0 (https://intercom.help/kognity)

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5. Calculus / 5.5 Introduction to integration

The big picture

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26293/print/) Assign

This video shows a 'leaf area meter', which is a tool that allows scientists to measure the area of a leaf.

The applet below shows how a leaf area meter might work. While scanning the leaf, the meter identifies the two points on the perimeter. To find the area of the leaf, it plots this distance as a function of the position. The area of the region below the graph is equal to the area of the leaf.

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Interactive 1. Finding the Area of the Leaf.

More information for interactive 1

This interactive demonstrates how a leaf area meter works by simulating the process of scanning a leaf. As the meter scans the leaf, it identifies two points on the perimeter and measures the distance between them. The distance between these two points represents the leaf's width at that specific position.

This distance is plotted as a function of the meter's position, creating a graph. The x-axis of the graph represents the position along the leaf's length, while the y-axis represents the width at each point.

The area under this graph corresponds to the area of the leaf. The graph effectively represents a series of narrow rectangles whose combined area gives an approximation of the leaf's total area, a fundamental principle of integral calculus. Mathematically, this is equivalent to integrating the function that describes the leaf's width along its length.

Users can adjust the position of the meter to see how the distance between the perimeter points changes and how this affects the graph. By exploring this simulation, users can gain a better understanding of how the area of irregular shapes, like leaves, can be calculated using integration techniques. Understanding leaf area is crucial in plant physiology studies, as it helps in analysing photosynthetic efficiency, transpiration rates, and overall plant health.

This interactive serves as an excellent tool for understanding and applying mathematical integration in practical biological measurements. It bridges the gap between theoretical calculus and its applications in plant sciences and environmental monitoring.

In this subtopic, you will learn about two seemingly unrelated concepts:

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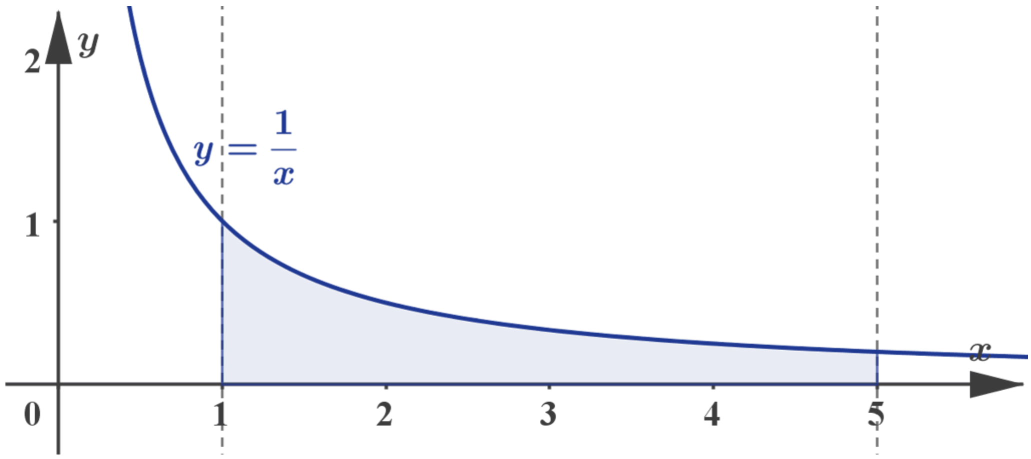
- How technology can be used to find the area of certain planar regions. For example, you will see that the area of the region bounded by the x-axis, the graph of $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 5$ is approximately 1.61 square units.



More information

This image is a graph showing the curve y = 1/x with a focus on the region between the lines x = 1 and x = 5. The x-axis ranges from 0 to 5, and the y-axis ranges from 0 to 2. The curve is plotted on a Cartesian coordinate system with the x-axis and y-axis clearly marked. The area under the curve from x = 1 to x = 5 is shaded in blue, indicating the region whose area is being considered. The curve decreases and asymptotically approaches the x-axis as x increases. The point at which the curve intersects with the y-axis is approaching zero as x increases beyond 5, highlighting the diminishing area covered by the curve as x values increase. This visual representation helps in understanding the concept of integration and area calculation under a curve in calculus.

[Generated by AI]

- Anti-differentiation, which is the method of finding a function from its derivative. For example, you will see that if $f'(x) = 3x^2$, then $f(x) = x^3$ or $f(x) = x^3 + 1$ or, in general, $f(x) = x^3 + c$ for some real number c.

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The notation for the area bounded by a graph and the notation for the anti-derivative are very similar. There is, of course, a reason for this, which you can explore in the investigations (/study/app/m/sid-122-cid-754029/book/investigation-id-26300/) for

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this subtopic.

Concept

While learning about the way in which technology can be used to find area, think about examples where knowing the area of certain planar regions can be useful. Think about how the **modelling** techniques you learned earlier can be extended to **approximate** areas.

Theory of Knowledge

As mathematics is based on axioms, rules that provide the foundation for mathematical inquiry, it is crucial that these axioms be correct. However there are many mathematical axioms that have not been proven. For example the basic axiom that all addition is commutative cannot be proven for all real numbers, though it is assumed to be accurate and valid based on inductive logic.

Some philosophers such as Alfred North Whitehead and Bertrand Russell believed that language got in the way of axioms being fully provable. Thus they sought to create a system of mathematics that was based in symbols in order to remove the confound of language.

This leads to the knowledge question, 'Must something be proven with evidence in order to be considered true?'

5. Calculus / 5.5 Introduction to integration

Area under a curve

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/area-under-a-curve-id-26294/print/) Assign

You will start by investigating how to use technology to find the area of the region introduced in The big picture. (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26293/)

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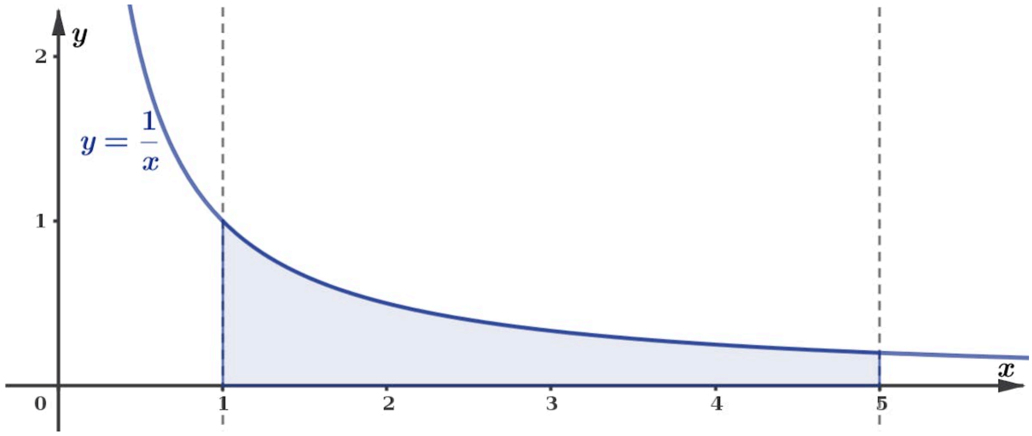

More information

The image is a graph of the function y = 1/x. The x-axis is labeled with tick marks at 1, 2, 3, 4, and 5, and the y-axis is labeled with numbers 1 and 2. The graph is a decreasing curve starting at y = 2 for x = 0 on the y-axis and approaching the x-axis as x increases. The region between the curve and the x-axis is shaded from x = 1 to x = 5. Dotted vertical lines denote the boundaries at x = 1 and x = 5, indicating the area of interest under the curve.

[Generated by AI]

All graphic display calculators have applications that can find an approximate value for this area.

The steps to follow, on all calculators, are:

1. Enter the expression that defines the curve.
2. Specify the most appropriate window to display the relevant part of the graph.
3. Select the application that finds the area of a region between a graph and the x-axis.
4. Specify the lower and upper limits of the region.

Here are instructions for using your calculator:

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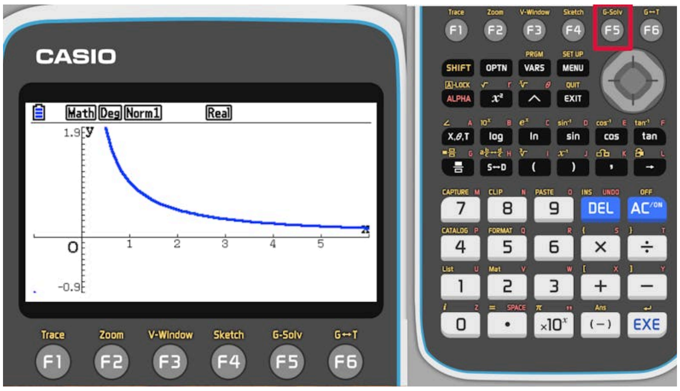
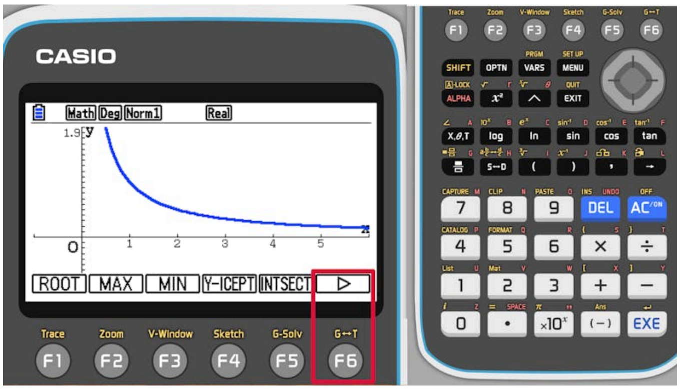
Steps	Explanation
These instructions assume that you have the graph of $y = \dfrac{1}{x}$ on the screen in the viewing window $-1 \leq x \leq 6$ and $-1 \leq y \leq 2$. You will see how to find the area below the graph over the interval $[1, 5]$. Press F5 (G-Solv) to open the options to analyse the graph.	
The option to find the area is not among the first five options, so press F6 to see the others.	

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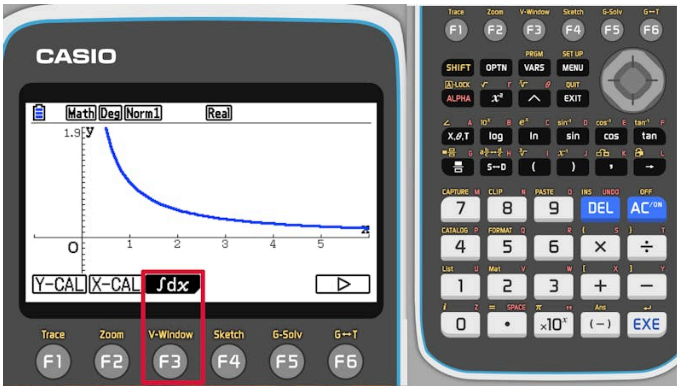
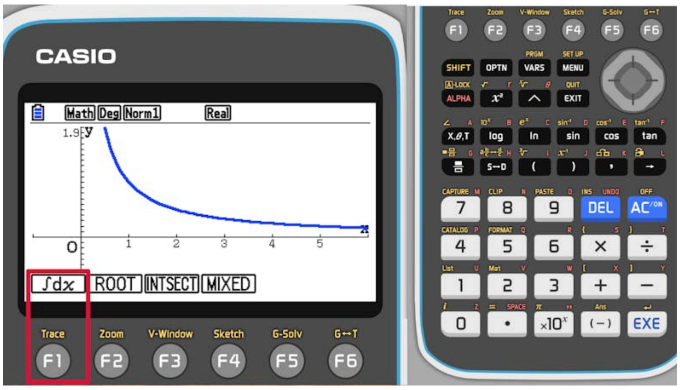
Steps	Explanation
Choose the option involving the integral sign.	
The calculator can find areas of different type of regions. In this case you will need the first option, so press F1.	

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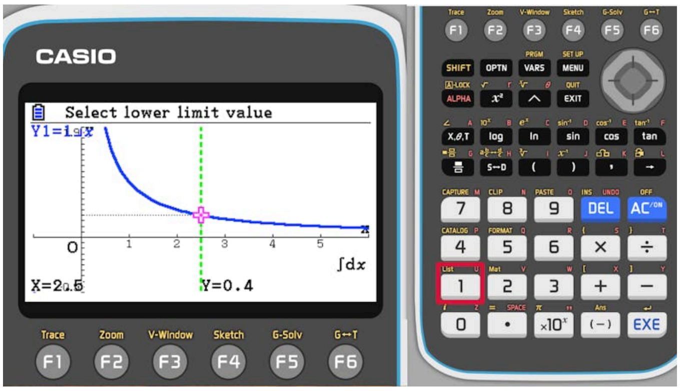
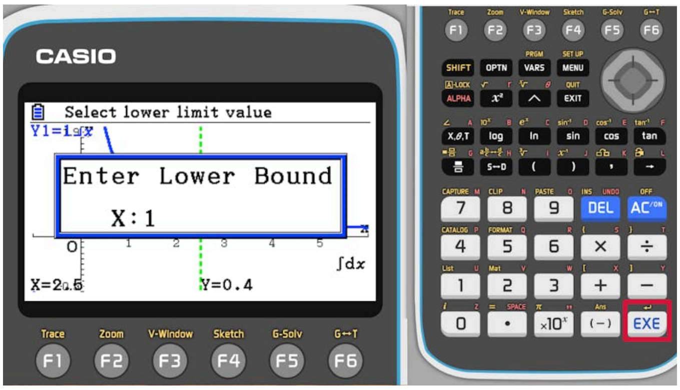
Steps	Explanation
You need to set the lower bound for the region. Start typing the value. In this example, press the number 1.	
Press EXE to confirm the value.	

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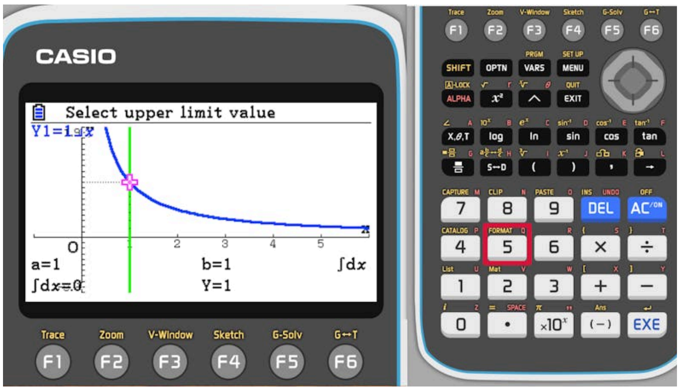
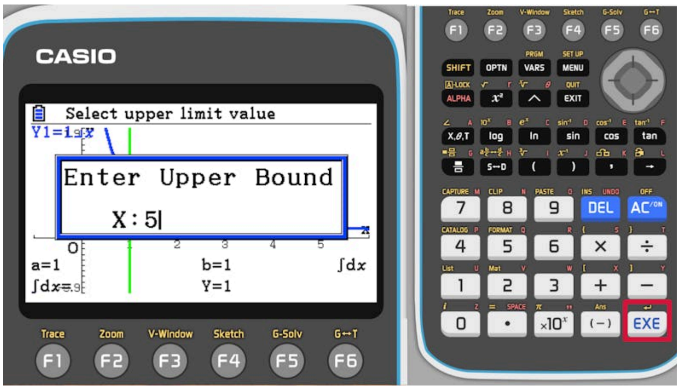
Steps	Explanation
The lower bound is now set, the calculator is waiting for the upper bound. Start typing the value. In this case, press 5.	
Press EXE to confirm the value.	

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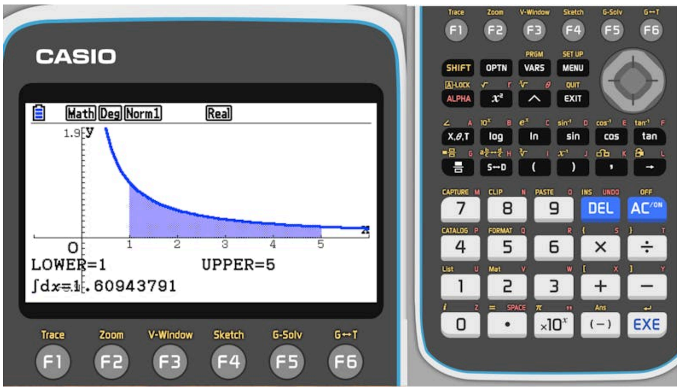
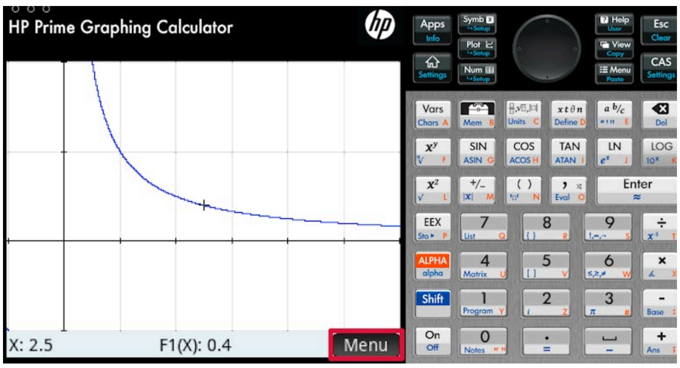
Steps	Explanation
The region is shaded on the diagram and the area is displayed on the screen.	

Steps	Explanation
These instructions assume that you have the graph of $y = \frac{1}{x}$ on the screen in the viewing window $-1 \leq x \leq 6$ and $-1 \leq y \leq 2$. You will see how to find the area below the graph over the interval $[1, 5]$. Open the menu.	

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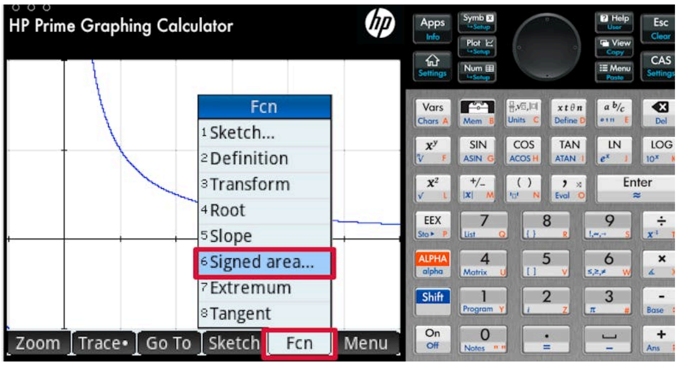
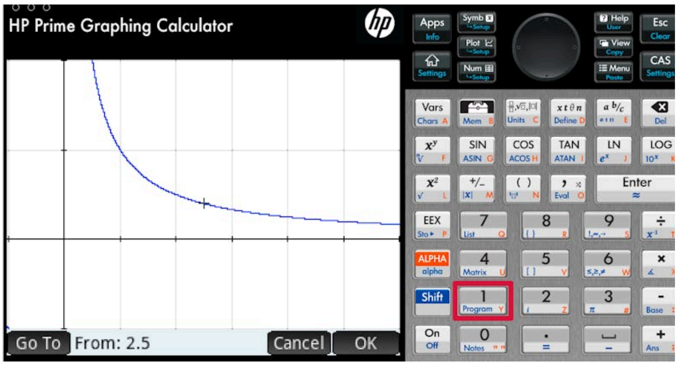
Steps	Explanation
Choose the option to find an area.	
You need to set the lower bound for the region. Start typing the value. In this example, press the number 1.	

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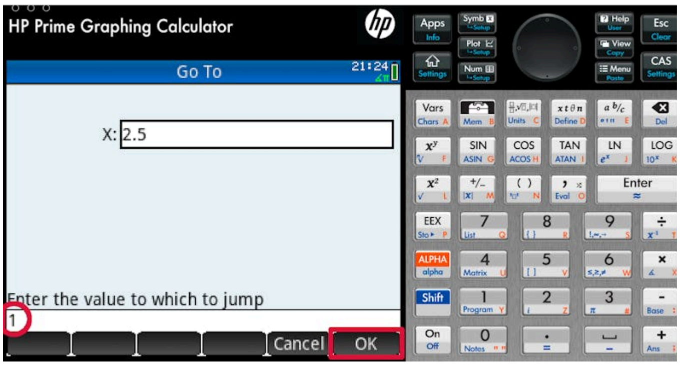
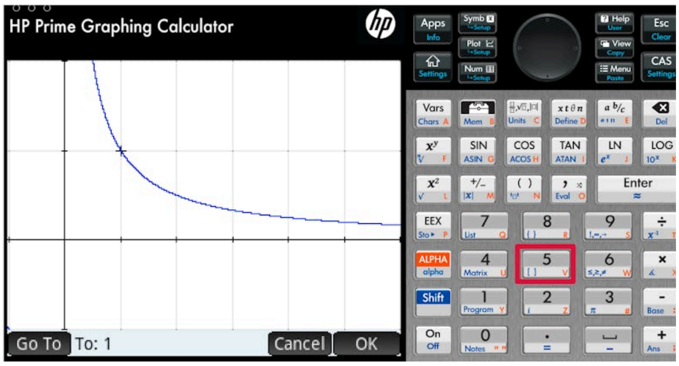
Steps	Explanation
Press OK or enter to confirm the value.	
The lower bound is now set, the calculator is waiting for the upper bound. Start typing the value. In this case, press 5.	

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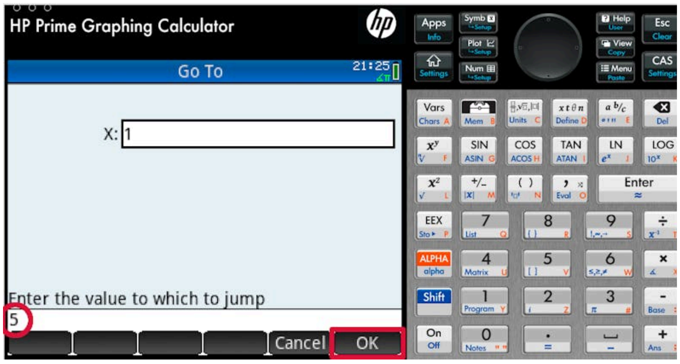
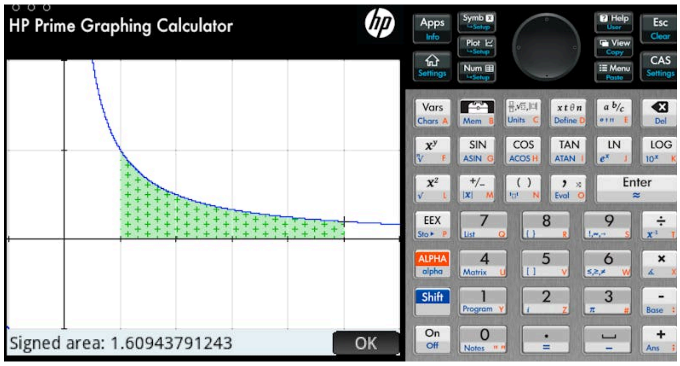
Steps	Explanation
Press OK or enter to confirm the value.	
The region is shaded on the diagram and the area is displayed on the screen.	

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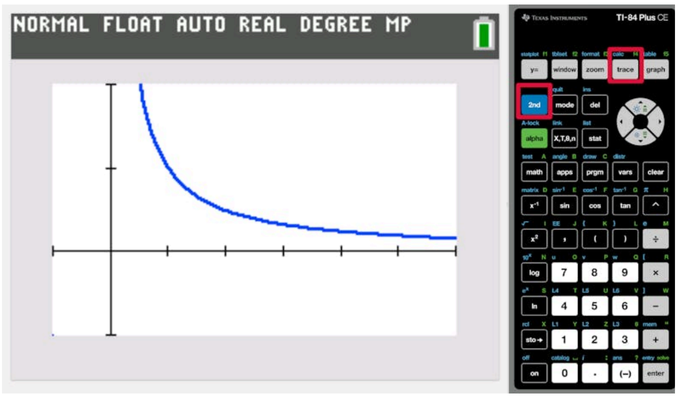
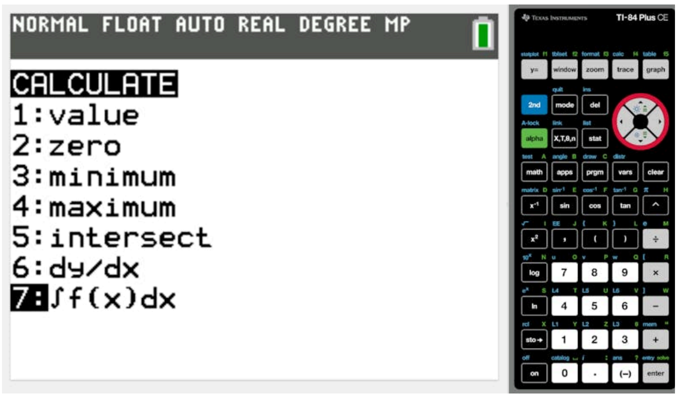
Steps	Explanation
These instructions assume that you have the graph of $y = \dfrac{1}{x}$ on the screen in the viewing window $-1 \leq x \leq 6$ and $-1 \leq y \leq 2$. You will see how to find the area below the graph over the interval $[1, 5]$. Open the options (2nd/calc) to analyse the graph	
Choose the option involving the integral sign.	

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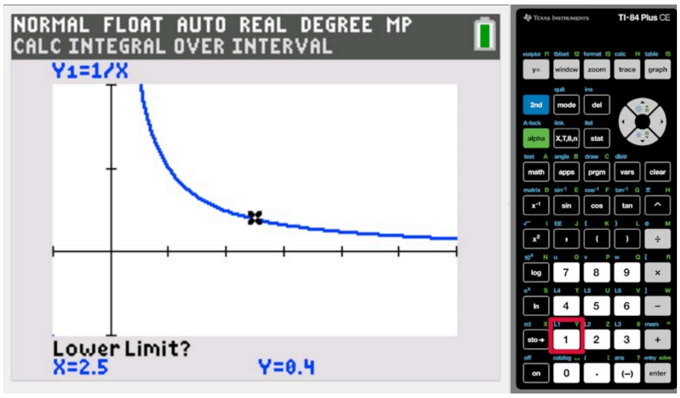
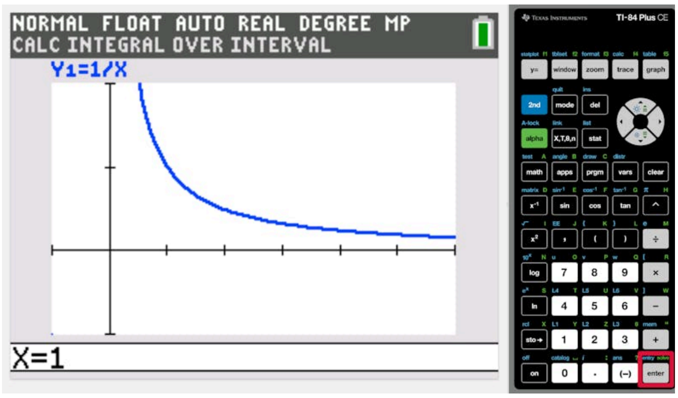
Steps	Explanation
You need to set the lower bound for the region. Start typing the value. In this example, press the number 1.	
Press enter to confirm the value.	

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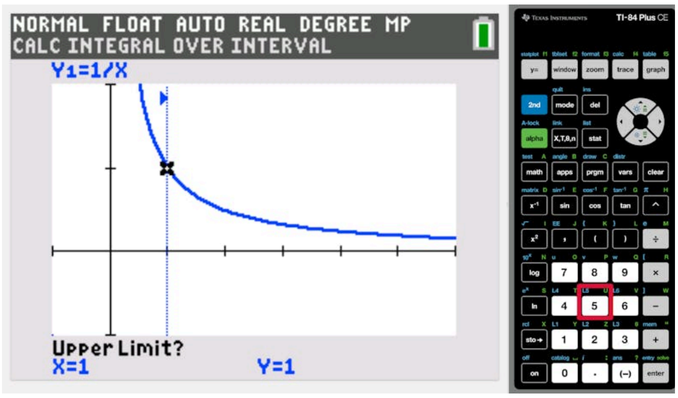
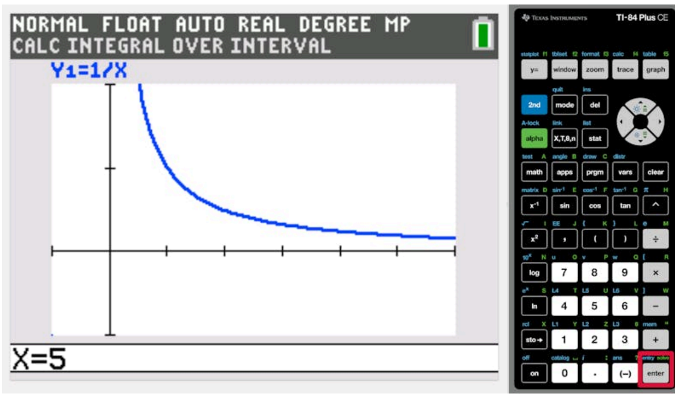
Steps	Explanation
The lower bound is now set, the calculator is waiting for the upper bound. Start typing the value. In this case, press 5.	
Press enter to confirm the value.	

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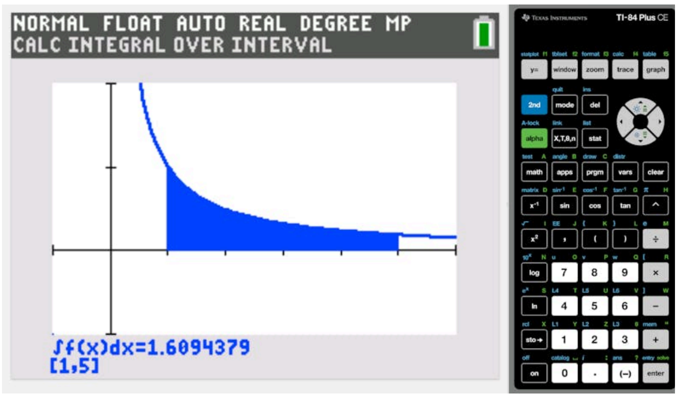
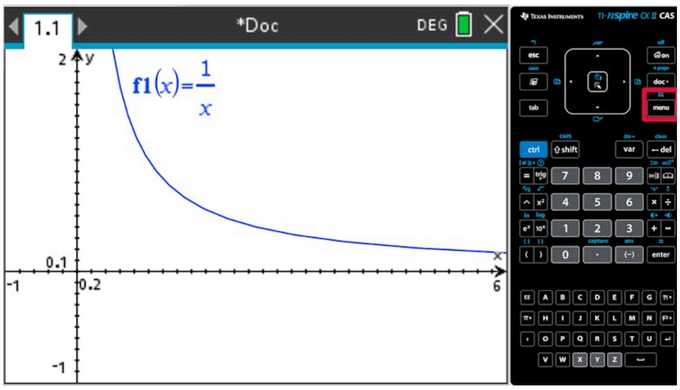
Steps	Explanation
The region is shaded on the diagram and the area is displayed on the screen.	

Steps	Explanation
These instructions assume that you have the graph of $y = \frac{1}{x}$ on the screen in the viewing window $-1 \le x \le 6$ and $-1 \le y \le 2$. You will see how to find the area below the graph over the interval $[1, 5]$. Open the menu ...	

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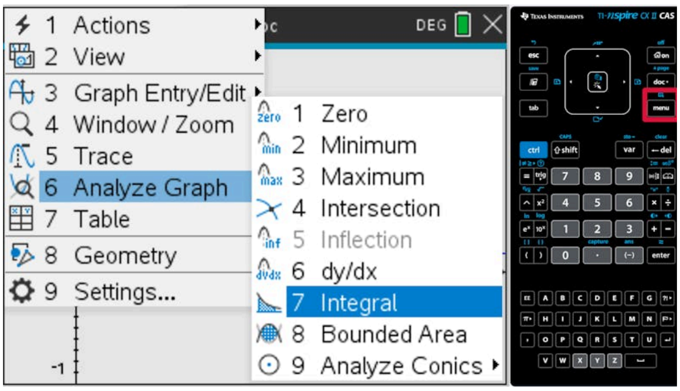
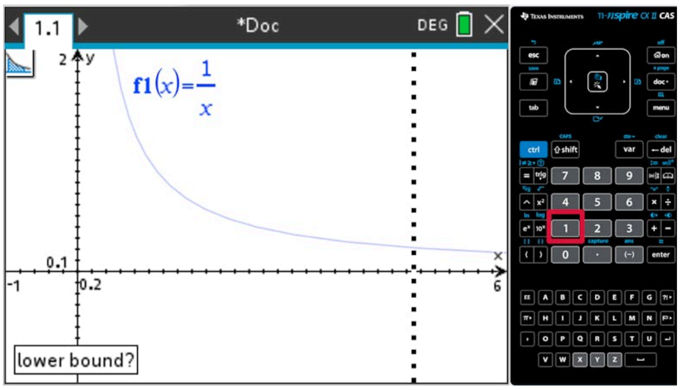
Steps	Explanation
... and choose the option to find an integral.	
You need to set the lower bound for the region. Start typing the value. In this example, press the number 1.	

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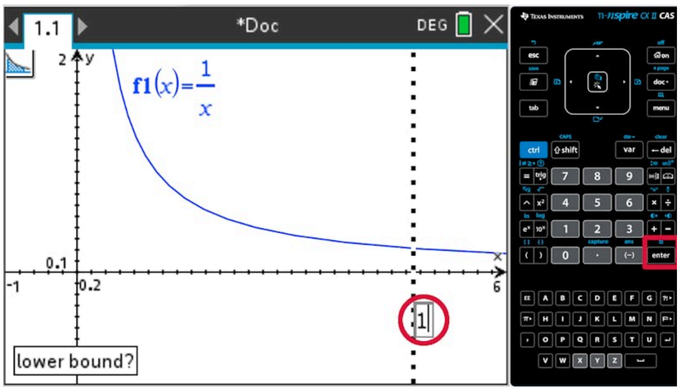
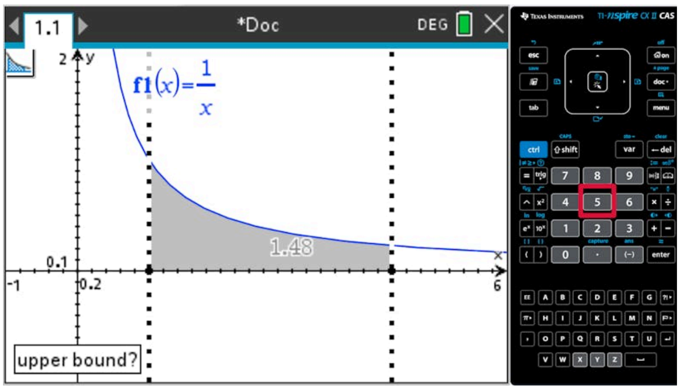
Steps	Explanation
Press enter to confirm the value.	
The lower bound is now set, the calculator is waiting for the upper bound. Start typing the value. In this case, press 5.	

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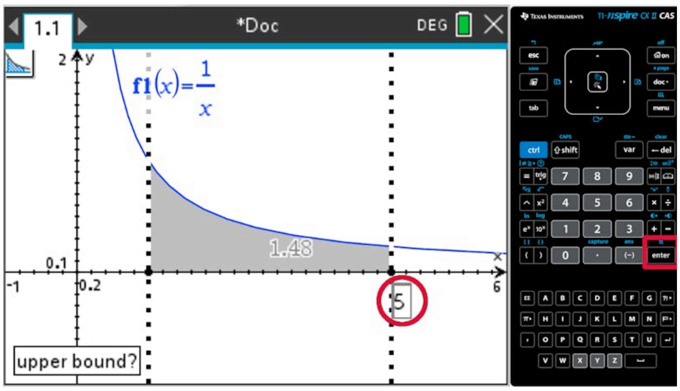
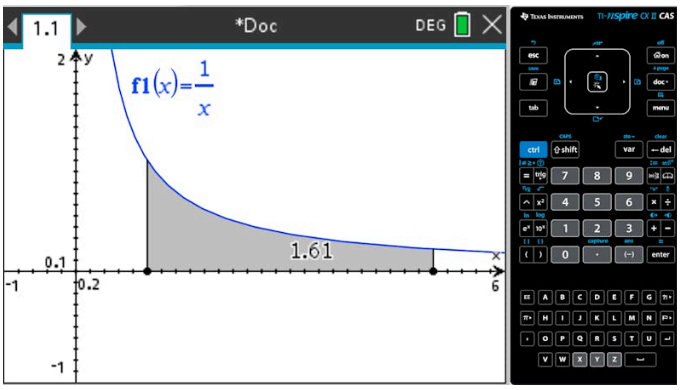
Steps	Explanation
Press enter to confirm the value.	
The region is shaded on the diagram and the area is displayed on the screen.	

Different makes of calculator will display results to differing levels of accuracy. Rounded to 3 significant figures, the area in the example above is 1.61 square units. Using more powerful tools, you can, of course, get answers with accuracy beyond the one displayed above:

$$1.6094379124341003746007593332261876395256013542685\ldots$$

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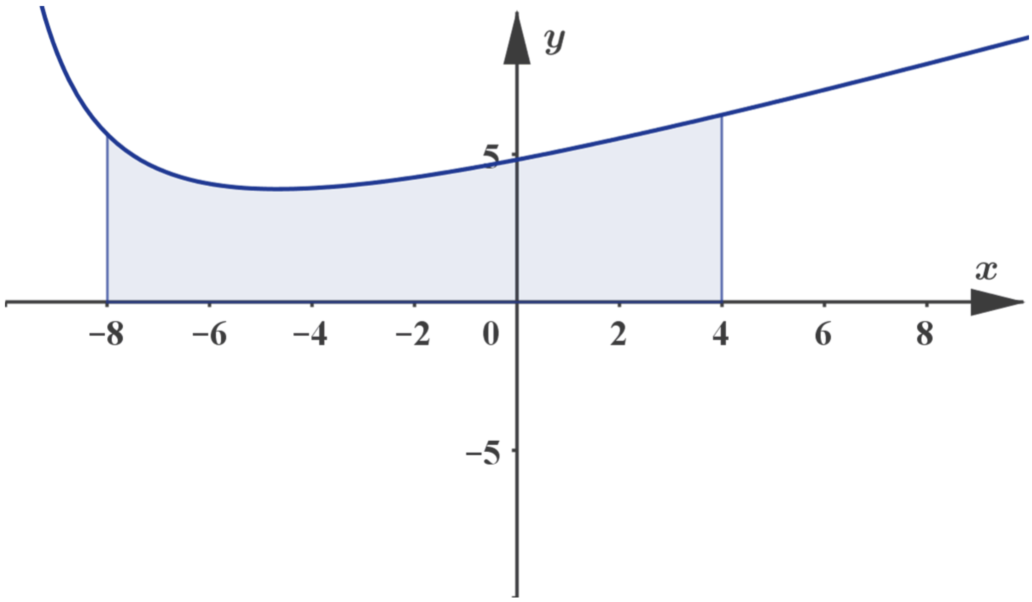
Example 1

★☆☆

Find the area of the region bounded by the graph of $y = \frac{x^2 + 17x + 106}{2(x + 11)}$, the x-axis and the lines $x = -8$ and $x = 4$.

You can use your calculator to draw the graph, identify the region and find the area. The default window for the graphing screen on your calculator will not need to be changed to find this area. The diagram below shows part of the graph in the window $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$. It also shows the region. You should see something similar on your calculator screen.



The area is approximately 56.2 square units.

Example 2

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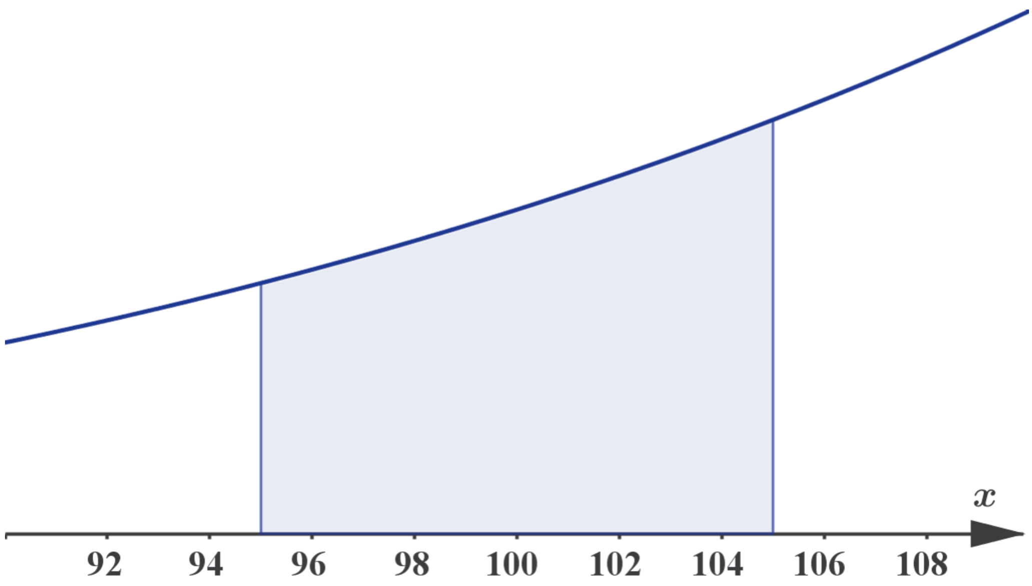
Find the area of the region bounded by the graph of $y = \dfrac{x^8 + x}{x^3 + x^2}$, the x-axis and the lines $x = 95$ and $x = 105$.

You can use your calculator to draw the graph, identify the region and find the area. However, on the default window settings of the calculator you will not see the relevant part of the graph, so you need to zoom out.

- To see the vertical bounds of the region, you need a horizontal range of numbers containing $x = 95$ and $x = 105$. You can choose, for example, to display the graph for $90 \leq x \leq 110$.
- You also need to modify the vertical range. Since the y-value corresponding to $x = 110$ is approximately 1.60×10^{10}, you can try setting the vertical range of the window to $-1 \times 10^9 \leq y \leq 1.6 \times 10^{10}$ (you need the negative lower bound to see the x-axis on the screen).

The diagram below shows the graph and the region using the window settings described above.



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The area of the region is approximately 9.98×10^{10} square units.

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Example 3

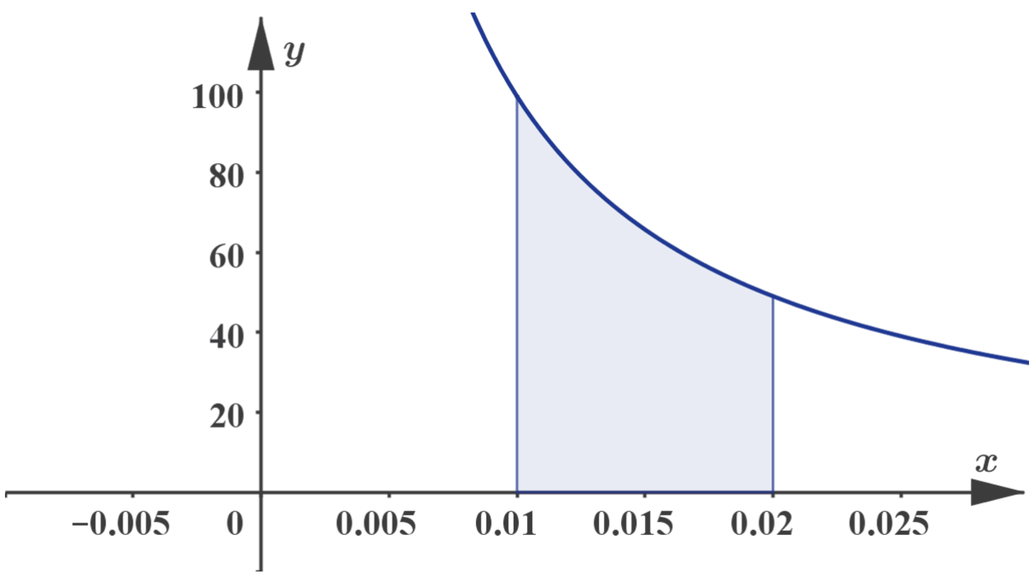
★★☆

Find the area of the region bounded by the graph of $y = \frac{1}{x^2 + x}$, the x-axis and the lines $x = 0.01$ and $x = 0.02$.

You can use your calculator to draw the graph, identify the region and find the area. However, on the default window settings of the calculator you will not see the relevant part of the graph, so you need to zoom in.

- To see the vertical bounds of the region, you need a horizontal range of numbers containing $x = 0.01$ and $x = 0.02$ but not going much beyond these values. You can choose, for example, to display the graph for $-0.01 \leq x \leq 0.03$.
- You also need to modify the vertical range. Since the y-value corresponding to $x = 0.01$ is approximately 99.0, the y-value corresponding to $x = 0.02$ is approximately 49.0, so you can try setting the vertical range of the window to $-20 \leq y \leq 120$ (you need the negative lower bound to see the x-axis on the screen).

The diagram below shows the graph and the region using the window settings described above.



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The area of the region is approximately 0.683 square units .

Be aware

Note that the upper bounding curve for the region needs to be defined for all values of x within the limits of the region. You will not learn methods to investigate, for example, the area between the graph of $y = \frac{1}{|x|}$ and the *x*-axis over the interval $[-1, 2]$. Can you think of a reason for this?

In the applet below, you can check your proficiency.

Interactive 1. Finding the Area of the Region Bounded by a Curve.

More information for interactive 1

This interactive allows users to practise finding the area of a region bounded by the x-axis, the graph of a given function, and two vertical lines. The interactive provides a visual representation of the graph and the bounded region, helping users understand the area they need to calculate. It allows users to explore how integration is used to find the exact area enclosed by a curve and given vertical boundaries.

By clicking "Click here for a new question", users can generate a new problem with a different function and vertical lines. After solving the problem, users can check their answer by clicking "Show the answer," which displays the correct area.

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Example: The interactive presents a mathematical function: $y = 8\sqrt{x + 10} + 2\sqrt{6 - x}$

The task is to find the area of the shaded region under the curve, bounded by the x-axis and the vertical lines at $(x = -3.1)$ and $(x = 5)$. The area of the shaded region is displayed as 248.44 units squared.

This interactive tool is a powerful tool for studying calculus and integration, offering both theoretical knowledge and mathematical problem-solving.

Looking ahead

Use WolframAlpha (http://www.wolframalpha.com) to find the area of the region in the opening example. Type the following into the search line:

area below 1/x for 1<x<5

There are two interesting things you can see when you take a look at the answer WolframAlpha gives:

- In addition to the approximate value, WolframAlpha also gives the exact area as $\log(5)$. In the HL extension of the applications and interpretation course and the SL/HL analysis and approaches course, you will learn how to find this exact value.
- In the answer WolframAlpha uses the symbol $\int$. In the following sections, you will learn what this symbol means in two different contexts.

3 section questions

5. Calculus / 5.5 Introduction to integration

Definite integrals

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/definite-integrals-id-26295/print/) Assign

To become familiar with the notation introduced in this section, start by experimenting a bit more with WolframAlpha (http://www.wolframalpha.com). Type one of the following into the search line:

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area below y=2^x for -1<x<1

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area below y=ln(x) for 2<x<5

area below y=sqrt(x) for 1<x<4

Activity

Experiment with a variety of different functions and domains.

Make sure to enter functions that are positive on the domain you choose.

What do you notice?

Did you notice that WolframAlpha uses notation similar to

$$\int_a^b f(x)\mathrm{d}x$$

to represent the area bounded by the graph of $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$?

The expression is read as the definite integral of $f(x)$ from $x = a$ to $x = b$ with respect to the variable x.

- The symbols $\int$ and $\mathrm{d}x$ indicate the start and the finish of the expression $f(x)$ that describes the upper bound of the region.
- The numbers a and b, written below and above the integral sign, indicate the bounds of the region from the left and the right.

Important

If $f(x) \geq 0$ for all $a \leq x \leq b$, then $\int_a^b f(x)\mathrm{d}x$ gives the area of the region bounded by the x -axis, the graph of $y = f(x)$ and the lines $x = a$ and $x = b$.

Exam tip

The formula booklet gives the following:

The area between a curve $y = f(x)$ and the x -axis, where $f(x) > 0$ is

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$$A = \int_a^b y \, dx.$$

Example 1

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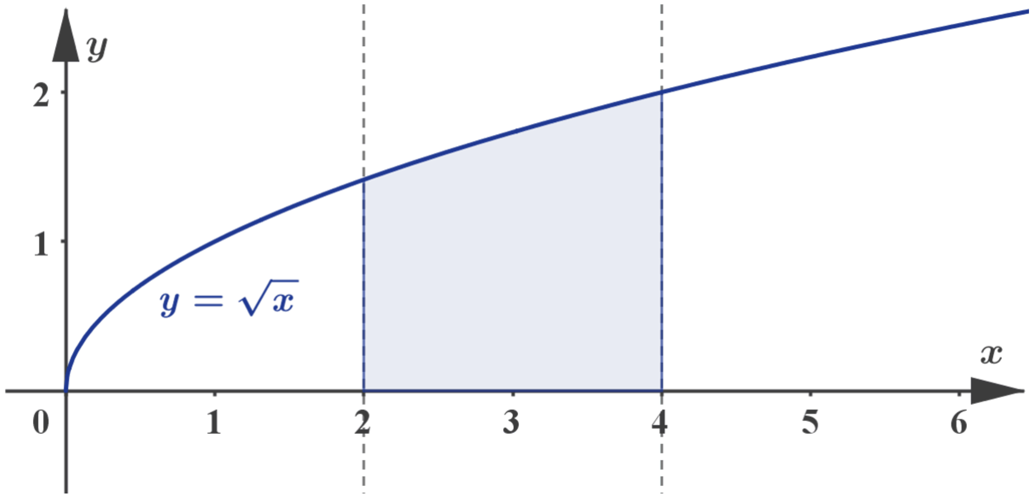
Describe the geometrical meaning of $\int_2^4 \sqrt{x} \, dx$.

Since $\sqrt{x} > 0$,the expression $\int_2^4 \sqrt{x} \, dx$gives the area of the region bounded

- from the left by the line $x = 2$
- from the right by the line $x = 4$
- from below by the x -axis
- from above by the graph of $y = \sqrt{x}$.

The diagram below shows this region.



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Graphic display calculators recognise this notation. You can access this feature on the calculation screen there is no need to graph the function.

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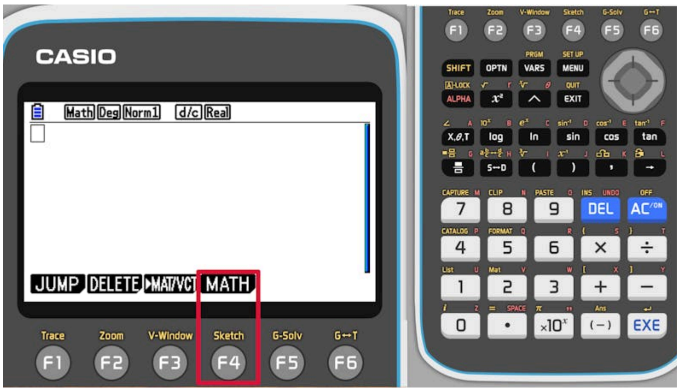
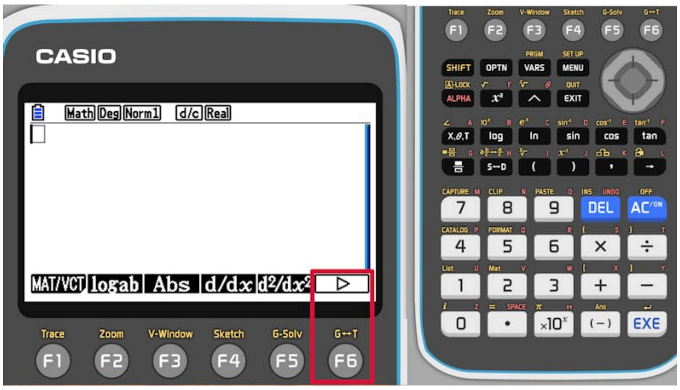
Steps	Explanation
On the calculation screen press F4 to select the math options.	
The option to find definite integrals is not among the first five, so press F6 to see the remaining options.	

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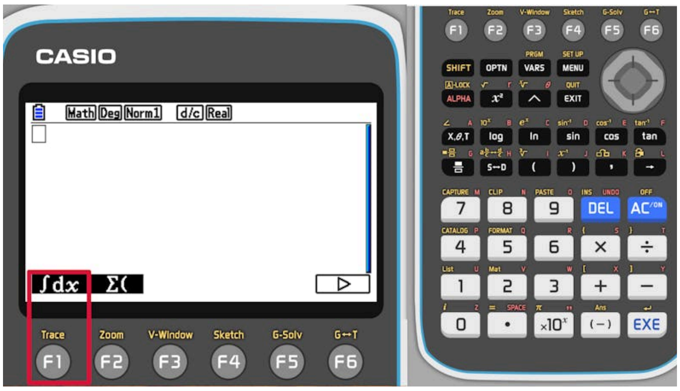
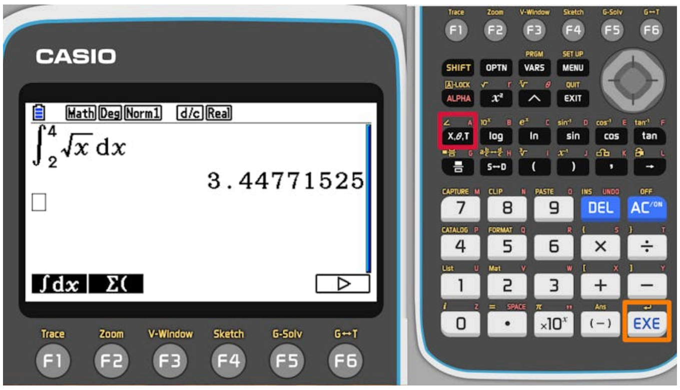
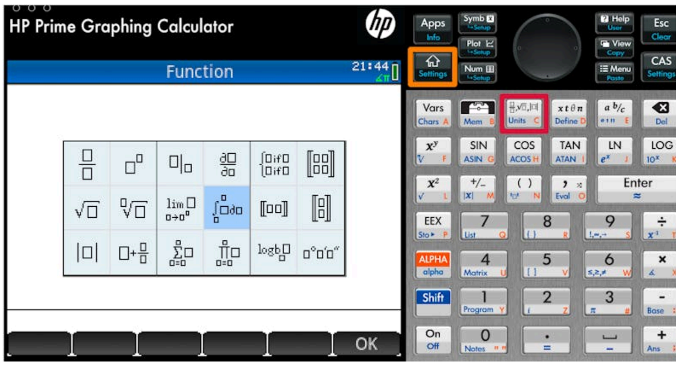
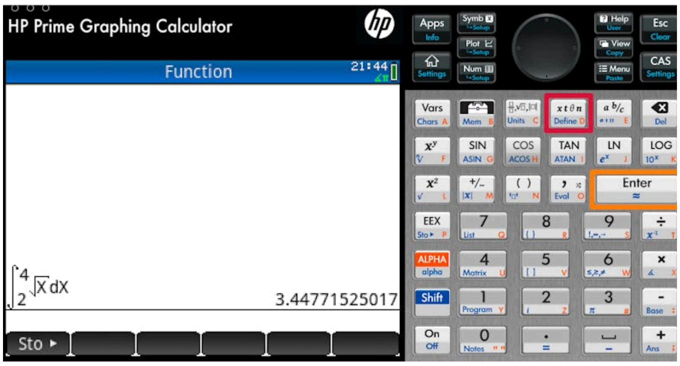
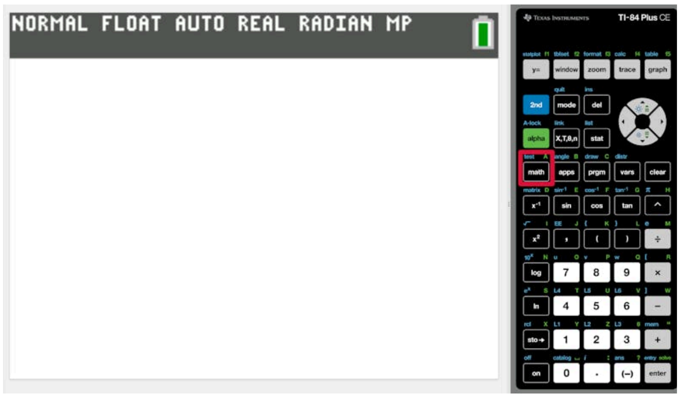
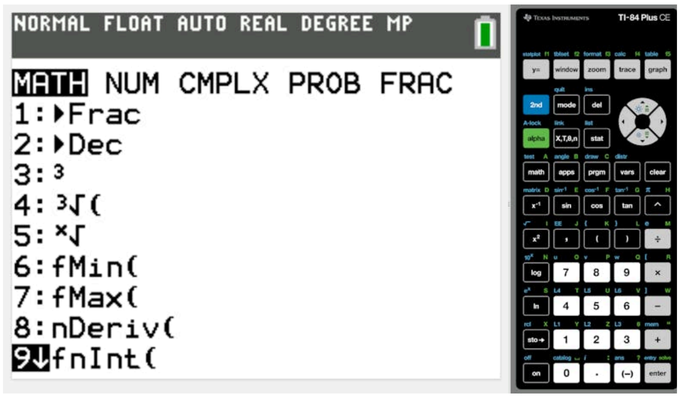
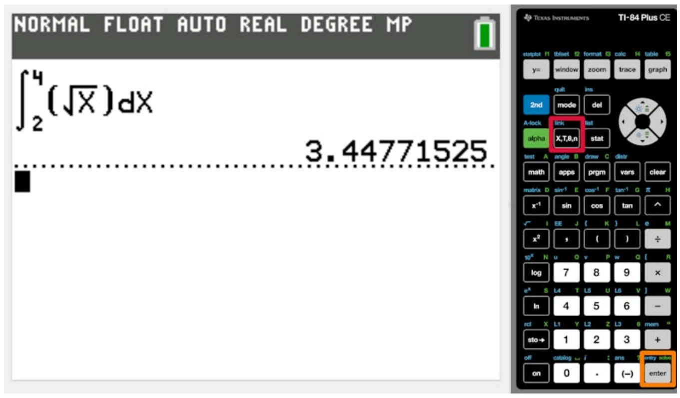
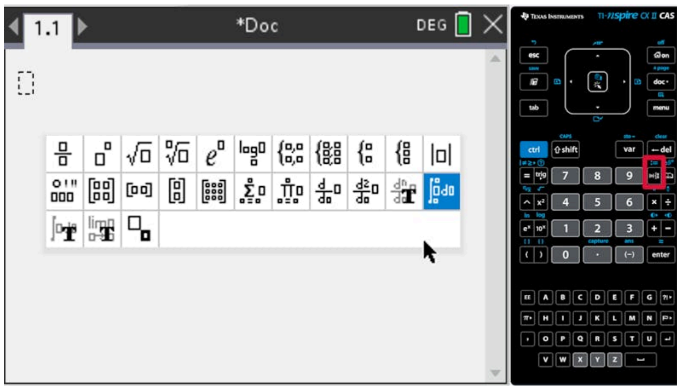
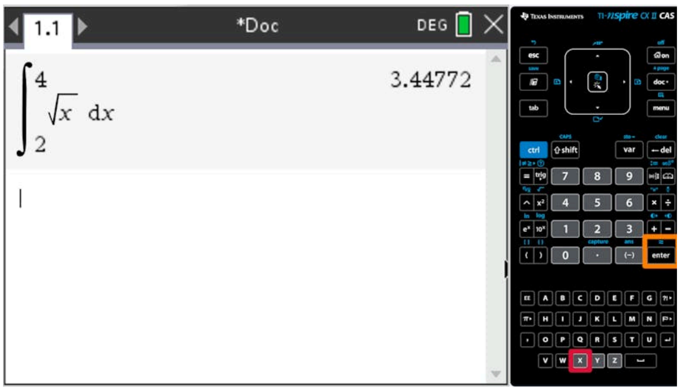
Steps	Explanation
Press F1 to choose the option to find a definite integral.	
Fill in the details in the template. Use the variable button to enter the unknown. After pressing EXE, you can see the value of the definite integral.	

Steps	Explanation
On the home screen of any application open the template menu and choose the template to find a definite integral.	
Fill in the details in the template. Use the variable button to enter the unknown. After pressing enter or OK, you can see the value of the definite integral.	

Steps	Explanation
On the calculation screen select the math options.	
Select the option (fnInt) to find a definite integral.	

Steps	Explanation
Fill in the details in the template. Use the variable button to enter the unknown. After pressing enter, you can see the value of the definite integral.	

Steps	Explanation
On a calculation screen, open the template menu and choose the template to find a definite integral.	

Steps	Explanation
Fill in the details in the template. After pressing enter, you can see the value of the definite integral.	

▷ **Which calculator models would you like Kognity to support? Let us know [here](https://goo.gl/forms/8FvI3K5Xbr7AXmkZ2) (https://goo.gl/forms/8FvI3K5Xbr7AXmkZ2).**

Example 2

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Find $\int_{-0.5}^{1} \mathrm{e}^{-x^2}\mathrm{d}x$and describe the geometric meaning.

Using a graphic display calculator, you can find the approximate value of this definite integral.

$$\int_{-0.5}^{1} \mathrm{e}^{-x^2}\mathrm{d}x \approx 1.21.$$

The definite integral gives the area of the region between the graph of $y = \mathrm{e}^{-x^2}$and the x-axis, over the interval $[-0.5, 1]$.

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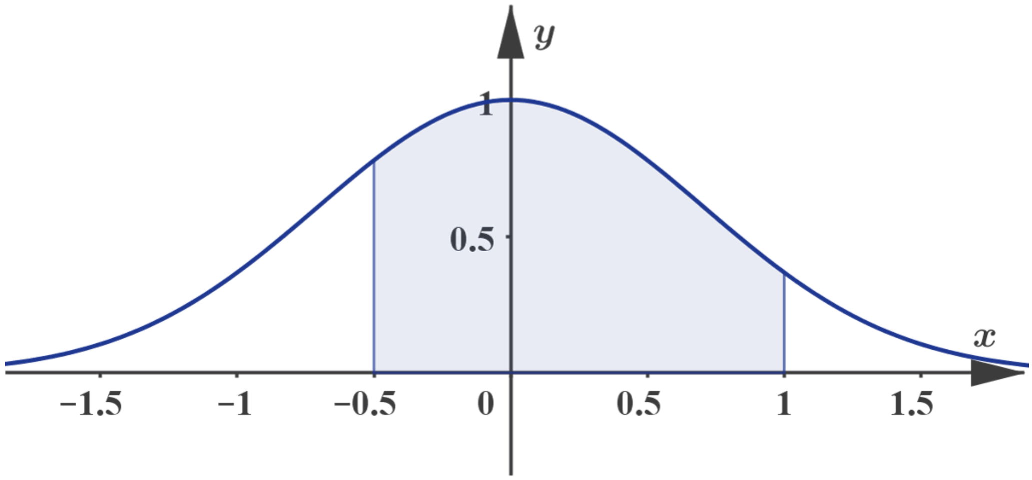

Example 3

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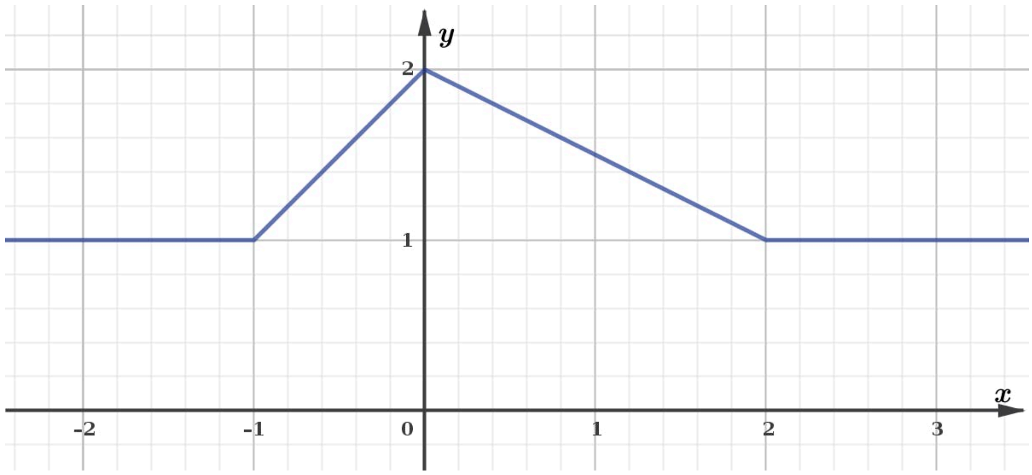
The diagram below shows the graph of $y = f(x)$.



More information

The graph of (y=f(x)) is depicted on a coordinate plane. The x-axis ranges from -3 to 3, with major tick marks at -3, -2, -1, 0, 1, 2, and 3. The y-axis ranges from 0 to 3, with major tick marks at 0, 1, 2, and 3.

The graph of the function begins on the x-axis at (x = -3), showing a constant y-value of 1 from (x = -3) to (x = -2). At (x = -2), the graph begins to rise linearly, reaching its peak at (x = 0), (y = 2). From the peak, the graph descends linearly back to a y-value of 1 at (x = 2). The graph remains constant with a y-value of 1 from (x = 2) to (x = 3).

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Overall, the function exhibits a peak at the origin and linear segments with the inflection points at (x = -2), (x = 0), and (x = 2). The area under the curve from (x = -2) to (x = 3) represents a bounded region that could be evaluated for integration.

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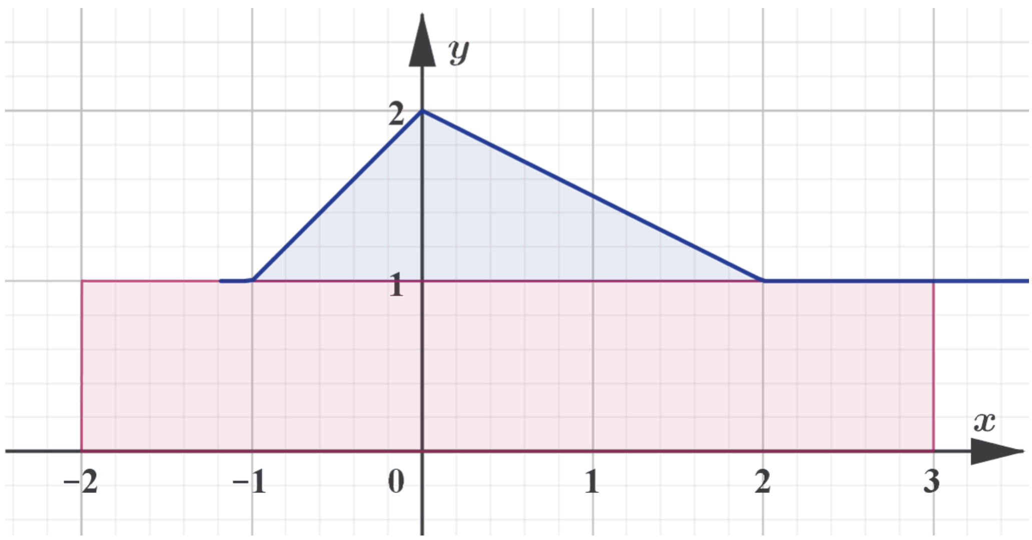
Find $\int_{-2}^{3} f(x)\mathrm{d}x$.

The value of the integral is the area of the region between the graph and the x-axis, over the interval $[-2, 3]$.

The diagram below shows this region as a combination of a rectangle and a triangle.



- The area of the rectangle is $5 \times 1 = 5$.
- The area of the triangle is $\frac{1}{2} \times 3 \times 1 = 1.5$.

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Hence, $\int_{-2}^{3} f(x)\mathrm{d}x = 5 + 1.5 = 6.5.$

Example 4

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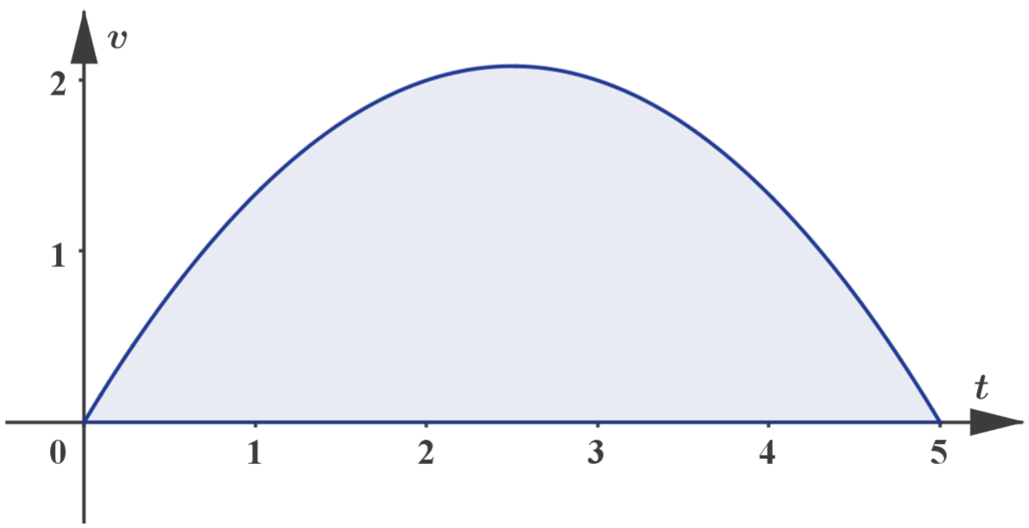
A model car is travelling on a straight road. Its velocity t s after the start of the movement is given by $v(t) = \frac{t(5-t)}{3}$ m s^{-1}.The car stops 5 s after the start of the movement.

How far is the car from the starting point when it stops?

Making connections

You may already know from your physics studies that the area below a velocity—time graph gives the distance travelled by the car.

The diagram below shows the velocity–time graph of the movement. The area of the shaded region gives the distance travelled.



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This area can be calculated as a definite integral, so the distance travelled is

$$\int_0^5 \frac{t(5-t)}{3} \mathrm{d}t \approx 6.94 \text{ m}.$$

Example 5

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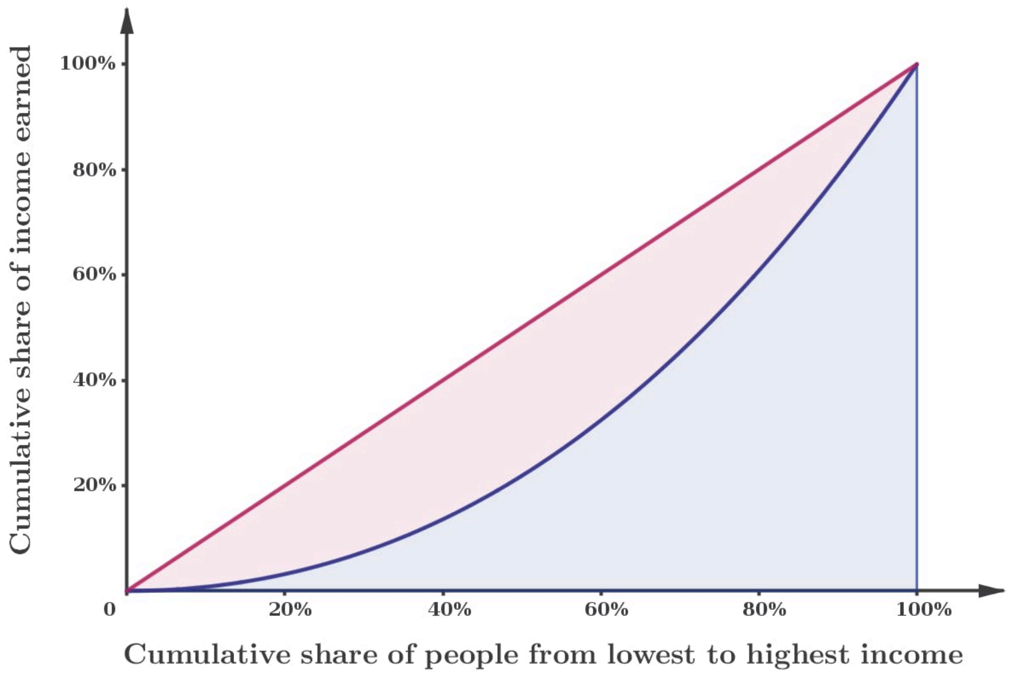
In economics, the Lorenz curve is a graphical illustration of wealth. The blue curve on the diagram below is a typical Lorenz curve.



More information

The image is a graph depicting a Lorenz curve, used to represent wealth distribution. The X-axis is labeled "Cumulative share of people from lowest to highest income," ranging from 0% to 100%. The Y-axis is labeled "Cumulative share of income earned," also ranging from 0% to 100%. The graph includes a blue curve, representing the Lorenz curve, showing the actual distribution of income, and a straight red line from the origin to the top-right corner, representing perfect equality. The area between the red line and the blue curve is colored, illustrating the inequality in income distribution and is related to the Gini index, calculated as the ratio of the area between the line and curve to the total area under the line.

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A related concept is the Gini index, which is a measurement of inequality. The Gini index is calculated as the ratio of the area of the red region on the diagram to the sum of the areas of the red and blue regions.

Find the Gini index if the equation of the Lorenz curve is $y = 75\left(\frac{x}{100}\right)^2 + 25\left(\frac{x}{100}\right)^3$, where $x\%$ is the cumulative share of people and $y\%$ is the cumulative share of income earned.

The blue and the red regions together form a triangle. The area of this triangle is

$$\frac{100 \times 100}{2} = 5000.$$

Using a calculator you find that the area of the blue region is

$$\int_0^{100} 75\left(\frac{x}{100}\right)^2 + 25\left(\frac{x}{100}\right)^3 \mathrm{d}x = 3125.$$

The area of the red region is therefore $5000 - 3125 = 1875$.

Hence, the Gini index is $\frac{1875}{5000} = 0.375 = 37.5\%$.

International Mindedness

The World Bank publishes estimates of the Gini index for different countries. Explore Gini index data and read about the methodology of the measure using the interactive below.

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Interactive 7. Gini Coefficient.

More information for interactive 7

This interactive leverages graphical representations, data visualizations, and mathematical applications (such as definite integrals) to illustrate how the index is calculated and interpreted.
This interactive tool provides a comprehensive visualization of global income inequality trends from 1963 to 2023 using the Gini coefficient, a widely recognized metric developed by Corrado Gini. The platform leverages data from the World Bank Poverty and Inequality Platform (2024) to offer insights into how income distribution has evolved across countries over six decades.
Users can explore the data through three primary formats: a color-coded map, a sortable table, and dynamic charts, each designed to highlight different dimensions of inequality. The Gini coefficient's scale (0 to 1) is clearly explained, with visual cues (e.g., color gradients) to intuitively represent inequality levels, from blue (low inequality) to red (high inequality).
The tool's standout feature is its time-lapse function, which animates changes in inequality year by year, revealing macroeconomic trends, policy impacts, or crises. Hovering over any country displays its specific Gini coefficient and trajectory, while clicking tabs switches between map, table, and chart views for tailored analysis. The table allows sorting by country or inequality level, and charts can isolate regional comparisons or temporal patterns. Methodological notes clarify whether data reflects post-tax income, consumption, or per capita metrics, ensuring transparency.
Designed for policymakers, researchers, and educators, this interactive simplifies complex data into actionable insights. The time-lapse feature, for instance, could help identify the effects of progressive taxation in Scandinavia or rising inequality in emerging economies. By toggling between visualizations, users gain multifaceted perspectives. For example, the map highlights geographic disparities, while charts reveal outliers like South Africa's persistently high inequality.
This interactive serves as an excellent educational resource by combining mathematical visualization (definite integrals) with real-world economic data. It helps users grasp how the Gini index is derived and why it matters in analyzing income distribution worldwide.

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2 section questions

Anti-derivatives of power functions

In subtopic 5.3 (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26281/) you saw how to find the derivatives of functions of the form $f(x) = x^n$(where n is an integer) and multiples and sums of these.

Important

Recall that if $f(x) = ax^n$, then $f'(x) = nax^{n-1}$.

Exam tip

The formula booklet gives the formula without the coefficient:

$$f(x) = x^n \implies f'(x) = nx^{n-1}.$$

In this section, you will learn how to find the function if the derivative is given.

Activity

Work in pairs.

- Think of a function of the form $f(x) = ax^n + c$ for some integer a, c and n.
- Find the derivative, $f'(x)$ and write it on a piece of paper.
- Exchange papers and attempt to find the function your partner thought of.

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Example 1

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Find a possible function such that $f'(x) = 7x^3$.

You know that differentiating a power function reduces the exponent by 1.

So, if you find x^3 in the derivative, the original function must have x^4 in it.

The derivative of $g(x) = x^4$ is $g'(x) = 4x^3$.

Since $f'(x)$ has 7 as the coefficient instead of 4, you get $f(x)$ by modifying $g(x)$.

$$f(x) = \frac{7}{4}g(x) = \frac{7}{4}x^4.$$

To check that this is indeed a correct choice, you can differentiate:

$$f'(x) = \frac{7}{4} \times 4x^3 = 7x^3.$$

Note that, since the derivative of a constant function is 0,

or $f(x) = \frac{7}{4}x^4 + \pi$or $f(x) = \frac{7}{4}x^4 + c$for any c

also gives $f'(x) = 7x^3 + 0 = 7x^3$.

Example 2

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Find possible functions that give the following derivatives.

$f'(x)$	$f(x)$
$3x^2$	

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$f'(x)$	$f(x)$
$6x^2$	
x^2	
$5x^4$	
$3x^7$	

It is good revision of differentiation to check that the derivatives of the functions given in the right-hand column of the following table do indeed give the expressions in the left-hand column.

$f'(x)$	$f(x)$
$3x^2$	x^3 or x^3+1 or x^3-7 or $\ldots$
$6x^2$	$2x^3$ or $2x^3+0.4$ or $2x^3-3.9$ or $\ldots$
x^2	$\frac{1}{3}x^3$ or $\frac{1}{3}x^3+5$ or $\frac{1}{3}x^3-\frac{8}{5}$ or $\ldots$
$5x^4$	x^5 or x^5+17 or x^5-138 or $\ldots$
$3x^7$	$\frac{3}{8}x^8$ or $\frac{3}{8}x^8+4.59$ or $\frac{3}{8}x^8-1.27$ or $\ldots$

Do you see a pattern in the table above? Can you formulate a rule? In the applet below you can check your conjecture and modify it if you need to.

Interactive 1. Finding the Anti-derivatives of Power Functions.

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More information for interactive 1

The interactive tool allows users to generate new questions related to finding the original function (antiderivative) from its derivative, with a focus on power functions (ax^n). Here, a is a constant, and n is an exponent.

At the top of the interface, there is a button labelled "Click here for a new question." When clicked, this generates a new mathematical expression for $f'(x)$, which represents the derivative of an unknown function $f(x)$.

Below the button, the interface displays a mathematical expression prompting the user to find a possible function $f(x)$ given its derivative $f'(x)$. The task is to determine the original function by computing the antiderivative.

Below the problem statement, there is a checkbox labelled "Show the answer." When checked, the tool displays the computed antiderivative $f(x)$. The correct expression for the function includes an integration constant, as the antiderivative of a function is not unique and can differ by a constant.

Example: If $f'(x) = \frac{8}{3}x^2$

Find a possible expression of $f(x)$.

Answer: $f'(x) = \frac{8}{3}x^3$ + any constant

This interactive system provides an excellent way to practice finding antiderivatives of power functions, ensuring a deeper grasp of fundamental calculus concepts.

You are now ready for the following definition and notation.

Important

The function F is an anti-derivative of the function f if $F'(x) = f(x)$.

The collection of all anti-derivatives of the function f is denoted by $\int f(x)\mathrm{d}x$.

This is also called the indefinite integral of f.

The following gives the formula for the anti-derivatives of power functions.

Important

$\int ax^n \mathrm{d}x = \frac{a}{n+1}x^{n+1} + c$, where c is any constant and $n \neq -1$ is an integer.

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Can you prove this claim?

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You can use the rules of differentiation to check the derivative of $F(x) = \frac{a}{n+1}x^{n+1} + c$,

$$F'(x) = \frac{a}{n+1}(n+1)x^{n+1-1} + 0 = ax^n.$$

To show that there are no other functions with the same derivative is beyond the syllabus.

Note that the claim above is also true for $n = 0$. Since $x^0 = 1$, this means that $\int a \mathrm{d}x = ax + c$.

Exam tip

The formula booklet gives the formula without the coefficient:

$$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

Example 3

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Find $\int 3x^4 \mathrm{d}x$ and $\int \frac{3}{x^4} \mathrm{d}x$.

Using the formula $\int ax^n \mathrm{d}x = \frac{a}{n+1}x^{n+1} + c$for $a = 3$ and $n = 4$ gives

$$\int 3x^4 \mathrm{d}x = \frac{3}{4+1}x^{4+1} + c = \frac{3}{5}x^5 + c.$$

To find the other integral, rewrite $\frac{3}{x^4}$ as $3x^{-4}$.Using this form,

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$$\begin{aligned}\int \frac{3}{x^4}\mathrm{d}x &= \int 3x^{-4}\mathrm{d}x\\ &= \frac{3}{-4+1}x^{-4+1}+c\\ &= \frac{3}{-3}x^{-3}+c = -\frac{1}{x^3}+c.\end{aligned}$$

Example 4

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Find the following indefinite integrals.

$f(x)$	$\int f(x)\mathrm{d}x$
$7x^3$	
$\frac{x^2}{5}$	
$4x^{-3}$	
$-5x$	
8	
$\frac{2}{x^4}$	
$\frac{x}{7x^3}$	

$f(x)$	$\int f(x)\mathrm{d}x$
$7x^3$	$\int 7x^3\mathrm{d}x = \frac{7}{4}x^4+c$
$\frac{x^2}{5}$	$\int \frac{x^2}{5}\mathrm{d}x = \int \frac{1}{5}x^2\mathrm{d}x = \frac{1}{5\times 3}x^3+c = \frac{1}{15}x^3+c$

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$f(x)$	$\int f(x)\mathrm{d}x$
$4x^{-3}$	$\int 4x^{-3}\mathrm{d}x = \frac{4}{-2}x^{-2} + c = -2x^{-2} + c$
$-5x$	$\int -5x\mathrm{d}x = \int -5x^{1}\mathrm{d}x = \frac{-5}{2}x^{2} + c$
8	$\int 8\mathrm{d}x = 8x + c$
$\frac{2}{x^4}$	$\int \frac{2}{x^4}\mathrm{d}x = \int 2x^{-4}\mathrm{d}x = \frac{2}{-3}x^{-3} + c = -\frac{2}{3x^3} + c$
$\frac{x}{7x^3}$	$\int \frac{x}{7x^3}\mathrm{d}x = \int \frac{1}{7}x^{-2}\mathrm{d}x = \frac{1}{7\times(-1)}x^{-1} + c = -\frac{1}{7x}$

You can check your understanding in the following applet .

Interactive 2. Finding the Definite Integrals.

More information for interactive 2

The interactive tool is designed to help users practice integration, specifically focusing on power functions, which form the foundation of integral calculus. The tool demonstrates how to integrate monomial terms using the basic power rule, allowing users to master this essential technique before moving on to more complex integrals. By practicing with simple power functions, users can develop a strong understanding of the integration process and build confidence in solving fundamental calculus problems.

The interactive tool provides focused practice on integrals of the form ax^n, where users can generate unlimited variations of these problems. The applet focuses on power functions, which are functions of the form:

$f(x) = ax^n$,

where a is a constant and n is a real number. The anti-derivative (or integral) of a power function follows the rule:

$\int ax^n\,dx = \frac{a}{n+1}x^{n+1} + c$,

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where c is the constant of integration.

By clicking the "Click here for a new question" button, the system creates a fresh integration problem, allowing learners to apply the power rule directly. After attempting the solution, users can verify their work by clicking "Show the Answer" to reveal the final answer.

For example, if you click "Click here for a new question," a problem like this might appear:

Find $\int 6x^5 dx$:

Solution:

1. Apply the power rule: $\int 6x^5 dx = \frac{6}{5+1}x^{5+1} + c$

2. Simplify: $x^6 + c$

Answer: $x^6 + c$

This approach ensures systematic practice, with immediate feedback to reinforce proper technique. The tool adapts to different skill levels by varying coefficients and exponents, building both competence and confidence in basic integration.

Making connections

- In the Applications and interpretation SL course, you will only learn about finding anti-derivatives of power functions of the form discussed above. In the next section, you will see how to deal with sums and differences of these type of functions.
- In the Applications and interpretation HL course and in the Analysis and approaches SL and HL courses, you will:
  - discover that the claim above is true for other exponents, not only integers
  - learn that $\int x^{-1}\mathrm{d}x = \int \frac{1}{x}\mathrm{d}x$
  - learn about anti-derivatives of other types of function (trigonometric, exponential, logarithmic).

4 section questions

5. Calculus / 5.5 Introduction to integration

Integral of the sum or difference

Student view

As for differentiation, you can integrate a sum or difference of functions term by term.

Important

$$\int f(x)+g(x)\mathrm{d}x=\int f(x)\mathrm{d}x+\int g(x)\mathrm{d}x$$

$$\int f(x)-g(x)\mathrm{d}x=\int f(x)\mathrm{d}x-\int g(x)\mathrm{d}x$$

Can you prove these results?

If F is an anti-derivative of f and G is an anti-derivative of g, then using the rules of differentiation,

$$(F+G)'(x)=F'(x)+G'(x)=f(x)+g(x)=(f+g)(x).$$

This means, that $F+G$ is an anti-derivative of $f+g$, so

$$\int f(x)+g(x)\mathrm{d}x=F(x)+G(x)+c=\int f(x)\mathrm{d}x+\int g(x)\mathrm{d}x.$$

The proof for the difference is similar.

Example 1

★☆☆

Find $\int x+1-\frac{1}{x^2}\mathrm{d}x$.

Using the sum rule and the difference rule,

$$\int x+1-\frac{1}{x^2}\mathrm{d}x=\int x\mathrm{d}x+\int 1\mathrm{d}x-\int\frac{1}{x^2}\mathrm{d}x.$$

In other words, you integrate term by term.

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- $\int x \, dx = \int x^1 dx = \frac{1}{2}x^2 + c.$
- $\int 1 \, dx = x + c.$
- $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{1}{-1}x^{-1} + c = -\frac{1}{x} + c.$

Hence,

$$\int x + 1 - \frac{1}{x^2} dx = \frac{1}{2}x^2 + x - \left(-\frac{1}{x}\right) + c = \frac{1}{2}x^2 + x + \frac{1}{x} + c.$$

Example 2

★★☆

Find the following indefinite integrals.

$f(x)$	$\int f(x) dx$
$5x^2 + 4x^{-3}$	
$\frac{x^5 - x}{7}$	
$7x + \frac{2}{x^3} - 6$	
$\frac{x^6 - x^2}{x^2}$	
$(2x+3)(x^2 - x - 4)$	

First, rewrite the expressions, then use the power rule and the rules for the sum or difference.

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$f(x)$	$\int f(x)\mathrm{d}x$
$5x^2+4x^{-3}$	$\int 5x^2+4x^{-3}\mathrm{d}x = \frac{5}{3}x^3+\frac{4}{-2}x^{-2}+c$ $= \frac{5}{3}x^3-2x^{-2}+c$
$\frac{x^5-x}{7}$	$\frac{x^5-x}{7} = \frac{1}{7}x^5-\frac{1}{7}x$ $\int \frac{x^5-x}{7}\mathrm{d}x = \frac{1}{7\times 6}x^6-\frac{1}{7\times 2}x^2+c$ $= \frac{1}{42}x^6-\frac{1}{14}x^2+c$
$7x+\frac{2}{x^3}-6$	$7x+\frac{2}{x^3}-6 = 7x+2x^{-3}-6$ $\int 7x+\frac{2}{x^3}-6\mathrm{d}x = \frac{7}{2}x^2+\frac{2}{-2}x^{-2}-6x+c$ $= \frac{7}{2}x^2-\frac{1}{x^2}-6x+c$
$\frac{x^6-x^2}{x^2}$	$\frac{x^6-x^2}{x^2} = x^4-1$ $\int \frac{x^6-x^2}{x^2}\mathrm{d}x = \frac{1}{5}x^5-x+c$
$(2x+3)(x^2-x-4)$	$(2x+3)(x^2-x-4) = 2x^3+x^2-11x-12$ $\int (2x+3)(x^2-x-4)\mathrm{d}x = \frac{2}{4}x^4+\frac{1}{3}x^3-\frac{11}{2}x^2-12x$ $= \frac{x^4}{2}+\frac{x^3}{3}-\frac{11x^2}{2}-12x+$

In the applet below you can check your understanding.

Interactive 1. Finding the Indefinite Integrals.

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More information for interactive 1

This interactive applet is designed to help users practice and understand the integration of sums of power functions. It provides automatically generated questions where the user must integrate an expression consisting of a sum of terms, reinforcing the linearity property of integration:

$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

This property allows the integral of a sum to be calculated term by term, making complex integrations more manageable. By practicing with polynomial and rational functions, users can develop a deeper understanding of how to apply this property effectively in various scenarios.

The interactive tool provides focused practice on integrals of the form $\frac{(ax^p + bx^k)}{cx^r}$, where users can generate unlimited variations of these problems. Here a, b and c are constants. Also p, k and r are real numbers.

By clicking the "Click here for a new question" button, the system creates a fresh integration problem, allowing learners to apply the linearity property to split and solve each term individually. After attempting the solution, users can verify their work by clicking "Show the Answer" to reveal the final answer.

For example, if you click "Click here for a new question," a problem like this might appear:

Find $\int \frac{4x^5 + 6x^3}{2x^4}\,dx$

Solution:

1. Simplify: $\frac{4x^5 + 6x^3}{2x^4} = \frac{4x^5}{2x^4} + \frac{6x^3}{2x^4} = 2x + 3x^{-1}$
2. Integrate term by term: $\int 2x\,dx + \int 3x^{-1}\,dx$
3. Compute: $x^2 + 3\ln|x| + C$

Answer: $x^2 + 3\ln|x| + C$

This approach ensures systematic practice, with immediate feedback to reinforce proper technique. The tool adapts to different skill levels by varying exponents and coefficients, building both competence and confidence in integration.

Example 3

★★☆

Consider the functions defined by $f(x) = x + \frac{1}{x^2}$ and $g(x) = x^3 + x^2$.

- Find $\int f(x)\mathrm{d}x$.

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- Find $\int g(x)\mathrm{d}x$.
- Find $\int f(x)g(x)\mathrm{d}x$.
- Comment on the results.

- $\int x + \frac{1}{x^2}\mathrm{d}x = \int x + x^{-2}\mathrm{d}x = \frac{1}{2}x^2 + \frac{1}{-1}x^{-1} + c = \frac{1}{2}x^2 - \frac{1}{x} + c$
- $\int x^3 + x^2\mathrm{d}x = \frac{1}{4}x^4 + \frac{1}{3}x^3 + c$

First, find the product:

$$f(x)g(x) = \left(x + \frac{1}{x^2}\right)(x^3 + x^2) = x^4 + x^3 + x + 1.$$

So,

$$\int f(x)g(x)\mathrm{d}x = \frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{2}x^2 + x + c.$$

Since

$$\left(\frac{1}{2}x^2 - \frac{1}{x}\right)\left(\frac{1}{4}x^4 + \frac{1}{3}x^3\right) = \frac{1}{8}x^6 + \frac{1}{6}x^5 - \frac{1}{4}x^3 - \frac{1}{3}x^2,$$

you can see that $\int f(x)g(x)\mathrm{d}x$ is not the same as the product of $\int f(x)\mathrm{d}x$ and $\int g(x)\mathrm{d}x$.

Be aware

The rule you learned in this section works for the sums and the differences, but it does not work for products or the quotients.

3 section questions

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Boundary conditions

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/boundary-conditions-id-26298/print/) Assign

In the previous section, you discovered the formula for how to find anti-derivatives of power functions and sums and differences of these. However, in the formula

$$\int ax^n \mathrm{d}x = \frac{a}{n+1}x^{n+1} + C,$$

the constant C can be any real number. There are infinitely many functions with the same derivative ax^n.

In this section, you will explore how additional information (besides the derivative) lets you find a particular function.

Example 1

★☆☆

Find $f(x)$ when $f'(x) = 3x^2$ and $f(0) = 5$.

You start by finding the anti-derivative of f':

$$\int 3x^2 \mathrm{d}x = x^3 + c$$

Since $f'(x) = 3x^2$, this means that $f(x) = x^3 + c$ for some value of c.

To find this particular value, you use the additional information that $f(0) = 5$.

$$\begin{aligned} f(x) &= x^3 + c \\ 5 &= 0^3 + c \\ c &= 5 \end{aligned}$$

Hence, $f(x) = x^3 + 5$.

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Example 2

★★☆

Find $f(x)$ when $f'(x)=\dfrac{3+2x^3}{3x^2}$ and $f(1)=0$.

You start by finding the anti-derivative of f':

$$\begin{aligned}\int\frac{3+2x^3}{3x^2}\,\mathrm{d}x&=\int\frac{1}{x^2}+\frac{2x}{3}\,\mathrm{d}x\\&=\int x^{-2}+\frac{2}{3}x\,\mathrm{d}x\\&=\frac{1}{-1}x^{-1}+\frac{2}{3\times 2}x^2+c=\frac{x^2}{3}-\frac{1}{x}+c\end{aligned}$$

Since $f'(x)=\dfrac{3+2x^3}{3x^2}$, this means that $f(x)=\dfrac{x^2}{3}-\dfrac{1}{x}+c$ for some value of c.

To find this particular value, you use the additional information that $f(1)=0$.

$$\begin{aligned}f(x)&=\frac{x^2}{3}-\frac{1}{x}+c\\0&=\frac{1^2}{3}-\frac{1}{1}+c\\c&=\frac{2}{3}\end{aligned}$$

Hence, $f(x)=\dfrac{x^2}{3}-\dfrac{1}{x}+\dfrac{2}{3}$.

Example 3

★★☆

Find the following functions.

$f'(x)$	**Boundary condition**	$f(x)$
$3x+2x^{-2}$	$f(2)=3$	

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$f'(x)$	Boundary condition	$f(x)$
$2+3x+4x^2$	$f(1)=4$	
$\frac{2x^2+3}{x^2}$	$f\left(\frac{1}{3}\right)=-5$	
$(1+x)(1+x^2)$	$f(1)=\frac{7}{3}$	

You can use the methods of the previous examples to find the particular solutions.

$f'(x)$	Boundary condition	$f(x)$
$3x+2x^{-2}$	$f(2)=3$	$f(x)=\frac{3x^2}{2}-\frac{2}{x}-2$
$2+3x+4x^2$	$f(1)=4$	$f(x)=2x+\frac{3}{2}x^2+\frac{4}{3}x^3-\frac{5}{6}$
$\frac{2x^2+3}{x^2}$	$f\left(\frac{1}{3}\right)=-5$	$f(x)=2x-\frac{3}{x}+\frac{10}{3}$
$(1+x)(1+x^2)$	$f(1)=\frac{7}{3}$	$f(x)=\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+\frac{1}{4}$

Example 4

★★☆

Find $f(x)$ when $f'(x)=(x^2-x)\left(1-\frac{1}{x}\right)$ and the point $(3,1)$ is on the graph of $y=f(x)$.

We start by expanding the expression given in the question and then find the anti-derivative of $f'(x)$.

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$$f'(x) = (x^2 - x)\left(1 - \frac{1}{x}\right)$$
$$f'(x) = x^2 - x - x + 1 = x^2 - 2x + 1$$
$$\int f'(x)\mathrm{d}x = \frac{1}{3}x^3 - \frac{2}{2}x^2 + x + c = \frac{x^3}{3} - x^2 + x + c$$

This means that $f(x) = \frac{x^3}{3} - x^2 + x + c$for some value of c.

To find this particular value, you use the additional information that the point $(3, 1)$ is on the graph, which means that $f(3) = 1$.

$$f(x) = \frac{x^3}{3} - x^2 + x + c$$
$$1 = \frac{3^3}{3} - 3^2 + 3 + c$$
$$1 = 9 - 9 + 3 + c$$
$$c = -2$$

Hence, $f(x) = \frac{x^3}{3} - x^2 + x - 2$.

Example 5

★★☆

Find the following functions.

$f'(x)$	**Point on graph of** $y = f(x)$	$f(x)$
$5x^3 - 6x^{-3}$	$(-1, 4)$	
$1 - x^2 + x^5$	$(2, 0)$	
$\frac{3x^{11} + 2 + 5x^7}{x^7}$	$(3, 99)$	
$(x - x^2)(1 + x^3)$	$(0, 5)$	

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You can use the methods of the previous examples to find the particular solutions.

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$f'(x)$	Point on graph of $y = f(x)$	$f(x)$
$5x^3 - 6x^{-3}$	$(-1, 4)$	$\frac{5x^4}{4} + \frac{3}{x^2} - \frac{1}{4}$
$1 - x^2 + x^5$	$(2, 0)$	$x - \frac{x^3}{3} + \frac{x^6}{6} - 10$
$\frac{3x^{11} + 2 + 5x^7}{x^7}$	$(3, 99)$	$\frac{3x^5}{5} - \frac{1}{3x^6} + 5x - 6$
$(x - x^2)(1 + x^3)$	$(0, 5)$	$-\frac{x^6}{6} + \frac{x^5}{5} - \frac{x^3}{3} + \frac{x^2}{2}$

Note that in the third row the value of the constant is only an approximate.

The exact value is $-\frac{675\,778}{10\,935}$.

You can check your understanding in the applet below.

Interactive 1. Integration with Boundary Conditions.

More information for interactive 1

The interactive is designed to help users practice finding specific functions $f(x)$ when given their derivatives $f'(x)$ and additional boundary conditions, such as a point $P(x, f(x))$ on the graph of the function. The process involves two key steps: First, finding the general antiderivative of $f'(x)$ by integrating term by term, which includes a constant of integration C. Second, the boundary condition provided by the point $P(x, f(x))$ is used to determine the specific value of C.

At the top of the interface, there is a button labelled "Click here for a new question." Clicking this button generates a

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new integration problem that involves solving for C using a given boundary condition. Below the problem, there is an option labelled "Show the answer." If selected, the correct solution is displayed, including the computed value of C. This allows users to verify their work and identify any errors in their calculations.

Example: Find $(f(x))$, if $f'(x) = 3x^4 + \frac{9}{x^7} + 8$, and the point $P(-1, 54)$ is on the graph of $f(x)$.

The answer is:

$f(x) = \frac{3}{5}x^5 - \frac{9}{6x^6} + 8x + 64.1$

This interactive tool provides an excellent tool for experiencing mastering integration with boundary conditions.

Example 6

★★☆

A car is travelling in a town with speed 13.5 m s^{-1}. When the car leaves the town, the driver starts accelerating with a constant acceleration $a = 4 \text{ m s}^{-2}$.Find the speed of the car 3 s after it starts accelerating.

Making connections

You may already know from your physics studies that the acceleration is the change in velocity:

$$a(t) = \frac{\mathrm{d}v}{\mathrm{d}t}.$$

Since the acceleration is the derivative of the velocity, the velocity is an anti-derivative of the acceleration,

$$v(t) = \int a(t)\mathrm{d}t = \int 4\mathrm{d}t = 4t + c, \text{for some value of } c.$$

The speed at the start of the acceleration is 13.5 m s^{-1},so $v(0) = 13.5$.

So,

$$\begin{aligned} 13.5 &= 4 \times 0 + c \\ c &= 13.5. \end{aligned}$$

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Hence, the velocity of the car t s after the start of the acceleration is given by

$$v(t) = 4t + 13.5.$$

So the speed 3 s after the driver starts accelerating is

$$v(3) = 4 \times 3 + 13.5 = 25.5 \text{ m s}^{-1}.$$

Example 7

★★☆

The line $y = 4x - 1$ is a tangent to the graph of $y = f(x)$. It is also given that $f'(x) = 2x$.

Find $f(x)$.

- First, find the point of tangency:
  The gradient of the line $y = 4x - 1$ is 4.
  Hence, if the point of tangency is (a, b), then $f'(a) = 4$.
  Since $f'(x) = 2x$, this means that $2a = 4$, so $a = 2$.
  Since the point of tangency is on the line $y = 4x - 1$, this means that
  $b = 4 \times 2 - 1 = 7$.
  Hence, the point of tangency is $(2, 7)$.
- Next, find $f(x)$.
  Since $\int 2x \, \mathrm{d}x = x^2 + c$,you know that $f(x) = x^2 + c$ for some value of c.
  Since the point of tangency is also on the graph of $y = f(x)$,
  $$\begin{aligned} f(2) &= 7 \\ 2^2 + c &= 7 \\ c &= 3. \end{aligned}$$
  Hence, $f(x) = x^2 + 3$.

Example 8

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The line $y = 4x + 9.875$ is normal to the graph of $y = f(x)$. It is also given that $f'(x) = \dfrac{2}{x^3}$.

Find $f(x)$.

- First, find the point where the given normal line intersects the graph of $y = f(x)$.
  The gradient of the line $y = 4x + 9.875$ is 4.
  Hence, if the point where this line is normal to the graph is (a, b), then $f'(a) = -\dfrac{1}{4}$.
  Since $f'(x) = \dfrac{2}{x^3}$, this means that
  $$\begin{aligned}\frac{2}{a^3} &= -\frac{1}{4}\\ -8 &= a^3\\ a &= -2.\end{aligned}$$
  Since the point where the normal meets the graph is on the given line, this means that
  $$b = 4 \times (-2) + 9.875 = 1.875.$$
  Hence, the point where the normal meets the graph is $(-2, 1.875)$.
- Next, find $f(x)$.
  Since $\displaystyle\int \frac{2}{x^3}\mathrm{d}x = \int 2x^{-3}\mathrm{d}x = \frac{2}{-2}x^{-2} + c = -\frac{1}{x^2} + c$, you know that
  $$f(x) = -\frac{1}{x^2} + c \text{ for some value of } c.$$
  Since the point $(-2, 1.875)$ is also on the graph of $y = f(x)$,
  $$\begin{aligned}f(-2) &= 1.875\\ -\frac{1}{(-2)^2} + c &= 1.875\\ c &= 2.125.\end{aligned}$$
  Hence, $f(x) = 2.125 - \dfrac{1}{x^2}$.

3 section questions

5. Calculus / 5.5 Introduction to integration

Checklist

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What you should know

By the end of this subtopic you should be able to:

- understand the connection between differentiation and anti-differentiation
- understand the notation $\int f(x)\mathrm{d}x$ for the indefinite integral (anti-derivative)
- work out anti-derivatives of functions of the form $f(x) = ax^n$ where $n \neq -1$ is an integer, and of sums and differences of this type of function
- understand the importance of the added constant in the indefinite integral and work out its value from a boundary condition
- understand the concept of a definite integral and be familiar with the notation $\int_a^b f(x)\mathrm{d}x$
- recognise the similarity in the notation for indefinite and definite integrals
- find definite integrals using technology
- appreciate the connection between a definite integral and the areas of a region bounded by the x-axis and the graph of a function.

Investigation

Part 1

The graph of $y = x^n$ (for $n = 1, 2, 3, \ldots$) goes through the points $(0, 0)$ and $(1, 1)$, so it cuts the unit square into two regions.

Philippe and Michael play the following game: They pick a random point in the unit square. Philippe gets a domino tile in every round. Michael gets a certain number of domino tiles if the point is in the region below the graph of $y = x^n$. They then put their dominoes on top of

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each other to build two towers.

Guinness world record: The tallest domino structure on ear...

The applet below simulates this game. You can change the exponent in the definition of the boundary curve. You can also change the number of dominoes Michael gets in each round for a point in the shaded region. Every click of the button generates 100 points (one for each of the 100 rounds of the game) and tells you the total number of dominoes Michael has after 100 rounds.

Interactive 1. Investigating Integration Concepts.

More information for interactive 1

This interactive applet simulates a game where users can explore the distribution of points relative to a boundary curve and track how adjustments affect the number of dominoes Michael earns.

The boundary curve is defined by the equation $y = x^n$, and users can adjust the exponent n to any integer value

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between 1 and 9, which changes the curve's shape on the graph. The graph displays 100 points within the unit square (where x and y range from 0 to 1), each point representing a round of the game.

The region below the curve $y = x^n$ is shaded, and points falling within this shaded region earn a user-specified number of dominoes per point. Users can select the number of dominoes awarded per point in the shaded region, with options ranging from 1 to 10. By clicking the "New points" button, the applet generates a new set of 100 random points, recalculates the number of points in the shaded region below the curve, and updates the total number of dominoes the user has based on the chosen reward per point. The total number of dominoes the user has after 100 rounds is displayed, allowing users to observe how changing n (which alters the curve and thus the size of the shaded region) and the number of dominoes awarded per point impacts Michael's total dominoes, providing insight into the relationship between these parameters and the game's outcome.

As n increases, the curve steepens, making it more likely for points to fall below the curve, favouring Michael. The multiplier setting for Michael can drastically affect the game's balance. The distribution of points and tile collection patterns can be analyzed statistically for different values of n.

Example: In the interactive applet, if we set to the curve $y = x^2$, dividing the unit square into two regions, the shaded region below the curve represents the area where Michael earns dominoes, while the unshaded region above the curve represents Philippe's territory.

The text on the left bottom indicates that Michael currently has 54 dominoes, based on the number of points that landed in his region and the multiplier of 2 dominoes per point.

The "New points" button at the bottom lets users generate a fresh set of 100 random points and observe how the results change.

By adjusting the exponent n and Michael's domino multiplier, users can observe how small mathematical changes impact the game's outcome over multiple rounds.

- Investigate what happens when you change the exponent.
- Investigate what happens when you change the number of dominoes Michael gets for a point in the shaded region.
- Who would you expect to have the taller tower, Philippe or Michael?
- Does this depend on the number of dominoes Michael gets for a point in the shaded region?
  - If yes, can you explain why?
  - If yes, can you explain how?

Part 2

In the applet below you can investigate the relationship between the geometrical concept of the area of a region and the process of differentiation/anti-differentiation.

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The applet shows the graph of a function f and the graph of its derivative, f'. It also lets you move a point on the x-axis and calculates the definite integral of $f'(x)$ over the interval from the origin to this moving point. If you use the notation p for the x-coordinate of this moving point, the applet also shows the point $\mathrm{P}\left(p, \int_0^p f'(x)\mathrm{d}x\right)$ and also shows the trace of this point.

Interactive 2. Exploring the Fundamental Idea of Integration.

More information for interactive 2

The interactive applet visually explores the relationship between differentiation and integration. The interactive allows users to explore the relationship between a function $f(x)$ and its derivative $f'(x)$. The applet displays the graph of $f(x)$ in blue and the graph of its derivative $f'(x)$ in red. There are two options for understanding the interactive. 'Trace point P' and 'Adjust Curve'

When 'Trace point P' in selected users can move a red point P along the x-axis, starting from the origin, and the applet calculates the definite integral of $f'(x)$ over the interval from 0 to p, where p is the x-coordinate of P. The point $P(p, \int_0^p f'(x)dx)$ traces its path as P moves, highlighted in light blue colour on the graph.

Users can move the red dot along the x-axis and note the real-time change in the equation and the coordinates of point P changing. For example, the integration of the derivative of the function $f(x)$, when the trace point, $P(1, 0.76)$, displays in the interactive as:

$\int_0^1 f'(x)dx = 0.76$

When the 'Adjust curve' is selected, six interactive red points appear on the blue graph, which, when dragged, move the whole curve according to the equation.

By observing the trace of point P, the user discovers that the computed integral reconstructs $f(x)$, demonstrating the

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Fundamental Theorem of Calculus. The final challenge asks the user to formalise this observation mathematically.

The applet provides an engaging, hands-on way to understand how differentiation and integration are inverse processes. Through this interactive applet, users gain an intuitive grasp of the Fundamental Theorem of Calculus.

- Move the red point on the x-axis.
- Do you notice anything about the trace of point P?
- Change the shape of the curve and repeat the process. Do you need to modify your observation?
- Challenge: Can you formalise this observation?