

# Package ‘CMDMeasure’

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**Title** Conditional Mean Dependence Measures via Energy Statistics

**Version** 1.0.0

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**Description** Implementation of conditional mean dependence measures and conditional mean independence tests in Shao, X., and Zhang, J. (2014) <doi:10.1080/01621459.2014.887012> and Park, T., et al. (2015) <doi:10.1214/15-EJS1047>.

**Depends** R (>= 3.4.0)

**Suggests** testthat (>= 2.0.0),  
energy (>= 1.7-0)

**License** GPL (>= 2)

**LazyData** true

**RoxygenNote** 6.0.1

**Collate** 'CMDMeasure-package.R'  
'functions.R'  
'pmdd.R'  
'mdd.R'  
'cmdm\_test.R'  
'mdc.R'  
'pmdc.R'

## R topics documented:

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**Description**

CMDMeasure: A package for conditional mean dependence measures via energy statistics

**Details**

The CMDMeasure package provides measures of conditional mean dependence and tests of conditional mean independence.

**Measuring conditional mean dependence**

The conditional mean dependence measures include:

- conditional mean dependence of  $Y$  given  $X$ 
  - martingale difference divergence
  - martingale difference correlation
- conditional mean dependence of  $Y$  given  $X$  conditioning on  $Z$ 
  - partial martingale difference divergence
  - partial martingale difference correlation

**Testing conditional mean independence**

The conditional mean independence tests include:

- conditional mean independence of  $Y$  given  $X$  conditioning on  $Z$ 
  - martingale difference divergence under a linear assumption
  - partial martingale difference divergence

The conditional mean independence tests based on the conditional mean dependence measures are implemented as permutation tests.

**Author(s)**

Ze Jin <zj58@cornell.edu>, Shun Yao <shunyao2@illinois.edu>,  
Xiaofeng Shao <xshao@illinois.edu>, David S. Matteson <matteson@cornell.edu>

cmdm\_test

*Conditional Mean Independence Tests***Description**

cmdm\_test tests conditional mean independence of Y given X conditioning on Z, where each contains one variable (univariate) or more variables (multivariate). All tests are implemented as permutation tests.

**Usage**

```
cmdm_test(X, Y, Z, num_perm = 500, type = "linmdd", compute = "C",
          center = "U")
```

**Arguments**

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Z	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
num_perm	The number of permutation samples drawn to approximate the asymptotic distributions of mutual dependence measures.
type	The type of conditional mean dependence measures, including <ul style="list-style-type: none"> <li>• linmdd: martingale difference divergence under a linear assumption;</li> <li>• pmdd: partial martingale difference divergence.</li> </ul>
compute	The computation method for martingale difference divergence, including <ul style="list-style-type: none"> <li>• C: computation implemented in C code;</li> <li>• R: computation implemented in R code.</li> </ul>
center	The centering approach for martingale difference divergence, including <ul style="list-style-type: none"> <li>• U: U-centering which leads to an unbiased estimator;</li> <li>• D: double-centering which leads to a biased estimator.</li> </ul>

**Value**

cmdm\_test returns a list including the following components:

stat	The value of the conditional mean dependence measure.
dist	The p-value of the conditional mean independence test.

**References**

- Shao, X., and Zhang, J. (2014). Martingale difference correlation and its use in high-dimensional variable screening. *Journal of the American Statistical Association*, 109(507), 1302-1318. <http://dx.doi.org/10.1080/01621459.2014.887012>.
- Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

## Examples

```
## Not run:
# X, Y, Z are vectors with 10 samples and 1 variable
X <- rnorm(10)
Y <- rnorm(10)
Z <- rnorm(10)

cmdm_test(X, Y, Z, type = "linmdd")

# X, Y, Z are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)
Z <- matrix(rnorm(10 * 2), 10, 2)

cmdm_test(X, Y, Z, type = "pmdd")

## End(Not run)
```

---

mdc

---

*Martingale Difference Correlation*


---

## Description

mdc measures conditional mean dependence of Y given X, where each contains one variable (univariate) or more variables (multivariate).

## Usage

```
mdc(X, Y, center = "U")
```

## Arguments

- |        |  |
|--------|--|
| X      | A vector, matrix or data frame, where rows represent samples, and columns represent variables.   |
| Y      | A vector, matrix or data frame, where rows represent samples, and columns represent variables.   |
| center | The approach for centering, including <ul style="list-style-type: none"> <li>• U: U-centering which leads to an unbiased estimator;</li> <li>• D: double-centering which leads to a biased estimator.</li> </ul> |

## Value

mdc returns the value of squared martingale difference correlation.

## References

- Shao, X., and Zhang, J. (2014). Martingale difference correlation and its use in high-dimensional variable screening. *Journal of the American Statistical Association*, 109(507), 1302-1318. <http://dx.doi.org/10.1080/01621459.2014.887012>.
- Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

## Examples

```
# X, Y are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)

mdc(X, Y, center = "U")
mdc(X, Y, center = "D")
```

---

mdd

*Martingale Difference Divergence*


---

## Description

mdd measures conditional mean dependence of Y given X, where each contains one variable (univariate) or more variables (multivariate).

## Usage

```
mdd(X, Y, compute = "C", center = "U")
```

## Arguments

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
compute	The method for computation, including <ul style="list-style-type: none"> <li>• C: computation implemented in C code;</li> <li>• R: computation implemented in R code.</li> </ul>
center	The approach for centering, including <ul style="list-style-type: none"> <li>• U: U-centering which leads to an unbiased estimator;</li> <li>• D: double-centering which leads to a biased estimator.</li> </ul>

## Value

mdd returns the value of squared martingale difference divergence.

## References

- Shao, X., and Zhang, J. (2014). Martingale difference correlation and its use in high-dimensional variable screening. *Journal of the American Statistical Association*, 109(507), 1302-1318. <http://dx.doi.org/10.1080/01621459.2014.887012>.
- Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. *Electronic Journal of Statistics*, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

### Examples

```
# X, Y are vectors with 10 samples and 1 variable
X <- rnorm(10)
Y <- rnorm(10)

mdd(X, Y, compute = "C")
mdd(X, Y, compute = "R")

# X, Y are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)

mdd(X, Y, center = "U")
mdd(X, Y, center = "D")
```

---

pmdc

*Partial Martingale Difference Correlation*


---

### Description

pmdc measures conditional mean dependence of Y given X conditioning on Z, where each contains one variable (univariate) or more variables (multivariate).

### Usage

```
pmdc(X, Y, Z)
```

### Arguments

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Z	A vector, matrix or data frame, where rows represent samples, and columns represent variables.

### Value

pmdc returns the value of squared partial martingale difference correlation.

### References

Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. Electronic Journal of Statistics, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

### Examples

```
# X, Y, Z are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)
Z <- matrix(rnorm(10 * 2), 10, 2)

pmdc(X, Y, Z)
```

---

pmdd

*Partial Martingale Difference Divergence*

---

### Description

pmdd measures conditional mean dependence of Y given X conditioning on Z, where each contains one variable (univariate) or more variables (multivariate).

### Usage

```
pmdd(X, Y, Z)
```

### Arguments

X	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Y	A vector, matrix or data frame, where rows represent samples, and columns represent variables.
Z	A vector, matrix or data frame, where rows represent samples, and columns represent variables.

### Value

pmdd returns the value of squared partial martingale difference divergence.

### References

Park, T., Shao, X., and Yao, S. (2015). Partial martingale difference correlation. Electronic Journal of Statistics, 9(1), 1492-1517. <http://dx.doi.org/10.1214/15-EJS1047>.

### Examples

```
# X, Y, Z are vectors with 10 samples and 1 variable
X <- rnorm(10)
Y <- rnorm(10)
Z <- rnorm(10)

pmdd(X, Y, Z)

# X, Y, Z are 10 x 2 matrices with 10 samples and 2 variables
X <- matrix(rnorm(10 * 2), 10, 2)
Y <- matrix(rnorm(10 * 2), 10, 2)
Z <- matrix(rnorm(10 * 2), 10, 2)

pmdd(X, Y, Z)
```

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