Index

The big picture

? Guiding question(s)

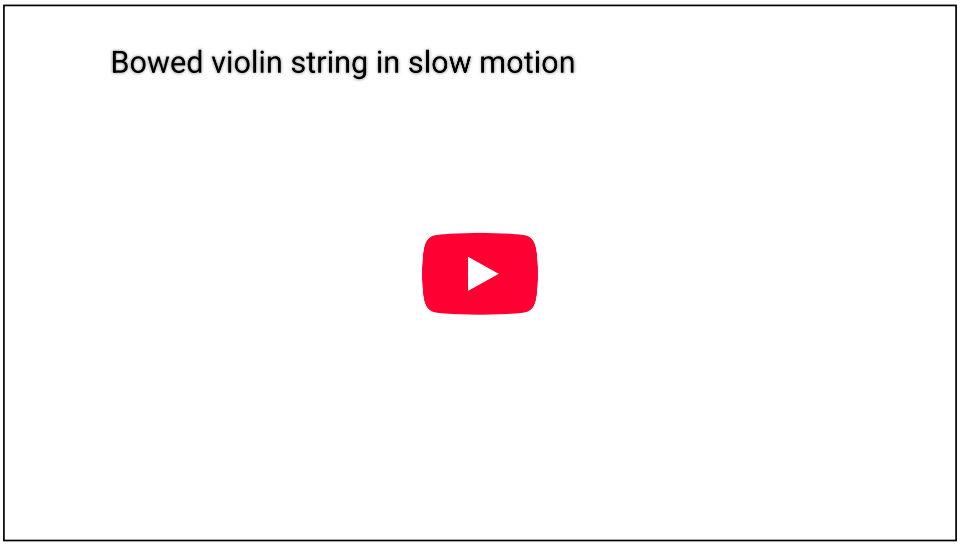
- What distinguishes standing waves from travelling waves?
- How does the form of standing waves depend on the boundary conditions?
- How can the application of force result in resonance within a system?

Keep the guiding questions in mind as you learn the science in this subtopic. You will be ready to answer them at the end of this subtopic. The guiding questions require you to pull together your knowledge and skills from different sections, to see the bigger picture and to build your conceptual understanding.

Look at **Video 1** to **Video 4**. They all show standing waves. But what is a standing wave? And how is a standing wave produced?

Standing waves are all around us, including music, and heating food using microwaves. The understanding of standing waves even contributes to our models of electrons existing in orbitals around nuclei.

Video 1. A Ruben's tube.



Video 2. Bowed Violin String in Slow Motion.

More information for video 2

Video 3. Chladni plate.

Video 4. Standing waves in water.

More information for video 4

The video shows an experiment in a large water tank, where waves are created to study how they behave. It starts by generating a wave group with a frequency of 0.6 Hz and a height of 4 inches. These waves travel down the tank until they hit a fixed wall at the end. When the waves bounce back, they overlap with new incoming waves, creating a pattern called a standing wave. Unlike regular traveling waves that move forward, standing waves appear to "vibrate in place" because the reflected waves and incoming waves combine. This doubling effect makes the wave peaks and troughs much taller and deeper.

The shape of the standing wave depends on the tank's boundaries. Since the wall doesn't move, the waves reflect perfectly, creating points where the water hardly moves (nodes) and points where it moves the most (antinodes). Over time, the wave maker adds more energy, increasing the wave height. The waves are held up by gravity and water

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

pressure but slowly lose energy due to the water's internal friction. Even though the tank walls cause some friction, it's not enough to stop the waves quickly. The video shows how the wave height doubles when incoming and reflected waves combine, forming a pattern described by the equation

$$\text{Amplitude} = \left(\frac{H}{2}\right)\text{Cos}(kx)\text{Cos}(\omega t)$$

The video also shows a phenomenon called tank seiche, where the standing wave sloshes back and forth after the wave maker is turned off. This back-and-forth motion is like water rocking in a bathtub. Eventually, after about half an hour, the waves fade away. To stop them faster, the wave maker acts like a "wave absorber" by producing new waves that cancel out the existing ones.

Through these visuals, viewers see firsthand how boundaries (like walls) influence wave behavior, how standing waves differ from traveling ones, and how precise energy input creates resonance.

Prior learning

Before you study this subtopic make sure that you understand the following:

- Transverse and longitudinal travelling waves (see subtopic C.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43778/))
- Superposition of waves and wave pulses (see subtopic C.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44900/))
- Wave behaviour at boundaries in terms of reflection, refraction and transmission (see subtopic C.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-44900/))

C. Wave behaviour / C.4 Standing waves and resonance

What are standing waves?

C.4.1: Nature and formation of standing waves   C.4.2: Nodes, antinodes, relative amplitude and phase difference

Student view

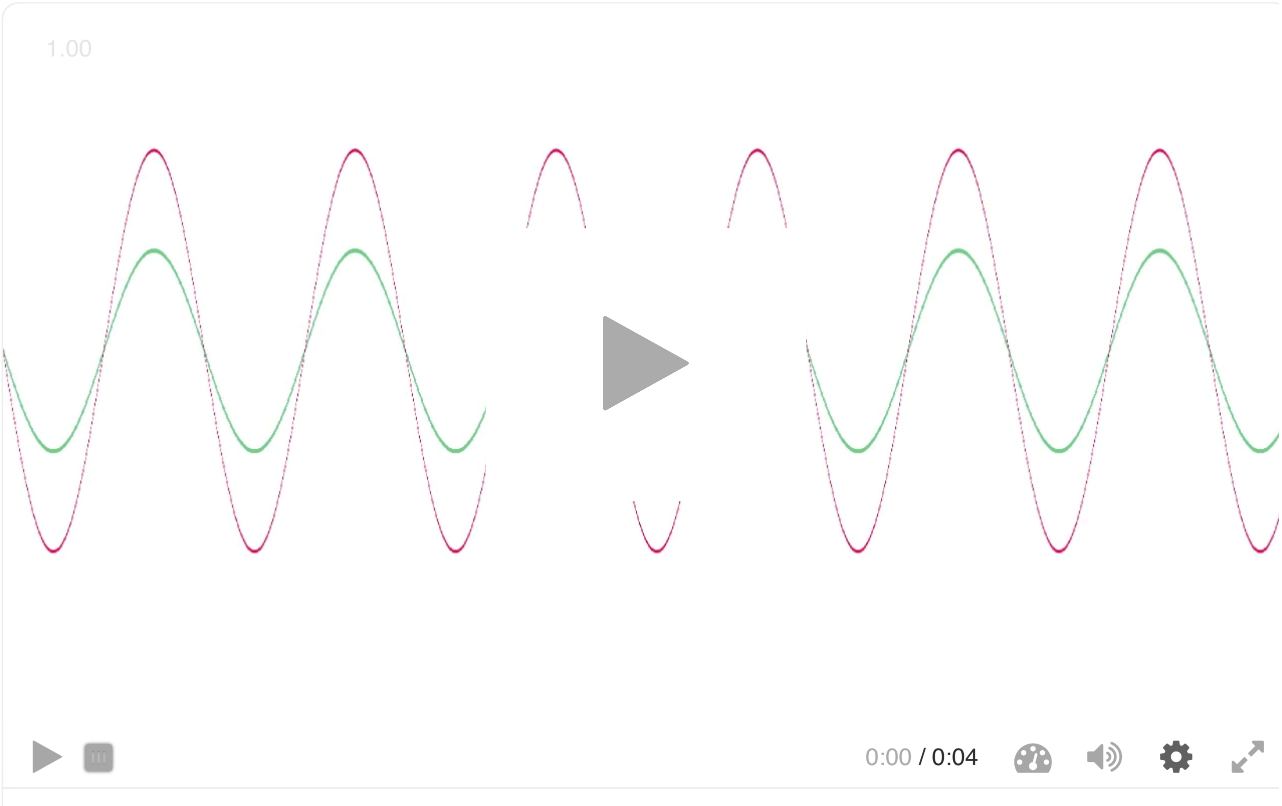
Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Learning outcomes

At the end of this section you should be able to:

- Understand the nature of standing waves and how they are formed.
- Understand what is meant by nodes and antinodes.
- Know the phase difference and relative amplitude of different points along a standing wave.

Interactive 1 shows the formation of a standing wave (red) as a result of the superposition of a wave travelling to the right (green) and a wave travelling to the left (blue). The two travelling waves have the same amplitude. They meet in phase at certain locations, where they interfere constructively. At these locations, their amplitudes add up, so that the amplitude of the resultant standing wave is double the amplitude of the travelling waves. At other locations, the blue and green waves meet out of phase (i.e. with a phase difference of 180°) and interfere destructively. At these locations, their amplitudes cancel each other completely, so that the resultant standing waves has zero amplitude here.



Interactive 1. The Formation of a Standing Wave (Red) as a Result of the Superposition of a Wave Travelling to the Right (Green) and a Wave Travelling to the Left (Blue).

More information for interactive 1

Student
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The animation illustrates the formation of a standing wave through the superposition of two traveling waves. A green wave moves to the right while a blue wave moves to the left. As they interact, they create a red standing wave, which results from their constructive and destructive interference. At certain points, where the waves meet in phase, the amplitudes add up, forming peaks and troughs. At other points, where the waves meet out of phase, they cancel each other out, creating fixed positions with zero displacement called nodes. The animation shows that while the individual waves continue to move, the standing wave appears stationary, with alternating points of maximum and zero displacement.

For a standing wave, the points of maximum displacement are known as antinodes, and the points of zero displacement are known as nodes.

From **Interactive 1**, we can see that the nodes and antinodes do not move along the wave, but always stay in the same horizontal position, with the antinodes only oscillating vertically. This is a characteristic of standing waves.

Revisit the simulation from section C.3.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/superposition-of-waves-and-young's-double-slit-interference-id-46613/) in **Interactive 2** to investigate the superposition of wave pulses. Use the sliders to change the height (amplitude) of the pulses and the width (wavelength) of the pulses. The superposition of travelling waves works in the same way. When two (or more) waves superpose, they can interfere.

Student view

Interactive 2. Superposition of wave pulses simulation.

Credit: Tom Walsh

More information for interactive 2

The interactive simulation titled, Superposition of wave pulses simulation, illustrates the superposition of wave pulses, allowing users to manipulate various parameters and observe how waves interact. It features two wave pulses traveling towards each other, with controls to adjust their heights (amplitudes) and widths (wavelengths). The pulses can be displayed individually, helping users visualize their contributions to the overall wave behavior.

A time control slider allows users to step through different moments of the interaction. The animation speed slider provides the option to slow down or speed up the motion, making it easier to observe the details of the interaction. When the pulses meet, their amplitudes combine according to the principle of superposition. Users can investigate how the pulses either constructively or destructively interfere. If two crests or two troughs overlap, the resultant wave has a higher amplitude. If a crest and a trough of the same amplitude and width meet, they cancel each other out, creating a momentary flat wave.

After the pulses pass through each other, they continue moving in their original directions, maintaining their shapes. This illustrates that wave pulses do not bounce off one another but pass through unchanged after interaction. The simulation encourages the exploration of different amplitude and width settings to analyze varying levels of constructive and destructive interference.

By adjusting the parameters and observing the interactions, users can develop an understanding of fundamental wave behavior. Through these observations, the simulation demonstrates key wave phenomena, such as interference and amplitude summation, reinforcing concepts in wave physics.

Producing a standing wave

Concept

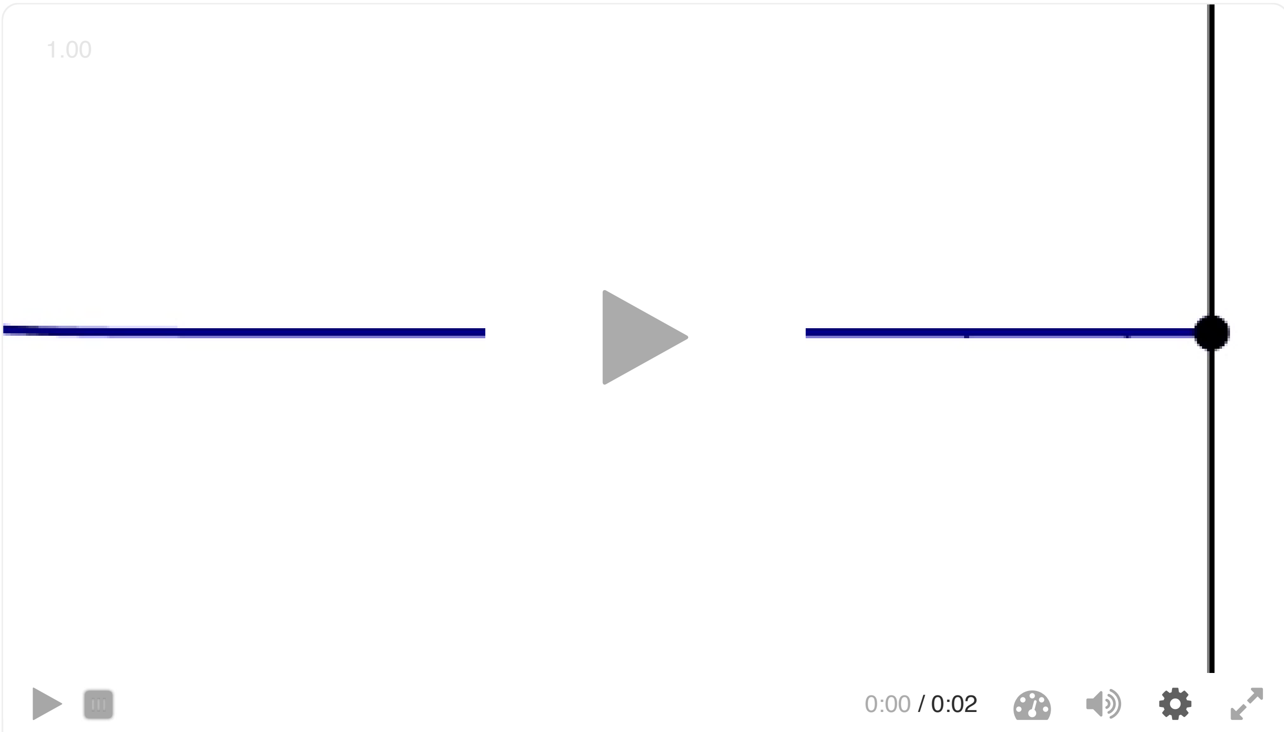
A standing wave is a wave where there is **no net transfer of energy**, or matter. Compare a standing wave with a travelling wave, which transfers energy, but not matter, in the direction of the wave.

When a wave in a string reflects off a hard boundary (it meets a more dense medium), it reflects and undergoes a π radian (180°) phase change (**Interactive 3**). The incoming and reflected waves in **Interactive 3** interfere just like the two travelling waves in **Interactive 2**. When they meet in phase (a crest meets a crest, or a trough meets a trough), they interfere constructively, and the resultant amplitude is double the amplitude of the incoming and

Overview (/study/ap aa- hl/sid- 423- cid- 762593/c

reflected waves; an antinode is formed here. When the two waves meet in antiphase (a crest meets a trough), they interfere destructively, and the resultant amplitude is zero; a node is formed here. This is how a standing wave is formed on a string.

As opposed to a progressive wave, a standing wave transfers no energy, hence the name ‘standing’.



Interactive 3. Wave Reflecting off a Hard Boundary.

More information for interactive 3

This interactivity features a short video demonstrating the formation of a standing wave on a string. Users can play or pause the video using the playback controls, and the timeline at the bottom allows them to navigate through the footage.

The wave travels along the string and reflects off a fixed boundary. As the wave reflects, its phase inverts—this means the crest becomes a trough and vice versa, a phenomenon clearly visible in the video. This inversion is a result of the wave hitting a rigid end, which forces the reflected wave to reverse direction and flip vertically.

Throughout the interaction, alternating regions called antinodes (which vibrate with maximum amplitude) and nodes (which remain still) become prominent. These patterns result from the interference of the incident and reflected waves, forming a stable standing wave structure.

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Making connections

A wave reflecting off a hard boundary is an example of Newton’s third law of motion (section A.2.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-forces-id-44732/)). As the wave pulse meets the wall, the string exerts an upwards force on the wall. The wall exerts an equal (and opposite) downwards force on the string, causing the pulse to invert, and a π radian (180°) phase change to occur.

Study skills

Note that destructive interference can only occur when the **amplitudes** of the outgoing wave and the reflected wave are **equal**. If one wave has a larger amplitude than the other wave, this will not lead to a zero amplitude resultant wave, even if the two waves have π radian phase difference.

Exercise 1

Click a question to answer

Look at the simulation in **Interactive 4**:

- adjacent antinodes in the standing wave (for example, A and F) are in antiphase, this means that their phase difference is π radians (i.e. 180°). The direction of the displacements of A and F is always opposite (e.g. when A moves up F moves down).
- E and F are in phase; their displacements always have the same direction.

Overview
(/study/ap
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hl/sid-
423-
cid-
762593/c

- **Interactive 4.** Formation of a standing wave.

More information for interactive 4

The interactive titled, Formation of a standing wave, demonstrates the formation of a standing wave by the superposition of two traveling waves moving in opposite directions. The individual waves are represented by color-coded dashed curves, Wave 1(pink color) and Wave 2 (blue color). They interfere and produce a resultant standing wave, shown as a solid green curve.

Nodes, marked as black dots, are points that do not oscillate because the two waves cancel each other out at these positions due to destructive interference. Antinodes are points where the amplitude of oscillation is maximum as a result of constructive interference. The interactive highlights how adjacent antinodes, such as points A and F, are in antiphase, meaning they always move in opposite directions with a phase difference of 180 degrees or π radians. If one moves up, the other moves down.

Conversely, points E and F are in phase, meaning they always move in the same direction. The interactive allows users to visualize the continuous oscillation of the standing wave by adjusting the slider for phase angle alpha that ranges from 0 to 720 degrees. At any given moment, the displacement of different points on the wave varies based on their position in the oscillation cycle.

This interactive simulation helps in understanding wave properties such as constructive and destructive interference, phase relationships, and resonance phenomena. It visually reinforces key wave concepts by illustrating how standing waves form through the combination of two traveling waves.

The phase difference of points in each part of a standing wave is shown in **Figure 1**, the red parts of the standing wave are in phase. The blue parts of the standing wave are also in phase. Any point in the red part is in antiphase with any point in the blue part.

Student
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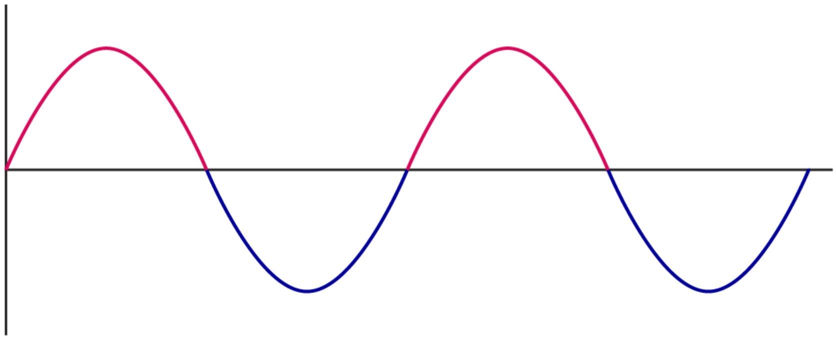

Figure 1. In this standing wave, the red parts are in antiphase with the blue parts.

More information for figure 1

The image depicts a graph of a sine wave showing alternating red and blue sections. The graph has no labeled axes, but it indicates a wave pattern with peaks and troughs. The red parts of the wave are in antiphase with the blue parts, meaning red and blue sections are out of phase with each other. This graphical representation illustrates the concept of phase difference in a standing wave, where points in red sections are in phase with each other and out of phase with points in blue sections.

[Generated by AI]

Concept

In a standing wave, there is a range of amplitudes along the medium, from maximum amplitude at the antinodes to zero amplitude at the nodes. However, all points on the wave fall into two **phase** groups. Half the points are in phase with one another, and in antiphase with the other half of the points. There are no $\frac{\pi}{2}$ or $\frac{3\pi}{4}$ phase relationships in a standing wave.

Worked example 1

Antinode A on a standing wave is separated from antinode B by three nodes.

1. How many wavelengths apart are A and B?
2. What is their phase relationship?

Overview
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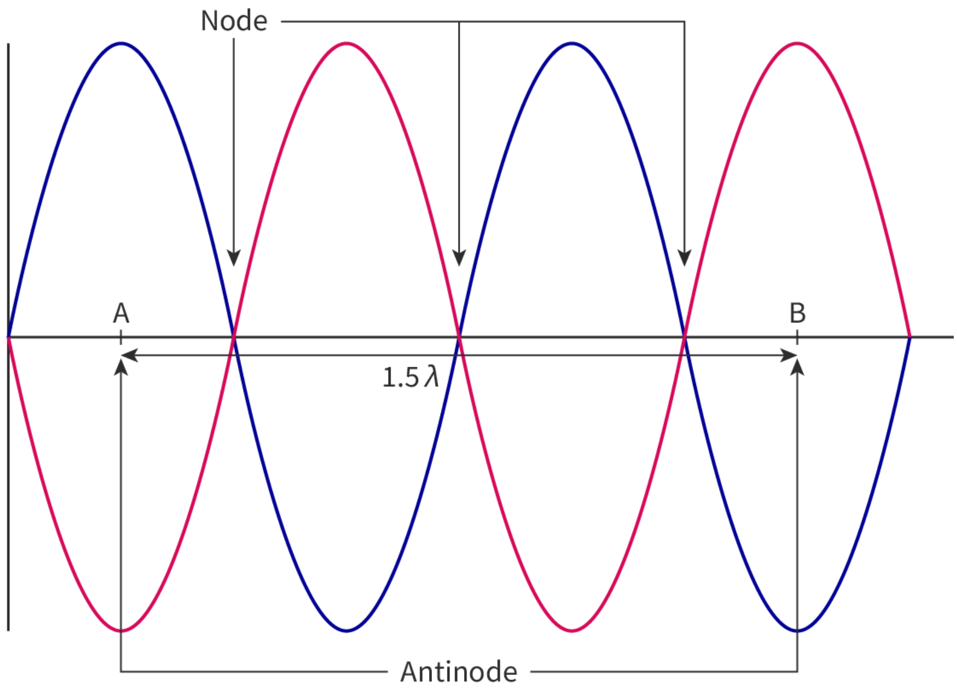
hl/sid-
423-
cid-
762593/c



1. If A is the first antinode, B is the fourth antinode. There are three antinode-to-antinode intervals. Each antinode-to-antinode interval is half a wavelength, so A and B are separated by three half wavelengths: $\left(\frac{3\lambda}{2}\right)$ or 1.5λ.
2. Every other antinode is in phase. The first and third antinodes are in phase. The second and fourth antinodes are in phase with one another, but in antiphase with the first and third antinodes. The first and fourth antinodes (A and B) are in antiphase, and have a phase difference of π radians.

You can see from **Interactive 4** that E, A, and F have the same amplitude. Look at **Figure 2.** You can see that red points have the same amplitude as yellow points. You can also see that red points have **opposite displacement** to yellow points.

Student
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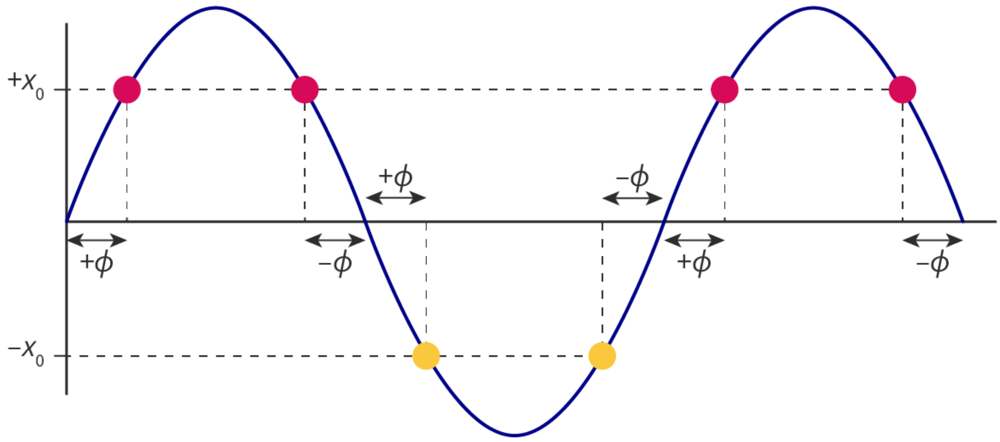


Figure 2. Amplitude and displacement.

More information for figure 2

The image displays a sine wave graph with red and yellow points plotted on the wave. The horizontal axis represents phase shift denoted by ϕ, ranging from negative to positive. The vertical axis shows displacement with values ranging between $-x_0$ and $+x_0$. Red points are located at the peaks of the wave, indicating maximum and minimum displacement, while yellow points lie at the troughs, indicating opposite displacement. Dashed lines indicate the amplitude at these points, showing consistent distances from the zero line to both red and yellow points.

[Generated by AI]

Can you locate the nodes and antinodes in **Interactive 5**? How far apart are adjacent nodes, in terms of number of wavelengths? How far apart are adjacent antinodes?

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Interactive 5. Find the Nodes and Antinodes on This Standing Wave.

More information for interactive 5

The interactive image displays a standing wave with multiple marked hotspots where users can click to learn more about nodes and antinodes. The wave has alternating peaks and troughs, with specific locations along the horizontal axis where the wave does not move, known as nodes. Between these nodes are points of maximum displacement known as antinodes. The interactive explains the spacing between adjacent nodes and antinodes in terms of wavelength and helps users identify their positions on the wave.

A hotspot at the leftmost point where the wave crosses the horizontal axis reads: Node: A region of zero displacement, where the wave experiences destructive interference.

A hotspot at the first peak above the horizontal axis reads: Antinode: A region of maximum displacement, where the wave experiences constructive interference.

A hotspot at the center where the wave crosses the horizontal axis again reads: Node: A region of zero displacement, where the wave experiences destructive interference.

A hotspot at the first trough below the horizontal axis reads: Antinode: A region of maximum displacement, where the wave experiences constructive interference.

A hotspot at the rightmost point where the wave crosses the horizontal axis again reads: Node: A region of zero displacement, where the wave experiences destructive interference.

Work through the activity to check your understanding of standing waves.

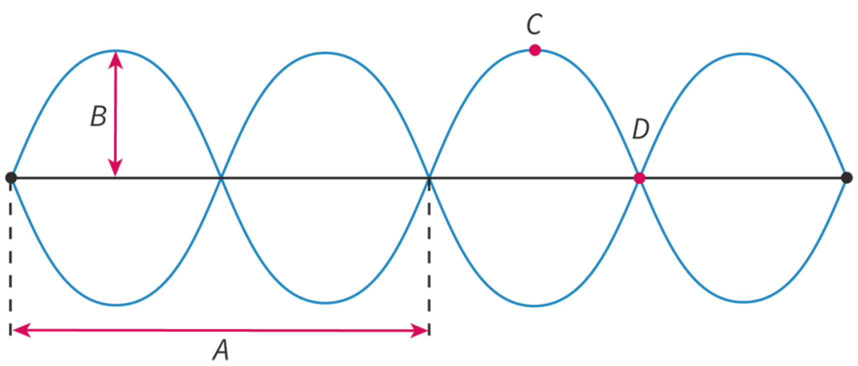
Student view

Activity

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

- **IB learner profile attribute:**
  - Knowledgeable
  - Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 10 minutes
- **Activity type:** Individual activity

Label each part of the standing wave in **Interactive 6** by dragging the labels to the correct boxes A—D.



A:

B:

C:

D:

Node

Wavelength

Antinode

Amplitude

Check

Interactive 6. Labelling a Standing Wave.

More information for interactive 6

This interactivity features a drag-and-drop labeling activity focused on the anatomy of a standing wave, as illustrated in the provided diagram. The image shows a sinusoidal wave fixed at both ends, with four labeled points A, B, C, and D each representing a specific wave property. Users are asked to drag the correct terms—Node, Antinode, Wavelength, and Amplitude—to their corresponding locations. Point A spans the distance between two adjacent nodes and represents the wavelength. Point B is shown as a vertical arrow from the centerline to the peak of the wave,

Student
view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

indicating the amplitude. Point C is located at the crest, where the displacement is maximum, representing an antinode. Point D lies on the central axis where the displacement is always zero, identifying it as a node. This interactive effectively reinforces key concepts in wave physics by requiring learners to match terminology with visual representations of wave properties.
In the image, the wave is illustrated with marked points for specific features. The horizontal line represents the distance, while the vertical dotted lines show the region within which the wavelength is spread across. The labels are designed to be matched with the corresponding parts of the standing wave. The user can check their answers by clicking the "Check" button, which verifies the correctness of their labeling. The correct labels are displayed, and users can retry if needed.
Solution:
Point A, Wavelength.

Point B, Amplitude.

Point C, Antinode.

Point D, Node.

The green checkmarks next to each option confirm the accuracy of the answers. Finally, the consistent green line along with a score of $4/4$ and a star indicates that all answers were correct.

Answer the following questions:

1. What is the phase difference between two adjacent antinodes?
2. What is the phase difference between an antinode and an adjacent node?
3. What is the phase difference between two points in the same ‘loop’?

5 section questions

Question 1

SL HL Difficulty:

A wave reflects off a hard boundary and travels in the opposite direction with the same frequency, but a decreased amplitude.

True or false?

The reflected wave interferes with the outgoing wave and a standing wave is formed.

False

Student
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Accepted answers

False, F, false, f

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Explanation

The waves have different amplitudes. It is impossible for nodes to form, as the crest of the reflected wave cannot sum to zero with the (larger amplitude) crest of the outgoing wave. A standing wave cannot be formed.

Question 2

SL HL Difficulty:

When waves interfere fully constructively, 1 antinodes ✓ are formed.

When waves interfere fully destructively, 2 nodes ✓ are formed

Accepted answers and explanation

#1 antinodes

#2 nodes

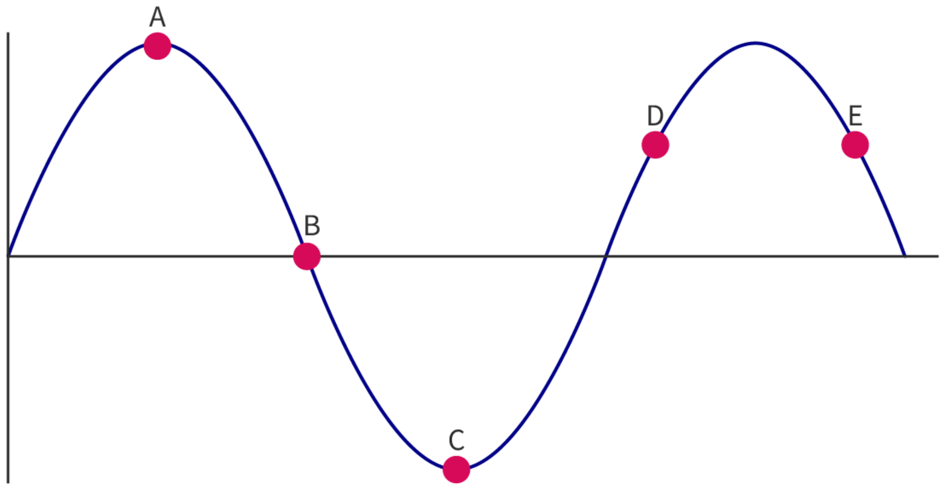
General explanation

Antinodes are regions of maximum displacement, where two waves meet and interfere constructively. Nodes are regions of zero displacement, where waves meet and interfere destructively.

Question 3

SL HL Difficulty:

The diagram shows a standing wave. Which pair of points are in antiphase, with the same amplitude?



More information

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

1 A and C ✓

2 D and E

3 C and D

4 C and E

Explanation

A and C have the same amplitude. As A moves down, C moves up, so they are in antiphase.

Question 4

SL HL Difficulty:

There are two points, A and B, on a standing wave. Neither of the points is at a node. The distance between A and B is (n + ½)λ. What is their phase difference in radians?

1 π ✓

2 2π

3 $\frac{\pi}{2}$

4 $\frac{\pi}{4}$

Explanation

As the separation is (n + ½)λ, particles A and B will always be in antiphase. The phase difference between A and B is π radians.

Question 5

SL HL Difficulty:

There are two points, A and B, on a standing wave. Neither of the points is at a node. The distance between A and B is 2.5λ. Which of the following must always be true?

1 A and B oscillate with the same amplitude ✓

2 A and B are always in phase

3 A and B always have the same displacement

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

4 The amplitude of the motion of A and B is equal to the amplitude of the wave

Explanation

As A and B are separated by 2.5λ, you can see from **Figure 2** that they will always be in antiphase. Their distance from the equilibrium position (amplitude) is the same.

The displacements of A and B will not be the same, as when A has positive displacement, B has negative displacement, and vice versa. The only time their displacements are the same is when A and B are both in the equilibrium position, which only happens twice per cycle.

The amplitude of A and B's displacements can be equal to the amplitude of the wave, but this is only true for antinodes of the wave. For any other pair of points, this is not true.

C. Wave behaviour / C.4 Standing waves and resonance

Standing waves in strings and pipes

C.4.3: Standing wave patterns in strings and pipes

Learning outcomes

At the end of this section you should be able to understand the standing wave patterns in strings and pipes when there are different boundary conditions.

Look at **Video 1**. What do you notice about the number of wavelengths? Is it possible for there to be any number of wavelengths? Are there any limitations? If so, how do these limitations affect the relationships between standing waves of different frequencies?

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

Video 1. Standing waves on a string.

Standing waves in strings

For the wave on the string in **Video 1**, you can see that certain rules have to be obeyed. The right-hand end of the string is being held firmly, so there is a node. The left-hand end of the string is oscillating, but the amplitude of oscillation is small, so we can also consider this point to be a node.

The boundary conditions for the string is two **fixed boundaries** – the ends of the string cannot move. This means that only certain wavelengths are allowed on the string. **Figure 1** shows the longest possible wavelength, known as the **first harmonic**.

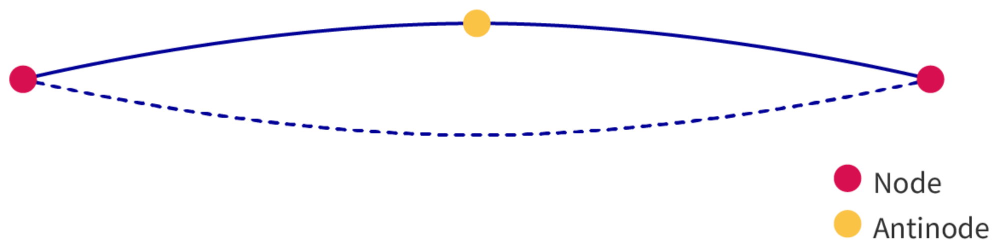


Figure 1. The first harmonic for a wave on a string with two fixed boundaries.

More information for figure 1

The image is a diagram representing the first harmonic of a wave on a string with two fixed boundaries. The diagram features a blue solid curve representing the wave, stretching between two red points marked as nodes on either end of the string. In the center of the wave, a yellow circle labels the antinode, where the wave reaches its maximum amplitude. The nodes are points where the wave has zero amplitude. The wave shape is a single curve (half a

Student
view

Overview
(/study/ap
aa-
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423-
cid-
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wavelength) with no crossings over the baseline, illustrating the longest possible wavelength that can fit on the string, known as the first harmonic. The caption on the right clarifies the symbols, with nodes indicated by red circles and the antinode by a yellow circle.

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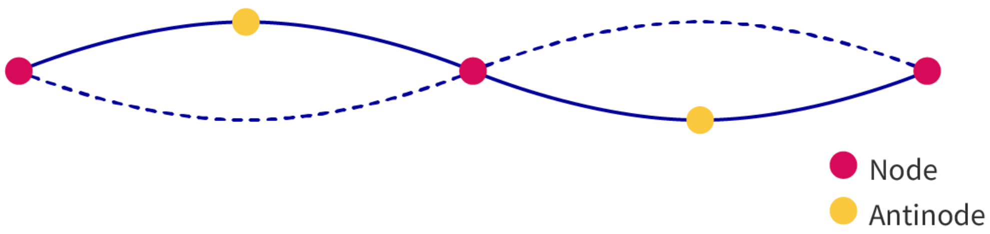
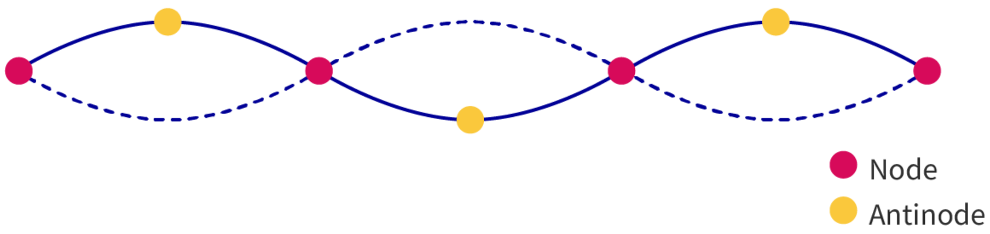
The second harmonic is the second longest possible wavelength. Sketch the second harmonic for a wave on a string with two fixed boundaries. Click on ‘Show or hide solution’ to see if you are correct.



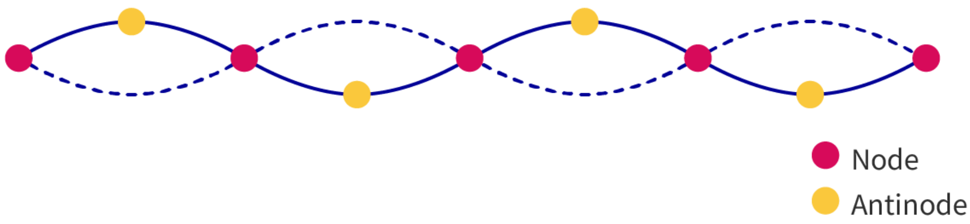
The second harmonic for a wave on a string with two fixed boundaries.

Now sketch the third and fourth harmonics. Click on ‘Show or hide solution’ to see if you are correct.



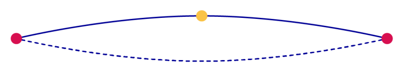
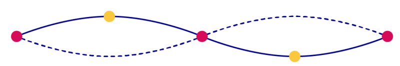
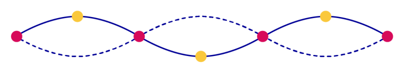
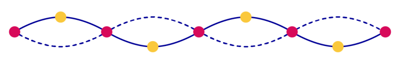
Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c



The third and fourth harmonics for a wave on a string with two fixed boundaries.

What is the relationship between the different harmonics and their wavelengths? Match each equation linking wavelength, λ, and string length, L, to the correct harmonic in **Interactive 1.**

Harmonic	Wave	Equation
First harmonic		
Second harmonic		
Third harmonic		
Fourth harmonic		

$\lambda = L$

$\lambda = \frac{1}{2}L$

$\lambda = \frac{2}{3}L$

$\lambda = 2L$

Check

Interactive 1. Match the Equation to the Harmonic.

Student view

More information for interactive 1

This is a drag-and-drop interactivity that involves matching wave diagrams of harmonics with their corresponding wavelength equations on a string fixed at both ends. The table has three columns labeled “Harmonic,” “Wave,” and “Equation.” The Harmonic column lists the different harmonics: “First harmonic,” “Second harmonic,” “Third harmonic,” and “Fourth harmonic.” The Wave column shows graphical representations of standing waves with nodes and antinodes marked by red and yellow dots, respectively. Each wave diagram represents a different mode of vibration that occurs along a string fixed at both ends.

In the “Equation” column, there are blank slots where users can drag and drop the correct equation from the set of available labels provided on the right. The draggable labels include $\lambda = L$, $\lambda = \frac{1}{2}L$, $\lambda = \frac{2}{3}L$, and $\lambda = 2L$.
The harmonics shown in the image represent different standing wave patterns on a string fixed at both ends. The first harmonic displays a single loop, with two nodes at the ends and one antinode in the center. The second harmonic shows two loops, indicating the presence of three nodes and two antinodes. The third harmonic has three loops, formed by four nodes and three antinodes. The fourth harmonic displays four loops, with five nodes and four antinodes, fitting exactly two full wavelengths along the length of the string. Each higher harmonic adds an additional loop, showing increasing frequency and decreasing wavelength.
Users interact with the simulation by dragging the equation labels and dropping them into the correct slots in the “Equation” column based on the wave patterns shown. The goal is to visually match each harmonic with its mathematical wavelength relation in terms of the string length *L*. This activity helps reinforce the concept of standing waves and harmonics in physics, particularly how different vibrational modes relate to string length and wavelength. Once all the labels are placed correctly, the user can press the “Check” button to confirm their answers.
Solution:
Here are the correct equations matched with each harmonic based on the standing wave patterns:
The first harmonic: $\lambda = 2L$
The second harmonic: $\lambda = L$
The third harmonic: $\lambda = \frac{2}{3}L$
The fourth harmonic: $\lambda = \frac{1}{2}L$
These correspond to how many half-wavelengths fit into the string length *L*:
The first harmonic has half a wave $(\lambda = 2L)$.
The second harmonic has a full wave $(\lambda = L)$.
The third harmonic has 1.5 waves $(\lambda = \frac{2}{3}L)$.
The fourth harmonic has two full waves $(\lambda = \frac{1}{2}L)$.
This interactive reinforces physics concepts related to wave behavior, vibration modes, and how harmonic frequencies relate to string length and wavelength.

Student view

If we write down the equations in **Interactive 1** with $2L$ as the numerator, we get the equations for wavelength in **Table 1**.

Table 1. Equations for wavelength in a string with two fixed boundaries.

Harmonic	Equation
First harmonic	$\lambda_1 = \frac{2L}{1}$
Second harmonic	$\lambda_2 = \frac{2L}{2}$
Third harmonic	$\lambda_3 = \frac{2L}{3}$
Fourth harmonic	$\lambda_4 = \frac{2L}{4}$

The general equation is shown in **Table 2**.

Table 2. The general equation for wavelength.

Equation	Symbols	Units
$\lambda_n = \frac{2L}{n}$	λ_n = wavelength	metres (m)
	L = length	metres (m)
	n = harmonic (first, second, etc.)	unitless

This equation is not given in the DP physics data booklet, and you do not need to know how to derive it.

Worked example 1

A 1.5 m long string is oscillating at the eighth harmonic with a wave speed of 4.2 m s^{-1}. Determine the frequency of the wave.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$L = 1.5\text{ m}$ $v = 4.2\text{ m s}^{-1}$ $n = 8$
Step 2: Determine the wavelength.	$\lambda_\text{n} = \frac{2L}{n} = \frac{(2\times1.5)}{8} = 0.375\text{ m}$
Step 3: Determine the frequency.	$v = f\lambda$ $f = \frac{v}{\lambda} = \frac{4.2}{0.375} = 11.2\text{ Hz} = 11\text{ Hz (2 s.f.)}$

Standing waves in pipes

Standing waves in pipes are very common in wind instruments, such as the flute (**Figure 2**).

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Figure 2. Flautist playing the flute.

Credit: Allison Michael Orenstein, Getty Images

A flute can be considered to be a pipe, open at both ends. The musician blows in one end. This creates a vibration, and a standing wave is formed.

When the end of a pipe (such as a flute) is open, an antinode is formed. So, the standing wave in a flute (open–open pipe) has an antinode at each end.

Figure 3 shows the displacement of air particles near the ends of an open–open pipe.

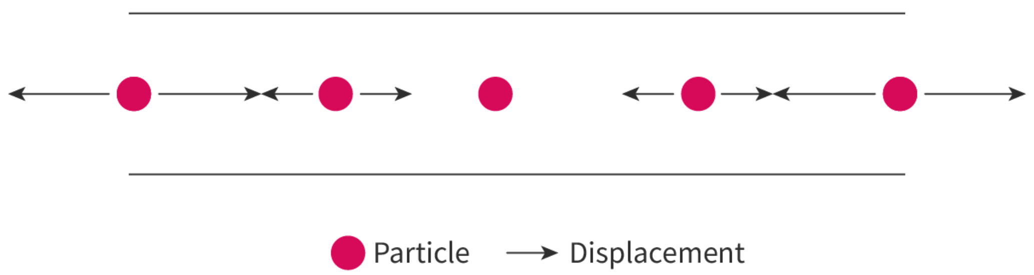


Figure 3. Displacement of air particles near the ends of an open—open pipe.

More information for figure 3

The image is a diagram illustrating the displacement of air particles within an open-open pipe. The diagram shows five red circles representing particles. These circles are arranged linearly between two parallel horizontal lines, representing the pipe boundaries. Displacement arrows indicate the direction of movement for each particle. Particles near the ends of the pipe have arrows pointing outward, indicating maximal displacement at these points, which are the antinodes of a standing wave. In the center, particles have arrows indicating no movement, representing a node where there is no displacement. A legend at the bottom indicates that the red circles symbolize 'Particle' and arrows represent 'Displacement.' This diagram visually conveys the concept of standing wave patterns in acoustics within a pipe with open ends.

[Generated by AI]

Student view

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The particles at the open ends of the pipe oscillate with maximum displacement. They are at the antinodes of the wave. The particles in the middle do not oscillate at all. They are at the node of the wave.

Figure 4 shows a displacement–distance diagram for the particles at a particular moment in time when displacement is at its maximum.

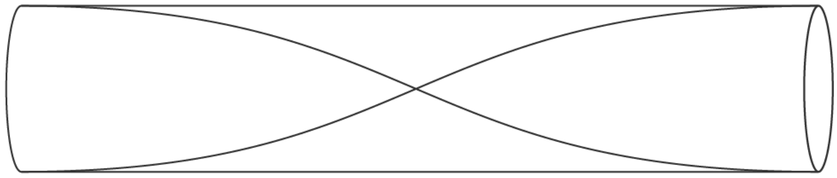

Figure 4. Displacement—distance graph for a standing wave in an open—open pipe.

More information for figure 4

The image shows a diagram representing a standing wave in an open-ended pipe. The pipe is drawn horizontally with two labeled open ends. Inside the pipe, a sine wave is depicted, illustrating the displacement of particles along the pipe's length. The wave intersects at the center of the pipe, indicating points of maximum displacement and nodes where there is no movement. This diagram emphasizes the wave's behavior in a longitudinal setting within the confines of the open-ended pipe.

[Generated by AI]

The displacement–distance graph in **Figure 4** looks like a transverse wave. However, a sound wave is a longitudinal wave. The particles oscillate along the length of the pipe, and not perpendicular to it.

Nature of Science

Aspect: Models

Student view

Overview
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When a displacement—distance graph of a standing wave in a pipe is drawn, it looks like a transverse wave, even though it is a longitudinal wave. Why do physicists present the wave in this way? Does this model have any implications for our understanding of standing waves in pipes?

Figure 4 shows the first harmonic for a standing wave in an open–open pipe. Sketch the second, third and fourth harmonics for an open–open pipe. Remember, each end is an antinode. Click on 'Show or hide solution' to see the answers in **Figure 5**.

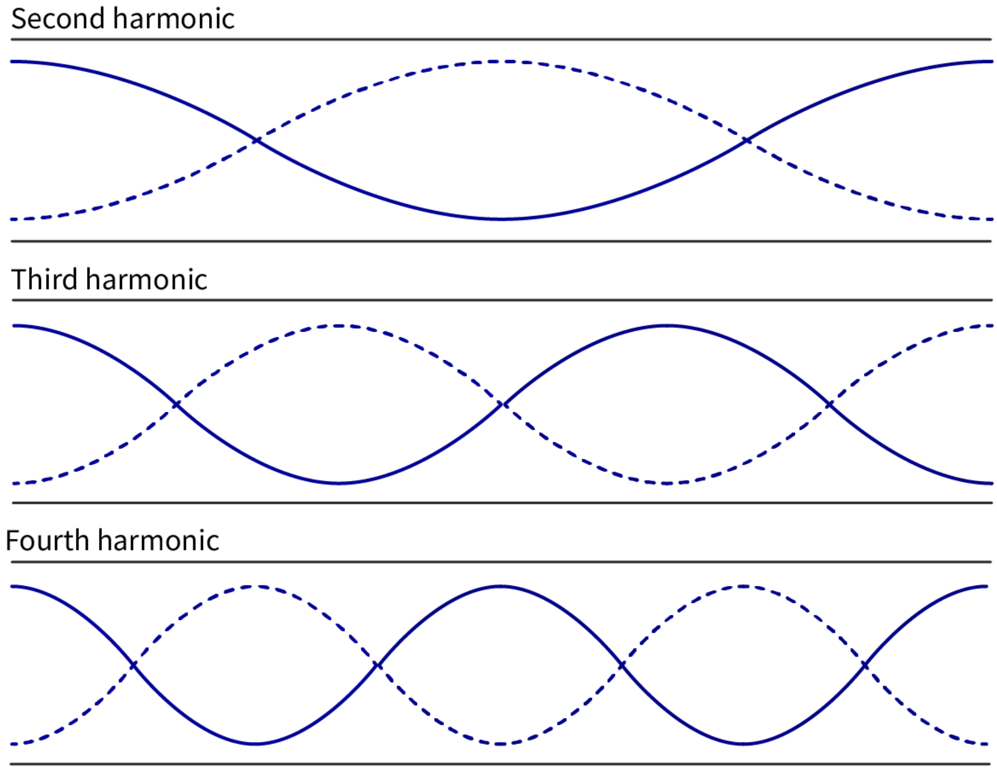


Figure 5. The second, third and fourth harmonics for an open—open pipe.

The wave forms in **Figure 4** and **Figure 5** also represent the harmonics in a string with **free boundary conditions at each end**. The string is free to move at the ends, and there are antinodes at each end.

Study skills

The node for a string occurs where there is a fixed boundary. The node for a pipe occurs where there is a closed end. The particles are unable to oscillate at the fixed boundary and the closed end.

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

An antinode for a string occurs where there is a free boundary. An antinode for a pipe occurs where there is an open end. The particles are able to oscillate at the free boundary and the open end.

Try and write down the wavelength equation for each harmonic in an open–open pipe, where L is the length of the pipe, and λ is the wavelength of the wave. Click on 'Show or hide solution' to see the answer.

Table 3. Equations for wavelength in a pipe with two open ends.

Harmonic	Equation
First harmonic	$\lambda_1 = \frac{2L}{1}$
Second harmonic	$\lambda_2 = \frac{2L}{2}$
Third harmonic	$\lambda_3 = \frac{2L}{3}$
Fourth harmonic	$\lambda_4 = \frac{2L}{4}$

Note that these are the same equations as for a string with fixed boundaries at each end (**Table 1**).

Concept

The wavelength equations in **Table 3** (or **Table 1**) can be used for the following situations:

- Standing waves in a pipe that is **open** at each end
- Standing waves in a string that has **fixed** boundaries at each end
- Standing waves in a string that has **open** boundaries at each end.

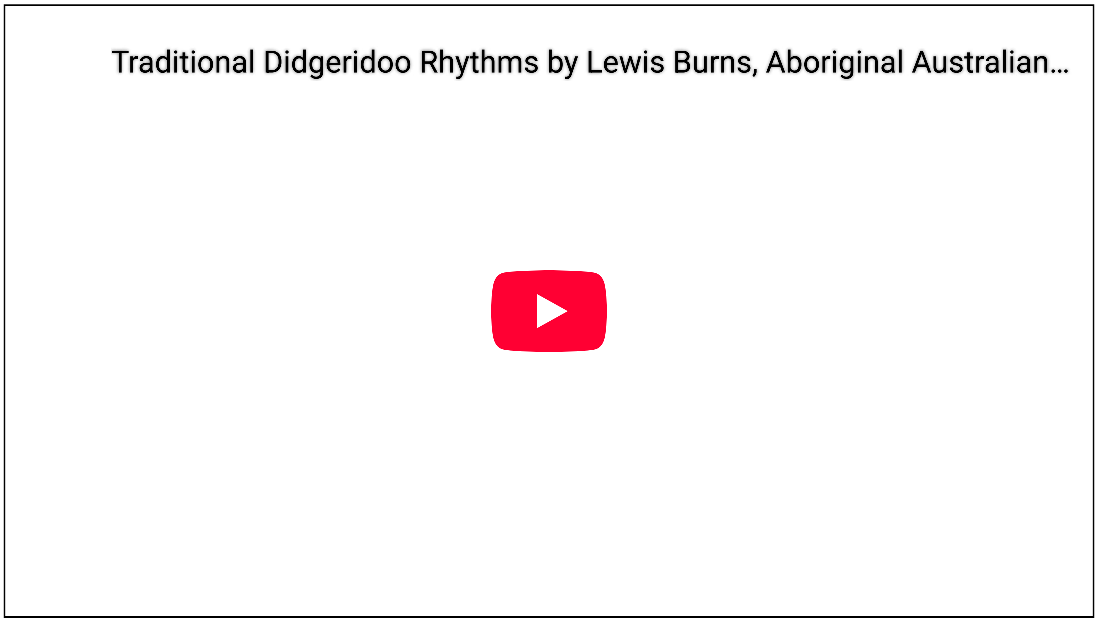
The general equation is:

$$\lambda_n = \frac{2L}{n}$$

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

The didgeridoo is a wind instrument traditionally played by Indigenous Australians. It is a long hollow pipe, open at each end. To play it, musicians place their lips on one end, forming an open–closed pipe (**Video 2**).



Video 2. Playing a didgeridoo.

The air particles are excited, with particles at the open end vibrating with the largest displacement (forming an antinode), and particles at the closed end not vibrating at all (forming a node). **Figure 6** shows the air particles.

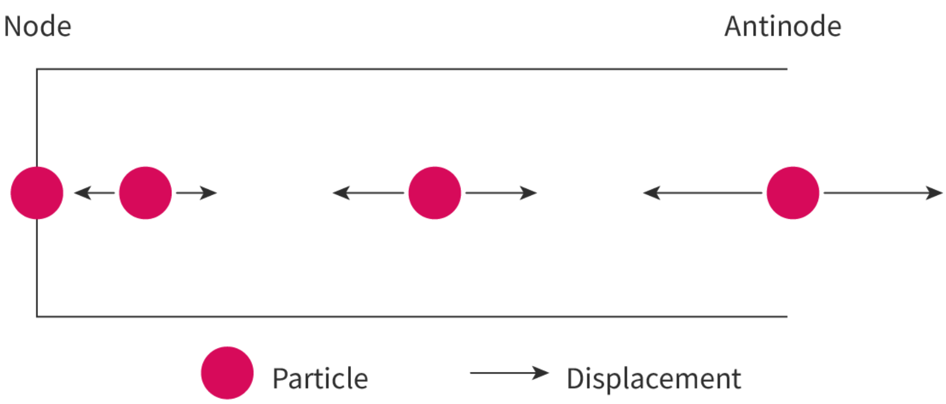


Figure 6. Motion of air particles in an open—closed pipe.

More information for figure 6

The diagram illustrates the motion of air particles in an open-closed pipe. It is segmented into three main parts. At the left end is the 'Node', where particles are closely packed and demonstrate little to no movement. Arrows depict the absence of displacement in these particles. Towards the right is the 'Antinode', showing particles vibrating with significant displacement. This area has larger arrows indicating the direction of particle movement. Between the node

Student view

and antinode, particles gradually exhibit increasing displacement, visualized through arrows of increasing length. A key at the bottom clarifies symbols used: a red circle represents a particle, and a double-headed arrow indicates displacement.

[Generated by AI]

Figure 7 shows a displacement–distance diagram for the particles at a particular moment in time when displacement is at its maximum

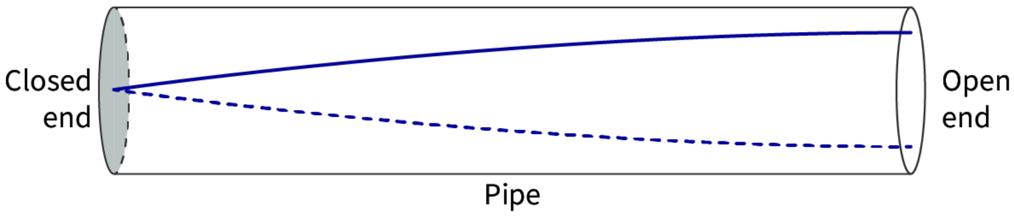


Figure 7. Displacement—distance graph for a standing wave in an open—closed pipe.

More information for figure 7

The image depicts a cylindrical pipe labeled 'Pipe', with a 'Closed end' on the left and an 'Open end' on the right. Inside the pipe, a standing wave is illustrated with two curves, one solid and one dashed, representing the displacement of particles. The solid line starts at the closed end and curves upwards towards the open end, indicating an antinode at the open end. The dashed line starts at the closed end and curves downwards, intersecting the horizontal centerline at the open end, indicating a node. This representation shows the first harmonic, where there is a single node at the closed end and an antinode at the open end.

[Generated by AI]

Figure 7 shows the first harmonic for a standing wave in an open–closed pipe. In an open–closed pipe, the harmonics are first, third, fifth, and so on. There are no even-numbered harmonics in an open–closed pipe.

Draw the next three harmonics for an open–closed pipe. Click on 'Show or hide solution' to see the answer.

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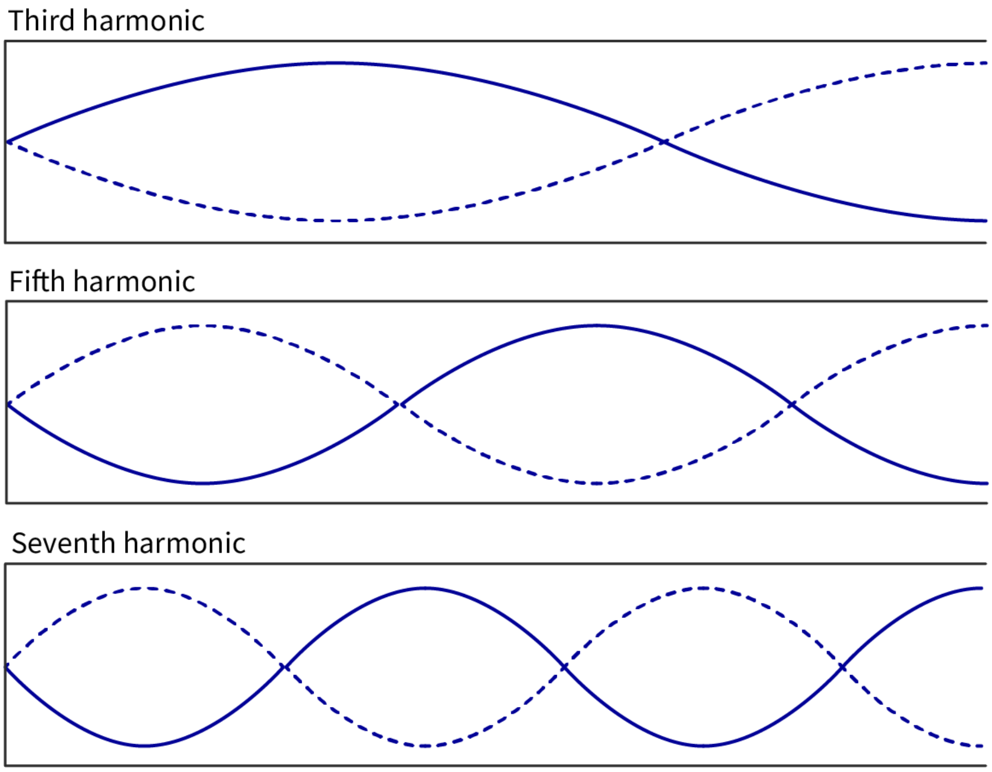

Third, fifth and seventh harmonics for an open—closed pipe.

Concept

In an open—closed pipe, the harmonics are first, third, fifth, and so on. There are no even-numbered harmonics in an open—closed pipe.

Going from the first harmonic to the next possible harmonic in an open—closed pipe, the wavelength decreases by a factor of 3, and so the frequency increases by a factor of 3. Hence, the next harmonic is the third harmonic. The harmonic is the ratio of its frequency to the frequency of the first harmonic.

In an open—open pipe or closed—closed pipe, the frequency change from the first harmonic to the next possible harmonic is a factor of 2, so we call that harmonic the second harmonic.

Match the equation linking wavelength, λ, and string length, L, to the correct harmonic in **Interactive 2**.

Student
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Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Interactive 2. Match the equation to the harmonic.

More information for interactive 2

This is a drag and drop interactive that involves identifying the correct wavelength equations for various harmonic modes on an open-closed pipe (pipe that is closed at one end and open at the other). The layout consists of a table with three columns labeled “Harmonic”, “Wave”, and “Equation”. The Harmonic column lists four different modes: “First harmonic”, “Third harmonic”, “Fifth harmonic”, and “Seventh harmonic”. In the Wave column, each row contains a graphical representation of a standing wave corresponding to the harmonic level. These waveforms consist of alternating solid and dotted curves showing the nodes and antinodes along the string. The number of loops in each wave pattern increases with the harmonic number.

The “Equation” column contains blank slots where users are required to drag and drop the correct wavelength equations. On the right side of the interactive, five draggable equation labels are provided: $\lambda = 4L$, $\lambda = \frac{4L}{3}$, $\lambda = \frac{4L}{5}$, and $\lambda = \frac{4L}{7}$. Each of these equations expresses the wavelength in terms of the string length L for the

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

given harmonic. The users interact with the activity by observing the wave pattern for each harmonic and then selecting and placing the appropriate equation label into the corresponding blank box in the table.

The first harmonic shows one-quarter of a full wavelength fitting into the pipe, with a node at the closed end and an antinode at the open end. The third harmonic fits three-quarters of a wavelength, forming a pattern with two nodes and two antinodes. The fifth harmonic includes five-quarters of a wavelength, with additional nodes and antinodes along the pipe. The seventh harmonic fits seven-quarters of a wavelength, creating the most complex standing wave pattern shown, with the highest frequency and shortest wavelength among the four harmonics. These waveforms follow the pattern of odd-numbered harmonics that naturally occur in pipes closed at one end.

This activity helps students connect visual representations of harmonic standing waves with their corresponding mathematical expressions. By completing the drag and drop correctly, users reinforce their understanding of how wavelength is related to the pipe length and the harmonic number in the case of odd harmonics. Once all the equations are placed into the table, users can click the “Check” button to verify their answers and receive feedback on their selections.

Solutions:

Here are the correct solutions for matching the wavelength equations to each harmonic shown in the image:

- The first harmonic shows $\frac{1}{4}$ of a full wavelength $(\lambda = 4L)$.
- The third harmonic fits $\frac{3}{4}$ of a wavelength $(\lambda = \frac{4L}{3})$.
- The fifth harmonic fits $1\frac{1}{4}$ wavelengths $(\lambda = \frac{4L}{5})$.
- The seventh harmonic fits $1\frac{3}{4}$ wavelengths $(\lambda = \frac{4L}{7})$.

This activity helps users connect the visual patterns of resonance in an open-closed pipe with the corresponding mathematical relationships between wavelength and pipe length.

If we write down the equations in **Interactive 2** with $4L$ as the numerator, we get the equations for wavelength in **Table 4**.

Table 4. Equations for wavelength in an open—closed pipe.

Harmonic	Equation
First harmonic	$\lambda_1 = \frac{4L}{1}$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Harmonic	Equation
Third harmonic	$\lambda_3 = \frac{4L}{3}$
Fifth harmonic	$\lambda_5 = \frac{4L}{5}$
Seventh harmonic	$\lambda_7 = \frac{4L}{7}$

The general equation is shown in **Table 5**.

Table 5. The general equation for wavelength in an open—closed pipe.

Equation	Symbols	Units
$\lambda_n = \frac{4L}{n}$	λ_n = wavelength	metres (m)
	L = length	metres (m)
	n = harmonic (first, third, etc.)	unitless

This equation is not given in the DP physics data booklet, and you do not need to know how to derive it.

Note that the equations in **Table 4** and **Table 5** are also the equations for a string with a fixed boundary at one end and a free boundary at the other end.

Concept

The wavelength equations in **Table 4** and **Table 5** can be used for the following situations:

- Standing wave in a pipe that is **open** at one end and **closed** at the other end
- Standing wave in a string that has a **fixed** boundary at one end and a **free** boundary at the other end.

The general equation is:

$$\lambda_n = \frac{4L}{n}$$

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Exercise 1

Click a question to answer

Note that it does not matter whether a standing wave is in a string or a pipe; the distance between adjacent nodes (or adjacent antinodes) is always half of one wavelength ($\lambda = 2 \times$ distance between adjacent nodes) regardless of the boundary conditions.

Worked example 2

A seventh harmonic standing wave is formed in an open–closed pipe. If the distance between adjacent nodes is 15 cm, determine the length of the tube.

Solution steps	Calculations
Step 1: Write out the values given in the question and convert the values to the units required for the equation.	$\text{node–node distance} = 15 \text{ cm}$ $= 0.15 \text{ m}$ $n = 7$
Step 2: Determine the wavelength.	$\lambda = 2 \times \text{distance between adjacent n}$ $= 2 \times 0.15$ $= 0.3 \text{ m}$
Step 3: Write out the equation and rearrange to find *L*.	$\lambda_n = \frac{4L}{n}$ $L = \frac{n\lambda_n}{4}$
Step 4: Substitute the values given.	$= \frac{(7 \times 0.3)}{4}$
Step 5: State the answer with appropriate units and the number of significant figures used in rounding.	$= 0.525 \text{ m} = 0.53 \text{ m (2 s.f.)}$

Student view

Work through the activity to check your understanding of standing waves in pipes.

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Activity

- **IB learner profile attribute:**
  - Inquirer
  - Thinker
- **Approaches to learning:** Thinking skills — Being curious about the natural world
- **Time required to complete activity:** 25 minutes
- **Activity type:** Individual activity

In this activity, you are going to make a wind instrument.

You will need: drinking straw, scissors, ruler

1. Flatten one end of the straw.
2. Cut the flattened end into a pointed shape, as in **Figure 8**. This end will act as a 'reed'.
3. Place the pointed end of the straw in your mouth and blow. You should hear a tone.

   When you blow through the pointed end of the straw, it starts to vibrate. This causes a standing wave to form in the straw. The standing wave is similar to that in an open—closed pipe. The tone is likely to be due to the first harmonic, which is usually the harmonic with the largest amplitude.

4. Cut the non-pointed end of the straw, so the straw becomes a little shorter. What happens to the frequency of the tone produced? Why do you think this happens?
5. C5 is the C note above middle C on a piano. It is about 524 Hz. Using the equation for the first harmonic in an open—closed pipe, calculate how long your straw needs to be to produce this frequency. Cut the straw to that length and use an online tone generator (https://onlinetonegenerator.com) to see how close your tone is to C5 (524 Hz).

The equation for the *n*th harmonic in an open—closed pipe is:

$$\lambda_n = \frac{4L}{n}$$

1. What happens if you cut finger holes in your straw? Can you play different notes? Does the angle of the 'reed' affect the quality of the note provided?

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

Can you change the volume or frequency of the note produced without changing the length of the straw?

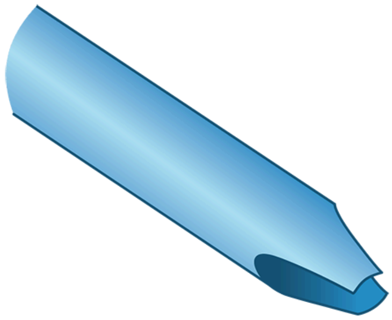


Figure 8. Straw cut into a 'reed'.

Theory of Knowledge

In this subtopic, you have come across many scenarios and devices that use standing waves. Would any of these make a suitable artifact for your TOK Exhibition? Choose an artifact related to waves and use it to prepare a 60-second verbal response to one of the Exhibition prompts (https://www.ibmastery.com/blog/the-theory-of-knowledge-exhibition-prompts).

5 section questions

Question 1

SL HL Difficulty:

An open—open pipe has 1 antinodes ✓ at each end.

A closed—closed pipe has 2 nodes ✓ at each end.

Accepted answers and explanation

#1 antinodes
anti-nodes
an antinode
anti nodes

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

#2 nodes

a node

General explanation

Particles are only free to vibrate where there is no physical barrier, so antinodes (regions of maximum displacement) are formed at the open ends of pipes. Where there is a closed end, the particles cannot vibrate, and nodes (regions of no displacement) are formed.

Question 2

SL HL Difficulty:

The first harmonic for an open—open tube is 55 Hz. If the speed of sound is 330 m s^{-1}, what is the length of the tube?

1 3.0 m ✓

2 6.0 m

3 12 m

4 1.5 m

Explanation

$f = 55\ \text{Hz}$

$v = 330\ \text{m s}^{-1}$

In an open—open tube, the first harmonic is at: $\lambda = 2L$

$$\begin{aligned} v &= f\lambda \\ v &= f \times 2L \\ L &= \frac{v}{2f} \\ &= \frac{330}{(2\times55)} \\ &= 3.0\ \text{m}\ (2\ \text{s.f.}) \end{aligned}$$

Question 3

SL HL Difficulty:

A clarinet is open at one end and closed at the other end. If the frequency of the third harmonic is 360 Hz, what is the frequency of the fifth harmonic?

1 600 Hz ✓

Student view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

2	720 Hz
3	180 Hz
4	220 Hz

Explanation

$f_3 = 360 \text{ Hz}$

Third harmonic:

$$\lambda_3 = \frac{4L}{3}$$

Fifth harmonic:

$$\lambda_5 = \frac{4L}{5}$$

$L = \frac{3\lambda_3}{4}$

$L = \frac{5\lambda_5}{4}$

$\frac{3\lambda_3}{4} = \frac{5\lambda_5}{4}$

$$\begin{aligned}\lambda_3 &= \frac{20\lambda_5}{12}\\ &= \frac{5\lambda_5}{3}\end{aligned}$$

$v = f_3\lambda_3$

$v = f_5\lambda_5$

$f_3\lambda_3 = f_5\lambda_5$

Student view

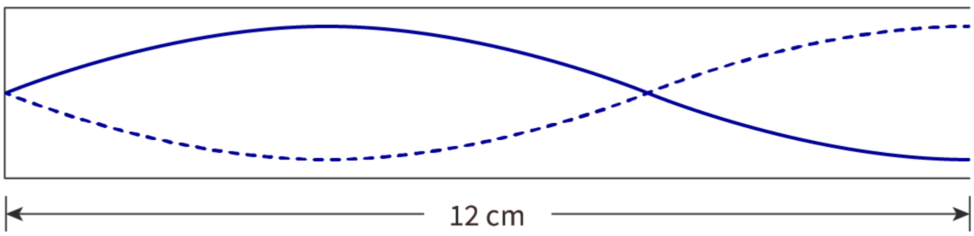
Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

$$
\begin{aligned}
f_5 &= \frac{(f_3\lambda_3)}{\lambda_5} \\
&= \frac{\left(f_3\left(\frac{5}{3}\right)\lambda_5\right)}{\lambda_5} \\
&= \left(f_3\left(\frac{5}{3}\right)\right) \\
&= 360\times\left(\frac{5}{3}\right) \\
&= 600\ \text{Hz}\ (2\ \text{s.f.})
\end{aligned}
$$

Question 4

SL HL Difficulty:

This wave is formed in an open—closed pipe.



More information

If the speed of sound is 330 m s^{-1}, what is the frequency of tone produced?

1 2100 Hz ✓

2 53 Hz

3 3700 Hz

4 30 Hz

Explanation

$$
\begin{aligned}
L &= 12\ \text{cm} \\
&= 0.12\ \text{m}
\end{aligned}
$$

$$v = 330\ \text{m s}^{-1}$$

$$
\begin{aligned}
\lambda &= \frac{4}{3}L \\
&= \frac{4}{3}\times 0.12 \\
&= 0.16\ \text{m}
\end{aligned}
$$

Student view

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$v = f\lambda$

$$
\begin{aligned}
f &= \frac{v}{\lambda} \\
&= \frac{330}{0.16} \\
&= 2062 \text{ Hz} \\
&= 2100 \text{ Hz } (2 \text{ s.f.})
\end{aligned}
$$

Question 5

SL HL Difficulty:

The frequency of the first harmonic wave in an open—open pipe is the same as the frequency of the first harmonic wave in an open—closed pipe. If the length of the open—open pipe is 1.2 m, what is the length of the open—closed pipe?

1 0.6 m ✓

2 1.2 m

3 2.4 m

4 0.3 m

Explanation

$L_{\text{oo}} = 1.2 \text{ m}$

Open—open:

$$\lambda_{\text{oo}} = 2L_{\text{oo}}$$

Open—closed:

$$\lambda_{\text{oc}} = 4L_{\text{oc}}$$

Both tones have the same frequency and the same wave speed, so they must also have the same wavelength:

$$\lambda_{\text{oo}} = \lambda_{\text{oc}}$$

$$2L_{\text{oo}} = 4L_{\text{oc}}$$

Student view

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$$\begin{aligned} L_{\mathrm{oc}} &= \frac{2L_{\mathrm{oo}}}{4} \\ &= \frac{(2\times 1.2)}{4} \\ &= 0.6\ \mathrm{m} \end{aligned}$$

C. Wave behaviour / C.4 Standing waves and resonance

Resonance and damping

C.4.4: Resonance C.4.5: Damping C.4.6: Light, critical and heavy damping

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-423-cid-762593/book/resonance-and-damping-id-45186/print/) Assign

Learning outcomes

At the end of this section you should be able to:

- Understand what causes resonance.
- Understand damping and how it affects maximum amplitude and resonant frequency.
- Know the effects of light, heavy and critical damping.

If you have ever pushed someone on a swing, you will know that the swing moves backwards and forwards with a particular frequency, and that there is a particular frequency of push that allows the swing to move with maximum amplitude (**Figure 1**).

This phenomenon is called resonance, and it is when an oscillating system is forced to vibrate close to the frequency at which it would vibrate naturally.

Controlled resonance is used in hospitals, precision timing, church organs and for other applications. What happens if resonance is not controlled?

Student
view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Figure 1. Experiencing resonance.

Credit: andreswd, Getty Images

Resonance

Natural frequency is the frequency with which an oscillator will move if left alone. Imagine a pendulum swinging without too much drag, or a bungee jumper bouncing up and down on the end of a piece of elastic. **Interactive 1** shows a pendulum and a mass on a spring and the equations for their frequency (section C.1.1a (/study/app/math-aa-hl/sid-423-cid-762593/book/simple-harmonic-motion-shm-id-44869/)).

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Overview
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Interactive 1. Pendulum on a Spring.

More information for interactive 1

The animation depicts the motion of a simple pendulum swinging back and forth around its equilibrium position. The pendulum consists of a mass attached to a string or rod, moving in a curved path. A dashed arc represents the trajectory of the pendulum's motion. Below the pendulum, an equation is displayed describing the frequency of the oscillation based on the length of the pendulum and gravitational acceleration. The equation shown in the animation is:

$$F = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

This equation represents the frequency F of a simple pendulum's oscillation. T is the period of the pendulum (the time for one complete swing). g is the acceleration due to gravity. l is the length of the pendulum.

The motion continues smoothly, demonstrating the concept of natural frequency in oscillatory systems.

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Interactive 2. A Mass on a Spring.

More information for interactive 2

The animation shows a mass-spring system oscillating vertically. A block is attached to the end of a coil spring, which is fixed to a horizontal support. The mass moves up and down in simple harmonic motion as the spring stretches and compresses. Below the animation, the equation for the natural frequency of the system is displayed, indicating that the frequency depends on the spring constant and the mass. The equation shown in the animation is:

$$F = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

This equation represents the frequency F of oscillations of a spring-mass system. T is the time period of the oscillation. k is the spring constant. m is the mass attached to the spring.

The animation visually demonstrates the periodic motion characteristic of a mass-spring system.

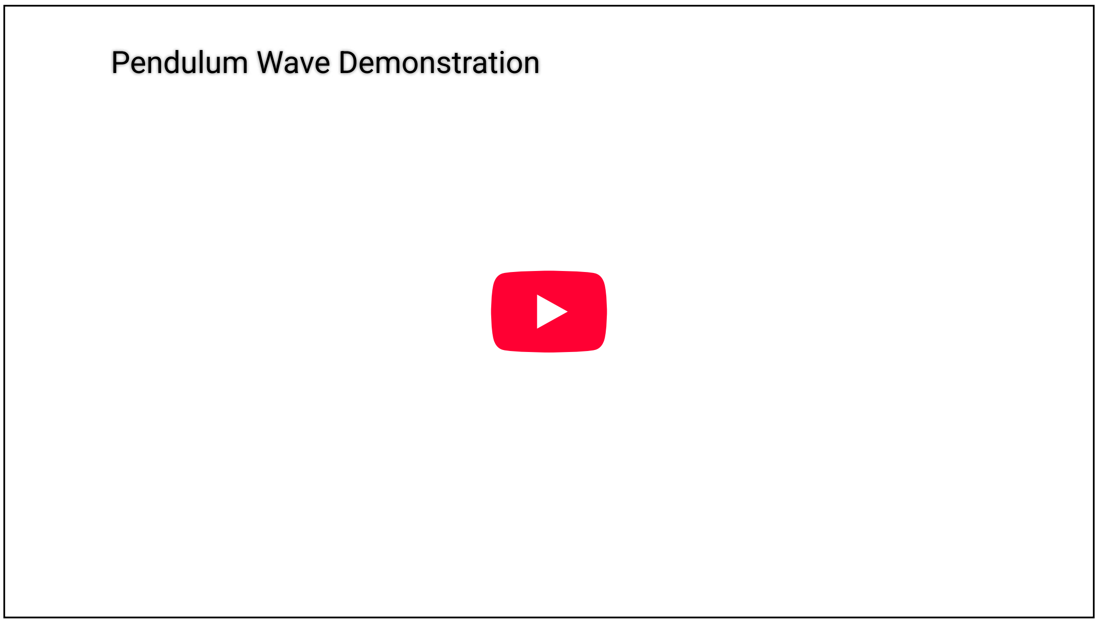
Student view

The natural frequency of the pendulum and the mass–spring system is given by the equations in **Interactive 1**.

Making connections

The type of motion in **Interactive 1** is simple harmonic motion (subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/)). Physicists like to look at a situation from different perspectives to fully understand the system and make accurate predictions.

Predict whether the length of a pendulum will affect its natural frequency. **Video 1** shows the effect of the length of a pendulum on its natural frequency. As you watch **Video 1**, see which oscillators have the highest natural frequency, and whether this agrees with your prediction.



Video 1. Pendulums with different natural frequencies.

As you saw in subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/), the time period of a pendulum (and hence its natural frequency of oscillation) can be calculated using:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

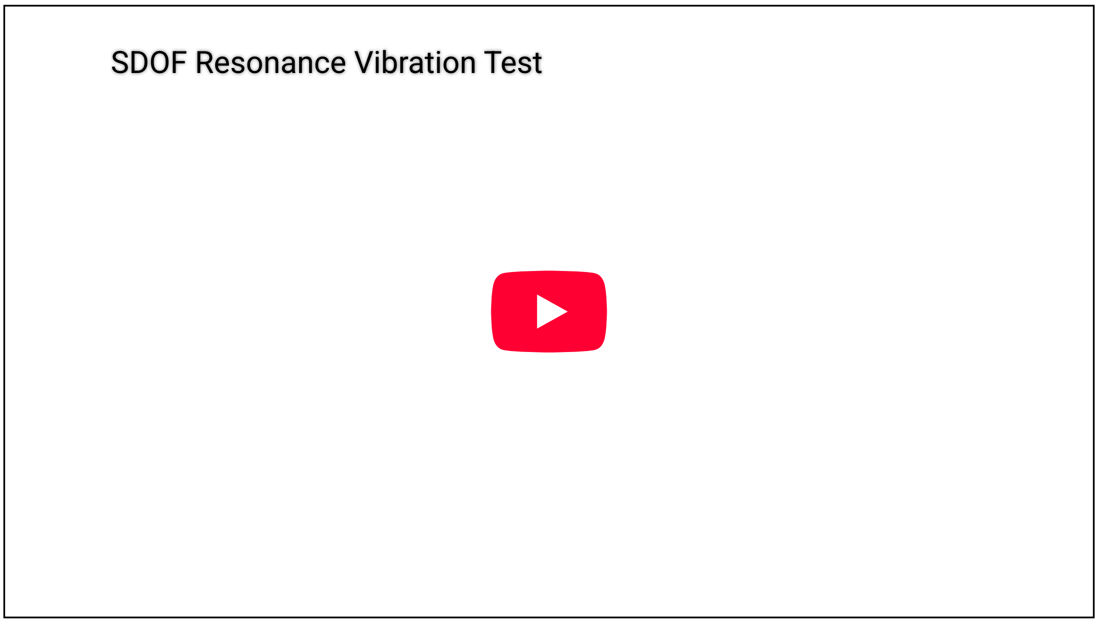
Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

An initial disturbance (the driving force) is needed to start an oscillator moving, then the oscillator moves freely. When the driving force continues over time, there is a driving frequency. If this driving frequency matches the natural frequency of the system, then the system oscillates with maximum amplitude. This is called resonance, and the resonant frequency is the same as the natural frequency.

Concept

Resonance is when the driving frequency matches the natural frequency of a system, causing the system to oscillate at maximum amplitude.

In **Video 2**, three different platforms are subjected to an external force, which causes them to vibrate. By changing the driving frequency, it is possible to identify the natural frequency of each block by observing when the amplitude of the vibration is at its maximum. This will occur when the block is resonating – its natural frequency is equal to the driving frequency.



Video 2. The relationship between driving frequency and natural frequency.

More information for video 2

This video shows an experiment with three simple mechanical systems (SDOF — Single Degree of Freedom) of increasing height from left to right in order, mounted on a vibration shaker table. The goal is to demonstrate resonance—a phenomenon where a system vibrates most strongly when it's shaken at its natural frequency. The table applies an external shaking force at different frequencies: 4.00 Hz, 6.35 Hz, and 11.30 Hz.

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The test begins by applying an external shaking force to each structure. As the shaking frequency changes, the blocks vibrate with varying intensities. At 4.00 Hz, the tallest structure shakes the most violently, indicating that this

Student view

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

frequency matches its natural rhythm—like pushing a swing perfectly in sync. This is called resonance, where the driving frequency equals the system's natural frequency, causing maximum vibration.

Next, the shaking frequency increases to 6.35 Hz. Now, the middle-sized structure starts shaking intensely, while the others calm down. This shows that every structure has its own natural frequency based on its design—here, the middle block resonates at this higher frequency. Finally, at 11.35 Hz, the shortest structure vibrates the most, completing the pattern. The shorter the structure, the higher its natural frequency, much like how a shorter guitar string produces a higher-pitched sound.

By adjusting the shaking frequency, the video visually explains how engineers identify natural frequencies in structures. Resonance is key—when external forces match these frequencies, vibrations amplify, which is crucial for designing earthquake-resistant buildings or avoiding unwanted vibrations in bridges. The experiment ends by emphasizing that understanding resonance helps predict and control how structures behave under dynamic forces, making it a foundational concept in engineering and physics.

Making connections

The phenomenon of resonance appears in many areas of physics. Global heating is caused by excess greenhouse gases in the atmosphere. These gas molecules have a natural frequency equal to the infrared radiation emitted from the Earth's surface. The greenhouse gas molecules resonate when excited by the infrared radiation, absorbing the energy and re-emitting it in all directions. Unfortunately, some of this re-emitted radiation is directed back towards the Earth, causing it to heat up (subtopic B.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43134/)).

Video 3 shows Barton's pendulum, where pendulums of different lengths are attached to a single string. One pendulum is displaced and begins to swing. This provides the driving force causing the other pendulums to move. The pendulums that swing with the greatest amplitude are the ones that have a similar length to the driving pendulum. The natural frequency of a pendulum depends on its length, so the pendulums that swing with the largest amplitude are the ones with a natural frequency close to that of the driving pendulum.

Student view

Video 3. Barton's pendulum.

More information for video 3

The video features a demonstration involving pendulums and energy transfer. The demonstration consists of two metal stands with a string tied to each stand and five pendulums or oscillator pendulums of varying lengths hanging from the string. A single pendulum, or the driver pendulum, also hangs on the string further from the five pendulums. The driver pendulum is released at the start of the demonstration. It swings back and forth at a specific frequency determined by its length. This motion causes vibrations in the horizontal support string, which in turn drives the motion of the other pendulums. Each of the hanging pendulums has a different natural frequency, because their lengths vary. The longer the pendulum, the slower it swings; the shorter it is, the faster it swings.

Over time, some of the pendulums begin to swing more than others. Specifically, the pendulums whose natural frequency is close to that of the driving pendulum begin to oscillate with greater amplitude. These pendulums are in resonance with the driving pendulum.

Pendulums with frequencies that are too different from the driver show minimal movement since they are not resonating with the driving force. As the energy continues to transfer through the system, the resonant pendulums exhibit increasingly large swings until friction and damping gradually reduce all motion.

The demonstration shows that a system responds most strongly to a driving frequency that matches its own natural frequency, a phenomenon known as resonance. It also visually illustrates how energy can be selectively transferred within a system of coupled oscillators.

Resonance can be a helpful phenomenon, but at other times it can have serious consequences. Click on each image in **Interactive 2** to learn more about the effects of resonance.

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Interactive 2. Helpful and Harmful Effects of Resonance.

More information for interactive 2

The interactive shows a collage of six images, representing the helpful and harmful effects of resonance. Each image has a hotspot represented by a plus sign. The hotspots are named hotspot 1, hotspot 2, hotspot 3, hotspot 4, hotspot 5, and hotspot 6. Clicking on these hotspots reveals information about the corresponding image and its effects on resonance.

Read below to learn about each image and the text in the hotspot:

Hotspot 1:

This hotspot is for the first image in the top left. The image shows a vintage-style radio with an antenna extended, featuring a wooden casing and an analog tuning dial. The hotspot text reads “Analogue radios rely on resonance to work. When the incoming radio wave’s frequency matches the natural frequency of the electrons in the aerial, resonance occurs. An amplifier is used to increase the strength of the signal, and information carried in the radio wave can be decoded.”

Student
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Overview
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Hotspot 2:

This hotspot is for the third image in the top right. This image features a silver wristwatch with a black dial, bold white numerals, and a rotating bezel. The hotspot text reads “Quartz timing devices work by exposing the quartz crystal to a electric potential difference. This causes the crystal to vibrate at its natural frequency. The frequency is stable and does not change with temperature. Most quartz watches have a crystal that oscillates 32 768 times per second.”

Hotspot 3:

This hotspot is for the second image in the top center. It displays multiple brain scans in blue shades, likely an MRI scan, arranged in a grid format. The hotspot text reads “A magnetic resonance imaging (MRI) scanner uses radio waves and oscillating magnetic fields to excite water molecules in the body. The radio waves have the same frequency as the oscillation of the magnetic field, causing resonance to occur. When the water molecules ‘relax’, they emit energy that can be detected and mapped by a computer.”

Hotspot 4:

This hotspot is located near the fourth image in the middle. The image showcases a global map with colorful atmospheric data, depicting air pollution or climate patterns across different regions. The hotspot text reads “UV and visible light from the Sun passes through the Earth’s atmosphere, warming the Earth's surface. As the Earth warms, infrared radiation is emitted, which has a similar frequency to the natural frequency of greenhouse gases, causing the greenhouse gases to resonate. The gas molecules re-emit the energy in all directions. Some of the re-emitted energy is directed back towards the Earth, heating it up.”

Hotspot 5:

This hotspot is for the fifth image in the bottom left. The image shows a busy bridge with multiple lanes of traffic, with vehicles moving in both directions. The hotspot text reads “Sometimes, buildings and bridges have a problematic resonant frequency. If the driving frequency (caused by wind, traffic or footfall) matches the structure’s natural frequency, resonance can occur, with dangerous results.”

Hotspot 6:

This hotspot is located on the sixth image in the bottom right. The image shows a large rocket on a launch pad, with smoke and flames at the base, indicating an imminent or ongoing launch. The hotspot text reads “NASA engineers noticed a series of damaging vibrations in launches of the Saturn V rocket. Oscillations were created along the length of the rocket due to vibrations in the fuel line. A team of engineers were tasked with solving the problem. They eventually changed the frequency of the engine oscillations, reducing the strain on the rocket.”

The interactive helps viewers learn about the various applications of resonance along with their helpful and harmful effects.

Student view

It has been said that some opera singers have such powerful voices and perfect pitch that they can break a crystal wine glass with their voice. If the pitch of the voice matches the natural frequency of the wine glass, the glass is forced to resonate, which causes it to break. This can be observed in the laboratory, using a frequency generator and loudspeaker (**Video 4**).

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Video 4. When a tone’s driving frequency matches the natural frequency of a wine glass, resonance can cause the glass to break.

On the morning of November 7, 1940, strong winds were blowing through the Tacoma Narrows strait in Pierce County, Washington, USA. The bridge connecting the city of Tacoma to the Kitsap Peninsula began to sway. Resonance occurred, with devastating consequences (**Video 5**).

Video 5. News footage of the Tacoma Narrows bridge collapse.

How can we deal with resonance that causes problems, such as glass breaking and bridges collapsing?

Student
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Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Damping

For simple harmonic motion (SHM) (see subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/)), it is assumed that the oscillations take place in the absence of any resistive forces (such as friction or air resistance) which, if present, will dissipate the energy in the oscillating system.

Imagine that a pendulum is displaced from its equilibrium position. In an ideal world, the pendulum will obey the equations of SHM and will swing forever with constant amplitude.

In reality, the amplitude of the pendulum slowly decreases until the pendulum comes completely to rest, all of its energy having been lost due to air resistance and friction at the pivot. This loss of energy due to resistive forces is called damping.

Concept

Damping is produced by the presence of resistive forces, which dissipate the energy stored in an oscillating system over a period of time. Damping causes the amplitude of an oscillation to decrease exponentially with time, while leaving the frequency mostly unchanged.

The amount of damping in a system will depend on the size of the resistive force. For example, a mass oscillating on the end of a spring in air will eventually come to rest, but if you place the same oscillating system in water, it will come to rest faster.

There are three types of damping:

- **Light damping:** There is a small amount of damping only. The system will continue to oscillate, but the amplitude of the oscillations decreases exponentially over time.
- **Heavy damping:** There is a large amount of damping. The system gradually dissipates all its energy. It does not oscillate, but returns very slowly to its equilibrium state.
- **Critical damping:** There is a very large amount of damping. The system returns to its equilibrium state as quickly as possible without any oscillations.

Concept

When light damping is applied to an oscillator, the amplitude of the oscillations

Student view

decreases. The resonant frequency also decreases.

Video 6 shows metal disks on the end of a spring oscillating in air and in a beaker of water. Water increases the resistance to movement and so provides light damping in comparison to the air, reducing the amplitude and frequency of the oscillations.

Video 6. Damping effect of water.

Figure 2 is a displacement–time graph showing the effect on amplitude of the three types of damping.

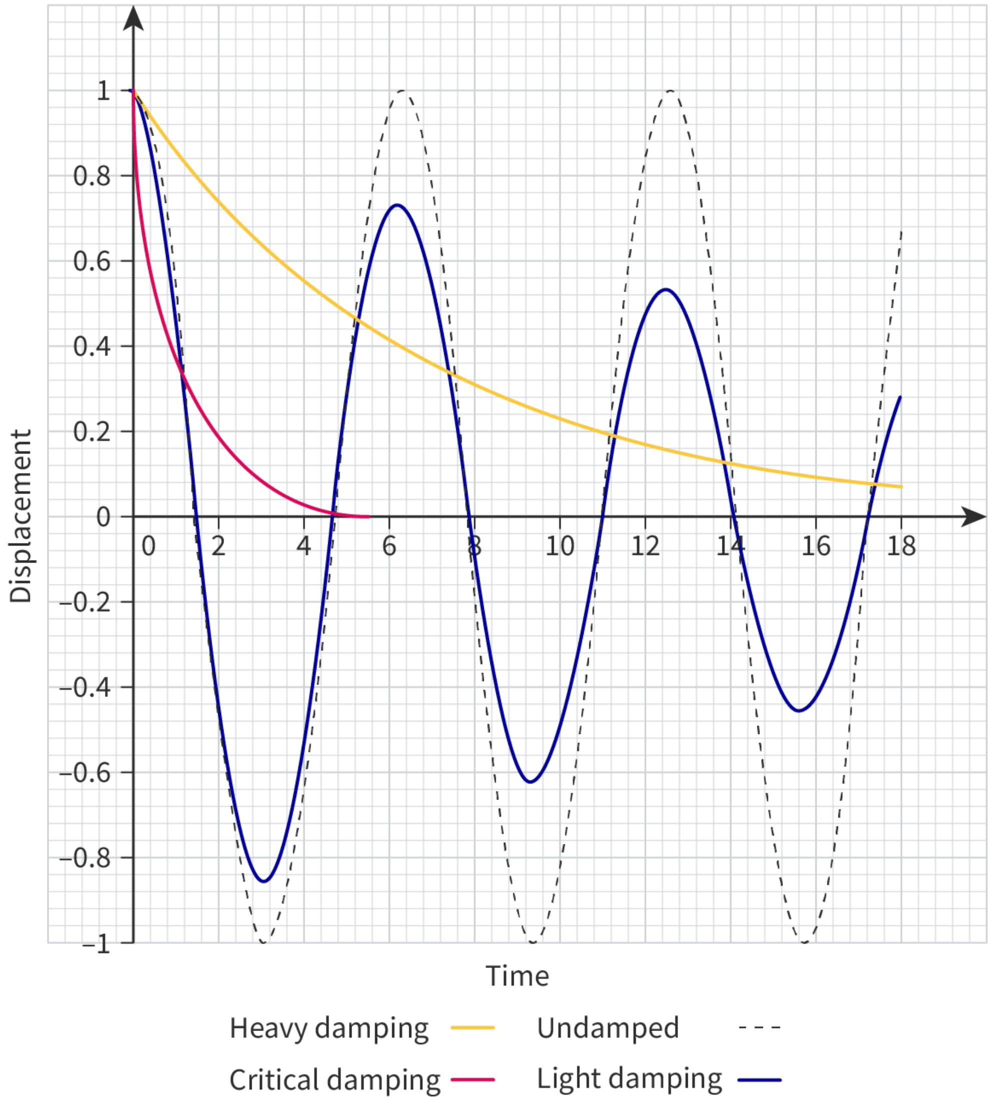

Figure 2. Displacement—time graph showing effects of different types of damping.

More information for figure 2

The image is a displacement-time graph illustrating the effects of different types of damping: heavy, critical, light, and undamped. The X-axis represents time, marked with intervals from 0 to 18. The Y-axis represents displacement, ranging from -1 to 1.

Four curves are shown. A yellow curve represents heavy damping, decreasing steadily from the top left starting point at (0, 1) towards zero. A red curve for critical damping rapidly drops initially from (0, 1) and settles to zero around time 6. A blue curve with oscillations indicates light damping, beginning at (0, 1) and showing increasingly reduced peaks approaching the X-axis over time; significant peaks occur around times 3, 9, and 15. A black dashed curve shows the undamped displacement, featuring continuous oscillations with consistent peak heights at times around 3, 9, and 15.

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Order the five images in **Interactive 3** from most heavily damped to least heavily damped.

Least damped **Most damped**





Check

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Interactive 3. Order the images from most heavily damped to least heavily damped.

Damping changes the frequency of an oscillator. As damping is increased, the resonant frequency decreases, and the amplitude decreases (**Figure 3**). Since $T = \frac{1}{f}$ (subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/)), the time period increases.

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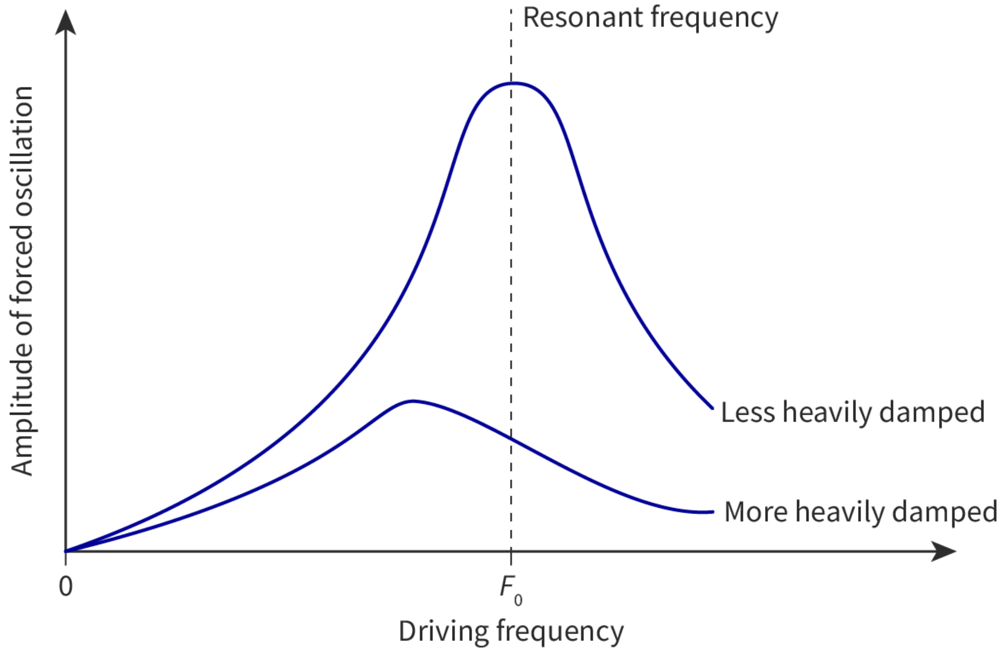

Figure 3. The effect of damping on an oscillator.

More information for figure 3

The graph illustrates the effect of damping on an oscillator. The X-axis represents the driving frequency, starting from zero, while the Y-axis represents the amplitude of forced oscillation. There are two curves depicted on the graph. The higher curve indicates less heavily damped oscillation, peaking sharply at the resonant frequency. The lower curve represents more heavily damped oscillation, peaking at a lower amplitude and showing a more gradual rise and fall around the resonant frequency. The resonant frequency is marked by a vertical dashed line at F_0, illustrating where the maximum amplitudes occur for each condition of damping.

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There are different types of damping systems, including shock absorbers, liquid damping and mass damping.

The suspension system of a vehicle, such as a car or motorbike, includes shock absorbers (**Figure 4**). Shock absorbers contain a piston mounted in oil that moves so the vehicle does not oscillate up and down when the wheels hit a bump in the road.

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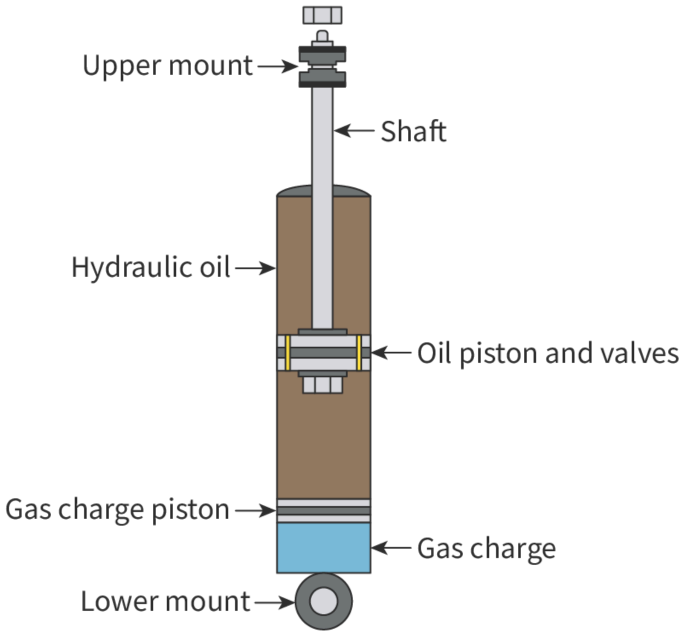

Figure 4. The damping mechanism in a shock absorber.

More information for figure 4

The diagram shows a shock absorber, featuring labeled components. At the top, there's the "Upper mount" connected to a "Shaft" that runs down the center. Below this, the "Hydraulic oil" area surrounds the shaft. In the middle section, there's an "Oil piston and valves" which controls the movement of the fluid. Beneath this is the "Gas charge piston," followed by the "Gas charge" area at the bottom. Finally, the "Lower mount" is at the base of the shock absorber. Each section is marked with arrows pointing from the labels to the respective parts.

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What is the degree of damping for the shock absorbers in a vehicle suspension system? Are they lightly damped, heavily damped or critically damped?

If the system was lightly damped, the vehicle body would oscillate every time the wheels hit a bump in the road. If the system was critically damped, the vehicle body would return to its equilibrium position in the shortest possible time without any oscillation.

The components of a suspension system degrade slightly with time, and a critically damped system will eventually become a lightly damped one. Shock absorbers are manufactured to be heavily damped.

Student view

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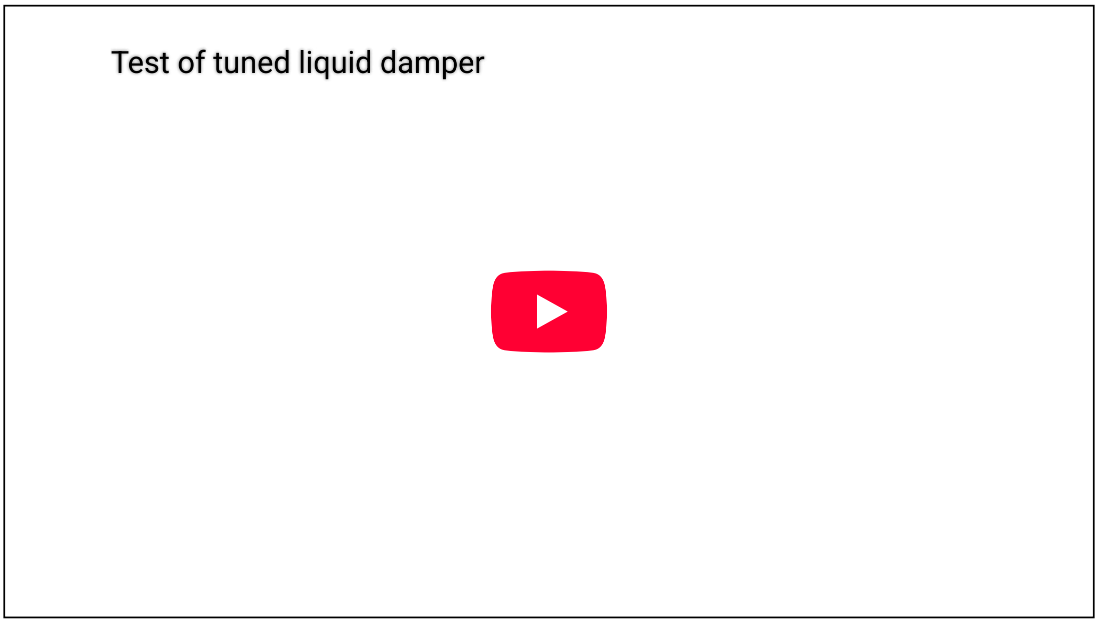
Bridges and skyscrapers need to include damping systems that safely dissipate the energy of oscillations caused by external driving forces, such as strong winds or earthquakes.

Shock absorbers at the base of a building are used to damp the oscillations from an earthquake. Vast tanks of water at the top of skyscrapers provide a restoring force, pulling a building back to its equilibrium position if it starts to sway.

Video 7 shows a tuned liquid damper, which uses the motion of liquid in a container to dissipate the energy of the oscillation.



Video 7. Tuned liquid damping.

More information for video 7

In this video demonstration, two identical vertical structures are displayed side by side on a flat, light wooden surface, each consisting of two vertical metal rods connected by off-white rectangular components at the top. A small, transparent container filled with light turquoise liquid is placed atop the structure on the right. The experiment visually contrasts the behavior of a basic oscillating structure with one that incorporates a tuned liquid damper to reduce vibrations.

A hand enters the frame and gently presses down on the top of the left structure, initiating vertical oscillation. As the pressure is released, the two structures start to sway. The structure on the left continues to sway slightly, and the rectangular element on top vibrates in response. The oscillations gradually dissipate over time due to inherent damping, but the motion remains visible. Meanwhile, the structure on the right, identical in build but equipped with the liquid-filled container, quickly stops swaying and remains completely motionless, with neither the structure nor the liquid showing any signs of movement.

The same exercise is repeated, and the same result is observed.

This demonstration effectively illustrates the concept of a tuned liquid damper (TLD), a passive damping system used

Student
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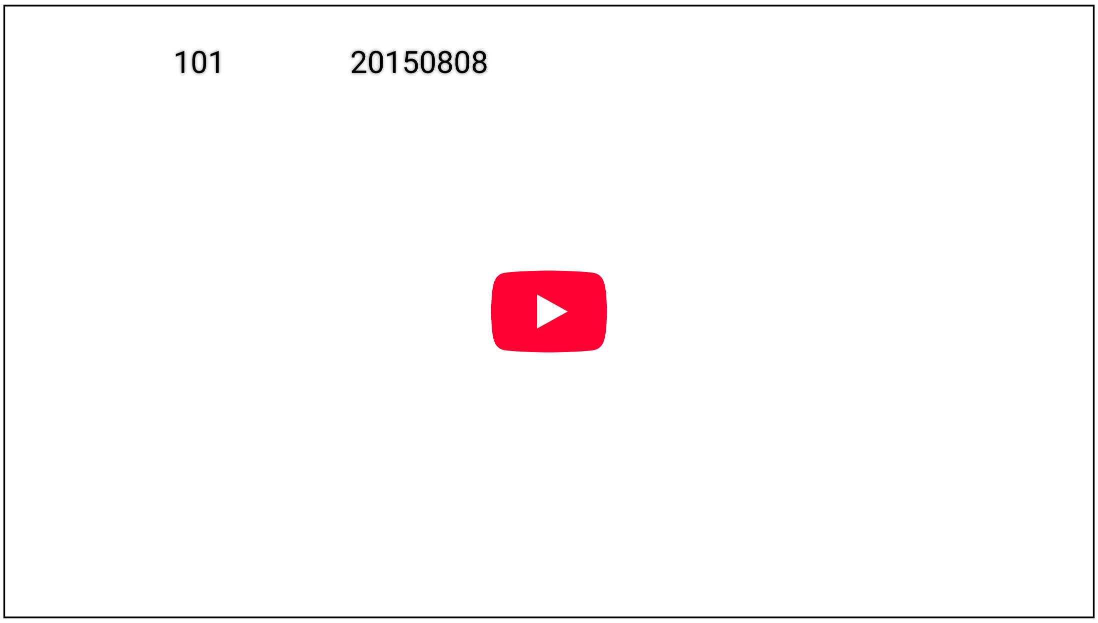
in civil engineering, especially in high-rise buildings and bridges. The turquoise liquid in the container is not merely for demonstration—its mass and movement are designed to interact with the motion of the structure. If the structure begins to oscillate, the liquid moves out of phase with that motion. The sloshing of the liquid absorbs kinetic energy and dissipates it through internal friction and wave motion, effectively reducing the amplitude of oscillations. In this demonstration, however, the liquid remains stationary because the structure itself is not disturbed; the focus remains on the comparative motion of the undamped and damped systems.

Skyscrapers can also have tuned mass dampers at the top of the building. The largest mass damper in the world is a 660 tonne steel ball suspended from floors 92 to 87 of the Taipei 101 skyscraper. Acting as a giant pendulum, it sways to counteract the building's movement and large shock absorbers attached to the underside of the sphere dissipate the energy (**Video 8**).



Video 8. A mass damper in action.

More information for video 8

In this video, you can observe a massive steel ball gently swinging inside the upper levels of the Taipei 101 skyscraper. This structure is known as a tuned mass damper (TMD), and it is one of the largest and most famous in the world. The damper weighs 660-ton and is suspended between the 87th and 92nd floors of the building by thick steel cables. The video shows the ball swaying, and as it does, it’s works to stabilize the building against natural forces like strong winds or even earthquakes. It acts like a giant pendulum, and its movement is a fascinating example of how physics is used in real-world engineering to keep tall buildings safe and comfortable.

The physics behind this lies mainly in the concepts of resonance, inertia, and energy dissipation. When wind or seismic activity causes a tall building like Taipei 101 to sway, it can enter a state of resonance—where the swaying increases and becomes dangerous if not controlled. The tuned mass damper is designed to have a natural frequency

Student
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Overview
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that is close to, but not exactly the same as, the frequency of the building's sway. This way, when the building moves in one direction, the damper swings in the opposite direction. The opposing motion helps cancel out or reduce the building's sway. This is called destructive interference, and it's a way of using physics to reduce vibrations.
Additionally, shock absorbers placed under the damper absorb and dissipate the energy from the ball's movement, converting mechanical energy into heat, which prevents the motion from getting out of control.
This use of a swinging damper is an impressive example of how engineers apply physics concepts to solve real problems. It also shows how structures must be designed not just to stand tall, but to stay stable and resist forces from the environment.

Work through the activity to check your understanding of resonance.

Activity

- **IB learner profile attribute:**
  - Knowledgeable
  - Thinker
- **Approaches to learning:** Thinking skills — Applying key ideas and facts in new contexts
- **Time required to complete activity:** 25 minutes
- **Activity type:** Individual activity

Look at the simulation in **Interactive 4.** There are three identical masses, hanging from springs with different spring constants.

Student view

Overview
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Interactive 4. Resonance simulation.

More information for interactive 4

This interactive allows users to explore forced vibrations in a system of springs and masses with different spring constants. The system consists of three springs, each attached to a mass of 2 kg. The springs have different spring constants: $k = 50\,\text{N/m}$, $k = 100\,\text{N/m}$, and $k = 200\,\text{N/m}$. The user can control the frequency of the applied force using a slider that ranges from low to high, with the current frequency displayed in Hz.

At the top of the interface, two buttons — “Run” and “Reset” — are provided. The "Run" button starts the simulation, allowing users to observe the system's oscillations based on the frequency they have selected. The “Reset” button resets the system, returning it to its initial state, making it easy to start over and experiment with different values.

A reference line is also provided, which users can move up or down to mark the low point of vibration. As the applied frequency is adjusted, the system vibrates accordingly, and users can observe how the system responds to different frequencies. The key feature of this interactive is to show how the frequency of the applied force affects the oscillation behavior of the mass-spring systems with different spring constants.

For example, in the case shown in the image, the frequency of the forced vibration is set to 1.40056 Hz. As the system oscillates, users can see how the different spring constants ($k = 50\,\text{N/m}$, $100\,\text{N/m}$, and $200\,\text{N/m}$) influence the vibration behavior of the mass-spring systems.

This interactive highlights the impact of spring constant (k) and driving frequency on oscillatory motion. By adjusting the applied frequency, users can observe how different springs react, leading to key insights into resonance—when the system oscillates with maximum amplitude at its natural frequency. It also demonstrates the relationship between stiffness and oscillation, where stiffer springs (higher k) tend to have higher natural frequencies.

The interactive is an excellent way to understand the relationship between frequency, spring

Student
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Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

constant, and mass in a system undergoing forced vibrations. By adjusting the frequency of the applied force and observing how the system responds, users can gain insight into the principles of oscillatory motion, resonance, and the effects of varying spring constants on vibration behavior.

Task 1

Calculate the natural frequency of each mass—spring system. You will need to use understanding from subtopic C.1 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43161/).

Use the equation $T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$ to find the time period, then use $f = \frac{1}{T}$ to find the frequency.

Task 2

Move the ‘Frequency of Forced Vibration’ slider to set the driving frequency as close as you can to the natural frequency for each mass. What do you observe?

Task 3

Use Hooke’s law to find the extension of each spring when the mass is added. You will need to use understanding from subtopic A.2 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43136/).

Use the equation $F_{\mathrm{H}} = -kx$ to find the extension.

Task 4 (Higher Level)

If the blue mass has a maximum extension of 30 cm when oscillating at its resonant frequency, what is the maximum elastic potential energy stored in the system? You will need to use understanding from subtopic A.3 (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43083/).

Use the equation $E_{\mathrm{T}} = \frac{1}{2}m\omega^2 x_0^2$ to find the elastic potential energy.

Task 5 (Higher Level)

If we assume that, when resonating, the yellow mass oscillates with the same total energy as the blue mass, what is the amplitude of the yellow mass when resonating? You will need to use understanding from subtopic A.3

Student view

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Task 1: Calculate the natural frequency of each mass spring system.

Blue:

$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$

so:

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= \frac{1}{2\pi}\sqrt{\frac{50}{2}} \\ &= \frac{5}{2\pi} \\ &= 0.80\ (\mathrm{Hz}) \end{aligned}$$

Pink:

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= \frac{1}{2\pi}\sqrt{\frac{100}{2}} \\ &= \frac{5\sqrt{2}}{2\pi} \\ &= 1.13\ (\mathrm{Hz}) \end{aligned}$$

Yellow:

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= \frac{1}{2\pi}\sqrt{\frac{200}{2}} \\ &= \frac{10}{2\pi} \\ &= 1.59\ (\mathrm{Hz}) \end{aligned}$$

Task 2: Use the driving frequency slider to set the driving frequency as close as you can to the natural frequency for each mass. What do you observe?

The masses should each oscillate at maximum amplitude.

Overview (/study/app/aa-hl/sid-423-cid-762593/c

Task 3: Use Hooke's law to find the extension of each spring when the mass is added.

Blue:

$F_{\mathrm{H}} = -kx$

so:

$$\begin{aligned} x &= \frac{F_{\mathrm{H}}}{-k} \\ &= \frac{2 \times 9.81}{50} \\ &= 0.39\ (\mathrm{m}) \end{aligned}$$

Pink:

$$\begin{aligned} x &= \frac{F_{\mathrm{H}}}{-k} \\ &= \frac{2 \times 9.81}{100} \\ &= 0.20\ (\mathrm{m}) \end{aligned}$$

Yellow:

$$\begin{aligned} x &= \frac{F_{\mathrm{H}}}{-k} \\ &= \frac{2 \times 9.81}{200} \\ &= 0.098\ (\mathrm{m}) \end{aligned}$$

(Note: the minus sign can be discarded as the extension of a spring when a mass is added is always positive.)

Task 4: If the blue mass has a maximum extension of 30 (cm) when oscillating at its resonant frequency, what is the maximum elastic potential energy stored in the system?

$$\begin{aligned} E_{\mathrm{T}} &= \frac{1}{2} m \omega^2 {x_{\mathrm{o}}}^2 \\ &= \frac{1}{2} m (2\pi f^2) {x_{\mathrm{o}}}^2 \\ &= \frac{1}{2} \times 2(2\pi \times 0.80^2) 0.3^2 \\ &= 0.36\ (\mathrm{J}) \end{aligned}$$

Task 5: If we assume that, when resonating, the yellow mass oscillates with the same amount of total energy as the blue mass in Task 4, what is the amplitude of the yellow mass when resonating?

Student view

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

$$E_{\mathrm{T}} = \frac{1}{2}m\omega^2{x_{\mathrm{o}}}^2$$
$$= \frac{1}{2}m(2\pi f^2){x_{\mathrm{o}}}^2$$
$$0.36 = \frac{1}{2}\times 2(2\times\pi\times 1.59^2){x_{\mathrm{o}}}^2$$
$$x_{\mathrm{o}} = 0.15\ (\mathrm{m})$$

5 section questions

Question 1

SL HL Difficulty:

An oscillator is released from a displaced position and undergoes several complete oscillations before eventually coming to rest in the equilibrium position. What kind of damping has occurred?

1 Light damping ✓

2 No damping

3 Heavy damping

4 Critical damping

Explanation

Light damping is where energy is gradually removed from a system, causing the amplitude of the oscillations to gradually decrease to zero.

Question 2

SL HL Difficulty:

A car suspension system is designed to absorb oscillations quickly, but without so much damping that the forces required make driving too uncomfortable for the driver. Which kind of damping is used in a car suspension system?

1 Heavy damping ✓

2 Light damping

Student
view

Overview
(/study/app/math-aa-hl/sid-423-cid-762593/c

3 No damping

4 Critical damping

Explanation

If the damping was critical, the forces required to stop the car from oscillating so quickly would be very strong, and the driver and passengers would have an uncomfortable experience. Light (or no) damping would mean the car continues to oscillate as it travels down the road, which would make it hard to control the vehicle.

Question 3 SL HL Difficulty:

An oscillating system has a damping system added. Which row correctly describes the effect on the amplitude and time period for oscillations at the new resonant frequency?

	Amplitude	Time period
A	Decreases	Increases
B	Increases	Decreases
C	Increases	Increases
D	Decreases	Decreases

1 A ✓

2 B

3 C

4 D

Explanation

If damping is applied to a system, the amplitude decreases.

The resonant frequency decreases, meaning the time period increases.

Student view

Question 4 SL HL Difficulty:

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

Which of the following is an effect of resonance?

1 The greenhouse effect ✓

2 Earth orbiting the Sun

3 A car suspension system reducing oscillations

4 Diffraction of light

Explanation

The greenhouse effect occurs when the frequency of infrared waves matches the natural frequency of greenhouse gases, causing resonance.

Question 5

SL HL Difficulty:

A mass on the end of a spring is attached to an oscillator. Resonance occurs, and the mass—spring system oscillates at maximum amplitude.

Two springs, each identical to the first, are attached in series. A new mass, half the size of the original mass, is attached to the springs. How does the new mass—spring system behave when subject to the same driving frequency?

1 The mass—spring system undergoes resonance and oscillates at its maximum amplitude. ✓

2 The mass—spring system does not oscillate at all.

3 The mass—spring system oscillates, but not at its maximum amplitude.

4 The mass—spring system oscillates with a very small frequency.

Explanation

The natural frequency of a mass—spring system is given by:

$$\begin{aligned} f &= \frac{1}{T} \\ &= \left(\frac{1}{2}\pi\right)\left(\sqrt{\left(\frac{k}{m}\right)}\right) \end{aligned}$$

When the two springs are added in series, the total extension for the same hanging mass will double. Therefore, the spring constant $\left(k = \frac{F}{x}\right)$ will halve. As the mass is also halved, there is no change to the natural frequency.

Student view

If the driving frequency causes resonance with the previous mass—spring system, it will also cause resonance (and maximum amplitude oscillations) with the new mass—spring system.

Summary and key terms

- Standing waves are formed when a wave is reflected and the reflected wave interferes with the original outgoing wave.
- Both outgoing and reflected waves must be identical for a standing wave to occur.
- Standing waves have nodes (where there is zero displacement) and antinodes (where there is maximum displacement).
- Standing waves can be formed in longitudinal and transverse waves.
- The wavelength and frequency of standing waves depend on several factors:
  - speed of the wave in the medium
  - driving frequency
  - boundary conditions
  - length of the string or pipe.
- The longest wavelength wave is known as the first harmonic.
- Standing waves can form in strings fixed at both ends and in pipes open at both ends. The relationship between the wavelength of the standing wave and the length of the string or length of the pipe depends on the order of the harmonic (i.e. first, second, etc.)
- In an open–closed pipe (or a string with one fixed boundary and one free boundary), the harmonics are first, third, fifth and so on. There are no even numbered harmonics.
- Resonance is when the driving frequency of an oscillator matches the natural frequency of a system, causing the system to vibrate at maximum amplitude.
- Damping is when energy is removed from a system, causing the amplitude to decrease over time.
- Light damping is when oscillations are allowed to continue for a period of time, as amplitude gradually decreases.
- Heavy damping is when the displacement of the oscillator is gradually brought back to zero without oscillating further.
- Critical damping is when the energy is removed from the system as quickly as possible, bringing the oscillator back to the equilibrium position with minimal further

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

oscillations.

Key terms

Student
view

Ineractive 1. Fill in the Blanks with the Correct Terms to Test Your Understanding of Oscillations and Wave Behavior.

Checklist

What you should know

After studying this subtopic, you should be able to:

- Understand the nature of standing waves and how they are formed.
- Understand what is meant by nodes and antinodes.
- Know the phase difference and relative amplitude of different points along a standing wave.
- Understand the standing wave patterns in strings and pipes when there are different boundary conditions.
- Understand what causes resonance.
- Understand damping and how it affects maximum amplitude and resonant frequency.
- Know the effects of light, heavy and critical damping.

Investigation

- **IB learner profile attribute:** Communicator
- **Approaches to learning:** Communication skills – Reflecting on the needs of the audience when creating engaging presentations
- **Time to complete activity:** 60 minutes

Overview (/study/app/math-aa-hl/sid-423-cid-762593/c

- **Activity type:** Group activity

Your task

Standing waves in wind and string instruments, resonance in the performance space, and damping to prevent external noise pollution entering the auditorium are just some of the issues sound engineers have to deal with when music is played live.

In small groups, create a slide presentation with information on the following:

- How music studios use damping to avoid interference from echoes when recording music
- In an orchestra, which instruments rely on:
  - standing waves in strings
  - standing waves in open–open pipes
  - standing waves in open–closed pipes.
- Situations in musical performances when resonance is helpful, and when it is not helpful
- Which musical instruments are capable of producing the highest and lowest pitched tones
- How damping is used in music to improve the experience of the musicians and listeners
- Examples of where light, heavy and critical damping can be found in musical instruments and musical performance venues.

Creativity, activity, service

Aspect: Activity
Learning outcome: Demonstrate the skills and recognise the benefits of working collaboratively

Live music, theatre or dance performances are a team effort. Scientists and engineers are crucial to the success of a performance.

Think how you could use your physics understanding to help your community. How does your understanding of resonance and damping apply to a live show?

Could you volunteer as a stage assistant, helping to arrange sound and lighting? Advise on the best locations for live music, based on your understanding of damping? Explain why recording spaces work best with softer materials on the

Student view

walls, rather than solid surfaces?

Is there a way of arranging the audience so they have the best audio and visual experience?

What more do you need to learn to better support your team in their production?

Reflection

Teacher instructions

The goal of this section is to encourage students to reflect on their learning and conceptual understanding of the subject at the end of this subtopic. It asks them to go back to the guiding questions posed at the start of the subtopic and assess how confident they now are in answering them. What have they learned, and what outstanding questions do they have? Are they able to see the bigger picture and the connections between the different topics?

Students can submit their reflections to you by clicking on 'Submit'. You will then see their answers in the 'Insights' part of the Kognity platform.

Reflection

Now that you’ve completed this subtopic, let’s come back to the guiding questions introduced in The big picture (/study/app/math-aa-hl/sid-423-cid-762593/book/the-big-picture-id-43788/).

- What distinguishes standing waves from travelling waves?
- How does the form of standing waves depend on the boundary conditions?
- How can the application of force result in resonance within a system?

With these questions in mind, take a moment to reflect on your learning so far and type your reflections into the space provided.

You can use the following questions to guide you:

Overview
(/study/ap
aa-
hl/sid-
423-
cid-
762593/c

- What main points have you learned from this subtopic?
- Is anything unclear? What questions do you still have?
- How confident do you feel in answering the guiding questions?
- What connections do you see between this subtopic and other parts of the course?

⚠ Once you submit your response, you won't be able to edit it.

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Submit

Rate subtopic C.4 Standing waves and resonance

Help us improve the content and user experience.

☆ ☆ ☆ ☆ ☆

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