5. Calculus / 5.3 Derivatives of power functions

The big picture

In the previous subtopics, you learned about some of the properties of the gradient function. You also learned how to use technology to find the approximate value of the gradient of a curve at any point. In this subtopic, you will learn analytic methods to find exact values of gradients to certain type of curves.

Theory of Knowledge

If an answer is only an approximation of something, can that knowledge be said to be truth? Consider other areas of knowledge such as history. To what extent can historical knowledge be said to have precision? This leads to the knowledge question, 'Does each area of knowledge necessitate its own standards by which knowledge is evaluated, or, is a singular metric of assessing knowledge quality possible?'

There is of course no 'correct answer' to the question above. Here are two (of the many) comments that are relevant.

- Even if approximation is acceptable in practice, you can think of looking for exact values as a mental exercise.
- Knowing theoretical reasons can improve the accuracy of the approximation. The video below about the theory behind GPS explains a situation like this.

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Concept

In section 5.3.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-power-rule-id-25554/), while looking for the power rule, you will be asked to investigate gradients of certain functions at certain points and state a general statement based on the pattern you observe.

5. Calculus / 5.3 Derivatives of power functions

The power rule

Activity

In the applet below you can investigate the gradient of a curve of the form $y = x^n$ at different points for some positive integer value of n.

- Use the applet or your graphic display calculator to find the gradient values missing from the tables below the applet.
- Try to identify the pattern and suggest a general formula for the derivative of functions of the form $f(x) = x^n$.
- Try your formula on examples not in the table. Does it give the right answer? If not, investigate different values until you can suggest another formula.

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Interactive 1. Investigating the Gradient of a Curve.

More information for interactive 1

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In this interactive, users can explore how the gradient of curves changes for functions of the form $f(x) = x^2$, where **n** is a positive integer ranging from **2 to 10**. By selecting different values of **n** and examining the gradient at various points on the curve, users can observe how the steepness of the function varies. The red point represents a **movable marker**, and as it moves along the curve, the gradient at that specific position updates dynamically.

For example, when $f(x) = x^2$, the gradient at $x = 1$ is displayed as $f'(1) = 2$. Users can **zoom in and out** to better visualize the curve and its derivative. The goal is to identify a pattern in how the gradient depends on n and suggest a general formula for differentiating functions of the $f(x) = x^n$ form. After proposing a formula, users can test it on examples not included in the table to verify its accuracy.

For example, if the exponent is set to **4**, the function becomes $f(x) = x^4$, and its derivative is $f'(x) = 4x^3$. At $x = 1$ the gradient is $f'(1) = 4$. If the point is moved to $x = 1.2$ on the graph, the gradient updates to $f'(1.2) = 6.91$, demonstrating how the slope of the function changes with x.

By actively interacting with the visualization, users develop a deeper understanding of the **relationship between exponents and derivatives**, reinforcing key concepts in differentiation through exploration and pattern recognition.

expression	x-value	gradient
$f(x) = x^2$	$x = 1$	$f'(1) =$
$f(x) = x^2$	$x = 3$	$f'(3) =$
$f(x) = x^2$	$x = -2$	$f'(-2) =$

Student view

expression	x-value	gradient
$f(x) = x^2$	$x = 9$	$f'(9) =$

expression	x-value	gradient
$f(x) = x^3$	$x = 1$	$f'(1) =$
$f(x) = x^3$	$x = 3$	$f'(3) =$
$f(x) = x^3$	$x = -2$	$f'(-2) =$
$f(x) = x^3$	$x = 9$	$f'(9) =$

expression	x-value	gradient
$f(x) = x^4$	$x = 1$	$f'(1) =$
$f(x) = x^5$	$x = 1$	$f'(1) =$
$f(x) = x^6$	$x = 1$	$f'(1) =$
$f(x) = x^7$	$x = 1$	$f'(1) =$

expression	x-value	gradient
$f(x) = x^2$	$x = 1$	$f'(1) = 2$
$f(x) = x^2$	$x = 3$	$f'(3) = 6$
$f(x) = x^2$	$x = -2$	$f'(-2) = -4$
$f(x) = x^2$	$x = 9$	$f'(9) = 18$
$f(x) = x^3$	$x = 1$	$f'(1) = 3$
$f(x) = x^3$	$x = 3$	$f'(3) = 27$

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expression	x-value	gradient
$f(x) = x^3$	$x = -2$	$f'(-2) = 12$
$f(x) = x^3$	$x = 9$	$f'(9) = 243$
$f(x) = x^4$	$x = 1$	$f'(1) = 4$
$f(x) = x^5$	$x = 1$	$f'(1) = 5$
$f(x) = x^6$	$x = 1$	$f'(1) = 6$
$f(x) = x^7$	$x = 1$	$f'(1) = 7$

Activity

Finding the values that are missing from the table and investigating other possibilities may have led you to the following claim.

Important

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

The claim above is based on investigating examples for positive integer exponents (indices). In the higher level extension of the analysis and approaches course, you will see how to prove it analytically. In the other courses, you will use this claim without proof.

Example 1

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Find the derivative of the functions defined by

a) $f(x) = x^2$

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b) $g(x) = x^5$

c) $h(x) = x^{2021}$.

Step	Explanation
a) $f'(x) = 2x^{2-1} = 2x^1 = 2x$ b) $g'(x) = 5x^{5-1} = 5x^4$ c) $h'(x) = 2021x^{2021-1} = 2021x^{2020}$	Use the formula for $n = 2$, $n = 5$ and $n = 2021$.

Activity

It is a natural question to ask whether the formula above is true for exponents other than positive integers.

- Use your calculator to check the formula for other exponents. Find the gradient of different power functions at different points. Check whether the application of the formula gives the same gradient that you get from your calculator.
- Use WolframAlpha (https://www.wolframalpha.com/) to find the derivative of different power functions. Type, for example, "derivative of x^(-3)" in the search line and interpret the answer WolframAlpha gives.

Exam tip

The formula above about the derivative of x^n is in the formula booklet in the form

$$f(x) = x^n \quad \Longrightarrow \quad f'(x) = nx^{n-1}.$$

It is not given explicitly in the formula booklet, but you should remember that this is true for any (not only positive integer) exponents.

Example 2

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Find the derivative of the functions defined by

a) $f(x) = x$

b) $g(x) = x^0$

c) $h(x) = x^{-2}$.

Step	Explanation
a) $f'(x) = 1x^{1-1} = x^0 = 1$	Since $x = x^1$, you can use the formula for $n = 1$.
b) $g'(x) = 0x^{0-1} = \frac{0}{x} = 0$	The formula is true for $n = 0$.
c) $h'(x) = -2x^{-2-1} = -2x^{-3}$	The formula is true for negative exponents, so you can use it for $n = -2$.

Example 3

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Find the derivative of the functions defined by

a) $f(x) = 1$

b) $g(x) = \frac{1}{x^3}$

c) $h(x) = \frac{x^5}{x^3}$

d) $l(x) = \frac{x^2}{x^6}$.

Student view

	Step	Explanation
a)	$f(x) = 1 = x^0$ so $f'(x) = 0x^{0-1} = 0$	You can also get the same result by noticing that the graph of f is a horizontal line, so the gradient is 0 at every point.
b)	$g(x) = \frac{1}{x^3} = x^{-3}$ so $g'(x) = -3x^{-3-1} = -3x^{-4} = -\frac{3}{x^4}$	Since x^3 is in the denominator, you apply the formula for $n = -3$.
c)	$h(x) = \frac{x^5}{x^3} = x^{5-3} = x^2$ so $h'(x) = 2x$	Note that you need to simplify the expression first. It is not correct to find the derivatives of the numerator and denominator separately.
d)	$l(x) = \frac{x^2}{x^6} = x^{2-6} = x^{-4}$, so $l'(x) = -4x^{-4-1} = -4x^{-5} = -\frac{4}{x^5}$	

You can check your understanding on the following applet.

Interactive 2. Power Rule Practice.

More information for interactive 2

In this interactive, users are presented with functions in the form

$f(x) = \frac{x^n}{x^m}$, where n and m are integers. To find the derivative, users must first simplify the function using exponent rules. After simplifying, they can apply the power rule for differentiation, which states that if $f(x) = x^k$, then

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$f'(x) = kx^{(k-1)}$.

For example, consider the function $f(x) = \frac{x^4}{x^2}$. Simplifying this function gives. $f(x) = x^{4-2} = x^2$ Applying the power rule to the simplified function results in $f'(x) = 2x^{2-1} = 2x$

Additionally, a **Click here for a new question** tab generates a new function with a similar structure, allowing further practice with different exponents. A **Show solution** checkbox is also available, enabling users to verify their calculations and reinforce their understanding.

By practicing with various functions, users can improve their skills in simplifying expressions and applying the power rule for differentiation.

3 section questions

Question 1

Difficulty:

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Find the derivative of $f(x) = x^5$.

1 $f'(x) = 5x^4$ ✓

2 $f'(x) = 5x^5$

3 $f'(x) = 4x^4$

4 $f'(x) = 4x^5$

Explanation

$f'(x) = 5x^{5-1} = 5x^4$

You can use the formula $f'(x) = nx^{n-1}$ for $n = 5$.

Question 2

Difficulty:

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Find the derivative of $f(x) = x^{-5}$.

1 $f'(x) = -\frac{5}{x^6}$ ✓

2 $f'(x) = -\frac{5}{x^4}$

3 $f'(x) = -\frac{6}{x^5}$

4 $f'(x) = -\frac{4}{x^5}$

Explanation

Steps	Explanation
$f'(x) = -5x^{-5-1} = -5x^{-6} = -\frac{5}{x^6}$	You can use the formula $f'(x) = nx^{n-1}$ for $n = -5$

Question 3

Difficulty:

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Find the derivative of $f(x) = \frac{x^2}{x^3}$.

1 $f'(x) = -\frac{1}{x^2}$ ✓

2 $f'(x) = \frac{1}{x^2}$

3 $f'(x) = -1$

4 $f'(x) = 1$

Explanation

Steps	Explanation
$f(x) = \frac{x^2}{x^3} = \frac{1}{x} = x^{-1}$, so $f'(x) = -1x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$.	After simplifying the expression to the form x^n, you can use the formula $f'(x) = nx^{n-1}$.

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5. Calculus / 5.3 Derivatives of power functions

The constant factor rule

Our next goal is to find the derivative of functions of the form $f(x) = ax^n$.

Activity

- Use your graphing calculator to find the gradient values missing from the table below.
- Try to identify the pattern and suggest a general formula for the derivative of $f(x) = ax^2$, and, in general, for the derivatives of functions of the form $f(x) = ax^n$.
- Try your formula on examples not in the table. Does it give the right answer? If not, investigate different values until you can suggest another formula.

Expression	x-value	Gradient
$f(x) = x^2$	$x = 1$	
$f(x) = 2x^2$	$x = 1$	
$f(x) = 3x^2$	$x = 1$	
$f(x) = x^2$	$x = 3$	
$f(x) = 2x^2$	$x = 3$	
$f(x) = 3x^2$	$x = 3$	

Expression	x-value	Gradient
$f(x) = x^2$	$x = 1$	$f'(1) = 2$

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Expression	x-value	Gradient
$f(x) = 2x^2$	$x = 1$	$f'(1) = 4$
$f(x) = 3x^2$	$x = 1$	$f'(1) = 6$
$f(x) = x^2$	$x = 3$	$f'(3) = 6$
$f(x) = 2x^2$	$x = 3$	$f'(3) = 12$
$f(x) = 3x^2$	$x = 3$	$f'(3) = 18$

Activity

Finding the values missing from the table and investigating other possibilities may have led you to the following claim.

Important

If $f(x) = ax^n$, then $f'(x) = nax^{n-1}$.

Another notation for the derivative $f'(x)$ is $\frac{\mathrm{d}y}{\mathrm{d}x}$, so this rule can also be expressed as:

If $y = ax^n$, then $\frac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$.

The claim above is based on investigating examples. In the higher level extension of the analysis and approaches course, you will see how to prove it analytically. In the other courses, you will use this claim without proof.

Example 1

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Find the derivatives.

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Step	Explanation
$y = 2x^4$	$\frac{dy}{dx} =$
$y = 5x^{-3}$	$\frac{dy}{dx} =$
$y = 7x$	$\frac{dy}{dx} =$
$y = 9$	$\frac{dy}{dx} =$
$y = \frac{3}{x}$	$\frac{dy}{dx} =$
$y = \frac{1}{2x^2}$	$\frac{dy}{dx} =$
$y = \frac{-5}{3x^7}$	$\frac{dy}{dx} =$
$y = \frac{-4x}{x^4}$	$\frac{dy}{dx} =$
$y = \frac{2x^3}{3x^2}$	$\frac{dy}{dx} =$
$y = \frac{6x^3}{2x^3}$	$\frac{dy}{dx} =$

First, simplify the expressions (if necessary), then use the formula to find the derivatives.

Step	Explanation
$y = 2x^4$	$\frac{dy}{dx} = 4 \times 2x^3 = 8x^3$
$y = 5x^{-3}$	$\frac{dy}{dx} = -3 \times 5x^{-4} = -15x^{-4}$
$y = 7x = 7x^1$	$\frac{dy}{dx} = 1 \times 7x^0 = 7$

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Step	Explanation
$y = 9 = 9x^0$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \times 9x^{-1} = 0$
$y = \frac{3}{x} = 3x^{-1}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -1 \times 3x^{-2} = -\frac{3}{x^2}$
$y = \frac{1}{2x^2} = \frac{1}{2}x^{-2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2 \times \frac{1}{2}x^{-3} = -\frac{1}{x^3}$
$y = \frac{-5}{3x^7} = -\frac{5}{3}x^{-7}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -7 \times \left(-\frac{5}{3}\right) x^{-8} = \frac{35}{3x^8}$
$y = \frac{-4x}{x^4} = -4x^{-3}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3 \times (-4)x^{-4} = \frac{12}{x^4}$
$y = \frac{2x^3}{3x^2} = \frac{2}{3}x$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{3}$
$y = \frac{6x^3}{2x^3} = 3$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

You can check your understanding with the following applet.

Interactive 1. The Constant Factor Rule Practice.

More information for interactive 1

This interactive tool presents functions in the form $f(x) = ax^n$, where a is a coefficient, and n is an integer. Users must first simplify the function using exponent rules and algebraic manipulation before applying differentiation. The interactive reinforces the power rule, which states that for $f(x) = x^k$, the derivative is $f'(x) = kx^{k-1}$.

Student view

For example, consider the function $f(x) = \frac{2x^4}{3x^2}$. Simplifying this function gives $f(x) = \frac{2}{3}x^{4-2} = \frac{2}{3}x^2$. Applying the power rule to the simplified function results in $f'(x) = \frac{2}{3} \cdot 2 \cdot x^{2-1} = \frac{4}{3}x$.

Users can generate a new function by clicking the "Click here for a new question" button, allowing further practice with different exponents and coefficients. Additionally, a "Show solution" checkbox enables users to verify their calculations and reinforce their understanding.

By working through multiple examples, users can develop confidence in simplifying algebraic expressions and applying differentiation techniques effectively.

3 section questions

Question 1

Difficulty:

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The derivative of $f(x) = 6x^3$ is $f'(x) = bx^k$.

Find the value of b. Give an exact answer as an integer.

18

Accepted answers

18, b=18

Explanation

$f'(x) = 3 \times 6x^2 = 18x^2$

Use the formula $f'(x) = nax^{n-1}$ with $a = 6$ and $n = 3$.

Question 2

Difficulty:

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The derivative of $f(x) = \frac{8}{2x^5}$ is $f'(x) = \frac{b}{x^k}$.

Find the value of b. Give an exact answer as an integer.

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Accepted answers

-20, b=-20

Explanation

$f(x) = \frac{8}{2}x^{-5} = 4x^{-5}$

Rewrite the expression in the form ax^n.

$f'(x) = -5 \times 4x^{-6} = \frac{-20}{x^6}$

Use the formula $f'(x) = nax^{n-1}$ with $a = 4$ and $n = -5$.

Question 3

Difficulty: ★★☆

The derivative of $f(x) = \frac{9x^3}{-6x^4}$ is $f'(x) = bx^k$.

Find the value of b. Give an exact answer either as a decimal or as a fraction (in the form n/m) in fully simplified form.

1.5 ✓

Accepted answers

1.5, 1,5, 3/2, b=1.5, b=1,5, b=3/2

Explanation

Steps	Explanation
$f(x) = \frac{9}{-6}x^{-1} = -\frac{3}{2}x^{-1}$	Rewrite the expression in the form ax^n.
$f'(x) = -1 \times \left(-\frac{3}{2}\right)x^{-2} = \frac{3}{2}x^{-2}$	Use the formula $f'(x) = nax^{n-1}$ with $a = -\frac{3}{2}$ and $n = -1$.

Student view

5. Calculus / 5.3 Derivatives of power functions

The sum rule

So far, you have learned how to find the derivative of $f(x) = ax^n$. It is natural to look for the derivative of the sum, difference, product and quotient of these type of expressions.

In this section, you will see how to deal with sums and differences.

Start with a general investigation.

Activity

Choose two functions and a number in the domain. Use your calculator to fill in the table.

$f(x)$	$g(x)$	a	$h(x) = f(x) + g(x)$	$f'(a)$	$g'(a)$	h'

Do this with different functions (not necessarily power functions) and different values of a.

- What do you notice? Of course, to notice a pattern you need to investigate several examples.
- Check your observation on more examples. Do these additional examples confirm your observation? If not, modify your conjecture.

Based on the observations, you may have found the following relationship.

Important

If $h(x) = f(x) + g(x)$, then $h'(x) = f'(x) + g'(x)$.

In fact, the following, more general claim, is also true. In this course, these claims are not proved, but you will need to be able to use these rules.

Example 1

★☆☆

Suppose that $h(x) = 2f(x) - 3g(x)$, $f'(x) = \ln x$ and $g'(x) = \dfrac{1}{x}$.

Find $h'(x)$.

Step	Explanation
$h'(x) = 2f'(x) - 3g'(x)$ $= 2\ln x - 3\dfrac{1}{x}$ $= 2\ln x - \dfrac{3}{x}$	You can use the formula $h'(x) = af'(x) - bg'(x)$ with $a = 2$ and $b = 3$.

Example 2

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Suppose that $h(x) = 3f(x) + g(x)$, $f'(-2) = 4$ and $g'(-2) = -5$.

Find $h'(-2)$.

Step	Explanation
$h'(x) = 3f'(x) + g'(x)$	You can use the formula $h'(x) = af'(x) + bg'(x)$ with $a = 3$ and $b = 1$.
$h'(-2) = 3f'(-2) + g'(-2)$ $= 3 \times 4 + (-5) = 7$	Substitute $x = -2$.

Example 3

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For some functions, f and g, the graph of f has gradient -2 at the point $(1, 3)$ and the graph of g has gradient 4 at the point $(1, -5)$.

Let $h(x) = 6g(x) - 8f(x)$.

a) Show that the graph of h passes through the point $(1, -54)$.

b) Find the gradient of the graph of h at the point $(1, -54)$.

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	Step	Explanation
a)	$f(1) = 3$	Since the point $(1, 3)$ is on the graph of f.
	$g(1) = -5$	Since the point $(1, -5)$ is on the graph of g.
	$h(1) = 6g(1) - 8f(1)$ $= 6 \times (-5) - 8 \times 3 = -54$	Substitute $x = 1$ in the definition of h.
	Hence, the point $(1, -54)$ is on the graph of h.	Interpret the result of the previous line.
b)	$h'(x) = 6g'(x) - 8f'(x)$	Use the formula for the difference of the derivative.
	$f'(1) = -2$	Since the graph of f has gradient -2 at the point $(1, 3)$.
	$g'(1) = 4$	Since the graph of g has gradient 4 at the point $(1, -5)$.
	$h'(1) = 6g'(1) - 8f'(1)$ $= 6 \times 4 - 8 \times (-2) = 40$	Substitute $x = 1$ in $h'(x)$.
	Hence, the gradient of the graph of h at the point $(1, -54)$ is 40.	Interpret the result of the previous line to answer the second question.

3 section questions

Question 1

Difficulty: ★☆☆

Suppose that $h(x) = 4f(x) + 3g(x)$, $f'(1) = 2$ and $g'(1) = -1$.

Find $h'(1)$.

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Accepted answers

5, h'(1)=5

Explanation

Steps	Explanation
$h'(x) = 4f'(x) + 3g'(x)$	You can use the formula $h'(x) = af'(x) + bg'(x)$ with $a = 4$ and $b = 3$.
$h'(1) = 4f'(1) + 3g'(1)$ $= 4 \times 2 + 3 \times (-1) = 5$	Substitute $x = 1$.

Question 2

Difficulty: ★☆☆

Suppose that $h(x) = f(x) - 2g(x)$, $f'(-1) = 3$ and $g'(-1) = -2$.

Find $h'(-1)$.

7

Accepted answers

7, h'(-1)=7

Explanation

Steps	Explanation
$h'(x) = f'(x) - 2g'(x)$	You can use the formula $h'(x) = af'(x) - bg'(x)$ with $a = 1$ and $b = 2$.
$h'(-1) = f'(-1) - 2g'(-1)$ $= 3 - 2 \times (-2) = 7$	Substitute $x = -1$.

Question 3

Difficulty: ★★☆

Student view

For some functions, f and g, the graph of f has gradient 2 at the point $(5, 4)$ and the graph of g has gradient -1 at the point $(5, -3)$.

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Let $h(x) = 2g(x) - 3f(x)$.

Find the gradient of the graph of h at the point, where $x = 5$.

-8

Accepted answers

-8

Explanation

Steps	Explanation
$h'(x) = 2g'(x) - 3f'(x)$	You can use the formula for the difference of the derivatives.
$f'(5) = 2$	Since the graph of f has gradient 2 at the point $(5, 4)$.
$g'(5) = -1$	Since the graph of g has gradient -1 at the point $(5, -3)$.
$h'(5) = 2g'(5) - 3f'(5)$ $= 2 \times (-1) - 3 \times 2 = -8$	Substitute $x = 5$ in $h'(x)$.
Hence, the gradient of the graph of h at the point where $x = 5$ is -8.	Interpret the result of the previous line to answer the question.

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Combination of the rules

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In the previous sections, you learned several rules. Here is a summary.

Exam tip

The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$.

This rule is in the formula booklet.

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Exam tip

- The derivative of $f(x) = ax^n$ is $f'(x) = nax^{n-1}$.
  An important special case (for $n = 0$): if $f(x) = a$, then $f'(x) = 0$.
  Another important special case (for $n = 1$): if $f(x) = ax$, then $f'(x) = a$.
- More generally, if $h(x) = af(x)$, then $h'(x) = af'(x)$.
  This is sometimes called the constant factor rule.
- If $h(x) = f(x) + g(x)$, then $h'(x) = f'(x) + g'(x)$.
  This is sometimes called the sum rule.
- The sum and constant factor rules together give:
  If $h(x) = af(x) \pm bg(x)$, then $h'(x) = af'(x) \pm bg'(x)$.

These are not in the formula booklet. You need to remember these rules and, more importantly, make sure that you know how to use them.

In this section, you will practise using these rules to find derivatives of combinations of power functions.

Example 1

★★☆

Find the derivatives.

	Step	Explanation
a)	$y = 5x - 4$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
b)	$y = 3x^2 + 5x^4$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
c)	$y = 3x + x^{-1}$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
d)	$y = 2 - \frac{1}{x}$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
e)	$y = x^2 + \frac{1}{x^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$

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	Step	Explanation
f)	$y = 3x^2 + 4x + 5$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
g)	$y = (2x + 1)(x + 3)$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
h)	$y = (1 - 2x)(3 + 4x^2)$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
i)	$y = \frac{x^2 + 1}{x}$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$
j)	$y = \frac{6x - 4x^3}{2x}$	$\frac{\mathrm{d}y}{\mathrm{d}x} =$

First, simplify the expression, if necessary, to write it as a sum or difference of expressions of the form ax^n. Then use the appropriate rules to find the derivative.

	Step	Explanation
a)	$y = 5x - 4$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - 0 = 5$
b)	$y = 3x^2 + 5x^4$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + 20x^3$
c)	$y = 3x + x^{-1}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - x^{-2}$
d)	$y = 2 - \frac{1}{x} = 2 - x^{-1}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 + x^{-2} = \frac{1}{x^2}$
e)	$y = x^2 + \frac{1}{x^2} = x^2 + x^{-2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 2x^{-3} = 2x - \frac{2}{x^3}$
f)	$y = 3x^2 + 4x + 5$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + 4 + 0 = 6x + 4$
g)	$y = (2x + 1)(x + 3) = 2x^2 + 7x + 3$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 7$
h)	$y = (1 - 2x)(3 + 4x^2) = 3 - 6x + 4x^2 - 8x^3$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -6 + 8x - 24x^2$

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	Step	Explanation
i)	$y = \frac{x^2+1}{x} = x + \frac{1}{x} = x + x^{-1}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2 -}{x^2}$
j)	$y = \frac{6x - 4x^3}{2x} = 3 - 2x^2$	$\frac{\mathrm{d}y}{\mathrm{d}x} = -4x$

Example 2

★★☆

Find the gradient of the graph at the given point on the graph.

	Expression	x-coordinate of the point	Gradient
a)	$f(x) = 3x^2 - 4x + 5$	$x = 2$	
b)	$f(x) = (2x - 1)(x + 3)$	$x = -5$	
c)	$f(x) = 2x + \frac{3}{x}$	$x = 1$	
d)	$f(x) = \frac{3x^3 - 5}{4x^2}$	$x = -1$	

First, simplify the expression, if necessary, to write it as a sum or difference of expressions of the form ax^n.

Then use the appropriate rules to find the derivative.

Finally, substitute the given value of x in the derivative to find the gradient.

	Simplified expression	Derivative	Gradient
a)	$f(x) = 3x^2 - 4x + 5$	$f'(x) = 6x - 4$	$f'(2) = 6 \times 2 -$

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	Simplified expression	Derivative	Gradient
b)	$f(x) = (2x-1)(x+3)$ $= 2x^2 + 5x - 3$	$f'(x) = 4x + 5$	$f'(-5) = 4 \times (-$
c)	$f(x) = 2x + \frac{3}{x} = 2x + 3x^{-1}$	$f'(x) = 2 - 3x^{-2} = 2 - \frac{3}{x^2}$	$f'(1) = 2 - \frac{3}{1^2} =$
d)	$f(x) = \frac{3x^3 - 5}{4x^2}$ $= \frac{3}{4}x - \frac{5}{4x^2}$ $= \frac{3}{4}x - \frac{5}{4}x^{-2}$	$f'(x) = \frac{3}{4} - (-2) \times \frac{5}{4}x^{-3}$ $= \frac{3}{4} + \frac{5}{2x^3}$	$f'(-1) = \frac{3}{4} + \frac{}{2}$ $= -\frac{7}{4}$

Example 3

★★☆

The graph of $y = 4x + \frac{1}{x}$ has a horizontal tangent at the point $\mathrm{P}(a, b)$, where $a > 0$.

Find the coordinates of P.

Step	Explanation
$y = 4x + \frac{1}{x} = 4x + x^{-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - x^{-2} = 4 - \frac{1}{x^2}$	The gradient of the tangent is the value of the derivative, so finding the derivative is a good first step.
$0 = 4 - \frac{1}{x^2}$ $\frac{1}{x^2} = 4$ $x^2 = \frac{1}{4}$ $x = \pm\frac{1}{2}$	The gradient of a horizontal line is 0, so look for the value of x, where the derivative is 0.

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Step	Explanation
$a = \frac{1}{2}$	Since $a > 0$, you need the positive solution.
$b = 4 \times \frac{1}{2} + \frac{1}{1/2} = 4$	To find the second coordinate of the point, substitute the value of a into the original expression.
Hence, the graph of $y = 4x + \frac{1}{x}$ has a horizontal tangent at the point $\mathrm{P}\left(\frac{1}{2}, 4\right)$.	

3 section questions

Question 1

Difficulty:

★☆☆

Which of the following is the derivative of $y = 3x^2 - \frac{5}{x}$?

1 $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + \frac{5}{x^2}$ ✓

2 $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x - \frac{5}{x^2}$

3 $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + 5$

4 $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x - 5$

Explanation

$y = 3x^2 - \frac{5}{x} = 3x^2 - 5x^{-1}$

Rewrite the expression.

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 \times 3x - (-1) \times 5x^{-2}$

Use the rules of differentiation.

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$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x + \frac{5}{x^2}$$

Simplify the result.

Question 2

Difficulty:

★★☆

Find the gradient of the graph of $y = (2 - 3x)(1 - x)$ at the point, where $x = 7$.

37

Accepted answers

37

Explanation

Steps	Explanation
$y = 2 - 5x + 3x^2$	Expand the expression.
$\frac{\mathrm{d}y}{\mathrm{d}x} = -5 + 6x$	Find the derivative.
$-5 + 6 \times 7 = 37$	Substitute $x = 7$ in the derivative. The result is the gradient asked for in the question.

Question 3

Difficulty:

★★☆

The graph of $y = x^2 - \frac{16}{x}$ has a horizontal tangent at the point $\mathrm{P}(a, b)$.

Find the coordinates of P.

Give your answer in the form (m,n), where you replace m and n with the coordinates you found.

(-2,12)

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Accepted answers

(-2,12), a=-2,b=12, -2,12

Explanation

Steps	Explanation
$y = x^2 - \frac{16}{x} = x^2 - 16x^{-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 16x^{-2} = 2x + \frac{16}{x^2}$	The gradient of the tangent is the value of the derivative, so finding the derivative is a good first step.
$2x + \frac{16}{x^2} = 0$ $2x^3 + 16 = 0$ $x^3 = -8$ $x = -2$	The gradient of a horizontal line is 0, so find the value of x, where the derivative is 0.
$a = -2$	The first coordinate of P is the solution found above.
$b = (-2)^2 - \frac{16}{-2} = 12$	To find the second coordinate of the point, substitute the value of a into the original expression.
Hence, the graph of $y = x^2 - \frac{16}{x}$ has a horizontal tangent at the point $\mathrm{P}(-2, 12)$.	

5. Calculus / 5.3 Derivatives of power functions

Checklist

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What you should know

By the end of this subtopic you should be able to:

- find the derivative of $y = x^n$, where $n \in \mathbb{Z}$
- use the constant factor rule to find the derivative of $y = ax^n$, where $n \in \mathbb{Z}$

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- use the sum and constant factor rules to find the derivative of combinations (sums and differences) of expressions of the form ax^n, where $n \in \mathbb{Z}$.

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5. Calculus / 5.3 Derivatives of power functions

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-25559/print/) Assign

In this subtopic, you learned the rule about finding the derivative of the sum and difference of functions. In this investigation, you are asked to look for a rule for the derivative of the product of functions.

Choose two functions, f and g, and a number a in the domain.

Use your calculator to fill in the details in the table.

$h(x) = f(x)g(x)$	a	$f(a)$	$g(a)$	$f'(a)$	$g'(a)$	$h'(a)$

- Do this for a variety of functions (not necessarily power functions) and different values of a.
- What do you notice? Of course, to notice a pattern you need to investigate several examples.
- Check your observations with more examples. Do these additional examples confirm your observations? If not, modify your conjecture.