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5. Calculus / 5.7 Optimisation



Notebook

# The big picture



Glossary



Reading  
assistance

In this subtopic you will learn about the application of differentiation in context.

Have you ever wondered about what type of function models the shape of hanging chains or cables?



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Credit: picturist Getty Images

### Hanging curves

These curves look like parabolas, but in fact they are not. In your physics studies you may have heard about this curve (or you may in your future studies). It is called a catenary. The name is derived from the Latin word for chain. It is the shape with lowest potential energy. Interestingly, this shape is often used in architecture. For example, the cross-section of the roof structure at Keleti railway station in Budapest, Hungary, can be modelled by such a curve.



Washington Dulles International Airport

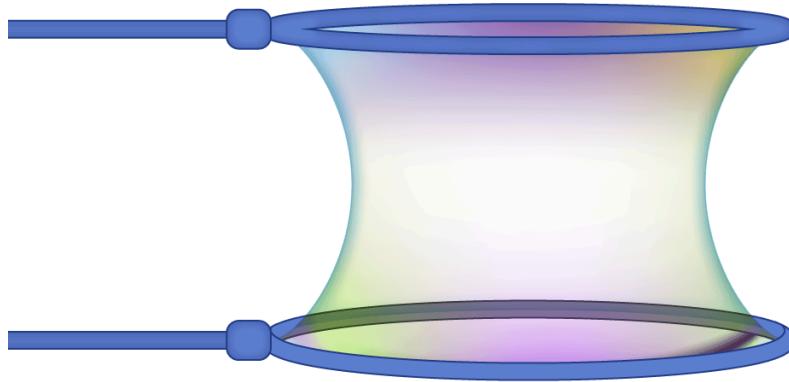
Credit: joeravi Getty Images



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Yet another interesting 3D connection is the shape of the soap film between two circular wire frames. The cross-section is a catenary.

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 More information

The image depicts a soap film stretched between two circular wire frames. The film assumes a saddle-like shape known as a catenary surface, which is a minimal surface connecting the two frame boundaries. The frames appear parallel to each other with the film smoothly spanning the space between them, demonstrating surface tension at work creating a minimal surface area configuration. The structure is symmetrical along a vertical axis, with the soap film narrowing near the midpoint and expanding towards the edges, illustrating the property of minimizing the surface area between the boundaries.

[Generated by AI]

The shape of the soap film has the property that it minimises the surface connecting the boundary. Watch this video to see some more minimum 3D shapes formed by soap film.



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Finally, take a look at the video below. It is an illustration of how nature finds the 2D shape that has the maximum area with a fixed perimeter.

### Soap Film Loops



#### Video 2. Soap Film Loops.

More information for video 2

The video titled "Soap-Film Loops" is used as an example to demonstrate the theme of optimisation, that is, finding minimum or maximum values. The video is an experiment that explores the behavior of soap films and their interaction with external objects.

The scene transitions to show a circular plastic loop with a handle being dipped in a dark liquid. When the loop is lifted from the liquid, a soap film stretched across the circular frame of the loop is formed. The handle of the loop with the soap film is then placed in a wooden holder.



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A thin thread with a small slip knot at its end is placed in the soapy film. The thread is carefully manipulated within the soap film, demonstrating the precision required to interact with such a fragile medium. When the thread is poked gently, the thread gets pulled and a circular hole is created in the soap film.

Another shorter thread with a small slip knot at the end is placed into the soap film. When the thread is poked gently, the thread gets pulled and a smaller circular hole is created in the soap film. The two circular holes created by the threads move when disturbed and get attracted to one another.

The threads are removed gently one after the other without breaking the soap film. The experiment illustrates how nature finds the 2D shape that has the maximum area with a fixed perimeter, which in this case is a circle.

In all of the examples above, the connecting theme is optimisation, that is, finding minima or maxima. However, the mathematical tools needed to work out these optimum shapes are beyond the scope of high school mathematics. In this subtopic, you will see how to use the tools that you have already learned to solve simpler problems. These will serve as a foundation for future studies where you may learn to tackle these more advanced questions.



## Concept

Calculus is an important tool for investigating **relationships** in real life. In this subtopic, you will see some examples of this, but, of course, there are plenty more that are not discussed here. While studying the examples in this book, look out for possible similarities with other applications where you could use calculus as a **modelling** tool to investigate a problem and find an optimal solution.

5. Calculus / 5.7 Optimisation

# Optimisation problems

In optimisation problems, you will need to be able to find the minimum or maximum value of a function. Here is a summary of what you have already seen in previous sections.



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## ✓ Important

If  $f$  is a differentiable function defined on the interval  $[a, b]$ , then the largest and smallest values of the function are either  $f(a)$ ,  $f(b)$  or the  $y$ -coordinate of one of the turning points.

Note also, that at turning points  $f'(x) = 0$ .

To see an example of an optimisation problem, take a look at this video.

Building aviary



Video 1. Building aviary.

More information for video 1

A roll of silver wire mesh stands upright against a weathered wooden fence, its metallic surface glinting faintly in the light. The roll is tall and cylindrical, its edges neatly coiled, suggesting it is new or only partially used. Atop the upright roll, a black plastic disc sits securely, fastened in place with white plastic ties that crisscross over its surface. The ties appear taut, ensuring the disc remains firmly in position, likely to protect the roll from dirt or moisture. To the right of the upright roll, another partial roll of wire mesh leans casually against the fence. This second roll is slightly obscured, its edges less defined, as though it has been used or left unattended for some time. To the left of the scene, green leafy plants grow vibrantly, their natural hues contrasting with the industrial appearance of the wire mesh and the rustic texture of the wooden fence.

Nearby, several long, light-colored wooden planks are scattered across a dark paved surface. The planks vary in length, their smooth surfaces and straight edges suggesting they have been recently cut or prepared for construction. They lie in a seemingly haphazard arrangement next to a brick wall, their pale tones standing out against the darker pavement. Against the brick wall, a large rectangular frame constructed from similar wooden planks leans at an angle. The frame is supported by additional wooden pieces, which prop it up securely, creating a



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slanted appearance. The frame's size and sturdy construction hint at its potential use in a larger project. In the background, a partially covered object is visible, its details obscured by a tarp or other material, adding an air of mystery to the scene.

In a nearby garden, a large aviary enclosure dominates the space. The aviary is constructed from reddish-brown wooden beams, their warm tones complementing the natural surroundings. The structure is covered with wire mesh, creating a secure yet breathable environment for its inhabitants. A metal ladder leans against the inside wall of the aviary, suggesting ongoing maintenance or adjustments. The enclosure is surrounded by lush greenery, with green grass stretching out in the foreground and various plants and a section of wooden fence visible in the background. The garden setting feels serene, a harmonious blend of natural and man-made elements.

As the camera moves closer to the aviary, the interior comes into focus, revealing its lively occupants: bright yellow canaries. The birds flit about energetically, their vibrant plumage standing out against the muted tones of the wire mesh and wooden frame. Branches are strategically placed within the enclosure, providing natural perches for the birds to rest on. The scene is alive with movement and the soft chirping of the canaries, adding a sense of vitality to the tranquil garden.

The camera lingers on a section of the aviary where darker wire mesh has been used, its finer texture creating a subtle contrast with the rest of the enclosure. Behind the aviary, ivy climbs the wooden fence, its green tendrils weaving through the slats and adding a touch of wild beauty to the scene. The interplay of light and shadow highlights the intricate patterns of the ivy leaves and the mesh, creating a visually striking composition.

Finally, the camera zooms in for a close-up of a single canary perched delicately on a metal feeder within the aviary. The bird's bright yellow feathers seem to glow in the light, their vivid color a testament to the health and care it receives. The feeder, simple yet functional, holds a small amount of food, and the canary appears content as it pauses to eat. The close-up captures the intricate details of the bird's feathers and the gentle curve of its beak, offering a moment of quiet intimacy in the bustling aviary.

If you would like to build an aviary similar to the one in the video, you will need some building material.

Of course, the bigger the aviary the more material you need. The next example will show you how to minimise the cost if you need a particular amount of space for your birds but have some flexibility in choosing the actual dimensions of the aviary.

## Example 1



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view

Suppose that you need 1 cubic metre of space for the birds and you already have the timber to build the frame. You decide that you would like the aviary to be a cuboid with a square base. The following table shows the price of 1 square metre of the different materials you need.

material	price per square metre in euros
floor	8
wire mesh for the sides	5
roofing	10

Other than having a square base, you do not have any preference about the shape. It can be a tall aviary with a small base or a flat aviary with a larger base. Your main goal is to minimise the cost of the materials. Start by introducing some variables:  $x$  for the length of the sides of the square base and  $y$  for the height of the cuboid (both measured in metres).

- a) Express the volume of the cuboid in terms of these variables.

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- b) Using the volume given in the question, express  $y$  in terms of  $x$ .

- c) Let  $C$  be the total cost of the material. Express  $C$  in terms of  $x$ .

- d) Find  $C'$ , the derivative of  $C$ .

- e) Use this derivative to find the  $x$ -value that gives the minimum cost.

- f) Find the amount of the different materials you need to build this optimum aviary.

- g) Find the cost of this optimum aviary.

- h) Confirm your calculation using a GDC.

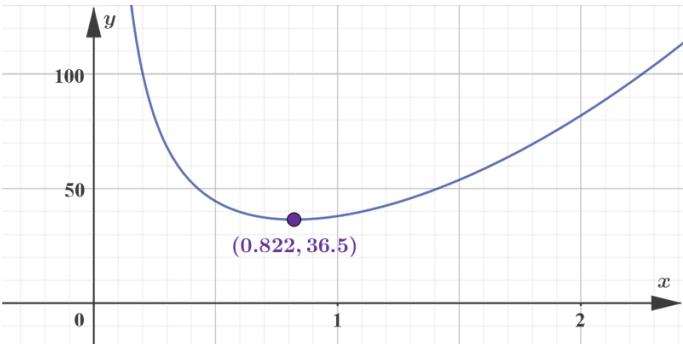


Section	Steps	Explanation
a)	$V = xxy = x^2y$	The volume of a cuboid is the product of its length, breadth and height.
b)	$x^2y = 1$ $y = \frac{1}{x^2}$	The question specifies that you need a cuboid of volume 1 cubic metre.
c)	The cost of the material for the floor is $8x^2$ euros.	The floor is an $x$ by $x$ square.  The price of the flooring is 8 euros per square metre.
	The cost of the material for the roof is $10x^2$ euros.	The roof is also an $x$ by $x$ square.  The price of the roofing is 10 euros per square metre.
	The cost of the material for the sides is $4 \times 5xy = 20x \frac{1}{x^2} = \frac{20}{x}$ euros.	There are four $x$ by $y$ rectangles on the side.  The price of the wire mesh for the sides is 5 euros per square metre.
	$C(x) = 18x^2 + \frac{20}{x}$	The sum of these individual costs is the total cost.
d)	$\begin{aligned} C(x) &= 18x^2 + \frac{20}{x} \\ &= 18x^2 + 20x^{-1} \\ C'(x) &= 18 \times 2x + 20 \times (-1)x^{-2} \\ &= 36x - \frac{20}{x^2} \end{aligned}$	



	Steps	Explanation
e)	$C'(x) = 0$ $36x - \frac{20}{x^2} = 0$ $36x = \frac{20}{x^2}$ $36x^3 = 20$ $x^3 = \frac{20}{36}$ $x = \sqrt[3]{\frac{20}{36}} \approx 0.822$	At turning points, the derivative is 0.
	The length of the side of the base square of the optimum aviary is approximately 0.822 metres.	
f)	The height of the optimum aviary is $y = \frac{1}{0.822^2} \approx 1.48$ metres.	
	You need $0.822^2 \approx 0.676$ square metres material for the floor.	The floor is an $x$ by $x$ square.
	You need $0.822^2 \approx 0.676$ square metres material for the roof.	The roof is also an $x$ by $x$ square.
	You need $4 \times 0.822 \times 1.48 \approx 4.87$ square metres material for the sides.	There are four sides of size by $y$ .
g)	The minimum cost to build an aviary that has 1 cubic metre space for the birds is $0.676 \times 8 + 4.87 \times 5 + 0.676 \times 10 \approx 36.5$ euros.	You can find the cost by adding the cost of the different materials.

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	Steps	Explanation
h)		Plot the cost function and find the minimum point.
		⑧
	The first coordinate of the turning point confirms the optimum value of $x$ .	
	The second coordinate of the turning point confirms the optimum cost.	

## 3 section questions ▾

5. Calculus / 5.7 Optimisation

# Optimisation in context

## Section

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In this section, you will see some problems that can be solved using the techniques you learned in the previous sections.

## Example 1



Student view

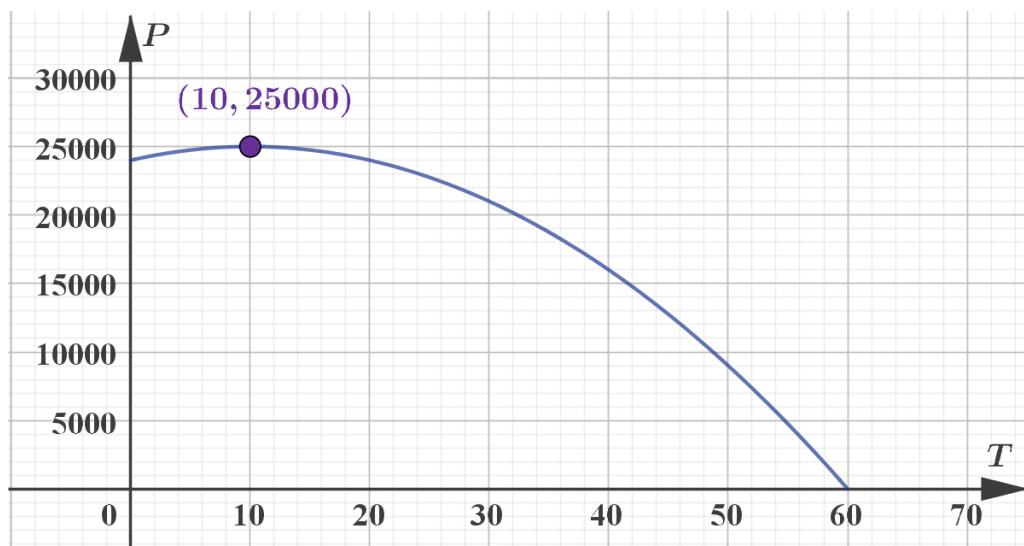
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There are 40 apple trees in an orchard. Each tree produces 600 apples in a year. For each additional tree planted in the orchard, the output of every tree drops by 10 apples. How many trees should be added to the existing trees in the orchard to maximise the total production of apples?

Steps	Explanation
<p>Let <math>P</math> be the production of the orchard (total number of apples).</p> <p>Let <math>T</math> be the number of trees added to the orchard.</p> <p>Let <math>A</math> be the number of apples on a tree.</p>	<p>Introducing variables is useful to translate the problem to the language of mathematics.</p> <p>The question gives the relationships between these variables.</p>
$P = (40 + T)A$	The production is the number of trees multiplied by the number of apples on a tree.
$A = 600 - 10T$	The number of apples on a tree decreases when new trees are planted.
$P = (40 + T)(600 - 10T) = 24\,000 + 200T - 10T^2$	The combination of these two relationships gives the production of the orchard in terms of the number of new trees.
$\frac{dP}{dT} = 200 - 20T = 0$ $T = 10$	At the optimum level of production the rate of change, $\frac{dP}{dT}$ , is 0.
$P(10) = (40 + 10)(600 - 10 \times 10) = 25\,000$ <p>The maximum production of the orchard is 25 000 apples in a year and it is achieved by planting 10 new trees in addition to the existing 40.</p>	Find the maximum production by substituting the optimum number of new trees into the expression for the production.

Instead of finding the derivative and setting it to 0, you can answer the question by graphing  $P$  as a function of  $T$  and finding the maximum point. It is a good idea to check your results using a graph even if you use the algebraic method to answer the question.

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Student view



## Example 2



A farmer wants to enclose a rectangular part of his field. He has 800 metres of fencing available to use. What is the largest possible area that he can enclose?

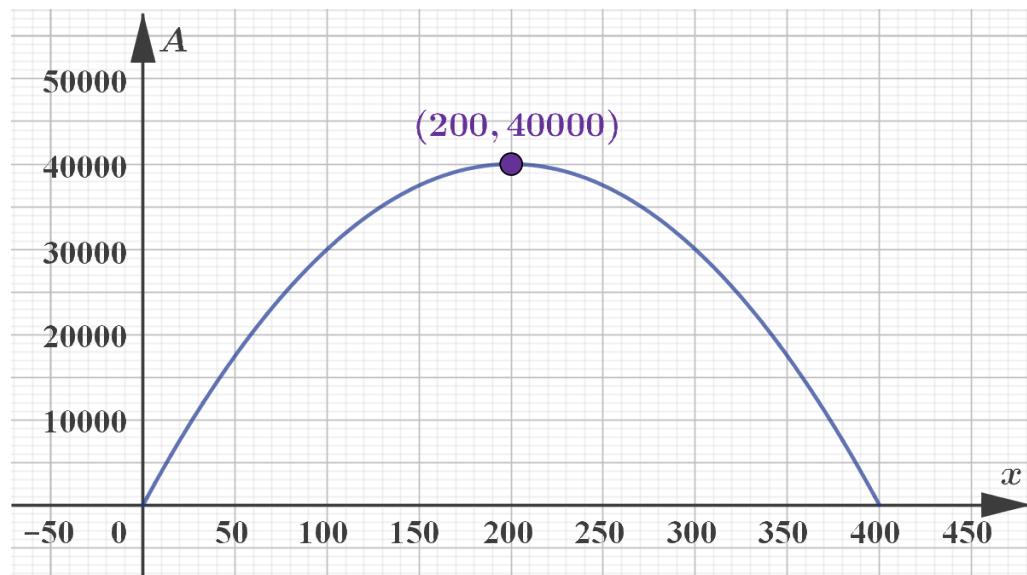
Steps	Explanation
Let $x$ and $y$ be the length of the two sides of the rectangle.	Introducing variables is useful to translate the problem to the language of mathematics.
$2x + 2y = 800$ $x + y = 400$ $y = 400 - x$	The given perimeter can be used to express $y$ in terms of $x$ . Since both $x$ and $y$ are positive, the meaningful $x$ -values are $0 < x < 400$ .



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Steps	Explanation
$\begin{aligned} A &= xy \\ &= x(400 - x) \\ &= 400x - x^2 \end{aligned}$	The area can also be expressed in terms of $x$ .
$\begin{aligned} \frac{dA}{dx} &= 400 - 2x = 0 \\ x &= 200 \\ \text{For } x = 200, \text{ the area is } A &= 200 \times 200 = 40\,000. \end{aligned}$	You can use the derivative to search for the maximum point.
Hence, the maximum rectangular area that the farmer can enclose with 800 metres of fencing is 40 000 square metre.	If there is exactly one turning point on the graph, then it gives the global maximum or minimum.

Note, that instead of using the derivative, you can get the same result by finding the vertex of the quadratic expression that gives the area in terms of  $x$ .



## Example 3

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Nasima is preparing for a craft fair. She needs to decide on the quantity of her product to take to the fair. To make the product, she needs to invest USD 30. After this initial investment the materials to make each item costs USD 10. From past experience, she estimates that if she charges USD  $p$  for each item, then she will sell  $100 - 2p$  items at the fair.

- How many items should she make and how much should she charge for each item to maximise her profit?
- What is her maximum profit?

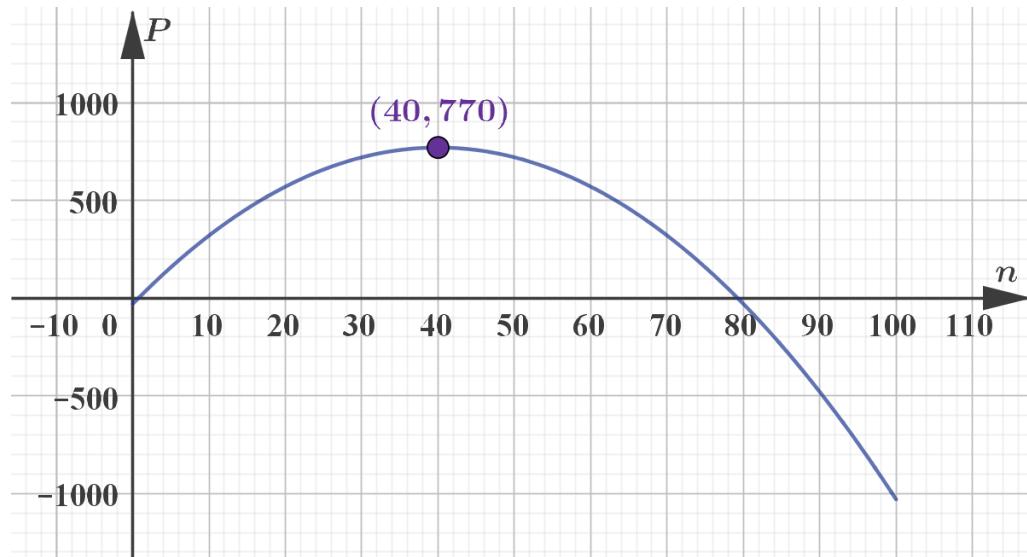
Steps	Explanation
Let $n$ be the number of items she takes to the fair and let $p$ be the price of each item.	Introducing variables is useful to translate the problem to the language of mathematics.
$\begin{aligned} n &= 100 - 2p \\ 2p &= 100 - n \\ p &= 50 - \frac{n}{2} \end{aligned}$	If she wants to sell all the items, she needs to price them according to her past experience.
$\begin{aligned} I(n) &= np \\ &= n \left(50 - \frac{n}{2}\right) \\ &= 50n - \frac{n^2}{2} \end{aligned}$	Her income at the fair is the product of the number of items sold and the price of each item.
$C(n) = 30 + 10n$	Her cost is the sum of the initial investment and the cost of the materials to make the items for the fair.
$\begin{aligned} P(n) &= I(n) - C(n) \\ &= \left(50n - \frac{n^2}{2}\right) - (30 + 10n) \\ &= -\frac{n^2}{2} + 40n - 30 \end{aligned}$	The profit is the cost subtracted from the income.
The maximum profit corresponds to $n = -\frac{40}{2 \times (-1/2)} = 40$ .	The turning point of the parabola $y = ax^2 + bx + c$ is on the axis of symmetry, $x = -\frac{b}{2a}$ .

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Steps	Explanation
So Nasima should take 40 items to the fair and sell each item for $p = 50 - \frac{40}{2} = 30$ USD.	
The maximum profit she can achieve is $P(40) = -\frac{40^2}{2} + 40 \times 40 - 30 = 770$ USD.	

As in the previous examples, finding the optimum using a graphical calculator is an alternative method.



There are several important things you should notice on this graph.

- The graph is only displayed for  $0 \leq n \leq 100$ . Since  $n$  is the number of items she is taking to the fair, it is clearly not negative. The upper bound is given by the condition in the question: for \$ $p$  for each item she will sell  $100 - 2p$  items.
- The graph has parts below the horizontal axis at both ends. This reflects the fact that she will lose money if she charges too much (because she may not sell any and lose the cost) and also if she charges too little (she may sell a lot but will not recover the cost of materials).

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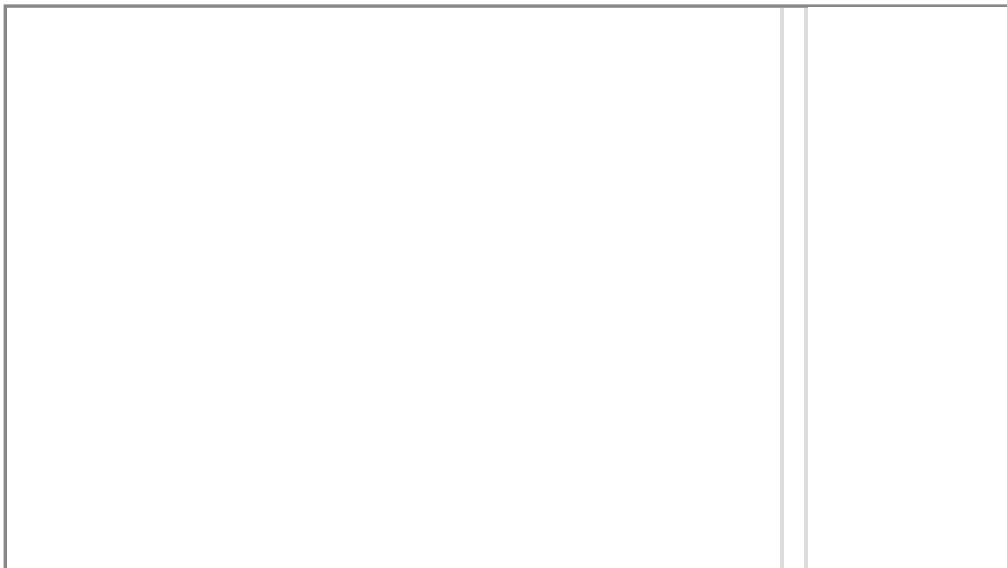


## Example 4

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- The applet below illustrates a method of making a cone from a paper disk. When a sector is cut from the disk and the cut sides are pulled together, the paper forms a cone.



### Interactive 1. Method of Making a Cone From a Paper Disk.

More information for interactive 1

The interactive allows users to explore the formation of a cone from a paper disc by cutting out a sector from a circular disc and joining the remaining edges.

As users change the cutout angle, the applet dynamically displays the updated cone shape. The curved boundary of the remaining disc becomes the circumference of the cone's base.

By adjusting the cutout angle, users can observe how changes in the sector size affect the cone's dimensions and ultimately its volume.

The interactive tool continuously displays the calculated volume of the cone at the bottom, enabling real-time observation of how changes in the sector angle impact the volume. Users can find a pattern in the volume with respect to the angle. If the angle of cutout increases, the volume of the cone decreases. The volume of the cone displays when the angle of cutout is  $66^\circ$  is  $0.40304R^3$ . Here, R is the radius of the circular sheet.

This applet provides an engaging way to visualize 3D shape formation from a 2D perspective, making it a valuable tool for students studying geometry, mathematics, and engineering concepts.

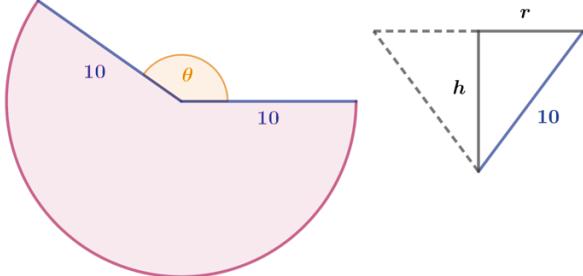
What is the maximum volume of the cone that can be formed from a disk of radius 10 cm?

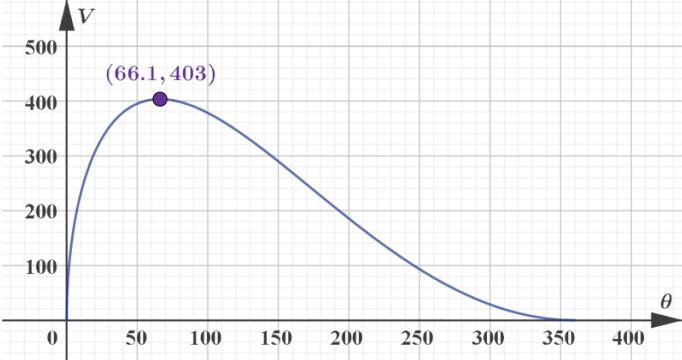


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Steps	Explanation
	<p>Drawing a sketch and introducing variables is useful to translate the problem to the language of mathematics.</p>
<p>The image consists of a diagram on the left showing a semicircle subdivided by two lines that converge, forming an angle labeled (<math>\theta</math>). Each of the two lines is labeled as "10". Above the angle, there is an orange sector also labeled (<math>\theta</math>). On the right, a triangle is depicted with a dotted line leading up to a vertical line, both of which split the triangle. The triangle also has a side labeled "10". The image seems designed to explore geometric relationships involving angles, circles, and triangles.</p> <p>[Generated by AI]</p>	
<p>Let the cut out angle be <math>\theta</math> (measured in degrees).</p> <p>Let the height of the corresponding cone be <math>h</math> and the radius of the base of the cone is <math>r</math>.</p>	
$2r\pi = \frac{360 - \theta}{360} \times 2\pi \times 10$ $r = \frac{360 - \theta}{36}$	<p>The length of the shaded arc and the circumference of the base circle of the cone are the same.</p>
$h^2 + r^2 = 10^2$ $h^2 = 100 - r^2$ $h = \sqrt{100 - r^2}$	<p>The slant height of the cone is the same as the radius of the paper disk, so you can use Pythagoras' theorem to express the height of the cone in terms of the radius of the base.</p>
$V = \frac{1}{3}r^2\pi h$	<p>The volume of the cone can be expressed in terms of the height and the radius of the base.</p>

Steps	Explanation
$  \begin{aligned}  V &= \frac{1}{3} r^2 \pi h \\  &= \frac{1}{3} r^2 \pi \sqrt{100 - r^2} \\  &= \frac{1}{3} \left( \frac{360 - \theta}{36} \right)^2 \pi \sqrt{100 - \left( \frac{360 - \theta}{36} \right)^2}  \end{aligned}  $	Putting all this together gives the volume in terms of the size of the cut out angle.
	GDCs have applications that can find the maximum of this expression for $0 \leq \theta \leq 360$ (these are the meaningful central angles of the cut).
<p>The image is a graph with a grid background, showing a curve reaching its peak and then descending. The X-axis is labeled with theta (<math>\theta</math>) and marked at intervals from 0 to 1000. The Y-axis represents variable v, with values marked from 0 to 500 in increments of 100. A point on the curve is highlighted at approximately (66.1, 403). The graph illustrates a peak around this point and then shows a decreasing trend.</p> <p>[Generated by AI]</p>	

So the maximum volume of the cone that can be formed starting with a disk of radius 10 cm is  $403 \text{ cm}^3$ .

Although it was not asked for in this question, the graph also indicates that this optimum cone can be formed by cutting out a sector with central angle approximately  $66^\circ$ .

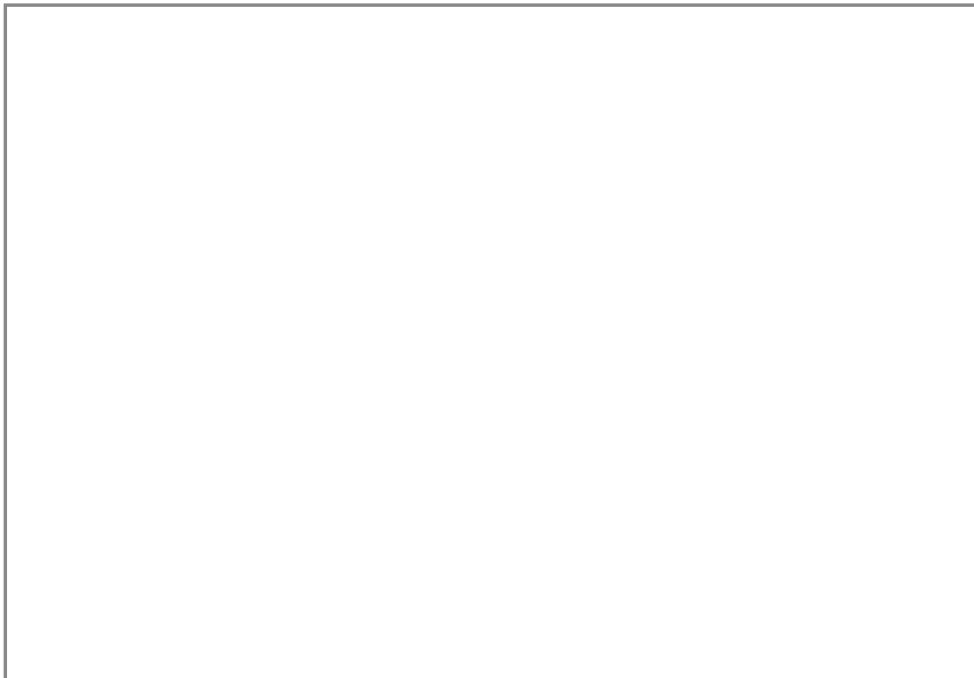
By solving  $V' = 0$  (although it is beyond the level of the standard level course), an exact solution can also be found. The exact cut out angle that gives the maximum volume is  $\theta = \frac{2}{3}(3 - \sqrt{6})\pi$  radians (or  $360 - 120\sqrt{6}$  degrees), and the

maximum volume is  $V = \frac{2000\pi}{9\sqrt{3}}$ .

## Example 5



A company manufactures robots that can travel both on land and in water. Different models have different speeds on land and in water. The robots are tested on a circular lake with diameter 300 m. The aim is to reach from a point on the shore to the opposite point at the end of the diameter. On the applet below you can investigate the total travel time needed with different speeds and different ways of crossing the lake.



**Interactive 2.** Total Travel Times at Various Speeds and Methods for Lake Crossing.

More information for interactive 2

This interactive lets users explore the optimal path for the shortest time for travelling across a circular lake. There are two windows. On the left side of the interface, there is a speed selector, in which the x-axis depicts speed in water, ranging from 0.38 m/s to 2.38 m/s and the y-axis represents the speed on land from 1 m/s to 3 m/s. Placing the red dot gives a combination of land and water speed.

On the right side of the interface, there is a circular lake in which the object has to move from point A to B via C. A represents the starting point on the shore. O is the center of the lake. B is the destination on the opposite shore.

The straight line represents movement in water and the curved line represents the movement on land. The objective of this interactive is to find the shortest time to travel from A to B using a perfect combination of AC and CB and land and water speed. The total time is displayed in real time on the bottom left, enabling users to simulate various



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crossing strategies, such as traveling directly across the lake or combining land and water routes to find the shortest time to travel.

For example, if we keep the speed selector on the extreme top right corner, making the fastest speed on land 2.38 m/s and water 3 m/s and the line from A to B completely straight. That is, point C on point B, using only the waterway, we will get the total time as 126.32 seconds. However, if we keep point C on point A, using only land, we will get a time of 157.08 seconds. Now the user needs to find a combination of land and water, giving the fastest way of traveling from point A to C

The purpose of this interactive applet is to explore the fastest possible route for amphibious robots to cross a circular lake while considering their different speeds on land and in water.

The company is testing two robots, both of which can travel on land with a speed of 3 metres per second.

- The first robot can travel in water with a speed of 1 metre per second.
- The second robot can travel in water with a speed of 2 metres per second.

What is the shortest time in which these robots can cross the lake? Which way should they go?

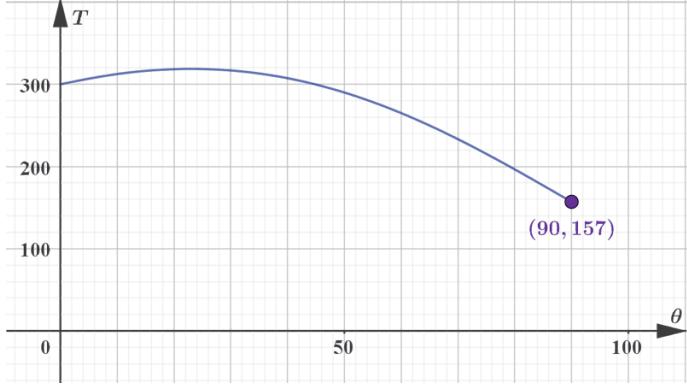
Steps	Explanation
<p>Let A be the starting point, let B the target, let O the centre of the circle and let C be the point the robot is aiming for on the other side of the lake.</p> <p>Let <math>\theta</math> be the angle <math>\hat{CAB}</math> (measured in degrees).</p> <p>The radius of the circle is <math>OA = OB = OC = 150</math>.</p>	Introducing variables is useful translate the problem to the language of mathematics.
$\hat{CAO} = \hat{ACO}$ , so $\hat{AOC} = 180^\circ - 2\theta$ . $\begin{aligned} AC^2 &= 150^2 + 150^2 - 2 \times 150 \times 150 \cos(180^\circ - 2\theta) \\ &= 45\,000 - 45\,000 \cos(180^\circ - 2\theta) \\ AC &= \sqrt{45\,000 - 45\,000 \cos(180^\circ - 2\theta)} \end{aligned}$	You can use the cosine rule in isosceles triangle AOC to express AC in terms of $\theta$ .
<p>The arc length formula gives the length of arc CB:</p> $\frac{2\theta}{360} \times 2\pi \times 150 = \frac{5\pi}{3}\theta.$	$\hat{COB} = 180^\circ - (180^\circ - 2\theta)$

Student view



To calculate the time needed for the robot to cross the lake, you can use

$$\text{time} = \frac{\text{distance}}{\text{speed}}.$$

Steps	Explanation
<p>The crossing time of the first robot is</p> $T(\theta) = \frac{\sqrt{45\ 000 - 45\ 000 \cos(180^\circ - 2\theta)}}{1} + \frac{5\pi\theta/3}{3}$	<p>The first robot has a speed of 3 metres per second on land and 1 metre per second in water.</p>
	<p>Investigating the graph gives the shortest time. The meaningful values of the angle are <math>0 \leq \theta \leq 90^\circ</math>.</p>
<p>The shortest time corresponds to <math>90^\circ</math> i.e. when the robot does not go in the water at all. The shortest crossing time for the first robot is approximately 157 seconds.</p>	<p>Note that the shortest time does not correspond to a turning point on the graph.</p>

If you experiment a bit with the applet, you will see that for this particular shape of lake the shortest time always corresponds to either moving all the way around the lake on land or swimming straight across. For this question, there is no way of getting a shorter time by swimming to some point first and then moving on land from there.

## 3 section questions ▼





# Checklist

Overview  
(/study/app/m/sid-122-cid-754029/)**Section**

Student... (0/0)

Feedback

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**Assign**

## What you should know

By the end of this subtopic you should be able to:

- use variables to translate an optimisation problem in context to the language of mathematics
- set up relationships between the variables you introduced according to the conditions of the problem
- find a function in one variable that expresses the quantity that needs to be optimised
- use differentiation to find the optimum values
- use technology to find the optimum values
- interpret the result of your work in the context of the question.

5. Calculus / 5.7 Optimisation

# Investigation

**Section**

Student... (0/0)

Feedback

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In the big picture section, you saw an example of how nature chooses the shape with maximal area when the perimeter is fixed. In [section 5.7.2 \(/study/app/m/sid-122-cid-754029/book/optimisation-in-context-id-27485/\)](#) you found the rectangle with the maximum area when the perimeter was fixed. In this investigation you will explore triangles.

## Activity

On the applet below move the vertices of the triangle and try to find a triangle that has maximal area. You can either move the points around freely or you can tick the box and restrict movement so that the perimeter does not change.

  
Student view



Overview  
(/study/app/  
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—

- What do you notice?
  - Can you prove what you noticed?
- 

### Rate subtopic 5.7 Optimisation

Help us improve the content and user experience.



Student  
view