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2. Functions / 2.9 Further modelling

The big picture

Mathematical modelling has many important real-world applications. Watch the video to hear some students talk about the modelling process.

What is Math Modeling? Video Series Part 1: What is Math Modeling?

Concept

Many areas of knowledge, such as science, technology and social science, need the assistance of mathematicians in **modelling** and analysing data. Mathematical **modelling** helps in understanding the trends in the data and the phenomena behind the data. Also, powerful technology plays a major role in the modern methods of **modelling** especially in big data analysis and simulation experiments.

2. Functions / 2.9 Further modelling

Exponential model of half-life

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Exponential functions can be used to model the half-life of a substance.

Making connections

You may have learned about the half-life of radioactive substances in your Chemistry or Physics studies.

First let's refresh your knowledge of exponential growth and decay models, which you studied in subtopic 2.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/the-big-picture-id-27838/). Look at the two figures below. The upper figure shows exponential increase (for example, the growth of bacteria) and the lower figure shows exponential decay(for example, radioactive decay).

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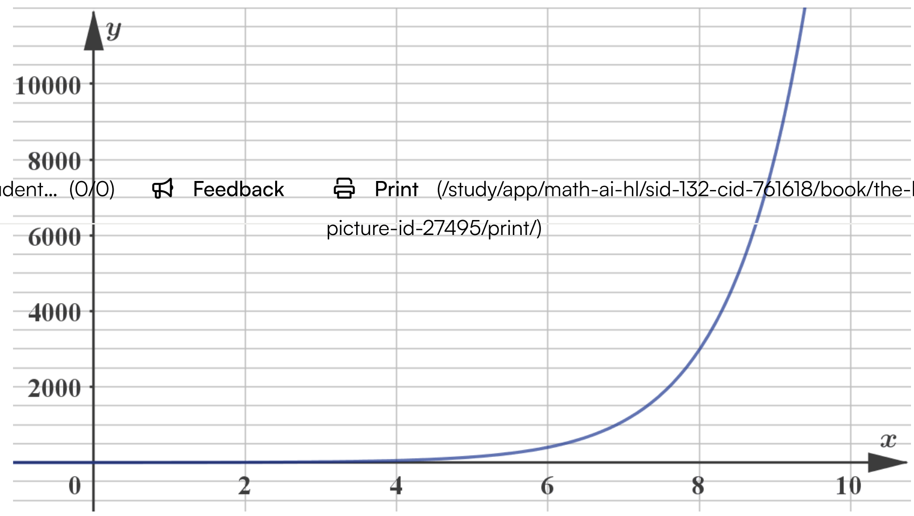

Graph of $y = e^x$

More information

The image is a graph depicting the exponential function (y = e^x). The X-axis, labeled as (x), ranges from 0 to 10. The Y-axis, labeled as (y), ranges from 0 to over 10,000. The graph shows a curve that starts from the origin (0,0), remaining close to the x-axis initially, and then rapidly increasing as it moves towards the right, demonstrating exponential growth. This represents the typical shape of an exponential function, with the curve becoming steeper as x increases.

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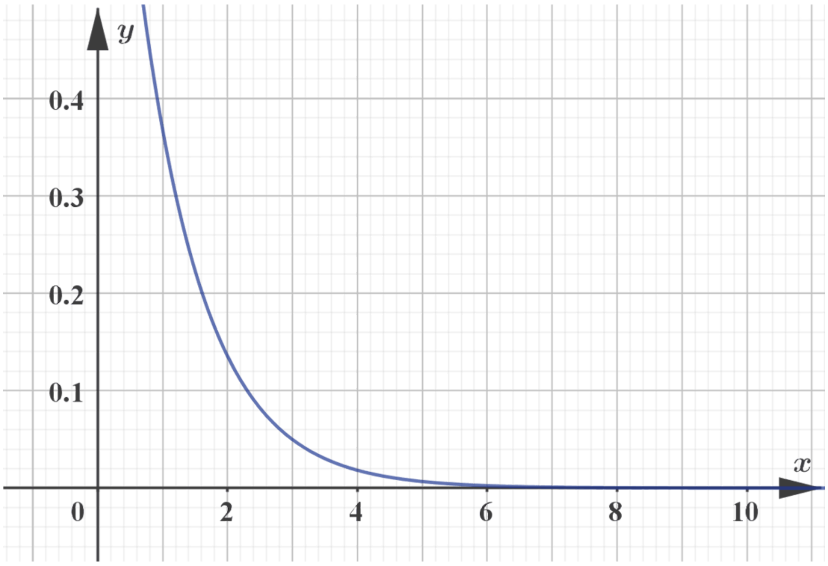

Graph of $y = e^{-x}$

More information

This image is a graph illustrating the function y = e^-x. The horizontal X-axis represents the values of x, starting at 0 and extending to 12. The vertical Y-axis represents the values of y, starting at 0.1 and extending upwards in increments of 0.1. The curve of the graph shows an exponential decay, starting at a higher y value when x = 0 and decreasing rapidly as x increases. As x approaches larger values, the curve flattens and approaches zero but never reaches it, demonstrating the characteristic shape of exponential decay. The gridlines provide a visual reference for reading values from both axes.

[Generated by AI]

When a radioactive isotope decays, the half-lifeis the time taken for the amount of the substance to decrease to half of its original amount.

The amount A after time t is given by $A = A_0\mathrm{e}^{-\lambda t}$, where A_0 is the initial amount and λ is the decay constant.

Using this formula, you can find the half-life of a substance. At the half-life, $A = \frac{A_0}{2}$, that is:

$$\begin{aligned}
\frac{A_0}{2} &= A_0\mathrm{e}^{-\lambda t} \\
\frac{1}{2} &= \mathrm{e}^{-\lambda t} \qquad \text{(cancelling } A_0 \text{ on both sides)} \\
\ln\left(\frac{1}{2}\right) &= -\lambda t \\
t &= \frac{\ln\left(\frac{1}{2}\right)}{-\lambda}
\end{aligned}$$

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Example 1

★★☆

After 1000 years, a sample of radium-226 has decayed to 64.7% of its original mass. Find the half-life of radium-226.

Let A be the mass of radium present at time t (where $t = 0$ corresponds to 1000 years ago). You want to find t when $A = \frac{1}{2}A_0$. However, first you need the value of λ in the formula $A = A_0 e^{-\lambda t}$. Once you know the value of λ, the decay constant, you can set $A = \frac{1}{2}A_0$ and solve for t .

We know that after 1000 years, the amount present is 64.7% of its original mass. That is, when $t = 1000$ years, $A = 0.647A_0$. Substituting these values into the formula for exponential decay, we obtain:

$$0.647A_0 = A_0 e^{-\lambda(1000)}$$

Dividing through by A_0 gives us:

$$0.647 = e^{-1000\lambda}$$

which is an exponential equation.

To solve this equation, apply the natural logarithm on both sides:

$$\begin{aligned}
\ln(0.647) &= \ln\left(e^{-1000\lambda}\right) \\
\ln(0.647) &= -1000\lambda \\
\lambda &= \frac{\ln(0.647)}{-1000} \\
\lambda &\approx 0.000435
\end{aligned}$$

Now that you know the decay constant, you can use the formula:

$$\begin{aligned}
t_{\frac{1}{2}} &= \frac{\ln\left(\frac{1}{2}\right)}{-\lambda} \\
t_{\frac{1}{2}} &= \frac{\ln\left(\frac{1}{2}\right)}{-0.000435} \\
t_{\frac{1}{2}} &\approx 1593 \text{ years}
\end{aligned}$$

The half-life is approximately 1593 years

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Be aware

If the formula is used as $A = A_0 e^{-\lambda t}$, the exponent is already negative. Hence the decay constant λ will be positive.

If the formula is given as $A = A_0 e^{\lambda t}$, then λ **will be negative** for exponential decay .

3 section questions

2. Functions / 2.9 Further modelling

More models

Logarithmic models

Logarithmic models are of the form $f(x) = a + b \ln x$. These functions can be useful to model data which is increasing, but the rate of increase is slowing down.

Let's look at for example the box office performance of the movie Frozen II in the first ten weeks after it was released in 2019.

source (https://www.the-numbers.com/movie/Frozen-II-(2019)#tab=box-office)

Week (after release)	Total gross (USD)
1	202 867 358
2	302 924 901
3	347 360 072
4	374 233 961
5	404 790 889
6	438 585 364
7	453 623 042
8	461 151 690

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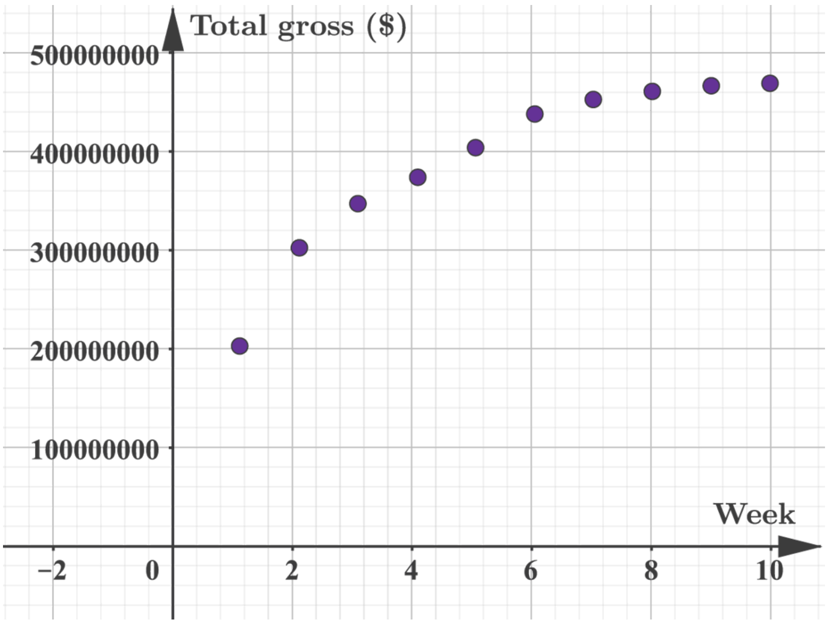
Week (after release)	Total gross (USD)
9	467 263 568
10	470 637 370

The following is a scatter plot of the data.



More information

This is a scatter plot representing total gross ($) over time, measured in weeks. The X-axis indicates the week number, ranging from 0 to 10. The Y-axis represents total gross, measured in increments of one billion dollars, ranging from 0 to 5 billion dollars. There are purple markers at various points, illustrating the data collected weekly. Starting from week 1, the data points rise gradually, peaking around week 8-9, and then show a slight leveling off towards week 10. The trend suggests overall growth in total gross over the observed period.

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In the model of the form $f(x) = a + b \ln x$ there are two parameters. Let us try to find these so that the model goes through the first and last point of the scatter plot.

Student view

- To match the first point, you need a and b such that $202\,867\,358 = a + b \ln 1$

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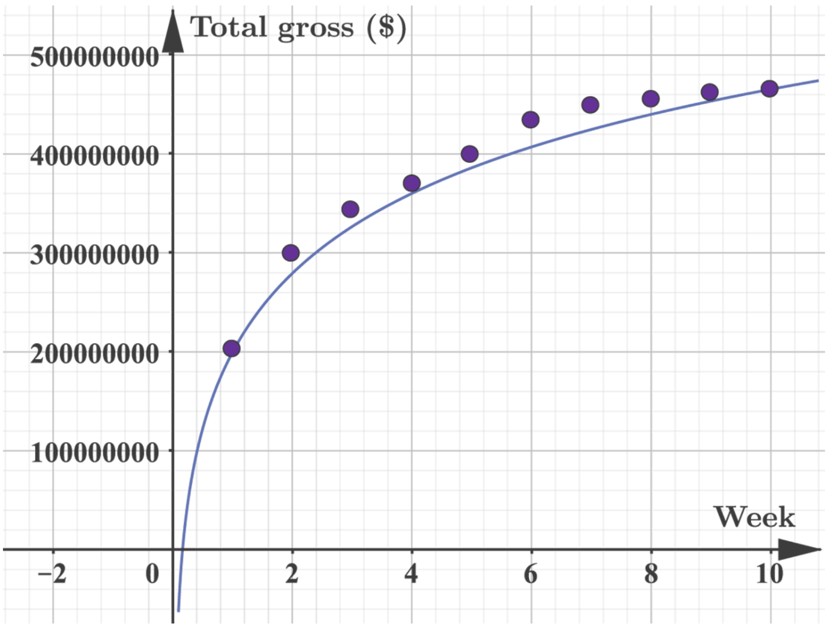
- To match the last point, you need a and b such that $470\,637\,370 = a + b\ln 10$

Since $\ln 1 = 0$, the first equation gives $a = 202\,867\,358$

Using this value we can solve the second equation for b.

$$\begin{aligned} 470\,637\,370 &= 202\,867\,358 + b\ln 10 \\ b\ln 10 &= 267\,770\,012 \\ b &= \frac{267\,770\,012}{\ln 10} \approx 116\,292\,037 \end{aligned}$$

You can check these values by drawing the graph of $f(x) = 202\,867\,358 + 116\,292\,037\ln x$ next to the scatter plot.



More information

The image is a graph depicting the total gross over time, represented as weeks. The X-axis is labeled 'Week,' ranging from 0 to 10. The Y-axis is labeled 'Total gross ($)' with intervals marked from $0 to $5,000,000,000.

The graph contains purple data points plotted in a scatter format. These points appear to follow a logarithmic trend, represented by a blue curve on the graph. The pattern suggests that total gross increases more rapidly initially before leveling out.

The graph shows a logarithmic curve, which is calculated by the function (f(x)=202,867,358+116,292,037\ln x). This curve fits through the initial and final data points, suggesting a model of how total gross changes over the weeks without using every single data point in its calculation. A calculator's logarithmic regression tool would include all points for a precise model.

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Student
view

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This graph of course goes through the first and last point, but the other data was not used to find the equation. The logarithmic regression tool of your calculator can find a model that uses all data points.

$$f(x) = 213\,186\,757 + 118\,612\,402 \ln x$$

Your calculator can show the scatter plot and the logarithmic regression curve on the same graph.

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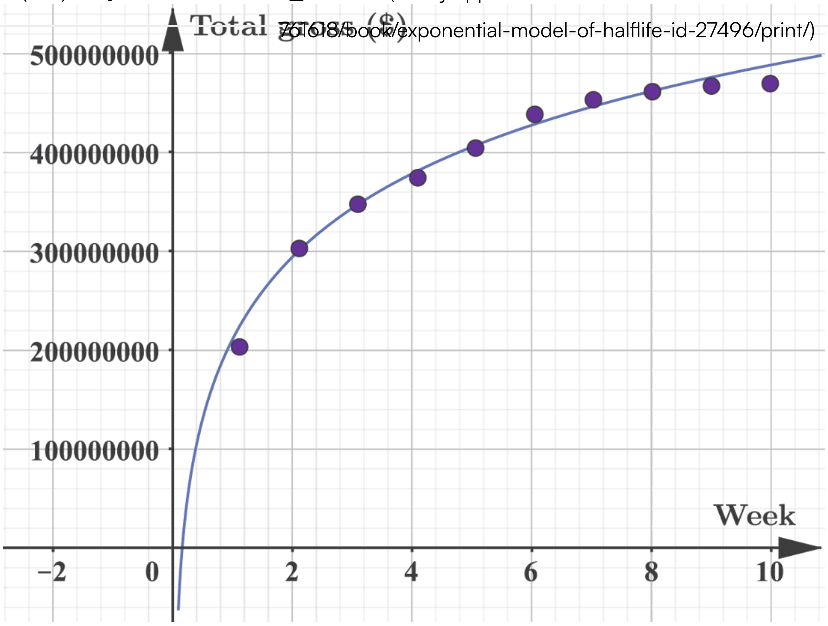

More information

The image is a graph showing a scatter plot with a series of data points and a blue logarithmic regression curve. The X-axis is labeled "Week" and ranges from 0 to 10, while the Y-axis is labeled "Total gross ($)" with values starting from $0 up to $500,000,000. The scatter plot points are shown in purple and follow an upward trend, initially rising steeply and then leveling off as they approach the higher values on the X-axis. The blue regression curve closely follows these data points, indicating the general trend over time.

[Generated by AI]

You can see that the trend is similar to the model you found algebraically, but the curve is overall closer to the data points.

Sinusoidal models

In section 2.8.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/reallife-applications-id-27625/) you already worked with periodic models of the form $f(x) = a\sin(b(x - c)) + d$. The key concepts to remember are the amplitude, the period, the equation of the axis and the phase shift.

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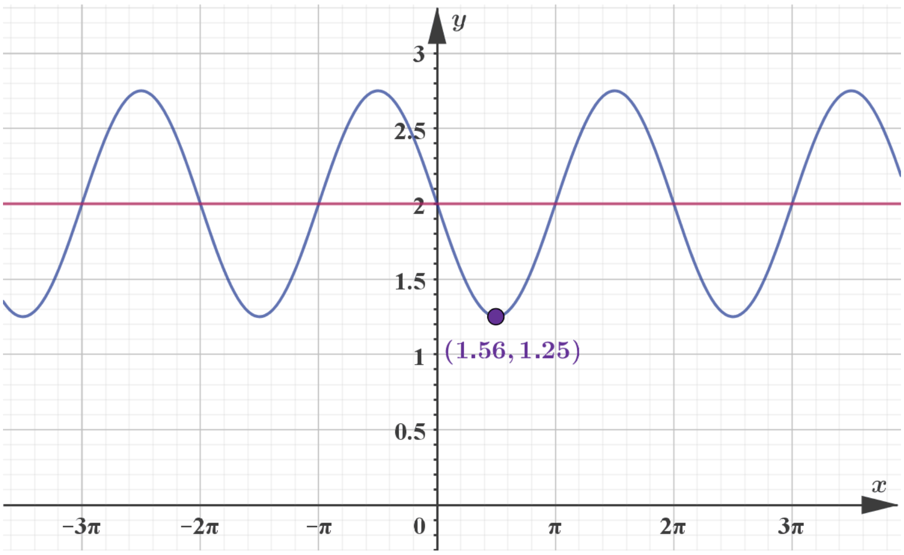
	Model $y = a\sin(b(x-c)) + d$	Example $y = 2\sin\left(3\left(x - \frac{\pi}{2}\right)\right) + 1$
Amplitude	$\|a\|$	2
Period	$\frac{2\pi}{\|b\|}$	$\frac{2\pi}{3}$
Axis	$y = d$	$y = 1$
Phase shift	c	$\frac{\pi}{2}$

You have also seen the example below to find the model from the graph.



More information

The image shows a graph illustrating a sine wave. The X-axis is represented by angles labeled in multiples of π, starting from -3π to 3π. The Y-axis is marked from 0 to 3, indicating amplitude. The sine wave oscillates between approximately 0.5 and 2.5. A specific point on the graph is highlighted, noted as (-1.56, 1.25), where the curve appears to reach a trough. The main trend is a periodic oscillation characteristic of a sine function, consistent with the formula provided: f(x) = 0.75 sin(x - π) + 2.

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The model for the graph above is $f(x) = 0.75\sin(x-\pi) + 2$. You can see the details of the calculation in section 2.8.5 (/study/app/math-ai-hl/sid-132-cid-761618/book/reallife-applications-id-27625/).

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Let's see how to fit a model to a data given by a table of values.

Example 1

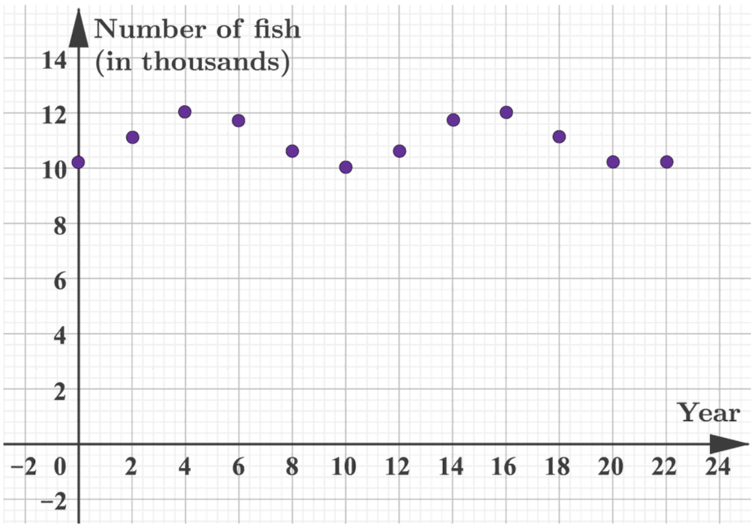
★★☆

The table shows the number (in thousands) of a species of fish at the start of each year over 22 years.

Year	0	2	4	6	8	10	12	14	16	18	20
Population	10.2	11.1	12.0	11.7	10.6	10.0	10.6	11.7	12.0	11.1	10.2

Why do the numbers oscillate? Find an appropriate sinusoidal model that fits the above data.

It is not possible to find the exact values of the parameters but they can be estimated. Let's draw a scatter plot of the data to visually see the pattern.



Looking at the data, the highest value is 12 and the lowest is 10, giving the vertical shift $\frac{10+12}{2} = 11$ and the amplitude $\frac{12-10}{2} = 1$.

So, $a = 1$ and $d = 11$.

Next, we will find the period and phase shift.

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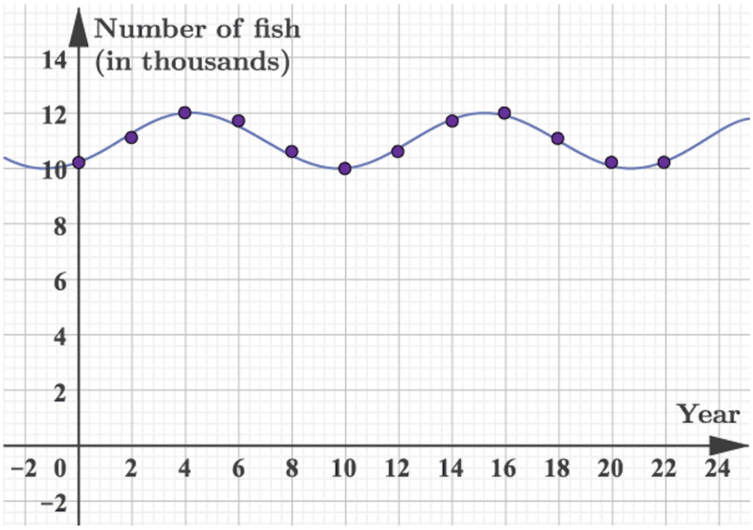
The minimum seems to be 10, and this occurs in year 10. There is another minimum between years 20 and 22. We can assume it occurred in year 21. Since, $21–10 = 11$, the period is 11 and so $b = \frac{2\pi}{11}$.

Since the vertical shift is 11, the axis is $y = 11$. The data suggests that the graph first intersects the axis between $t = 0$ and $t = 2$, closer to $t = 2$. Let's use, $c = -1.5$.

Hence, the model is

$$p(t) = \sin\left(\frac{2\pi}{11}(t - 1.5)\right) + 11$$

If you put the model on the graph, you can see that it indeed follows the oscillating pattern pretty closely.



To create the model only a few data points were actually used. Graphic display calculators have options to fit a sinusoidal model using all information given in the data set. Using this option gives the following model.

$$p(t) = 1.005\sin(0.5753t - 1.041) + 11.03$$

For this data set the two models are very close.

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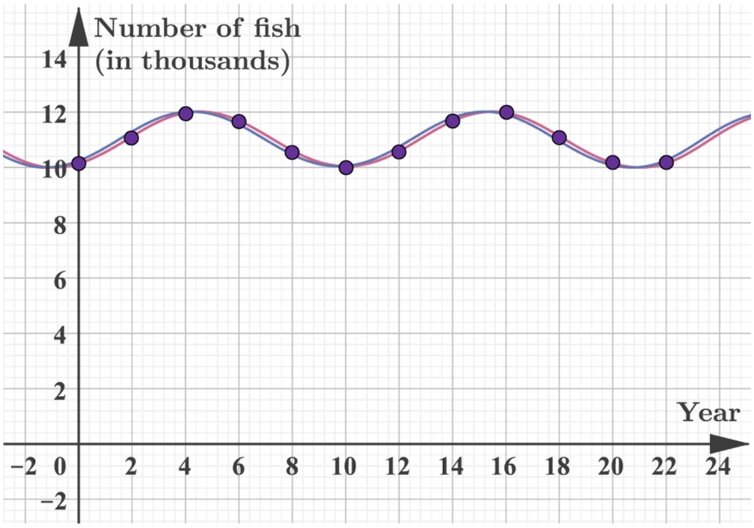

Logistic models

The next model you explore is the following.

$$f(t) = \frac{L}{1 + C\mathrm{e}^{-kt}}, \quad L,\ C,\ k > 0$$

Use the applet given below to investigate the impact of the parameters L, C and k on the graph by changing their values using the sliders.

What do you conclude?

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Interactive 1. Logistic Model: Investigate the Impact of the Parameters L, C and k on the Graph.

More information for interactive 1

The interactive allows users to explore the effects of the parameters *L*, *C*, and *k* on the graph of the logistic function $f(t) = \frac{L}{1+Ce^{(-kt)}}$, where *L*, *C*, and *k* are positive constants.

A graph with a blue line of logistic function. On the top left side of the graph, there are 3 horizontal sliders for the parameters, *L*, *C*, and *k* each, ranging from 0 to 5.

The parameter *L* vertically stretches the curve and shifts the horizontal asymptotes up or down, also affecting the y-intercept. The parameter *C* changes the horizontal position of the curve and alters the y-intercept. The parameter *k* affects the slope of the curve without changing the y-intercept.

By adjusting the sliders for each parameter, users can observe how changes in *L*, *C*, and *k* impact the shape and position of the graph. This interactive tool provides a hands-on way to understand how each parameter influences the logistic function, helping users visualize and analyze the behavior of the curve under different conditions.

Example 2

What is the impact of:

1. L
2. C
3. k

Student view

1. L stretches the curve vertically. It changes the position of the y-intercept, but more importantly it shifts the horizontal asymptote vertically up or down. It is also called the carrying capacity.

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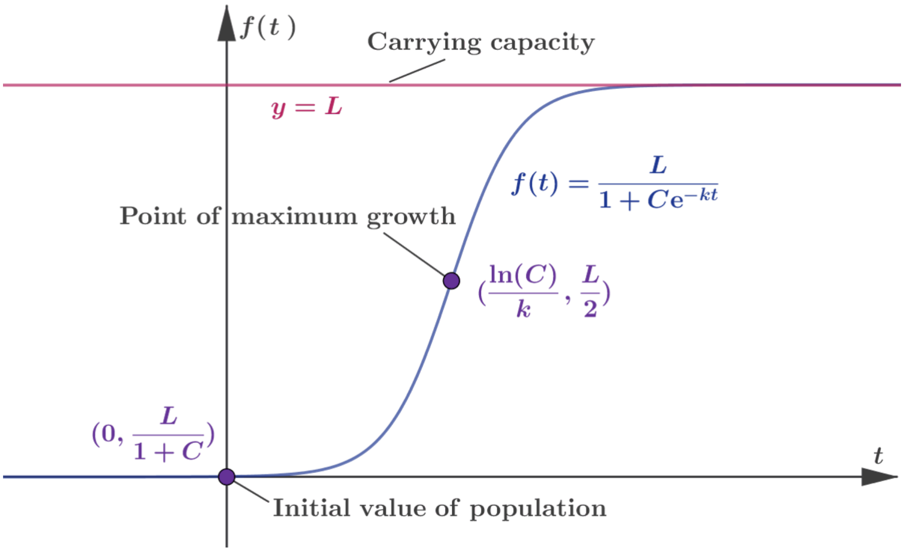
2. C changes the horizontal position of the curve. Since the curve is increasing, this also means that the y-intercept also changes.
3. k changes the slope of the curve. The y-intercept does not move when only k is changed.

The above model is called a logistic model. The important features of a logistic curve are shown below.



More information

The image shows a logistic model graph which presents a sigmoidal curve illustrating population growth over time. The curve starts near the 'Initial value of population' at the bottom left and increases steeply at the 'Point of maximum growth,' reaching a plateau at the 'Carrying capacity'. The X-axis represents time (t), while the Y-axis represents the population function f(t). A key formula is shown: (f(t) = \frac{L}{1 + Ce^{-kt}}). Various points are labeled: the initial value of population (\left(0, \frac{L}{1+C}\right)) at the curve's base, and the 'Point of maximum growth' (\left(\frac{\ln(C)}{k}, \frac{L}{2}\right)) towards the steepest part of the curve. The carrying capacity is denoted as (y = L), where the graph shows a horizontal line indicating the maximum population level. This line is labeled as "Carrying capacity" and crosses the Y-axis at (f(L)). The graph effectively communicates the concept of logistic growth, depicting how the growth rate accelerates rapidly before slowing down as it approaches the upper limit.

[Generated by AI]

International Mindedness

The logistic equation was first used by Pierre-François Verhulst and later modified by many scientists such as A.G. McKendrick, Raymond Pearl and Lowell Reed. It was mainly used for biological growth models. However, this model is useful in other aspects of contemporary science including neural networks and machine learning. It is important in economics, such as for the diffusion of innovations. How do you think this model could be useful in the future?

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Example 3

The following data is part of what Verhulst published in his second paper on using the logistic curve to model population growth.

source: Verhulst, Pierre-François (1847). "Deuxième mémoire sur la loi d'accroissement de la population". Mémoires de l'Académie Royale des Sciences, des Lettres et des Beaux-Arts de Belgique (https://gdz.sub.uni-goettingen.de/id/PPN129323659_0020). 20: 1—32.

Year	Population of Belgium
1815	3 627 253
1825	4 024 855
1830	4 247 113
1835	4 404 220
1840	4 608 776
1845	4 800 861

(a) Draw a scatter plot and comment on the trend.

(b) Use the population data in 1815, 1830 and 1845 to find the parameters of a logistic curve that goes through these three points.

(c) Draw the model on your scatter plot. Extend the viewing window to see the key features of the logistic curve. Comment on the the trend you see.

(d) Use the logistic regression option of your calculator to find a model that also considers the other data points. Comment on the difference between the two models.

(e) According to the Worldometer data (https://www.worldometers.info/world-population/belgium-population/), the population of Belgium at the time of writing this (in 2020) is $11\ 589\ 623$. How does this information change the model?

(a) Note that on the scatter plot below the coordinate axes do not cross at $(0, 0)$.

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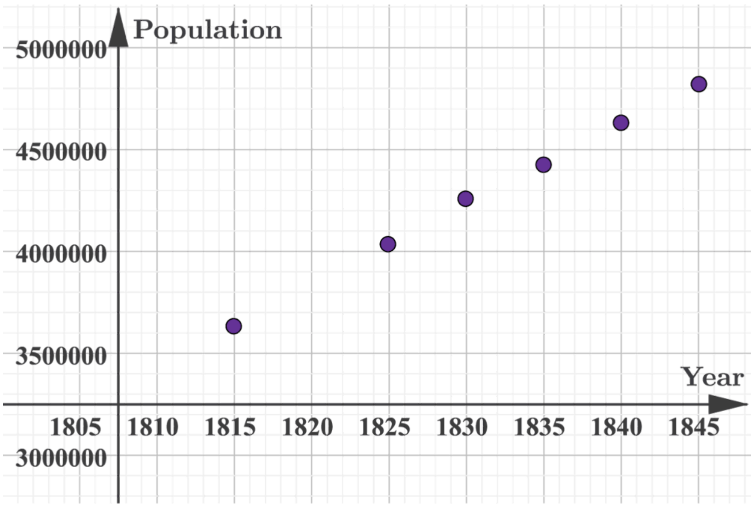

The data shows an increasing trend. It is close to linear, but it is not a perfect linear fit.

(b) The general form of the logistic model is $f(t) = \frac{L}{1 + Ce^{-kt}}$. Substituting the values for population data in 1815, 1830 and 1845 gives an equation system to solve for L, C and k.

$$f(1815) = \frac{L}{1 + Ce^{-1815k}} = 3\,627\,253$$
$$f(1830) = \frac{L}{1 + Ce^{-1830k}} = 4\,247\,113$$
$$f(1845) = \frac{L}{1 + Ce^{-1845k}} = 4\,800\,861$$

Multiplying each equation with the denominators give three expressions for L. You can set these equal to each other to eliminate L and get two equations in C and k.

$$3\,627\,253\left(1 + Ce^{-1815k}\right) = 4\,247\,113\left(1 + Ce^{-1830k}\right)$$
$$4\,247\,113\left(1 + Ce^{-1830k}\right) = 4\,800\,861\left(1 + Ce^{-1845k}\right)$$

Let's use now the first equation to express C in terms of k.

$$3\,627\,253 + 3\,627\,253Ce^{-1815k} = 4\,247\,113 + 4\,247\,113Ce^{-1830k}$$
$$C\left(3\,627\,253e^{-1815k} - 4\,247\,113e^{-1830k}\right) = 619\,860$$
$$\frac{619\,860}{3\,627\,253e^{-1815k} - 4\,247\,113e^{-1830k}} = C$$

Similar calculation gives an expression for C from the second equation.

$$\frac{553748}{4\,247\,113e^{-1830k} - 4\,800\,861e^{-1845k}} = C$$

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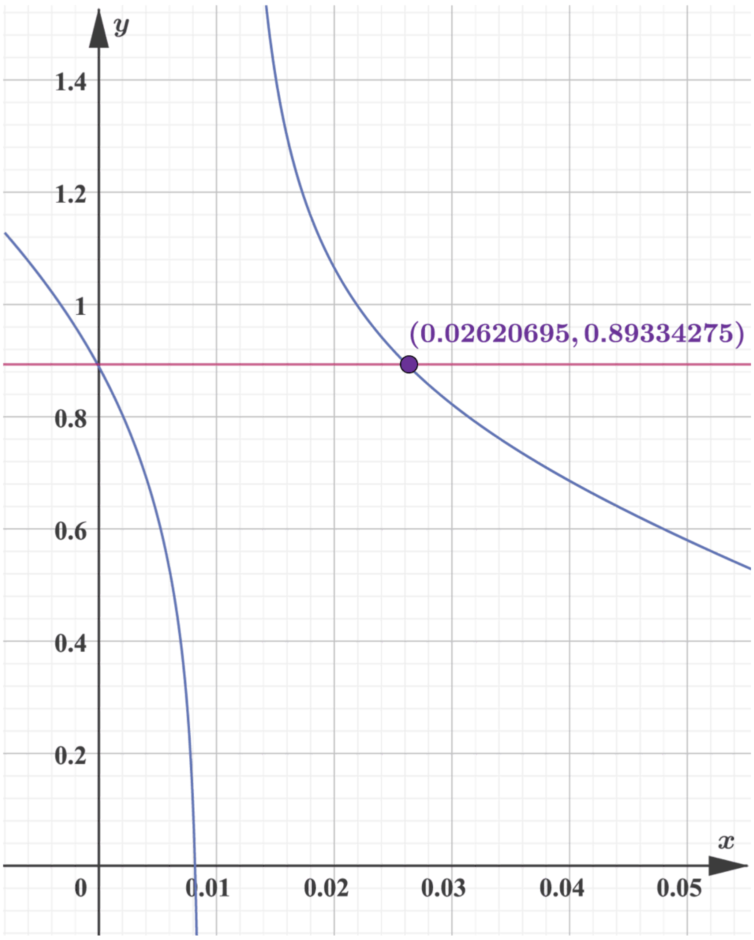
Set these expressions equal to each other and use your calculator to solve the equation. Because of the large numbers involved, use the following rearrangement, graph the left and right hand side of the equation and find the intersection point.

$$\frac{553748}{619\,860} = \frac{4\,247\,113\mathrm{e}^{-1830\mathrm{k}} - 4\,800\,861\mathrm{e}^{-1845\mathrm{k}}}{3\,627\,253\mathrm{e}^{-1815\mathrm{k}} - 4\,247\,113\mathrm{e}^{-1830\mathrm{k}}}$$

You may need to adjust the window several times before you get a clear view of the intersection.



The solution is $k \approx 0.02620695$

Use this value to find $C \approx 3.70344 \times 10^{20}$

Using these values you can get $L \approx 6\,583\,218$

(c) The graph below shows the model $f(t) = \dfrac{6\,583\,218}{1 + 3.70344 \times 10^{20} \times \mathrm{e}^{-0.02620695t}}$

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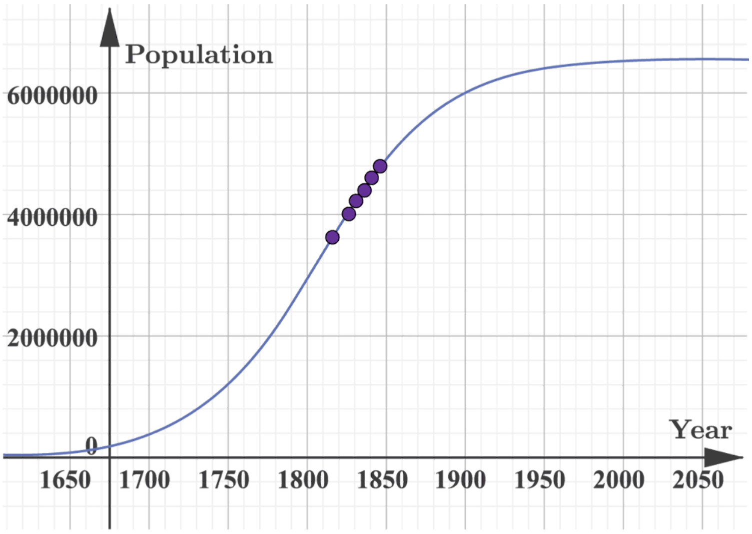
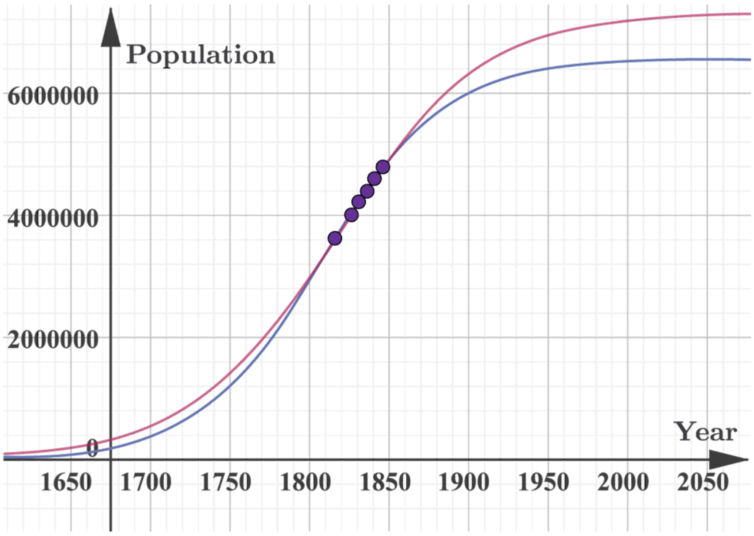


The model based on the information available in 1849 predicted an increase of population which by 2000 stabilises a bit below seven million people in Belgium. This actually did not happen. Compare recent data with the prediction of this model.

(d) The logistic regression tool of the calculator gives the following model.

$$f(t) = \frac{7\,347\,823}{1 + 1.7401524 \times 10^{17} \times \mathrm{e}^{-0.0218585542t}}$$

Let's add this model to the graph.



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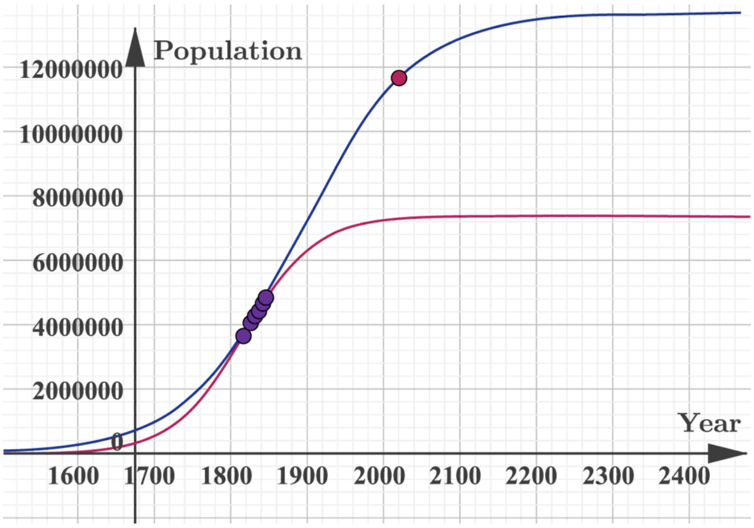
The model that the calculator gave has a similar trend to the manually calculated model, but it predicts a bit higher carrying capacity.

(e) The current (2020) population in Belgium is higher than the carrying capacity predicted by the previous models. Add this data to the list and ask the calculator to find the logistic regression with this extra information. You will get the following model.

$$f(t) = \frac{13\,618\,708}{1 + 1.0009706 \times 10^{11} \times \mathrm{e}^{-0.0134019063t}}$$

Add this to the graph:



This new model predicts the increase in the population for some time from now with stabilisation around 13.6 million in a few hundred years from now.

Theory of Knowledge

We often try to predict the future using mathematical models. Such models are commonly found by simulation on a computer. However, many things are happening more quickly in the modern era compared to the past. Do you think these models help us to better predict such changes with less errors of approximation?

Exam tip

Student view

Overview (/study/app/math-ai-hl/sid-132-cid-761618/ov

You should be able to recognise the different kinds of models and understand what the parameters mean and how to find them.

2 section questions

2. Functions / 2.9 Further modelling

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/checklist-id-27498/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- Model a data set using one of the following forms:
  - exponential decay model
  - logarithmic model
  - logistic model
  - sinusoidal model
  - piecewise model.

2. Functions / 2.9 Further modelling

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-ai-hl/sid-132-cid-761618/book/investigation-id-27499/print/) Assign

Here is a picture of a car. Can you model the outline of the car so that it can be exactly reproduced in a different scale?

Student view

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Credit: Stratol Getty Images

The shape of a car affects its aerodynamics.

Use graphing software to model the outer shape (the red line) and find the best fit as a piecewise function.

This video demonstrates how to create piecewise functions in GeoGebra.

Authoring & Analyzing PIECEWISE FUNCTIONS in GeoGebra GC: Yes, ...

Authoring & Analyzing PIECEWISE FUNCTIONS in GeoGebra GC: Yes, It's THAT EASY!

More information

This video provides a step-by-step demonstration of how to create and analyze **piecewise functions** using GeoGebra, a dynamic graphing software. The screen is divided into two sections: on the left, a list of variables and expressions is displayed, while on the right, a Cartesian coordinate plane dynamically updates to reflect changes in the entered functions.

The presenter begins by typing a **piecewise function** into the input field, defining different expressions for different intervals of x. The expression entered is "$\text{If}(x < -6, 3, -6 < x < 0, x + 9)$".

As the equation is entered, the corresponding graph updates in real time. Initially, a **horizontal line segment** appears at $y = 3$ for $x < -6$, while a sloped line segment forms between $x < -6$ and $x = 0$.

The function is then extended to include another segment, making it a continuous function across multiple intervals. The presenter prefixes the function with "$f(x) =$", ensuring that GeoGebra recognizes it as a formal mathematical function.

Student view

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Next, a **sine wave** is introduced as part of a new piecewise function.

The piecewise function will be:

$$f(x) = \begin{cases} 3 & : \quad x < -6 \\ x + 9 & : \quad -6 \leq x < 0 \\ 9 - 2\sin(x) : & \quad x \geq 0 \end{cases}$$

The presenter enters an equation that generates a sine curve for $x < -2\pi$, which is then modified to transition into a horizontal segment for $-2\pi \leq x \leq 2\pi$. These updates illustrate how different mathematical expressions can be combined to construct complex **piecewise models.**

The presenter then defines a new function, "$g(x)$", which is plotted on the graph in a brownish-red color.

$$g(x) = \begin{cases} 3\sin(x) - 4 & : \quad x < -6.283 \\ -4 & : \quad -6.283 \leq x \leq 6.283 \\ -\frac{8}{10-2\pi}|x - 10| + 4 : & \quad \text{otherwise} \end{cases}$$

Following this, the presenter types "$c = \text{Integral Between}[f, g, a, b]$" to define "c" as the definite integral between the functions "f" and "g" over the interval [a, b].

Sliders for the variables "a" and "b" appear on the left side of the interface, allowing the user to adjust their values within specified ranges (-10 to 10 for "a" and -10 to 20 for "b"). On the graph, points A and B are initially positioned at (-10, 0), and a shaded region representing the integral appears between the two functions. At this stage, the shaded area is minimal because both "a" and "b" are set to -10.

As the presenter increases the value of "b" using its slider, point B moves along the x-axis, and the shaded area between the functions expands. The value of the integral, "c", is displayed near the top right of the screen and updates dynamically as the shaded area changes. The presenter continues to adjust the slider for "b", demonstrating how the integral value increases or decreases depending on the size of the shaded region. As point B moves beyond the initially visible range of the x-axis, the graph automatically scrolls to the right to accommodate the new position of point B, ensuring that the visualization remains clear and interactive.