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The big picture

Differential equations are very important in science and mathematics, both theoretically and because they have many applications. For example:

- Can you predict over time the temperature of a hot liquid as it cools to room temperature (Newton’s law of cooling)?
- Can you predict the age of an artefact by analysing the amount of carbon-14 remaining in the material?
- Can you predict the future population of bacteria (exponential growth)?
- Can you predict the future radioactivity of uranium (exponential decay)?
- Can you predict the future reaction rate of a first-order chemical reaction?

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Scientists can predict the exponential decay of uranium using differential equations

Credit: abadonian Getty Images

The answer to all of these questions is yes, through differential equations.

In general, the solution to a differential equation is not unique, as there may be several functions satisfying the same equation. For example , (undefined) $y = \sin x + c$ is a solution of $y' = \cos x$, for any real constant, c. Sometimes, however, you are interested in finding a specific solution satisfying some additional conditions.

You could study an entire course on just ordinary differential equations. In college, many of you may do so. However, this course is limited to just a handful of topics:

- Numerical approximations of first-order differential equations using Euler's method.
- Numerical solution of certain type of differential equation systems.
- Numerical solution of certain type of second order differential equations.

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- Exact solutions of separable first order differential equations using separation of variables.
- Graphical illustration of certain differential equations, equation systems and their solution.

Concept

Throughout this section, you will be expanding your understanding of differential and integral calculus to solve problems related to science and mathematics involving derivatives and rates of change. Just as you solved related rate problems in subtopic 5.9 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-28208/) you will now **model** more complex problems with more advanced techniques. As you go through this chapter, think about how these methods can be applied to scientific concepts.

Theory of Knowledge

Your goal throughout this course has been to get the 'right' answer. Did it ever occur to you that mathematics may be unique to all the other areas of knowledge in this regard? Is there a right answer in history? Art? Human sciences? Consider the methodology by which areas of knowledge establish validity of knowledge.

Knowledge Question: What is unique about the methodology of mathematics that allows for such certainty?

5. Calculus / 5.14 Differential equations

First-order differential equations

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This section will formally define differential equations and how to classify them by order and linearity. As you go through this section, consider the following example problem.

The function $y = \mathrm{e}^{x^2}$ is differentiable on the interval $(-\infty, \infty)$. Using the chain rule, you can find $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\mathrm{e}^{x^2}$. If you replace e^{x^2} with y, the derivative becomes $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy$.

What if you only had the derivative function $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy$ and were asked to solve for the unknown function $y = f(x)$?

Classifying differential equations

As you learned in subtopic 5.3 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-26147/), the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ of a function $y = f(x)$ is itself a function $f'(x)$. The equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy$ is an example of a differential equation.

✓ **Important**

A **differential equation** is an equation containing the derivatives of one or more dependent variables with respect to one or more independent variables.

As you have already seen, there are many different ways of writing a derivative. All of them are still acceptable here, for example, $\frac{\mathrm{d}y}{\mathrm{d}x}$, $f'(x)$, y', $\dot{y}$.

There are many ways to classify, or describe, differential equations. Although this course limits discussions of differential equations considerably, it is good to know what assumptions are made before you start. The most common classifications are by type, order and linearity.

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There are two main types of differential equations: ordinary differential equations (ODEs) and partial differential equations (PDEs). Ordinary differential equations deal with functions of a single variable and ordinary derivatives, the type that you have already worked with in this course. Partial differential equations deal with multivariable equations and their partial derivatives. For example, if you have a multivariate function

$$f(x, y) = x^2y + 2y$$

you can find partial derivatives by treating other variables as constant. If you treat y as a constant, you can find the partial derivative with respect to x,

$$\frac{\partial f}{\partial x} = 2xy$$

and the partial derivative with respect to y,

$$\frac{\partial f}{\partial y} = x^2 + 2\,.$$

However, you will only deal with ordinary differential equations in this course. Some examples of ordinary differential equations include:

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 8y = \sin x$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = 5y + x$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\mathrm{d}y}{\mathrm{d}t} = x + y$$

The **order of a differential equation** is the highest order *derivative* in the equation. You worked with second derivatives in subtopic 5.10 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27511/). Some examples include:

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Steps	Explanation
$\frac{\mathrm{d}y}{\mathrm{d}x}+2y=\mathrm{e}^x$	First-order ODE.
$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-8y=3x$	First-order ODE (the first derivative is squa
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}=5y+x$	Second-order ODE.

Finally, differential equations can be classified by their **linearity**. A differential equation is said to be linear if all of the terms with dependent variables are first-order. Some examples of linear and non-linear differential equations include:

Steps	Explanation
$y+4x\frac{\mathrm{d}y}{\mathrm{d}x}=x$	First-order linear ODE.
$(1-y)\frac{\mathrm{d}y}{\mathrm{d}x}+2y=\sin x$	First-order non-linear ODE (y in coefficien
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\frac{\mathrm{d}y}{\mathrm{d}x}-y=0$	Second-order linear ODE.
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\cos y=0$	Second-order non-linear ODE ($\cos y$).
$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+x\frac{\mathrm{d}y}{\mathrm{d}x}+7y=\ln x$	Third-order linear ODE.

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Important

An **ordinary differential equation** is an equation containing only ordinary derivatives of one or more dependent variables with respect to a single independent variable.

The **order of a differential equation** is the highest order derivative in the equation.

An ordinary differential equation is said to be **linear** if:

- the dependent variable *y* and all of its derivatives are first degree
- the coefficients of all terms with the dependent variable and derivatives depend only on the independent variable x.

So, back to the initial example: $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy$ can be classified as a first-order linear ordinary differential equation.

4 section questions

5. Calculus / 5.14 Differential equations

Exact solution: separable equations

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Solving through separation of variables

Later on in this course, you will learn Euler's method for finding a numerical approximation to a first-order ordinary differential equation. You may find that numerical methods using computer technology give good enough solutions for many purposes, but there may come a time when you want the exact answer without having to consider the error that occurs when you find an approximate

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answer using a numerical method. A closed-form exact answer at times is much more valuable than a numerical solution that slowly moves away from being correct.

The first technique you are going to study is for finding exact solutions of separable equations ; specifically, first-order separable equations.

Important

A first-order differential equation is **separable** if it can be written in the form $y' = f(x)g(y)$.

For separable equations, it is more useful to use the $\frac{\mathrm{d}y}{\mathrm{d}x}$ notation for the derivative, so you can write the equation as $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$. This allows you to separate the variables in the equation completely by moving all of the x-variables on to one side and the y-variables on the other. More formally, the steps leading to exact solutions of a separable equation are:

1. **Separate the variables** to write the equation in the form $\frac{1}{g(y)}\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)$.
2. **Integrate** both the left-hand side and right-hand side (with respect to x) to get $\int \frac{1}{g(y)}\mathrm{d}y = \int f(x)\mathrm{d}x$.
3. Evaluate $\int \frac{1}{g(y)}\mathrm{d}y = \int f(x)\mathrm{d}x$ on both sides. Do not forget the constant of integration. This will lead to an implicit equation for y.
4. Most of the time, you will be asked to express y in terms of x, but sometimes this is not necessary (or possible), and the implicit equation is also fine as an answer.

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This course will use this process without going into details about the conditions on f and g under which this method gives all the solutions of the differential equation. This would be beyond the syllabus. For example, if $g(y)$ is 0, then it is

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not valid to divide by it, but these subtleties will be left for a future course.

When you first learned integration, you learned about indefinite and definite integrals. Indefinite integrals resulted in a family of functions, all looking very similar other than a constant added at the end. A '$+c$' accounts for all of these functions. For a definite integral, some initial conditions allows you to find the value of the constant and produce the one correct result.

Differential equations are similar. A general solution is one that still has a constant resulting from the integration in Step 3. A particular solution is obtained from the general solution by specifying some set of initial conditions.

✓ Important

A **general solution** to the differential equation $y' = F(x, y)$ is a solution that involves an essential arbitrary constant.

A **particular solution** to the differential equation $y' = F(x, y)$ is the specific solution that also satisfies some initial condition $y(x_0) = y_0$.

The method to solve separable first-order differential equations requires you to find two integrals. Sometimes, these integrals are not too difficult to find, but in exams you can get questions when a specific method is needed for the integration.

Exam tip

Make sure that you review the techniques of integration. You will need to be efficient in finding antiderivatives when you are asked to solve differential equations.

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Example 1

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Separate the variables and write down the integrals you need to find to solve the following differential equations:

1. $\frac{y'}{x} = \frac{e^x}{y}$.
2. $y' = \cos^2 x \cos^2 y$.
3. $xy' \ln y = \ln x$.
4. $\sqrt{1-x^2}\,y' = 1 + y^2$.

1. $yy' = xe^x$
   $\int y \,\mathrm{d}y = \int xe^x \mathrm{d}x$
2. $\frac{1}{\cos^2 y} y' = \cos^2 x$
   $\int \sec^2 y \,\mathrm{d}y = \int \cos^2 x \,\mathrm{d}x$
3. $y' \ln y = \frac{\ln x}{x}$
   $\int \ln y \,\mathrm{d}y = \int \frac{\ln x}{x} \mathrm{d}x$
4. $\int \frac{1}{1+y^2} \mathrm{d}y = \int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x$

Example 2

★☆☆

Determine whether $y' = x + y$ is separable. If so, find the general solution to the differential equation.

$y' = x + y$ is not separable.

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Example 3

★★☆

Determine whether $y' = xy^2$ is separable. If so, find the general solution to the differential equation.

Rewrite the derivative as $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = xy^2$$

Separate the variables:

$$\frac{1}{y^2}\mathrm{d}y = x\mathrm{d}x$$

Integrate both sides:

$$\int \frac{1}{y^2}\mathrm{d}y = \int x\mathrm{d}x$$

Evaluate both integrals:

$$-\frac{1}{y} = \frac{x^2}{2} + c_1 = \frac{x^2 + 2c_1}{2}$$

Solve for y:

$$y = -\frac{2}{x^2 + 2c_1}$$

Redefine a new constant $c = 2c_1$:

$$y = -\frac{2}{x^2 + c}$$

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Notice the last step. Although not required, it is a good idea to simplify constants when possible. Any value you have for c_1 maps directly to a specific value for c.

Example 4

★★☆

Determine whether $xy' - y = 1 - y'$ is separable. If so, find the general solution to the differential equation.

$$xy' + y' = 1 + y$$

Factorise:

$$(x+1)y' = 1 + y$$

Separate the variables:

$$\int \frac{1}{1+y}\mathrm{d}y = \int \frac{1}{1+x}\mathrm{d}x$$

Integrate: recall that $\int \frac{1}{y}\mathrm{d}y = \ln|y| + c$:

$$\ln(1+y) = \ln(x+1) + c$$

$$1 + y = e^{\ln(x+1)+c}$$

Solve for y:

$$y = e^c e^{\ln(x+1)} - 1$$

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Redefine a new constant $c = \mathrm{e}^{c}$:

$$y = c(x+1) - 1$$

Example 5

★★☆

Find the particular solution of the differential equation $xy' - y = 1 - y'$ that satisfies $y(1) = -2$.

From **Example 5**, you have already found the general solution to be $y = c(x+1) - 1$. By applying the initial conditions, you get:

Using $x = 1$, $y = –2$:

$$-2 = c(1+1) - 1$$

$$-1 = 2c$$

Calculate c:

$$c = -\frac{1}{2}$$

Substitute for c in the general solution:

$$y = -\frac{1}{2}(x+1) - 1$$

This is the particular solution:

$$y = -\frac{1}{2}x - \frac{3}{2}$$

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3 section questions

5. Calculus / 5.14 Differential equations

Applications

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At the beginning of this section, several real-world scenarios were mentioned. The following examples will address a few of these topics.

Example 1

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A hard-boiled egg at $100^{\circ}\mathrm{C}$ is cooled in running water at $20^{\circ}\mathrm{C}$. After 5 minutes, the temperature of the egg is $43^{\circ}\mathrm{C}$. When will the temperature reach $30^{\circ}\mathrm{C}$?

First define the variables:

- T Temperature
- $T_s = 20$ Temperature of surroundings
- $T_0 = 100$ Starting temperature
- r Time constant

Change in temperature over time:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -r(T - T_s)$$

Separate the variables and substitute known value:

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$$\frac{\mathrm{d}T}{T-20} = -r\mathrm{d}t$$

Integrate:

$$\ln(T-20) = -rt + C^*$$

Solve for T:

$$T = \mathrm{e}^{-rt+C^*} + 20$$

Let $C = e^{C^*}$:

$$T = C\mathrm{e}^{-rt} + 20$$

Apply initial conditions $T(0) = 100$:

$$100 = C\mathrm{e}^{-r(0)} + 20$$

Solve for C:

$$C = 80$$

Apply initial conditions $T(5) = 43$:

$$T = 80\mathrm{e}^{-rt} + 20$$

$$43 = 80\mathrm{e}^{-r(5)} + 20$$

Solve for r:

$$r = 0.2493$$

$$T = 80\mathrm{e}^{-0.2493t} + 20$$

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Apply values for question:

$$30 = 80e^{-0.2493t} + 20$$

Solve for t:

$$t = 8.34 \text{ min}$$

This is known as Newton's law of cooling. It is often written as $T - T_S = (T_0 - T_S)e^{-rt}$ where T_0 is the initial temperature, T_S is the surrounding temperature, and t is time.

Example 2

★★★

Carbon dating is a technique used to determine the age of anything containing organic material. Carbon-14 is in the Earth's atmosphere. It enters plants through photosynthesis and enters animals when they eat plants (or other animals). Once the plants and animals die, they stop absorbing new carbon-14, and the atoms decay naturally to become nitrogen-14. The term half-liferefers to the amount of time it takes for half of the carbon-14 to decay. By measuring the amount of carbon-14 in a sample and comparing this with the amount expected before death, an approximate age can be determined.

Carbon-14 has a half-life of about 5700 years. Find the age of a fossil where 80% of the carbon-14 remains.

First define the variables:

- A Amount
- A_0 Initial amount
- k Time constant

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Change in amount over time:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = -kA$$

Separate variables and substitute known value:

$$\frac{\mathrm{d}A}{A} = -k\mathrm{d}t$$

Integrate:

$$\ln(A) = -kt + C^*$$

Solve for A:

$$A = \mathrm{e}^{-kt+C^*}$$

Let $C = \mathrm{e}^{C^*}$:

$$A = C\mathrm{e}^{-kt}$$

Apply initial conditions $A(0) = A_0$:

$$A_0 = C\mathrm{e}^{-k(0)}$$

Solve for C:

$$C = A_0$$

$$A = A_0\mathrm{e}^{-kt}$$

Apply half-life conditions $A(5700) = \frac{1}{2}A_0$:

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$$\frac{1}{2}A_0 = A_0\mathrm{e}^{-k(5700)}$$

Solve for k:

$$k = \frac{\ln 2}{5700}$$

$$A = A_0\mathrm{e}^{-\frac{t\ln 2}{5700}}$$

Apply values for question:

$$0.8A_0 = A_0\mathrm{e}^{-\frac{t\ln 2}{5700}}$$

Solve for t:

$$t = 1835 \text{ years}$$

This specific example is known as the radioactive half-life formula. More generally, it is referred to as the exponential decay model. It can be used for carbon dating or prediction of the decay of other radioactive elements. It is commonly written as $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ where A is the amount remaining, A_0 is the initial amount, t is the time and h is the half-life. Can you follow the algebraic conversion from the equation in the example to this one using the laws of exponents?

Example 3

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A colony of bacteria grows exponentially over time. At the end of 3 hours, there are 5000 bacteria. At the end of 5 hours, there are $20\,000$ bacteria. How many were there initially?

First define the variables:

- P Amount
- P_0 Initial amount
- r Growth rate

Change in amount over time:

$$\frac{dP}{dt} = rP$$

Separate variables and substitute known value:

$$\frac{dP}{P} = rdt$$

Integrate:

$$\ln(P) = rt + C^*$$

Solve for P:

$$P = \mathrm{e}^{rt+C^*}$$

Let $C = \mathrm{e}^{C^*}$:

$$P = C\mathrm{e}^{rt}$$

Apply initial conditions $P(0) = P_0$:

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$$P_0 = C\mathrm{e}^{r(0)}$$

Solve for C:

$$C = P_0$$

$$P = P_0\mathrm{e}^{rt}$$

Apply first condition $P(3) = 5000$:

$$5000 = P_0\mathrm{e}^{r(3)}$$

Apply second condition $P(5) = 20\,000$:

$$20\,000 = P_0\mathrm{e}^{r(5)}$$

Divide second equation by first:

$$\frac{20\,000}{5000} = \frac{\mathrm{e}^{5r}}{\mathrm{e}^{3r}}$$

Simplify:

$$4 = \mathrm{e}^{5r}\mathrm{e}^{-3r} = \mathrm{e}^{2r}$$

Solve for r:

$$r = 0.6931$$

Substitute into first equation

$$5000 = P_0\mathrm{e}^{0.6931(3)}$$

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Solve for P_0:

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$$P_0 = 625$$

This is generally referred to as the exponential growth model. It is not only used in modelling population growth, but is also used in economics when modelling continuous compounding. The equation used in economics is $A = Pe^{rt}$, where P is the amount of money invested with continuous compounding at an interest rate of $r\%$ for t units of time (typically years) resulting in in a final amount of A. This is sometimes abbreviated to the PERT model.

3 section questions

5. Calculus / 5.14 Differential equations

Checklist

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What you should know

By the end of this subtopic you should:

- be able to identify and classify a differential equation by type, order, and linearity
- be able to work with first-order differential equations of the form $y' = F(x, y)$
- know that a general solution is a function y that satisfies the differential equation and that there are usually infinitely many solutions
- be able to find an exact solution of a separable differentiable equation
- know that a solution that satisfies an initial condition $y(x_0) = y_0$ is called a particular solution of the differential equation
- understand that the first step of the solution for separable differentiable equation is separating the variables

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view

- be able to rearrange a separable differentiable equation into the form $y' = f(x)g(y)$.

Investigation

In this subtopic, you learned about first-order differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$ and techniques to solve them through separation of variables $\int \frac{1}{g(y)}\mathrm{d}x = \int f(x)\mathrm{d}x$.

Activity

In the applet below, you can investigate separable functions of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$.

- Compose a separable function of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$.
- Work through the problem, finding the general and particular solutions.
- Open the applet and enter the equations represented by $f(x)$ and $g(x)$. In the applet, they are labelled as \x part' and 'y part'.
- Set the initial conditions (x_0, y_0).
- Does your solution match the solution on the applet?
- Click and drag the starting point on the graph. What happens to the solution curve?

Interactive 1. Exploring Solution Curves in Differential Equations.

More information for interactive 1

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This interactive is a graph that allows the user to investigate the process of solving separable functions of the form $\frac{dy}{dx} = f(x)g(y)$.

The screen is divided into two halves. On the top half, a graph of xy axes is displayed which ranges from -7 to 7 on the x-axis and -1 to 5 on the y-axis. On the bottom half, the users can manually enter the functions $f(x)$ and $g(x)$ and the initial conditions for x and y in their respective text boxes. The function is represented by a blue curve which projects on the graph with a purple dot, which can be moved to modify the blue curve and its parameters.

If the input of the function is $f(x) = x$ and $g(y) = \frac{1}{y}$. The initial conditions are given as $x = 1$ and $y = 1$. This means we are looking for a solution to the differential equation that passes through the point $(1, 1)$. The graph displays a V-shape, which is the characteristic graph of the function $y = |x|$. The purple dot is at the point $(1, 1)$. This point lies on the graph of $y = |x|$.

Separate variables indicate: The equation $y\,dy = x\,dx$ is obtained by multiplying both sides of the original equation by $y\,dx$. This group the terms involving y with dy and the terms involving x with dx.

Integrate indicates: Integrating both sides of the separated equation gives $\int y\,dy = \int x\,dx$.

Evaluate indicates: The integration yields $\frac{y^2}{2} = \frac{x^2}{2} + c$, where (c) is the constant of integration.

Solve for c: Using the initial conditions $x = 1$ and $y = 1$, substitute these values into the evaluated equation: $\frac{1^2}{2} = \frac{1^2}{2} + c \Rightarrow c = 0$. The calculation shows that the constant of integration (c) is equal to 0.

Rewrite with c value indicate: Substituting $c = 0$ back into the evaluated equation gives $\frac{y^2}{2} = \frac{x^2}{2} + c$, which simplifies to $\frac{y^2}{2} = \frac{x^2}{2}$.

Solve for y indicate: Multiplying both sides by 2 gives $y^2 = x^2$. Taking the square root of both sides yields $y = |x|$.

This interactive will help users find the solution to a differential equation with a given initial condition corresponding to a specific curve passing through that point on a graph.

Student view

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