NEWGIZA UNIVERSITY

Precalculus
Revision Sheet: Trigonometry

Dr. Laila H. Afify

1 Summary

- Trigonometric functions are either even or odd, meaning that they are symmetric around the y-axis or origin, respectively.
- The definitions of even and odd functions can be used to derive symmetry identities that correspond to each of the six trigonometric functions. The symmetry identities can be used to find the trigonometric functions of negative values.

Even	Odd
Cosine and Secant	Sine, Cosecant, Tan, Cotan
$\cos(\theta) = \cos(-\theta)$ $\sec(\theta) = \sec(-\theta)$	$\sin(\theta) = -\sin(-\theta)$ $\csc(\theta) = -\csc(-\theta)$ $\tan(\theta) = -\tan(-\theta)$ $\cot(\theta) = -\cot(-\theta)$

Sum/difference Formula for Sine (sine is ”sumthing” that switches) “sine goes with cosine and cosine goes with sine”	$\sin(x+y) = \overbrace{\sin x}\underbrace{\cos y} + \underbrace{\cos x}\overbrace{\sin y}$ $\sin(x-y) = \overbrace{\sin x}\underbrace{\cos y} - \underbrace{\cos x}\overbrace{\sin y}$
Sum/difference Formula for Cosine	$\cos(x+y) = \underbrace{\cos x \cos y} - \overbrace{\sin x \sin y}$ $\cos(x-y) = \underbrace{\cos x \cos y} + \overbrace{\sin x \sin y}$
Sum/difference Formula for Tan	$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$ $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$
Cofunction Identities	$\sin(\frac{\pi}{2} - x) = \cos x$ $\cos(\frac{\pi}{2} - x) = \sin x$ $\tan(\frac{\pi}{2} - x) = \cot x$ $\cot(\frac{\pi}{2} - x) = \tan x$ $\sec(\frac{\pi}{2} - x) = \csc x$ $\csc(\frac{\pi}{2} - x) = \sec x$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\tan^2(\theta) + 1 = \sec^2(\theta)$$
$$1 + \cot^2(\theta) = \csc^2(\theta)$$

1.1 Solving Trigonometric Equations

Solve $\sin(\theta) = q$

- If $q < -1$ or $q > 1$, then the equation $\sin\theta = q$ has no solution.
- If $q = -1$, the equation $\sin\theta = q$ has one solution between 0 and 2π: $= \frac{3\pi}{2}$.
- If $q = 1$, the equation $\sin\theta = q$ has one solution between 0 and 2π: $= \frac{\pi}{2}$.
- If $-1 < q < 1$, the equation $\sin\theta = q$ has two solutions between 0 and 2π:
  - The first is obtained by typing [$\sin^{-1} q$] on the calculator; the other is $\pi-$**"(the first answer)"**
  - Do not forget, adding/subtracting multiples of 2π gives new angles (greater than 360 or less than 0. Then, adding $2n\pi$, where $n = \cdots, -2, -1, 0, 1, 2, \cdots$ to any answer still gives a valid answer.

Solve $\cos(\theta) = q$

- If $q < -1$ or $q > 1$, then the equation $\cos\theta = q$ has no solution.
- If $q = -1$, the equation $\cos\theta = q$ has one solution between 0 and 2π: $= \pi$.
- If $q = 1$, the equation $\sin\theta = q$ has one solution between 0 and 2π: $= 0$.
- If $-1 < q < 1$, the equation $\cos\theta = q$ has two solutions between 0 and 2π:
  - The first is obtained by typing [$\cos^{-1} q$] on the calculator; the other is **$2\pi-$"(the first answer)"**
  - Do not forget, adding/subtracting multiples of 2π gives new angles (greater than 360 or less than 0. Then, adding $2n\pi$, where $n = \cdots, -2, -1, 0, 1, 2, \cdots$ to any answer still gives a valid answer.

2 Notes

- For visualizing the addition formulae:
  https://www.geogebra.org/m/HK967BRB

3 Summary

3.1 Given a trigonometric identity, how to verify that it is true:

1. Work on one side of the equation. It is usually better to start with the more complex side, as it is easier to simplify than to build.
2. Look for opportunities to factor expressions, square a binomial, or add fractions.
3. Noting which functions are in the final expression, look for opportunities to use the identities and make the proper substitutions.
4. If these steps do not yield the desired result, try converting all terms to sines and cosines.

3.2 How to derive the identities including double (2α) or half the angle $(\alpha/2)$?

1. Start from $\sin(x + y)$ or $\cos(x + y)$..
2. Let $x = \alpha$ and $y = \alpha \rightarrow$ For the identities of double the angle
3. Let $x = \dfrac{\alpha}{4}$ and $y = \dfrac{\alpha}{4} \rightarrow$ For the identities of half the angle

3.3 How to derive the identities including double (3α) ?

1. Start from $\sin(x + y)$ or $\cos(x + y)$..
2. Let $x = 2\alpha$ and $y = \alpha$.

4 Problems

1. Evaluate:

   (a) $3\sin^3(t)\csc(t) + \cos^2(t) + 2\cos(-t)\cos(t) - \tan(-x)\cot(-x)$

2. Prove the following identities:

   (a) $(1 + \sin x)[1 + \sin(-x)] = \cos^2 x$

   (b) $\dfrac{\sec^2(x) - 1}{\sec^2(x)} = \sin^2(x)$

   (c) $\dfrac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos(\theta) - \sin(\theta)$

   (d) $\dfrac{\sin^2(x) - 1}{\tan(x)\sin(x) - \tan(x)} = \dfrac{\sin(x) + 1}{\tan(x)}$

   (e) $(1 - \cos^2(x))(1 + \cot^2(x)) = 1$

   (f) $\csc^2(x) - \cot^2(x) = 1$

   (g) $\sin(x + y) + \sin(x - y) = 2\sin(x)\cos(y)$

   (h) $\dfrac{\sin(\alpha - \beta)}{\cos(\alpha)\cos(\beta)} = \tan(\alpha) - \tan(\beta)$

   (i) $\tan(\pi - \theta) = -\tan(\theta)$

   (j) $\sin(2x) = 2\sin(x)\cos(x)$

   (k) $\cos(x) = \cos^2\left(\dfrac{x}{2}\right) - \sin^2\left(\dfrac{x}{2}\right)$

   (l) $\tan(4x) = \dfrac{2\tan(2x)}{1 - \tan^2(2x)}$