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0 (https://intercom.help/kognity)

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TRIANGLE TRIGONOMETRY

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The big picture

The strongest shape: triangle

There are many types of triangles. You can classify them according to their side lengths or angles, as seen in the following table.

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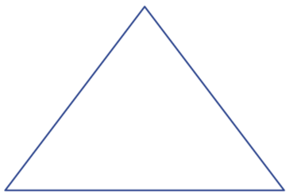
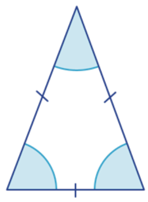
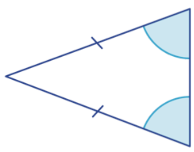
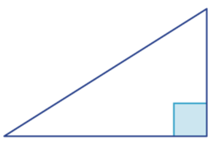
Classifying triangles

By side lengths	By angles
 More information The image shows an equilateral triangle with all sides labeled. The sides are labeled as A, B, and C, with A at the bottom, B on the left, and C on the right. Each of these sides is equal in length, indicating the equilateral property of the triangle. The angles of the triangle, located at each vertex, are also equal, each measuring 60 degrees. This image visually represents the text, "Equilateral triangle: All side lengths and angles are equal." [Generated by AI] Equilateral triangle: All side lengths and angles are equal.	 More information The image is a diagram of an acute triangle, labeled to indicate that all its angles are acute, meaning each is less than 90 degrees. The triangle is shown with its three internal angles highlighted in blue. Each side of the triangle appears to be marked to indicate equal length, suggesting it may be an equilateral triangle. The text accompanying the image specifies, "Acute triangle: All angles are acute (less than 90°)." [Generated by AI] Acute triangle: All angles are acute (less than 90°).
 Isosceles triangle: Two side lengths and corresponding angles are equal.	 Right-angled triangle: One right angle and two acute angles.

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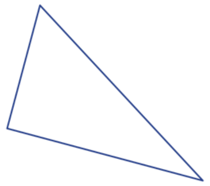
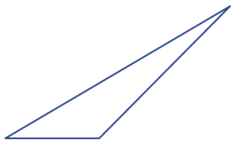
Classifying triangles	
By side lengths	**By angles**
	
Scalene triangle: All side lengths and angles are different.	Obtuse triangle: One obtuse (more than $90°$) angle.

Triangles are the building blocks of many three-dimensional structures. They are considered the strongest shape for building and are therefore used when a structure is required to bear a lot of weight. A triangular-shaped structure will only collapse because of material fatigue, not as a result of the shape becoming distorted.

Triangles are used, for example, in bridges, geodesic domes and skyscrapers. Look out for them in the structures around you. The Montreal Biosphere, shown in the photograph below, is designed using equilateral triangles and is one of the finest geodesic domes built in recent history.

The Montreal Biosphere, Montreal, Canada

Credit: jnnault Getty Images

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view

Concept

Properties of shapes are dependent on the dimensions of their components. Triangles are defined by their three sides and corresponding three angles. Using these measurements, you can find missing sides, missing angles and the area of the triangles. Use of right-angled triangles along with trigonometric ratios and basic properties of triangles enables you to solve problems in 2D space.

Theory of Knowledge

In your Theory of Knowledge studies, you will come across various ‘truth theories’. One such theory is referred to as The correspondence theory of truth (https://plato.stanford.edu/entries/truth-correspondence/). In short, the theory states that which is true corresponds to a fact in reality — that truth is a relational property. This theory relates to mathematics in so far as mathematics corresponds to reality.

In the examples provided in this section, mathematics is used to build buildings. The correspondence theory of truth would hold that if the buildings don’t fall down, the mathematics used to create them is true. In social sciences, this is referred to as having predictive validity. Consider the knowledge question, ‘Does math necessitate real-world correspondence in order to be valid?’

Trigonometric ratios

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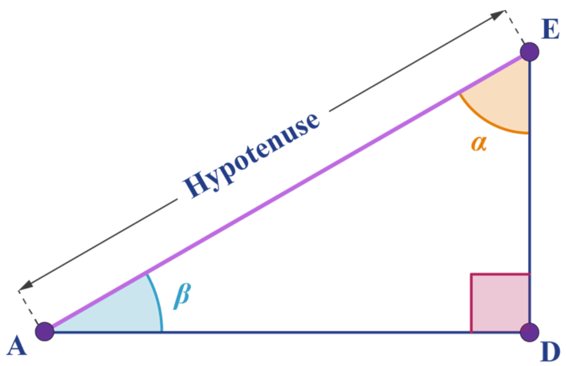
Identifying sides of right-angled triangles

A right-angled triangle contains a right angle, and the other two angles are acute angles. The longest side opposite the right angle is called the hypotenuse. In the diagram below, the acute angles add up to $90°$, so

$$\alpha + \beta = 90°$$



More information

The image depicts a right triangle labeled with points A, D, and E at the vertices. The hypotenuse is marked as side AE, and segments AD and DE represent the other sides. The angle at vertex E is denoted as α (alpha), and the angle at vertex A as β (beta). A right angle is indicated at vertex D by a small square. The triangle shows the relationship between the angles and sides in a right triangle, with α and β identified as complementary, indicating that α + β = 90 degrees. The sides are labeled to show the hypotenuse, with arrows pointing outward from the angles indicating their aspects relative to the sides.

[Generated by AI]

The other two sides, ED and AD, are identified with respect to their positions relative to the acute angles. In the diagram below, you can see how the other two sides are labelled with respect to these relative positions.

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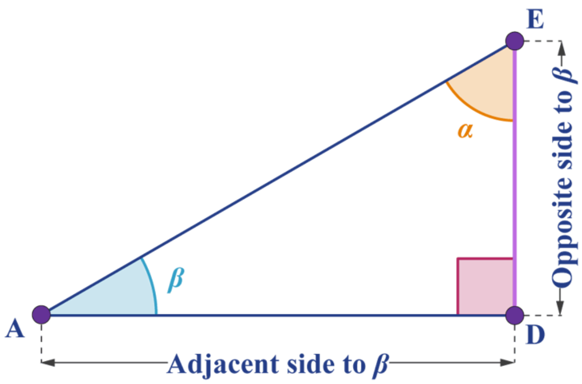

More information

The image is a geometric diagram of a right triangle labeled with points A, D, and E. The triangle has two acute angles, labeled with Greek letters α (alpha) and β (beta). The hypotenuse, AE, is the side opposite to the right angle at D. Line segment AD is labeled as the "Adjacent side to β," and line segment DE is labeled as the "Opposite side to β." The right angle is marked with a small square at point D.

The vertices are distinctly labeled: - A is the initial point of the hypotenuse AE. - D marks the right angle, forming the base AD and the vertical DE. - E is the endpoint for both the hypotenuse AE and perpendicular side DE.

The diagram visually demonstrates how the triangle’s sides are defined relative to the acute angles, emphasizing their geometric relationships.

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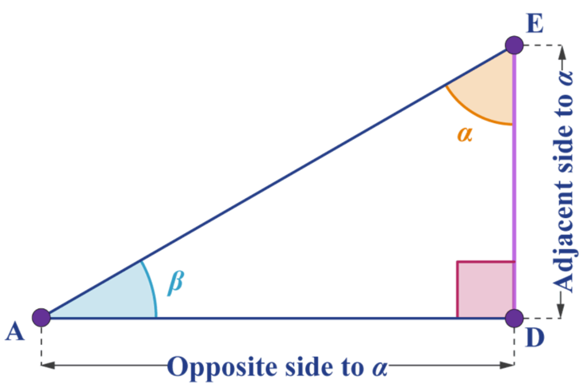
Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-big-picture-id-26206/print/) Assign



More information

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view

The diagram shows a right triangle labeled with points A, D, and E. The hypotenuse is labeled as AE. The angle at A is marked with β (beta), and the angle at E is marked with α (alpha). The side AD is labeled as the opposite side to α, while the vertical side ED is labeled as the adjacent side to α. Each point is marked with a circular dot.

- The base of the triangle is the line segment AD, labeled "Opposite side to α."
- The vertical side DE is labeled "Adjacent side to α."
- The hypotenuse AE is unlabeled with a specific measurement but connects points A and E.
- At angle E, the side ED has a small rectangular shape indicating it's a perpendicular.
- No specific units or measurements are provided.

[Generated by AI]

Exam tip

In an IB exam, a right-angled triangle will be identified as right-angled in the written information in a question. Even if a triangle *appears* to be right-angled, it should be treated as right-angled only if the question states that it is or, if the square symbol, to represent a 90° angle, is present in the diagram.

Similar right-angled triangles

If two right-angled triangles have an acute angle of the same size, or two sides with the same ratio then these two triangles are similar.

Consider the triangles below. Triangle ABC is similar to triangle DEF because both triangles are right-angled and the side lengths of triangle DEF are twice the side lengths of triangle ABC.

Therefore, the corresponding angles are also equal:

$$\text{angle } A = \text{angle } D \text{ and angle } C = \text{angle } F\,.$$

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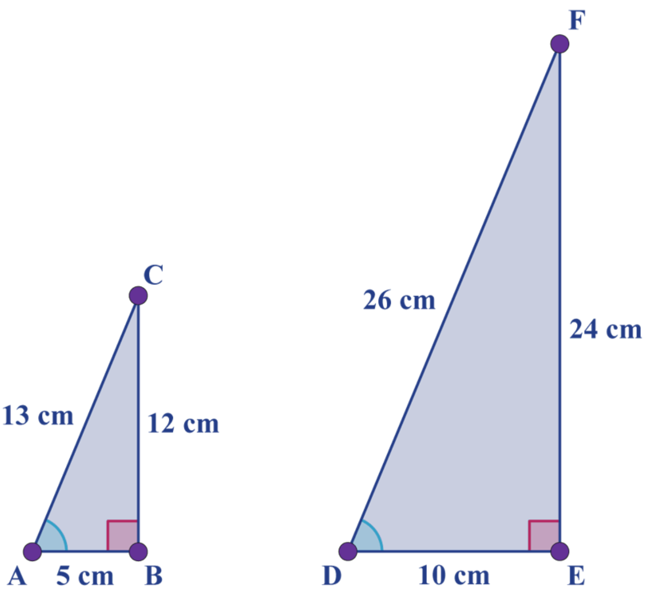

More information

The image depicts two right triangles adjacent to each other. The smaller triangle on the left, labeled as triangle ABC, has its vertices marked A, B, and C. The sides of triangle ABC are labeled as 5 cm (AB), 12 cm (BC), and 13 cm (AC), with a 90-degree angle at vertex B. Triangle DEF is the larger triangle on the right. Its vertices are labeled D, E, and F. The sides are labeled as 10 cm (DE), 24 cm (EF), and 26 cm (DF), with a 90-degree angle at vertex E. Angles A and D are marked similarly to indicate that they are equal.

[Generated by AI]

Important

If two triangles are similar, the ratios of corresponding sides in both triangles are equal.

This is demonstrated in the table.

Triangle ABC	Triangle DEF
$\dfrac{\text{opposite side to } \angle A}{\text{adjacent side to } \angle A} = \dfrac{12}{5}$	$\dfrac{\text{opposite side to } \angle D}{\text{adjacent side to } \angle D} = \dfrac{24}{10} = \dfrac{12}{5}$

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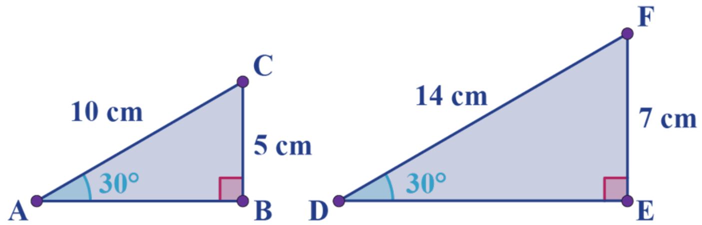
Triangle ABC	Triangle DEF
$\frac{\text{opposite side to } \angle A}{\text{hypotenuse of triangle ABC}} = \frac{12}{13}$	$\frac{\text{opposite side to } \angle D}{\text{hypotenuse of triangle DEF}} =$
$\frac{\text{adjacent side to } \angle A}{\text{hypotenuse of triangle ABC}} = \frac{5}{13}$	$\frac{\text{adjacent side to } \angle D}{\text{hypotenuse of triangle DEF}} =$

Trigonometric ratios

The word trigonometry comes from the Greek meaning ‘triangle measurement’. The study of trigonometry uses the fact that, in similar right-angled triangles, the ratios of corresponding sides are equal.

For example, the diagram below shows two similar right-angled triangles with an acute angle of $30°$.



More information

The diagram depicts two similar right-angled triangles, each with an acute angle of 30 degrees. The first triangle is labeled with points A, B, and C. Side AB is 10 cm, BC is 5 cm, and angle CAB is 30 degrees at point A, with a right angle at point B. This makes the triangle's orientation ABC, with AB as the base and BC as the height.

The second triangle, similar to the first, is labeled with points D, E, and F. Side DE is 14 cm, EF is 7 cm, and angle DEF is 30 degrees at point D, with a right angle at point E. The orientation DEF mirrors that of the first triangle, maintaining the proportional relationship between corresponding sides and angles.

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In both triangles the ratios of sides relative to the 30° angle will be

$$\frac{\text{opposite side to } \angle A}{\text{hypotenuse of triangle ABC}} = \frac{\text{opposite side to } \angle D}{\text{hypotenuse of triangle DEF}} = \frac{5}{10} = \frac{7}{14} = \frac{1}{2}$$

You can make the generalisation

$$\frac{\text{opposite side to } 30^\circ}{\text{hypotenuse of triangle}} = \frac{1}{2}.$$

Therefore, in a right-angled triangle, when one of the angles is 30° the length of the side opposite this angle is always half the length of the hypotenuse.

On the applet below you can check this ratio for different right-angled triangles.

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view

Interactive 1. Right Triangle Trigonometry: Intro.

Credit: GeoGebra (https://www.geogebra.org/m/Z3wcyhcB) Tim Brzezinski

More information for interactive 1

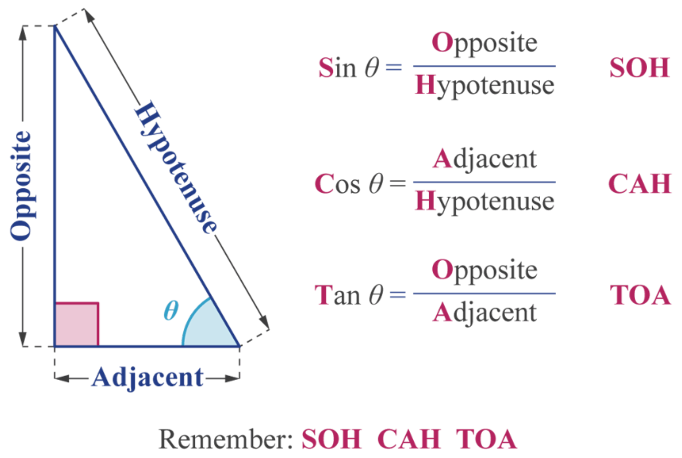
This interactive applet provides a dynamic way to explore the ratios of sides in different right-angled triangles. Users can explore a right-angled triangle by adjusting the angle α from 0° to 90°, using a slider, or by repositioning the corners of the triangle. As the angle or the triangle's shape changes, the lengths of the sides and the ratios between them are dynamically calculated and displayed. The tool provides real-time feedback, showing how the ratio of the opposite side to the hypotenuse evolves with each adjustment, helping users understand the relationships within the triangle.

This interactive tool illustrates the fundamental trigonometric relationship where the ratio of the opposite side to the hypotenuse ($\sin(\alpha) = \frac{\text{opp}}{\text{hyp}}$) remains constant for a given angle α, regardless of the triangle's size.

For example, if $\alpha = 25^\circ$, the sine ratio is consistently approximately 0.42, demonstrating that $\sin(25^\circ) \approx \frac{4.69}{11.09} \approx \frac{3.48}{8.24} \approx 0.42$. So, when angle α is fixed, moving point B changes the side lengths proportionally, keeping their ratio $\frac{\text{opp}}{\text{hyp}}$ unchanged at $\sin(\alpha)$.

This illustrates that trigonometric ratios depend solely on the angle measure, not the triangle's dimensions. By dynamically adjusting the triangle, users can observe how these ratios remain consistent for a fixed angle, reinforcing key concepts in trigonometry.

In this section, you will look at the three trigonometric ratios in right-angled triangles only. These ratios are defined in the diagram below, with respect to angle θ in a right-angled triangle.



More information

The diagram illustrates the three primary trigonometric ratios within a right-angled triangle, focusing on angle (\theta). The triangle features three sides labeled as "Opposite," "Adjacent," and "Hypotenuse." The diagram shows: (\text{Sin} \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}) (SOH), (\text{Cos} \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}) (CAH), (

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\text{Tan} \theta = \frac{\text{Opposite}}{\text{Adjacent}}) (TOA). These equations are part of the mnemonic SOHCAHTOA, which helps in remembering the ratios. The angle (\theta) is indicated in the bottom corner adjacent to the hypotenuse and opposite side.

[Generated by AI]

Important

When you say that $\sin 30^{\circ} = \frac{1}{2}$ it is the same as saying that, in a right-angled triangle with an angle of 30°, the *ratio of the length of the side opposite the* 30° *angle to the length of hypotenuse is* $\frac{1}{2}$.

However, writing these ratios as formulae is a more efficient way of giving this information.

You can use the following applet to see the meaning of trigonometric ratios in a right-angled triangle.

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Interactive 2. True Meanings of the 3 Trigonometric Ratios Within a Right Triangle Context.

Credit: GeoGebra (https://www.geogebra.org/m/maK2WwP4) Tim Brzezinski

More information for interactive 2

This interactive allows users to explore the three primary trigonometric ratios, sine, cosine, and tangent, in the context of a right-angled triangle. On the screen, one triangle with a red base named the Adjacent leg, a green side named the Opposite leg, and a blue Hypotenuse is present. All vertices have purple dots, which you can drag to change the sides and angles of the triangle.

By clicking on the tabs for each ratio (sine, cosine, and tangent), users can visualize the physical meaning of these ratios with respect to an angle θ in the triangle. The applet dynamically updates to show how each ratio corresponds to the relationship between the sides of the triangle: the opposite side, the adjacent side, and the hypotenuse.

For example, if you choose “The Tangent Ratio: Show True Meaning,” the green opposite leg line gets thickened and rotates at the base and drops on the adjacent side. For an angle $\theta = 22.281°$, a dotted arrow points to the opposite leg lying on the adjacent side from the text mentioning:

“$\tan(22.281)° = \frac{\text{opposite leg}}{\text{opposite leg}} \approx 40.97$

The Opposite leg is ≈ 40.97% the length of the adjacent leg.”

The “RESET” button lets the screen reset to all three options, where the user can choose between sine, cosine, and tangent. Similar action takes place with cos and sin when selected. This interactive tool provides a clear and engaging way to visualize trigonometric relationships, making it an essential resource for students studying trigonometry.

Exam tip

A common error reported by examiners is that students do not always properly consider whether or not a triangle is right-angled. To use either Pythagoras or SOH CAH TOA, a right-angled angle must be present in the triangle you are working with.

You can use Pythagoras when the question involves sides only and trigonometry when angles are involved in the question.

Example 1

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In right-angled triangle ABC, $\cos A = \frac{3}{4}$. Find $\sin A$ and $\tan A$.

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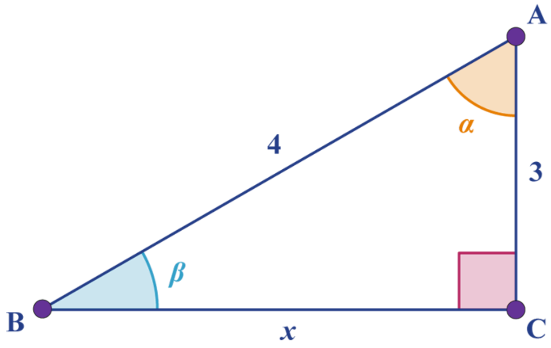


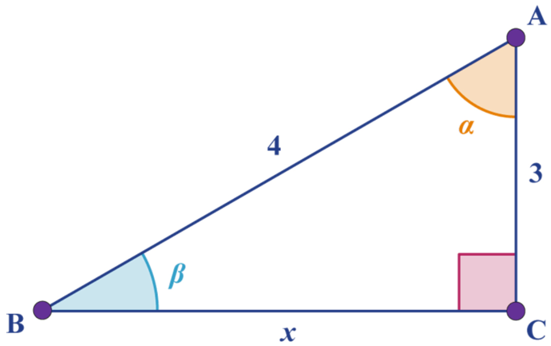
More information

The image depicts a right-angled triangle marked as triangle ABC. Angle A is marked as alpha (α) and angle B is marked as beta (β). The hypotenuse is labeled as AB. The side opposite angle A is AC, measuring 3 units, and the adjacent side BC is marked as 4 units. The third side, which forms angle C with the hypotenuse, is labeled x. Point C indicates the right angle of the triangle. A small square is placed at angle C to denote it as a right angle.

[Generated by AI]

$$\cos A = \frac{\text{adjacent side to } A}{\text{hypotenuse of triangle ABC}} = \frac{3}{4}$$

Draw the right-angled triangle:



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Using Pythagoras' theorem

$$4^2 = 3^2 + x^2$$

$$x = \sqrt{4^2 - 3^2}$$

$$x = \sqrt{7}$$

Write the trigonometric ratios. Remember SOH CAH TOA.

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{7}}{4} = 0.661 \text{ (3 significant figures)}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{7}}{3} = 0.882 \text{ (3 significant figures)}$$

✓ **Important**

When trigonometric ratios are given, it is important to note that they do not represent the actual length of the sides of the triangle. These are just ratios of the sides, so the actual side lengths can be any multiple of the trigonometric ratios.

Example 2

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In the figure below, right-angled triangle ABC has a hypotenuse of 22 cm.

If $\sin A = \dfrac{3}{5}$, find the lengths of other sides.

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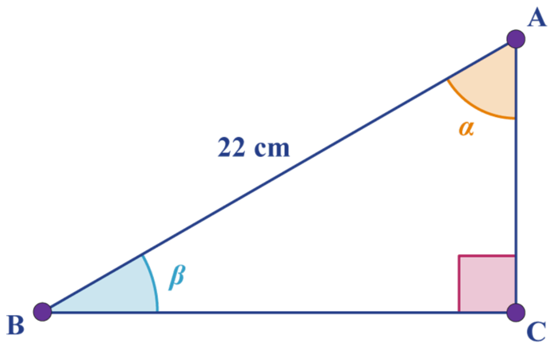

More information

The image depicts a right triangle ABC. Triangle sides are denoted by points A, B, and C, forming a right angle at C. The hypotenuse, which is the side opposite the right angle, is labeled as AC with a length of 22 cm.

Angle α is adjacent to side BC and is located at vertex A, while angle β is adjacent to side AB at vertex B. The angle at vertex C is a right angle of 90 degrees. The side BC is the base, and AB is the perpendicular height. The triangle represents a standard trigonometry problem where one side length is given (AC = 22 cm) and the task might be to find other sides or angles using the sine provided in the text before the image.

[Generated by AI]

Let $BC = x$

$$\sin A = \frac{3}{5} = \frac{x}{22}$$

Solving the equation $\frac{3}{5} = \frac{x}{22}$ gives $x = \frac{66}{5} = 13.2$ cm

Let $AC = y$

Using Pythagoras' theorem in triangle ABC:

$$AB^2 = BC^2 + AC^2$$

$$22^2 = 13.2^2 + y^2$$

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$y = 17.6 \text{ cm}$

2 section questions

3. Geometry and trigonometry / 3.2 Triangle trigonometry

Right-angled triangles

Finding a missing side

Any problem related to finding a missing side in a right-angled triangle can be solved in three steps:

1. Label the triangle and identify which of the three ratios to use.
2. Substitute the values you are given into the correct ratio.
3. Rearrange and solve.

Here are four examples where a side length is unknown.

Be aware

When the angles are given in degrees, your calculator needs to be in 'degrees' mode, otherwise your answers will not be correct.

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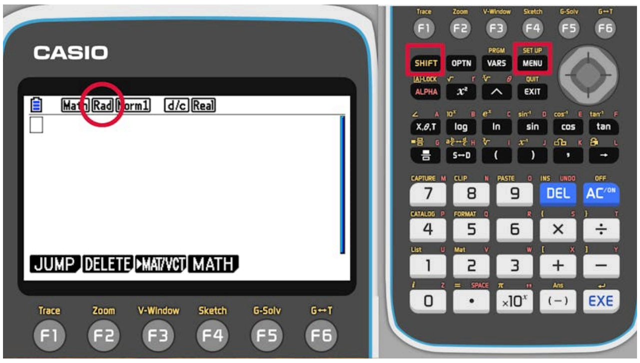
Steps	Explanation
You can change the settings from any mode.	
Notice, that the setting is displayed on the calculator screen. Currently the calculator works with radian angle measures. To change this, bring up the set up options.	

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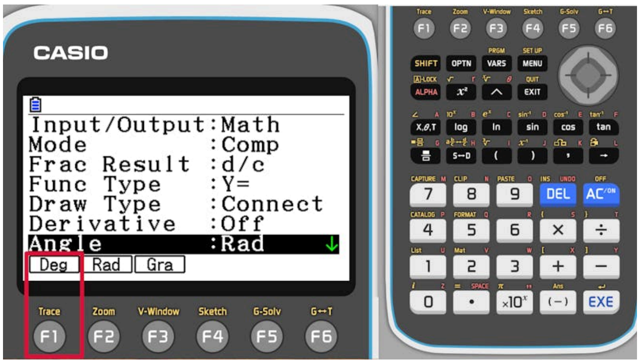
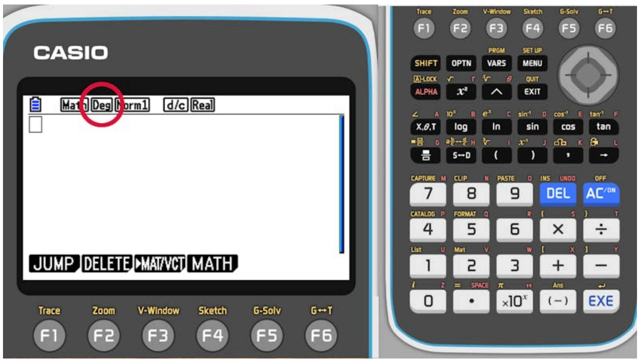
Steps	Explanation
Move to the line related to angle measurements and use F1 or F2 to change the mode. You will not need F3, it corresponds to gradiant measure, which is not used in the IB syllabus.	
Confirm the change of mode on the calculator screen.	

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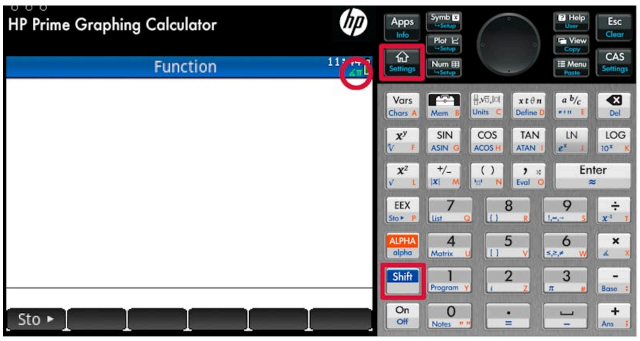
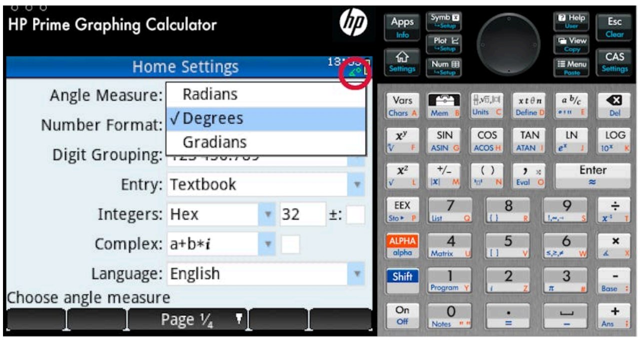
Steps	Explanation
Enter the home screen of any of the applications.	
Notice, that the setting is displayed on the calculator screen. Currently the calculator works with radian angle measures. This is indicated by π next to the angle symbol $\angle$. To change this, bring up the screen where you can change the settings.	

Steps	Explanation
The angle measure setting is the first of the optios. Currently the calculator works with radian angle measures. To change this, open the pull-down list to se the other options.	
Choose either degrees or radians. You will not need the third option. It corresponds to gradiant measure, which is not used in the IB syllabus. Notice the change of symbol from $\angle\pi$ to $\angle°$. The little circle next to the angle symbol indicates that the calculator now works with degrees.	

<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>Notice, that the setting is displayed on the calculator screen. Currently the calculator works with radian angle measures. To change this, press moode to bring up the screen where you can change several options.</td><td>
</td></tr>
<tr><td>Move to the line with the radian/degree options and confirm your choice by pressing enter.

When done, quit this mode selection screen.</td><td>
</td></tr>
</table>

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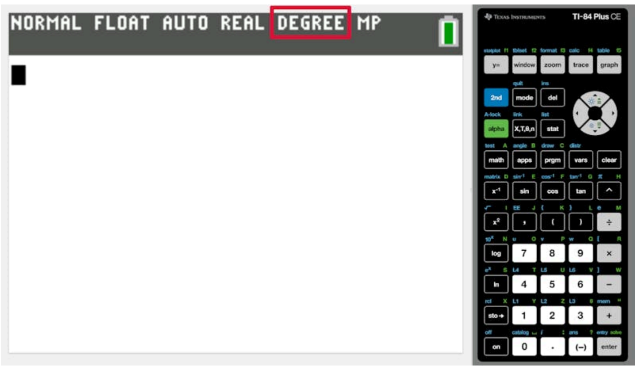
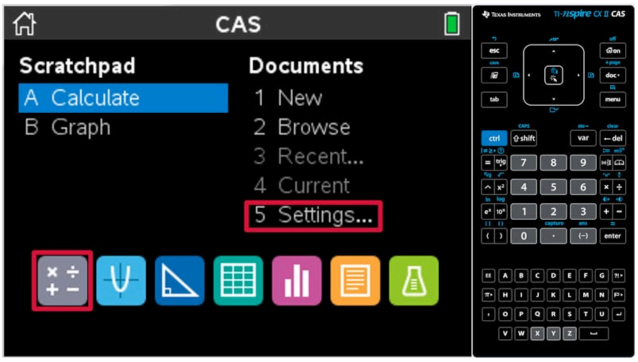
Steps	Explanation
Confirm the change of mode on the calculator screen.	

Steps	Explanation
You can access document settings either directly from the home screen or through an open document. Here you will see the method through opening a calculator page.	

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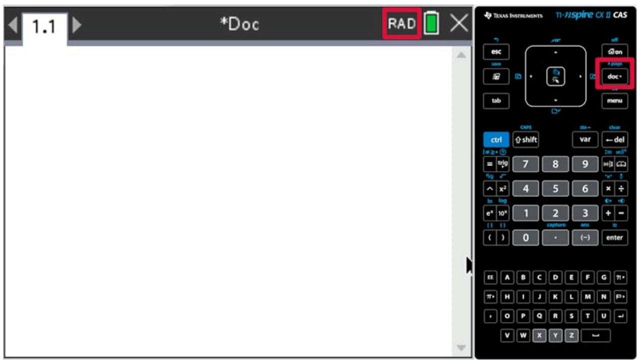
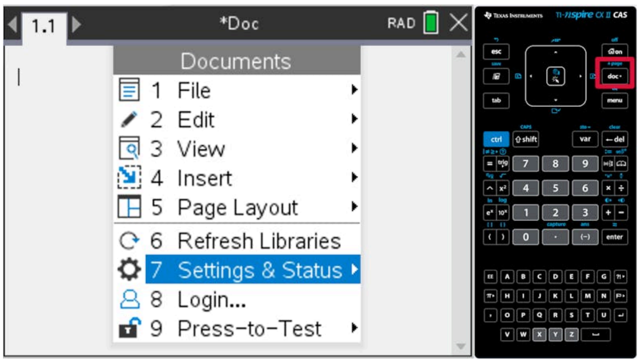
Steps	Explanation
Notice, that the setting is displayed on the calculator screen. Currently the calculator works with radian angle measures. To change this, press doc to see the document settings options.	
In the pop-up menu, choose the settings option ...	

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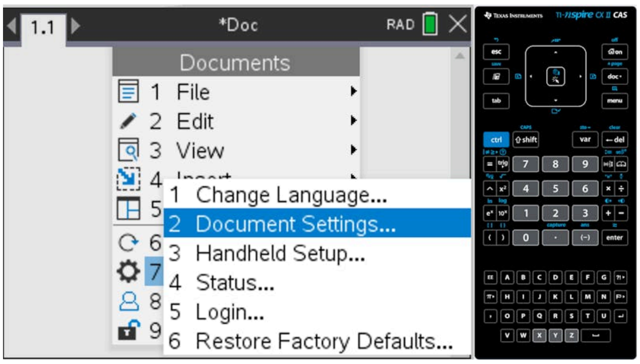
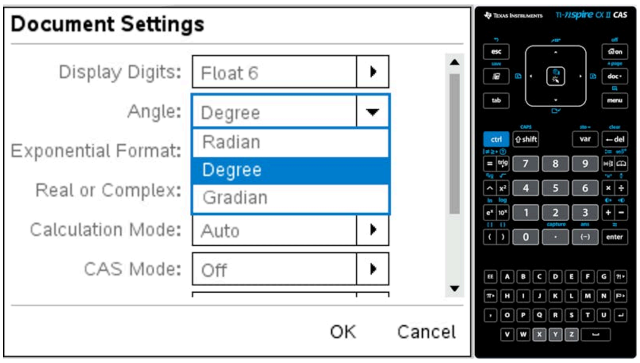
Steps	Explanation
... and then the document settings.	
Expand the options for angle measurements and choose the one you would like to use. You will not need the third option, it corresponds to gradiant measure, which is not used in the IB syllabus.	

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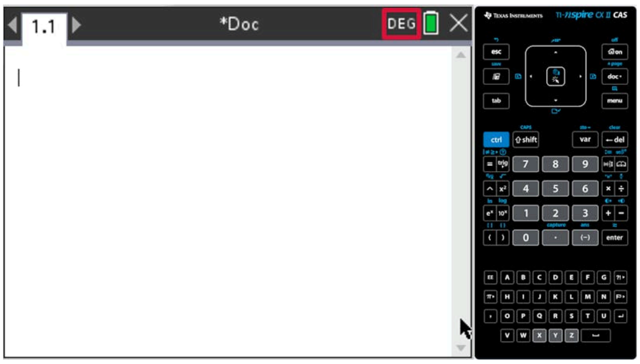
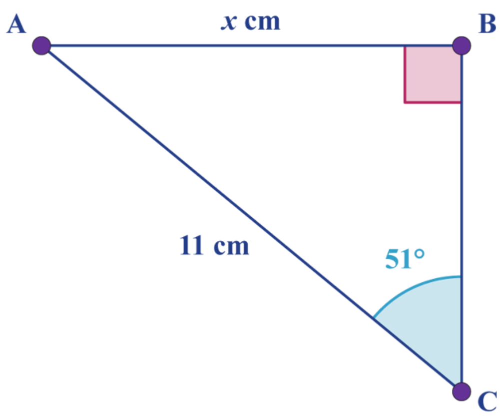
Steps	Explanation
Confirm the change of mode on the calculator screen.	

Example 1

★☆☆

ABC is a right-angled triangle in which angle $C = 51^\circ$, angle $B = 90^\circ$ and $AC = 11$ cm.

Find the length of side AB.



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More information

The image is a diagram of a right triangle labeled ABC. The angle at vertex C is marked as 51 degrees, and point B is the right-angle vertex. Side AB is labeled as (x) cm and side BC is perpendicular to AB and labeled with a red square to indicate the right angle. The side AC, which is the hypotenuse, is labeled as 11 cm. The triangle illustrates the task of finding the length of side AB. This image may be used to illustrate a trigonometry problem or application, such as using the cosine or sine of angle C to find the missing length of AB.

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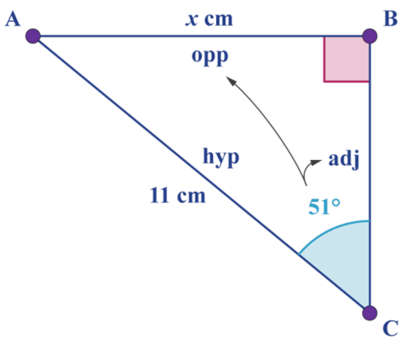
Steps	Explanation
	First, label the triangle, identifying which side is the hypotenuse, and then which sides are opposite or adjacent to the given angle (51 °). The sides of interest are the side marked *x* (because you are required to calculate it), and the side marked 11 cm (because you are given it). Those sides are the sides labelled opp and hyp.
$\sin 51 = \frac{\text{opp}}{\text{hyp}}$ $= \frac{x}{11}$	Referring to **SOH** CAH TOA, you see that SOH has o and h for opp and hyp; therefore you need to use the sine ratio to solve this problem.
$\sin 51 = \frac{x}{11}$ $x = 11 \sin 51$ $= 8.548\ldots$ So AB = 8.55 cm (to 3 significant figures).	Multiply both sides by 11.

Making connections

You are already familiar with Pythagoras' theorem in right-angled triangles, which states:

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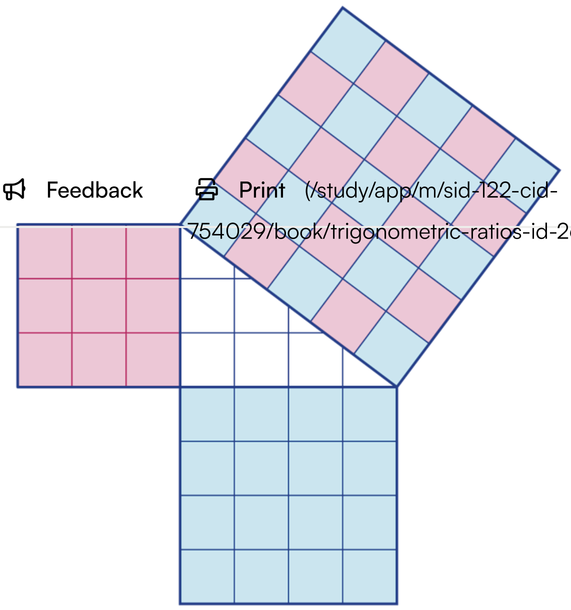
'In a right-angled triangle: the square of the hypotenuse is equal to the sum of the squares of the other two sides.'

This means that the sum of the areas of the squares formed on the two shorter sides will be equal to the area of the square formed on the hypotenuse. In the diagram below, you can see the basic proof demonstrated using coloured squares.

Is this claim true for other shapes formed around the sides of a right-angled triangle? What if you draw semicircles, or some random shapes instead of squares?

See the video below where this idea is discussed.

A Mathematical Fable - Numberphile



Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/trigonometric-ratios-id-26207/print/) Assign

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More information

The image is a geometric representation of the Pythagorean Theorem. It features three squares arranged in a manner that resembles a right triangle. The square on the left is colored solidly. The top square is positioned at a diagonal and shows a checkerboard pattern with alternating colors. The bottom square, directly below the left square, has a grid pattern without any coloration. This layout visually demonstrates that the area of the square built on the hypotenuse (the diagonal position) is equal to the sum of the areas of the other two squares. There are no numerical values or specific labels on the diagram, but the arrangement clearly illustrates the theorem's concept.

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Example 2

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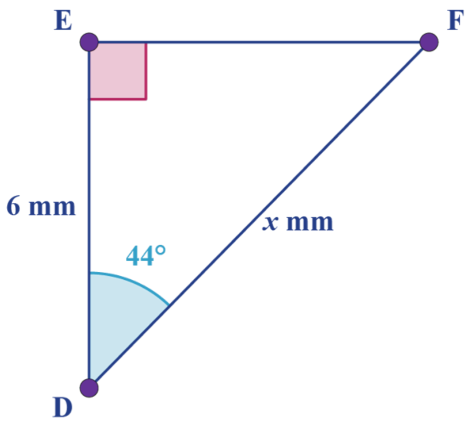
In the diagram below DEF is a right-angled triangle in which angle $D = 44°$, angle $E = 90°$, and $DE = 6\text{ mm}$.

Find the length of side DF.



More information

The image is a diagram of a right triangle. The triangle has three vertices labeled D, E, and F. Side DE is vertical and measures 6 mm. Side DF is horizontal and is marked as 'x mm,' which is unknown. The angle at vertex D is 44 degrees, situated at the base of the horizontal side DF. The right angle is located at vertex E, indicating that DE is the height and DF the base. The hypotenuse of the triangle is the side connecting vertices E and F. The diagram includes marked segments and angle to depict a trigonometry problem aimed at finding the length of side DF.

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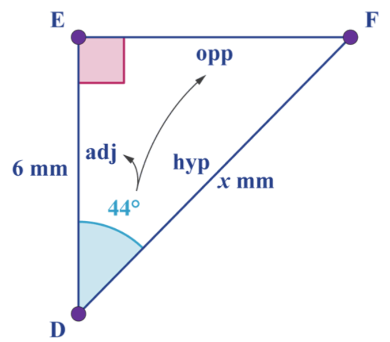
Steps	Explanation
	Label the triangle DEF.
$\cos 44 = \dfrac{\text{adj}}{\text{hyp}}$	Referring to SOH **CAH** TOA, the cosine ratio requires you to know the adjacent side and the hypotenuse.
$\cos 44 = \dfrac{6}{x}$	Substituting values from the labelled triangle.
$\begin{aligned} \cos 44 &= \frac{6}{x} \\ x \cos 44 &= 6 \\ &= \frac{6}{\cos 44} \\ &= 8.3409\ldots \end{aligned}$ So *x* = 8.34 mm (to 3 significant figures).	Divide by $\cos 44$.

Example 3

Student view

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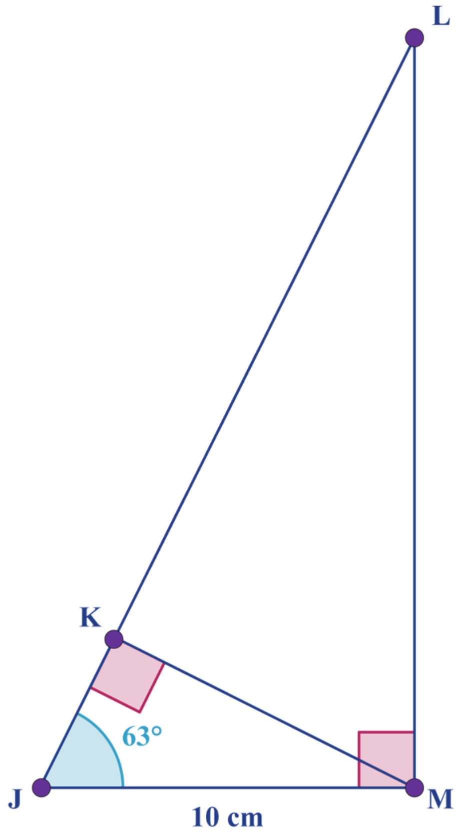
JML and KML are right-angled triangles with angle JML = 90° and angle LKM = 90°.

If JM = 10 cm and angle KJM = 63°, find the lengths of the sides KM, JK, ML and JL.



More information

The image is a geometric diagram featuring a triangle and a right-angled triangle. It consists of four key points labeled J, K, L, and M. Angle KJM is marked as 63 degrees, and the segment JM is labeled as 10 cm long. There is a right-angle indicator at point K, suggesting triangle KJM is a right triangle. The image implies relationships between several geometric elements, prompting the calculation of the lengths of sides KM, JK, ML, and JL.

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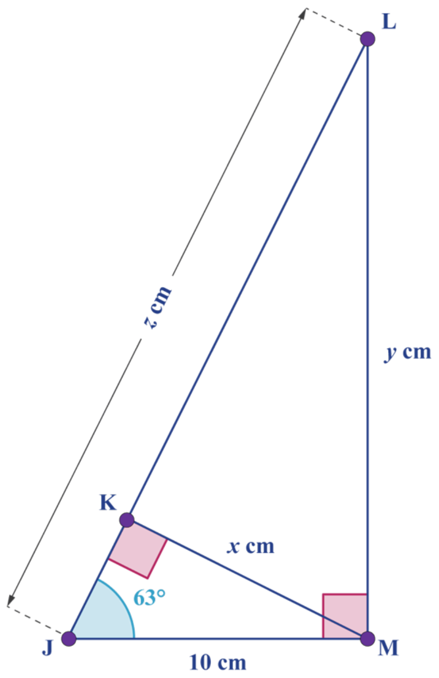
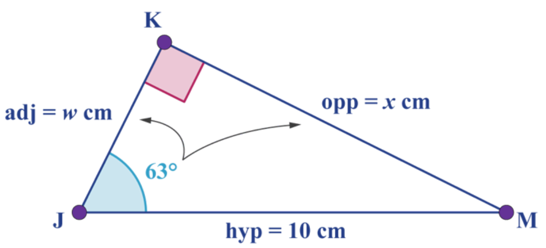
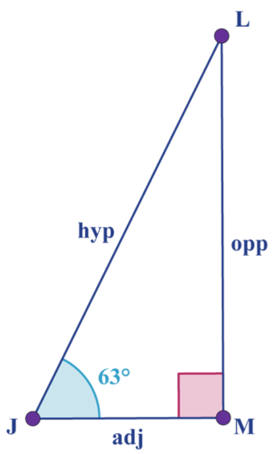
Steps	Explanation
	Label the sides as $KM = x$ cm, $ML = y$ cm and $JL = z$ cm.
	Focus on the smaller triangle, JKM. Isolate this triangle from the diagram, and draw a separate sketch of it. You may now use the labels opp, adj and hyp without confusion. Label side JK w cm.

Student
view

Steps	Explanation
$\sin 63 = \frac{\text{opp}}{\text{hyp}}$ $= \frac{x}{10}$ $10 \sin 63 = x$ $x = 8.91 (3\,\text{s. f.})$ $\cos 63 = \frac{\text{adj}}{\text{hyp}}$ $= \frac{w}{10}$ $10 \cos 63 = w$ $w = 4.54\,(3\,\text{s. f.})$ So the length of KM is 8.91 cm (to 3 significant figures) and the length of JK is 4.54 cm (to 3 significant figures).	You have been given the hypotenuse; therefore you should use sine to find *x* and cosine to find *w* .
	Sketch triangle JML.
$\tan 63 = \frac{\text{opp}}{\text{adj}}$ $10 \tan 63 = y$ $\tan 63 = \frac{y}{10}$ $y = 19.6\,(3\,\text{s. f.})$ So the length of ML is 19.6 cm (to 3 significant figures).	Note that the side JM was the hypotenuse for the smaller triangle, but it is now the adjacent side for the larger triangle.

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Steps	Explanation
$10^2 + y^2 = z^2$	Finally, to calculate the value of z, you can use the cosine ratio or Pythagoras' theorem. Here the Pythagoras' theorem is used.
Therefore, $z = 22.0$ (to 3 significant figures). So the length of JL is 22.0 cm (to 3 significant figures).	To avoid using a rounded answer in this calculation, you should use the full calculator value for y.

Exam tip

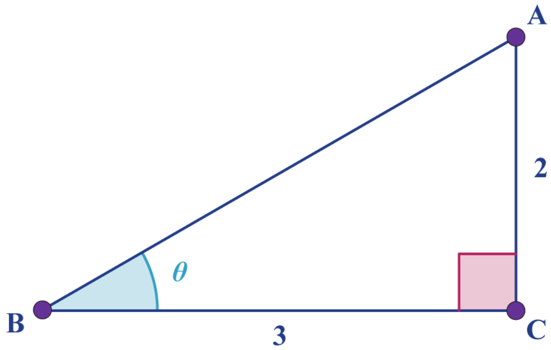
Use the 'Ans' function on your calculator to use exact values while solving the problem.

Round *only* your final answer to avoid losing marks for accuracy.

Finding an unknown angle

You are familiar with using inverse functions when solving equations. You know that the inverse operation of addition is subtraction and that the inverse operation of multiplication is division. In this subsection, you will use the inverse operation of a trigonometric function.

For example, in the triangle below, $\tan\theta = \dfrac{2}{3}$.



More information

Student
view

The image shows a right triangle labeled ABC. The right angle is at point C. The triangle's base (BC) measures 3 units, and its height (AC) measures 2 units. The angle θ is located at point B, opposite the side AC and adjacent to the side BC. The hypotenuse (AB) extends from point A to point B. The image demonstrates a basic trigonometric example where (\tan\theta = \frac{opposite}{adjacent} = \frac{2}{3}).

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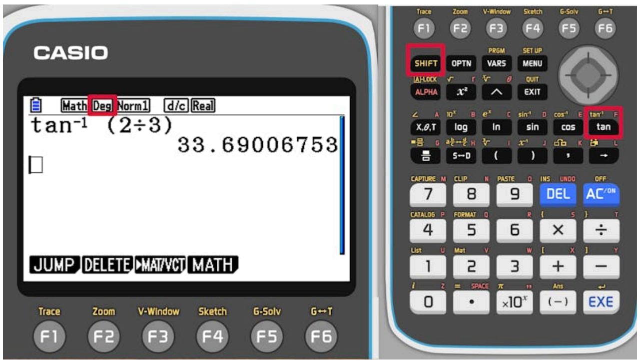
If you want to find angle θ then you would need to use the inverse of $\tan\theta$ which is written as $\theta = \tan^{-1}\left(\frac{2}{3}\right)$ and often referred as arctan.

Which means that θ is the angle with a tan ratio of $\frac{2}{3}$.

Using your calculator gives $\theta = \tan^{-1}\left(\frac{2}{3}\right) = \arctan\left(\frac{2}{3}\right) \approx 34^{\circ}$ to the nearest degree.

Steps	Explanation
Make sure that your calculator is in degree mode if you would like to get an angle measure in degrees as an answer. Remember, the angle mode of your calculator is indicated on the calculation screen. Inverse tangent is printed as $\tan^{-1}$ on your calculator and you can access it using the shift button.	

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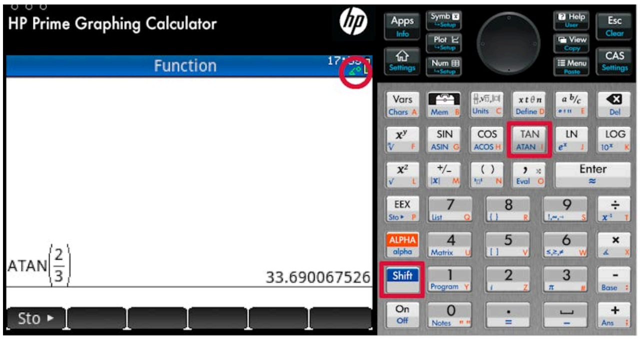
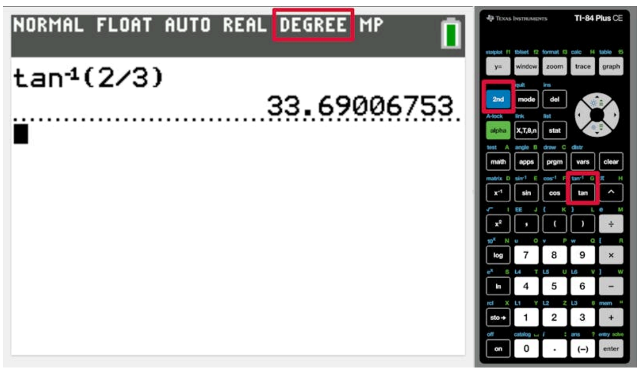
Steps	Explanation
Make sure that your calculator is in degree mode if you would like to get an angle measure in degrees as an answer. Remember, the angle mode of your calculator is indicated on the calculation screen. Inverse tangent is printed as ATAN on your calculator and you can access it using the shift button.	

Steps	Explanation
Make sure that your calculator is in degree mode if you would like to get an angle measure in degrees as an answer. Remember, the angle mode of your calculator is indicated on the calculation screen. Inverse tangent is printed as $\tan^{-1}$ on your calculator and you can access it using the 2nd button.	

Student
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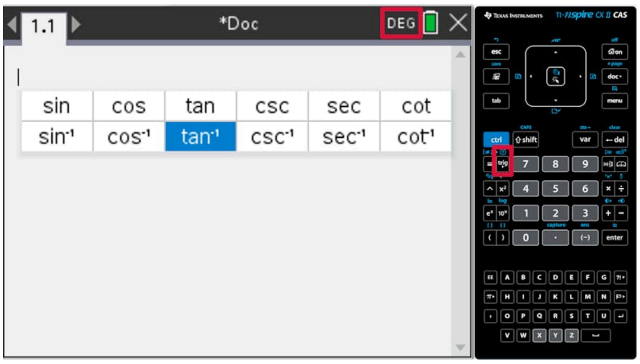
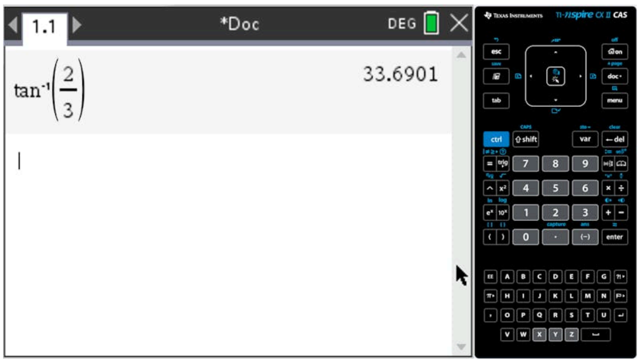
Steps	Explanation
Make sure that your calculator is in degree mode if you would like to get an angle measure in degrees as an answer. Remember, the angle mode of your calculator is indicated on the calculation screen. You can access inverse tangent using the button that brings up all trigonometric ratios on the screen. Inverse tangent is displayed as $\tan^{-1}$.	
	

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Exam tip

To avoid losing marks due to incorrect rounding, include the unrounded answers in your working. For example, if you find that $x = \sqrt{77}$, write the line x = 8.77496... (include the three dots) before your final answer of x = 8.77 (to 3 significant figures).

If the correct unrounded answer is seen, you will obtain the final answer mark even if you make a mistake the rounding. And, if units are used in the question, make sure you use them in the answer.

Example 4

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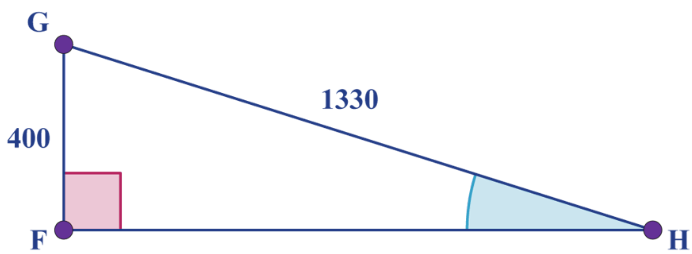
In the triangle below, angle $F = 90°$. When $GF = 400$ cm and $GH = 1330$ cm, find the size of angle GHF.



More information

The image is a diagram of a right triangle labeled as (\triangle GFH). The right angle is at point (F), creating a vertical line segment (GF) and a horizontal line segment (FH). The hypotenuse of the triangle is (GH). The side (GF) is labeled with a length of 400 cm, and (GH) is labeled with a length of 1330 cm. The triangle has point (G) at the top, point (F) at the bottom left forming the right angle, and point (H) at the bottom right. The diagram visually represents the relationship between the three sides in a right triangle.

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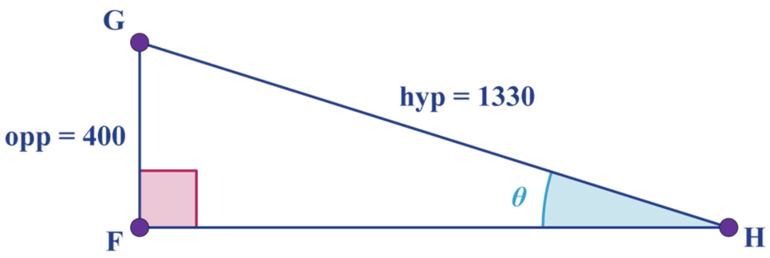
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>
</td><td>First, label the sides that you have been given. It can also be useful to label the angle with a single letter. It is traditional but not necessary to use Greek letters to represent angles. Let angle FHG be θ .</td></tr>
<tr><td>$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$ $$\sin\theta = \frac{400}{1330}$$ $$\theta = \sin^{-1}\frac{400}{1330}$$ $$\theta = 17.5$$ The angle FHG = 17.5° (to 3 significant figures).</td><td>You need to use the sine ratio, as you are given the opposite side and the hypotenuse.</td></tr>
</table>

2 section questions

3. Geometry and trigonometry / 3.2 Triangle trigonometry

The area of a triangle

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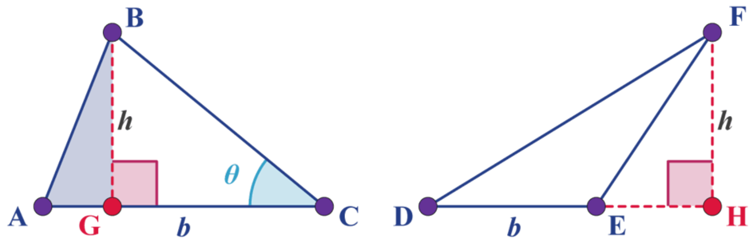
You are familiar with the area of a triangle as $A = \frac{1}{2}\text{base} \times \text{height}$ or

$$A = \frac{1}{2}b \times h$$

Student view

You can see the base and corresponding height marked in two different triangles in the diagram below.



More information

The image consists of two right-angled triangles, labeled on the base and height. The first triangle, labeled ABC, has a base marked as 'b' and a perpendicular height marked from point B to point G as 'h'. The angle θ is marked at point C. The second triangle, labeled DEF, also has a base marked as 'b', and a perpendicular height is drawn from point F to point H, also marked as 'h'. Each triangle has a right angle indicated and different colors are used for the height and base for distinction.

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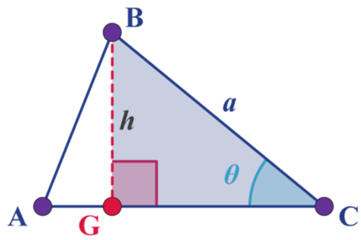
In each case, you form right-angled triangles where you draw the perpendicular height.

These right-angled triangles can be used to establish a relationship between the angles and areas of triangles.



More information

The image is a diagram of a right-angled triangle labeled BCG. Point B is the vertex of the right angle, with points A, G, and C forming the other angles of the triangle. The line segment BC is labeled 'a'. The angle at C is denoted as θ, while the perpendicular segment from B to the line AC is marked with 'h'. The point where the perpendicular meets AC is labeled as G. The triangle is visually divided into smaller segments to indicate the relationships between the sides and angles.

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In triangle BCG above,

$$\sin C = \frac{h}{a}$$

Making h the subject gives

$$h = a \sin C$$

Substituting this expression for h into the formula for the area of a triangle, $A = \frac{1}{2}b \times h$

$$A = \frac{1}{2}b \times a \sin C$$

When this is rearranged it becomes

$$A = \frac{1}{2}ab \sin C$$

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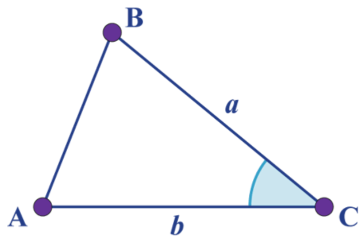

More information

The image is a diagram of a triangle labeled ABC with sides 'a', 'b', and an included angle 'C'. Side 'BC' is labeled 'a', side 'AC' is labeled 'b', and the angle at 'C' is highlighted. The diagram illustrates the formula for the area of a triangle as being half the product of two sides and the sine of the included angle. Vertexes A, B, and C are marked with points, and the included angle C is shaded to indicate it as a key feature in the formula for calculating area.

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You can generalise this rule by saying ‘the area of a triangle is half of the product of the lengths of two sides and the sine of the included angle’.

Exam tip

The formula booklet gives you the following formula for the area of a triangle:

$$A = \frac{1}{2}ab\sin C$$

Example 1

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Find the area of triangle ABC .

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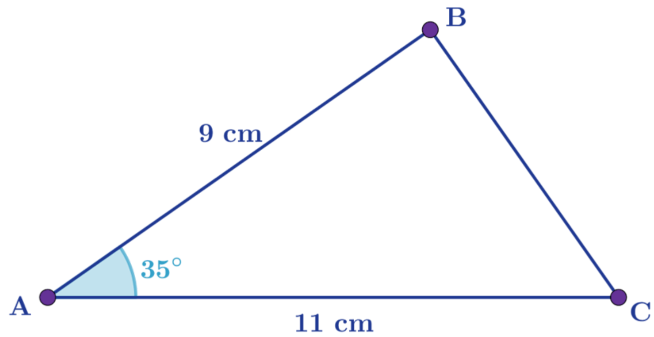

More information

The image shows triangle ABC. Side AC is 11 cm long, and side AB is 9 cm. The angle at A, labeled angle BAC, is 35 degrees. The triangle consists of three points A, B, and C, with AB sloping upwards from left to right, AC being horizontal, and BC connecting B and C. The triangle appears to be scalene as all sides have different lengths.

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Steps	Explanation
$\text{Area} = \frac{1}{2} \times 11 \times 9 \times \sin 35^\circ$ $= 28.392\ldots$ So, area of triangle ABC is $28.4\ \text{cm}^2$.	To find the area of the triangle, it is enough that two sides and the angle in between are given in the figure. Use $\text{Area} = \frac{1}{2}bc\sin \text{A}.$

Example 2

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The area of triangle ABC is 69 centimetre squared. Find the length of side AB.

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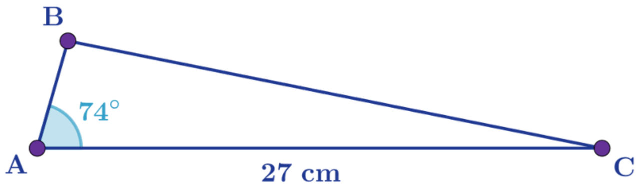

More information

The image shows a geometric diagram of triangle ABC. The triangle is outlined with points labeled A, B, and C at its vertices. The side AC is labeled as 27 cm, and the angle at vertex A, which is angle BAC, is labeled as 74 degrees. The image is likely used to aid in finding the length of side AB, given the area and angle measures provided in accompanying text.

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Steps	Explanation
$69 = \frac{1}{2} \times 27 \times \text{AB} \times \sin 74°$ $69 = 12.97703\ldots \times \text{AB}$ $\text{AB} = \frac{69}{12.97703\ldots} = 5.317086\ldots$ So, AB is approximately 5.32 centimetre long.	Use the area formula $\text{Area} = \frac{1}{2}bc\sin \text{A}.$

Example 3

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The area of both triangles on the diagram is 98 centimetre squared. Find the angle at A for both triangles.

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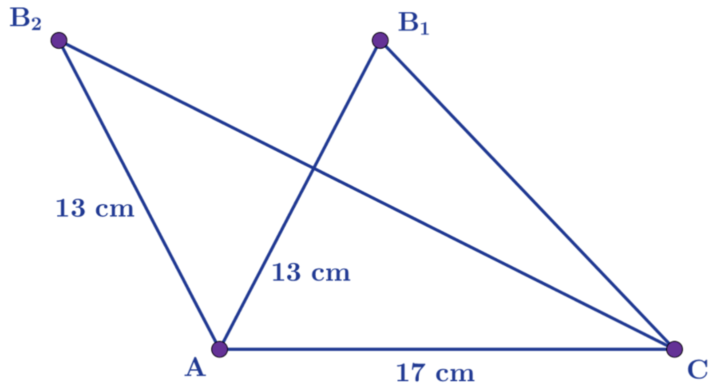

More information

The image shows a geometric diagram featuring two triangles that share a common side and a vertex. Triangle ABC is formed with the points A, B1, and C, while the points A, B2, and C form another triangle. Both triangles share the side AC, which measures 17 cm. The side AB1 and AB2 both measure 13 cm each. Vertex A is a common point for both triangles. The area for each triangle is given as 98 cm². The image is a visual depiction requiring users to find the angle at point A for both triangles.

[Generated by AI]

Steps	Explanation
$$98 = \frac{1}{2} \times 17 \times 13 \times \sin \mathrm{A}$$ $$98 = 110.5 \times \sin \mathrm{A}$$ $$\sin \mathrm{A} = \frac{98}{110.5} = 0.886877828\ldots$$	Notice that in both triangles you can use the same area formula $$\text{Area} = \frac{1}{2}bc\sin \mathrm{A}.$$

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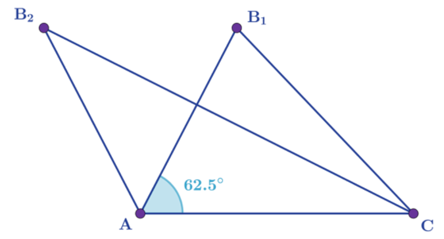
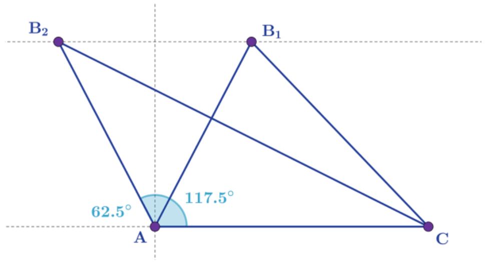
Steps	Explanation
$\sin^{-1}(0.886877828\ldots) \approx 62.5°$ The angle at A in triangle $\mathrm{ACB_1}$ is acute, this is what you found. 	Use $\sin^{-1}$ on your calculator and interpret the result.
The angle at A in triangle $\mathrm{ACB_2}$ is approximately $180° - 62.5° = 117.5°$. 	The area of the two triangles are the same, so $\mathrm{B_1}$ and $\mathrm{B_2}$ are in symmetrical position. You can use this symmetry to find the obtuse angle at A in triangle $\mathrm{ACB_2}$

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/rightangled-triangles-id-26208/print/) Assign

Example 4

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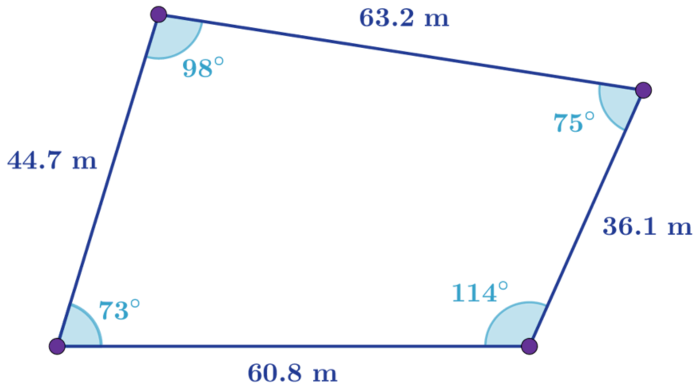
A land surveyor is asked to find the area of a piece of land. They walked around the land and made the following sketch including the length of the sides and the angles of the quadrilateral.

Student view



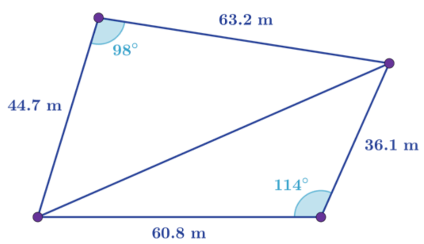
More information

The image is a sketch of a quadrilateral with labeled sides and angles, created by a land surveyor to find the area of a piece of land. The four sides of the quadrilateral have lengths of 63.2 meters, 36.1 meters, 60.8 meters, and 44.7 meters. The angles at the vertices are 98°, 75°, 114°, and 73°, connecting the respective sides.

The shape is irregular, indicating that the two opposite sides are not parallel, and the angles vary, suggesting no right angles present in the quadrilateral. The lengths and angles provide a complete understanding of the four-sided figure for calculating its area.

[Generated by AI]

Find the area of the land.

Steps	Explanation
	Draw a diagonal and split the quadrilateral in two triangles.

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Steps	Explanation
$\frac{1}{2} \times 44.7 \times 63.2 \times \sin 98° = 1398.77\ldots$ $\frac{1}{2} \times 36.1 \times 60.8 \times \sin 114° = 1002.56\ldots$	Use the area formula to find the area of both triangles.
$1398.77\ldots + 1002.56\ldots = 2401.33\ldots$ The area of the land is approximately 2400 squared metre (rounded to three significant figures).	Add the two areas together.

- If you split the quadrilateral along the other diagonal, you will get the same approximate answer. Try it!
- Although the two answers are approximately the same, they do not match perfectly. The reason for this is that the measurements of the surveyor are only approximate. There is no quadrilateral with exactly these measurements. In fact, measuring three sides and the included two angles would give enough information for the surveyor to make a scaled sketch of the land and find the area. You will learn the tools to carry out the calculation using this limited information in the next sections.

1 section question

3. Geometry and trigonometry / 3.2 Triangle trigonometry

The sine rule

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/the-sine-rule-id-26210/print/) Assign

Even if there are no right-angled triangles in the given triangle, you can draw height from one of the vertices to the opposite side as seen in the diagram below.

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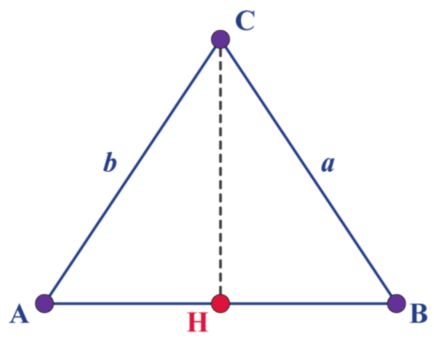

More information

The diagram shows triangle ABC, an isosceles triangle with sides labeled as 'a' and 'b'. The vertices are marked as A, B, and C, each with purple dots. A perpendicular line, labeled as height CH, is drawn from vertex C to the base AB, dividing triangle ABC into two right triangles, ACH and CHB. The base AB is horizontal and has a red dot at point H, where the height intersects the base. The triangle is symmetric along height CH.

[Generated by AI]

Now you have two right-angled triangles: ACH and CHB. In the previous section, you used these triangles to find the area of the triangle ABC, using the side lengths and included angle.

You can also establish the relationship between side lengths and corresponding angles as both triangles have a common side CH.

In triangle ACH, $\sin A = \frac{\text{CH}}{b} \Rightarrow \text{CH} = b \sin A$

In triangle CHB , $\sin B = \frac{\text{CH}}{a} \Rightarrow \text{CH} = a \sin B$

Thus, $b \sin A = a \sin B$

Rearranging the equation you get $\frac{a}{\sin A} = \frac{b}{\sin B}$.

You can also establish the relationship between side AB and sin C by drawing the height from A to the side BC.

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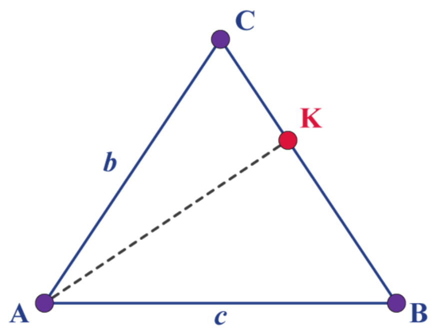

More information

The image depicts a triangle labeled ABC with vertices A, B, and C. A point labeled K is located on side BC. A dashed line from point A to point K indicates the height of the triangle. The sides are labeled as follows: b for side AC and c for side AB. The position of point K divides the base BC, forming two smaller right-angled triangles, ABK and ACK. The diagram visually represents the relationship between side AB and the sine of angle C, as well as the relationship between side AC and the sine of angle B, supporting the equation (\frac{b}{\sin B} = \frac{c}{\sin C}).

[Generated by AI]

Using the right-angled triangle s ABK and ACK, $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

When you combine the first relationship with this one you get the sine rule.

The sine rule states that in any general triangle,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or, equivalently,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

The first version of the rule is useful when finding side lengths and the second version is useful when finding an angle.

Thus, the ratios of the sine of the angle at a vertex to its opposite side are identical in a triangle.

Student
view

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This also holds in a right-angled triangle (because $\sin A = \sin 90° = 1$ in which case, $\frac{1}{a} = \frac{\sin B}{b}$ or $\sin B = \frac{b}{a}$ which is the same as the expression given by the sine ratio).

The sine rule is useful in two circumstances:

1. When you have **two angles** and one of the **sides opposite** one of these angles, you can use the sine rule to find the other opposite **side**.
2. When you have **two sides** and one **angle opposite** one of these sides, you can use the sine rule to find the other opposite **angle**.

Then, once you have two angles in a triangle, you can easily find the third angle, since all interior angles in a triangle add up to 180 ° You can find the last remaining side if necessary.

Exam tip

The formula booklet gives you the following formula for the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Finding a side length using the sine rule

Example 1

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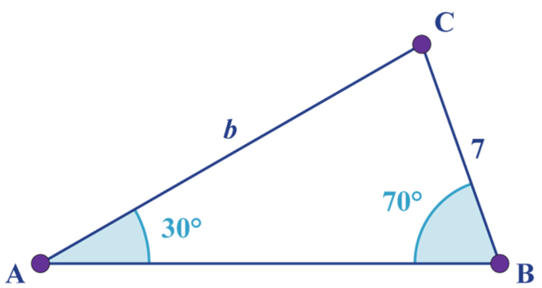

More information

Student
view

Overview (/study/ap 122- cid- 754029/

The image shows triangle ABC with angles labeled and sides indicated. Angle A is 30 degrees, and angle B is 70 degrees. The vertices are labeled as follows: vertex A at the bottom left, vertex B at the bottom right, and vertex C at the top. Side AC is labeled with the variable 'b', and side AB has a length of 7. The problem asks to find the value of 'b'. This is a geometry problem involving a non-right triangle with given angles and side measurements.

[Generated by AI]

In triangle ABC, shown above, find the value of b.

You can use the sine rule because you have two angles and one opposite side:

$$A = 30^\circ,\ B = 70^\circ \text{and } a = 7$$

Thus, choosing the more useful expression of the sine rule:

$$\begin{aligned}
\frac{b}{\sin B} &= \frac{a}{\sin A} \\
b &= \frac{a}{\sin A} \times \sin B \\
b &= \frac{7}{\sin 30^\circ} \times \sin 70^\circ \\
b &= 13.155
\end{aligned}$$

So b is 13.2 (to 3 significant figures).

Exam tip

Although you should use your calculator to find the solution, you must write down all the steps. Make sure your GDC is in ‘degrees’ mode (see section 3.2.2 (/study/app/m/sid-122-cid-754029/book/rightangled-triangles-id-26208/) for instructions).

Finding an angle using the sine rule

Example 2

★☆☆

Student view

In triangle DEF, $DF = 7\,\text{cm}$ and $DE = 6\,\text{cm}$.

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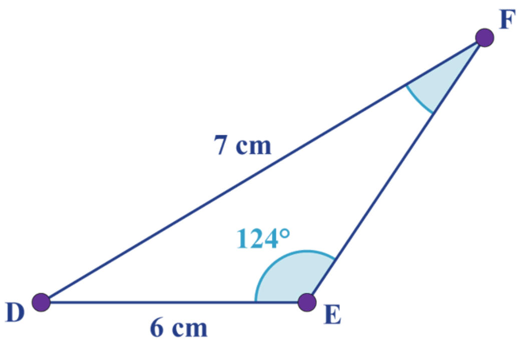
Find angle F if angle $DEF = 124°$.



More information

The image depicts a triangle labeled DEF. The point D is at the bottom left, E is at the bottom right, and F is at the top. Angle DEF is marked as 124 degrees. Side DE measures 6 cm, and side EF measures 7 cm. The diagram shows the triangle's angles and sides clearly annotated with dimensions and labels.

[Generated by AI]

In this case you need to find an angle, which you can call x :

$$\frac{\sin x}{6} = \frac{\sin 124°}{7}$$

$$\sin x = \frac{\sin 124° \times 6}{7}$$

using your GDC angle $F = 45.3°$ (to 3 significant figures).

Use the applet below to test your understanding.

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view

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Interactive 1. Sine Rule — Find an Angle.

Credit: GeoGebra (https://www.geogebra.org/m/KK9tTH43) David T

More information for interactive 1

This interactive allows users to explore and apply the sine rule in a triangle by manipulating side lengths and angles. Users can change the length of side ‘a’ (opposite to angle A) and side ‘b’ (opposite to angle B) within a range of 0° to 10° units. They can also fix angle B between 0 and 180. Moving to Page 2, they can visualize how to find angle A, using the sine rule, applying the relationship $\frac{\sin A}{a} = \frac{\sin B}{b}$. On Page 3, users can reveal whether their calculated angle A is correct, providing immediate feedback on their understanding and calculations.

Additionally, by sliding the pairings tab, users can visualize which side is opposite to which angle, reinforcing the relationship between sides and angles in a triangle.

3 section questions

3. Geometry and trigonometry / 3.2 Triangle trigonometry

The cosine rule

Student view

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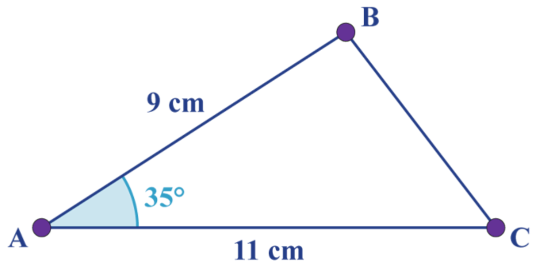
In previous section, you found the area of the triangle, ABC, below.



More information

The image is a diagram of a triangle labeled ABC. Point A is at the bottom left, point B at the top, and point C at the bottom right, forming a triangle. The angle at point A is marked as 35 degrees. The side between points A and B is labeled 9 cm, and the side between points A and C is labeled 11 cm. Points A, B, and C are marked with small purple circles. The diagram illustrates the angles and side lengths of the triangle.

[Generated by AI]

What about finding the side length of BC, or the sizes of other two angles B and C?

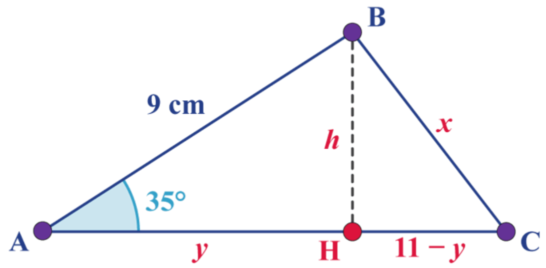
If you use the sine rule for angle ABC .

$$\frac{BC}{\sin 35} = \frac{11}{\sin B} = \frac{9}{\sin C}$$

you can see that there is not enough information to solve the triangle with the sine rule.

Similar to the previous strategy, you can find the missing lengths by drawing the height and dividing angle ABC . into two right-angled triangles, as shown in the diagram below.

Student
view



More information

The image depicts a triangle labeled ABC with a perpendicular line from point B to line AC, meeting at point H. The triangle is divided into two right-angled triangles by the height line BH. The following labels are visible in the diagram:

- Line AB is labeled with a length of 9 cm.
- Angle BAC is marked as 35 degrees.
- Line segment AH is labeled as y, and HC is labeled as 11 - y.
- The height from B to line AC is labeled as h.
- Line segment BC is labeled as x.

The diagram aids in solving the lengths of the missing segments by using the properties of right-angled triangles and trigonometric ratios, leveraging the given angle and side lengths.

[Generated by AI]

In triangle ABH

$$AH = y = 9\cos 35 \ldots\ldots(1)$$

using Pythagoras' theorem,

$$y^2 + h^2 = 9^2$$

Rearranging gives

$$h^2 = 9^2 - y^2 \ldots\ldots(2)$$

In triangle BHC, using Pythagoras' theorem,

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$$h^2 + (11 - y)^2 = x^2$$

Expanding the brackets gives

$$h^2 + 11^2 - 2 \times 11 \times y + y^2 = x^2 \ldots\ldots(3)$$

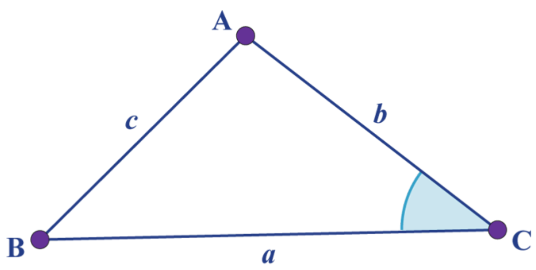
Now, you can substitute (1) and (2) into equation (3)

$$9^2 - y^2 + 11^2 - 2 \times 11 \times 9 \cos 35 + y^2 = x^2$$

Rearranging and solving for x,

$$\begin{aligned} x^2 &= 9^2 + 11^2 - 2 \times 11 \times 9 \cos 35 \\ x &= 6.31 \quad (\text{to 3 significant figures}) \,. \end{aligned}$$

This method can be generalised to find a side length of triangle ABC when the lengths of two sides and the included angle are given, as in the diagram below.



More information

The image is a geometric diagram of triangle ABC. It shows a triangle with three vertices labeled A, B, and C. The sides opposite these vertices are labeled as follows: the side opposite vertex A is labeled 'a', the side opposite vertex B is 'b', and the side opposite vertex C is 'c'. The angle at vertex C is shaded and marked as (\angle ACB). The diagram is intended to illustrate a situation for applying the cosine rule, which requires two side lengths and the included angle.

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view

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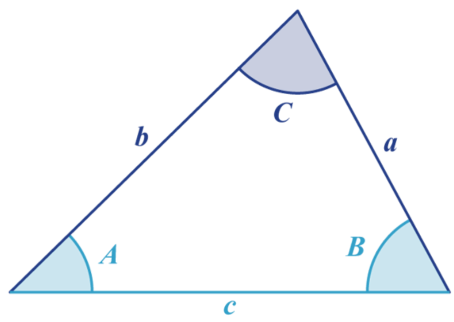
This general rule is called the cosine rule which states that in any general triangle,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

where a, b, and c represent the sides of a triangle opposite their respective vertices A, B and C. There are four variables in the cosine rule. Thus, to use it, you must know the values of three of them, and then the cosine rule can be used to obtain the fourth.

The cosine rule is useful in two circumstances:

1. When you have **one angle**, say C , and the **two sides adjacent** to this angle, a and b (see **the first triangle below**), you may obtain the side, c , opposite angle C using the cosine rule.
2. When you have the values of **all the sides** of a triangle (see **the second triangle below**), you may obtain the **angles** A , B and C . using the cosine rule.



More information

This is a diagram of a triangle with three sides labeled as 'a,' 'b,' and 'c,' and three angles labeled as 'A,' 'B,' and 'C.' Angle A is opposite side 'a,' angle B is opposite side 'b,' and angle C is opposite side 'c.' This diagram visually represents the concept of using known angles and adjacent sides to determine the third side of a triangle, as noted in the accompanying text. If one angle and the adjacent sides are known, you can calculate the third side accordingly.

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If one angle and the adjacent sides are known you can find the third side.

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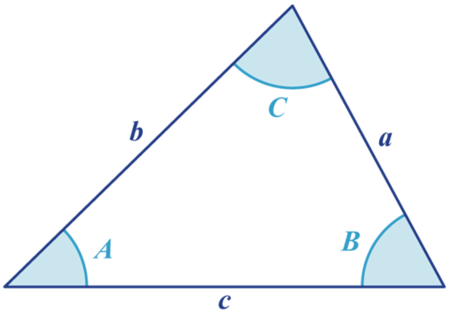

More information

The image is a diagram of a triangle with three angles and three sides labeled. The triangle is labeled with angles A, B, and C, and the sides opposite these angles are labeled as a, b, and c, respectively. Angle A is in the bottom left corner, angle B is in the bottom right corner, and angle C is at the top vertex of the triangle. The side opposite angle A is labeled 'a', the side opposite angle B is labeled 'b', and the side opposite angle C is labeled 'c'. This diagram is used to illustrate the relationship between the angles and sides of a triangle, which is important in trigonometry and geometry for solving problems when some of these measurements are known.

[Generated by AI]

If all the sides are known you can find the three angles.

The cosine rule also holds in a right-angled triangle, in which case, it becomes Pythagoras' theorem (as cos 90° = 0).

There was nothing special about choosing angle C and the adjacent sides a and b ; the cosine rule applies equally to the angle A , for example

$$a^2 = b^2 + c^2 - 2bc\cos A$$

and angle B , for example

$$b^2 = a^2 + c^2 - 2ac\cos B.$$

Student
view

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Finding a side length using the cosine rule

Exam tip

The formula booklet gives you the following formula for the cosine rule:

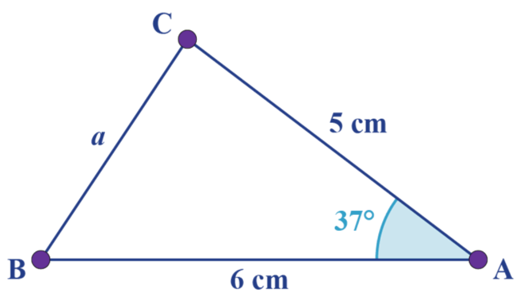
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example 1

★☆☆

In triangle ABC, $CA = 5$ cm and $AB = 6$ cm and angle $A = 37°$.

Find the length of side BC.



More information

The image depicts a triangle labeled as triangle ABC. The vertices are labeled with points A, B, and C, with point A on the right, B on the bottom left, and C at the top.

- Side AB is labeled as 6 cm.
- Side AC is labeled as 5 cm.
- The angle at vertex A is labeled as 37 degrees, marked with an arc.
- Side BC is labeled with "a," indicating the unknown side length to find.

The image is a geometrical diagram, focusing on the given measurements and the task to calculate the length of side BC, using the included angle and side lengths information.

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Steps	Explanation
$a^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos 37°$	Using the cosine rule.
$a = 3.6168\ldots$ So the length of *BC* is 3.62 cm (to 3 significant figures).	Take the square root of the value you got in the previous step.

Be aware

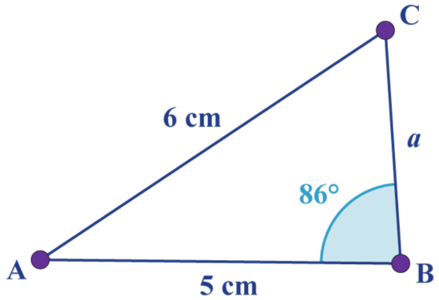
Avoid rounding errors: do not round cos 37, or any other answer, until the last step of the solution. Only round the final answer.

Example 2

★★☆

For triangle ABC, $AB = 5\text{cm}$, $AC = 6\text{cm}$ and angle $\angle B = 86°$.

Find the length of the side BC.



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More information

The image shows a triangle labeled ABC with the following details: \n- Side AB is 5 cm long.\n- Side AC is 6 cm long.\n- Angle at B is 86 degrees.\n- The triangle is oriented with point A on the left, B on the bottom right, and C on the top right.\n- Side BC, labeled as 'a,' is opposite the angle at A, and its length is to be determined.

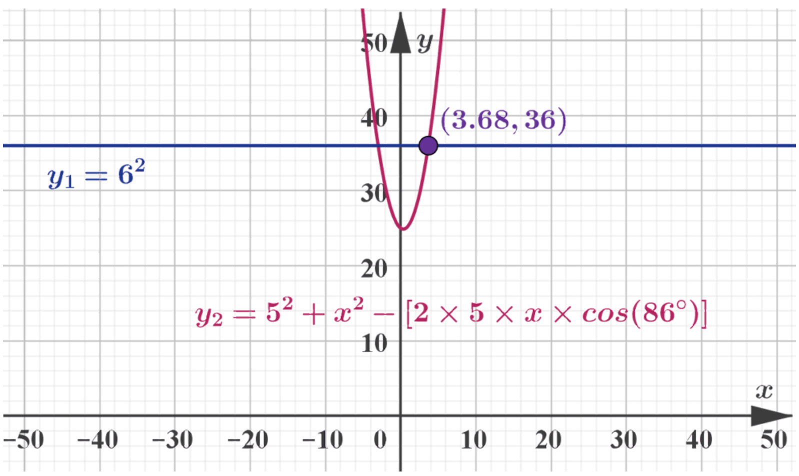
[Generated by AI]

Steps	Explanation
$6^2 = 5^2 + a^2 - 2 \times 5 \times a \times \cos 86$	Using the cosine rule.
$a = 3.6836\ldots$ So the length of *BC* is 3.68 cm (to 3 significant figures).	Using a calculator.

Be aware

When you graph it is important to use the correct values for the intersection point.

As a is a length, find only the positive value.



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Finding angles using the cosine rule

Given the values of all three sides of a triangle, the angles are given by

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right),$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

and

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right).$$

Example 3

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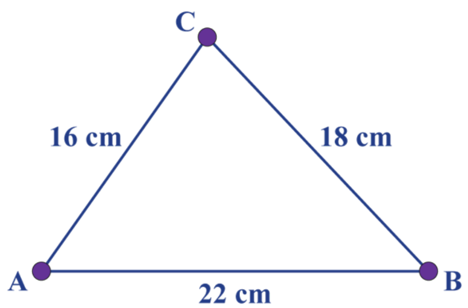

More information

The image shows a triangle labeled ABC, with the following details:

- Vertex A is located at the bottom left.
- Vertex B is at the bottom right.

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- Vertex C is at the top center, forming the apex of the triangle.

The sides have the following measurements: - Line segment AB is the base of the triangle, measuring 22 cm. - Line segment AC measures 16 cm. - Line segment BC measures 18 cm.

The task associated with this triangle, as indicated by the surrounding text, is to find the size of angle ABC, which is at vertex B, to the nearest degree. This triangle is not drawn to scale.

[Generated by AI]

In triangle ABC (not to scale), find the size of angle ABC . Give your answer correct to the nearest degree.

$$\begin{aligned} \cos B &= \frac{18^2 + 22^2 - 16^2}{2 \times 18 \times 22} \\ \cos B &= \frac{23}{33} \qquad\qquad\qquad\qquad \text{[must be between -1 and 1!]} \\ B &= \cos^{-1}\left(\frac{23}{33}\right) \\ B &= 45.8156\ldots^\circ \end{aligned}$$

So the size of angle ABC is 46° to the nearest degree.

Exam tip

After writing the initial line of working, you can use your calculator to find the angle.
Make sure your calculator is in degree mode.

3 section questions

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view

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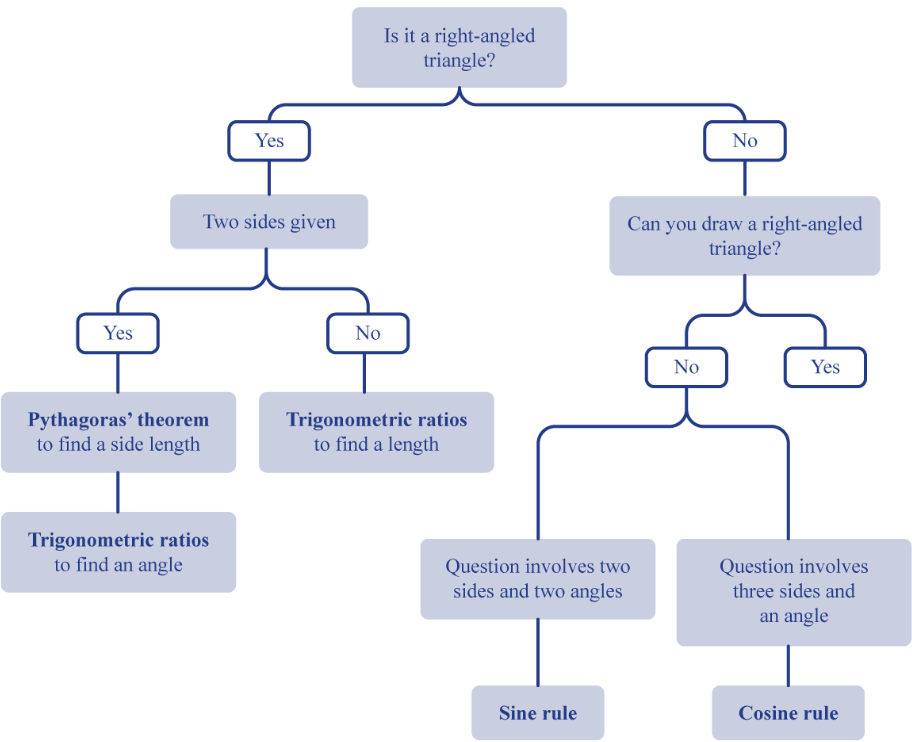
Miscellaneous problems

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/miscellaneous-problems-id-26212/print/) Assign

Choosing the correct rule to solve problems

As you have seen, there are several rules when it comes to solving triangles. The first thing you should look for is whether it is a right-angled triangle or not. If it is a right-angled triangle, then you can either use Pythagoras' theorem or trigonometric ratios.

If the triangle given in the question doesn't have a right angle, then see if you can divide it up to give a useful right-angled triangle. If you cannot, then, depending on the given values, you should use either the sine rule or the cosine rule The flowchart below will help you to decide which rule to use.



More information

This flowchart helps decide whether to use the sine rule or the cosine rule for solving triangle problems. It starts with the decision node asking whether the triangle has a right angle. If "Yes," it suggests using trigonometric ratios. If "No," the chart splits based on whether the known values are angles and sides, leading to either the sine rule or the cosine rule.

1. The first decision node asks if the triangle has a right angle.
2. If "Yes," it leads to a process box instructing to use trigonometric ratios.

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3. If "No," it asks if two angles and one side are known.
4. If "Yes," follow the path to use the sine rule.
5. If "No," it checks if two sides and the included angle are known.
    - If "Yes," follow the path to use the cosine rule.
    - If "No," it checks if all three sides are known, leading to the cosine rule.

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Example 1

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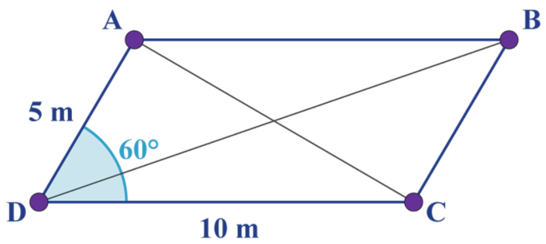


More information

The image is a diagram of a parallelogram labeled ABCD. The side AD measures 5 meters, and the side DC measures 10 meters. The angle at vertex D is labeled 60 degrees. The diagram includes diagonal lines connecting opposite vertices A to C and B to D. All vertices are labeled, and small circles highlight each corner of the parallelogram. The image visually represents the specified angles and side lengths within the geometric shape.

[Generated by AI]

ABCD is a parallelogram with side lengths $AD = 5\text{ m}$ and $DC = 10\text{ m}$.

If angle $ADC = 60^{\circ}$, find the lengths of diagonals AC and BD.

Student
view

Overview (/study/app/m/sid-122-cid-754029/

Steps	Explanation
$\text{AC}^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos 60°$ $\text{AC}^2 = 100 + 25 - 100 \times \frac{1}{2}$ $\text{AC}^2 = 75$ $\text{AC} = 8.660\ldots = 8.66 \text{ m (to 3 significant figures)}$	Note that here all the steps are shown but in the exam you can use your calculator after you have written the first line to find the unknown lengths. As you know the angle *D* and the adjacent sides, you apply the cosine rule.
$\text{BD}^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos 120°$ $\text{BD}^2 = 100 + 25 - 100 \times \left(-\frac{1}{2}\right)$ $\text{BD}^2 = 175$ $\text{BD} = 13.228\ldots = 13.2 \text{ m (to 3 significant figures)}$	You also know that the figure is a parallelogram. Thus $BC = AD = 5$ m and $C = 180–60 = 120$. Again you know the size of angle *C* and two adjacent sides so you apply the cosine rule once more.

⚠ **Be aware**

1. You will not need to measure any variables in a triangle with your protractor. Diagrams are not drawn accurately. Hence, do not be misled by the scale of any triangle. You must always calculate angles. Consider only the values given.
2. Remember that $\sin\theta$ and $\cos\theta$ must always lie between − 1 and 1 for any θ. Hence, if you find, e.g. cos $C = 1.1$, you know that you have done something wrong (check your signs!).
3. Clearly, you do not need to consider any negative solutions for the length of a side of a triangle.

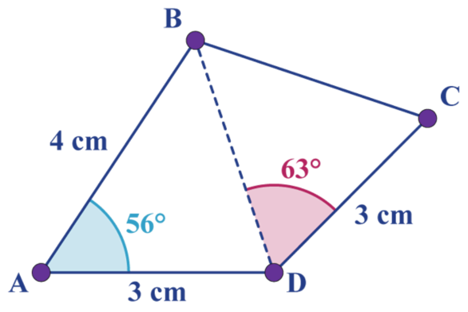
Example 2

★★☆

Quadrilateral ABCD has side lengths $AB = 4\,\text{cm}$, $AD = 3\,\text{cm}$ and $DC = 3\,\text{cm}$.

If angle $BAD = 56°$ and angle $BDC = 63°$, find the area of the quadrilateral ABCD.

Student view



More information

The image is a geometric diagram depicting a quadrilateral named ABCD. It contains labeled angles and sides: (\angle BAD = 56^\circ) and (\angle BDC = 63^\circ). The side lengths are labeled as follows: side AB is 4 cm, side AD is 3 cm, and side DC is 3 cm. Additionally, there is a dashed line BD in the quadrilateral. Points A, B, C, and D are marked.

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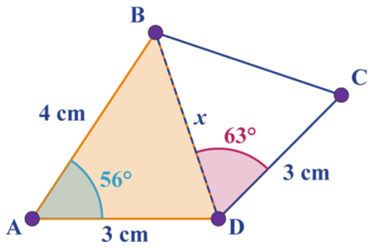
Steps	Explanation
$\text{Area of triangle } BAD = \frac{1}{2} \times 4 \times 3 \times \sin 56$ $= 4.974\ldots \text{cm}^2$	The area of the quadrilateral is the sum of the areas of two triangles BAD and BDC. As you do not have the heights of either triangle, you should use the area formula with the included angle. Make sure your calculator mode is in degrees and record more than 3 significant figures.
$\text{Area of triangle } BDC = \frac{1}{2} BD \times DC \times \sin BDC$ $\text{Area of triangle } BDC = \frac{1}{2} BD \times 3 \times \sin 63$	Now you can look at area of the second triangle, BDC.

Overview (/study/ap 122- cid- 754029/

Steps	Explanation
	You need the length of BD to find the area of the second triangle. If you look at triangle ABD, you are given two sides and the included angle.
$x^2 = 4^2 + 3^2 - 2 \times 4 \times 3\cos 56$ $x = 3.402\ldots \text{ cm}$	You need to find the length of third side. Following the flowchart, this means you will use the cosine rule.
Area of triangle $BDC = \frac{1}{2} \times 3.402 \times 3 \times \sin 63$ Area of triangle $BDC = 4.5479\ldots \text{ cm}^2$	Now you can calculate the area of triangle BDC.
Area of triangle $ABC = 4.974\ldots + 4.5479\ldots$	Area of $ABCD =$ Area of triangle $BAD +$ Area of triangle BDC
Therefore, the total area of the quadrilateral will be $\text{Area of ABCD} \approx 9.52 \text{ cm}^2 (\text{to 3 significant figures})$.	

Example 3

★★★

The following diagram shows quadrilateral ABCD.

$AD = 7 \text{ cm}$, $DC = 3 \text{ cm}$, angle BDA $= 41°$, angle DBC $= 28°$ and angle BCD $= 107°$.

Student view

Overview
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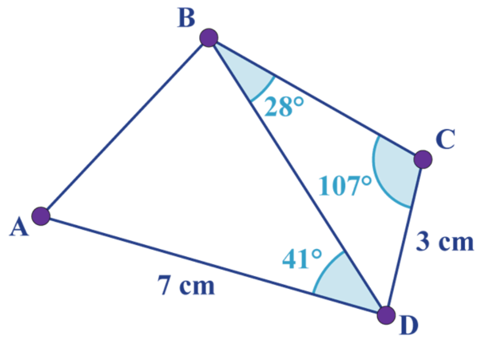

More information

The image shows quadrilateral ABCD. Point A is connected to B and D, forming line segments AB and AD, with the length of AD labeled as 7 cm. Point D connects to C, forming line segment DC, labeled as 3 cm. The angles in the figure are labeled with angle BDA as 41 degrees, angle DBC as 28 degrees, and angle BCD as 107 degrees. The shape seems to consist of two triangles, ADB and BDC, sharing the segment BD, which is the hypotenuse of both triangles.

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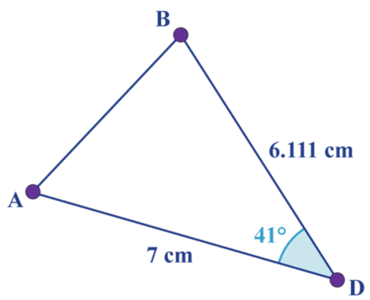
a) Find the length of DB

b) Find the length of AB.

Give your answers correct to 3 significant figures.

	Steps	Explanation
a)	$\frac{DB}{\sin 107} = \frac{3}{\sin 28}$	You have a combined shape. Instead of looking at the quadrilateral as a whole, focus on each triangle separately. Start with angle BCD as it has two angles and a side given. If you follow the flow chart, you will see you can use the sine rule here.

Student view

	Steps	Explanation
	$DB = \dfrac{3\sin 107}{\sin 28}$	Rearrange to make DB the subject.
	$DB \approx 6.110\ldots$ So the length of DB is 6.11 cm (to 3 significant figures).	
b)		Now you can focus on the second triangle, ABD. You have two sides and one angle given and you want to calculate the length of the third side, AB. Following the flow chart, you will see that you should use the cosine rule.
	$AB^2 = 7^2 + 6.11^2 - 2 \times 7 \times 6.11\cos 41$ $AB = \sqrt{7^2 + 6.110\ldots^2 - 2 \times 7 \times 6.110\ldots \times \cos 41}$ $AB \approx 4.6667$	
	So the length of AB is 4.67 cm (to 3 significant figures).	

Exam tip

Even though you have all the steps to solve this equation algebraically in the exam, remember to use your calculator after you write the first equation.

Example 4

★★★

Student view

Overview
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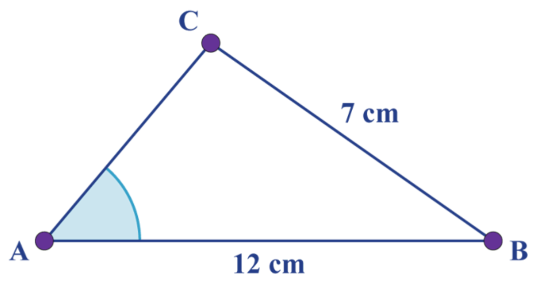
Triangle ABC has side lengths $BC = 7$ cm and $AB = 12$ cm. It is also given that $\tan A = \frac{5}{7}$.

There are two triangles like this. Find the area of both.



More information

The diagram shows a triangle labeled as ABC. Point A is on the left, point B is on the right, and point C is at the top, forming a triangle. The base of the triangle AB measures 12 cm. The line from point B to the top point C measures 7 cm. An angle at point A is highlighted with a blue arc. All three vertices (A, B, and C) are marked with small purple circles.

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Student
view

Overview
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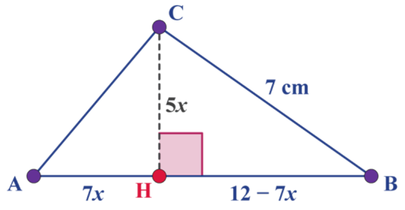
Steps	Explanation
	If you use the flow chart, there is no right-angled triangle, but you can draw a right-angled triangle to use the given tangent ratio. Draw the height CH.
$\tan A = \frac{CH}{AH} = \frac{5}{7}$ $CH = 5x$ and $AH = 7x$	Now, you can use the right-angled triangle ACH trigonometric ratio.
$HB = 12 - 7x$	Total length of $\text{AB} = 12\text{ cm}$.
$(5x)^2 + (12 - 7x)^2 = 7^2$	Using Pythagoras' theorem in triangle CHB.
$x = 1.066$ and $x = 1.204$ (to 3 significant figures)	Using your calculator. Use extra decimal places; do not round to 3 significant figures here to avoid rounding errors.
$\text{Area} = \frac{1}{2} \times 12 \times 5.33 = 31.98$ or $\text{Area} = \frac{1}{2} \times 12 \times 6.02 = 36.12$	$\text{Area of triangle ABC} = \frac{1}{2} \times 12 \times 5x$.
$\text{Area} = 32.0\text{ cm}^2$ or $\text{Area} = 36.1\text{ cm}^2$ (to 3 significant figures)	Round the final answer to 3 significant figures.

Student
view

Overview
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3 section questions

3. Geometry and trigonometry / 3.2 Triangle trigonometry

Checklist

Section Student... (0/0) Feedback Print (/study/app/m/sid-122-cid-754029/book/checklist-id-26213/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- calculate an unknown side using a trigonometric ratio (sine, cosine or tangent)
- calculate an unknown angle using inverse trigonometric functions ($\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$)
- use the cosine rule:

$$
\begin{aligned}
c^2 &= a^2 + b^2 - 2ab\cos C \\
b^2 &= a^2 + c^2 - 2ac\cos B \\
a^2 &= b^2 + c^2 - 2bc\cos A
\end{aligned}
$$

- use the sine rule:

$$
\begin{aligned}
\frac{\sin A}{a} &= \frac{\sin B}{b} = \frac{\sin C}{c} \\
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C}
\end{aligned}
$$

- use the formula for the area of a triangle:

$$\text{area} = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$$

- choose the correct rule from trigonometric ratios in a right-angled triangle, the sine rule and the cosine rule to find missing lengths and/or angles in any given triangle.

3. Geometry and trigonometry / 3.2 Triangle trigonometry

Investigation

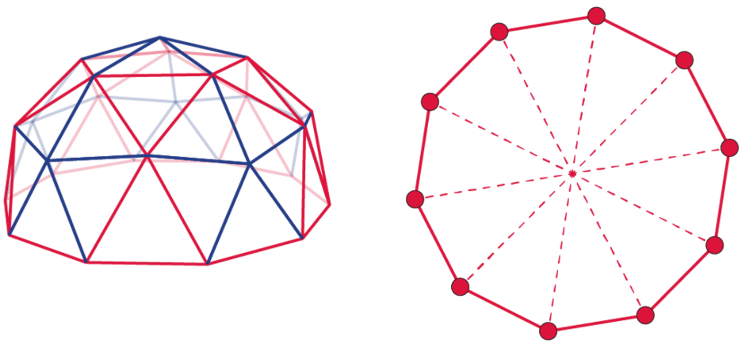
Student view

With the rapid increase in world population and decreasing farm land, urban farming is becoming a more and more popular way to create sustainable gardens in cities. Apart from providing fresh seasonal vegetables and flowers, it is also an escape from busy city life and helps people to get closer to nature.

In this investigation, you will work out the cost of creating a geodesic dome urban garden.

A geodesic dome urban garden

Credit: theasis Getty Images



More information

The image shows two geometric structures:

1. The shape on the left is a semi-spherical structure made up of a network of intersecting blue and red lines forming triangular and quadrilateral segments. Each segment contributes to creating a dome-like shape with a complex pattern.

2. The shape on the right is a circular formation with red outlines. It features evenly spaced red points connected by solid lines around the perimeter, forming a polygon. Inside the circle, red dashed lines emanate from a central point to each vertex, creating a star-like pattern.

These shapes demonstrate structural designs often found in architecture and mathematics, illustrating symmetry and geometric principles.

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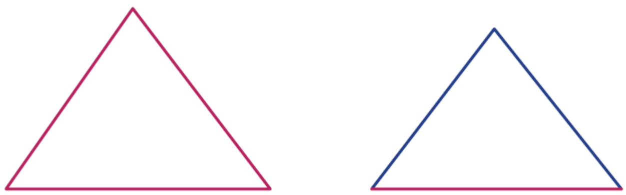
The diagram on the above left shows the dome, and the figure on the right shows the10-sided regular polygon base.

Here you can find the link to 3D model of the dome (http://acidome.com/lab/calc/#1/2_Cone_2V_R3_beams_50x40).

- If the base frame is made of a 10-sided regular polygon, calculate:
  - the length of the red and blue edges
  - the total length of material needed for the frame
  - the total length of the frame of the base
  - the total area needed for the base.
- There are two types of triangles formed on the surface of the dome, see the diagram below.



- Calculate the material needed to cover each triangle.
- Considering the total number of triangles of each type needed, calculate the total surface area needed to cover the dome.

- Decide what type of material you will use to:
  - create the frame
  - cover the surfaces.

- How much would it cost to create the greenhouse dome?
- Is there any other shape which would cover the same base area but have a lower cost? What if the base was octagonal?

Rate subtopic 3.2 Triangle trigonometry

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