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The big picture

Think about an animated movie or a computer game. Why do the characters appear to be moving? A film sequence is made up of still pictures or frames. Each frame is slightly different from the previous one. Do they draw each and every one by hand? Well, in the early years of animated movies, they did.

Nowadays, not only cartoons but also many live action movies, use vector geometry. Scenes with real actors are shot in front of blue or black screens and the background is developed by computer animations using vectors. Vectors are used to represent forces, acceleration, velocity and momentum and enable the motion of an object to be predicted and described.

This video describes how hand drawings and vector animation can be combined.

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Concept

Vectors help you to quantify positions and movements. They provide the tools for enhancing your spatial awareness in two and three dimensions. This topic provides you with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Theory of Knowledge

Knowledge and Technology: The comprehension of vectors plays a large role in the development and innovation in regard to mechanised motion. Mathematics, perhaps more than any other area of knowledge, has a very diverse application array. Given that mathematics is the foundation for so many other technologies and sets of knowledge, does this imply that mathematics is the 'best' area of knowledge?

Knowledge Question: Should society place a greater emphasis on some areas of knowledge over others?

3. Geometry and trigonometry / 3.12 Vectors

The concept of a vector

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Scalar and vector quantities

In daily life, you regularly use quantities such as temperature measured in Fahrenheit or Celsius, mass in kilograms or pounds and distances in kilometres or miles. All these quantities are scalar quantities. They have magnitude but no direction. In contrast, other measurements such as velocities, displacements and forces need both a magnitude and a direction to describe them fully.

There is a difference between walking 2 km and walking 2 km north. The first case represents a distance, which is a scalar quantity because it only gives the magnitude (size) of how far you walked. The second case gives both a magnitude and a direction and is an example of a displacement. A quantity that has both direction and magnitude(size) is called a vector quantity.

Take a look at the villain in the video below.

Despicable Me | Clip: "Vector's Introduction" | Illumination

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Example 1

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Categorise each quantity as a scalar or vector.

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a) 3 km north east

b) 2000 calories

c) 80 km per hour

d) $15°\text{C}$

e) Running towards the finishing line at 7 km per hour.

	Steps	Explanation
a)	3 km north east	vector
b)	2000 calories	scalar
c)	80 km per hour	scalar
d)	$15°\text{C}$	scalar
e)	Running towards the finishing line at 7 km per hour	vector

Directed line segments, vectors and displacement vectors

As vectors have both magnitude and direction, both quantities need to be represented in the notation used. When you write a line segment you only need to use end points such as AB or BA as the length of both segments are equal. But when you represent a directed line segment it needs to have a direction, which means a starting point and an end point.

The usual notation for a *directed* line segment which starts at A and ends at point B is $\overrightarrow{\text{AB}}$. The arrow above the letters is used to distinguish it from the segment AB.

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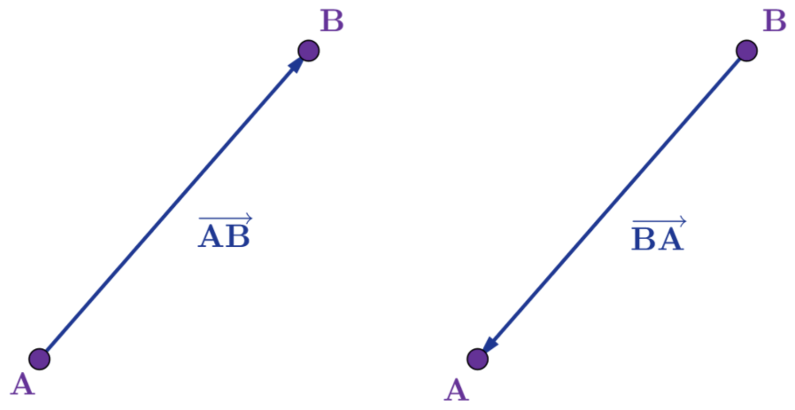
What would $\overrightarrow{BA}$ represent?



More information

The image contains two diagrams depicting vectors. On the left side, there is a vector represented as (\overrightarrow{AB}) starting from point A and ending at point B. The vector is depicted with an arrow pointing from A to B. On the right side, the vector is shown as (\overrightarrow{BA}), starting from point B and ending at point A, with the arrow pointing from B to A. These diagrams illustrate the concept of directionality in vectors, where (\overrightarrow{AB}) and (\overrightarrow{BA}) represent opposite directions along the same path.

[Generated by AI]

Important

$\overrightarrow{AB} \neq \overrightarrow{BA}$ as the starting and end points are different.

Making connections

You might need to revisit some of the definitions and notations of Euclidean geometry.

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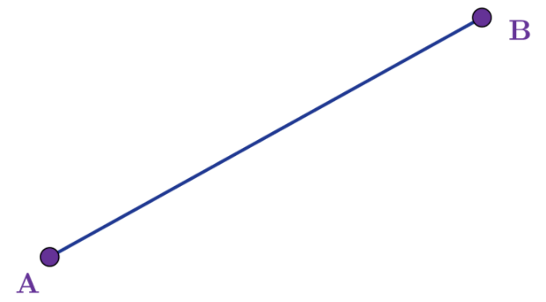
Point : This does not have any dimensions. It is used to represent a unique location in Euclidian space. It is usually represented by a capital letter, for example, A.

Line : A line is one-dimensional and consists of infinitely many points. It is either represented by a small italic letter, for example l or given by two points that are on the line. There is one and only one straight line passing through two points.

Line segment: This is any straight segment joining two points.



More information

The image is a diagram showing a straight line segment labeled as AB. The segment runs diagonally from the bottom left to the top right, connecting two points labeled A and B. Point A is at the bottom left, while point B is at the top right. Both points are marked with small circles, and the line segment is drawn in blue. The points and labels are in purple.

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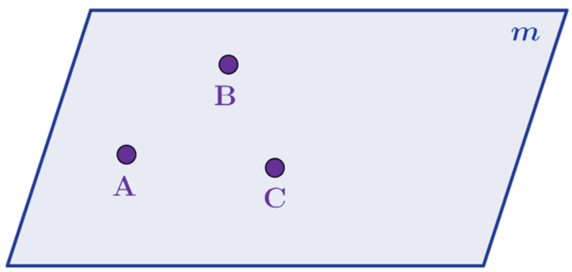
Plane : This is a flat two-dimensional surface. Three non-collinear points or a point and a line determine a plane.



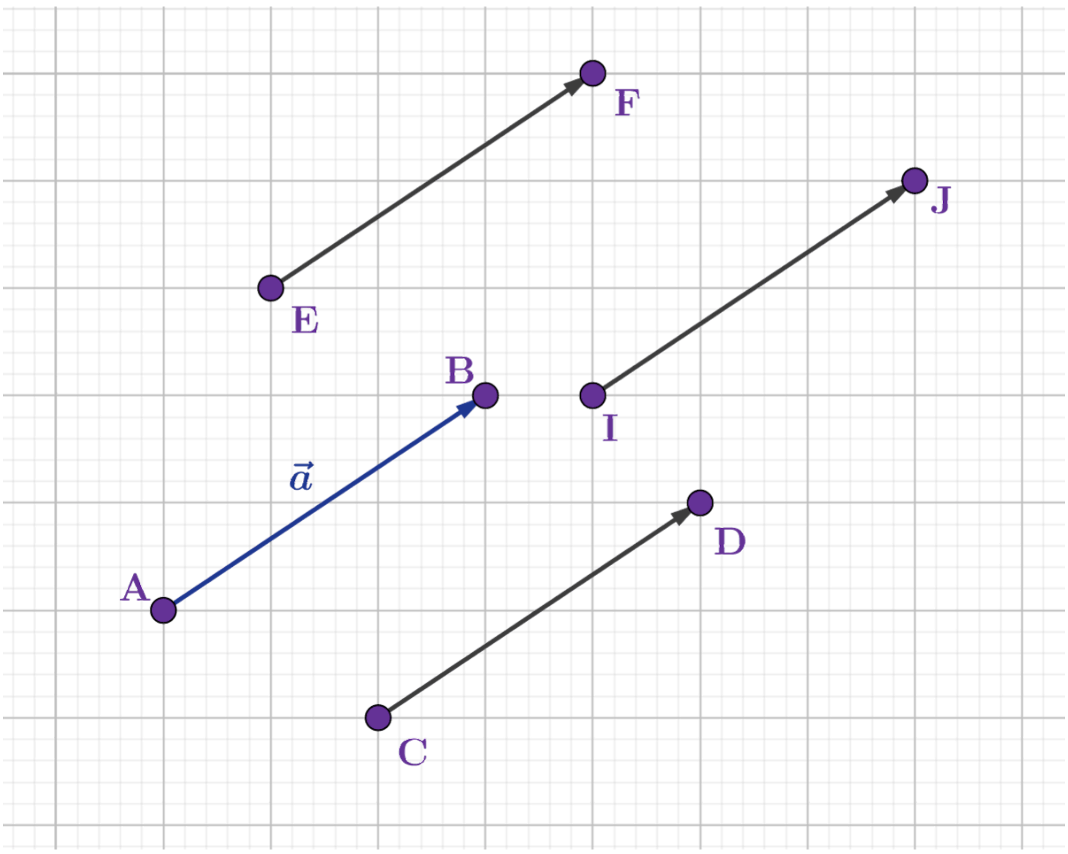
Student view

More information

The image is a diagram of a plane, depicted as a slanted parallelogram. There are three distinct points labeled A, B, and C within the plane. It is marked with a stylized curve at the top-right corner, labeled with 'm' as the edge or boundary line of the plane. These non-collinear points A, B, and C define the plane, illustrating how they are positioned within a two-dimensional space.

[Generated by AI]

Two directed line segments are congruent if they have the same direction and magnitude. In the diagram below, all the directed line segments are congruent because each starts from some initial point and moves 3 units to the right and 2 units up. This relationship is represented by the symbol $\cong$. In the diagram below, $\overrightarrow{\mathrm{AB}} \cong \overrightarrow{\mathrm{EF}} \cong \overrightarrow{\mathrm{CD}} \cong \overrightarrow{\mathrm{IJ}}$.



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More information

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The diagram shows a grid with four directed line segments represented. Each line segment starts from one point and moves 3 units to the right and 2 units up, indicating that they are congruent. The points are labeled as follows: A to B, C to D, E to F, and I to J. Line AB is highlighted in blue, and the others are in black, showing congruence using direction and magnitude.

[Generated by AI]

Important

A vector is represented by a directed line segment. All line segments which have the same magnitude (length) and direction are congruent. A vector can be denoted by a bold lower case letter, for example $\boldsymbol{a}$, or by upper case letters with an arrow on top.

$$\boldsymbol{a} = \overrightarrow{\mathrm{AB}} \cong \overrightarrow{\mathrm{EF}} \cong \overrightarrow{\mathrm{CD}} \cong \overrightarrow{\mathrm{IJ}}$$

Exam tip

The IB examination papers and text books use the notation $\boldsymbol{a}$, to denote a vector. But it is not possible to write in bold, so when you are writing vectors you *must* identify them as such by underlining, $\underline{a}$, or by writing an arrow on top, $\vec{a}$, otherwise you might be penalised for not using the correct notation.

Important

One application of vectors is to show displacements. The directed line segment $\overrightarrow{\mathrm{AB}}$ could represent a displacement vector where A is the initial point and B is the terminal point .

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International Mindedness

There are various forms of notation for vectors.

- In a printed text, bold type is used to denote a vector, e.g. $\boldsymbol{v}$ or $|\boldsymbol{v}|$.
- Vectors may be written in component form, e.g. $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors parallel to the x and y axes, or column vector format, e.g. $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
- When you are writing a vector by hand, you can use $\overrightarrow{v} = \overrightarrow{i} + 2\overrightarrow{j}$, or column vector format $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
- You may also see <1,2>
- When you are writing the magnitude of a vector by hand, you can write $\left|\overrightarrow{v}\right|$, or $\|\vec{v}\|$.

Why do these different forms of notation exist in the language of mathematics if it is a universal language?

Magnitude of a vector

The magnitude of a vector is given by the length of the directed line segment. The magnitude of the displacement vector $\overrightarrow{\mathrm{AB}}$ is denoted by $\left|\overrightarrow{\mathrm{AB}}\right|$ and the magnitude of vector $\boldsymbol{a}$ is written as $|\boldsymbol{a}|$.

Example 2

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Find the magnitude of the vector $\boldsymbol{a}$ in the diagram below.

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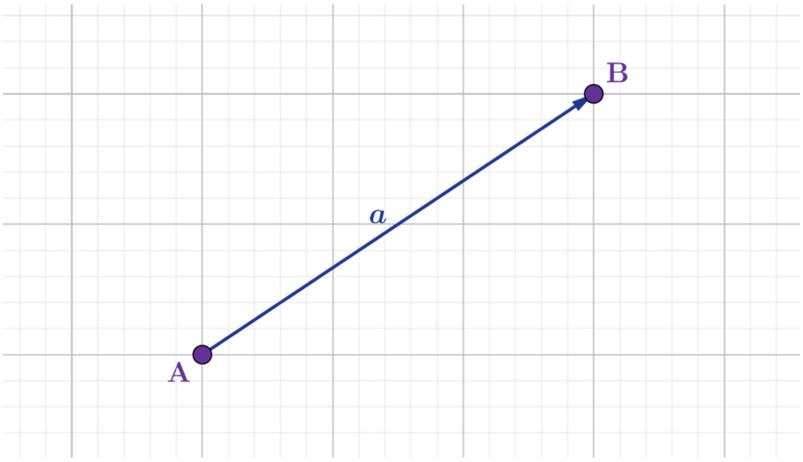
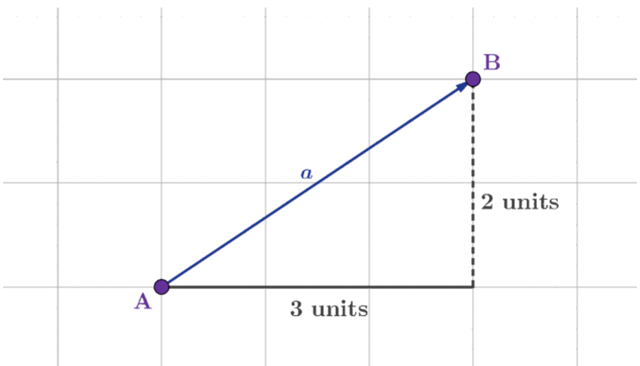


More information

The image is a diagram of a vector (\boldsymbol{a}) displayed on a grid. The vector originates from point A, marked on the grid, and extends to point B. The direction of the vector is indicated by an arrow pointing from point A towards point B. The vector is shown on a two-dimensional plane with equal grid spacing, allowing for determination of vector magnitude by counting units or using mathematical calculations. The diagram includes a label "a" next to the vector, indicating it is the vector of interest.

[Generated by AI]

Steps	Explanation
	Draw the right-angled triangle. Vector $\overrightarrow{AB}$ represents a movement of 3 to the right and 2 units up. These are the components of the vecto

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Steps	Explanation
$\|\boldsymbol{a}\| = \sqrt{3^2 + 2^2}$	Use Pythagoras theorem
Therefore, the magnitude of the vector is $\sqrt{13}$ units	

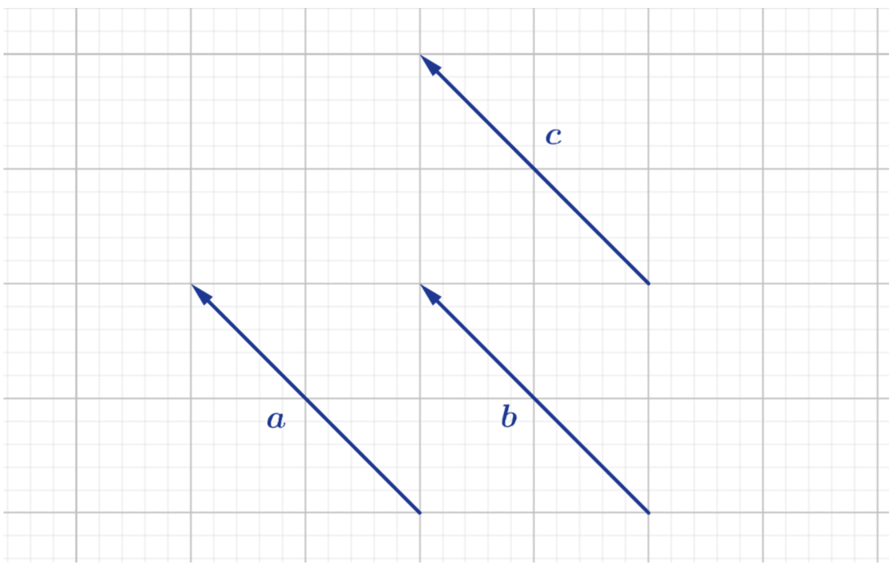
Zero vector

If the magnitude of a vector is 0, then it is called the zero vector. Geometrically it is the vector which has the same starting point and end point. Its direction is undefined. Therefore, the displacement is zero.

Equal vectors

If two vectors have the same direction and magnitude then they are equal vectors regardless of their starting point. In the diagram below, although $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ start and finish at different points they are all equal because their magnitudes and directions are equal.

$$\boldsymbol{a} = \boldsymbol{b} = \boldsymbol{c}$$



Student view

More information

The image is a graph with a grid background, displaying three diagonal parallel lines labeled as 'a', 'b', and 'c'. The lines are evenly spaced apart and run from the top right to the bottom left. Each line has an arrowhead at the end, indicating direction. The lines are meant to be equidistant from each other, suggesting equality (a = b = c) as per the preceding text description. There are no specific numerical values or scales visible on the grid background, focusing more on the spatial and directional relationship between the lines.

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Example 3

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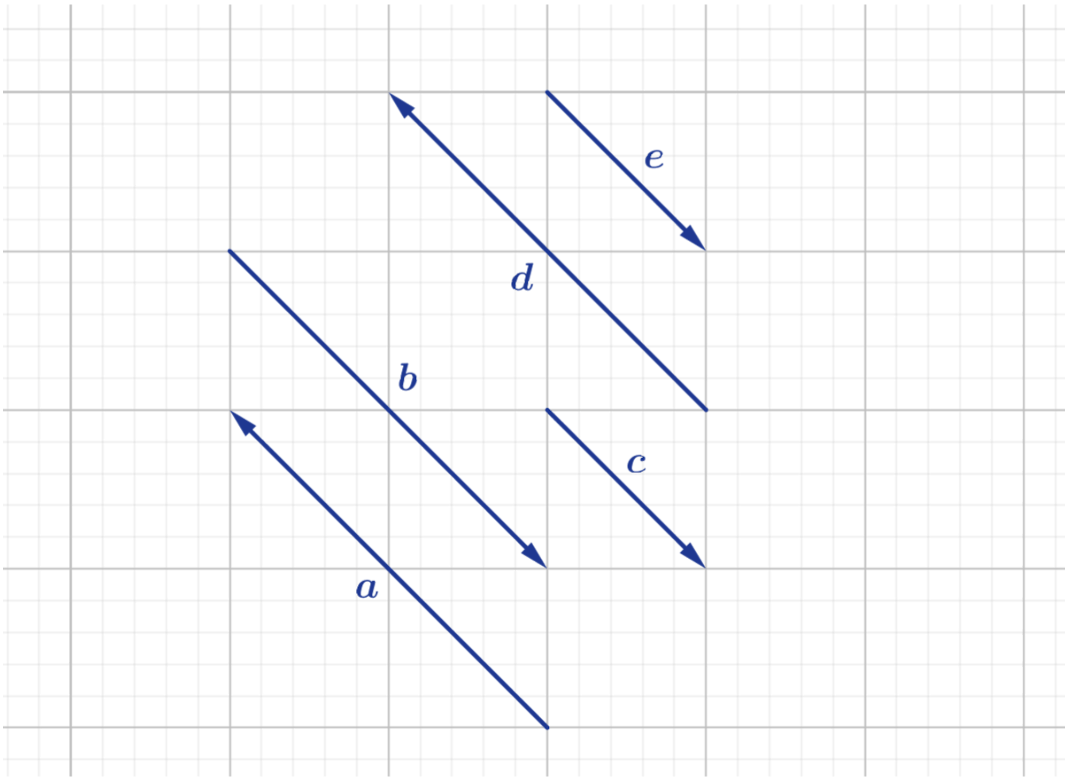
Which of the following vectors are equal?



More information

Overview
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The image is a diagram displaying five vectors labeled a, b, c, d, and e on a grid. Each vector is represented by a line with an arrow indicating its direction. Vectors a, b, and d are parallel and point from top left to bottom right, with vector a being the longest, followed by b and d. Vector c is shorter and points almost horizontally to the right with a slight downward tilt, while vector e points in the opposite direction from the bottom left to the top right. The vectors are positioned at various angles, suggesting a comparison of magnitude and direction to determine their equality.

[Generated by AI]

Steps	Explanation
$\boldsymbol{a} = \boldsymbol{d}$	$\lvert\boldsymbol{a}\rvert = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$ $\lvert\boldsymbol{d}\rvert = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$ Both vectors have the same direction and magnitude, $2\sqrt{2}$. Why is $\boldsymbol{a} \neq \boldsymbol{b}$?
$\boldsymbol{c} = \boldsymbol{e}$	$\lvert\boldsymbol{c}\rvert = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\lvert\boldsymbol{e}\rvert = \sqrt{1^2 + 1^2} = \sqrt{2}$ Both vectors have the same direction and magnitude, $\sqrt{2}$.

Opposite vectors

If two vectors are parallel, have the same magnitude but not point in the same directions, then one vector is **opposite** to the other. These pairs are represented by $\boldsymbol{a}$ and $-\boldsymbol{a}$.

3 section questions

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Question 1

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Which one of the following can be represented as a vector quantity?

1 Driving $30\,\text{km}$ west ✓

2 A 9 cubic metre box next to a wall

3 Burning 300 calories during a workout

4 Running 300 metres in under one minute

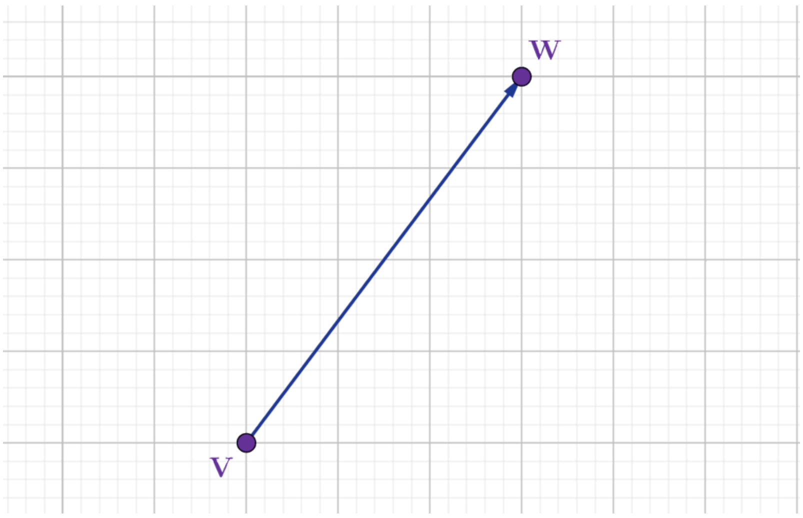
Explanation

Driving $30\,\text{km}$ west is a vector because it has direction (west) and magnitude $(30\,\text{km})$.

Question 2

★☆☆

Find the magnitude of the displacement vector $\overrightarrow{\mathrm{VW}}$.



More information

1 5 units ✓

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2 3 units

3 4 units

4 2 units

Explanation

Using Pythagoras' theorem

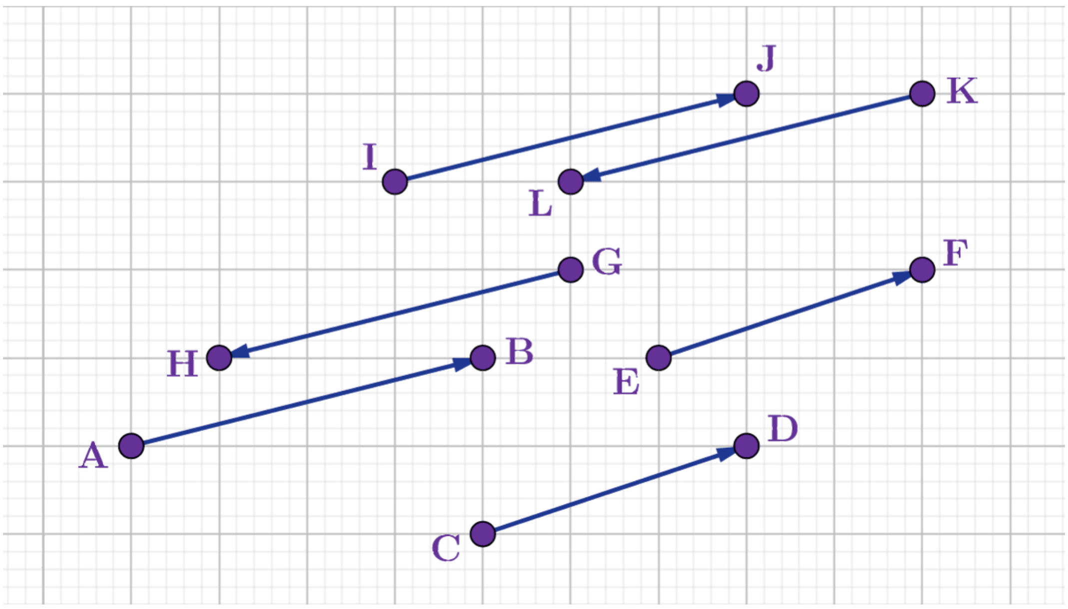
$$\overrightarrow{\mathrm{VW}} = \sqrt{3^2 + 4^2} = 5$$

Therefore, the magnitude is 5 units.

Question 3

★☆☆

Which of the following vectors are equal to $\overrightarrow{\mathrm{AB}}$?



More information

1 $\overrightarrow{\mathrm{IJ}}$ ✓

2 $\overrightarrow{\mathrm{GH}}$

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3 $\overrightarrow{\text{KL}}$

4 $\overrightarrow{\text{CD}}$

Explanation

$\overrightarrow{\text{IJ}}$ has the same direction and magnitude as $\overrightarrow{\text{AB}}$.

So the correct answer is $\overrightarrow{\text{IJ}}$

3. Geometry and trigonometry / 3.12 Vectors

Vector algebra and geometrical applications

Journeys and the triangle rule

Suppose you are travelling from Istanbul in Turkey to Milan in Italy. You start in Istanbul, go to Sofia in Bulgaria, then go to Athens in Greece and finally end your journey in Milan.

How would you represent this journey using displacement vectors?

How could you describe your starting and finishing points?

Student
view

More information

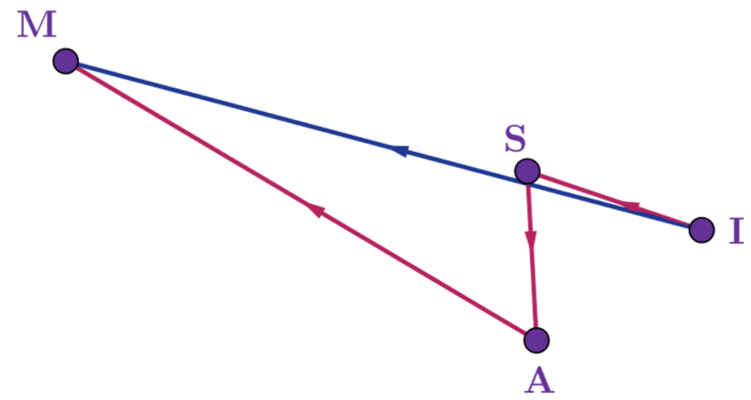
The image is a map depicting a travel route across parts of Europe. It shows the cities of Istanbul, Sofia, Athens, and Milan, connected by straight lines, representing the journey's path.

The path starts at Istanbul, Turkey, marked by a dot and labeled with its name, and proceeds northwest to Sofia, Bulgaria. Another line extends southwest from Sofia to Athens, Greece. Finally, the route concludes by extending northwest from Athens to the destination city of Milan, Italy. All lines are connected to show the journey's flow sequentially.

Each city name is labeled clearly on the map, and the countries Turkey, Bulgaria, Greece, and Italy are also labeled. The path illustrates the vector summation explained in the accompanying text, showing how the journey combines to form a single displacement.

[Generated by AI]

First, assign a letter to each city: Istanbul-I, Sofia-S, Athens-A and Milan-M. You can represent each leg of the journey by a displacement vector. So, in order, the journey would be the sum of vectors $\overrightarrow{\mathrm{IS}} + \overrightarrow{\mathrm{SA}} + \overrightarrow{\mathrm{AM}}$. The total displacement is from Istanbul to Milan so $\overrightarrow{\mathrm{IS}} + \overrightarrow{\mathrm{SA}} + \overrightarrow{\mathrm{AM}} = \overrightarrow{\mathrm{IM}}$, as seen in the diagram below.



More information

The diagram illustrates a journey as a series of displacement vectors between four cities: Istanbul (I), Sofia (S), Athens (A), and Milan (M). The journey consists of three vectors representing the paths between these cities: (\overrightarrow{IS}), (\overrightarrow{SA}), and (\overrightarrow{AM}). The diagram shows these vectors as arrows placed between points labeled with the city initials. Additionally, there is a resultant vector, (\overrightarrow{IM}), representing the direct displacement from Istanbul to Milan. This vector is shown as a blue arrow, while the other vectors (\overrightarrow{IS}), (\overrightarrow{SA}), and (\overrightarrow{AM}) are red arrows, demonstrating the sum of the journey's paths. Each arrow connects the corresponding city points, with segment labels placed beside the vectors to indicate the direction of travel.

[Generated by AI]

If you were to fly back directly from Milan to Istanbul after your trip, you would end up where you started. Therefore the resultant displacement would be zero.

You could represent this by

$$\overrightarrow{\mathrm{IS}}+\overrightarrow{\mathrm{SA}}+\overrightarrow{\mathrm{AM}}+\overrightarrow{\mathrm{MI}}=\overrightarrow{\mathrm{II}}=0$$

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Important

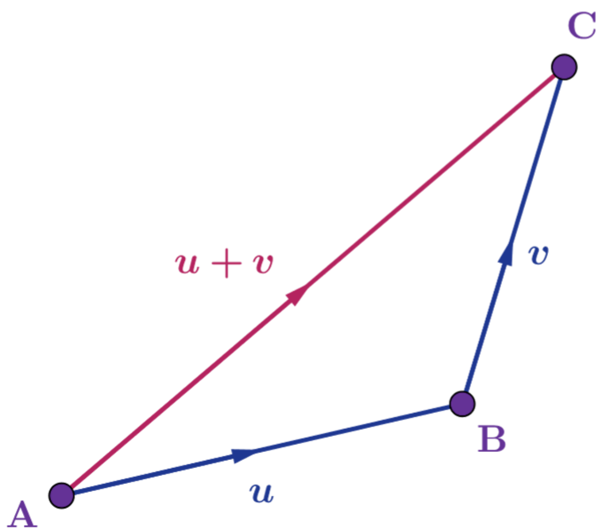
Triangle rule for addition:

Vector addition can be represented by joining the initial point of the second vector to the end of the first vector as seen in the diagram below.

$$\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{AC}}$$

This is often referred to as drawing the vectors 'nose to tail'.

Vector addition is commutative, i.e. it does not matter in which order the vectors are added.



More information

The image is a diagram illustrating the commutative property of vector addition with vectors labeled 'u', 'v', and 'u+v'. The diagram shows three points labeled A, B, and C. Vector 'u' is drawn from point A to point B, and vector 'v' is drawn from point B to point C. The resultant vector 'u+v' is drawn from point A to point C, forming a triangle. Arrows indicate the direction from A to B to C and then back to C from B, reflecting the rule of vector addition (u + v = v + u). The labels are placed on the arrows to denote each vector clearly.

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If you add three or more vectors and end up where you started, the resulting vector would be zero. For example,

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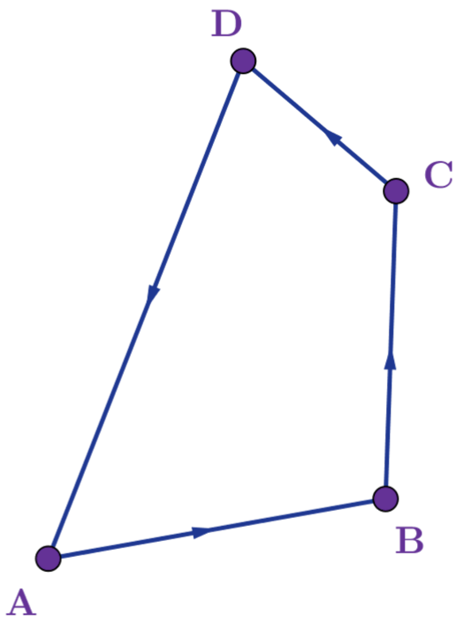
$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0$$



More information

The image is a vector diagram representing four points labeled A, B, C, and D connected by arrows. The arrows indicate the following vectors: (\overrightarrow{AB}), (\overrightarrow{BC}), (\overrightarrow{CD}), and (\overrightarrow{DA}). The diagram illustrates the concept that the sum of these vectors equals zero: (\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 0). This equates to the path forming a closed polygon, signifying a net zero displacement. This visual demonstrates a basic principle of vector addition where the sequence of vectors results in a return to the original position.

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Making connections

When solving problems related to position and movement, a displacement vector can be used to represent the position of one object relative to another. If the magnitude of the displacement vector is zero, the objects will have the same position.

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Vectors can also be used to represent forces. If the sum of two or more forces is zero, then the forces are in equilibrium.

Example 1

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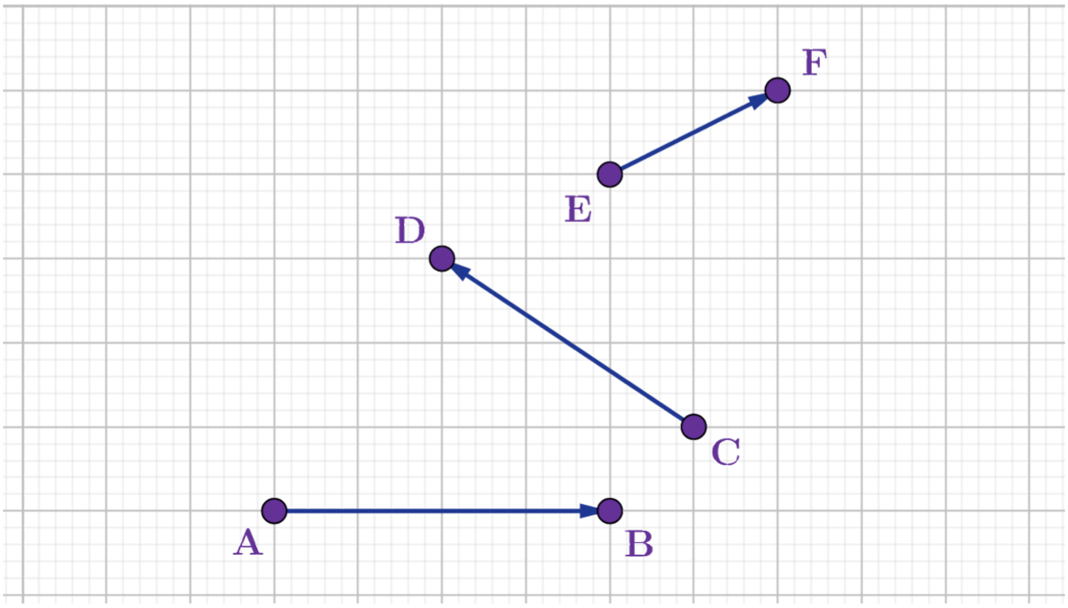
Draw the vector representing the sum $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF}$



More information

The image displays a vector diagram over a grid. It includes three vectors represented as directed line segments. The vectors are labeled (\overrightarrow{AB}), (\overrightarrow{CD}), and (\overrightarrow{EF}), with each segment having a starting point and an endpoint:

1. (\overrightarrow{AB}) - A horizontal vector originating from point A on the left and ending at point B on the right.
2. (\overrightarrow{CD}) - A diagonal vector starting at point C, located midway on the grid, and ending at point D, directed northwest.
3. (\overrightarrow{EF}) - Another diagonal vector starting from point E and terminating at point F, directed northeast.

These vectors are part of an exercise to calculate their sum. The diagram also displays a grid in the background.

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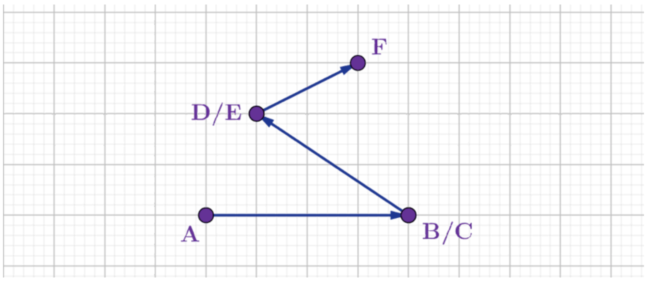
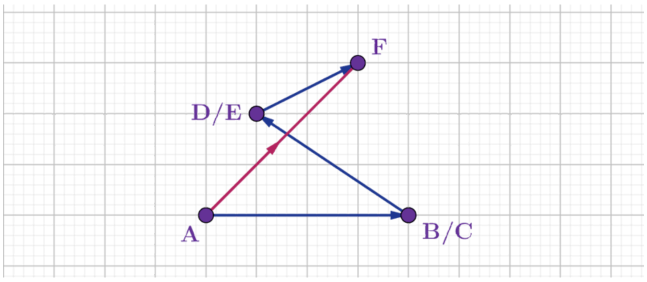
Steps	Explanation
	Draw vector representing $\overrightarrow{\text{AB}}$. Transla vector $\overrightarrow{\text{CD}}$ so that it follows on from $\overrightarrow{\text{A}}$ Then translate $\overrightarrow{\text{EF}}$ so that it follows on $\overrightarrow{\text{CD}}$. The vector from A to F represen sum $\overrightarrow{\text{AB}} + \overrightarrow{\text{CD}} + \overrightarrow{\text{EF}}$.
	$\overrightarrow{\text{AF}}$ is the resultant of the vectors $\overrightarrow{\text{AB}}$, and $\overrightarrow{\text{EF}}$.

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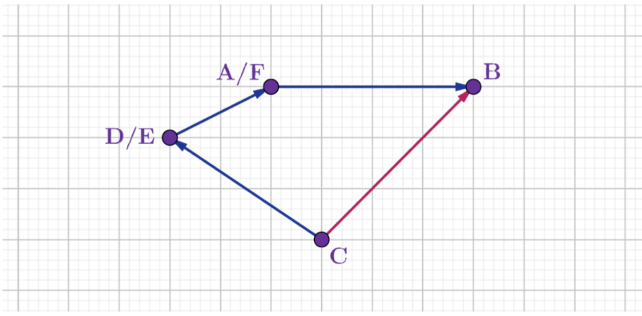
Steps	Explanation
	Note that there are many possible way answer the question. It does not matter which vector you st with and in which order you draw the vectors. The resultant will have the sar magnitude and direction.

Important

The sum of two or more vectors is called the resultant . Always draw the vectors 'nose to tail' when finding the resultant. You may need to use a scale drawing when adding vectors. The resultant is often indicated by a double arrowhead on the vector.

Using parallelograms for adding and subtracting vectors

Two vectors may have the same starting point.

Consider the diagram below. How would you add $\overrightarrow{AB}$ and $\overrightarrow{AC}$?

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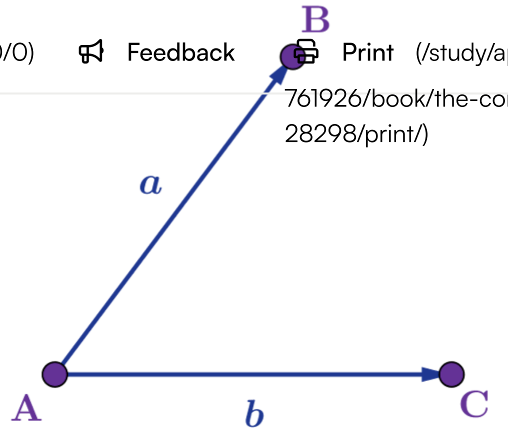

More information

The diagram illustrates a vector addition technique using the parallelogram method. It shows a two-dimensional plane with three labeled points: A, B, and C. Point A is the origin. From A, there are two vectors: one pointing towards B and the other towards C. (\overrightarrow{{AB}}) is depicted with a diagonal arrow pointing upwards to the left, while (\overrightarrow{{AC}}) is represented with an arrow extending horizontally to the right. The diagram suggests completing a parallelogram to visually add these vectors. Vectors are marked with labels 'a' (AB) and 'b' (AC) along the arrows, showing direction and orientation. To add these vectors, a parallelogram can be drawn, completing the shape using these vectors and their duplicates to find the resultant vector.

[Generated by AI]

Draw a parallelogram according to the following:

$$\overrightarrow{\mathrm{AB}} \cong \overrightarrow{\mathrm{CD}} \text{ so } \overrightarrow{\mathrm{CD}} = \boldsymbol{a}$$
$$\overrightarrow{\mathrm{AC}} \cong \overrightarrow{\mathrm{BD}} \text{ so } \overrightarrow{\mathrm{BD}} = \boldsymbol{b}.$$

Draw BD parallel to AC and CD parallel to AB forming a parallelogram.

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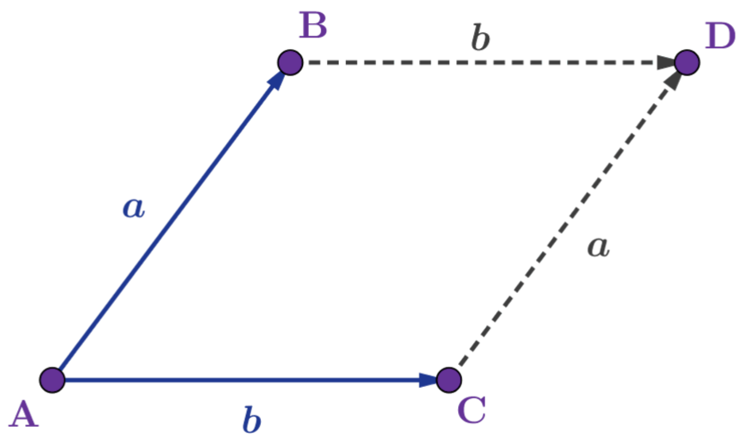

More information

The image shows a parallelogram with points labeled A, B, C, and D at each vertex. There are vectors labeled (a) and (b) indicating directions between the points. Line (AB) is marked with vector (a), pointing from A to B. Line (BC) is shown with vector (b), pointing from B to C. Similarly, line (CD) continues the line parallel to (AB), and line (DA) continues the line parallel to (BC), forming a complete parallelogram. The text indicates that (\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}), implying a relationship among vectors in the parallelogram.

[Generated by AI]

Using the parallelogram, $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{AC}} \cong \overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BD}}=\overrightarrow{\mathrm{AD}}$.

So, $\overrightarrow{\mathrm{AB}}+\ \overrightarrow{\mathrm{AC}}=\overrightarrow{\mathrm{AD}}$ which is the diagonal of the parallelogram ABCD.

Student view

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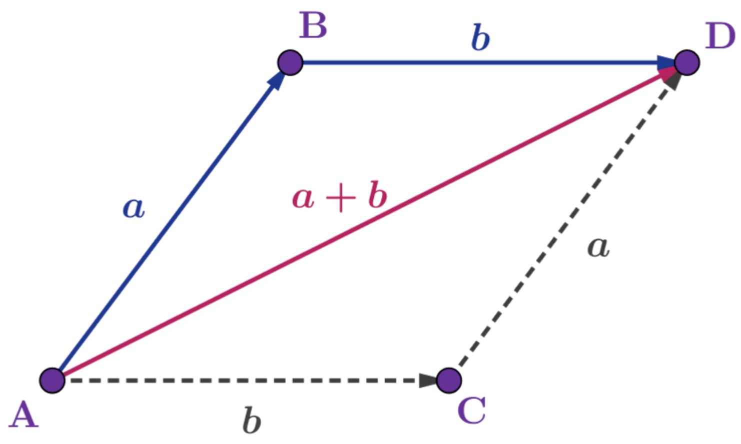

More information

The image is a diagram showing a parallelogram labeled ABCD. The point A is connected to point B with a vector labeled 'a'. Point B is connected to point D with a vector labeled 'b'. The vector going directly from A to D is labeled 'a + b'. The sides of the parallelogram opposite to vectors 'a' and 'b' are dotted lines, indicating that these represent the same magnitude and direction but shifted in space to complete the parallelogram shape. The points A, B, C, and D are marked with dots.

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Example 2

★★☆

Use the diagram below, where ABCD is a parallelogram, to represent $\overrightarrow{\mathrm{AB}} - \overrightarrow{\mathrm{AC}}$.

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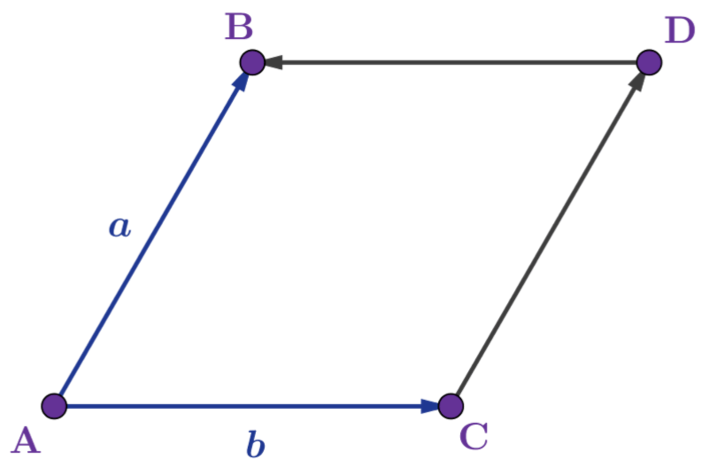

More information

The diagram shows a parallelogram labeled as ABCD. The corners of the parallelogram are labeled with points A, B, C, and D. The vector (\overrightarrow{AB}) is drawn from point A to point B, labeled with 'a,' and the vector (\overrightarrow{AC}) is drawn from point A to point C, labeled with 'b.' Arrows indicate the direction of the vectors. The task is to represent (\overrightarrow{AB} - \overrightarrow{AC}) using this parallelogram setup.

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Student view

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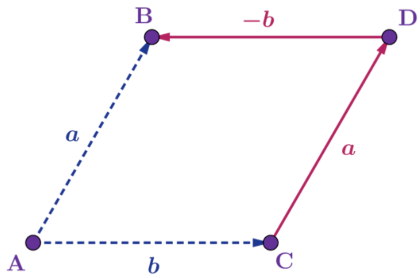
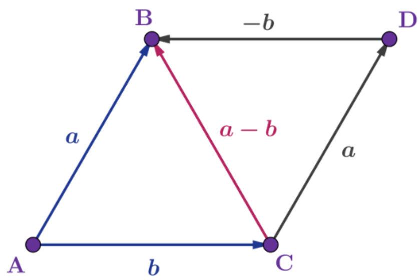
Steps	Explanation
 $\overrightarrow{AB} - \overrightarrow{AC} \cong \overrightarrow{CD} + \overrightarrow{DB}$	$\overrightarrow{CD} \cong \overrightarrow{AB}$ as they have same magnitu and direction. $\overrightarrow{DB} \cong -\overrightarrow{AC}$ as they have the same magnitude but opposite directions.
$\overrightarrow{AB} - \overrightarrow{AC} \cong \overrightarrow{CD} + \overrightarrow{DB} = \overrightarrow{CB}$	Adding
	

Student view

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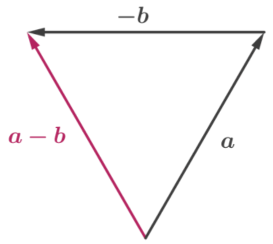
Steps	Explanation
	There are other ways of drawing the diagram.

Be aware

Make sure the vectors follow on 'nose to tail' when subtracting one vector from another.

Subtraction of vectors is not commutative.

Multiplying vectors by scalars: parallel vectors

Consider the sum of the vectors in the diagram below: $\boldsymbol{a} + \boldsymbol{a} + \boldsymbol{a} + \boldsymbol{a}$.

The resultant vector is in the same direction and 4 times as long as $\boldsymbol{a}$.

Thus, $\boldsymbol{a} + \boldsymbol{a} + \boldsymbol{a} + \boldsymbol{a} = 4\boldsymbol{a}$

Student view

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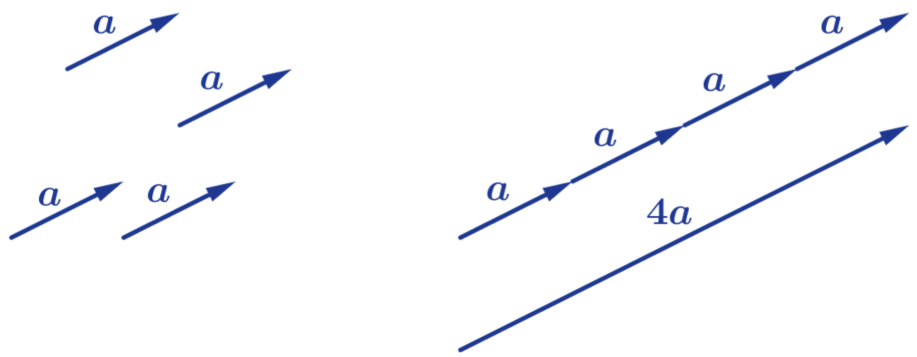

More information

The image consists of two diagrams side by side. The left diagram shows four separate vector arrows, each labeled with an 'a', arranged in a scattered pattern. The right diagram illustrates the vector addition of these four vectors. It shows them aligned end-to-end, along with an additional longer arrow labeled '4a', representing the resultant vector. This demonstrates the concept of vector addition where adding four vectors 'a' results in a single vector '4a'.

[Generated by AI]

The sum of the vectors in the first diagram below can be written as $4\boldsymbol{a}$ as shown in the diagram on the right.

Important

When a vector $\boldsymbol{a}$ is multiplied by a scalar k, the resultant vector is $\boldsymbol{v} = k\boldsymbol{a}$ which is parallel to $\boldsymbol{a}$ and has a magnitude $|\boldsymbol{v}| = k\,|\boldsymbol{a}|$. If $k < 0$, then the direction of $\boldsymbol{v}$ is opposite to $\boldsymbol{a}$.

If $\boldsymbol{v} = k\boldsymbol{a}$ then $\boldsymbol{v}$ and $\boldsymbol{a}$ are parallel vectors, that is, $\boldsymbol{v} \parallel \boldsymbol{a}$

Two vectors are parallel if they are scalar multiples of the same vector.

Student view

Example 3

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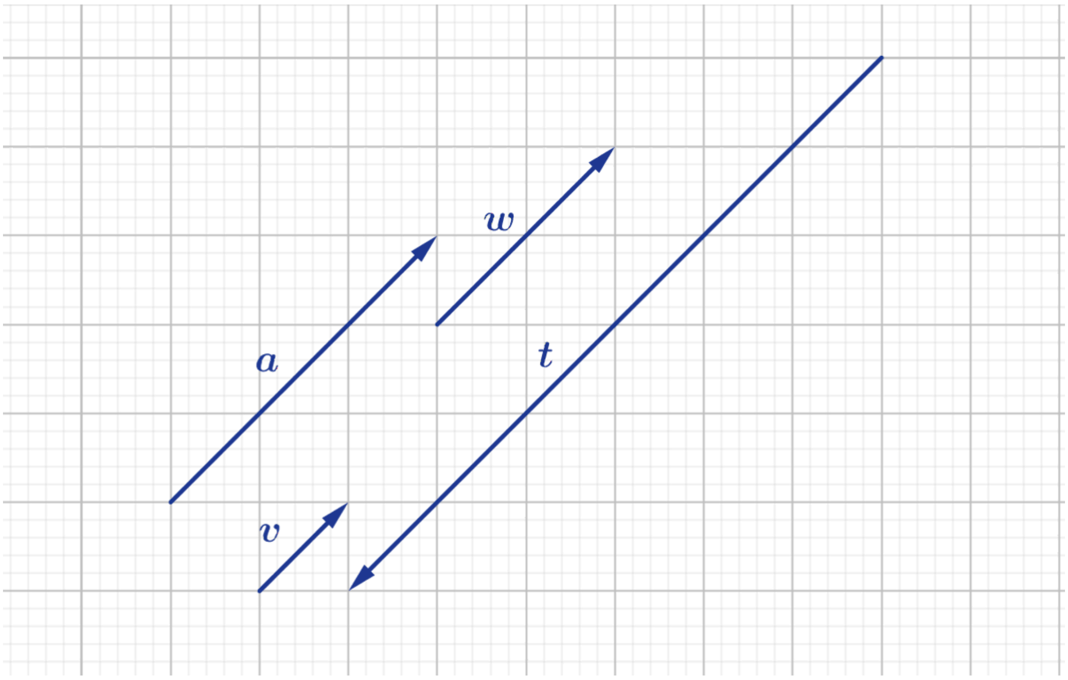
Write the following vectors in terms of $\boldsymbol{a}$



More information

The image is a diagram illustrating three vectors labeled as (\boldsymbol{a}), (\boldsymbol{b}), and (\boldsymbol{c}) on a grid. The grid serves as a coordinate plane. Each vector is represented by an arrow indicating both direction and magnitude.

- Vector (\boldsymbol{a}) is directed upwards and to the left.
- Vector (\boldsymbol{b}) is directed upwards and to the right, shorter in magnitude compared to (\boldsymbol{a}).
- Vector (\boldsymbol{c}) represents a combination or another direction in the same grid, pointing upwards and to the right but with a different magnitude.

The vectors are placed in such a way they might be intended to illustrate relationships or transformations among them within the grid pattern, which is typical in vector addition or transformational geometry.

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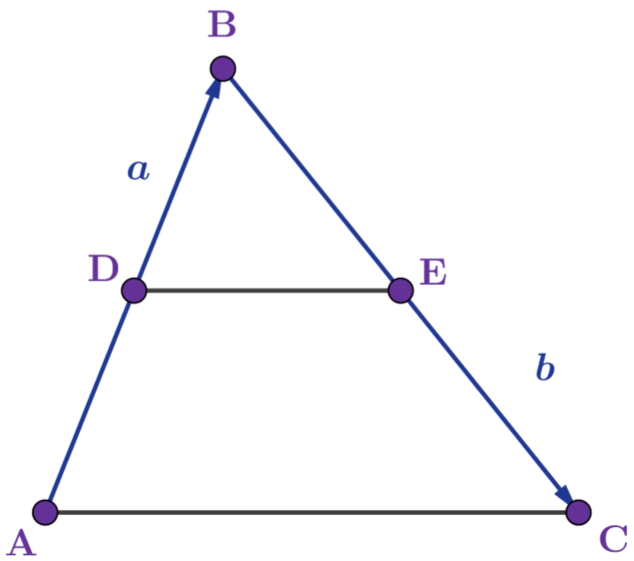
Steps	Explanation
$\boldsymbol{v} = \frac{1}{3}\boldsymbol{a}$	$\boldsymbol{v}$ has the same direction as $\boldsymbol{a}$ and its magnitude is $\frac{1}{3}$ the magnitu $\boldsymbol{a}$.
$\boldsymbol{w} = \frac{2}{3}\boldsymbol{a}$	$\boldsymbol{w}$ has the same direction as $\boldsymbol{a}$ and its magnitude is $\frac{2}{3}$ the magnitu $\boldsymbol{a}$.
$\boldsymbol{t} = -2\boldsymbol{a}$	$\boldsymbol{t}$ is in the opposite direction to $\boldsymbol{a}$ and its magnitude is twice that o

Example 4

★★☆

In triangle ABC, points D and E are midpoints of AB and BC respectively. If $\overrightarrow{\mathrm{AB}} = \boldsymbol{a}$ and $\overrightarrow{\mathrm{BC}} = \boldsymbol{b}$, show that DE is parallel to AC.



Student
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More information

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

The image is a diagram of triangle ABC. Point D is the midpoint of line segment AB, and point E is the midpoint of line segment BC. The vector (\overrightarrow{AB}) is labeled as (\boldsymbol{a}), and the vector (\overrightarrow{BC}) is labeled as (\boldsymbol{b}). Between points D and E is line segment DE, which is parallel to line segment AC. The points are connected by lines forming the triangle, and the diagram visually demonstrates the parallel nature of DE to AC as required by the geometry problem stated before the image.

[Generated by AI]

Steps	Explanation
$\overrightarrow{\mathrm{AC}} = \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} = \boldsymbol{a} + \boldsymbol{b}$	Triangle rule.
$\overrightarrow{\mathrm{DE}} = \overrightarrow{\mathrm{DB}} + \overrightarrow{\mathrm{BE}}$	Triangle rule.
$\overrightarrow{\mathrm{DE}} = \frac{1}{2}\overrightarrow{\mathrm{AB}} + \frac{1}{2}\overrightarrow{\mathrm{BC}} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$	As D and E are midpoints, $\overrightarrow{DB} = \frac{1}{2}\overrightarrow{AB}$ and $\overrightarrow{\mathrm{BE}} = \frac{1}{2}\overrightarrow{\mathrm{BC}}$.
$\overrightarrow{\mathrm{DE}} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$	Factorise.
$\overrightarrow{\mathrm{DE}} = \frac{1}{2}\left(\overrightarrow{\mathrm{AC}}\right)$	$\overrightarrow{\mathrm{DE}}$ is a scalar multiple of $\overrightarrow{\mathrm{AC}}$ so they are parallel.
Therefore, $$\mathrm{DE} \parallel \mathrm{AC}$$	

Example 5

Student view

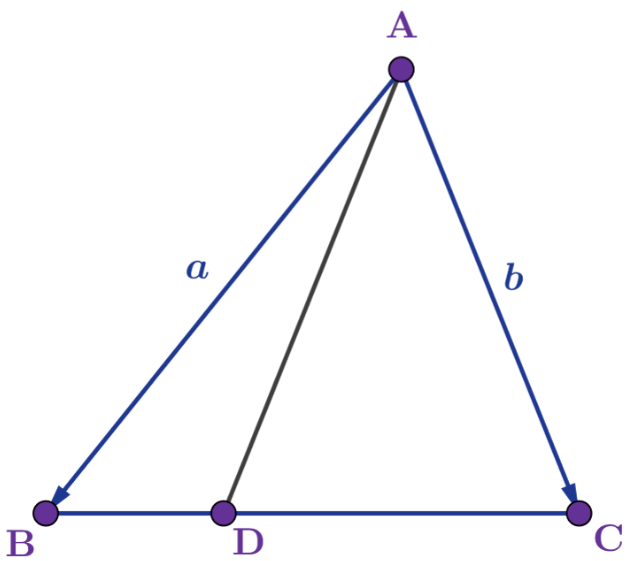
★★☆

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

In triangle ABC, point D lies on BC. If $3BD = 2DC$, $\overrightarrow{\mathrm{AB}} = \boldsymbol{a}$ and $\overrightarrow{\mathrm{AC}} = \boldsymbol{b}$, write the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

a) $\overrightarrow{\mathrm{BC}}$

b) $\overrightarrow{\mathrm{AD}}$



More information

This is a geometric diagram depicting triangle ABC. The triangle has vertices labeled A, B, and C. Point D is located on side BC, dividing the line into segments BD and DC. The line segments AB and AC are labeled with the variables 'a' and 'b' respectively. The diagram shows AD as a median or altitude from point A to side BC. Arrowheads on lines indicate direction or order among the points.

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Overview
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	Steps	Explanation
a)	$\overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AC}}$	Triangle rule.
	$\overrightarrow{BC} = -\boldsymbol{a} + \boldsymbol{b}$	$\overrightarrow{\mathrm{BA}} = -\overrightarrow{\mathrm{AB}}$
	So, the answer is $-\boldsymbol{a} + \boldsymbol{b}$	
b)	$\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BD}}$	Triangle rule.
	$\overrightarrow{\mathrm{AD}} = \boldsymbol{a} + \frac{2}{5}(-\boldsymbol{a} + \boldsymbol{b})$	$3BD = 2DC$ so $BD : DC = 2 : 3$ $\overrightarrow{\mathrm{BD}} = \frac{2}{5}\overrightarrow{\mathrm{BC}}$
	$\overrightarrow{\mathrm{AD}} = \frac{3}{5}\boldsymbol{a} + \frac{2}{5}\boldsymbol{b}$	Simplify.

4 section questions

Question 1

★☆☆

Select which of the following represents the sum of the two vectors shown in the diagram.

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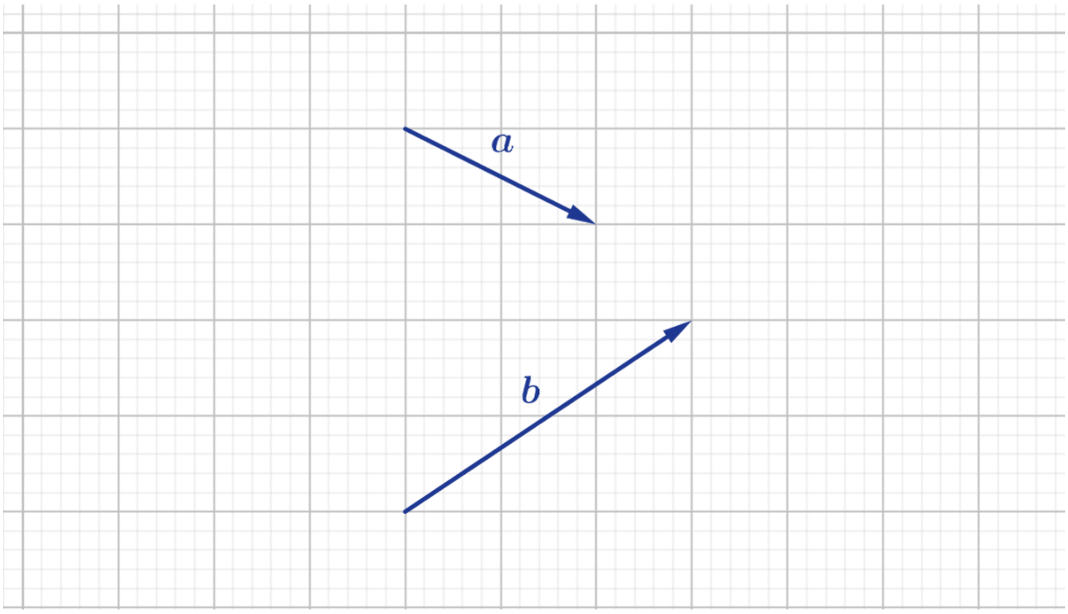
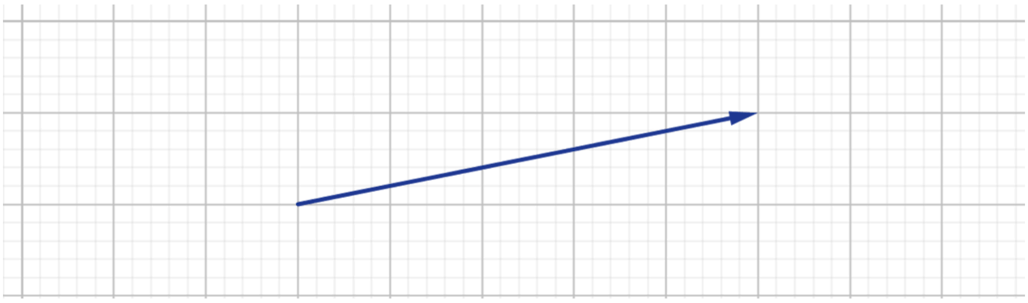


More information

1



✓

More information

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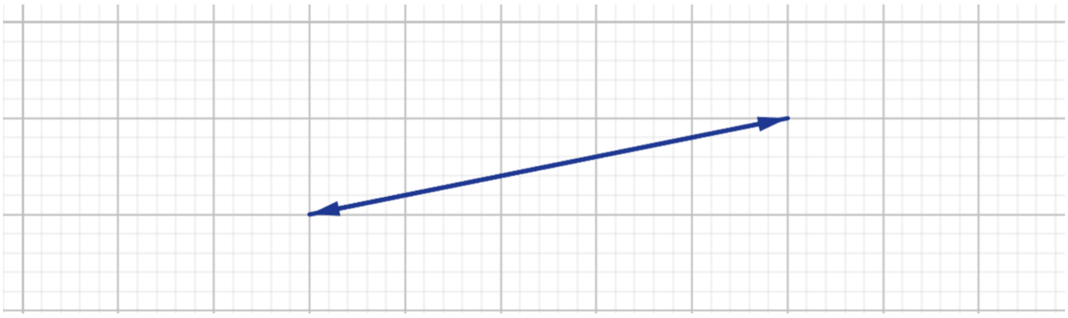

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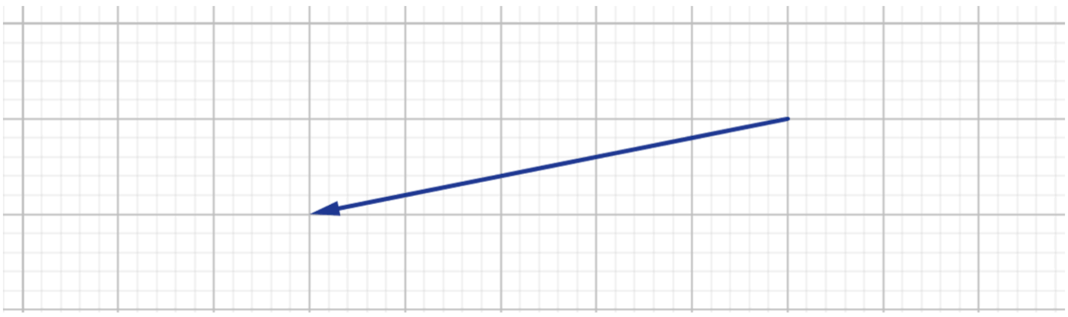

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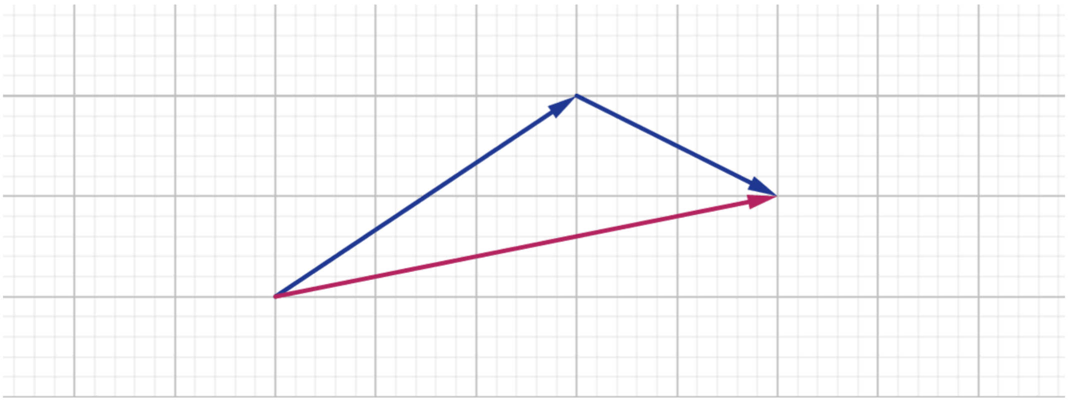
More information

Explanation



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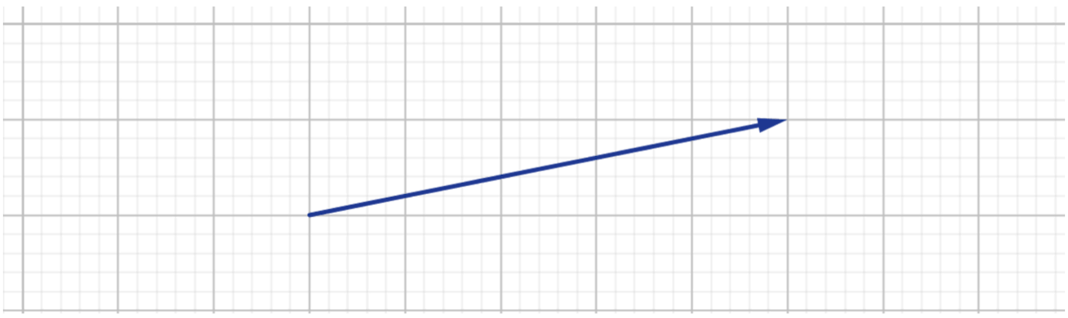
Or the vectors could have been added in the opposite order, which would give the other half of a parallelogram. The resultant would be the same.

Therefore, the correct answer is



More information

Student
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Question 2

Overview
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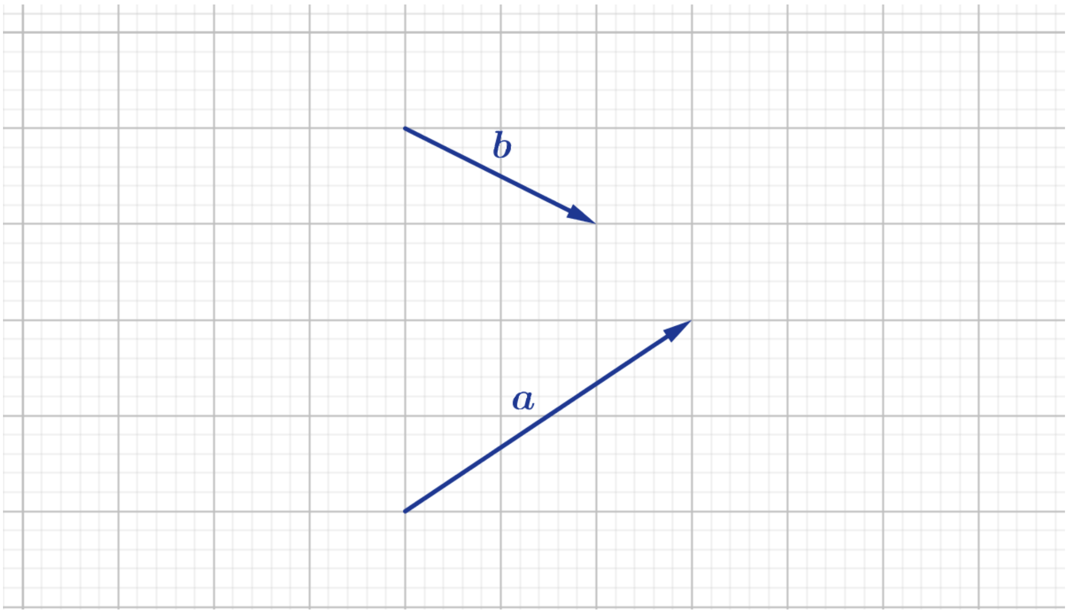
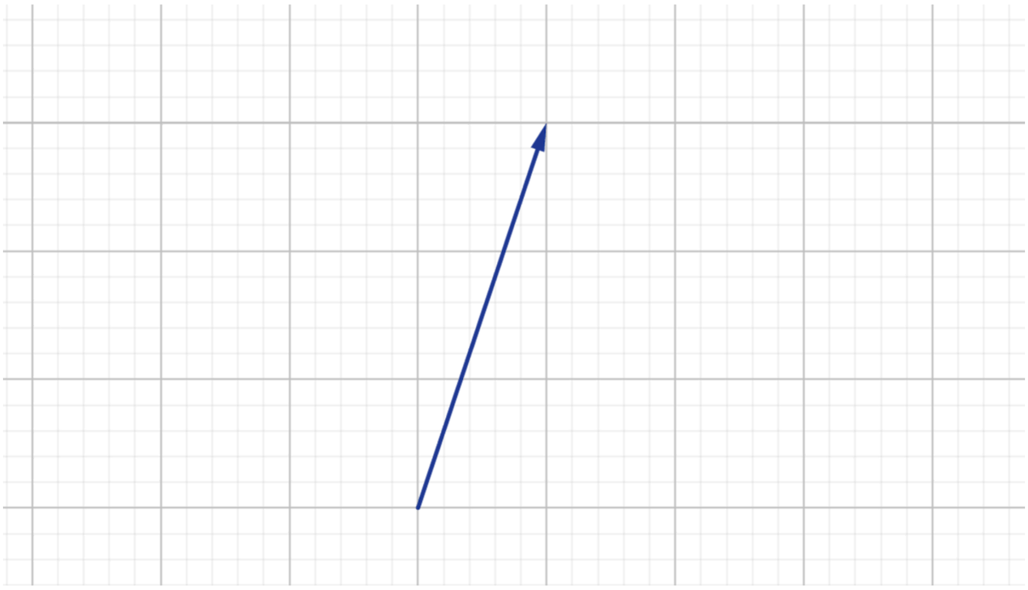
★☆☆

Select which of the following represents $\boldsymbol{a} - \boldsymbol{b}$.



More information

1



✓

More information

Student view

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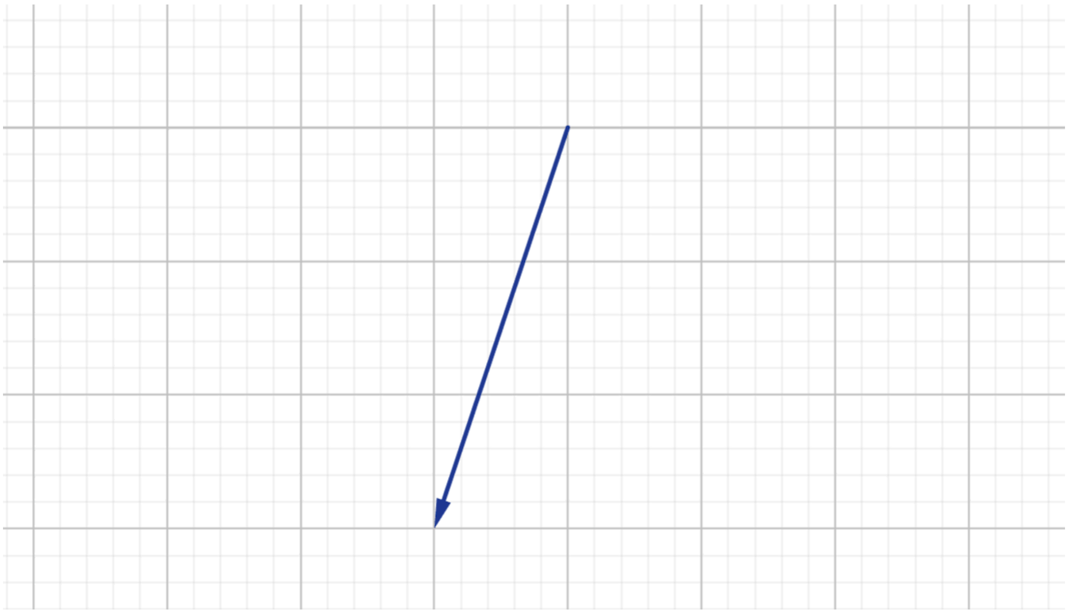

More information

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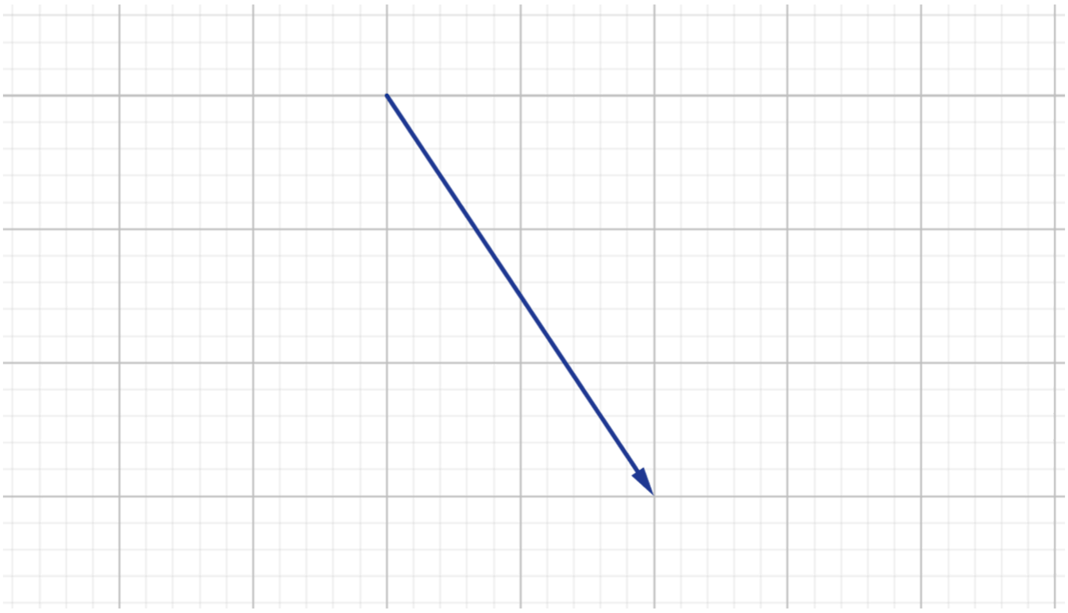

More information

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Student view

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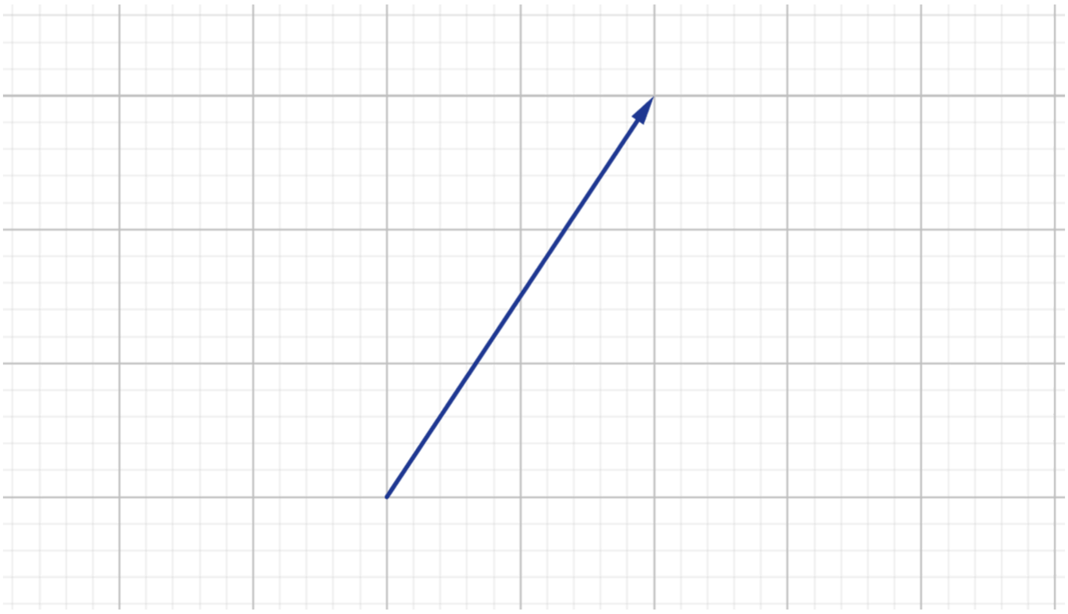

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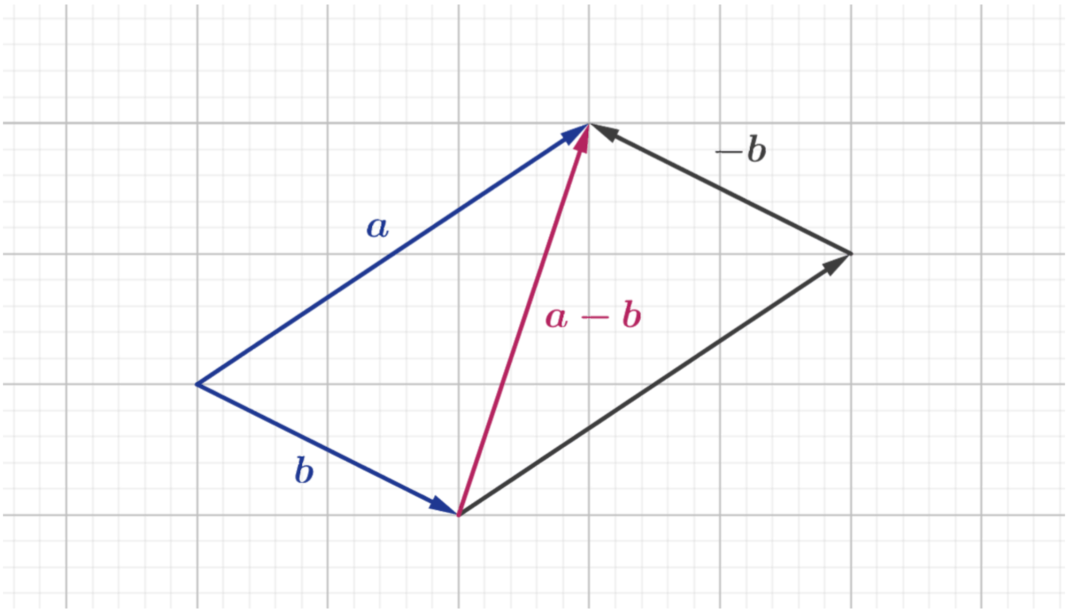
Explanation

Draw the parallelogram



More information

Or this could have been drawn with $-\boldsymbol{b}$ following on from $\boldsymbol{a}$.

So, the correct answer is

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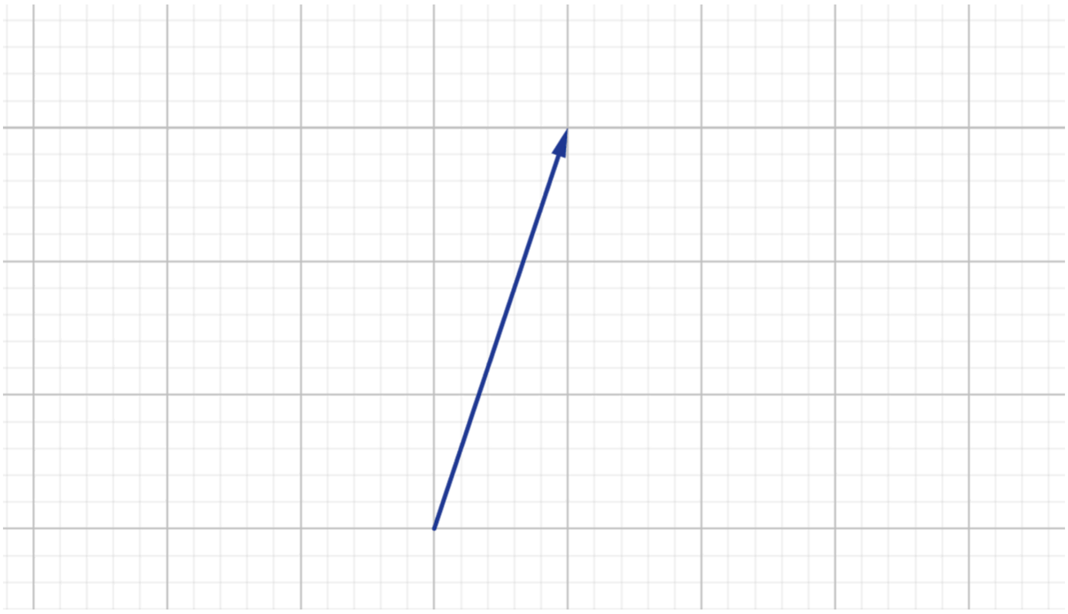

More information

Question 3

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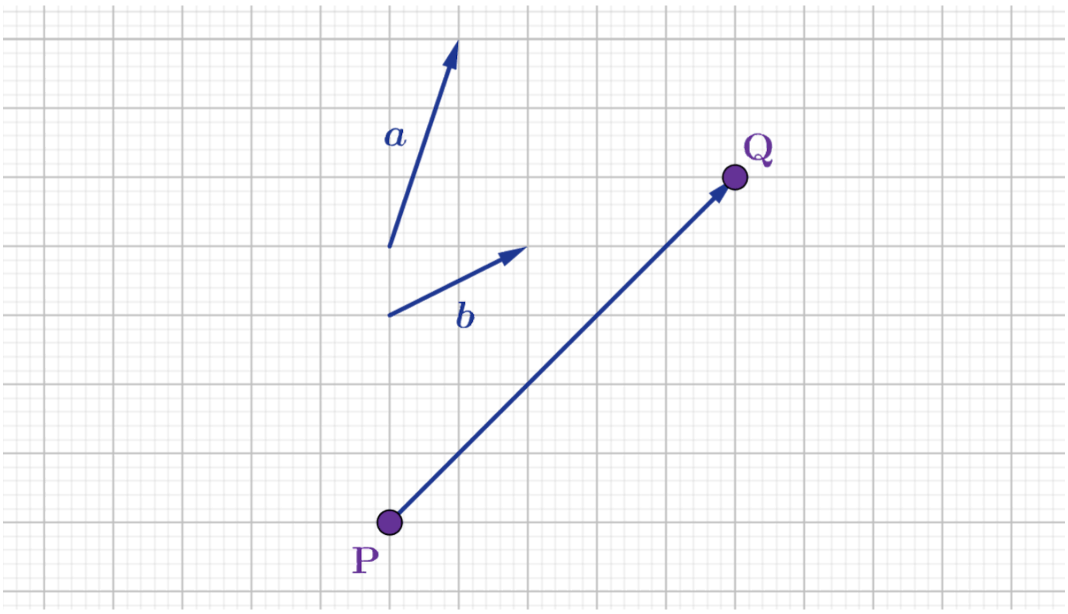
Find the vector $\overrightarrow{\mathrm{PQ}}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ shown in the diagram.



More information

1 $2\boldsymbol{b}+\boldsymbol{a}$

2 $\boldsymbol{a}+\boldsymbol{b}$

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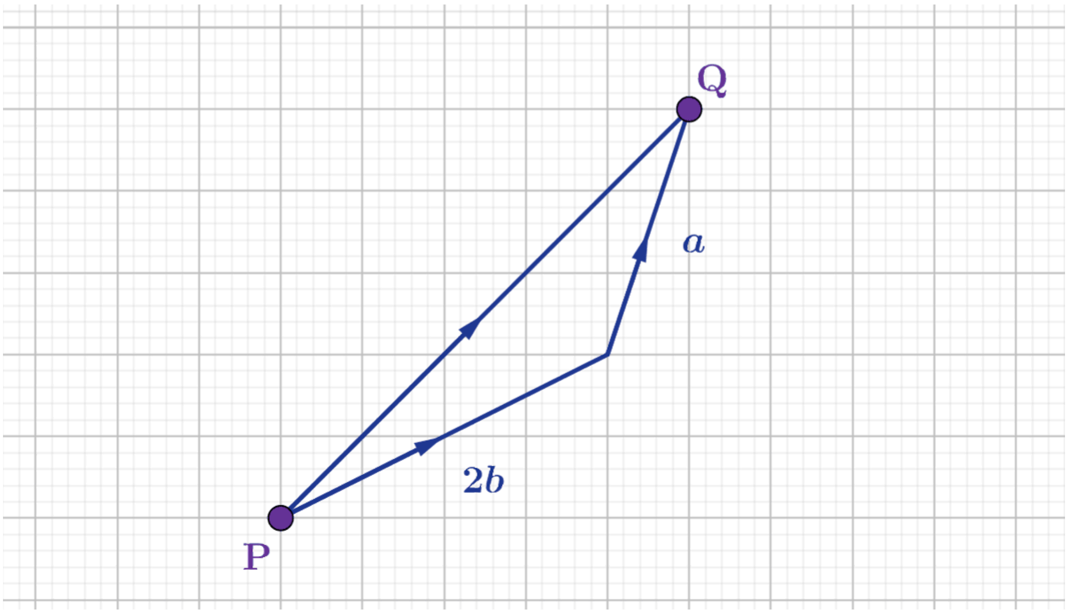
3 $2\boldsymbol{a}+\boldsymbol{b}$

4 $2\boldsymbol{a}+2\boldsymbol{b}$

Explanation



More information

Question 4

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In triangle ABC, point D is on the line segment BC with $\frac{BD}{BC}=\frac{2}{3}$, and point E is on the line segment AD with $\frac{AE}{ED}=\frac{2}{3}$.

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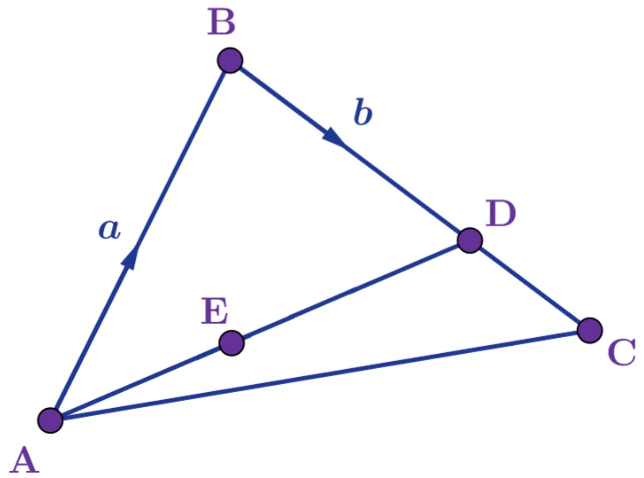

More information

If $\overrightarrow{\mathrm{AB}} = \boldsymbol{a}$ and $\overrightarrow{\mathrm{BD}} = \boldsymbol{b}$ write $\overrightarrow{\mathrm{EC}}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

1 $\frac{3}{5}\boldsymbol{a} + \frac{11}{10}\boldsymbol{b}$ ✓

2 $\frac{7}{5}\boldsymbol{a} - \frac{19}{10}\boldsymbol{b}$

3 $\frac{7}{5}\boldsymbol{a} + \frac{19}{10}\boldsymbol{b}$

4 $\frac{7}{5}\boldsymbol{a} - \frac{11}{10}\boldsymbol{b}$

Explanation

$\overrightarrow{\mathrm{EC}} = \overrightarrow{\mathrm{EA}} + \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}}$

As $\frac{BD}{BC} = \frac{2}{3}$, $\overrightarrow{\mathrm{BC}} = \frac{3}{2}\overrightarrow{\mathrm{BD}}$

$$\overrightarrow{\mathrm{EC}} = \overrightarrow{\mathrm{EA}} + \boldsymbol{a} + \frac{3}{2}\boldsymbol{b} \qquad (1)$$

As $\frac{\mathrm{AE}}{\mathrm{ED}} = \frac{2}{3}$, $\overrightarrow{\mathrm{AE}} = \frac{2}{5}\overrightarrow{\mathrm{AD}}$

$$\overrightarrow{\mathrm{AE}} = \frac{2}{5}\left(\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BD}}\right) = \frac{2}{5}(\boldsymbol{a} + \boldsymbol{b})$$

Student
view

As $\overrightarrow{\mathrm{EA}} = -\overrightarrow{\mathrm{AE}}$

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$$\overrightarrow{EA} = -\frac{2}{5}\left(\boldsymbol{a} + \boldsymbol{b}\right) \qquad \textbf{(2)}$$

Substitute **(2)** into **(1)**

$$\overrightarrow{\mathrm{EC}} = -\frac{2}{5}\left(\boldsymbol{a} + \boldsymbol{b}\right) + \boldsymbol{a} + \frac{3}{2}\boldsymbol{b}$$

Simplify

$$\overrightarrow{\mathrm{EC}} = \frac{3}{5}\boldsymbol{a} + \frac{11}{10}\boldsymbol{b}$$

Therefore, the correct answer is

$$\frac{3}{5}\boldsymbol{a} + \frac{11}{10}\boldsymbol{b}$$

3. Geometry and trigonometry / 3.12 Vectors

Component form

Linear combination of vectors

Any vector in a plane can be written as a combination of two non-parallel vectors.

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non-parallel vectors. Then the linear combination of the two vectors would be

$$\boldsymbol{c} = \lambda\boldsymbol{a} + \mu\boldsymbol{b}$$

where λ and μ are scalars.

Example 1

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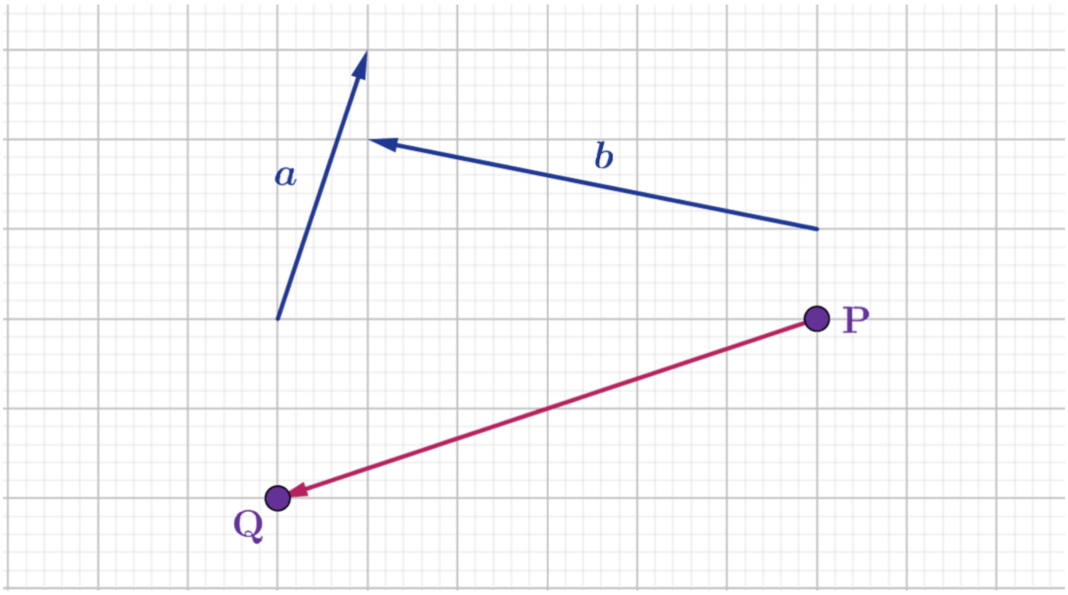
Write $\overrightarrow{PQ}$ as linear combination of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.



More information

The image is a diagram showing vectors on a grid, used to illustrate the problem of writing the vector (\overrightarrow{PQ}) as a linear combination of the vectors (\boldsymbol{a}) and (\boldsymbol{b}). - The diagram features two main points, labeled (P) and (Q), connected by a vector line, (\overrightarrow{PQ}). - Vectors (\boldsymbol{a}) and (\boldsymbol{b}) are depicted as arrows originating from different grid points, providing reference directions. - The vector (\overrightarrow{PQ}) is drawn in a distinct color compared to (\boldsymbol{a}) and (\boldsymbol{b}), emphasizing its role in the problem. - The background shows a grid which helps in representing the spatial orientation and length of the vectors visually. - The goal is to express (\overrightarrow{PQ}) in terms of (\boldsymbol{a}) and (\boldsymbol{b}), indicating the proportions or coefficients needed for each vector to achieve the same result in linear algebraic terms.

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Draw the vectors.

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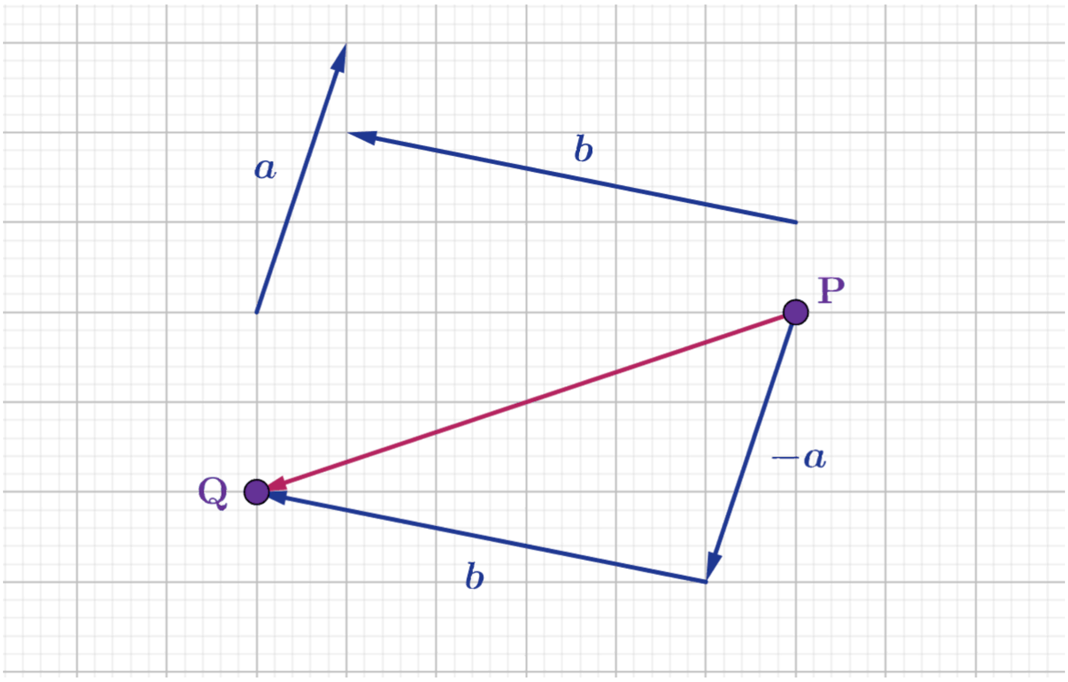

The scalars are -1 and 1.

Therefore

$$\overrightarrow{PQ} = -\boldsymbol{a} + \boldsymbol{b}$$

$\boldsymbol{i}$ and $\boldsymbol{j}$ unit vectors

Any vector in a 2D plane can be written as a linear combination of a vector parallel to the positive x-axis and a vector parallel to the positive y-axis.

A vector one unit long parallel to the x-axis is denoted by $\boldsymbol{i}$ and a vector one unit long parallel to the y-axis is denoted by $\boldsymbol{j}$.

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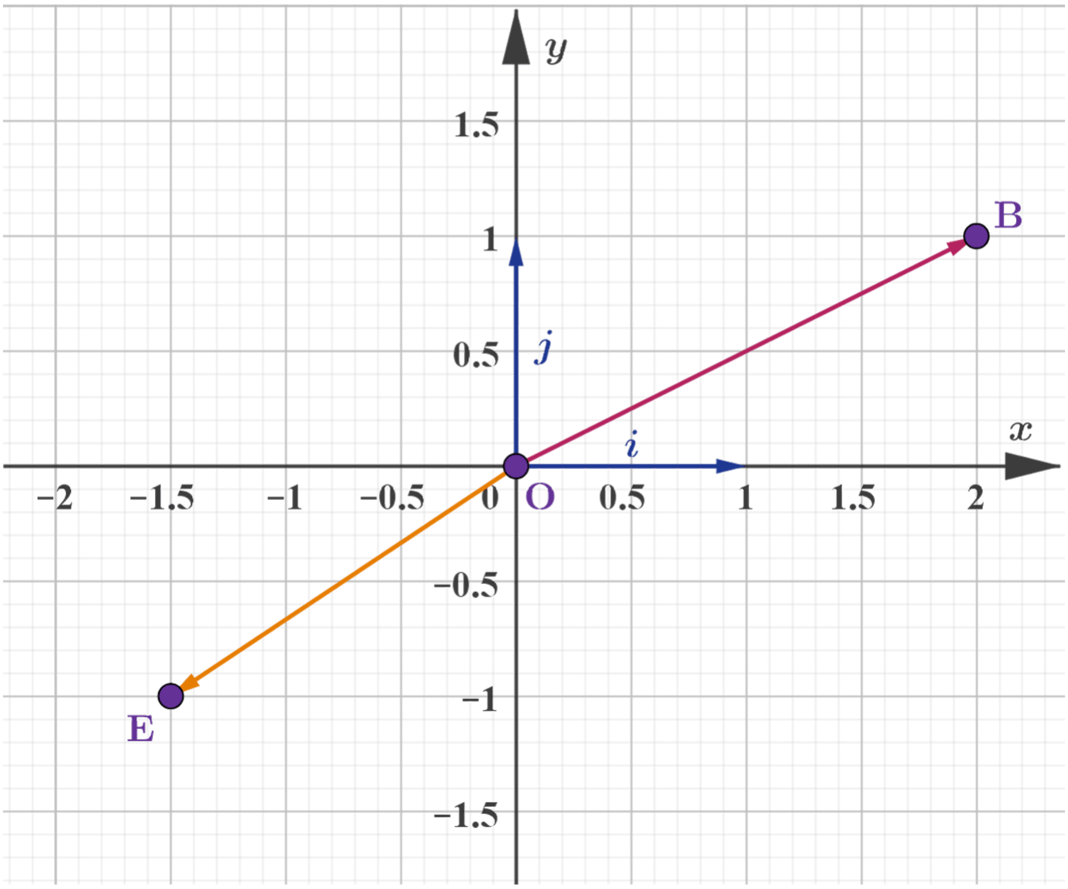
The diagram below shows these vectors. All the other vectors in the Cartesian plane can be written as linear combinations of $\boldsymbol{i}$ and $\boldsymbol{j}$, for example, $\overrightarrow{\mathrm{OB}} = 2\boldsymbol{i} + \boldsymbol{j}$ and $\overrightarrow{\mathrm{OE}} = -1.5\boldsymbol{i} - \boldsymbol{j}$.

The multiples of $\boldsymbol{i}$ and $\boldsymbol{j}$ are called the components of the vector.



More information

The image is a graph with an overlaid grid, displaying vectors labeled as components of the vector, specifically mentioning i and j. The X-axis, labeled as i, ranges from -2 to 2 in increments of 0.5. The Y-axis, labeled as j, similarly spans from -2 to 2, emphasizing the points i = 2, j = 1.5, and a point labeled as B at (2, 1). The vector components form a right triangle, with the origin point being O at (0,0). The graph visually demonstrates the relationship of these components in relation to each other in a Cartesian plane, highlighting vector magnitudes and directions.

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Important

Although any vector can be written as a linear combination of any two other non-parallel vectors, by convention vectors are written in terms of the two perpendicular unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$. These vectors are often called base vectors .

They are unit vectors because they are one unit long.

Vectors can be represented in variety of ways

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\boldsymbol{i} + \boldsymbol{j}$$

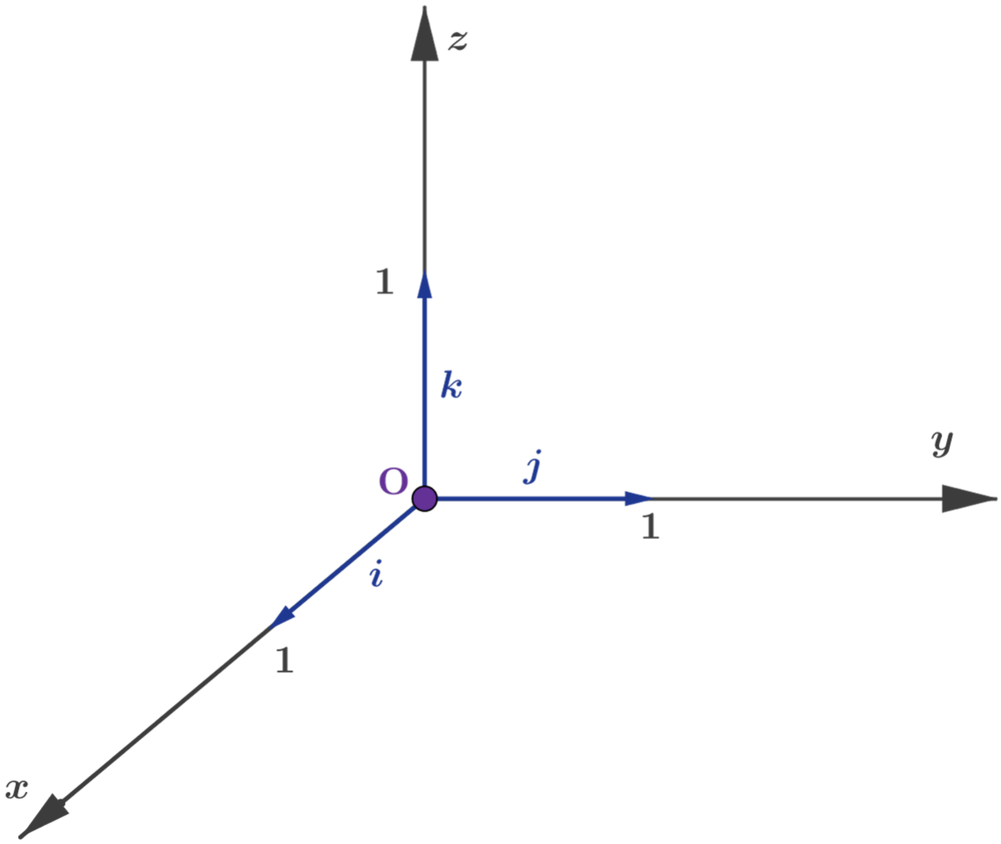
Depending on the question, you can choose the most suitable notation.

When a vector is written in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, you do not write $\boldsymbol{i}$ and $\boldsymbol{j}$ with the scalars to indicate the components of the vector. The vector bracket is sufficient to denote a vector. The top number represents the horizontal component and the bottom number gives the vertical component.

The vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ can also be used to denote unit vectors due east and due north, respectively. The context of the question will make this clear.

In 3D, you would define the base vectors as $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ as shown below. These vectors are parallel to the x, y and z coordinate axes, respectively, and are therefore perpendicular.

Student view



More information

The image is a 3D coordinate system diagram showing the unit vectors (\boldsymbol{i}), (\boldsymbol{j}), and (\boldsymbol{k}). The (\boldsymbol{i}) vector is aligned with the x-axis, (\boldsymbol{j}) with the y-axis, and (\boldsymbol{k}) with the z-axis. Each axis is labeled, and the unit distance of 1 is marked on each axis. The origin is labeled as (O) where all three vectors meet. (\boldsymbol{i}), (\boldsymbol{j}), and (\boldsymbol{k}) are perpendicular to each other, forming a 90-degree angle between each pair. This setup illustrates the Cartesian coordinate system in three dimensions, highlighting the independence of each axis from one another, which is fundamental for 3D vector calculations.

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Important

In 3D, base vectors are $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

All the other vectors in 3D can be written as linear combinations of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

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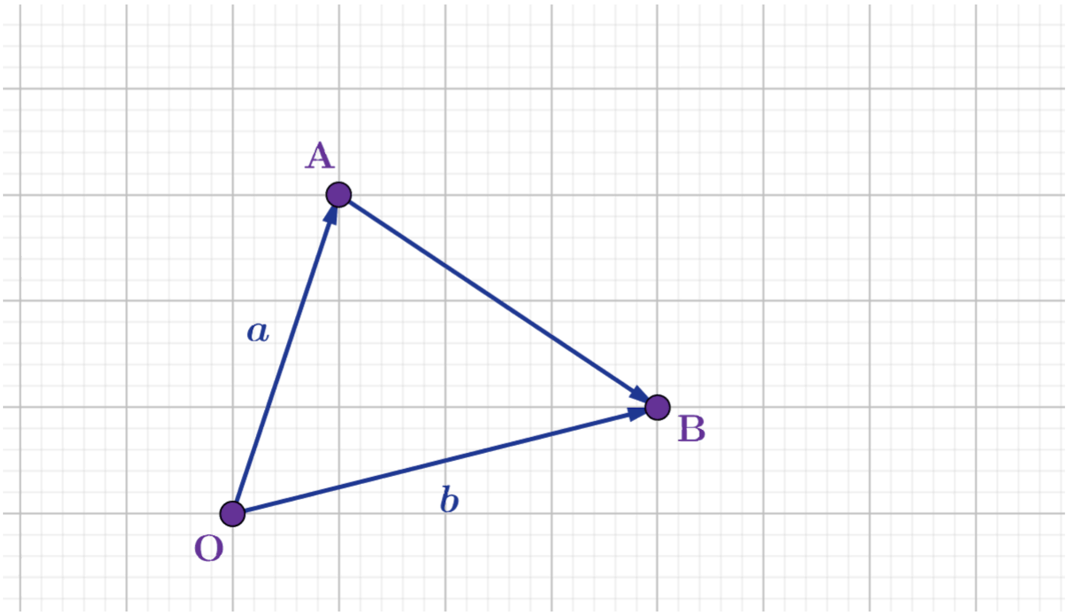
Example 2

★☆☆

Points A and B have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, relative to a fixed origin O.



More information

The diagram depicts a triangle on a grid with points O (origin), A, and B. Each point is marked by a purple dot. Vector (\boldsymbol{a}) is represented by an arrow pointing from origin O to point A, and vector (\boldsymbol{b}) is represented by an arrow pointing from origin O to point B. There is another arrowed line indicating the vector (\overrightarrow{\text{AB}}) between point A and point B. The entire arrangement is overlaid on a rectangular grid, emphasizing position vectors in a coordinate system. The task is to express vector (\overrightarrow{\text{AB}}) in terms of (\boldsymbol{a}) and (\boldsymbol{b}).

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Student view

Write vector $\overrightarrow{\text{AB}}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

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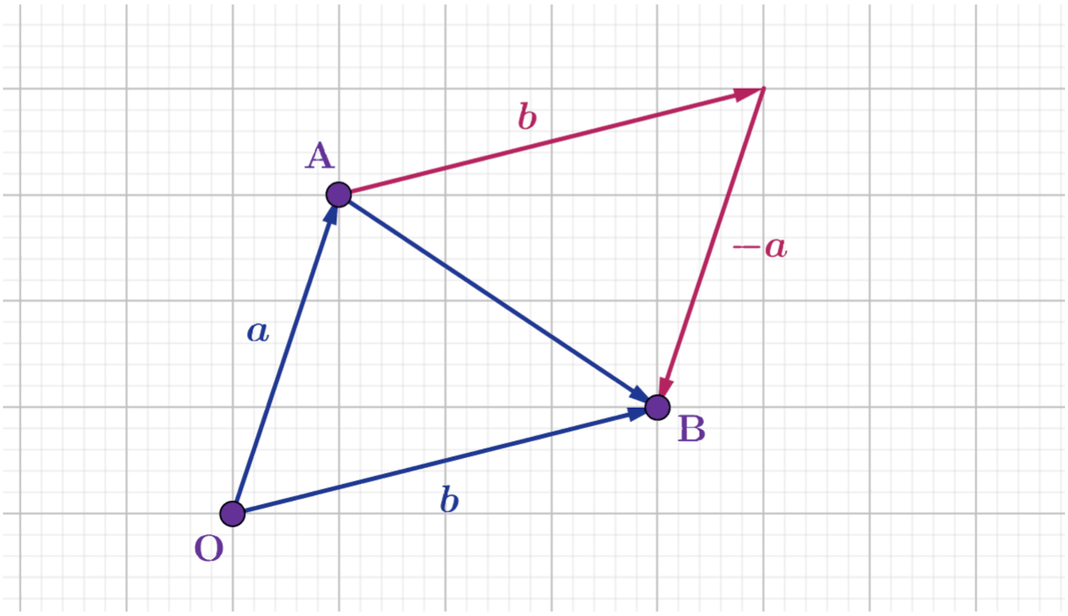
Extend triangle OAB to a parallelogram and use the parallelogram rule to find $\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a}$.



In the example above you proved the following property.

Important

If points A and B have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, then

$$\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a} \text{ or } \overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$$

Activity

In this activity, you will investigate the algebra of vectors written in component form using GeoGebra.

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- Create two vectors using the vector command for example $\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.
- Input $\boldsymbol{u} + \boldsymbol{v}$, $\boldsymbol{u} - \boldsymbol{v}$, $2\boldsymbol{u}$, $-2\boldsymbol{v}$ using the algebra input. What do you notice about the components of each vector?

Algebra of vectors in component form

In 2D: For two vectors $\boldsymbol{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} c \\ d \end{pmatrix}$

- $\boldsymbol{u} \pm \boldsymbol{v} = \begin{pmatrix} a \pm c \\ b \pm d \end{pmatrix}$
- $k\boldsymbol{u} = k \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$, where $k \in \mathbb{R}$

In 3D: $\boldsymbol{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

- $\boldsymbol{u} \pm \boldsymbol{v} = \begin{pmatrix} a \pm d \\ b \pm e \\ c \pm f \end{pmatrix}$
- $k\boldsymbol{u} = k \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$, where $k \in \mathbb{R}$

Example 3

★☆☆

If $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, find

a) $\boldsymbol{u} + \boldsymbol{v}$

b) $\boldsymbol{u} - 2\boldsymbol{v}$

Student view

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	Steps	Explanation
a)	$\boldsymbol{u}+\boldsymbol{v}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\2\\0\end{pmatrix}$	$\boldsymbol{u}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$ and $\boldsymbol{v}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ It is easier to work with vectors whe they are written in column notation
	Therefore $\boldsymbol{u}+\boldsymbol{v}=3i+2j$	
b)	$\boldsymbol{u}-2\boldsymbol{v}=\begin{pmatrix}2\\3\\-1\end{pmatrix}-\begin{pmatrix}2\\-2\\2\end{pmatrix}=\begin{pmatrix}0\\5\\-3\end{pmatrix}$	$2\boldsymbol{v}=2\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}2\\-2\\2\end{pmatrix}$
	$\boldsymbol{u}-2\boldsymbol{v}=5j-3k$	

Example 4

★★☆

Points A and B have position vectors $\begin{pmatrix}1\\1\\1\end{pmatrix}$ and $\begin{pmatrix}2\\5\\-4\end{pmatrix}$ respectively.

Point C lies on the line segment AB. If $AC : CB = 1 : 2$, find the position vector of C.

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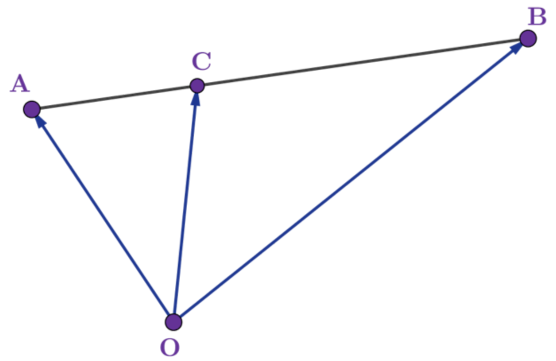
Steps	Explanation
	Draw a diagram.
$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$	$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$
$\overrightarrow{\mathrm{AC}} = \frac{1}{3}\overrightarrow{\mathrm{AB}} = \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$	$AC : CB = 1 : 2$ so $\overrightarrow{\mathrm{AC}} = \frac{1}{3}\overrightarrow{\mathrm{AB}}$
$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$	$\overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AC}}$
$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} \frac{4}{3} \\ \frac{7}{3} \\ \frac{-2}{3} \end{pmatrix}$	Simplify.

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view

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Steps	Explanation
Therefore, the position vector of point C is $\begin{pmatrix} \frac{4}{3} \\ \frac{7}{3} \\ \frac{-2}{3} \end{pmatrix}$	

Example 5

★☆☆

If the vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ a \\ b \end{pmatrix}$ are parallel, find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$.

As the two vectors are parallel $\boldsymbol{v} = k\boldsymbol{a}$.

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = k\begin{pmatrix} -6 \\ a \\ b \end{pmatrix} = \begin{pmatrix} -6k \\ ak \\ bk \end{pmatrix}$$

Respective components will be equal.

$$-6k = 3 \Rightarrow k = -\frac{1}{2}$$

$$ak = 2 \Rightarrow a = -4$$

$$bk = 1 \Rightarrow b = -2$$

Therefore,

$$a = -4 \text{ and } b = -2$$

Student view

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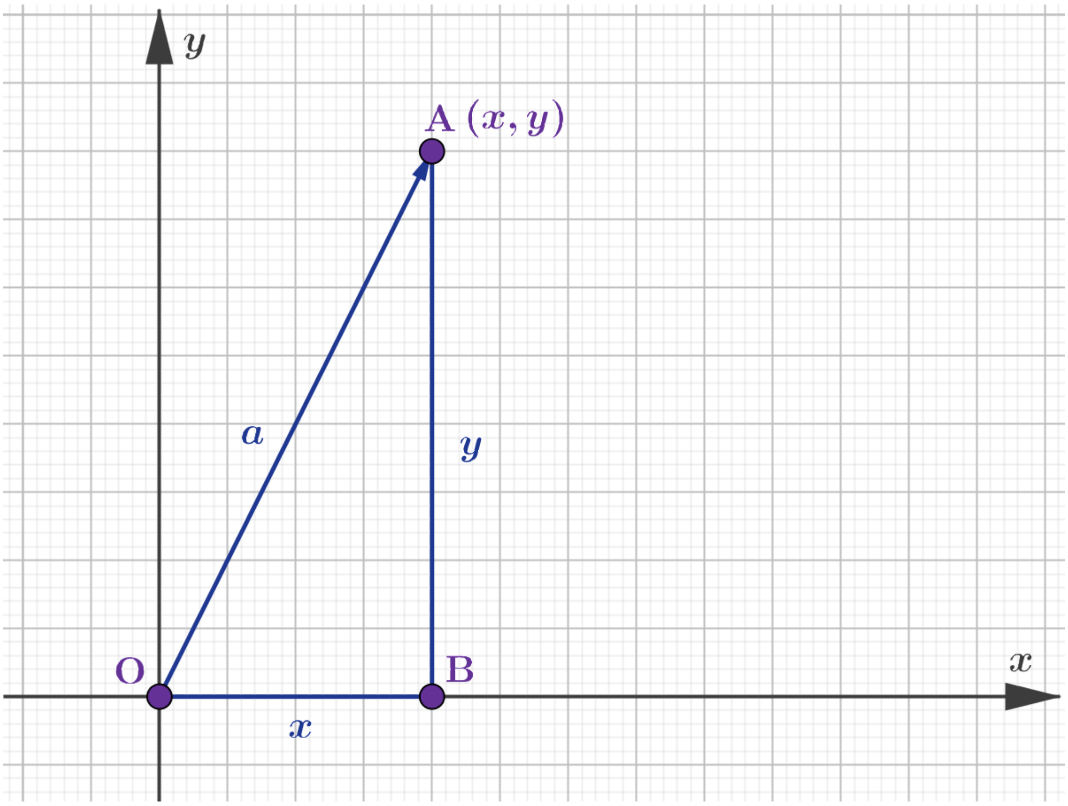
Magnitude of vectors in component form

Point (x, y), in the diagram below, has position vector $\boldsymbol{a}$.

You can use Pythagoras theorem to find the magnitude: $|\boldsymbol{a}| = \sqrt{x^2 + y^2}$



More information

The diagram displays a right triangle on a 3D grid, illustrating the application of the Pythagorean theorem to find the magnitude of a vector. It shows three points labeled O, B, and A. The horizontal axis is labeled as "x" with the segment OB, the vertical axis is labeled as "y" with the segment AB, and the hypotenuse OA is diagonally extended into 3D space, labeled "a". The point A is marked as ((x, y)). The diagram visually represents calculating the magnitude (|\boldsymbol{a}| = \sqrt{x^2 + y^2}) using the lengths on the axes. This visualization aids in spatial understanding of vector magnitudes in three-dimensional space.

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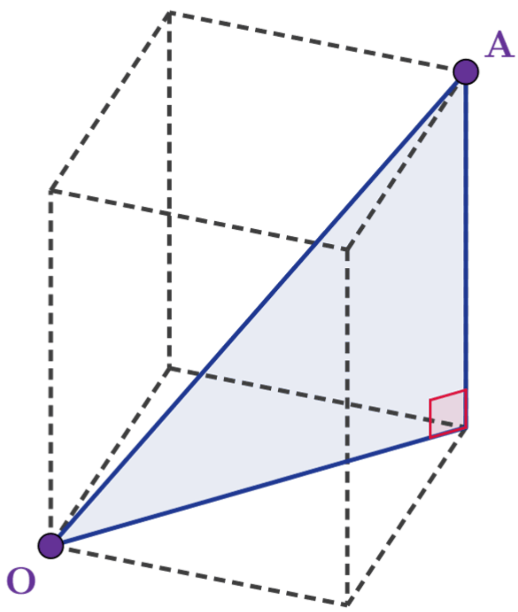
You can find the distance between two points in 3D using Pythagoras' theorem twice as shown in the diagram below. You can apply this to finding the magnitude of a vector in 3D.

Point (x, y, z) in the diagram below has position vector $\boldsymbol{a}$. Using Pythagoras theorem, $|\boldsymbol{a}| = \sqrt{x^2 + y^2 + z^2}$



More information

The image shows a three-dimensional diagram with a cube and a right triangle. The cube is outlined with dashed black lines, and the points labeled O and A are connected by a solid blue line representing the vector (\boldsymbol{a}). The right triangle is formed by this vector and two more dashed lines from points O to A, lying in the plane of the cube's face. The triangle visually illustrates the Pythagorean theorem in a 3D space with its right angle marked in red, connecting the vector's components along the x, y, and z axes. The diagram helps depict how the position vector's magnitude is calculated using Pythagoras' theorem as (|\boldsymbol{a}| = \sqrt{x^2 + y^2 + z^2}).

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Important

$$\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow |\boldsymbol{a}| = \sqrt{x^2 + y^2}$$

$$\boldsymbol{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow |\boldsymbol{b}| = \sqrt{x^2 + y^2 + z^2}$$

Example 6

★☆☆

Find the magnitude of the vector $\boldsymbol{w} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$

Sum the square of the components, then take the square root:

$$|\boldsymbol{w}| = \sqrt{(-2)^2 + 3^2 + 6^2} = \sqrt{49} = 7$$

Exam tip

In the IB formula booklet, the formula for the magnitude of a vector is given as $|\boldsymbol{v}| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}$, where $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

Making connections

Vectors can be used to represent any quantity that has both magnitude and direction.

What is the difference between speed and velocity?

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In everyday language the words can be interchanged, but in maths and physics they have different meanings. Speed is a scalar quantity — it measures how fast an object is moving. Velocity is a vector quantity. It not only measures how fast an object is moving, but it also gives the direction, e.g. 4 m s^{1} west. The magnitude of a velocity vector gives the speed.

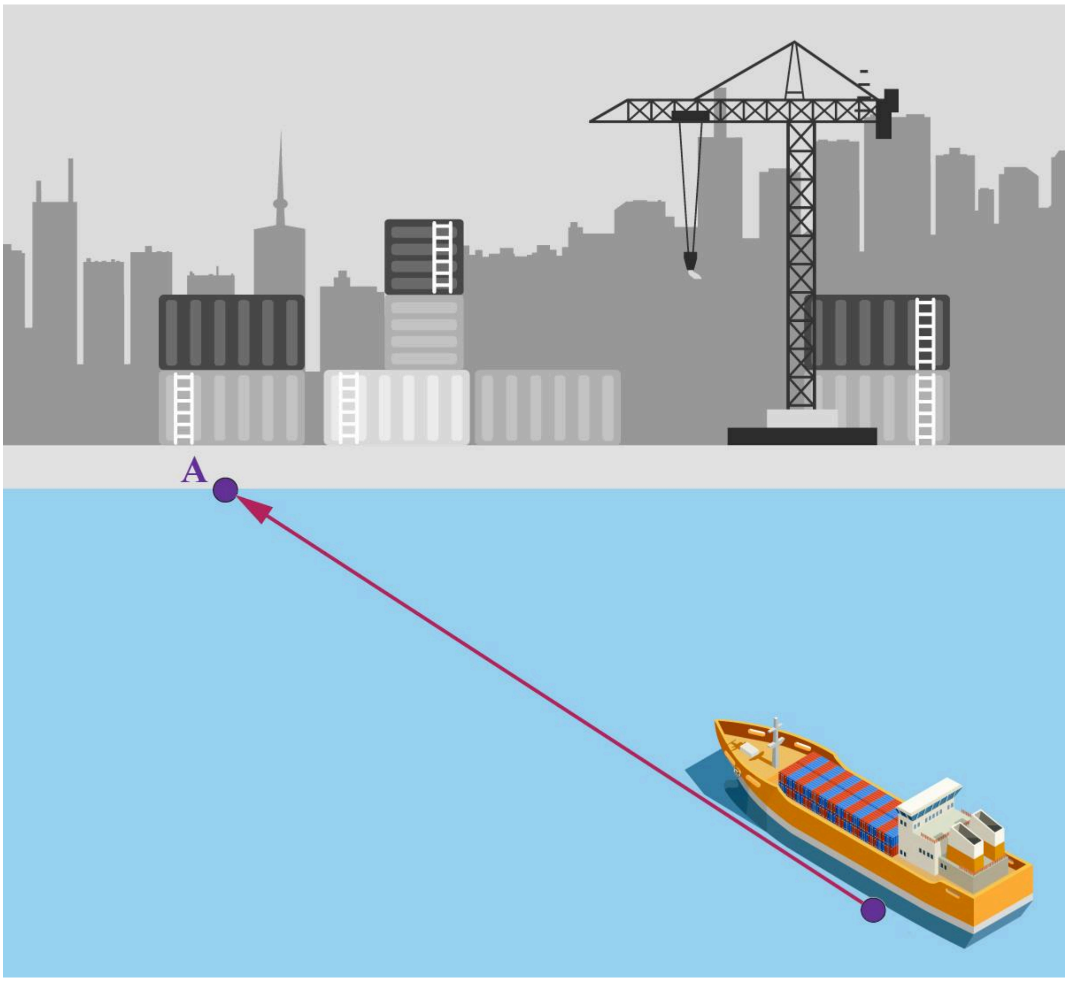
Example 7

★★☆

A ship is travelling towards port A with velocity $\begin{pmatrix} -40 \\ 30 \end{pmatrix}$ kilometres per hour.

a) How far will the ship be from the port 10 hours before it reaches the port?

b) Write the coordinates of the port relative to the ship.



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More information

The image shows a diagram of a port with several stacked cargo containers and a crane on the dockside. A ship is in the water, sailing towards the port from the bottom right of the image, represented by an orange vessel carrying multiple colored containers on its deck. The ship is connected to a point labeled 'A' on the dock through a red arrow, indicating a path towards the port. The skyline of a city with buildings is visible in the background.

[Generated by AI]

	Steps	Explanatic
a)	$\boldsymbol{v} = \begin{pmatrix} -40 \\ 30 \end{pmatrix} \Rightarrow \lvert\boldsymbol{v}\rvert = \sqrt{(-40)^2 + 30^2} = 50\,\mathrm{km\,h^{-1}}$	The magnitude of the vector is the speed of
	$\text{distance} = 10 \times 50 = 500\,\mathrm{km}$	The speed is constan $\text{distance} = \text{spee}$
b)	$\boldsymbol{p} = 10 \times \begin{pmatrix} -40 \\ 30 \end{pmatrix}$	$\text{Displacement} = \text{vel}$
	$\boldsymbol{p} = \begin{pmatrix} -400 \\ 300 \end{pmatrix}$	
	So the coordinates are $(-400, 300)$	The port is $400\,\mathrm{km}$ w $300\,\mathrm{km}$ north of the s

International Mindedness

There is a global decline in the number of bees. Bees are very important for pollination of food crops. Without them many food crops would not grow. Around the globe, wild flower gardens are being built in cities to provide

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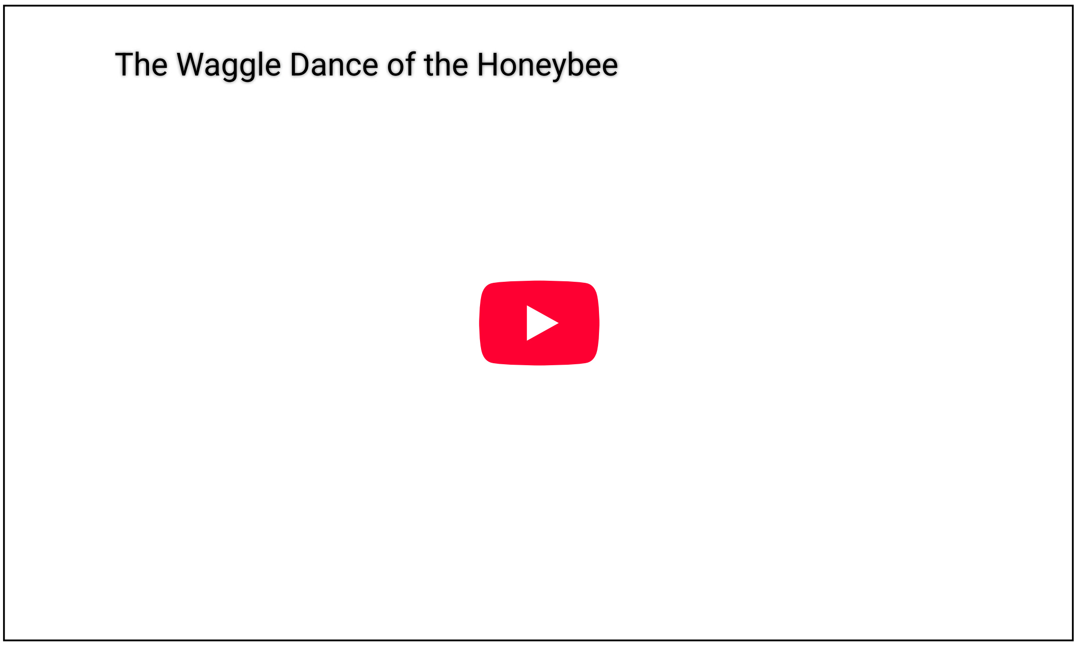
suitable habitats for bees to help them to survive and to halt the decline. You could help by planting some wild flowers in a pot or in your garden. Find out what kinds of plants are likely to attract bees.

Try to observe their behaviour. But, how do bees know where the flowers are?

Watch the following video to find out the answer.



6 section questions

Question 1

★☆☆

What is the vector $\overrightarrow{\mathrm{AB}}$ if A is the point $(3, 2, -2)$ and B the point $(-1, 2, 5)$?

1 $\begin{pmatrix} -4 \\ 0 \\ 7 \end{pmatrix}$ ✓

2 $\begin{pmatrix} 4 \\ 0 \\ -7 \end{pmatrix}$

3 $\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$

4 $\begin{pmatrix} -3 \\ -1 \\ 7 \end{pmatrix}$

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view

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Explanation

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -1-3 \\ 2-2 \\ 5-(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 7 \end{pmatrix}$$

Question 2

★☆☆

Consider a square with a church tower with its bells 120 m above the level of the town square. On the square is an ice cream shop.

To get to the ice cream shop from the base of the tower, you need to go 60 m east and then 20 m north.

Take the east-west direction as the x-direction with east being positive.

Take the north-south direction as the y-direction with north as positive.

What is the vector from the ice cream shop to the bell, using metres (m) as your unit?

1 $\begin{pmatrix} -60 \\ -20 \\ 120 \end{pmatrix} \text{m}$ ✓

2 $\begin{pmatrix} -60 \\ -20 \\ -120 \end{pmatrix} \text{m}$

3 $\begin{pmatrix} 60 \\ -20 \\ -120 \end{pmatrix} \text{m}$

4 $\begin{pmatrix} 60 \\ 20 \\ 120 \end{pmatrix} \text{m}$

Explanation

We can take the base of the church tower to be the origin of the axis system. Then the ice cream shop is at the point $(60 \text{ m}, 20 \text{ m}, 0)$ and the bell at $(0, 0, 120 \text{ m})$.

Then the vector from the ice cream shop to the bell is $\begin{pmatrix} 0-60 \\ 0-20 \\ 120-0 \end{pmatrix} = \begin{pmatrix} -60 \\ -20 \\ 120 \end{pmatrix} \text{m}$

Student view

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Question 3

★☆☆

If $\boldsymbol{u} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$, work out $\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w}$.

1 $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ ✓

2 $\begin{pmatrix} 2 \\ -4 \\ 9 \end{pmatrix}$

3 $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

4 $\begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix}$

Explanation

$$\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -1+2-1 \\ 2-3-(-3) \\ 4+1-4 \end{pmatrix}$$

Question 4

★☆☆

Select the value of λ for which these two vectors are parallel:

$$\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} -3 \\ \lambda \\ 12 \end{pmatrix}$$

1 $\lambda = 6$ ✓

2 $\lambda = \frac{2}{3}$

3 $\lambda = 2$

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4 $\lambda = -6$

Explanation

For these two vector to be parallel, there must be a scalar multiple between them, i.e.

$$\begin{pmatrix} -3 \\ \lambda \\ 12 \end{pmatrix} = k \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$$

This means that

$$\begin{aligned} -3 &= k \times (-1) \\ \lambda &= k \times 2 \\ 12 &= k \times 4 \end{aligned}$$

So, $k = 3$, which also means that $\lambda = k \times 2 = 3 \times 2 = 6$.

Question 5

★☆☆

Select the **exact** magnitude of $\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$

1 $\sqrt{29}$ ✓

2 1

3 $-\sqrt{29}$

4 5.39

Explanation

$\sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$

Student view

Question 6

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★★☆

Let $\boldsymbol{a} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $|\boldsymbol{a} + \lambda \boldsymbol{b}| = \sqrt{2}$.

Select which of the following could be the value of λ.

1 -1 ✓

2 1

3 -2

4 $-\sqrt{2}$

Explanation

$$\boldsymbol{a} + \lambda \boldsymbol{b} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2+\lambda \\ \lambda \end{pmatrix}$$

$$|\boldsymbol{a} + \lambda \boldsymbol{b}| = \sqrt{(2+\lambda)^2 + \lambda^2} = \sqrt{2}$$

Solve $\sqrt{(2+\lambda)^2 + \lambda^2} = \sqrt{2}$ using a graphic display calculator

$$\lambda = -1$$

3. Geometry and trigonometry / 3.12 Vectors

Unit vectors

You have already seen the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ that are used as base vectors. A unit vector is a vector that is 1 unit long in a specified direction.

Student view

To find a unit vector you need to divide the components of a vector by its magnitude.

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Consider the vector $\boldsymbol{v} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

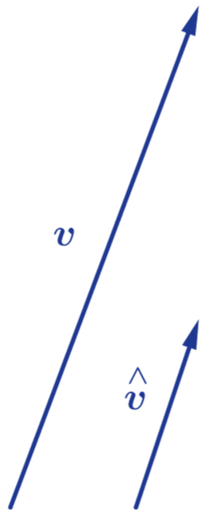
This has magnitude $|\boldsymbol{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$.

To find a unit vector parallel to $\boldsymbol{v}$ you need to divide each component of $\boldsymbol{v}$ by $\sqrt{5}$.

Unit vectors are denoted by $\widehat{\boldsymbol{v}}$.

A unit vector parallel to $\boldsymbol{v}$ is given by $\widehat{\boldsymbol{v}} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ 0 \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

$$\boldsymbol{u} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ 0 \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$



More information

Student view

The image shows a diagram illustrating the projection of a vector (\mathbf{v}) onto another vector (\mathbf{u}). Vector (\mathbf{v}) is represented as a long arrow pointing diagonally upward and to the right. The projection of (\mathbf{v}) onto (\mathbf{u}) is shown as a smaller arrow parallel to (\mathbf{u}) and is labeled as (\hat{\mathbf{v}}). There is also a perpendicular component of (\mathbf{v}) depicted, which indicates the error between (\mathbf{v}) and its projection.

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Important

For a vector $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, the unit vector in the same direction as $\boldsymbol{v}$ is $\widehat{\boldsymbol{v}}$

Example 1

★☆☆

Find the unit vector in the direction of the vector $\boldsymbol{w} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$

This is given by

$$\widehat{\boldsymbol{w}} = \frac{1}{|\boldsymbol{w}|}\boldsymbol{w} = \frac{1}{\sqrt{(-2)^2 + 3^2 + 6^2}} \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} \frac{-2}{7} \\ \frac{3}{7} \\ \frac{6}{7} \end{pmatrix}$$

Example 2

Student view

★★☆

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Consider the points $\mathrm{A}(1,-3,2)$ and $\mathrm{B}(-4,0,3)$.

Find the vector $\overrightarrow{\mathrm{AB}}$, the distance between the points A and B, and the unit vector in the direction $\overrightarrow{\mathrm{AB}}$.

The position vectors of the two points are

$$\overrightarrow{\mathrm{OA}} = \boldsymbol{a} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \text{ and}$$

$$\overrightarrow{\mathrm{OB}} = \boldsymbol{b} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$$

Then,

$$\overrightarrow{\mathrm{AB}} = -\boldsymbol{a} + \boldsymbol{b} = -\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$$

Thus, the distance points A and B is the magnitude of vector $\overrightarrow{\mathrm{AB}}$, i.e.

$$\left|\overrightarrow{\mathrm{AB}}\right| = \left|\begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}\right| = \sqrt{(-5)^2 + 3^2 + 1^2} = \sqrt{35}$$

Lastly, the unit vector in the direction of $\overrightarrow{\mathrm{AB}}$ is given by

$$\widehat{\mathrm{AB}} = \frac{1}{\sqrt{35}} \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$$

Student view

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Example 3

★★☆

Consider the points $\mathrm{A}(1, k, 2)$ and $\mathrm{B}(4, -2, 1)$. If the length of the vector $\overrightarrow{\mathrm{AB}}$ is $\sqrt{46}$, find k and then the unit vector in the direction $\overrightarrow{\mathrm{AB}}$.

The position vectors of the two points are $\overrightarrow{\mathrm{OA}} = \boldsymbol{a} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \boldsymbol{b} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

Then,

$$(\overrightarrow{\mathrm{AB}} = -\boldsymbol{a} + \boldsymbol{b} = -\begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2-k \\ -1 \end{pmatrix}$$

Thus, the distance between points A and B is the magnitude of vector $\overrightarrow{\mathrm{AB}}$, i.e.

$$\left|\overrightarrow{AB}\right| = \left|\begin{pmatrix} 3 \\ -2-k \\ -1 \end{pmatrix}\right| = \sqrt{3^2 + (-2-k)^2 + (-1)^2} = \sqrt{10 + (2+k)^2}$$

And as it is given that $\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{46}$, we have

$$\begin{aligned} \sqrt{10 + (2+k)^2} &= \sqrt{46} \\ 10 + (2+k)^2 &= 46 \\ (2+k)^2 &= 36 \\ 2 + k &= \pm 6 \\ k &= 4 \text{ or } \mathrm{k} = -8 \end{aligned}$$

Student
view

Thus, either

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$$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ -6 \\ -1 \end{pmatrix} \text{ or } \overrightarrow{\mathrm{AB}} = \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix}$$

with relative unit vectors

$$\widehat{\mathrm{AB}} = \frac{1}{\sqrt{46}} \begin{pmatrix} 3 \\ -6 \\ -1 \end{pmatrix} \text{ or } \widehat{\mathrm{AB}} = \frac{1}{\sqrt{46}} \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix}$$

Scaling

When you multiply a vector by a scale factor you create a new vector parallel to the original one. The two vectors differ only in their magnitudes. They will have the same direction. Multiplying a vector by a scalar is called scaling.

✓ **Important**

When a vector $\boldsymbol{a}$ is multiplied by a scalar k, the resultant vector is $\boldsymbol{v} = k\boldsymbol{a}$ which is parallel to $\boldsymbol{a}$ and has a magnitude $|\boldsymbol{v}| = k\,|\boldsymbol{a}|$. If $k < 0$, then the direction of $\boldsymbol{v}$ is opposite to $\boldsymbol{a}$.

If you need to find a vector in the same direction as a given vector but with a different magnitude, you can find the unit vector and multiply it by the scale factor to find the new vector. This process is called rescaling.

Example 4

★★☆

Find a vector 2 units long in the direction of $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Student view

$$\boldsymbol{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow |\boldsymbol{v}| = \sqrt{3}$$

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Unit vector in the direction of $\boldsymbol{v}$

$$\hat{\boldsymbol{v}}$$

Vector in the same direction as the unit vector with a magnitude of 2.

$$\boldsymbol{w} = 2 \times \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Therefore, the answer is

$$\begin{pmatrix} \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix}$$

3 section questions

Question 1

★☆☆

Find the unit vector that is in the opposite direction to $\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

1 $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$ ✓

2 $\begin{pmatrix} \frac{-1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \end{pmatrix}$

Student view

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3 $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$

4 $\begin{pmatrix} \frac{-1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix}$

Explanation

$$\boldsymbol{v} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \Rightarrow |\boldsymbol{v}| = \sqrt{6}$$

The unit vector with the same direction is $\hat{\boldsymbol{v}}$

As you are looking for the unit vector in the opposite direction to $\boldsymbol{v}$, it is

$$-\hat{\boldsymbol{v}} = -\frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$$

Therefore, the correct answer is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$

Question 2

★☆☆

Select which of the following is the unit vector in the direction of $\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$.

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view

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1 $\frac{1}{\sqrt{29}}\begin{pmatrix}3\\-4\\2\end{pmatrix}$ ✓

2 $-\frac{1}{\sqrt{29}}\begin{pmatrix}3\\-4\\2\end{pmatrix}$

3 $\frac{1}{\sqrt{29}}\begin{pmatrix}-3\\4\\-2\end{pmatrix}$

4 $\frac{1}{\sqrt{-3}}\begin{pmatrix}3\\-4\\2\end{pmatrix}$

Explanation

A unit vector has a magnitude of 1.

Since $\left|\begin{pmatrix}3\\-4\\2\end{pmatrix}\right| = \sqrt{29}$, we have that the vector of length 1 in the direction of the given vector is given by $\frac{1}{\sqrt{29}}\begin{pmatrix}3\\-4\\2\end{pmatrix}$.

Question 3

If $\boldsymbol{a} = \begin{pmatrix}1\\2\\0\end{pmatrix}$, which of the following is the vector in the opposite direction to $\boldsymbol{a}$, with a magnitude of 2.

1 $\begin{pmatrix}\frac{-2}{\sqrt{5}}\\\frac{-4}{\sqrt{5}}\\0\end{pmatrix}$ ✓

2 $\begin{pmatrix}\frac{2}{\sqrt{5}}\\\frac{4}{\sqrt{5}}\\0\end{pmatrix}$

3 $\begin{pmatrix}\frac{2}{\sqrt{5}}\\\frac{-4}{\sqrt{5}}\\0\end{pmatrix}$

Student
view

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4 $\begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{4}{\sqrt{5}} \\ 0 \end{pmatrix}$

Explanation

$\boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \Rightarrow |\boldsymbol{a}| = \sqrt{5}$

The unit vector in the same direction is $\hat{\boldsymbol{a}} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

A vector in the opposite direction with a magnitude of 2.

$$\hat{\boldsymbol{b}} = -2 \times \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{-4}{\sqrt{5}} \\ 0 \end{pmatrix}$$

3. Geometry and trigonometry / 3.12 Vectors

Checklist

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What you should know

By the end of this subtopic you should be able to:

- recall that vectors have a size (magnitude) and a direction
- represent a displacement between two points as a vector
- decompose a 2D vector into its components in the x- and y-directions and a 3D vector into its components in the x-, y- and z-directions
- write a vector in column form, e.g. $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ or base vector form $\boldsymbol{v} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$ where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit base vectors in the x-, y-

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and z-directions, respectively: $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

- recall that two vectors are equal if and only if all their components are equal
- add vectors by adding their components: e.g. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\boldsymbol{v} + \boldsymbol{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_3 + w_3 \end{pmatrix}$
- recall that addition of vectors is commutative: $\boldsymbol{v} + \boldsymbol{w} = \boldsymbol{w} + \boldsymbol{v}$
- recall that if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, then $-\boldsymbol{v} = -\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -v_1 \\ -v_2 \\ -v_3 \end{pmatrix}$
- recall that $\boldsymbol{v} + (-\boldsymbol{v}) = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector
- know how to subtract one vector from another by the adding the negative components of the vector being subtracted: i.e. if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$, then $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w}) = \begin{pmatrix} v_1 - w_1 \\ v_2 - w_2 \\ v_3 - w_3 \end{pmatrix}$
- recall that subtraction of vectors is not commutative: i.e. $\boldsymbol{v} - \boldsymbol{w} \neq \boldsymbol{w} - \boldsymbol{v}$
- be able to multiply a vector by a scalar, for example if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is multiplied by scalar $k \in \mathbb{R}$ the result is $k\boldsymbol{v} = \begin{pmatrix} kv_1 \\ kv_2 \\ kv_3 \end{pmatrix}$
- recall that $k\boldsymbol{v}$ is a vector in the same direction as $\boldsymbol{v}$ but with a different magnitude (unless $k = \pm 1$)
- recall that two vectors are parallel if they are scalar multiples of the same vector: i.e. $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel if there exist m, n and $\boldsymbol{u}$ such that $\boldsymbol{v} = m\boldsymbol{u}$ and $\boldsymbol{w} = n\boldsymbol{u}$
- find the magnitude of a vector using Pythagoras' theorem: if $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, the magnitude of $\boldsymbol{v}$ is denoted by $|\boldsymbol{v}|$ and is given by $|\boldsymbol{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- know the properties of the magnitude of a vector:
  - $|\boldsymbol{v}| \geqslant 0$, i.e. it is never negative

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- $|\boldsymbol{v}| = |-\boldsymbol{v}|$
- in general, $|\boldsymbol{v} + \boldsymbol{w}| \neq |\boldsymbol{v}| + |\boldsymbol{w}|$

- Recall that a unit vector in a specified direction is denoted by $\widehat{\boldsymbol{v}}$
- Calculate a unit vector in the direction of a given vector using $\widehat{\boldsymbol{v}} = \frac{1}{|\boldsymbol{v}|}\boldsymbol{v}$
- write the position vector of point A relative to a fixed origin O: if point A has coordinates (x, y, z) then the position vector of A is denoted by $\overrightarrow{\text{OA}} = \boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$
- recall that the displacement vector between two points A and B is given in terms of their position vectors $\overrightarrow{\text{OA}} = \boldsymbol{a}$ and $\overrightarrow{\text{OB}} = \boldsymbol{b}$ by $\overrightarrow{\text{AB}} = \overrightarrow{\text{OB}} - \overrightarrow{\text{OA}} = -\boldsymbol{a} + \boldsymbol{b}$
- be able to calculate displacement vectors between two points: for example if A has coordinates (x_A, y_A, z_A) and B has coordinates (x_B, y_B, z_B) then $\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix}$

3. Geometry and trigonometry / 3.12 Vectors

Investigation

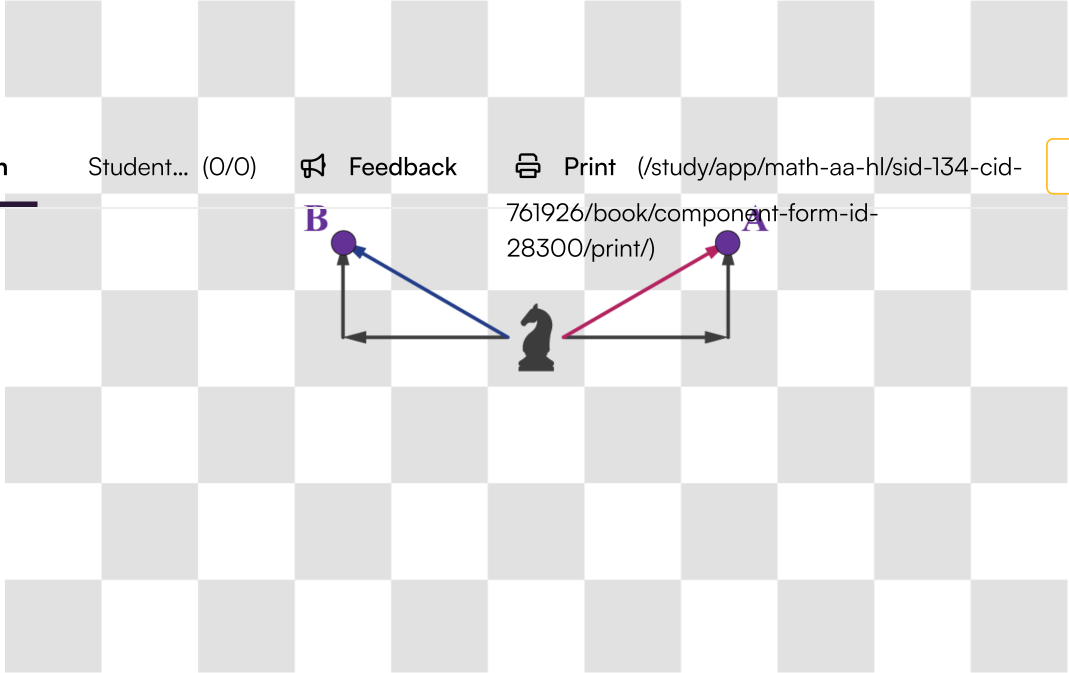
Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-28300/print/) Assign

In a chess game, the knight is one of the powerful pieces as it can hop over other pieces and move freely from black squares to white squares.

However, its moves follow a certain rule: they must be L shaped. A knight can move two squares right/left and 1 square up/down, or 1 square right/left followed by 2 squares up/down.

The knight in the diagram below could move 2 squares to the right and 1 square up to reach square A, or 2 squares left and 1 square up to reach square B.

Student view



More information

The image depicts a section of a chessboard, highlighting the knight's possible movements from its current central position. There are two potential moves illustrated by arrows. One arrow points to the right and slightly upwards, leading to square A; another arrow points to the left and slightly upwards, leading to square B. The diagram visually demonstrates two L-shaped paths, typical of a knight's movements in chess: moving two squares in one direction and then one square perpendicular. Each path is marked with a letter (A and B) at its endpoint, indicating the target squares based on the knight's potential moves.

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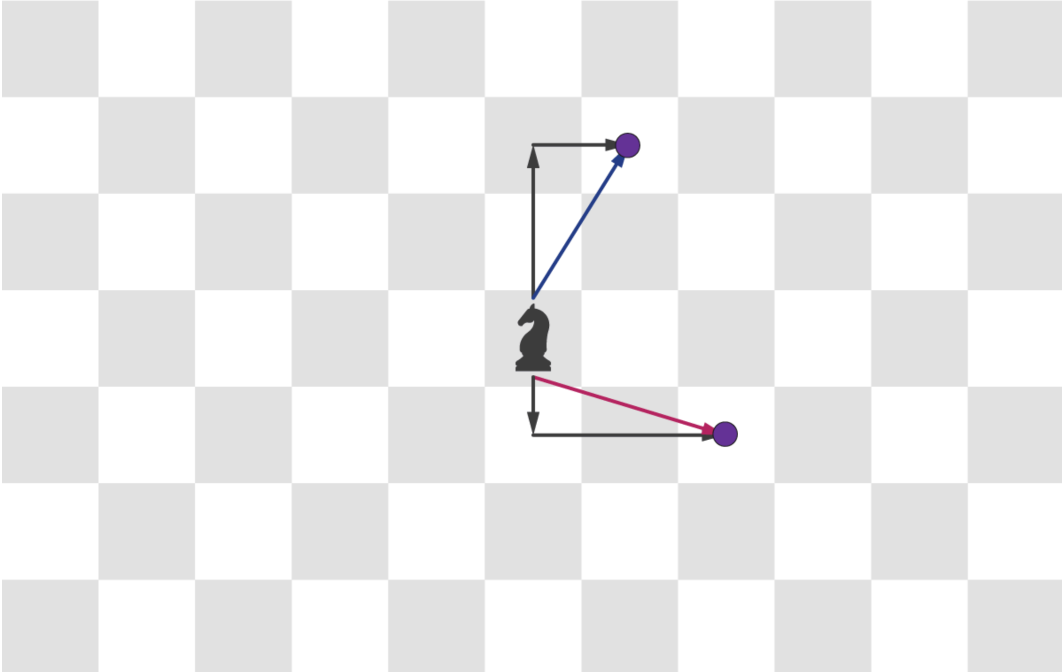
How can you represent these two moves using vectors?

How can you represent the moves shown in the diagram below?

Student view



More information

The image depicts a chessboard with a knight piece at the center. Two L-shaped arrows indicate the possible moves of the knight. One arrow shows the knight moving two squares up and one square to the right, while the other arrow displays the knight moving two squares down and one square to the left. These directions illustrate the unique movement pattern of a knight in chess, where it moves in an 'L' shape, either two squares in one direction and then one square perpendicular or one square in one direction and then two squares perpendicular.

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All the moves a knight can make are shown in the diagram below.

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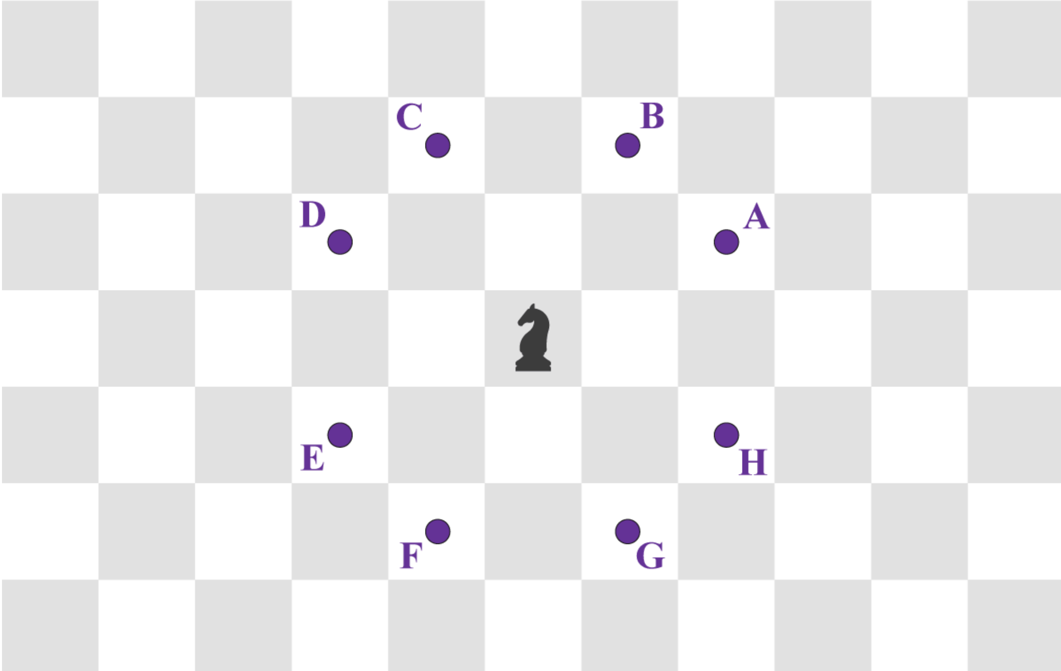

More information

This diagram illustrates a standard 8x8 chessboard, highlighting all possible moves of a knight from a central position. The chessboard is composed of alternating white and gray squares, following the standard layout for a chessboard. A knight is located in the center square. The potential squares the knight can move to are marked with purple dots labeled A through H.

- Square A is located two squares up and one square right from the knight.
- Square B is two squares up and one square left.
- Square C is two squares left and one square up.
- Square D is two squares left and one square down.
- Square E is two squares down and one square left.
- Square F is two squares down and one square right.
- Square G is two squares right and one square down.
- Square H is two squares right and one square up.

The diagram demonstrates the 'L' shape move pattern that is characteristic of a knight's move in chess, reflecting its ability to jump over other pieces on the board.

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Student
view

In a chess board, there are $8 \times 8 = 64$ black and white squares. If a knight starts at one of the corners of the board, what is the smallest number of moves it needs to make to reach the corner which is diagonally opposite?