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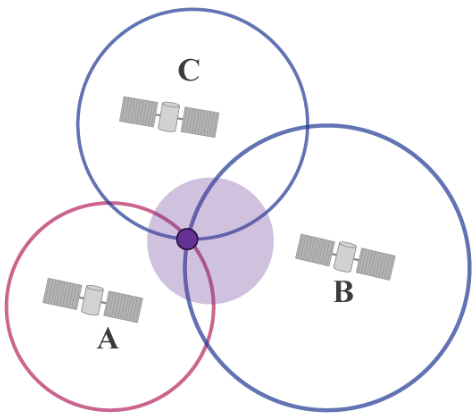
The big picture

What do you do when you are lost and want to know your location? You probably use the Global Position System (GPS) on your phone to locate your position and to guide you to your destination. But how does GPS work?

The GPS uses approximately 30 satellites orbiting the Earth. Whatever your location, at least four GPS satellites are 'visible' to your GPS device (e.g. phone) at any time. Each satellite sends out information at regular time intervals about its position. Your GPS device receives these signals and calculates how far away it is from each satellite.

Your GPS device requires information from at least three satellites to identify your location. It uses a process called trilateration. Watch the video in section 5.3.0 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-26147/) if you want to learn more about how GPS works.



More information

The diagram shows a trilateration process involving three satellites, labeled A, B, and C. Each satellite is represented by an icon with solar panels and is located at the center of a circle. The circles are color-coded and overlap at a central point, highlighted with a purple area indicating the location determined by the GPS. Satellite A, positioned at the left, intersects with satellite B on the bottom right and satellite C at the top right. The central overlapping region symbolizes how GPS triangulates a position using signals from at least three satellites. This diagram is a visual representation of the trilateration process used in GPS technology to pinpoint precise locations.

[Generated by AI]

GPS data can also be used plot vectors that represent the movement of the Earth's tectonic plates so as to predict the likelihood of earthquakes. You can r ead about this application of vectors in this article (https://spotlight.unavco.org/how-gps-works/gps-and-tectonics/gps-data.html).

In this subtopic, you will learn how to use vectors to form an alternative version of the equation of a straight line. You will see how this can be applied to kinematics problems involving motion in a straight line.

Concept

Different representations of lines help you to analyse different aspects of directional motion. Under what circumstances might a vector equation be more useful than a Cartesian equation? What extra information does it give?

Theory of Knowledge

Writing lines in various forms could be considered a type of 'model'. Models are important in knowledge production in all areas of knowledge because they serve to represent knowledge. A key knowledge issue with models, however, is that they are simplified representations of a thing — not the thing itself.

Student view

Knowledge Question: Do mathematical models hold a higher level of validity than models from other areas of knowledge?

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3. Geometry and trigonometry / 3.11 Vector equation of a line

Vector equation of a straight line

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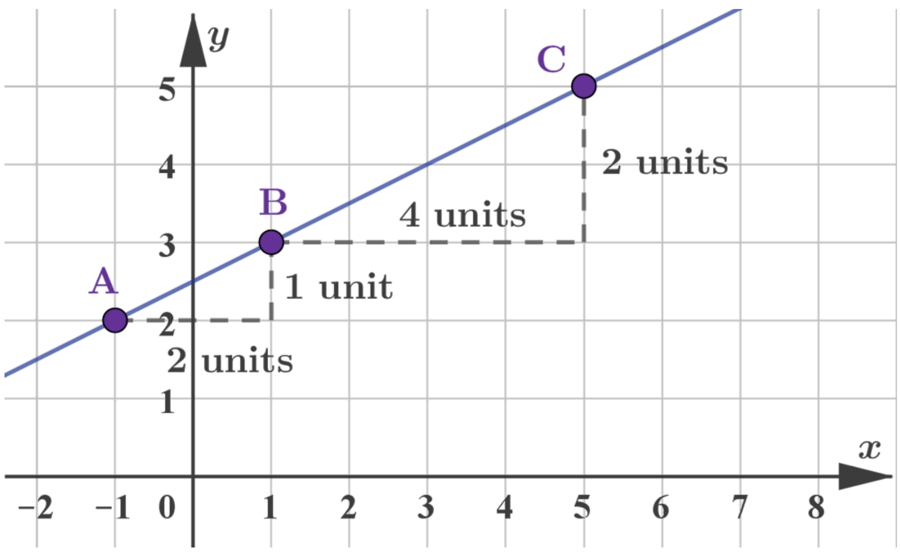
Consider the straight line shown below, which passes through the points A, B and C.

The gradient is 0.5 and the y - intercept is 2.5 so, in slope–intercept form, the equation of the line is $y = 0.5x + 2.5$. The coordinates of any point on the line satisfy this equation.



More information

The image is a graph that illustrates a straight line with a slope (gradient) of 0.5 and a y-intercept of 2.5, consistent with the equation (y = 0.5x + 2.5). The X-axis ranges from -2 to 8, while the Y-axis ranges from 1 to 5.

The line passes through three labeled points, A, B, and C: - Point A is located at (-1, 2), shown on the graph as a purple dot. - Point B is positioned at (2, 3), also denoted by a purple dot, with textual indications showing "2 units" vertically and "1 unit" horizontally from the origin. - Point C is at (6, 5), with a label showing "2 units"

Student view

vertically between points B and C, indicating a consistent vertical change. "4 units" horizontally indicates the distance from B to C.

The graph's main focus is on the relationship between the points, demonstrating how each position satisfies the linear equation, emphasizing slope and intercept through units of distance between points.

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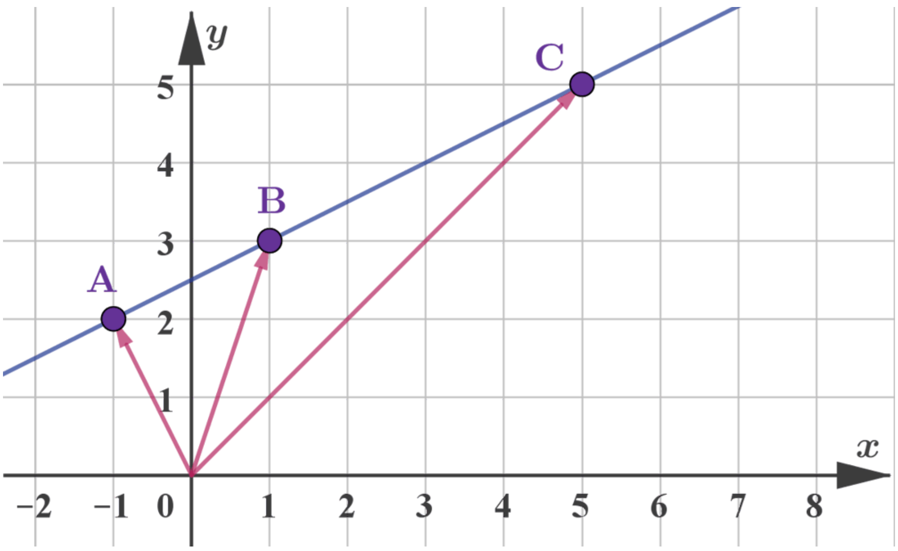

More information

The image is a graph displaying a line and three labeled points: A, B, and C. The X-axis is labeled from -2 to 8, and the Y-axis is labeled from 0 to 5.

Point A is located at (0, 1), point B is located at (1, 3), and point C is located at (3, 4). The blue line passes through these points, showing a linear relationship. Each point has a vector pointing away from the origin (0, 0) towards the point on the line.

The vectors represent direction and magnitude, illustrating how the position of each point along the line can be described using a vector representation. The text above notes that instead of using a gradient and y-intercept, a line can be represented using direction and position vectors for a known point.

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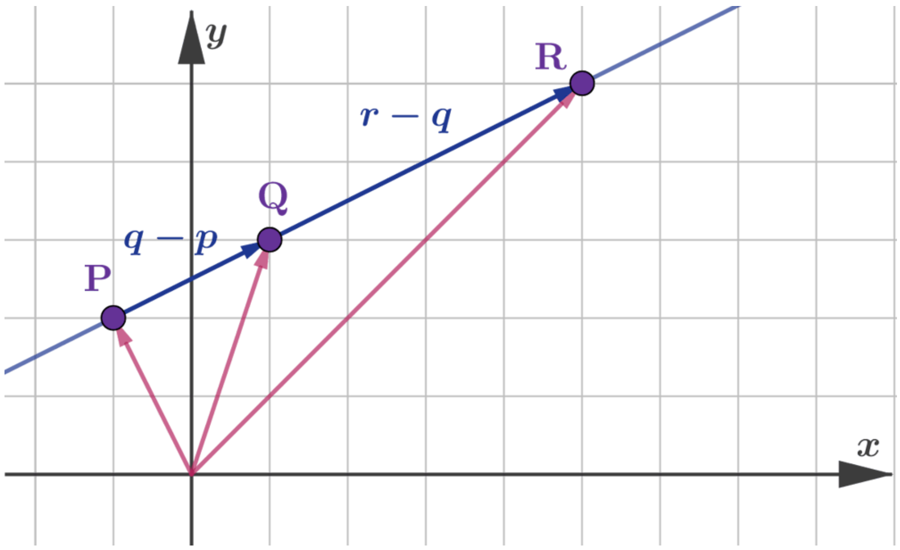
[Generated by AI]

The same straight line can be represented using vectors. Instead of the gradient, you use a direction vector and instead of the y-intercept you use the position vector of a known point on the line.

Consider the line below, where P, Q and R are collinear and $\overrightarrow{\mathrm{OP}}$, $\overrightarrow{\mathrm{OQ}}$ and $\overrightarrow{\mathrm{OR}}$ are the position vectors of these points, respectively.

As $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{QR}}$ are parallel vectors,

$$\overrightarrow{\mathrm{PQ}} = \lambda\overrightarrow{\mathrm{QR}}, \text{ where } \lambda \in \mathbb{R}$$



More information

The image represents a vector diagram on a 2D Cartesian plane with a grid. There are points labeled P, Q, and R along a line. Vectors are depicted between these points:

1. Vector from P to Q labeled 'q - p'.
2. Vector from Q to R labeled 'r - q'.

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The x-axis and y-axis are marked at the bottom right and top left corners respectively. The background grid aids in visualizing the position and direction of vectors relative to each other. The arrows indicate direction from point P to Q and from Q to R.

[Generated by AI]

$$\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{OQ}} - \overrightarrow{\mathrm{OP}} = \boldsymbol{q} - \boldsymbol{p} \text{ and } \overrightarrow{\mathrm{QR}} = \overrightarrow{\mathrm{OR}} - \overrightarrow{\mathrm{OQ}} = \boldsymbol{r} - \boldsymbol{q}.$$

$$\overrightarrow{\mathrm{QR}} = \lambda\overrightarrow{\mathrm{PQ}} \Rightarrow \boldsymbol{r} - \boldsymbol{q} = \lambda(\boldsymbol{q} - \boldsymbol{p})$$

Rearranging gives

$$\boldsymbol{r} = \boldsymbol{q} + \lambda(\boldsymbol{q} - \boldsymbol{p}).$$

Now let $\boldsymbol{q} - \boldsymbol{p} = \boldsymbol{b}$, where $\boldsymbol{b}$ is a vector in the same direction as $\overrightarrow{\mathrm{PQ}}$.

Therefore, the position vector of any point on the line should satisfy the equation

$$\boldsymbol{r} = \boldsymbol{q} + \lambda\boldsymbol{b}$$

This equation represents the vector equation of a straight line, where $\boldsymbol{q}$ is the vector representing the position vector of a point on the line and $\boldsymbol{b}$ is the direction vector.

Example 1

★☆☆

a) Write a vector equation of the line passing through points $\mathrm{A}(1, 1, 1)$ and $\mathrm{B}(-1, 2, 3)$.

b) C is the point $(2, -2, 2)$. Use your answer to part **a** to determine whether A, B and C are collinear.

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view

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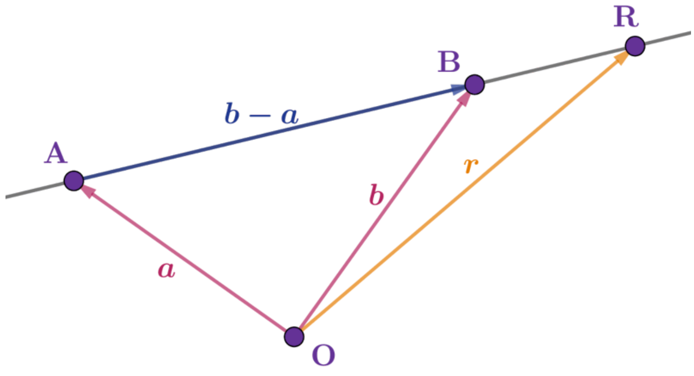
	Steps	Explanation
a)		Sketch the points and positi vectors.
	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}-2\\1\\2\end{pmatrix}$	$\overrightarrow{OA}=\begin{pmatrix}1\\1\\1\end{pmatrix}$ and $\overrightarrow{OB}=\Big($
	$\boldsymbol{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}$ or $\boldsymbol{r}=\begin{pmatrix}-1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}$	$\boldsymbol{r}=\overrightarrow{OA}+\lambda\overrightarrow{AB}$ or $\boldsymbol{r}=\overrightarrow{OB}+\lambda\overrightarrow{AB}$
b)	$\overrightarrow{OC}=\begin{pmatrix}2\\-2\\2\end{pmatrix}$	Position vector of point C.
	$\begin{pmatrix}2\\-2\\2\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}$	If point C is on the line, its p vector will satisfy the vector equation of the line
	$2=1-2\lambda\Rightarrow\lambda=-\frac{1}{2}$ $-2=1+\lambda\Rightarrow\lambda=-3$ $2=1+2\lambda\Rightarrow\lambda=\frac{1}{2}$	$\begin{pmatrix}2\\-2\\2\end{pmatrix}=\begin{pmatrix}1-2\lambda\\1+\lambda\\1+2\lambda\end{pmatrix}$

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	Steps	Explanation
	As λ is different for each component, point C is not on line AB, so A, B and C are not collinear.	

Important

If three points A, B and C with respective position vectors $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$ are collinear, then there will be a unique k which satisfies the equation

$$\boldsymbol{a} = k\left(\boldsymbol{b} - \boldsymbol{c}\right),\ k \in \mathbb{R}$$

Be aware

The vector equation of a line is not unique.

$$\boldsymbol{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\2\end{pmatrix},\ \boldsymbol{r} = \begin{pmatrix}-1\\2\\3\end{pmatrix} + \mu\begin{pmatrix}-2\\1\\2\end{pmatrix} \text{ or}$$

$$\boldsymbol{r} = \begin{pmatrix}-1\\2\\3\end{pmatrix} + t\begin{pmatrix}4\\-2\\-4\end{pmatrix}$$

all represent the same line using different points and equivalent direction vectors.

Activity

Use the following applet to investigate how using different parameters in the equation changes the line and the points on the line.

The vector equation of the line is given by $\boldsymbol{r} = \begin{pmatrix}a\\b\end{pmatrix} + t\begin{pmatrix}c\\d\end{pmatrix}$.

Use the sliders to change the values of a, b, c, d and t and see how this changes the position of point P on the line.

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Interactive 1. Investigating How Different Parameters Affect the Line and Its Points.

More information for interactive 1

This interactive allows users to explore how different parameters affect a line's position and orientation in 2D space through its vector equation. The visualization demonstrates the relationship between the algebraic representation $r = (a, b) + t(c, d)$ and its geometric manifestation, enabling users to manipulate the equation's components and immediately observe their impact on the line's behavior in the coordinate plane.

The display presents a 2D coordinate system with x and y axes, showing a line generated by the current vector equation. Users can adjust five interactive sliders controlling parameters a and b in blue (the position vector components ranging from -3 to 3), c and d orange (the direction vector components from -3 to 3), and t in orange (a scalar parameter from -2 to 2). A movable point P marks the specific location on the line corresponding to the current t-value, with all components updating in real-time as sliders are manipulated. The current vector equation appears visibly above the graph, changing dynamically with each adjustment.

By manipulating the sliders, users can observe various line transformations. For example: setting $(a, b) = (1, 2)$ and $(c, d) = (3, 1)$ with $t = 0$ places point P at $(1, 2)$; increasing t to 1 moves P to $(4, 3)$; while changing (c, d) to $(-2, 2)$ with $t = 1.5$ reorients the line's direction and moves P to $(-2, 5)$. The t-slider specifically shows how the parameter traces different points along the infinite line.

Through this exploration, users develop a concrete understanding of how vector equations represent lines in 2D space. They learn that (a, b) determines a fixed point on the line while (c, d) controls its direction, and ‘t’ acts as a scalar that generates all points on the infinite

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line. The activity clarifies why changing the direction vector's components alters the line's slope and how different parameter combinations can produce identical lines through distinct equations.

Exam tip

In IB examinations the vector equation of a line will be given as

$$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$$

where $\boldsymbol{a}$ is the position vector of any point on the line and $\boldsymbol{b}$ is the direction vector.

Be aware

Although the vector equation is given as $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, you will need to use the handwriting notation $\vec{r} = \vec{a} + \lambda\vec{b}$ or in the exam. Otherwise you might be penalised.

Example 2

★☆☆

Consider the two points $\mathrm{A}(1, 2, -1)$ and $\mathrm{B}(11, -2, -7)$.

a) Find the vector equation of the line containing these two points.

b) Show that the point $\mathrm{S}(-14, 8, 8)$ lies on the line while the point $\mathrm{T}\ (6, -7, 5)$ does not lie on the line.

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	Steps	Explanation
a)	$\boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	Write the position vector a of A.
	$\boldsymbol{b} = -\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 11 \\ -2 \\ -7 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ -6 \end{pmatrix}$	Find the direction vector $\boldsymbol{b} =$
	The vector equation is $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ -4 \\ -6 \end{pmatrix}$ $(\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ -3 \end{pmatrix}$	The λ parameter is an arbitra constant and can be any num you can replace $2 \times \lambda$ by a n parameter, which can be represented by μ. This has nc on the direction of the vector
b)	If point $\mathrm{S}(-14, 8, 8)$ lies on the line, $\begin{pmatrix} -14 \\ 8 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ -3 \end{pmatrix}$	If point $\mathrm{S}(-14, 8, 8)$ lies on th then there is a unique value o satisfies the equation.
	Using the x-coordinate, $\mu = \frac{-14 - 1}{5} = -3$ The third equation becomes $-1 + (-3)(-3) = 8$ So $\mathrm{S}(-14, 8, 8)$ lies on the line.	Substituting $\mu = -3$ into the two equations, The value of μ must be the sa all three coordinates if the po on the line.
	Consider point $\mathrm{T}(6, -7, 5)$. Using the x-coordinate, $\mu = \frac{6 - 1}{5} = 1$ but this does not satisfy the y-coordinate: $2 + 1 \times (-2) \neq -7$ so $\mathrm{T}(6, -7, 5)$ does not lie on the line.	

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Example 3

★☆☆

Write the equation of a line that passes through point $\mathrm{D}(1, -4)$ and is parallel to the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

You are given the position vector of a point on the line.

Let $\boldsymbol{e}$ be the direction vector.

$$\boldsymbol{d} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

The direction of the line is $\boldsymbol{e} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Hence, the line has a vector equation given by

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

or, equivalently,

$$\boldsymbol{r} = \boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{t}(2\boldsymbol{i} - 3\boldsymbol{j})$$

Example 4

★★☆

Find a vector equation of a line passing through the points $\mathrm{K}(1, -1, 1)$ and $\mathrm{L}(0, 0, 2)$. Hence find the possible coordinates of a point R on the line which satisfies $\mathrm{KR} = 3\mathrm{RL}$.

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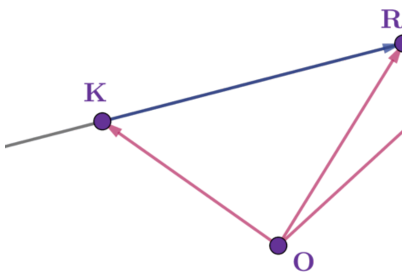
Steps	Explanation
$\overrightarrow{\mathrm{KL}}=\overrightarrow{\mathrm{OL}}-\overrightarrow{\mathrm{OK}}=\begin{pmatrix}0\\0\\2\end{pmatrix}-\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}-1\\1\\1\end{pmatrix}$	$\overrightarrow{\mathrm{OK}}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\overrightarrow{\mathrm{OL}}=($
$\boldsymbol{r}=\begin{pmatrix}1\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$	
Position of point R is $$\overrightarrow{\mathrm{OR}}=\begin{pmatrix}1-\lambda\\-1+\lambda\\1+\lambda\end{pmatrix}$$	
$\begin{pmatrix}1-\lambda\\-1+\lambda\\1+\lambda\end{pmatrix}-\begin{pmatrix}1\\-1\\1\end{pmatrix}=3\left[\begin{pmatrix}0\\0\\2\end{pmatrix}-\begin{pmatrix}1-\lambda\\-1+\lambda\\1+\lambda\end{pmatrix}\right]$	$\overrightarrow{\mathrm{KR}}=3\overrightarrow{\mathrm{RL}}\Rightarrow\overrightarrow{\mathrm{OR}}-\overrightarrow{\mathrm{OK}}=$
$\begin{pmatrix}-\lambda\\\lambda\\\lambda\end{pmatrix}=\begin{pmatrix}-3+3\lambda\\3-3\lambda\\3-3\lambda\end{pmatrix}$	Simplify both sides.
$-\lambda=-3+3\lambda\Rightarrow\lambda=\dfrac{3}{4}$	Solve for λ.

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Steps	Explanation
$\overrightarrow{\mathrm{OR}} = \begin{pmatrix} 1-\frac{3}{4} \\ -1+\frac{3}{4} \\ 1+\frac{3}{4} \end{pmatrix}$ $\overrightarrow{\mathrm{OR}} = \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \\ \frac{7}{4} \end{pmatrix}$	
Therefore, the coordinates of point R are $\left(\frac{1}{4}, -\frac{1}{4}, \frac{7}{4}\right)$	

Be aware

The coordinates of a point A are given in the form $\mathrm{A}(x, y, z)$, while the position vector of point A relative to a fixed point O is given in vector format as $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Make sure you give the answer in the correct form in the exam.

4 section questions

3. Geometry and trigonometry / 3.11 Vector equation of a line

Parametric and Cartesian forms of the equation of a straight line

Student view

The equation of a straight line can be written in several equivalent ways.

For a line with equation in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where $\boldsymbol{a} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$

you can write

$$\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} x_0 + \lambda l \\ y_0 + \lambda m \\ z_0 + \lambda n \end{pmatrix}$$

Therefore

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 + \lambda l \\ y_0 + \lambda m \\ z_0 + \lambda n \end{pmatrix}$$

where (x, y, z) are the coordinates of a general point on the line.

As the components on the left-hand side are equal to those on the right,

$$x = x_0 + \lambda l, y = y_0 + \lambda m \text{ and } z = z_0 + \lambda n$$

This is the parametric form of the equation of a straight line.

✓ **Important**

If a parametric equation of a line is

$$x = x_0 + \lambda l, \quad y = y_0 + \lambda m \text{ and } z = z_0 + \lambda n$$

then a vector equation of the line is

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$$\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

Example 1

★☆☆

Write the parametric equation of a line passing through $\text{A}(-1, 2, 0)$ and $\text{B}(3, 1, 1)$.

Vector form:

$$\begin{aligned} \boldsymbol{r} &= \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \lambda \left(- \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \right) = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} \end{aligned}$$

where A is a point on the line and vector $\overrightarrow{\text{AB}}$ is a direction vector.

Which gives the parametric form:

$$\begin{aligned} x &= -1 + 4\lambda \\ y &= 2 - \lambda \\ z &= \lambda \end{aligned}$$

Exam tip

The IB formula booklet gives the parametric form of the equation of a line as

$x = x_0 + \lambda l, y = y_0 + \lambda m$ and $z = z_0 + \lambda n$

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Cartesian form

In 2D, you write the Cartesian form of an equation as $y = mx + c$. How could you write the Cartesian form of the equation of a straight line in 3D?

As the Cartesian equation is the relationship between the x, y and z components, you can use the parametric form of the equation of a line to find it

$$x = x_0 + \lambda l, y = y_0 + \lambda m \text{ and } z = z_0 + \lambda n$$

Rearranging each expression and solving for λ gives

$$\lambda = \frac{x - x_0}{l}$$
$$\lambda = \frac{y - y_0}{m}$$
$$\lambda = \frac{z - z_0}{n}$$

As all the equations are equal to λ, you can write

$$\lambda = \frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

This shows the relationship between the $x-,\ y-$ and $z-$ coordinates.

Exam tip

The Cartesian equation of a line is not in the IB formula booklet and you will not be tested on it. However, it is a useful equation to work with.

Example 2

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The vector equation of a line is $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

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Write the equation in Cartesian form.

Steps	Explanation
$\frac{x-(-2)}{1}=\frac{y-1}{-1}=\frac{z-3}{-2}$	The direction vector is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$.
or $\frac{x+2}{1}=\frac{1-y}{1}=\frac{3-z}{2}$	The position vector of the point on the line is $\begin{pmatrix}-\\1\\3\end{pmatrix}$

Example 3

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The Cartesian equation of a line is $\frac{1+x}{3}=\frac{1-y}{2}=\frac{2+z}{1}$. Write its vector equation.

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Steps	Explanation
$\frac{1+x}{3}=\frac{1-y}{2}=\frac{2+z}{1}$ $\frac{x-(-1)}{3}=\frac{y-1}{-2}=\frac{z-(-2)}{1}$	Rearrange into the form $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$
$\boldsymbol{r}=\begin{pmatrix}-1\\1\\-2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}$	The direction vector is $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. The position vector of the point on the line $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$.

Example 4

★☆☆

If the point $\mathrm{A}(2, a, b)$ is on the line $\frac{1+x}{3}=\frac{1-y}{2}=\frac{2+z}{1}$, find $a+b$

As the point is on the line, it will satisfy the equation.

$$\frac{1+2}{3}=\frac{1-a}{2}=\frac{2+b}{1}$$

Solve for a

$$\frac{1-a}{2}=\frac{1+2}{3}=1$$

$a=-1$

Solve for b

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$$\frac{2+b}{1} = 1$$

$b = -1$

Therefore,

$$a + b = -2$$

4 section questions

3. Geometry and trigonometry / 3.11 Vector equation of a line

Checklist

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What you should know

By the end of this subtopic you should be able to:

- write the vector equation of a straight line as $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where $\boldsymbol{a}$ is the position vector of a point on the line, $\boldsymbol{b}$ is a vector describing the direction of the line and the parameter λ is a scalar
- write the position of a point on a straight line with coordinates (x, y, z) in terms of a vector equation: $\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$
- write the equation of a straight line in vector form, parametric form and Cartesian form: given a point (x_0, y_0, z_0) and a direction vector $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$, the equation of a straight line can be written:
    - in vector form as $\boldsymbol{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$
    - in parametric form as $x = x_0 + \lambda l, y = y_0 + \lambda m, z_0 + \lambda n$

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- in Cartesian form as $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$

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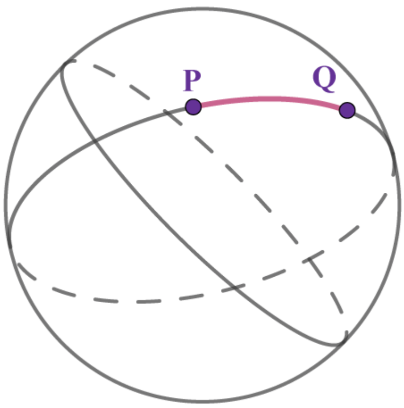
3. Geometry and trigonometry / 3.11 Vector equation of a line

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-28362/print/) Assign

The orthodromic distance is the shortest distance between two points on a sphere. It is also called the great circle distance. As the Earth is almost a sphere, the distances between points on Earth can be found using the principles of orthodromic distances. The orthodromic distance between two points marked on a sphere is shown in the figure below.



More information

The image shows a sphere with a great circle drawn on it. There is a dashed line around the circumference of the sphere indicating the equator and another perpendicular dashed line representing a meridian. Two points, labeled P and Q, are marked on the sphere along the great circle. The shortest path, which is the orthodromic distance, is represented by a bold line connecting P and Q. This path highlights the great circle distance between the points across the surface of the sphere.

[Generated by AI]

Student view

Read this article (https://undergroundmathematics.org/trigonometry-triangles-to-functions/lost-but-lovely-the-haversine) from *Underground Mathematics* about calculating orthodromic distance.

Try to prove the formulae given in the article.

Find the coordinates of two places you would like to visit and calculate the orthodromic distance between these two locations.