

# Package ‘CMDMeasure’

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**Title** Conditional Mean Dependence Measures via Energy Statistics

**Version** 1.0.0

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**Description** Implementation of conditional mean dependence measures and conditional mean independence tests in Shao, X., and Zhang, J. (2014) <doi:10.1080/01621459.2014.887012> and Park, T., et al. (2015) <doi:10.1214/15-EJS1047>.

**Depends** R (>= 3.4.0)

**Suggests** testthat (>= 2.0.0),  
energy (>= 1.7-0)

**License** GPL (>= 2)

**LazyData** true

**RoxygenNote** 6.0.1

**Collate** 'CMDMeasure-package.R'  
'functions.R'  
'cmdm\_test.R'  
'mdc.R'  
'mdd.R'  
'pmdc.R'  
'pmdd.R'

## R topics documented:

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**Description**

CMDMeasure: A package for mutual dependence measures via energy statistics

**Details**

The CMDMeasure package provides measures of conditional mean dependence and tests of conditional mean independence.

**Measuring conditional mean dependence**

The conditional mean dependence measures include:

- asymmetric measure  $\mathcal{R}_n$  based on distance covariance  $\mathcal{V}_n$
- symmetric measure  $\mathcal{S}_n$  based on distance covariance  $\mathcal{V}_n$
- complete measure  $\mathcal{Q}_n$  based on complete V-statistics
- simplified complete measure  $\mathcal{Q}_n^*$  based on incomplete V-statistics
- asymmetric measure  $\mathcal{J}_n$  based on complete measure  $\mathcal{Q}_n$
- simplified asymmetric measure  $\mathcal{J}_n^*$  based on simplified complete measure  $\mathcal{Q}_n^*$
- symmetric measure  $\mathcal{I}_n$  based on complete measure  $\mathcal{Q}_n$
- simplified symmetric measure  $\mathcal{I}_n^*$  based on simplified complete measure  $\mathcal{Q}_n^*$

**Testing conditional mean independence**

The conditional mean independence tests based on the conditional mean dependence measures are implemented as permutation tests.

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**Description**

mdc measures conditional mean dependence of  $Y$  given  $X$ , where each contains one variable (univariate) or more variables (multivariate).

**Usage**

```
mdc(X, Y, center = "U")
```

**Arguments**

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
center	The approach for centering, including <ul style="list-style-type: none"> <li>• U: U-centering which leads to an unbiased estimator;</li> <li>• D: double-centering which leads to a biased estimator.</li> </ul>

**Value**

mdc returns the value of squared martingale difference correlation.

**References**

- Shao, X., and Zhang, J. (2014). Martingale difference correlation and its use in high-dimensional variable screening. *Journal of the American Statistical Association*, 109(507), 1302-1318. <http://dx.doi.org/10.1080/01621459.2014.887012>.
- Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

**Examples**

```
# X, Y are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)

mdc(X, Y, center = "U")
mdc(X, Y, center = "D")
```

mdd

*Martingale Difference Divergence***Description**

mdd measures conditional mean dependence of Y given X, where each contains one variable (univariate) or more variables (multivariate).

**Usage**

```
mdd(X, Y, compute = "C", center = "U")
```

**Arguments**

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
compute	The method for computation, including

- C: computation implemented in C code;
  - R: computation implemented in R code.
- center      The approach for centering, including
- U: U-centering which leads to an unbiased estimator;
  - D: double-centering which leads to a biased estimator.

## Value

mdd returns the value of squared martingale difference divergence.

## References

Shao, X., and Zhang, J. (2014). Martingale difference correlation and its use in high-dimensional variable screening. *Journal of the American Statistical Association*, 109(507), 1302-1318. <http://dx.doi.org/10.1080/01621459.2014.887012>.

Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

## Examples

```
# X, Y are vectors with 10 samples and 1 variable
X <- rnorm(10)
Y <- rnorm(10)

mdd(X, Y, compute = "C")
mdd(X, Y, compute = "R")

# X, Y are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)

mdd(X, Y, center = "U")
mdd(X, Y, center = "D")
```

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pmdc

*Partial Martingale Difference Correlation*

---

## Description

pmdc measures conditional mean dependence of Y given X conditioning on Z, where each contains one variable (univariate) or more variables (multivariate).

## Usage

```
pmdc(X, Y, Z)
```

**Arguments**

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Z	A vector, matrix or data frame, where rows represent samples, and columns represent variables.

**Value**

pmdc returns the value of squared partial martingale difference correlation.

**References**

Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. Electronic Journal of Statistics, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

**Examples**

```
# X, Y, Z are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)
Z <- matrix(rnorm(10 * 2), 10, 2)

pmdc(X, Y, Z)
```

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pmdd

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*Partial Martingale Difference Divergence*


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**Description**

pmdd measures conditional mean dependence of Y given X conditioning on Z, where each contains one variable (univariate) or more variables (multivariate).

**Usage**

```
pmdd(X, Y, Z)
```

**Arguments**

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Z	A vector, matrix or data frame, where rows represent samples, and columns represent variables.

**Value**

pmdd returns the value of squared partial martingale difference divergence.

## References

Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

## Examples

```
# X, Y, Z are vectors with 10 samples and 1 variable
X <- rnorm(10)
Y <- rnorm(10)
Z <- rnorm(10)

pmdd(X, Y, Z)

# X, Y, Z are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)
Z <- matrix(rnorm(10 * 2), 10, 2)

pmdd(X, Y, Z)
```

xxx

*Mutual Dependence Measures*

## Description

cmdm measures mutual dependence of all components in  $X$ , where each component contains one variable (univariate) or more variables (multivariate).

## Usage

```
xxx(x, y)
```

## Arguments

<code>x</code>	A matrix or data frame, where rows represent samples, and columns represent variables.
<code>y</code>	A matrix or data frame, where rows represent samples, and columns represent variables.

## Value

cmdm returns a list including the following components:

<code>stat</code>	The value of the mutual dependence measure.
<code>dist</code>	The distances between all components from all samples.

## References

Shao, X., and Zhang, J. (2014). Martingale difference correlation and its use in high-dimensional variable screening. *Journal of the American Statistical Association*, 109(507), 1302-1318. <https://arxiv.org/abs/1709.02532>.

Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <https://arxiv.org/abs/1709.02532>.

**Examples**

```
# X, Y is a 10 x 3 matrix with 10 samples and 3 variables
X <- matrix(rnorm(10 * 3), 10, 3)
Y <- matrix(rnorm(10 * 3), 10, 3)
```

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