Index

The big picture

Imagine that you leave your room right now and go for a walk. However, instead of planning where you will walk, you let probability decide where you will go. Each time you come to an intersection in the walkway or road, you use some type of random model (a coin flip for example) to determine in which direction to continue.

If you continued to randomly determine the direction you would travel in, what is the probability that you would make it back to your room? Is it guaranteed that you would some day return?

Concept

The patterns that emerge from randomness allow us to approximate the future conditions of a model.

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Theory of Knowledge

It is important to consider the impact of culture on the individual knower; however, it seems at first glance that mathematics is immune from cultural bias. Is such a statement accurate? Is this immunity a key factor in creating mathematical authority in regard to knowledge validity? Or on the contrary, do you believe mathematics is also influenced by culture?

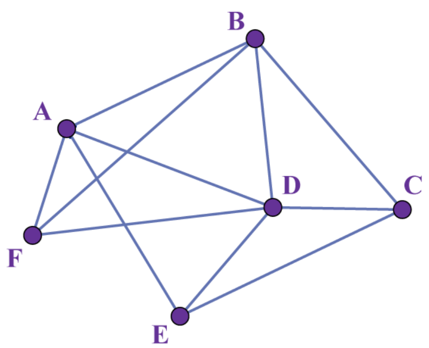
Knowledge Question: Is it possible to abstract knowledge from the culture in which it was produced?

3. Geometry and trigonometry / 3.15 Further matrices

Adjacency tables

As an introduction to adjacency tables, have a go at this activity.

Activity



More information

The image is a graph structure composed of six nodes labeled A, B, C, D, E, and F. These nodes are connected with multiple edges. Node A is connected to nodes B, C, D, E, and F. Node B has connections with nodes A, D, and C. Node C connects with nodes B, D, and E. Node D links to nodes A, B, C, E, and F. Node E shares edges with nodes A, C, D, and F. Finally, node F connects with nodes A, D, and E. The graph is a complete graph, meaning every pair of distinct nodes is connected by a unique edge.

[Generated by AI]

1. Make an ordered list containing all the edges in the graph shown above.
2. Reflect on the order of your list from the previous step. How did you decide to order the edges?

Student view

3. How can you verify that you have included all the edges from the graph in your list?
4. Would your method still be viable for a graph containing a large number of vertices and edges?

As you saw in the activity above, as graphs become larger, listing and ordering the edges become more difficult. An adjacency table can help you to see all the relationships in a graph clearly.

Important

An adjacency table, A, shows the number of edges that join each pair of vertices.

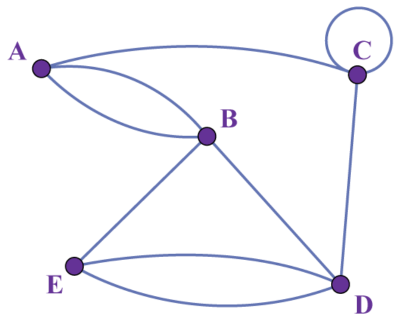
	A	B	C	D	E	F
A	0	1	0	1	1	1
B	1	0	1	1	0	1
C	0	1	0	1	1	0
D	1	1	1	0	1	1
E	1	0	1	1	0	0
F	1	1	0	1	0	0

The adjacency table shown above is for the graph you analysed in the activity.

Have all the edges in your list been included in the adjacency table?

Example 1

★☆☆



Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

More information

The image is a diagram of a directed graph with five vertices labeled A, B, C, D, and E. The vertices are connected by directed edges, creating a complex network.

- Vertex A is connected to vertices B and C.
- Vertex B is connected to C and D.
- Vertex C has a loop back to itself and is also connected to D.
- Vertex D is connected to vertices E and C.
- Vertex E is connected to B and D.

The graph's structure forms various paths, illustrating possible routes and connections between different points. This diagram represents data in the form of nodes and arcs, useful for creating an adjacency table.

[Generated by AI]

Create an adjacency table for the graph shown above.

Create the table by counting the edges between each pair of vertices. Note that the loop on vertex C counts as 2 in the adjacency table

	A	B	C	D	E
A	0	2	1	0	0
B	2	0	0	1	1
C	1	0	2	1	0
D	0	1	1	0	2
E	0	1	0	2	0

Activity

Consider the two adjacency tables for the two graphs shown above.

1. How does the sum of the values in a column for a vertex relate to the sum of the values in the row for that vertex? For example, how does the sum of the values in column A relate to the sum of the values in row A?

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

2. What does the sum of the columns or rows tell you about the vertex?
3. Find the sum of all the values in the adjacency table for the first graph. How does that sum compare to the total number of edges in the graph? Does the same relationship hold for the sum of the values in the adjacency table for the second graph?

✓ **Important**

The degree of a vertex can be found by finding the sum of the values in its column or row of an adjacency table.

The sum of all the values in the adjacency table of an undirected graph is equal to twice the number of edges in the corresponding graph. A loop is counted as 2 to maintain this relationship within an adjacency table.

Note that in other sources you may find a different approach to represent loops in the adjacency matrix. Some authors prefer to count the loop as one edge and enter a 1 in the diagonal. Both approaches can be used, but it is important to be consistent.

Example 2

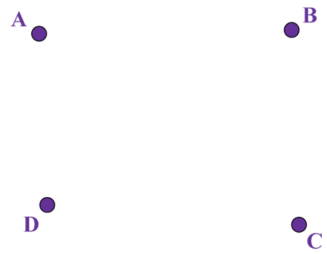
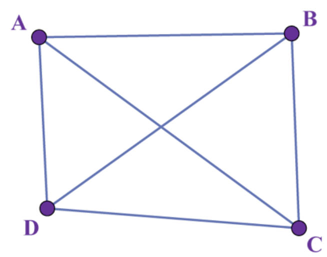
★☆☆

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0

Create a graph for the adjacency table shown above.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Steps	Explanation
Begin by drawing the four vertices:	
Then draw the edges as prescribed by the adjacency table:	

Here are the two adjacency tables again:

	A	B	C	D	E	F
A	0	1	0	1	1	1
B	1	0	1	1	0	1
C	0	1	0	1	1	0
D	1	1	1	0	1	1

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

	A	B	C	D	E	F
E	1	0	1	1	0	0
F	1	1	0	1	0	0

Activity

Consider the diagonals in the adjacency tables above formed by the cells for AA, BB, CC, etc. This diagonal is called the leading diagonal .

1. Recall that the adjacency table on the left was for the simple graph shown in the Important box, above.
   What are the values in its leading diagonal?
2. Recall that the adjacency table on the right was for the multigraph shown in **Example 1** above.
   What are the values of its leading diagonal?
3. In each adjacency table, how do the values on the right of the leading diagonal compare to the values on the left of the leading diagonal?

Important

In a simple graph, all values in the leading diagonal will be zero. All other entries in the adjacency table will either be 1 or 0. All adjacency tables of **undirected** graphs are symmetric about the leading diagonal.

Exam tip

Note that the answer for **Example 2** was a complete graph .

The adjacency tables of complete graphs contain a 0 in every cell of the leading diagonal.

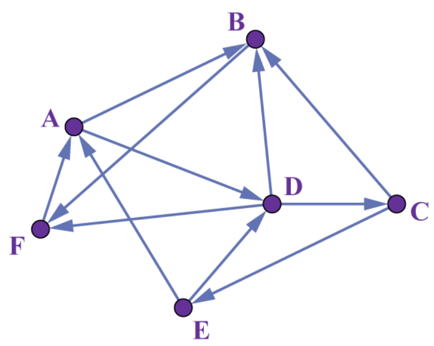
All other cells will contain a 1.

Adjacency tables for directed graphs

The graph from the start of this section is shown again below. However, the graph is now a directed graph since each of the edges has a specific direction. How would you construct an adjacency table for a directed graph?

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov



More information

The image depicts a directed graph with six nodes labeled as A, B, C, D, E, and F. The nodes are connected by directed edges, indicating specific directions.

- Node A has outgoing edges pointing to nodes B, C, and F.
- Node B has outgoing edges pointing to nodes D and E.
- Node C has an outgoing edge pointing to node D.
- Node D has outgoing edges pointing to nodes B and E.
- Node E has outgoing edges pointing to nodes C and F.
- Node F has an outgoing edge pointing to node A.

These edges create a network of paths where directionality is crucial in understanding the connectivity and flow from one node to another.

[Generated by AI]

Making connections

Recall from subtopic 1.14 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27428/) that the notation a_{ij} refers to the value in a matrix that is in the i^{th} row and the j^{th} column.

Important

The value of A_{ij} in the adjacency table for a directed graph is the number of edges that allow you to travel from vertex i to vertex j .

Student
view

Example 3

★★☆

Construct the adjacency table for the directed graph shown above.

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

As the graph is sim ple (there is at most one edge between each vertex and no loops), all of the values in the adjacency table will either be 1 or 0.

Note that this matrix is not symmetric to the main diagonal. For example, there is a 1 in the fourth coloumn of the first row, because the is an edge from A to D, but there is a 0 in the first coloumn of the fourth row, because the is no edge from D to A.

	A	B	C	D	E
A	0	1	0	1	0
B	0	0	0	0	0
C	0	1	0	0	1
D	0	1	1	0	0
E	1	0	0	1	0
F	1	0	0	0	0

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

Exam tip

Note that the adjacency table for a directed graph is not necessarily symmetric across the leading diagonal. This is because it is no longer possible to travel in both directions along an edge.

Adjacency tables for weighted graphs

More information

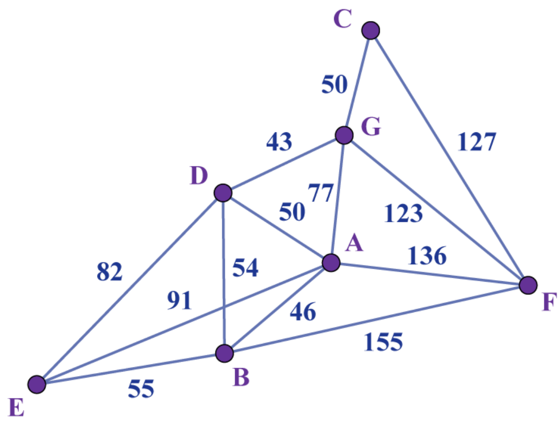
The map depicts a section of Germany with seven highlighted locations that Enrique plans to visit. These locations are Crivitz, Grabow, Dannenberg, Arendsee, Beetzendorf, Eldingen, and Fehrbellin. Each place is marked with a purple dot. The map shows a network of roads and rivers connecting these locations. Crivitz is positioned towards the north, while Fehrbellin is to the east. Dannenberg is situated centrally among the plotted locations. The map provides a visual representation of the geographical layout and possible driving routes between the destinations for Enrique's trip.

[Generated by AI]

The map above shows seven places that Enrique would like to visit during a trip to Germany. He will be driving a rental car to get to each place. He has created the graph, below, to help him decide how to order his trip.

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov



More information

The graph displays seven points labeled A, B, C, D, E, F, and G, representing places Enrique wants to visit in Germany. The distances between each point are labeled in kilometers. Starting from point E:

- E to B: 55 km
- E to D: 82 km
- E to A: 91 km

Continuing:

- B to A: 54 km
- B to D: 91 km
- B to F: 155 km

Other distances:

- D to A: 50 km
- D to G: 43 km

Connected from G:

- G to C: 50 km
- G to A: 77 km
- C to F: 127 km

Lastly:

- A to F: 136 km
- A to G: 123 km

The layout does not follow a typical linear or circular path but links each point through direct lines.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

[Generated by AI]

Example 4

★★☆

Construct an adjacency table for the weighted graph shown above.

For weighted graphs, the values in the adjacency table are the weights given to the edge connecting the two vertices:

	A	B	C	D	E	F
A	0	46	0	50	91	136
B	46	0	0	54	55	155
C	0	0	0	0	0	127
D	50	54	0	0	82	0
E	91	55	0	82	0	0
F	136	155	127	0	0	0
G	77	0	50	43	0	123

For his graph, Enrique decided to weight each edge with the distance of the route between the two towns. What other factors could Enrique have used to give a weight to each edge?

Exam tip

Common examples used for the weights of edges in a weighted adjacency table include cost, distance and time.

International Mindedness

Recall from the video in section 3.14.0 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-28226/) that Euler's development of graph theory originated in his desire to solve the seven bridges of Königsberg problem. The city of Königsberg was mostly destroyed by extensive

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

bombing during World War II. It was rebuilt and renamed as Kaliningrad by the Soviet Union and is now part of Russia. You can still see some of the original bridges in online maps.

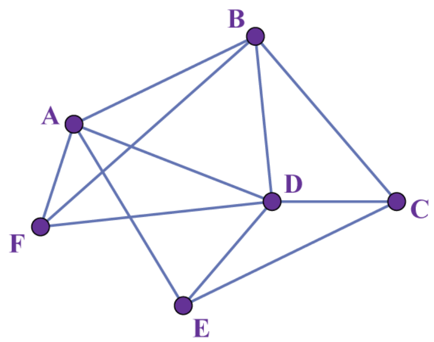
If someone were to ask you to list some cities that no longer exist, which ones would you include? Troy? Angkor? Other ancient cities? A Mental Floss article found here (http://mentalfloss.com/article/518369/30-cities-around-world-no-longer-exist)discusses other cities that have recently ceased to exist.

2 section questions

3. Geometry and trigonometry / 3.15 Further matrices

Walks

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/walks-id-28235/print/) Assign



More information

The image is a diagram representing a graph with vertices labeled as A, B, C, D, E, and F. Each vertex is connected by edges to some of the other vertices. Specifically,\n\n- Vertex A is connected to vertices B, D, F, and E.\n- Vertex B is connected to vertices A, C, and D.\n- Vertex C is connected to vertices B, D, and E.\n- Vertex D is connected to vertices A, B, C, and E.\n- Vertex E is connected to vertices A, C, D, and F.\n- Vertex F is connected to vertices A and E.\n\nThe arrangement allows for multiple possible paths or "walks" between any two vertices, and the edges indicate possible connections within the network.

[Generated by AI]

Consider the graph above. It is the first graph we looked at in section 3.15.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/adjacency-tables-id-28234/) ; however, this time we will think about the different possible walks we can take from one vertex to another.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

✓ **Important**

A walk is a sequence of connected edges.

From the adjacency table that was produced earlier, it should be clear that there is only one edge that directly connects vertices A and D. Therefore, there is only one walk between these vertices with a length of 1. However, what if you did not want to go directly from A to D?

Example 1

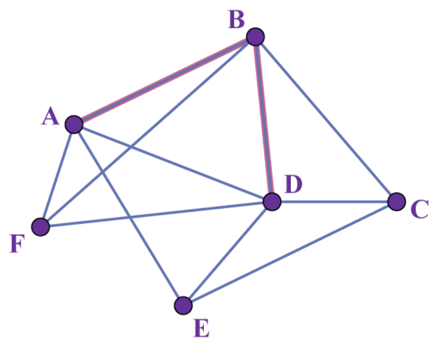
★☆☆

For the graph shown above, find the number of walks of length 2 between vertex A and vertex D.

Find the walks by looking at the graph to find sequences of two edges that begin at vertex A and end at vertex D:

The following walks from vertex A to vertex D are of length 2:

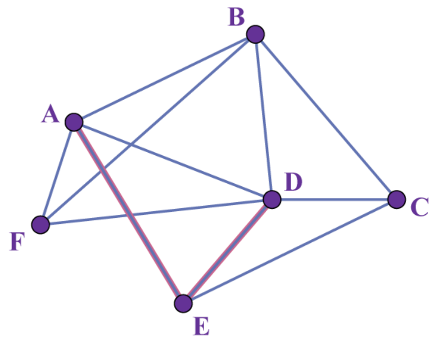
- AB , BD



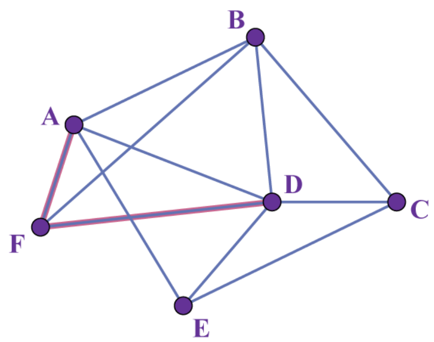
- AE , ED

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov



- AF , FD



Therefore, there are three walks of length 2 that begin at vertex A and end at vertex D .

Making connections

Recall from subtopic 1.14 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27428/) , that the notation $[a_{ij}]$ refers to a matrix with i rows and j columns. You may also want to refer to that section to review how to multiply matrices.

Activity

Consider again the vertices A and D in the graph shown at the start of this section.

Student view

Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

1. By choosing an appropriate method, find the number of walks between vertex A and vertex D that are of length 3.
2. Find how many walks between vertex A and vertex D are of length 4.
3. How can you be sure that your results from Steps 1 and 2 are correct?

Recall the adjacency table, A , for this graph:

	A	B	C	D	E	F
A	0	1	0	1	1	1
B	1	0	1	1	0	1
C	0	1	0	1	1	0
D	1	1	1	0	1	1
E	1	0	1	1	0	0
F	1	1	0	1	0	0

4. Input the values of the adjacency table into your GDC as a 6×6 matrix.
5. Using your GDC, find A^2. How does the value of ${A^2}_{4,1}$ (the value in the fourth row and first column) compare to the answer to **Example 1** from above?
6. Using your GDC find A^3 and compare ${A^3}_{4,1}$ to your answer from Step 1.
7. How can you verify your answer to Step 2?
8. Using your GDC, find the number of walks between vertex A and vertex D that are of length 5.
9. Now create a graph with at least 5 vertices and derive the adjacency table for it. Pick two vertices and check whether your method for finding the number of walks of different lengths from above works with this new graph.

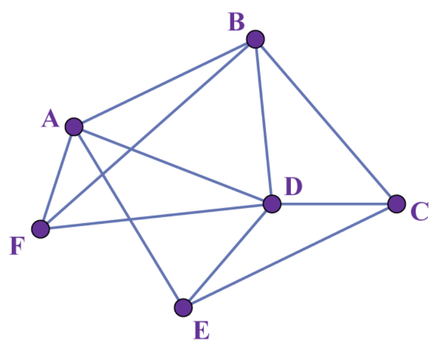
✓ **Important**

Given an adjacency table A , the number of k -length walks connecting vertex i and vertex j is the value of entry ${A^k}_{i,j}$.

Example 2

★★☆

Student
view



More information

The image is a graph consisting of six vertices labeled A, B, C, D, E, and F. These vertices are connected by edges, forming a network of paths. Vertex A is connected to B, F, and D. Vertex B is connected to A, D, and C. Vertex C is connected to B, D, and E. Vertex D is connected to A, B, C, and E. Vertex E is connected to D and C. Vertex F is connected to A. This structure allows various paths between the vertices, including from B to E, which is relevant to the context of the problem presented in the text.

[Generated by AI]

For the graph shown above, find the number of walks from vertex B to vertex E that are of length 6.

Begin by entering the adjacency table into your GDC as a 6×6 matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Use your GDC to find A^6 as you are looking for a walk of length 6:

$$A^6 = \begin{bmatrix} 569 & 537 & 446 & 643 & 414 & 462 \\ 537 & 569 & 414 & 643 & 446 & 462 \\ 446 & 414 & 352 & 498 & 320 & 362 \\ 643 & 643 & 498 & 753 & 498 & 534 \\ 414 & 446 & 320 & 498 & 352 & 362 \\ 462 & 462 & 362 & 534 & 362 & 391 \end{bmatrix}$$

The number of walks of length 6 from vertex B to vertex E is the entry in the second row and fifth column:

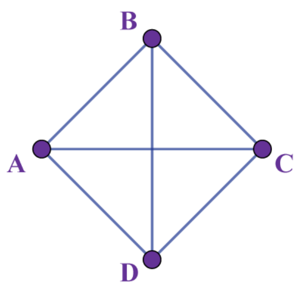
Overview
(/study/ap
ai-
hl/sid-
132-
cid-
761618/ov

$$A^6{}_{2,5} = 446$$

$\therefore$ there are 446 walks of length 6 from vertex B to vertex E.

Example 3

★★☆



More information

The diagram is a graph representing a quadrilateral structure with labeled vertices A, B, C, and D. These vertices are connected by edges forming a diamond shape. The specific connections are: vertex A is connected to vertices B, C, and D; vertex B is connected to vertices A, C, and D; vertex C is connected to vertices A, B, and D; vertex D is connected to vertices A, B, and C. The structure shows all vertices interconnected in a complete graph manner.

[Generated by AI]

By creating an appropriate adjacency table, find the number of walks of length 3 or less between vertex C and vertex D, in the graph shown above.

Create the appropriate adjacency table, A , for the graph:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Since you are finding all walks of length 3 or less, you need to find A^2 and A^3 :

Student
view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

$$A^2 = \begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{bmatrix}$$

Use the values in A , A^2 , and A^3 to find the number of walks with length 1, 2, or 3 between vertex C and vertex D:

Number of walks of length 3 or less:

$$\begin{aligned} \sum_{k=1}^{3} {A^k}_{3,4} &= {A^1}_{3,4} + {A^2}_{3,4} + {A^3}_{3,4} \\ &= 1 + 2 + 7 \\ &= 10 \end{aligned}$$

$\therefore$ there are ten walks of length 3 or less between vertex C and vertex D.

Random walks

Interactive 1. Random Walks.

More information for interactive 1

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

This interactive simulates a random walk on an isometric grid, allowing users to observe how a dot moves randomly across a grid that resembles a hexagonal layout, where each step is taken in one of six symmetrical directions. The grid is arranged so that the distances between points are equal in each direction, forming an evenly spaced pattern. In the bottom left corner of the screen, there are two buttons labeled “Start” and “Reset.” When “Start” is pressed, the dot begins moving randomly across the grid. Pressing “Reset” returns the dot to its starting position and clears its path. In the bottom right corner, a horizontal slider lets users adjust the speed of the dot’s motion, while two buttons labeled “Zoom plus” and “Zoom minus” allow users to zoom in and out on the grid to view the walk more closely. As the dot moves, several real-time statistics are displayed to help analyze its path: the total distance covered in steps shows how many moves the dot has made; the straight-line distance from the starting point indicates how far the dot is from its original position in a direct line; the number of lattice points visited counts how many grid points the dot has stepped on, including repeated visits; and the number of unique lattice points visited tells how many distinct points have been visited, which gives a sense of how widely the dot has explored. Users can restart the simulation as many times as they like to observe different random walk paths and compare how these statistics change with each run. This hands-on tool offers an intuitive and engaging way to understand randomness, distance, and spatial exploration in a structured grid environment, helping users build foundational insight into stochastic processes and probability.

Credit: GeoGebra (https://www.geogebra.org/m/j2hkxccy) Steve Phelps

Activity

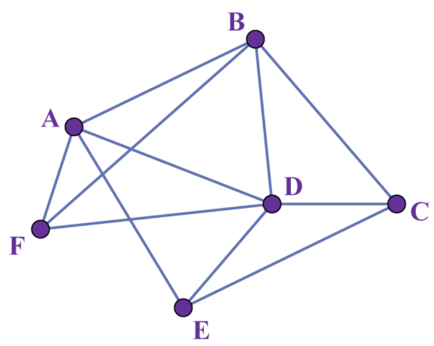
In the applet shown above, begin by pressing the reset button. Then press start and watch the dot move around the screen for a few moments. (If it is too fast or too slow, change the speed. You can also zoom in and out.)

- What is an isometric grid?
- In how many different directions can the computer choose to move the dot each time? (Hint: consider what isometric means.)
- How does the computer decide which way to step each time?
- Reset the applet and run the program a few more times. Is there a discernible pattern in how the dot is moving?

In the activity above, you saw that at each step the dot had six different options for the direction it could move in. As the probability was the same for each direction, the dot had a one in six chance of moving in any direction. Since the direction was chosen randomly at each step, the motion shown in the applet is an example of a random walk.

How can you use adjacency tables and your knowledge of multiplying matrices to calculate the probability of a random walk of k steps ending up at a specific vertex?

Student view



More information

The image shows a graph depicting a network of six vertices labeled A, B, C, D, E, and F. These vertices are connected by edges representing possible paths.

Vertex A is connected to vertices B, D, F, and E. Vertex B connects to A, D, and C. Vertex C is linked to B, D, and E. Vertex D connects with all vertices A, B, C, E, and F, acting as a central node. Vertex E connects to A, C, and D. Lastly, vertex F is connected to A and D.

The structure appears to display a random walk scenario where movements between vertices are represented by 1 unit of step probability, facilitating probability calculations of moving from one vertex to another in a defined number of steps, k.

[Generated by AI]

Let us consider again the graph shown above. You can see that four edges are incident to vertex A. Therefore, a random walk going through vertex A would have a probability of 0.25 of continuing to any of the four vertices connected to this vertex.

Exam tip

When calculating probabilities for random walks, there will be an equal probability of leaving a vertex via any available edge.

Be aware

In directed graphs, an edge must be directed away from the current vertex to be an option for continuing the random walk.

A transition matrix can now be created by determining the probability of a random walk continuing from a vertex to a connected vertex.

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

✓ **Important**

A transition matrix contains the probabilities of moving from one vertex to another vertex.

Example 4

★★☆

Create a transition matrix for the graph shown above.

Create a table by determining the probability of randomly moving between each vertex:

	A	B	C	D	E	F
A	0	0.25	0	0.25	0.25	0.25
B	0.25	0	0.25	0.25	0	0.25
C	0	$0.\dot{3}$	0	$0.\dot{3}$	$0.\dot{3}$	0
D	0.20	0.20	0.20	0	0.20	0.20
E	$0.\dot{3}$	0	$0.\dot{3}$	$0.\dot{3}$	0	0
F	$0.\dot{3}$	$0.\dot{3}$	0	$0.\dot{3}$	0	0

Then simply write the table as a matrix:

$$T = \begin{bmatrix} 0 & 0.25 & 0 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0 & 0.\dot{3} & 0 & 0.\dot{3} & 0.\dot{3} & 0 \\ 0.20 & 0.20 & 0.20 & 0 & 0.20 & 0.20 \\ 0.\dot{3} & 0 & 0.\dot{3} & 0.\dot{3} & 0 & 0 \\ 0.\dot{3} & 0.\dot{3} & 0 & 0.\dot{3} & 0 & 0 \end{bmatrix}$$

Making connections

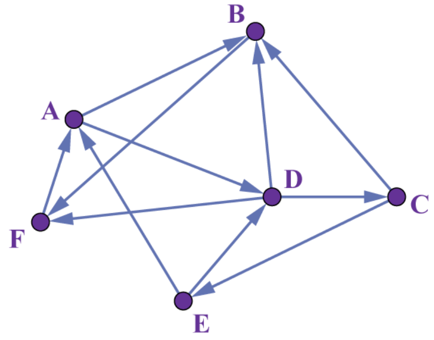
Transition matrices are discussed in detail in section 4.19.1 (/study/app/math-ai-hl/sid-132-cid-761618/book/transition-matrices-id-28004/) and in section 4.19.2 (/study/app/math-ai-hl/sid-132-cid-761618/book/markov-chains-id-28005/) they are also used to calculate probabilities using Markov chains.

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

Example 5

★★★



More information

The image is a directed graph with six vertices labeled from A to F. The vertices are interconnected by directed edges or arrows, indicating the direction of movement between the vertices.

- Vertex A has outgoing edges to vertices B, D, and F.
- Vertex B connects to vertices C and D.
- Vertex C has an arrow leading to vertex D.
- Vertex D has outgoing edges to vertices B, C, E, and F.
- Vertex E connects to vertices A and F.
- Vertex F has an arrow going to vertex E.

The arrows indicate the hierarchy of potential movements or connections between the different vertices.

[Generated by AI]

For the directed graph above, find the probability that a random walk that begins on vertex A will end on vertex E after three steps.

Begin by creating a table with the probabilities of moving from one vertex to another:

	A	B	C	D	E	F
A	0	0.5	0	0.5	0	0

Student view

Overview
(/study/app/math-ai-hl/sid-132-cid-761618/ov

	A	B	C	D	E	F
B	0	0	0	0	0	1
C	0	0.5	0	0	0.5	0
D	0	$0.\dot{3}$	$0.\dot{3}$	0	0	$0.\dot{3}$
E	0.5	0	0	0.5	0	0
F	1	0	0	0	0	0

Write the table as a matrix:

$$T = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0.\dot{3} & 0.\dot{3} & 0 & 0 & 0.\dot{3} \\ 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

As you are looking for the probabilities after three steps, you need to find T^3 :

$$T^3 = \begin{bmatrix} 0.66667 & 0.08333 & 0.00000 & 0.00000 & 0.08333 & 0.16667 \\ 0.00000 & 0.50000 & 0.00000 & 0.50000 & 0.00000 & 0.00000 \\ 0.50000 & 0.20833 & 0.08333 & 0.12500 & 0.00000 & 0.08333 \\ 0.41667 & 0.16667 & 0.00000 & 0.25000 & 0.00000 & 0.16667 \\ 0.16667 & 0.16667 & 0.08333 & 0.00000 & 0.08333 & 0.50000 \\ 0.00000 & 0.16667 & 0.16667 & 0.00000 & 0.00000 & 0.66667 \end{bmatrix}$$

The final answer is the value in the first row and fifth column as this corresponds to beginning on vertex A and ending on vertex E:

The probability of beginning on vertex A and ending on vertex E after three steps is 0.08333.

Note that **Example 5** could have also been solved by multiplying the individual probabilities for each of the steps in the only three-step walk from vertex A to vertex E:

$$\mathrm{P}\left(\mathrm{A \text{ to } E \text{ in three steps}}\right) = \mathrm{P}\left(AD\right) \cdot \mathrm{P}\left(DC\right) \cdot \mathrm{P}\left(CE\right) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$$

However, usually there will be several possible walks and finding them can be difficult, which is what makes the transition matrix so helpful.

Student view

3 section questions

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

3. Geometry and trigonometry / 3.15 Further matrices

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-28236/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- create an adjacency table for undirected graphs, directed graphs, and weighted graphs
- find the degree of a vertex by summing the values in its column or row of an adjacency table
- state that the sum of all values in the adjacency table of an undirected graph is equal to twice the number of edges in the corresponding graph
- describe the symmetry of the adjacency table for an undirected graph
- find the number of edges that allow you to travel from vertex i to vertex j by stating the value A_{ij} in the adjacency table for the directed graph
- calculate the number of k -length walks (or less than k -length walks) between two vertices
- construct a transition matrix for a random walk.

3. Geometry and trigonometry / 3.15 Further matrices

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-28237/print/) Assign

In 1998, Larry Page and Sergey Brin released an algorithm for ordering web pages called Page Rank, which became the core of the search engine they would name Google.

Today, Google and other popular search engines are an important factor in helping us find some order in the massive amount of information available on the internet.

However, how do these search engines know where all the information is and how do they make it readily available to us?

Student view

After watching the video shown above, consider the information found here (http://www.eprisner.de/MAT103/PageRank.html). Now consider the spider bot used to explore different web sites and produce a brief report that investigates its similarities (and differences) with a random walk.

Rate subtopic 3.15 Further matrices

Help us improve the content and user experience.