5.7 Teacher view

Further graph properties

Index

The big picture

In the previous sections you learned about the derivative of a function. This subtopic moves a step forward; here, you will work with the derivative of a derivative. Remember that the derivative describes a rate of change. It is sometimes useful to know how the change is changing, which is why you may need the second derivative. A typical application is the description of the movement of an object. Speed describes the rate of change of position; acceleration describes the rate of change of speed, so is the rate of change of the rate of change of the position. Another application is the description of the shape of curves. You can investigate this on the applet below.

Activity

The applet below shows the graph of a function and a point on the graph with both the tangent and the normal drawn at this point. The applet also gives the values of the first and second derivatives at the point. In addition to this information, the applet also shows either a parabola or a circle as an approximation to the curve around the fixed point.

- First move the points on the left and right of the fixed point to create a parabola through the three points. Note that this is similar to how you created the tangent as a limit of secant lines. What do you notice?
- Next, move the points to create a circle that passes through the three points. What do you notice?

Overview
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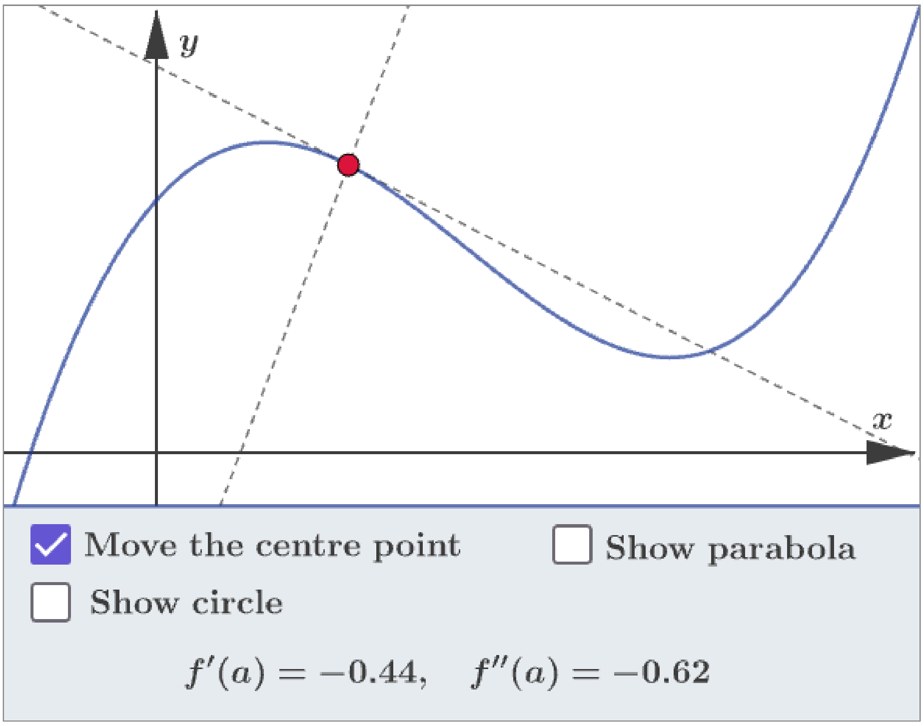

Interactive 1. Graph of a Function With Tangent and Normal Lines at a Specific Point.

More information for interactive 1

This interactive allows users to explore the graph of a function by examining the tangent, normal, and second derivative at a given point. It provides three toggle options to enhance understanding. The first option, "**Move the centre point**," enables users to drag the red center point along the curve, dynamically observing how the tangent, normal, and second derivative change. The interactive also displays the values of the first and second derivatives at the selected point. For example, when the point is positioned at the extreme left, a possible case is f'(a)=1.86 and f''(a)=−2, demonstrating how these values vary with position.

The second option, "**Show Parabola**," allows users to visualize a parabolic approximation of the curve around a fixed point. By moving points on either side of the fixed point, they can create a parabola passing through three points, reinforcing the concept of a tangent as the limit of secant lines. Similarly, the third option, "**Show Circle**," provides a circular approximation of the curve. Users can adjust the points to create a circle passing through three selected points, helping them understand how curvature and concavity relate to local approximations.

Through this interactive, users gain hands-on experience with graphing functions and develop a deeper understanding of how the first and second derivatives influence the shape of a function's graph.

Take a look at the video below. It shows an interesting application for the investigation of curvature of curves drawn on two-dimensional surfaces.

Student
view

Concept

When graphing functions, think about the relationship between the sign of the first and second derivatives and the shape of the graph of the function.

5. Calculus / 5.7 Further graph properties

Increasing/decreasing behaviour revisited

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/increasingdecreasing-behaviour-revisited-id-27789/print/) Assign

In subtopic 5.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25549/) you saw the relationship between the sign of the derivative and the shape of the graph. Here is a reminder of what you have observed there.

Important

- If $f'(x) > 0$ on an interval $]a, b[$, then f is increasing on $]a, b[$.
- If $f'(x) < 0$ on an interval $]a, b[$, then f is decreasing on $]a, b[$.
- If the function f is differentiable and $(a, f(a))$ is a turning point of the graph of f, then $f'(a) = 0$.

Now that you are able to find derivatives, you can move one step forward. In this section, you will see examples where the function is given and you are asked to sketch the graph. You should be able to draw these sketches without the use of graphic display calculators.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Example 1

★★☆

Consider the function defined by $f(x) = x + \frac{1}{x}$ for $0 < x \leq 5$.

- Identify the intervals where f is increasing and decreasing.
- Identify the turning point on the graph of f.
- Sketch the graph of f.

Steps	Explanation
$f(x) = x + x^{-1}$ $f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$ $= \frac{x^2-1}{x^2}$	To use the observations above, you need the derivative of f.
Since the denominator is positive, • $f'(x) > 0$ if $x^2 > 1$, so for $1 < x \leq 5$. • $f'(x) < 0$ if $x^2 < 1$, so for $0 < x < 1$.	You need to identify the intervals within the domain $0 < x \leq 5$, where the derivative is negative and where it is positive.
Hence, f is • increasing on $]1, 5]$ and • decreasing on$]0, 1[$.	
The graph arrives at $(1, f(1)) = (1, 2)$ as a decreasing curve and leaves this point as an increasing curve. The point $(1, 2)$ is a minimum point of the graph.	At the turning point the derivative is 0 and changes sign.
• $f(0)$ is not defined and the $x = 0$ line is a vertical asymptote. • $f(5) = 5 + \frac{1}{5} = 5.2$, so the point $(5, 5.2)$ is the endpoint of the graph.	To sketch the graph, you need to investigate the behaviour at the endpoints of the domain.

Student view

Overview
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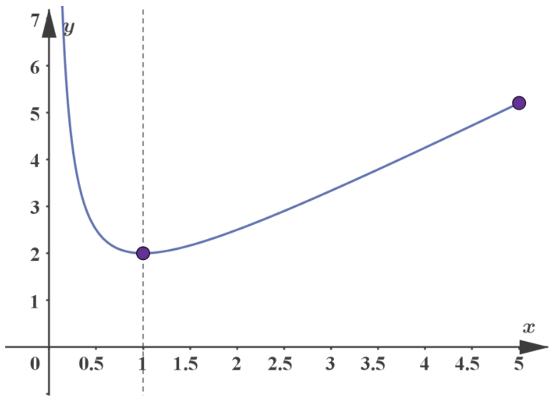
Steps	Explanation
	The sketch should match the observations above.

Example 2

★★☆

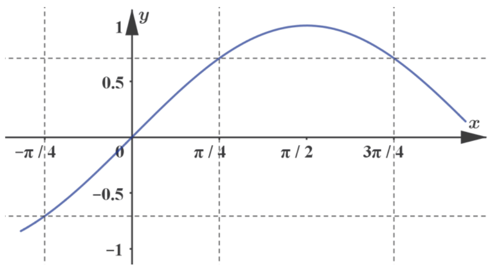
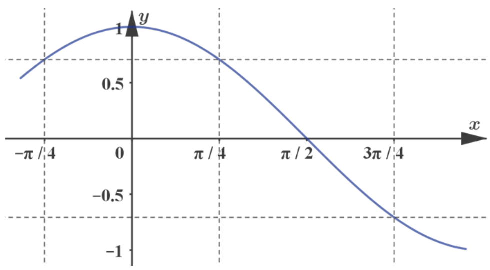
Consider the function defined by $f(x) = \mathrm{e}^x \sin x$ for $-1 \leq x \leq 3$.

- Identify the values of x where $f'(x) = 0$.
- Identify the intervals where f is increasing and decreasing.
- Identify the turning points on the graph of f.
- Find the axes intercepts of the graph of f.
- Given that $f(-1) \approx -0.3$ and $f(3) \approx 2.8$, sketch the graph of f.

Steps	Explanation
$\begin{aligned} f(x) &= \mathrm{e}^x \sin x \\ f'(x) &= \mathrm{e}^x \cos x + \mathrm{e}^x \sin x \\ &= \mathrm{e}^x (\cos x + \sin x) \end{aligned}$	You can use the product rule to find the derivative.
$\begin{aligned} f'(x) &= 0 \\ \mathrm{e}^x (\cos x + \sin x) &= 0 \\ \cos x + \sin x &= 0 \\ \cos x &= -\sin x \end{aligned}$	$\mathrm{e}^x \neq 0$

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
$x = -\frac{\pi}{4}$ $x = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$	This equation has two solutions between -1 and 3.
• For $-1 \leq x < -\frac{\pi}{4}$, $\sin x < -\frac{1}{\sqrt{2}}$ and $\cos x < \frac{1}{\sqrt{2}}$, so $\cos x + \sin x < 0$ • For $-\frac{\pi}{4} < x < \frac{\pi}{4}$ $\sin x > -\frac{1}{\sqrt{2}}$ and $\cos x > \frac{1}{\sqrt{2}}$, so $\cos x + \sin x > 0$ • For $\frac{\pi}{4} < x < \frac{3\pi}{4}$ $\sin x > \frac{1}{\sqrt{2}}$ and $\cos x > -\frac{1}{\sqrt{2}}$, so $\cos x + \sin x > 0$ • For $\frac{3\pi}{4} < x \leq 3$ $\sin x < \frac{1}{\sqrt{2}}$ and $\cos x < -\frac{1}{\sqrt{2}}$, so $\cos x + \sin x < 0$	You need to identify the intervals within the domain $[-1, 3]$, where the derivative is negative and positive. Since $e^x > 0$ for any x, only the sign of $\cos x + \sin x$ needs to be checked. A sketch of the sine and cosine curve on the given domain can be helpful.  
Hence, f is • increasing on $\left]-\frac{\pi}{4}, \frac{3\pi}{4}\right[$ and • decreasing on $\left]-1, -\frac{\pi}{4}\right[$ and $\left]\frac{3\pi}{4}, 3\right[$	

Student view

Overview
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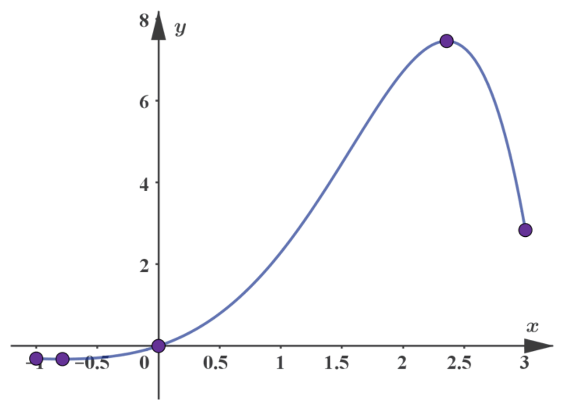
Steps	Explanation
The point $\left(-\frac{\pi}{4}, f\left(-\frac{\pi}{4}\right)\right)$ is a local minimum point.	At $x = -\frac{\pi}{4}$ the derivative is changing from negative to positive, so at $\left(-\frac{\pi}{4}, f\left(-\frac{\pi}{4}\right)\right)$ the curve is changing from decreasing to increasing.
The point $\left(\frac{3\pi}{4}, f\left(\frac{3\pi}{4}\right)\right)$ is a local maximum point.	At $x = \frac{3\pi}{4}$ the derivative is changing from positive to negative, so at $\left(\frac{3\pi}{4}, f\left(\frac{3\pi}{4}\right)\right)$ the curve is changing from increasing to decreasing.
$f(0) = \mathrm{e}^0 \sin 0 = 0 \times 0 = 0$, so the y-intercept is $(0, 0)$.	The y-intercept is the point with x-coordinate 0.
$f(x) = 0$ $\mathrm{e}^x \sin x = 0$	The x-intercepts are the points with y-coordinate 0.
$\sin x = 0$ The only solution of this equation in $[-1, 3]$ is $x = 0$. Hence, the only point where the graph intersects the coordinate axes is the origin of the coordinate system.	$\mathrm{e}^x \neq 0$
 	The sketch should match the observations above.

3 section questions

Student view

Second derivative

In this section you will see some examples of how the derivative of a function is used. You will also go one step further: you will explore examples where the derivative of the derivative is also needed.

Example 1

★★☆

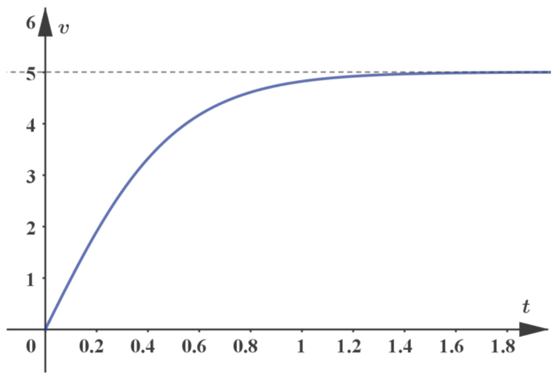
According to a simplified model, the velocity, $v(t)$, (measured in metres per second) of a skydiver t seconds after jumping out of a plane satisfies the equation $m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - \frac{1}{2}\rho C_d A v^2$, where m is the mass of the skydiver, g is the gravitational acceleration, ρ is the air density, C_d is the drag coefficient and A is the cross-sectional area of the skydiver.

With certain values of the parameters, this equation becomes $100\frac{\mathrm{d}v}{\mathrm{d}t} = 1000 - 40v^2$.

- Show that $v(t) = \frac{5(\mathrm{e}^{4t} - 1)}{\mathrm{e}^{4t} + 1}$ satisfies this equation.
- Sketch the graph of v and comment on the shape in the context of the question.

Steps	Explanation
$\begin{aligned}\frac{\mathrm{d}v}{\mathrm{d}t} &= \frac{5(4\mathrm{e}^{4t} - 0)(\mathrm{e}^{4t} + 1) - 5(\mathrm{e}^{4t} - 1)(4\mathrm{e}^{4t} + 0)}{(\mathrm{e}^{4t} + 1)^2} \\ &= \frac{20\mathrm{e}^{8t} + 20\mathrm{e}^{4t} - 20\mathrm{e}^{8t} + 20\mathrm{e}^{4t}}{(\mathrm{e}^{4t} + 1)^2} \\ &= \frac{40\mathrm{e}^{4t}}{(\mathrm{e}^{4t} + 1)^2}\end{aligned}$	The quotient rule can be used to find the derivative.
$100\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{4000\mathrm{e}^{4t}}{(\mathrm{e}^{4t} + 1)^2}$	Left-hand side.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
$1000 - 40v^2 = 1000 - 40 \times \left(\frac{5(e^{4t}-1)}{e^{4t}+1}\right)^2$ $= \frac{1000(e^{4t}+1)^2 - 40 \times 25(e^{4t}-1)^2}{(e^{4t}+1)^2}$ $= \frac{1000((e^{8t}+2e^{4t}+1)-(e^{8t}-2e^{4t}+1))}{(e^{4t}+1)^2}$ $= \frac{4000e^{4t}}{(e^{4t}+1)^2}$	Right-hand side.
Since the left- and right-hand sides simplify to the same expression, $v(t) = \frac{5(e^{4t}-1)}{e^{4t}+1}$ indeed satisfies the equation.	
	You can use your graphic display calculator to help you sketch the graph. Notice, that there is a horizontal asymptote.
According to this model, the velocity of the skydiver approaches, but does not reach 5 metres per second.	The graph has a horizontal asymptote, $v = 5$.

In **Example 1**, the equation you needed to check involved the velocity function and its derivative. Equations like this are called differential equations. You were asked to verify the given solution. In the higher level extension of this course you will learn methods of solving some types of differential equations. For the moment, if you are interested in other solutions, you can type

solve 100v'=1000-40v^2

into the search line of WolframAlpha (http://www.wolframalpha.com)and interpret the result.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

The example above investigates the velocity, $v(t)$, of the skydiver. It is a natural to ask if there is a way to investigate the position instead of the velocity. For this, you need an equation for $s(t)$. Since the velocity is the change of position, $s(t)$, we can replace $v(t)$ with $s'(t)$ in the equation above. The equation also contains $v'(t)$, the derivative of $v(t)$. You will now learn a notation that can help you express $v'(t)$ in terms of $s(t)$, and as a result will give you an equation for $s(t)$ instead of the equation for $v(t)$.

✓ **Important**

The derivative of the derivative of $y = f(x)$ is called the second derivative.

Different forms of notation used for the second derivative are, for example, y'', f'' and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

WolframAlpha (http://www.wolframalpha.com)understands these notations. If you are interested in the function describing the motion of the skydiver, type

solve {100s''=1000-40(s')^2, s'(0)=0, s(0)=-3000}

into the search line. The solution will provide the position of a skydiver, having jumped from a plane at an altitude of 3000 metres, with an initial speed of 0 metres per second.

Before taking a look at some further applications, here is an exercise to practise finding the second derivative.

Example 2

★★☆

Find the first and second derivatives.

$f(x)$	$f'(x)$	$f''(x)$
$mx + c$		
$ax^2 + bx + c$		
$ax^3 + bx^2 + cx + d$		
$\sqrt{x}$		
$\ln x$		

Student view

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e^{-x}		
$\sin x$		

$f(x)$	$f'(x)$	$f''(x)$
$mx + c$	m	0
$ax^2 + bx + c$	$2ax + b$	$2a$
$ax^3 + bx^2 + cx + d$	$3ax^2 + 2bx + c$	$6ax + 2b$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$-\frac{1}{4x\sqrt{x}}$
$\ln x$	$\frac{1}{x}$	$-\frac{1}{x^2}$
e^{-x}	$-e^{-x}$	e^{-x}
$\sin x$	$\cos x$	$-\sin x$

The next example is related to the activity in section 5.7.0 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27788/).

Example 3

★★☆

The radius of the best approximating circle of a curve at a given point is called the radius of curvature. For the graph of $y = f(x)$ this radius is given by

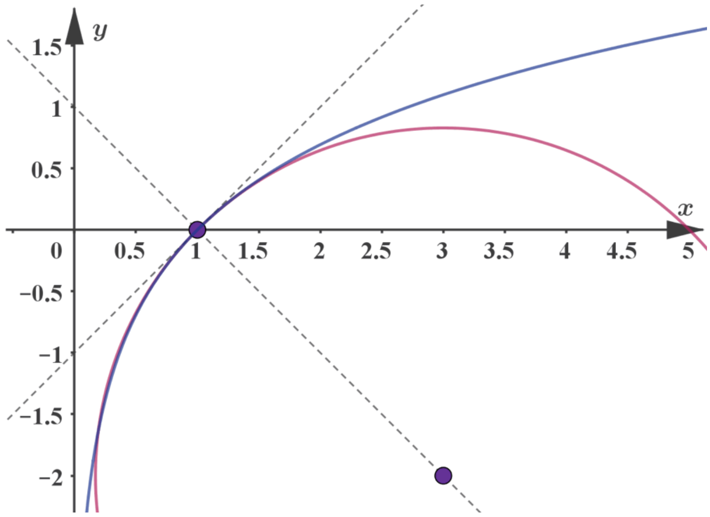
$$R = \left| \frac{\left(1 + y'^2\right)^{\frac{3}{2}}}{y''} \right|.$$

The centre of this best approximating circle is on the normal to the curve.

Find the radius and the centre of the best approximating circle to the graph of $y = \ln x$ at the x-intercept.

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o



More information

The image is a graph displaying the function y = ln x. The x-axis ranges from 0 to 5, and the y-axis ranges from -2 to 1.5. A circle approximates the function at the x-intercept, where x is slightly over 1.

The graph shows the natural logarithm curve, starting from the origin at (1, 0) on the x-axis and moving upwards slowly. A circle intersects this point, visualizing the approximation. Dashed lines are drawn from the point of intersection to highlight the tangent, suggesting the curvature at the intersection. The circle is plotted over the curve to represent the best approximating circle at the identified point. Both x and y axes are labeled, with tick marks and values indicated on each axis.

[Generated by AI]

Steps	Explanation
The x-intercept is $(1, 0)$.	$\ln 1 = 0$
$y'(x) = \frac{1}{x}$ $y'(1) = \frac{1}{1} = 1$ $y''(x) = -\frac{1}{x^2}$ $y''(1) = -\frac{1}{1^2} = -1$	To find the radius, you need the first and second derivative at $x = 1$.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
$R = \left\lvert \frac{\left(1+y'^2\right)^{\frac{3}{2}}}{y''} \right\rvert = \left\lvert \frac{\left(1+1^2\right)^{\frac{3}{2}}}{-1} \right\rvert = 2\sqrt{2}$	
The equation of the normal is $y = 1 - x$.	The gradient of the tangent is $y'(1) = 1$, so the gradient of the normal is $\frac{-1}{1} = -1$.
The centre of the best approximating circle is $(3, -2)$.	Since the angle between the normal and the x-axis is $45°$ and since the radius is $2\sqrt{2}$, the centre is 2 units to the right and 2 units down from the x-intercept.

Activity

Before the next example take a look at this applet (https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html). It illustrates the movement of an object when you hang it on a spring. The applet can illustrate the movement in an ideal situation when there is no friction (so the object would move up and down forever) or in a more realistic situation, when the amplitude of the movement is decreasing.

- Can you suggest a model that might describe the position of the object in an ideal situation?
- Can you modify this model so that it might be applicable in the realistic scenario?

Example 4

★★☆

According to the laws of physics, in an ideal situation (with no friction), the position, $y(t)$, of an object placed at the end of a vertical spring can be described by the differential equation $my'' + ky = 0$, where m is the mass of the object and k is a constant determined by the properties of the spring.

Student view

For certain values of the constants, this equation becomes $8y'' + 16y = 0$.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

A solution of this equation is $y(t) = \sin ct$, where $c > 0$.

Find the value of c.

Steps	Explanation
$y(t) = \sin ct$ $y'(t) = c\cos ct$ $y''(t) = -c^2 \sin ct$	To use the equation, you need the second derivative.
$8 \times (-c^2 \sin ct) + 16\sin ct = 0$ $(16 - 8c^2)\sin ct = 0$	Substituting in the equation.
$16 - 8c^2 = 0$ $c^2 = 2$	Since $c > 0$ and the equality needs to be true for any value of t.
$c = \sqrt{2}$	The question asked for the positive solution.

In **Example 4** you were asked to verify a given solution for the differential equation. If you are interested in other solutions, you can type

solve 8y''+16y=0

or

solve m*y''+k*y=0

into the search line of WolframAlpha (http://www.wolframalpha.com)and interpret the result.

Example 5

★★☆

In a more realistic model, the position, $y(t)$, of an object placed at the end of a vertical spring can be described by the differential equation $my'' + cy' + ky = 0$, where m is the mass of the object and c and k are constants determined by the properties of the spring and the surroundings.

For certain values of the parameters, this equation becomes $2y'' + 3y' + y = 0$.

Student view

Show, that $y(t) = ae^{-\frac{t}{2}} + be^{-t}$ is a solution of this differential equation for any value of a and b.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Steps	Explanation
$y'(t) = -\frac{a}{2}\mathrm{e}^{-\frac{t}{2}} - b\mathrm{e}^{-t}$ $y''(t) = \frac{a}{4}\mathrm{e}^{-\frac{t}{2}} + b\mathrm{e}^{-t}$	To check the equality, you need the first and second derivative.
$2y'' + 3y' + y = 2\left(\frac{a}{4}\mathrm{e}^{-\frac{t}{2}} + b\mathrm{e}^{-t}\right) + 3\left(-\frac{a}{2}\mathrm{e}^{-\frac{t}{2}} - b\mathrm{e}^{-t}\right) + \left(a\mathrm{e}^{-\frac{t}{2}} + b\mathrm{e}^{-t}\right) = \left(\frac{2a}{4} - \frac{3a}{2} + a\right)\mathrm{e}^{-\frac{t}{2}} + (2b - 3b + b)\mathrm{e}^{-t} = 0 \times \mathrm{e}^{-\frac{t}{2}} + 0 \times \mathrm{e}^{-t} = 0$	

If you are interested in the solution of the general equation, you can type

solve m*y''+c*y'+k*y=0

into the search line of WolframAlpha (http://www.wolframalpha.com)and interpret the result.

Making connections

In this section you learned about the second derivative and some of its applications. You might ask, 'why stop here, why not take the derivative of the second derivative?' Higher order derivatives are indeed useful, for example, for finding more and more accurate approximations to functions. You have already seen the tangent (a linear approximation) and in section 5.7.0 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27788/) a quadratic approximation. You will learn about higher order polynomial approximations in AAHL subtopic 5.19. To find more and more accurate approximations, you will use higher order derivatives, which you will learn about in AAHL subtopic 5.12.

3 section questions

5. Calculus / 5.7 Further graph properties

Concavity

Student view
Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/convexity-id-27791/print/) Assign

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

In previous subtopics you have seen how you can use the sign of the derivative to identify intervals where a graph is increasing and decreasing. This was also helpful in identifying turning points on the graph. In this section you will see how to use the second derivative to get more information on the shape of a curve.

Activity

On the applet below you see the graph of a function. You can move a point on the graph and the applet shows you the tangent to the graph and the value of the second derivative at that point. You can also ask the applet to show you a parabola approximating the curve at the given point. Two parts of the curve are drawn using different colours.

- Describe the connection between the value of the second derivative, the approximating parabola, the colouring and the shape of the curve.

Interactive 1. Connection Between the Value of the Second Derivative, the Approximating Parabola, the Colouring and the Shape of the Curve.

More information for interactive 1

This interactive makes the user understand how the value of the second derivative relates to the approximating parabola, and the overall shape of the curve.

The screen is divided into two halves. The top half displays a graph on the XY-axis, with the x-axis ranging from 0 to 4 and the y-axis ranging from -1 to 1. A yellow upward parabolic curve is projected on the graph intersecting the Y-axis and cubic function curve. The cubic function curve plots a concave upward curve intersecting the Y-axis up to around $x = 1.8$ followed by a concave downward curve. Two highlighted points, a red dot, and a blue dot are marked on the cubic curve. A dotted line at the red dot appears to be a tangent line to both the parabola and cubic curve. The red dot on the curve allows users to modify an approximating curve, which is the second derivative of the function $f(x)$, with its tangent line drawn as a straight dashed line with a negative slope.

Student view

On the bottom half, users can click on the “Adjust curve” checkbox to modify the curve using two red dots moving along the y-axis. Users can also click on the checkbox “Show approximating parabola” to see a parabola approximating the curve at the given point.

If the users adjust the curve in such a way that it passes through the x-axis at point 2, the value $f''(a) = 2.7$, displayed on the graph represents the second derivative of a function at point ‘a’.

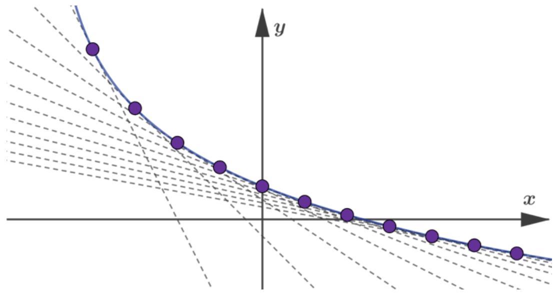
In the interactive users will notice that the section of the curve to the left of the blue dot $(x = 2)$ is concave down while the section of the curve to the right of the blue dot $(x = 2)$ is concave up. While at point $x = 2$ the concavity changes in this case and therefore it becomes the inflection point.

Thus, giving users a better view and insight into how concavity changes in different cases.

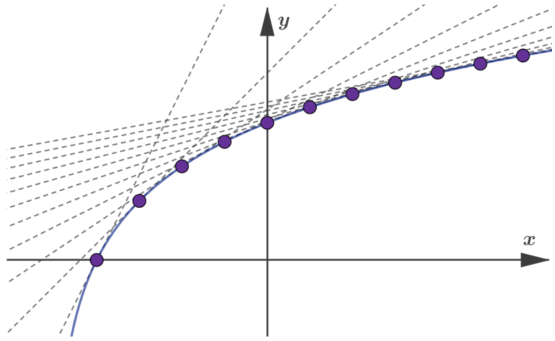
The following is a summary of the terminology related to the second derivative and the shape of the graph of a function.

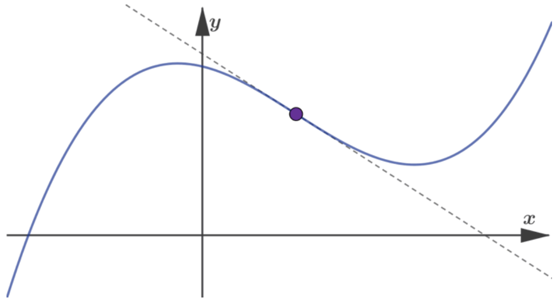
Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Important

Steps	Explanation
The graph of a differentiable function is concave up on an interval $$]a,b[$$ if the graph is above all of the tangents.	 More information The image is a graph showing a curve with data points. The X-axis is labeled "x" and the Y-axis is labeled "y". The curve is blue, running from the top left to the bottom right. Purple data points are plotted on the curve, demonstrating a decreasing trend. Several dotted lines are present, connecting the data points to the axes, indicating specific values. The graph suggests an inverse relationship between the X and Y values, with the Y value decreasing as the X value increases. [Generated by AI]

Student view

Steps	Explanation
The graph of a differentiable function is concave down on an interval $]a, b[$ if the graph is below all the tangents.	 More information The image is a graph with labeled axes “x” and “y”. A curve is drawn from the lower left to the upper right, demonstrating an increasing trend. Along the curve, there are multiple purple points marked, indicating specific data points on the trajectory. From each of these points, dashed tangent lines radiate outward in different directions, illustrating the concept of tangents and potentially the derivative or rate of change at each point. The image visually demonstrates the relationship between the curve and its tangents, commonly used in calculus to show how a function's slope changes across its domain. [Generated by AI]

Steps	Explanation
A point on a graph where the concavity is changing is called a point of inflexion.	 More information The image is a graph featuring a smooth, oscillating curve crossing the X and Y axes. The X-axis is labeled as 'x' and the Y-axis as 'y.' There is a visible point on the curve marked with a small purple circle. This graph shows the behavior of a function, with an inflection point where the curve transitions direction. The axes intersect at the origin, providing a reference for the position of the curve and the marked point. The overall trend shows an initial increase followed by a decrease and subsequent increase. [Generated by AI]

The following is a summary of how you can use the second derivative to investigate concavity.

✓ **Important**

- If $f''(x) > 0$ for every $a < x < b$, then the graph of f is concave up on $]a, b[$.
- If $f''(x) < 0$ for every $a < x < b$, then the graph of f is concave down on $]a, b[$.
- If $f''(a) = 0$ and $f''(x)$ changes sign at $x = a$, then the point $(a, f(a))$ is a point of inflexion of the graph.

Example 1

★☆☆

- Show that the graph of $y = \mathrm{e}^x$ is concave up on $]-\infty, \infty[$.
- Show that the graph of $y = \ln x$ is concave down on $]0, \infty[$.

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- Show that the graph of $y = \sqrt{x}$ is concave down on $]0, \infty[$.

Steps	Explanation
$y' = \mathrm{e}^x$ $y'' = \mathrm{e}^x$	You can investigate concavity using the second derivative.
Since $y''(x) > 0$ for all real numbers, the graph of $y = \mathrm{e}^x$ is concave up on $]-\infty, \infty[$.	e^x is positive for all real number x.
$y' = \frac{1}{x}$ $y'' = -\frac{1}{x^2}$	You can investigate concavity using the second derivative.
Since $y''(x) < 0$ for all positive real numbers, the graph of $y = \ln x$ is concave down on $]0, \infty[$.	A square is positive for all positive real number.
$y' = \frac{1}{2\sqrt{x}}$ $y'' = -\frac{1}{4x\sqrt{x}}$	You can investigate concavity using the second derivative.
Since $y''(x) < 0$ for all positive real numbers, the graph of $y = \sqrt{x}$ is concave down on $]0, \infty[$.	A square root of a positive real number is positive.

Example 2

★☆☆

- Find the intervals where the graph of $y = \sin x$ is concave up and the intervals where the graph is concave down.
- Find the points of inflexion on the graph of $y = \sin x$.

Steps	Explanation
$y' = \cos x$ $y'' = -\sin x$	You can investigate concavity using the second derivative.

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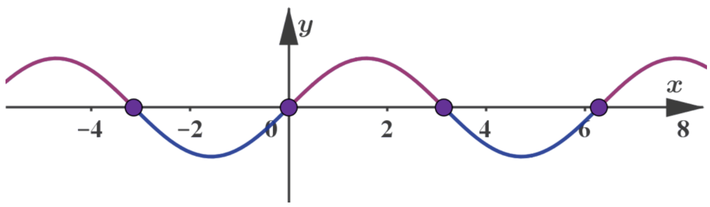
Steps	Explanation
$y''(x) > 0$ in $](2k-1)\pi, 2k\pi[$ for $k = 0, \pm 1, \pm 2, \ldots$	$\sin x$ is negative in the third and fourth quadrant of the unit circle.
The graph of $y = \sin x$ is concave up in the intervals $](2k-1)\pi, 2k\pi[$ for $k = 0, \pm 1, \pm 2, \ldots$	
$y''(x) < 0$ in $]2k\pi, (2k+1)\pi[$ for $k = 0, \pm 1, \pm 2, \ldots$	$\sin x$ is positive in the first and second quadrant of the unit circle.
The graph of $y = \sin x$ is concave down in the intervals $]2k\pi, (2k+1)\pi[$ for $k = 0, \pm 1, \pm 2, \ldots$	
The points of inflexion are where $y''(x)$ is 0 and changes sign, so $(0, 0), (\pm\pi, 0), (\pm 2\pi, 0), \ldots$	$\sin x = 0$ for $x = 0, \pm\pi, \pm 2\pi, \ldots$



Example 3

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Find the two points of inflexion on the graph of $y = x \cos x$ that are shown on the diagram below.

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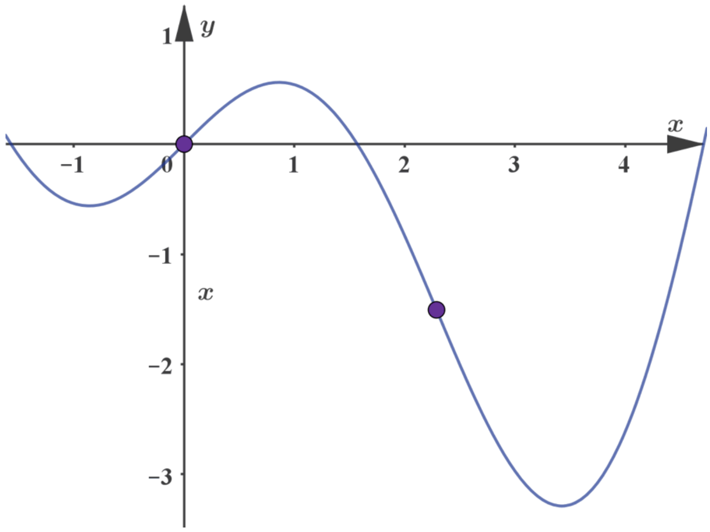
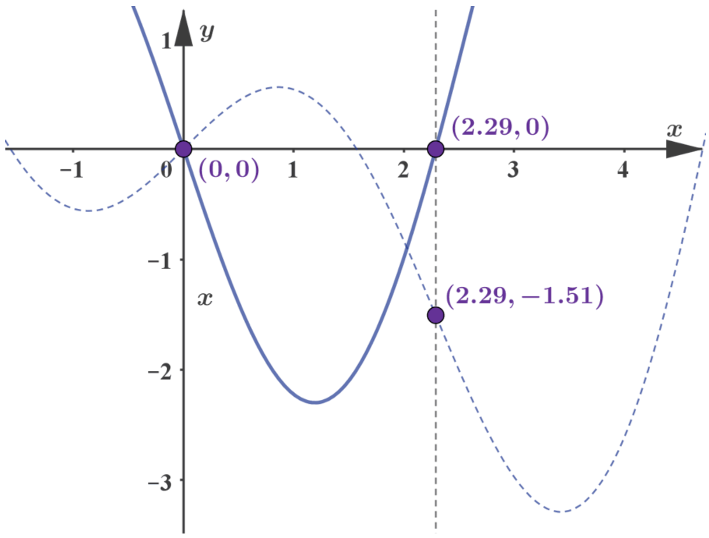


More information

The image is a graph showing the function y = x cos x. The X-axis ranges from -2 to 5, and the Y-axis ranges from -3 to 2. The graph is a curve that oscillates, passing through the origin (0,0). There are two marked points of inflection on the graph. The first inflection point is near (-1, 1), and the second is near (2, -2). The curve peaks and dips as it passes through these inflection points. The graph shows a pattern of waves, decreasing in amplitude as x increases.

[Generated by AI]

Steps	Explanation
$\begin{aligned} y' &= \cos x + x(-\sin x) \\ &= \cos x - x\sin x \\ y'' &= -\sin x - (\sin x + x\cos x) \\ &= -2\sin x - x\cos x \end{aligned}$	You can identify points of inflexion by investigating the second derivative.
$-2\sin x - x\cos x = 0$	At points of inflexion, the second derivative is 0.

Steps	Explanation
The solutions within the domain on the diagram in the question are $x = 0$ and $x = 2.29$. The corresponding points of inflexion are $(0, 0)$ and $(2.29, -1.51)$. 	Graphic display calculators have applications to solve equations like this.

Example 4

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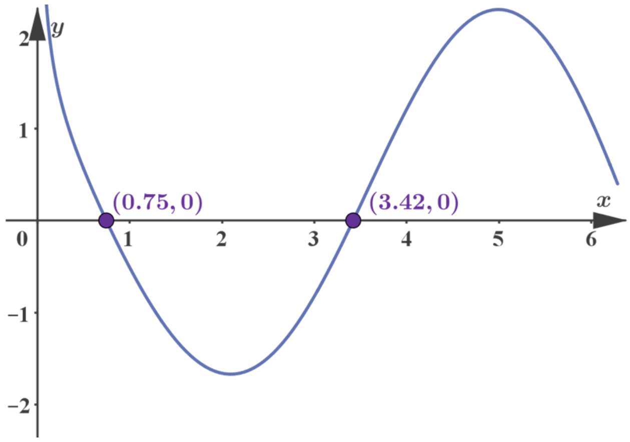
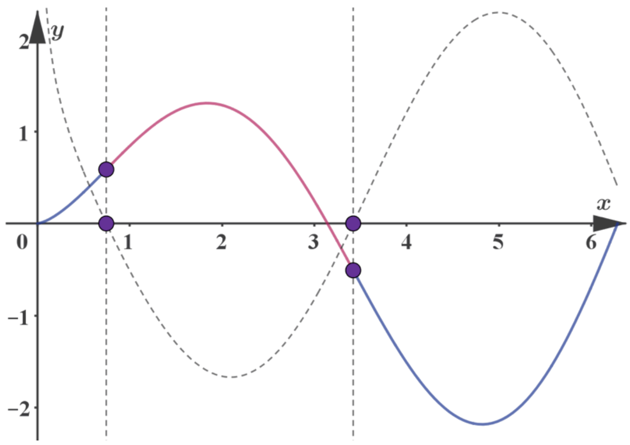
Find the interval where the graph of $y = \sqrt{x} \sin x$ is concave down in the interval $[0, 2\pi]$.

Steps	Explanation
$$\begin{aligned} y' &= \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x \\ &= \frac{\sin x + 2x\cos x}{2\sqrt{x}} \end{aligned}$$ $$\begin{aligned} y'' &= \frac{(3\cos x - 2x\sin x)2\sqrt{x} - (\sin x + 2x\cos x)\dfrac{1}{\sqrt{x}}}{4x} \\ &= \frac{6x\cos x - 4x^2\sin x - \sin x - 2x\cos x}{4x\sqrt{x}} \\ &= \frac{4x\cos x - (4x^2+1)\sin x}{4x\sqrt{x}} \end{aligned}$$	You can investigate concavity using the second derivative.

Steps	Explanation
$$\frac{4x\cos x - (4x^2+1)\sin x}{4x\sqrt{x}} = 0$$	The second derivative is 0 when concavity changes.
The solutions of this equation between 0 and 2π are $x = 0.746$ and $x = 3.42$. 	Graphic display calculators have applications to solve equations like this.
The graph of $y = \sqrt{x}\sin x$ is concave down on $]0.746, 3.42[$. 	The graph is concave down on the interval where the second derivative is negative.

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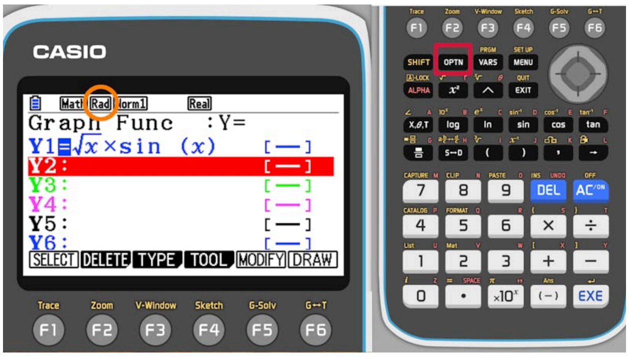
<table>
<tr><th>Steps</th><th>Explanation</th></tr>
<tr><td>In these instructions you will see how to find the solution of $f''(x) = 0$ for $f(x) = \sqrt{x}\sin x$ on the interval $0 \leq x \leq 2\pi$.

From the main menu, choose the graph option.</td><td>
</td></tr>
<tr><td>Enter the definition of the function and make sure your calculator is in radian mode.

The next step is telling the calculator that the second function is the second derivative of Y1. Press OPTN to find the template for the second derivative ...</td><td>
</td></tr>
</table>

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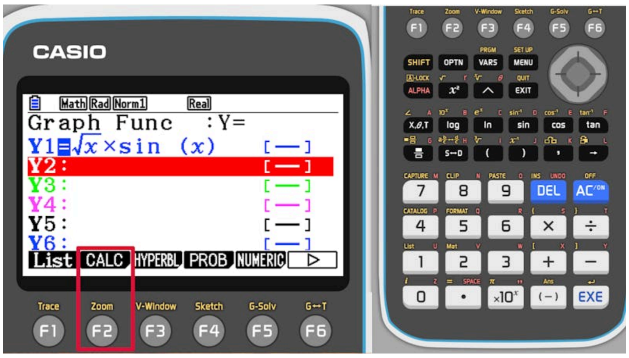
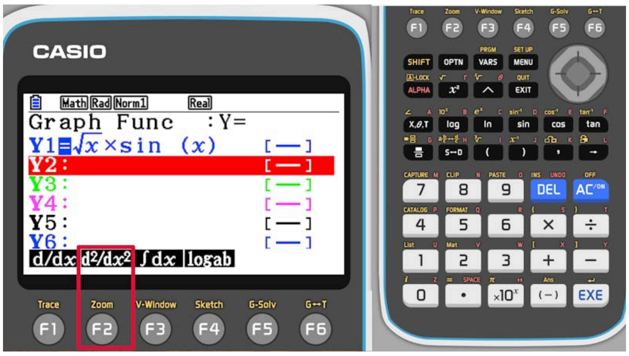
Steps	Explanation
... press F2 to bring up the calculus related options ...	
... and press F2 again to insert the second derivative template.	

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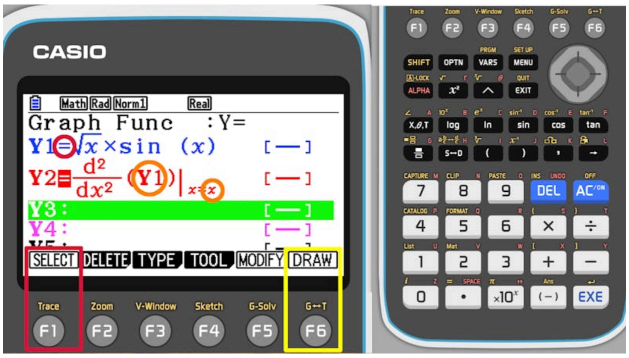
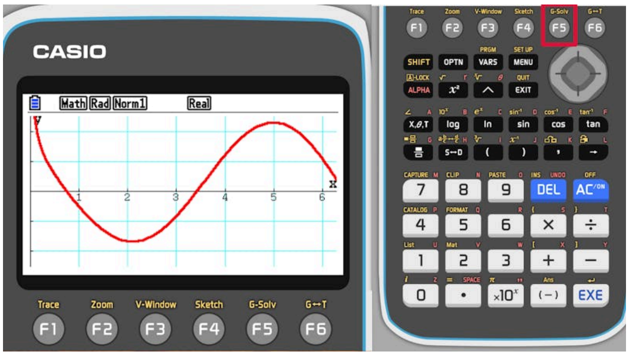
Steps	Explanation
Use the function name instead of retyping the expression. Use the variable x to indicate, that you are defining a function, not evaluating the second derivative at a given point. Make sure you unselect the function itself (Y1) so that you do not see it on the graph view. Once done, press F6 to draw the graph.	
You probably will need to adjust the window so that you only see the graph of this second derivative for $0 \leq x \leq 2\pi$. Once done, press F5 (G-Solve) to bring up options to analyze the graph.	

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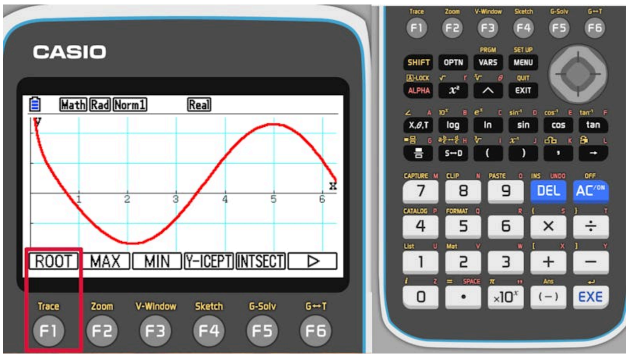
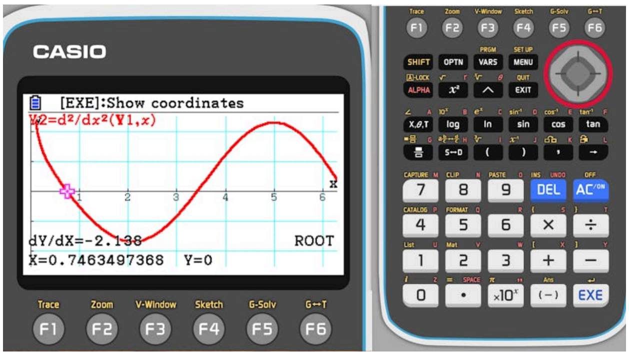
Steps	Explanation
You are interested in the x-intercepts, so press F1 to find the roots.	
The calculator moves the cursor to one of the x-intercepts, and displays the coordinates. You can move left and right to move between the roots visible on the screen.	

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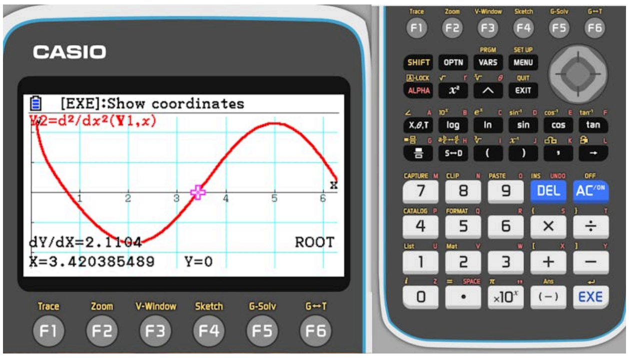
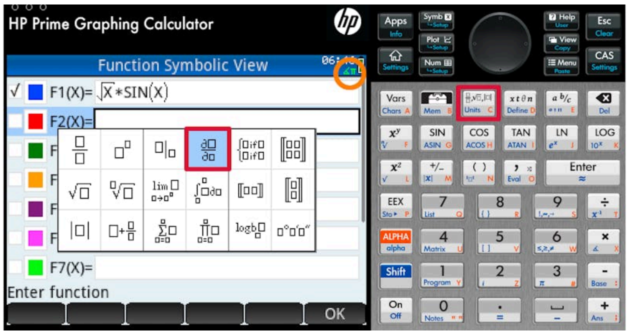
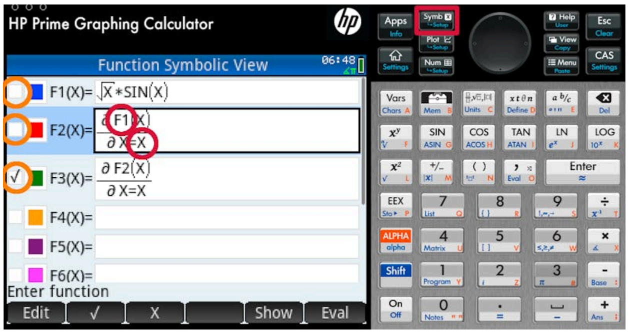
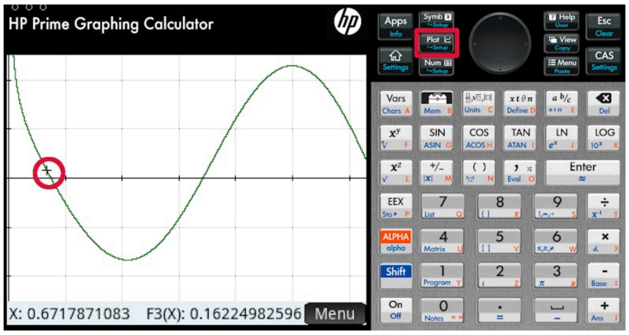
Steps	Explanation
On this screenshot you can see the coordinates of the second x-intercept.	

Steps	Explanation
In these instructions you will see how to find the solution of $f''(x) = 0$ for $f(x) = \sqrt{x}\sin x$ on the interval $0 \leq x \leq 2\pi$. Choose the function application.	

Steps	Explanation
In symbolic view, enter the definition of the function and make sure your calculator is in radian mode. The next step is telling the calculator that you would like to graph the second derivative of F1. Bring up the templates and choose the template for the derivative.	
Use the function name instead of retyping the expression. Use the syntax $x = x$ to indicate, that you are defining a function, not evaluating the derivative at a given point. You will need two steps, the derivative of F1 and the derivative of this derivative. Make sure you unselect the function and the derivative (F1 and F2) so that you do not see these in the plot view.	
Change now to plot view. You probably will need to adjust the window so that you only see the graph of this second derivative for $0 \leq x \leq 2\pi$. You are interested in the x-intercepts, so move the cursor close to one of these.	

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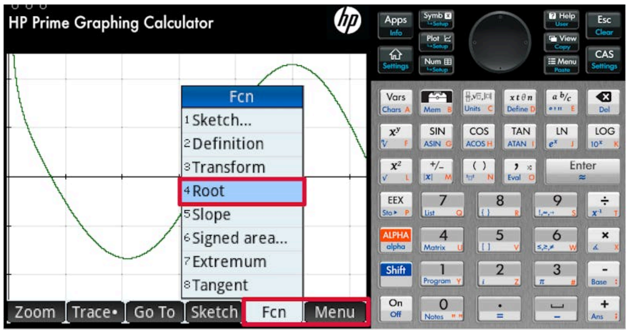
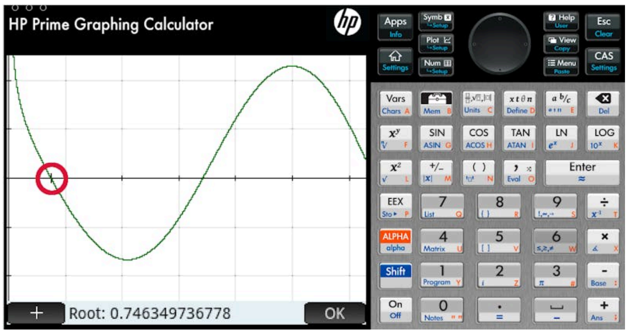
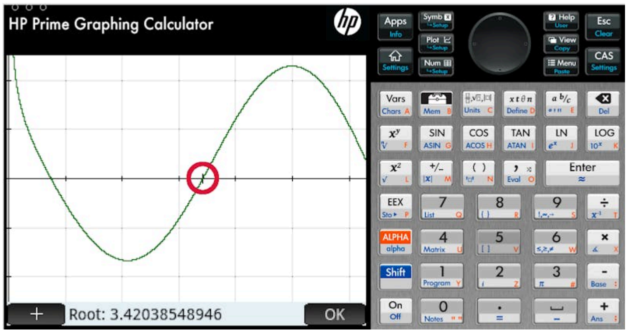
Steps	Explanation
Tap on Menu and select to find the roots among the options to analyze a function.	
The calculator moves the cursor to the x-intercept and displays the first coordinate.	
If you search for the roots with the cursor close to the other x-intercept, the calculator will find that one.	

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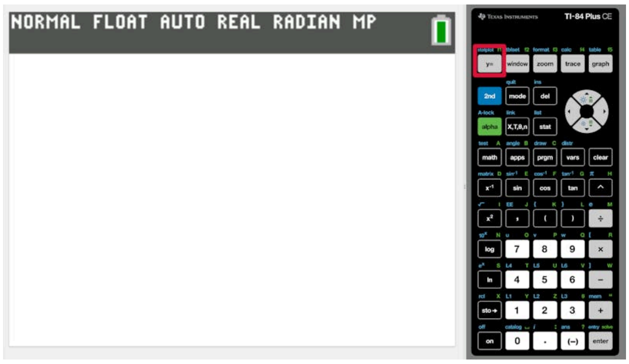
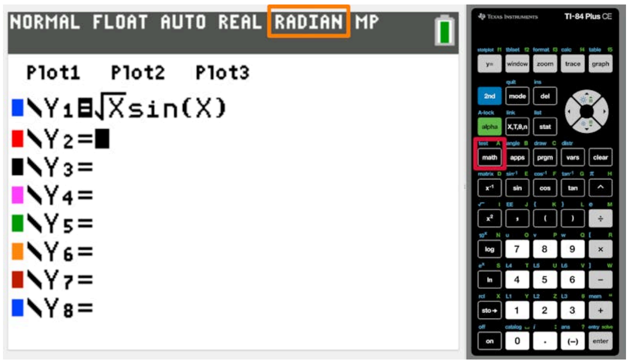
Steps	Explanation
In these instructions you will see how to find the solution of $f''(x) = 0$ for $f(x) = \sqrt{x}\sin x$ on the interval $0 \leq x \leq 2\pi$. Enter the function definition screen.	
Enter the definition of the function and make sure your calculator is in radian mode. The next step is telling the calculator that you would like to graph the second derivative of Y1. Press math ...	

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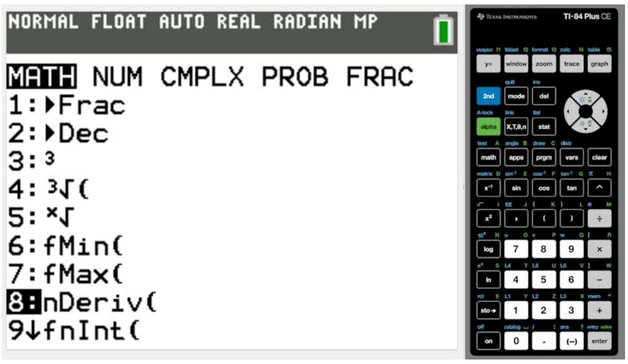
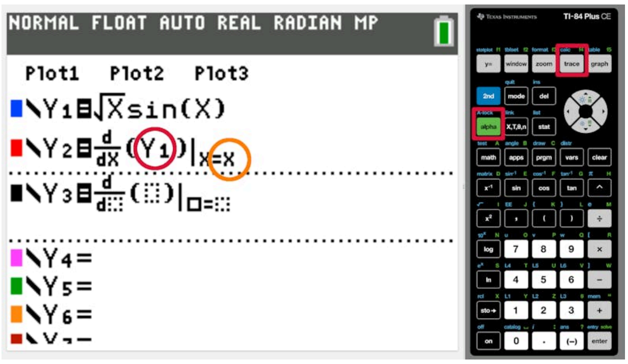
Steps	Explanation
... and choose the option (nDeriv) to find the numerical derivative.	
Use the function name instead of retyping the expression (you can access it either by pressing the vars button or through alpha/f4). Use the syntax $x = x$ to indicate, that you are defining a function, not evaluating the derivative at a given point. You will need two steps, the derivative of Y1 and the derivative of this derivative.	

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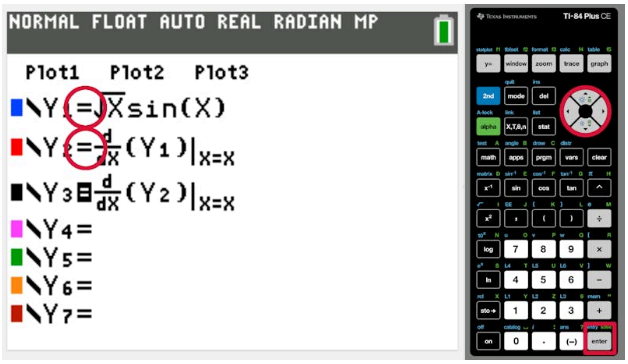
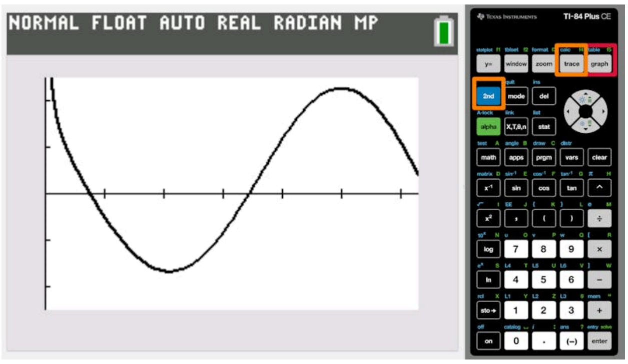
Steps	Explanation
Make sure you unselect the function and the derivative (Y1 and Y2) so that you do not see these in the plot view. You can unselect these by moving over the equality sign and pressing enter.	
Press graph to bring up the plot. You probably will need to adjust the window so that you only see the graph of the second derivative for $0 \le x \le 2\pi$. Once done, bring up options to analyze the graph (2nd calc).	

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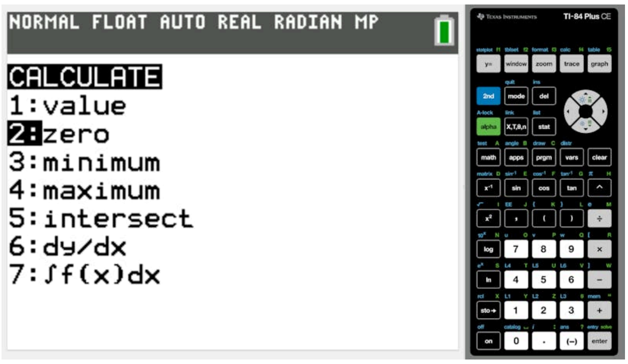
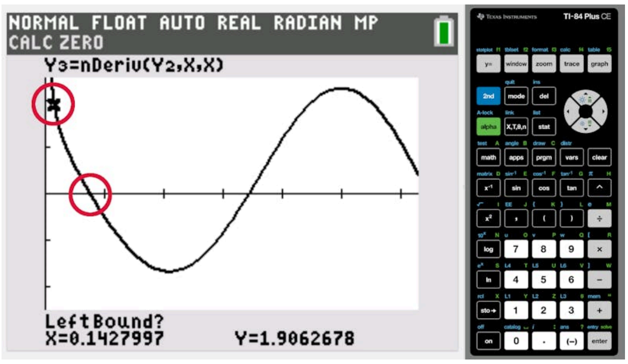
Steps	Explanation
You are interested in the x-intercepts, so choose the option to find the zeros.	
The calculator needs more information, so it asks questions. First it asks for a left bound, so move the cursor to the left of the x-intercept you woudl like to find and press enter.	

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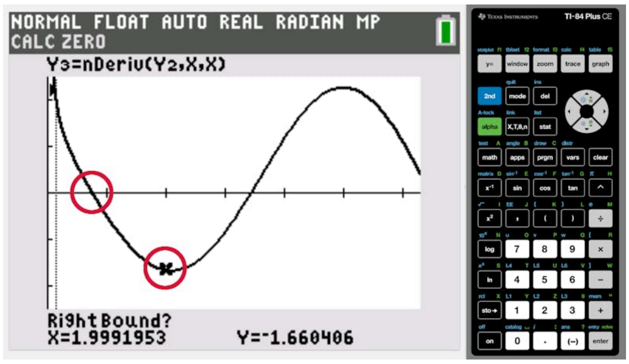
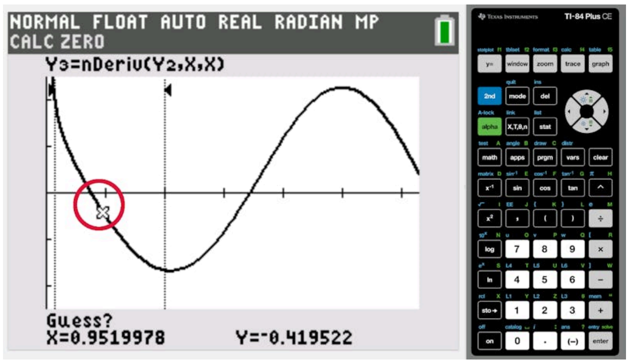
Steps	Explanation
The next step is to give an upper bound for the zero.	
Finally, move the cursor close to the x-intercept to tell the calculator your guess (it needs this as a starting point for the numerical algorithm) and press enter.	

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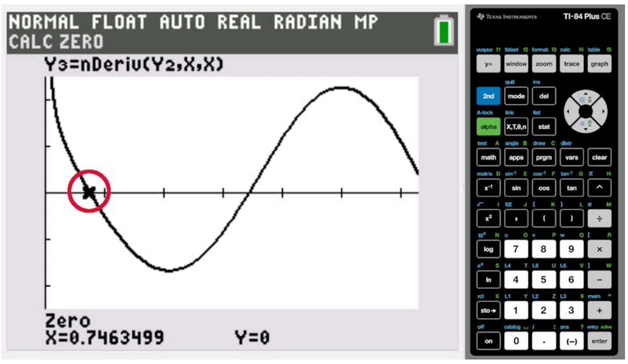
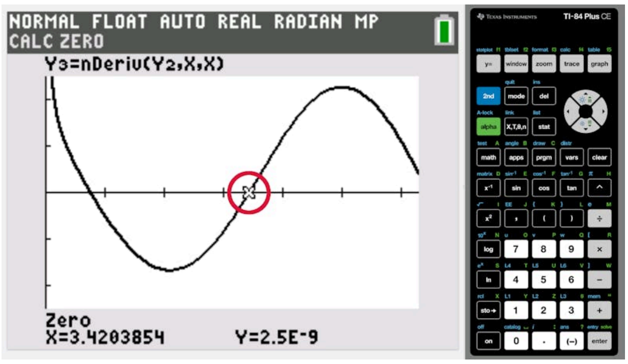
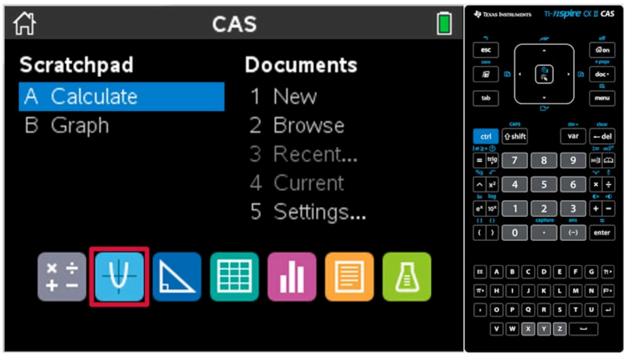
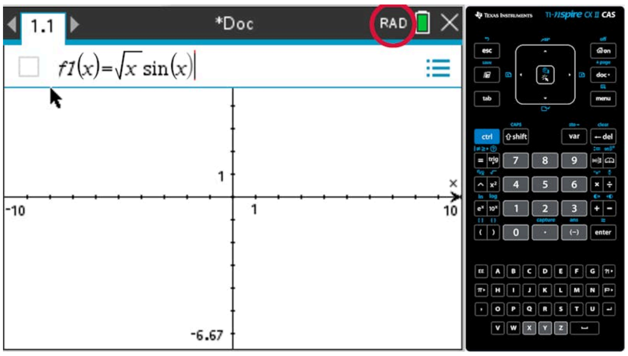
Steps	Explanation
The calculator moves the cursor the x-intercept, and displays the coordinates.	
Follow the same steps (but specify different left and right bounds) to find the other x-intercept.	

Steps	Explanation
In these instructions you will see how to find the solution of $f''(x) = 0$ for $f(x) = \sqrt{x}\sin x$ on the interval $0 \leq x \leq 2\pi$. From the home screen, open a graphing page.	
Enter the definition of the function and make sure your calculator is in radian mode. In the newest operating system you can change between degree and radian mode by simply clicking on the word.	

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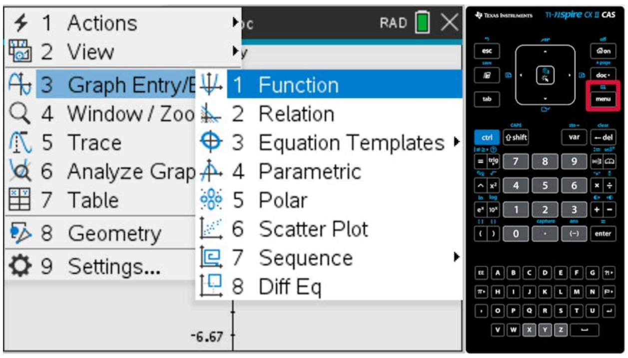
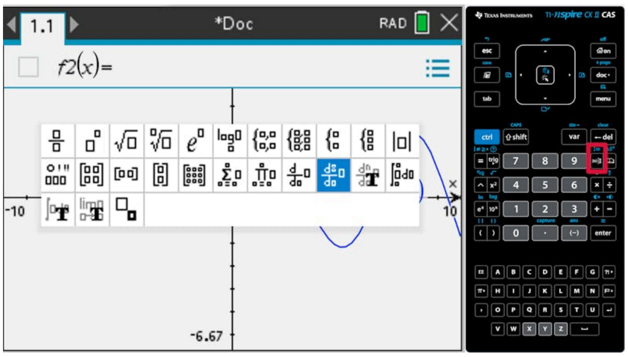
Steps	Explanation
The next step is telling the calculator that you would like to see the graph of the second derivative of the function you just defined. Add a new function to the document. One way of doing this is through the menu.	
Press the button to bring up the template options and find the template for the second derivative.	

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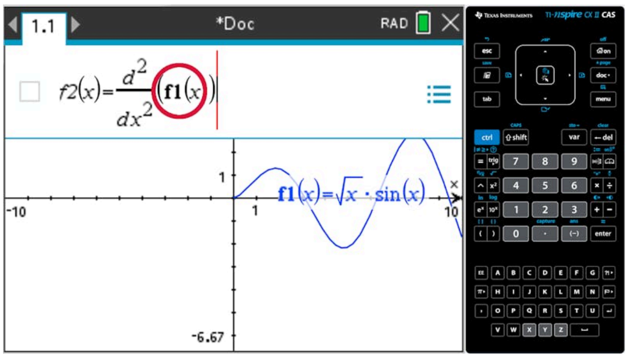
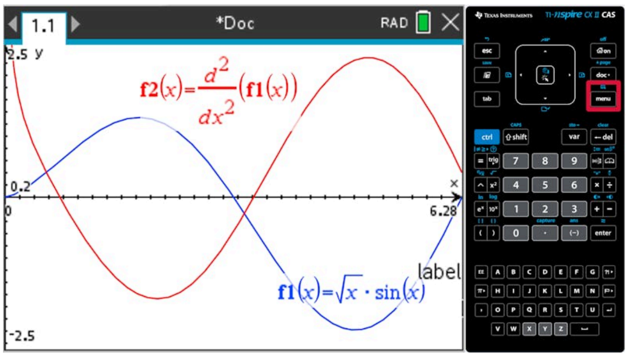
Steps	Explanation
In filling the blanks in the template, use the function name instead of retyping the expression.	
You can now see the graph of the function and the second derivative. You probably will need to adjust the window so that you only see the graph of this second derivative for $0 \leq x \leq 2\pi$. If you want to hide the graph of the function (which is not needed for finding the solutions of $f''(x) = 0$), press Menu ...	

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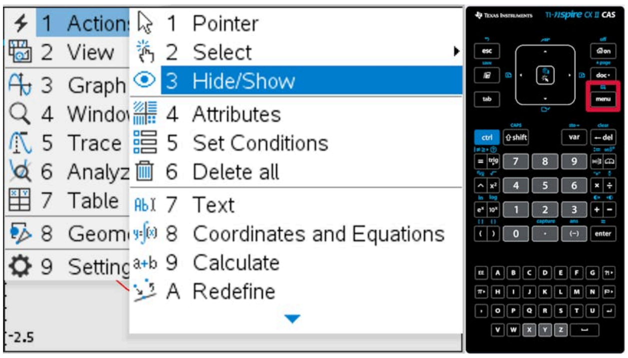
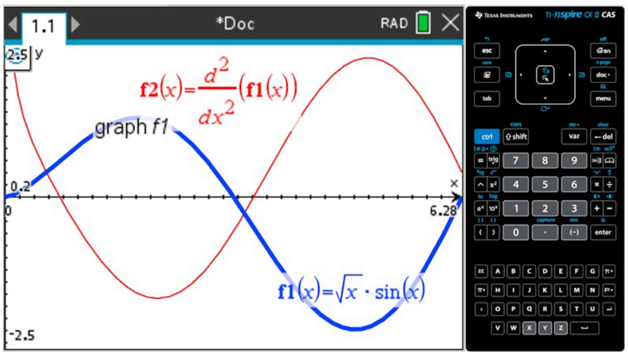
Steps	Explanation
... and find the option to hide/show objects.	
If you move over the graph of the function and press enter, it will remove it from the view.	

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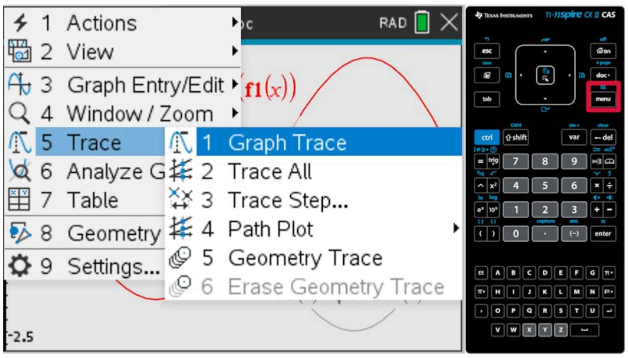
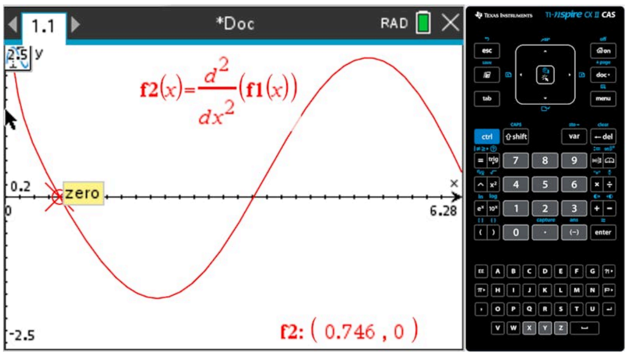
Steps	Explanation
You are interested in the x-intercepts. There are several ways of finding these. Probably the quickest is through tracing the graph.	
Tracing the graph has the feature, that when you move close to an important point (like the x-intercept), the calculator will jump to that point, shows the type and displays the coordinates.	

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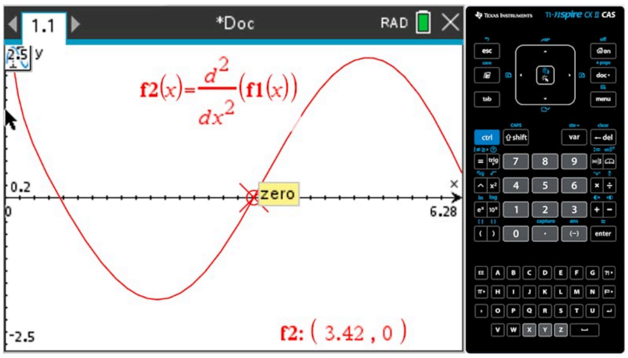
Steps	Explanation
To find the other x-intercept, all you need to do is to move the cursor close to it.	

Example 5

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Find the x-coordinate of the point of inflexion on the graph of $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

Steps	Explanation
$y' = 3ax^2 + 2bx + c$ $y'' = 6ax + 2b$	You can identify points of inflexion by investigating the second derivative.
$6ax + 2b = 0$ $6ax = -2b$ $x = -\frac{2b}{6a} = -\frac{b}{3a}$	At the point of inflexion, $y'' = 0$.
So the x-coordinate of the point of inflexion on the graph of $y = ax^3 + bx^2 + cx + d$ is $-\frac{b}{3a}$.	

3 section questions

Relationship between graphs and derivative graphs

In this section you will see a summary of the relationship between the graph of a function and its first and second derivative.

Activity

On the applet below you can see the graph of the derivative of a certain function.

- Can you sketch the graph of the function itself, based on the information the derivative graph gives you?

By moving the red point the applet will show you the graph and give some information about the shape. This information is not new; you have seen all these in previous sections.

Interactive 1. Graph of the Derivative of a Certain Function.

More information for interactive 1

This interactive allows users to visualize the graph of the derivative of a given function. By moving a red dot along the $x-\text{axis}$, users can observe the graph of $y=f(x)$. As the red dot moves to the right, the behavior of the derivative, $f'(x)$, is revealed. Also as we move the red point the necessary information about the behavior of the function and its derivative appears below the graph.

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When $f'(x)$ is positive, the function $f(x)$ is increasing. If the graph of $f'(x)$ is increasing, the second derivative, $f''(x)$, is positive, and the graph of $f(x)$ is concave up. If the graph of $f'(x)$ reaches a local maximum, this indicates a point of inflection for $f(x)$. Moving the red dot further to the right, if $f(x)$ continues increasing but the graph of $f'(x)$ starts decreasing, $f''(x)$ becomes negative, and the graph of $f(x)$ is concave down. When the graph of $f'(x)$ reaches zero without changing the sign, it marks an inflection point where $f(x)$ pauses its increase or decrease.

In another scenario, when the red dot moves forward and the graph of $f'(x)$ reaches zero while changing from positive to negative, this indicates a local maximum for $f(x)$. As the dot moves further, if $f'(x)$ is negative, $f(x)$ is decreasing, and if $f'(x)$ is decreasing, $f''(x)$ is negative, making the graph of $f(x)$ concave down. When $f'(x)$ reaches a local minimum, it marks an inflection point for $f(x)$. Moving the dot further, if $f(x)$ continues decreasing but $f'(x)$ starts increasing, $f''(x)$ becomes positive, making the graph of $f(x)$ concave up. When $f'(x)$ changes from negative to positive, this indicates a local minimum for $f(x)$.

This interactive tool helps users visualize how the graph of the derivative changes as the function changes.

Here is a summary of the properties of the graph of a function that you can determine from the graph of the derivative.

✓ **Important**

- If $f'(x) > 0$ for all $a < x < b$, then f is increasing on $]a, b[$.
- If $f'(x) < 0$ for all $a < x < b$, then f is decreasing on $]a, b[$.
- The x-intercepts of f' correspond to points on the graph of f where the tangent is horizontal. These are the stationary points on the graph of f.
  - If the graph of f' crosses the x-axis from above, the corresponding point is a local maximum point on the graph of f.
  - If the graph of f' crosses the x-axis from below, the corresponding point is a local minimum point on the graph of f.
  - If f' is not changing sign at the point where the graph of f' reaches the x-axis, then the corresponding point on the graph of f is a horizontal point of inflexion.
- The local maximum and minimum points of the graph of f' correspond to points of inflexion on the graph of f.

Note: The last two claims (about points of inflexion and horizontal points of inflexion) are not true in general, but are true for functions where the derivative only changes sign a finite number of times. Since in this course you will only meet functions like this, there is no need to state the more general claim.

Example 1

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Student view

The diagram below shows the graphs of f, f' and f'' for some function f.

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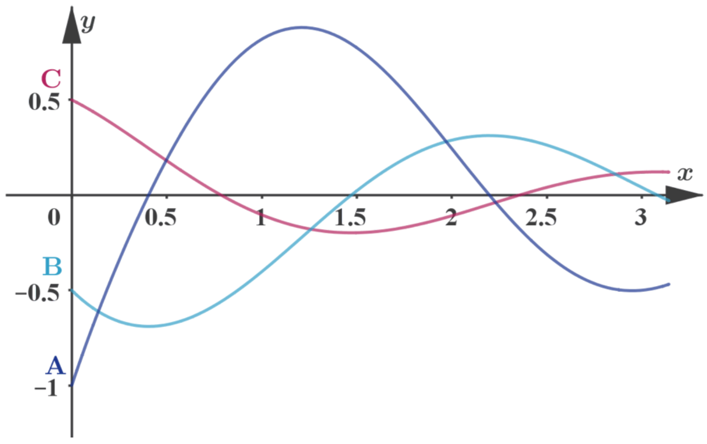

More information

The image presents a graph displaying three curves labeled as A, B, and C. The X-axis is labeled 'x' and ranges from 0 to 3, divided into increments of 0.5. The Y-axis is labeled 'y', ranging from -1 to 1, divided at 0.5 intervals. Curve C starts around Y=0.5 and moves downward, passing near (1,0), peaking slightly into the positive region near (1.5,0.5), and then gently moving back towards the X-axis around the right end of the graph. Curve B begins at -0.5 on the Y-axis, rises quickly to intersect the X-axis near (0.7,0), then it descends passing through the X-axis again near (2.3,0), and ends slightly negative on the far right. Finally, curve A starts below the minimum value on the Y-axis at around -1, ascends sharply, peaking around (1,0.8), then descends back towards the X-axis before rising slightly again towards the end of 3 on the X-axis. The curves represent the function, its first derivative, and its second derivative.

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Identify which is the graph of the function, which is the derivative and which is the second derivative.

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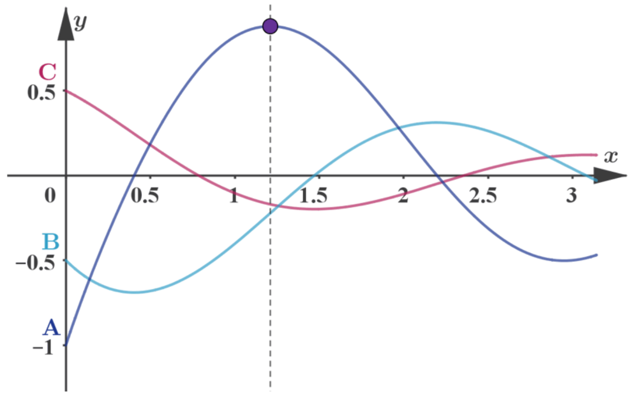
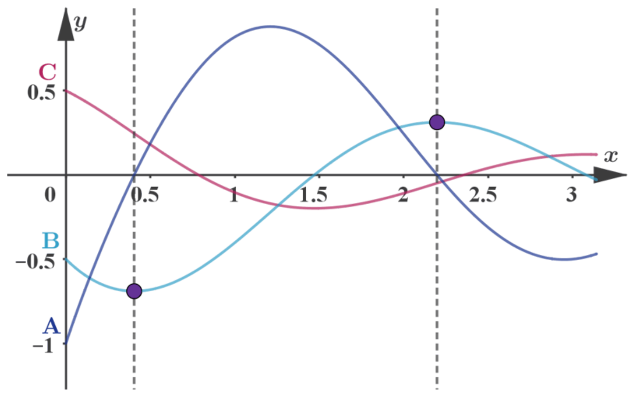
Steps	Explanation
Graph A is the second derivative. 	Graph A has a local maximum about halfway between 1 and 1.5, so the derivative of function A must be 0 approximately halfway between 1 and 1.5. Since neither graph B nor graph C intersect the x-axis approximately halfway between 1 and 1.5, neither of these is the graph of the derivative of function A.
Graph B is the first derivative. 	The local minimum and maximum points of graph B correspond to the x-intercepts of graph A (and this is not true for graph C), so function A is the derivative of function B.
Graph C is the function.	The local minimum of graph C corresponds to the x-intercept of graph B. Also, the concavity of graph C changes according to the sign of graph A.

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Example 2

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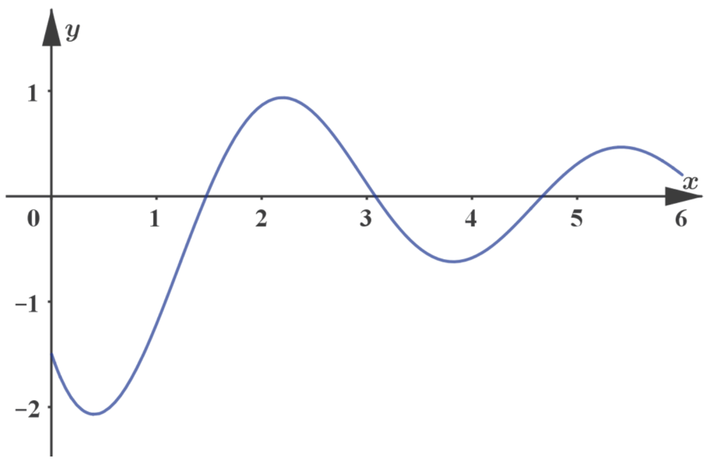
The diagram below shows the graph of $y = f'(x)$ for some function f defined on $0 \leq x \leq 6$.

- Find the number of turning points on the graph of $y = f(x)$. Also specify the nature of these turning points.
- Find the number of points of inflexion on the graph of $y = f(x)$.



More information

The graph depicts a sine wave plotted on a coordinate plane. The X-axis extends from 0 to 6, while the Y-axis ranges from -2 to 1. The curve starts at the origin, declines to its minimum point near (1.57, -2), then rises to a peak near (3.14, 1), followed by descending and ascending again to a smaller peak around (5, 0.5). The graph illustrates the periodic and oscillatory nature of the sine function within one cycle.

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Steps	Explanation
There are three turning points on the graph of $y = f(x)$. Two of the turning points are local minima and one is a local maximum.	There are three x-intercepts of the graph of $y = f'(x)$. At two of these, f' is changing from negative to positive and at one of these f' is changing from positive to negative.

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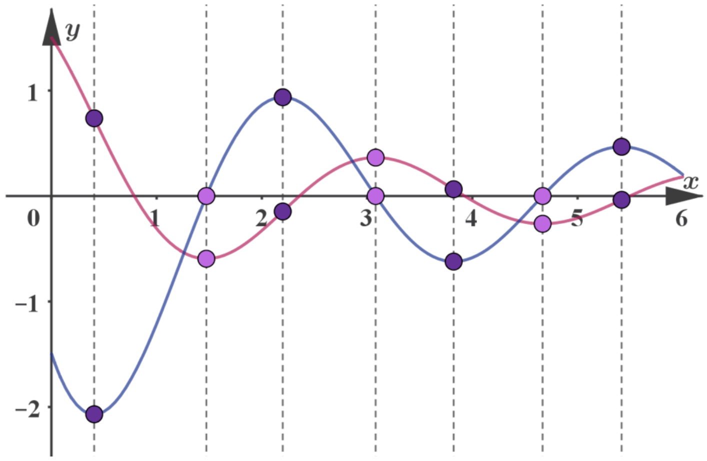
Steps	Explanation
There are four points of inflexion on the graph of $y = f(x)$.	There are four turning points on the graph of $y = f'(x)$.

The diagram below shows the graph of both $y = f'(x)$ and $y = f(x)$, and the points used to find the answer to the questions.



5 section questions

5. Calculus / 5.7 Further graph properties

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27793/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- find the second derivative of a function and apply it in context
- understand the different forms of notation used for the second derivative
- use a calculator to graph the first and second derivative without algebraically finding these derivatives
- understand the relationship between the sign of the derivative and the increasing/decreasing behaviour of the graph of a function.

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- understand the relationship between the sign of the second derivative and the concavity of the graph of a function
- identify stationary points and be aware of the different types
- identify points of inflexion
- understand the relationship between the x-intercepts and the turning points on the graph of the derivative function and the features of the original function
- draw sketches based on information about the first and second derivative of a function.

Investigation

On the applet below you can investigate the derivatives of $y = xe^x$, $y = x^2e^x$ and $y = x^3e^x$.

Activity

- Before you move the slider, find the first and second derivatives. You can even go further and differentiate the second derivative to find the third derivative.
- Finding further derivatives by hand gives you practice, but it is a very repetitive process. Move the slider to see the first 50 derivatives.
  - The applet uses the notation you will learn in subtopic 5.12 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-26489/). Can you see the pattern in the notation?
- Can you find a pattern in the derivatives? Can you find the hundredth derivative?

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Interactive 1. Graph of First, Second, and Third Derivatives.

More information for interactive 1

This interactive allows the users to investigate the derivatives of $y = xe^x$, $y = x^2e^x$ and $y = x^3e^x$. The screen is divided in two halves vertically. On the right there is an equation $y = x^1 * e^x$; on the left it is mentioned “Exponent” with three options to select, namely, 1, 2, and 3. A slider is also provided to control which derivative is being displayed, ranging from the first derivative to the 50th derivative.

The applet uses differential notation commonly seen in calculus, such as $\frac{d^n y}{dx^n}$, which helps users become familiar with mathematical notation. By observing successive derivatives, users can identify patterns in differentiation.

As users move the slider, the function updates to show the corresponding derivative.

$$\frac{d^1 y}{dx^1} = (x + 1)e^x$$

(first derivative)

$$\frac{d^2 y}{dx^2} = (x + 2)e^x$$

(second derivative)

$$\frac{d^{50} y}{dx^{50}} = (x + 50)e^x$$

(50th derivative)

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When 2 is selected, the equation changes to $y = x^2 * e^x$, and the derivatives change accordingly.

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$$\frac{d^1y}{dx^1} = (x^2 + 2x)e^x$$

(first derivative)

$$\frac{d^2y}{dx^2} = (x^2 + 4x + 2)e^x$$

(second derivative)

$$\frac{d^{50}y}{dx^{50}} = (x^2 + 100x + 2450)e^x$$

(50th derivative)

Lastly, when 3 is selected, the derivative appears accordingly.

That is, for the function $y = x^3e^x$

$$\frac{d^1y}{dx^1} = (x^3 + 3x^2)e^x$$

(first derivative)

$$\frac{d^2y}{dx^2} = (x^3 + 6x^2 + 6x + 0)e^x$$

(second derivative)

$$\frac{d^{50}y}{dx^{50}} = (x^3 + 150x^2 + 7350x + 117600)e^x$$

(50th derivative)

The users will find a pattern when they find the derivatives in order and hence will understand how the pattern can be given a general notation. Additionally helping them in finding the derivatives easily.

You can use WolframAlpha (https://www.wolframalpha.com) to check the hundredth derivative you found. Type, for example,

100th derivative of x^3*e^x

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into the search line.

Rate subtopic 5.7 Further graph properties

Help us improve the content and user experience.