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3. Geometry and trigonometry / 3.15 Relative positions of lines

The big picture

Have you ever looked up at the sky and seen two aircraft that seem to be on a collision course? Mid-air collisions are very rare thanks to air-traffic control systems. How can this software chart the motion of two aircraft moving in 3D space and predict whether or not they are going to collide?

The motion of an object moving in 3D space can be described by a vector equation, which gives the line along which the object is moving. By analysing the relative positions of these lines in space and time it is possible to predict whether they will be at the same place at the same time.

In this subtopic, you will explore the relative positions of vectors in 3D.

Concept

This subtopic focuses on the relationshipsbetween lines in terms of their relative positions. It uses the vector representation of the equation of a straight line to identify lines that intersect in 3D space and those that don't. As you work through the subtopic think about how the methods you are learning can be applied to real-world applications such as computer gaming, air traffic control and ballistics.

3. Geometry and trigonometry / 3.15 Relative positions of lines

Relative positions of lines

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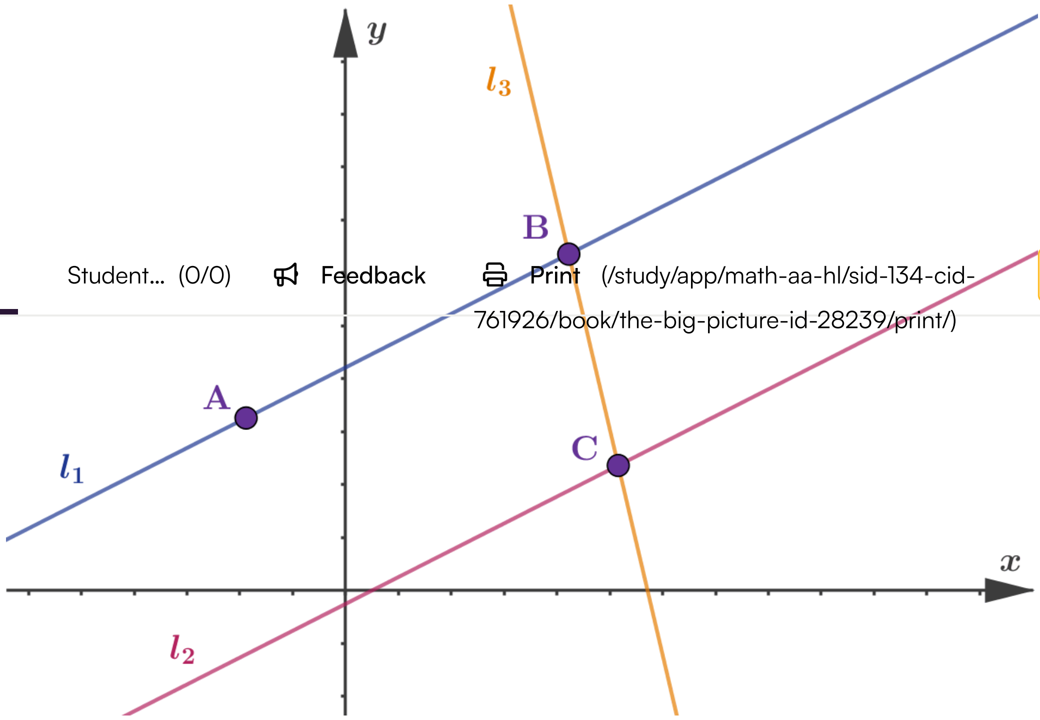
In a 2D plane, the relative positions of lines can be described as parallel, intersecting or coincident. In the diagram below.

- l_1 and l_2 are parallel, so there is no intersection point.
- l_1 and l_3 intersect at one point, as do l_2 and l_3.
- l_1 and AB are coincident, which means they have infinitely many intersection points.



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More information

The image is a graph displaying three intersecting lines on an X and Y coordinate plane. The X-axis is labeled 'x,' and the Y-axis is labeled 'y.' The lines are labeled as 'l1,' 'l2,' and 'l3.' Line 'l1' is blue, line 'l2' is magenta, and line 'l3' is orange. Three intersection points are labeled on the graph: Point A is on line 'l1,' Point B is at the intersection of lines 'l1' and 'l3,' and Point C is at the intersection of lines 'l2' and 'l3.' The graph features tick marks along both axes but no specific numerical values are provided. The lines and points appear to represent a geometric relationship between the sets of lines.

[Generated by AI]

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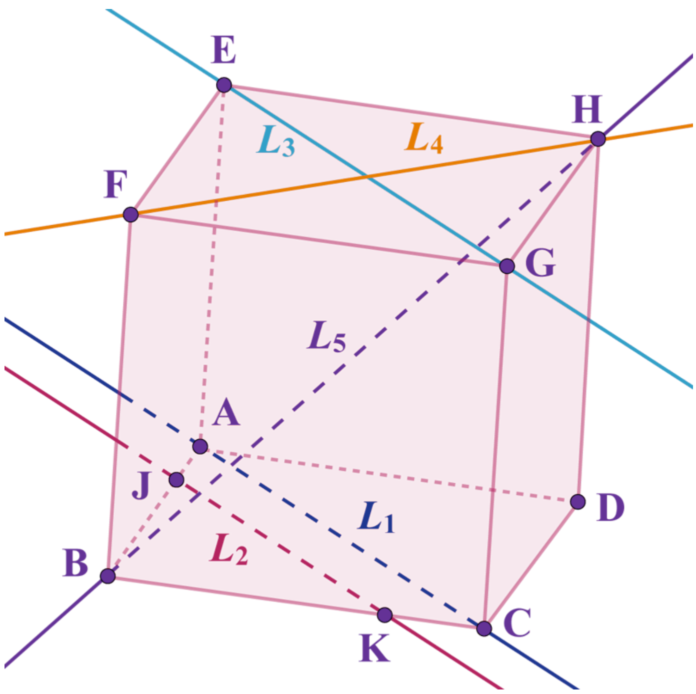
Making connections

In subtopic 2.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-24414/). you studied the properties of lines in 2D and Cartesian form.

- If two lines are parallel, then their gradients are equal, i.e. $m_1 = m_2$, and they do not have an intersection point.
- If two lines are perpendicular, then their gradients are negative reciprocals, $m_1 = -\frac{1}{m_2}$, and they intersect at one point.
- If two lines are coincident, then they have the same gradient and they have infinitely many intersection points.

Consider the lines drawn over the faces of the cuboid in the diagram below.

How would you describe their positions relative to one another?



More information

The image is a geometric diagram showing a red quadrilateral with vertices labeled as B, C, D, and E. There are several lines labeled L1, L2, L3, L4, and L5 intersecting and passing through different points. Line L3 in blue and line L4 in orange intersect at a point labeled as F. Lines L1 and L2 are parallel to each other; L1 passes through points J and C, and L2 is not shown but can be inferred to be parallel. Line L5 is a dashed line crossing from J through point

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A, diagonally across the quadrilateral. The quadrilateral has additional intersection points labeled as H, G, and K. Each line is distinguished by different colors and intersects at various labeled points around and within the quadrilateral shape.

[Generated by AI]

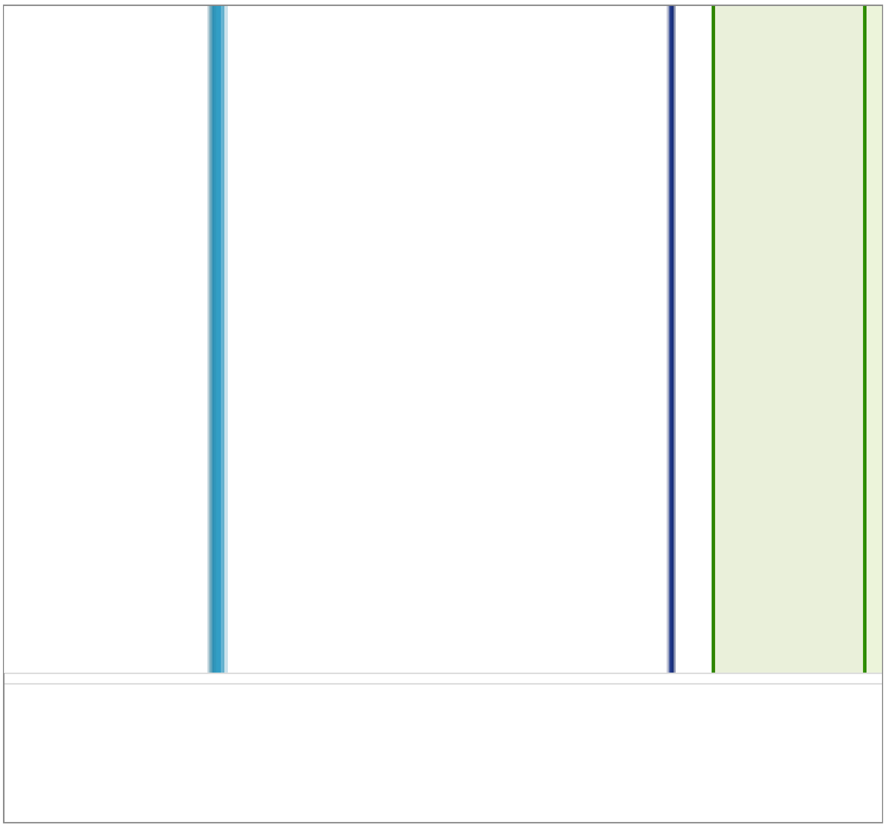
For example, lines L_3 and L_4 intersect at one point while lines L_1 and L_2 are parallel to each other and have no intersection point.

What about lines L_4 and L_1? Are they parallel? Do they intersect?

Activity

Use the following applet to explore the relative positions of lines.

You can drag the vertices to observe the lines from different perspectives.



Interactive 1. Exploring Line Positions.

Student
view

More information for interactive 1

This interactive tool helps users explore how lines behave in 3D space by adjusting just two points of a cube, demonstrating four key relationships between lines: parallel (never meeting), intersecting (crossing at one point), coincident (overlapping completely), and skew (not parallel and not intersecting).
The display shows a cube with side *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*. The display shows multiple lines ($L1$, $L2$, $L3$, $L4$, $L5$) in 3D space, with only Vertices *A* and *B* of a cube movable. $L1$ passes through vertices *A* and *C*, $L2$ passes parallel to $L1$ though edges *AB* and *BC*, $L3$ passes through vertices *E* and *G*, $L4$ passes through vertices *F* and *H* and $L5$ passes through vertices *H* and *B*,
Users can drag these points to change line orientations while observing real-time updates. The "Zoom in and Zoom out button helps examine close interactions and observe overall.
This interactive visualization demonstrates the possible relationships between lines in 3D space. For example, parallel lines like $L1$ and $L4$ that run in identical directions without ever intersecting, intersecting lines such as $L3$ and $L4$ that cross at a single point, and skew lines including $L3$ and $L5$ that exist in different planes - neither parallel nor intersecting.
By experimenting, users learn to classify line relationships in 3D, and distinguish subtle differences like skew versus parallel.

As the applet shows, you can describe relative positions of lines in 3D in four different ways.

Lines in 3D can be

- parallel
- intersecting
- coincident or
- none of these.

In this subtopic you will use vectors to describe the equations of straight lines and to predict the relationship between them.

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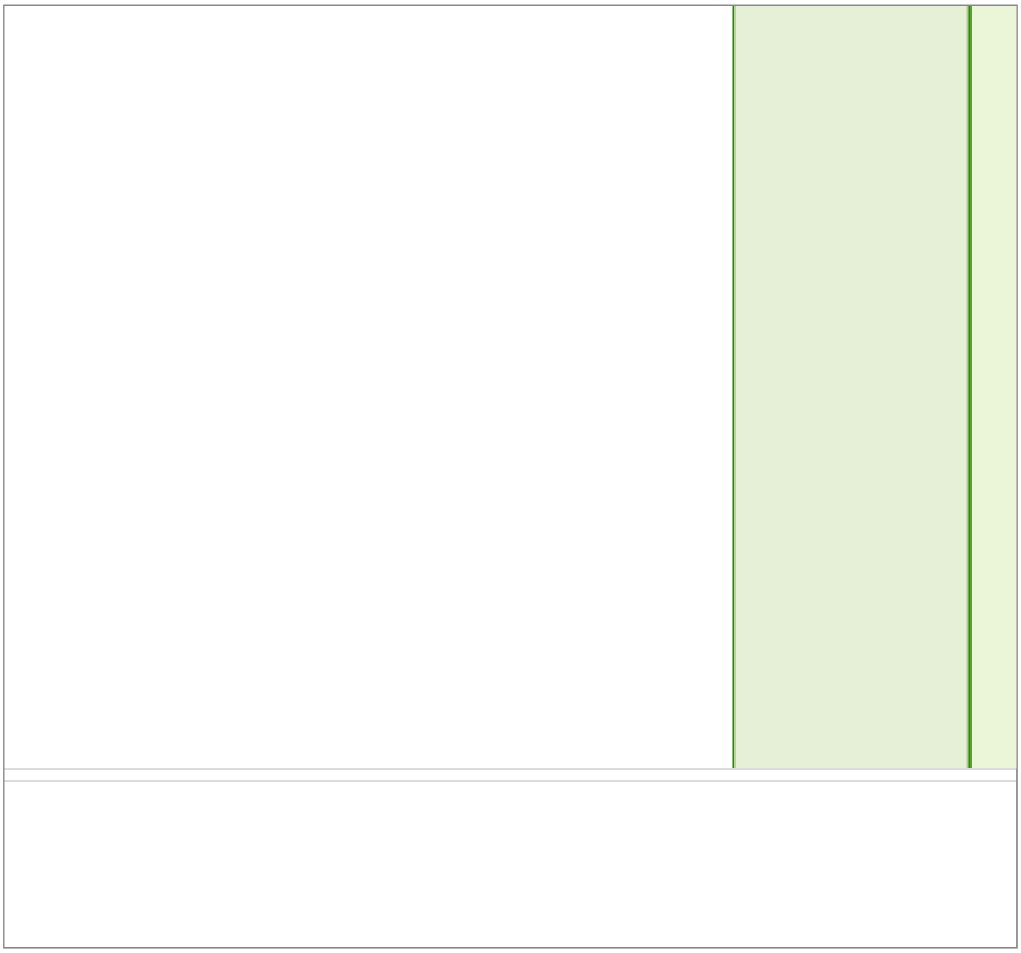
Important

Given a point (x_0, y_0, z_0) and a direction vector $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$, the equation of a straight line can be written in vector form as $\mathbf{r} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$

Parallel lines

Parallel lines

- have no intersection point and
- their direction vectors are parallel (i.e. they are the same or are scalar multiples of the same vector).



Interactive 2. Parallel Lines and Their Properties.

More information for interactive 2

This interactive tool enables users to investigate the properties of parallel lines in 3D space. By manipulating a geometric cube and observing vector relationships, users can explore how parallel lines maintain constant separation and identical direction regardless of their position in space. The activity demonstrates the fundamental characteristics that define parallel lines in a three-dimensional geometric cube.

The display shows a cube with side *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*.

Line crossing from vertices *A* and *C* is $\mathrm{L}1$ with vector v, from vertices *E* and *G* is $\mathrm{L}3$ with vector u and through *J* a point on line *AB* and *K* a point on line *BC* is line $\mathrm{L}2$ with vector w. $\mathrm{L}1$ and $\mathrm{L}2$ are parallel, users can move points on the geometric cube such as *A* and *C* to adjust line positions while observing how the parallel relationship persists. The real-time display maintains vector proportionality indicators, showing when direction vectors remain scalar multiples of each other. Zoom in and Zoom out button on the bottom allows detailed examination of vector alignment and the consistent distance between parallel lines.

Through this interactive experience, users develop an understanding of how parallel lines are defined by their direction vectors in 3D space. They observe that parallel lines never intersect and maintain proportional direction vectors regardless of translation. This knowledge is essential for applications in computer graphics, architectural design, and engineering, where maintaining parallel relationships is crucial for structural integrity and visual accuracy. The tool reinforces theoretical concepts by providing immediate visual feedback during user manipulation.

In the above diagram, parallel lines have parallel direction vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$.

Example 1

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Find a vector equation of the line passing through the point $\mathrm{P}(1, -1, 1)$ that is parallel to the line with vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$

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If two lines are parallel their direction vectors are parallel. Remember that vector equations are not unique and you could use any vector parallel to the vector $\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$, i.e. any vector that is a scalar multiple of $\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$.

$$\mathbf{b} = \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

Use the position vector of point P in the vector equation

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

Make sure you use a different parameter as you are already given λ in the question.

$$\mathrm{L}_2 : \ \mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

Therefore, the equation of the new line is

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

Example 2

★★☆

Determine whether or not the following two lines are parallel or coincident.

$$\mathrm{L}_1 : \ \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \text{and } \mathrm{L}_2 : \ \mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$$

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The direction vectors of the two lines are parallel as they are scalar multiples of the same vector:
$\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$$

If the two lines are coincident, then all the points on one line will also be on the other line. So check whether the position vector of the known point on L_1 also lies on L_2.

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1-4t \\ 3+2t \\ 2+4t \end{pmatrix}$$

Equate each component and solve for t.

$$1 = -1 - 4t \Rightarrow t = -0.5$$

$$1 = 3 + 2t \ \Rightarrow t = -1$$

$$1 = 2 + 4t \ \Rightarrow t = -0.25$$

The values of t are not the same.

Point $(1, 1, 1)$ does not lie on L_2.

As they have parallel direction vectors and no common point.

Therefore, the lines L_1 and L_2 are parallel.

Student view

Intersecting lines

Two lines can:

- intersect at one point or
- intersect at infinitely many points.

Lines that intersect at infinitely many points are called coincident lines.

In the diagram below,

- Lines L_3 and L_4 intersect at point P, and lines L_4 and L_5 intersect at point H.
- Lines L_3 and EP intersect at infinitely many points because they are coincident lines.

Interactive 3. Intersecting and Coincident Lines.

More information for interactive 3

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

This interactive allows users to explore the concept of intersecting and coincident lines in 3D space. By manipulating points and observing the resulting configurations, users can visualize how lines may intersect at a single point or coincide infinitely. The activity highlights the key differences between these two types of intersecting lines and their unique geometric properties.

The display shows a cube with side *A, B, C, D, E, F, G* and *H*. Three distinct lines, L 3, L 4, and L 5. L 4 passes through vertices *H* and *F*, L 3 passes through vertices *E* and *G* and L 5 passes through *H* and *B*. L 3 and L 4 intersect at point *P* on the plane *EFGH*. Users can drag points *A*, and *B* to reposition the lines dynamically. The tool provides real-time updates, allowing users to see how the intersection points move as lines are adjusted. A special case is shown with line *EP*, which coincides completely with L 3, demonstrating infinite intersection points. The “zoom out” and “Zoom in“ button at the bottom of the screen feature enables closer examination of these intersections.

Through this exploration, users learn to distinguish between lines that intersect at a single point and coincident lines that share all points. They observe how direction vectors determine whether lines will intersect or coincide, and how these relationships persist when lines are moved in 3D space.

Example 3

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Find the point where these two lines intersect:

$$\mathrm{L}_1:\ \mathbf{r}=\begin{pmatrix}3\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}\text{and }\mathrm{L}_2:\ \mathbf{r}=\begin{pmatrix}-1\\2\\1\end{pmatrix}+t\begin{pmatrix}-1\\2\\2\end{pmatrix}$$

The two lines are not parallel as their direction vectors are not the same or scalar multiples of the same vector:

$$\begin{pmatrix}-1\\1\\1\end{pmatrix}\text{ is not a scalar multiple of }\begin{pmatrix}-1\\2\\2\end{pmatrix}$$

$$\mathrm{L}_1:\quad \mathbf{r}=\begin{pmatrix}3\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}=\begin{pmatrix}3-\lambda\\1+\lambda\\\lambda\end{pmatrix}$$

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$$\mathrm{L}_2: \quad \mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -1-t \\ 2+2t \\ 1+2t \end{pmatrix}$$

If the two lines intersect there will be a common point on both lines which can be found by solving this equation.

$$\begin{pmatrix} 3-\lambda \\ 1+\lambda \\ \lambda \end{pmatrix} = \begin{pmatrix} -1-t \\ 2+2t \\ 1+2t \end{pmatrix}$$

Equate each component and rearrange to simplify.

$$\begin{aligned} 3-\lambda = -1-t &\Rightarrow \lambda - t = 4 \\ 1+\lambda = 2+2t &\Rightarrow \lambda - 2t = 1 \\ \lambda = 1+2t &\Rightarrow \lambda - 2t = 1 \end{aligned}$$

Solve the first pair of simultaneous equations.

$$\left.\begin{array}{l} \lambda - t = 4 \\ \lambda - 2t = 1 \end{array}\right\} \lambda = 7 \text{ and } t = 3$$

Check that the solution satisfies the 3rd equation.

$$\lambda - 2t = 1 \Rightarrow 7 - 2(3) = 1 \text{ is correct}$$

Therefore, the two lines intersect at the point corresponding to $\lambda = 7$ on the first line and $t = 3$ on the second line.

$$\begin{pmatrix} 3-7 \\ 1+7 \\ 7 \end{pmatrix} = \begin{pmatrix} -1-3 \\ 2+2\times 3 \\ 1+2\times 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \\ 7 \end{pmatrix}$$

The point of intersection is $(-4, 8, 7)$.

Example 4

Student view

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Determine whether the following two lines intersect.

$$\mathrm{L}_1 : \ \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 10 \\ 0 \end{pmatrix} \text{and } \mathrm{L}_2 : \ \mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -10 \\ -2 \\ 7 \end{pmatrix}$$

The two lines are not parallel as their direction vectors are not the same or scalar multiples of the same vector:

$\begin{pmatrix} -2 \\ 10 \\ 0 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} -10 \\ -2 \\ 7 \end{pmatrix}$

$$\mathrm{L}_1 : \quad \mathbf{r} = \begin{pmatrix} -2\lambda + 2 \\ 10\lambda - 3 \\ 0 \end{pmatrix}$$

$$\mathrm{L}_2 : \quad \mathbf{r} = \begin{pmatrix} 2 - 10t \\ 3 - 2t \\ 7t \end{pmatrix}$$

If the two lines intersect there will be a common point on both lines which can be found by solving this equation.

$$\begin{pmatrix} -2\lambda + 2 \\ 10\lambda - 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 - 10t \\ 3 - 2t \\ 7t \end{pmatrix}$$

Equate the components and rearrange to simplify.

$$\begin{aligned} -2\lambda + 2 = 2 - 10t &\Rightarrow \lambda - 5t = 0 \\ 10\lambda - 3 = 3 - 2t &\Rightarrow 5\lambda + t = 3 \\ 0 = 7t &\Rightarrow t = 0 \end{aligned}$$

Substituting $t = 0$ in the first equation gives $\lambda = 0$.

Substituting these in the left hand side of the second equation gives $5 \times 0 + 0 \neq 3$, so the two lines do not intersect.

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view

If two lines do not intersect and are not parallel, then how can you describe them?

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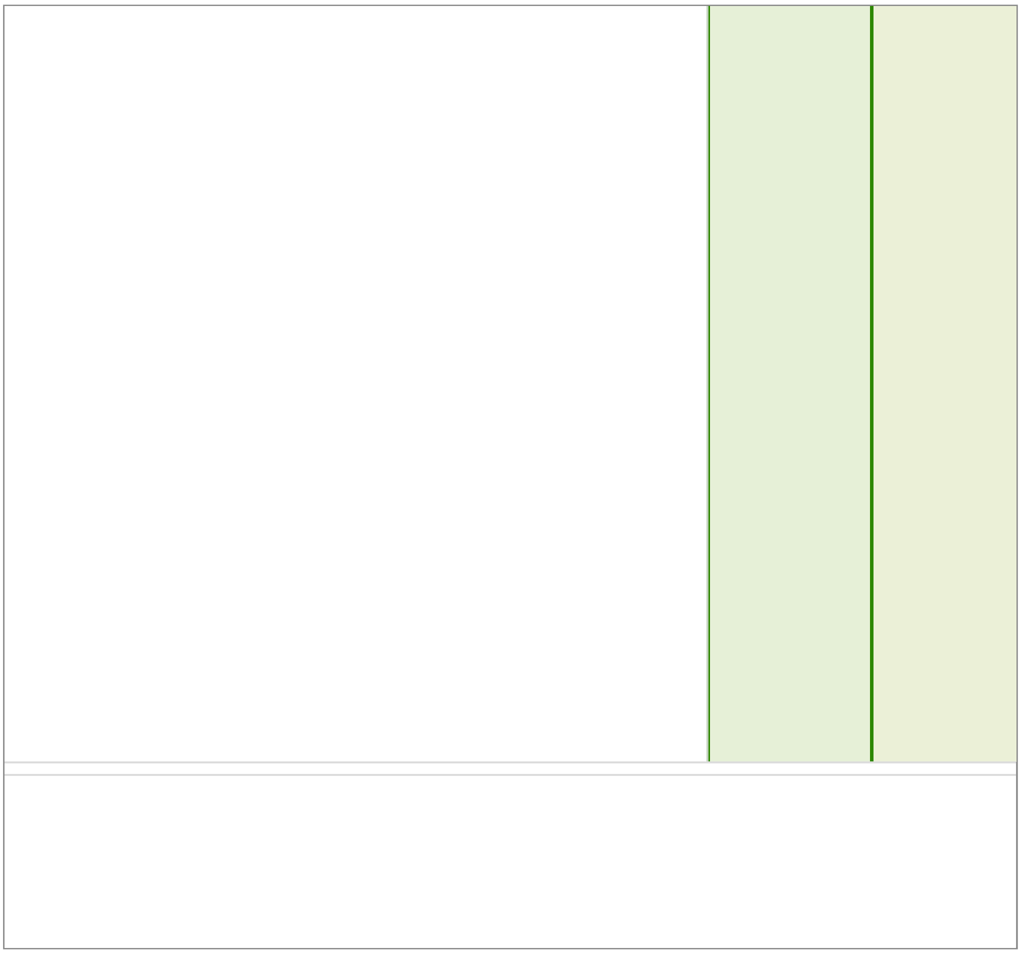
Skew lines

Skew lines

- have no intersection points, and
- their direction vectors are not parallel (i.e. they are not the same or scalar multiples of the same vector).

In the diagram below, lines L_1 and L_5 have no intersection point. The angle between the respective direction vectors, $\boldsymbol{u}$ and $\boldsymbol{v}$ is not 0, therefore the lines are not parallel.



Interactive 4. Non-Intersecting Lines in 3D Geometry.

More information for interactive 4

This interactive tool allows users to explore the concept of skew lines in 3D space—lines that neither intersect nor run parallel. By manipulating points and observing the resulting configurations, users can visualize how skew lines maintain their non-parallel, non-intersecting relationship regardless of their position in space. The activity highlights the unique properties of skew lines, which differ fundamentally from parallel or intersecting lines.

The display shows a cube with side *A, B, C, D, E, F, G* and *H*. Two distinct lines, L_1 and L_5, with their direction vectors u and v are represented as arrows. Users can drag vertices *A*, *B* and point *Q* (located on the edge *BC*) to reposition the lines dynamically. The tool provides real-time updates, ensuring users see how the lines remain

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skewed—never intersecting nor becoming parallel—as they adjust their positions. The zoom feature allows a closer inspection of the lines' orientations, emphasizing their lack of intersection and non-parallel direction vectors. Additionally, the 'Zoom in/out' feature helps users explore detailed interactions or view the bigger picture more easily.

Through this exploration, users learn to identify skew lines by their defining characteristics: no intersection point and non-parallel direction vectors. They observe how the angle between the direction vectors (neither $0°$ nor indicative of an intersection) reinforces the skew relationship.

Lines L_1 and L_5 do not have an intersection point and they are not parallel so they are skew.

Important

In 3D there are three relative positions for straight lines

- Intersecting lines: two straight lines can intersect at one point or intersect at infinitely many points (coincident lines).
- Parallel lines: these have no intersection point and their direction vectors are parallel.
- Skew lines: these have no intersection points and their direction vectors are not parallel.

Example 5

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Consider the lines L_1 and L_2:

$$\mathrm{L}_1: \quad x = 2 + 4t, y = 6 - 2t, z = -2 + 5t$$

and

$$\mathrm{L}_2: \quad x = -1 + 2s, y = 2 + 2s, z = 1 + 4s$$

Investigate whether the lines are parallel, intersecting or skew.

Student view

First investigate whether the lines are parallel.

Since the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$ is parallel to L_1 and vector $\begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ is parallel to L_2 and the two vectors are not scalar multiples of each other, then the vectors and thus, the lines are not parallel.

Consider whether they intersect. If they intersect, they must do so at a point. Give this point the coordinates (x_1, y_1, z_1) , which satisfies the parametric equations of the two lines for some t and s, i.e.

$$\begin{aligned} x_1 &= 2 + 4t = -1 + 2s \\ y_1 &= 6 - 2t = 2 + 2s \\ z_1 &= -2 + 5t = 1 + 4s \end{aligned}$$

This leads to the simultaneous equations:

$$\begin{aligned} 2 + 4t &= -1 + 2s \\ 6 - 2t &= 2 + 2s \\ -2 + 5t &= 1 + 4s \end{aligned}$$

Solving for t and s using the first two equations, we find $t = \frac{1}{6}$ and $s = \frac{11}{6}$. Substitute these into the third equation:

$$-2 + 5 \times \frac{1}{6} \neq 1 + 4 \times \frac{11}{6}$$

This is not an equality and so this system of linear equations is not consistent, which implies that the lines represented by these equations do not intersect. These lines are not parallel either, hence they are skew.

5 section questions ^

Question 1

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Select the option that is parallel to the line $\boldsymbol{r} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ -3 \\ 2 \end{pmatrix}$

1 $\boldsymbol{r} = \begin{pmatrix} 0 \\ 6 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix}$ ✓

2 $\boldsymbol{r} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{1}{2} \\ 3 \\ 2 \end{pmatrix}$

3 $\boldsymbol{r} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} \frac{3}{2} \\ -9 \\ 8 \end{pmatrix}$

4 $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 6 \\ 9 \end{pmatrix}$

Explanation

Since the direction vectors are scalar multiples of each other: $\begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix} = -2 \begin{pmatrix} \frac{1}{2} \\ -3 \\ 2 \end{pmatrix}$

Question 2

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Select which of the following is coincident with $\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$

1 $\begin{pmatrix} \frac{1}{2} \\ -\frac{7}{2} \\ 4 \end{pmatrix} + t \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$ ✓

2 $\begin{pmatrix} \frac{1}{2} \\ -\frac{7}{2} \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$

3 $\begin{pmatrix} 1 \\ -7 \\ 8 \end{pmatrix} + t \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$

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4 $\begin{pmatrix}1\\-7\\8\end{pmatrix}+t\begin{pmatrix}3\\3\\2\end{pmatrix}$

Explanation

Since the direction vectors are parallel and for $\lambda=\frac{1}{2}$, the point $\left(\frac{1}{2},-\frac{7}{2},4\right)$ is on the line given by the equation $\begin{pmatrix}-1\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-3\\2\end{pmatrix}$

Question 3

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Select the correct values of a and b given that the following two lines are parallel:

$$\mathrm{L}_1: \quad \boldsymbol{r}_1=\begin{pmatrix}-2\\0\\7\end{pmatrix}+s\begin{pmatrix}-3\\5\\6\end{pmatrix}$$

and

$$\mathrm{L}_2: \quad \boldsymbol{r}_2=\begin{pmatrix}3\\-2\\4\end{pmatrix}+t\begin{pmatrix}a\\-10\\b\end{pmatrix}$$

1 $6,-12$ ✓

2 $-6,12$

3 $-\frac{3}{2},3$

4 $\frac{3}{2},-3$

Explanation

For $\begin{pmatrix}-3\\5\\6\end{pmatrix}$ to be parallel to $\begin{pmatrix}a\\-10\\b\end{pmatrix}$, you must choose a scalar multiple k so that $\begin{pmatrix}-3\\5\\6\end{pmatrix}=k\begin{pmatrix}a\\-10\\b\end{pmatrix}$

Therefore, $5=-10k$ and $k=-\frac{1}{2}$.

Since, $-3-ak$ then $-3=-\frac{1}{2}a$ giving $a=6$.

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Also, $6 = kb$ so $6 = -\frac{1}{2}b$ giving $b = -12$.

Question 4

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Select which one of the following statements is true.

1. If two lines are not parallel then they can have a common point. ✓
2. If two lines are not parallel then they must have a common point.
3. If two lines are not parallel then they cannot have a common point.
4. If two lines are not parallel then they can have exactly two common points.

Explanation

If they are in 2D, yes, they must intersect, but if they are in 3D they can be skew, in which case they do not intersect and the question does not specify whether the case was 2D or 3D.

Question 5

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Consider the lines L_1 and L_2 in their parametric form:

$$L_1 : x = 4 + 3t, y = 2 + t, z = -1 - 2t$$

and

$$L_2 : x = -1 + s, y = 4 + 2s, z = 1 + s$$

Decide whether the lines are parallel, intersecting or skew and give your answer by writing ‘parallel’ or ‘intersecting’ or ‘skew’.

skew ✓

Accepted answers

skew, ‘skew’, ‘Skew’

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Explanation

We first investigate whether the lines are parallel:Since the vector $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ is parallel to L_1 and vector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is parallel to L_2 and the two vectors are not scalar multiples of each other, then the vectors and thus, the lines are not parallel.

Let us consider whether they intersect. If they intersect, they must do so at a point. Let us give this point the coordinates (x_1, y_1, z_1), which satisfies the parametric equations of the two lines for some t and s, i.e.

$$x_1 = 4 + 3t = -1 + s$$

$$y_1 = 2 + t = 4 + 2s$$

$$z_1 = -1 - 2t = 1 + s$$

This leads to the simultaneous equations:

$$4 + 3t = -1 + s$$

$$2 + t = 4 + 2s$$

$$-1 - 2t = 1 + s$$

Solving for *t* and *s* using the first and the third equation, we find $t = -\frac{7}{5}$ and $s = \frac{4}{5}$.

Substituting these into the second equation, we find

$$2 - \frac{7}{5} \neq 4 + 2 \times \frac{4}{5}$$

which is not an equality and so this system of linear equations is not consistent, which implies that the lines represented by these equations do not intersect.

These lines are not parallel either, hence they are skew.

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Points of intersection and angles between lines

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view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

In the [last section (/study/app/math-aa-hl/sid-134-cid-761926/book/relative-positions-of-lines-id-28240/)] you saw how the vector equation of a line can help you to identify the relative position of two lines.

In this section, you will use the properties of vector equations to find the angle between two lines and the coordinates of the intersection point if it exists.

Example 1

★★☆

The equations of lines L_1 and L_2 are

$$\mathrm{L}_1: \quad r_1 = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

and

$$\mathrm{L}_2: \quad r_2 = \begin{pmatrix} -8 \\ 1 \\ 7 \end{pmatrix} + t \begin{pmatrix} 10 \\ 2 \\ 0 \end{pmatrix}$$

a) Show that the two lines are skew.

b) Find the acute angle between the two lines.

Give your answer in degrees correct to one decimal place.

	Steps	Explanation
a)	$\left.\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = k \begin{pmatrix} 10 \\ 2 \\ 0 \end{pmatrix}\right\\} \Rightarrow k = 0$ and $k = \frac{1}{2}$	The direction vectors of the two lir are not parallel as there is no uniqu k, i.e. they are not scalar multiples the same vector.

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	Steps	Explanation
	$\text{L}_1: \quad r_1 = \begin{pmatrix} -1 \\ -2+\lambda \\ 0 \end{pmatrix}$ $\text{L}_2: \quad r_2 = \begin{pmatrix} -8+10t \\ 1+2t \\ 7 \end{pmatrix}$	Write the position vectors of point on each line.
	$\begin{pmatrix} -1 \\ -2+\lambda \\ 0 \end{pmatrix} = \begin{pmatrix} -8+10t \\ 1+2t \\ 7 \end{pmatrix}$	If the lines have a common point t equation will have a unique solutio
	$-1 = -8+10t \Rightarrow t = \frac{7}{10}$ $-2+\lambda = 1+2t \Rightarrow \lambda - 2t = 3$ $0 = 7 \Rightarrow \text{no solution}$	Equate the components and rearrange to simplify. Solve the equations simultaneousl
	There is no intersection point.	As there is no solution to the simultaneous equations.
	Therefore the two lines are skew as they are not parallel and have no intersection point.	
b)	$u \cdot v = 0 \times 10 + 1 \times 2 + 0 \times 0 = 2$ $\lvert u \rvert = \sqrt{0^2+1^2+0^2} = 1$ $\lvert v \rvert = \sqrt{10^2+2^2+0^2} = \sqrt{104}$	Find the scalar product of the direction Vectors $\boldsymbol{u} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 10 \\ 2 \\ 0 \end{pmatrix}$
	$\cos\theta = \frac{u \cdot v}{\lvert u \rvert \lvert v \rvert} = \frac{2}{1 \times \sqrt{104}} = \frac{2}{\sqrt{104}}$ $\theta = \cos^{-1}\left(\frac{2}{\sqrt{104}}\right) = 78.7°$	
	Therefore the angle between the two lines is $78.7°$	

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Making connections

In subtopic 3.13 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-28034/). you studied the scalar product of two vectors. When you want to find the angle between two lines you can use the scalar product of the direction vectors.

Exam tip

The formula for the angle between two vectors is given in the IB formula booklet as $\cos\theta = \dfrac{v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3}{|v|\,|w|}$, where $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ and θ is the angle between the two vectors.

Example 2

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Find the coordinates of the closest point on the line with the equation $\boldsymbol{r} = i + 2\boldsymbol{j} - \boldsymbol{k} + \lambda(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ to the point $\mathrm{A}(1, -1, 1)$.

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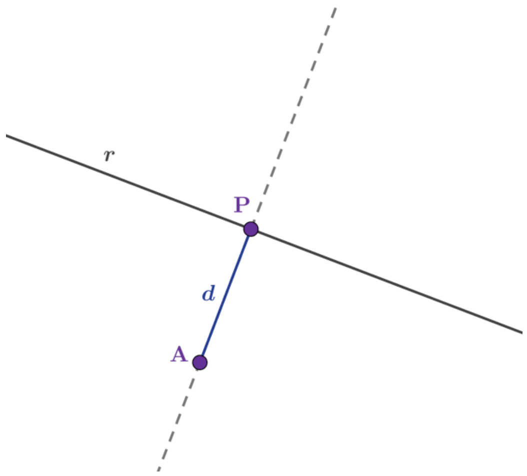
Steps	Explanation
	Sketch the lines to see the relative positions. The closest point will be the point of intersection of the lines $\boldsymbol{r}$ and $\overrightarrow{\mathrm{AP}}$, whe $\overrightarrow{\mathrm{AP}}$ is perpendicular to $\boldsymbol{r}$.
Position vector of point P relative to fixed point O is $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 1+\lambda \\ 2-\lambda \\ -1+\lambda \end{pmatrix}$	$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
$\overrightarrow{\mathrm{AP}} = \overrightarrow{\mathrm{OP}} - \overrightarrow{\mathrm{OA}} = \begin{pmatrix} 1+\lambda \\ 2-\lambda \\ -1+\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\overrightarrow{\mathrm{AP}} = \begin{pmatrix} \lambda \\ 3-\lambda \\ -2+\lambda \end{pmatrix}$	In the diagram, $\boldsymbol{d} = \overrightarrow{\mathrm{AP}}$.
$\overrightarrow{\mathrm{AP}} \cdot u = \begin{pmatrix} \lambda \\ 3-\lambda \\ -2+\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $= \lambda + (-3+\lambda) + (-2+\lambda)$ $\overrightarrow{\mathrm{AP}} \cdot u = 3\lambda - 5 = 0$	Let the direction vector of the line $\boldsymbol{r}$ be i.e. $\boldsymbol{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\boldsymbol{u}$ is perpendicular to $\overrightarrow{\mathrm{AP}}$ so $\overrightarrow{\mathrm{AP}} \cdot \boldsymbol{u} = 0$.

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Steps	Explanation
$\lambda = \frac{5}{3}$	
$\overrightarrow{OP} = \begin{pmatrix} 1+\frac{5}{3} \\ 2-\frac{5}{3} \\ -1+\frac{5}{3} \end{pmatrix} = \begin{pmatrix} \frac{8}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$	$\overrightarrow{OP} = \begin{pmatrix} 1+\lambda \\ 2-\lambda \\ -1+\lambda \end{pmatrix}$
So the coordinates of point P are $\left(\frac{8}{3}, \frac{1}{3}, \frac{2}{3}\right)$.	The question asks for the coordinates.

⚠ Be aware

In the IB examinations make sure you give the coordinates if this is what the question asks for. If you leave your answer as a position vector you might lose marks.

Example 3

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a) Find a vector equation of the line L_1 that passes through the points $\mathrm{A}\,(-3, -1, 2)$ and $\mathrm{B}\,(2, -3, 3)$.

Line L_2 has equation $r = \begin{pmatrix} 19 \\ -5 \\ 12 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

b) Find the angle between the two lines L_1 and L_2. Give your answer in degrees to 1 decimal place.

c) Find the point of intersection of the two lines L_1 and L_2.

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	Steps	Explanation
a)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$	Find the direction vector.
	$\mathrm{L}_1 : \quad r_1 = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$	Using the position vector Alternatively, you could us position vector of A and t equation would be $\mathrm{L}_1 : \ r_1 = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} + \lambda \Bigg($
b)	$\cos\theta = \frac{u \cdot v}{\|u\| \ \|v\|} = \frac{5 \times 1 + (-2) \times 2 + 1 \times 3}{\sqrt{14} \times \sqrt{30}}$ $= \frac{4}{\sqrt{14} \times \sqrt{30}}$ $\theta = \cos^{-1}\left(\frac{4}{\sqrt{14} \times \sqrt{30}}\right) = 78.7°$	Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be the directi vectors of the lines L_1 an $\boldsymbol{u} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ The angle between the tw lines can be found using t formula $\cos\theta = \frac{u \cdot v}{\|u\| \ \|v\|}$
c)	$r_1 = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 + 5\lambda \\ -3 - 2\lambda \\ 3 + \lambda \end{pmatrix}$ $r_2 = \begin{pmatrix} 19 \\ -5 \\ 12 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 19 + t \\ -5 + 2t \\ 12 + 3t \end{pmatrix}$	Write the position vectors points on each line.

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	Steps	Explanation
	$\begin{pmatrix} 2+5\lambda \\ -3-2\lambda \\ 3+\lambda \end{pmatrix} = \begin{pmatrix} 19+t \\ -5+2t \\ 12+3t \end{pmatrix}$	
	$2+5\lambda = 19+t \Rightarrow 5\lambda - t = 17$ $-3-2\lambda = -5+2t \Rightarrow 2\lambda + 2t = 2 \text{or} \lambda + t = 1$ $3+\lambda = 12+3t \Rightarrow \lambda - 3t = 9$	Equate the components a rearrange.
	$\left.\begin{array}{l} 5\lambda - t = 17 \\ \lambda + t = 1 \end{array}\right\} \lambda = 3$ and $t = -2$	Solve the first two equatic simultaneously.
	$\lambda - 3t = 9$ $3 - 3 \times -2 = 3 + 6 = 9$	Check that the solution al satisfies the 3rd equation.
	$\lambda = 3$ and $t = -2$	The two lines intersect.
	$r_1 = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} + 3\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+15 \\ -3-6 \\ 3+3 \end{pmatrix} = \begin{pmatrix} 17 \\ -9 \\ 6 \end{pmatrix}$ Therefore, the coordinates of the intersection point are $(17, -9, 6)$	You can substitute $\lambda = 3$ the equation for L_1 or $t =$ into the equation for L_2

Exam tip

In IB examinations, you can use a graphic display calculator in papers 2 and 3. So if you are asked to find the point of intersection in one of these papers you can use the simultaneous equations app on your calculator to solve the systems of equations.

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Example 4

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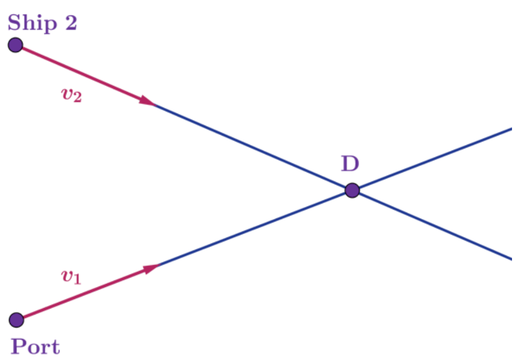
A ship starts its journey from a port and moves with velocity $v_1 = (3\boldsymbol{i} + 4\boldsymbol{j})\ \text{kmh}^{-1}$.

Another ship which is $5\ \text{km}$ north of the port starts its journey at the same time and travels with velocity $v_2 = (\boldsymbol{i} - 4\boldsymbol{j})\ \text{kmh}^{-1}$.

Determine whether or not the two ships will collide.

Steps	Explanation
$r_1 = \begin{pmatrix}0\\0\end{pmatrix} + t\begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}3t\\4t\end{pmatrix}$ $r_2 = \begin{pmatrix}0\\5\end{pmatrix} + t\begin{pmatrix}1\\-4\end{pmatrix} = \begin{pmatrix}t\\5-4t\end{pmatrix}$	Write a vector equation for both ships using $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$. Let the port be the point $\begin{pmatrix}0\\0\end{pmatrix}$ a the point that is $5\ \text{km}$ due north of the port have position vector $\begin{pmatrix}0\\5\end{pmatrix}$
$\begin{pmatrix}3t\\4t\end{pmatrix} = \begin{pmatrix}t\\5-4t\end{pmatrix}$	 If the two ships meet, they need to be at poin at the same time.
$\begin{cases}3t = t\\4t = 5-4t\end{cases}$	Examine the simultaneous equations. There is no t for which both equations are true.

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Steps	Explanation
Therefore, the two ships do not meet.	Even though both ships pass through the poi D, it is at different times.

Making connections

In the world of science, you often read about how two scientists collaborated on a solution to a particular problem. On one of these occasions, two professors, John Conway and Simon Kochen, proved 'if we have free will' using vectors. This theorem now famously called 'The free will theorem' and it is one of the important theorems of quantum physics. You can find more about the theorem and its development in this article (https://plus.maths.org/content/john-conway-discovering-free-will-part-i).

3 section questions

Question 1

★★★

Select which of the following is the point of intersection of the two lines

$$\mathrm{L_A}: \quad \boldsymbol{r}_\mathrm{A} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

and

$$\mathrm{L_B}: \quad \boldsymbol{r}_\mathrm{B} = \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

1 $(8, -2, 5)$ ✓

2 $(-8, 2, -5)$

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3 $(-8, 2, 5)$

4 $(8, 2, 5)$

Explanation

$$\mathrm{L_A}: \quad \boldsymbol{r}_\mathrm{A} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

and

$$\mathrm{L_B}: \quad \boldsymbol{r}_\mathrm{B} = \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

As in the 2D case, you should find the values for the parameters, here λ and μ, which give the same point of each line, if a point of intersection exists.

Any two of the x, y and z equations may be used.

Consider the equations in x and y.

This leads to the following system of equations:

$$\begin{aligned} -1 + 3\lambda &= 4 + 2\mu \\ 4 - 2\lambda &= 4 - 3\mu \end{aligned}$$

or, for line one $\times 2$ and line two $\times 3$:

$$\begin{aligned} 2(-1 + 3\lambda) &= 2(4 + 2\mu) \\ 3(4 - 2\lambda) &= 3(4 - 3\mu) \end{aligned}$$

giving

$$\begin{aligned} -2 + 6\lambda &= 8 + 4\mu \\ 12 - 6\lambda &= 12 - 9\mu \end{aligned}$$

Adding

$$\begin{aligned} 10 &= 20 - 5\mu \\ \mu &= 2 \end{aligned}$$

and thus,

$$\lambda = 3.$$

You need the values for *both* λ and μ for you need to substitute these into $\mathrm{L_A}$ and $\mathrm{L_B}$ respectively, to see whether the same z-coordinate results. If not, and the lines are not parallel, they are said to be skew (see below).

Substituting in $\mathrm{L_A} : 2 + 3 \times 1 = 5$ and $\mathrm{L_B} : 1 + 2 \times 2 = 5$ shows that, indeed, these two lines intersect.

Then, using either λ or μ to find the x- and y-coordinates of the point of intersection, you can find that $\mathrm{L_A}$ and $\mathrm{L_B}$ intersect at $(8, -2, 5)$.

Question 2

★★☆

Select which of the following is the point of intersection between the lines given by $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 8 \end{pmatrix}$ and $\boldsymbol{s} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

1 $(-83, -215)$ ✓

2 $(83, 215)$

3 $(75, 25)$

4 $(-7, -25)$

Explanation

Write the lines in their parametric form:

Line $\boldsymbol{r}$:

$$\begin{aligned} x &= -2 + 3\lambda \\ y &= 1 + 8\lambda \end{aligned}$$

Line $\boldsymbol{s}$:

$$\begin{aligned} x &= 3 + 2t \\ y &= 5t \end{aligned}$$

For them to intersect, lines $\boldsymbol{r}$ and $\boldsymbol{s}$ have to share a common point where the x-components and the y-components of both are equal to each other. Setting the equations equal to each other allows you to solve the simultaneous equations:

$$\begin{aligned} -2 + 3\lambda &= 3 + 2t \\ 1 + 8\lambda &= 5t \end{aligned}$$

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This gives either $\lambda = -27$ or $t = -43$, and putting these values back into their respective equations gives the point of intersection $(-83, -215)$.

Question 3

★★★

Two lines L_1 and L_2 are given by the equations $\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, respectively, and intersect at a point R.

The point $Q(11, 7, 4)$ lies on L_2.

A point P lies on L_1 such that $|QR| = |PR|$.

Select which of the following are the coordinates of P.

1. $(7, -9, 8)$ and $(-1, 7, -8)$ ✓
2. $(7, -9, 8)$
3. $\left(3 + \frac{\sqrt{3}}{2}, -1 + \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(3 - \frac{\sqrt{3}}{2}, -1 - \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$
4. $\left(3 + \frac{\sqrt{3}}{2}, -1\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$

Explanation

It is obvious that the point of intersection R is $(3, -1, 0)$ as it gives as a point on both lines in their given equations.

The distance QR is

$$\sqrt{(11-3)^2 + (7-(-1))^2 + (4-0)^2} = \sqrt{144} = 12$$

Let the point P have coordinates (x, y, z).

Because it lies on L_1, the coordinates can also be written as $(3 + \lambda, -1 - 2\lambda, 2\lambda)$.

The distance PR must be also 12.

Thus,

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$$\begin{aligned}
|PR|^2 = 144 \quad & = ((3+\lambda)-3)^2 + ((-1-2\lambda)-(-1))^2 + ((2\lambda)-0) \\
\Rightarrow \quad & = \lambda^2 + 4\lambda^2 + 4\lambda^2 \\
\Rightarrow \quad & = 9\lambda^2 \\
\Rightarrow 144 = 9 \times 16 \quad & = 9\lambda^2 \Leftrightarrow \lambda \pm 4
\end{aligned}$$

Hence, the point P has coordinates $(3 \pm 4, -1 - 2 \times \pm 4, 2 \times \pm 4)$ which gives the points $(7, -9, 8)$ and $(-1, 7, -8)$.

3. Geometry and trigonometry / 3.15 Relative positions of lines

Checklist

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What you should know

By the end of this subtopic you should be able to:

- use the vector equations of two straight lines to determine whether the lines are parallel, coincident, intersecting or skew
- recall that if two straight lines are parallel:
  - they have the same direction vector or the direction vectors are scalar multiples of the same vector
  - they have no points of intersection
- recall that if two straight lines intersect, then they will have one point in common; if two lines in 2D space are non-parallel, then they will always intersect at one point
- recall that if two lines have infinitely many points of intersection, then they are coincident
- recall that if two lines in 3D are non-parallel and have no points of intersection, then they are skew
- find the coordinates of the point of intersection of two lines that are not parallel, coincident or skew.

Student view

Investigation

How can you model the trajectory of a ball when you kick it? Can you estimate the location of the ball after it is kicked by a player so that you can intercept it? If you are the goalkeeper trying to prevent a goal being score from a penalty kick, how can you predict where the ball will go?

You can answer these questions using vectors, kinematics and maybe some calculus. Read this article (https://plus.maths.org/content/os/issue40/features/bray/index) about the maths behind kicking a ball.

You might need help from your friends: one person to be a goalkeeper, one to kick the ball and one person to record the result.

Ask a friend to kick the ball and get the goalkeeper to catch it. Record the motion of the ball and the goalkeeper. Work out an estimate of the speed of the ball and the angle at which it starts to move. Model the motion of the ball and the goalkeeper using vector equations and try to predict whether they will be able to intercept the ball.

Compare your model with the actual result. Why there might be differences between the model and the actual result? Will the ball move in a straight line?

What assumptions do you have to make when setting up your model? How could you make it more realistic?

Will the goalkeeper make the save?

Credit: Klaus Vedfelt Getty Images

Find out how the electronic system Hawk-Eye is used in sports such as tennis, cricket and football to predict the motion of a ball and plot its most-likely path.