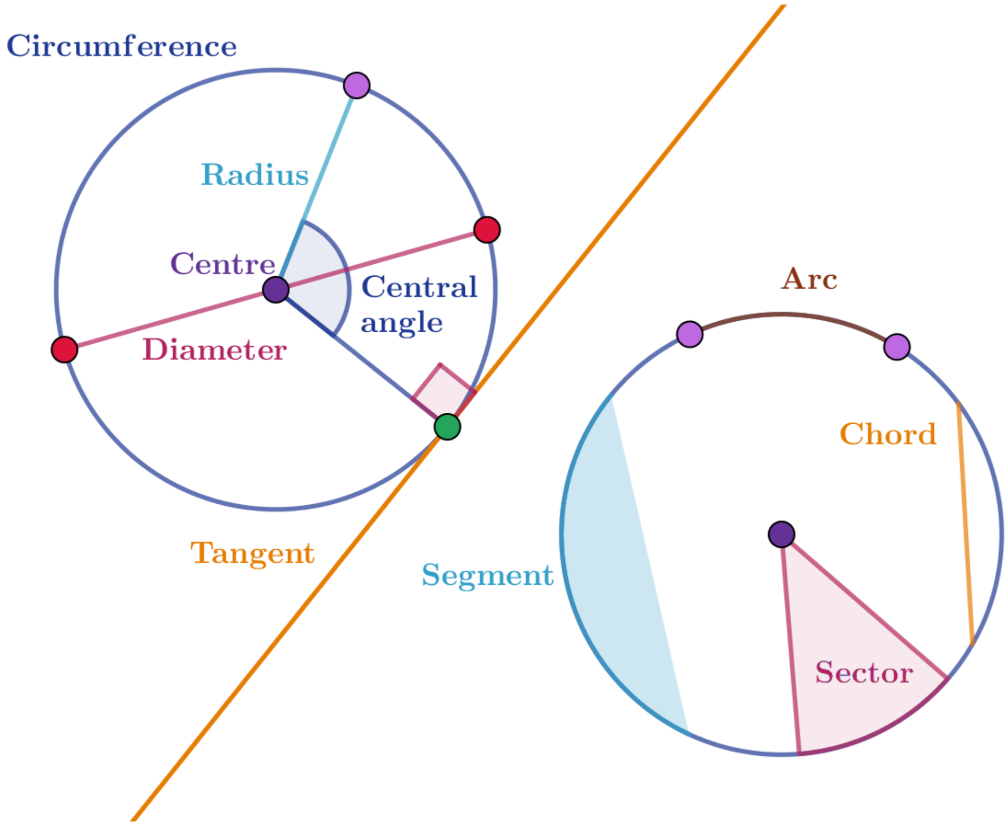
Overview
(/study/ap
122-
cid-
754029/
Teacher view

3. Geometry and trigonometry / 3.4 Circle

The big picture

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-27399/print/) Assign

A circle is a collection of points that are equidistant from a fixed point. The distance is called the radius and the fixed point is called the centre of the circle. Although a circle appears to have no parts, we can divide one into various sections using chords , arcs and central angles, as shown in the diagrams below.



More information

The image consists of two diagrams demonstrating different parts of a circle.

Student view

Overview
(/study/ap
122-
cid-
754029/

The first diagram (left side) shows a circle labeled with: - Centre: A point at the circle's center. - Radius: A line from the center to the circumference. - Diameter: A line passing through the center, touching two points of the circumference. - Circumference: The outer boundary of the circle. - Central Angle: An angle formed between two radii. - Tangent: A line touching the circle at exactly one point.

The second diagram (right side) displays: - Chord: A line segment whose endpoints lie on the circle. - Arc: A curved section of the circle's circumference. - Segment: The area between a chord and the arc it subtends. - Sector: The area enclosed by two radii and the arc between them.

These diagrams illustrate how different geometric parts form a circle.

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Concept

In this subtopic, you will study the relationships between the parts of a circle and their corresponding angles, lengths and areas. Why are all lengths and areas in circles approximate ? Why do we use the irrational number π ?

3. Geometry and trigonometry / 3.4 Circle

Length of an arc

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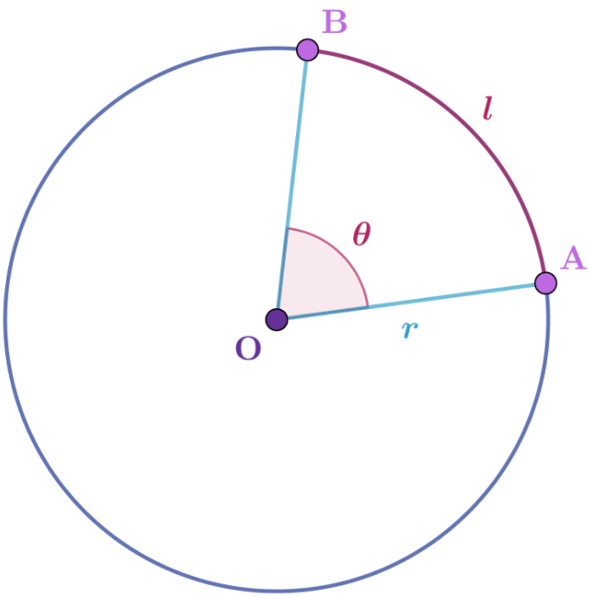
Ratios of length

An arc is part of a circle's circumference. The diagram below shows a circle with centre O and radius r . The points A and B on the circumference form two arcs, both called AB . The shorter one is called the minor arc as it is smaller than half of

Student view

Overview
(/study/ap
122-
cid-
754029/

the circumference of the circle. It subtends the central angle θ and is labelled l . The longer arc is called the major arc as it is more than half of the circumference of the circle.



More information

The diagram illustrates a circle centered at point O with a radius labeled r. There are two points, A and B, on the circumference of the circle that form two arcs. The shorter arc is labeled l, representing the minor arc, and it subtends a central angle θ at point O. The major arc goes the longer route around the circle from point A to point B. The diagram visually represents these concepts with arcs and a sector denoting the area covered by angle θ.

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Important

The circumference of a circle is $2\pi r$.

Student
view

Overview
(/study/ap
122-
cid-
754029/

The ratio of angle θ to the full rotation, 360° , is the same as the ratio for the arc length l to the circumference, $2\pi r$.

Thus,

$$\frac{l}{2\pi r} = \frac{\theta}{360^\circ}.$$

Rearranging and simplifying, you get the length of the arc in terms of the angle and radius as

$$l = \frac{\theta}{360^\circ} \times 2\pi r.$$

Exam tip

In IB examinations, the formula booklet gives the formula for the length of an arc as $l = \frac{\theta}{360} \times 2\pi r$ where r is the radius and θ is the angle measured in degrees.

Example 1

★☆☆

A circle with radius 5 cm has an arc that subtends an angle of 60° .

Find the length of the arc accurate to 3 significant figures.

Steps	Explanation
$l = \frac{60^\circ}{360^\circ} \times 2 \times \pi \times 5$	Use the formula $l = \frac{\theta}{360^\circ} \times 2\pi r$.

Student view

Overview
(/study/ap
122-
cid-
754029/

Steps	Explanation
$l \approx 5.24 \text{ cm}$	Use your graphic display calculator and roun significant figures.

Important

In IB examinations do not use rounded values of π like 3.14 or $\frac{22}{7}$. Use the button marked π on your calculator and only round final answers to avoid accuracy errors.

Be aware

Pay attention to the measure of the angle and set the mode of your calculator correctly.

- If you are studying this course at Standard Level you will encounter angles only in degrees.
- If you are studying at Higher Level you will use both degrees and radians.

Example 2

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Given that a circle has a radius of 4 cm and the length of an arc subtended by the central angle θ is 8 cm , find the measure of the angle θ to the nearest degree.

Student
view

Overview
(/study/ap
122-
cid-
754029/

Steps	Explanation
$8 = \frac{\theta}{360} \times 2 \times \pi \times 4$	Use the formula $l = \frac{\theta}{360} \times 2\pi r$.
$\theta = \frac{8 \times 360}{8\pi}$	Rearrange to make θ the subject..
$\theta = 115^\circ$	Round to the nearest degree.

2 section questions

3. Geometry and trigonometry / 3.4 Circle

Area of a sector

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/area-of-a-sector-id-27401/print/) Assign

Ratios of area

Just as the length of an arc is a fraction of the circle's circumference, so the area of a sector is a fraction of a circle's area, πr^2 , where r is the radius of the circle .

Student
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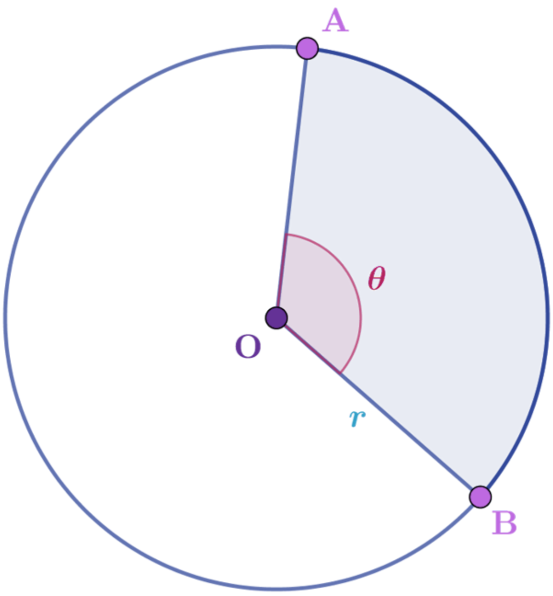
Overview
(/study/ap
122-
cid-
754029/



More information

The image is a diagram of a circle with a highlighted sector. The circle's center is labeled as 'O', and two points on the circle forming the sector are labeled 'A' and 'B'. The sector area is shaded in blue, and an angle θ is marked between the radius lines OA and OB. The radius of the circle, denoted as 'r', is labeled along line OB. The diagram illustrates how the sector's area is a fraction of the entire circle's area related to the angle θ.

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Writing this as ratios:

$$\frac{\text{area of the sector}}{\pi r^2} = \frac{\theta}{360}$$

Rearrange to get:

$$\text{area of the sector} = \frac{\theta}{360} \times \pi r^2.$$

Of course, the angle θ here is in degrees.

Student view

Overview
(/study/ap
122-
cid-
754029/

Exam tip

In IB examinations, the formula booklet gives the formula for the area of a sector as

$$A = \frac{\theta}{360} \times \pi r^2$$

where r is the radius and θ is the angle measured in degrees.

Example 1

★☆☆

Given a circle of radius 5 cm and a central angle of $60°$, find the area of the sector for this central angle. Give your answer accurate to 3 significant figures.

Steps	Explanation
$A = \frac{60}{360} \times \pi \times 5^2$	$A = \frac{\theta}{360} \times \pi r^2$
$A \approx 13.089969$	Use a graphic display calculator.
$A = 13.1 \text{ cm}^2 \text{ (3 significant figures)}$	Give your answer correct to the number of significant figures requested in the questio

Making connections

Circles are frequently seen in real life, like camera lenses, pizzas, cake pans and Ferris wheels such as the London Eye. One of the most common uses is for tyres on cars and other vehicles. How do you think a speedometer determines the speed of a car?

Student view

A car speedometer

Credit: Robert Rowe / EyeEm Getty Images

More information

This image is a close-up of a car speedometer. The speedometer displays speed readings ranging from 0 to 260 km/h, marked in increments of 20. The needle is pointing at approximately 15 km/h. The face of the speedometer is black, with white numbers and tick marks for each increment. A small fuel gauge icon is visible below the needle's base, indicating fuel levels, with markings ranging from empty to full.

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Example 2

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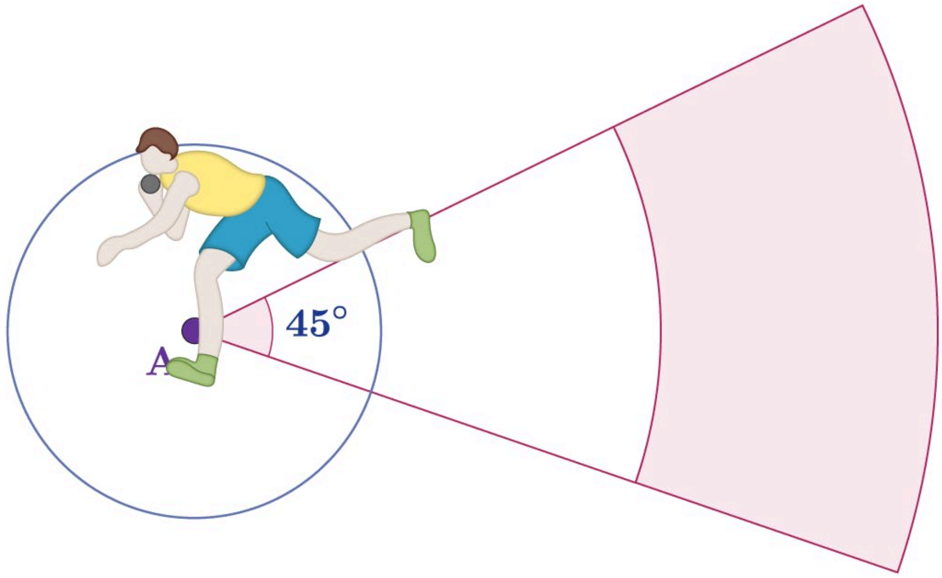
Students in a high school would like to create a shot-put field with a centre circle of radius 2 m for their track and field practice. The second ring would have a radius of 4 m and the third ring would have a radius of 6 m. They need to plant grass seeds between these rings. If the central angle is 45° , calculate the area of the shaded region to the nearest square metre.

Overview
(/study/ap
122-
cid-
754029/



More information

The image illustrates a shot put athlete positioned on a circular field for track and field practice. The field has concentric circles with a central circle of radius 2 meters, a second ring of 4 meters, and a third ring of 6 meters. The illustration highlights a sector with a central angle of 45 degrees, marked in blue. The athlete is in a throwing stance within the central circle. The shaded area between the second and third rings represents the region where grass seeds will be planted. This area forms a sector with boundaries at 4 meters and 6 meters from the center, covering a 45-degree angle.

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Steps	Explanatio
$\text{shaded area} = \text{larger sector area} - \text{smaller sector area}$	Largest circle has 6 r and middle circle ha radius.

Student view

Overview
(/study/ap
122-
cid-
754029/c

Steps	Explanatio
$A = \frac{45}{360} \times \pi \times 6^2 - \frac{45}{360} \times \pi \times 4^2$	
$A \approx 7.853981634$	Use your graphic dis calculator.
$A = 8\ \text{m}^2$	Round to the neares metre.

Activity

Work in pairs.

Use the following GeoGebra applet to challenge your friend by changing the central angle. Check your friend's answer.

Student
view

Overview
(/study/ap
122-
cid-
754029/c

Interactive 1. Practice Exercise to Find the Area of Sector for Different Central Angles.

Credit: GeoGebra (https://www.geogebra.org/m/pJUFaTWU) Tim Brzezinski

More information for interactive 1

This interactive allows users to explore the relationship between the central angle, radius, arc length, and area of a sector in a circle. Users can choose any central angle by either dragging a slider or entering a value into the input box. They can also adjust the radius by moving a red point on the circle. Based on the selected angle and radius, users can calculate the arc length and the area of the sector.
Users can then use the checkboxes to reveal the correct arc length and sector area, allowing them to check whether their calculations are accurate. This interactive tool helps users practice and understand the formulas for arc length and sector area:

$\text{Arc Length} = \frac{\theta}{360} \times 2\pi r$, $\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$

where θ is the central angle in degrees and r is the radius of the circle.

Student view

Overview
(/study/ap
122-
cid-
754029/

For example, if a user selects a central angle of 137° and sets the radius to 5.2 units, the interactive will calculate and display the arc length as approximately 12.43 units and the area of the sector as approximately 32.33 square units.

3 section questions

3. Geometry and trigonometry / 3.4 Circle

Checklist

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/checklist-id-27402/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- find the length of an arc using the formula

$$l = \frac{\theta}{360} \times 2\pi r$$

where r is the radius and θ is the angle measured in degrees

- find the area of a sector using the formula

$$A = \frac{\theta}{360} \times \pi r^2$$

where r is the radius and θ is the angle measured in degrees.

3. Geometry and trigonometry / 3.4 Circle

Investigation

Student view

All around India, rangoli patterns are drawn on the floor by the entrance of a house to bring good fortune and blessings into the house. It is believed that the intricate and complicated designs trap negative energy and let only good energy into the house. They are made from natural materials only, like rice, chalk, flour and flowers, as shown below.

Rangoli patterns

Credit: balaji chennai Getty Images

Using a sheet of A4 paper, design your own Rangoli pattern based on circles.

Calculate how much of each colour you will need for the actual design.