

1. The Cauchy distribution is a continuous distribution with two parameters that is popular among physicists, which came from the Cauchy function described by Fermat in the 17th century.
 - a. Given a Cauchy distributed random variable X with $x_0 = 6$ and $\gamma = 1$. Use the inverse transform method to generate 1000 sample points from this distribution! Display the MLE for the two parameters by using the *mlcauchy(...)* function. Are the MLEs similar to those defined above?
 - b. Generate 20 sample points from the same distribution defined in (1a). Display the MLEs.
 - c. Generate 20000 sample points from the same distribution defined in (1a). Display the MLEs.
 - d. What can you say about the results found in (1a), (1b), and (1c)?
 - e. Is it possible to generate X from normally distributed random variable?

Note: The CDF of $X \sim \text{Cauchy}(x_0, \gamma)$ is given as

$$F(x) = \frac{1}{\pi} \arctan\left(\frac{x - x_0}{\gamma}\right) + \frac{1}{2}, \quad x_0 \in \mathbb{R}, \quad \gamma \in \mathbb{R}^+$$

and the *mlcauchy(...)* function may be found within the *univariateML* package.

2. Let X be a random variable with logistic distribution with $\mu = 95$ and $s = 0.6$. The CDF is given as follow.

$$F(x; \mu, s) = \frac{1}{1 + e^{-\frac{x - \mu}{s}}}, \quad \mu \in \mathbb{R}, \quad s \in \mathbb{R}^+$$

- a. Generate 10000 sample points of X using the inverse transform method.
 - b. Generate 10000 sample points of X using any other method (you may use any R function).
 - c. Compare the MLEs from two methods above using *mllogis(...)* function available in *univariateML* package.
 - d. Compute the variance of the sample points. The theoretical variance of X is $\frac{s^2\pi^2}{3}$. Comment on the result.
3. Dimiliki suatu sampel random berukuran 100 dari distribusi Normal dalam file norm.xlsx. Diketahui probability density function dari distribusi Normal adalah:

$$f(x|\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Perintah

- Carilah estimasi parameter μ dan σ menggunakan metode maximum likelihood estimator dengan pendekatan newton-raphson. Gunakan inisiasi nilai awal μ adalah 76 dan σ adalah 9.
- Untuk memastikan kembali periksa dengan menggunakan fungsi `fitdistr()` dari paket MASS.
- Setelah didapatkan estimasi parameternya, lakukan uji Kolmogorov-Smirnov untuk melihat apakah benar sampel berdistribusi Normal dengan parameter tersebut.
- Periksa rata-rata dan standar deviasi dari data, lalu bandingkan dengan hasil estimasi parameter yang diperoleh dengan metode sebelumnya, apakah hasil yang didapatkan sama atau relatif mirip? Apa yang dapat anda simpulkan dari hal ini?