Index

The big picture

What does it mean for something to be random or occur randomly? Given the depth of our current scientific understanding, is anything actually truly random? Consider these questions as you watch the following video.

As you will explore in the following sections, events that we often consider to be random have predictable tendencies when observed over a period of time.

Concept

We can analyse the outcomes of models of seemingly random events to find patterns that can be used to make predictions.

Theory of Knowledge

As you have inevitably noticed, mathematics is built upon itself conceptually. Lower-level axioms provide the basis upon which more complex mathematics and subsequent axioms are built. This methodology gives maths its epistemic clout.

To what extent are other areas of knowledge built in a similar fashion? If you were to rank the AOKs in terms of validity of knowledge produced, would the most axiomatic AOKs be at the top of your list?

Knowledge Question: To what extent is new knowledge a result of prior knowledge?

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

4. Probability and statistics / 4.14 Continuous random variables

Revisiting variance and standard deviation

Variance

In section 4.3.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/measures-of-dispersion-id-25519/) you explored the concept of how measures of dispersion can help you analyse the spread of a certain data set. In this section you will continue to explore that concept through hand calculations of the different measures.

Let us consider an example of a teacher, Somchai, who has recently given an assessment. After marking the papers, he finds that his students have the following scores:

Class A			Class B		
92	69	76	58	95	78
77	86	90	74	83	97
69	81	55	58	58	74
70	75	67	72	65	95

To begin analysing the scores, Somchai decides to calculate the mean of each class. Using the mean formula from section 4.3.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/measures-of-central-tendency-id-25518/), Somchai finds:

$$\bar{x}_{\text{Class A}} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{92+77+69+70+69+86+81+75+76+90+55+67}{12} = 75.58\bar{3}$$

$$\bar{x}_{\text{Class B}} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{58+74+58+72+95+83+58+65+78+97+74+95}{12} = 75.58\bar{3}$$

How does this information help Somchai compare the results of the two classes?

Somchai wants to further compare the classes. He decides to calculate the spread of the scores in each class using the variance, σ^2:

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27256/print/) Assign

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Making connections

In section 4.3.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/measures-of-dispersion-id-25519/), you learned about using variance as a measure of the spread, such that using the squares of the differences ensures that the positive and negative differences do not cancel each other out. Look at the formula and see how it gives the mean square deviation from the mean.

Using the equation shown above, Somchai finds the following:

$$\sigma^2_{\text{Class A}} = \frac{\left(92 - 75.58\bar{3}\right)^2 + \left(77 - 75.58\bar{3}\right)^2 + \ldots + \left(55 - 75.58\bar{3}\right)^2 + \left(67 - 75.58\bar{3}\right)^2}{12} = 102.743\ldots$$

$$\sigma^2_{\text{Class B}} = \frac{\left(58 - 75.58\bar{3}\right)^2 + \left(74 - 75.58\bar{3}\right)^2 + \ldots + \left(74 - 75.58\bar{3}\right)^2 + \left(95 - 75.58\bar{3}\right)^2}{12} = 194.243\ldots$$

How does this new information about the scores of the two classes help Somchai understand their performance on the assessment?

Example 1

★☆☆

Somchai gives the assessment to a third class and they achieve the following scores:

Class C		
99	54	62
54	90	70
92	91	66
83	59	57

Find the mean and variance of the scores.

Step	Explanation
$\bar{x}_{\text{Class C}} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{99+54+92+83+54+90+91+59+62+70+66+57}{12} = 73.08\bar{3}$	Use the formulae from above first find the mean.

Student view

Step	Explanation
$\sigma^2{}_{\text{Class C}} = \dfrac{\sum\limits_{i=1}^{n}(x_i-\mu)^2}{n}$ $= \dfrac{\left(99-75.08\bar{3}\right)^2+\left(54-75.08\bar{3}\right)^2+\ldots+\left(57-75.08\bar{3}\right)^2}{12}$ $= 258.576\ldots$	Then you find the variance.

Standard deviation

Look back at the values that you have calculated for variance so far. How does the value of the variance relate to the students' scores on the assessment?

As you can see, since you squared the differences between each score and the class mean, the value is much larger than the actual scores on the assessment. To give the measure more meaning within the context, you can take the square root of the variance to find the standard deviation:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum\limits_{i=1}^{n}(x_i-\mu)^2}{n}}$$

Example 2

★☆☆

Find the standard deviation of each Somchai's classes.

To find the standard deviation of each class, you simply need to take the square root of each variance you have found so far.

$$\begin{aligned}\sigma_{\text{Class A}} &= \sqrt{\sigma^2{}_{\text{Class A}}}\\ &= \sqrt{102.743\ldots}\\ &= 10.1362\ldots\end{aligned}$$

$$\begin{aligned}\sigma_{\text{Class B}} &= \sqrt{\sigma^2{}_{\text{Class B}}}\\ &= \sqrt{194.243\ldots}\\ &= 13.9371\ldots\end{aligned}$$

$$\begin{aligned}\sigma_{\text{Class C}} &= \sqrt{\sigma^2{}_{\text{Class C}}}\\ &= \sqrt{258.576\ldots}\\ &= 16.0803\ldots\end{aligned}$$

3 section questions ^

Student view

Question 1

Difficulty:

★☆☆

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Without a calculator, find the variance of the following data set:

$$11 \quad 10 \quad 8 \quad 4 \quad 7$$

6 ✓

Accepted answers

6

Explanation

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{11+10+8+4+7}{5} = \frac{40}{5} = 8$$

$$\begin{aligned}\sigma^2 &= \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}\\ &= \frac{(11-8)^2+(10-8)^2+(8-8)^2+(4-8)^2+(7-8)^2}{5}\\ &= \frac{9+4+0+16+1}{5}\\ &= \frac{30}{5}\\ &= 6\end{aligned}$$

Question 2

Difficulty:

★★☆

Find the values of p and q in the list below given that the mean of the numbers is 3, the variance is $\frac{10}{3}$ and $p < q$.

$$1 \quad 2 \quad 4 \quad 6 \quad p \quad q$$

1 $p = 1,\ q = 4$ ✓

2 $p = 3,\ q = 4$

3 $p = 2,\ q = 5$

4 $p = 4,\ q = 1$

Explanation

Begin by using the fact that the mean of the data is 3 to find the value of q in terms of p:

$$\begin{aligned}\bar{x} &= 3\\ \frac{1+2+4+6+p+q}{6} &= 3\\ p+q+13 &= 18\\ p+q &= 5\\ q &= 5-p\end{aligned}$$

Then substitute into the variance formula:

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

$$\begin{aligned}
\sigma^2 &= \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}\\
\frac{10}{3} &= \frac{(p-3)^2+(1-3)^2+(2-3)^2+(4-3)^2+(q-3)^2+(6-3)^2}{6}\\
20 &= (p-3)^2+(-2)^2+(-1)^2+(1)^2+((5-p)-3)^2+(3)^2\\
20 &= (p-3)^2+(2-p)^2+15\\
20 &= p^2-6p+9+4-4p+p^2+15\\
0 &= 2p^2-10p+8\\
0 &= 2(p-1)(p-4)\\
\therefore\ p &= 1 \text{ or } p=4
\end{aligned}$$

As $p<q$, choose $p=1$ and $q=4$.

Question 3

Difficulty:

★★★

Consider the following set of data:

$$4 \quad k \quad 3 \quad 5 \quad 7$$

Find the sum of the two possible values of μ when the variance of the data equals 2. Give the exact answer in decimal form.

9.5

Accepted answers

9.5

Explanation

First find an expression for k in terms of μ:

$$\begin{aligned}
\mu &= \frac{\sum_{i=1}^{n}x_i}{n}\\
\mu &= \frac{4+k+3+5+7}{5}\\
\mu &= \frac{19+k}{5}\\
k &= 5\mu-19
\end{aligned}$$

Substitute into the variance formula:

$$\begin{aligned}
\sigma^2 &= \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}\\
2 &= \frac{(4-\mu)^2+(k-\mu)^2+(3-\mu)^2+(5-\mu)^2+(7-\mu)^2}{5}\\
10 &= 5\mu^2-38\mu+99-2\mu k+k^2
\end{aligned}$$

Substitute in the expression for k:

$$\begin{aligned}
10 &= 5\mu^2-38\mu+99-2\mu(5\mu-19)+(5\mu-19)^2\\
10 &= 20\mu^2-190\mu+460\\
0 &= 20\mu^2-190\mu+450\\
\mu &= 5 \text{ or } 4.5
\end{aligned}$$

Therefore, the sum of the possible values of μ is 9.5

Student view

4. Probability and statistics / 4.14 Continuous random variables

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Variance and standard deviation for frequency tables

Calculations from frequency tables

In the previous section (/study/app/math-aa-hl/sid-134-cid-761926/book/revisiting-variance-and-standard-deviation-id-27257/), you found the variance and standard deviation of a list of data. However, what if the data is given to you in a frequency table rather than a list?

Making connections

In section 4.3.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/measures-of-central-tendency-id-25518/), you learned to find the mean from a frequency table using the formula:

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \text{ where } n = \sum_{i=1}^{k} f_i$$

Using the same concept of repeated values, you can extend the formulae for variance and standard deviation to include the frequencies:

$$\text{Variance} = \sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$$

$$\text{Standard deviation} = \sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}}$$

Be aware

Note that the upper bound of these sums is k , which is the number of unique values or class intervals. In the equations from the previous section, the upper bound was n , the total number of data values.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/revisiting-variance-and-standard-deviation-id-27257/print/) Assign

Exam tip

The formulae for variance and standard deviation given in the formula booklet include the frequencies.

Example 1

★☆☆

The following table shows the number of days each week that Somchai had students stay after school for tutorials.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Number of days student stayed after school in a given week	Frequency
0	10
1	9
2	10
3	2
4	6
5	5

Manually calculate the mean, variance and standard deviation of the number of days students stay behind.

Step	Explanation
$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n} = \frac{10\times 0+9\times 1+10\times 2+2\times 3+6\times 4+5\times 5}{42} = \frac{84}{42} = 2$	First, find the mean using the formula from section 4.3 .1
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i-\mu)^2}{n} = \frac{10(0-2)^2+9(1-2)^2+10(2-2)^2+2(3-2)^2+6(4-2)^2+5(5-2)^2}{42} = \frac{40+9+0+2+24+45}{42} = \frac{120}{42} = 2.85714\ldots$	Then find the variance using the calculated mean.
$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i (x_i-\mu)^2}{n}} = \sqrt{2.85714\ldots} = 1.69030\ldots$	Finally, take the square root of the variance to find the standard deviation.

An alternative formula for variance

Recall the equation for the mean calculated from a frequency table, given at the beginning of this section:

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \text{ where } n = \sum_{i=1}^{k} f_i$$

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Important

The notations $\bar{x}$ and μ both represent the mean. $\bar{x}$ is used when calculating the mean of a sample and μ is used when calculating the mean of a population. In both cases, the formula used to calculate the mean is the same.

Let us explore how we can use this formula to algebraically manipulate the formula for variance.

Step	Explanation
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i(x_i-\mu)^2}{n}$	
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i\left(x_i{}^2 - 2x_i\mu + \mu^2\right)}{n}$ $= \frac{\sum_{i=1}^{k} \left(f_i x_i{}^2 - 2f_i x_i\mu + f_i\mu^2\right)}{n}$	Begin with the formula for variance.
$\sigma^2 = \frac{\sum_{i=1}^{k} \left(f_i x_i{}^2 - 2f_i x_i\mu + f_i\mu^2\right)}{n}$ $= \frac{\sum_{i=1}^{k} f_i x_i{}^2 - \sum_{i=1}^{k} 2f_i x_i\mu + \sum_{i=1}^{k} f_i\mu^2}{n}$ $= \frac{\sum_{i=1}^{k} f_i x_i{}^2}{n} - \frac{\sum_{i=1}^{k} 2f_i x_i\mu}{n} + \frac{\sum_{i=1}^{k} f_i\mu^2}{n}$	Expand the binomial and distribute in f_i .
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i{}^2}{n} - 2\mu\frac{\sum_{i=1}^{k} f_i x_i}{n} + \mu^2\frac{\sum_{i=1}^{k} f_i}{n}$	Use the rules of sigma notation, split this into separate fractions.
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i{}^2}{n} - 2\mu \times \mu + \mu^2\frac{n}{n}$ $= \frac{\sum_{i=1}^{k} f_i x_i{}^2}{n} - 2\mu^2 + \mu^2$ $= \frac{\sum_{i=1}^{k} f_i x_i{}^2}{n} - \mu^2$	Since the 2 and μ are not dependent on i, they can be brought outside the sum.

Exam tip

This formula for variance is also included in the formula booklet. A good mnemonic is 'the mean of the squares minus the square of the mean.'

Example 2

★☆☆

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Recalculate the variance for the data from **Example 1** using this new formula for variance.

Number of days student stayed after school in a given week	Frequency
0	10
1	9
2	10
3	2
4	6
5	5

Steps	Explanation
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i {x_i}^2}{n} - \mu^2$ $= \frac{10\,(0)^2 + 9(1)^2 + 10(2)^2 + 2(3)^2 + 6(4)^2 + 5(5)^2}{42} - 2^2$ $= \frac{9 + 40 + 18 + 96 + 125}{42} - 4$ $= \frac{288}{42} - \frac{168}{42}$ $= \frac{120}{42} = 2.85714\ldots$	Begin with the formula and make the appropriate substitutions.

International Mindedness

The design of your typical day at school very much depends on where you live! How long you study each day, the subjects you study, and the days of week you go to school vary from country to country. Have a look at the infographic found here (http://ncee.org/2018/02/statistic-of-the-month-how-much-time-do-students-spend-in-school/)to see a few of the differences you might experience.

3 section questions

Question 1

Difficulty:

★☆☆

The committee of a village museum are trying to decide whether it is worth keeping the museum open in winter. They collect data on the number of visitors each day.

Find the standard deviation of the data in the following frequency table, giving your answer correct to three significant figures.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Number of people	Frequency
0	2
1	5
2	7
3	8
4	9
5	6

1.45

Accepted answers
1.45

Explanation
To begin, find the mean of the data:

$$\begin{aligned}\bar{x} &= \frac{\sum_{i=1}^{k} f_i x_i}{n}\\ &= \frac{(2)(0)+(5)(1)+(7)(2)+(8)(3)+(9)(4)+(6)(5)}{2+5+7+8+9+6}\\ &= \frac{109}{37} = 2.94594\ldots\end{aligned}$$

Then use the formula for variance to find the standard deviation:

$$\begin{aligned}\sigma^2 &= \frac{\sum_{i=1}^{k} f_i {x_i}^2}{n} - \mu^2\\ &= \frac{(2)(0)^2+(5)(1)^2+(7)(2)^2+(8)(3)^2+(9)(4)^2+(6)(5)^2}{37} - (2.94594\ldots)^2\\ &= 2.10518\ldots\\ \sigma &= \sqrt{\sigma^2} = \sqrt{2.10518\ldots} \approx 1.45\end{aligned}$$

Question 2

Difficulty:

★★☆

A road safety campaign is monitoring traffic at the times when children leave a nearby school. Find the variance of the data listed in the following frequency table.

Note: as you don't know the individual values of each piece of data, you can use the middle of the class interval to find the expectation.

Number of cars passing an intersection	Frequency
0–9	5
10–19	2
20–29	8

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Number of cars passing an intersection	Frequency
30–39	6
40–49	9
50–59	6

1 258.3 ✓

2 251.4

3 261.7

4 259.6

Explanation

Method 1 (GDC)

Use the mid-interval values and the one-variable statistics application of the calculator to find the standard deviation.

$$\sigma \approx 16.07275127$$

The variance is the square of the standard deviation.

$$\sigma^2 \approx 16.07275127^2 \approx 258.3$$

Method 2 (using the formula)

Use the middle of each class interval to find the expectation:

$$\begin{aligned}\bar{x} &= \frac{\sum_{i=1}^{k} f_i x_i}{n} \\ &= \frac{(4.5)(5) + (14.5)(2) + (24.5)(8) + (34.5)(6) + (44.5)(9) + (54.5)(6)}{5+2+8+6+9+6} \\ &= \frac{1182}{36} \approx 32.83\end{aligned}$$

Then you use the mean to find the variance:

$$\begin{aligned}\sigma^2 &= \frac{\sum_{i=1}^{k} f_i {x_i}^2}{n} - \mu^2 \\ &= \frac{(5)(4.5)^2 + (2)(14.5)^2 + \ldots + (9)(44.5)^2 + (6)(54.5)^2}{36} - \left(\frac{1182}{36}\right)^2 \\ &= 258.333\ldots \approx 258.3\end{aligned}$$

Question 3

Difficulty:

★☆☆

Find the standard deviation of the data shown below, giving your answer correct to three significant figures.

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Number of jelly beans in a bag	Frequency
24	3
25	5
26	6
27	10
28	11
29	7

1.48

Accepted answers

1.48

Explanation

To begin, find the mean of the data:

$$\begin{aligned}\bar{x} &= \frac{\sum_{i=1}^{k} f_i x_i}{n}\\ &= \frac{(3)(24)+(5)(25)+(6)(26)+(10)(27)+(11)(28)+(7)(29)}{3+5+6+10+11+7}\\ &= \frac{1134}{42} = 27\end{aligned}$$

Then use the formula for variance:

$$\begin{aligned}\sigma^2 &= \frac{\sum_{i=1}^{k} f_i (x_i-\mu)^2}{n}\\ &= \frac{3(24-27)^2+5(25-27)^2+\ldots+11(28-27)^2+7(29-27)^2}{42}\\ &= \frac{27+20+6+0+11+28}{42}\\ &= \frac{92}{42}\\ &= 2.19047\ldots\\ \sigma &= \sqrt{2.19047\ldots}\\ \sigma &= 1.4800\ldots\\ &= 1.48 \text{ (to 3 significant figures)}\end{aligned}$$

4. Probability and statistics / 4.14 Continuous random variables

Discrete random variables and measures of dispersion

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Revisiting discrete random variables

Making connections

Recall from section 4.7.0 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25656/) that a random variable is a variable whose value is the outcome of a statistical experiment. A discrete random variable is one whose possible outcomes are countable.

Marit is a student who has been given the task of creating a game based on probability for an upcoming fundraising event at her school. Marit decides to use a pair of 12-sided dice similar to the one shown below.

A 12-sided die

Credit: Getty Images, Paulo Jose Lima Gomes

For her game, Marit decides that each player will pay $2 to play the game and the prizes will be:

- $1 if the sum of their dice is odd
- $10 if their sum is either 2 or 24.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/variance-and-standard-deviation-id-27258/print/) Assign

Example 1

★★☆

Construct a probability distribution table for the outcomes of Marit’s game.

Student view

Total number of outcomes = $12 \times 12 = 144$

First, find the total number of possible outcomes.

For this distribution, X represents a player's winnings. There is only one way of getting either a 2 or a 24, therefore:

$$\mathrm{P}\left(X=2\right)=\frac{1}{144} \text{ and } \mathrm{P}\left(X=24\right)=\frac{1}{144}$$
$$\therefore \mathrm{P}\left(X=\left(2\cup 24\right)\right)=\frac{2}{144}$$

Find the probability of getting a 2 or a 24.

	1	2	3	4	5	6	7	8	9	10	11	12
1		√		√		√		√		√		√
2	√		√		√		√		√		√	
3		√		√		√		√		√		√
4	√		√		√		√		√		√	
5		√		√		√		√		√		√
6	√		√		√		√		√		√	
7		√		√		√		√		√		√
8	√		√		√		√		√		√	
9		√		√		√		√		√		√
10	√		√		√		√		√		√	
11		√		√		√		√		√		√
12	√		√		√		√		√		√	

$$\mathrm{P}\left(X=\text{odd}\right)=\frac{72}{144}$$

Find the probability of the sum of the dice being odd using a sample space.

$$\mathrm{P}\left(X=\left(2\cup 24\right)\right)+\mathrm{P}\left(X=\text{odd}\right)+\mathrm{P}\left(X=\left(\text{even}\cap\left(2\cup 24\right)'\right)\right)=1$$
$$\frac{2}{144}+\frac{72}{144}+\mathrm{P}\left(X=\left(\text{even}\cap\left(2\cup 24\right)'\right)\right)=1$$
$$\mathrm{P}\left(X=\left(\text{even}\cap\left(2\cup 24\right)'\right)\right)=\frac{70}{144}$$

Find the probability of the player losing given that the probabilities of each outcome must sum to 1.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Total number of outcomes = $12 \times 12 = 144$	**First, find the total number of possible outcomes.**

X	\$0	\$1	\$10
$\mathrm{P}(\mathrm{X}=\mathrm{x})$	$\frac{70}{144}$	$\frac{72}{144}$	$\frac{2}{144}$

Expectation (mean) of a discrete random variable

You studied the expectation of discrete random variables in section 4.7.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/expected-value-of-a-discrete-random-variable-id-25658/). Let us review it and make the connection between mean and expectation.

Step	Explanation
$\bar{x}=\frac{\sum_{i=1}^{k} f_i x_i}{n}$	Begin with the mean formula for a data set.
$\bar{x}=\frac{1}{n}\sum_{i=1}^{k} f_i x_i = \sum_{i=1}^{k} \frac{f_i}{n} x_i$	Rearrange to move n into the summation notation.
$\mathrm{E}(X)=\sum_{i=1}^{k} x_i \mathrm{P}(X=x_i)$	Recognise that the experimental probability $\frac{f_i}{n}$ is an approximation of the theoretical probability $\mathrm{P}(X=x_i)$.

✓ **Important**

As you can see, $\bar{x}$ and $\mathrm{E}(X)$ are very close in their meaning. Although they both relate to the average, you will use $\mathrm{E}(X)$ when working with random variables and $\bar{x}$ when working with data.

Example 2

★☆☆

Using the probability distribution you worked out in **Example 1**, calculate the expectation for Marit's game.

Begin with the expectation formula from above and insert the values from the probability distribution table.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

$$\begin{aligned}\mathrm{E}(X) &= \sum x\,\mathrm{P}(X=x)\\ &= (0)\left(\frac{70}{144}\right)+(1)\left(\frac{72}{144}\right)+(10)\left(\frac{2}{144}\right)\\ &= 0+\frac{72}{144}+\frac{20}{144}\\ &= \frac{92}{144}\\ &\approx 0.64\end{aligned}$$

Making connections

How can you determine whether Marit's game is fair? Refer to section 4.7.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/expected-value-of-a-discrete-random-variable-id-25658/) if necessary.

Variance of a discrete random variable

Until now, the formulae you have used to calculate variance have been based on the frequency of the data. As random variables are based on the probability of outcomes occurring, you will need new formulae to represent this change.

Important

So far you have used the notation σ^2 to represent the variance of data. Below you will encounter the notation $\mathrm{Var}(X)$, which is used for variance when working with random variables.

Using a method similar to that used in connecting mean and expectation, you can manipulate the variance formula to one that can be used with random variables:

Steps	Explanation
$\sigma^2 = \frac{\sum_{i=1}^{k} f_i(x_i-\mu)^2}{n}$	Begin with the variance formula for a data set. In this formula μ is the mean of the data set.
$\sigma^2 = \frac{1}{n}\sum_{i=1}^{k} f_i(x_i-\mu)^2 = \sum_{i=1}^{k}\frac{f_i}{n}(x_i-\mu)^2$	
$\sigma^2 = \sum_{i=1}^{k}(x_i-\mu)^2\mathrm{P}(X=x_i)$	Recognise that the experimental probability $\frac{f_i}{n}$ is an approximation of the theoretical probability $\mathrm{P}(X=x_i)$.

Notice that the formula in the last line of the table calculates the expected value of the discrete random variable $(X-\mu)^2$.

Student view

$$\mathrm{E}(X)=\sum_{i=1}^{k} \overset{\text{The value of the } i^{\text{th}} \text{ outcome}}{x_i}\ \underbrace{\mathrm{P}(X=x_i)}_{\text{The probability of the } i^{\text{th}} \text{ outcome occuring}}$$

$$\mathrm{Var}(X)=\sum_{i=1}^{k} \overbrace{(x_i-\mu)^2}^{\text{The value of the } i^{\text{th}} \text{ variation}}\ \underbrace{\mathrm{P}(X=x_i)}_{\text{The probability of the } i^{\text{th}} \text{ variation occuring}}$$

More information

The image displays two mathematical formulas related to probability and statistics. The first formula reads as follows: E(X) equals the sum from i equals 1 to k of x sub i multiplied by P(X equals x sub i). This formula is described as representing the value of the i-th outcome and the probability of the i-th outcome occurring.

The second formula is for Var(X) and is expressed as the sum from i equals 1 to k of the quantity (x sub i minus mu) squared, multiplied by P(X equals x sub i). This is described as the value of the i-th variation and the probability of the i-th variation occurring.

[Generated by AI]

Changing the notation for the expected value of X from μ to $\mathrm{E}(X)$ and for the variance from σ^2 to $\mathrm{Var}(X)$gives the definition of the variance of the random variable X.

$$\mathrm{Var}(X)=\mathrm{E}\left((X-\mathrm{E}(X))^2\right)$$

Just like the formula for the variance of a data set, this formula can also be written in a different form.

Steps	Explanation
$\sigma^2=\dfrac{\sum_{i=1}^{k} f_i {x_i}^2}{n}-\mu^2$ $=\sum_{i=1}^{k}\left({x_i}^2\cdot\dfrac{f_i}{n}\right)-\mu^2$	Recall the alternate formula for the variance of a data set.
$\sigma^2=\sum_{i=1}^{k}\left({x_i}^2\,\mathrm{P}\left(X=x_i\right)\right)-\mu^2$	Recognise that the experimental probability $\dfrac{f_i}{n}$ is an approximation of the theoretical probability $\mathrm{P}\left(X=x_i\right)$.

Notice that the first part is the expected value of X^2. Changing the notation gives the alternative formula for the variance of the random variable X.

$$\mathrm{Var}\left(X\right)=\mathrm{E}\left(X^2\right)-\left[\mathrm{E}\left(X\right)\right]^2$$

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Exam tip

The formula for the variance of a random variable X is in the formula booklet in the following form.

$$\mathrm{Var}(X) = \mathrm{E}\left[(X-\mu)^2\right] = \mathrm{E}(X^2) - \left[\mathrm{E}(X)\right]^2$$

The formula booklet also has the form useful for discrete random variables.

$$\mathrm{Var}(X) = \sum (x-\mu)^2\,\mathrm{P}(X=x) = \sum x^2\,\mathrm{P}(X=x) - \mu^2$$

Example 3

★☆☆

Calculate the variance in payout for Marit's game from **Example 1**.

Begin with the variance formula from above.

Recall that the mean is the expected value and then substitute the values in from the probability distribution table.

$$\begin{aligned}\mathrm{Var}\left(X\right) &= \sum_{i=1}^{k}\left(x_i - \mathrm{E}(X)\right)^2 \mathrm{P}\left(X = x_i\right)\\ &= \left(0-\frac{92}{144}\right)^2\left(\frac{70}{144}\right)+\left(1-\frac{92}{144}\right)^2\left(\frac{72}{144}\right)+\left(10-\frac{92}{144}\right)^2\left(\frac{2}{144}\right)\\ &= \left(\frac{8464}{20736}\right)\left(\frac{70}{144}\right)+\left(\frac{2704}{20736}\right)\left(\frac{72}{144}\right)+\left(\frac{1\,817\,104}{20736}\right)\left(\frac{2}{144}\right)\\ &= \frac{4\,421\,376}{2985\,984}\\ &= 1.48070\ldots\end{aligned}$$

3 section questions

Question 1

Difficulty:

★☆☆

The probability distribution of the random variable X is shown below. Find the mean and the standard deviation.

x	0	1	2	3	4
$\mathrm{P}\left(X = x\right)$	0.25	0.15	0.15	0.20	0.25

1 $\mu = 2.05\,,\quad \sigma = 1.53$ ✓

2 $\mu = 2.05\,,\quad \sigma = 2.35$

3 $\mu = 2.20\,,\quad \sigma = 1.53$

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

4 $\mu = 2.20$, $\sigma = 2.35$

Explanation

Use the mean formula first:

$$\begin{aligned}\mathrm{E}(X) &= \sum_{i=1}^{k} x_i \, \mathrm{P}(X = x_i)\\ &= (0)(0.25) + (1)(0.15) + (2)(0.15) + (3)(0.20) + (4)(0.25)\\ &= 0.15 + 0.30 + 0.60 + 1\\ &= 2.05\end{aligned}$$

Then choose a variance formula to use and take the square root of the result to obtain the standard deviation:

$$\begin{aligned}\mathrm{Var}(X) &= \mathrm{E}\left(X^2\right) - [\mathrm{E}(X)]^2\\ &= (0)^2(0.25) + (1)^2(0.15) + (2)^2(0.15) + (3)^2(0.20) + (4)^2(0.25) - (2.05)^2\\ &= 0 + 0.15 + 0.6 + 1.8 + 4 - 4.2025\\ &= 2.3475\\ \sigma &= \sqrt{\mathrm{Var}(X)}\\ &= \sqrt{2.3475}\\ &\approx 1.53\end{aligned}$$

Question 2

Difficulty: ★☆☆

A fair six-sided dice with sides numbered 0, 1, 2, 2, 5 and 6 is thrown once. Calculate the expected value of the score.

1 $\frac{8}{3}$ ✓

2 1

3 $\frac{7}{3}$

4 $\frac{15}{16}$

Explanation

First, understand that since it is a fair dice each side has a probability of $\frac{1}{6}$ of occurring. Then substitute into the expectation formula:

$$\begin{aligned}\mathrm{E}(X) &= \sum_{i=1}^{k} x_i \, \mathrm{P}(X = x_i)\\ &= (0)\left(\frac{1}{6}\right) + (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{6}\right)\\ &= 0 + \frac{1}{6} + \frac{2}{6} + \frac{2}{6} + \frac{5}{6} + \frac{6}{6}\\ &= \frac{16}{6} = \frac{8}{3}\end{aligned}$$

Question 3

Difficulty: ★★☆

Consider the probability density distribution shown below. Given that $\mathrm{E}(X) = 2.5$, find the values of p and q.

x	1	2	3	4

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

$\mathrm{P}(X = x)$	0.2	p	q	0.2

1 $p = 0.3,\ q = 0.3$ ✓

2 $p = 0.25,\ q = 0.35$

3 $p = 0.45,\ q = 0.15$

4 $p = 0.375,\ q = 0.225$

Explanation

Since the probabilities must sum to 1, you know that:

$$\begin{aligned} 0.2 + p + q + 0.2 &= 1 \\ p + q &= 0.6 \ \therefore\ p = 0.6 - q \end{aligned}$$

Next, use the expectation formula to get a second equation:

$$\begin{aligned} \mathrm{E}(X) &= \sum_{i=1}^{k} x_i \, \mathrm{P}(X = x_i) \\ 2.5 &= (1)(0.2) + 2p + 3q + (4)(0.2) \\ 2p + 3q &= 1.5 \end{aligned}$$

Finally, using substitution you obtain:

$$\begin{aligned} 2(0.6 - q) + 3q &= 1.5 \\ 1.2 - 2q + 3q &= 1.5 \\ q &= 0.3 \ \therefore\ p = 0.6 - q = 0.3 \end{aligned}$$

4. Probability and statistics / 4.14 Continuous random variables

Probability density functions

In this section, you will move from discrete random variables and begin to analyse continuous random variables.

Making connections

Recall from section 4.1.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/data-types-and-sources-id-25507/) that discrete data is typically obtained by counting and has specific values within a given range. Continuous data is obtained by measuring and can have an infinite number of values within a given range.

Since the outcomes of a continuous random variable are infinite in number, they cannot be listed in a frequency distribution table. Instead, you will often see them represented with a probability density function. The area of the region below the graph of the probability density function between the vertical lines $x = a$ and $x = b$ represent the probability $\mathrm{P}(a < X < b)$.

Student view

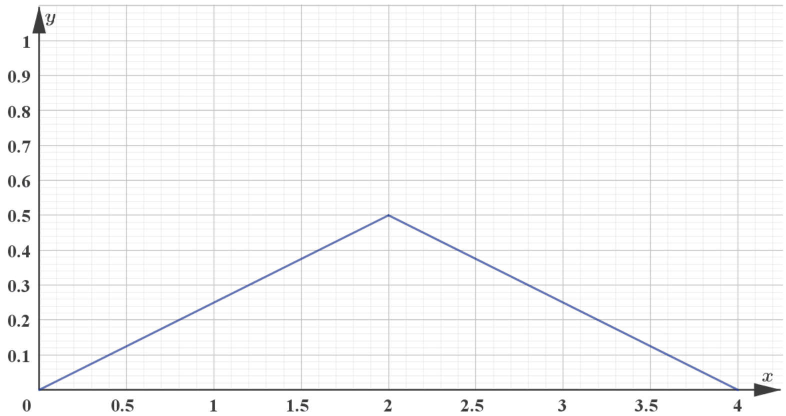
Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Exam tip

Questions will often use PDF as an abbreviation for probability density function.

Consider a continuous random variable whose PDF is defined by the function shown below.

$$f(x) = -\left|\frac{1}{4}(x-2)\right| + \frac{1}{2},\ 0 \leqslant x \leqslant 4$$



More information

The image is a graph of the function (f(x) = -\left|\frac{1}{4}(x - 2)\right| + \frac{1}{2}) over the domain (0 \leq x \leq 4). The graph is plotted on a coordinate plane with the x-axis ranging from 0 to 4 and labeled at intervals of 0.5. The y-axis ranges from 0 to 1.2 and is labeled at intervals of 0.1. The function creates a triangular shape within these axes, starting from the origin (0, 0), reaching a peak at (2, 0.5), and returning to (4, 0). This triangular pattern reflects the absolute value form of the function that causes the graph to peak at x = 2, with the maximum y-value being 0.5.

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Before you begin analysing the distribution of this graph, you must first understand how to determine whether a given function is a valid probability density function.

Important

Since the total probability of all outcomes must sum to 1, the area under the graph of $f(x)$ must be equal to 1 for it to be a valid probability density function.

Be aware

As there are an infinite number of possible outcomes within the domain of the probability density function, the probability of any specific outcome occurring is zero.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Example 1

★☆☆

Calculate the area between the x-axis and the graph of $f(x)$ to ensure it is equal to 1.

Use the formula for the area of a triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4 \times \frac{1}{2} \\ &= 1 \end{aligned}$$

Important

As all probabilities must be positive, the value of $f(x) \geqslant 0$ for its entire given domain.

Looking at the graph, you can see that the values of $f(x)$ are indeed all positive. Therefore $f(x)$ has passed both of the tests needed to be a valid probability density function.

Activity

Consider the graph shown in the following applet.

Interactive 1. Consider the Graph and Explain How to Verify That $f(x)$ Is a Valid Probability Density Function.
Credit: GeoGebra (https://www.geogebra.org/m/Ss6T54Dz) Mark Willis

More information for interactive 1

This interactive graph will help the users to understand the probability density function and calculate the probability within a specific interval. The x axis ranges from 0 to 9 and the y axis ranges from 0 to $\frac{1}{2}$. The bottom of the screen has two sliders allowing users to adjust the values for a and b.
The probability density function is defined as

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

$f(x) = \frac{3}{32}x^2$ when $0 \leq x < 2$

$f(x) = \frac{3}{32}(6 - x)$ when $2 \leq x \leq 6$

$f(x) = 0$ otherwise

The shaded area under the curve between a and b represents the probability of X falling within this interval. It is expressed as $P(a \leq X \leq b)$, where the users can use the slider to choose the values of a from 0 to 5 with the difference of 0.05 and b from 0 to 6 with the difference of 0.05. At a = 5 and b = 6 the area comes out to be 0.0469. In case where a > b, like a = 5 and b = 4.1 the area comes out to be −0.1223.

Hence, by checking different values of a and b users can check if the given function is a valid probability density function or not.

Explain how you can verify that $f(x)$ is a valid probability density function.

1. In your own words, explain what a and b represent.
2. Describe what happens to the area when $a > b$.
3. Making connections to what you learned in section 5.5.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/definite-integrals-id-25566/).

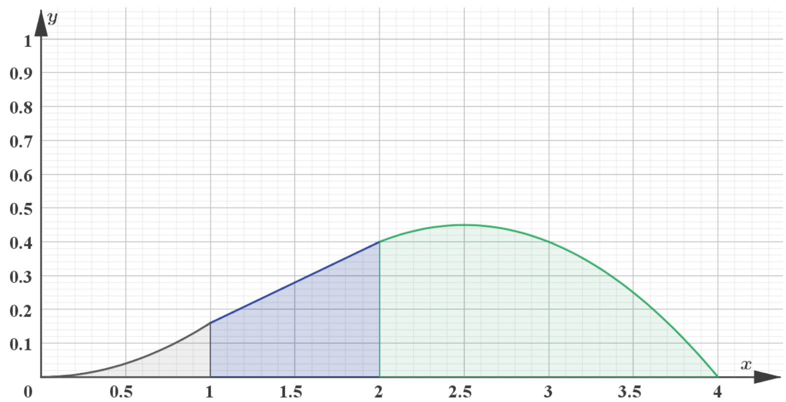
Example 2

★★☆

Determine whether the following piecewise function is a valid probability density function:

$$f(x) = \begin{cases} \frac{4}{25}x^2 & \text{if } 0 \leqslant x < 1 \\ \frac{6}{25}x - \frac{2}{25} & \text{if } 1 \leqslant x < 2 \\ -\frac{1}{5}x^2 + x - \frac{4}{5} & \text{if } 2 \leqslant x \leqslant 4 \end{cases}$$

First, consider the graph of $f(x)$.



Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/discrete-random-variables-id-27259/print/) Assign

Student view

As the graph is defined by quadratics (section 5.5.2), you need to use integrals to find the area under the curve.

Overview
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$$
\begin{aligned}
\text{Area} =\ & \text{grey area} + \text{blue area} + \text{green area} \\
&= \int_0^1 \frac{4}{25}x^2 \mathrm{d}x + \int_1^2 \frac{6}{25}x - \frac{2}{25}\ \mathrm{d}x + \int_2^4 -\frac{1}{5}x^2 + x - \frac{4}{5}\ \mathrm{d}x \\
&= \left[\frac{4}{75}x^3\right]_0^1 + \left[\frac{3}{25}x^2 - \frac{2}{25}x\right]_1^2 + \left[-\frac{1}{15}x^3 + \frac{1}{2}x^2 - \frac{4}{5}x\right]_2^4 \\
&= \left(\frac{4}{75} - 0\right) + \left(\frac{12}{25} - \frac{4}{25} - \left(\frac{3}{25} - \frac{2}{25}\right)\right) + \\
&\quad \left(-\frac{64}{15} + 8 - \frac{16}{5} - \left(-\frac{8}{15} + 2 - \frac{8}{5}\right)\right) \\
&= \frac{4}{75} + \frac{7}{25} + \frac{10}{15} \\
&= \frac{4}{75} + \frac{21}{75} + \frac{50}{75} \\
&= 1
\end{aligned}
$$

Therefore, since the area under the curve is 1 and all values of $f(x)$ are positive, the function is a valid probability density function.

Important

The two validity tests for a probability density function can be formally displayed as

$0 \leqslant f(x)$ and $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1$.

Exam tip

Whereas the integral shown above has infinity as its upper and lower bounds, it is often the case that the problem places limits on the possible values of x . You will see such situations represented as

$\int_a^b f(x)\,\mathrm{d}x = 1$

where a and b are the upper and lower limits of the given domain of $f(x)$.

Making connections

Does the normal distribution curve from section 4.9.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-normal-distribution-id-25667/) pass the two tests shown above?

4 section questions

Question 1

Difficulty:

★☆☆

Given that $f(x) = x^3,\ 0 \leqslant x \leqslant a$ is a valid probability distribution function, find the value of a.

Give your answer correct to three significant figures.

Student
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1.41

Overview
(/study/ap
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hl/sid-
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761926/o

Accepted answers

1.41

Explanation

Given that the area under the curve must equal 1 for the given domain of $f(x)$, set the integral equal to 1 and solve:

$$\begin{aligned}\int_0^a x^3\,\mathrm{d}x &= 1\\ \left[\frac{1}{4}x^4\right]_0^a &= 1\\ \frac{1}{4}a^4 - \frac{1}{4}(0)^4 &= 1\\ a^4 &= 4\\ a &= \sqrt[4]{4}\\ &= 1.41421\ldots\\ &= 1.41 \text{ (to 3 significant figures)}\end{aligned}$$

Question 2

Difficulty:

★☆☆

Given that $f(x) = 3x^2,\ 0 \leqslant x \leqslant 1$ is a valid probability distribution function, find $\mathrm{P}\left(x = \dfrac{1}{2}\right)$. Give the exact answer.

0 ✓

Accepted answers

0

Explanation

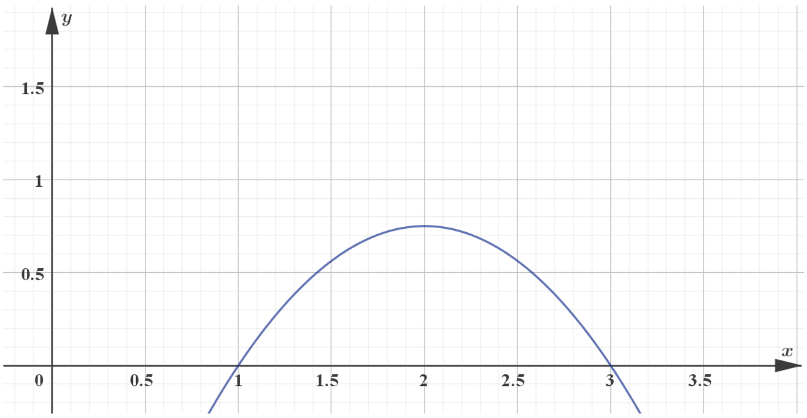
As there are an infinite number of possible outcomes between 0 and 1 for this function, the probability of any specific outcome must be zero.

Question 3

Difficulty:

★★☆

Given that the graphed function shown below is a valid probability density function, find the value of a when the function is expressed in the form of $f(x) = ax^2 + bx + c$.



More information

1 $-\dfrac{3}{4}$ ✓

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

2 $-\frac{1}{8}$

3 $-\frac{4}{5}$

4 $-\frac{2}{3}$

Explanation

With your knowledge of quadratics, you can see from the graph that the equation must be in the form of $f(x) = a(x-1)(x-3)$. As you want this to be a valid probability function, you set the definite integral of this function from $x = 1$ to $x = 3$ equal to 1 and solve:

$$\int_1^3 a(x-1)(x-3)\,\mathrm{d}x = 1$$
$$a\int_1^3 x^2 - 4x + 3\,\mathrm{d}x = 1$$
$$a\left[\frac{1}{3}x^3 - 2x^2 + 3x\right]_1^3 = 1$$
$$a\left(\frac{1}{3}(3)^3 - 2(3)^2 + 3(3) - \left(\frac{1}{3}(1)^3 - 2(1)^2 + 3(1)\right)\right) = 1$$
$$-\frac{4}{3}a = 1$$
$$a = -\frac{3}{4}$$

Question 4

Difficulty:

★★★

Consider the function $f(x) = a\sin^2 x$. Given that $x \geqslant 0$, find the value of a that is needed for the first period of the function to be a valid probability density function.

1 $\frac{2}{\pi}$ ✓

2 π

3 $\frac{\pi}{2}$

4 $\frac{1}{\pi}$

Explanation

You see that the first period of the function ends when $x = \pi$, therefore you set up the following integral:

$$\int_0^\pi a\sin^2 x\,\mathrm{d}x = 1$$
$$a\int_0^\pi \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)\mathrm{d}x = 1$$
$$a\left[\frac{1}{2}x - \frac{1}{4}\sin(2x)\right]_0^\pi = 1$$
$$a\left[\left(\frac{\pi}{2} - \frac{1}{4}\sin(2\pi)\right) - \left(\frac{1}{2}(0) - \frac{1}{4}\sin(0)\right)\right] = 1$$
$$\frac{\pi}{2}a = 1$$
$$a = \frac{2}{\pi}$$

4. Probability and statistics / 4.14 Continuous random variables

Student view

Mode and median of a continuous random variable

Overview
(/study/ap
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hl/sid-
134-
cid-
761926/o

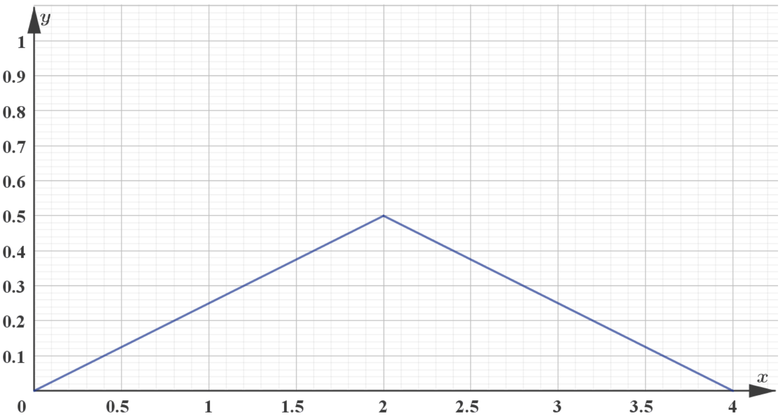
The mode of a continuous random variable

✓ **Important**

The mode of a continuous random variable is defined as the value at which the probability density function (PDF) of the variable is at a maximum.

Consider the continuous random variable from the previous section (/study/app/math-aa-hl/sid-134-cid-761926/book/probability-density-functions-id-27260/):

$$f(x) = -\left|\frac{1}{4}(x-2)\right| + \frac{1}{2},\ 0 \leqslant x \leqslant 4$$



More information

The image is a line graph representing the function (f(x) = -\left| \frac{1}{4}(x - 2) \right| + \frac{1}{2}). The X-axis ranges from 0 to 4, and the Y-axis ranges from 0 to 1. The graph starts at the point (0, 0), reaches a maximum at (2, 0.5), and ends at (4, 0). The function is represented by a V-shaped curve on the graph, with the vertex at the maximum point (2, 0.5). This implies the graph is symmetrical around x = 2, where the maximum value of 0.5 is attained. As x moves further away from 2 on either side, the function value decreases consistently, indicating a decrease in height of the graph.

[Generated by AI]

As you can see from the graph, $f(x)$ is at a maximum when $x = 2$, therefore 2 is the mode of this continuous random variable.

Example 1

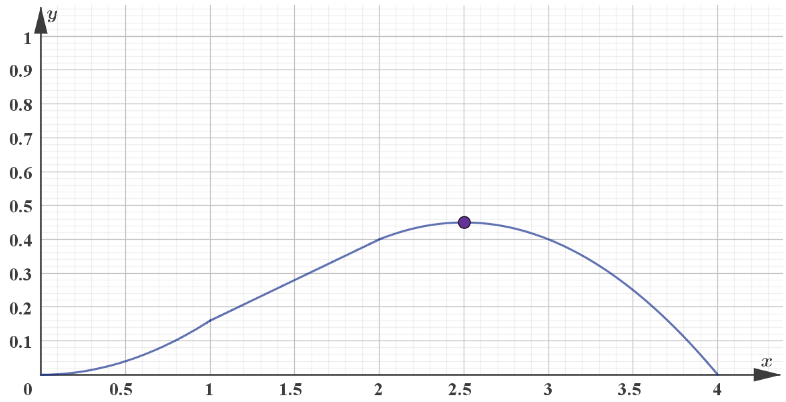
★★☆

Student
view

Find the mode of the continuous random variable whose PDF is defined as:

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

$$f(x) = \begin{cases} \frac{4}{25}x^2 & \text{if } 0 \leqslant x < 1 \\ \frac{6}{25}x - \frac{2}{25} & \text{if } 1 \leqslant x < 2 \\ -\frac{1}{5}x^2 + x - \frac{4}{5} & \text{if } 2 \leqslant x \leqslant 4 \end{cases}$$



As you can see from the graph, the maximum occurs somewhere in the third section of the piecewise function.

Find the derivative of $f(x)$ in that section.

$$f(x) = -\frac{1}{5}x^2 + x - \frac{4}{5}$$
$$f'(x) = -\frac{2}{5}x + 1$$

Set $f'(x) = 0$ to find the maximum.

$$-\frac{2}{5}x + 1 = 0$$
$$-\frac{2}{5}x = -1$$
$$x = 2.5$$
$$\therefore \text{ the mode of } f(x) = 2.5$$

The median of a continuous random variable

✓ **Important**

The median of a continuous random variable is defined as the point where the area under the probability density curve reaches $\frac{1}{2}$ or when $\int_{-\infty}^{m} f(x)\, \mathrm{d}x = \frac{1}{2}$.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Again, consider $f(x) = -\left|\frac{1}{4}(x-2)\right| + \frac{1}{2},\ 0 \leqslant x \leqslant 4$. Looking at its graph you can see that the area of the triangle to the right of when $x = 2$ is equal to the area of the triangle to the left of when $x = 2$. Therefore, 2 is the median of $f(x)$.

Example 2

★★★

Find the median of the continuous random variable in **Example 1** above, whose PDF is defined as

$$f(x) = \begin{cases} \frac{4}{25}x^2 & \text{if } 0 \leqslant x < 1 \\ \frac{6}{25}x - \frac{2}{25} & \text{if } 1 \leqslant x < 2 \\ -\frac{1}{5}x^2 + x - \frac{4}{5} & \text{if } 2 \leqslant x \leqslant 4 \end{cases}$$

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/probability-density-functions-id-27260/print/) Assign

Steps	Explanation
$\text{Area} = \int_0^1 \frac{4}{25}x^2\,dx$ $= \left[\frac{4}{75}x^3\right]_0^1$ $= \left(\frac{4}{75} - 0\right)$ $= \frac{4}{75} \approx 0.0533$	Find the area under the first piece of the function.
$\text{Area} = \frac{4}{75} + \int_1^2 \frac{6}{25}x - \frac{2}{25}\,dx$ $= \frac{4}{75} + \left[\frac{3}{25}x^2 - \frac{2}{25}x\right]_1^2$ $= \frac{4}{75} + \left(\frac{12}{25} - \frac{4}{25} - \left(\frac{3}{25} - \frac{2}{25}\right)\right)$ $= \frac{4}{75} + \frac{7}{25}$ $= \frac{1}{3}$	As $0.0533 < 0.5$, add the area under the second piece.
$0.5 - \frac{1}{3} = \frac{1}{6}$ $\frac{1}{6} = \int_2^m -\frac{1}{5}x^2 + x - \frac{4}{5}\,dx$ $\frac{1}{6} = \left[-\frac{1}{15}x^3 + \frac{1}{2}x^2 - \frac{4}{5}x\right]_2^m$ $\frac{1}{6} = \left(-\frac{1}{15}m^3 + \frac{1}{2}m^2 - \frac{4}{5}m - \left(-\frac{8}{15} + 2 - \frac{8}{5}\right)\right)$ $\frac{1}{6} = -\frac{1}{15}m^3 + \frac{1}{2}m^2 - \frac{4}{5}m + \frac{2}{15}$ $0 = -\frac{1}{15}m^3 + \frac{1}{2}m^2 - \frac{4}{5}m - \frac{1}{30}$ $m = -0.0406,\ 2.39, \text{ and } 5.15$	As $0.333 < 0.5$, you need to use the third piece of $f(x)$ for the remaining area needed.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
$\therefore\ 2.39$ is the median of $f(x)$	Choose the value of m that falls in the domain you need.

Exam tip

Exam questions may ask for the interquartile range of a probability density function. In this case, you would need to find the values of x that relate to where the area under the curve is 0.25 and 0.75.

Making connections

What are the mode and the median of the normal distribution curve?

3 section questions

Question 1

Difficulty: ★☆☆

Calculate the median of the probability density function that is defined as $f(x) = \dfrac{1}{x \ln 9},\ 1 \leqslant x \leqslant 9.$

3

Accepted answers

3

Explanation

Recall that the median occurs when the area under the curve equals 0.5 and set up the following integral:

$$\begin{aligned}\int_1^a \frac{1}{x\ln 9}\mathrm{d}x &= \frac{1}{2}\\ \frac{1}{\ln 9}\int_1^a \frac{1}{x} &= \frac{1}{2}\\ [\ln x]_1^a &= \frac{1}{2}\ln 9\\ \ln a - \ln 1 &= \ln\sqrt{9}\\ \ln a &= \ln 3\\ a &= 3\end{aligned}$$

Question 2

Difficulty: ★☆☆

Find the mode of the probability density function that is defined as

$$f(x) = -\frac{3}{128}x(x-4)(x+2),\ 0 \leqslant x \leqslant 4$$

giving your answer correct to three significant figures.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

2.43

Accepted answers

2.43

Explanation

As the mode is defined as when the PDF is at a maximum, you need to find when the derivative is equal to zero.

$$\begin{aligned} f(x) &= -\frac{3}{128}x(x-4)(x+2) \\ &= -\frac{3}{128}x^3 + \frac{3}{64}x^2 + \frac{3}{16}x \\ f'(x) &= -\frac{9}{128}x^2 + \frac{3}{32}x + \frac{3}{16} \\ 0 &= -\frac{9}{128}x^2 + \frac{3}{32}x + \frac{3}{16} \\ x &= -1.097 \text{ or } 2.43 \qquad \text{(using GDC)} \end{aligned}$$

As the domain of the function is $0 \leqslant x \leqslant 4$, you choose 2.43.

Question 3

Difficulty:

★★☆

Find the interquartile range of the probability function that is defined as $f(x) = \frac{1}{(x+1)^2},\ x \geqslant 0$.

1 $\frac{8}{3}$

2 $\frac{1}{2}$

3 $\frac{9}{4}$

4 $\frac{3}{7}$

Explanation

As the question asks for the interquartile range, you need to find the values of x for when the area equals to $\frac{1}{4}$ and $\frac{3}{4}$:

$$\begin{aligned} \int_0^a \frac{1}{(x+1)^2}\,\mathrm{d}x &= \frac{1}{4} \\ \left[-\frac{1}{x+1}\right]_0^a &= \frac{1}{4} \\ -\frac{1}{a+1} + 1 &= \frac{1}{4} \\ a &= \frac{1}{3} \quad \therefore \quad \mathrm{Q}_1 = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \int_0^a \frac{1}{(x+1)^2}\,\mathrm{d}x &= \frac{3}{4} \\ \left[-\frac{1}{x+1}\right]_0^a &= \frac{3}{4} \\ -\frac{1}{a+1} + 1 &= \frac{3}{4} \\ a &= 3 \quad \therefore \quad \mathrm{Q}_3 = 3 \end{aligned}$$

Therefore the interquartile range is $\mathrm{Q}_3 - \mathrm{Q}_1 = 3 - \frac{1}{3} = \frac{8}{3}$.

Student view

4. Probability and statistics / 4.14 Continuous random variables

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Mean and variance of a continuous random variable

Mean of a continuous random variable

✓ **Important**

As with discrete random variables, the mean of a continuous random variable is the same as the expectation where $\mathrm{E}(X) = \int_a^b x\ f(x)\ \mathrm{d}x$.

Let us again consider the continuous random variable whose PDF is defined as $f(x) = -\left|\frac{1}{4}(x-2)\right| + \frac{1}{2},\ 0 \leqslant x \leqslant 4$. Since you need to find the integral now, let's first express the absolute value function as a piecewise function:

$$f(x) = \begin{cases} \frac{1}{4}x & \text{if } 0 \leqslant x < 2 \\ -\frac{1}{4}x + 1 & \text{if } 2 \leqslant x \leqslant 4 \end{cases}$$

Example 1

★☆☆

Find the value of $\mathrm{E}(X)$ for the continuous random variable whose PDF is defined with $f(x)$ shown above, that is:

$$f(x) = \begin{cases} \frac{1}{4}x & \text{if } 0 \leqslant x < 2 \\ -\frac{1}{4}x + 1 & \text{if } 2 \leqslant x \leqslant 4 \end{cases}$$

Begin with the expectation formula.

$$\mathrm{E}(X) = \int_a^b x\ f(x)\ \mathrm{d}x$$

Replace the upper and lower bounds with the values for the domain of the function, split the integral and integrate.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

$$
\begin{aligned}
\mathrm{E}(X) &= \int_0^4 x\, f(x)\ \mathrm{d}x \\
&= \int_0^2 x\left(\frac{1}{4}x\right)\mathrm{d}x + \int_2^4 x\left(-\frac{1}{4}x+1\right)\mathrm{d}x \\
&= \int_0^2 \frac{1}{4}x^2\mathrm{d}x + \int_2^4 -\frac{1}{4}x^2 + x\mathrm{d}x \\
&= \left[\frac{1}{12}x^3\right]_0^2 + \left[-\frac{1}{12}x^3 + \frac{1}{2}x^2\right]_2^4 \\
&= \left(\frac{1}{12}(8) - \frac{1}{12}(0)\right) + \left(-\frac{1}{12}(4^3) + \frac{1}{2}(4^2) - \left(-\frac{1}{12}(2^3) + \frac{1}{2}(2^2)\right)\right) \\
&= \frac{8}{12} - \frac{64}{12} + 8 + \frac{8}{12} - 2 \\
&= 2
\end{aligned}
$$

You can check this value with your calculator.

Making connections

You have now found that the mean, median, and mode of the continuous random variable whose PDF is defined as $f(x) = -\left|\frac{1}{4}(x-2)\right| + \frac{1}{2}$, $0 \leqslant x \leqslant 4$ all have a value of 2.

What does this tell you about the distribution of the variable's PDF?

How does that compare to the distribution of the normal distribution curve?

Example 2

★☆☆

Find the value of $\mathrm{E}(X)$ for the continuous random variable whose PDF is defined as

$$
f(x) = \begin{cases} \frac{4}{25}x^2 & \text{if } 0 \leqslant x < 1 \\ \frac{6}{25}x - \frac{2}{25} & \text{if } 1 \leqslant x < 2 \\ -\frac{1}{5}x^2 + x - \frac{4}{5} & \text{if } 2 \leqslant x \leqslant 4 \end{cases}
$$

Substitute and integrate.

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/mode-and-median-of-a-continuous-random-variable-id-27261/print/) Assign

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

$$
\begin{aligned}
\mathrm{E}(X) &= \int_0^4 x\ f(x)\ \mathrm{d}x \\
&= \int_0^1 x\left(\frac{4}{25}x^2\right)\mathrm{d}x + \int_1^2 x\left(\frac{6}{25}x - \frac{2}{25}\right)\mathrm{d}x + \int_2^4 x\left(-\frac{1}{5}x^2 + x - \frac{4}{5}\right)\mathrm{d}x \\
&= \int_0^1 \frac{4}{25}x^3\mathrm{d}x + \int_1^2 \frac{6}{25}x^2 - \frac{2}{25}x\mathrm{d}x + \int_2^4 -\frac{1}{5}x^3 + x^2 - \frac{4}{5}x\mathrm{d}x \\
&= \left[\frac{4}{100}x^4\right]_0^1 + \left[\frac{6}{75}x^3 - \frac{2}{50}x^2\right]_1^2 + \left[-\frac{1}{20}x^4 + \frac{1}{3}x^3 - \frac{4}{10}x^2\right]_2^4 \\
&= \left(\frac{4}{100}\right) + \left(\frac{48}{75} - \frac{8}{50} - \left(\frac{6}{75} - \frac{2}{50}\right)\right) + \left(-\frac{256}{20} + \frac{64}{3} - \frac{64}{10} - \left(-\frac{16}{20} + \frac{8}{3} - \frac{16}{10}\right)\right) \\
&= \frac{4}{100} + \frac{11}{25} + \frac{28}{15} \\
&= \frac{176}{75} \\
&\approx 2.347\ \therefore\ \text{the expectation of the continuous random variable is } 2.347
\end{aligned}
$$

This is a very time consuming calculation. On an exam when calculator use is allowed, you can find the same answer by using the calculator to evaluate the integrals.

Variance of a continuous random variable

✓ **Important**

Recall how the variance of a discrete random variable was defined using the formula

$$\mathrm{Var}(X) = \sum_{i=1}^{k}(x_i - \mu)^2\mathrm{P}(X = x_i)$$

Similarly, the variance of a continuous random variable can be defined using the formula

$$\mathrm{Var}(X) = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,\mathrm{d}x$$

To find the variance of continuous random variables you need to use calculus. You can find the definite integrals with the use of a graphic display calculator (see section 5.5.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/definite-integrals-id-25566/)).

Example 3

★★☆

Calculate the variance and standard deviation of the continuous random variable in **Example 1**, whose PDF is defined as

$$f(x) = \begin{cases} \frac{1}{4}x & \text{if } 0 \leqslant x < 2 \\ -\frac{1}{4}x + 1 & \text{if } 2 \leqslant x \leqslant 4 \end{cases}$$

Recall the expectation (mean) for this PDF.

$$\mu = \mathrm{E}(X) = 2$$

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Substitute into the variance formula and integrate.

$$\begin{aligned}\mathrm{Var}\left(X\right) &= \int_0^2 (x-2)^2\left(\frac{1}{4}x\right)\mathrm{d}x + \int_2^4 (x-2)^2\left(-\frac{1}{4}x+1\right)\mathrm{d}x\\ &= \int_0^2 \left(\frac{1}{4}x^3 - x^2 + x\right)\mathrm{d}x + \int_2^4\left(-\frac{1}{4}x^3+2x^2-5x+4\right)\mathrm{d}x\\ &= \left[\frac{1}{16}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_0^2 + \left[-\frac{1}{16}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 + 4x\right]_2^4\\ &= \left(1-\frac{8}{3}+2\right) + \left(-16+\frac{128}{3}-40+16-\left(-1+\frac{16}{3}-10+8\right)\right)\\ &= \frac{2}{3}\end{aligned}$$

Take the square root to find the standard deviation.

$$\begin{aligned}\sigma &= \sqrt{\mathrm{Var}\left(X\right)}\\ &= \sqrt{\frac{2}{3}}\\ &\approx 0.816\end{aligned}$$

Example 4

★★☆

Calculate the variance and standard deviation of the continuous random variable whose PDF is defined as

$$f\left(x\right) = \begin{cases} \frac{4}{25}x^2 & \text{if } 0 \leqslant x < 1\\ \frac{6}{25}x - \frac{2}{25} & \text{if } 1 \leqslant x < 2\\ -\frac{1}{5}x^2 + x - \frac{4}{5} & \text{if } 2 \leqslant x \leqslant 4\end{cases}$$

Recall the mean for this PDF

$$\mu = \mathrm{E}\left(X\right) = \frac{176}{75}$$

Substitute into the variance formula and integrate.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

$$\begin{aligned}\mathrm{Var}(X) &= \int_0^1 x^2\left(\frac{4}{25}x^2\right)\mathrm{d}x + \int_1^2 x^2\left(\frac{6}{25}x - \frac{2}{25}\right)\mathrm{d}x \\ &\quad + \int_2^4 x^2\left(-\frac{1}{5}x^2 + x - \frac{4}{5}\right)\mathrm{d}x - \left(\frac{176}{75}\right)^2 \\ &= \int_0^1 \frac{4}{25}x^4\mathrm{d}x + \int_1^2 \frac{6}{25}x^3 - \frac{2}{25}x^2\mathrm{d}x + \int_2^4 -\frac{1}{5}x^4 + x^3 - \frac{4}{5}x^2\mathrm{d}x - \left(\frac{176}{75}\right)^2 \\ &= \left[\frac{4}{125}x^5\right]_0^1 + \left[\frac{6}{100}x^4 - \frac{2}{75}x^3\right]_1^2 + \left[-\frac{1}{25}x^5 + \frac{1}{4}x^4 - \frac{4}{15}x^3\right]_2^4 - \left(\frac{176}{75}\right)^2 \\ &= \frac{4}{125} + \left(\frac{96}{100} - \frac{16}{75} - \left(\frac{6}{100} - \frac{2}{75}\right)\right) \\ &\quad + \left(-\frac{1024}{25} + 64 - \frac{256}{15} - \left(-\frac{32}{25} + 4 - \frac{32}{15}\right)\right) - \frac{30\,976}{5625} \\ &= \frac{4}{125} + \frac{107}{150} + \frac{808}{150} - \frac{30\,976}{5625} \\ &\approx 0.625\end{aligned}$$

Take the square root to find the standard deviation.

$$\begin{aligned}\sigma &= \sqrt{\mathrm{Var}(X)} \\ &= \sqrt{0.62515555\ldots} \\ &\approx 0.791\end{aligned}$$

Note that because of the large numbers involved, on an exam it is unreasonable to expect that you carry out this calculation without the help of a calculator. It is a good practice to work out the variance using antiderivatives, but on an exam use your calculator to find the definite integrals when it is allowed.

2 section questions ^

Question 1

Difficulty:

★☆☆

Find the expectation and variance of the random variable X whose probability density function is:

$$f(x) = \begin{cases} \frac{1}{3}2^x \ln 2 & 0 \leqslant x \leqslant 2 \\ 0 & \text{otherwise}\end{cases}$$

Note: you can use a graphic display calculator for this calculation.

1 $\mathrm{E}(X) = 1.22,\ \mathrm{Var}(X) = 0.304$ ✓

2 $\mathrm{E}(X) = 1.35,\ \mathrm{Var}(X) = 0.304$

3 $\mathrm{E}(X) = 1.22,\ \mathrm{Var}(X) = 0.297$

4 $\mathrm{E}(X) = 1.35,\ \mathrm{Var}(X) = 0.297$

Explanation

Student view

Overview
(/study/ap aa-hl/sid-134-cid-761926/o

$$\begin{aligned}\mathrm{E}(X) &= \int_a^b x\, f(x)\,\mathrm{d}x \\ &= \int_0^2 x\left(\frac{1}{3}2^x \ln 2\right)\mathrm{d}x = 1.22397\ldots \\ &\approx 1.22 \quad \text{(using GDC)}\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(X) &= \int_a^b x^2\, f(x)\,\mathrm{d}x - \mu^2 \\ &= \int_0^2 x^2\left(\frac{1}{3}2^x \ln 2\right)\mathrm{d}x - (1.22397\ldots)^2 = 0.30359\ldots \\ &\approx 0.304 \quad \text{(using GDC)}\end{aligned}$$

Question 2

Difficulty:

★★★

Find the variance of the random variable X whose PDF is defined,

$$f(x) = \frac{12x^2 - 4x^3}{27},\ 0 \leqslant x \leqslant 3,$$ giving the exact answer.

0.36 ✓

Accepted answers

0.36, .36, 9/25, 0,36

Explanation

To begin, you need to find the expectation:

$$\begin{aligned}\mathrm{E}(X) &= \int_a^b x\, f(x)\,\mathrm{d}x \\ &= \int_0^3 x\,\frac{12x^2 - 4x^3}{27}\,\mathrm{d}x \\ &= \frac{9}{5} \quad \text{(using GDC)}\end{aligned}$$

Then you use the expectation in the variance formula:

$$\begin{aligned}\mathrm{Var}(X) &= \int_a^b x^2 f(x)\,\mathrm{d}x - \mu^2 \\ &= \int_0^3 x^2\frac{12x^2 - 4x^3}{27}\,\mathrm{d}x - \mu^2 \\ &= 3.6 - 3.24 \ \text{(using GDC)} \\ &= 0.36\end{aligned}$$

4. Probability and statistics / 4.14 Continuous random variables

The effect of linear transformations of X

Example 1

★☆☆

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

For a class project, Alberto creates a game where players can receive one of $x = 5,\ 10,\ 15$, or 20 points in each round. The probability distribution for receiving each point value is shown below:

x	5	10	15	20
$\mathrm{P}(X = x)$	$\frac{6}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

Calculate the expectation and variance for one round of Alberto's game.

Use the expectation formula.

$$\begin{aligned} \mathrm{E}(X) &= \sum_{i=1}^{k} x_i\, \mathrm{P}(X = x_i) \\ &= (5)\left(\frac{6}{15}\right) + (10)\left(\frac{4}{15}\right) + (15)\left(\frac{3}{15}\right) + (20)\left(\frac{2}{15}\right) \\ &= \frac{30}{15} + \frac{40}{15} + \frac{45}{15} + \frac{40}{15} \\ &= \frac{31}{3} = 10.\bar{3} \end{aligned}$$

Use a variance formula.

$$\begin{aligned} \mathrm{Var}(X) &= \sum_{i=1}^{k} \left({x_i}^2\, \mathrm{P}(X = x)\right) - \mu^2 \\ &= \left(\left(5^2\right)\left(\frac{6}{15}\right) + \left(10^2\right)\left(\frac{4}{15}\right) + \left(15^2\right)\left(\frac{3}{15}\right) + \left(20^2\right)\left(\frac{2}{15}\right)\right) - \left(\frac{31}{3}\right)^2 \\ &= \left(\frac{150}{15} + \frac{400}{15} + \frac{675}{15} + \frac{800}{15}\right) - \left(\frac{961}{9}\right) \\ &= \frac{254}{9} = 28.\bar{2} \end{aligned}$$

Example 2

★☆☆

Alberto decides that the expected score of 10.3 is too low and he adds 5 points to each score to create the new probability distribution shown below:

x	10	15	20	25
$\mathrm{P}(X = x)$	$\frac{6}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

Calculate the expectation and variance of Alberto's new scoring system.

Using the expectation formula.

Student view

$$\begin{aligned}
\mathrm{E}(X) &= \sum_{i=1}^{k} x_i \, \mathrm{P}(X = x_i) \\
&= (10)\left(\frac{6}{15}\right) + (15)\left(\frac{4}{15}\right) + (20)\left(\frac{3}{15}\right) + (25)\left(\frac{2}{15}\right) \\
&= \frac{60}{15} + \frac{60}{15} + \frac{60}{15} + \frac{50}{15} \\
&= \frac{46}{3} = 15.\bar{3}
\end{aligned}$$

Using a variance formula.

$$\begin{aligned}
\mathrm{Var}(X) &= \sum_{i=1}^{k} \left({x_i}^2 \, \mathrm{P}(X = x)\right) - \mu^2 \\
&= \left((10^2)\left(\frac{6}{15}\right) + (15^2)\left(\frac{4}{15}\right) + (20^2)\left(\frac{3}{15}\right) + (25^2)\left(\frac{2}{15}\right)\right) - \left(\frac{46}{3}\right)^2 \\
&= \left(\frac{600}{15} + \frac{900}{15} + \frac{1200}{15} + \frac{1250}{15}\right) - \left(\frac{2116}{9}\right) \\
&= \frac{254}{9} = 28.\bar{2}
\end{aligned}$$

Making connections

What effect did the addition of 5 points have on the expectation and variance of the scoring system?

Example 3

★☆☆

Alberto decides to try something different and this time doubles the original scores to create the new probability distribution shown below.

x	10	20	30	40
$\mathrm{P}(X = x)$	$\frac{6}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

Calculate the expectation and variance of this third scoring system.

Using the expectation formula.

$$\begin{aligned}
\mathrm{E}(X) &= \sum_{i=1}^{k} x_i \, \mathrm{P}(X = x_i) \\
&= (10)\left(\frac{6}{15}\right) + (20)\left(\frac{4}{15}\right) + (30)\left(\frac{3}{15}\right) + (40)\left(\frac{2}{15}\right) \\
&= \frac{60}{15} + \frac{80}{15} + \frac{90}{15} + \frac{80}{15} \\
&= \frac{310}{15} = 20.\bar{6}
\end{aligned}$$

Using a variance formula.

$$
\begin{aligned}
\mathrm{Var}(X) &= \sum_{i=1}^{k} \left(x_i{}^2\, \mathrm{P}(X=x)\right) - \mu^2 \\
&= \left((10^2)\left(\frac{6}{15}\right) + (20^2)\left(\frac{4}{15}\right) + (30^2)\left(\frac{3}{15}\right) + (40^2)\left(\frac{2}{15}\right)\right) - \left(\frac{310}{15}\right)^2 \\
&= \left(\frac{600}{15} + \frac{1600}{15} + \frac{2700}{15} + \frac{3200}{15}\right) - \left(\frac{96100}{225}\right) \\
&= \frac{1016}{9} = 112.\bar{8}
\end{aligned}
$$

Here you can see that the doubling of the points also doubled the expected value from the original scoring system in **Example 1**. But what about the variance?

Important

The effects of linear transformations on the random variable X can be summarised:

$$\mathrm{E}(aX+b) = a\mathrm{E}(X) + b \qquad \mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$$

Note that, as in **Example 2** above, the transformation of adding b does not affect the variance of X .

1 section question

Question 1

Difficulty:

★☆☆

Given that $\mathrm{E}(X) = \frac{23}{7}$ and $\mathrm{Var}(X) = \frac{1}{12}$, find $\mathrm{E}(4X-2)$ and $\mathrm{Var}(4X-2)$.

1 $\mathrm{E}(X) = \frac{78}{7},\ \mathrm{Var}(X) = \frac{4}{3}$ ✓

2 $\mathrm{E}(X) = \frac{78}{7},\ \mathrm{Var}(X) = \frac{2}{3}$

3 $\mathrm{E}(X) = \frac{92}{7},\ \mathrm{Var}(X) = \frac{4}{3}$

4 $\mathrm{E}(X) = \frac{92}{7},\ \mathrm{Var}(X) = \frac{2}{3}$

Explanation

Since $\mathrm{E}(X) = \frac{23}{7}$ then $\mathrm{E}(4X-2) = 4\left(\frac{23}{7}\right) - 2 = \frac{78}{7}$

Since $\mathrm{Var}(X) = \frac{1}{12}$ then $\mathrm{Var}(4X-2) = 4^2\left(\frac{1}{12}\right) = \frac{4}{3}$

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27264/print/) Assign

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

What you should know

By the end of this subtopic you should be able to:

- state that discrete random variables are usually obtained from a counting experiment and that continuous random variables arise in experiments that involve measurement
- state that the expected value of a random variable X with a probability distribution $\mathrm{P}\left(X=x\right)$ is written as $\mathrm{E}\left(X\right)$ or μ
- calculate the mean, standard deviation and variance of both discrete and continuous random variables
- prove that $f\left(x\right)$ is a valid probability density function by showing that, within its given domain, $f\left(x\right)\geqslant 0$ and the area under its curve is equal to 1
- calculate the mode of a continuous random variable by finding the value of x for which the probability density function is at a maximum
- calculate the median of a continuous random variable by finding the value of m for which $\int_a^m f\left(x\right)\mathrm{d}x=\frac{1}{2}$ where a is the lower limit of the domain
- calculate the effects of the linear transformations of the random variable X on its expectation and variance.

4. Probability and statistics / 4.14 Continuous random variables

Investigation

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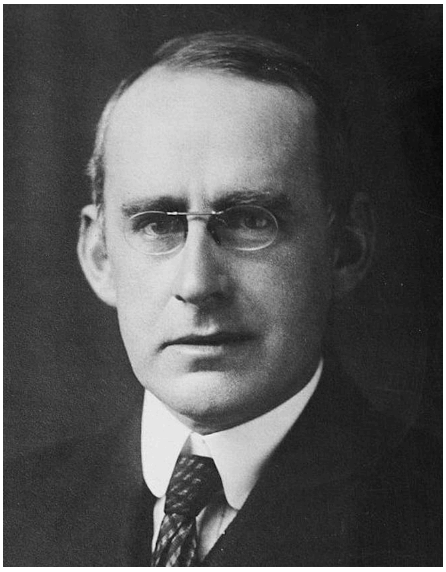

Arthur Eddington

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Ronald Fisher

During the early 1900's, the astronomer Arthur Eddington and the statistician Ronald Fisher debated the merits of both the mean absolute deviation and the standard deviation. A summary of their discussions can be found here (https://web.archive.org/web/20210606100434/https://www.leeds.ac.uk/educol/documents/00003759.htm).

By exploring the concepts of 'ideal circumstances' and 'repeated simulations', explore the benefits of each of these measures of dispersion. How can anyone decide on which measure more accurate reflects their own collected data?

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