Index

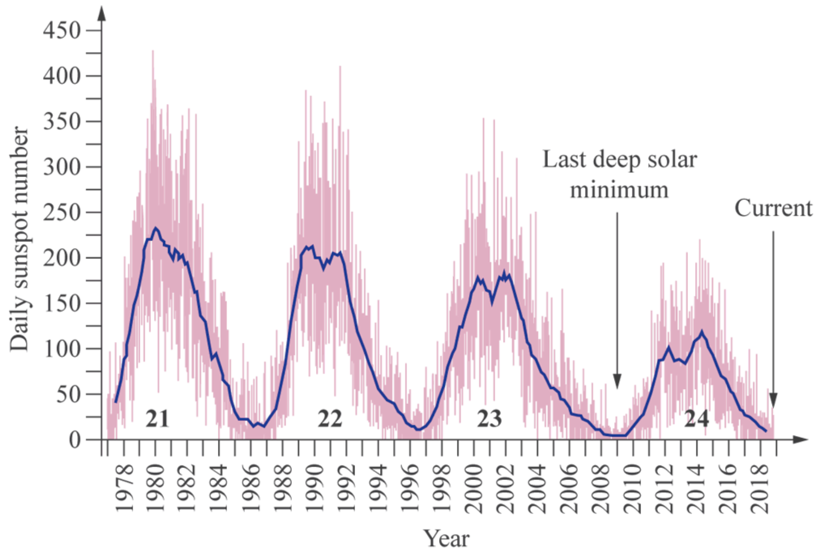
The big picture

So far in trigonometry you have learned about geometric problems concerning mostly triangles. When you start treating sine and cosine as functions of a variable, you will see wavy graphs with intriguing properties. These wavy graphs are created by functions with a cyclic nature, which are used to model cyclic phenomena.

Making connections

The Earth's climate varies without human interference, and variations in climate occur over multiple timescales such as years, decades and centuries. Studying these patterns, and variations from the patterns because of human impact, helps us to understand climate change and its impact on earth. The graph below shows daily observations of the number of sunspots since 1 January 1977. You can see the cyclic nature of the graph, but there is a drastic change to these cycles at the last peak.

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o



More information

The graph displays the daily sunspot numbers from 1978 to 2018. The X-axis represents the Year, spanning from 1978 to 2018, with key decadal markers. The Y-axis represents the Daily Sunspot Number, ranging from 0 to 450. Over the 40-year period, four significant peaks are visible, indicating cyclic phenomena in sunspot numbers. The peaks occur approximately every 11 years, with the highest point approaching 400 sunspots. The last deep solar minimum is marked around 2008, followed by a rise leading to the term 'Current' by 2018. The graph highlights cycles 21, 22, 23, and 24, showing clear cyclic patterns in solar activity as documented by the variations in sunspot numbers.

[Generated by AI]

In this subtopic, you will encounter further examples of cyclic phenomena, such as simple harmonic motion, water waves and sound waves.

Concept

Trigonometric functions allow you to model the physical world and cyclic phenomena.

Study of trigonometric functions provides you with the tools to analyse, measure and transform the quantities, movements and relationships of cyclic (periodic) functions.

What are the properties of cyclic phenomena? How does trigonometry help you to model them?

Models can include a single function or a combination of functions.

Student
view

How can you check the validity of your models?

Periodicity and frequency

Graphs of the sine and cosine functions

Asia's highest observation wheel, the Singapore Flyer, is a giant circle with a diameter of 150 metres and 28 carts which take you on a journey to view the skyline of the city of Singapore, and the ocean.

The highest point of the wheel is 165 metres, and the total journey takes 30 minutes. So, every 30 minutes the cart will complete one cycle and the journey will start again.

Singapore Flyer

Credit: Andrey Krav Getty Images

If you create a graph of the height of a cart versus time you will see the graph repeating itself over 30-minute time periods. Have a closer look at the motion of the Ferris wheel in the

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

following activity.

Activity

Use the applet below to investigate how changing certain parameters of a Ferris wheel affects the graph of a cart's height versus time.

What do you notice about the graph?

Student
view

Interactive 1. Ferris Wheel (2): Modeling With Trigonometric Functions.
Credit: GeoGebra (https://www.geogebra.org/m/dQNWHC7S) Tim Brzezinski

More information for interactive 1

This interactive allows users to explore how different parameters of a Ferris wheel influence the graph of a cart's height over time. The graph plots the height (in meters) on the y-axis against time (in minutes) on the x-axis, displaying a sinusoidal curve (with red draggable point) that represents the cart's vertical motion as the wheel rotates. Users can adjust three key parameters: the height of the lowest car (H), the diameter of the Ferris wheel (D), and the rotation period in minutes (slider given below the graph). Each adjustment dynamically alters the graph, providing immediate visual feedback on how these changes affect the cart's movement.

The height function, shown as h(x) = −D/2 × cos(2 × π × x/period(min)) + D/2 + H, updates in real-time as users modify the inputs. For instance, increasing the diameter (D) enlarges the amplitude of the graph, reflecting a taller Ferris wheel with more extreme height variations. Adjusting the period stretches or compresses the graph horizontally, representing slower or faster rotations. Meanwhile, changing the height of the lowest car (H) shifts the entire graph up or down, setting a new baseline for the cart's minimum height.

A red point can be dragged along the graph to observe the cart's height at any specific time. As the point moves, the applet also displays the corresponding position of the cart on the Ferris wheel's circular path. This dual visualization helps users connect the abstract graph to the physical motion of the wheel, reinforcing the relationship between mathematical models and real-world systems.

For example, if a user sets the diameter to 150 meters, the lowest car height to 15 meters, and the period to 58.6 minutes, the graph will show a curve peaking at 165 meters (75 meters above the midpoint) and dipping to 15 meters, with one full cycle completed every 58.6 minutes. By experimenting with these parameters, users can gain a deeper understanding of how sinusoidal functions model periodic motion, such as the rotation of a Ferris wheel.

Describe how the graph changes when you input different values.

Important

If a function repeats itself in a regular interval it is called a periodic function.

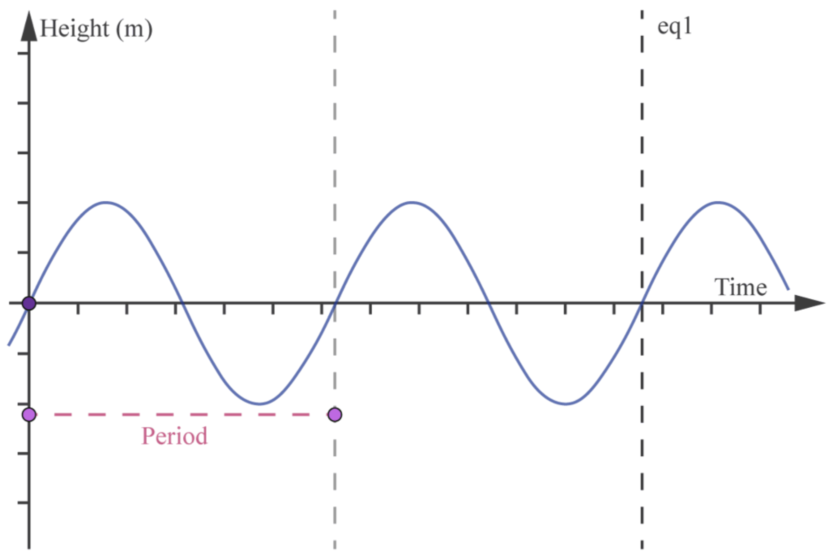
For a function to be periodic it must have a consistent repetition of the values of $f(x)$ over the domain of the function.

If f is a periodic function, then $f(x) = f(x + p)$ where p is the period of $f(x)$.

Student view

In the diagram below, you can see the graph of the height of a cart over time. It looks like a continuous wave. The time it takes to complete one cycle on the Ferris wheel is called the period . It is the length of one complete wave (oscillation) on the graph.

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o



More information

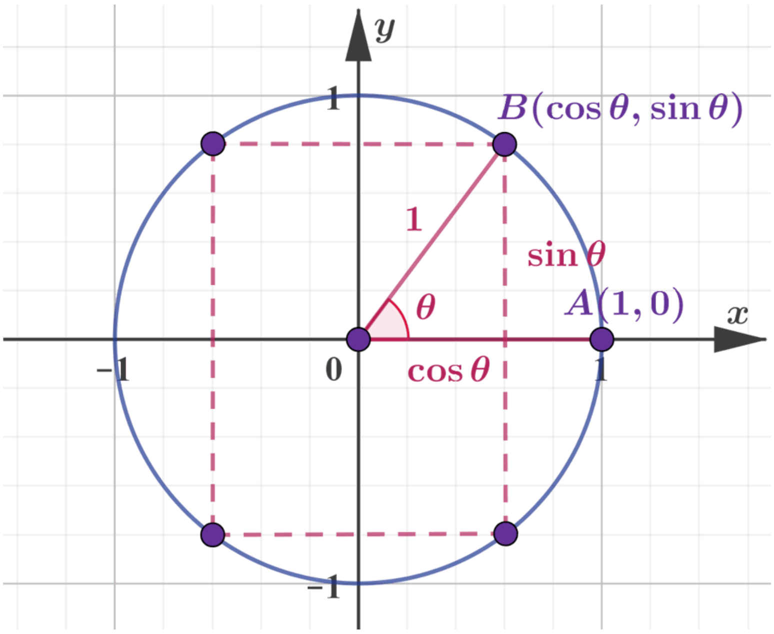
The image is a graph depicting the height of a cart over time, resembling a continuous wave. The X-axis represents time, marked at regular intervals. The Y-axis indicates height in meters. The graph shows a repetitive wave pattern that oscillates between positive and negative heights, indicating the periodic motion of the cart, similar to a Ferris wheel ride. The time it takes to complete one cycle is labeled as "Period," indicating the length of one full oscillation in the wave. Dashed vertical and horizontal lines emphasize parts of the wave related to its period. The graph provides visual insights into the concept of periodic motion and function using a wave structure.

[Generated by AI]

Section 3.5.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-unit-circle-id-27742/) defined the coordinates of any point on a unit circle, using sine and cosine of the rotation angle, see below. As you might have noticed, you could use the unit circle to define this periodic function.

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o



More information

The image is a diagram illustrating a unit circle on a coordinate plane. The unit circle is drawn in blue and centered at the origin (0,0) with a radius of 1. The X and Y axes are labeled "x" and "y" respectively, with markings at -1 and 1.

A triangle is formed inside the circle with a rotation angle θ, highlighted in red. Point A (1,0) is located on the circle's intersection with the X-axis, and point B (cosθ, sinθ) is where the ray at angle θ intersects the circle. The adjacent side of the triangle is labeled "cos θ" and the opposite side is labeled "sin θ". The hypotenuse is labeled "1" indicating the radius of the circle. Dashed purple lines connect the points and extend from B to the X and Y axes, creating a right triangle that visualizes sine and cosine values of angle θ.

[Generated by AI]

If you define the function where x values are the angles θ, and y values are $\sin\theta$, you would get a periodic function $f(x) = \sin\theta$, as you can see in the following applet.

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Interactive 2 . Unit Circle and Sine Graph — GeoGebra Materials

Credit: GeoGebra (https://www.geogebra.org/m/S2gMrkbD) Anthony OR

More information for interactive 2

This interactive applet allows users to visualize the relationship between the angle and the sine function, f (x) = sin, in both a unit circle and a graph. The x-axis represents the angle (in degrees), ranging from 0 to 360, while the y-axis shows the corresponding value of sin. A slider at the top of the screen lets users adjust the angle θ, to observe how the sine value changes dynamically, creating a smooth, periodic wave on the graph. Also, users can reset the view using the circular arrow button.

On the unit circle, the angle is measured from the positive x-axis, and a purple point marks the position of the angle on the circumference. The y-coordinate of this point corresponds to sin, which is simultaneously plotted as a purple point on the sine graph. This dual representation helps users connect the geometric interpretation of sine (as the vertical component of the unit circle) with its graphical representation as a wave.

The sine graph displays key characteristics of the function, such as its periodic nature with a period of 360, its amplitude of 1, and its symmetry about the origin. As increases, users can see the sine wave oscillate between -1 and 1, peaking at 90 and 270 and crossing zero at 0, 180, and 360. The applet reinforces these properties by highlighting the angle and its sine value in real-time.

For example, when =120, the purple point on the unit circle is located in the second quadrant, and its y-coordinate (sine value) is 0.866. On the sine graph, this corresponds to the point (120, 0.866), which lies between the peak at 90 and the zero-crossing at 180. By experimenting with different angles, users can observe how the sine function behaves across all four quadrants, deepening their understanding of trigonometric concepts.

Student view

Of course, with a similar approach you can graph $y = \cos x$.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Exam tip

The applets above use degrees, but when you are graphing trigonometric functions on your graphic display calculator, ensure that it is in radian mode, not degree mode.

Refer to the calculator instructions in section 3.2.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/rightangled-triangles-id-25419/) for further detail.

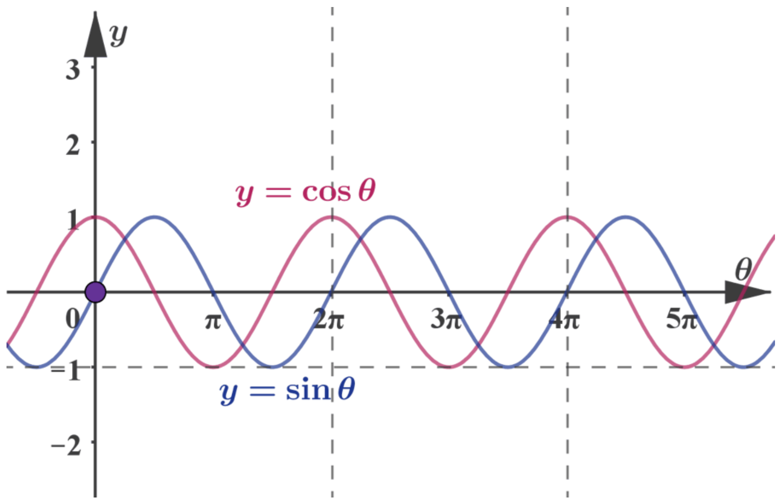
Important

As you can see in the diagram below, both

$y = \sin\theta$ and $y = \cos\theta$ have a period of 2π or $360°$.

Their values are between -1 and 1 . Or you can write

$-1 \leqslant \sin\theta \leqslant 1$ and $-1 \leqslant \cos\theta \leqslant 1$.



More information

The image is a graph depicting the sine (y = sin θ) and cosine (y = cos θ) functions plotted against the angle θ on the x-axis. The graph features two wavy lines: a blue wave representing the sine function and a red wave representing the cosine function. The x-axis is labeled with values in terms of π, ranging from 0 to 5π, and the y-axis ranges from -2 to 3. Both functions have a periodic pattern, intersecting the y-axis at distinct points, and peak at different intervals

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

indicated by π, 2π, 3π, etc. The sine wave starts from the origin (0,0), rises to 1 at π/2, returns to 0 at π, and has a negative peak of -1 at 3π/2. The cosine wave begins at 1 when θ is 0 and follows a similar pattern but shifted by π/2 in comparison to the sine wave.

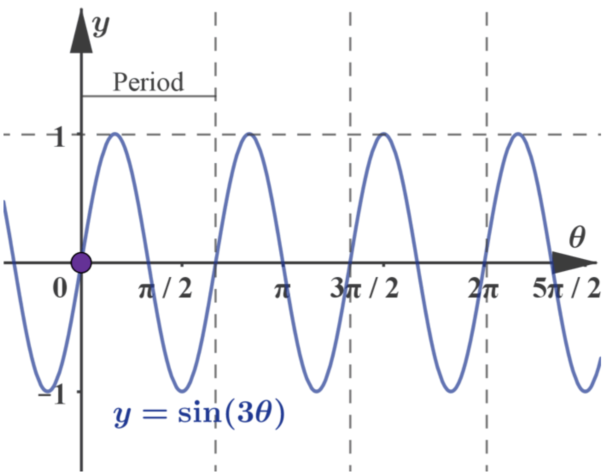
[Generated by AI]

Important

The number of oscillations or complete waves in a given interval of a periodic function is called the frequency . In trigonometric functions $y = \sin x$ and $y = \cos x$ both have complete waves in 2π . Therefore, their frequency is 1 in the interval [0, 2π].

As shown in the diagram below, $y = \sin 3\theta$ has 3 oscillations in 2π , therefore its frequency is 3. If you divide the 2π by 3, you get the length of one complete wave.

This is the period, $p = \frac{2\pi}{3}$.



More information

The image is a graph of the sine function represented as (y = \sin(3\theta)). It features a sine wave that oscillates along the horizontal axis labeled (\theta) and the vertical axis labeled (y). The range of the horizontal axis is marked from 0 to (5\pi/2), while the vertical axis ranges from -1 to 1. The amplitude peaks are at 1 and -1. The graph shows a complete cycle of the sine function repeating every (2\pi/3) units on the horizontal axis, labeled as the 'Period'. The sine wave passes through the zero point at the origin and peaks at (\pi/2), (3\pi/2), and (5\pi/2), and has a trough at (\pi) and (2\pi). A point is marked at the origin, and the section of the graph indicates where (\theta = 0) and where the sine wave takes its maximum and minimum values, displaying periodic behavior.

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

[Generated by AI]

Important

In the trigonometric functions $y = \sin b\theta$ and $y = \cos b\theta$, $|b|$ is the frequency of the function and $\frac{2\pi}{|b|}$ is the period of the function.

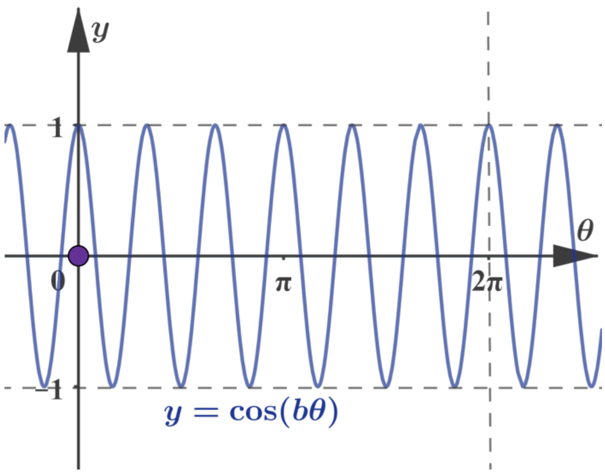
Example 1

★☆☆

The graph of $y = \cos b\theta$ is given below. Find

a) the frequency of the function

b) the period of the function.



More information

The image is a graph of the function y = cos(bθ). The X-axis represents θ with labeled points at 0, π, and 2π. The Y-axis represents the values of y, ranging from -1 to 1. The graph displays oscillations of the cosine function, showing multiple cycles between 0 and 2π. A point is highlighted at θ = 0 on the Y-axis at y = 1. The graph visualizes the period of oscillations, depicting the regular pattern of peaks and troughs.

Student
view

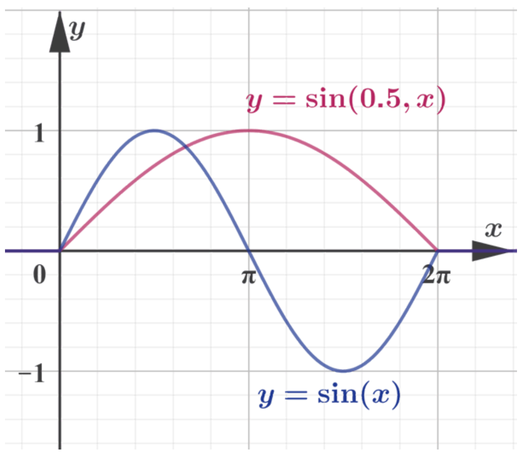
[Generated by AI]

Steps	Explanation
$b = 6$	There are 6 oscillations in 2π.
$\text{frequency} = 6$	Frequency is the number of oscillations in 2π.
$\text{period} = \frac{2\pi}{6} = \frac{\pi}{3}$	If there are 6 oscillations in 2π then the length of one wave will be $\frac{2\pi}{6}$

Example 2

★☆☆

Sketch the graph of $y = \sin 0.5\theta$ for $0 \leqslant x \leqslant 2\pi$

Steps	Explanation
	In $y = \sin bx$. is the number complete wav in $0 \leqslant x \leqslant 2\pi$ As $b = 0.5$ it is only a half wave.

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Example 3

★★☆

If $y = \cos(\pi x)$:

a) What is the frequency of the function?

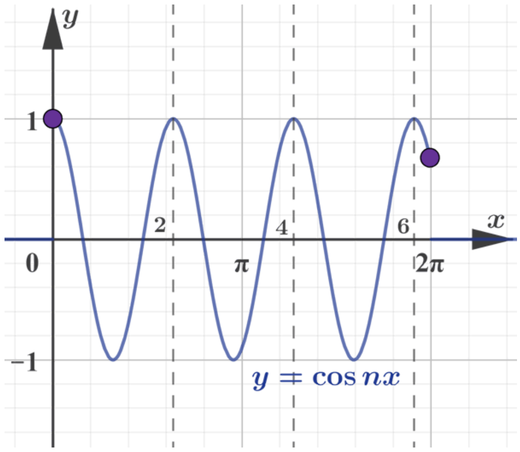
b) What is the period of the function?

c) Sketch the graph of the function for $0 \leqslant x \leqslant 2\pi$.

	Steps	Explanation
a)	$\text{Frequency} = \pi$	In $y = \cos bx$ is the frequency.
b)	$\text{Period} = \frac{2\pi}{\pi} = 2$	The period is the length of one complete oscillation in 2 .

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
c)		Since the period is 2, th cosine graph will repeat eve 2 units. Make sure to mark the end points of $0 \leqslant x \leqslant 2\pi$.

Graph of the tangent function

Activity

Use the applet below to graph $y = \tan\theta$ for $0 \leqslant \theta \leqslant 360$.

Interactive 3. Tangent Graph
Credit: GeoGebra Anthony OR

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

More information for interactive 3

This interactive allows users to explore the relationship between the angle and the tangent function, y = tan over the interval [0,360]. The x-axis represents the angle (in degrees), while the y-axis displays the corresponding value of tan. Users can adjust using a slider at the top and observe how the tangent value changes dynamically, creating a distinct, discontinuous curve on the graph. As the slider moves, a point traces out the tangent value on the graph and simultaneously updates a visual representation on the unit circle. The orange segment shows the tangent line from the terminal side of the angle, and the calculated tangent value (e.g., −0.9 at θ = 138) is displayed both geometrically and numerically.
On the unit circle, the angle is measured from the positive x-axis, and a colored point marks its position. The tangent of is geometrically represented as the ratio of the y-coordinate to the x-coordinate tan = sin(θ)/cos(θ). As approaches 90 or 270, where cos = 0. The tangent function exhibits vertical asymptotes, causing the graph to shoot toward infinity or negative infinity. These asymptotes divide the graph into distinct branches, each corresponding to a period of radians (or 180).
The graph highlights key properties of the tangent function, such as its periodicity (repeating every 180), its symmetry about the origin (odd function), and its undefined values at = 90+ k180 (where k is an integer). Users can drag a point along the curve to see how tan behaves in different quadrants—positive in the first and third quadrants, and negative in the second and fourth.
For example, when θ = 45, The tangent value is 1, which appears as a point at (45,1) on the graph. As θ approaches 90, the tangent values grow rapidly toward infinity, creating a vertical asymptote. At = 135, tan = −1, reflecting the function's periodicity and sign changes. This visualization helps users understand why the tangent graph has its characteristic shape and discontinuities.

Use the slider to vary the value of θ .

What do you notice about the graph?

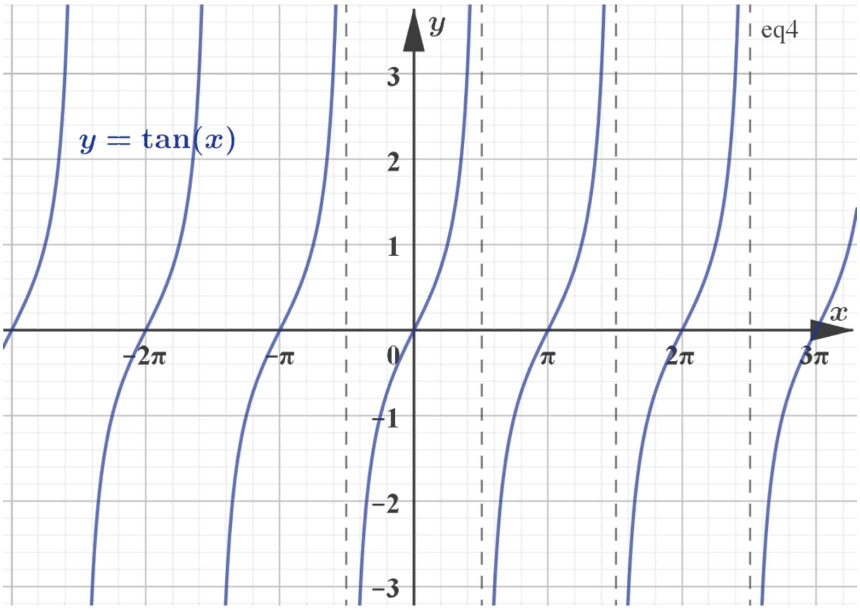
What is the period of $\tan\theta$?

When does $\tan\theta$ tend to ∞ or $-\infty$? Why?

As you saw in section 3.5.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-unit-circle-id-27742/), $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Therefore, when $\cos\theta = 0$ the tangent is undefined. This creates vertical asymptotes for the graph of $\tan\theta$.

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o



More information

The image shows the graph of the tangent function, y = tan(x), with x-values ranging from -2π to 2π. The horizontal axis represents the angle x in radians, marked at intervals of π/2, π, 3π/2, and so on. The vertical axis represents the tan(x) values, with tick marks at intervals of 1, 2, and 3. The graph features periodic curves, demonstrating the periodic nature of the tangent function with cycles repeating at intervals of π. There are vertical asymptotes displayed at points where cos(θ) = 0, such as at x = -3π/2, -π/2, π/2, 3π/2, where the function is undefined. The asymptotes are shown as dashed vertical lines, dividing the graph into multiple sections where the tangent curve moves upwards and then quickly jumps from negative to positive infinity, repeating this behavior regularly.

[Generated by AI]

✓ **Important**

The period of $\tan\theta$ is π because the graph repeats itself in intervals of π.

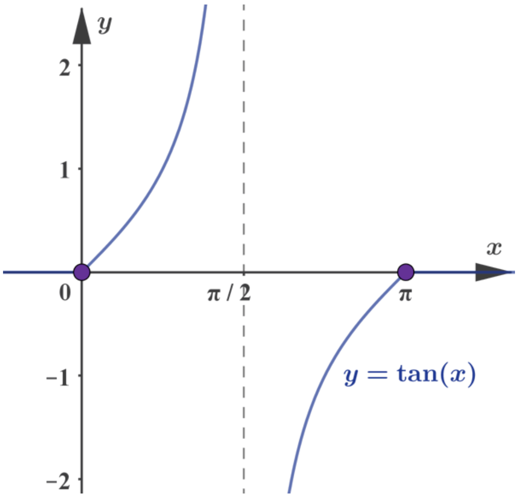
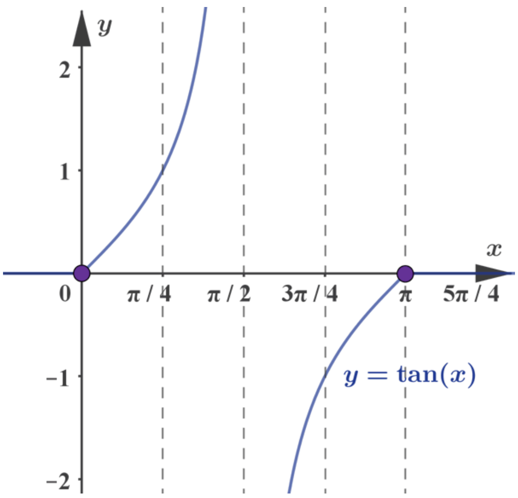
The period of $y = \tan b\theta$ is $\dfrac{\pi}{|b|}$.

Example 4

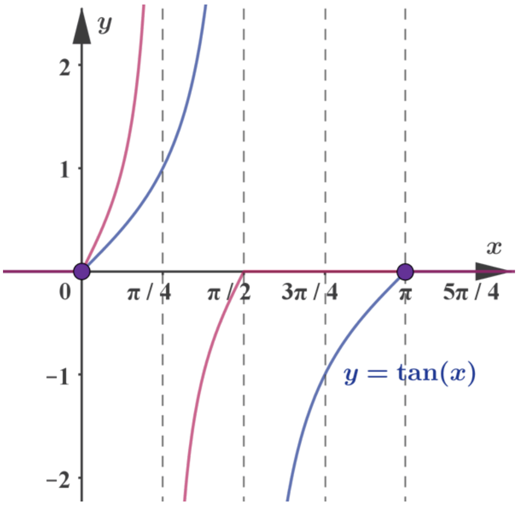
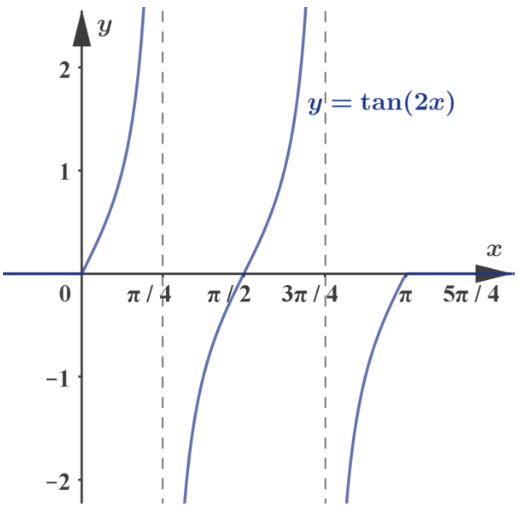
★★☆

Student view

Sketch the graph of $y = \tan 2x$ for $0 \leqslant x \leqslant \pi$, marking the vertical asymptotes clearly.

Steps	Explanation
	Sketch the grap of $y = \tan x$ in the interval $0 \leqslant x \leqslant \pi$.
	As $y = \tan 2x$ has frequency 2 divide each of tl intervals into 2 and mark the lines.

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Steps	Explanation
	Sketch the grap for 0 to π.
	And complete tl graph for $0 \leqslant x \leqslant \pi$ Make sure to mark the end points and the asymptotes.

3 section questions

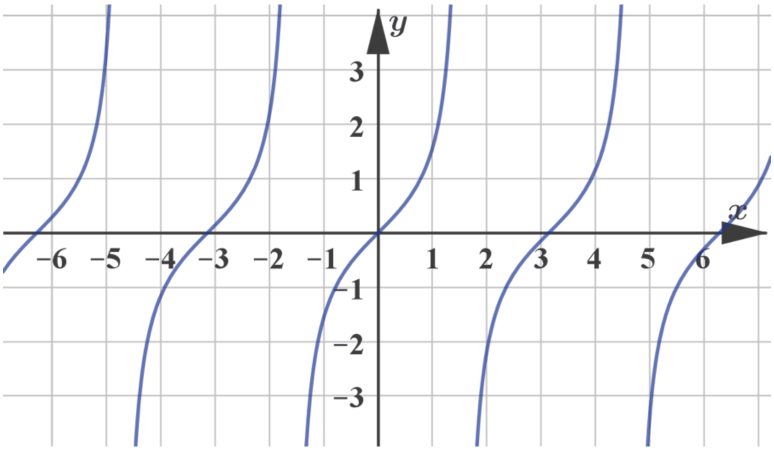
Student view

Question 1

★☆☆

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Select the function that best matches the graph shown below.



More information

1 $y = \tan x$ ✓

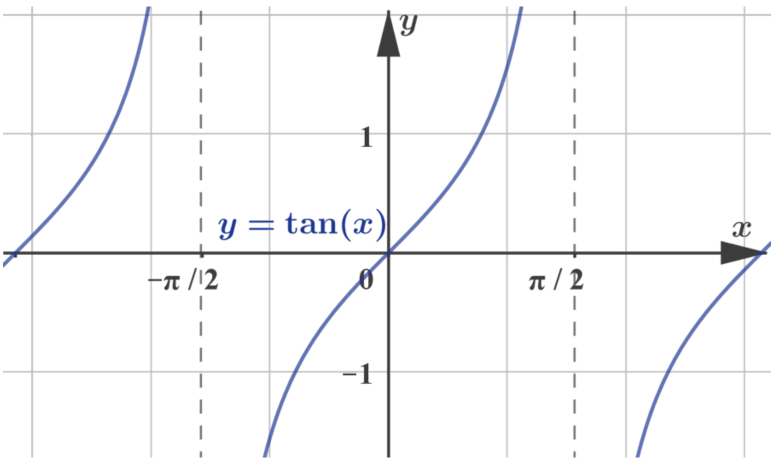
2 $y = \tan 2x$

3 $y = \tan \pi x$

4 $y = \tan 2\pi x$

Explanation

The graph of $y = \tan x$ is



More information

and $\pi = 180^\circ$, so the correct answer is $y = \tan x$

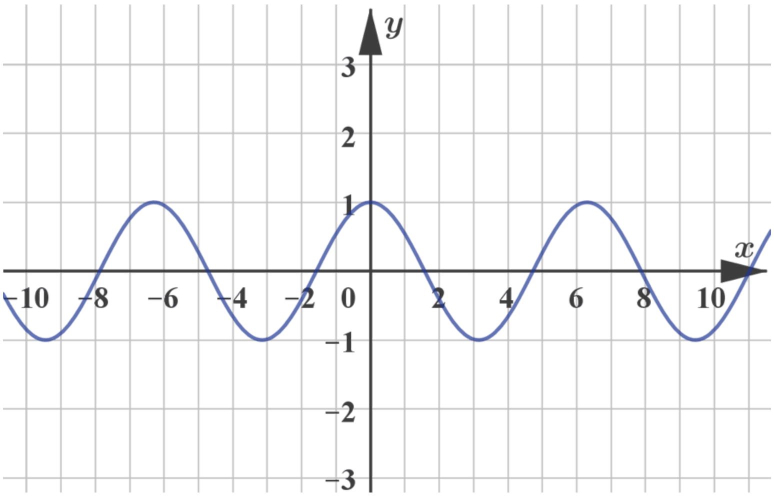
Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Question 2

★★☆

Which one of the following is the function for the graph shown below?



More information

1 $y = \cos\left(\frac{2\pi x}{6.3}\right)$ ✓

2 $y = \sin\left(\frac{x}{6.3}\right)$

3 $y = \sin\left(\frac{6.3}{\pi}x\right)$

4 $y = \cos\ (6.3\pi x)$

Explanation

A complete cosine wave is in the interval $0 \leq x \leq 6.3$

The period is 6.3. The frequency is $\frac{2\pi}{\text{period}} = \frac{2\pi}{6.3}$

Therefore, the correct answer is $y = \cos\left(\frac{2\pi x}{6.3}\right)$

Question 3

★☆☆

What is the period of the function $y = \sin 3\pi x$?

Student
view

1 $\frac{2}{3}$

2 3π

3 $\frac{3\pi}{2}$

4 $\frac{4}{3}$

Explanation

$y = \sin bx$ and where b is the frequency

So,

for $y = \sin 3\pi x$,

$$\text{frequency} = b = 3\pi$$

and

$$\text{frequency} = \frac{2\pi}{\text{period}}$$

So, the period is

$$\begin{aligned}\frac{2\pi}{b} &= \frac{2\pi}{3\pi}\\ &= \frac{2}{3}\end{aligned}$$

Amplitude

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/amplitude-id-27754/print/) Assign

Have you ever been told to lower the volume when you are watching TV or listening to your favourite song?

Anything that creates a disturbance in the air creates a pulse. This pulse travels through the air and reaches your ear drum, which vibrates so that you can hear the sound.

Student view

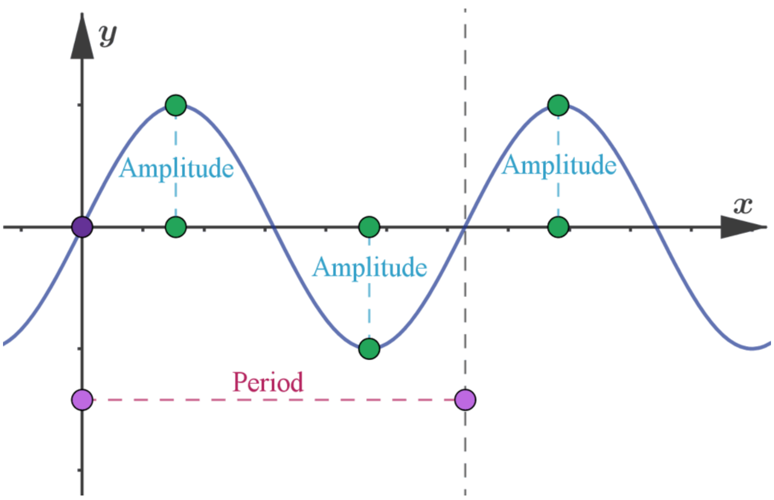
Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

The vibration of the sound source transmits energy, which creates the vibration that travels through air. When the energy in the source is higher, the molecules in the air move more vigorously, which creates a louder sound.

$$\text{frequency} = \frac{2\pi}{\text{period}}$$

In the video below, a group of physics and chemistry students created a pyro board using this property of sound and turning it into an audio-visual masterpiece.

As sound travels in waves, it is modelled using trigonometric functions (as observed in the video above). A sound wave is modelled by a sine function. The distance from the reference line to the peak of the wave is called the amplitude of the function, as seen in the diagram below. The higher the amplitude, the louder the sound.



Student view

More information

The image is a diagram of a sine wave, clearly illustrating its amplitude and period. The wave is plotted on an xy coordinate plane, with the x-axis denoted as "x" and the y-axis as "y". The sine wave oscillates above and below the x-axis, reaching peaks and troughs. The amplitude is marked from the x-axis to the peak at several points along the wave, with vertical dashed lines labeled "Amplitude" and marked by green circles at the peaks. The period of the wave is also indicated with a horizontal dashed line showing the distance between two corresponding points on consecutive cycles, labeled "Period" and marked by purple circles at these points. The overall structure of the wave demonstrates the periodic nature of sound waves, emphasizing the relationship between the amplitude and loudness.

[Generated by AI]

Activity

In the following applet you can explore the changes in the graph of

$y = a\sin\theta$ and $y = a\cos\theta$ as a changes.

Student
view

interactive 1. Transforming Sine and Cosine Functions (1): Dynamic Illustrator.

Credit: GeoGebra (https://www.geogebra.org/m/VeaygmQV) Tim Brzezinski

More information for interactive 1

This interactive applet allows users to explore the effects of changing the amplitude aa in the sine and cosine functions $y = a\sin x$ and $y = a\cos x$. Users can select either the sine or cosine function by checking the corresponding checkboxes labeled “Sine function” and “Cosine function. The amplitude a can be adjusted using a slider at the bottom, which ranges from -10 to 10. As a increases or decreases, the height of the waves in the graph changes accordingly, demonstrating how amplitude affects the function's behavior.

Additionally, the applet provides an animation feature labeled "Slide to graph." When activated, this animation first draws the standard sine or cosine function with an amplitude of 1 and then gradually transforms it into the function $y = a\sin x$ or $y = a\cos x$ based on the chosen amplitude value. This dynamic visualization helps users understand how the function stretches or compresses as the amplitude changes, reinforcing key concepts in trigonometry.

For example, in the given screenshot, the sine function is selected, and the amplitude is set to $a = 4$, resulting in the equation $y = 4\sin x$ The graph shows a sine wave that reaches a maximum of 4 and a minimum of -4, instead of the standard range of -1 to 1. The original sine

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

function with amplitude 1 is also shown as a dashed curve for comparison, making it easy to see how the transformation occurs.

How does parameter a affect the graph of $y = a\sin\theta$ and $y = a\cos\theta$ when $a > 0$?

When $a < 0$?

✓ **Important**

In a sine or cosine function, perpendicular distance from the reference line to the peak of the wave is called the amplitude of the function.

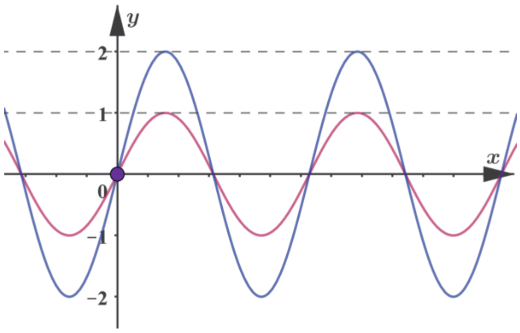
In both $y = \mathrm{a}\sin x$ and $y = \mathrm{a}\cos x$, $|a|$ represents the amplitude.

Example 1

★☆☆

If a sound wave is modelled by $y = \sin x$, what would be the function if the sound was twice as loud?

Student
view

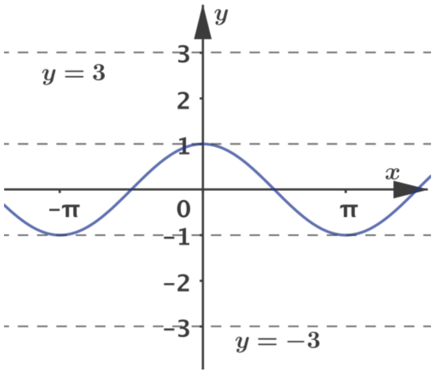
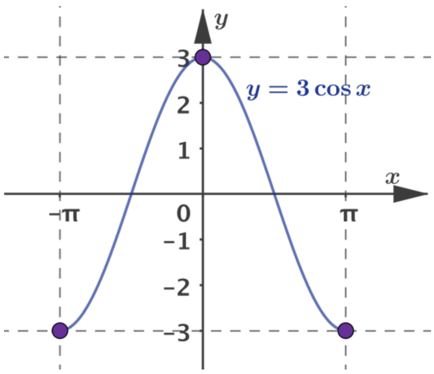
Steps	Explanation
	The graph of $y = \sin x$ has an amplitude of 1, as the amplitude represent the loudness (volume) of the sound it needs to be twice as much which gives you $y = 2\sin x$
Therefore the new function is $$y = 2\sin x$$	

Example 2

★☆☆

Sketch the graph of $y = 3\cos x$ in the interval $-\pi \leq x \leq \pi$.

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Steps	Explanation
	Sketch the graph of $y = \cos x$. Mark $y = 3$ and $y = -$ lines to indicate the highest and lowest value for $y = 3\cos x$.
	Sketch $y = 3\cos x$ in th interval $-\pi \leq x \leq \pi$, labelling the key points clearly.

Making connections

As discussed earlier, sound waves are modelled by the sine function. If you are a musician, you can create new sounds using combinations of different notes.

Is it possible to identify which notes are used by just looking at the graph of the sound waves? This could be possible using Fourier analysis , which is the study of the way general functions may be represented or approximated by sums of simpler trigonometric functions.

Student
view

Overview (/study/ap aa-hl/sid-134-cid-761926/o

In the words of Oli Freke:

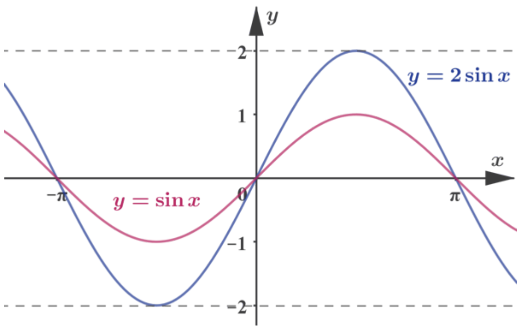
'Sine waves are unique in that they are the only sound in nature not to contain any harmonics beyond their fundamental frequency — they are the vampires of the sound world casting no harmonic shadow or reflections.' You can read the full article here (https://plus.maths.org/content/sine-language).

It is not only trigonometric functions which are related to sound modelling. Pythogoras investigated the ratios of musical notes. Euclid's algorithm and calculus can also be used to investigate sound waves. Listen to this composition (https://www.soundslice.com/slices/8klcc/) based on the Fibonacci sequences.

Example 3

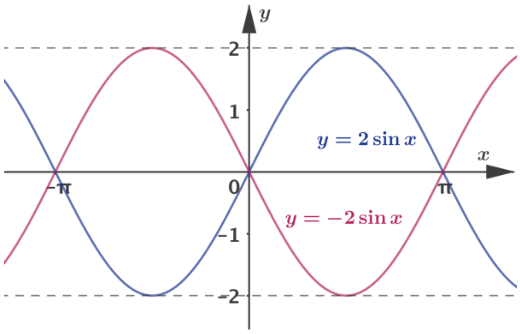
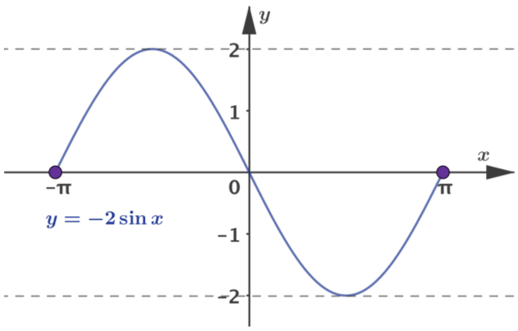
★☆☆

Sketch the graph of $y = -2\sin x$ in the interval $-\pi \leq x \leq \pi$.

Steps	Explanation
	Sketch the graph of $y = 2\sin x$ using $y = \sin x$.

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Steps	Explanation
	Reflect the graph of $y = 2\sin x$ along the x axis .
Therefore, the sketch of $y = -2\sin x$ is 	Clearly label the graph an end points.

2 section questions

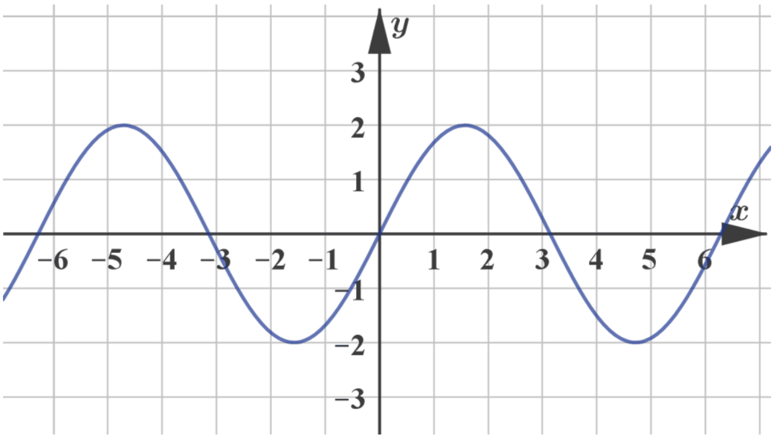
Question 1

★☆☆

Select the graph that best matches the equation $y = 2\sin x$.

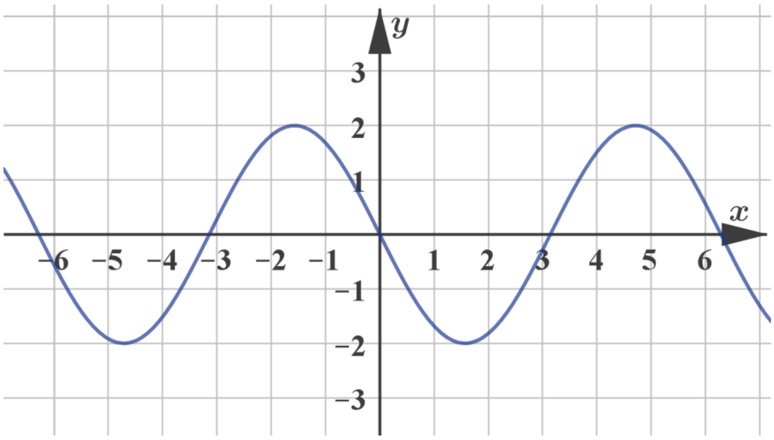
Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

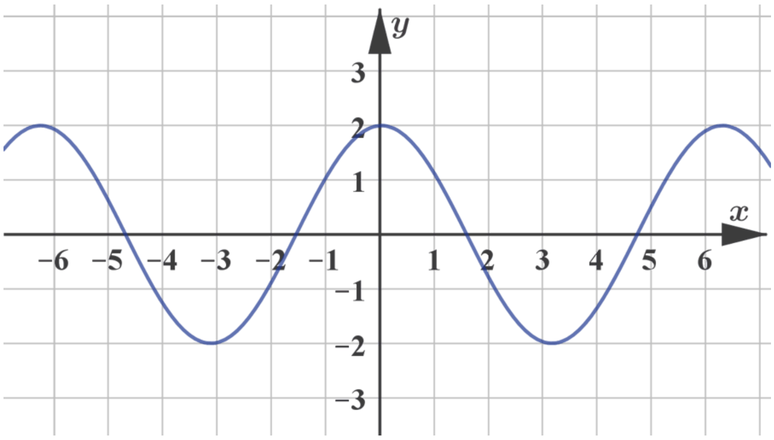
1 

✓

More information

2 

More information

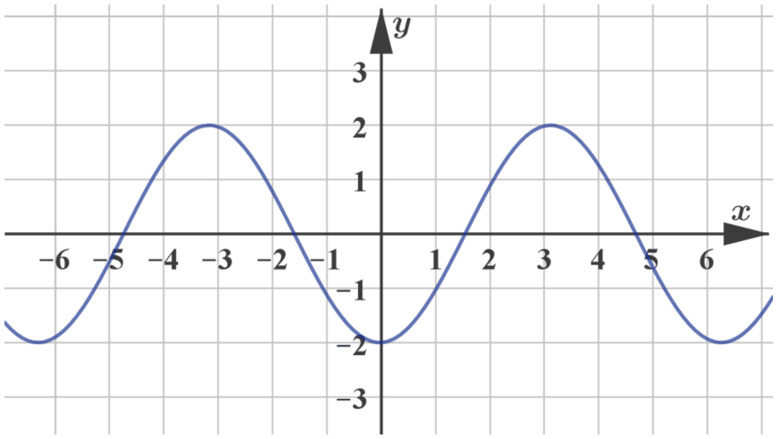
3 

More information

Student view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

4

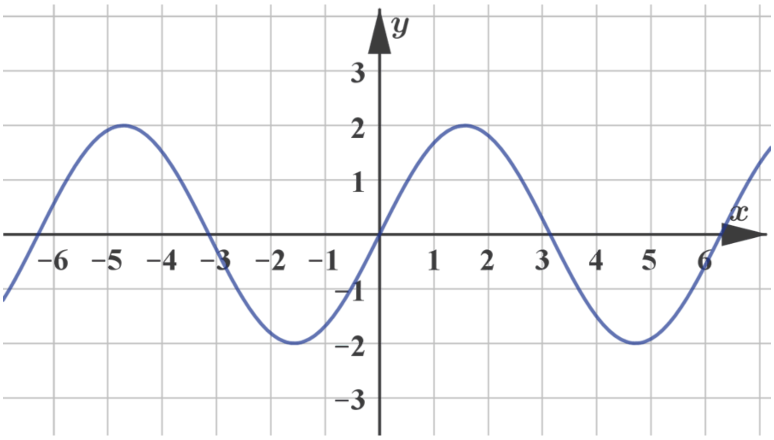


More information

Explanation

The amplitude of $y = 2\sin x$ is 2 which is twice $y = \sin x$.

Therefore, the correct answer is



More information

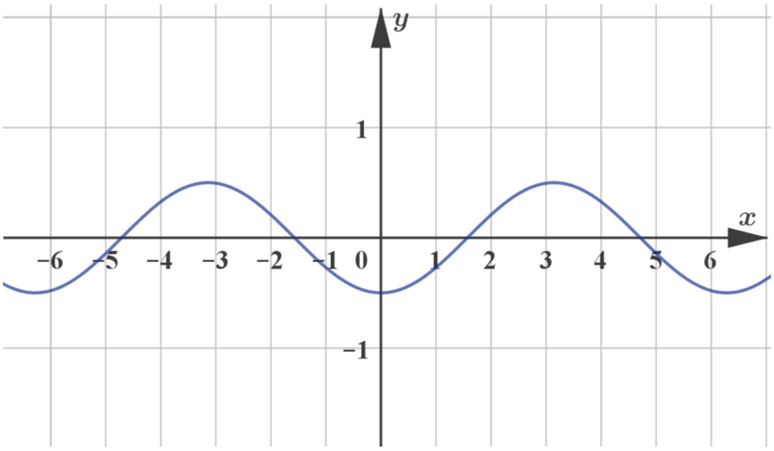
Question 2

★☆☆

Select the graph that best matches the equation $y = -\dfrac{1}{2}\cos x$.

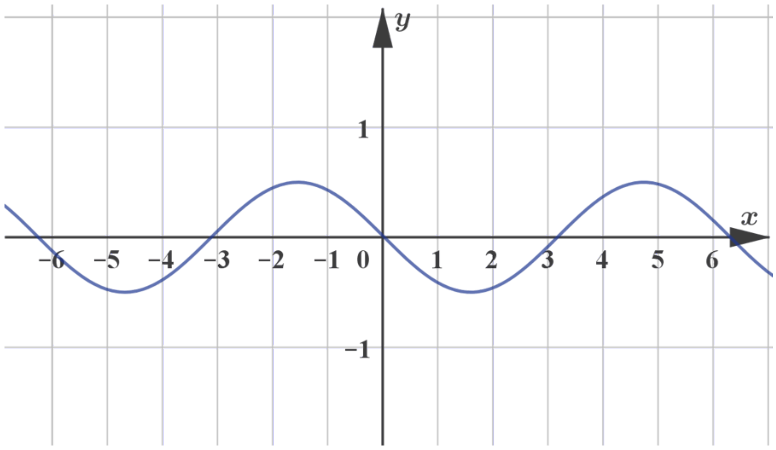
Student view

1



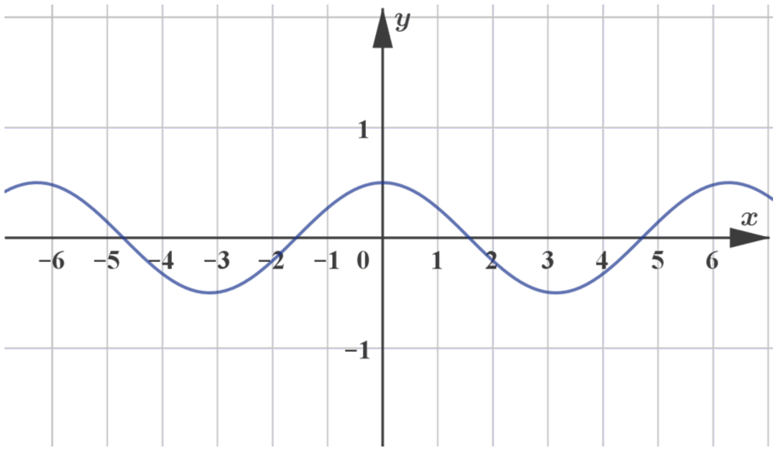
More information

2



More information

3

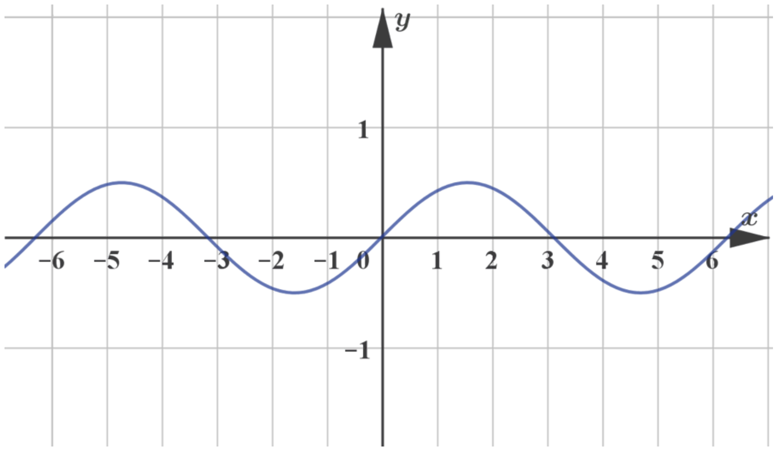


More information

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

4

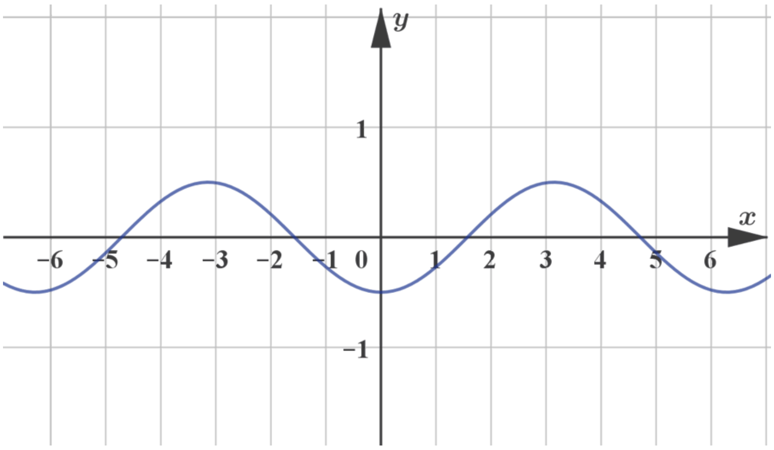


More information

Explanation

The amplitude of $y = -\dfrac{1}{2}\cos x$ is $\dfrac{1}{2}$, that is 0.5 of $y = \cos x$, and it is reflected along x-axis.

Therefore, the correct answer is



More information

3. Geometry and trigonometry / 3.7 Circular functions

Transformations

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/transformations-id-27755/print/) Assign

Revisiting function transformations

In subtopic 2.11 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-27728/) , you studied transformations of functions.

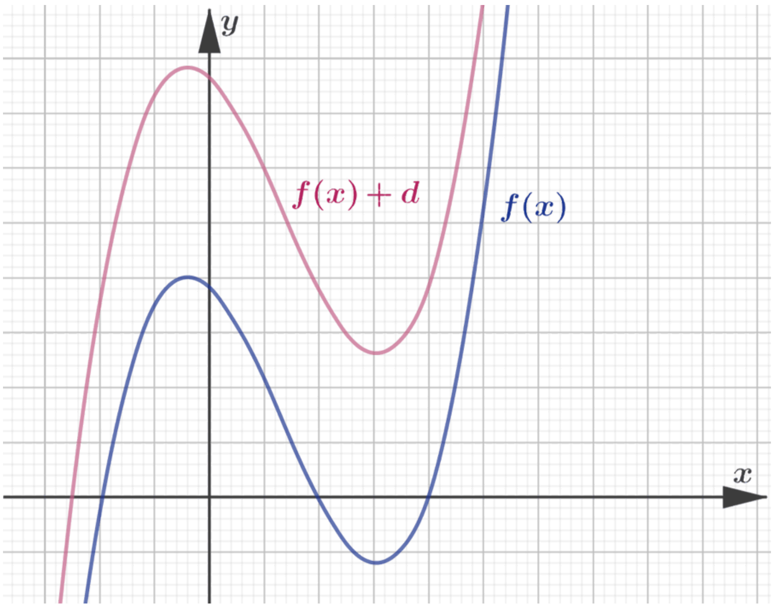
Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Here is a summary of the transformations you encountered.

Translations

- $y = f(x) + d$ vertical translation d units.



More information

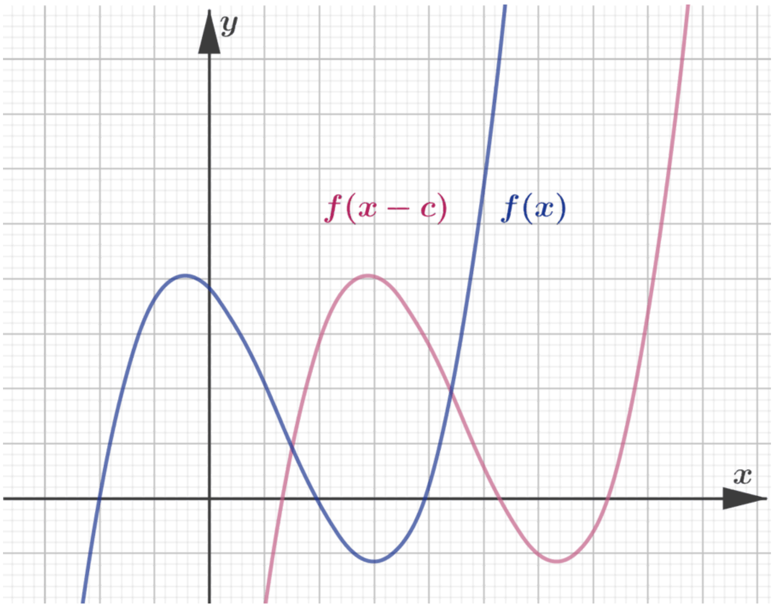
The image is a graph displaying two curves on a grid. The X-axis is labeled, but the label isn't fully visible in the image. The Y-axis is also labeled with a variable (not fully visible). One curve is represented in red and follows a wave-like pattern with peaks and troughs, and it is labeled as "f(x) + d." Another curve is in blue and is labeled "f(x)." This curve also exhibits wave-like characteristics, but with different peaks and troughs compared to the red curve. The graph includes a grid for precision in reading values, but specific values and units are not visible due to the limited view of the axes.

[Generated by AI]

- $y = f(x - c)$ horizontal translation of c units.

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o



More information

The image shows a graph with two mathematical functions plotted on a grid. The x-axis is labeled 'x' and the y-axis is labeled 'y'. There are two functions displayed: one is labeled 'f(x)' in blue, and the other 'f(x-c)' in red. The graph illustrates how the function 'f(x-c)' is a horizontal shift of the function 'f(x)' to the right, by a distance 'c'. Both functions are sinusoidal, exhibiting wave-like forms with different phases, but they share common amplitude and wavelength characteristics. The grid has a consistent scale, which is typical for graphs depicting such functions.

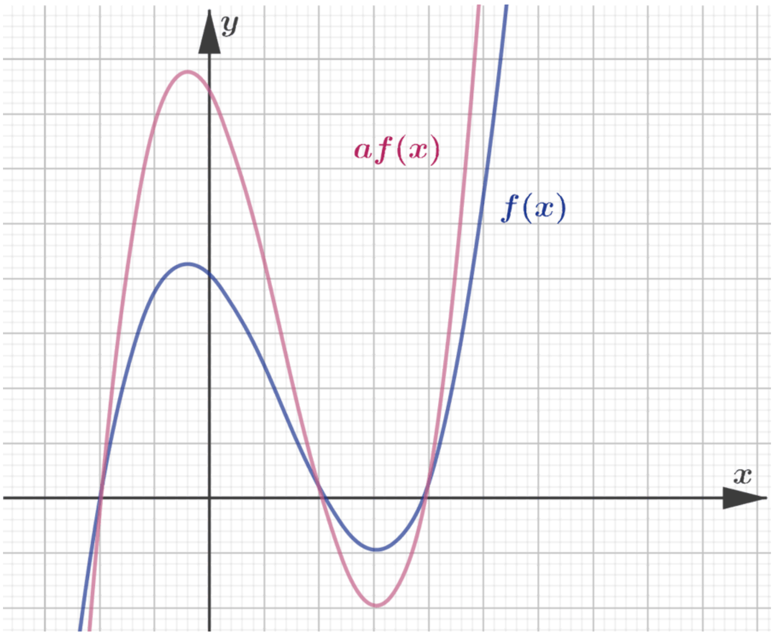
[Generated by AI]

Stretches

- $y = af(x)$ vertical stretch with scale factor a.

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o



More information

The image is a graph showing two functions, labeled as (a \cdot f(x)) and (f(x)). The graph is plotted on a grid with black axes marked as (x) for the horizontal axis and (y) for the vertical axis. (y) is labeled with an upward arrow and (x) with a rightward arrow, with each axis having intervals marked along the grid lines.

The function (f(x)) is represented by a blue curve. It exhibits periodic oscillations, beginning with an upward slope, peaking, then falling below the x-axis, hitting a local minimum, rising again, and finally increasing steeply beyond the grid.

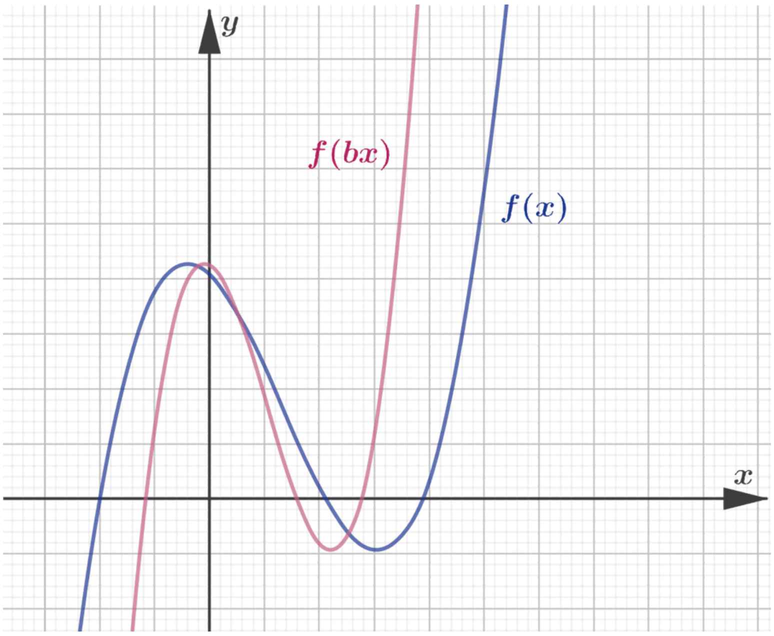
In contrast, (a \cdot f(x)) is portrayed using a pink curve. This function starts at the same point as (f(x)), but with steeper ascent and descents, showing larger peaks and troughs. The peaks and troughs of (a \cdot f(x)) occur at the same x-values as (f(x)), but with greater amplitude, demonstrating that the multiplier (a) affects the magnitude of (f(x)) without changing the frequency.

Overall, the graph portrays the effect of scaling a function in terms of its amplitude while keeping the frequency constant.

[Generated by AI]

- $y = f(bx)$ horizontal stretch with scale factor $\frac{1}{b}$.

Student
view



More information

The image is a graph depicting two trigonometric functions, labeled as f(x) in blue and f(bx) in red. The X-axis represents the variable x, while the Y-axis represents the variable y. Both axes are marked with an arrow indicating positive direction. The graph has a regular grid background. The function f(x) is represented by a blue curve with multiple inflection points, showing typical wave-like patterns commonly seen in trigonometric functions. The function f(bx) is shown in red and appears to have undergone a horizontal transformation, changing its frequency compared to f(x). Both functions start at the bottom left of the graph, intersect near the vertical center, and continue off the top right of the grid. The key transformation observed is the horizontal compression or expansion demonstrated by the differing widths of the repeated patterns in f(bx).

[Generated by AI]

In this section you will be looking at the impact of these transformations on trigonometric graphs.

Activity

Use the following applet to investigate the parameters of $y = a\sin(b(x+c)) + d$

Student
view

Interactive 1. Organic Sine Graph and Equation

Credit: GeoGebra (https://www.geogebra.org/m/avmSmXvW) Steve Phelps

More information for interactive 1

This interactive allows user to explore how the parameters a, b, c, and d shape the graph of the sine function $y = a\sin(b(x + c)) + d$. By dragging points A, B, and C, you can adjust the amplitude, frequency, phase shift, and vertical shift of the wave in real-time. Point A controls the amplitude (a) and vertical shift (d), stretching or compressing the wave vertically and moving it up or down. Point B influences the frequency (b) and phase shift (c), altering how many waves appear in a given interval and shifting the graph left or right. Point C serves as a reference to observe the combined effects of these transformations.

As you move these points, the equation updates dynamically, providing immediate visual feedback. A dashed horizontal line marks the midline $y = d$, and arrows indicate the amplitude. Adjusting the amplitude makes the peaks and troughs taller or shorter, while changing the frequency stretches or compresses the wave horizontally. The phase shift slides the entire graph sideways, and the vertical shift raises or lowers it. This interactive approach helps build an intuitive understanding of how each parameter affects the sine function's behavior.

For example If you drag point A upward, increasing the amplitude to $a = 3$, the wave's peaks and troughs become more pronounced. Moving point B to the right might set the phase shift to $c = -2$, shifting the graph left by 2 units. Meanwhile, adjusting the vertical shift to $d = 1$ raises the entire wave. The updated equation, such as $y = 3\sin(4(x + 2) + 1)$, reflects these changes, showing a taller, shifted sine wave with a moderate horizontal stretch. This hands-on experimentation makes it easy to see how each parameter transforms the graph.

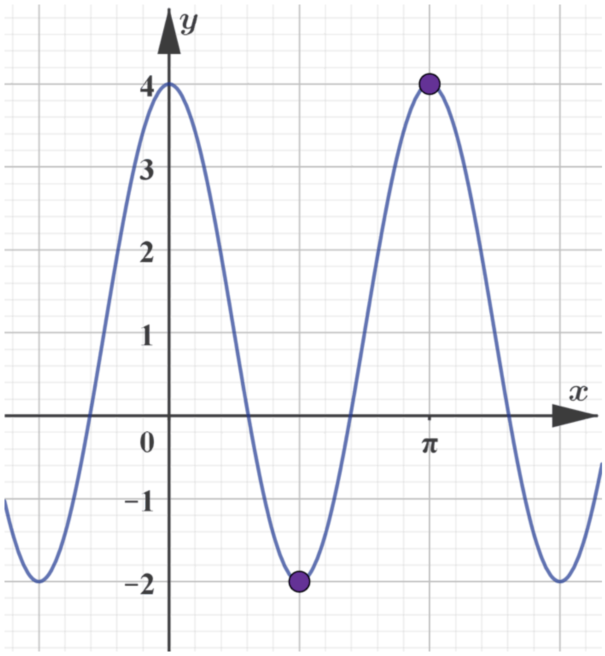
Move the points to change the shape of the graph.

E xample 1

★☆☆

The diagram below shows the part of the graph of f .

The graph has a minimum at $\left(\dfrac{\pi}{2}, -2\right)$ and a maximum at $(\pi, 4)$.



More information

The image is a graph depicting a trigonometric function which seems to represent a cosine wave. The X-axis is labeled with increments of (\pi) (0, (\pi), 2(\pi), etc.) and represents the angle in radians. The Y-axis is labeled with integers ranging from -2 to 4 and represents the function value. The function shows a maximum point at (\left(\pi, 4 \right)) and a minimum point at (\left(\dfrac{\pi}{2}, -2 \right)). The wave clearly oscillates between these maximum and minimum values, defining a typical cosine wave pattern.

[Generated by AI]

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

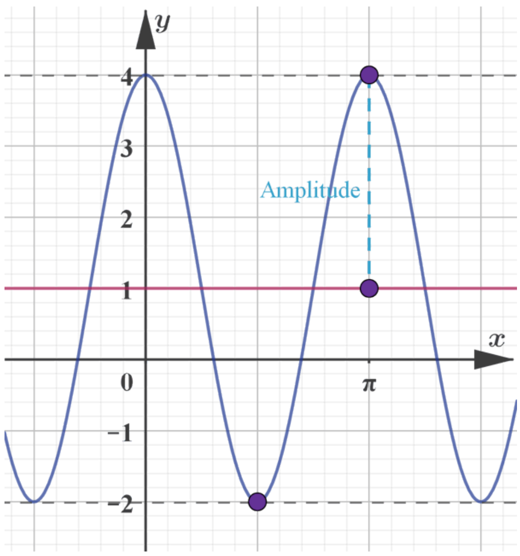
The function can be written in the form $f(x) = p\cos(qx) + d$.

Find the value of

a) p

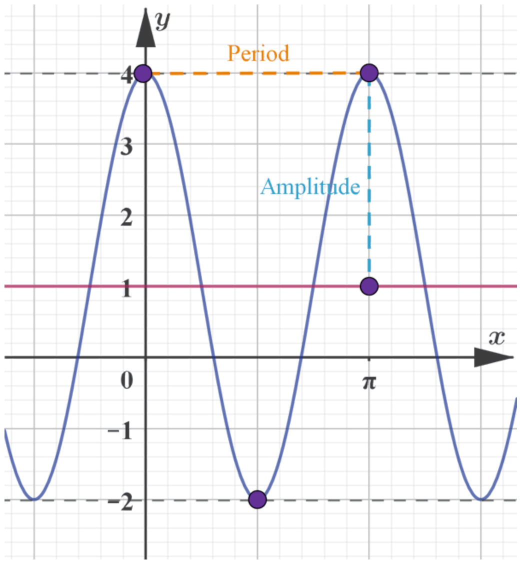
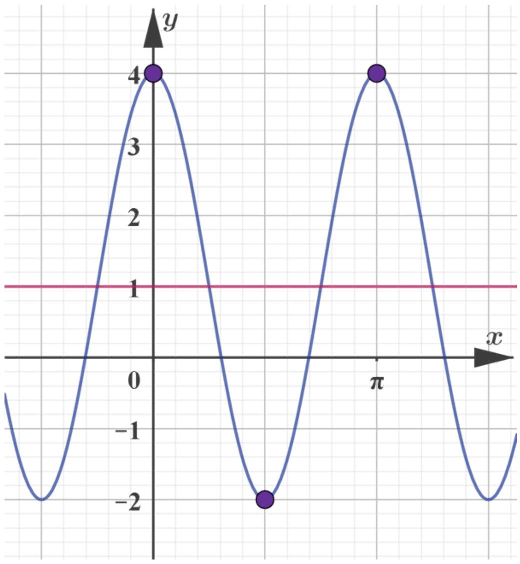
b) q

c) d.

	Steps	Explanation
a)	 $p = 3$	Maximum value of y is 4 and the minimum value is -2, therefore ha way through is $x = 1$. The amplitude then is 3 units.

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
b)	 $q = 2$	A complete cosine wave is π , therefore the frequency is $\frac{2\pi}{\pi} = 2$
c)	 $d = 1$	Reference line is $y = 1$ which is up unit from original $y = 0$.

Student view

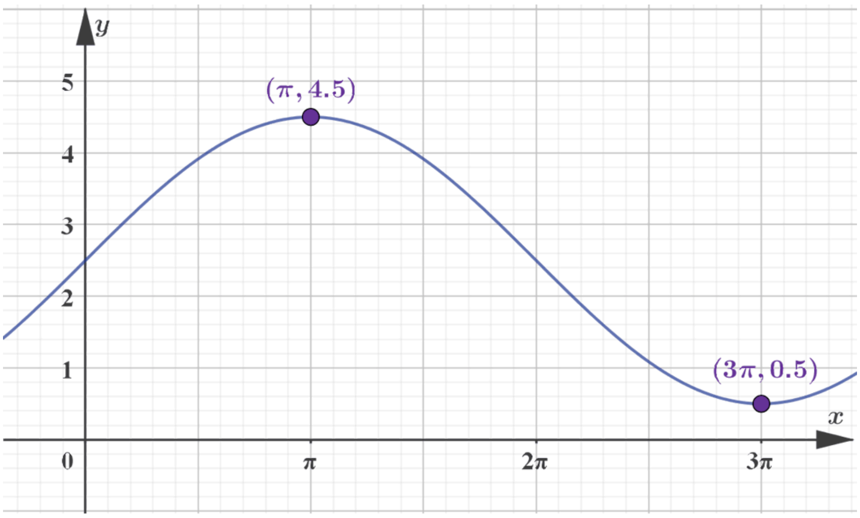
Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Example 2

★★☆

The following diagram shows part of the graph of $y = p\sin(qx) + r$.

A $(\pi, 4.5)$ is a maximum point and B $(3\pi, 0.5)$ is a minimum point.



More information

The image is a graph representing a sinusoidal curve plotted on a grid. The X-axis is labeled with values denoted as (\pi), (2\pi), and (3\pi). The Y-axis is labeled with numerical values ranging from 0 to 7. The graph has two highlighted points: point A at ((\pi, 4.5)), which is a maximum point, and point B at ((3\pi, 0.5)), which is a minimum point. The curve starts from the lower left, rises to a peak at ((\pi, 4.5)), descends to a trough at ((3\pi, 0.5)), and continues beyond that.

[Generated by AI]

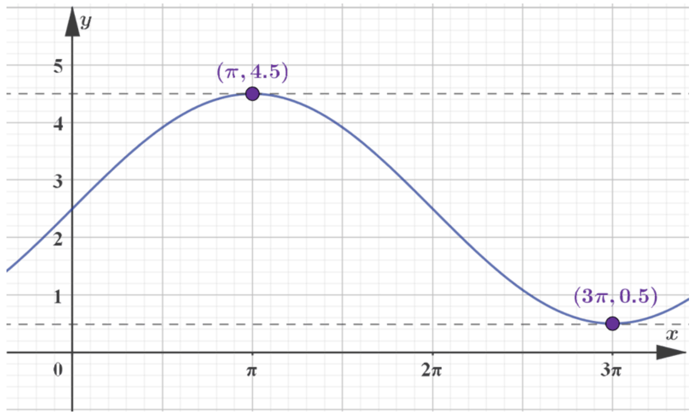
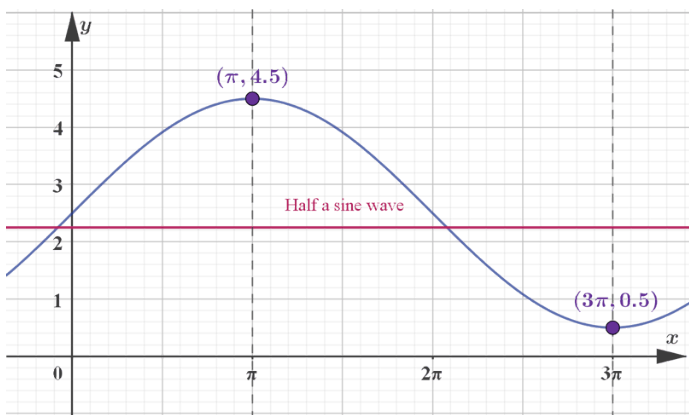
Find the values of

a) p

b) q

Student
view

c) r

	Steps	Explanation
a)	 $p = 2$	Half the length between the highest and lowest points is 2 units.
b)	 $q = 0.5$	From maximum point to minimum point on the graph a half a sine wave. $$3\pi - \pi = 2\pi$$ So, the full length of the wave is 4π frequency = $\frac{2\pi}{\text{period}} = \frac{2\pi}{4\pi} = 0.5$.

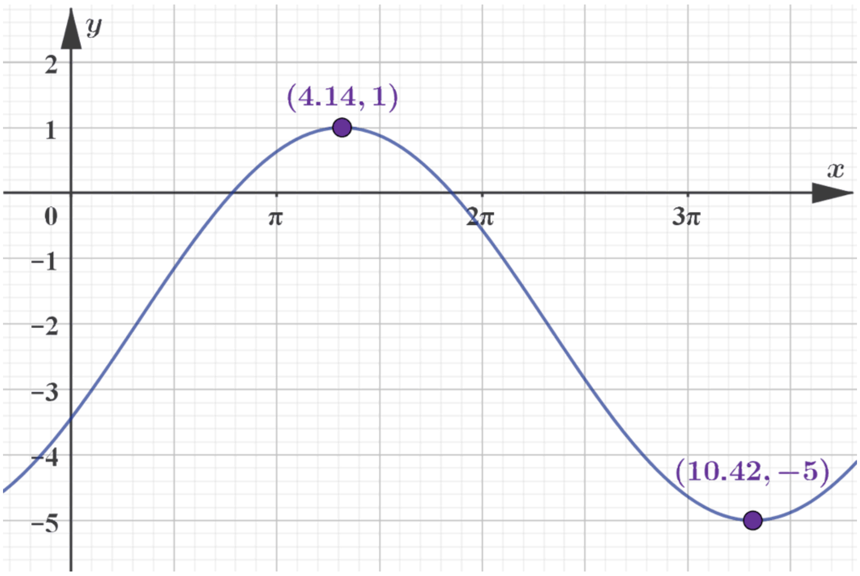
Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
c)	$r = 2.5$	The reference line is at midwa from highest to lowest points on the graph So, $\dfrac{4.5 + 0.5}{2} = 2.5$.

Example 3

★★☆

Let $f(x) = a\cos b(x + c) + d$. The following diagram shows a part of f.



More information

The diagram represents a graph of the function (f(x) = a \cos b(x + c) + d). It shows a sinusoidal curve on a grid background. The X-axis is labeled with multiples of (\pi), ranging from (0) to (2\pi). The Y-axis ranges from (-5) to (5). The graph displays a peak at the point ((4.14, 1)) and a trough at ((10.42, -5)).

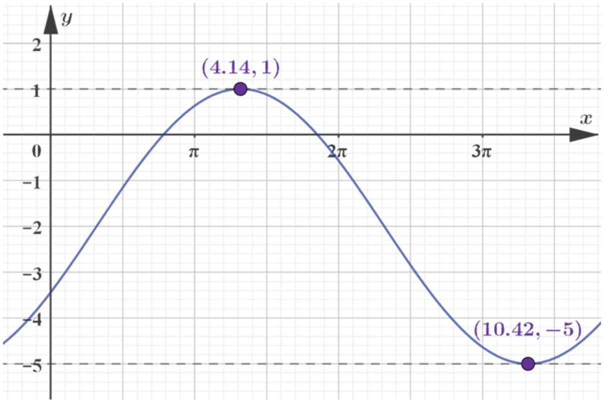
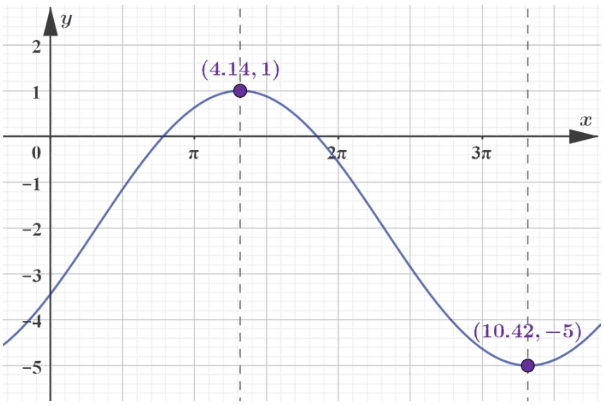
[Generated by AI]

Student
view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

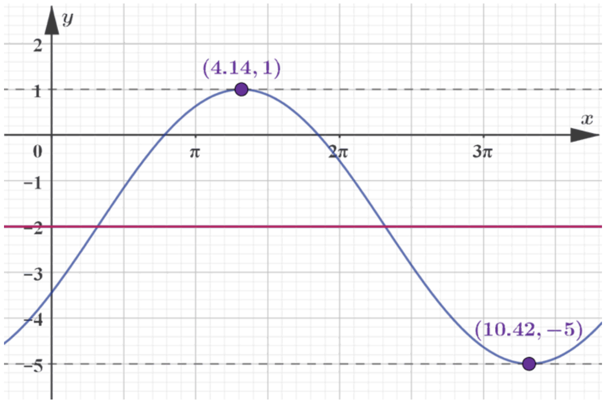
The graph has a maximum at $(4.14, 1)$ and a minimum at $(10.42, -5)$.

Find a, b, c and d.

Steps	Explanation
 $a = 3$	Half the distance between highest and lowest points is 3 units.
 $b = 0.500$ (3 significant figures)	From maximum point to minimum point c the graph is a half a sine wave. $$10.42 - 4.14 = 6.28$$ So, the full length of the wave is 12.56. Frequency is $\frac{2\pi}{12.56} \approx 0.50025$.
$c = -4.14$	As the cosine graph moved to right 4.14 units.

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

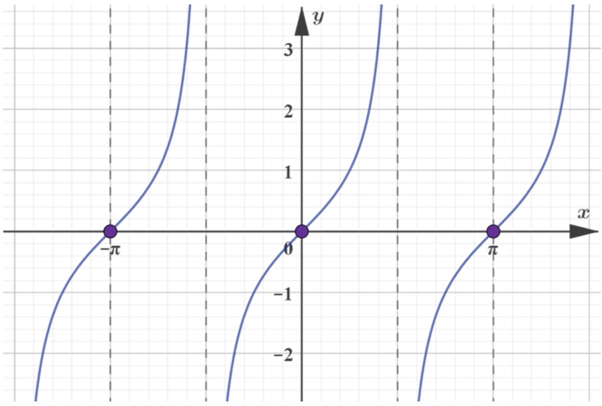
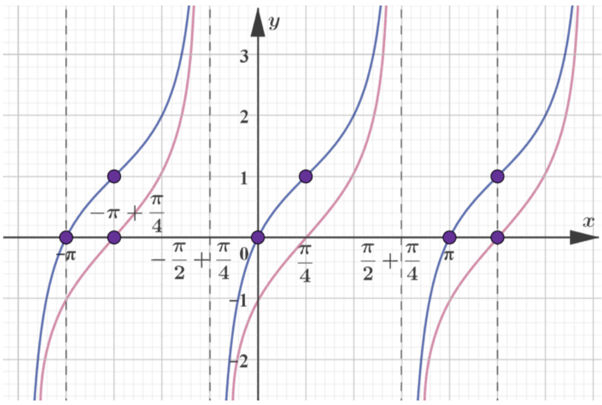
Steps	Explanation
 $d = -2$	Half way through the maximum and minimum values of y is $$\frac{1+(-5)}{2} = -2.$$

Example 4

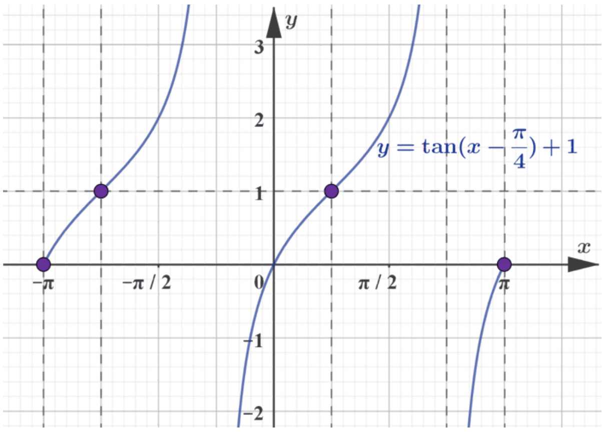
★★☆

Sketch the graph of $y = \tan\left(x - \frac{\pi}{4}\right) + 1$, - $\pi \leq x \leq \pi$.

Student
view

Steps	Explanation
	Sketch $y = \tan x$.
 >	$y = \tan\left(x - \frac{\pi}{4}\right) + 1$ is the transformation of $y = \tan x$ by the vecto $\begin{pmatrix} \frac{\pi}{4} \\ 1 \end{pmatrix}$ which is $\frac{\pi}{4}$ units to the right and unit up. Note that since $y = \tan\left(-\frac{\pi}{4}\right) = -1$, th transformed graph will go though the origin.

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Steps	Explanation
Therefore, the sketch of $y = \tan\left(x - \frac{\pi}{4}\right) + 1$ is 	Label the sketch clearly and make sure yc only draw the graph on the domain mentioned in the question.

Activity

Use the following applet to match the given graph by changing the parameters *a* , *b* , *c* , and *d* using the input boxes and/or sliders for $y = a\sin\left(b\left(x + c\right)\right) + d$

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Interactive 2. Sine and Cosine Modeling

Credit: GeoGebra (https://www.geogebra.org/m/BJCaZ5tm) Tim Brzezinski

More information for interactive 2

The applet allows users to match a given graph by adjusting the parameters a, b, c, and d in the sine or cosine function $y = a\sin(b(x + c)) + d$ or $y = a\cos(b(x + c)) + d$. Users can input values manually. The goal is to align the generated curve (displayed as a dotted line) with the target graph. The applet also provides the coordinates of five points (A, B, C, D, E) to guide the user in achieving an accurate match.

The applet also includes a "Generate New Wave" button, which provides a new target wave and five updated reference points for continued practice say ($A(-5, -5)$, $B(4, -5)$, $C(-0.5, -10.5)$, $D(8.5, 0.5)$, and $E(13, -5)$). This feature encourages repeated exploration and strengthens understanding of how each parameter affects the shape and position of sine and cosine graphs. Your task is to adjust the cosine function $y = a\cos(bx - c) + d$ by analyzing these points. For instance, the vertical shift (d) should be -5.5, calculated as the midpoint between the highest (D at $y = 0.5$) and lowest (C at $y = -10.5$)

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

points. The amplitude (a) is 6, representing half the distance between these extremes. The frequency (b) is approximately 0.36π, determined by the wave's cycle length between repeating points like A and E. Finally, the phase shift (c) is around 1.45π, aligning the first peak with point D. As you tweak these parameters, the applet updates the graph in real-time. Once your function perfectly matches the target, you'll receive a "Correct" confirmation, reinforcing how each parameter shapes the wave's position and behavior. This hands-on approach helps visualize the direct relationship between coordinates and trigonometric functions.

Be aware

In the activity above you were asked to look for the equation in the form $y = a\sin(b(x+c)) + d$.

Compare this to the form $y = A\sin(Bx + C) + D$, which some graphing calculators use when they return a trigonometric model.

What is the relationship between the values of the two sets of coefficients $a,\ b,\ c,\ d$ and $A,\ B,\ C,\ D$?

3 section questions

Question 1

★★☆

For the function $y = 3\cos\frac{1}{6}x - 2$, x in radians, what are the amplitude, period and maximum value of y?

1 $3, 12\pi, 1$ ✓

2 $3, 12\pi, 5$

3 $3, \frac{\pi}{3}, 1$

4 $-3, 12\pi, -5$

Explanation

The vertical stretch factor is $a = 3$, hence the amplitude is 3.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

The horizontal stretch factor is $b = \frac{1}{6}$, hence the period is given by $T = \frac{2\pi}{\frac{1}{6}} = 12\pi$.

The vertical translation is given by $d = -2$, hence the maximum value along the curve is given by $-2 + 3 = 1$.

Question 2

★★☆

For the function $y = 2\cos\left[2\left(x + \frac{\pi}{2}\right)\right] + 7$, where x is measured in radians, what are the amplitude, period and maximum value of y?

1 $2, \pi, 9$ ✓

2 $2, 4\pi, 9$

3 $-2, \pi, 5$

4 $2, -\pi, 9$

Explanation

The vertical stretch factor is $a = 2$, hence the amplitude is 2.

The horizontal stretch factor is $b = 2$, hence the period is $T = \frac{2\pi}{2} = \pi$.

The vertrical translation is $d = 7$, hence the maximum value along the curve is $7 + 2 = 9$.

Question 3

★★☆

For the function $y = \frac{1}{2}\tan\frac{1}{2}x + \frac{1}{2}$, x in radians, what are the amplitude, period and maximum value of y?

1 $\infty, 2\pi, \infty$ ✓

2 $\infty, 4\pi, \infty$

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

3 $\frac{1}{2}, 2\pi, \infty$

4 $\frac{1}{2}, 4\pi, \infty$

Explanation

There is no well-defined amplitude for the tangent function. Its oscillations are unbounded, hence, its amplitude is ∞ or, alternatively, not defined.

The horizontal stretch factor is $b = \frac{1}{2}$, hence the period is given by $T = \frac{\pi}{\frac{1}{2}} = 2\pi$.

The tangent graph is unbounded, hence its maximum displacement is ∞.

3. Geometry and trigonometry / 3.7 Circular functions

Applications

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/applications-id-27756/print/) Assign

Simple harmonic motion

Anything that repeats its motion to and from its rest point will create a simple harmonic motion. For example, a mass spring system, a swing, or a simple pendulum such as this one.

Student view

Video 1. Simple Pendulum and Its Period.

More information for video 1

This interactive video demonstrates the simple harmonic motion of a pendulum, allowing users to observe and analyze its oscillations. By watching the pendulum swing back and forth around its equilibrium position, creating a repeating oscillating pattern.

This interactive provides an opportunity to understand how factors like gravity and tension influence the pendulum's motion. Through observation, users will gain insights into the principles of simple harmonic motion and how these apply to real-world oscillatory systems.

Simple harmonic motion can be modelled using trigonometric functions.

Example 1

★★☆

A ball on a spring is attached to a fixed point O.

The ball is pulled down and released, so that it moves back and forth vertically.

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

More information

The image shows a vertical spring with a ball attached at the bottom. At the top of the spring, there is a label 'O'. The ball is depicted as hanging from the spring. There is a double-headed arrow next to the spring, denoting the vertical movement of the ball, marked with the variable 'd'. The arrow suggests that the ball can move up and down, indicating oscillation around the point 'O'. This setup represents a classic physics diagram for harmonic motion.

[Generated by AI]

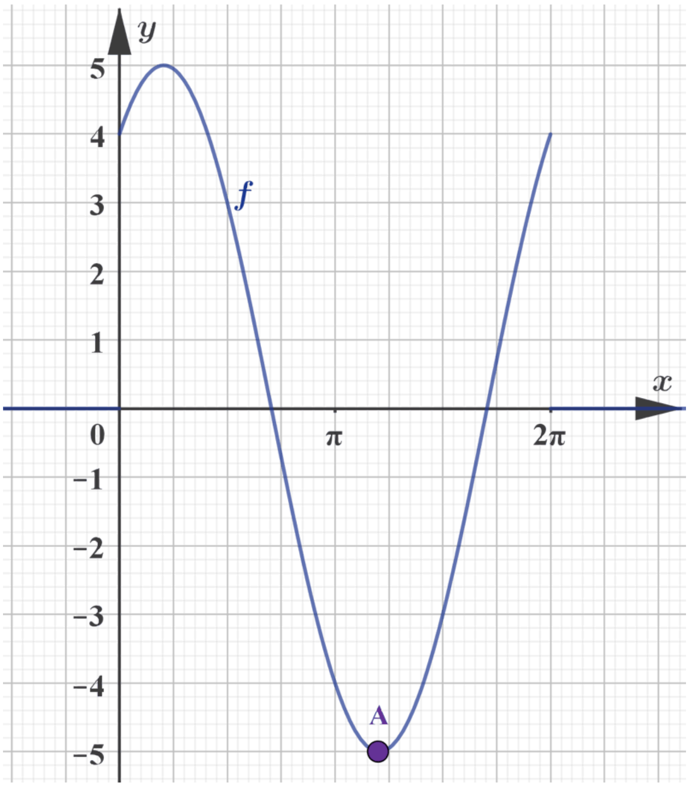
The distance, d centimetres, of the centre of the ball from O at time t seconds, is given by

$$d\,(t) = f\,(t) + 17\,,\ 0 \leq t \leq 20$$

where $f\,(x) = 4\cos x + 3\sin x$ for $0 \leq x \leq 2\pi$. Below is the graph of f .

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o



More information

The image is a graph of the function (f(x) = 4\cos x + 3\sin x) over the interval (0 \le x \le 2\pi). The X-axis represents the angle (x) in radians and ranges from 0 to (2\pi). The Y-axis represents the function values and is labeled with values ranging from -5 to 5. The graph shows a trigonometric wave that starts at the Y-axis maximum value of 5, descending to a minimum near (-5) around (\pi), and then rising back towards 5 at (2\pi). The curve has a labeled point 'A', indicating the minimum value of the function.

[Generated by AI]

The maximum value of f is 5 and there is a minimum value at point A.

a) Find the coordinates of the point A.

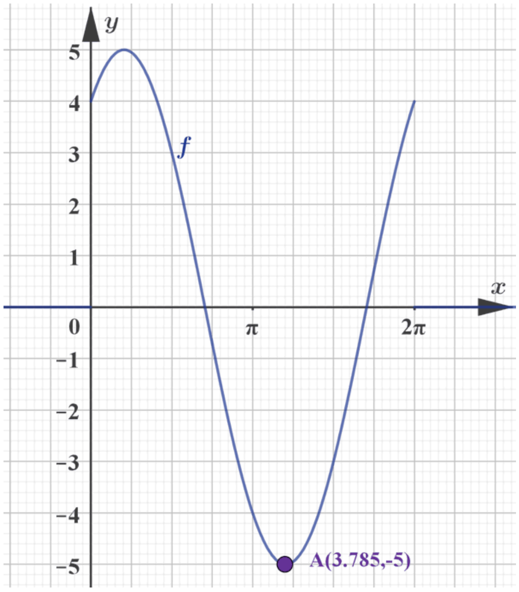
b) Write down the amplitude of f.

c) Write down the period of f .

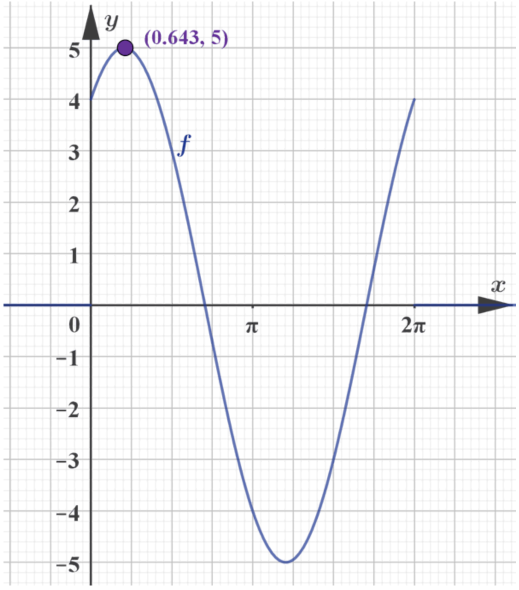
d) Hence, write f in the form $p\cos(x-r)$.

e) Find the maximum distance between the centre of the ball and point O.

Student
view

	Steps	Explanation
a)	 $\mathrm{A}\,(3.785, -5)$	Using GDC minimum point.
b)	$\text{amplitude} = 5$	Highest point is $y = 5$ and the minimum is at $y = -5$.

Overview (/study/ap aa-hl/sid-134-cid-761926/o

	Steps	Explanation
c)	 period $= 6.282$	The maximum is at $x = 0.644$ anc minimum at $x = 3.785$. There is a half an oscillation betwe these two values, so $\text{period} = 2(3.785 - 0.644) = 6.2$ Note that the exact period is 2π, s this is the period of both sine and cosine, which are used in the defir of the function.
d)	$f(x) = 5\cos(x - 0.644)$	The graph of $y = \cos x$ is stretche vertically by 5 and shifted to the ri by 0.644.
e)	$d = 22$	As $d(t) = f(t) + 17$, maximum will be at maximum $f(t)$, which is So the maximum distance is $5 + 17 = 22$.

Water waves

Many of the currents in water are created by differences in temperature, salinity, global wind patterns and the earth's rotation. Although there are many factors creating the tides, the gravitational pull between the moon, earth and the sun are the main factors. The moon exerts about 2.2 times more pull factor than the sun. Since the rotation of the moon around the earth is periodic, the tides around the world are also periodic.

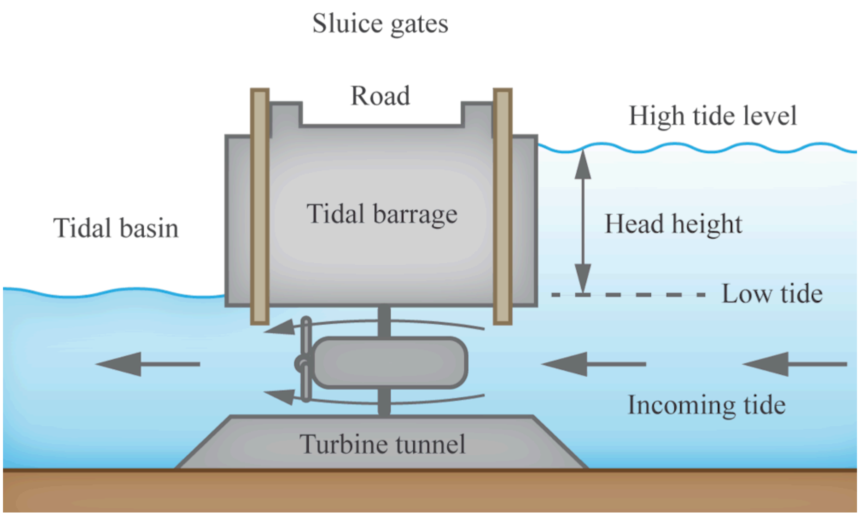
Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

International Mindedness

Tidal power or tidal energy is a form of hydropower which converts the energy obtained from tides to other forms of energy; mostly electrical energy. Although tide mills have been used since the Middle Ages, the world's first large-scale tidal power plant became operational in 1966: the Rance Tidal Power Station in France.

Engineers around the globe are working to develop this energy on a larger scale to help the world's climate change and energy crisis. There are varying opinions about the tidal power stations. Some argue that the power plants may cause more damage than expected on marine life, and create changes in marine life.



More information

This diagram illustrates a tidal barrage system for generating energy from tidal power. At the top center, two sluice gates control the flow of water over a road that runs across a tidal barrage structure. Below the road and gates is the tidal barrage, which creates a barrier between the tidal basin on the left and the incoming tide on the right. Arrows indicate the direction of the water flow during high tide and low tide.

The head height is marked with a double-headed arrow, showing the difference between high tide and low tide levels. Water flows from the tidal basin into a turbine tunnel located at the base of the tidal barrage, where the force of the incoming tide turns the turbines and generates power. This diagram includes labels for the tidal basin, sluice gates, road, tidal barrage, high tide level, low tide level, head height, turbine tunnel, and incoming tide, providing a clear depiction of a tidal power station's components and flow direction.

[Generated by AI]

Student
view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

Example 2

★★☆

The depth of water in a port is modelled by $d(t) = p\cos(qt) + r$ for $0 \leq t \leq 24$, where t is the number of hours after a high tide.

At high tide, the depth of water is 10 m, and after 8 hours, at low tide, the depth of water is 3.2 m.

a) Find the value of p.

b) Find the value of q.

c) Find the value of r.

d) Use the model to find the depth of water 7 hours after high tide.

	Steps	Explanation
a)	$p = 3.4$	The difference between high tide and low tide is $10 - 3.2 = 6.8$. This is twice the amplitude, so $p = \frac{6.8}{2} = 3.4$
b)	$\text{period} = 2 \times 8 = 16$ $q = \frac{2\pi}{\text{period}} = \frac{2\pi}{16} = \frac{\pi}{8}$ $q = 0.393$ (3 significant figures)	The time between high tide and low tide is hours, which is a half oscillation. $\text{frequency} = \frac{2\pi}{\text{period}}$
c)	$r = \frac{10 + 3.2}{2} = 6.6$	The mid-value, r, is halfway between high and low tide.

Student
view

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

	Steps	Explanation
d)	$d(7) = 3.4\cos\left(\frac{\pi}{8} \times 7\right) + 6.6$ $= 3.46 \text{ m (3 significant figures)}$	$d(t) = 3.4\cos\left(\frac{\pi}{8}t\right) + 6.6$

Container port

Credit: thitivong Getty Images

Example 3

★★☆

The depth of water level, d m, t hours after midnight in the port can be modelled by

$$d(t) = 4\cos\left(\frac{\pi}{3}(t-4)\right) + 7$$

a) Find the highest and lowest depth of water

b) How many hours after midnight is high tide?

c) What is the depth of water at 12 noon?

Student view

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

	Steps	Explanation
a)	$\text{Maximum depth} = 11\text{ m}$ $\text{Minimum depth} = 3\text{ m}$	As $\lvert\cos a\rvert \leq 1$, highest value for depth $$d(t) = 4 \times 1 + 7 = 11\text{ m}$$ and lowest value for depth $$d(t) = 4 \times (-1) + 7 = 3\text{ m}.$$
b)	4 hours	As the graph of $y = \cos x$ is shifted 4 units to the right, the first high tide will happen 4 hours after midnight.
c)	$$\begin{aligned} d(12) &= 4\cos\left(\frac{\pi}{3}(12-4)\right) + 7 \\ &= 4\cos\left(\frac{8\pi}{3}\right) + 7 \\ &= 4 \times \frac{-1}{2} + 7 \\ &= 5 \end{aligned}$$ Therefore at 12 noon the depth of the water will be 5 metres.	$t = 12$ $$\cos\left(\frac{8\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

3 section questions

Question 1

★★☆

The depth of water level, d m, t hours after midnight in a port can be modelled by

$$d(t) = 2\sin\left(\frac{\pi}{3}t\right) + 7, 0 \leq t \leq 24$$

How many hours after midnight is the first high tide?

Do not give units with your answer.

Student view

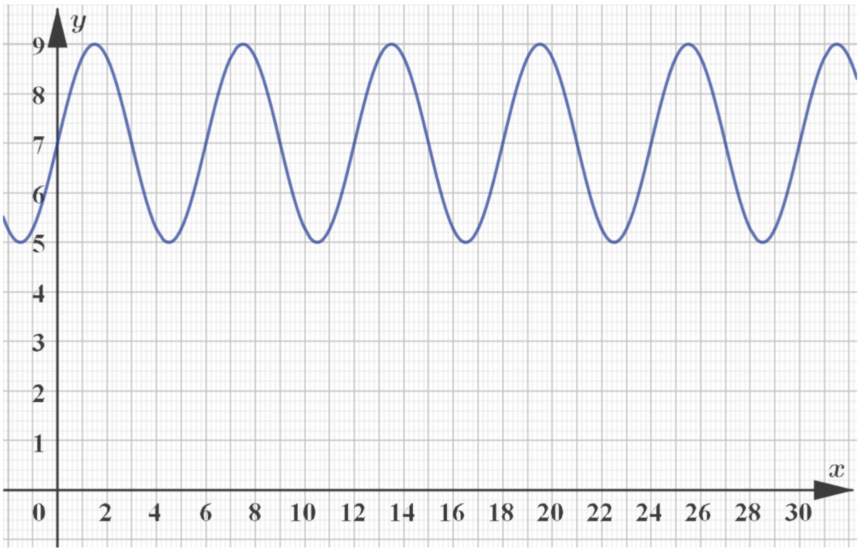
1.5

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

Accepted answers

1.5, 1,5, 3/2

Explanation



More information

High tide is when $d(t)$ is at a maximum so $t = 1.5$.

Question 2

★★☆

The depth of water level, d metres, t hours after midnight in a port can be modelled by

$$d\left(t\right) = 2\sin\left(\frac{\pi}{3}t\right) + 7, 0 \leq t \leq 24$$

What are the highest and lowest depths of water?

1 9 m and 5 m ✓

2 7 m and 5 m

3 9 m and 7 m

4 8 m and 5 m

Student view

Explanation

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

$\left|\sin\left(\frac{\pi}{3}t\right)\right| \leq 1$

so when $\sin\left(\frac{\pi}{3}t\right) = -1$, $d(t)$ is minimum $d_{\min} = -2 + 7 = 5$

and when $\sin\left(\frac{\pi}{3}t\right) = 1$, $d(t)$ is maximum $d_{\max} = 2 + 7 = 9$

Therefore, the highest and lowest depth of water are 9 m and 5 m respectively.

Question 3

★★☆

The depth of water level, d metres, t hours after midnight in a port can be modelled by

$$d(t) = 2\sin\left(\frac{\pi}{3}t\right) + 7,\ 0 \leq t \leq 24$$

What is the depth of the water at noon?

Do not give units with your answer.

7 ✓

Accepted answers

7

Explanation

Noon is 12 hours after midnight.

$$d(12) = 2\sin\left(\frac{\pi}{3}\cdot 12\right) + 7 = 2\sin(4\pi) + 7 = 7$$

Therefore the depth of the water is 7 m

3. Geometry and trigonometry / 3.7 Circular functions

Checklist

Student view

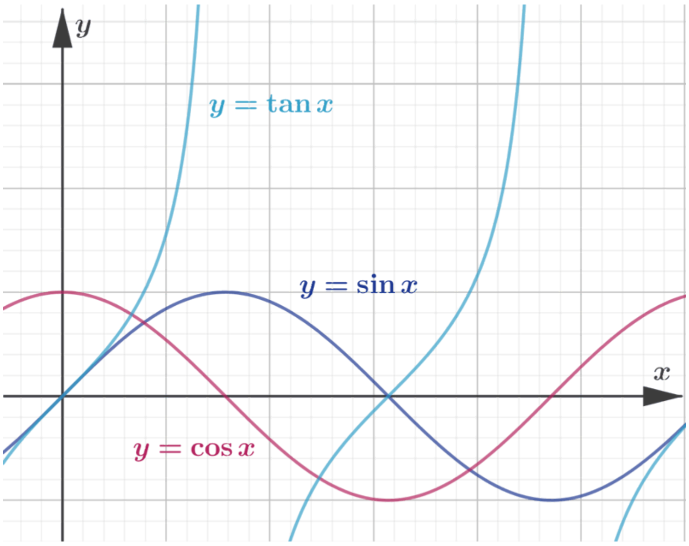
Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-27757/print/) Assign

Overview
(/study/ap
aa-
hl/sid-
134-
cid-
761926/o

What you should know

By the end of this subtopic you should be able to:

- recognise and interpret the graphs of the circular functionssine, cosine and tangent:



More information

The image is a graph displaying the trigonometric functions sine, cosine, and tangent on a grid. The X-axis represents the angle in radians, generally ranging from negative to positive values, while the Y-axis represents the function values. The grid provides a visual guide for scale. The graph includes three curves: the blue curve for y = sin(x), the pink curve for y = cos(x), and the cyan curve for y = tan(x). The sine and cosine curves are smooth and periodic, displaying regular wave patterns with regular intervals. The tangent curve has vertical asymptotes where the function is undefined. Labels are present next to each curve to denote the respective function.

[Generated by AI]

- use sine, cosine and tangent to solve problems involving the amplitude and period periodic functions
- use composite functions of the form $f(x) = a\sin(b(x+c)) + d$
- apply transformations to circular functions, including stretch and translation
- use trigonometric functions in real-life contexts.

Student view

Investigation

Sound waves create pockets of higher and lower air pressure. When these pockets reach your ear, the vibration created in your ear drum makes you hear the sound. Experiments show that a healthy young person can hear frequencies between 20 and 20 000 hertz.

International Mindedness

In 1939, 440 Hz was generally accepted as the standard frequency, or pitch . Most musicians will tune to this pitch, which is the note A above middle C. Some musicians specialising in different historical eras will tune to a different pitch, such as 415 Hz for the baroque period.

Theory of Knowledge

In the words of James Sylvester:

'May not music be described as the mathematics of the sense, mathematics as music of the reason? The musician feels mathematics, the mathematician thinks music: music the dream, mathematics the working life.'

To what extent do you agree with the statement?

What does this tell us about the relationship between music and mathematics?

Musical tones are a set of sound frequencies that we call notes. Adjusting the frequency alters the pitch of the sound and the period of the graph. Adjusting the volume of the sound alters the amplitude of the graph.

The equal tempered scale is the common musical scale used for tuning pianos and other instruments of relatively fixed scale. It divides the octave into 12 equal semitones.

The table below shows the frequency (in Hz) of these notes in different octaves.

Frequency table based on $A_4 = 440$

C_0 to B_8 (middle C = C_4)

	Octave number								
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
C	16.35	32.70	65.41	130.81	261.63	523.25	1046.50	2093.00	4186.01
C#	17.32	34.65	69.30	138.59	277.18	554.37	1108.73	2217.46	4434.92
D	18.35	36.71	73.42	146.83	293.66	587.33	1174.66	2349.32	4698.64
D#	19.45	38.89	77.78	155.56	311.13	622.25	1244.51	2489.02	4978.03
E	20.60	41.20	82.41	164.81	329.63	659.26	1318.51	2637.02	5274.04
F	21.83	43.65	87.31	174.61	349.23	698.46	1396.91	2793.83	5587.65
F#	23.12	46.25	92.50	185.00	369.99	739.99	1479.98	2959.96	5919.91
G	24.50	49.00	98.00	196.00	392.00	783.99	1567.98	3135.96	6271.93
G#	25.96	51.91	103.83	207.65	415.30	830.61	1661.22	3322.44	6644.88
A	27.50	55.00	110.00	220.00	440.00	880.00	1760.00	3520.00	7040.00
A#	29.14	58.27	116.54	233.08	466.16	932.33	1864.66	3729.31	7458.62
B	30.87	61.74	123.47	246.94	493.88	987.77	1975.53	3951.07	7902.13

To use these frequency values, round to the nearest integer

More information

The image is a table titled 'Frequency table based on A4 = 440, C0 to B8 (middle C = C4)'. The table consists of musical notes listed vertically, C to B, along with their frequencies in Hertz across octave numbers 0 to 8.

Each row corresponds to a note, such as C, C#, D, D#, E, F, F#, G, G#, A, A#, and B, showing how the frequency doubles with each ascending octave. For example, the note C has frequencies starting from 16.35 Hz in octave 0 to 4186.01 Hz in octave 8.

The octaves are labeled at the top, 0 through 8, and the frequencies are arranged in columns beneath these octave labels. The bottom of the table advises, 'To use these frequency values, round to the nearest integer.'

[Generated by AI]

For example, C_1 has a frequency of 33 Hz (rounded to nearest integer), which means when this note is played, 33 pockets of higher air pressure will vibrate against your ear each second.

Activity

By adding the functions modelling two different notes, you can analyse the sound that occurs when the notes are played together.

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

- Write functions that model the notes $\mathrm{C_1}$, $\mathrm{E_1}$ and $\mathrm{B_1}$, using software to graph them.
- How many cycles of each function occur in 1 second?

Create a new function to model the sound when two notes are played together.

- What do you notice? How many cycles of each function occur in 1 second?
- What is the frequency of each function?
- Is there any change in the amplitude?
- How can we determine if a combination of notes is consonant or dissonant?

What other links can you find between mathematics and music?

Rate subtopic 3.7 Circular functions

Help us improve the content and user experience.

Student view