TOPIC 4
PROBABILITY AND STATISTICS

SUBTOPIC 4.6
PROBABILITY CALCULATIONS

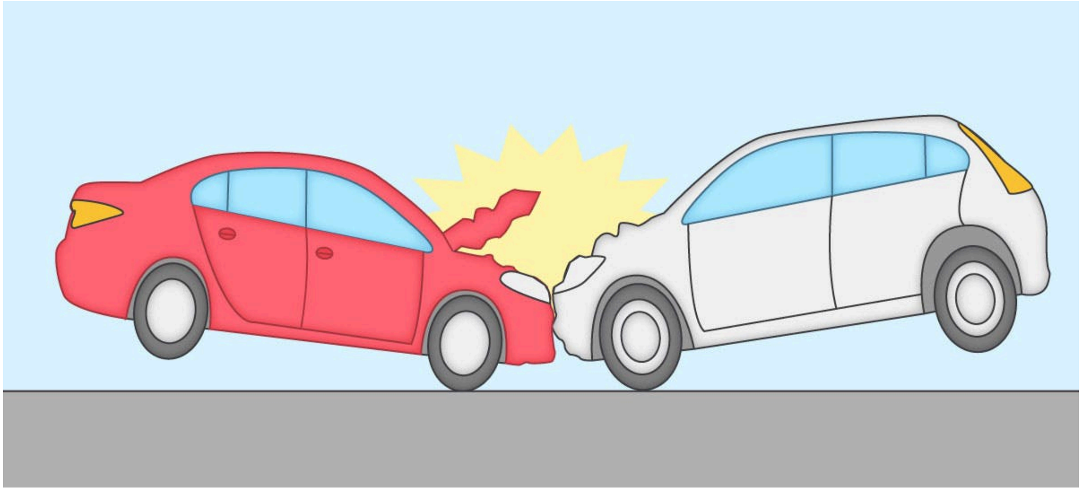
Index





# The big picture

For car insurance company A to make a profit, the value of their clients' claims (what the insurance company pays to its clients) must be less than the total value of its clients' premiums (what each client pays to the company).



When disaster strikes.

However, an insurance company cannot tell who or how many of its clients will be involved in a car accident and need to claim. It needs to be able to predict the future.

Not only that, but car insurance company B is in competition with car insurance company A. Both want to offer competitive rates. However, neither wishes to offer premiums that are so low that they do not cover the cost of insuring clients. They need to be able to predict the future with a reasonable amount of accuracy.

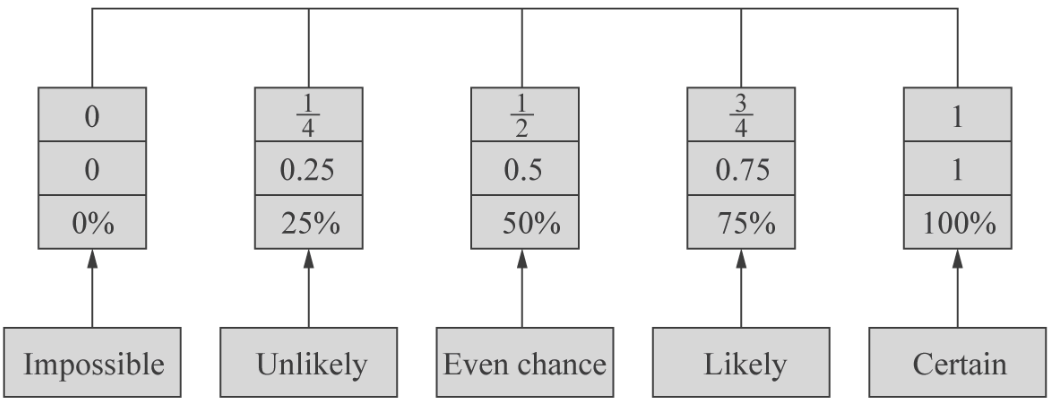
In the study of probability, we use the theoretical world to predict what will happen in the practical world.

Interestingly, the very first insurance company was the idea of two Scottish churchmen, Robert Wallace and Alexander Webster, in the middle of the 18th century. It was deemed unchristian that the widow of a deceased minister (a clergyman) should have to live in poverty. Therefore, Wallace and Webster set up an account for all clergy to pay into, such that their widows could live comfortably should their husbands suffer an early death. Thus, the first life insurance company was born.

Insurance companies play a significant role in modern society. The groundwork for this mathematical development was laid by Pascal, Fermat and Huygens in the 17th century. The work of these mathematicians on theoretical probability was motivated by games of chance. This is an interesting example of how new developments in mathematics in one century might seem irrelevant to practical living, but in the next century become central to our society.

# The probability scale

In this course, we use either fractions or decimals to record a probability , although it is also acceptable to give a probability as a percentage. Any probability takes a value between   (impossible) and   (certain), as illustrated in the figure below . Terms such as ‘unlikely’ and ‘likely’ do not have fixed definitions in terms of probabilities, but they can be useful for you to consider as you interpret problems involving probability.




The probability scale.

Student view

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The image depicts a diagram representing the probability scale with different sections labeled from "Impossible" to "Certain." There are five main divisions:

1. **Impossible**: Displays "0" in fractional form, "0" decimal form, and "0%" as a percentage.
2. **Unlikely**: Shows "1/4" in fractional form, "0.25" in decimal form, and "25%" as a percentage.
3. **Even chance**: Indicates "1/2" in fractional form, "0.5" in decimal form, and "50%" as a percentage.
4. **Likely**: Contains "3/4" in fractional form, "0.75" in decimal form, and "75%" as a percentage.
5. **Certain**: Displays "1" in fractional form, "1" in decimal form, and "100%" as a percentage.

Each section is visually aligned top to bottom, with arrows pointing from the values to the corresponding descriptive terms at the bottom, which describe the likelihood of the occurrence ranging from impossible to certain.

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In this subtopic, we introduce the basic ideas of probability: ways to record all possible outcomes and basic probability calculations.

# Fractions

Work on probability requires some fluency with fractions. The skills required are the ability to add, subtract, multiply, divide and simplify fractions . If confidence regarding fractions is not optimal, you may either brush up your fraction skills here (http://www.mathsisfun.com/fractions-menu.html) or review how to input and read fractions on your calculator.

### Concept

The study and application of probabilities is dependent on the concept of **patterns**. To apply probabilities, we must assume that events in the future will occur much as they have occurred in the past. This is particularly true when considering multiple events occurring together. A simple example is the assumption that a coin will never land on its edge; more complex assumptions include those about humans making choices similar to those they have made before. If, for whatever reason, we cannot assume that the

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events will continue to occur in this way, then we try to determine how they will change and account for that in our predictions, if possible. How can events in real life differ from the probability of events we consider from the safety of the classroom? How realistic are the assumptions we make?

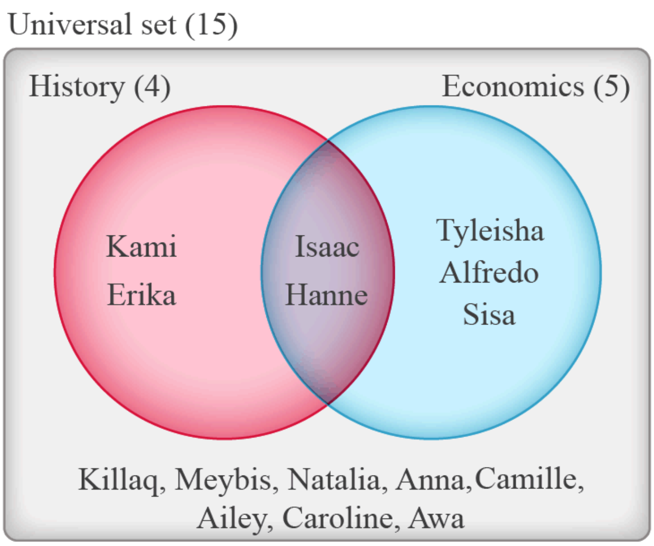
4. Probability and statistics / 4.6 Probability calculations

# Modelling related events

## Modelling events

### Venn diagrams

A Venn diagram is a model illustrating two or more sets of data using overlapping circles to show the elements of the sets. If elements are members of multiple sets, then they are located in the part of the diagram where the circles for those sets overlap. Some examples of Venn diagrams are shown in the three figures below.

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This Venn diagram shows a group of 15 students and their participation in two classes.

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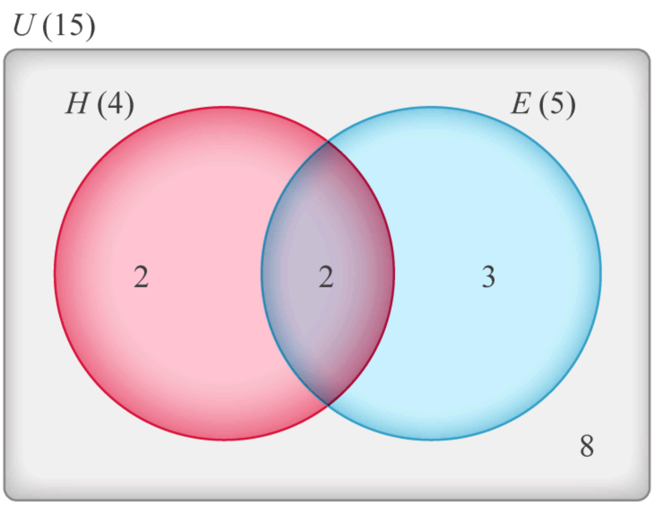
More information

The image is a Venn diagram representing a group of 15 students and their participation in two classes: History and Economics. There are two overlapping circles, with one labeled "History (4)" and the other "Economics (5)." The overlapping section shows the students Isaac and Hanne, indicating that they are enrolled in both classes. In the History-only circle, the students Kami and Erika are listed. In the Economics-only circle, the students Tyleisha, Alfredo, and Sisa are listed. Outside the circles, additional students Killaq, Meybis, Natalia, Anna, Camille, Ailey, Caroline, and Awa are mentioned, representing those not participating in either class. The universal set is indicated with a total of 15 students.

[Generated by AI]




The names of the students have been replaced with the number of students in each area of the diagram. This is more useful for calculating probabilities.

More information

The image is a Venn diagram containing two overlapping circles. The left circle is labeled H (4) and the right circle is labeled E (5). The intersection of the two circles contains the number 2, indicating elements common to both sets. The left circle, representing set H, contains the number 2, and the right circle, representing set E, contains the number 3. Outside the circles, the number 8 is written, likely representing elements outside sets H and E but within the universal set U, which is labeled as U (15) at the top left corner of the image.

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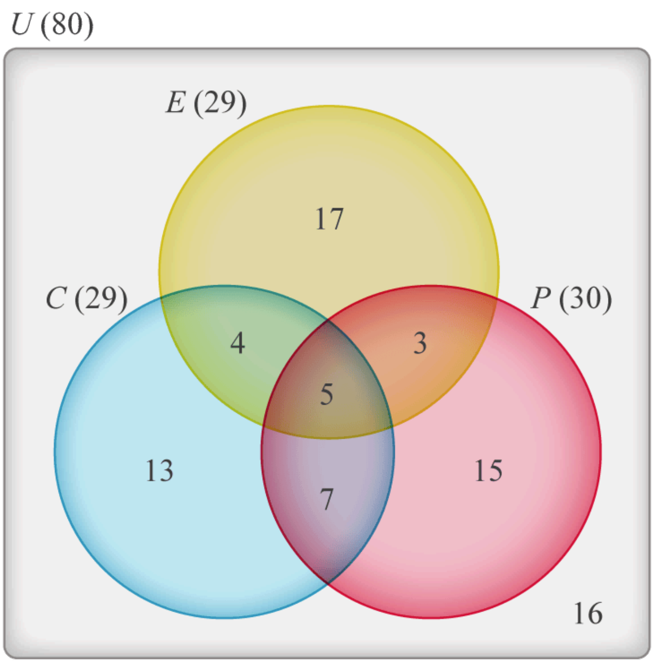


This Venn diagram shows a group of 80 students studying economics (E), chemistry (C) and physics (P).

More information

The image is a Venn diagram representing the distribution of 80 students studying economics, chemistry, and physics. The diagram consists of three overlapping circles labeled as follows: E (Economics), C (Chemistry), and P (Physics). Each circle represents a group of students in these subjects:

- Circle E has 29 students total, with 17 studying only economics, 4 studying both economics and chemistry, 3 studying both economics and physics, and 5 studying all three subjects.
- Circle C has 29 students total, with 13 studying only chemistry, 4 studying both chemistry and economics, 5 studying all three subjects, and 7 studying both chemistry and physics.
- Circle P has 30 students total, with 15 studying only physics, 3 studying both physics and economics, 7 studying both physics and chemistry, and 5 studying all three subjects.

The universal set, U, has a total of 80 students. The diagram helps visualize how many students are involved in the individual subjects, overlapping subjects, and all three subjects together.

[Generated by AI]

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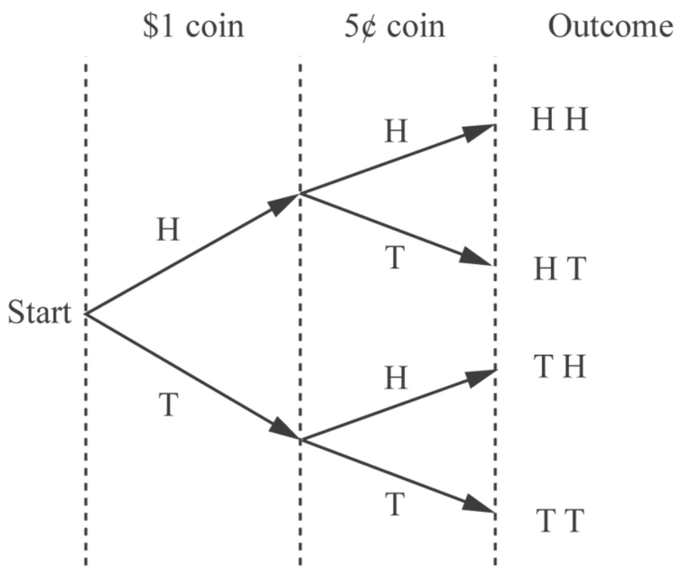
# Tree diagrams

Tree diagrams can be used to model situations in which a sequence of related events each has multiple outcomes. The tree diagram below shows the possible outcomes of flipping a 1 dollar coin and a 5 cent coin.




A tree diagram showing a sequence of flipping a 1 dollar coin and a 5 cent coin.

More information

The image is a tree diagram illustrating the possible outcomes of flipping a $1 coin followed by a 5¢ coin. The diagram starts with a single point labeled "Start." From this point, two branches emerge, representing the outcome of the $1 coin flip: "H" (heads) or "T" (tails).

From each of these outcomes, two additional branches emerge for the 5¢ coin, also labeled "H" or "T." This results in four final outcome branches. The results at the end of these branches are labeled: - For $1 coin head, 5¢ coin head: "HH" - For $1 coin head, 5¢ coin tail: "HT" - For $1 coin tail, 5¢ coin head: "TH" - For $1 coin tail, 5¢ coin tail: "TT"

This tree structure clearly shows all possibilities of two sequential coin flips, modeling how each flip affects the potential outcomes.

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These are particularly useful when there are more than two events, as you can visualise all the outcomes easily. Later we will see how these can be used to calculate probabilities of combined events.

# Related events

## Intersection and union

Intersection and union are two ways to combine related sets of events. The intersection combines sets by looking only for the elements that are in **both** sets, while the union is formed by combining all the elements in the two sets. Logically, the intersection can never have more elements than the smaller of the sets that are being combined, and the union must have at least as many elements as the larger of the sets. Why is this the case?

Mutually exclusive events are those that have no intersection. These relationships are illustrated in the table below.

Venn diagrams may help us to visualise the probabilities of combined events.

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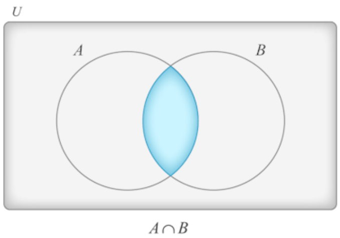
| Set theory | Venn diagram | Probability theory |
|---|---|---|
| $A \cap B$ is the intersection of sets $A$ and $B$. It contains the elements that belong to both set $A$ and set $B$. | <br><br>More information<br>The image is a Venn diagram depicting two intersecting circles labeled 'A' and 'B'. The overlapping area, representing the intersection of sets A and B, is highlighted in blue and labeled 'A ∩ B'. The entire space, referred to as the universal set, is marked with a 'U' at the top left corner. The diagram illustrates the concept of set intersection, where elements common to both A and B are included in the highlighted central section. The circles themselves are contained within a rectangular boundary, which symbolizes the universal set 'U'.<br>[Generated by AI] | $\mathrm{P}(A \cap B)$ is the probability of both $A$ and $B$ occurring. |

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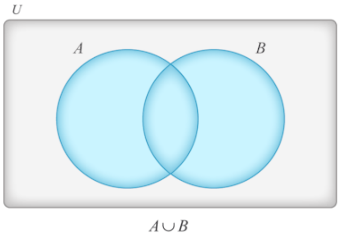
| Set theory | Venn diagram | Probability theory |
|---|---|---|
| $A \cup B$ is the union of sets $A$ and $B$. It is the set of elements that belong to $A$ or $B$ or both $A$ and $B$. | <br>More information<br>The image is a Venn diagram consisting of two overlapping circles labeled A and B. The circles are filled with a light blue color. The overlapping region represents the union of sets A and B, which is denoted below the circles as A ∪ B. The entire diagram is enclosed within a rectangle labeled U, representing the universal set. The components demonstrate the relationship between the two sets, illustrating common elements in the overlap while maintaining distinct sections for each set. This basic Venn diagram helps visualize the concept of union in set theory.<br>[Generated by AI] | $\mathrm{P}(A \cup B)$ is the probability that even event $B$ or both $A$ and $B$ occurs. Thi leads to the **addition rule:** $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap$ which practically says that from the su of $\mathrm{P}(A)$ and $\mathrm{P}(B)$, we need to subtra $\mathrm{P}(A \cap B)$ so that we do not double count the intersection. |

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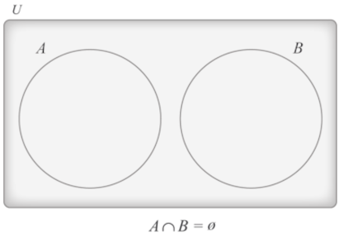
| Set theory | Venn diagram | Probability theory |
|---|---|---|
| If $A \cap B = \emptyset$, the sets $A$ and $B$ are **mutually exclusive** and have no elements in common. |  More information<br>The image is a Venn diagram within a rectangular universal set labeled 'U'. Inside the rectangle are two separate circles labeled 'A' and 'B'. There is no overlap between the circles, and at the bottom of the rectangle, it shows the notation 'A ∩ B = Ø', indicating that the intersection of sets A and B is empty.<br>[Generated by AI] | If $A$ and $B$ are mutually exclusive eve they cannot both occur. Thus, $n(A \cap B) = 0 \Rightarrow \mathrm{P}(A \cap B) = 0$. Therefore, $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(E$ |

As you can see from the third column of the table, Venn diagrams are used to develop and visualise rules for probabilities. In the row illustrating $A \cup B$, you are given the addition rule. Why do you think you need to subtract the intersection rather than simply finding $\mathrm{P(A)} + \mathrm{P(B)}$?

## Important

Understanding how to construct and interpret Venn diagrams is essential. As you determine how many elements fall into each region of the diagram, you can use the probability formula to find probabilities of various combinations. We will explore the probabilities of these events in the next section.

## Activity

Student view

Construct a Venn diagram of your own using two or three things that you and your classmates have in common. They could be classes you take, sports or instruments you play or foods you like to eat. Almost anything can work. Once you have selected the sets you want to create, survey your classmates and create a Venn diagram for the data you find. Are there any surprising facts about your class that you can infer from your diagram?

Before trying some examples, use the applet below to visualise the various relationships between sets with a Venn diagram. Drawing a Venn diagram to visualise the situation often makes finding probabilities much easier.

## Be aware

Don’t forget to account for all the elements in the sample space in your Venn diagrams. If an element does not belong to the sets for the events you have circles for, then it goes on the diagram outside the circle, in the universal set.

**Interactive 1.** Visualising Venn Diagrams.

Credit: GeoGebra (https://www.geogebra.org/m/gsSpeVpz) shanlee

More information for interactive 1

This interactive allows users to visualize and explore the relationships between sets through a dynamic Venn diagram. Users can choose from various set operations including the complement of set A $(A')$, the complement of set B $(B')$, the union of sets A and $B(A \cup B)'$, the intersection of sets A and $B(A \cap B)$, the complement of the union of set A and $B(A \cap B)'$, the intersection of the complement of A with $B \cap (A' \cap B)$, the intersection of A with the complement of $B(A \cap B')$ and the complement of the intersection of sets A and $B \cap (A \cup B)'$. As users tick the checkboxes for these operations, the corresponding shaded regions appear in the Venn diagram, clearly representing the selected set relationships.

For instance, selecting the union of A and B highlights the entire area covered by both circles A and B, showing all elements that belong to either set or both.

This visual representation helps users better understand how different set operations influence the layout and content of the Venn diagram. By interacting with the diagram and seeing the immediate effect of each operation, users develop a deeper grasp of foundational set theory concepts such as unions, intersections, and complements.

## Example 1

★★☆

A bag contains tokens with positive whole numbers to $25$. A token is drawn at random. Find the probability that the number on the token is divisible by $3$ or by $4$ .

Let $T$ be the event that the number is divisible by $3$ , such that $T = \{3, 6, 9, 12, 15, 18, 21, 24\}$ and $n(T) = 8$ . Let $F$ be the event that the number is divisible by $4$ , such that $F = \{4, 8, 12, 16, 20, 24\}$ and $n(F) = 6$ . Hence, the set $T \cap F = \{12, 24\}$ and $n(T \cap F) = 2$ .

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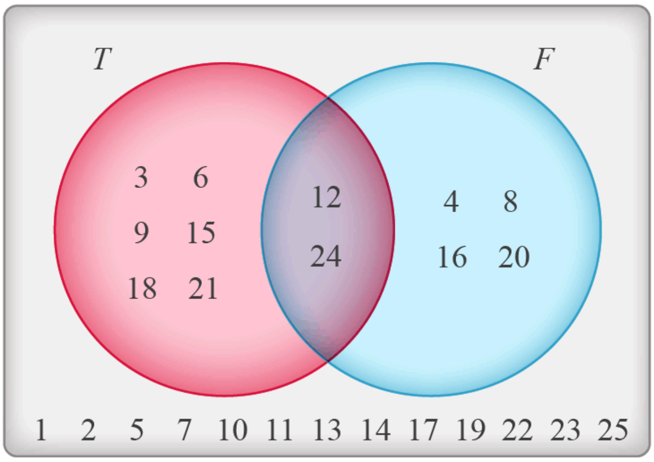


Numbers up to 25 divisible by 3 or 4.

As we are asked to consider the probability of $T$ **or** $F$, we use the equation

$$\mathrm{P}(\mathrm{T} \cup \mathrm{F}) = \mathrm{P}(\mathrm{T}) + \mathrm{P}(\mathrm{F}) - \mathrm{P}(\mathrm{T} \cap \mathrm{F}).$$

As the sample space, $U$, is all numbers 1 to 25, we have that $n(U) = 25$. Thus,

$$\begin{aligned}\mathrm{P}(\mathrm{T}) &= \frac{n(T)}{n(U)} = \frac{8}{25}\\ \mathrm{P}(\mathrm{F}) &= \frac{n(F)}{n(U)} = \frac{6}{25}\\ \mathrm{P}(\mathrm{T} \cap \mathrm{F}) &= \frac{n(T \cap F)}{n(U)} = \frac{2}{25}\end{aligned}$$

and hence, using the expression for $\mathrm{P}(\mathrm{T} \cup \mathrm{F})$,

$$\mathrm{P}(\mathrm{T} \cup \mathrm{F}) = \frac{8}{25} + \frac{6}{25} - \frac{2}{25} = \frac{12}{25}.$$

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# Example 2

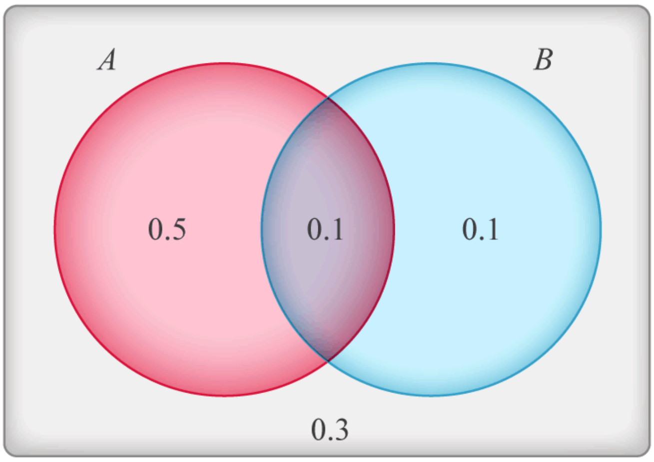
★★☆

Students were surveyed about whether they have eaten apples ($A$), bananas ($B$) or both in the last month. The survey found the following:

$$\begin{aligned} \mathrm{P}(A) &= 0.6 \\ \mathrm{P}(B) &= 0.2 \\ \mathrm{P}(A \cap B) &= 0.1 \end{aligned}$$

Find $\mathrm{P}(\mathrm{A} \cup \mathrm{B})$, $\mathrm{P}(\mathrm{A}')$ and $\mathrm{P}(\mathrm{A}' \cap \mathrm{B})$ and explain what each represents.

The Venn diagram for this situation looks like this.




Do students eat more apples or bananas?

1. We use the formula:
   $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = 0.6 + 0.2 - 0.1 = 0.7$. This means there is a probability of $0.7$ that a student has recently eaten at least one of the two fruits.
2. We apply the expression for the complementary event:
   $\mathrm{P}(\mathrm{A}') = 1 - \mathrm{P}(\mathrm{A}) = 1 - 0.6 = 0.4$. This means there is a probability of

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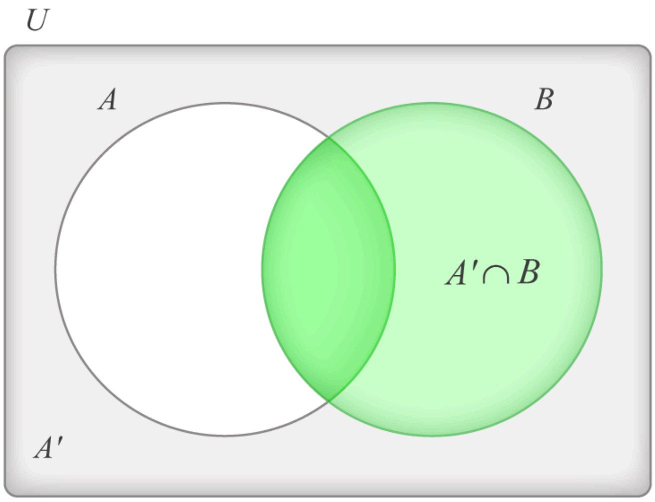
0.4 that a student has not eaten any apples in the last month.

3. The set $A' \cap B$ contains all the elements that are **not** in $A$ **but** are in $B$. See the Venn diagram above for a better visualisation. As $A \cap B$ and $A' \cap B$ are mutually exclusive and $(A \cap B) \cup (A' \cap B) = B$, we have $\mathrm{P(A' \cap B) + P(A \cap B) = P(B)}$ or $\mathrm{P(A' \cap B) = P(B) - P(A \cap B)} = 0.2 - 0.1 = 0.1$. This means there is a probability of $0.1$ that a student has not eaten any apples but has eaten bananas.




$A' \cap B$ contains all the elements **not** in $A$ (i.e. elements in the grey area) that are also in $B$ (deeper green area). It is the pale green area.

## Example 3

★★★

A card is drawn from a deck of 52 cards. The deck is made up of half red cards and half black cards. Two of the black cards and two of the red cards have an 'A' on them, while six other black cards and six other red cards have a picture of a face on them. Find the probability that the card drawn is (1) a red card or a card that has a face on it, and (2) a red card or a card with an 'A' on it.

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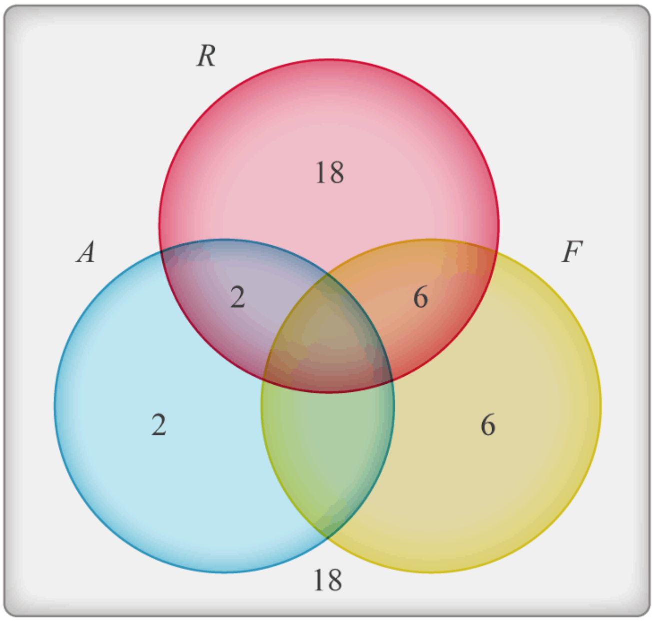
Let $R$ be the event of a red card drawn, such that $\mathrm{P(R)} = \frac{26}{52}$. As we will be adding and subtracting fractions, it is easiest to leave them unchanged. Let $F$ be the event that a card with a face on it is drawn, such that $\mathrm{P(F)} = \frac{12}{52}$. Let $A$ be the event that a card with an 'A' on it is drawn, such that $\mathrm{P(A)} = \frac{4}{52}$. Of the 52 cards, 6 cards are red and have faces on them, hence $\mathrm{P(R \cap F)} = \frac{6}{52}$. Similarly, 2 cards are red and have an 'A' on them, thus, $\mathrm{P(R \cap A)} = \frac{2}{52}$.




Are the cards red, or do they have As or faces on them?

1. $$\begin{aligned}\mathrm{P(R \cup F)} &= \mathrm{P(R) + P(F) - P(R \cap F)} \\ &= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} \\ &= \frac{32}{52} \\ &= \frac{8}{13}\end{aligned}$$

2. $$\mathrm{P(R \cup A)} = \mathrm{P(R) + P(A) - P(R \cap A)} = \frac{26+4-2}{52} = \frac{7}{13}$$

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## Dependent and independent events

When considering combinations of events, we need to remember that sometimes when one event occurs, it affects the probability of the other event – that is, the second event is a dependent event. In **Example 3** above, we saw that $\mathrm{P}(\mathrm{R} \cap \mathrm{A}) = \frac{2}{52}$. If the first card you draw is red and has an 'A' on it, and you select a second card without putting the first one back (without replacement), $\mathrm{P}(\mathrm{R} \cap \mathrm{A})$ for the second card is now $\frac{1}{51}$. All of the other probabilities we found that involved a card with an 'A' or a red card would also change for the second card. Thus, the events that occur when you draw the second card are dependent on the identity of the first card you draw. On the other hand, if the occurrence of one event has **no effect** on the probability of the other event occurring, the events are independent events. If you were to replace the card in the deck before drawing the second card in the example above, then the probabilities would all remain the same and each draw would be independent.

### Be aware

Certain rules and formulae we will explore in later subtopics only apply to independent events, so learning to recognise whether events are independent or dependent is essential. Why do you think it is important to consider the conditions of the theorems you learn?

## 3 section questions

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# Probabilities of related events

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# The probability of a union

We have already looked at how to find the number of elements in a union of sets $A$ and $B$ using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, so finding the probability of a union can be found using a similar formula. Let's derive it using the basic probability formula.

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{n(A \cup B)}{n(U)}$$

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{n(A) + n(B) - n(A \cap B)}{n(U)}$$

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)} - \frac{n(A \cap B)}{n(U)}$$

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$$

If $A$ and $B$ are mutually exclusive, then $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = 0$ and $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B})$.

## Example 1

★★☆

If you draw a card out of a standard deck of cards, find the probability that it is a face card or a spade.

Let $F$ represent drawing a face card and $S$ represent drawing a spade.

Start with the formula $\mathrm{P}(\mathrm{F} \cup \mathrm{S}) = \mathrm{P}(\mathrm{F}) + \mathrm{P}(\mathrm{S}) - \mathrm{P}(\mathrm{F} \cap \mathrm{S})$

There are 12 face cards, so $\mathrm{P}(\mathrm{F}) = \dfrac{12}{52}$

There are 13 spades, so $\mathrm{P}(\mathrm{S}) = \dfrac{13}{52}$

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There are $3$ spades that are also face cards, so $\mathrm{P}(\mathrm{F} \cap \mathrm{S}) = \frac{3}{52}$

Hence, $\mathrm{P}(\mathrm{F} \cup \mathrm{S}) = \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$

### Exam tip

Waiting to reduce fractions until the end of your work can save you time when solving problems, because you often have a common denominator to begin with. Time is valuable when you take your exam, so working efficiently is a key to success!

# Conditional probability

### International Mindedness

Construct a Venn diagram by surveying your classmates to find all the languages spoken. Do you see any associations — for example, are people who speak Flemish more likely to be able to speak French too than those who speak Italian?

Conditional probability occurs when we have some knowledge about the outcome of an experiment. Considering rolling a fair dice. Without any other knowledge about the experiment, you know that t he probability of obtaining a $2$ is $\frac{1}{6}$. However, what if you knew the outcome of a particular throw was an even number ? In that case, what is the probability that it is a $2$? Knowing that the outcome is an even number **shrinks** the sample space from $\{1, 2, 3, 4, 5, 6\} \rightarrow \{2, 4, 6\}$, i.e. from a set of $6$ values to a set of $3$ values. Therefore the *probability of rolling a $2$ with a fair dice **given** that the outcome is an even number* is $\frac{1}{3}$ , which is larger than if we did not have that additional knowledge.

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### Be aware

Conditional probability shrinks the sample space and, therefore, increases the probability of an event occurring, unless the given information renders the event impossible. For example, the probability of obtaining a 2 given that the outcome of a throw is an odd number is $0$.

What is the probability of an event $A$ occurring *given* that an event $B$ has occurred, which is written as $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$? If we know that $B$ has occurred, we know that the sample space now contains all the elements of $B$ but no more, i.e. $n(B)$. Now we select from that sample space the events that fall in $A$ , i.e. $n(A \cap B)$. Then, using our definition of

$$\mathrm{P}(A \mid B) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{\frac{n(A \cap B)}{n(U)}}{\frac{n(B)}{n(U)}}$$

$$= \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

## Example 2

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There are 60 executives attending a regional conference in South Africa. The Venn diagram below shows those who do some business in Angola ($A$) and those who do some business in Mozambique ($M$).

Find the probability that one of the people has done business in Mozambique given that they have **not** done business in Angola.

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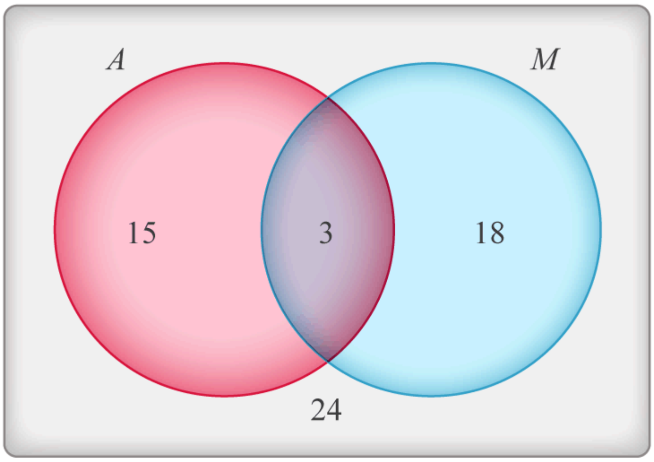


More information

The image shows a Venn diagram with two overlapping circles. The left circle is labeled A, containing the number 15, and the right circle is labeled M, containing the number 18. The overlapping section of the circles, which represents the intersection of sets A and M, contains the number 3. Below the circles, the total value displayed is 24, indicating the total number of elements in the union of sets A and M.

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To solve the problem algebraically, use the conditional probability formula:

$$\mathrm{P}(\mathrm{M} \mid \mathrm{A}') = \frac{\mathrm{P}(\mathrm{M} \cap \mathrm{A}')}{\mathrm{P}(\mathrm{A}')}$$

$$\mathrm{P}(\mathrm{M} \cap \mathrm{A}') = \frac{18}{60}$$

$$\mathrm{P}(\mathrm{A}') = \frac{42}{60}$$

$$\mathrm{P}(\mathrm{M} \mid \mathrm{A}') = \frac{18/60}{42/60} = \frac{18}{42} = \frac{3}{7}$$

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view

To use the Venn diagram, recognise that $A'$ represents the $18 + 24 = 42$ people that are not in the red circle representing $A$. The sample space therefore is now only those $42$ people.

Now notice that $M \cap A'$ is the $18$ people that are in $M$ but *not* in $A$.

Therefore, $\mathrm{P}(M \mid A') = \frac{18}{42} = \frac{3}{7}$.

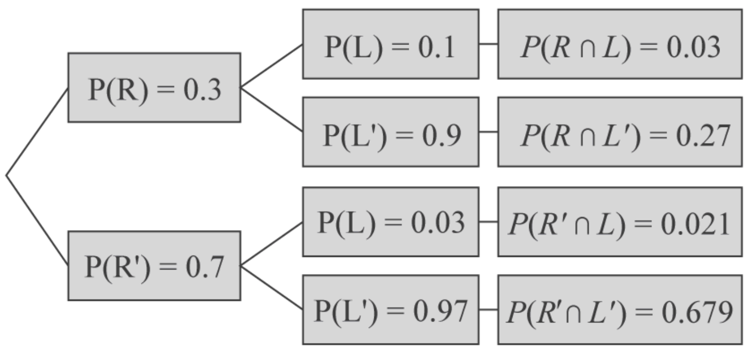
You can also use tree diagrams to find conditional probability. When you look at the probabilities of consecutive events on a branch of the tree diagram, multiply the probabilities together to find the probability of both events happening.

## Example 3

★★★

Lauren has noticed that when it rains, she has a harder time making it to work on time. It rains $30\%$ of the time. Lauren is late to work $10\%$ of the time when it rains, but only $3\%$ of the time when it does not rain. Find the probability that it is raining given that she is on time to work.

Construct a tree diagram to illustrate the possible outcomes for Lauren.




Rain (R) can affect Lauren’s chances of getting to work on time (L).

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To find the probability that it is raining given that Lauren is on time to work, we need to calculate $\mathrm{P}(\mathrm{R} \mid \mathrm{L}')$.

Use the formula for conditional probability: $\mathrm{P}(\mathrm{R} \mid \mathrm{L}') = \dfrac{\mathrm{P}(\mathrm{R} \cap \mathrm{L}')}{\mathrm{P}(\mathrm{L}')}$

From the tree diagram, we see:

$$\mathrm{P}(\mathrm{R} \cap \mathrm{L}') = 0.3 \times 0.9 = 0.27 \text{ and}$$
$$\mathrm{P}(\mathrm{L}') = \mathrm{P}(\mathrm{R} \cap \mathrm{L}') + \mathrm{P}(\mathrm{R}' \cap \mathrm{L}') = 0.27 + 0.679 = 0.949$$

Thus, $\mathrm{P}(\mathrm{R} \mid \mathrm{L}') = \dfrac{\mathrm{P}(\mathrm{R} \cap \mathrm{L}')}{\mathrm{P}(\mathrm{L}')} = \dfrac{0.27}{0.949} \approx 0.285.$

## The probability of independent events

Earlier we learned that $A$ and $B$ are independent events if the occurrence of $A$ has no effect on the probability of $B$ occurring. Mathematically, we can demonstrate this relationship with the equation $\mathrm{P}(\mathrm{B} \mid \mathrm{A}) = \mathrm{P}(\mathrm{B})$ and $\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \mathrm{P}(\mathrm{A})$.

For any events $A$ and $B$, $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{B} \mid \mathrm{A})$. Therefore, $A$ and $B$ are independent if and only if $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{B})$.

The following example illustrates independence. If I throw a coin, the chance of throwing **heads** is the same each time. Suppose I throw a coin $5$ times and obtain $5$ heads. On the $6$ th throw, the probability of obtaining heads is still $0.5$ , since each throw is independent of the throws that have already occurred. A person might **feel** that the $6$ th throw cannot also be heads as they have already thrown $5$ heads. However, that feeling is known as the gambler's fallacy. The probability of throwing heads is $0.5$ for every throw of a fair coin.

Typically, events are independent when a person or a thing is chosen from a group and then put back into the group such that the same person or thing might be chosen again on the next trial.

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Suppose a teacher asks 10 questions each lesson and picks a student's name at random from a hat each time. Suppose the teacher replaces each name chosen, so that all the students' names are always in the hat. Each student has the same probability of being asked for every question, even if they have already been asked. The probability of being called does not change, regardless of how many questions a student has already answered.

## Example 4

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A game is played whereby a dice and a coin are thrown. If the score on the dice is 6 and the coin is heads up, the player wins \$20.

If the score on the dice is not 6 but the coin is heads up, the player will win \$1.

Otherwise, the player does not win anything.

1. Display the probabilities on a tree diagram.
2. What is the probability that the player will win \$20?
3. What is the probability that the player will win \$1?
4. What is the probability that the player will not win anything?

1. To draw the tree, we may draw the subtrial of throwing a dice on the first set of branches, and the subtrial of throwing a coin on the second set of branches. Notice that we are interested only in whether the dice lands with a 6 or not a 6. Therefore, we need only two branches for the dice. Similarly, the coin is either heads or tails, therefore we only need two branches for the coin.

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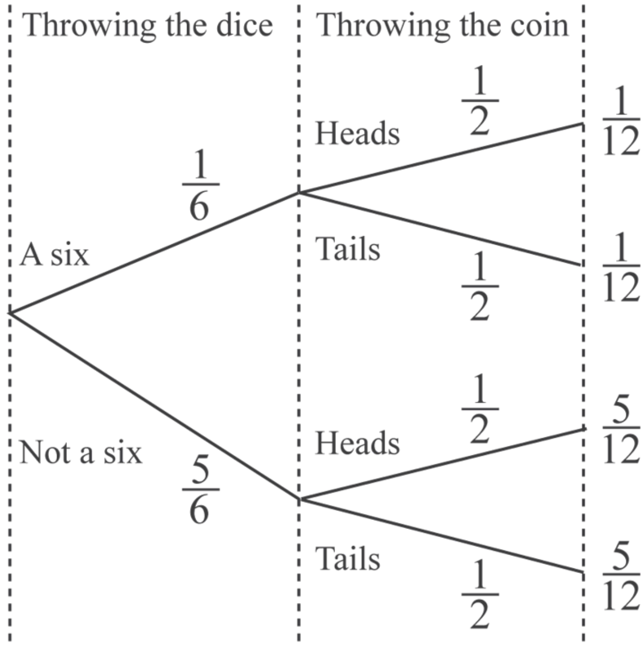


The completed tree diagram.

$$2.\ \mathrm{P}(\text{win } \$20) = \mathrm{P}(6 \text{ and heads}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

$$3.\ \mathrm{P}(\text{win } \$1) = \mathrm{P}(\text{not } 6 \text{ and heads}) = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$

$$4.\ \mathrm{P}(\text{win } \$0) = 1 - \left(\frac{1}{12} + \frac{5}{12}\right) = 1 - \frac{6}{12} = \frac{1}{2}$$

## Making connections

Gambling, carnival games and other games of chance like the one described in **Example 4** are designed with careful attention to the probabilities involved. Game designers make rules for betting and set pay-outs at just the right level so the odds are just barely in their favour. What do you think would happen if the odds were in their favour too much? What if they were too much in favour of the player?

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## Theory of Knowledge

Think about how much probabilities specifically relate to gambling. Do game designers have an ethical responsibility to enable players to win some of the time? What do you think would happen if they did not? How much of a responsibility do they have to try to discourage players who might become addicted to gambling?

## Exam tip

These probability formulae are in the formula booklet that you can use for the exam:

Combined events (union): $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$

Mutually exclusive events: $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B})$

Conditional probability: $\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \dfrac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})}$

Independent events: $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A})\mathrm{P}(\mathrm{B})$

## Theory of Knowledge

Mathematics can be represented visually as shown in the Venn diagrams in this subtopic as well as in the chaos equations in the video below. A knowledge question emerges: ‘Is mathematics art?’

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Chaos Equations - Simple Mathematical Art

## 3 section questions

4. Probability and statistics / 4.6 Probability calculations

# Checklist

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/checklist-id-26253/print/) Assign

### What you should know

By the end of this subtopic you should be able to:

- construct a Venn diagram to illustrate the outcome of two or three related events
- construct a tree diagram to illustrate outcomes of consecutive events
- identify the sample space for a random experiment
- recognise notation indicating intersection, union, complement and given events
- understand the difference between dependent and independent events and the difference in how their probabilities are calculated
- use conditional probability to calculate the probability of an event given that another event has occurred.

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4. Probability and statistics / 4.6 Probability calculations

# Investigation

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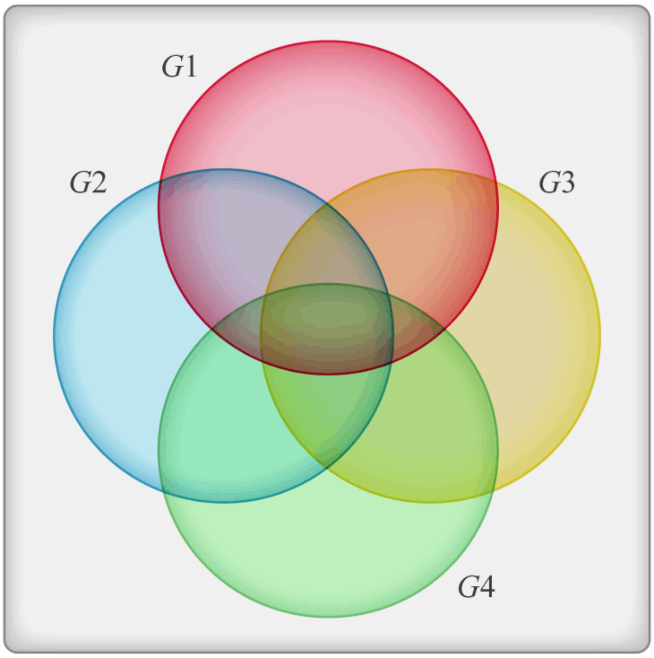
Survey your classmates to see whether they are taking SL courses for subject groups other than mathematics in the diploma program (Studies in language and literature, Language acquisition, Individuals and societies and Sciences, respectively). Once you have collected your data, try to construct a Venn diagram for all four subject groups. Is the template below general enough to include all combination you have in your data?




Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/modelling-related-events-id-26251/print/) Assign

How many of your classmates are taking SL courses in subject groups $1 - 4$?

More information

The image shows a Venn diagram consisting of four overlapping circles, each labeled with a different group name: G1, G2, G3, and G4. The circles overlap in such a way that there are multiple areas where two or more circles intersect. The diagram illustrates the relationships and shared areas between these groups. Each circle

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is uniquely colored, and the overlapping regions blend the colors together. The layout suggests comparisons between the four groups, common characteristics, or shared quantities, typical of Venn diagrams.

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Once you have your Venn diagram drawn, trade data and diagrams with a classmate to provide peer feedback on each other's work. Did you both construct your Venn diagrams correctly for the data you collected?

Once you are both confident that your diagrams are correct, challenge each other to express specific regions as unions or intersections of various sets and to find the probability that a randomly selected classmate is a member of that group. For example, which part of the diagram represents $(G1 \cap G3 \cap G2')$, and what is $\mathrm{P}(G1 \cap G3 \cap G2')$?

Finally, use conditional probability to find these values:

- $\mathrm{P}(G2 \mid G1)$, $\mathrm{P}(G3 \mid G1)$ and $\mathrm{P}(G4 \mid G1)$
- $\mathrm{P}(G1 \mid G2)$, $\mathrm{P}(G3 \mid G2)$ and $\mathrm{P}(G4 \mid G2)$
- $\mathrm{P}(G1 \mid G3)$, $\mathrm{P}(G2 \mid G3)$ and $\mathrm{P}(G4 \mid G3)$
- $\mathrm{P}(G1 \mid G4)$, $\mathrm{P}(G2 \mid G4)$ and $\mathrm{P}(G3 \mid G4)$.

Do any of these probabilities indicate anything to you about what classes students choose to take? Are there any conclusions about course selection that you can draw from your results?

**Rate subtopic 4.6 Probability calculations**

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