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The big picture

All sciences use mathematics, essentially, to study relationships. Physicists, chemists, engineers, biologists and, increasingly, economists, psychologists, and other social scientists, all seek to recognise relationships amongst the various elements of their chosen fields and to arrive at a clearer understanding of why these elements behave the way they do. The concept of a function is a central idea in the mathematical study of relationships because they describe how one quantity depends on another.

It is often useful to understand how quantities are related within the context of a particular problem – in chemistry, public policy, or mathematics itself. Functions are very useful tools because they give us a precise language with which to describe these relationships. Relationships appear as connections between properties, objects or elements; including human association with the world in which we live. Changes in relationships have impacts – some of which occur on a small scale while others may affect large systems such as human societies and the planetary ecosystem.

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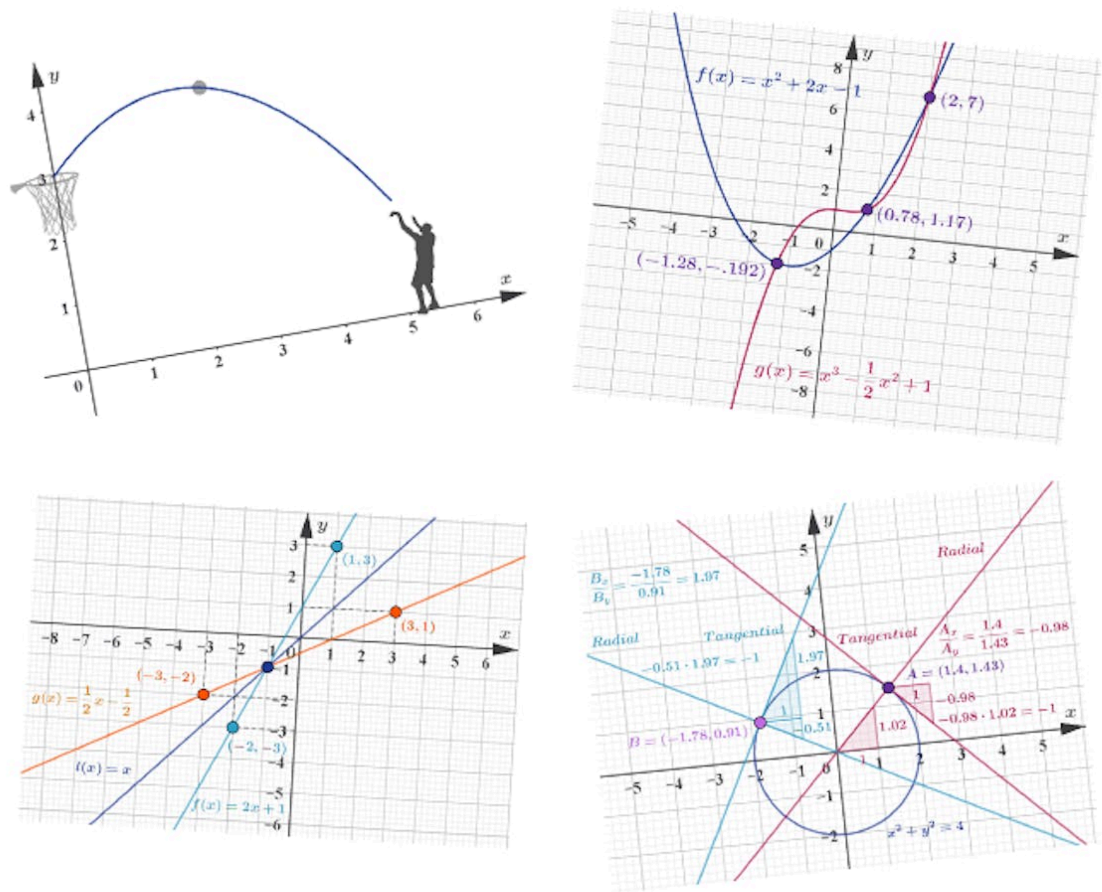

More information

The image displays four distinct mathematical graphs, likely illustrating different functions or equations.

1. **Top Left**: A graph depicting a parabolic curve representing the trajectory of a basketball shot. The graph includes a basketball hoop and player, with the x-axis ranging from 0 to 6, and the y-axis ranging from 0 to 5.

2. **Top Right:** Two intersecting parabolic curves are plotted on a graph with both x- and y-axes, which show values ranging from approximately -5 to 6. Points along the curves are labeled, such as (2,7) and the curve equations are given as f(x) = x^2 + 2x and g(x) = 1/(2x^2 + 1).

3. **Bottom Left**: A graph showing intersections of three linear functions with distinct slopes. The x-axis has a range of -8 to 6, while the y-axis also ranges from -8 to 6. Specific points are labeled, such as (-3,2), and equations are provided like y = x and y = 2x + 1.

4. **Bottom Right:** A complex graph featuring radial and tangential lines with labels like B_0 = -1.78 and B_1 = 0.91. The plot is intricate with intersections and annotations showing specific coordinates on a grid.

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In this subtopic, you will learn about:

- the concept of a function in mathematics
- how functions can be used to model real-life phenomena
- the concept of an inverse function.

Concept

This subtopic will demonstrate how functions give us a language with which to create mathematical models that describe real-life situations. Models involve creating functions to represent relationships between quantities and can lead to a better understanding of how systems work. Mathematical modelling is a logical process by which you start with real-life problems and arrive at quantitative solutions using the tools of functions.

While you are learning about functions through various representational forms such as mappings, equations and graphs, reflect on whether all mathematical relationships represent functions. Think about different ways to express one quantity as a function of another quantity. How can you use functions to model real-life phenomena and make predictions?

2. Functions / 2.2 Functions

The concept of a function

Relations

Relations as sets of ordered pairs

When architects make decisions about how buildings are designed, they must consider factors such as how much sunlight the building will receive at various times throughout the day. The relationship between the size and location of the windows in a building, for example, will be a significant factor in the design of the electric lighting. Likewise, as the sun rises and warms a location, the air temperature typically increases, which has an impact on the amount of heating or cooling needed in the building.

Student
view

The following table is a record of the sunset time in Anchorage, Alaska of the first ten days during April, 2020.

Table 1. Sunset Times in Anchorage, Alaska for the First Ten Days of April 2020.

Day	1	2	3	4	5	6	7	8	9
Sunset time (p.m.)	8:47	8:50	8:52	8:55	8:57	9:00	9:02	9:05	9:08

This table specifies a set of ordered pairs, the first component in each pair being the day and the second component the sunset time.

✓ Important

Any set of ordered pairs is called a relation . A relation is a rule that expresses a relationship between two elements. The set of all first elements of the ordered pairs is the domain of the relation; the set of all second elements is the range of the relation.

Thus, the data obtained in the table above defines a relationship between each day and the sunset time, which you can write using a set of ordered pairs:

$$\{(1, 8{:}47), (2, 8{:}50), (3, 8{:}52), (4, 8{:}55), (5, 8{:}57),\\ (6, 9{:}00), (7, 9{:}02), (8, 9{:}05), (9, 9{:}08), (10, 9{:}10)\}$$

The domain of the relationship is the set of the ten days during April and the range is the set of the corresponding sunset times, as shown:

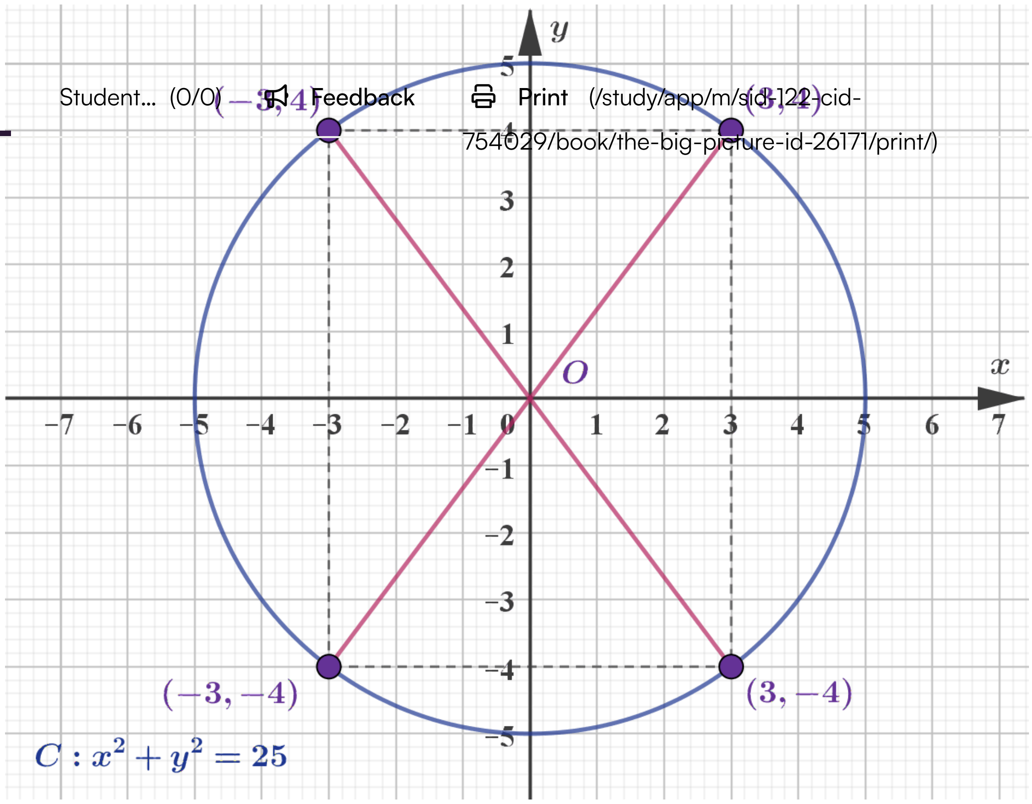
Domain = $\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$

Range = $\{8{:}47,\ 8{:}55,\ 8{:}52,\ 8{:}55,\ 8{:}57,\ 9{:}10,\ 9{:}02,\ 9{:}05,\ 9{:}08,\ 9{:}10\}$

Relations as equations and graphs

A relationship is a rule that links two sets of numbers, and equations are useful tools to express relationships. Equations in coordinate geometry are fundamental representational forms that describe analytically the shape of curves. For example, the equation

$x^2 + y^2 = 25$, links $x = 3$ to $y = 4$ and $x = -3$ to $y = 4$, but it does not link $x = 3$ to $y = 0$. The diagram shows that the position of all points on the circle with centre at O (0, 0) and radius 5 are determined by the equation $x^2 + y^2 = 25$.



More information

The image is a graph depicting a circle centered at the origin point O (0, 0) on a coordinate grid. The circle has a radius of 5 and is represented by the equation (x^2 + y^2 = 25). Several points are marked on the circle's circumference: at coordinate (-3, 4), point (3, 4), and (0, -5). These points demonstrate solutions to the circle's equation. The grid lines represent integer values on the X and Y axes, covering a range from -7 to 7 on both axes. Diagonal lines within the circle align with these points, indicating intersections of the radius with the circle at specific coordinates.

[Generated by AI]

Functions

The concept of function

A function is a special kind of mathematical relationship. The difference between relationships and functions is explained in the video below. Functions (and in many cases, relationships) have two general characteristics:

1. A function is a rule.
2. A function deals with numbers.

For example, consider the area, A, of a circle that depends on the radius, r. The rule that connects A and r is given by the formula $A = \pi r^2$. With each positive number r there is associated one value of A, and you say that the area A is a function of the radius r.

In the following video, you can explore these concepts related to functions:

- Functions are mappings from one number to another number.
- Functions relate a set of input numbers, the domain, to a set of output numbers, the range.
- The image of a relationship is the result of a mapping from one number to another, which thus forms an ordered pair.
- Not all relationships are functions; see the vertical line test.

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Video 1. Concepts Related to Functions.

More information for video 1

1

00:00:00,834 --> 00:00:03,270

narrator: In this video,

we're going to look at functions,

2

00:00:03,337 --> 00:00:04,938

our first look at functions,

3

00:00:05,305 --> 00:00:07,407

and we are going

to see what functions are.

4

00:00:07,474 --> 00:00:10,410

The way I like to think about

function is like a grinder,

5

00:00:10,611 --> 00:00:13,380

and it's a good analogy

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because a grinder has an input

6

00:00:13,547 --> 00:00:15,249

and an output just like functions do,

7

00:00:15,315 --> 00:00:17,284

but in between the input and output,

8

00:00:17,351 --> 00:00:19,453

you're doing something to it,

you're grinding it.

9

00:00:20,020 --> 00:00:23,824

So what goes into a function

grinder is a number

10

00:00:23,891 --> 00:00:27,361

and outcomes a number

let's denote them by x's and y's.

11

00:00:27,628 --> 00:00:33,467

So x_1 to y_1, x_2 to y_2,

and another one x_3 to y_3.

12

00:00:33,867 --> 00:00:38,705

Now this function, this grinder

is a mapping from x to y,

13

00:00:38,772 --> 00:00:42,676

and we can indicate

it as $f : x \rightarrow y$ or $y = f(x)$.

14

00:00:42,976 --> 00:00:45,913

So $y_1 = f(x_1)$, $y_2 = f(x_2)$,

15

00:00:46,113 --> 00:00:47,981

and $y_3 = f(x_3)$.

16

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view

00:00:48,849 --> 00:00:50,651

Now the collection of numbers

17

00:00:50,784 --> 00:00:53,620

that go into the grinder into the function

18

00:00:53,687 --> 00:00:58,091

we call the domain, and then this leads

to a collection of numbers

19

00:00:58,158 --> 00:01:00,694

that comes out of the

function or the grinder,

20

00:01:00,761 --> 00:01:02,296

and that's called the range.

21

00:01:02,663 --> 00:01:06,800

So really we have an x value

that gets mapped to a y value

22

00:01:07,401 --> 00:01:10,237

and those two values

correspond to each other.

23

00:01:13,440 --> 00:01:16,810

And of course then we have an entire

collection of numbers to domain

24

00:01:18,178 --> 00:01:23,016

that are mapped

to an entire sectional numbers

25

00:01:23,083 --> 00:01:26,653

called the range under some function f .

26

00:01:28,889 --> 00:01:34,094

And we can write this as $y = f(x)$,

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where y is called the image

27

00:01:34,294 --> 00:01:36,997

of x under the function f .

28

00:01:37,531 --> 00:01:40,701

Often we graph these

because we can think of those

29

00:01:40,767 --> 00:01:44,872

corresponding x and y values

as a set of ordered pairs

30

00:01:44,938 --> 00:01:48,242

and can plot them

on a xy co-ordinate axis.

31

00:01:48,509 --> 00:01:52,145

Now there are four types

of relation between numbers

32

00:01:52,312 --> 00:01:53,847

and we're gonna distinguish between them.

33

00:01:53,914 --> 00:01:56,216

The first one is a one to one relation.

34

00:01:56,283 --> 00:02:00,754

So in terms of graphs,

you have a graph where 1x

35

00:02:00,988 --> 00:02:02,656

leads to 1y.

36

00:02:02,789 --> 00:02:06,126

So it is 1x leads to a 1y relation,

37

00:02:06,727 --> 00:02:09,696

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but you can also have one to many

38

00:02:10,364 --> 00:02:12,799

and a graph that you can explore this

39

00:02:13,100 --> 00:02:14,434

with looks something like this.

40

00:02:14,501 --> 00:02:18,205

So you've got $1x$ that corresponds to $2y$'s.

41

00:02:19,206 --> 00:02:23,477

So $1x$ to multiple,

at least $2y$'s is a one to many.

42

00:02:23,610 --> 00:02:26,480

Of course, you can then have a many

to one relation as well.

43

00:02:26,780 --> 00:02:30,250

You can think of that

as a very famous graph

44

00:02:30,317 --> 00:02:32,719

that we will study in detail like this.

45

00:02:32,853 --> 00:02:36,523

So you can get that this y

by this x by $x2$ by $x3$,

46

00:02:36,590 --> 00:02:38,225

or even by this fourth x.

47

00:02:38,392 --> 00:02:41,728

So many to one is multiple x's lead to $1y$.

48

00:02:42,029 --> 00:02:44,431

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And lastly, of course,

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you can have a many to many,

49

00:02:44,498 --> 00:02:48,635

and a graph that goes with this

can be thought of as something like this.

50

00:02:48,735 --> 00:02:51,905

So this 1 y you can get to via 2x's,

51

00:02:52,372 --> 00:02:54,842

but those x's lead to multiple y's.

52

00:02:55,042 --> 00:02:57,911

So it's multiple x's to multiple y's.

53

00:02:58,779 --> 00:03:01,615

Now, only two of those relations

are functions,

54

00:03:01,682 --> 00:03:04,184

and that is the one-to-one

and the many-to-one,

55

00:03:05,052 --> 00:03:09,223

and those are going to be the topic

of much studying in this course.

56

00:03:09,790 --> 00:03:11,825

The way to distinguish between functions

57

00:03:11,892 --> 00:03:13,727

and non-function

is the vertical line test.

58

00:03:13,861 --> 00:03:15,796

So if we draw a vertical line anywhere,

59

00:03:15,896 --> 00:03:18,765

Student view

if it cuts it only once,

then it's a function.

60

00:03:19,666 --> 00:03:22,569

This is an important and easy to use test.

61

00:03:22,736 --> 00:03:25,472

So let's reiterate it. Vertical line test.

62

00:03:25,572 --> 00:03:29,209

If a vertical line

anywhere cuts the graph,

63

00:03:29,676 --> 00:03:33,180

not more than once,

then you have a function.

64

00:03:33,981 --> 00:03:37,284

So let's look in an example

$y^2 = x$,

65

00:03:37,351 --> 00:03:40,153

which means that $y = \pm\sqrt{x}$.

66

00:03:40,621 --> 00:03:44,892

Now the graph of this is that branch

and this branch,

67

00:03:45,058 --> 00:03:48,028

and you can immediately see that

if a vertical line is drawn there,

68

00:03:48,095 --> 00:03:49,263

it cuts it twice.

69

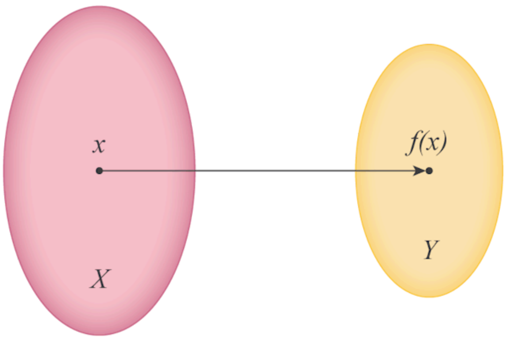
00:03:49,329 --> 00:03:50,731

It's not a function.

Important

A function f, or a mapping f, is a relationship between two sets of elements, where each element x in the first set X, the domain, corresponds to exactly one element y in the second set Y, the range.

You can visualise a function as a mapping diagram between two sets, as shown in below.



You say that *f* maps *x* to *f(x)* or *f(x)* is the image of *x* under *f*

More information

The image is a mapping diagram illustrating a function between two sets, X and Y. Set X is depicted as a pink oval on the left, labeled 'X' with an element 'x' inside. The set Y is represented as a yellow oval on the right, labeled 'Y' with an element 'f(x)' inside. There is a single arrow pointing from 'x' in set X to 'f(x)' in set Y, showing the mapping of the element under the function f. The text below the diagram reads "You say that f maps x to f(x) or f(x) is the image of x under f." This indicates the mapping relationship in the function.

[Generated by AI]

Not all mapping diagrams represent functions. The mapping diagram shown below does **not** represent a function, as one element of set X is associated to two elements of set Y .

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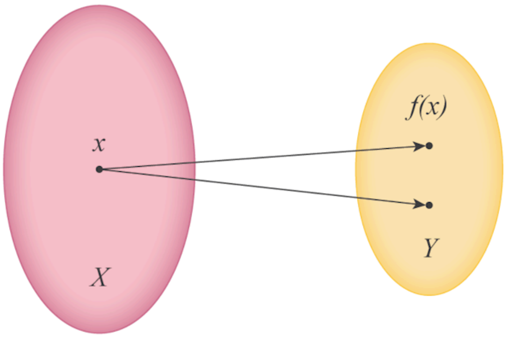

More information

The image is a mapping diagram showing sets X and Y. Set X, on the left, is represented by a pink oval, containing a single element labeled 'x'. Arrows extend from this element pointing to two elements in set Y, on the right, which is represented by a yellow oval. These elements in set Y are labeled 'f(x)'. This map illustrates an instance where one element in set X is associated with two different elements in set Y, indicating that the mapping does not represent a function according to mathematical definitions.

[Generated by AI]

Important

A mapping diagram represents a function if and only if each element of the domain X has a unique image in range Y.

Function notation

You can express the rule of function f in two ways:

You write	You say
$y = f(x)$	y is the image of x under function f.
$f : X \mapsto Y$	f assigns each element of set X to a unique element of set Y.

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International Mindedness

There has been a debate within the mathematical community regarding the time in history when the concept of function was introduced. A commonly accepted view is that the real notion of a function, with the use of coordinates, was initially expressed clearly and publicly by René Descartes. Descartes, Newton, Leibniz, Euler and other mathematicians of the 16 th and 17 th centuries contributed to the development of calculus. German mathematician, Leibniz, was the first to use the word 'function' (from the Latin verb fungor, which means operate), in his book *Methodus tangentium inversa, seu de functionibus* . Leibniz introduced words such as 'variables', 'constants' and 'parameters'. Years later, the Swiss mathematician Euler was the first to express a function using the notation $y = f(x)$, which is commonly used today.

Functions as mapping diagrams

Functions as graphs

Recall that a function f can be thought of as a machine which takes an input, x, and returns an output, y, that depends on the rule of f and the value of the input x. The graph of a function is the set of all points (x, y) on the plane that satisfy the equation $f(x) = y$.

Activity

The applet below shows how a graph gives a visual representation of the relationship between inputs and outputs for the rule given by function f .

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Interactive 1. Functions as Graphs.

More information for interactive 1

This interactive visually demonstrates the relationship between inputs and outputs for a given function, represented graphically. By interacting with the graph, users can observe how different functions behave and change as the input value varies.
Slider “Slide to Change Example” is visible in the interface (above the graph), users can use a slider to explore four different functions and their corresponding graphs. The four different functions are:

1. $f(x) = x$
2. $f(x) = -x^2 + 4$
3. $f(x) = x^3 - 3x + 1$
4. $f(x) = 4\sin x$

By moving the red point on the x-axis, users can observe how the output value y changes based on the input value x. The input value of x can be varied from -1 to 5 by moving the red dot along the x-axis. The interactive displays the function's equation and allows users to verify the output value for a chosen input value using the function's equation.
For example, for the function $f(x) = x$, setting $x = 2$ gives $y = 2(f(2) = 2)$, For the function $f(x) = -x^2 + 4$, when the output is $f(1) = 3$, plotted as point (1, 3) on the parabola.
Users can slide through different x-values to see how the output changes, visually reinforcing the relationship between inputs and outputs in quadratic functions. The real-time updates help connect the algebraic equation with its graphical representation.

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- Use the slider to visualise the graphs and equations of various functions.
- Move the input (red point on x -axis) with your mouse to see how the output value changes.
- Display a function of your choice. Verify the output value for your chosen input value, by using the equation of the function.

Important

The graph of a relationship is the set of points (x, y) in the Cartesian plane, where x is linked to one or more y values according to the rule describing the relationship.

The graph of a function f is the set of points (x, y) in the plane, where every x is linked to a single y value according to the rule $y = f(x)$.

Remember that not all relationships between numbers are considered to be functions. For a relationship to be a function, it must pass the vertical line test. This test states that if a vertical line drawn anywhere on the graph of the relationship cuts the graph no more than once, the relationship is a function.

On the following applet, you can use the vertical line test to determine whether a graph of a relationship represents a function. Tick the relationship boxes one at a time to display the graph of each relationship. You can apply the vertical line test by dragging the corresponding slider to move the vertical line. Observe the number of intersection points between the graph of the relationship and the vertical line to determine whether the relationship is a function.

Check your assumptions by clicking the ‘Answer box’. Before you move on to the next relationship, put a tick in the ‘Answer box’.

Student
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Interactive 2. Using the Vertical Line Test to Identify Functions.

More information for interactive 2

This interactive helps users determine whether a given relationship is a function by applying the vertical line test.

A graphing area displaying different relationships in the form of plotted graphs. Users can select and display the graphs of four different relations one at a time by clicking one of the checkboxes given above the graph. A vertical line appears by clicking the first check box and by dragging a vertical line across the graph using a slider, they observe how many times it intersects the curve. If the vertical line touches only one point on the graph at any given x-value, the relationship is a function. If the vertical line intersects multiple points for a single x-value, the relationship is not a function.

To verify their conclusions, users can check the "Answer" box for each relation. A message will appear stating:

- "This relation is a function" (if the vertical line intersects the graph at most once).
- "This relation is not a function" (if the vertical line intersects the graph more than once).

This tool provides a practical way to understand and apply the vertical line test, enhancing comprehension of what defines a function in mathematical terms.

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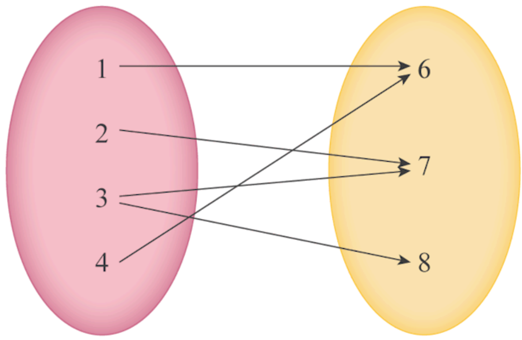
Example 1

★☆☆

Determine whether the following mapping diagram is a function. If it is a function, write the domain and range. Explain your reasoning.



More information

The image is a mapping diagram showing two sets of numbers contained within two separate ovals. The left oval, depicted in pink, contains the numbers 1, 2, 3, and 4. The right oval, shown in yellow, contains the numbers 6, 7, and 8. Arrows are used to connect these numbers: number 1 from the left oval points to number 6 in the right oval; number 2 points to number 7; number 3 points to number 7; and number 4 points to number 8. The mapping suggests the relationships between elements of the domain (left oval) and the range (right oval). This diagram can help determine whether a mapping represents a function by observing if there is a one-to-one correspondence between the domain and range.

[Generated by AI]

This relationship is not function because number 3 (element of the domain) corresponds to both 7 and 8 (elements of the range). To be a function, each element, x, in the domain must correspond to exactly one element, y, in the range.

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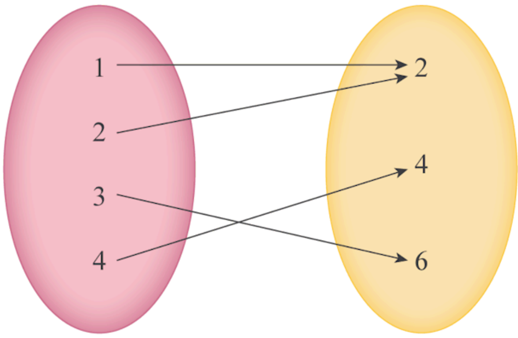
Example 2

★☆☆

Determine whether the following mapping diagram is function. If it is function, write the domain and range. Explain your reasoning.



More information

The mapping diagram consists of two ovals. The left oval contains the numbers 1, 2, 3, and 4, while the right oval contains the numbers 2, 4, and 6. Arrows extend from the numbers in the left oval to the numbers in the right oval as follows: 1 points to 2, 2 points to 4, 3 points to 4, and 4 points to 6. Each number in the left oval has a unique arrow pointing to a number in the right oval except for number 3, which points to the same number, 4, as number 2. This setup asks if this mapping qualifies as a function, with each input having a single output, and suggests writing the domain (set of inputs: 1, 2, 3, 4) and range (set of outputs: 2, 4, 6).

[Generated by AI]

This relationship is a function because each number of the domain corresponds to exactly one number of the range. Although two elements of the domain have the same image it is still a function.

Domain = $\{1,\ 2,\ 3,\ 4\}$

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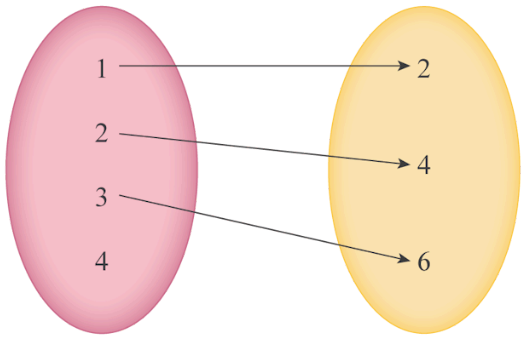
Range = $\{2,\ 4,\ 6\}$

Example 3

★☆☆

Determine whether the following mapping diagram is a function. If it is a function, write the domain and range. Explain your reasoning.



More information

The image shows a mapping diagram with two sets represented by ovals. The first set (on the left) contains the numbers 1, 2, 3, and 4 arranged vertically. The second set (on the right) contains the numbers 2, 4, and 6 arranged vertically.

There are arrows indicating a mapping from each element in the first set to one element in the second set: - The number 1 maps to 2. - The number 2 maps to 4. - The number 3 maps to 6.

No mappings for the number 4 are indicated. This mapping suggests functionality since each element from the left set is mapped to exactly one element in the right set, except for number 4 which has no mapping displayed. To determine if this constitutes a function, check whether multiple left numbers map onto a single right number, which doesn't occur in this case.

The domain of this function is the set ({1, 2, 3}) and the range is the set ({2, 4, 6}).

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This relationship is not a function. To be a function, every element of the domain must have an image, but 4 does not.

Example 4

★☆☆

Determine whether the following relationship is function. If it is a function, write the domain and range. Explain your reasoning.

$$\{(3,\ 4)\,,(3,\ 5)\,,(2,\ 5)\}$$

This relationship is a not function, since number 3 (an element of the domain) corresponds to both 4 and 5 (elements of the range).

Example 5

★★☆

Determine whether the relationship $x^2 + y^2 = 25$ is a function. If it is a function, write the domain and range. Explain your reasoning.

Solving the equation $x^2 + y^2 = 25$ for y gives:

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}.$$

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One input gives two associated values of y and thus the relationship is not function. For example, the relationship maps $x = 3$ to both $y = 4$ and $y = -4$.

Finding the domain and range of functions graphically

This section takes a closer look at the domain of a function. Can the domain can be any value, i.e. any real number, $x \in \mathbb{R}$, or is it restricted to certain values only?

There are two reasons why the domain may not contain all the real numbers:

1. The function may be explicitly defined for a particular context only. For example, if you are modelling the growth of the number of bacteria in a given volume as a function of time, then you would not consider negative time.
2. The function may be of a form that implicitly excludes certain numbers as inputs. Examples of domain restriction could be the accepted mathematical rules: 'do not divide by zero' and 'do not take square roots of negative numbers'.

The most straightforward way of thinking about the range of a function is to consider the graph of the function. The range is the set of all possible y values that result from the function that maps all the x values in the domain.

Activity

On the applet below move any of the purple points to change the shape of the graph of the function, and click the buttons to display the domain and range. The domain and range of the function are expressed in interval notation.

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Interactive 3. Graphical Approach to Finding the Domain and Range of Function.

More information for interactive 3

This interactive tool lets users explore domain and range concepts by manipulating different types of function graphs. Users can switch between linear (two-point) and non-linear (three-point) graphs using the "Change Example" button. By dragging the adjustable points (purple), they reshape the graph and instantly see how modifications affect its behavior. Three main buttons present at the top left of the interface are:

1. "Change Example" — Loads a new function to analyze.
2. "Click for Domain" — Highlights the set of x-values for the function.
3. "Click for Range" — Highlights the set of y-values for the function.

By clicking the " Click for Domain" and "Click for Range" buttons, users can display the domain and range in interval notation. For example, when users click on the “Click for Domain” button, the domain is displayed in interval notation as $[-3, 2]$ means the function exists from $x = -3$ to $x = 2$ When users click on the “Click for Range” button, the range is also displayed in interval notation as $[-3.41, 1]$ means the function’s output varies from $y = -3.41$ to $y = 1$.
The tool visually demonstrates the connection between a graph's shape and its domain (all possible x-values) and range (all possible y-values), making abstract concepts tangible through direct manipulation.

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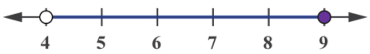
Important

The domain of a function is the complete set of possible values of the independent variable x.

The range of a function is the complete set of all possible values of the dependent variable y, that result from the function mapping all the x values in the domain.

Domain and range notation

The domain of a function and the range of function are often expressed using the notations shown in the table below.

Number line	Inequality notation	Set notation	Interval notation
	$4 < x \leq 9$	$\{x \in \mathbb{R} \mid 4 < x \leq 9\}$	$(4,\ 9]$ or $]4,\ 9]$

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Number line	Inequality notation	Set notation	Interval notation
 More information The image is a diagram featuring a horizontal line marked with numbers from 4 to 9. The line has arrowheads on both ends, indicating it's a number line that extends indefinitely. Circle markers indicate positions on the line at numbers 4 and 9. The marker at 4 is filled and purple, and the marker at 9 is empty. The line is divided into equal segments corresponding to the numbers, with vertical ticks and the numbers 5, 6, 7, and 8 also displayed beneath the respective divisions. [Generated by AI]	$4 \leq x < 9$	$\{x \in \mathbb{R} \| 4 \leq x < 9\}$	$[4,\ 9)$ or $[4,\ 9[$
	$x < 9$	$\{x \in \mathbb{R} \| x < 9\}$	$(-\infty, 9)$ or $]-\infty, 9[$

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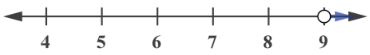
Number line	Inequality notation	Set notation	Interval notation
 More information This image features a number line ranging from 4 to 9. The number line is marked with ticks at each integer value from 4 to 9. An arrow indicates directionality towards the right past the number 9, suggesting continuity in that direction. A white marker is placed directly at the number 9, highlighting it as a specific point of interest on the number line. [Generated by AI]	$x > 9$	$\{x \in \mathbb{R} \vert\, x > 9\}$	$(9, +\infty)$ or $]9, +\infty[$

Note that when a number is not included in the domain or range, you use the parenthesis $(\ ,\ )$ or $]\ ,\ [$.

Finding the domain and range of functions

To find the domain of a function, algebraically, you consider the equation of the function and the input values of x for which the expression of $f(x)$ is not defined. Think about the number of restrictions of division or for when you take the square root of a number.

For example, consider the function $f(x) = \sqrt{x-1}$.

You know that the square root function is defined for only positive numbers and number zero.
So,

$$x - 1 \geq 0 \Leftrightarrow x \geq 1.$$

Therefore, the domain of function f is $D_f = \{x \in \mathbb{R} \mid x \geq 1\}$ or, using interval notation,

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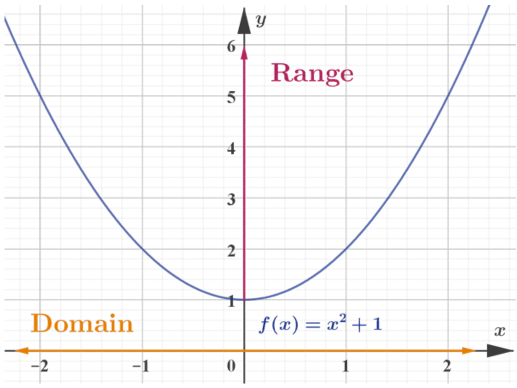
$x \in [1, +\infty)$.

The range of f is the set of output values $y = f(x)$. Since the square root of any positive number or zero can only result in a positive number or zero, respectively, the range of f is written as $R_f = \{y \in \mathbb{R} \mid y \geq 0\}$.

Shown here are four functions that explore the different scenarios for the domain: unrestricted, i.e. all real numbers, restricted explicitly and restricted implicitly as a consequence of the particular function.



More information

The image is a graph depicting the quadratic function (f(x) = x^2 + 1). The graph is plotted on a grid with two axes: the horizontal x-axis and the vertical y-axis. The x-axis, labeled as "Domain," ranges from -2 to 2, with 0 at the center. The y-axis, labeled as "Range," ranges from 0 to 6, with each line representing an increment of 1 unit.

The function is represented by a parabola opening upwards with its vertex at the point (0, 1). The parabola passes through the point (1, 2) on the right and (-1, 2) on the left. There are additional grid lines intersecting at integer intervals, providing a detailed background for plotting the curve.

Text labels include "Range" along the y-axis and "Domain" along the x-axis. The function is stated at the bottom right of the graph as (f(x) = x^2 + 1).

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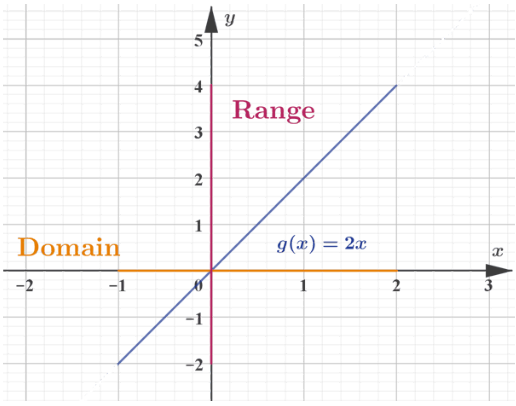

More information

The graph shows two main lines on a Cartesian plane with labeled axes. The X-axis represents the real numbers and extends from -3 to 3. The Y-axis represents another real number line, displayed from 0 to 5.

- A blue line, labeled with the equation (q(x) = 2x), is plotted, starting from the origin and progressing diagonally upwards, crossing through points such as (1, 2) and (2, 4). This line represents a linear function.
- The horizontal range of the function (f(x) = x^2 + 1), as described in the accompanying text, is depicted in orange, highlighting all values along the X-axis.
- A vertical pink line defines the range boundary on the Y-axis, labeled "Range," indicating that the range of this function is all values greater than or equal to 1.

The text on the graph also includes keywords and labels such as "Domain," "Range," and the equation itself.

[Generated by AI]

The domain of the function $f(x) = x^2 + 1$ is all real numbers, i.e. $x \in \mathbb{R}$. As shown in the graph, the range comprises all the y values that are greater than or equal to 1; thus the range is the set $\{y \mid y \in \mathbb{R}, y \geq 1\}$.

The domain of the function $g(x) = 2x$ has been explicitly restricted to $-1 \leq x \leq 2$. You write that $g(x) = 2x, -1 \leq x \leq 2$. As a consequence, the range is $\{y \mid y \in \mathbb{R}, -2 \leq y \leq 4\}$.

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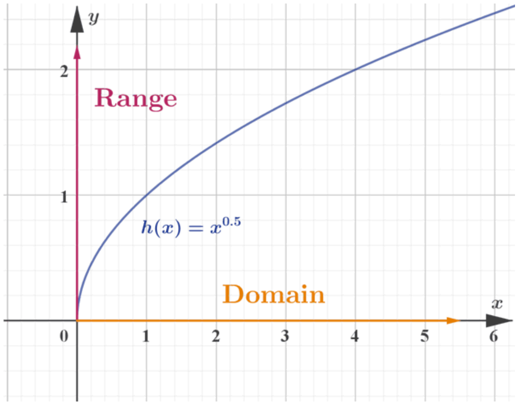

More information

This image is a graph of the function h(x) = x^0.5. The graph is plotted on a grid with labeled axes. The x-axis, labeled 'Domain,' stretches horizontally and is marked from 0 to 6. The y-axis, labeled 'Range,' extends vertically and is marked from 0 to approximately 3.9. A curve representing h(x) = x^0.5 starts at the origin (0,0) and rises upwards to the right, indicating that as x increases, the value of h(x) also increases gradually. The text on the graph highlights the concepts of domain and range related to this function.

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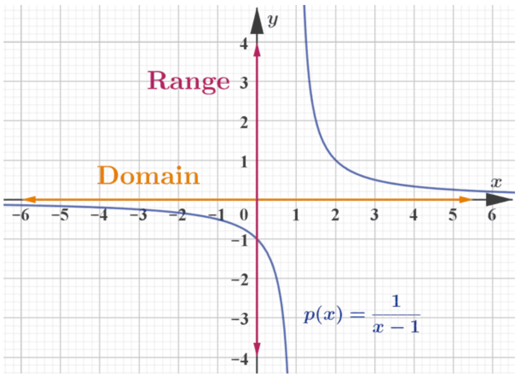

More information

The image is a graph that represents the function h(x) = √x (x to the power of 1/2). The x-axis represents the domain of the function, labeled as extending horizontally from negative to positive. The y-axis denotes the range, extending vertically. Text notes that the domain of h(x) is restricted to real numbers where x ≥ 0, and the range is real numbers where y ≥ 0. A blue curve is plotted to represent y = √x, illustrating that as x increases, y also increases. There are arrows labeled 'Domain' and 'Range' to further clarify the respective axes. The equation P(x) = 1/(m - 1) is present on the graph, likely as an equation for comparison or context. Grid lines provide a clear coordinate system for interpreting the graph.

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The function $h(x) = \sqrt{x} = x^{\frac{1}{2}}$ is implicitly restricted to the domain $\{x \mid x \in \mathbb{R}, x \geq 0\}$. Hence, the range becomes $\{y \mid y \in \mathbb{R}, y \geq 0\}$.

The function $p(x) = \frac{1}{x-1}$ allows any number for x except $x = 1$, which makes the denominator equal to zero. Hence the domain is $\{x \mid x \in \mathbb{R}, x \neq 1\}$. And for any value x from the domain, the formula $p(x) = \frac{1}{x-1}$ will never be equal to zero, so the range excludes the value $y = 0$ and thus the range is given by $\{y \mid y \in \mathbb{R}, y \neq 0\}$.

Exam tip

There are many ways of writing domain and range notation. Do not worry about notation too much! You can even write in your examination a statement such as 'All x 's are allowed except π ', which could also be written as:

$$x \in (-\infty, \pi) \cup (\pi, +\infty)$$

$$x \in]-\infty, \pi[\cup]\pi, +\infty[$$

$x < \pi$ or $\pi < x$.

4 section questions

2. Functions / 2.2 Functions

Functions as models

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What is mathematical modelling?

Modelling with functions

Models describe ideas to explain our observations of the world. Mathematical models transform our observations into the language of mathematics. This has many advantages because mathematics is a very precise language, which helps to formulate ideas and identify underlying assumptions. In mathematical modelling, functions are fundamental tools used to describe how quantities are related.

When modelling with functions, there are no fast rules or recipes that will solve a problem with any certainty. However, there are some useful guiding principles for the process. Consider the four steps outlined below.

✓ **Important**

1. **Express the model in words** . Identify the quantity you want to model and express it, in words, as a function of the other quantities in the problem.
2. **Choose the variables** . Identify all the variables used to express the function in Step 1. Assign a symbol, such as x , to one variable.
3. **Set up the model** . Express the other variables in terms of x. Express the function using the language of algebra by writing it as a function of the single variable, x , that was chosen in Step 2.
4. **Use the model** . Use the function to answer the questions posed in the problem.

In the following examples, imagine you are working for a team of designers who are experimenting with the design of a variety of products.

Example 1

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You need to make an open rectangular box with volume $2\ \mathrm{m}^3$ and a square base, as shown.

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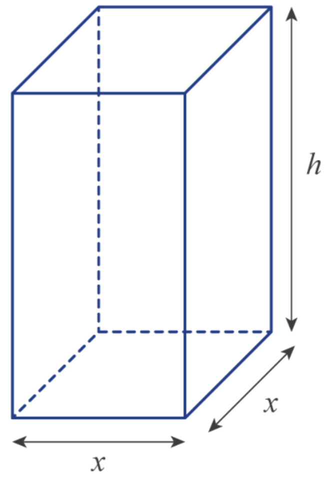

More information

The image is a diagram illustrating an open rectangular box with a square base. The diagram shows a three-dimensional rectangular structure with the base defined as a square. The square base is labeled with sides of length 'x'. The height of the box is marked as 'h'. The perspective lines show the depth and height, giving it a 3D appearance, with some edges depicted with dashed lines to indicate visibility from this perspective. The image is designed to help understand how to create a box with a specific volume and dimensions from a square base.

[Generated by AI]

1. Show that $h = \frac{2}{x^2}$.
2. Find a model to express the surface area of the box as a function of the length of a side of the base.
3. Find the surface area of the box when the length of the side of the square base is equal to 10 m.

1. The volume of the box is given by the formula $V = x^2h$.

Steps	Explanation
$V = 2$	Equating the volume to 2 (given information).

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Steps	Explanation
$x^2h = 2$	Use the volume formula.
$h = \frac{2}{x^2}$	Solve for h .

2. The surface area of the open box is the sum of the area of the base and the four sides.

Steps	Explanation
$A = x^2 + 4xh$	Add the area of the base, x^2, and the area of all other four faces.
$A = x^2 + 4x\left(\frac{2}{x^2}\right)$	Substitute the height h by $\frac{2}{x^2}$, (as shown in 1).
$A = x^2 + \frac{8}{x}$	Simplify the expression.

3. Use your function for the model to find the surface area of the box when $x = 10$.

Steps	Explanation
$A = x^2 + \frac{8}{x}$	
$A = 10^2 + \frac{8}{10}$	Substitute x = 10.
$A = 100.8$ m ^{2}	

Example 2

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You have been asked by a team of architects to set up a model for the area of a window. You must consider the following design requirements.

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- The window should be in the shape of a rectangle with a semicircle placed on the top of it.
- The window should have perimeter equal to 30 ft.

a) Draw a sketch of the window, where x is the width of window, h is the height of the rectangular part and r is the radius of semi-circular part of the window.

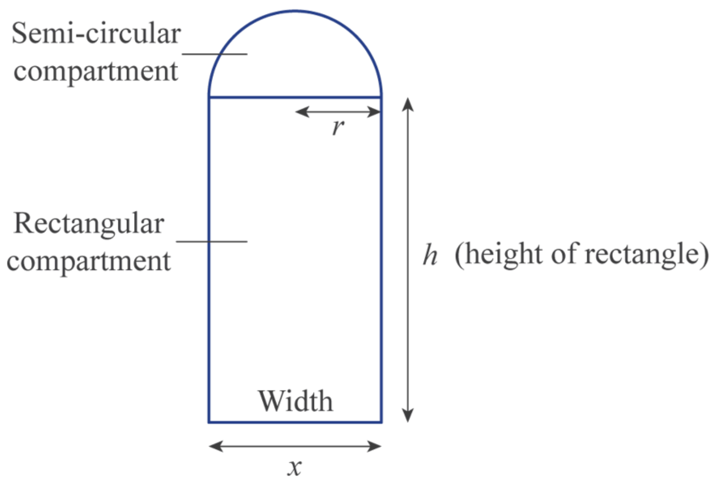
b) Express the radius r of the semi-circular part of the window as a function of the width, x.

c) Show that the height of the rectangular part of the window can be expressed as $h = 15 - \frac{(2+\pi)\,x}{4}$.

d) Create a mathematical model that describes the area of the window as a function of its width, x.

e) Find the area of the window when its width is 4 ft.

a) Make a sketch of the window, as shown.



b) The radius r is half of the width of the window and therefore its expression in terms of x is given by $r = \frac{x}{2}$.

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c) The perimeter of the window is equal to 30 ft, so form the following two equations:

$$P = P_{\text{ rectangle}} + P_{\text{ semicircle}} \text{ and } P = 30.$$

Therefore,

Steps	Explanation
$P_{rectangle} + P_{semicircle} = 30$	Make the two equations equal.
$\Rightarrow (2h + x) + \pi r = 30$	πr is the perimeter of semicircle and $2h + x$ is the perimeter o the rectangular part.
$\Rightarrow 2h + x + \pi \left(\frac{x}{2}\right) = 30$	Substitute $r = \frac{x}{2}$.
$\Rightarrow 2h = 30 - x - \frac{\pi x}{2}$	Isolate h on the LHS of the equation.
$\Rightarrow h = 15 - \frac{x}{2} - \frac{\pi x}{4}$	Solve the equation for h; divide by 2.
$\Rightarrow h = 15 - \frac{(2 + \pi)\,x}{4}$	Simplify the expression.

d) Set up your model for the total area of the window, A, by adding the rectangular and semi-circular area functions:

Steps	Explanation
$A = A_{rectangle} + A_{semicircle}$ $A = xh + \frac{1}{2}\pi r^2$	Where xh the area of rectangular part and $\frac{1}{2}$ $(\pi r$ the area of semi-circular part.
$A = x\left[15 - \frac{(2 + \pi)\,x}{4}\right] + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$	Substitute $r = \frac{x}{2}$.
$A = x\left[15 - \frac{(2 + \pi)\,x}{4}\right] + \frac{\pi x^2}{8}$	Simplify the expression.

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e) The area of the window, when $x = 4$ is given by

Steps	Explanation
$A = x\left[15 - \frac{(2+\pi)\,x}{4}\right] + \frac{\pi x^2}{8}$	
$A = 4\left[15 - \frac{(2+\pi)\,4}{4}\right] + \frac{\pi 4^2}{8}$	Substitute x = 4.
$A = 4\,[15 - (2+\pi)] + \frac{16\pi}{8}$ $A = 4\,[13 - \pi] + 2\pi$ $A = 52 - 4\pi + 2\pi$ $A = 52 - 2\pi \text{ units}^2$	Simplify the function formula.

Making connections

Linear models are used to represent how one quantity is a function of another quantity, where every change in the independent variable results in a constant change in the dependent variable.

Linear models are useful, for example, in estimating the amount of time it takes to complete a road trip, by assuming that the traffic conditions are optimal and that the driver is travelling at a constant speed. The equation $v = \frac{s}{t}$ can be used to predict the total distance, s travelled or time, t needed to complete the trip at a constant speed, v.

Banking institutions determine the amount of simple interest accumulated on an account by using the linear model $I = Prt$, where I is the amount of interest, P is the initial capital, r is the interest rate, and t is the time in years in which the interest has been accumulating.

In this video (http://www.ted.com/talks/irina_kareva_math_can_help_uncover_cancer_s_secrets?language=en#t-410377), Irina Kareva describes how mathematical models can be used in biology.

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Theory of Knowledge

'The map is not the territory' is a famous quote credited to Alfred Korzybski which reminds us of the representative nature of models. Apply this quote to mathematical modelling, can the same be said of mathematical modelling? Or do mathematical models differ from maps?

Knowledge Question: What **justification** provides mathematical models with their **certainty** ?

3 section questions

2. Functions / 2.2 Functions

The concept of an inverse function

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An introduction to the inverse function

Finding the inverse function informally

To understand the concept of an inverse function, consider, informally, the following question:

‘If you add number 5 to any number x, how do you get the number x back?’

Of course, you ‘undo’ adding 5 by taking 5 away. This is the inverse operation called subtraction. What is the inverse of multiplication? It is called division.

Now consider the function $f(x) = 2x$. How can you ‘undo’ the action of $f(x)$.

Since you are seeking a second function, you get

$$x \xrightarrow{f} 2x \xrightarrow{?} x.$$

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The answer is easy to see:

$$x \xrightarrow{f} 2x \xrightarrow{\frac{1}{2}(2x)} x.$$

Thus, you 'undo' multiplying by 2 by dividing by 2 (or by multiplying by $\frac{1}{2}$).

So, setting up a new function $g(x) = \frac{1}{2}x$:

$$f(1) = 2 \qquad g(2) = 1$$

$$f(2) = 4 \qquad g(4) = 2$$

$$f(3) = 6 \qquad g(6) = 3$$

$$f(4) = 8 \qquad g(8) = 4$$

Important

If function f and g satisfy the property

$$f(x) = y \Leftrightarrow g(y) = x$$

then you call function $g(x)$ the inverse function of $f(x)$.

You denote the inverse function of f as f^{-1}.

Look back at section 2.2.1 (/study/app/m/sid-122-cid-754029/book/the-concept-of-a-function-id-26172/), where you expressed a function by a set of ordered pairs. For example, consider a function f that adds 5 to each number of its domain:

$$f(x) = x + 5 : \quad \{(1,\ 6),\ (2,\ 7),\ (3,\ 8),\ (4,\ 9)\}.$$

The domain of f is set $A = \{1,\ 2,\ 3,\ 4\}$ and the range of f is set $B = \{6,\ 7,\ 8,\ 9\}$. In this case, by interchanging the first and second components of each of these order pairs, you can form the inverse function of f. It is a function from set B to set A, and can be written as follows:

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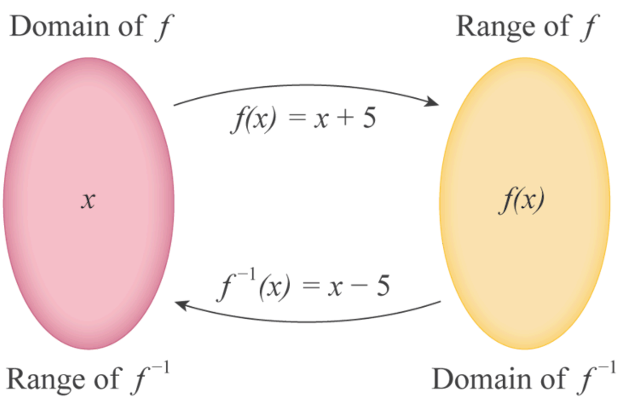
$$f^{-1}(x) = x - 5: \quad \{(6,\ 1),\ (7,\ 2),\ (8,\ 3),\ (9,\ 4)\}.$$

In other words, solving $f(x) = 6$ is equivalent to finding $f^{-1}(6)$. That is:

$$f(x) = 6 \Leftrightarrow x + 5 = 6 \Leftrightarrow x = 1. \text{ Therefore, } f^{-1}(6) = 1.$$

In our example, note that the domain of f is equal to the range of f^{-1}, and vice versa, as shown in the diagram. Also note that the function f and f^{-1} have the effect of 'undoing' each other. Function f adds 5 to each number of its domain while function f^{-1}subtracts 5 from each element of its domain.



More information

The image is a diagram that illustrates the relationship between a function (f) and its inverse (f^{-1}). It consists of two ellipses. The ellipse on the left is labeled "Domain of (f)" and contains the variable (x), while the ellipse on the right is labeled "Range of (f)" and contains (f(x)). An arrow from the left ellipse to the right is labeled (f(x) = x + 5). Another arrow from the right ellipse back to the left is labeled (f^{-1}(x) = x - 5). This reflects the fact that the inverse function (f^{-1}) reverses the effect of function (f), and vice versa.

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Be aware

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- It is extremely important that you do not think of f^{-1} as 'one over f'.
  In other words: $f^{-1}(x) \neq \frac{1}{f(x)}$.
- Note that $\frac{1}{f(x)}$ is written as $[f(x)]^{-1}$.

Example 1

★★☆

Find the inverse function of $f(x) = 5x$ informally. Then find $f^{-1}(2)$.

Steps	Explanation
$f(x) = 5x$	Function f multiplies each input by 5.
$f^{-1}(x) = \frac{x}{5}$	To 'undo' function f , you divide each input by 5.

To find $f^{-1}(2)$, either solve the equation $f(x) = 2$ (Method 1) or substitute $x = 2$ into the function $f^{-1}(x)$ (Method 2).

Method 1

Steps	Explanation
$f(x) = 2$	Set function f equal to 2.
$5x = 2$	Substitute f by its expression.
$x = \frac{2}{5}$	Solve for x.

Method 2

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Steps	Explanation
$f^{-1}(x) = \frac{x}{5}$	Write the inverse function f^{-1}.
$f^{-1}(2) = \frac{2}{5}$	Substitute x by 2.

Example 2

★★☆

Find the inverse function of $f(x) = \frac{2}{3}x + 1$ informally. Then find $f^{-1}(5)$.

Steps	Explanation
$f(x) = \frac{2}{3}x + 1$	The function f *multiplies* each input by $\frac{2}{3}$ and then adds 1.
$f^{-1}(x) = \frac{3}{2}(x-1)$	To 'undo' function f, you first subtract 1 and then divide by $\frac{2}{3}$ (or, equivalently, multiply by $\frac{3}{2}$).

To find $f^{-1}(3)$, either solve the equation $f(x) = 3$ (Method 1) or substitute $x = 3$ into the function $f^{-1}(x)$ (Method 2).

Method 1

Steps	Explanation
$f(x) = 5$	Set function f equal to 5.
$\frac{2}{3}x + 1 = 5$	Substitute f by its expression.

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Steps	Explanation
$\frac{2}{3}x = 4$ $2x = 12$ $x = 6$	Rearrange terms and solve for x.

Therefore, $f^{-1}(5) = 6$.

Method 2

Substituting $x = 5$ into the function $f^{-1}(x) = \frac{3}{2}(x - 1)$ gives

$$f^{-1}(5) = \frac{3}{2}(5 - 1) = \frac{12}{2} = 6.$$

The graph of the inverse function as a reflection in the line $y = x$

The graphs of function f and its inverse function f^{-1} are related to each other in a special way. Remember that the inverse function f^{-1} interchanges the x and y components of each ordered pair of function f. Therefore, if a point (x, y) lies on the graph of f, then the point (y, x) must lie on the graph of f^{-1}. In the video below, you can explore some graphical implications of the inverse function and find an easy method to graph f^{-1} given f.

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Video 1. Graphical Implications of the Inverse Function.

More information for video 1

1

00:00:00,534 --> 00:00:03,403

narrator: In this video, we're going to

investigate inverse functions

2

00:00:03,470 --> 00:00:06,039

and remember what inverse function are.

3

00:00:06,139 --> 00:00:07,975

They undo what another function does.

4

00:00:08,041 --> 00:00:09,643

So if x goes through f

5

00:00:10,010 --> 00:00:11,912

and the image is $y = f(x)$,

6

00:00:11,979 --> 00:00:14,648

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view

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if put that through f^{-1},

I get x back,

7

00:00:14,715 --> 00:00:16,183

or of course the other way around.

8

00:00:16,250 --> 00:00:19,586

If x goes through f^{-1}

producing $y = f^{-1}(x)$

9

00:00:19,653 --> 00:00:22,289

of x, then bring it through f,

I get x back.

10

00:00:22,623 --> 00:00:26,627

In other words, x through f,

the result through f^{-1} is x

11

00:00:26,693 --> 00:00:29,129

and the opposite directions true too.

12

00:00:29,296 --> 00:00:30,831

So what does it mean in terms of graphs?

13

00:00:30,931 --> 00:00:32,733

Well let's you take a general graph,

14

00:00:33,000 --> 00:00:38,739

then x gets mapped to y,

so it goes through point $(x, f(x))$.

15

00:00:39,273 --> 00:00:41,008

Now if I take it y value,

16

00:00:41,375 --> 00:00:45,679

then if that goes

through f^{-1}, it should produce x.

17

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00:00:45,746 --> 00:00:49,149

So the point $(y, f^{-1}(y))$,

18

00:00:49,449 --> 00:00:53,086

but $f^{-1}(y)$

is really $f^{-1}(f(x))$.

19

00:00:53,153 --> 00:00:56,356

And that should of course

produce that value x.

20

00:00:56,557 --> 00:00:59,660

Now here we have a line

$f(x) = 2x + 1$.

21

00:00:59,726 --> 00:01:01,428

Now let's investigate this.

22

00:01:01,795 --> 00:01:06,233

So if I take a point $(1, 0)$,

then it gets mapped

23

00:01:06,633 --> 00:01:08,335

through $f(x)$.

24

00:01:08,669 --> 00:01:11,305

Now, if I take a value 3,

if I put that through

25

00:01:11,371 --> 00:01:14,641

and the inverse function,

it should produce a value 1.

26

00:01:14,775 --> 00:01:18,011

Now let's take one more

point $(-2, 0)$.

27

00:01:18,078 --> 00:01:20,013

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That's get mapped through f

28

00:01:20,214 --> 00:01:22,883

to the point $y = -3$.

29

00:01:23,250 --> 00:01:25,886

So if I take -3

through the inverse function,

30

00:01:25,953 --> 00:01:28,288

that should get mapped to -2,

31

00:01:28,589 --> 00:01:30,991

'cause that's what I started

with in the blue mapping.

32

00:01:31,225 --> 00:01:32,659

Well now we've got two points

33

00:01:32,826 --> 00:01:36,797

and inverse function of a line

is another line, and here it is.

34

00:01:36,864 --> 00:01:39,766

The inverse of $2x + 1$

is $\frac{1}{2}x - \frac{1}{2}$.

35

00:01:40,400 --> 00:01:44,171

Now let's take one more point of interest

and I'm gonna take -1,

36

00:01:44,238 --> 00:01:46,139

which get mapped to -1

37

00:01:46,206 --> 00:01:49,543

through blue, but it also gets

mapped to -1 through

38

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00:01:49,877 --> 00:01:50,978

the inverse function.

39

00:01:51,211 --> 00:01:53,380

It goes, it is an intersection point.

40

00:01:53,747 --> 00:01:56,817

Now very great importance is the fact that

41

00:01:56,884 --> 00:02:01,522

f and f $^{-1}$ are reflections

in the y = x axis.

42

00:02:02,122 --> 00:02:05,526

That will certainly help you to plot them

and it also helps you that

43

00:02:05,592 --> 00:02:09,696

if f intersects y = x,

44

00:02:09,763 --> 00:02:13,800

then f $^{-1}$ will intersect

y = x at exactly the same point

45

00:02:13,901 --> 00:02:15,269

as seen here.

In the video, you investigated the inverse function of $f\left(x\right) = 2x + 1$ and found it to be $f^{-1}\left(x\right) = \frac{1}{2}x - \frac{1}{2}$, which is shown in the diagram below.

The light blue line is the function $f\left(x\right) = 2x + 1$; the inverse function is the orange line, $f^{-1}\left(x\right) = \frac{1}{2}x - \frac{1}{2}$. Note how these graphs are symmetrical about the dark blue line, $y = x$. Also, their point of intersection lies on the line $y = x$.

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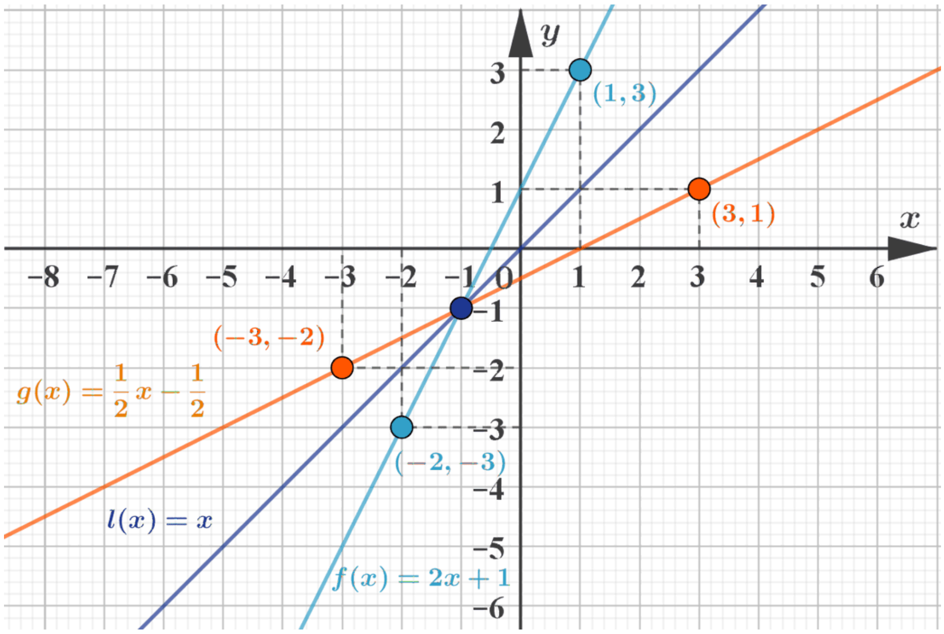

More information

The image displays a graph with three lines: the light blue line representing the function (f(x) = 2x + 1), the orange line depicting the inverse function (f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}), and the dark blue line showing the line of symmetry (y = x). These graphs intersect at the point ((0, 0)), which lies on the line (y = x).

Key points marked include ((1, 3)), ((2, 3)), and ((3, 1)). Another point, ((-3, -2)), is indicated along the inverse function line. The axes are labeled with tick marks ranging from -8 to 4 on the x-axis and -5 to 7 on the y-axis. The graph is on a grid background to help visualize the slope and intersections.

[Generated by AI]

Important

- On a graph, f and f^{-1} are symmetrical about the line $y = x$, meaning that if you reflect f in $y = x$ then you obtain f^{-1}, and vice versa.
- This symmetry property implies that if the graph of f intersects the line $y = x$ at a point, say (a, a), then the graph of f^{-1}intersects the line $y = x$ at that point as well. In other words, the point (a, a) lies on the graphs of both f and f^{-1}.
- Also note that the range of f is the domain of f^{-1}, and the domain of f is the range of f^{-1}.

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Activity

In the following applet, entering a rule for the function f shows the graph of f and its reflection graph in the line $y = x$. Try out rules for linear, quadratic, cubic and quartic functions. Do all reflection graphs represent functions? Use the sliders 'left' and 'right' to adjust the domain of the function. What do you notice?

Interactive 1. Exploring Function Reflections and Their Validity.

Credit: GeoGebra (https://www.geogebra.org/m/S9mgw5vv) Malin Christersson

More information for interactive 1

This interactive allows users to explore the graph of a function f and its reflection across the line $y = x$ which represents the inverse of the function.

In mathematics, reflecting a function across the line $y = x$ involves interchanging the input and output values. Given a function $y = f(x)$, its reflection graph consists of all points (b, a) where (a, b) lies on the original function's graph. This transformation provides insight into inverse functions and their behavior.

Users can manually type in the rule for any function, and observe how the graph of (f) in blue and its inverse in orange are displayed. The applet allows users to input various types of functions, including linear functions, quadratic functions, cubic and quartic functions. By adjusting the "left" and "right" sliders (at the bottom), users can modify the domain of the function simultaneously within the range of -10 to 10.

Here, the entered function is an exponential function: $y = 2^x$

The function graph rises quickly, passing through $(0, 1)$

The reflected graph represents the inverse function $f^{-1}(x) = \log_2(x)$

The inverse function is only defined for positive x values and has a vertical asymptote at $x = 0$.

This tool helps users understand the relationship between a function and its inverse and whether the inverse graph represents a valid function. By experimenting with the sliders and observing the

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changes in the graph, you can gain a deeper understanding of the concept of inverse functions through reflection.

Example 3

★☆☆

Find the inverse function of $f(x) = 4x - 2$ informally. Then, plot the graphs of f and f^{-1} to show that the graph of f^{-1} is a reflection of f in the line $y = x$.

You can find the inverse function f^{-1} informally, by reversing the order of operations of f.

Steps	Explanation
$f(x) = 4x - 2$	Function f multiplies each input x by 4 and then it subtracts 2.
$f^{-1}(x) = \frac{1}{4}(x + 2)$	The inverse function f^{-1} adds 2 in each input x and then divides by 4.

By plotting the two functions on the same coordinate system (see below), you see that the graph of the inverse function f^{-1} is reflection of function f.

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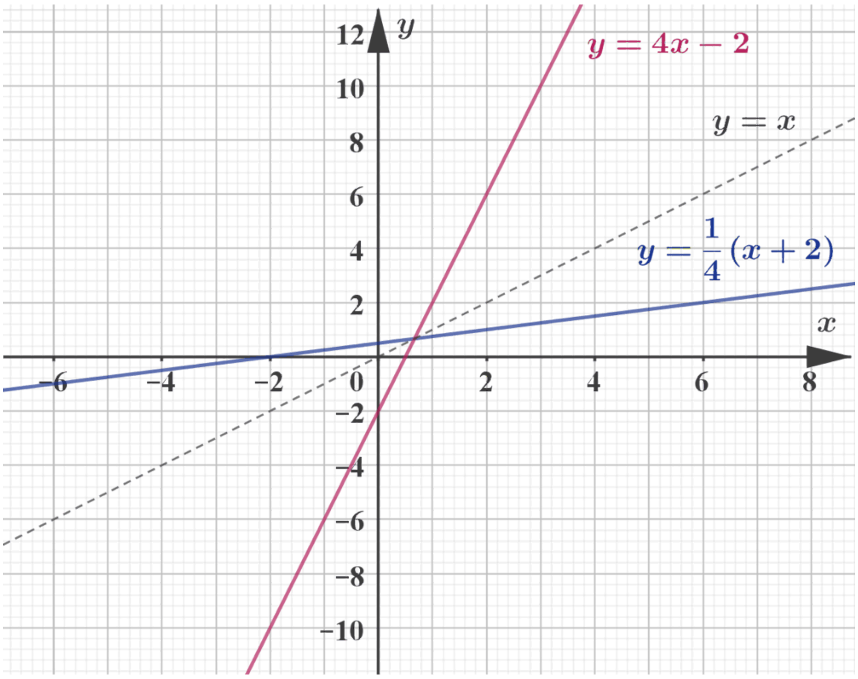

You can further verify this reflective property by testing a few points on each graph.

On the graph of $f(x) = 4x - 2$ you have points $(1,\ 2)$ and $(2,\ 6)$.

On the graph of $f^{-1}(x) = \frac{1}{4}(x + 2)$ you have points $(2,\ 1)$ and $(6,\ 2)$.

Therefore, the function $f(x) = 4x - 2$ has inverse $f^{-1}(x) = \frac{1}{4}(x + 2)$.

3 section questions

2. Functions / 2.2 Functions

Checklist

What you should know

By the end of this subtopic you should be able to:

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- determine whether a relationship, expressed as a set of ordered pairs and as a mapping diagram, is a function
- use the vertical line test
- find the domain of a function implicitly by considering restrictions on the input values
- use the graph of a function to find its range
- evaluate the output value of a function for various input values by using the graph and the formula of the function
- set up mathematical models to solve real-life problems
- find the formula of the inverse function f^{-1} informally
- use the property of reflection in the line $x = y$ to determine whether two functions are the inverse of each other.

Investigation

In the following investigation you will explore the condition for a function to have an inverse function. Follow the steps below and discuss the questions with your fellow students.

- Enter the rule of the function in the input box $f(x)$. You can enter any rule that you want.
- Set the domain of the function by dragging the endpoints of the blue line at the bottom of the window.
- Determine whether $f(x)$ is a one-to-one function. Recall that a function f is one-to-one if, for every a and b in its domain, $f(a) = f(b)$ implies $a = b$. Click on the 'Horizontal line test' and drag the vertical slider to check whether f is one-to-one.
- Click on the 'Reflect $f(x)$ ON/OFF' button.
- Drag the horizontal slider to run the 'Vertical line test'. Determine whether the reflection graph is a function.
- Drag the endpoints of the blue segment under the graph to restrict the domain of $f(x)$ such that the reflected graph is a graph of a function.

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- Click 'Show corresponding points'. Drag the orange point along the graph of $f(x)$ and notice its reflection on the graph of the inverse function.
- Formulate a general rule for a function to have an inverse.

Interactive 1. Exploring Function Inverses.

More information for interactive 1

This interactive allows users to explore the concept of one-to-one functions and their inverses. Users can enter any function rule $f(x)$ and set its domain by adjusting the endpoints of the blue line at the bottom of the window. The interactive provides tools to determine whether$f(x)$ is one-to-one using the horizontal line test, which checks if every horizontal line intersects the graph at not more than one point.

A function must be one-to-one (injective) to have an inverse. A function is one-to-one if different inputs always produce different outputs, meaning $f(a) = f(b)$ only if $a = b$.

To verify this property, users can perform the Horizontal Line Test by clicking on the corresponding button and dragging the vertical slider. If any horizontal line intersects the function at more than one point, the function is not one-to-one and does not have an inverse over that domain.

Users can also reflect the graph of $f(x)$ across the line $y = x$ to visualize its inverse by clicking on the "Reflect f(x) ON/OFF" button.

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By running the vertical line test on the reflected graph, users can determine if the inverse is a function. After reflection, the applet allows users to drag a horizontal slider to check whether the reflected graph is still a function by performing the Vertical Line Test. If any vertical line crosses the reflection at more than one point, the reflection is not a function, meaning the original function does not have an inverse unless its domain is restricted.

To make the inverse function valid, users can further restrict the domain by adjusting the endpoints of the blue segment under the graph. This ensures that the reflected graph passes the Vertical Line Test and remains a function. Clicking the "Show corresponding points" button enables users to track specific points on the function and their corresponding reflections. Users can drag the orange point along the function's graph and observe how its reflection appears on the inverse graph, illustrating the concept of inverse function pairs $(x, y) \leftrightarrow (y, x)$.

After interacting with various functions and their reflections, users can formulate a general rule: A function must be one-to-one to have an inverse function. If it is not one-to-one over its original domain, restricting the domain can help ensure the existence of an inverse.

The interactive allows users to restrict the domain of $f(x)$ to ensure that its inverse is also a function. Additionally, users can observe corresponding points on the graph of $f(x)$ and its inverse, helping them understand the relationship between a function and its inverse.

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