.6c$ relative to Z? In this case we find that the speed of Y relative to X is $1.2c$. This cannot be correct, because no observer ever measures an object's velocity to be greater than the speed of light.

In situations where relative speeds are significant fractions of the speed of light, Galilean relativity does not give accurate results. Instead we need to use relativistic velocity addition equations. These can be derived from the Lorentz equations and are shown in **Table 3**. (The derivation is not required in the DP physics course.)

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Table 3. Equation for relativistic velocity addition.

Equation	Symbols	Units
$u' = \dfrac{u - v}{1 - \dfrac{uv}{c^2}}$	u' = velocity of X relative to Y	metres per second (m s^{-1})
	u = velocity of X relative to Z	metres per second (m s^{-1})
	v = velocity of Y relative to Z	metres per second (m s^{-1})
	c = speed of light ($3.00 \times 10^8\ \text{m s}^{-1}$)	given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet

Worked example 4

A spaceship is travelling away from Earth at a speed of $0.60c$ relative to Earth. The spaceship fires a rocket in the forward direction (away from Earth). The rocket travels at speed $0.80c$ relative to the spaceship.

Determine the speed of the rocket, in terms of c, in the reference frame of the Earth.

Solution steps	Calculations
Step 1: Consider the quantities involved.	Consider the two known velocities relative to the object the in common — the spaceship. Taking the velocity of the rocket relative to the spaceship tc positive, the velocity of the Earth relative to the spaceship i: negative because it is in the opposite direction: $u = \text{velocity of rocket relative to spaceship} = 0.80c$ $v = \text{velocity of Earth relative to spaceship} = -0.60c$ $u' = \text{velocity of Earth relative to rocket}$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Solution steps	Calculations
Step 2: Use the relativistic velocity addition equation to calculate the relative speed of the rocket and Earth.	$u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$ $= \dfrac{0.80c-(-0.60c)}{1-\dfrac{0.80c \times -0.60c}{c^2}}$ $= \dfrac{1.40c}{1+0.48}$ $= 0.95c\ (2\ \text{s.f.})$ So the speed of the rocket relative to the Earth is $0.95c$.

Study skills

Do not worry too much about which quantity is u and which quantity is v. Swapping these quantities will give an answer of the same magnitude but opposite direction (+ instead of — and vice versa). Most questions will only be concerned with the **magnitude** of the relative velocity. If you need to know the direction, you can work it out by considering Galilean relativity.

An alternative way to deal with signs when using the relativistic velocity addition equation is to treat all of the quantities as speeds (ignoring direction and therefore making them all positive), but then consider how you would calculate the required speed if you were applying Galilean relativity:

$u' = v + u$ (for v and u in opposite directions)

or

$u' = v - u$ (for v and u in the same direction)

If you would use the first equation, then use the relativistic equation shown in **Table 3**. If you would use the second equation, then use the relativistic equation with the signs changed:

$$u' = \frac{u+v}{1+\dfrac{uv}{c^2}}$$

The space—time interval

An **invariant quantity** is one that is constant across all reference frames. For example, the speed of light, c, proper length, L_0, and proper time interval, Δt_0, are all invariant quantities.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

From the Galilean transformations, we know that position, *x*, is not invariant across all reference frames. At relativistic speeds, time is also no longer invariant, as shown by the Lorentz transformations for time. But *c* is constant in all reference frames.

Drag and drop the quantities in **Interactive 1** into the correct columns in the table to show whether they are variant or invariant.

Check

Interactive 1. Classifying Variables as Variant or Invariant.

More information for interactive 1

A drag-and-drop interactivity consists of two labeled sections, "Variant" and "Invariant," each represented by an empty box. Below these sections are draggable tiles labeled with various physical quantities, including time interval (Δt), proper length (L_0), length (L), speed of light (c), position (x), and proper time interval (Δt_0). Users are expected to drag and drop these items into the appropriate category. A "Check" button at the bottom left allows users to verify their choices.
An invariant quantity is the quantity that remains a constant in all reference frames, and a variant quantity is one that varies in all reference frames. The invariant quantities are proper length (L_0), speed of light (c), and proper time interval (Δt_0). The variant quantities are time interval (Δt), length (L), and position (x).
The user can interact with the interface by attempting to place the tile named “Time interval (Δt)" for example, into one of the categories. The draggable items are still available for selection, and the interface remains neutral, awaiting further input. Once the user has completed a sorting attempt and pressed the "Check" button. Feedback is provided in the form of color-coded responses: correct answers are highlighted in green with a checkmark, while incorrect ones are marked in red with a cross. A progress indicator at the bottom shows, for example, "2/6," suggesting that the user has correctly sorted two out of six items. A "Retry" button allows the user to make corrections and try again.

Although position and time are variant quantities, they can be combined in a particular way to produce an invariant quantity called the space–time interval.

A space–time interval, Δs, is not a period of time, or a distance in space, but a combination of the two. It is a measurement of **space–time**. **Figure 5** shows a time interval and a space interval.

Student view

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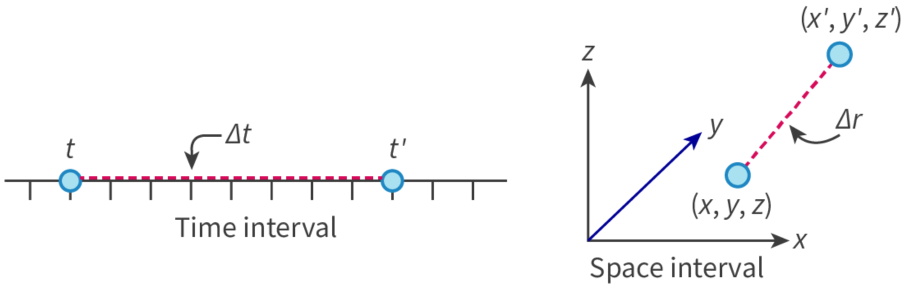


Figure 5. A time interval and a space interval.

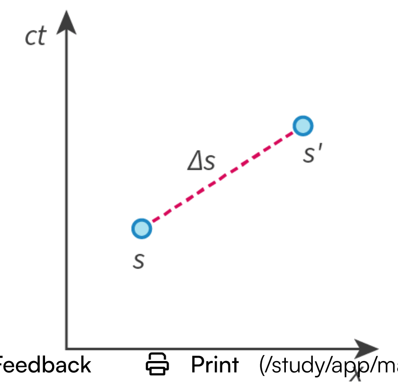
More information for figure 5

The image is a diagram with two main elements illustrating time and space intervals. On the left, a horizontal line labeled 'Time interval' has two points marked 't' and 't'' connected by a dashed red line labeled 'Δt'. This represents a time interval. On the right, there is a three-dimensional axis labeled with 'x', 'y', and 'z' representing spatial dimensions. Two points are plotted with coordinates '(x, y, z)' and '(x', y', z')' connected by a dashed red line labeled 'Δr', indicating a space interval. The diagram is designed to visually demonstrate the concept of space-time intervals.

[Generated by AI]

A space–time interval is a change experienced in the space of the Universe through which we are constantly moving.

Figure 6 shows a space-time interval, Δs, on a space–time diagram. You will learn more about space-time diagrams in section A.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/space-time-diagrams-and-simultaneity-hl-id-46607/). We use ct for the time variable of space–time, where c is a unitless speed of light (3.00×10^8) and t is the proper time in seconds (s).



Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/special-relativity-and-time-dilation-and-length-contraction-hl-id-46605/print/) Assign

Figure 6. A space-time diagram showing a space-time interval.

More information for figure 6

The image is a space-time diagram with two axes. The horizontal axis is labeled 'x' and the vertical axis is labeled 'ct', indicating the product of the speed of light and time. There are two points labeled 's' and 's'' on the diagram. The points are connected by a dashed red line representing a space-time interval, labeled as 'Δs'. The

Student view

Overview
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point 's' is positioned nearer to the origin, and 's'' is further along both axes, indicating a progression in both space and time.

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Figure 7 shows a space-time diagram with two space–time intervals, Δs_1 and Δs_2.

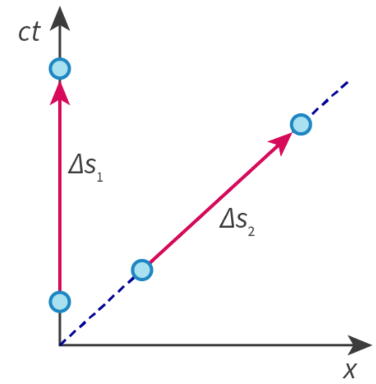

Figure 7. Space—time diagram with two space—time intervals, Δs_1 and Δs_2.

More information for figure 7

The image is a space-time diagram with two primary intervals labeled Δs1 and Δs2. The diagram features an X-axis, denoted by 'x', and a Y-axis, shown as 'ct', indicating time. The interval Δs1 is represented by a vertical line parallel to the Y-axis, indicating a constant position with changing time. In contrast, Δs2 is depicted as a diagonal line, suggesting a movement in both space and time. The lines have arrows indicating direction, starting from the bottom-left corner. Each interval has circular markers at endpoints. This diagram emphasizes the spatial and temporal relationships.

[Generated by AI]

We can see that space–time interval Δs_1 is constant in position (in our reference frame) and varying in time. This could be an object at rest relative to us, viewed at two different times (time is on the y-axis in a space–time diagram).

Space–time interval Δs_2 is a line at 45° to either axis. The value for Δx is the same as the time $c\Delta t$. This means that the object is travelling 1 light year in 1 year, which means that the object is travelling at the speed of light. This will be explained in more detail in section A.5.4 (/study/app/math-aa-hl/sid-423-cid-762593/book/space-time-diagrams-and-simultaneity-hl-id-46607/).

The equation for the space–time interval between two events is shown in **Table 4**.

Table 4. Equation for the space—time interval between two events.

Student
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Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Equation	Symbols	Units
$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$	Δs = space—time interval	metres (m)
	c = speed of light (3.00×10^8 m s^{-1})	given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) of the DP physics data booklet
	Δt = change in time	seconds (s)
	Δx = change in position	metres (m)

For space–time interval, Δs_1:

$$(\Delta s_1)^2 = (c\Delta t)^2 - (0)^2$$

(as there is no change in position)

Therefore:

$$\Delta s_1 = c\Delta t$$

(the space–time interval that has elapsed between the two points in time that the object is recorded)

For space–time interval, Δs_2:

$$(\Delta s_2)^2 = (c\Delta t)^2 - (\Delta x)^2$$

As the object is travelling at the speed of light:

$$(c\Delta t)^2 = (\Delta x)^2$$

Therefore:

$$(\Delta s)^2 = 0$$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Concept

For an object travelling at the speed of light, such as a photon, the space—time interval Δs, is always zero:

$$\Delta s = 0$$

For an object travelling at less than the speed of light, the space—time interval, Δs, is always greater than 0:

$$\Delta s > 0$$

Exercise 1

Click a question to answer

Higher level (HL)

Worked example 5

Izumi observes two events, separated by Δx = 5 light years and Δt = 10 years. Rebecca, travelling very fast relative to Izumi, observes the same two events, but with $\Delta x'$ = 4 light years. Determine the time interval (t') that Rebecca observes between the two events. Give your answer to 2 significant figures.

As Δs is an invariant quantity, the events will have the same space—time interval, regardless of whether they are observed by Izumi or Rebecca.

For Izumi:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

For Rebecca:

$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2$$

Using the unit of years for Δt, light years for c*Δt and c for the speed of light:

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$(1 \times 10)^2 - (5)^2 = (1 \times t')^2 - (4)^2$$

$$t' = 9.5 \text{ years } (2 \text{ s.f.})$$

Student view

Overview
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Work through the activity to check your understanding of Lorentz transformations.

Activity

- **IB learner profile attribute:**
  - Knowledgeable
  - Inquirer
- **Approaches to learning:**
  - Thinking skills — Applying key ideas and facts in new contexts
  - Research skills — Using search engines and libraries effectively
- **Time required to complete activity:** 20 minutes
- **Activity type:** Individual activity

You are going to think about how relativistic effects impact our everyday lives.

1. Imagine two rugby players are sprinting towards each other at 10 m s^{-1}.

   (a) What is their relative speed to one another in the Newtonian calculation?
   (b) What is their relative speed to one another in the relativistic calculation?
   (c) What is the percentage difference between these two values?

2. Two fighter jets are doing a fly past, travelling in opposite directions, at a speed of 400 m s^{-1}.

   (a) What is their relative speed to one another in the Newtonian calculation?
   (b) What is their relative speed to one another in the relativistic calculation?
   (c) What is the percentage difference between these two values?

3. On Earth, we struggle to get close even to tiny relativistic effects. Despite this, scientists do have to account for relativistic effects. GPS relies on very accurate clocks on satellites as they orbit the Earth. As the satellites are moving relative to the Earth, scientists have to constantly adjust the times on the satellites to account for relativistic time dilation.

   Research one other practical impact of relativistic effects on technology. Write a short summary to express your findings in a concise way.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

1. (a) 20 m s^{-1}
   (b) 19.999999999999999 m s^{-1}
   (c) Almost zero
2. (a) 800 m s^{-1}
   (b) 799.999999999993 m s^{-1}
   (c) Almost zero

5 section questions

Question 1

HL Difficulty:

Which quantities are invariant?

1 Space—time interval ✓

2 Length

3 Position

4 Time

Explanation

The space—time interval is an invariant quantity. It is the same for all observers in all reference frames.

Question 2

HL Difficulty:

Which is the correct rearrangement of a Lorentz transformation equation?

1 $x = \gamma\left(x' + vt'\right)$ ✓

2 $x = \gamma\left(x' - vt\right)$

3 $x = \gamma\left(vt - x'\right)$

4 $x = \dfrac{\left(x' + vt\right)}{\gamma}$

Student view

Explanation

$x' = \gamma(x - vt)$ can be rearranged to $x = \gamma(x' + vt')$.

Overview
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Question 3

HL Difficulty:

An event occurs in Amartya's reference frame at t = 0.1 s and x = 10^7 m. What is the position x' of the event in Felix's reference frame if Felix is travelling at 0.2c (in the positive direction)

1 4×10^6 m ✓

2 4×10^5 m

3 4×10^7 m

4 4×10^8 m

Explanation

$t = 0.1$ s

$x = 10^7$ m

$$\begin{aligned} v &= 0.2c \\ &= 6.0 \times 10^7 \text{ m s}^{-1} \\ &= \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \\ &= \frac{1}{\sqrt{\left(1 - \frac{0.04c^2}{c^2}\right)}} \\ &= 1.02 \end{aligned}$$

$$\begin{aligned} x' &= \gamma (x - vt) \\ &= 1.02 \times \left[\left(10^7 - \left(6.0 \times 10^7 \times 0.1\right)\right]\right. \\ &= 4.08 \times 10^6 \text{ m} \\ &= 4 \times 10^6 \text{ m (1 s.f.)} \end{aligned}$$

Question 4

HL Difficulty:

Nia is in a spaceship that is stationary relative to the Earth. Emily is flying at a velocity of +0.80c relative to Nia. Yasmin is flying in a spaceship at a velocity −0.60c relative to Nia. What is Emily's speed relative to Yasmin?

1 $0.95c$ ✓

2 $0.20c$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

3 $1.4c$

4 $1.0c$

Explanation

$u = +0.8c$

$v = -0.6c$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$
$$= \frac{0.8c - (-0.6c)}{1 - \frac{(0.8c \times -0.6c)}{c^2}}$$
$$= \frac{1.4c}{1.48}$$

$$u' = 0.9459c$$
$$= 0.95c \text{ (2 s.f.)}$$

Question 5

HL Difficulty:

Misa is travelling in a spaceship at 0.50*c* relative to the Earth. Haruto is on the Earth observing Misa's journey. What is Misa's space—time interval, Δ*s*, after one year, as observed by Haruto?

1 0.87 light years ✓

2 0.50 light years

Explanation

As Misa is travelling at half the speed of light (0.5*c*), in one year, she has travelled 0.5 light years.

$$x = 0.5 \text{ light years}$$

$$t = 1 \text{ year}$$

Using the unit of years for *Δt*, light years for *Δx* and *c* for the speed of light:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$(\Delta s)^2 = (1 \times 1)^2 - (0.5)^2$$

$$\Delta s = 0.8660 \text{ light years}$$
$$= 0.87 \text{ light years (2 s.f.)}$$

Student view

As space—time intervals are invariant quantities, the space—time interval in Misa's frame of reference will be the same as the space—time interval in Haruto's frame of reference.

Space-time diagrams and simultaneity (HL)

Higher level (HL)

Learning outcomes

- Describe space—time diagrams.
- Explain the relativity of simultaneity.

Space—time diagrams

For a distance–time graph, distance goes on the *y*-axis and time goes on the *x*-axis (see subtopic A.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43128/)). For a space–time diagram, time goes on the *y*-axis (see section A.5.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/lorentz-transformations-and-the-space-time-interval-hl-id-46606/)). This is because time is no longer an invariant quantity, and so no longer needs to go on the *x*-axis. Position, on the *x*-axis, is also not invariant.

The symbol for time in a space–time diagram is not *t*. This is because we multiply time values by *c*, the speed of light, where speed has the unit of *c* (a beam of light has a speed of 1*c*). The speed of light is a constant for all reference frames, so multiplying the time by *c* has no effect on the shape of the diagram. We do this because we want to move away from the concept of time and distance as being two separate entities.

Figure 1 shows a space–time diagram.

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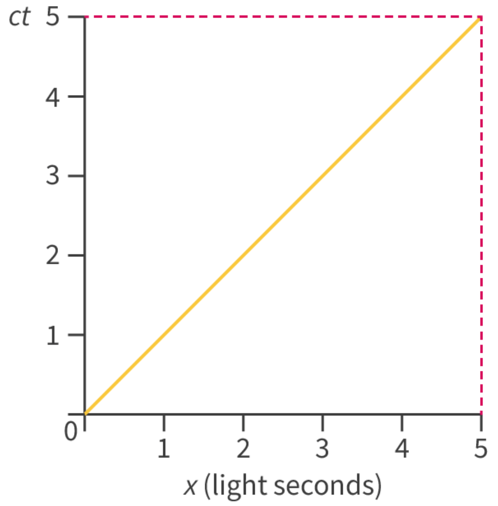

Figure 1. A space—time diagram.

More information for figure 1

The image is a space-time diagram featuring a plot with two axes. The X-axis, labeled as 'x (light seconds)', ranges from 0 to 5. The Y-axis, labeled as 'ct', ranges from 0 to 5 as well. The diagram shows a diagonal yellow line starting at the origin (0, 0) and going to a point at (5, 5), forming a 45-degree angle with the axes. This line represents an object traveling a distance equivalent to one light second per second, depicting the relationship between space and time. The diagram includes dotted lines that extend from the end of the diagonal line horizontally and vertically to the respective axes.

[Generated by AI]

The line at 45° means that the object travels a distance of 3.00×10^8 m in one second. One light second is the distance travelled by light in one second. So we can consider one light second of distance as being equivalent to one second of time for a beam of light.

If the object represented by the 45° line travels 3.00×10^8 m in one second, then it is travelling at the speed of light. As we know, this speed is the fastest anything in the Universe can travel. So the line $y = x$ (at 45° to both axes), is the **smallest** gradient we will see on a space–time diagram. Nothing can travel faster, so nothing can have a gradient smaller than 1.

Worked example 1

Which line (A, B, C, D or E) on the space–time diagram shows an object with the greatest speed?

Student
view

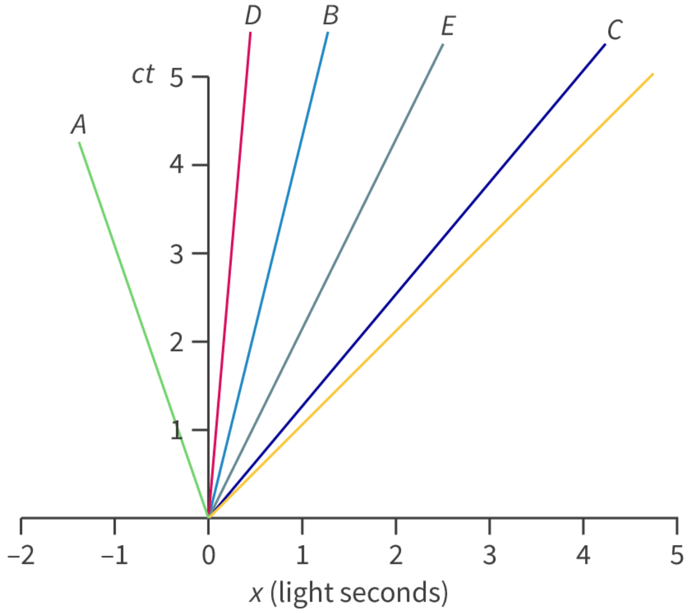

Figure 2. Identify the object with the greatest speed.

More information for figure 2

The image is a space-time diagram with X-axis labeled 'x (light seconds)' ranging from -2 to 5 and Y-axis labeled 'ct' ranging from 0 to 5. There are five lines labeled A, B, C, D, and E. Line A is the steepest and appears on the left side, representing a slower speed. Line B is less steep, closer to the vertical axis than A. Line C is almost vertical, suggesting the greatest speed. Line D is slightly steeper than C but less than B. Line E is between B and C in steepness. All lines originate from the same point at x=0, ct=0, indicating they start from the same event in space-time.

[Generated by AI]

For a space—time diagram, time is on the *y*-axis and position is on the *x*-axis. The speed is given by

$$\frac{1}{\text{gradient}}$$

Therefore, the smallest gradient is the greatest speed.

Line C has the smallest gradient and the greatest speed.

Imagine you are travelling through space–time in a spaceship. Your spaceship is travelling at $0.2c$. **Figure 3** shows the space–time diagram for your spaceship. The axes x and ct are based on the reference frame of a scientist on the Earth. (The yellow line represents the path of a light ray through space–time.)

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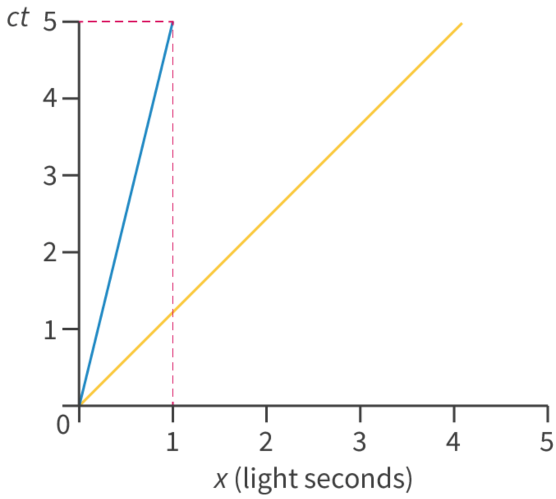

Figure 3. A space—time diagram for a spaceship.

More information for figure 3

The image is a space-time diagram with axes labeled 'ct' for time in seconds on the vertical axis and 'x' for distance in light seconds on the horizontal axis. The time axis ranges from 0 to 5 units, while the distance axis ranges from 0 to 5 light seconds. A yellow line with a gradient of 1 represents the path of a light ray, starting from the origin and extending diagonally upwards to the right. A blue line indicates the path or world line of a spaceship, starting from the origin, rising more vertically than the yellow light ray line, and reaching a point slightly beyond the 1 light second mark on the x-axis by a ct value of 5.

[Generated by AI]

The line followed by your spaceship is called a **world line**. The world line of the scientist is a line $x = 0$ (a line running along the y-axis). Light is represented by a world line of gradient 1.

From the reference frame of your spaceship, you are not moving, and it is the scientist on the Earth who is moving backwards at $0.2c$. Your world line, therefore, would be the line at $x = 0$.

It is possible to represent more than one reference frame on the same space–time diagram. We just need to draw some extra axes, as shown in **Figure 4**. The scientist's axes are ct and x, and your axes are ct' and x'.

Student view

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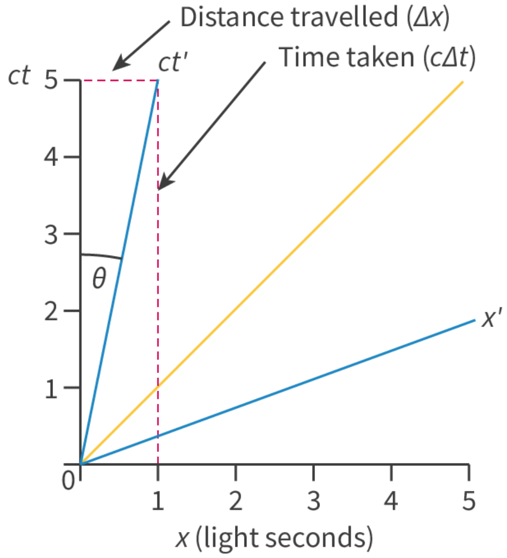

Figure 4. A space-time diagram for a spaceship showing more than one frame of reference.

More information for figure 4

The image is a space-time diagram displaying several frames of reference, primarily marked by the axes labeled ct, ct', x, and x'. On the vertical axis labeled ct, the scale ranges from 0 to 5, indicating time in light seconds. The horizontal axis represents x, also in light seconds, ranging from 0 to 5. A significant feature of this diagram is the angled lines depicting different frames of reference.

The line labeled ct' is at an angle from the ct axis, demonstrating a change in time dilation, while the line labeled x' is angled from the x axis, representing length contraction. This configuration visually expresses how both time and length measurements differ depending on the observer's frame of reference, a central aspect of relativity theory.

A dotted vertical line at x = 1 highlights a position in space, and an angle θ is formed between the ct axis and the ct' line. Notably, annotations near the ct' line indicate 'Distance travelled (Δx)' and 'Time taken (cΔt)', emphasizing the fundamental concepts of time and space intervals in relativistic physics.

[Generated by AI]

Notice that it is not only the ct-axis that changes angle, but the x-axis changes angle as well. The axes are no longer at right angles to each other, but have moved towards the 45° line, by the same angle, θ. This matches our understanding of length contraction and time dilation – length and time change depending on the frame of reference (see section A.5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/special-relativity-and-time-dilation-and-length-contraction-hl-id-46605/)).

Let us look at angle θ in the space–time diagram in **Figure 4**. The adjacent is time taken in c seconds, or the time it takes light to travel one metre. The opposite is the distance travelled in metres.

Student view

Using trigonometry:

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$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{\text{distance}}{\text{speed of light} \times \text{time}}$$
$$= \frac{\text{speed of object}}{\text{speed of light}}$$

This gives the relationship shown in **Table 1**.

Table 1. Equation for relationship between θ and speed of object.

Equation	Symbols	Units
$\tan\theta = \frac{v}{c}$	θ = angle between world line and time axis	degrees (°)
	v = speed of object	metres per second (m s^{-1})
	c = speed of light (3.00 × 10^{8} m s^{-1}) given in section 1.6.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/fundamental-constants-id-45155/) the DP physics data booklet	metres per second (m s^{-1})

Tan θ gives us the value of the speed of the object, expressed in c. Note that this only applies to the angle between the *ct* and *ct*' axes. Because of this, the largest the angle can be is 45°, as this gives:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{1}{1}$$
$$= c\ (\text{speed of light})$$

Exercise 1

Click a question to answer

Student view

Higher level (HL)

Drag and drop each value for velocity next to the correct space–time diagram in **Interactive 1.**

Check

Interactive 1. Identifying Velocity Using Space-Time Diagrams.

More information for interactive 1

This is a drag-and-drop interactivity that presents a set of space-time diagrams labeled (a), (b), (c), and (d), each showing the trajectory of an object through space over time. The horizontal axis of each diagram is labeled x (light seconds), representing position in space, and the vertical axis is labeled ct, representing the time coordinate scaled by the speed of light. Each diagram contains a blue line, known as a wordline, that illustrates the path of an object. On the right side of the activity, four draggable velocity labels are displayed. These labels are;

1. $v = +0.8c$, 2. $v = -0.5c$, 3. $v = +1.0c$, and 4. $v = 0.0c$.
   The task is to correctly drag and drop each velocity label next to the space-time diagram that corresponds to the object's motion shown by the blue line.

In diagram (a), the blue line has a negative slope, showing the object moving toward the origin with a velocity less than the speed of light in the negative x-direction. In diagram (b), the blue line is vertical, indicating that the object remains at a constant position over time, meaning it is at rest. In diagram (c), the blue line rises diagonally and ends at a point with $x = 4$ and $ct = 5$, showing a slope of $\frac{4}{5}$, which corresponds to a positive velocity less than the speed of light. In diagram (d), the blue line is at a 45-degree angle, indicating the object is moving at the speed of light (c).

By dragging and placing each velocity label next to the appropriate diagram, learners visually associate velocity values with corresponding worldlines in special relativity. This interactive exercise reinforces the concept that the slope of a line in a space-time diagram represents the object's velocity and helps to distinguish between different relative motions, including rest, subluminal, and light-speed motion.

Solution:
Here is the correct matching of the velocity labels with the space-time diagrams:
Diagram (a):
The object is moving in the negative x-direction at a moderate speed (the line slopes downward to the right).
Answer: $v = -0.5c$
Diagram (b):
The worldline is vertical, meaning the object is stationary in space; it doesn't move over time.
Answer: $v = 0.0c$
Diagram (c):
The object moves in the positive x-direction, with a slope less steep than diagram d, meaning it is slower than

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the speed of light.

Answer: $v = +0.8c$

Diagram (d):

The line forms a 45-degree angle, indicating motion at the speed of light.

Answer: $v = +1.0c$

This activity visually reinforces how the slope of a worldline in a space-time diagram represents an object's velocity, helping learners distinguish between rest, slower-than-light motion, and light-speed motion in the context of special relativity.

When we read off values for time and position on a space–time diagram, we read the time values from lines that are parallel to the position (x) axis (the red lines in **Figure 5**). We read the position values from lines that are parallel to the time (ct) axis (the blue lines in **Figure 5**). Each red line represents one point in time, each blue line represents one point in space.

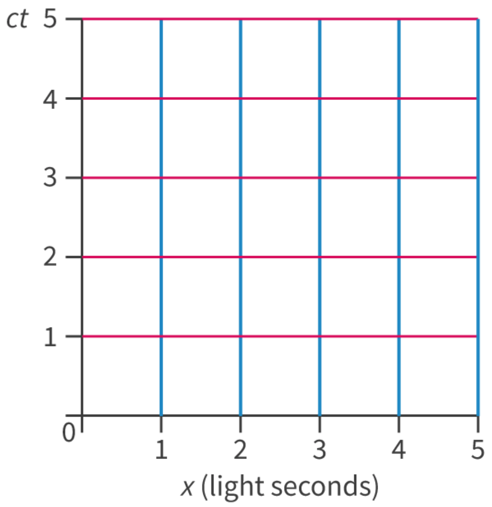

Figure 5. Red lines show points in time and blue lines show points in space.

More information for figure 5

The image is a space-time diagram with a grid. The horizontal axis is labeled 'x (light seconds)' ranging from 0 to 5, and the vertical axis is labeled 'ct' ranging from 0 to 5. The grid consists of red horizontal lines representing points in time and blue vertical lines representing points in space. The red lines are parallel to the x-axis, while the blue lines are parallel to the ct-axis, forming a grid that helps identify space-time coordinates by reading time from red lines and position from blue lines.

[Generated by AI]

When the x' and ct' axes are at an angle to the x and ct axes, the rules are the same (**Figure 6**). Space–time coordinates at the same point in time are read from lines that are parallel to the position axis. Space–time coordinates at the same point in space are read from lines that are parallel to the time axis. (The yellow line again represents the path of a light ray through space–time.)

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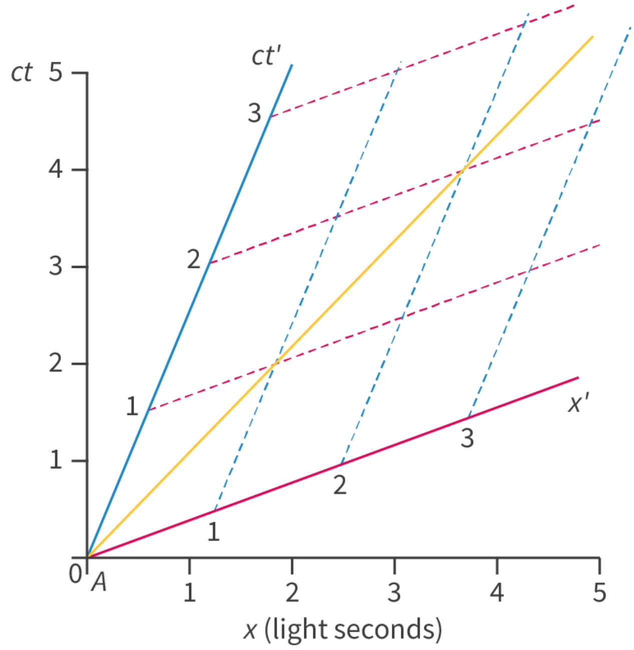

Figure 6. Red lines show points in time and blue lines show points in space.

More information for figure 6

The image is a graph illustrating space-time coordinates with tilted axes labeled x and ct (light seconds) and x' and ct'. The x and ct axes are the traditional horizontal and vertical axes, respectively. The axes, x' and ct', are at an angle to the original axes, showing transformed coordinates due to relativistic effects. The graph includes equally spaced lines parallel to these axes representing constant time (in space dimension) and constant space (in time dimension) lines. The colored lines (blue for ct', red for x', and yellow for the path of a light ray) show how time and space intersect relative to these axes. The yellow line indicates the light ray bisecting the x and ct axes, showing the principle of light traveling at the same speed in all frames.

[Generated by AI]

Worked example 2

In the space–time diagram, which points occur at the same time and which points occur at the same position in space in the moving reference frame?

Student
view

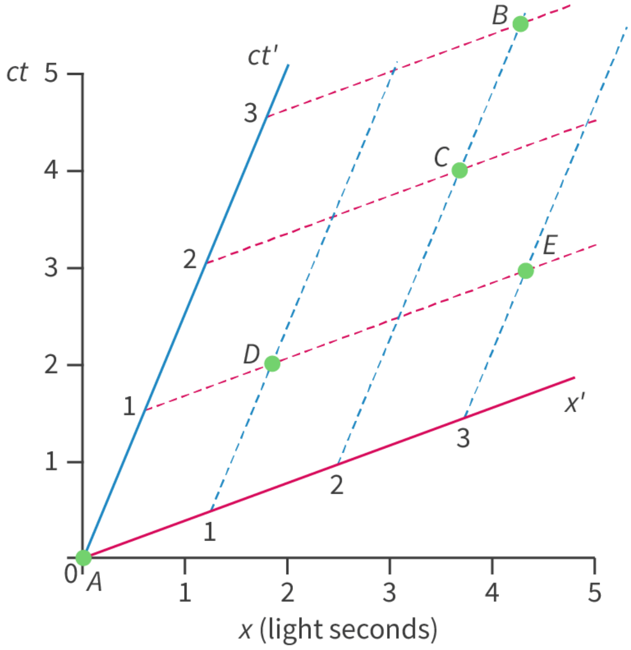

Figure 7. Which points occur at the same time and which points occur at the same position?

More information for figure 7

The image is a space-time diagram representing a moving reference frame. The diagram features two primary axes: the horizontal axis labeled 'x' (light seconds), representing spatial dimensions, and the vertical axis labeled 'ct', representing time. There are several diagonal lines and marked points labeled A, B, C, D, and E on the diagram. These points are positioned on or near the lines, indicating different events occurring at specific positions and times. Each axis has corresponding grid lines that help illustrate the coordinates of these events.

The diagonal lines might represent paths of objects as they move through space and time. Blue and magenta dashed lines intersect at points B, C, D, and E, while the origin is marked as 0A at the bottom left corner. The x' and ct' lines form diagonals, suggesting transformation in the reference frame. The grid and lines indicate how events might transform between different frames of reference.

[Generated by AI]

D and E lie on a line parallel to the *x*' axis, so they occur at the same time $(ct' = 1)$.

B and C lie on a line parallel to the *ct*' axis, so they occur at the same position $(x' = 2)$.

Length contraction and time dilation on space—time diagrams

Imagine a spaceship moving away from the Earth with a speed of $0.75c$. Its gamma factor is 1.5. We can see from the space–time diagram in **Figure 8**, that a 3 m stick on the Earth, as measured by the spaceship, is only 2 m long.

The length measurement involves finding the position of each end of the stick simultaneously. So a measurement is made to locate each end of the stick, with the two measurements made at the same time (in that reference frame).

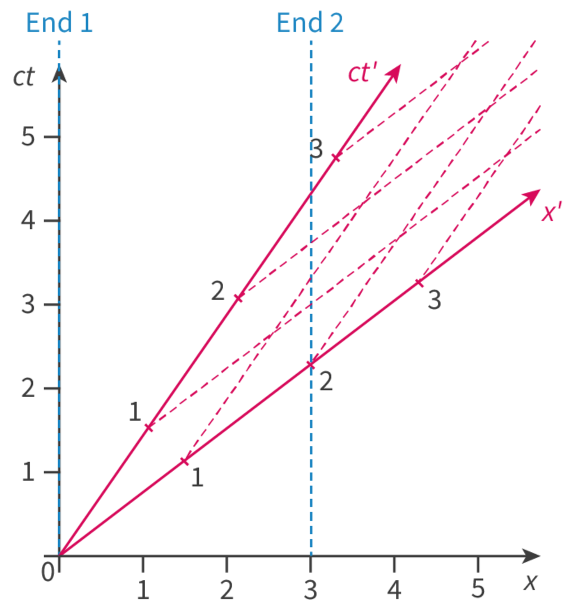

Figure 8. A 3 m stick in reference frame x is observed as a 2 m stick in reference frame x'.

More information for figure 8

This diagram illustrates the perceived length of a stick in two different reference frames in spacetime. It includes two axes: the x-axis and the ct-axis (time axis), both with scales marked from 0 to 5. There are also additional lines labeled x', ct', and points labeled End 1 and End 2.

In the original reference frame (x, ct), a stick's ends are at coordinates x=0 and x=3. In the transformed reference frame (x', ct'), these ends are observed along the lines of x' and ct'. The point where the transformed line x' intersects with the horizontal scale represents the perceived reduced stick length in the second reference frame. Solid lines represent the original reference frame's measurements, and dashed lines represent the transformed one.

This diagram helps illustrate the effects of relativistic transformations on the measurements of length due to differences in reference frames, showing that measurements depend on the observer's frame of reference, particularly evident in high-speed conditions close to the speed of light, as demonstrated within the context of spacetime geometry.

[Generated by AI]

In this case, both readings fall on the x' axis (at time $t' = 0$). We could make the same length measurement at any other time, as long as the measurement at each end takes place simultaneously in that reference frame (parallel to the x' axis).

Note that the proper length of the stick is 3 m, so on the space–time diagram, we draw the world line of each end of the stick (end 1 and end 2), being sure to draw them in the correct reference frame.

As the spaceship is moving relative to the stick (and the Earth), we will see that in the reference frame of the spaceship, the stick length is contracted.

From the space–time diagram, the world lines of the two ends of the stick indicate that, in the reference frame of the spaceship (x'), the stick is measured to be 2 m long.

Mathematically, we can confirm this:

$$\begin{aligned} L &= \frac{L_0}{\gamma} \\ &= \frac{3}{1.5} \\ &= 2 \text{ m} \end{aligned}$$

What would happen if the stick was on board the spaceship? In this case, we would draw the world lines of the two ends of the stick based on the reference frame of the stick (**Figure 9**).

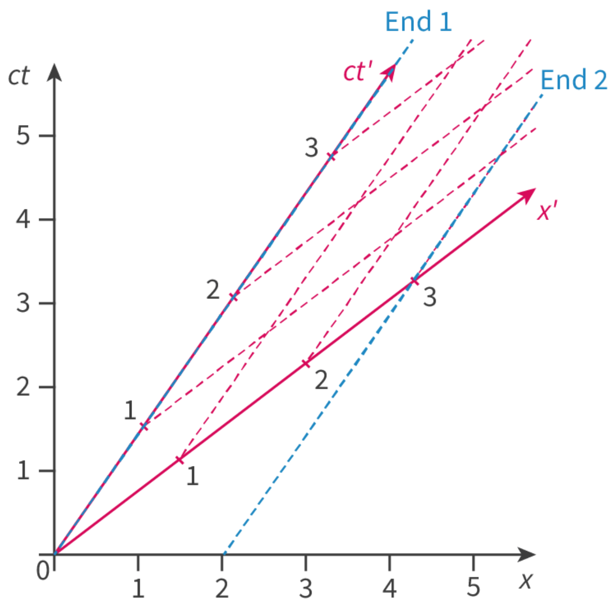

Figure 9. A 3 m stick in reference frame x' is observed as a 2 m stick in reference frame x.

Overview
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More information for figure 9

The image is a graph showing the world lines of a stick observed in two different reference frames, labeled x and x'. The x-axis is labeled from 0 to 5, and the ct-axis is labeled from 0 to 5, representing time. There are two main diagonal lines, one labeled as x' and the other as ct', both crossing through grid intersections.\n\nThe line labeled x' runs from the origin towards the top right, with marked points at (1, 1), (2, 2), and (3, 3) indicating measurements of 1, 2, and 3 units. The line labeled ct' also runs from the origin towards the top right, parallel to x', with similar markings. \n\nTwo blue dashed lines represent the ends of the stick, labeled "End 1" and "End 2", diverging from a common point at the origin and spreading outwards as time progresses. Each end of the stick is demarcated by dashed intersections at regular intervals with other lines across the graph.\n\nThis illustration demonstrates how a stick's length appears different when observed from another reference frame.

[Generated by AI]

This time, the proper length is measured by the x' reference frame axes, and the world lines of the two ends of the ruler show that the person observing in the x reference frame axes would measure the stick to be 2 m long.

Note that the positions of each end of the stick are being measured simultaneously. If you measured one end and then the other end, the position would change in the time between measurements, giving incorrect measurements.

For time dilation, if an observer on the Earth is watching a spaceship travelling at 0.75c away from the Earth, we can show a time interval by drawing two lines parallel to the appropriate x-axis (**Figure 10**). The observer on the Earth experiences an elapsed time of 1.5 seconds.

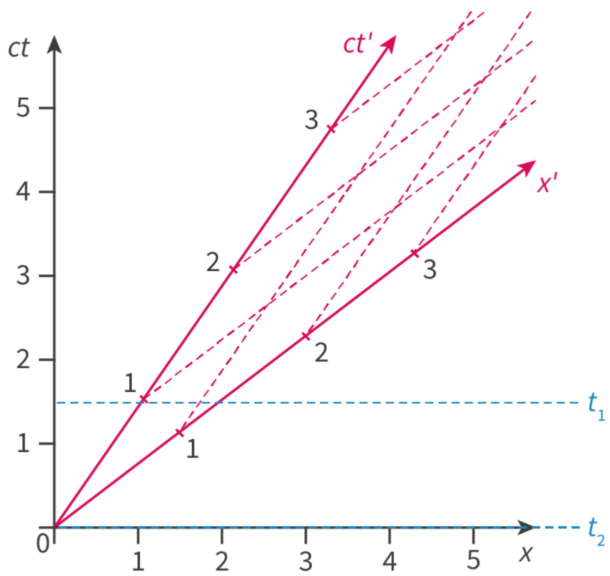

Figure 10. 1.5 seconds elapsing on the Earth is 1.0 seconds elapsing on a spaceship travelling away at 0.75c.

Student view

More information for figure 10.1.5

The image is a graph illustrating the concept of time dilation. It features two axes: the horizontal axis labeled 'x' representing space and the vertical axis labeled 'ct' representing a time interval scaled by the speed of light. The graph contains multiple lines.

There are solid arrows labeled 'x'' and 'ct'', indicating new reference frames. The x' arrow is angled more steeply, suggesting movement in a different space-time reference frame. Alongside the arrows, dashed lines run parallel and are marked with numbers 1, 2, and 3, denoting intervals or divisions in the space-time diagram.

The diagram illustrates how time intervals differ between an observer on Earth and an observer moving at a high velocity relative to the first. A horizontal dotted line labeled 't1' indicates a specific time frame for the stationary observer, and another dotted line labeled 't2' represents the moving observer's time frame. This visualization shows how, as per the theory of relativity, time elapses differently depending on the relative motion of the observer, supporting the explanation in the surrounding text.

[Generated by AI]

The clock on the spaceship is only going to show 1.0 s.

Verifying this mathematically using the time dilation equation (recalling that the spaceship's velocity relative to Earth is $0.75c$ and hence $\gamma = 1.5$):

$$\frac{t'}{\gamma} = t_0$$

$$\frac{1.5}{1.5} = 1.0\ s$$

What does the person on the spaceship observe? This time, the x' reference frame axes are measuring the proper time as we are talking about the reference frame of this observer (**Figure 11**).

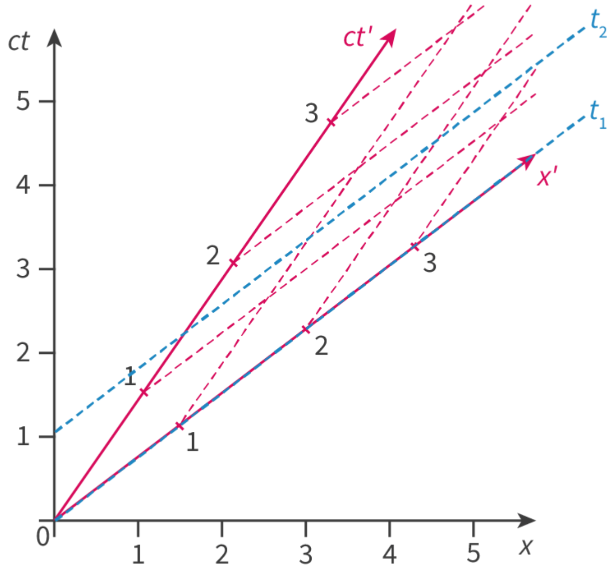

Figure 11. 1.5 seconds elapsing on a spaceship travelling at 0.75c is 1.0 seconds elapsing on the Earth.

More information for figure 11.1.5

The diagram depicts a space-time graph with the vertical axis labeled 'ct' (representing time) and the horizontal axis labeled 'x' (representing space). Two solid red lines extend diagonally across the graph: one labeled 'ct'',

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From the reference frame of the person on the spaceship, they measure a time interval of 1.5 seconds. Looking back at a clock on the Earth, they will see that it is the Earth's clock that is running slow and has only measured 1.0 seconds.

So, the observer on the spaceship sees the Earth's clock as running slow, and the observer on the Earth sees the spaceship's clock as running slow. This is possible because the speed of light is invariant and measurements of time and position depend on your reference frame.

Worked example 3

The space–time diagram shows the reference frame of an observer on a space station, watching an astronaut travelling away from the space station. (The yellow line represents the path that a light ray would take through spacetime.)

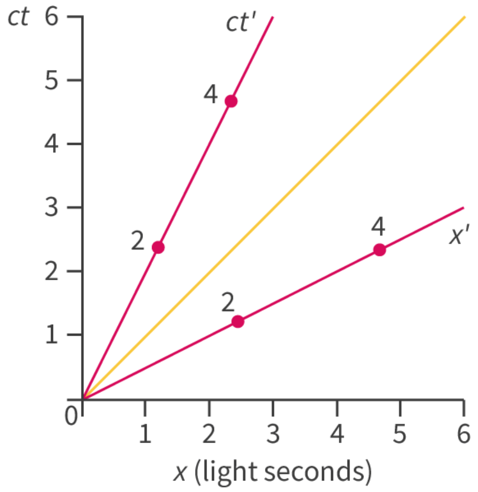

Figure 12. A space-time diagram showing an astronaut travelling away from the space station.

More information for figure 12

The image is a space-time diagram illustrating a reference frame for an observer on a space station. The diagram features two axes: the vertical axis is labeled 'ct' and ranges from 0 to 6, and the horizontal axis is labeled 'x (light seconds)' also ranging from 0 to 6. A yellow line represents the path of a light ray as it traverses space-time, originating from the origin (0,0) and passing through the graph at equal gradients. Pink lines labeled 'ct'' and 'x'' represent the astronaut's trajectory. The 'ct'' line runs steeper than the yellow light ray, indicated by the points at (2,2) and (4,4). The 'x'' line extends more horizontally, marked by points at (2,2) and (4,4). These lines suggest the astronaut is moving away from the space station at a speed slower than the speed of light, as the light ray maintains a constant diagonal path. The plotted points on the lines anchor this relationship between spatial distance and time in the astronaut's journey relative to the observer's frame.

[Generated by AI]

1. How fast is the astronaut travelling?

The astronaut wants to measure the length of the space station. The proper length is 4.6 m.

2. Using the space–time diagram, determine the length that the astronaut measures.

The astronaut takes two photographs, 2.0 s apart.

3. Using the space–time diagram, determine how much time has passed for the observer between the two photographs.

1. The speed is determined from the gradient of the x' reference frame axes as read from the *x* reference frame axes. The world line of the astronaut on the *x* reference frame axes will follow the same line and gradient as the ct' axis (vertical time axis from the reference frame of the astronaut). Taking the ct' axis as the world line, the distance travelled is 3 light seconds and the time is 6 seconds.

Overview
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$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{3}{6}\\ &= 0.5c\end{aligned}$$

2. From the space—time diagram, the proper length of the space station (as measured from the space station, x reference frame axes) is 4.6 m. Draw a vertical line up from 4.6 on the x axis. This meets the x'axis at 4.0. On the axes, the distances are measured in light seconds, but because this is a ratio, the units do not matter. The moving astronaut will measure the space station as 4.0 m.
3. The astronaut is measuring the time, and experiences proper time (because they are not moving relative to the camera). Draw a horizontal line from 2 on the ct' axis. It intercepts the ct axis at around 2.3, so the observer will measure this time as 2.3 s. (Notice that this is the same ratio as the length contraction in part 2.)

Space—time interval on a space—time diagram

As well as c, what else is constant across all reference frames in a space–time diagram? The space–time interval, Δs, is constant (see section A.5.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/lorentz-transformations-and-the-space-time-interval-hl-id-46606/)). As Δs is an invariant quantity, it does not change when viewed on different axes.

In the space-time diagram in **Figure 13**, the same space–time interval, Δs, is shown in two different reference frames: Δx and Δct, as well as $\Delta x'$ and $\Delta ct'$ are all different values. The value for the space–time interval is found by:

$$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$$

The value of Δs is invariant, and therefore the same in all reference frames.

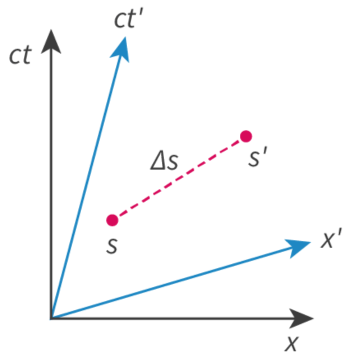

Figure 13. A space—time interval represented on two different space—time axes.

More information for figure 13

Student view

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The image is a space-time diagram illustrating two sets of axes. The black axes are labeled as 'ct' (vertical) and 'x' (horizontal). A second set of axes, represented in blue, is labeled 'ct'' (vertical) and 'x'' (horizontal), which are rotated relative to the black axes. There are two points marked on the diagram: 's' and 's'', connected by a red dashed line labeled 'Δs'. This line indicates a space-time interval between the two points in the diagram. The diagram visually represents the relationship between different space-time coordinates.

[Generated by AI]

Look at the axes on the space–time diagram in **Figure 14**. We have identified the point P, with coordinates ($x = 1$, $t = 0$) on the (x, ct) axes.

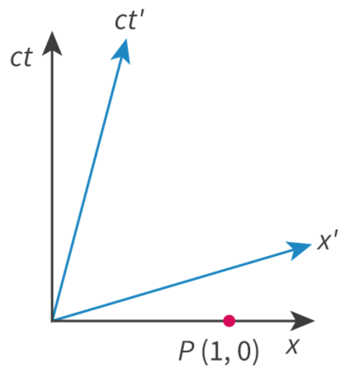

Figure 14. A space—time diagram with a coordinate P plotted.

More information for figure 14

The image is a space-time diagram representing a set of axes labeled 'x', 'x'', 'ct', and 'ct''. The diagram displays two axes meeting at the origin, where the horizontal axis represents 'x', and the vertical axis represents 'ct'. Additionally, there are two diagonal axes labeled 'x'' and 'ct'' extending from the origin in a V-shape. The axes 'x'' and 'ct'' depict changed coordinates due to some transformation.

There's a point P marked on the diagram at (1, 0) on the 'x' axis, illustrating the position in space-time coordinates. The point P is highlighted with a small red dot to indicate its specific location. This diagram is used to demonstrate spacetime events in a visual format, showcasing the relationship between space and time coordinates. The inclusion of 'ct' signifies that time is multiplied by the speed of light to ensure dimensional consistency with space coordinates.

[Generated by AI]

To calculate a value for Δs between P and the origin, we can use the equation:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

As $x = 1$ and $t = 0$, this gives:

$$(\Delta s)^2 = (c \times 0)^2 - (1)^2 = -1$$

Student
view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

A particular space–time interval will always give a hyperbolic curve on a space–time diagram. We can draw the hyperbolic space–time interval, which passes through the point $x = 1$ and $t = 0$ (**Figure 15**). This line joins together all points in the space–time diagram with $(\Delta s)^2 = -1$.

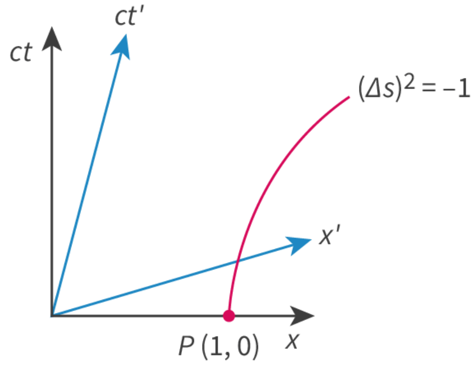

Figure 15. A space-time diagram showing a hyperbolic space—time interval.

More information for figure 15

The image is a space-time diagram, which features a hyperbolic curve. The diagram includes labeled axes: the x-axis is marked as 'x', and the y-axis as 'ct'. There are additional primed axes, 'x'' and 'ct''. A red hyperbolic curve passes through the point marked as P(1, 0) on the x-axis, and it is labeled as (Δs)^2 = -1. This curve joins together all the points in the space-time diagram that satisfy the equation.

[Generated by AI]

Concept

The hyperbolic space—time interval joins together all points with the same space—time interval. It does not matter which reference frame the interval is read from, as the space—time interval is invariant across all reference frames.

Let us now consider the x' and ct' axes. If we look at point Q in **Figure 16**, we can see that it lies on the same line, Δs. As this line links together all points with the same space–time interval, we can say that at Q, $(\Delta s)^2$ is also equal to –1.

Student view

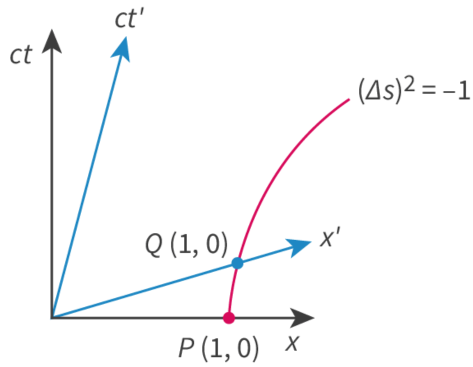

Figure 16. A space-time diagram showing a hyperbolic space—time interval.

More information for figure 16

The diagram illustrates a hyperbolic space-time interval with a set of axes including "ct" and "ct'" axes going upward and to the right, respectively. The horizontal axis is labeled with "x" and "x'" extending to the right. There is a curve labeled "(Δs)^2 = -1" which represents a hyperbolic line. The diagram includes a point labeled "Q (1, 0)" at the intersection of a line parallel to the "x'" axis and the above curve, showing the location where the derived space-time interval is equal to negative one. Another point labeled "P (1, 0)" is marked along the "x" axis. The axes and the curve depict the mathematical relationship and transition between different frames in the space-time diagram, commonly used in relativity theory.

[Generated by AI]

We can also see that Q lies on the line $ct' = 0$. Therefore, at Q, *t'* = 0.

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

Substituting in $(\Delta s)^2 = -1$ and *t'* = 0:

$$(-1) = (0)^2 - (\Delta x)^2$$

$$x = 1$$

This helps us plot a scale on our x' axis, as we know that at the origin, $x' = 0$, and at Q, $x' = 1$. We can also see that the scale on the (x', ct') axes is larger than the scale on the (x, ct) axes.

Worked example 4

In the space–time diagram, the point U has coordinates ($x = 0$, $t = 1$) in the non-moving reference frame. The red line represents the invariant space–time hyperbola for $(\Delta S)^2 = 1$.

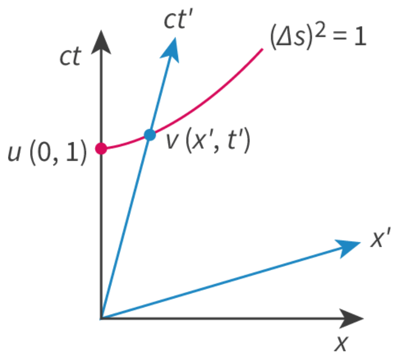

Figure 17. Determine the coordinates (x', t') for point V in this space-time graph.

More information for figure 17

This diagram depicts a space-time graph with four axes labeled ct, ct', x, and x'. The time axis (ct) is vertical while the horizontal axis represents space (x). There's an additional set of angled axes—ct' for time and x' for space—indicative of a moving reference frame. The point U is marked with coordinates (x=0, t=1) on the ct axis, and a red hyperbola is drawn through U indicating invariant space-time for (ΔS)^2=1. Another point, V, is located on the transformed axes ct' and x', suggesting coordinates (x', t').

[Generated by AI]

Determine the coordinates (x', t') for point V.

The invariant space—time hyperbola joins together all points of equal space—time interval, as measured from the origin. At Q, the space—time interval is given by:

$$(\Delta s)^2 = 1$$

V lies on the *ct*' axis, so value for x' = 0

$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$1 = (c\Delta t)^2 - (0)^2$$

$$c\Delta t = 1 \text{ so } \Delta t = 1 \text{ (c} = 1 \text{ on the } ct' \text{ axis)}$$

coordinates for V:

$$(x' = 0, ct' = 1)$$

Simultaneity

As humans, we have experience of events that are simultaneous for one observer and not for another observer. Consider a storm above you, with thunder crashing and lightning flashing. At your location, the lightning and the thunder will reach you at

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roughly the same time. Your friend in the next town will hear the thunder after they see the lightning, because the sound of the thunder travels much slower than the speed of light.

When we talk about relativistic simultaneity, events are not affected by different speeds of light. As we have seen, the speed of light is the same for all observers, regardless of their relative motion. Simultaneous events in the reference frame of one observer are not necessarily simultaneous in the reference frame of another observer.

Imagine your friend is riding past in a train and you are watching them from the platform (**Figure 18**). Your friend is in the middle of the carriage and there are light detectors at each end of the carriage. At the instant your friend is opposite you, they light a torch.

In the reference frame of your friend on the train, the light beam travels equal distances *d* to the light detectors at each end of the carriage. Therefore, the light detectors are triggered simultaneously in the friend's reference frame.

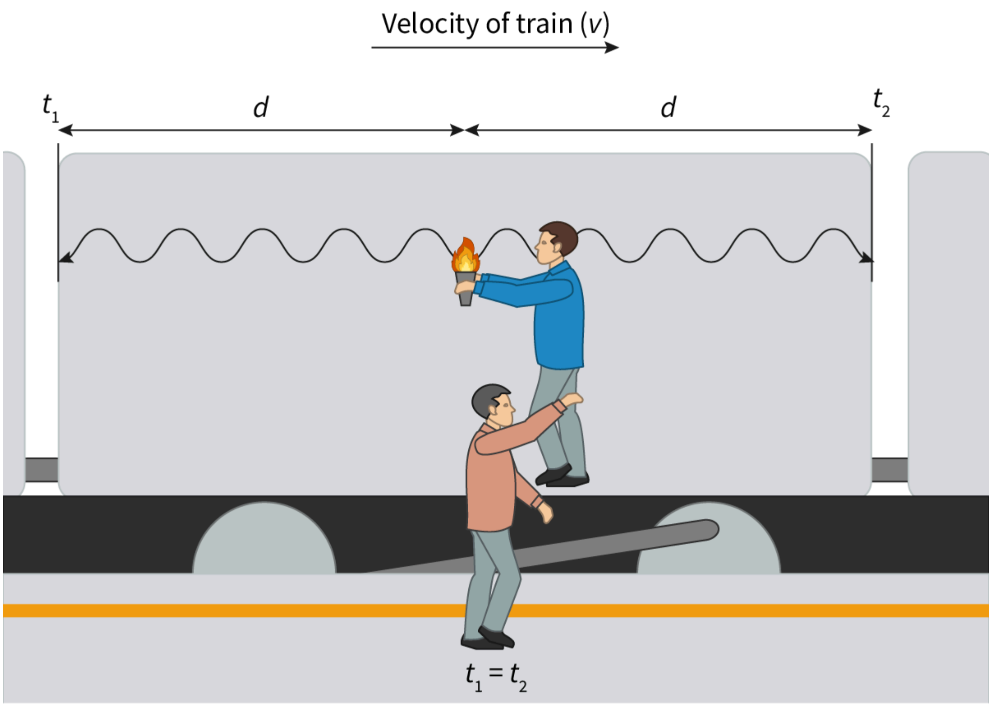

Figure 18. For your friend on the train, the light reaches the two light detectors simultaneously.

More information for figure 18

The image depicts a scenario illustrating light and motion on a train. It shows two men: one standing on the train platform and the other inside a moving train carriage holding a light source. The understanding is that both light detectors at each end of the carriage are triggered simultaneously due to equal travel distances for the light in the friend's reference frame. The diagram indicates the velocity of the train, marked as 'v'. Arrows show equal distances marked as 'd' going from each detector to the light source, with points marked as 't1' and 't2' indicating time, where t1 equals t2. The scene illustrates the concept of relative simultaneity in physics.

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You are watching from the platform. **Figure 19** shows what you observe.

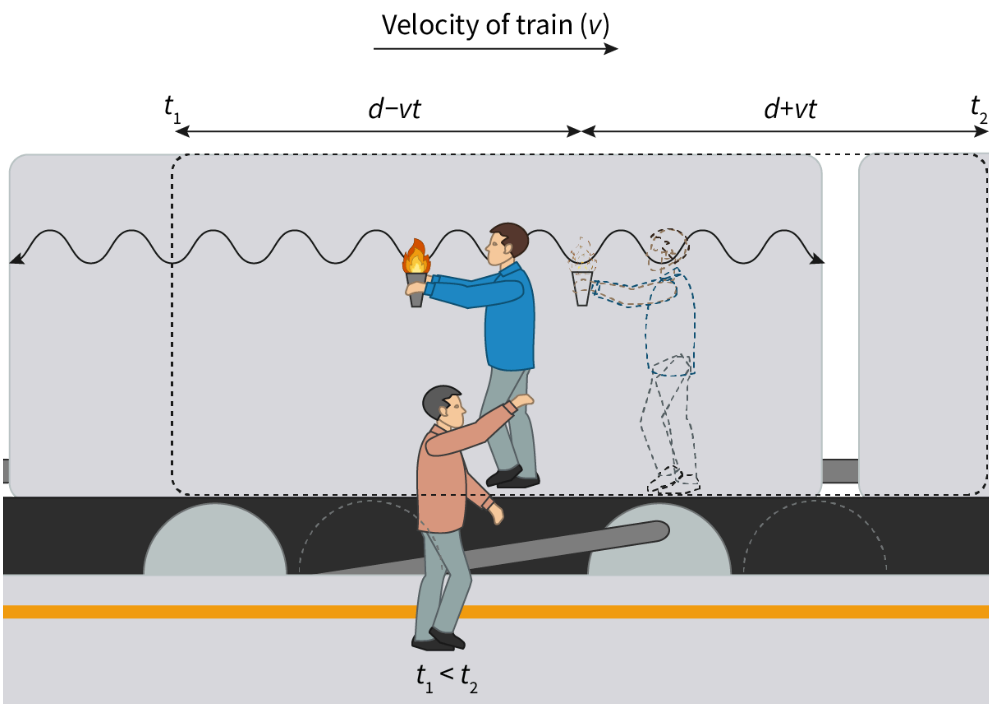

Figure 19. For you on the platform, the light does not reach the two light detectors simultaneously.

More information for figure 19

The diagram illustrates a light wave traveling within a train carriage moving from left to right at velocity v. Two people are shown inside the carriage. The person at the left is shown receiving or detecting light earlier than the person on the right, indicating time differences t1 and t2, where t1 < t2. The train's velocity is indicated by an arrow and label at the top, demonstrating relative movement of the detectors towards and away from the light source. The light wave is represented as a sinusoidal curve. The distances d-vt and d+vt indicate the respective positions of the detectors in relation to the light source over time.

[Generated by AI]

In the time, t, it takes the light to travel through the carriage, the detector on the left has moved to the right, towards the oncoming light beam, by a distance vt. The detector on the right has also moved to the right, away from the beam, by a distance vt.

This means that:

- the light beam moving left only has to move a distance: $d - vt$
- the light beam moving right has to move a larger distance: $d + vt$

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The second postulate of special relativity says that the speed of light is the same for all observers, regardless of their relative motion (see section A.5.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/special-relativity-and-time-dilation-and-length-contraction-hl-id-46605/)). As the light beam moving left travels less distance than the light beam moving right, you on the platform see the left light detector trigger before the right light detector. The two events are simultaneous for your friend on the train, but not for you on the platform.

Concept

Events at different points in space—time that are simultaneous for one observer will not necessarily be simultaneous for an observer in a different reference frame.

In the space–time diagram in **Figure 20**, the black ct and x axes represent your reference frame on the platform, and the red ct' and x' axis represents your friend's reference frame on the train. Event A is when the light beam hits the left-hand detector in the train carriage. Event B is when the light beam hits the right-hand detector. The two events occur at the same time, t', for the observer on the train but at different times (t_1 and t_2) for the observer on the platform.

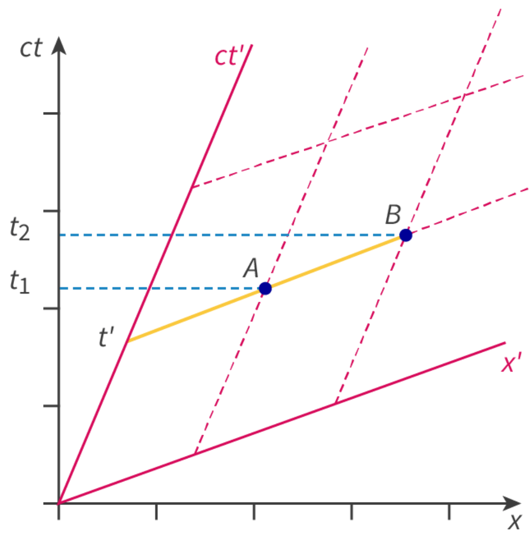

Figure 20. Events A and B are simultaneous for the moving observer, but not simultaneous for the stationary observer.

More information for figure 20

The space-time diagram illustrates the relationship between two frames of reference. The black axes labeled ct and x represent the stationary observer's reference frame on the platform, while the red axes labeled ct' and x' represent the moving observer's frame on the train. Two events, labeled A and B, are marked along a yellow line. Event A is when the light beam hits the left-hand detector, and Event B is when the light beam hits the

Student view

Overview
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right-hand detector. The events are simultaneous at time t' for the moving observer on the train but occur at different times, t1 and t2, for the stationary observer on the platform. This demonstrates how events can be simultaneous in one frame but not in another, illustrating the relativity of simultaneity in space-time physics.

[Generated by AI]

Now imagine that light beams are generated at each end of the carriage and travel towards your friend on the train. From the reference frame of your friend, the light beams reach them (in the middle of the carriage) at the same time (**Figure 21**), the detection of each light source is simultaneous.

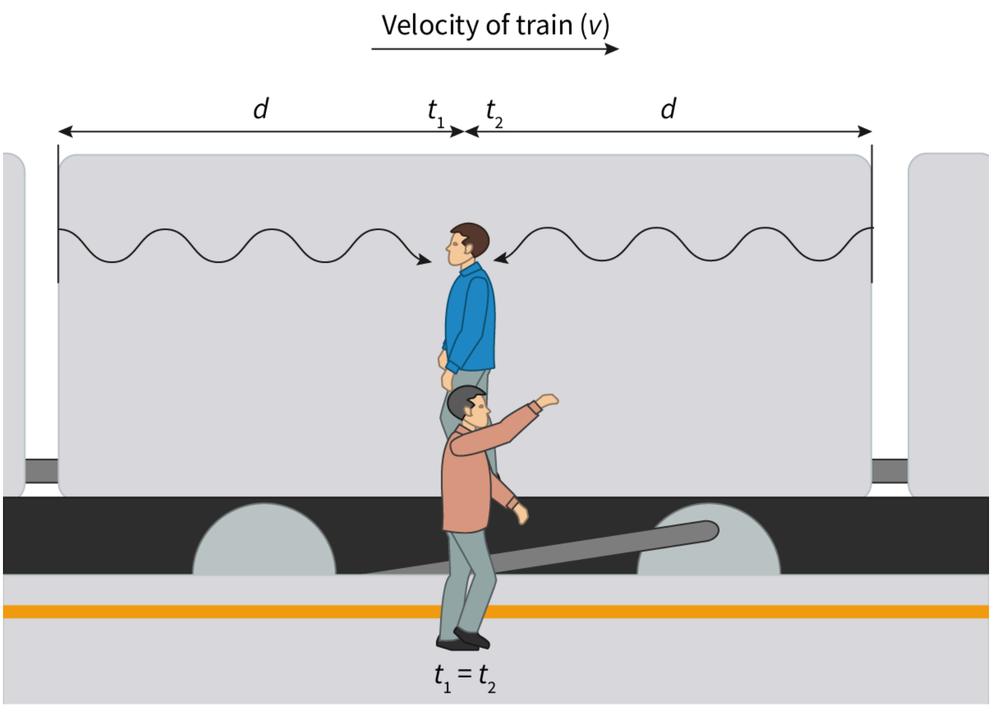

Figure 21. For your friend on the train, the light is emitted and reaches them simultaneously.

More information for figure 21

The image is a diagram illustrating the concept of light beams reaching an observer simultaneously. It shows a train car with two individuals; one is inside the train and the other on the platform. Above the train car, there's a label 'Velocity of train (v)' with a direction arrow. Two lines labeled 'd' point from the train ends towards the person inside, who is located in the middle. There's a wave pattern indicating simultaneous light paths reaching the person inside. Below the train, there's another person on the platform, pointing upwards with equations 't1 = t2' suggesting that both light beams reach the observer simultaneously in both frames of reference. The setup visualizes the relativity principle regarding the perception of light.

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Will the light beams also reach your friend on the train simultaneously in your reference frame, standing on the platform? The answer is yes. You also see the light beams reach your friend on the train simultaneously.

Student view

Overview
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Concept

Any simultaneous event **at one point in space—time** for an observer will also be a simultaneous event for that observer viewed from all other reference frames.

But what has happened to the idea about simultaneity not being observed under relativistic effects?

The light beams take time to travel to your friend from each end of the carriage. From your point of view, your friend moves to the right during this time. They move **towards** the right-hand beam and **away from** the left-hand beam. So the left-hand beam has to travel a greater distance to reach your friend than the right-hand beam.

Both light beams travel at the same speed, c, so the only logical conclusion is that the left-hand beam takes more time to reach your friend on the train than the right-hand beam (**Figure 22**). As the light beams reach your friend on the train simultaneously in their reference frame and in your reference frame, the left-hand beam must have started sooner than the right-hand beam. This means that, for you on the platform, the light beams reach your friend on the train simultaneously. However, for you, the light beams are not **emitted** simultaneously.

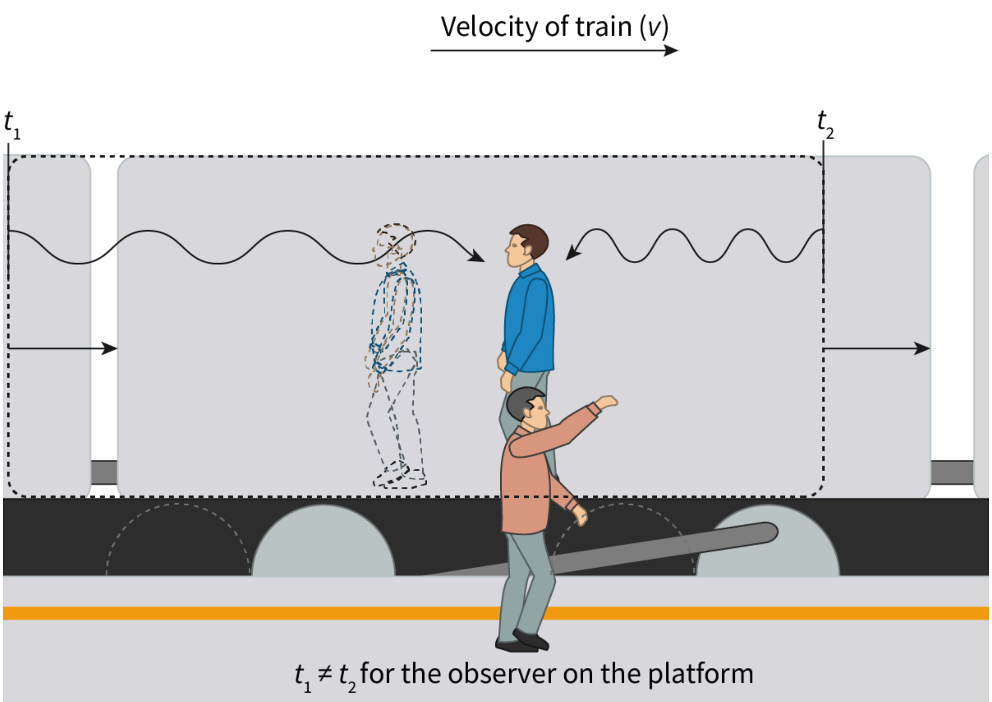

Figure 22. For you on the platform, the light beams reach your friend on the train simultaneously but the light beams are not emitted simultaneously.

More information for figure 22

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The image is a diagram illustrating the concept of light beams reaching an observer on a platform. At the top, it indicates the velocity of the train as v. Two light beams, represented by wavy lines, are shown entering the train at times t1 and t2. There is a depiction of a person inside the train, and another person outside on the platform pointing towards the train. The text at the bottom reads "t1 ≠ t2 for the observer on the platform," highlighting that for this observer, the times at which the light beams reach are different due to the motion of the train. The overall diagram helps to visualize the difference in reference frames between an observer on the platform and within the train.

[Generated by AI]

In the space–time diagram in **Figure 23**, the black ct and x axes represent your reference frame on the platform, and the red ct' and x' axis represents your friend's reference frame on the train. On this diagram, the two beams are both emitted at time t' = 0 in the train's reference frame, and they are detected by your friend at position x' = 0 simultaneously some time later. Their paths through space–time are shown as yellow lines.

In your reference frame, the beams are not emitted at the same time. (In this diagram, beam 1 is emitted at a negative time, which simply means a time before your stopwatch showed 0.) However, they reach your friend simultaneously even though they both travel at the speed of light. This is possible because they travel different distances in your reference frame. Beam 1 travels a distance Δx_1 in time Δt_1, while beam 2 travels a shorter distance Δx_2 in a shorter time Δt_2.

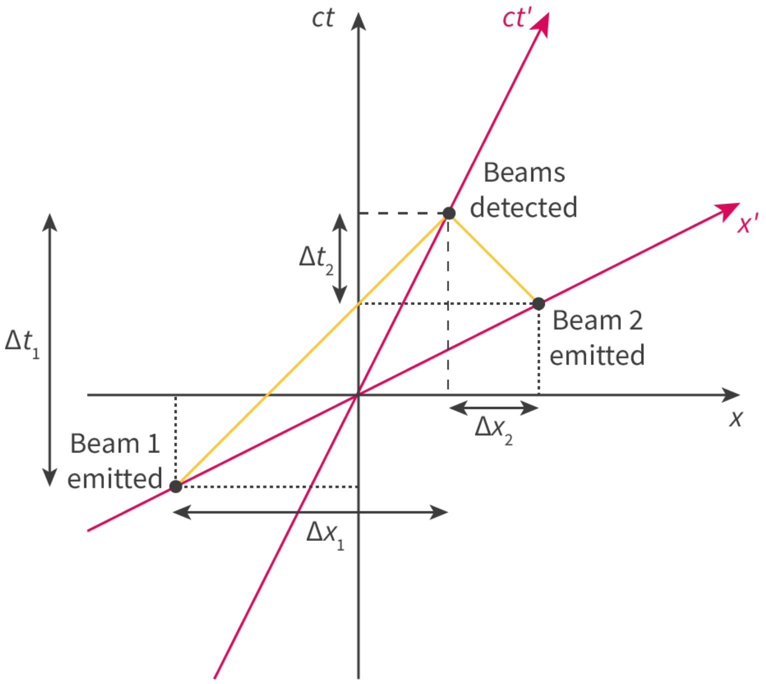

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Figure 23. For both observers, the two light beams travel at speed c and are detected by your friend simultaneously, but from your point of view they are emitted at different times and travel different distances.

More information for figure 23

This diagram is a space-time graph illustrating the emission and detection of two light beams. The graph features two axes: the x-axis representing space and the ct-axis representing time. There is a ct' red diagonal line extending from the origin.

On the graph, two events are labeled, one as 'Beam 1 emitted' and another as 'Beam 2 emitted,' both on the x-axis at different distances from the origin. From the point labeled 'Beam 1 emitted,' a thick dotted horizontal line extends until it intersects with the vertical ct-axis at a point labeled Δt1. From this intersection point, a vertical dotted line drops down to the x-axis directly below the emission point of Beam 1.

Another point is marked where 'Beam 2 emitted,' with a similar visual treatment as Beam 1's event but at a farther point on the x-axis, labeled Δx2. From this emission point, a line follows a similar path to the ct-axis where Δt2 is marked.

Additionally, there are two beams represented as yellow and red diagonals beginning at each emission point and converging at a single point labeled 'Beams detected' on a dotted line extending from another diagonal labeled ct'. This convergence point illustrates the simultaneity of the detection of the beams despite having different emission times and traveling different distances.

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Work through the activity to check your understanding of space–time diagrams and simultaneity.

Activity

- **IB learner profile attribute:** Knowledgeable
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 10 minutes
- **Activity type:** Individual activity

You are going to use the simulation in **Interactive 2** to investigate space—time diagrams and simultaneity.

Interactive 2. Space—time diagram simulation.

More information for interactive 2

The interactive simulation, “Space—time diagram simulation”, explores special relativity concepts, including time dilation, length contraction, and simultaneity. The black axes represent the reference frame of an observer on Earth. The blue axes represent the reference frame of a spaceship moving away from Earth at a velocity v relative to the Earth observer. The red line at 45 degrees corresponds to the trajectory of a light signal emitted from Earth at t=0 s.

Users can set the spaceship’s velocity and observe its impact on measurements in the moving frame. The gamma factor (γ) which quantifies time dilation and length contraction, is calculated using the formula,

$$Y = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

For v=0.3 c, gamma is approximately 1.098. Here, c is the speed of light in vacuum.

Users can move event A in the diagram and record its coordinates in both reference frames. The event's position in the Earth frame is given by its distance d and time t. In the spaceship’s frame, event A has coordinates d' and t', determined by the Lorentz transformation equations. The consistency of gamma can be checked by comparing the equations, $Y = \frac{t'}{t}$ and $Y = \frac{d'}{d}$

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A second event, B, can be introduced. By adjusting B such that A and B are simultaneous in the Earth frame, the time difference in the spaceship's frame can be observed, demonstrating the relativity of simultaneity. Conversely, when A and B are simultaneous in the spaceship's frame, the time difference for the Earth observer is examined.
The effect of velocity on time differences can be explored by adjusting v. Increasing v leads to greater discrepancies in simultaneity between frames, emphasizing that simultaneity is not absolute. The simulation provides an interactive approach to understanding Lorentz transformations and relativistic effects, allowing users to visualize and quantify how different observers perceive space and time.

The black (perpendicular) axes represent the frame of reference of an observer on Earth (neglecting Earth's orbital and rotational motions). The blue axes represent the frame of reference of a spaceship flying away from Earth. At the instant the spaceship leaves Earth, $t = t' = 0$. The red line at 45° represents the motion of a ray of light which leaves Earth at the same time as the spaceship.

Set the speed of the spaceship relative to Earth $v = 0.3$. Calculate the gamma factor, γ.

Move event A to a position where it is above the 45° line and below the t' axis. Record the distance d and time t of the event according to an observer on Earth. Record the distance d' and time t' of event A according to an observer on the spaceship. Using these data, determine the gamma factor, γ, for the moving observer. Check that the values of γ calculated in different ways are the same.

Add a second event, B. Move B so A and B happen simultaneously for the stationary observer. What is the time difference between A and B for the moving observer?

Now, move B so A and B happen simultaneously for the moving observer. What is the time difference between A and B for the stationary observer?

Repeat with a different speed, v. How does speed affect the time differences?

4 section questions

Question 1

HL Difficulty:

Which of the world lines (A, B, C or D) shows a body with the lowest speed?

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Overview
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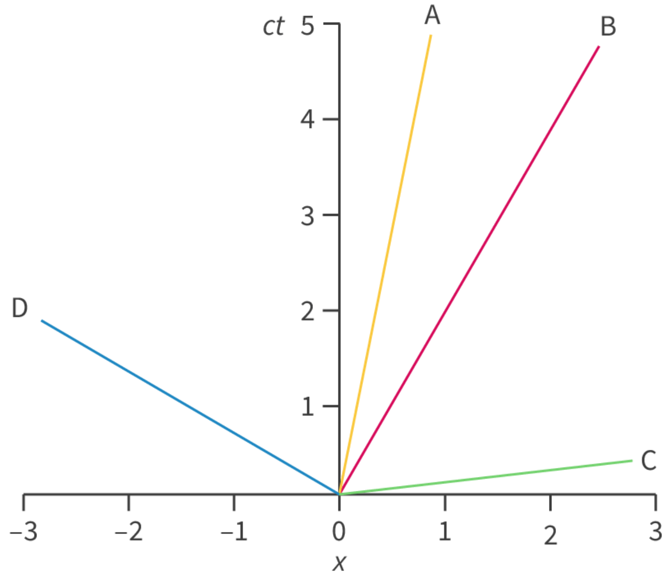

More information

1 Line A

2 Line B

3 Line C

4 Line D

Explanation

The steeper gradient has the lowest speed.

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{v}{c}\end{aligned}$$

As the line becomes steeper, the value for $\frac{\text{opposite}}{\text{adjacent}}$ descreases, and so does $\frac{v}{c}$. This means speed v decreases, and so world line A shows the lowest speed.

Question 2

HL Difficulty:

Spaceship X (blue axes, *x*’ and *ct*’) is travelling at speed *v* in the positive x-direction with respect to spaceship Y (black axes, *x* and *ct*).

Student view

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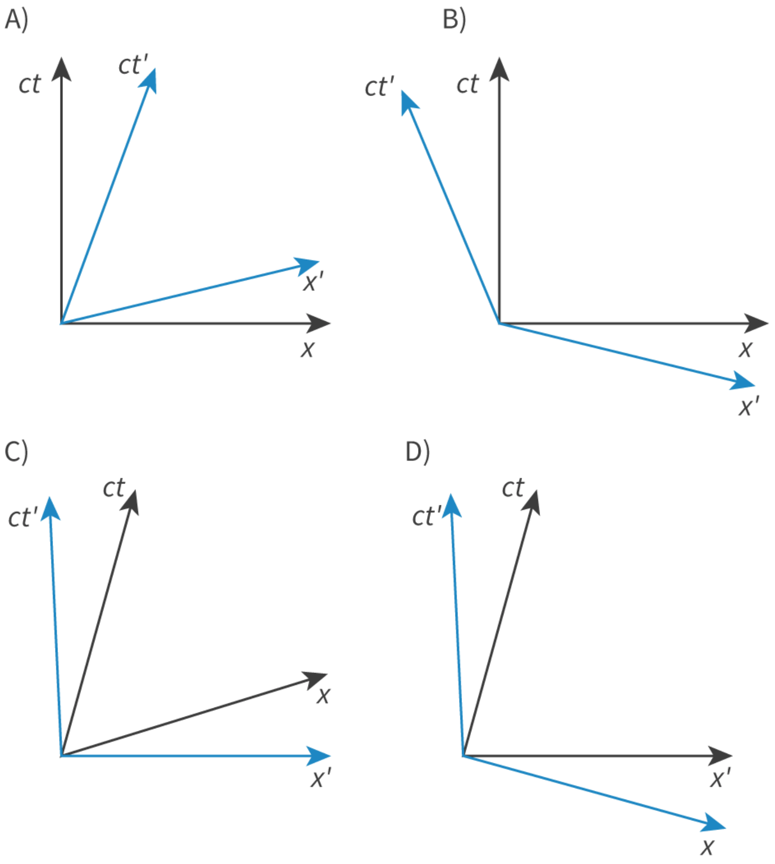

More information

Which space—time diagram (A, B, C or D) shows the correct relationship between spaceship X and spaceship Y?

1 A ✓

2 B

3 C

4 D

Explanation

As spaceship Y is travelling in the *x*-direction relative to spaceship X, its space—time axes will be displaced from X's by an angle θ, where:

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{v}{c}\end{aligned}$$

As the direction travelled is positive, the angle θ will also be positive.

Student
view

Question 3

HL Difficulty:

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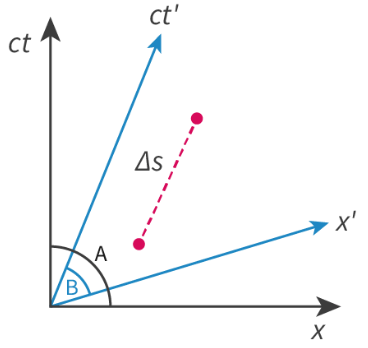
A space—time interval, Δs, is shown in two reference frames.



More information

Which statement about the space—time interval is correct?

1 Δs is the same in reference frame A and reference frame B ✓

2 Δs is smaller in reference frame A than in reference frame B

3 Δs is greater in reference frame A than in reference frame B

4 It is not possible to tell from the space—time diagram the relative size of the space—time interval in each reference frame

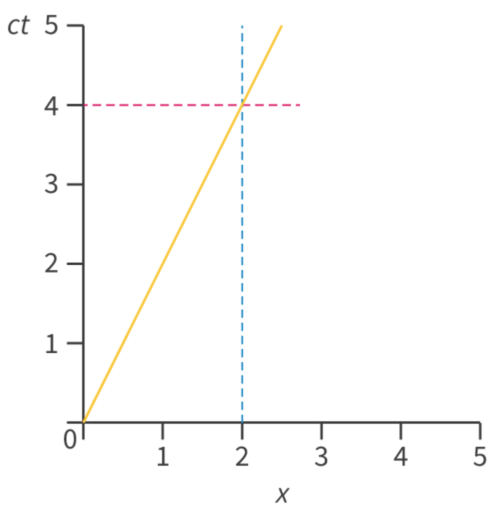
Explanation

As space—time intervals are invariant quantities, the space—time interval is the same in both reference frames.

Question 4

HL Difficulty:

What is the speed of the object represented in the space—time diagram?



Student view

Overview
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More information

1 0.5c ✓

2 2.0c

3 1.0c

4 1.5c

Explanation

Angle θ in the relationship:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{v}{c}$$

is the angle between the ct' axis and the ct axis.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{v}{c}$$

$$v = 0.5c$$

A. Space, time and motion / A.5 Galilean and special relativity (HL)

Summary and key terms (HL)

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/summary-and-key-terms-hl-id-46608/print/) Assign

Higher level (HL)

- A reference frame is a coordinate system used to describe the motion of a body in space—time. An inertial reference frame is one that is not accelerating.
- In Galilean relativity, Newton's laws of motion are the same in all inertial reference frames.
- The first postulate of special relativity is that the laws of physics are the same in all inertial reference frames. The second postulate is that the speed of light is the same in

Student view

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all reference frames.

- The proper time interval is the duration of an event, as measured by an observer who is stationary relative to the event being timed. Proper length is the length of an object as measured by an observer who is stationary relative to the object.
- Time dilation is where the duration of an event, as measured by an observer moving relative to the event, is found to be less than the proper time interval. Length contraction is where the length of an object, as measured by an observer moving relative to the object, is found to be less than the proper length.
- Muon decay is evidence of length contraction and time dilation, observed when muons are created in the upper atmosphere and detected at the surface of the Earth.
- Lorentz transformations allow us to find the position and time of an event in the reference frame of a moving observer, based on the position and time of the event in the reference frame of a stationary observer.
- The space—time interval is the interval between two events happening at different points in space and/or time. The space—time interval is invariant across all reference frames.
- A space—time diagram represents events happening in space—time in multiple reference frames. A line at 45° to either axis represents an object travelling at the speed of light. The angle θ, measured from the time axis to the world line for the moving object, allows us to find the speed of the moving object.
- Simultaneity is when two events happen at the same time in the reference frame of a particular observer. Events that are simultaneous in the reference frame of one observer may not be simultaneous in the reference frame of another observer.

Student
view

Key terms

Interactive 1. Exploring Motion, Time, and Space in Different Frames.

Check

Checklist (HL)

Higher level (HL)

What you should know

After studying this subtopic, you should be able to:

- Outline reference frames.
- Explain Galilean relativity and use the Galilean transformation equations:

$$x\prime = x - vt \text{ and } t\prime = t$$

- Know and use the velocity addition equation:

$$u\prime = u – v$$

- Know the two postulates of special relativity.
- Describe the concepts of proper time interval and time dilation and use the equation:

$$t = \gamma t_0$$

- Describe the concepts of proper length and length contraction and use the equation:

$$L = \frac{L_0}{\gamma}$$

- Recognise that muon decay provides evidence for time dilation and length contraction.
- Determine that the postulates of special relativity lead to the Lorentz transformation equations:

$$x' = \gamma (x - vt) \text{ and } t' = \gamma \left(t - \frac{vx}{c^2}\right)$$

- Explain that the Lorentz transformation equations lead to the relativistic velocity addition equation:

Overview
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$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

- Demonstrate that the space—time interval is an invariant quantity and use the equation:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

- Describe space—time diagrams.
- Explain the relativity of simultaneity.

A. Space, time and motion / A.5 Galilean and special relativity (HL)

Investigation (HL)

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/investigation-hl-id-46610/print/) Assign

Higher level (HL)

- **IB learner profile attribute:** Thinker
- **Approaches to learning:** Communication skills — Using digital media for communicating information
- **Time required to complete activity:** 40 minutes
- **Activity type:** Group activity

You have been asked by a film company to consult with their scriptwriters on their next superhero movie, which is expected to feature several scenes of space travel at relativistic speeds. The film producers would like those scenes to be presented as accurately as possible.

Creativity, activity, service

Strand: Creativity
Learning outcome: Demonstrate that challenges have been undertaken, developing new skills in the process

Student view

Overview (/study/ap aa- hl/sid- 423- cid- 762593/c

A huge amount of creativity is required for scientists to imagine relativity and relativistic speeds. It is even harder to represent relativistic speeds in popular media when most of the audience may not have an understanding of relativity. There are many ways in which this has been represented in science fiction, including 'warp speed' in *Star Trek* and 'light speed' in *Star Wars*.

Create a digital or physical artistic image of light speed travel. What would it look like to move this fast? What would it feel like?

Your task

You are going to fulfill the first bullet point instruction below, and then choose at least two of the other instructions to carry out.

- Explain time dilation and length contraction, in such a way that a non-physicist will understand the concepts. Ideally, make use of space—time diagrams to communicate your ideas. Space—time diagrams themselves may need to be explained.
- Research and describe the ‘twin paradox’, and explain it in terms of special relativity. The film producers are keen to incorporate this paradox into the movie, with humorous consequences.
- Advise on likely travel times for intergalactic travel, using our nearest galaxy, Andromeda, as a reference point.
- Explain why travelling at speeds greater than c is impossible, even for a superhero.
- Give examples of relativistic effects that occur in our human experience, such as the slowing of atomic clocks or muon detection.
- Explain why images of distant galaxies are snapshots in time, and why it is impossible to see real-time images of galaxies that are very far away.

You could explain your ideas:

- in writing, with pictures and text
- as a video, with a voiceover describing images, text and simulations (such as the ones you have seen in subtopic A.5)
- as a podcast, where you record episodes based on each section of your brief
- using stop-motion animation
- as a web page
- as a poster, with annotated images.

Remember that some of the ideas in special relativity are very strange. Use simple language and lots of visuals, and try to be concise.

Reflection (HL)

Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.

HL Extension

Reflection

Now that you've completed this subtopic, let's come back to the guiding questions introduced in The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-hl-id-45344/).

- How do observers in different reference frames describe events in terms of space and time?
- How does special relativity change our understanding of motion compared to Galilean relativity?
- How are space—time diagrams used to represent relativistic motion?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

- What main points have you learned from this subtopic?
- Is anything unclear? What questions do you still have?
- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

Once you submit your response, you won't be able to edit it.

0/2000

Submit

Rate subtopic A.5 Galilean and special relativity (HL)

Help us improve the content and user experience.