Index

The big picture

Success or failure: binary events have only two possible outcomes.

Credit: Wendy/Jeff Sparks/Torquemada GettyImages

Success or failure. True or false. Functional or defective.

In this subtopic, we are going to explore probabilities related to binary events– events that have only two possible outcomes. While this might seem like a small subset of events, even the most complex situations can often be interpreted in a binary way – getting a certain outcome or not getting that outcome. For example, while there are six sides on a dice, giving six possible outcomes, you can consider rolling a 5 one outcome and rolling anything else as the other outcome to make rolling a dice a binary event. The two possible outcomes will always be complementary events, meaning their probabilities will always add up to 1.

But more than just exploring binary events, this subtopic deals with binomial experiments , which consist of a finite number of repeated binary trials that are identical and independent **.** While rolling a dice to see if we roll a 5 or not is a binary event, rolling the same dice 50 times to see how many 5s are rolled would be a binomial experiment. The exploration of these types of experiments leads us to the first of the named probability distributions we will study in this course, the discrete binomial distribution.

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Concept

The named probability distributions in mathematics are an example of how mathematicians make use of **relationships** in constructing knowledge. Beginning with the basic components of a probability distribution that we looked at in subtopic 4.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25656/), mathematicians use similarities and differences of different distributions to recognise the ones that occur frequently. This enables them to categorise them with others that share certain characteristics to create a **system** that makes exploring probabilities simpler.

4. Probability and statistics / 4.8 The binomial distribution

The binomial distribution

Notation and parameters for the binomial distribution

Will it rain?

Credit: BeyondImages GettyImages

Suppose you have collected weather data from the past several years and found that in June there is a 0.3 experimental probability that it rains on any given day. How would you work out the probability that it would not rain at all in June, or that there would be only one or two rainy days?

The binomial distribution describes the probability distribution of different outcomes of repeated binary events. Since there are only two outcomes, we use p to represent the probability of success, meaning the probability that the event we are looking for occurs. It deals with discrete data, or data that can be counted. In particular, it is used to describe situations in which you know the probability of a single event but are trying to determine the probability of that event occurring a certain number of times in a binomial experiment.

Student view

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The notation $X \sim$ denotes that a random variable X is distributed a certain way. The binomial distribution is therefore expressed

$$X \sim B(n, p)$$

where the B stands for the name of the distribution, binomial, and the values in the parentheses represent the key parameters of the distribution. In this case, n represents the number of trials in the binomial experiment and p represents the probability of success.

So you can use the binomial distribution to find out the probability of a certain number of rainy days in June, given a 0.3 experimental probability that it rains on any given day. Assuming that the weather each day is independent of the weather on other days, you could express a binomial distribution for the number of rainy days like this: $X \sim B(30, 0.3)$. Notice that $n = 30$ because there are 30 days in June, and $p = 0.3$ because that is the probability of rain on each day.

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Another value often considered is the probability of failure, denoted q. Since we are dealing with binary events, $q = 1 - p$.

As we saw in subtopic 4.7 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-big-picture-id-25656/), each probability distribution has an expected value,

$$\mathrm{E}(X) = \mu = \sum_{i=1}^{N} x_i \, \mathrm{P}(X = x_i)$$

Since the probability of success for each event is equal in the binomial distribution, we can simplify this formula to $\mathrm{E}(X) = np$.

Exam tip

The formula for the expected value of a binomial random variable $X \sim \mathrm{B}(n, p)$ is in the formula booklet.

$$\mathrm{E}(X) = np$$

In addition, the formula for the variance is also in the formula booklet.

$$\mathrm{Var}(X) = np(1 - p)$$

Example 1

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For the example stated above with $X \sim B(30, 0.3)$, find the expected number of rainy days in June and the variance of the distribution.

Student view

The binomial distribution given has $n = 30$ and $p = 0.3$, so we can substitute these into the formulae.

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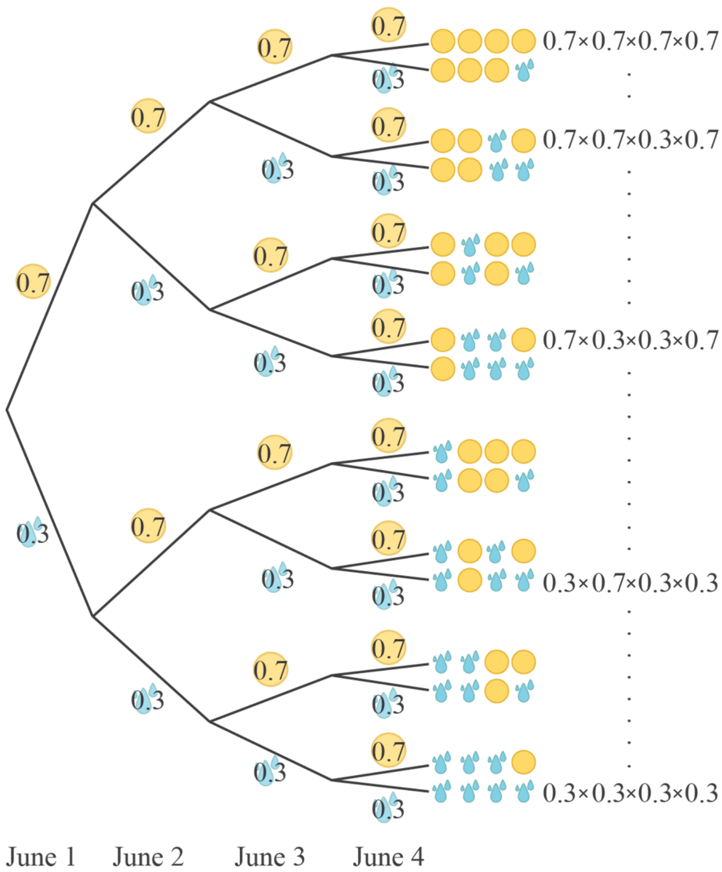
- First, $\mathrm{E(X)} = \mathrm{np} = 30 \times 0.3 = 9$, so one would expect 9 days of rain in June.
- Second, $Var(X) = np(1-p) = 30 \times 0.3(1-0.3) = 30 \times 0.3 \times 0.7 = 6.3$.

✓ **Important**

Remember that the binomial distribution deals with binary events. In the example above, you could classify the weather numerous ways, such as sunny, partly cloudy, mostly cloudy, light rain and heavy rain. However, in order to use the binomial distribution, you must cluster the specific categories into just two, more general, categories that encompass all the possibilities.

Probabilities within the binomial distribution

Now that we understand the basics of the distribution and the notation it uses, we can examine how probability is calculated. For example, imagine you are considering camping for the first four days of June. How can we calculate the probability that two or more of those days will have rain?



How many times will it rain in the first four days of June?

More information

The image is a tree diagram illustrating the probability of rain for the first four days of June. Each branch represents the outcome for one day, with several layers corresponding to each day.

- The tree starts with the first day (June 1) branching into two options, each labeled with probabilities: 0.7 for a sunny day (represented by a yellow circle) and 0.3 for a rainy day (represented by blue raindrops).

Student
view

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- Each node for subsequent days (June 2, 3, and 4) continues to branch into two similar outcomes with the same probabilities (0.7 for sunny, 0.3 for rainy).
- By the fourth day, there are multiple paths, each representing a specific combination of sunny and rainy days.
- Next to each end of the path, there is a multiplication of probabilities, such as 0.7 × 0.7 × 0.7 × 0.7 or 0.3 × 0.3 × 0.3 × 0.3, indicating the overall probability for that specific combination of weather over the four days.
- At the bottom, the days are labeled sequentially as June 1, June 2, June 3, and June 4, aligning with the branches above.

[Generated by AI]

Examining the tree diagram above and multiplying along the branches as we have done before ((undefined) section 4.6.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/modelling-related-events-id-25651/) and section 4.6.2 (/study/app/math-aa-hl/sid-134-cid-761926/book/probabilities-of-related-events-id-25652/)) , it is possible to find the probability that it rains on two or more days during your camping trip. Notice from the tree diagram that there are 6 different ways that it could rain on exactly 2 days, 4 different ways it could rain on exactly 3 days and 1 way it could rain on all 4 days. Therefore, if D is the number of days it rains, we calculate the probability this way:

$$\begin{aligned}\mathrm{P}(D \geqslant 2) &= \mathrm{P}(D=2)+\mathrm{P}(D=3)+\mathrm{P}(D=4)\\ &= 6(0.7\times0.7\times0.3\times0.3)+4(0.7\times0.3\times0.3\times0.3)+(0.3\times0.3\times0.3\times0.3)\\ &= 0.2646+0.0756+0.0081\\ &= 0.3483\approx 0.348\end{aligned}$$

While this process works, it is tedious and would be difficult to scale up to larger values of n. The generalization of the process above gives the following formula.

Making connections

A random variable that is binomially distributed and takes on the value of the number of successes in n trials is written as $X \sim B(n,p)$, where p is the probability of success in one trial. Then, the probability that X takes on an actual value of x successes is given by:

$$X \sim B(n,p) \Rightarrow \mathrm{P(X=x)} = \binom{n}{x}\mathrm{p^x(1-p)^{n-x}}\,,\ x=0,1,2,3,\ldots,n,$$

where $\binom{n}{x}$ is the binomial coefficient, also written as ${}^{n}C_{x}$, given by $\dfrac{n!}{x!(n-x)!}$.

Exam tip

This formula is not in the formula booklet and you do not need to know it for your exam. It is included here for completeness.

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Example 2

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A binomial experiment has a 0.8 probability of success. Find the probability of having success exactly once in 5 trials.

The probability of the first trial is successful and the other four are not is

$$0.8 \times (1 - 0.8)^4 = 0.00128.$$

The probability that the second trial is successful and the other four are not is

$$(1 - 0.8) \times 0.8 \times (1 - 0.8)^3 = 0.00128$$

The successful trial can also be the third, the fourth and the fifth. These also have the same probabilities, so the probability of having one success in 5 trials is the sum of these.

$$5 \times 0.8 \times (1 - 0.8)^4 = 0.0064.$$

Example 3

★★★

An accountant’s new clerk has a 25 % chance of making a mistake on an assignment. If 8 assignments are selected from this clerk, find the probability that **at least** 2 are incorrect.

We start by defining the random variable that enumerates success: let X be the number of incorrect assignments in a sample of 8. Also, the number of trials is $n = 8$ and $\mathrm{P}(\text{mistake}) = 0.25$. Hence, $X \sim \mathrm{B}(8, 0.25)$.

The question asks you to find $\mathrm{P}(X \geqslant 2)$.

First, notice that $\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X \leqslant 1)$, so

$$\begin{aligned} \mathrm{P}(X \geqslant 2) &= 1 - \mathrm{P}(X \leqslant 1) \\ &= 1 - (\mathrm{P}(X = 0) + \mathrm{P}(X = 1)) \end{aligned}$$

The probability that the clerk has no incorrect assignments is

$$\mathrm{P}(X = 0) = (1 - 0.25)^8 \approx 0.10011$$

Student view

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To find the probability that the clerk has one incorrect assignment, you can use the method presented in Example 2.

$$\mathrm{P}(X=1)=8\times 0.25\times(1-0.25)^7\approx 0.26697$$

Hence, the probability that the clerk has at least two incorrect assignments is

$$\mathrm{P}(X\geqslant 2)\approx 1-(0.10011+0.26697)\approx 0.633$$

Below is an applet that shows the histogram of a binomial distribution in relation to the number of trials and the probability p of success. Note that when you make p smaller (i.e. there is a small probability of success in each one of n trials), the peak of the histogram moves to the left, i.e. the biggest bars of the histogram, which indicate the biggest probabilities, are now found above smaller numbers of successes. This makes sense, as a small probability of success p in each trial increases the probability of getting fewer successes over n trials. On the other hand, if p increases, the peak moves to the right, as it is now more probable to get more successes in n trials. We will examine how to find these probabilities with calculator functions in the next section.

Interactive 1. How Does the Binomial Probability Change When You Change n And p?

More information for interactive 1

This interactive graph allows users to explore and understand binomial distribution across a range of up to 20 trials. The x-axis represents the number of successes, while the y-axis shows the probability of each outcome, ranging from zero to one. At the top of the graph, there are two sliders that let users adjust key parameters in real time. One slider controls the number of trials, which can be set between one and twenty. The other slider sets the probability of success for each individual trial, ranging from zero to one.

As users adjust these sliders, the bar graph below responds dynamically, updating to show the new binomial distribution. When the probability of success is low, the peak of the histogram shifts to the left, showing that fewer successes are more likely. As the probability increases, the peak moves to the right, indicating a higher chance of more successes.

This interactive provides a hands-on way for users to see how changing the probability of success and the number of trials affects the distribution. It helps users gain a clear understanding of the relationship between probability and outcomes in binomial experiments, offering a visual and intuitive approach to a key statistical concept.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

How does the binomial probability change when you change n and p?

2 section questions

Question 1

Difficulty:

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Given that $X \sim \mathrm{B}(16, 0.8)$, what is $\mathrm{E}(X)$ and $\mathrm{Var}(X)$?

1 $\mathrm{E}(X) = 12.8, \mathrm{Var}(X) = 2.56$ ✓

2 $\mathrm{E}(X) = 12.8, \mathrm{Var}(X) = 1.6$

3 $\mathrm{E}(X) = 2.4, \mathrm{Var}(X) = 2.56$

4 $\mathrm{E}(X) = 2.4, \mathrm{Var}(X) = 1.6$

Explanation

If $X \sim \mathrm{B}(16, 0.8)$ then $n = 16$ and $p = 0.8$. Using the definition for the expected value and the variance, we obtain

$$\begin{aligned}\mathrm{E}(X) &= np \\ &= 16 \times 0.8 \\ &= 12.8\end{aligned}$$

and

$$\begin{aligned}\mathrm{Var}(X) &= np(1-p) \\ &= 12.8 \times 0.2 \\ &= 2.56.\end{aligned}$$

Question 2

Difficulty:

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The probability that a young driver has a road accident in their first year of driving is $1/5$. Assuming each driver's probability of an accident is independent of any other driver's, find the probability that out of 10 such drivers, exactly one has an accident.

1 0.268 ✓

2 0.00268

3 2.68

4 0.0268

Explanation

For the situation we are given, X is the number of young drivers who have an accident their first year of driving, and $X \sim B(10, 0.2)$.

The probability that each particular driver has an accident and the others do not is $0.2 \times 0.8^9 = 0.026853\ldots$

Student view

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There are 10 drivers, so the probability we are looking for is 10 times this value.

$$\mathrm{P}(\mathrm{X}=1)=10\times0.2\times0.8^{9}\approx0.268$$

Calculating binomial probabilities with calculator functions

Finding binomial probabilities on the calculator

You have already learned how to find the probability of a specific number of successes in the binomial distribution using the formula, but your graphic display calculator (GDC) likely has the binomial probability distribution function, or binomial PDF, pre-programmed. The binomial PDF requires you to enter the number of trials, n, the probability of success, p, and a specific value of x to return the $\mathrm{P}(\mathrm{X}=\mathrm{x})$.

Activity

Use the simulation below to try finding different theoretical probabilities with your calculator and compare them to the experimental probabilities found in the simulation. You can also use the formulae for $\mathrm{E}(\mathrm{X})$ and $Var(X)$ that you learned in the previous section. Adjust the number of trials and the probability of success to see how the distribution changes. Is there anything you can do to make the experimental and theoretical probabilities closer to the same value?

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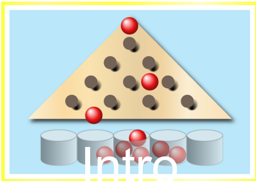

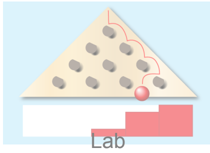

Interactive 1. Explore Binomial Probabilities with Different Values.

More information for interactive 1

This interactive simulation, *Plinko Probability*, provides users with a hands-on way to explore probability distributions, expected value, variance, and the Central Limit Theorem through a digital Galton board. It features two main tabs: Intro and Lab, each offering varying levels of control and detail.

In the Intro tab, a digital Galton board appears with a funnel at the top and 13 bins at the bottom. Users can control how balls are dropped—either one at a time, ten at a time, or all at once—via a play button on the right. As balls fall from the funnel, they bounce off pegs in a random pattern and accumulate in the bins, visually forming a distribution. A counter below the play button tracks the total number of dropped balls. Below that, there are toggles to turn sound on or off and to reset the simulation.

On the bottom right, three toggle buttons allow users to switch between different views: numerical counters on the bins, a graphical representation (with bin numbers on the x-axis and counts on the y-axis), and an eraser to reset the graph. As more balls are dropped—especially 100 or more—users will notice that the distribution tends to center around the middle bins, reflecting a normal distribution and reinforcing concepts of expected value and variance.

The Lab tab provides a more advanced exploration environment while retaining the Galton board layout. Additional controls appear for deeper investigation: users can select how the ball paths are visualized (show balls, show paths, or hide both), adjust the number of rows (from 1 to 26) with a slider, and set the binary probability of a ball falling left or right using another slider (ranging from 0 to 1).

Statistical information is displayed in real time, including total number of balls dropped, sample and theoretical mean, sample and theoretical standard deviation, and the standard error of the mean. Users can enable the “Ideal” toggle to overlay the theoretical distribution on the histogram for comparison. The play button here allows either single ball drops or a continuous stream of balls, offering flexibility in observing short-term randomness and long-term distribution patterns.

Together, both tabs offer an engaging platform for visualizing core probability concepts and understanding how randomness and sample size contribute to statistical regularities.

Student view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

It is very tedious to find the probability of a range of successes, such as $\mathrm{P}(\mathrm{X} \geqslant \mathrm{x})$, unless x is close to 0 or n (see **Example 3** in section 4.8.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-binomial-distribution-id-25662/)). Thankfully, the calculator also has a binomial cumulative distribution function, or binomial CDF, that will return the probability of a range of values.

It is important to understand exactly how these functions work in order to use them properly. Consider the difference in using the binomial CDF on the TI-nspire and the TI-84.

- The TI-nspire requires you to enter n, p, a lower bound and an upper bound and returns $\mathrm{P}(\text{lower bound} \leqslant \mathrm{X} \leqslant \text{upper bound})$.
- The TI-84 requires you to enter n, p and an upper bound and returns $\mathrm{P}(0 \leqslant \mathrm{X} \leqslant \text{upper bound})$.

Take note of the difference as we revisit an example from the previous section and work through it using these calculator functions.

Example 1

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Look again at **Example 3** from section 4.8.1 (/study/app/math-aa-hl/sid-134-cid-761926/book/the-binomial-distribution-id-25662/), using the calculator this time. An accountant's new clerk has a 25% chance of making a mistake on an assignment. If 8 assignments are selected from this clerk, find the probability that 2 are incorrect and the probability that **at least** 2 are incorrect.

Solution

A GDC can be used to evaluate these values.

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Step	Explanation
These instructions will show you how to calculate $\mathrm{P}(X=2)$ and $\mathrm{P}(X \geqslant 2)$ for a binomially distributed random variable $X \sim \mathrm{B}(8, 0.25)$. Choose the statistics mode.	
Press F5 to select the distribution menu.	
Press F5 again to select the binomial distribution.	

Student
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Overview
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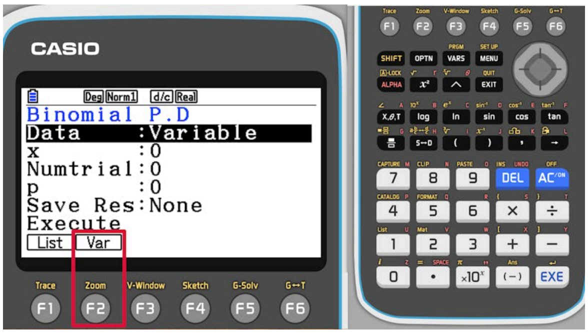
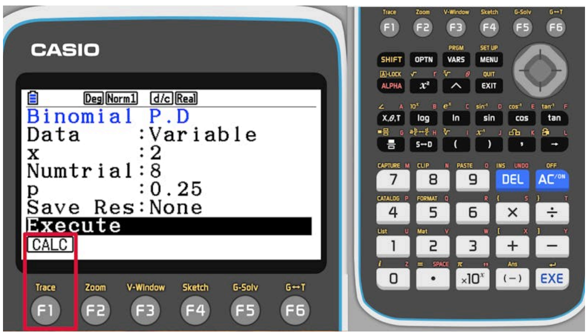
<table>
<tr><th>Step</th><th>Explanation</th></tr>
<tr><td>To calculate $\mathrm{P}(X=2)$ select the binomial probability distribution (Bpd) option.</td><td>
</td></tr>
<tr><td>Make sure that your data type is "variable".</td><td>
</td></tr>
<tr><td>Enter the parameters of the distribution:
number of trials: 8
probability of success: 0.25
The x-value indicates that you are interested in the probability, when $X=2$.

Once you entered the information, press F1 to calculate the probability.</td><td>
</td></tr>
</table>

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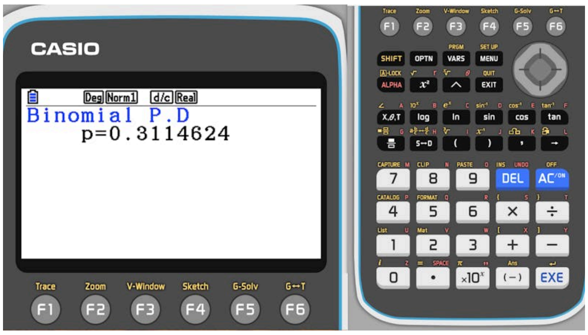
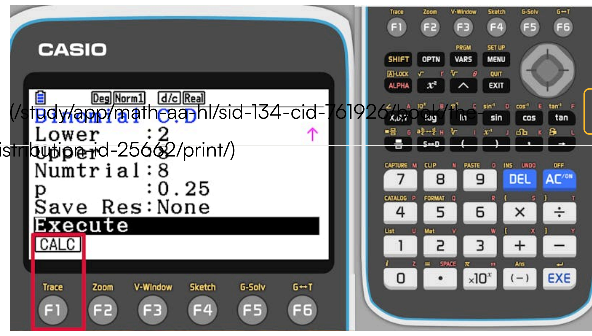
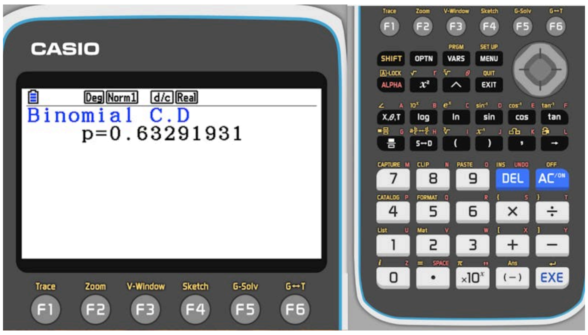
Step	Explanation
The calculator tells you the probability.	
To calculate $\mathrm{P}(X \geqslant 2)$, start the same way, but choose the cumulative binomial probability (Bcd) option.	
This option of the calculator will find probabilities of the form $\mathrm{P}(a \leqslant X \leqslant b)$, where a is the lower limit (in this case 2) and b is the upper limit. In this case there is no upper limit, so you need to enter the maximal possible outcome of a binomial experiment, which is the number of trials (so in this case, 8).	

Step	Explanation
	

Step	Explanation
These instructions will show you how to calculate $\mathrm{P}(X=2)$ and $\mathrm{P}(X \geqslant 2)$ for a binomially distributed random variable $X \sim \mathrm{B}(8, 0.25)$. Enter the home screen of any application.	

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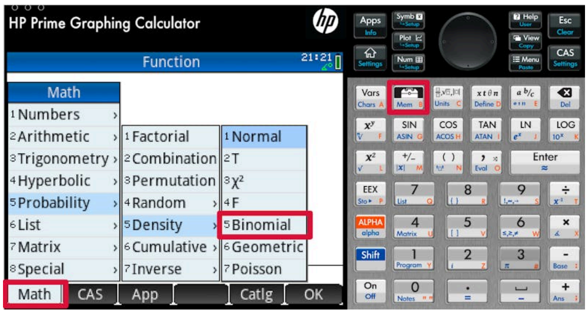
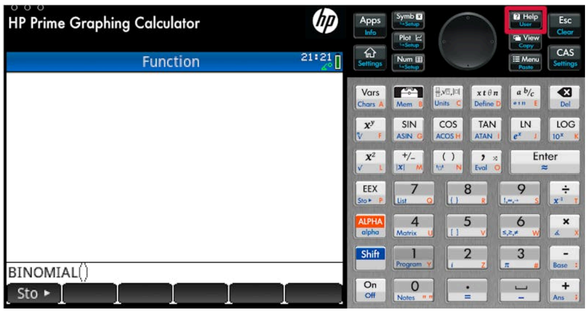
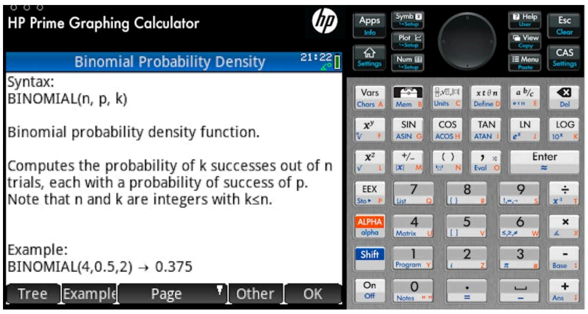
Step	Explanation
Press the toolbox button and find the binomial probability density option through the submenus.	
On this calculator there is no graphical interface to help you enter the parameters. However, there is a help button that you can always press if you forget what the calculator expects.	
This is the help screen for the binomial probability density function.	

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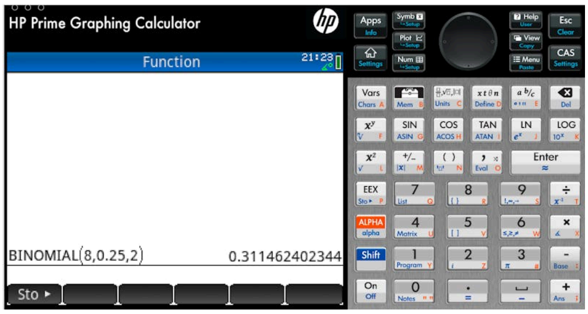
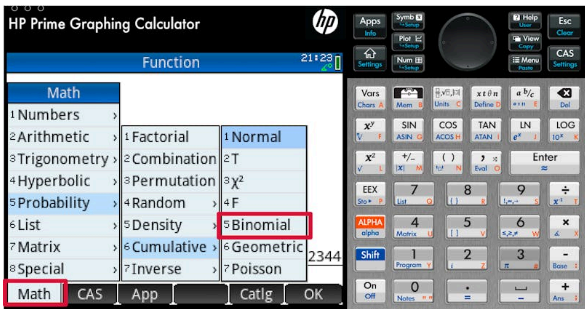
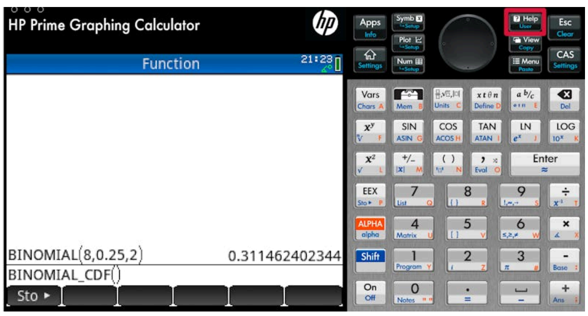
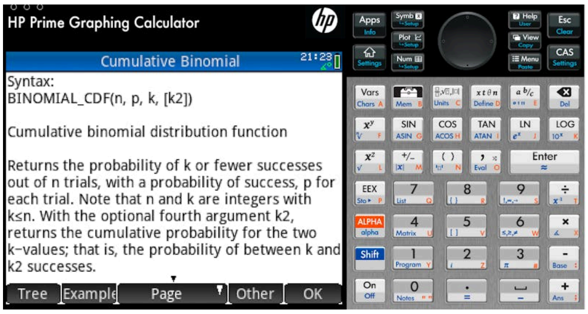
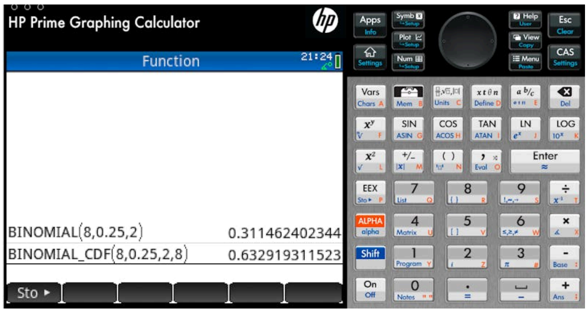
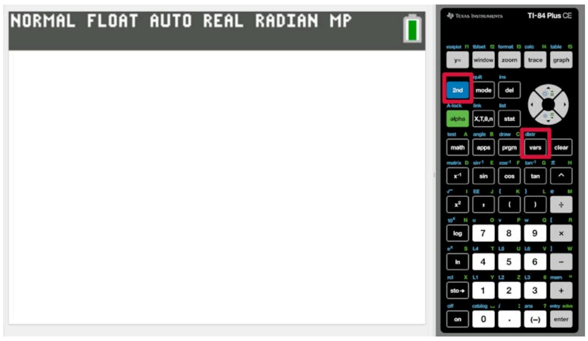
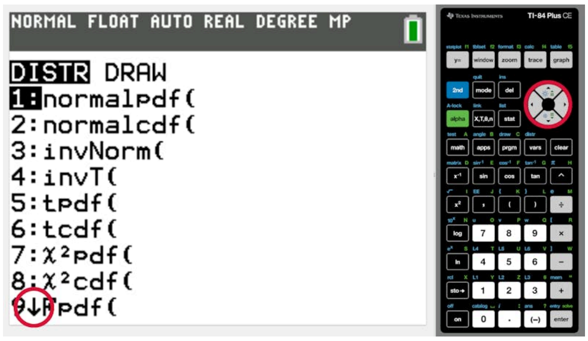
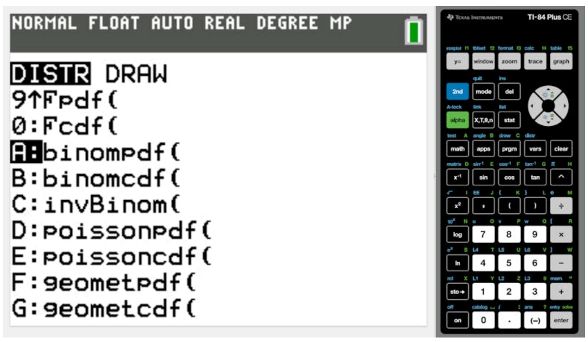
Step	Explanation
Enter the parameters of the distribution (in this order): • number of trials: 8 • probability of success: 0.25 The last value indicates that you are interested in the probability, when $X = 2$.	
To calculate $\mathrm{P}(X \geqslant 2)$, start the same way, but choose the cumulative binomial probability option.	
You can again pull up the help page in case you are not sure about the order of the parameters.	

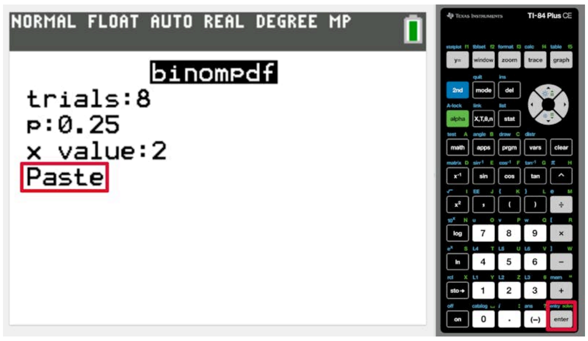
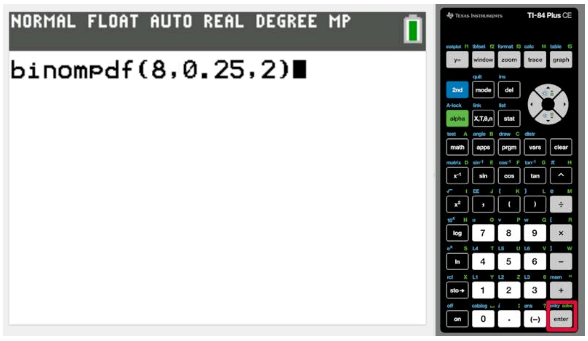
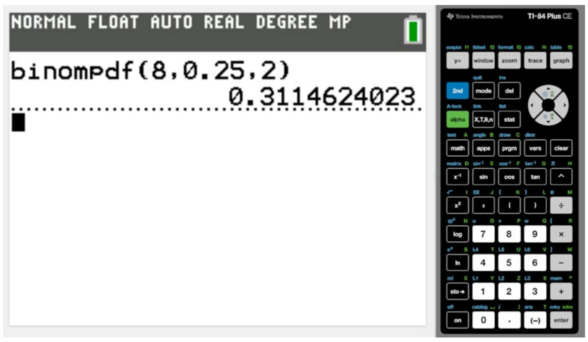
Student view

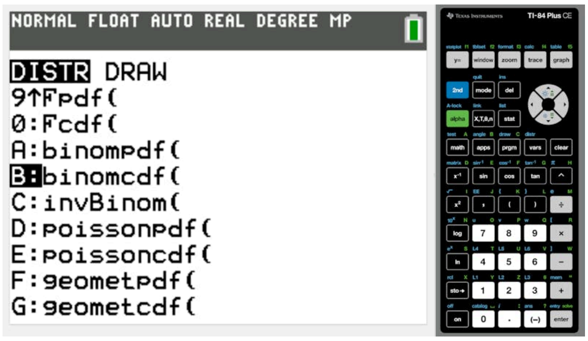
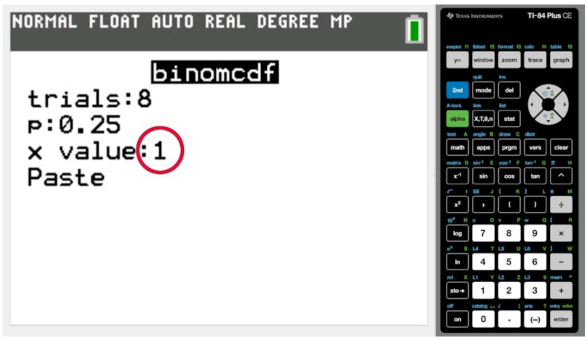
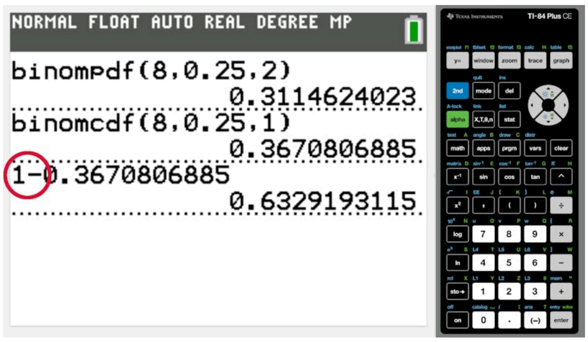
Step	Explanation
	
This option of the calculator will find probabilities of the form $\mathrm{P}(a \leqslant X \leqslant b)$, where a is the lower limit (in this case 2) and b is the upper limit. In this case there is no upper limit, so you need to enter the maximal possible outcome of a binomial experiment, which is the number of trials (so in this case, 8). $a = 2$ and $b = 8$ are the last two parameters.	

Overview (/study/ap aa-hl/sid-134-cid-761926/o

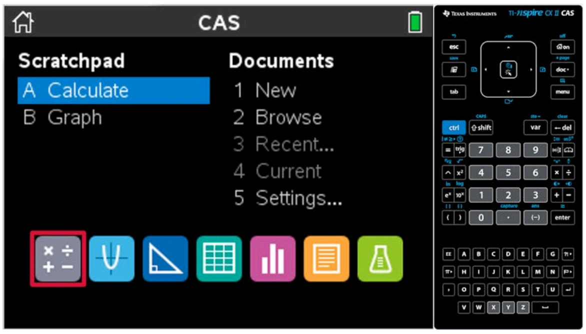
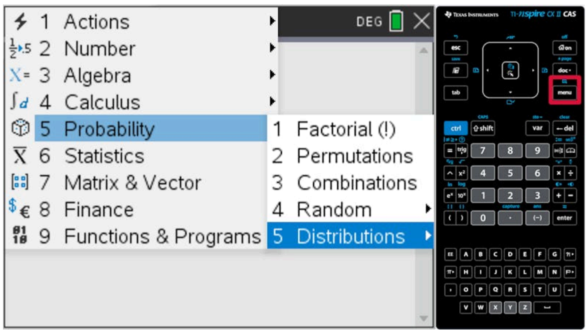
Step	Explanation
These instructions will show you how to calculate $\mathrm{P}(X=2)$ and $\mathrm{P}(X \geqslant 2)$ for a binomially distributed random variable $X \sim \mathrm{B}(8, 0.25)$. To start, enter the distribution menu.	
The options for binomial distribution are not on the first page, so you need to scroll down.	
To calculate $\mathrm{P}(X=2)$ select the binomial probability distribution (binompdf) option.	

Student view

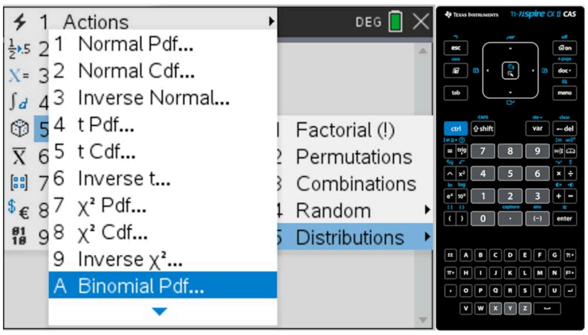
Step	Explanation
Enter the parameters of the distribution: • number of trials: 8 • probability of success: 0.25 The x-value indicates that you are interested in the probability, when $X = 2$. Once you entered the information, scroll down to "paste" and press enter to calculate the probability.	
You need to press enter again to confirm the parameters.	
The calculator tells you the probability.	

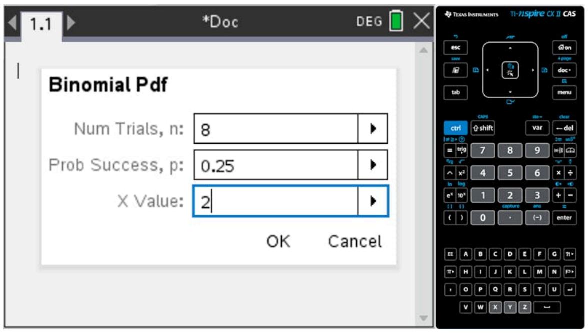
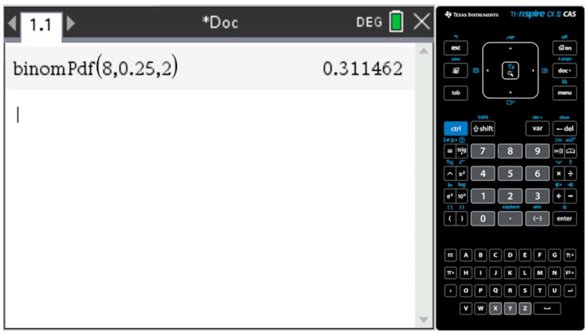
Step	Explanation
To calculate $\mathrm{P}(X \geqslant 2)$, start the same way, but choose the cumulative binomial probability (binomcdf) option.	
This calculator can only calculate cumulative probabilities of the form $\mathrm{P}(x \leqslant c)$. This means, that in this example, you should use the conversion $\mathrm{P}(x \geqslant 2) = 1 - \mathrm{P}(x \leqslant 1)$ In this step, $\mathrm{P}(x \leqslant 1)$ is calculated (the x-value is the upper limit).	
As explained in the previous step, you need to subtract the result of the calculation of $\mathrm{P}(x \leqslant 1)$ from 1 to get the value of $\mathrm{P}(X \geqslant 2)$.	

<table>
<tr><th>Step</th><th>Explanation</th></tr>
<tr><td>These instructions will show you how to calculate $\mathrm{P}(X=2)$ and $\mathrm{P}(X \geqslant 2)$ for a binomially distributed random variable $X \sim \mathrm{B}(8, 0.25)$.

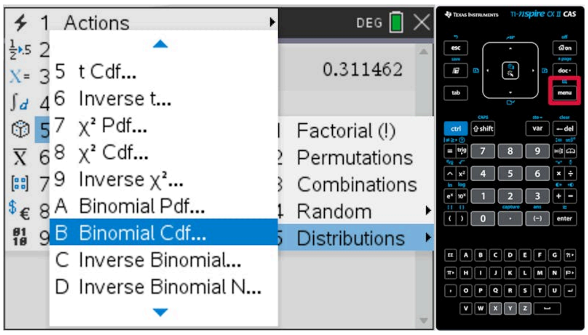
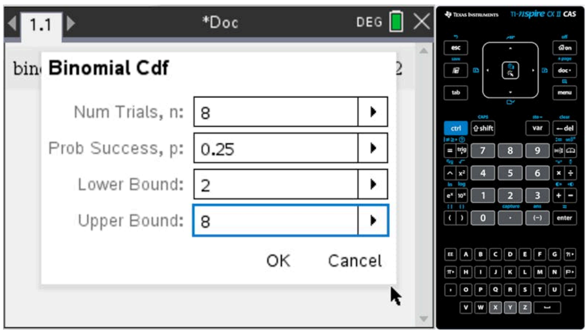
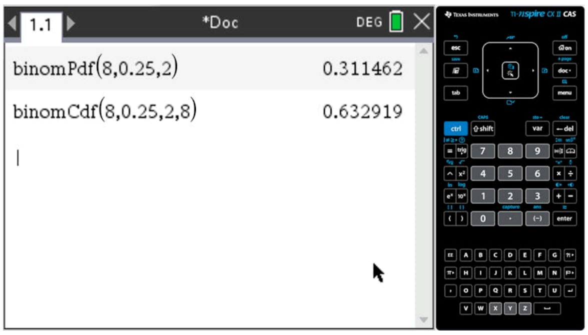
Start by opening a calculator page.</td><td>
</td></tr>
<tr><td>Using the menu, choose probability distributions ...</td><td>
</td></tr>
</table>

Overview
(/study/app/math-aa-hl/sid-134-cid-761926/o

<table>
<tr><th>Step</th><th>Explanation</th></tr>
<tr><td>To calculate $\mathrm{P}(X=2)$ select the binomial probability distribution (Pdf) option.</td><td>
</td></tr>
<tr><td>Enter the parameters of the distribution:
number of trials: 8
probability of success: 0.25
The x-value indicates that you are interested in the probability, when $X=2$.

Once you have entered the information, press OK.</td><td>
</td></tr>
<tr><td>The calculator tells you the probability.</td><td>
</td></tr>
</table>

Student view

Step	Explanation
To calculate $\mathrm{P}(X \geqslant 2)$, start the same way, but choose the cumulative binomial probability (Cdf) option.	
This option of the calculator will find probabilities of the form $\mathrm{P}(a \leqslant X \leqslant b)$, where a is the lower bound (in this case 2) and b is the upper bound. In this case there is no upper bound, so you need to enter the maximal possible outcome of a binomial experiment, which is the number of trials (so in this case, 8).	
	

Probability of 2 incorrect: $\mathrm{P}(\mathrm{X} = 2) \approx 0.311$.

Probability of at least 2 incorrect: $\mathrm{P}(\mathrm{X} \geq 2) \approx 0.633$.

One challenge is finding the probability of a range of values that does not start at 0 or end at n. To find $\mathrm{P}(\mathrm{a} \leqslant \mathrm{X} \leqslant \mathrm{b})$ with the TI-nspire, you can simply use the lower bound and upper bound, but with the TI 84 you have to be a bit more creative. There are two possibilities for finding this:

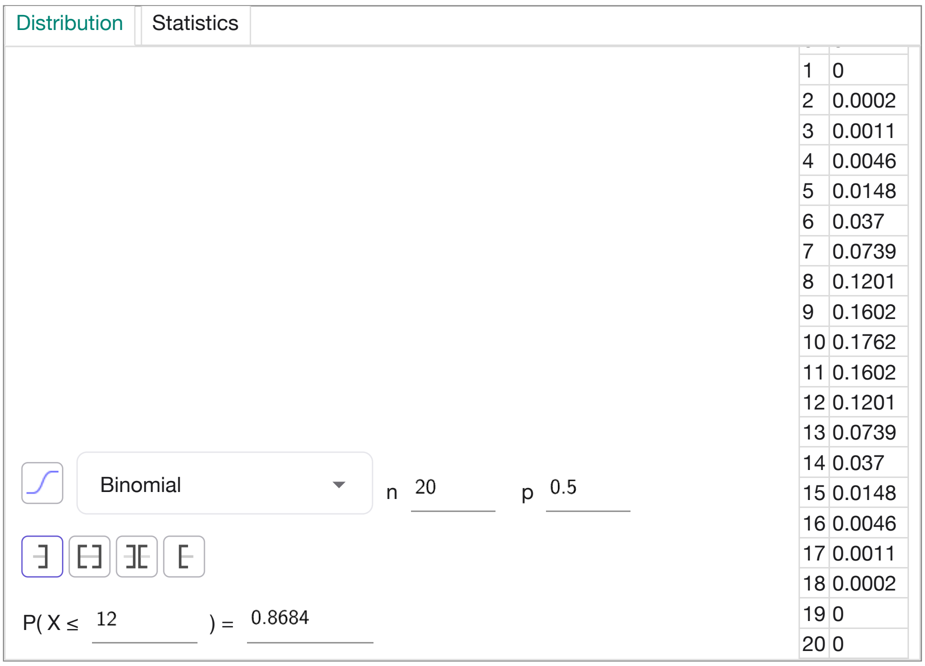
1. Since the binomial CDF on the TI 84 always gives $\mathrm{P}(0 \leqslant \mathrm{X} \leqslant \mathrm{b})$, you can subtract the sum of the terms less than a.
   Hence, $\mathrm{P}(\mathrm{a} \leqslant \mathrm{X} \leqslant \mathrm{b}) = \mathrm{P}(0 \leqslant \mathrm{X} \leqslant \mathrm{b}) - \mathrm{P}(0 \leqslant \mathrm{X} \leqslant \mathrm{a} - 1)$.

Be aware

Be sure to use the value **one less than the lower bound** when subtracting. Otherwise, you will be missing $\mathrm{P}(\mathrm{X} = \mathrm{a})$ in your answer.

2. Another method is to use the binomial PDF with the summation function. This will allow you to set the lower and upper bounds of the sum and assign the value of X to the same variable used as the index of the sum.

Both methods are shown in **Example 2**.



Interactive 2. Binomial Distribution Interactive

Credit: GeoGebra (https://www.geogebra.org/m/mx4qqb7s) Vivax Solutions

More information for interactive 2

This interactive tool is designed to help users understand and calculate binomial probabilities through a dynamic visual representation. The primary focus is on the binomial distribution, illustrated using a histogram where each bar's height represents the probability of achieving a specific number of successes. The image depicts a scenario with 20 trials ($n = 20$) and a probability of success per trial set at $p = 0.5$. Users can manipulate these values to explore different probability distributions and observe how changes affect the overall shape and spread of the distribution.

At the top right of the screen, there are toggles that allow users to switch between different graphical representations, including a line graph, step graph, bar chart, and an overlay of the normal curve. This feature enables a more comprehensive understanding of how binomial distributions behave and how they relate to other probability models. Below the graph, a drop-down menu provides options for different probability distributions, with the current selection set to Binomial. Users can manually enter the number of trials and probability of success using an on-screen keyboard, allowing for customization of the data being analyzed.

To the right of the graph, a probability table presents two columns: one displaying the number of successes k and the other showing the probability of obtaining exactly that number of successes $P(X = k)$. The selected probability values in the table are highlighted in purple, corresponding to the range set by the slidable arrows along the x-axis of the histogram. This interactive element enables users to focus on specific outcomes and better interpret probability values in context.

In the top-left corner, key statistical measures are displayed, providing deeper insights into the binomial distribution. The mean μ, calculated as $n \times p$, represents the expected number of successes, which in this case is 10. The standard deviation σ, which measures the spread or variability of the distribution, is computed using the formula $\sigma = \sqrt{n \times p(1 - p)}$, resulting in a value of 2.2361. Additionally, at the bottom left, the tool calculates cumulative probabilities for a specified range. In the given example, it shows $P(X \leq 12) = 0.8684$, indicating that there is an 86.84% probability of obtaining 12 or fewer successes in 20 trials.

On the left bottom you can calculate the probability of the number of successes falling within a specific range. In the image, it's set to $P(0 \leq X \leq 18) = 1$. This means the probability of getting between 0 and 18 successes (inclusive) is 1 (or 100%).

By visually representing the binomial distribution and allowing users to interact with the data, this tool bridges the gap between theoretical probability concepts and their practical applications. Users can adjust the number of trials and probability of success, instantly observing how these changes influence the shape of the distribution. The option to overlay a normal curve further reinforces the relationship between binomial and normal distributions, especially for large sample sizes. This interactive approach not only enhances statistical intuition but also provides a more engaging way to grasp probability concepts, making it an invaluable resource for students and learners looking to strengthen their understanding of binomial probability in real-world scenarios.

Example 2

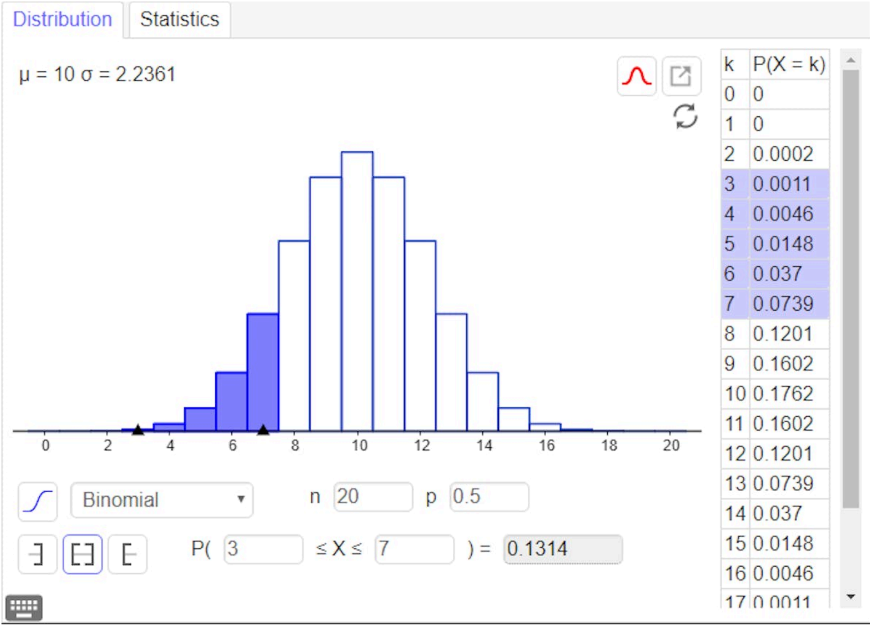
★★☆

If $X \sim B(11, 0.25)$, find $\mathrm{P}(3 \leqslant \mathrm{X} \leqslant 7)$.

Solution

It may be helpful to visualise the situation first. Sketching a histogram can be helpful, even if you do not know how large to make each bar.

Student view



More information

This is a histogram displaying a binomial distribution with parameters n = 20 and p = 0.5. The X-axis represents values from 0 to 20, indicating the number of successes in a binomial experiment. The Y-axis represents the probability P(X = k) for each corresponding value of k. Bars range from 0 to about 0.18 on the probability scale.

In the distribution, the mean (\mu = 10) and standard deviation (\sigma = 2.2361) are given. Among the bars, those representing values 3 to 7 are highlighted in blue. The probability calculated for (P(3 \leq X \leq 7)) is approximately 0.1314, as shown adjacent to the highlighted range. A table beside the graph lists values of k from 0 to 20 along with their associated probabilities, with k = 3 to 7 highlighted.

[Generated by AI]

Some calculators provide the option to find cumulative binomial probabilities of the form $\mathrm{P}(\mathrm{a} \leqslant \mathrm{X} \leqslant \mathrm{b})$, where both lower and upper bound can be specified. On these models (like the Casio fx-CG50, the HP Prime and the TI-nspire CX), you can use this option to find directly that $\mathrm{P}(3 \leqslant \mathrm{X} \leqslant 7) \approx 0.544$. You can find guidance to access this option after Example 1.

On calculators where only the option to find $\mathrm{P}(\mathrm{X} \leqslant \mathrm{b})$ is available, such as the TI-84 Plus CE, use one of the two methods below to find the probability asked in the question.

Method 1

Overview
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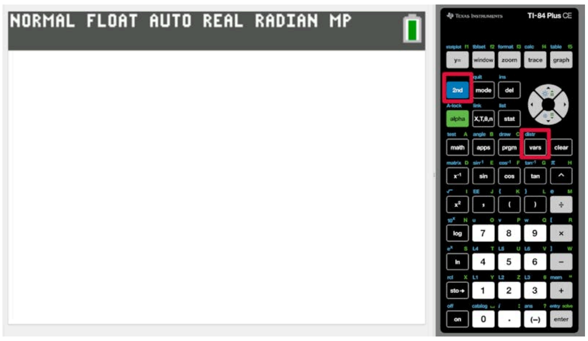
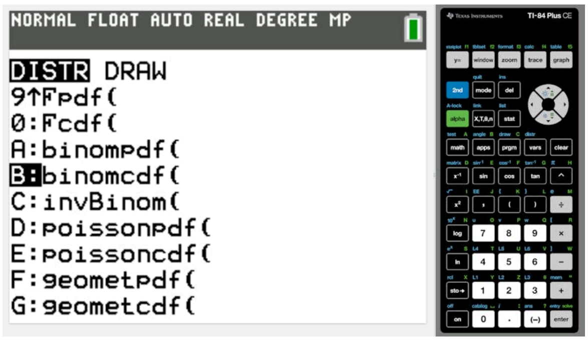
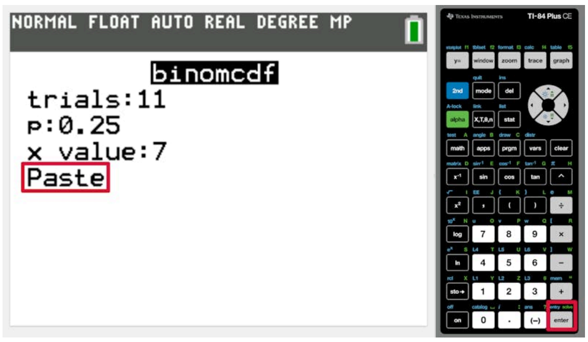
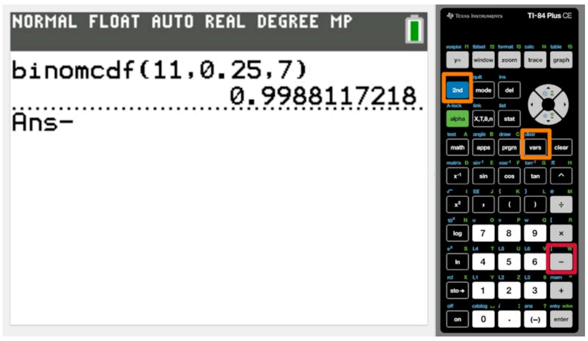
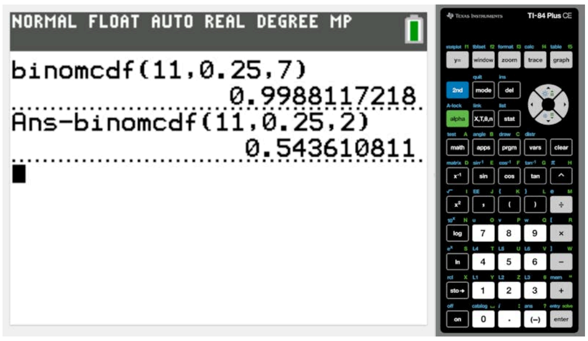
Step	Explanation
These instructions show you how to find $\mathrm{P}(3 \leqslant X \leqslant 7)$ as a difference $\mathrm{P}(X \leqslant 7) - \mathrm{P}(X \leqslant 2)$ for $X \sim \mathrm{B}(11, 0.25)$. This is a useful approach on calculators, where the option to find $\mathrm{P}(a \leqslant X \leqslant b)$ directly is not available. Start with accessing the distribution menu.	
Open the cumulative binomial probability calculation page.	
Enter the parameters to find $\mathrm{P}(X \leqslant 7)$ first.	

Student
view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Step	Explanation
Press the minus sign (you want to subtract something from this value) and bring up the distribution menu again.	
Enter the parameters to find $\mathrm{P}(X \leqslant 2)$ and subtract the result from the previous answer.	

Method 2

Student view

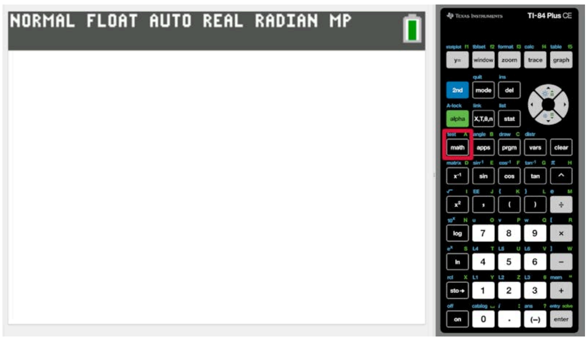
<table>
<tr><th>Step</th><th>Explanation</th></tr>
<tr><td>

These instructions show you how to find $\mathrm{P}(3 \leqslant X \leqslant 7)$ as a sum $\sum_{x=3}^{7} \mathrm{P}(X = x)$ for $X \sim \mathrm{B}(11, 0.25)$.

This is a useful approach on calculators, where the option to find $\mathrm{P}(a \leqslant X \leqslant b)$ directly is not available.

Start with accessing the math menu.

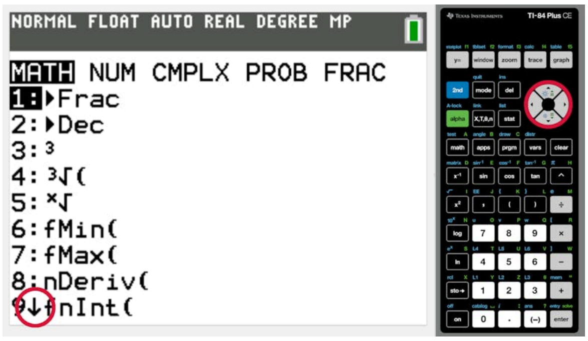
</td><td>



</td></tr>
<tr><td>

The summation option is not on the first screen, so you need to scroll down.

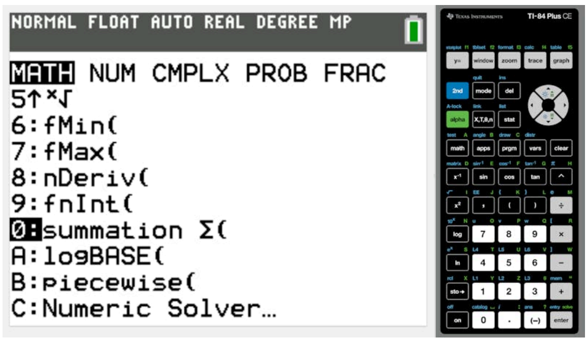
</td><td>



</td></tr>
<tr><td>

Choose the summation option.

</td><td>



</td></tr>
</table>

Overview
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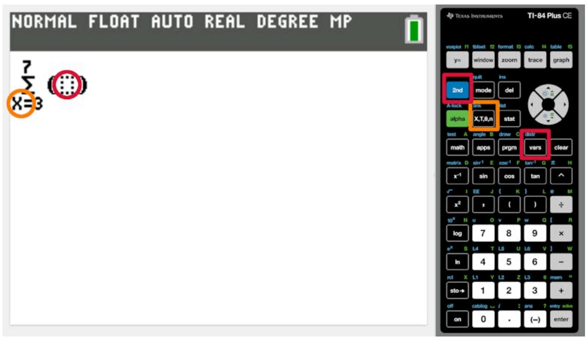
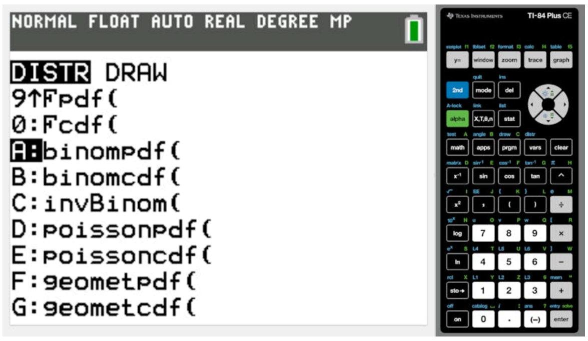
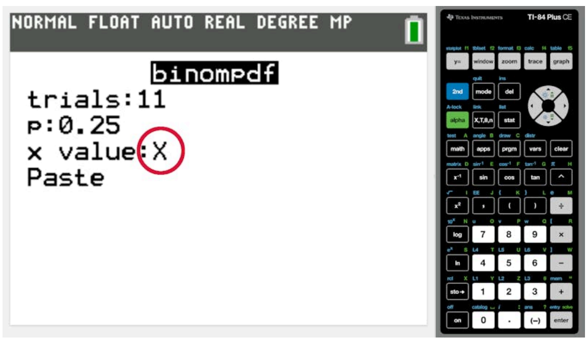
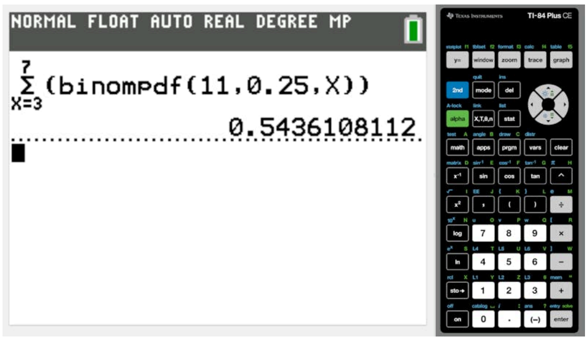
761926/o

Step	Explanation
Use the variable button to enter the counter variable and enter 3 and 7 as the lower and upper limits. Once done, bring up the distribution menu to fill the fourth empty box.	
You need exact probabilities, so open the binomial probability distribution (binompdf) option.	
Enter the parameters of the distribution. The x-value is the one that is changing in the summation from$x = 3$ to $x = 7$, so use the variable button to indicate this.	

Student
view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

Step	Explanation
	

Example 3

★★☆

In a large city, 45% of the population supports the proposal for a new mass transit system . If a random poll of 20 people who live in the city is taken and all 20 people give an answer, find:

1. The expected number of respondents who support the new transit system.
2. The probability (to 4 decimal places) that exactly 5 people support the new transit system.
3. The probability (to 5 decimal places) that at least 2 people support the new transit system.
4. The probability (to 2 decimal places) that at most 6 people support the new transit system.

Let X be the number of respondents in the sample of 20 who support the new transit system . Then $X \sim B(20, 0.45)$. Thus,

1. The expected number is $20 \times 0.45 = 9$.
2. $\mathrm{P}(\mathrm{X}=5) = \binom{20}{5}(0.45)^5(0.55)^{15} \approx 0.0365$ [using the GDC].
3. $\mathrm{P}(\mathrm{X} \geq 2) = 1 - \mathrm{P}(\mathrm{X} < 2) = 1 - (\mathrm{P}(\mathrm{X}=0) + \mathrm{P}(\mathrm{X}=1)) \approx 1 - 0.00011 = 0.99989$ [using the GDC].
4. $\mathrm{P}(\mathrm{X} \leq 6) \approx 0.13$ [using the GDC].

Student view

Theory of Knowledge

Overview
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As you've certainly appreciated by now, mathematics is an incredibly **powerful** area of knowledge. Mathematical equations are used in many ways that benefit humanity but can also be used to build weapons or carry out cost-benefit analysis of collateral damage in war. A very important knowledge question is, 'Does knowledge have an inherent value?'

4 section questions

Question 1

Difficulty: ★★☆

Suppose you roll a six-sided dice 15 times and count the number of 5s rolled. Find the probability of rolling at least three 5s. Round your answer to 3 significant figures.

0.468 ✓

Accepted answers

0.468, .468, 0,468

Explanation

Let X be the event where a 5 is rolled. Then $X \sim B(15, 1/6)$.

Using the formula would be very tedious, given that

$$\mathrm{P(X \geqslant 3) = P(X = 3) + P(X = 4) + P(X = 5) + \ldots + P(X = 15)},$$

or quite tedious with the complement $\mathrm{P}(X \geqslant 3) = 1 - \mathrm{P}(X = 2) - \mathrm{P}(X = 1) - \mathrm{P}(X = 0)$.

Using technology instead, find $\mathrm{P}(X \geqslant 3)$using the complement and the binomial CDF:

$$\mathrm{P(X \geqslant 3) = 1 - P(X \leqslant 2) \approx 0.468}.$$

Question 2

Difficulty: ★★☆

The same baseball player, with a batting average of 0.275, comes up to bat 6 times in one game. Assuming his chances of getting a hit do not change, find the probability that he gets 2, 3 or 4 hits in the game.

1 0.517 ✓

2 0.204

3 2.73

4 0.993

Student view

Overview
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Explanation

You can use the formula or technology to find

$$P(2 \leqslant X \leqslant 4) = P(X=2)+P(X=3)+P(X=4)$$
$$P(2 \leqslant X \leqslant 4) = \binom{6}{2}(0.275)^2(0.725)^4 + \binom{6}{3}(0.275)^3(0.725)^3 + \binom{6}{4}(0.275)^4(0.725)^2$$
$$P(2 \leqslant X \leqslant 4) \approx 0.517$$

Question 3

Difficulty: ★★☆

The probability of a defective part coming off an assembly line is 0.12. If a sample of 24 parts are randomly chosen from this assembly line, what is the probability that none are defective and no more than 4 are defective?

1 $P(\text{none defective}) = 0.0465, P(\text{no more than 4 are defective}) = 0.847$ ✓

2 $P(\text{none defective}) = 0.152, P(\text{no more than 4 are defective}) = 0.847$

3 $P(\text{none defective}) = 0.0465, P(\text{no more than 4 are defective}) = 0.676$

4 $P(\text{none defective}) = 0.152, P(\text{no more than 4 are defective}) = 0.676$

Explanation

Let X be the binomially distributed random variable of defective parts from an assembly line with $n = 24$ and $p = 0.12$, i.e. $X \sim B(24, 0.12)$.

Then for no parts defective, we must evaluate

$$P(X=0) = 0.0465. \quad \text{(using GDC)}$$

And for no more than 4 defective parts, we evaluate

$$\begin{aligned} P(X \leqslant 4) &= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4) \\ &= 0.847 \quad \text{(using GDC)} \end{aligned}$$

Question 4

Difficulty: ★★☆

A baseball player has a batting average of 0.275. This means when he comes up to bat, his probability of getting a hit is 0.275. Identify which of the following statements is true.

1 If he bats 120 times, you would expect him to get 33 hits. ✓

2 If he bats 100 times, he is certain to get a hit at least 1 time.

Student view

Overview
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3 He will get exactly 22 hits if he bats 80 times.

4 All of these choices are true.

Explanation

Since $\mathrm{E(X)} = \mathrm{np}$ for a binomial distribution, if the player bats $n = 120$ times and has $p = 0.275$ probability of hitting the ball, then you would expect $\mathrm{E(X)} = 120 \times 0.275 = 33$ hits.

The other two possibilities are both likely, but are stated as certain, something that probability cannot guarantee.

4. Probability and statistics / 4.8 The binomial distribution

Checklist

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/checklist-id-25664/print/) Assign

What you should know

By the end of this subtopic you should be able to:

- identify a binomial experiment as an experiment with these characteristics:
  - there are a finite number of trials
  - the trials are binary events
  - the trials are identical and independent
- recognise the notation $X \sim B(n, p)$ as a discrete random variable X being distributed binomially with n trials and a probability of success equal to p
- find the expected mean number of outcomes, the variance and the standard deviation of a binomial distribution
- use technology to find the probability of a certain number of outcomes or a range of outcomes in a binomial distribution.

4. Probability and statistics / 4.8 The binomial distribution

Investigation

Section Student... (0/0) Feedback Print (/study/app/math-aa-hl/sid-134-cid-761926/book/investigation-id-25665/print/) Assign

Do you have any brothers or sisters? Are they the same sex or different from you? Have you ever seen a family with many children that are all the same sex? Assuming that every baby theoretically has a 50% chance of receiving an X or a Y chromosome from their father, the global human population should be half male and half female, shouldn't it?

Student
view

Overview (/study/app/math-aa-hl/sid-134-cid-761926/o

According to World Bank data for the year 2017, 49.556% of the global population is female. This seems close to what we would expect. However, the number reflects a decrease from 1960, when 49.985% of the population was female. What do you think might account for the change? Consider the data from the World Bank (https://data.worldbank.org/indicator/sp.pop.totl.fe.zs), examining in particular how the proportion varies from country to country. In some countries it has increased, while in others it has decreased. Pick a country and dig a little deeper to see if there might be economic, social or biological issues that affect the proportion.

Finally, find the population of the country (or countries) you chose and use the percentage to find the approximate number of women and girls living in the country. Then use the binomial probability formula or technology to find the probability of having that many women and girls out of the total population if the probability of being female is 0.5. What do you think about your results?

Student view