Index

The big picture

Heron of Alexandria came across square roots of negative numbers in the first century while doing calculations of volumes of pyramids.

These numbers make their next big appearance in maths in the 16th century with the Italian mathematician, Girolamo Cardano, who was working on finding solutions to cubic equations.

Roots of negative numbers made mathematicians uncomfortable and seemed to present no real-world application and were thus called ‘imaginary numbers’. Interestingly, Cardano was able to use imaginary numbers to find real solutions to cubic equations. You can learn more about this by watching the video below.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Imaginary numbers were developed further in the 18th and 19th centuries by Leonard Euler, Carl Fredrich Gauss and William Rowan Hamilton.

Now complex numbers are used in many applied fields such as electrical engineering, making electricity, mobile phones and computers possible in your daily life.

Concept

Initially, mathematicians had two problems with complex numbers: the inability to see their connection to the real world and lack of structures to represent these kinds of numbers. As you work through this section, consider how the various forms of representation of complex numbers enhance your understanding of their nature.

1. Number and algebra / 1.12 Introduction to complex numbers

Cartesian form of complex numbers

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/cartesian-form-complex-numbers-id-26985/print/) Assign

Complex numbers

Defining complex numbers allows you to solve equations that involve square roots of negative numbers.

Making connections

Quadratics where the discriminant is negative ($\Delta < 0$) will have solutions that use imaginary numbers. You can get more information about solving quadratics and the discriminant by clicking here to navigate to subtopic 2.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27705/).

Important

The number i is defined as $\mathrm{i} = \sqrt{-1}$ or as the solution to $\mathrm{i}^2 = -1$.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Euler, who lived in the 1700s, was the first mathematician to use $\mathrm{i}=\sqrt{-1}$.

Example 1

★☆☆

Simplify each of the following:

a) i^2

b) i^3

c) i^4

Steps	Explanation
a)	$\mathrm{i}^2=-1$
b)	$\mathrm{i}^3=\mathrm{i}^2\times\mathrm{i}=-1\times\mathrm{i}=-\mathrm{i}$
c)	$\begin{aligned}\mathrm{i}^4&=\mathrm{i}^3\times\mathrm{i}\\&=-\mathrm{i}\times\mathrm{i}\\&=-\mathrm{i}^2\\&=-(-1)\\&=1\end{aligned}$

Making connections

Although surds are not defined for negative values, it is useful to remember that working with $\sqrt{-1}$ follows the same rules as for surds.

Some useful rules for surds are:

$$\sqrt{a}\times\sqrt{a}=a$$

$$\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$$

$$b\sqrt{a}+c\sqrt{a}=(b+c)\sqrt{a}$$

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

where $a \geq 0$ and $b \geq 0$.

✓ **Important**

The set of complex numbers is represented by $\mathbb{C}$ and in Cartesian form a complex number is defined as $z = a + b\mathrm{i}$, where $a, b \in \mathbb{R}$ and $\mathrm{i} = \sqrt{-1}$.

The complex number, z , is made up of two parts:

1. The real part which is represented by: $\mathrm{Re}\,(z) = a$
2. The imaginary part which is represented by: $\mathrm{Im}\,(z) = b$

If z is a purely real number , $\mathrm{Im}\,(z) = 0$ and $z = a$.

If z is a purely imaginary number , $\mathrm{Re}\,(z) = 0$ and $z = b\mathrm{i}$.

Example 2

★☆☆

Given that $z = 3 - 2\mathrm{i}$, find $\mathrm{Re}\,(z)$ and $\mathrm{Im}\,(z)$.

Steps	Explanation
$\mathrm{Re}\,(z) = 3$ $\mathrm{Im}\,(z) = -2$	Using: $z = a + b\mathrm{i}$ $\mathrm{Re}\,(z) = a$ $\mathrm{Im}\,(z) = b$

Example 3

★★☆

Consider the complex number $z = 3x\mathrm{i} + y^2 + 1$, where x and y are real.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Find the values of x and y when $\text{Re}\,(z) = 2$ and $\text{Im}\,(z) = 9$.

Steps	Explanation
$\text{Re}\,(z) = y^2 + 1 = 2$ $\text{Im}\,(z) = 3x = 9$	$z = 3x\text{i} + y^2 + 1 = \underbrace{y^2 + 1}_{a} + \underbrace{3x}_{b}\,\text{i}$
$3x = 9 \quad \Leftrightarrow \quad x = 3$	
$y^2 + 1 = 2 \quad \Leftrightarrow \quad y^2 = 1 \quad \Leftrightarrow \quad y = \pm\sqrt{1}$ $\Leftrightarrow \quad y = \pm 1$	

Activity

Use what you know about surds to simplify fully each of the following (the first one has been done for you).

$$\left(3 + 2\sqrt{5}\right) + \left(-1 + 3\sqrt{5}\right) = 2 + 5\sqrt{5}$$

$$\left(4 - \sqrt{2}\right) - \left(7 + 3\sqrt{2}\right) =$$

$$\left(2 + 3\sqrt{3}\right) \times \left(5 - \sqrt{3}\right) =$$

$$\frac{-3 + 2\sqrt{6}}{1 - \sqrt{6}} =$$

Hence, deduce the rules for addition, subtraction, multiplication and division of complex numbers.

Important

Let $z_1 = a + b\text{i}$ and $z_2 = c + di$. Then the following rules apply to addition, subtraction, multiplication, division and equality.

Addition

$$z_1 + z_2 = (a + c) + (b + d)\,\text{i}$$

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Subtraction

$$z_1 - z_2 = (a - c) + (b - d)\,\mathrm{i}$$

Multiplication

$$z_1 \times z_2 = (a + b\mathrm{i})\,(c + d\mathrm{i}) = ac + ad\mathrm{i} + bc\mathrm{i} + bd\mathrm{i}^2 = (ac - bd) + (ad + bc)\,\mathrm{i}$$

Division

$$\begin{aligned}\frac{z_1}{z_2} &= \frac{a + b\mathrm{i}}{c + d\mathrm{i}} \times \frac{c - d\mathrm{i}}{c - d\mathrm{i}} = \frac{ac - ad\mathrm{i} + bc\mathrm{i} - bd\mathrm{i}^2}{c^2 - cd\mathrm{i} + cd\mathrm{i} - d^2\mathrm{i}^2} \\ &= \frac{(ac + bd) + (bc - ad)\,\mathrm{i}}{c^2 + d^2} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}\mathrm{i}\end{aligned}$$

Equality

$z_1 = z_2$ if and only if $a = c$ and $b = d$

Exam tip

The rules for operations with complex numbers are not in the IB formula booklet but they are the same as those for working with surds so you are already familiar with them.

Example 4

★★☆

Find the real numbers a and b if $(2 - 3\mathrm{i})\,(a + b\mathrm{i}) = 4 + 7\mathrm{i}$.

Steps	Explanation
$\begin{aligned}(2 - 3\mathrm{i})\,(a + b\mathrm{i}) &= 2a + 2b\mathrm{i} - 3a\mathrm{i} - 3b\mathrm{i}^2 \\ &= 2a + 3b + (2b - 3a)\,\mathrm{i}\end{aligned}$	Using the multiplication rule. Remember that $\mathrm{i}^2 = -1$.

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Steps	Explanation
$2a+3b+(2b-3a)\,\mathrm{i}=4+7\mathrm{i}$	Using equality.
$\begin{cases}2a+3b=4\\-3a+2b=7\end{cases}$ $a=\dfrac{4-3b}{2}$ $-3\left(\dfrac{4-3b}{2}\right)+2b=7$ $-6+\dfrac{13}{2}b=7$ $13=\dfrac{13}{2}b$ $b=2$ $a=\dfrac{4-3b}{2}=\dfrac{4-3\,(2)}{2}=-1$	Solve the system of equations by using elimination or substitution.

Example 5

★★☆

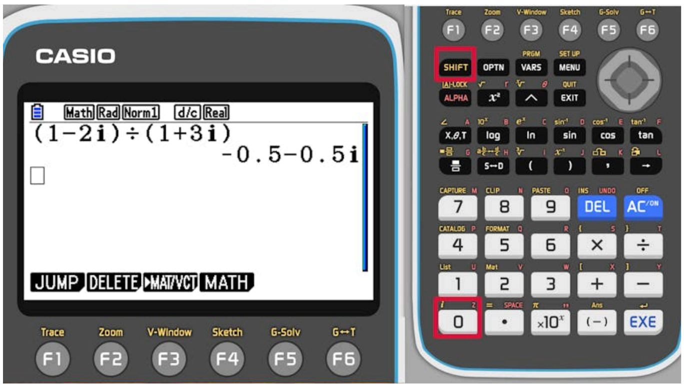
Write $\dfrac{1-2\mathrm{i}}{1+3\mathrm{i}}$ in $a+b\mathrm{i}$ form.

Steps	Explanation
$\begin{aligned}\frac{1-2\mathrm{i}}{1+3\mathrm{i}}&=\frac{1-2\mathrm{i}}{1+3\mathrm{i}}\times\frac{1-3\mathrm{i}}{1-3\mathrm{i}}\\&=\frac{1-3\mathrm{i}-2\mathrm{i}+6\mathrm{i}^2}{1+9}\\&=\frac{-5-5\mathrm{i}}{10}=-\frac{1}{2}-\frac{1}{2}\mathrm{i}\end{aligned}$	Multiply the numerator and denominator by the **complex conjugate** of $(1+3i)$ which is $(1–3i)$. This simplifies the denominator to a real number. (Compare this with rationalising the denominator of a surd.)

Student view

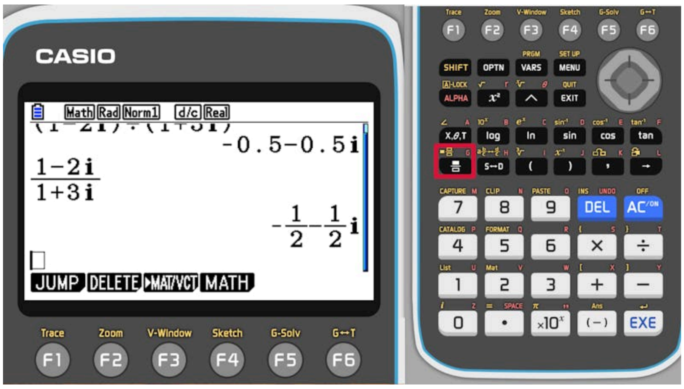
In addition to doing these calculations analytically, you should also be able to use your calculator to add, subtract, multiply and divide complex numbers.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
To work with complex numbers, open the calculator option.	
Use the imaginary unit in your expressions, otherwise you can use the operations that you use for real numbers.	

Student view

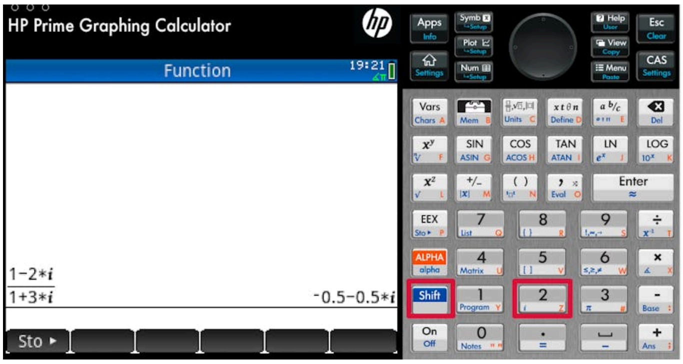
Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

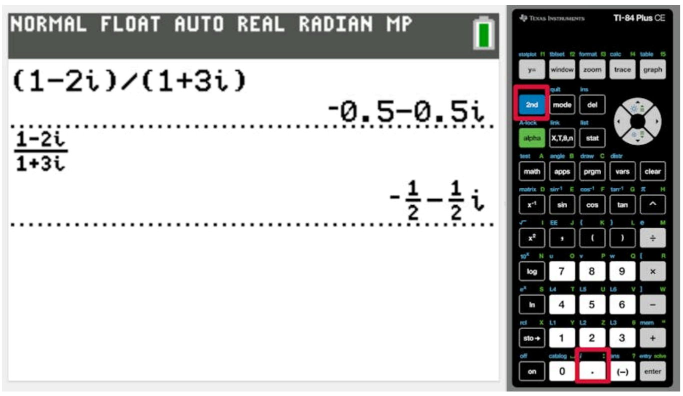
Steps	Explanation
If you use fractions, you will get the answer in different form.	

Steps	Explanation
To work with complex numbers, enter the home screen of any application.	

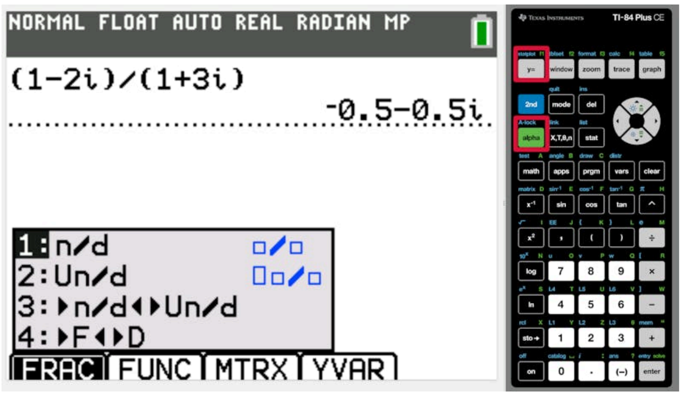
Student
view

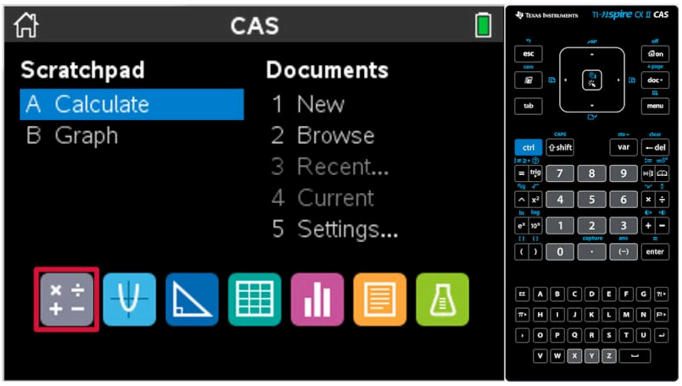
Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

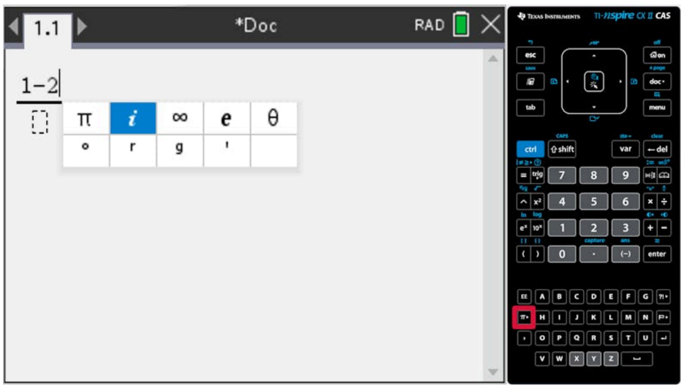
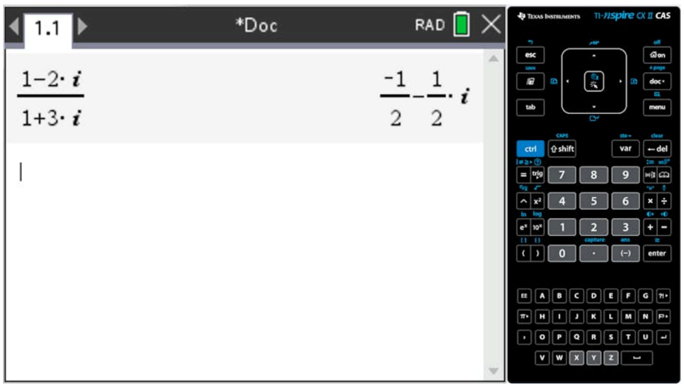
Steps	Explanation
Use the imaginary unit in your expressions, otherwise you can use the operations that you use for real numbers.	

Steps	Explanation
Use the imaginary unit in your expressions, otherwise you can use the operations that you use for real numbers. Note, that you get your answer in different format when you enter your expression in different format.	

Student view

Steps	Explanation
You can access the fraction form (the second form on the previous screen) using alpha/f1.	

Steps	Explanation
To work with complex numbers, open a calculator page.	

Steps	Explanation
Use the imaginary unit in your expressions, otherwise you can use the operations that you use for real numbers.	
	

Important

The numbers $c+d\text{i}$ and $c-d\text{i}$ that are used in division of complex numbers such that $\frac{z_1}{z_2}=\frac{a+b\text{i}}{c+d\text{i}}\times\frac{c-d\text{i}}{c-d\text{i}}$ are named complex conjugates.

If $z=a+b\text{i}$, then its complex conjugate is $z^*=a-b\text{i}$.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Example 6

★★☆

For two complex numbers z and w, show that $(z+w)^* = z^* + w^*$.

Let $z = a + bi$ and $w = c + d\mathrm{i}$

$$\begin{aligned}\text{LHS} &= (z+w)^* \\ &= ((a+b\mathrm{i}) + (c+d\mathrm{i}))^* \\ &= ((a+c) + (b+d)\,\mathrm{i})^* \\ &= (a+c) - (b+d)\,\mathrm{i}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= (a-b\mathrm{i}) + (c-d\mathrm{i}) \\ &= (a+c) + (-b-d)\,\mathrm{i} \\ &= (a+c) - (b+d)\,\mathrm{i}\end{aligned}$$

$\text{LHS} = \text{RHS}$

Example 7

★★★

Solve the system of equations

$$z + 2\mathrm{i} = \mathrm{i}\,(3-w)$$

$$7\mathrm{i}\,(z-1) = 2\,(z+2w)$$

where $z, w \in \mathbb{C}$.

Steps	Explanation
From the first equation: $$z = 3\mathrm{i} - w\mathrm{i} - 2\mathrm{i} = \mathrm{i} - w\mathrm{i}$$	Solve the system of equations by substitution.

Student view

Steps	Explanation
Substitute $z = \text{i} - w\text{i}$ into the second equation: $$7\text{i}\,(\text{i} - w\text{i} - 1) = 2\,(\text{i} - w\text{i} + 2w)$$ $$-7 + 7w - 7\text{i} = 2\text{i} - 2w\text{i} + 4w$$ $$3w + 2w\text{i} = 7 + 9\text{i}$$ $$w\,(3 + 2\text{i}) = 7 + 9\text{i}$$ $$w = \frac{7 + 9\text{i}}{3 + 2\text{i}} = 3 + \text{i}$$	
$z = \text{i} - (3 + \text{i})\,\text{i} = 1 - 2\text{i}$	Substitute $w = 3 + \text{i}$ into $z = \text{i} - w\text{i}$ to fin z .

6 section questions

1. Number and algebra / 1.12 Introduction to complex numbers

Graphing in the complex plane

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/graphing-in-the-complex-plane-id-26986/print/) Assign

Graphing complex numbers

Complex numbers can be represented on a two-dimensional plane called the Argand diagram. Similar to the Cartesian plane that you have used up to now for graphing, the Argand diagram has a vertical and a horizontal axis.

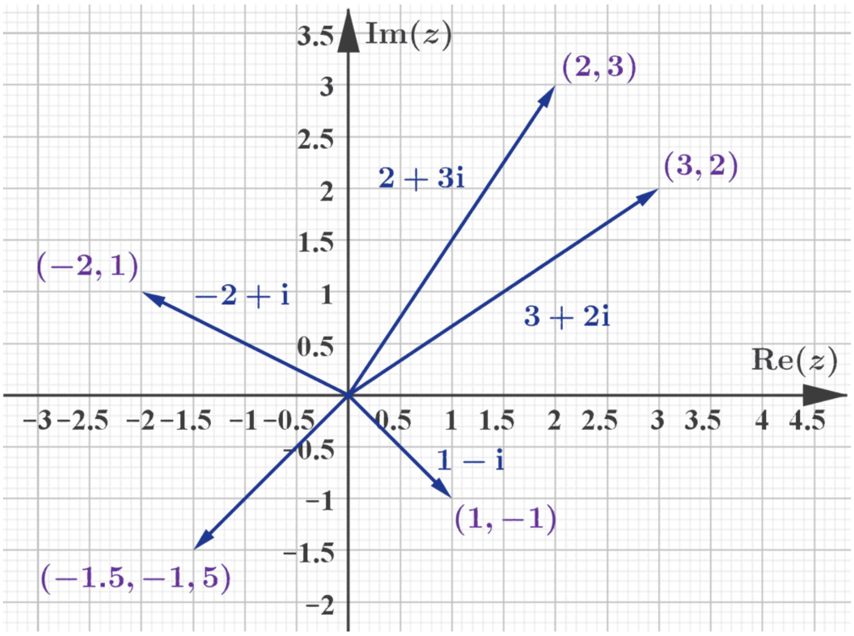
The numbers on the horizontal axis represent the real part of the complex number and the numbers on the vertical axis represent the imaginary part.

Student view

International Mindedness

John-Robert Argand and Carl Friedrich Gauss developed the concept of the complex plane during roughly the same time period. With one working in France and the other in Germany, they were able to come up with similar ideas independently. Hence, the Argand diagram is also referred to as the Gaussian plane.

Complex numbers are represented by vectors on the Argand plane. You will learn more about vectors in **topic 3**, but for now you just need to know how to plot complex numbers on the Argand plane. Examples are shown in the figure below.



More information

The image is a diagram showing an Argand plane, which is a way to represent complex numbers as vectors. The diagram has a grid background with horizontal and vertical axes. The horizontal axis is labeled as the real axis, and the vertical axis is labeled as the imaginary axis. Several vectors are drawn from the origin (0,0) to different points on the plane, each representing a complex number. The points are labeled with their corresponding complex numbers: (2, 3) representing 2+i3, (3, -2) representing 3-i2, (-2, 1) representing -2+i, (-1.5, -1.5) representing -1.5-i1.5, and (1, -1) representing 1-i1. Each vector is drawn as an arrow indicating direction from the origin to the respective point, illustrating how complex numbers can be visualized on the plane.

[Generated by AI]

As shown above, plotting $z = a + b\text{i}$ on an Argand diagram requires you to plot a point corresponding to a units on the real axis and b units on the imaginary axis. Connecting this point to the origin with a line allows you to visualise the complex number as a vector. Vectors are always drawn with an arrow indicating direction. You can see this notation in the solution to **Example 1**. However, the arrow is sometimes omitted when graphing complex numbers.

Example 1

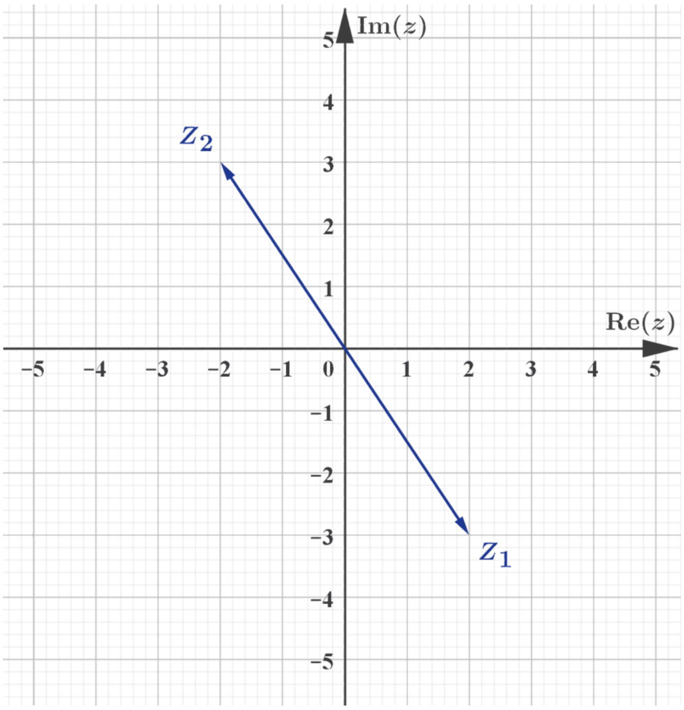
★☆☆

a) Plot $z_1 = 2 - 3\text{i}$ and $z_2 = -2 + 3\text{i}$ on the Argand diagram.

b) Plot $w_1 = \text{i}$ and $w_2 = -\text{i}$ on the Argand diagram.

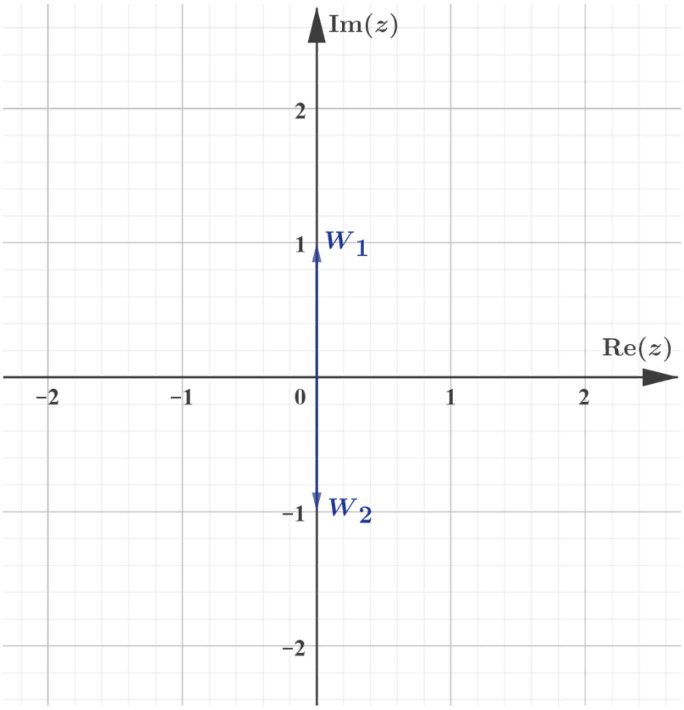
c) Hence, describe the graphical relationship between z and $-z$.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

	Steps	Explanation
a)		z_1 is plotted at 2 on the real axis and -3 on the imaginary axis. $z_2$is plotted at -2 on the real axis and 3 on the imaginary axis.

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
b)		w_1 is plotted at 0 on the real axis and 1 on the imaginary axis. w_2 is plotted at 0 on the real axis and -1 on the imaginary axis.
c)	$-z$ is the image of z rotated through 180° about the origin. $-z$ is the image of z after a reflection in the horizontal and vertical axis.	There are two ways to describe the relationship.

In addition to describing $z = a + b\text{i}$ on the Argand plane by giving the coordinates (a, b), you can also describe the same complex number by defining its distance from the origin and the angle that the line representing the complex number makes with the positive horizontal axis.

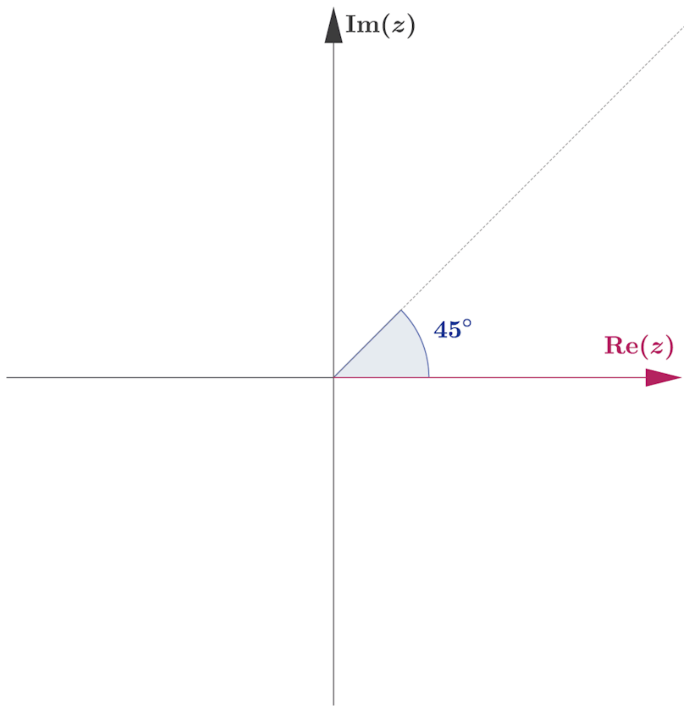
Example 2

★☆☆

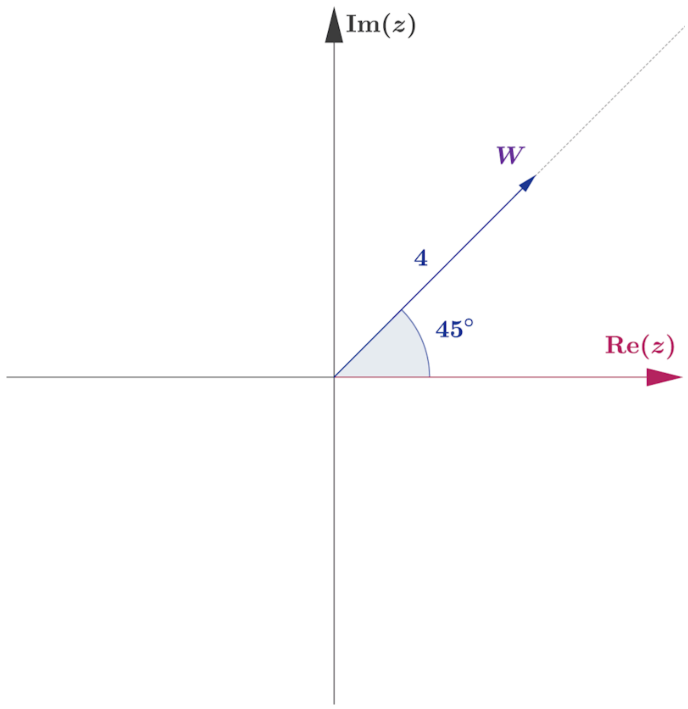
Sketch the graph for the complex number w on the Argand plane, given that it is 4 units away from the origin and makes an angle of 45° with the positive horizontal axis.

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Steps	Explanation
	The positive horizontal axis is shown in red. An angle of approximately $45°$ is drawn using a dashed line.

Student
view

Steps	Explanation
	Approximately 4 units are measured out along the line making a 45° angle with the positive horizontal axis.

Exam tip

A question that uses the command term 'sketch' is asking you to make a graph or a diagram that gives a general idea of the shape and proportions. Important information such as length, angles, coordinates of points of intersection, equations of curves, and so on, should be labelled. A sketch does not need to be made on graph paper with a precise scale.

Important

The distance from the origin on the Argand plane is called the modulus of a complex number. It is represented as $\text{mod}\,(\text{z}) = |z|$.

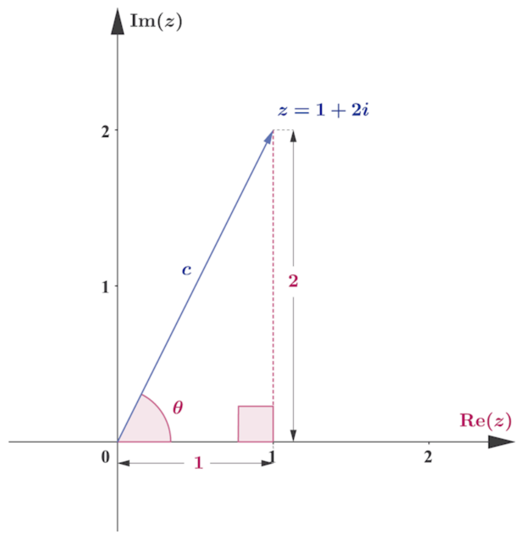
The angle with the positive horizontal axis is called the argument of a complex number . It is represented as $\arg\,(\text{z})$.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Example 3

★☆☆

Given that $z = 1 + 2\text{i}$, find $|z|$ and $\arg(z)$.

Steps	Explanation
Using Pythagoras' theorem: $$\lvert z\rvert = \sqrt{1^2 + 2^2} = \sqrt{5}$$ Using $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$: $$\arg(z) = \theta$$ $$\tan\theta = \frac{2}{1} = 2$$ $$\theta = \tan^{-1}2 = 63.4° \quad (3\text{ significant figures})$$ $$\arg(z) = 63.4°$$	You can plot z on an Argand plane to visualise what you are working with.  A right-angled triangle can be drawn with perpendicular sides of length 1 and 2 units.

Important

For a complex number given by $z = x + y\text{i}$, $\text{mod}(\text{z}) = |z| = \sqrt{x^2 + y^2}$.

Activity

Find the argument of each of the following:

Student view

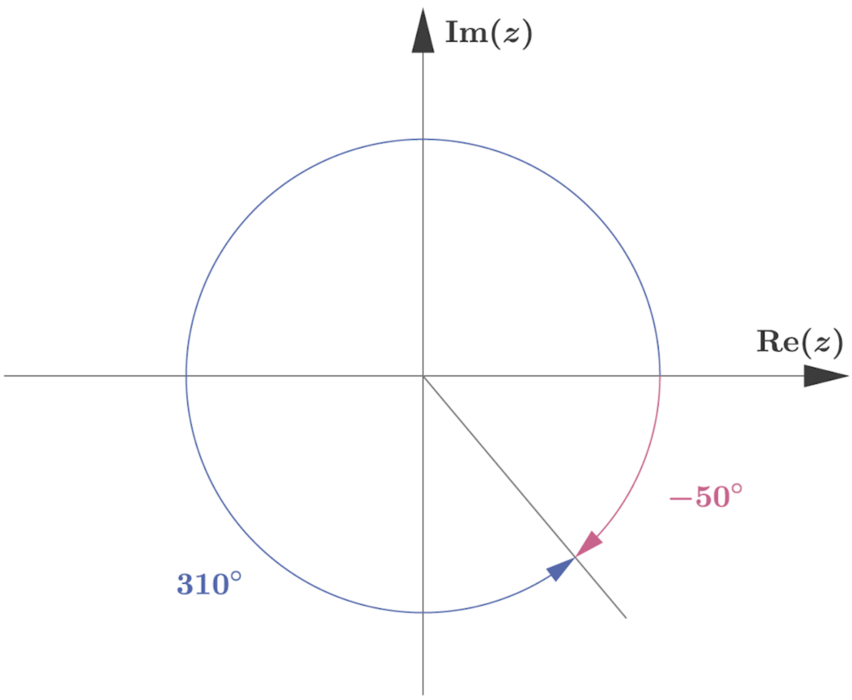
$2 + 3\text{i}$ $-1 + \sqrt{2}\text{i}$ $-3 - 5\text{i}$ $3 - 2\text{i}$

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

What difficulties do you encounter in your calculations when the complex number is not in the first quadrant?

Suggest some solutions to the problems you describe.

When you state the value of an angle there are a few equivalent possibilities. For instance, the angle of $310°$ is equivalent to $-50°$, as shown below. You will learn more about this in subtopic 3.5 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27741/).



More information

The image is a diagram of a unit circle with labeled axes Re(z) and Im(z) representing the real and imaginary components, respectively. The circle is centered at the origin of a Cartesian coordinate system. Two angles are indicated on the circle: 310° is measured clockwise, and -50° is measured counter-clockwise. Both angles correspond to the same point on the circle, showing their equivalence. The circle's pressure is depicted by arrows and lines connecting the angle labels to the perimeter, illustrating the direction of the angles.

[Generated by AI]

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Activity

The accepted convention for working with the argument of complex numbers is shown in the applet below. Drag the blue point through the quadrants and note the relationship between $\arg(z)$ and $\tan^{-1}\left(\frac{y}{x}\right)$. Describe what you notice.

Interactive 1. Graphing Complex Numbers in the Argand Plane.

More information for interactive 1

This interactive demonstrates the argument "*arg z*" of a complex number $z = x + iy$ and its relation to $\arctan(\frac{y}{x})$. The purple point represents the complex number and can be dragged across quadrants to observe how the argument changes. In the first quadrant, $\arg z = \arctan(\frac{y}{x})$. However, in the second and third quadrants, where x is negative, the argument is adjusted by adding 180° and subtracting 180°, respectively. In the fourth quadrant, the argument is negative and follows standard conventions. As the point moves, the argument smoothly transitions between -180° and 180°, illustrating how angle adjustments are necessary to correctly represent complex numbers in polar form.

For example, in the first quadrant, if we have a complex number $z = 2.82 + 2.26i$, then both x and y are positive. The argument is calculated directly using the inverse tangent function: $\arg z = \arctan(\frac{y}{x}) = \arctan\frac{(2.26}{2.82})38.71^{\circ}$. This shows how the argument of z is simply the angle between the positive real axis and the line connecting the origin to the point $(2.82, 2.26)$ in the complex plane.

Student
view

Important

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

For a complex number given by $z = x + y\text{i}$, the principal value of the argument is as follows:

- In the first and fourth quadrants when $Re\,(z) > 0$; $\arg\left(\text{z}\right) = \tan^{-1}\left(\frac{y}{x}\right)$.
- In the second quadrant when $Re\,(z) < 0$ and $Im\,(z) > 0$; $\arg\left(\text{z}\right) = 180° + \tan^{-1}\left(\frac{y}{x}\right)$.
- In the third quadrant when $Re\,(z) < 0$ and $Im\,(z) < 0, ; \arg\left(\text{z}\right) = -180° + \tan^{-1}\left(\frac{y}{x}\right)$.

Be aware

The argument of a complex number can be given in degrees or in radians. In this subtopic all questions will be in degrees as these are familiar to you. You will study radians later in the course.

Example 4

★★☆

Find the modulus and argument for

a) $2 + 3\text{i}$

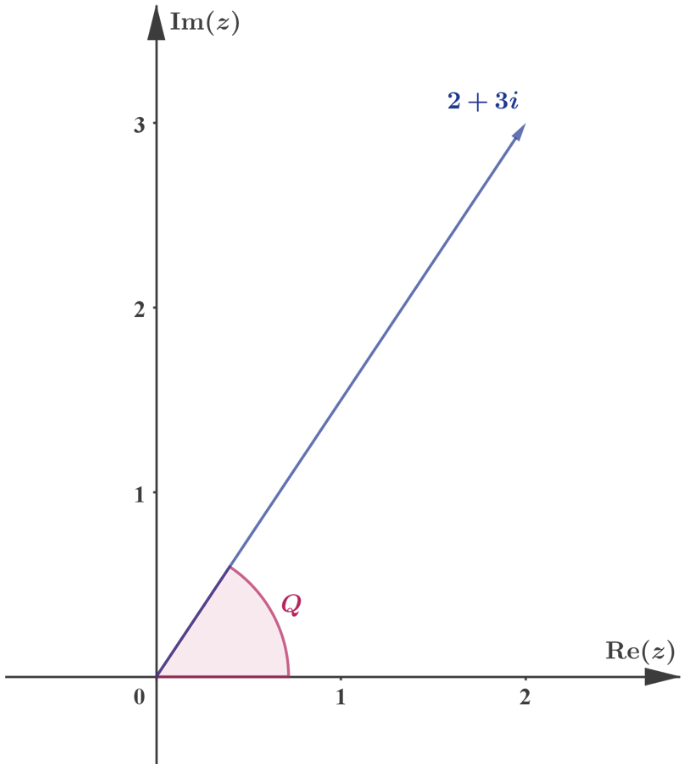
b) $-1 + \sqrt{2}\text{i}$

c) $-3 - 5\text{i}$

d) $3 - 2\text{i}$.

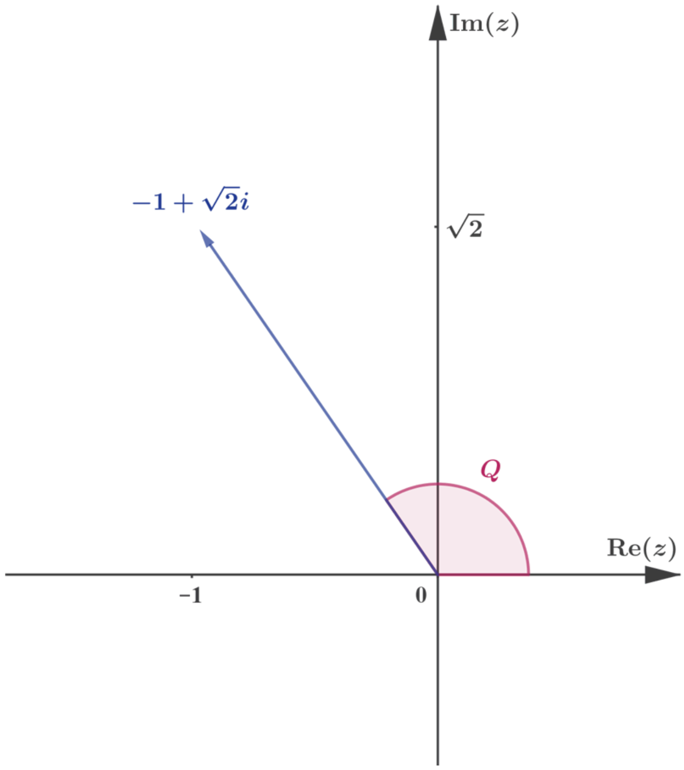
Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
a)	 $\lvert z\rvert = \sqrt{2^2+3^2} = \sqrt{13}$ $\arg(z) = \tan^{-1}\frac{3}{2}$ $= 56.3^\circ$ (3 significant figures)	You should always make a sketch of the Argand diagram for the complex number if you are working with the argument and modulus. Check that your final answer fc the modulus matches the diagram you have drawn.

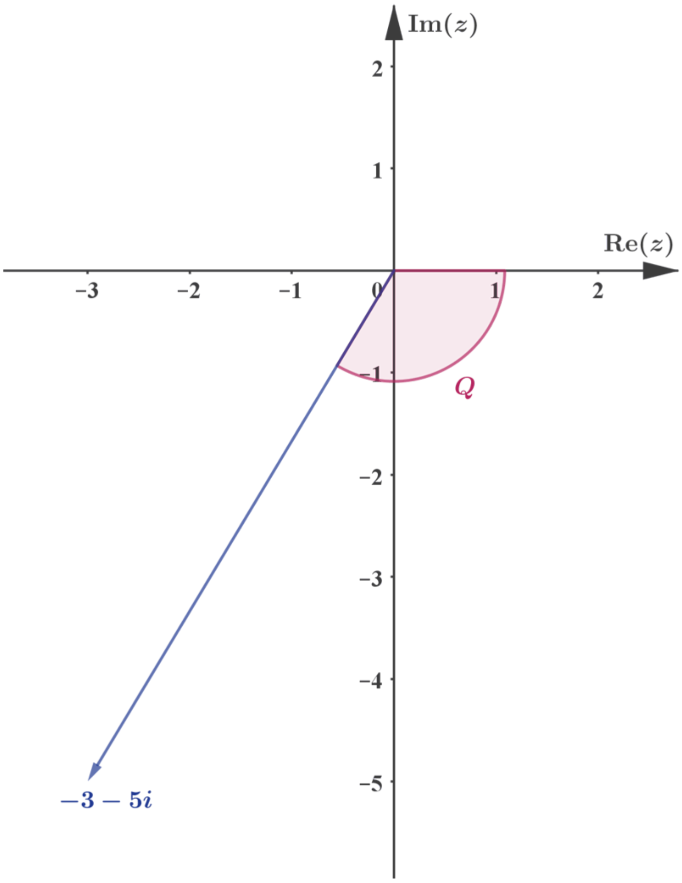
Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

	Steps	Explanation
b)	 $\|z\| = \sqrt{(-1)^2 + \sqrt{2}^2} = \sqrt{3}$ $\arg(z) = 180 + \tan^{-1}\frac{\sqrt{2}}{-1} = 180 + (-54.7356) = 125^\circ$ (3 significant figures)	

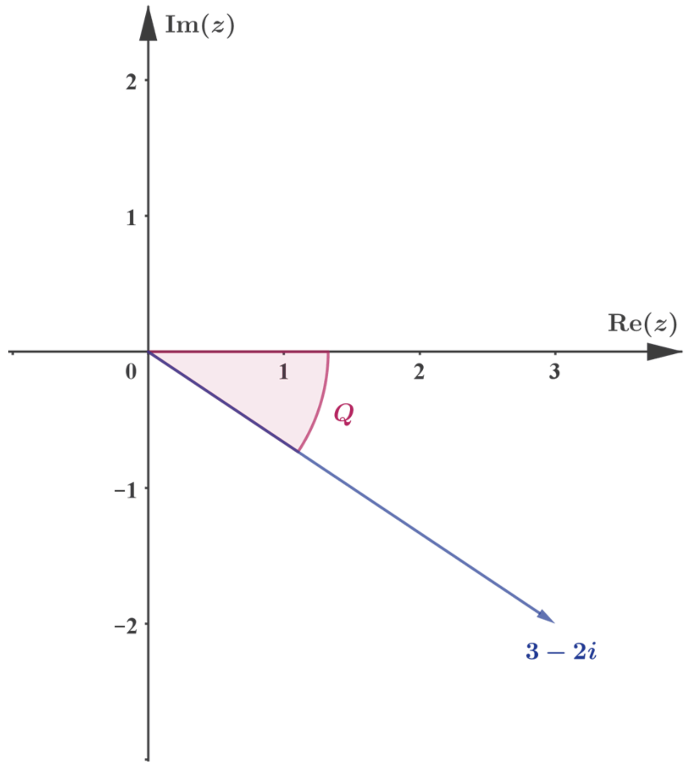
Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
c)	 $\|z\| = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}$ $\arg(\mathrm{z}) = -180 + \tan^{-1}\frac{-5}{-3}$ $= -180 + 59.0362$ $= -121^\circ$ (3 significant figures)	

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
d)	 $\lvert z\rvert = \sqrt{3^2 + (-2)^2} = \sqrt{13}$ $\arg(\mathrm{z}) = \tan^{-1}\frac{-2}{3}$ $= -33.7^\circ$ (3 significant figures)	

Be aware

You should always make a sketch of the Argand diagram for the complex number if you are working with the argument and modulus.

At this point it is hard to see the benefits of working with complex numbers using the modulus and argument as it is clearly easier to graph them in $a + b\mathrm{i}$ form. However, you will learn the benefits of the modulus-argument form in the next two subtopics.

Student
view

4 section questions

Checklist

What you should know

By the end of this subtopic you should be able to:

- identify that $\mathrm{Re}\,(z) = a$ and $\mathrm{Im}\,(z) = b$, given $z = a + b\mathrm{i}$
- add, subtract, multiply and divide complex numbers analytically and using a calculator
- write the complex conjugate of a complex number and use it to perform division
- use the equality property of complex numbers to solve for variables
- solve systems of equations with complex number coefficients and solutions
- represent complex numbers on the Argand plane
- find the modulus and argument of a complex number and use these to graph complex numbers.

Investigation

Part 1

Simplify each of the following.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

$$\mathrm{i}^2 \quad \mathrm{i}^3 \quad \mathrm{i}^4 \quad \mathrm{i}^5 \quad \mathrm{i}^6 \quad \mathrm{i}^7$$

Summarise any patterns that you find. You may need to complete further examples (e.g. i^8 and so on) if you have trouble finding the pattern in the ones given.

Use the patterns that you found to simplify the following.

(You can check your answers using the calculator.)

$$\mathrm{i}^{375} \quad \mathrm{i}^{2702} \quad \mathrm{i}^{3756}$$

Write a general rule for i^n where. $n \in \mathbb{N}$.

Part 2

Let $z_1 = 2 + \mathrm{i}$ and $z_2 = -1 - \mathrm{i}$. Find

$$(z_1 + z_2)^* \quad (z_1 - z_2)^* \quad (z_1 \times z_2)^* \quad \left(\frac{z_1}{z_2}\right)^* \quad z_1^* \times z_2^*$$

$$z_1^* - z_2^* \quad \frac{z_1^*}{z_2^*} \quad z_1^* + z_2^*$$

Comment on any patterns that you notice. Show that your observations hold true for any two complex numbers.

Student view