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5. Calculus / 5.8 Area of a region

The big picture

The following two applets illustrate possible ways of finding an approximate area of a given region using areas of known shapes.

In the first applet you will draw a grid and consider the number of squares inside the region that is drawn in an 8 cm by 8 cm square.

Interactive 1. Finding an Approximate Area of a given Region Using Areas of Squares.

More information for interactive 1

The interactive allows users to explore methods for approximating the area of a given region within an $10\text{cm} \times 10\text{cm}$ square.

The screen is divided in two parts. The right side of the screen is divided in a square of $10\text{cm} \times 10\text{cm}$. It consists of an organic shape with 9 red dots. On the left of the screen there are 4 check boxes namely, Adjust region, Adjust grid and Approximation type Inner and Outer. When the Adjust region is selected users can adjust the shape and size of the region

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by dragging the red dots. Below Adjust region the area of the region is displayed in real time. When the Adjust grid is selected a slider appears that can be slid from a grid of $2\text{cm} \times 2\text{cm}$ to $0.5\text{cm} \times 0.5\text{cm}$.

The inner approximation calculates the area based on the number of grid squares entirely within the region, while the outer approximation includes squares that partially overlap the region. Both approximations are displayed alongside the actual area of the region, allowing users to compare the accuracy of each method. Users are encouraged to investigate how adjusting the grid size affects the accuracy of the approximations and to consider ways to improve the precision of the area estimation. This interactive tool provides a hands-on way to understand geometric approximation techniques and their applications.

Activity

- Investigate what happens when you adjust the grid.
- How does the inner and outer approximation relate to the area?
- Can you suggest something that would improve the accuracy of the approximation?

In the second applet you pick points on the boundary of the region and consider the area of the polygon that has these points as vertices.

Interactive 2. Finding an Approximate Area of a given Region Using Area of the Polygon.

More information for interactive 2

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This interactive allows users to approximate the area of a given region by constructing a polygon with vertices placed on the boundary of the region. The screen is divided in two halves. The right side has an organic shape marked by a closed blue line and 9 red dots on the line.

On the left side there are 2 buttons, namely ‘Adjust region’ and ‘Adjust polygon’. When the "Adjust Region" button is selected, the red dots can be dragged to modify the shape. Below it, the "Adjust Polygon" button allows the user to modify the shape of a polygon with vertices placed on the boundary of the region. Below the “Adjust polygon” button, The number of vertices, ranging from 3 to 9, can be selected from a drop-down menu, forming a closed polygon with the chosen number of sides. Below all the buttons, the area of the region and the area of the polygon are displayed in blue and red.

By placing the vertices on the boundary, the applet calculates the area of the polygon and compares it to the actual area of the region.

This comparison helps users understand how increasing the number of vertices improves the accuracy of the approximation. For example, if the area of the region is 33.92 cm^2, the area of the polygon varies depending on the number of vertices selected. When three vertices are chosen, the polygon's area is 17.92 cm^2, while selecting nine vertices results in an area of 30.65 cm^2.

This interactive provides a hands-on way to explore geometric approximation techniques, demonstrating how polygons can be used to estimate the area of irregular shapes.

Activity

- Move the points to see how close you can get to the actual area of the region.
- Does increasing the number of points increase the accuracy?

In the following sections you will see another way of approximating areas. However, you will only consider regions between the x-axis and the graph of a function.

Concept

While you are investigating the new method, think about how you would **generalise** the method to get **approximation** of areas of other regions.

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Theory of Knowledge

The trapezoidal rule allows for an approximation. This is a knowledge-based approximation, though it is an approximation none the less. Consider other areas of knowledge (AOK) and whether or not they allow for approximations. Take history, for example. Does history allow for approximation more or less so than mathematics? Does this difference in methodology strengthen or weaken history as an AOK?

Knowledge Question: Can approximation be considered knowledge?

5. Calculus / 5.8 Area of a region

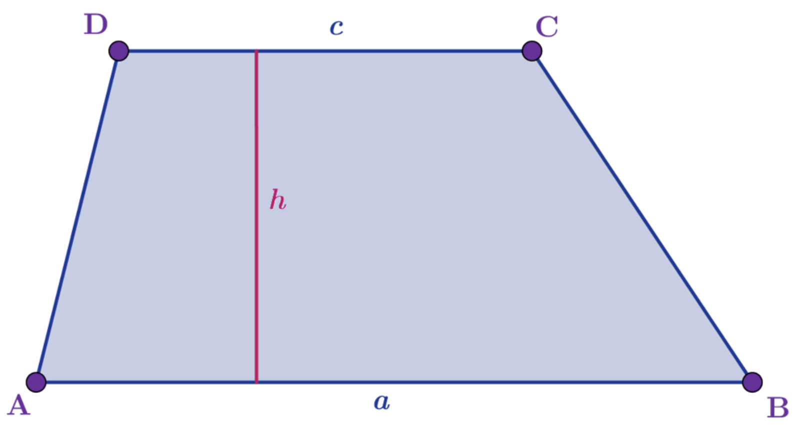
Trapezoidal rule with non-uniform spacing

International Mindedness

In this book youwill follow the terminology of the IB mathematics syllabus and use the term **trapezoid** for the quadrilateral that has one pair of parallel sides. In some countries the term **trapezium** is used for the same shape.

Area of a trapezoid

On the diagram below, the parallel sides of the trapezoid $\text{ABCD}$are $\text{AB} = a$and $\text{CD} = c$. The height of the trapezoid is h.



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More information

This diagram depicts a trapezoid labeled ABCD. The trapezoid has two parallel sides labeled AB and CD with lengths 'a' and 'c', respectively. The height of the trapezoid is denoted by the line segment 'h', which is perpendicular to the parallel sides.

Point A is located at the bottom left corner, B at the bottom right, C at the top right, and D at the top left. The side AB runs horizontally at the bottom, and side CD runs horizontally at the top, parallel to AB. The line indicating the height, 'h', is drawn from side CD to side AB.

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The area of the trapezoid is given by $\frac{(a+c)h}{2}$.

Area of a region below a graph

In section 5.5.2 (/study/app/m/sid-122-cid-754029/book/definite-integrals-id-26295/) you saw how to use technology to find an approximation to the area of a region bounded by the x-axis and the graph of a function over a finite interval. There are several ways a computer can be programmed to find this value. Now you will learn about one of the possible approaches.

The applet below shows a region bounded by the x-axis and a curve over the interval $[0, 5]$.The applet gives the area of the region. It also shows six trapezoids and the total area of these six trapezoids. You can change the shape of the curve and the position of the trapezoids.

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Interactive 1. Trapezoidal Rule with Uneven Intervals.

More information for interactive 1

The interactive allows users to explore the approximation of the area under a curve using trapezoids. The screen is divided in two halves. The top half contains a graph with X-axis ranging from 0 to 5 and a Y-axis ranging from 0 to 2. It contains a curve marked by a blue line. There are 5 red dots on the X-axis of the graph. On the bottom half of the screen, there are two buttons, namely, Adjust curve and Adjust the trapezoids. When the Adjust curve is selected, the red dots appear on the XY plane, which can be dragged along the y-axis by the user to modify the curve. Below the Adjust curve button, the area of the curve region is mentioned.

When Adjust the Trapezoids is selected, five red dots appear on the x-axis with red lines projecting onto the graph. When connected by a straight line, each segment forms a trapezoid. These red dots can be dragged along the x-axis to modify the trapezoids. A total of six trapezoids are displayed on the graph. The applet provides both the exact area of the region and the approximate area calculated using the trapezoids. Below the Adjust the Trapezoids button, the approximated area of the trapezoids is displayed. Users can modify the shape of the curve and adjust the positions of the trapezoids to observe how these changes affect the accuracy of the approximation. This interactive tool provides a hands-on way to understand numerical integration techniques, specifically the trapezoidal rule, and how it can be used to estimate the area under a curve.

Activity

- Can you explain how the trapezoids were constructed?
- The total area of the trapezoids is close to the area of the region. Can you suggest a way to get an even closer approximation?

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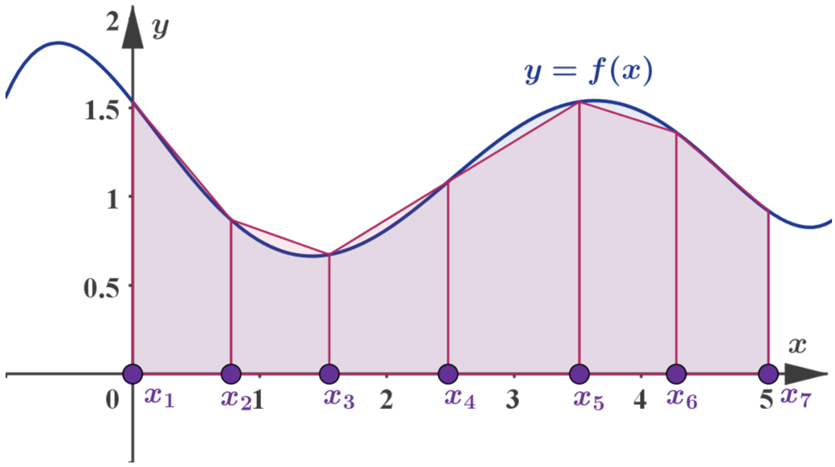
This method of finding the area under a curve will now be formalised by introducing some notation. Let $y = f(x)$ be the equation of the graph and let $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ be the points on the x-axis as shown in the diagram below.



More information

The graph depicts the function (y = f(x)) with the x-axis ranging from 0 to 5, marked with points (x_1) through (x_7). The y-axis is labeled from 0 to 2. The curve of (y = f(x)) is shown in blue, with peaks approximately at (x_1) and (x_5), and a noticeable trough around (x_4). The area under the curve is divided into trapezoidal sections between the x-axis points, highlighted in purple. For example, between (x_1) and (x_2), a trapezoid is formed with heights (f(x_1)) and (f(x_2)). This process continues across the horizontal sections defined by the x-points.

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The first trapezoid has parallel sides of length $f(x_1)$ and $f(x_2)$ and height $x_2 - x_1$, so the area is

$$\frac{(f(x_1) + f(x_2))(x_2 - x_1)}{2}.$$

Writing similar expressions for the areas of the other trapezoids, gives an approximation of the area of the region.

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Important

Consider a function f defined over the interval $[a, b]$ with the property that $f(x) > 0$ for all $a \leq x \leq b$. Consider also some numbers

$$a = x_1 < x_2 < \cdots < x_n = b.$$

Then the sum

$$\frac{(f(x_1) + f(x_2))(x_2 - x_1)}{2} + \cdots + \frac{(f(x_{n-1}) + f(x_n))(x_n - x_{n-1})}{2}$$

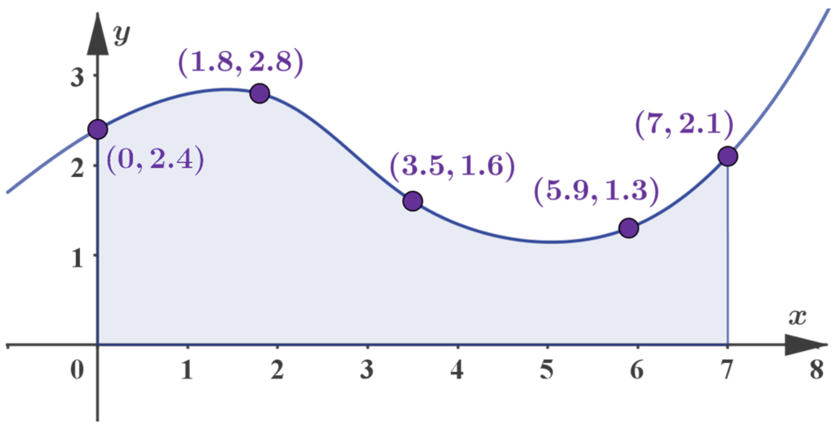
is an approximation of the area of the region bounded by the x-axis and the graph of f over the interval $[a, b]$.

It is beyond the scope of this course to investigate the difference between the actual area and the estimate obtained by using this formula.

Example 1

★☆☆

The diagram below shows a region below the graph of a function along with some points on the graph.



More information

The image is a graph representing a function with a shaded region below the curve. The X-axis ranges from 0 to 8 and the Y-axis from 0 to 4. Several points are labeled on the graph: (0, 2.4), (1.8, 2.8), (3.5, 1.6), (5.9, 1.3), and (7, 2.1). The graph is a smooth curve that rises from the origin towards the right. It's labeled with the points where the curve reaches

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or changes direction, indicating peaks and troughs in the function. The Y-axis represents the function's value, and the X-axis represents the domain of input values.

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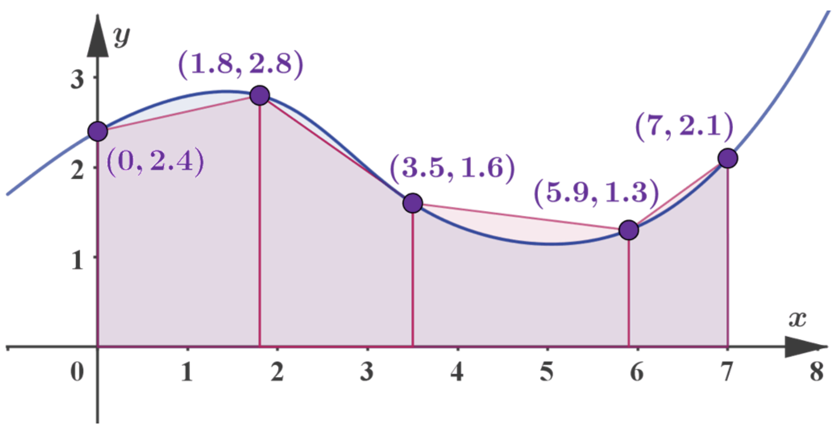
The information about the points is also summarised in the table below.

x	0	1.8	3.5	5.9	7
$f(x)$	2.4	2.8	1.6	1.3	2.1

Estimate the area of the shaded region.

The diagram below shows the trapezoids used for the estimate.



- The area of the first trapezoid is $\frac{(2.4 + 2.8)(1.8 - 0)}{2} = 4.68$.
- The area of the second trapezoid is $\frac{(2.8 + 1.6)(3.5 - 1.8)}{2} = 3.74$.

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- The area of the third trapezoid is $\dfrac{(1.6+1.3)(5.9-3.5)}{2}=3.48$.

- The area of the fourth trapezoid is $\dfrac{(1.3+2.1)(7-5.9)}{2}=1.87$.

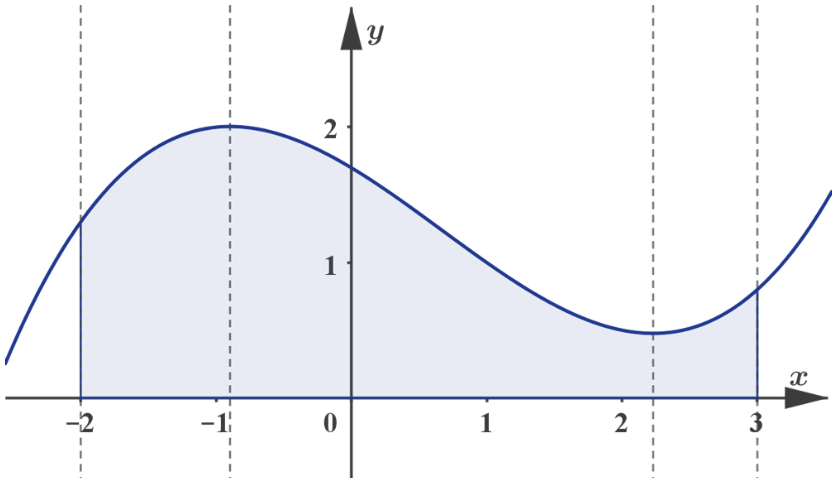
The approximate area of the region is the sum of these four areas,

$$4.68+3.74+3.48+1.87=13.77\text{squared units.}$$

Example 2

★★☆

The diagram below shows part of the graph of $y=0.1x^3-0.2x^2-0.6x+1.7$ and a region bounded by the graph, the x-axis and the vertical lines $x=-2$ and $x=3$.



More information

The image is a graph showing the function (y=0.1x^3-0.2x^2-0.6x+1.7). The X-axis ranges from (x=-2) to (x=3), with the curve intersecting the X-axis around these points, forming a closed region. The Y-axis is labeled, with approximate markings at 1 and 2. The graph shows a cubic curve starting from the left, rising to a peak, then dipping to a minimum, and rising again towards the right. The region under the curve, between (x=-2) and (x=3), and above the X-axis is shaded. Vertical dotted lines are drawn at (x=-2), (x=0), and (x=3), indicating boundaries and critical points. These lines also mark the maximum and minimum points on the curve within the shaded region.

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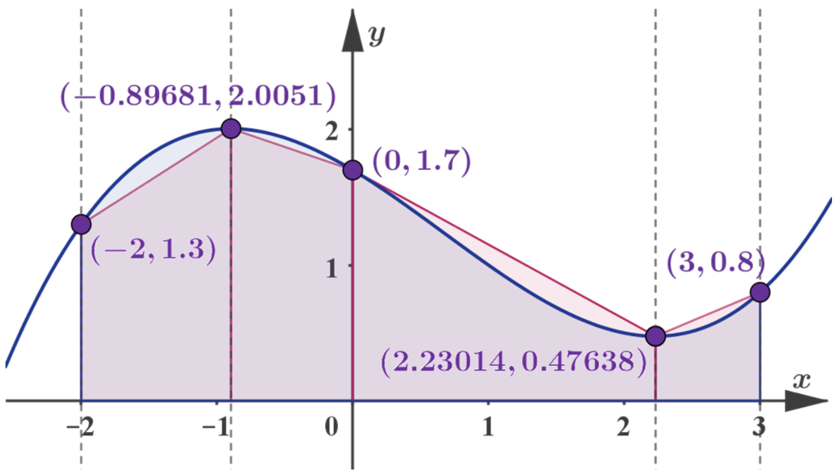
The diagram also shows the vertical lines through the local maximum and minimum points of the graph.

- Using the five vertical lines on the diagram, draw four trapezoids that can be used to estimate the area of the shaded region.
- Find an estimate of the area using these trapezoids.

Graphic display calculators have applications that can find the y-intercept and the local maximum and minimum points on a graph. The diagram below shows the trapezoids to use for the approximation and the coordinates of the vertices of these trapezoids.



- The area of the first trapezoid is $\dfrac{(2.0051+1.3)(-0.89681-(-2))}{2} \approx 1.82308.$
- The area of the second trapezoid is $\dfrac{(2.0051+1.7)(0-(-0.89681))}{2} \approx 1.66139.$
- The area of the third trapezoid is $\dfrac{(1.7+0.47638)(2.23014-0)}{2} \approx 2.42682.$
- The area of the fourth trapezoid is $\dfrac{(0.47638+0.8)(3-2.23014)}{2} \approx 0.49132.$

The approximate area of the region is the sum of these four areas,

$$1.82308 + 1.66139 + 2.42682 + 0.49132 \approx 6.40 \text{squared units.}$$

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Example 3

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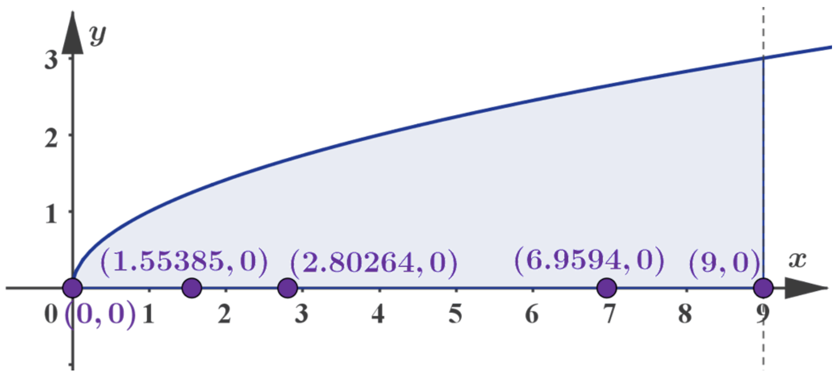
The diagram below shows the region bounded by the graph of $y = \sqrt{x}$, the x-axis and the line $x = 9$.



More information

The image is a graph depicting the region bounded by the curve of (y=\sqrt{x}), the (x)-axis, and the vertical line (x=9). The graph has a black horizontal (x)-axis labeled with numbers from 0 to 9, with significant points labeled. The (y)-axis is vertical and labeled with numbers 0, 1, 2, and 3. The curve (y=\sqrt{x}) begins at the origin ((0,0)) and gradually rises to approximate the coordinate ((9,3)), but it actually intersects the line (x=9) at the coordinate ((9,0)). The shaded region between the curve and the (x)-axis is highlighted in blue. Key x-intercepts on the curve (y=\sqrt{x}) and x-axis, such as ((1.55385,0)), ((2.80264,0)), and ((6.9594,0)), are marked. This illustrates the area under the curve between (x=0) and (x=9).

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- Find the area of the shaded region.
- Use your calculator to generate three random numbers between 0 and 9. Use 0, 9 and these three random numbers to draw four trapezoids to estimate the area of the shaded region. Find an estimate of the area using these trapezoids.
- What is the percentage error of this estimate?

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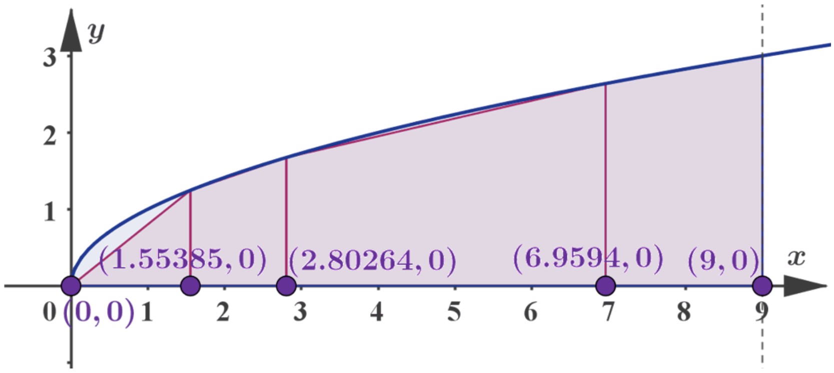
- The area of the shaded region is $\int_0^9 \sqrt{x}\mathrm{d}x = 18$.
- The diagram below shows three random numbers on the number line and the four trapezoids corresponding to these numbers. Note that the leftmost trapezoid is actually a triangle, since the length of one of the sides is $\sqrt{0} = 0$.



The table below contains all information needed to find the areas of the trapezoids. The numbers in your table and in the subsequent calculation will be different if your calculators generated different random numbers.

x	0	1.55385	2.80264	6.9594
$f(x) = \sqrt{x}$	0	1.24654	1.67411	2.63807

- The area of the triangle is $\frac{(0+1.24654)(1.55385-0)}{2} \approx 0.96847$.
- The area of the second trapezoid is $\frac{(1.24654+1.67411)(2.80264-1.55385)}{2} \approx 1.82364$.
- The area of the third trapezoid is $\frac{(1.67411+2.63807)(6.9594-2.80264)}{2} \approx 8.96235$.
- The area of the fourth trapezoid is $\frac{(2.63807+3)(9-6.9594)}{2} \approx 5.75252$.

  The approximate area of the region is

  $$0.96847 + 1.82364 + 8.96235 + 5.75252 \approx 17.5.$$

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- The percentage error of the estimate is $\left|\frac{17.5-18}{18}\right| \times 100\% \approx 2.78\%$.

Example 4

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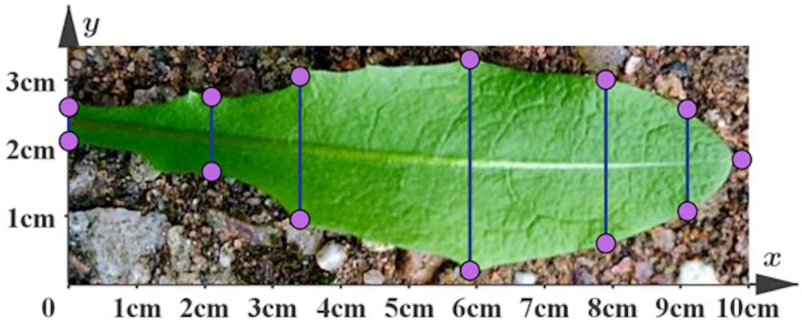
The diagram below shows some measurements of a leaf.



More information

The diagram presents a measurement layout of a leaf using a coordinate system with x and y axes. The x-axis is horizontal, labeled from 0 cm to 10 cm, and the y-axis is vertical, labeled from 0 cm to 3 cm. Both axes have arrows indicating positive directions. The leaf is positioned diagonally across the image. There are multiple purple dots along the leaf's edges at even intervals, connected vertically by blue lines to denote measurement points. The measurements are taken at 1 cm intervals along the x-axis till 10 cm. Along the y-axis, the heights of the measurement points vary, intersecting the leaf's linearly from the left edge at approximately 1 cm height and reaching up to 3 cm height on the right side, indicating varying widths of the leaf across its length.

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The table below shows the position and values of the measurements.

position (cm)	0	2.1	3.4	5.9	7.9	9.1	9.9
width (cm)	0.5	1.1	2.1	3.1	2.4	1.5	0

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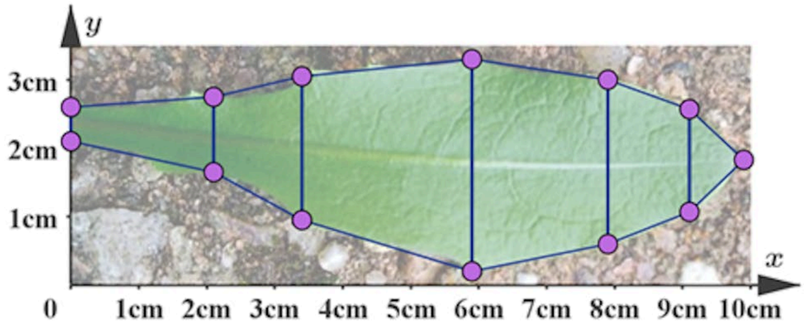
Estimate the surface area of the leaf.

You can connect the endpoints of the line segments on the diagram to form trapezoids. The sum of the areas of the trapezoids gives an estimate of the surface area of the leaf.



- The area of the leftmost trapezoid is $$\frac{(2.1-0)(1.1+0.5)}{2}=1.68\text{ cm}^2.$$
- The area of the next trapezoid is $$\frac{(3.4-2.1)(2.1+1.1)}{2}=2.08\text{ cm}^2.$$
- The area of the next trapezoid is $$\frac{(5.9-3.4)(3.1+2.1)}{2}=6.5\text{ cm}^2.$$
- The area of the next trapezoid is $$\frac{(7.9-5.9)(2.4+3.1)}{2}=5.5\text{ cm}^2.$$
- The area of the next trapezoid is $$\frac{(9.1-7.9)(1.5+2.4)}{2}=2.34\text{ cm}^2.$$
- The area of the triangle on the right is $$\frac{(9.9-9.1)\times 1.5}{2}=0.6\text{ cm}^2.$$

Hence, an estimate for the area of the leaf is

$$1.68+2.08+6.5+5.5+2.34+0.6=18.7\text{ cm}^2$$

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Trapezoidal rule with uniform spacing

Instead of arbitrarily choosing the division points on the x-axis, you can do this according to a pattern. Take a look at the applet below.

Interactive 1. Trapezoidal Rule with Uniform Spacing.

More information for interactive 1

The interactive allows users to explore the approximation of the area under a curve over using trapezoids. The screen is divided in two halves. The top half contains a graph with X-axis ranging from 0 to 5 and a Y-axis ranging from 0 to 2. It contains a curve marked by a blue line. On the bottom half of the screen, there are two buttons, namely, Adjust curve and Number the trapezoids. When the Adjust curve is selected, the 5 red dots appear on the XY plane, which can be dragged along the y-axis by the user to modify the curve. Below the buttons, the area of the curve region is mentioned.
When the Number of trapezoids button is selected, a horizontal slider appears on the right side for the user to modify the number of trapezoids displayed, ranging from 1 to 10, projecting onto the graph. The Trapezoids are equally distributed on the X-axis. The applet provides both the exact area of the region and the approximate area calculated using the trapezoids. Below the Number of Trapezoids button, the approximated area of the trapezoids is displayed. Users can modify the shape of the curve and adjust the number of the trapezoids to observe how these changes affect the accuracy of the approximation. This interactive tool provides a hands-on way to understand numerical integration techniques, specifically the trapezoidal rule, and how it can be used to estimate the area under a curve.

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Activity

- Can you explain how the trapezoids were constructed?
- The total area of the trapezoids is close to the area of the region. Can you suggest a way to get an even closer approximation?

In the applet above the points are chosen so as to divide the interval $[0, 5]$ evenly.

Important

Consider a function f defined over the interval $[a, b]$ with the property that $f(x) > 0$ for all $a \leq x \leq b$. Consider also some numbers

$$a = x_0 < x_1 < \cdots < x_n = b$$

so that

$$x_1 - x_0 = x_2 - x_1 = \cdots = x_n - x_{n-1} = \frac{b-a}{n}.$$

Then the sum

$$\frac{b-a}{n}\left(\frac{f(x_0)+f(x_1)}{2} + \frac{f(x_1)+f(x_2)}{2} + \cdots + \frac{f(x_{n-1})+f(x_n)}{2}\right)$$

is an approximation of the area of the region bounded by the x-axis and the graph of f over the interval $[a, b]$.

This sum can also be written in the form

$$\frac{1}{2}\frac{b-a}{n}\Big((f(x_0)+f(x_n)) + 2(f(x_1)+\cdots+f(x_{n-1}))\Big)$$

Can you prove this?

$$\begin{aligned}
&\frac{b-a}{n}\left(\frac{f(x_0)+f(x_1)}{2} + \frac{f(x_1)+f(x_2)}{2} + \cdots + \frac{f(x_{n-1})+f(x_n)}{2}\right) \\
&= \frac{1}{2}\frac{b-a}{n}\Big((f(x_0)+f(x_1) + (f(x_1)+f(x_2)) + \cdots + (f(x_{n-1})+f(x_n))\Big) \\
&= \frac{1}{2}\frac{b-a}{n}\Big((f(x_0)+f(x_n)) + 2(f(x_1)+f(x_2)+\cdots+f(x_{n-1}))\Big)
\end{aligned}$$

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Exam tip

In the formula booklet, the trapezoidal rule is given as

$$\int_a^b y\mathrm{d}x \approx \frac{1}{2}h\big((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\big), \text{ where } h = \frac{b-a}{n}.$$

Example 1

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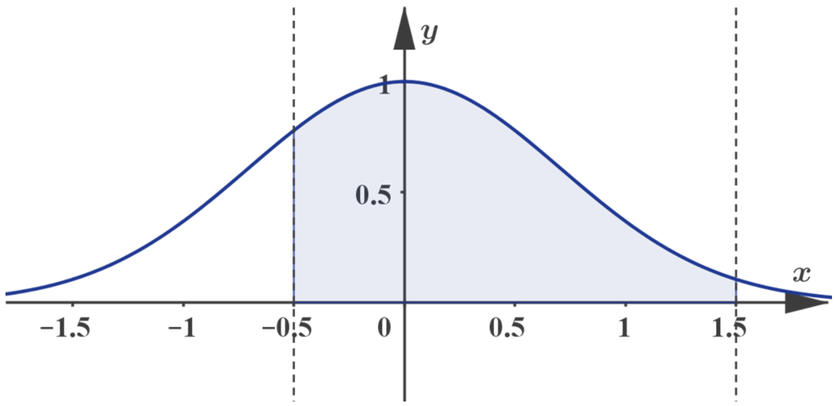
The diagram below shows part of the graph of $y = \mathrm{e}^{-x^2}$ and the region bounded by this graph, the
x-axis and the lines $x = -0.5$ and $x = 1.5$.



More information

The image is a graph of the function y = e^(-x^2). The x-axis is marked with values ranging from -1.5 to 1.5, while the y-axis is marked from 0 to 1. The curve depicted is a bell-shaped curve that peaks at x = 0, where y = 1, and gradually decreases symmetrically on either side.

The highlighted region is bounded by the curve y = e^(-x^2), the x-axis, and the vertical lines x = -0.5 and x = 1.5. This region forms a curved shape that is symmetrical along the y-axis. The curve crosses the x-axis at approximately x = -1.5 and x = 1.5. The overall shape illustrates the concepts of integration or area under a curve, specifically demonstrating the bounded area from x = -0.5 to x = 1.5 under the described function.

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- Find the area of the shaded region using a graphic display calculator. Round your answer to five decimal places.
- Find four numbers (not including the endpoints) that divide the interval $[-0.5, 1.5]$ evenly.
- Use the trapezoids corresponding to this division to approximate the area of the shaded region. Use the same accuracy as in the first part of the question.
- What is the percentage error made by this approximation?

- The GDC gives the area as $\int_{-0.5}^{1.5} \mathrm{e}^{-x^2} \mathrm{d}x \approx 1.31747$.
- The width of each dividing interval is $\frac{1.5-(-0.5)}{5} = 0.4$, so the four numbers are
$$-0.5+0.4=-0.1, -0.1+0.4=0.3, 0.3+0.4=0.7, 0.7+0.4=1.1.$$
Hence, the approximate area is
$$\begin{aligned} 0.4\Bigg(&\frac{\mathrm{e}^{-(-0.5)^2}+\mathrm{e}^{-(-0.1)^2}}{2}+\frac{\mathrm{e}^{-(-0.1)^2}+\mathrm{e}^{-0.3^2}}{2} \\ &+\frac{\mathrm{e}^{-0.3^2}+\mathrm{e}^{-0.7^2}}{2}+\frac{\mathrm{e}^{-0.7^2}+\mathrm{e}^{-1.1^2}}{2} \\ &+\frac{\mathrm{e}^{-1.1^2}+\mathrm{e}^{-1.5^2}}{2}\Bigg) \approx 1.30276. \end{aligned}$$
You can also get the same answer using the formula from the formula booklet:
$$\begin{aligned} \frac{1}{2} \times 0.4\Big(&\left(\mathrm{e}^{-(-0.5)^2}+\mathrm{e}^{-1.5^2}\right) \\ &+2\left(\mathrm{e}^{-(-0.1)^2}+\mathrm{e}^{-0.3^2}+\mathrm{e}^{-0.7^2}+\mathrm{e}^{-1.1^2}\right)\Big) \approx 1.30276. \end{aligned}$$
- The percentage error of this estimate is
$$\left|\frac{1.30276-1.31747}{1.31747}\right| \times 100\% \approx 1.12\%.$$

Using technology

Some GDCs can work with spreadsheets. Other calculators can work with sequences. Calculators also understand the sigma notation to add a sequence of numbers. All these options can be used to find the sum in the trapezoidal rule. These options are useful, when you need to use a lot of trapezoids.

Student view

For a few number of trapezoids the manual method in Example 1 can also be carried out.

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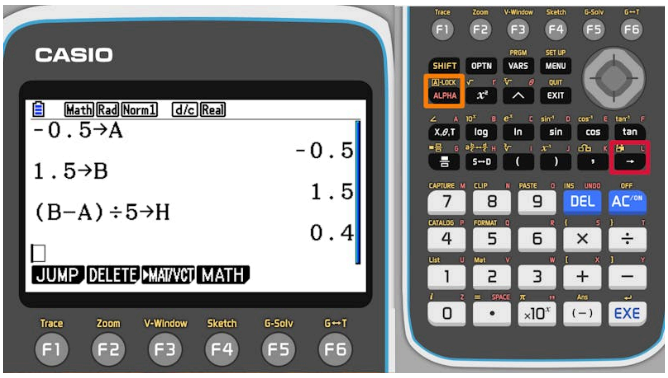
Below are instructions for different models of calculator that explain how you can use your calculator to find the approximations. The method is between the manual calculation of Example 1 and the full use of the sequence/spreadsheet capabilities of the calculators.

Steps	Explanation
These instructions will show you one way to find the solution to Example 1. We use the formula from the formula booklet. An alternative way is to find the areas of the individual trapezoids and add them up. Start the work in calculator mode.	
It is not necessary, but it can be helpful to store the bounds of the integral and the width of the dividing intervals in the memory. Note, that the denominator of the interval width is 5, because in this example you are asked to use five intervals. Press the store button ($\rightarrow$) to store the values and use the ALPHA button to access the letters.	

Student view

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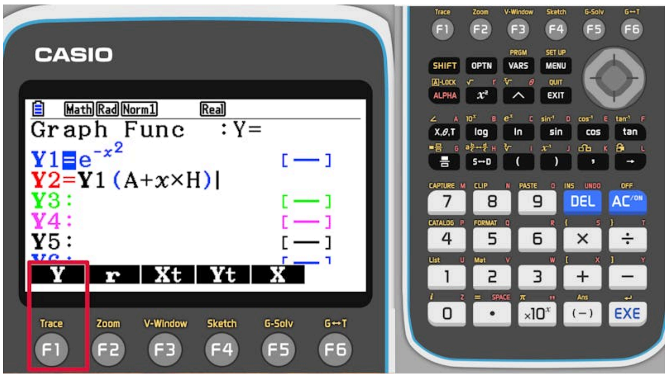
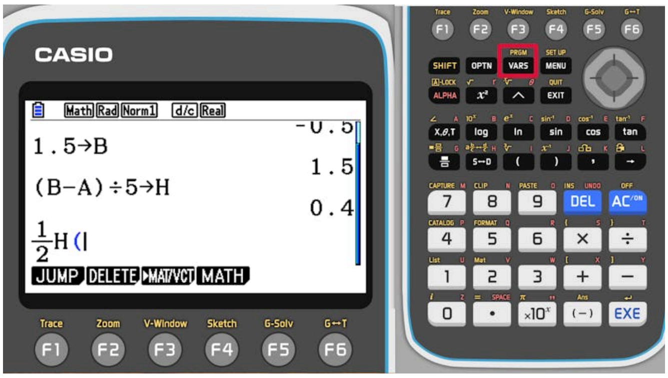
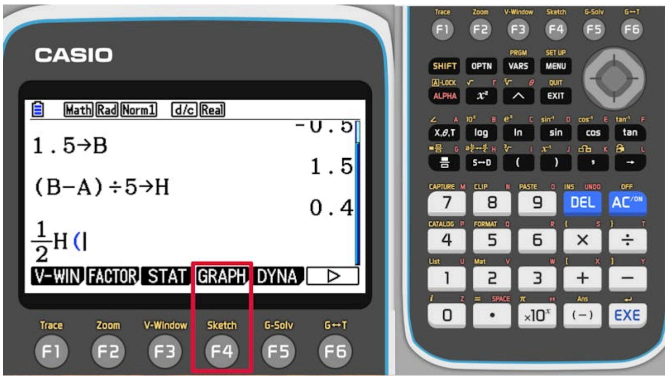
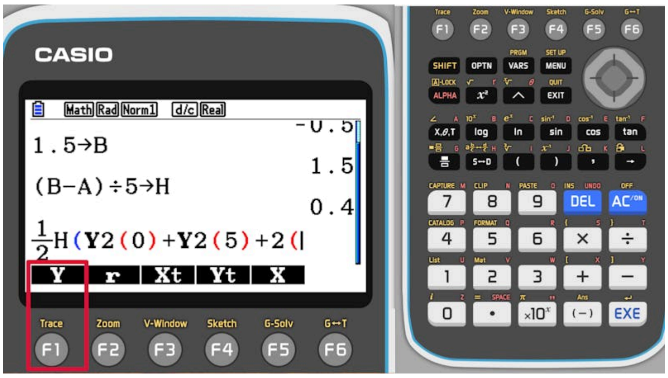
Steps	Explanation
Go back to the main menu and select the function mode to tell the calculator the function you want to work with.	
In $Y1$ (or in any other line), enter the definition of the function. Function $Y2$ calculates the values $$y_0, y_1, y_2, \ldots$$ Note, that A is the lower bound of the integral, H is the step size (you stored the values in the memory in the previous step), so $A + k \times H$ is x_k, the kth dividing point. $Y1(A + k \times H)$ is $y_k = f(x_k)$. These are the values you need in the formula. Note also, that in the definition of $Y2$ you needed to use the name of the first function. You can access this by pressing F1.	

Student
view

Steps	Explanation
Go back to the main menu and choose the calculator mode again.	
Start typing in the formula. When you reach the point to enter y_0, you need the name of $Y2$ that you defined in the previous step. To access this name, press VARS ...	

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>... and press F4 to access the variables related to graphs.</td><td>
</td></tr>
<tr><td>You can use F1 to enter the function name.

In this example $n = 5$ (you use five intervals).

Remember, you defined $Y2$ in the previous step to give you the values $y_0, y_1, ...$</td><td>
</td></tr>
</table>

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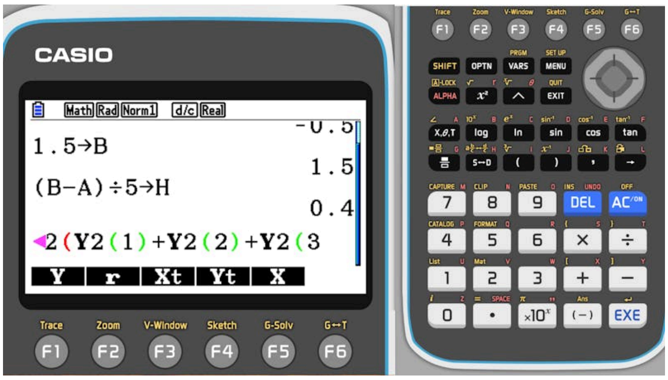
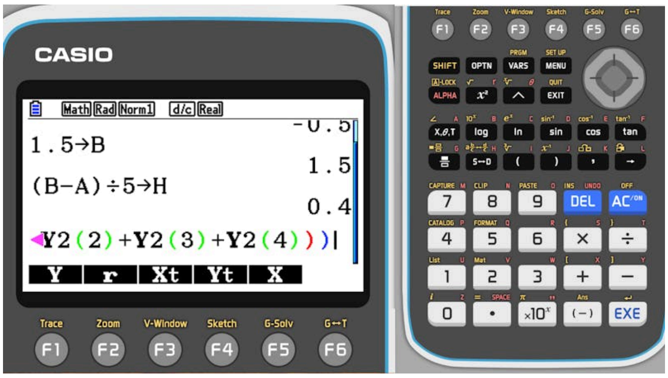
Steps	Explanation
... keep on typing, the formula is too long to fit on one screen ...	
When you finished the formula, make sure you close all the parentheses. The colors will help you with the number of closing parentheses you need.	

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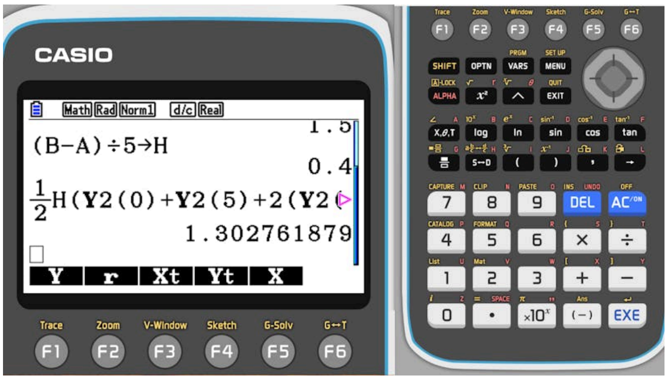
Steps	Explanation
After pressing enter you get the answer.	

Steps	Explanation
These instructions will show you one way to find the solution to Example 1. We use the formula from the formula booklet. An alternative way is to find the areas of the individual trapezoids and add them up. Start the work by opening the function application.	

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view

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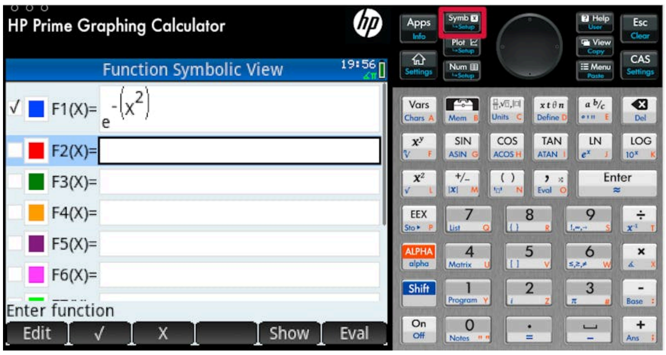
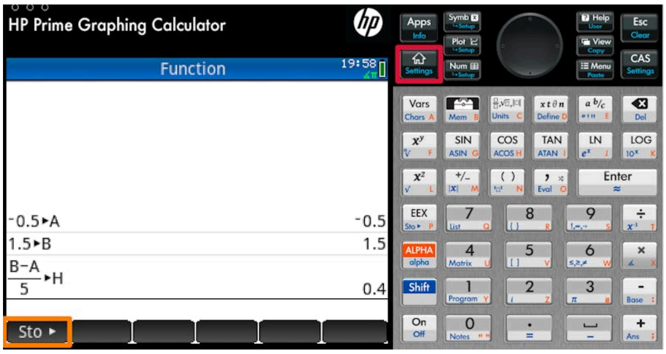
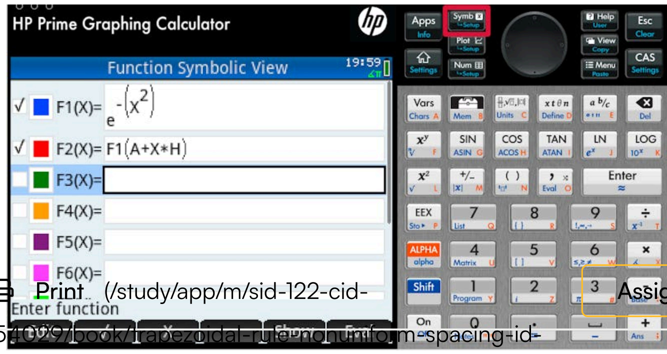
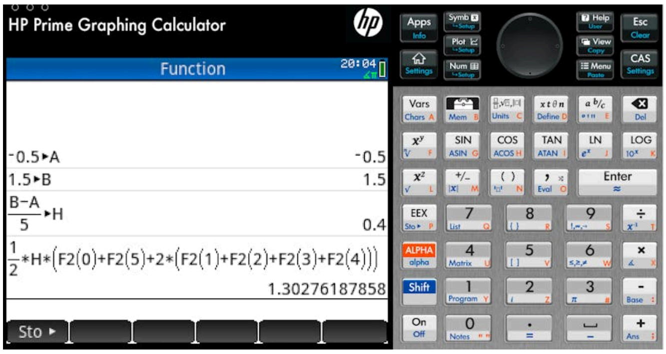
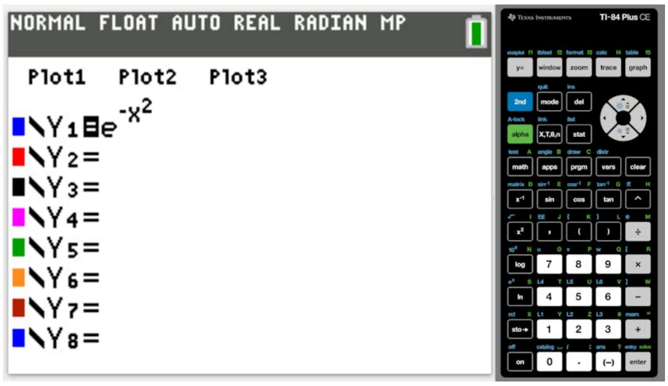
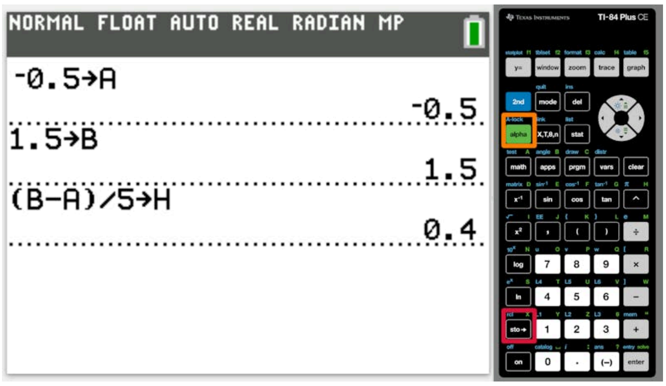
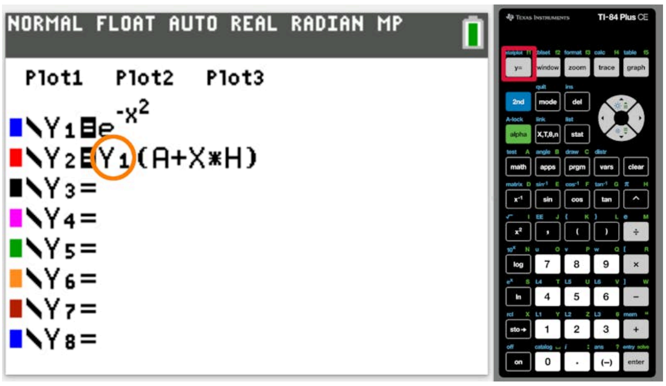
Steps	Explanation
In symbolic view enter the definition of the function.	
Enter the home screen and store the bounds of the integral and the width of the dividing intervals in the memory. This is not necessary, but it can help if you get used to storing values that you may want to use multiple times. Note, that the denominator of the interval width is 5, because in this example you are asked to use five intervals. Use the ALPHA key to access variable names.	

Student
view

Steps	Explanation
Go back to symbolic view and define a second function. Function $F2$ calculates the values $y_0, y_1, y_2, \ldots$ Note, that A is the lower bound of the integral, H is the step size (you stored the values in the memory in the previous step), so $A + k \times H$ is x_k, the kth dividing point. $F1(A + k \times H)$ is $y_k = f(x_k)$. These are the values you need in the formula.	
Type in the formula and press enter to see the answer. In this example $n = 5$ (you use five intervals). Remember, you defined $F2$ in the previous step to give you the values $y_0, y_1, \ldots$	

Steps	Explanation
These instructions will show you one way to find the solution to Example 1. We use the formula from the formula booklet. An alternative way is to find the areas of the individual trapezoids and add them up. Start the work by opening the function editor ...	
... and enter the definition of the function.	

Steps	Explanation
Go back to the main calculator screen. It is not necessary, but it can be helpful to store the bounds of the integral and the width of the dividing intervals in the memory. Note, that the denominator of the interval width is 5, because in this example you are asked to use five intervals. Press the store button (sto$\rightarrow$) to store the values and use the alpha button to access the letters.	
Go back to the function editor and define a second function. Function Y_2 calculates the values $$y_0, y_1, y_2, \ldots$$ Note, that A is the lower bound of the integral, H is the step size (you stored the values in the memory in the previous step), so $A + k \times H$ is x_k, the kth dividing point. $Y_1(A + k \times H)$ is $y_k = f(x_k)$. These are the values you need in the formula. To enter this expression, you need the name of the function you defined first. The next screeshot shows you how to access this.	

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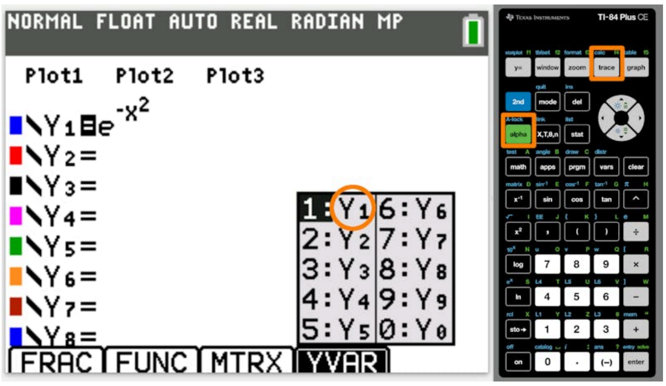
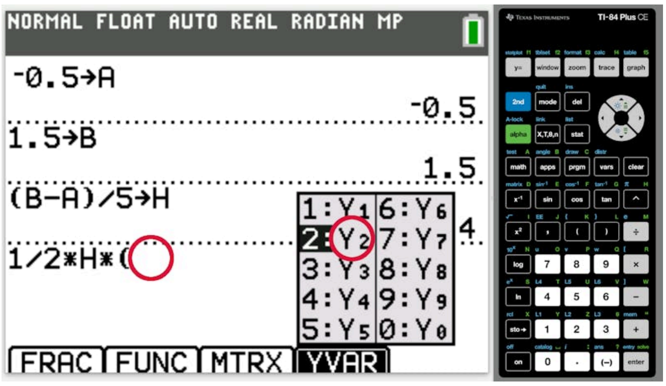
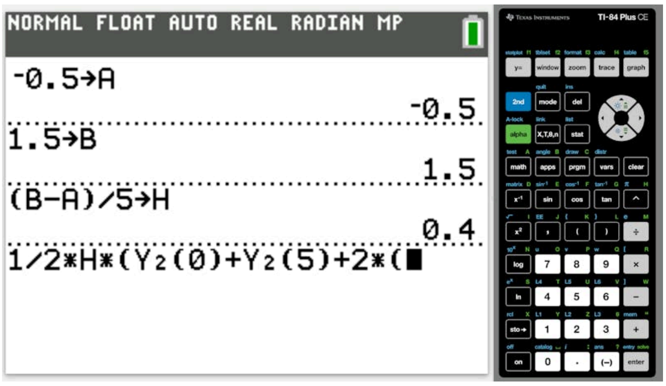
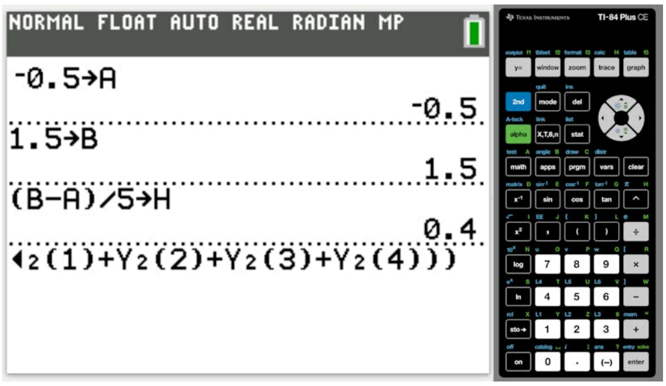
Steps	Explanation
This screenshot is a help on how to access the function name you need in entering the definition of Y_1.	
You are now ready to find the answer to the question in Example 1. Go back to the main screen and start typing the formula. When you reach the point to enter y_0, you need the name of Y_2 that you defined in the previous step.	

Student view

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>... keep on typing, the formula is too long to fit on one screen ...</td><td>
</td></tr>
<tr><td>When you finished the formula, make sure you close all the parentheses.</td><td>
</td></tr>
</table>

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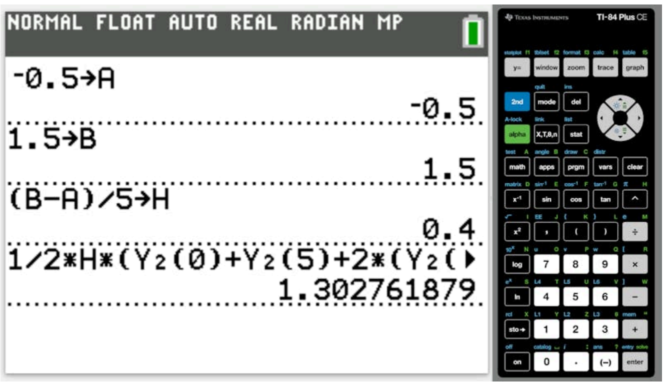
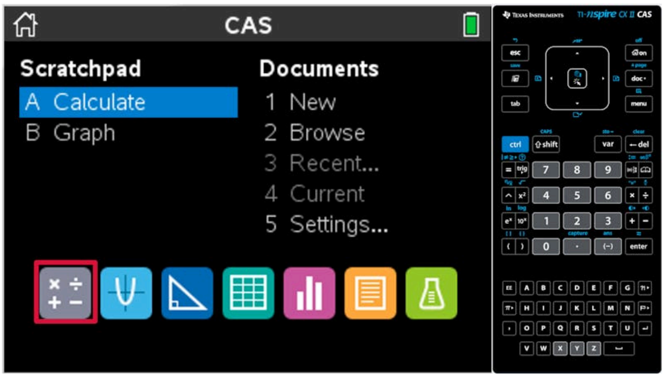
Steps	Explanation
After pressing enter you get the answer.	

Steps	Explanation
These instructions will show you one way to find the solution to Example 1. We use the formula from the formula booklet. An alternative way is to find the areas of the individual trapezoids and add them up. Open a calculator page.	

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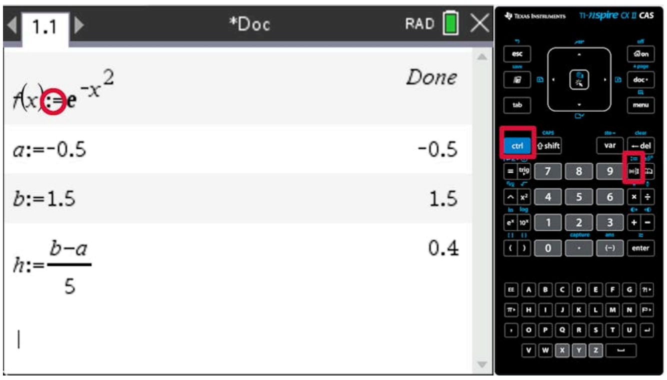
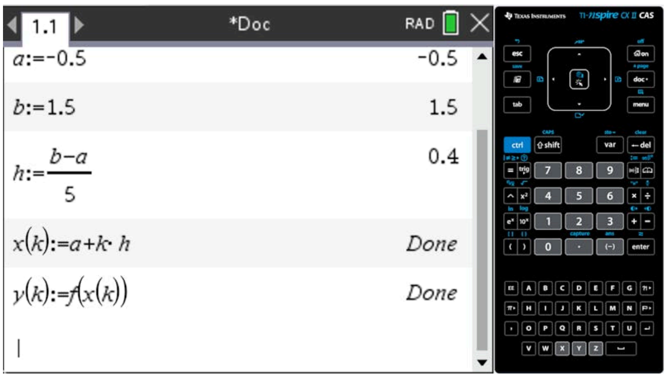
Steps	Explanation
You can store the definition of the function, the limits of the integral and the step size. Make sure you use the colon/equal notation for the definitions. Note, that the denominator of the interval width is 5, because in this example you are asked to use five intervals.	
You can also store the definition of the dividing points, and the value of the function at those dividing points. $$y_k = f(x_k)$$	

Student
view

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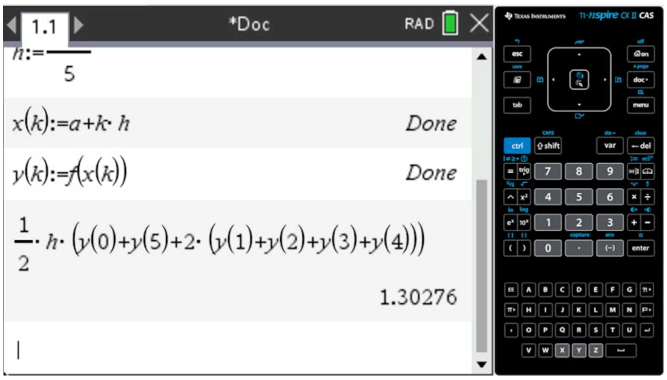
Steps	Explanation
Type in the formula (use $y(k)$ for y_k) and press enter to see the answer.	

More advanced technology can get this approximation more quickly. If you are familiar with GeoGebra, you can experiment with the TrapezoidalSum command (https://wiki.geogebra.org/en/TrapezoidalSum_Command). This, of course, cannot be used in exams, but it can give you a quick check of the answers that you get using your calculator.

Example 2

★★☆

Each line of the table below gives a description of a region. Find an approximate area between the bounding curve and the x-axis using the specified number of trapezoids. Give your answer rounded to 5 significant figures.

Bounding curve	Domain	Number of trapezoids	Approximate area
$y = \ln(x+3) + (\ln(9-x))^2$	$[-2, 8]$	2	
$y = \ln(x+3) + (\ln(9-x))^2$	$[-2, 8]$	5	
$y = \ln(x+3) + (\ln(9-x))^2$	$[-2, 8]$	10	

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Bounding curve	Domain	Number of trapezoids	Approximate area
$y = \sqrt{(x-10)^2(x-20)^4+1000}$	$[9, 22]$	3	
$y = \sqrt{(x-10)^2(x-20)^4+1000}$	$[9, 22]$	6	
$y = \sqrt{(x-10)^2(x-20)^4+1000}$	$[9, 22]$	9	

The method is the same as in the previous example. The table below shows the results.

Bounding curve	Domain	Number of trapezoids	Approximate area
$y = \ln(x+3) + \ln(9-x)^2$	$[-2, 8]$	2	45.380
$y = \ln(x+3) + \ln(9-x)^2$	$[-2, 8]$	5	46.673
$y = \ln(x+3) + \ln(9-x)^2$	$[-2, 8]$	10	46.829
$y = \sqrt{(x-10)^2(x-20)^4+1000}$	$[9, 22]$	3	1278.9
$y = \sqrt{(x-10)^2(x-20)^4+1000}$	$[9, 22]$	6	1167.6
$y = \sqrt{(x-10)^2(x-20)^4+1000}$	$[9, 22]$	9	1108.4

You can check your understanding using the applet below.

Interactive 1. Find an approximate area using the specified number of trapezoids.

More information for interactive 1

Student view

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This interactive tool allows users to practice approximating the area under a curve using the trapezoidal rule. Users can generate new questions automatically by clicking on “click here for a new question” at the top, which will prompt them to use a specified number of trapezoids n to approximate the area of a region over a given interval $[a, b]$, bounded by the graph of a function $f(x)$ and the x-axis. After attempting the problem, users can verify whether their calculated answer is correct by comparing it with the tool's solution. The tool provides immediate feedback, allowing users to assess their accuracy and understand any errors.

For example:

To approximate the area under the curve $f(x) = 4x^2 + \frac{6}{x^3} + 5$ over the interval $[0.150, 1.150]$ using 5 trapezoids,

Step 1: Determine the width of each trapezoid ($\triangle x$)

$x_0 = 0.150$

$x_1 = 0.150 + 0.200 = 0.350$

$x_2 = 0.350 + 0.200 = 0.550$

$x_3 = 0.550 + 0.200 = 0.750$

$x_4 = 0.750 + 0.200 = 0.950$

$x_5 = 0.950 + 0.200 = 1.150$

Step 3: Compute $f(x)$ at each x_i

$$f(x) = 4x^2 + \frac{6}{x^3} + 5$$

$$f(0.150) = 4(0.150)^2 + \frac{6}{(0.150)^3} + 5 \approx 0.090 + 1777.778 + 5 = 1782.868$$
$$f(0.350) = 4(0.350)^2 + \frac{6}{(0.350)^3} + 5 \approx 0.490 + 139.841 + 5 = 145.331$$
$$f(0.550) = 4(0.550)^2 + \frac{6}{(0.550)^3} + 5 \approx 1.210 + 36.036 + 5 = 42.246$$
$$f(0.750) = 4(0.750)^2 + \frac{6}{(0.750)^3} + 5 \approx 2.250 + 14.222 + 5 = 21.472$$
$$f(0.950) = 4(0.950)^2 + \frac{6}{(0.950)^3} + 5 \approx 3.610 + 7.004 + 5 = 15.614$$
$$f(1.150) = 4(1.150)^2 + \frac{6}{(1.150)^3} + 5 \approx 5.290 + 3.942 + 5 = 14.232$$

Student view

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Step 4: Apply the trapezoidal rule formula

$$\text{Area} \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5)]$$

$$\approx \frac{0.200}{2}[1782.868 + 2(145.331) + 2(42.246) + 2(21.472) + 2(15.614) + 14.232]$$

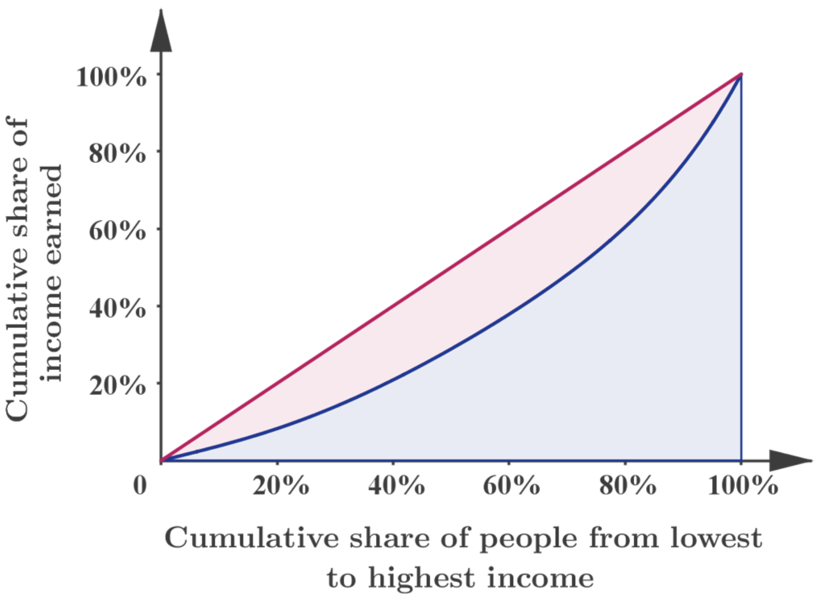
$$\text{Area} \approx 0.100[1782.868 + 290.662 + 84.492 + 42.944 + 31.228 + 14.232]$$

$$\text{Area} \approx 0.1002246.426 \approx 224.6426$$

Example 3

★★☆

In economics, the Lorenz curve is a graphical illustration of wealth. The blue curve on the diagram below is a typical Lorenz curve.



More information

The image is a graph representing a Lorenz curve, which illustrates the distribution of income across a population. The X-axis is labeled 'Cumulative share of people from lowest to highest income,' with intervals marked at 0%, 20%, 40%, 60%, 80%, and 100%. The Y-axis is labeled 'Cumulative share of income earned,' with similar markings from 0% to 100%.

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The graph features a blue curve, which represents a typical Lorenz curve, showing how income distribution deviates from perfect equality. The area under the blue curve represents the actual distribution of income. The red region above the blue curve highlights the disparity between perfect equality and the current distribution, which corresponds to the Gini index. The larger the red area, the greater the inequality in the distribution of income.

[Generated by AI]

A related concept is the Gini index, which is a measurement of inequality. The Gini index is calculated as the ratio of the area of the red region on the diagram to the sum of the areas of the red and blue regions.

The following table contains data about the distribution of income in Albania in 2012 using data from the World Bank (http://povertydata.worldbank.org/poverty/country/ALB).

Percentage share of income				
lowest 20%	second 20%	third 20%	fourth 20%	highest 20%
8.9	13.2	17.3	22.8	37.8

Find an approximation of the Gini index of Albania in 2012.

First create the table of the cumulative percentage share.

Cumulative percentage share of income				
lowest 20%	lowest 40%	lowest 60%	lowest 80%	100% of population
8.9	$8.9 + 13.2$ $= 22.1$	$22.1 + 17.3$ $= 39.4$	$39.4 + 22.8$ $= 62.2$	$62.2 + 37.8$ $= 100.0$

Using these values, you can approximate the area of the blue region by considering trapezoids for the function with values

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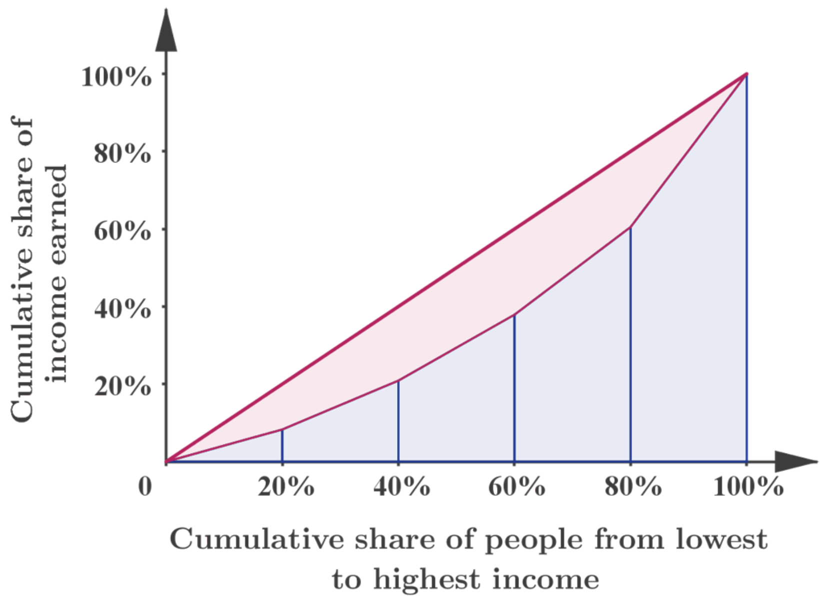
$$f(0) = 0$$
$$f(20) = 8.9$$
$$f(40) = 22.1$$
$$f(60) = 39.4$$
$$f(80) = 62.2$$
$$f(100) = 100$$



Find the total area of the blue trapezoids:

$$89 + 310 + 615 + 1016 + 1622 = 3652$$

The blue and red region together is a triangle, so the area is $\frac{100 \times 100}{2} = 5000$

Hence, the approximation of the Gini index is $\frac{5000 - 3652}{5000} = 0.2696 = 27.0\%$

3 section questions

Student view

5. Calculus / 5.8 Area of a region

Checklist

What you should know

By the end of this subtopic you should be able to:

- use trapezoids to estimate the area of a region below a graph of a function.
  - if the function is given
  - if only certain values of the function are given.

Investigation

Here are more approaches for approximating the area of a region.

Investigation 1

In the applet below, the region is inside an $8 \text{ cm} \times 8 \text{ cm}$ square. The applet gives the area of the region and also gives this area as a percentage of the area of the square. The applet generates random points inside this square and tells you how many of these points are inside the region. Can you find a way to estimate the area of the region using the data generated by the applet?

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Interactive 1. Find a Way to Estimate the Area of the Region Using the Data Generated.

More information for interactive 1

The interactive allows users to estimate the area of a region inside an organic shape. It shows the exact area of the region and what percentage this area is of the organic shape. The screen is divided into two halves. The right side has an organic shape marked by a closed blue line.

On the left side, there is an "Adjust Region" button. When selected, nine red dots appear on the boundary of the organic shape, allowing the user to modify its form by dragging them. Below the button, the area of the region and its percentage are displayed in blue. Below it, there is a "New Points" button that generates a green point at a random position on the right side of the screen. Below the button, a horizontal slider allows users to generate additional green points, ranging from 1 to 100. Below the slider, the number of points inside the region and the percentage of points landing inside are displayed in green.

Users can create different sets of random green points and see what percentage of these points land inside the region. By recording these percentages over several tries, users can analyze the data to estimate the region's area.

For example, if a user adjusts the organic shape to have an exact area of 36.57cm^2 (57.14% of the total shape) and then generates 60 random green points using the slider, they might find that 25 points land inside the region (41.67%). By repeating this process multiple times—say, generating 60 points five separate times—they could observe percentages like 38.33%, 43.33%, 41.67%, 45.00%, and 40.00%. Averaging these results (41.67%) and comparing them to the exact percentage (57.14%) helps users understand how random sampling can approximate an area while also revealing the limitations of Monte Carlo methods with small sample sizes.

Student
view

Activity

- Use the button to generate different sets of random points and record the percentage of the points that are inside the region.

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- What do you notice?
- Use statistical methods to investigate the data you generated.

Investigation 2

The applet below shows the graph of a function and the region bounded by this graph, the x-axis and the vertical lines $x = 0$ and $x = 5$. The applet also generates random numbers in the interval $[0, 5]$ and shows the average of the function values at the generated points. Can you find a way to approximate the area of the region using these average values?

Interactive 2. Exploring Area Approximation Using Random Points.

More information for interactive 2

Activity

- Use the button to generate different sets of random numbers and record the average of the function values.
- Consider also the width of the region.
- What do you notice?
- Use statistical methods to investigate the data you generated.

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