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1.2 Teacher view
Arithmetic sequences and series

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Patterns are everywhere in the world around you – in the distribution of petals on a flower, the position of cracks in dried mud, or the wind-shaped crests of sand on the beach. Patterns are powerful because they allow for prediction and are visually beautiful.

A sequence is a list of numbers. It may be as simple as a list of positive integers 1, 2, 3, 4, 5, ... or it may follow a more complex pattern such as 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The second sequence is a well-known sequence called the Fibonacci sequence, which is related to the Golden Ratio and models the patterns of the spirals in the nautilus shells.



**Nautilus shell spiral**

Credit: FlamingPumpkin Getty Images

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In this video, a mathematician uses toothpicks to illustrate the beauty and predictive power for some sequences. Note how sequences are both aesthetically pleasing and mathematically fascinating.

### Terrific Toothpick Patterns - Numberphile



A simpler version of the toothpick patterns is demonstrated in the picture below, which is an example of an arithmetic sequence. You will study arithmetic sequences in this subtopic.



### Concept

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In this subtopic, you will study how patterns are used to make rules that define a sequence and can be used to make predictions about the sequence. Using a pattern to make predictions requires a high degree of certainty that this pattern continues. As



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you work through this section, for each scenario given, consider how certain you are that the pattern continues.

# Sequences and their notation

Section

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## Identifying a sequence

Consider this example of a sequence. What do you notice? Can you predict the next number in this list?

2, 4, 6, 8, ...

Thus, a sequence is an ordered list of terms. A term is one entry in the list (often a number) that is separated from other entries by a comma.

The terms of a sequence are usually connected to each other by a pattern. For instance, in 1, 3, 9, 27, ... each term is multiplied by three to get the next one.

Being able to predict terms in a sequence is a useful skill that we will work on throughout this topic.

### Example 1



Determine the next term in each of these sequences.

a) -5, -2, 1, 4, ...

b)  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

c) 1, 1, 2, 3, 5, ...

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- a) In this sequence you add 3 to the numbers to get to the next number. The next term is  $4 + 3 = 7$ .
- b) In this sequence you divide the numbers by 2 or multiply by  $\frac{1}{2}$  to get to the next number. The next term is  $\frac{1}{16} \times \frac{1}{2} = \frac{1}{32}$ .
- c) The pattern is more complicated in this sequence. To get the next term you add together the two previous terms:

$$1 + 1 = 2, \quad 1 + 2 = 3, \quad 2 + 3 = 5.$$

The next term is  $3 + 5 = 8$ .

This is a very famous sequence called the Fibonacci sequence. The numbers of this sequence appear in many areas of maths and the natural world.

## Sequence notation

Consider the sequence  $-2, 0, 2, 4, \dots$

How would you describe the rule for this sequence?

Describing sequences in symbols is much easier than using words. First, we need to define some symbols:

✓ **Important**

A sequence can be written as

$$u_1, u_2, u_3, \dots, u_{n-1}, u_n, u_{n+1}, \dots$$

The subscript tells you the number of the term.

So  $u_3$  is the third term,  $u_n$  is the nth term where  $n$  is a positive integer,  $u_{n-1}$  is the term before  $u_n$ , and  $u_{n+1}$  is the term after  $u_n$ .

## Example 2

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Given the sequence 10, 20, 40, 80, ..., write down the value of  $u_3$  and describe this sequence using an algebraic expression for  $u_n$  in terms of  $u_{n-1}$ .

Steps	Explanation
$u_3 = 40$	$u_3$ is the third term in the sequence.
$u_1 = 10$ and $u_n = 2 \times u_{n-1}$ for $n > 1$	To get the next term ( $u_n$ ), multiply the previous term( $u_{n-1}$ ) by 2.

## Defining a sequence

Generally, there are two ways to define a sequence:

Recursive rule : defines each new term in the sequence based on the previous terms. This rule is also sometimes called term-to-term. For example:

$$u_n = u_{n-1} + 3.$$

Deductive rule : defines each term based on its position in the sequence. This rule is often referred to as the general term , the  $n$  th term, or position-to-term. For example:

$$u_n = 3n - 10.$$

## Example 3



Given  $u_n = 5 \times u_{n-1} + 2$  and  $u_1 = 2$ , find  $u_2$ ,  $u_3$ ,  $u_4$  and  $u_7$  and state the rule used to define this sequence.



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Steps	Explanation
$u_2 = 12$	$u_2 = 5u_1 + 2 = 5(2) + 2 = 12$
$u_3 = 62$	$u_3 = 5u_2 + 2 = 5(12) + 2 = 62$
$u_4 = 312$	$u_4 = 5u_3 + 2 = 5(62) + 2 = 312$
$u_7 = 39062$	<p>Since this is a recursive rule, you will need to find all the terms up to <math>u_7</math> to find <math>u_7</math>. <math>u_5 = 5u_4 + 2 = 5(312) + 2 = 1562</math></p> $u_6 = 5u_5 + 2 = 5(1562) + 2 = 7812$ $u_7 = 5u_6 + 2 = 5(7812) + 2 = 39062.$
This is a recursive rule.	You can tell that this is a recursive rule by noting that $u_{n-1}$ is in the definition for $u_n$ .

## Example 4



Given  $u_n = n^2 + 2n$ , find  $u_2$ ,  $u_3$ ,  $u_4$  and  $u_7$  and state the rule used to define this sequence.



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Steps	Explanation
$u_2 = 8$	$u_2 = 2^2 + 2(2) = 8$
$u_3 = 15$	$u_3 = 3^2 + 2(3) = 15$
$u_4 = 24$	$u_4 = 4^2 + 2(4) = 24$
$u_7 = 63$	Since this is a deductive rule, you can find $u_7$ directly by substituting $n = 7$ into the rule. $u_7 = 7^2 + 2(7) = 63$
This is a deductive rule.	You can tell that this is a deductive rule as you only need the value of $n$ to find $u_n$ .

## Theory of Knowledge

As seen in the magnificent video by ViHart below, the Fibonacci sequence is frequently occurring in nature. Observations like this have led to the knowledge question, ‘Does mathematics exist independent of human construction?’

Nature by Numbers | The Golden Ratio and Fibonacci Num...



In short, is mathematics ‘out there’ serving as the foundational fabric holding together our very elegant universe? Or alternatively, is mathematics created by humans in the same way that we have created language?

These two competing views are referred to as, Platonism — the belief that mathematics exists independent of humanity and Formalism — the belief that mathematics is created by humanity.



Discuss with a partner arguments in favour of both Formalism and Platonism.



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## 3 section questions ▾

1. Number and algebra / 1.2 Arithmetic sequences and series

# Arithmetic sequences

### Section

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### Activity

1.  $2, 4, 6, 8, \dots$
2.  $\frac{-1}{5}, 0, \frac{1}{5}, \frac{2}{5}, \dots$
3.  $x, 5x, 9x, 13x, \dots$
4.  $1 + \sqrt{2}, 1 + 2\sqrt{2}, 1 + 3\sqrt{2}, 1 + 4\sqrt{2}, \dots$

The above sequences are all arithmetic.

Compare them and describe any similarities that are shared by all of them.

An arithmetic sequence is a sequence where you add or subtract the same value from a previous term to get the next term. Recursively this means that  $u_n = u_{n-1} + d$ , where  $d$  is a constant called the common difference.

A common difference is found by using  $d = u_n - u_{n-1}$ .

### Exam tip

In the exam you may be asked to show that a given sequence is arithmetic. To do this you should find the difference between each pair of consecutive terms and show that this difference is constant.



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## Example 1



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Show that  $-7, -9, -11, -13, \dots$  is an arithmetic sequence.

Steps	Explanation
$-13 - (-11) = -2$ $-11 - (-9) = -2$ $-9 - (-7) = -2$	Find $d = u_4 - u_3$ , and $d = u_3 - u_2$ , and so on, for all of the terms that you were given.
The difference between all consecutive terms is constant. Therefore, $d = -2$ and the sequence is arithmetic.	When you are asked to show that a statement is true you should explain how your findings support the conclusion that the statement is true.

## Example 2



a) Given that  $u_1, u_2, u_3, u_4, \dots$  is an arithmetic sequence, write down an expression for  $u_2, u_3$ , and  $u_4$  in terms of  $u_1$  and  $d$ .

b) Hence, deduce an expression for the  $n$  th term of an arithmetic sequence.

	Steps	Explanation
a)	$u_2 = u_1 + d$	Using the recursive rule.
	$u_3 = u_2 + d$	Using the recursive rule.
	$= (u_1 + d) + d = u_1 + 2d$ $u_4 = u_3 + d = (u_1 + 2d) + d = u_1 + 3d$	Substituting $u_2 = u_1 + d$ from the previous result.



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	Steps	Explanation
b)	$n$	$u_n$
	2	$u_1 + d$
	3	$u_1 + \frac{2d}{d} u_2, u_3, \text{ and } u_4$
	4	$u_1 + 3d$ $u_n$ $n$
	$u_n = u_1 + (n - 1) d$	You can observe this based on the pattern revealed by the table.

### ① Exam tip

In an exam question the command word ‘hence’ means that you need to use the previous result in this part of the question. Getting the correct answer using a different method will not get you any credit on the exam.

### ✓ Important

The  $n$  th term of an arithmetic sequence is given by  $u_n = u_1 + (n - 1) d$  where  $n$  is a positive integer.

You can define  $n$  as a positive integer using the notation  $n \in \mathbb{Z}^+$ . You will often see this in test questions.

The fact that  $u_n = u_1 + (n - 1) d$  is defined only for  $n \in \mathbb{Z}^+$  ( $n$  is a positive integer), means  $n = 1, 2, 3, \dots$  which is consistent with the fact that  $n$  represents the number of the term in a sequence. It does not make sense to talk about the zeroth term or the 2.5 th term.

### ① Exam tip

The formula booklet gives the formula for the  $n$  th term of an arithmetic sequence:

$$u_n = u_1 + (n - 1) d$$

## Example 3

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- 761926/o Nudrat is training for a marathon by running 1 kilometre further each week than she did the previous week. Determine how far Nudrat will run in the 20th week of training if she runs 2 km in the first week.



Steps	Explanation
$2, 3, 4, 5, \dots$ $u_1 = 2$ and $d = 1$	You should recognise that this training schedule follows an arithmetic sequence.
$u_n = 2 + (n - 1)(1) = 2 + n - 1 = 1 + n$ $u_{20} = 1 + 20 = 21$ Nudrat will run 21 km in the 20th week.	For the 20th week $n = 20$ . The general rule will help you to find the 20th term.

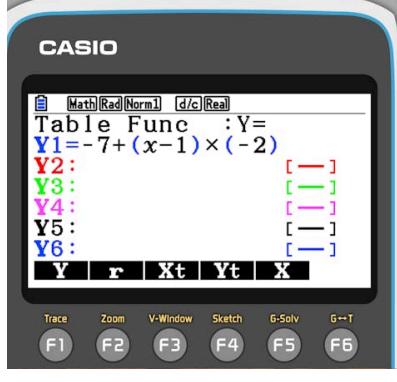
While using the  $n$  th term equation allows you to quickly obtain the value of any term in an arithmetic sequence, you should also be aware that you can use your calculator to generate as many terms of an arithmetic sequence as you need. The illustrations below show how you can do this on a GDC.

**Using the deductive rule**  $u_n = -7 + (n - 1) \times (-2)$ .



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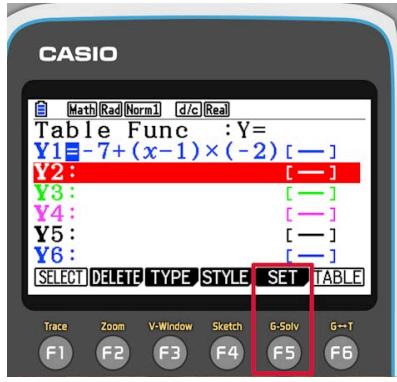
Steps	Explanation
From the main menu, choose the 'table' option.	 
Use the variable button (so use $x$ instead of $n$ as the unknown) to enter the deductive rule for the sequence.	 



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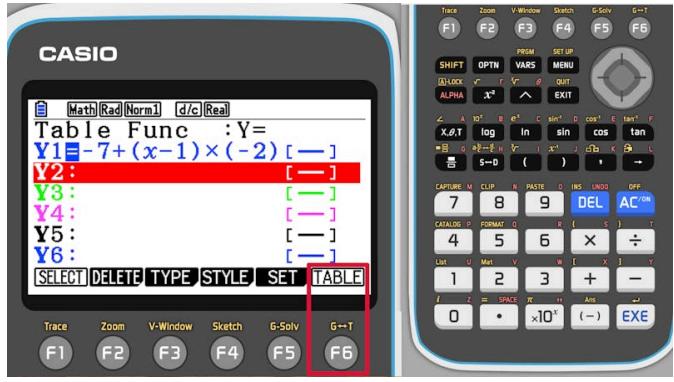
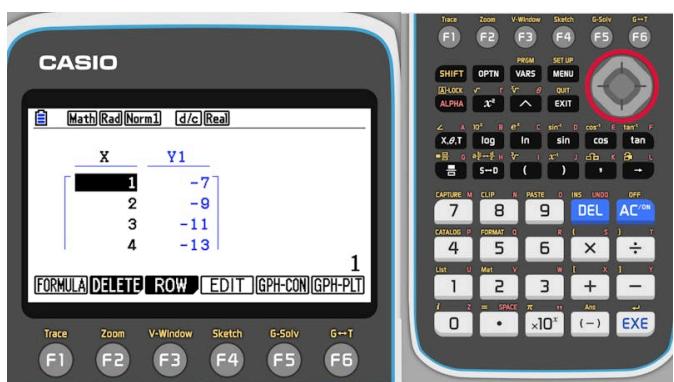
Steps	Explanation
You need to set some parameters to view the sequence.	 
You can specify the index of the first and last term. If you want to see every term, choose 1 for the step size.	 



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Steps	Explanation
To view the terms of the sequence, select 'table'.	
You can move up and down to see the terms of the sequence.	

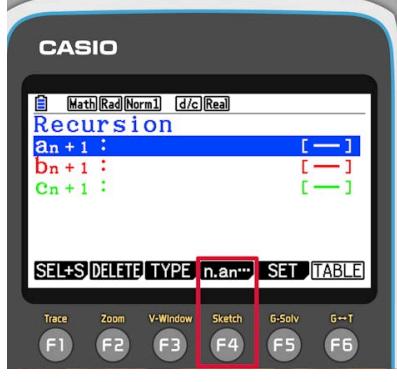
Using the recursive rule  $u_n = u_{n-1} - 2$ .



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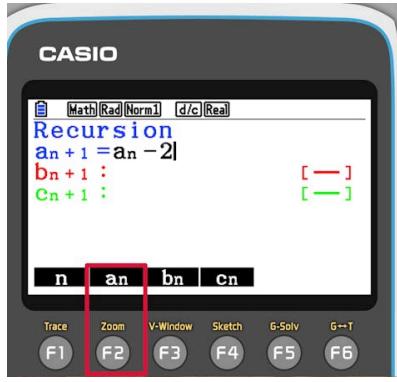
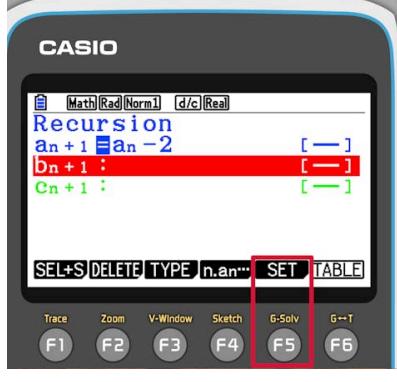
Steps	Explanation
To generate elements of a sequence when you only know the recursive rule, choose the 'recursion' option from the main menu.	 
Select the option that brings up the editor screen.	 



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Steps	Explanation
To enter a recursive rule for $a_{n+1}$ , you will need to use $a_n$ , the previous term of the sequence.	 
When you are done editing the rule, you need to set which terms of the sequence you would like to see.	 



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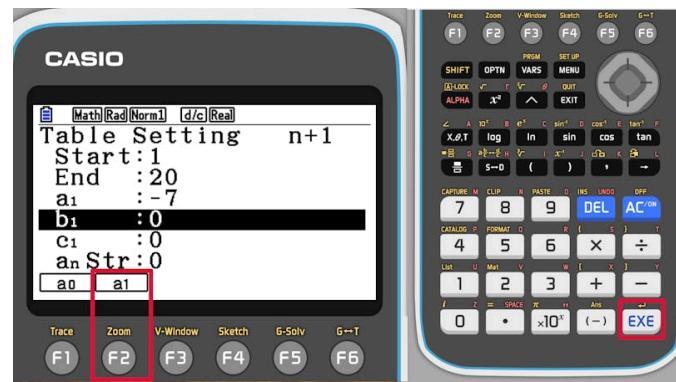


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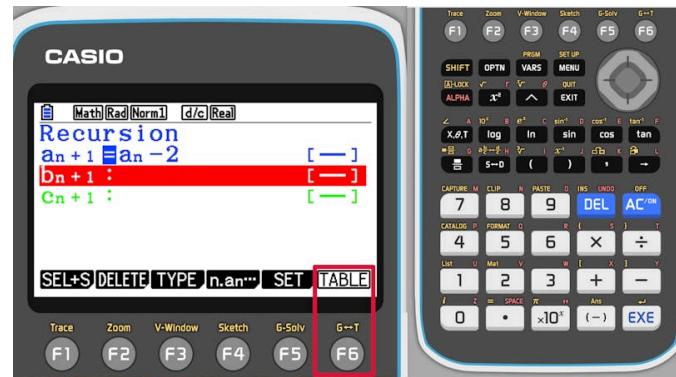
## Steps

In the settings menu there is an option to choose which is the first term of the sequence. Sequences in IB questions usually start with  $a_1$ .

## Explanation



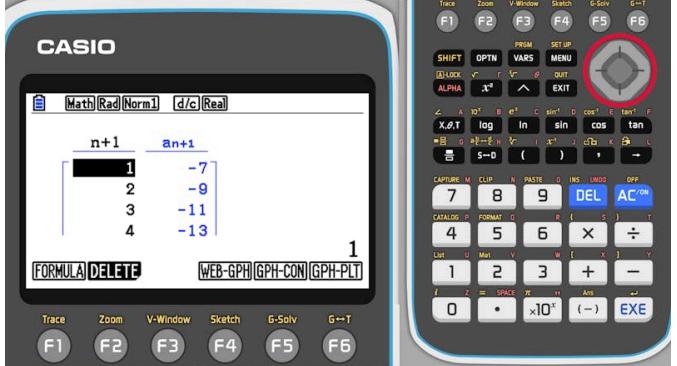
Once you are done editing the rule and the settings, you can choose the 'table' option to view the sequence.



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Steps	Explanation
<p>You can move up and down to see the terms of the sequence.</p>	

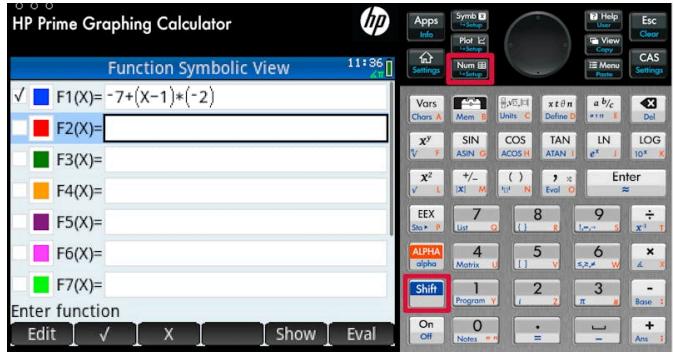
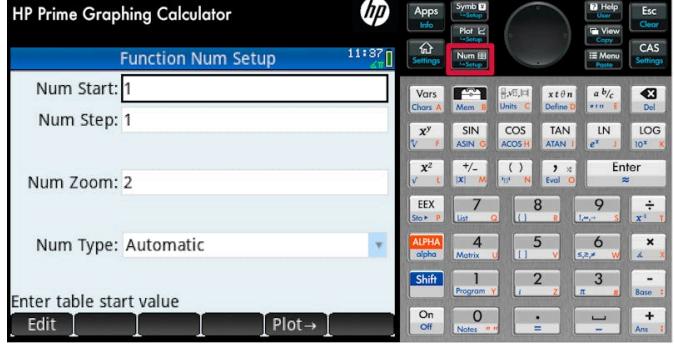
**Using the deductive rule**  $u_n = -7 + (n - 1) \times (-2)$ .

Steps	Explanation
<p>From the application library, choose the function application.</p>	

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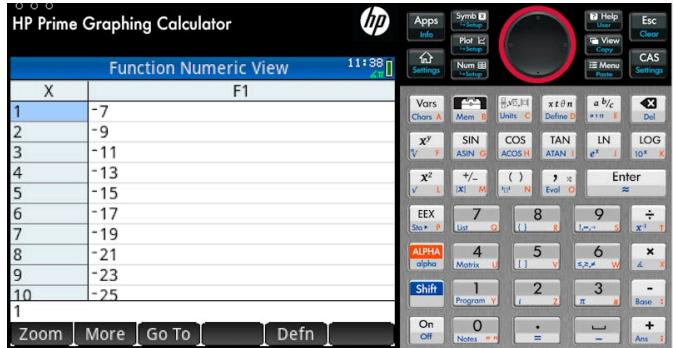
Steps	Explanation
<p>Use the variable button (so use <math>x</math> instead of <math>n</math> as the unknown) to enter the deductive rule for the sequence. Once you entered the rule, choose the setup of the numeric view.</p>	 <p>The screenshot shows the HP Prime Graphing Calculator in Function Symbolic View. The screen displays the equation <math>F1(X) = -7 + (X-1)(-2)</math>. Below the equations, there are fields for <math>F2(X) =</math>, <math>F3(X) =</math>, <math>F4(X) =</math>, <math>F5(X) =</math>, <math>F6(X) =</math>, and <math>F7(X) =</math>. At the bottom, there are buttons for Edit, <math>\sqrt{ }</math>, <math>X</math>, Show, and Eval. A red box highlights the 'Num' button on the top right of the calculator's interface.</p>
<p>You can specify the index of the first term. If you want to see every term, choose 1 for the step size. Once done, choose the numeric view to see the sequence.</p>	 <p>The screenshot shows the HP Prime Graphing Calculator in Function Num Setup mode. It has fields for Num Start (set to 1), Num Step (set to 1), Num Zoom (set to 2), and Num Type (set to Automatic). At the bottom, there is a button for Enter table start value and a Plot button. A red box highlights the 'Num' button on the top right of the calculator's interface.</p>



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Steps	Explanation
You can move up and down to see the terms of the sequence.	

Using the recursive rule  $u_n = u_{n-1} - 2$ .

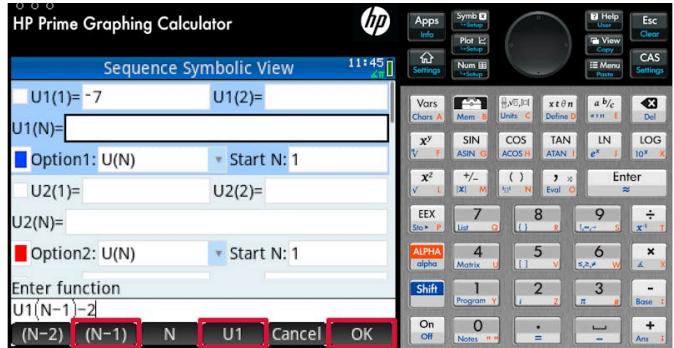
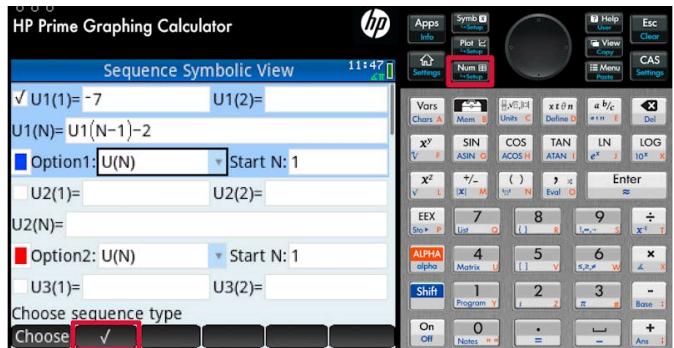
Steps	Explanation
To generate elements of a sequence when you only know the recursive rule, choose to work with sequences option from the application library.	



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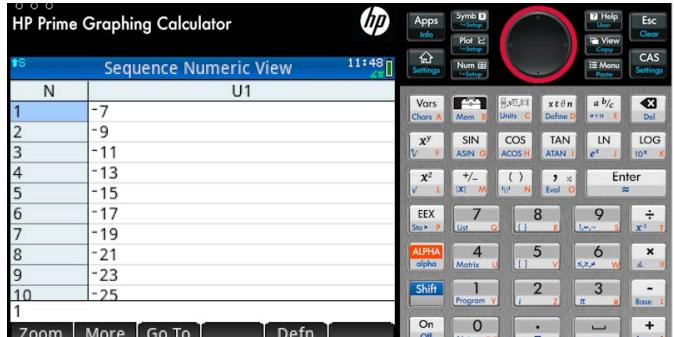
Steps	Explanation
<p>Specify the first term of the sequence and enter the recursive rule. Use the shortcuts available in the bottom row of the screen, and don't forget to press OK when you are done.</p>	
<p>Once you are done editing (don't forget about activating the sequence with the tick mark), you can enter the numeric view to see the terms.</p>	



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Steps	Explanation
<p>You can move up and down to see the terms of the sequence.</p>	

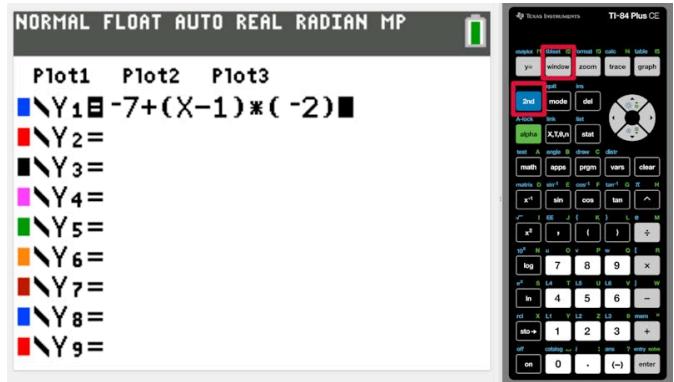
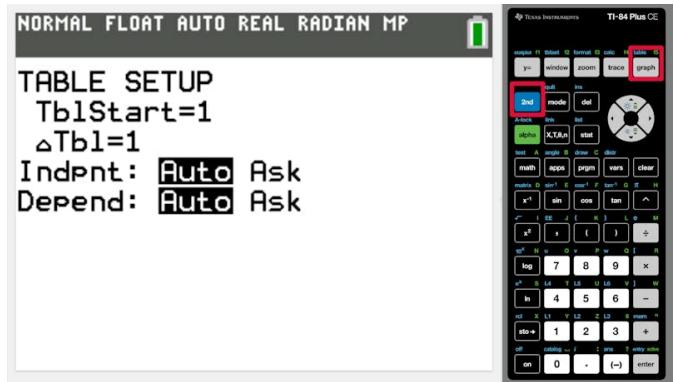
Using the deductive rule  $u_n = -7 + (n - 1) \times (-2)$ .

Steps	Explanation
<p>From the main screen choose the option that lets you enter the general term of the sequence.</p>	

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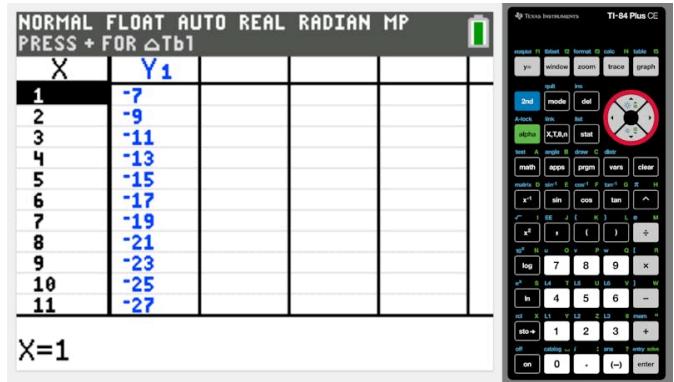
Steps	Explanation
<p>Use the variable button (so use <math>x</math> instead of <math>n</math> as the unknown) to enter the deductive rule for the sequence. Once you entered the rule, choose the option that lets you set some parameters to view the sequence.</p>	
<p>You can specify the index of the first term. If you want to see every term, choose 1 for the step size (<math>\Delta Tbl</math>). Once done, choose the option that brings up the table view to see the sequence.</p>	



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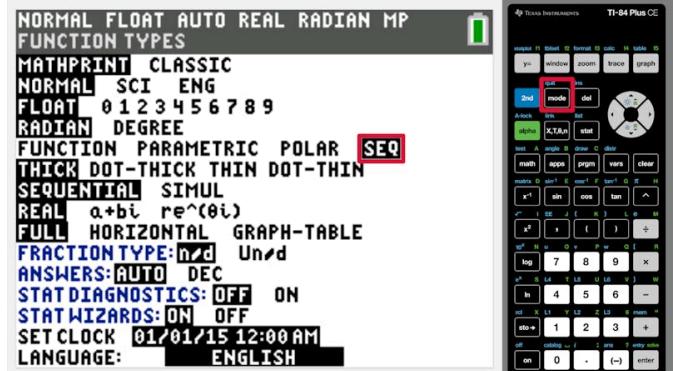
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Steps	Explanation																								
You can move up and down to see the terms of the sequence.	 <p>NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR <math>\Delta</math>Tb1</p> <table border="1"> <thead> <tr> <th>X</th> <th>Y1</th> </tr> </thead> <tbody> <tr><td>1</td><td>-7</td></tr> <tr><td>2</td><td>-9</td></tr> <tr><td>3</td><td>-11</td></tr> <tr><td>4</td><td>-13</td></tr> <tr><td>5</td><td>-15</td></tr> <tr><td>6</td><td>-17</td></tr> <tr><td>7</td><td>-19</td></tr> <tr><td>8</td><td>-21</td></tr> <tr><td>9</td><td>-23</td></tr> <tr><td>10</td><td>-25</td></tr> <tr><td>11</td><td>-27</td></tr> </tbody> </table> <p>X=1</p>	X	Y1	1	-7	2	-9	3	-11	4	-13	5	-15	6	-17	7	-19	8	-21	9	-23	10	-25	11	-27
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**Using the recursive rule**  $u_n = u_{n-1} - 2$ .

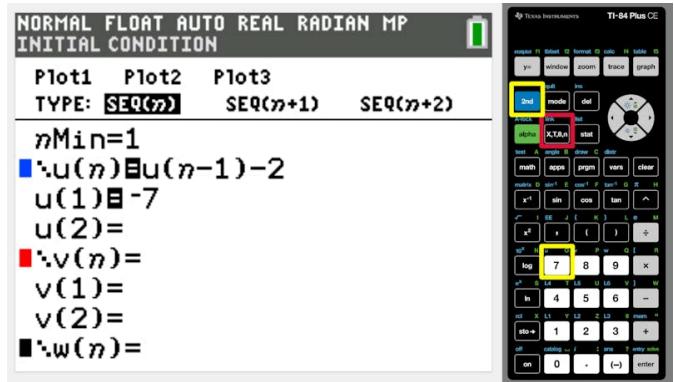
Steps	Explanation
To generate elements of a sequence when you only know the recursive rule, bring up the screen where you can change the settings of the calculator and change from the usual function mode to sequence mode.	 <p>NORMAL FLOAT AUTO REAL RADIAN MP FUNCTION TYPES MATHPRINT CLASSIC NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE FUNCTION PARAMETRIC POLAR SEQ THICK DOT-THICK THIN DOT-THIN SEQUENTIAL SIMUL REAL <math>a+bi</math> <math>r e^{(\theta)i}</math> FULL HORIZONTAL GRAPH-TABLE FRACTION TYPE: <math>\frac{a}{b}</math> <math>\frac{a}{b}</math> Un/d ANSWERS: AUTO DEC STAT DIAGNOSTICS: OFF ON STAT WIZARDS: ON OFF SET CLOCK 01/01/15 12:00 AM LANGUAGE: ENGLISH</p>



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Steps	Explanation
<p>Specify the first term of the sequence and enter the recursive rule. In sequence mode the variable button displays an <math>n</math> on the screen (instead of the <math>x</math>, which is used in function mode). Once done editing, bring up the table view.</p>	

You can move up and down to see the terms of the sequence.

NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR $\Delta$ Tbl	
$n$	$u(n)$
1	-7
2	-9
3	-11
4	-13
5	-15
6	-17
7	-19
8	-21
9	-23
10	-25
11	-27
$n=1$	

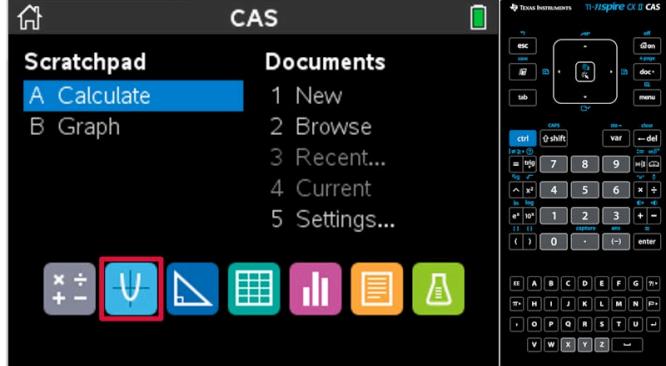
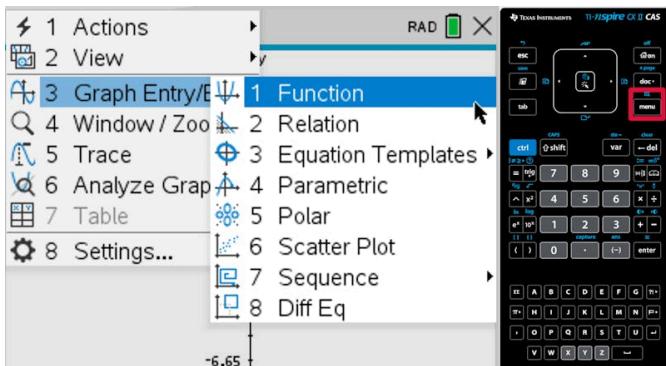
Using the deductive rule  $u_n = -7 + (n - 1) \times (-2)$ .



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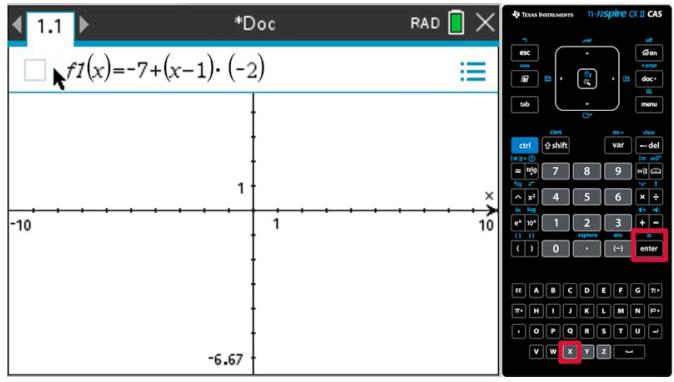
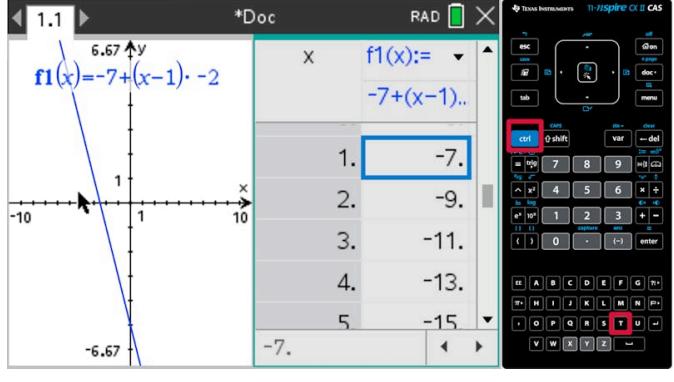
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Steps	Explanation
<p>From the main menu, choose the graphing option.</p>	
<p>To enter the deductive rule, bring up the option where you can define a function.</p>	



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Steps	Explanation
<p>Use <math>x</math> (instead of <math>n</math>) as the unknown to enter the deductive rule for the sequence.</p>	
<p>Pressing 'ctrl T' will split the screen and besides the graph you will also see the table containing the terms of the sequence. Pressing 'ctrl T' again will close the part containing the table.</p>	

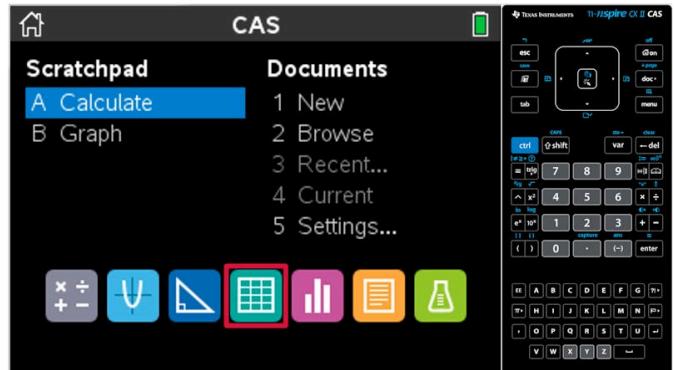
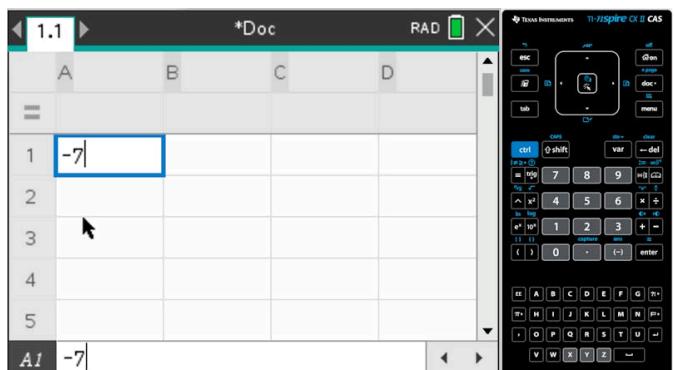
**Using the recursive rule**  $u_n = u_{n-1} - 2$ .



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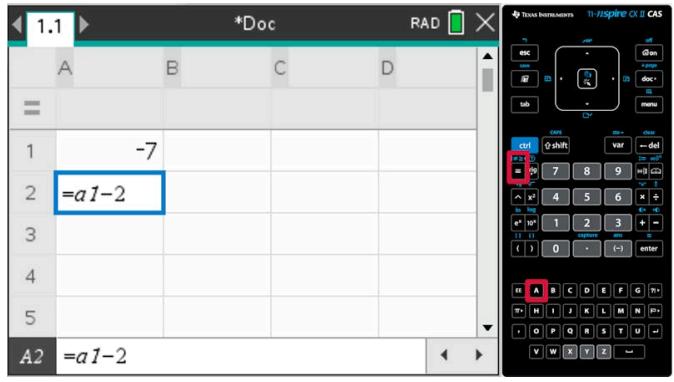
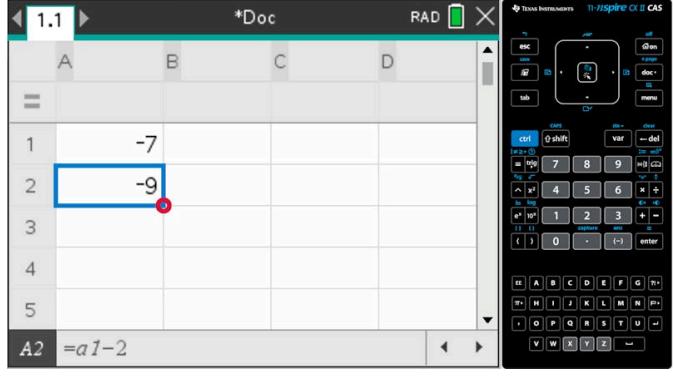
Steps	Explanation
<p>One way of generating elements of a sequence when you only know the recursive rule is to use the spreadsheet option.</p>	
<p>Specify the first term of the sequence.</p>	



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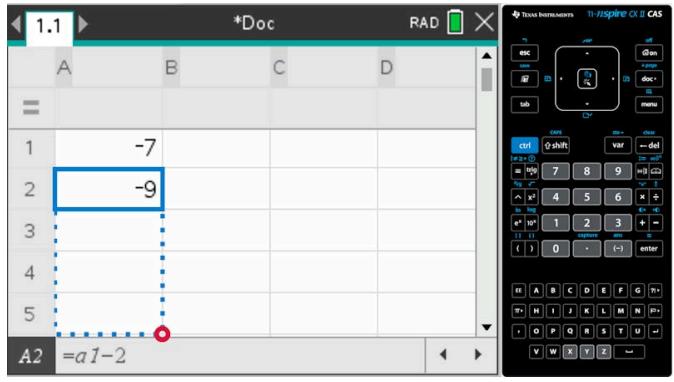
Steps	Explanation
<p>In the second row of the table tell the calculator how that value is generated from the value above.</p>	
<p>Click on the lower right corner of the second cell, ...</p>	



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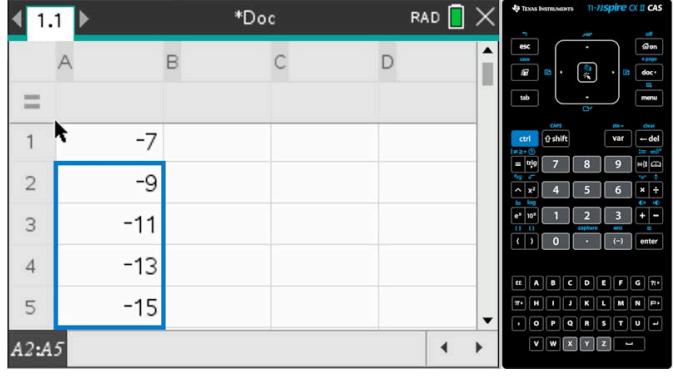
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Steps	Explanation
<p>... and pull the corner down. This will copy the generating rule to the other cells.</p>	



If you want to see more elements, simply extend the rectangle by pulling the lower right corner further down (the screen will scroll).

<p>If you want to see more elements, simply extend the rectangle by pulling the lower right corner further down (the screen will scroll).</p>	
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In addition to finding the  $n$  th term and a specific term in a given arithmetic sequence, the  $n$  th term equation can also be used to determine the position of a given term in a sequence.

Consider **Example 4**.

## Example 4

Student view

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The number of seats in a stadium follows an arithmetic progression. The first row has 40 seats while the second row has 45 seats. If this pattern continues, determine which row will have 100 seats.

Steps	Explanation
$40, 45, 50, \dots$ $d = 5, u_1 = 40$ $u_n = 40 + (n - 1)5 = 40 + 5n - 5 = 35 + 5n$	Use $u_n = u_1 + (n - 1)d$ to find an equation for the $n$ th term.
$u_n = 100$	You know that 100 is a term in this sequence, but you do not know which. Therefore, you can label it as $u_n$ .
$100 = 35 + 5n$ $65 = 5n$ $n = \frac{65}{5} = 13$	From previous work, you know that $u_n = 100$ and $u_n = 35 + 5n$ . Write this as an equation and solve for $n$ .
There are 100 seats in the 13th row.	As $u_{13} = 100$ corresponds to the 13th row.

## Example 5



An arithmetic sequence has a 5th term of 18 and an 11th term of 27.

Find the common difference, first term and general term.

Steps	Explanation
$18 = u_1 + 4d$ $27 = u_1 + 10d$	Using $u_5 = 18$ and $u_{11} = 27$ .

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Steps	Explanation
$\begin{aligned} 18 &= u_1 + 4d \\ - (27 &= u_1 + 10d) \\ -9 &= -6d \\ -\frac{9}{-6} &= d \\ d &= \frac{3}{2} \end{aligned}$	Solve the system of equations for $d$ by using elimination or by using the simultaneous equation solver option of your calculator.
$\begin{aligned} 18 &= u_1 + 4 \left( \frac{3}{2} \right) \\ u_1 &= 12 \end{aligned}$	Substitute $d = \frac{3}{2}$ into one of these: $18 = u_1 + 4d$ $27 = u_1 + 10d$ and solve for the first term.
$u_n = 12 + (n - 1) \left( \frac{3}{2} \right) = \frac{21}{2} + \frac{3}{2}n$	Using $u_n = u_1 + (n - 1)d$ .

In the following applet, you can practise questions similar to **Example 5** with numbers given up to 2 decimal places. However, in the exam you may be given the numbers as fractions and be asked for answers in the form of fractions (exact values).

### Interactive 1. Determination of the Common Difference and the First Term.

More information for interactive 1



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Given that the first three terms of an arithmetic sequence are  $2k + 2$ ,  $4k - 4$ ,  $3k + 2$ , find the value of  $k$ .

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Steps	Explanation
$d = 4k - 4 - (2k + 2) = 2k - 6$	Use $d = u_n - u_{n-1}$ to get $d = u_2 - u_1$ .
$d = 3k + 2 - (4k - 4) = -k + 6$	$d = u_3 - u_2$
$2k - 6 = -k + 6$	Set the expressions for $d$ equal to each other.
$3k = 12$ $k = \frac{12}{3}$ $k = 4$	Solve for $k$ .

## 6 section questions ▾

1. Number and algebra / 1.2 Arithmetic sequences and series

# Sigma notation

## Section

Student... (0/0)

Feedback



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# Series and sigma notation

A series is a sum of a sequence.

Thus, given a sequence  $\{u_n\}$ , the sum of the first 12 terms, written as  $S_{12}$ , is given by

$$S_{12} = u_1 + u_2 + u_3 + \dots + u_{11} + u_{12}.$$

## ✓ Important

The sum of the first  $n$  terms of a sequence is written as

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$$S_n = u_1 + u_2 + u_3 + \dots + u_{n-1} + u_n.$$

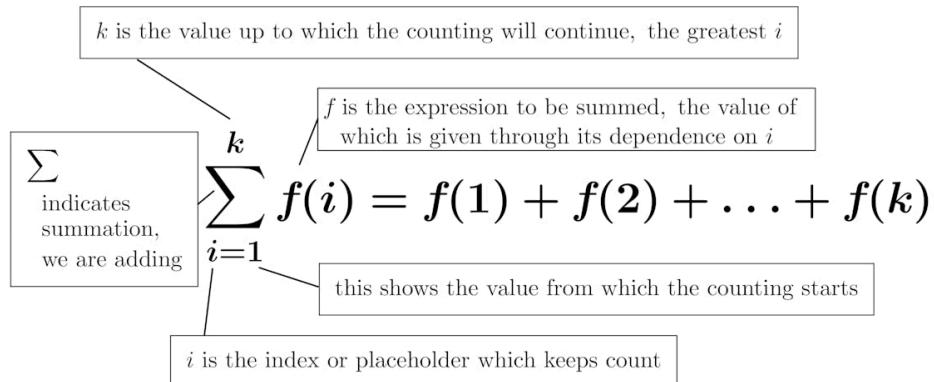
However, this is an inefficient way of writing a sum. There is an alternative, shorthand way, known as sigma notation.

It is called ‘sigma notation’ because it is denoted by the Greek symbol *sigma*,  $\Sigma$ .

Using the sigma notation,

$$S_n = \sum_{i=1}^n u_i = u_1 + u_2 + u_3 + \dots + u_{n-1} + u_n$$

An explanation of the various parts of sigma notation are shown below.



More information

The image is a diagram explaining sigma notation used in mathematics, with various labeled parts indicating the components involved in the expression. The central part of the diagram shows the sigma notation formula: ( $\sum_{i=1}^k f(i) = f(1) + f(2) + \dots + f(k)$ ). Annotations around the formula describe each component: ( $\Sigma$ ) signifies summation, indicating an additive process. ' $i = 1$ ' points to the value from which the counting starts, where ' $i$ ' is the index or placeholder keeping count. ' $k$ ' indicates the value up to which counting will continue, representing the greatest value of ' $i$ '. ' $f(i)$ ' represents the expression being summed, with its value dependent on ' $i$ '. This annotated visual guide clarifies each part of the mathematical notation.



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Thus, if you have the sequence  $u_n$  then

$$S_n = \sum_{i=1}^n (i^2 - 1).$$

For example, the sum of the first 12 terms becomes

$$S_{12} = \sum_{i=1}^{12} (i^2 - 1) = (1^2 - 1) + (2^2 - 1) + (3^2 - 1) + \dots + (12^2 - 1) = 638.$$

### Be aware

Do not confuse  $u_n$  and  $S_n$  when solving questions.

- $u_n$  is the general rule for finding the  $n$ th term of the sequence.
- $u_{10}$ , for example, is the tenth term of the sequence.
- $S_n$  is the sum of the first  $n$  terms of a sequence.
- $S_{10} = u_1 + u_2 + u_3 + \dots + u_9 + u_{10}$ .

$$u_n \neq S_n$$

## Example 1



Expand and simplify  $\sum_{i=1}^5 (2i + 1)$ .

Steps	Explanation
$\begin{aligned} & \sum_{i=1}^5 (2i + 1) \\ &= (2(1) + 1) + (2(2) + 1) + (2(3) + 1) + (2(4) + 1) + (2(5) + 1) \end{aligned}$	Substitute $i = 1, i = 2, \dots, 5$
$\begin{aligned} &= 3 + 5 + 7 + 9 + 11 \\ &= 35 \end{aligned}$	Simplify.



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## Example 2



Find  $\sum_{i=4}^8 (i^3 - i^2)$ .

Steps	E <sub>s</sub>
$\sum_{i=4}^8 (i^3 - i^2) = (4^3 - 4^2) + (5^3 - 5^2) + (6^3 - 6^2) + (7^3 - 7^2) + (8^3 - 8^2) = 1070$	E <sub>s</sub> ev th

## 4 section questions ▾

1. Number and algebra / 1.2 Arithmetic sequences and series

# Arithmetic series

### Section

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Feedback



Print (/study/app/math-aa-hl/sid-134-cid-761926/book/arithmetic-series-id-25039/print/)

Assign

An arithmetic series is a sum of the terms in an arithmetic sequence.

### ✓ Important

There are two formulae that can be used to find the sum of the first  $n$  terms of an arithmetic sequence,  $S_n$ , depending on the information you are given about the sequence.

1. If you know the first term and the common difference use

$$S_n = \frac{n}{2} (2u_1 + (n-1)d).$$

2. If you know the first and last terms to be summed use

$$S_n = \frac{n}{2} (u_1 + u_n).$$



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 The proof of the sum of an arithmetic sequence is shown in this video.

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### Video 1. Understanding Arithmetic Series: A Step-by-Step Guide.

 More information for video 1

1

00:00:00,100 --> 00:00:04,667

narrator: In this video, we're going  
to prove the arithmetic series equation,

2

00:00:04,733 --> 00:00:06,968.13

which we've seen in the text.

3

00:00:07,233 --> 00:00:08,367

And this was, of course,

4

00:00:08,433 --> 00:00:12,300

the sum of the first  $n$  terms

$$\text{is } \frac{n}{2}(2u_1 + (n - 1)d)$$

5

00:00:12,367 --> 00:00:17,000

$$\text{or } \frac{n}{2}(u_1 + u_n).$$

6

00:00:17,833 --> 00:00:19,500

Now, let's start in general.

7

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00:00:19,867 --&gt; 00:00:23,733

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In general, of course,

the sum of the first n terms  $S_n$

8

00:00:23,800 --> 00:00:26,433

is given by  $u_1 + u_2,$

9

00:00:26,533 --> 00:00:30,033

et cetera,  $+ u_{n-1} + u_n.$

10

00:00:30,133 --> 00:00:31,800

So the sum of the first n terms.

11

00:00:31,867 --> 00:00:34,800

Then for our particular arithmetic series,

12

00:00:34,867 --> 00:00:40,067

we have that  $S_n$  is  $u_1 + (u_1 + d),$

13

00:00:40,133 --> 00:00:44,100

'cause that is  $u_2$  plus

all the way to the n minus 1th term,

14

00:00:44,167 --> 00:00:46,500

which is  $u_1 + (n - 2)d.$

15

00:00:46,800 --> 00:00:50,400

And the last term,

which is  $u_1 + (n - 1)d.$

16

00:00:51,000 --> 00:00:52,800

Now let's rewrite this,

but in reverse order.

17

00:00:52,933 --> 00:00:56,733

So we start with the nth term

$u_1 + (n - 1)d,$

18

00:00:57,067 --> 00:01:01,333

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then an n minus 1th term,  
u<sub>1</sub> + (n - 2)d, et cetera,  
19  
00:01:01,400 --> 00:01:04,333  
all the way to the second term  
and the first term.  
—

20  
00:01:04,867 --> 00:01:07,933  
And now let's add up those two sums.

21  
00:01:08,533 --> 00:01:12,100  
So we've got two times S<sub>n</sub>.

Now let's look at the first couple.  
22  
00:01:12,167 --> 00:01:15,133  
You've got two u<sub>1</sub>'s  
and we've got (n - 1)d.

23  
00:01:15,433 --> 00:01:20,500  
So the first two terms is added up  
as 2u<sub>1</sub> + (n - 1)d.

24  
00:01:20,933 --> 00:01:24,767  
The second one is again, two u<sub>1</sub>'s.  
And we've got (n - 2)d + d.

25  
00:01:25,067 --> 00:01:28,400  
So again, we end up  
with 2u<sub>1</sub> + (n - 1)d

26  
00:01:28,600 --> 00:01:29,833  
as the sum of those two terms.

27  
00:01:30,533 --> 00:01:32,033  
And then we go to the end,

28  
00:01:32,100 --> 00:01:34,900  
which of course  
is also 2u<sub>1</sub> + (n - 1)d.

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00:01:35,000 --&gt; 00:01:39,400

And the last one also happens  
to be  $2u_1 + (n - 1)d$ .

30

00:01:40,200 --&gt; 00:01:43,800

Now this is a sum of  $n$  terms,  
so we've got  $n$  times

31

00:01:43,867 --&gt; 00:01:47,300

this factor  $2u_1 + (n - 1)d$ .

32

00:01:47,533 --&gt; 00:01:53,333

In other words, we've got  $2S_n$   
is  $n(2u_1 + (n - 1)d)$ ,

33

00:01:53,400 --&gt; 00:01:56,033

or in other words,  $S_n$  is  $\frac{n}{2}$ 

34

00:01:56,467 --&gt; 00:01:59,500

into  $2u_1 + (n - 1)d$ .

35

00:02:00,067 --&gt; 00:02:03,700

Now we need to remember that  $u_n$   
of course is  $u_1 + (n - 1)d$ .

36

00:02:03,900 --&gt; 00:02:08,267

So we can also write this sum as  $\frac{n}{2}$ 

37

00:02:08,433 --&gt; 00:02:10,233

into  $u_1 + u_n$ .

38

00:02:10,333 --&gt; 00:02:13,833

And so we've demonstrated  
that which we needed to do.

39

00:02:13,900 --&gt; 00:02:17,333

And I think that's a very pleasing  
proof for the equation,

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00:02:17,400 --&gt; 00:02:21,200

for the sum of the first  
 nth terms of an arithmetic series.

The proof given in the video is summarised below.

$$\begin{aligned}
 S_n &= U_1 + U_1 + d + \dots + U_1 + (n-2)d + U_1 + (n-1)d \\
 S_n &= U_1 + (n-1)d + U_1 + (n-2)d + \dots + U_1 + d + U_1 \\
 \hline
 2S_n &= \underbrace{2U_1 + (n-1)d + 2U_1 + (n-1)d + \dots + 2U_1}_{n \text{ times}} + (n-1)d + 2U_1 + (n-1)d \\
 2S_n &= n(2U_1 + (n-1)d) \\
 \text{or} \\
 S_n &= \frac{n}{2}(2U_1 + (n-1)d) & U_n = U_1 + (n-1)d \\
 &= \frac{n}{2}(U_1 + U_n) & \text{QED}
 \end{aligned}$$

 More information

This image shows a mathematical proof demonstrating the formula for the sum of an arithmetic sequence. It starts with the equation ( $S_n = U_1 + U_1 + d + \dots + U_1 + (n-1)d$ ), where ( $U_1$ ) is the first term and ( $d$ ) is the common difference. Then, rewriting the sequence backwards as ( $S_n = U_1 + (n-1)d + U_1 + (n-2)d + \dots$ ), it shows both sequences aligned and added to get ( $2S_n = 2U_1 + (n-1)d + 2U_1 + (n-1)d + \dots$ ), repeated ( $n$ ) times. This implies ( $2S_n = n(2U_1 + (n-1)d)$ ), ultimately leading to the conclusion ( $S_n = \frac{n}{2}(U_1 + U_n)$ ) with ( $U_n = U_1 + (n-1)d$ ). The QED symbol signifies the end of the proof.

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## Example 1

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Find the sum of the arithmetic sequence 3, 15, 27, 39, ..., 135 (12 terms).



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Steps	Explanation
$u_1 = 3$ and $d = 15 - 3 = 12$	
$S_n = \frac{n}{2} (2 u_1 + (n - 1) d)$	Since you know the first term and the common difference.
$S_{12} = \frac{12}{2} (2 \times 3 + (12 - 1) \times 12) = 828$	

## Example 2



An arithmetic sequence is given by the general term  $u_n = -121 + 18n$ .

Find the sum of the first 10 terms.

Steps	Explanation
$u_1 = -121 + 18 = -103$ , $u_{10} = -121 + 18 \times 10 = 59$	Find the first and the last term to use $S_n = \frac{n}{2} (u_1 + u_n)$ .
$S_{10} = \frac{10}{2} (-103 + 59) = -220$	Fill in the values for $n$ , $u_1$ , and $u_{10}$ and evaluate the sum.

## Example 3



An arithmetic sequence has a first term of 20 and a common difference of 13.



Student view

Identify which term is equal to 228. Find the sum of all the terms 20, 33, ..., 228.

Steps	Explanation
$u_n = u_1 + (n - 1)d = 7 + 13n$	Write an equation for the general term.
$228 = 7 + 13n$ $n = \frac{228 - 7}{13} = 17$	Since you do not know which term is 228 set it equal to the general term and solve for $n$ .
$S_{17} = \frac{17}{2}(20 + 228) = 2108$	Since 228 is the 17th term, find the sum of the first 17 terms.

## Example 4

★★★

The arithmetic sum of the first  $n$  terms is given by  $S_n = \frac{n}{2}(5n - 11)$ . Find the first term and the common difference.

Steps	Explanation
$u_1 = \frac{1}{2}(5 \times 1 - 11) = -3$	Using the fact that $u_1 = S_1$ .
$u_2 = S_2 - u_1 = \frac{2}{2}(5 \times 2 - 11) - (-3) = 2$	Using the fact that $S_2 = u_1 + u_2$ .
$d = u_2 - u_1 = 2 - (-3) = 5$	

### ① Exam tip

Both formulae for the sum of the first  $n$  terms of an arithmetic sequence,  $S_n$ , are given in the IB formula booklet. You do not need to know the derivation of these equations.

It is useful to remember that  $u_1 = S_1$  and  $u_2 = S_2 - u_1$ .

For  $n > 1$ , the latter can be generalised to get  $u_n = S_n - S_{n-1}$ .



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## Example 5

An arithmetic sequence has third term 11 and the sum of the first 10 terms is 160.

Find the first term, the common difference and the general term of this sequence.

Steps	Explanation
$11 = u_1 + (3 - 1)d$ $160 = \frac{10}{2}(2u_1 + (10 - 1)d)$	Using the formulae $u_n = u_1 + (n - 1)d$ and $S_n = \frac{n}{2}(2u_1 + (n - 1)d)$ .
$u_1 + 2d = 11$ $10u_1 + 45d = 160$	Simplify to get these two equations.
The first term is $u_1 = 7$ and the common difference is $d = 2$ .	Solve the equations algebraically or using your GDC.
$u_n = 7 + (n - 1) \times 2 = 2n + 5$	Substitute $u_1$ and $d$ into $u_n = u_1 + (n - 1)d$ to get the general term.

### 🔗 Making connections

When solving a question that gives you information about the sum and some terms in an arithmetic sequence, you will often need to create a system of two equations for the two unknowns that need to be found.

Systems of equations can be solved by substitution, elimination, or graphing.



Student view

On the following applet you can practise questions similar to the previous examples. The numbers given along with the answers are rounded to two decimal places. However, in the exams, you may be asked to give your answers as fractions (exact values).



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### Interactive 1. Finding the Common Difference and the First Term.



 More information for interactive 1

This interactive is designed to help users practice solving problems related to arithmetic sequences. Random questions are generated by providing one specific term and the sum of a certain number of terms in an arithmetic sequence. Users are tasked with finding the common difference and the first term based on the given term and sum.

Users can generate new practice questions by clicking the "New Question" button, which presents a different arithmetic sequence problem based on a given term and sum. After attempting to solve for the common difference and first term, they can click "Show Answer" to reveal the correct solution.

For example, when a user clicks New Question, they might see:

“For an arithmetic sequence,

$$u_4 = 2.8, S_9 = -5.4$$

Find the common difference and the first term."

**Solution:**

Using the formulae:

$u_n = u_1 + (n - 1)d$ , where:  $u_1$  = first term,  $d$  = common difference

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d)$$

## Step 1: Set Up the Equations

### Step 2: Simplify the Equations

From equation (1):  $2.8 = u_1 + 2d$

From equation (2):  $-1.2 = 2u_1 + 8d$

### Step 3: Solve the System of Equations

$$2(\text{eq.1}) - \text{eq.2}$$

$$2(u_1 + 2d) - (2u_1 + 8d) \equiv 2(2.8) - (-1.2) \Rightarrow -4d \equiv 6.8 \Rightarrow d \equiv -1.7$$

Substitute  $d = -1.7$  into equation (1):

$$2.8 = y_1 + 2(-1.7) \Rightarrow y_1 = 2.8 + 3.4 = 6.2$$



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**Final Answer:**

$$u_1 = 6.2 \text{ and } d = -1.7$$

After solving, clicking "Show Answer" will display:

"The common difference is  $d = -1.7$

The first term is  $u_1 = 6.2$ "

If the user clicks "New Question" again, another example might appear with different values.

By generating new problems and providing immediate feedback, the tool allows users to repeatedly practice finding these key sequence properties, reinforcing their understanding through varied examples while developing problem-solving skills for arithmetic sequences that involve both individual terms and sums of terms.

## Example 6



Consider the sequence,  $S_n = 2n^2 - 3n$ .

Find an expression for the general term,  $u_n$ , and show that this is an arithmetic sequence.

Steps	Explanation
$\begin{aligned} u_n &= S_n - S_{n-1} \\ &= (2n^2 - 3n) - (2(n-1)^2 - 3(n-1)) \\ &= (2n^2 - 3n) - (2n^2 - 4n + 2 - 3n + 3) \\ &= (2n^2 - 3n) - (2n^2 - 7n + 5) \\ &= 2n^2 - 3n - 2n^2 + 7n - 5 \\ &= 4n - 5 \end{aligned}$	Use the result highlighted in the exam tip box above.
$\begin{aligned} u_{n+1} - u_n &= (4(n+1) - 5) - (4n - 5) \\ &= 4n + 4 - 5 - 4n + 5 \\ &= 4 \end{aligned}$ <p><math>d = 4</math> the sequence is arithmetic.</p>	This difference is independent of $n$ , so it is common, and hence the sequence is arithmetic.

## 4 section questions ▾



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# Applications

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## Perfectly arithmetic sequences and series

As you travel through the world you will encounter examples of perfect arithmetic sequences.

The height of each step in a staircase is usually the same; if this is true, then the height above the ground for each step follows an arithmetic progression.

### Example 1



The height of each step is 20 cm . The first step is 25 cm above ground level.

How many steps would you need to climb to be 205 cm above ground level?

Steps	Explanation
$u_1 = 25, d = 20$ $u_n = 25 + (n - 1) 20 = 5 + 20n$	<p>The distance from the ground increases by 20 cm for each step. Therefore, the height above the ground follows an arithmetic progression.</p> <p>Here are the heights of the first few steps:</p> <p>25, 45, 65, 85, ...</p>
$u_n = 205$ $u_n = 5 + 20n$ $205 = 5 + 20n$ $200 = 20n$ $n = 10$	<p>Since you don't know which step is 205 cm above the ground you call it <math>u_n</math> .</p>
<p>You would need to be on the 10th step to be 205 cm above ground level.</p>	

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Some subscription plans have a set monthly membership fee and a smaller amount that is charged each time that you use the service. The amount of money that you pay follows an arithmetic sequence.

## Example 2



It costs an initial 20 zlotis for a plumber to come to your house and an additional 3 zlotis for every hour that the plumber spends on the repairs.

- Write an expression for the total cost of the plumber's visit that includes  $n$  hours of repairs.
- Hence, calculate the total amount that you will have to pay if the repair takes 5 hours.

	Steps	Explanation
a)	$u_1 = 23, d = 3$ $u_n = 23 + (n - 1) 3 = 20 + 3n$	<p>This is an example of an arithmetic sequence because the price is increased by adding 3 for each hour of repairs.</p> <p>Note, that the first term is <math>u_1 = 20 + 3 = 23</math>, because the plumber charges the 20 zlotis to come and in addition 3 zlotis to work in the first hour.</p>
b)	$n = 5$ $u_5 = 20 + 3(5) = 35$	<p>'Hence' means that you must use the result obtained in the previous part of the question.</p> <p>So, you will use <math>u_n = 20 + 3n</math>.</p>
	The cost is 35 zlotis.	<p>Don't forget to clearly answer the question.</p> <p>Finding <math>u_5</math> was just a tool for answering the question about the price.</p>





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## 🌐 International Mindedness

It is important to keep in mind that different cultures have different norms about how customers are charged for a service. In some cultures, it is acceptable to charge an initial flat fee, such as in the plumber example, while in others it is only acceptable to charge money specifically for the services provided (in the plumber example, this would only be the time spent on repairs).

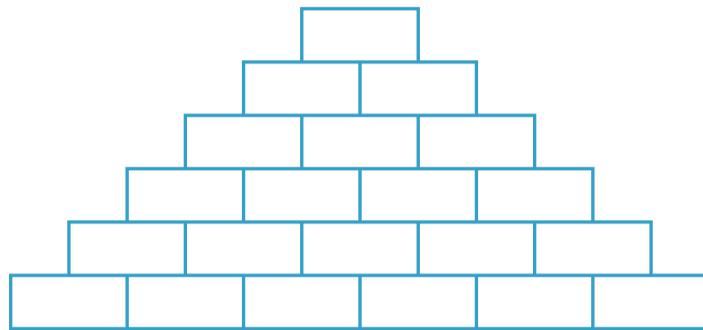
While the first two examples deal with arithmetic sequences, there are also many examples of arithmetic series. Consider **Example 3**.

### Example 3



A supermarket display of biscuit boxes is arranged into a pyramid shape as shown in the diagram. The base of the pyramid has 28 boxes and each row above has 3 fewer boxes.

If the top of the pyramid has one box, calculate the total number of boxes used in this display.



Student  
view



Steps	Explanation
$S_n = \frac{n}{2} (u_1 + u_n)$ $u_1 = 28, u_n = 1$ $S_n = \frac{n}{2} (28 + 1)$	The question asks for the total number of boxes, so you need to find a sum.  The number of boxes in each row follows an arithmetic progression: 28, 25, 22, ..., 1.  However, the number of terms is unknown and must be found to evaluate the sum.
$u_1 = 28, d = -3$ $u_n = 28 + (n - 1)(-3) = 31 - 3n$	Find an expression for $u_n$ first.
$1 = 31 - 3n \Leftrightarrow 30 = 3n \Leftrightarrow n = 10$	Use $u_n = 1$ to find $n$ and therefore the total number of rows.
$S_{10} = \frac{10}{2} (28 + 1) = 145$	Now that you know $n = 10$ you can evaluate the sum.
There are 145 boxes in the display.	

## Example 4



Janelle makes an investment that offers 4% of her initial investment to be added to her account at the end of every year that she keeps the money in the account.

Her initial investment was 4000 roubles.

- Write down the amount that is in her account at the end of each of the first 5 years.
- Write a general equation for  $A$ , the amount of money that she has in her account, in terms of  $t$ , the number of years.
- Deduce how the equation from part b is connected to an arithmetic sequence.



	Steps	Explanation
a)	$4000 \times 0.04 = 160$	160 is 4% of the initial investment and is therefore the amount added to her bank account each year.
	End 1st year: 4160 End 2nd year: 4320 End 3rd year: 4480 End 4th year: 4640 End 5th year: 4800	160 is added to the account at the end of each year.
b)	$A = 4000 + 160t$	Each year, 160 roubles are added to 4000. To represent this, you can multiply 160 by $t$ .
c)	$A = 4000 + 160t$ is an example of an arithmetic sequence. You can see its terms listed in part a.  $u_1 = 4160, d = 160$ $u_n = 4160 + (n - 1) 160 = 4160 + 160n - 160 = 4000 + 160n$ $A = 4000 + 160t$ and $u_n = 4000 + 160n$ are equivalent expressions.	This example, where the same amount of interest is added each year, is an example of simple interest which is easily modelled by an arithmetic sequence.

## ① Exam tip

Simple interest problems such as the one presented in **Example 4** can be modelled using the equation  $A = P(1 + rt)$ , where:

- $A$  amount in bank after time  $t$
- $P$  is the amount of the initial investment
- $r$  is the percent rate in decimal form
- $t$  is the amount of time that the investment is in the bank (units for  $t$  should match the units for  $r$ ).



However, the equation for simple interest is not given in the IB formula booklet. You can memorise this formula, or use the fact that simple interest is easily modelled as an arithmetic sequence by using  $u_n = u_1 + (n - 1) d$ .

## Modelling using arithmetic sequences

So far, you have looked at application questions dealing with examples of perfect arithmetic sequences where  $d$  was constant.

However, not all real-life examples involve addition or subtraction of a constant value.

Consider the following situation. You have ten bags of potatoes and you put the first six on a scale one at a time and record the following data.

Number of bags of potatoes		1	2	3	4	5	6
Mass (kg)		6.3	12.1	18.3	24.3	30.2	36.4

Your goal is to find the mass of all ten bags without making any more measurements. You can do this by investigating the data and asking and answering some questions.

**Question:** Does the mass of the bags follow an arithmetic progression?

This can be checked by calculating the difference between consecutive values.

$$36.4 - 30.2 = 6.2$$

$$30.2 - 24.3 = 5.9$$

$$24.3 - 18.3 = 6$$

$$18.3 - 12.1 = 6.2$$

$$12.1 - 6.3 = 5.8$$

The difference is not exactly the same, so this is not technically an arithmetic sequence.



**Question:** Even though this sequence is not arithmetic, is it reasonable to assume that it is approximately arithmetic?

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The differences of the measurements give the mass of the second, third, fourth, fifth and sixth bag of potatoes. These are approximately the same and it is reasonable to assume that the other bags will also have approximately the same mass. You are justified to assume that the difference is approximately constant, so the sequence is approximately arithmetic.

**Question:** What can be used as a common difference for the approximating arithmetic sequence?

Since the total mass of six bags is 36.4 kilograms, the average,  $d = \frac{36.4}{6} = 6.07$  can be used as the common difference of the approximating arithmetic sequence.

**Question:** What is the model and what is the estimated mass of 10 bags?

To model the total mass of  $n$  bags of potatoes, you can use the expression  $M = 6.07n$ .

This model gives  $6.07 \times 10 = 60.7$  kilograms as an estimate for the mass of 10 bags of potatoes.



## Theory of Knowledge

This section of the book deals with approximation, which is an important topic in maths.

How would you determine when it is appropriate to use approximation to obtain information about real-world quantities?



## Example 5

Student view



Without any passengers, an aeroplane has a mass of 4200 kg . The plane has the capacity to carry 120 passengers, and when there are 40 passengers on the plane, the total mass of the plane and passengers is 7480 kg .

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a) Calculate the mean mass of a passenger.

b) Hence, write an expression for  $u_n$  that gives the total mass of the plane and passengers when there are  $n$  passengers on board.

c) State the values of  $n$  for which your model is valid.

	Steps	Explanation
a)	$\text{Mass of 40 passengers} = 7480 - 4200$ $= 3280$  $\text{Mean mass} = \frac{3280}{40}$ $= 82 \text{ kg}$	$\text{Mean mass of passengers} = \frac{\text{total}}{\text{number of}}$
b)	$d = 82, u_1 = 4200 + 82 = 4282$  $u_n = u_1 + (n - 1)d$ $= 4282 + (n - 1)82$ $= 4200 + 82n$	If $n$ is the number of passengers, $u_1$ corresponds to $n = 1$ .
c)	The model is valid for $0 \leq n \leq 120$ .	The model works for predicting the mass from starting with 0 passengers and going up to the maximum capacity of the plane which you can fit 120.

## Example 6



Student view

You want to set the heating to come on before you get home. You know that with the heat off the house temperature is 15 °C and that on average the house temperature rises 0.35 °C for every minute that the heat is on.

- ❖ a) Write an expression for  $u_n$  which represents the approximate temperature of your house  $n$  minutes after you turn on the heat.

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- b) Hence, calculate how many minutes before your arrival the heat should turn on so that the house has warmed up to approximately 21 °C when you get home.

- c) Determine whether this model is appropriate if you want to work out the temperature of your house if you leave the heat on for 5 hours.

	Steps	Explanation
a)	$u_0 = 15, d = 0.35$ $u_n = 15 + 0.35n$	
b)	$21 = 15 + 0.35n \Leftrightarrow 6 = 0.35n$ $n = \frac{6}{0.35} \approx 17$ The heater needs to be turned on approximately 17 minutes before you get home.	Solve for $n$ when $u_n = 21$ . .
c)	$n = 60 \times 5 = 300$ $u_{300} = 15 + 0.35 \times 300 = 120$ The model predicts a temperature of 120 °C which is unrealistically high. This model does not work for large periods of time.	Don't forget to convert hours to minutes. In reality heaters turn off when they reach a set temperature. Even if the heater does not turn off automatically, the temperature cannot increase without bound.

## Example 7



Eggs are sold in cartons that can hold ten eggs. The following table shows the mass of a carton containing a few eggs.

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Number of eggs in the carton	1	2	3	4	5	6
Mass (in grams)	95	156	220	273	332	395

- a) What is the mass of an egg?
- b) What is the mass of the carton?
- c) What is the mass of a carton of ten eggs?



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	Steps	Explanation
a)	<p>Second egg: <math>156 - 95 = 61</math> grams</p> <p>Third egg: <math>220 - 156 = 64</math> grams</p> <p>Fourth egg: <math>273 - 220 = 53</math> grams</p> <p>Fifth egg: <math>332 - 273 = 59</math> grams</p> <p>Sixth egg: <math>395 - 332 = 63</math> grams</p> <p>The average mass of these five eggs is</p> $\frac{61 + 64 + 53 + 59 + 63}{5} = 60 \text{ grams}$ <p>The mass of an egg varies, but you can say that based on the data, a reasonable estimate is 60 grams.</p>	The mass of the second, third, fourth, fifth, sixth eggs are the differences of the consecutive numbers given in the table.
b)	<p>The exact mass cannot be determined from the data.</p> <p>The approximate mass of the carton is</p> $95 - 60 = 35 \text{ grams}$	<p>The first entry in the table is the total mass of the carton and the first egg.</p> <p>The approximate mass of the first egg is the result of part (a).</p>
c)	<p>The exact answer cannot be determined from the data.</p> <p>The approximate mass of a carton with <math>n</math> eggs is</p> $M = 35 + 60n \text{ grams.}$ <p>The approximate mass of a carton of ten eggs is</p> $35 + 60 \times 10 = 635 \text{ grams.}$	You can use the results in part (a) and (b) to set up a model for the mass of the carton with $n$ eggs.

## ⓐ Making connections

In the examples above you investigated relationships which are approximately linear. You will learn more about linear models in subtopic 4.4.





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# Checklist

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### What you should know

By the end of this subtopic you should be able to:

- recognise sequence notation such as  $u_1, u_2, u_{n-1}, u_n, u_{n+1}$
- know that  $u_{n-1}, u_n, u_{n+1}$  are three consecutive terms in a sequence
- write a recursive rule for a sequence and use the recursive rule to generate the first few terms of a sequence
- show that a sequence is arithmetic by proving that  $d = u_n - u_{n-1}$  is constant for all terms of a sequence
- write a recursive rule for an arithmetic sequence by using  $u_n = u_{n-1} + d$
- write an  $n$ th term or deductive rule for an arithmetic sequence by using  $u_n = u_1 + (n - 1)d$
- find the sum of an arithmetic sequence by using

$$S_n = \frac{n}{2} (2u_1 + (n - 1)d) \text{ or } S_n = \frac{n}{2} (u_1 + u_n)$$

- identify real-world situations which follow a perfectly arithmetic progression
- apply  $u_n = u_1 + (n - 1)d$ ,  $S_n = \frac{n}{2} (2u_1 + (n - 1)d)$ , and  $S_n = \frac{n}{2} (u_1 + u_n)$  to solve real-world application questions with perfectly arithmetic progressions
- identify real-world situations which are not perfectly arithmetic but are similar enough to be modelled using arithmetic sequences
- use the mean value for  $d$  in application questions involving sequences that are not perfectly arithmetic to create a model for  $u_n$  and to approximate its values for specific values of  $n$
- interpret sigma notation to write out and evaluate a given sum
- write an equivalent form of a sum in sigma notation.



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# Investigation

Section

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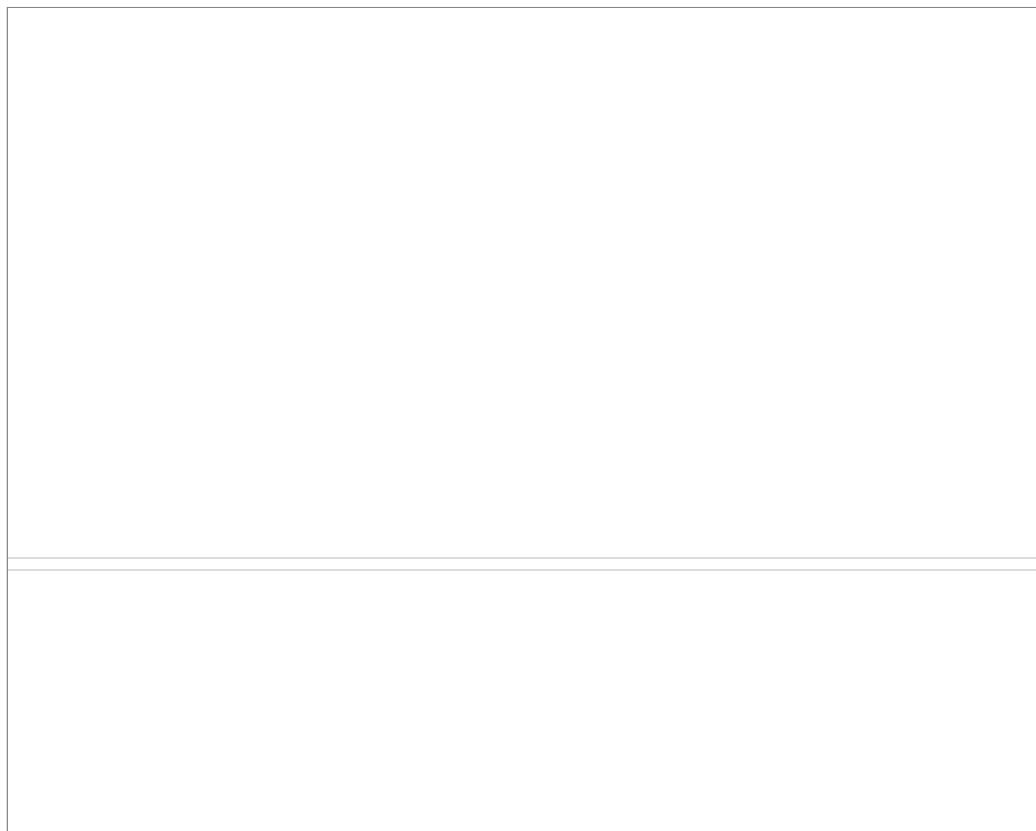


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## Investigation 1

The terms of an arithmetic sequence can be represented by points on a graph. The applet below shows you what the graph looks like for various examples of arithmetic sequences.



**Interactive 1. Graph for Various Examples of Arithmetic Sequences.**

Credit: [GeoGebra](https://www.geogebra.org/m/ABbaQFfY) (https://www.geogebra.org/m/ABbaQFfY) Lee W Fisher

More information for interactive 1

This interactive allows users to explore arithmetic sequences through graphical representation. The interactive displays the terms of an arithmetic sequence as points on a graph, helping users visualize the sequence's behavior. Users are prompted to find the general term ( $u_n$ ) for the displayed sequence. By clicking the "Show Equation" button, users can verify their solution, which reveals the correct formula for the sequence. Users can generate new sequences with the "New Sequence" button, providing fresh examples. The applet displays the first term and common difference, helping



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users understand the sequence's terms and their graphical representation.

Example:

The given sequence is 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

The first term is 3 and the common difference is 3.

So the equation is  $u_n = 3n$  and the points between 0 – 10 are plotted on the graph.

Start by finding the general term for the sequence shown in the applet. You can check your work by clicking the 'Show Equation' box and you can generate a new sequence by pressing the 'New' button.

The points generated by the sequence can be joined to create a line graph. The equation of this line can be expressed as  $y = mx + b$ .

Investigate how the equation of  $u_n$  corresponds to the equation of the line, in  $y = mx + b$  form, drawn through the points in the sequence.

Your findings should allow you to answer the following questions:

1. How would a graph for a sequence with  $d = 2$  compare with a sequence with  $d = 5$ ?  
What about  $d = 2$  and  $d = -2$ ?
2. In which sequence would you expect to have a larger first term, one with the line  $y = 2x + 4$  connecting the points or  $y = 2x + 10$ ?

Consider how your findings could be applied to sequences that are not perfectly arithmetic as studied in [section 1.2.5 \(/study/app/math-aa-hl/sid-134-cid-761926/book/applications-id-25040/\)](#)?

## Investigation 2

Make up an arithmetic sequence and write out the first 11 terms.

Find  $u_1 + u_{11}$ ,  $u_2 + u_{10}$ ,  $u_3 + u_9$ , ...

What do you notice?

Do these observations hold true when you write out an even number of terms at the beginning?

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Explore this property further with other examples of arithmetic sequences and try to generalise your findings.

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Reflect on how your findings are linked to what you learned about arithmetic sequences and series in this topic.

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### Rate subtopic 1.2 Arithmetic sequences and series

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