M647 Spring 2012 Assignment 9, due Friday April 20

1a. [5 pts] The ODE

$$\frac{dy}{dx} = x^2 y^2; \quad y(0) = 1,$$

has the solution $y(x) = 1/(1 - x^3/3)$. Solve this equation with MATLAB's built-in solver *ode45.m*, and adjust *RelTol* and *AbsTol* to obtain the smallest error possible on the value of y when $x = (3(1 - 10^{-6}))^{1/3}$.

1b. [5 pts] The van der Pol ODE

$$\frac{d^2y}{dt^2} - \mu(1 - y^2)\frac{dy}{dt} + y = 0$$

is known to be *stiff* for large values of μ. In order to gain an appreciation of stiff ODE solvers, use MATLAB's *tic* and *toc* commands to compute the length of time required to solve this equation using *ode45.m*, *ode15s.m*, *ode23s.m*, *ode23t.m*, and *ode23tb.m*. Take $\mu = 1000$, $y(0) = 2$, $y'(0) = 0$, and $t \in [0, 10]$. Using your choice of solver, solve the van der Pol ODE for $t \in [0, 5000]$, and include a plot of your solution.

2. [10 pts] This problem refers to the object described in Problem 1 of Assignment 8. The following experiments were carried out on a sponge dart in order to obtain values for b and $|\vec{v}_0|$: In order to compute the dart's coefficient of air resistance b, it was dropped from a height of 4.06 meters. It hit the ground after .95 seconds. In order to compute the dart's initial speed $|\vec{v}_0|$, it was fired straight up from a height of .39 meters. It hit the ground after 2.13 seconds.

2a. Find a value for b.

2b. Find a value for $|\vec{v}_0|$.

Note. If you write your force due to air resistance as $\vec{F} = -k\rho S|\vec{v}|\vec{v}$, then $b = k\rho S/m$.

3. [10 pts] Continuing with the dart from Problem 2, experiments were carried out in which the dart was fired at various angles and the dart's horizontal distance was recorded (see Table 1). Referring to the figure in Assignment 8, the height of the ramp was $h_0 = .18$ meters.

Angle of inclination	5	10	15	20	25	30	35	40	45
Distance traveled	4.37	5.23	6.95	7.84	8.17	8.69	8.81	8.99	8.95
Angle of inclination	50	55	60	65	70	75	80	85	
Distance traveled	8.83	8.19	7.84	7.12	6.38	5.08	3.34	2.13	

Table 1: Angles of inclination and distance traveled.

Use event location to write a MATLAB function M-file that takes as input an angle of inclination θ and returns the distance you predict according to your model from Problem 1 of Assignment 8. Plot your predicted values along with the values obtained experimentally, and also compute the error

$$E = \sqrt{\frac{1}{17}\sum_{j=1}^{17}(D_j - \hat{D}_j)^2},$$

where D_j denotes experimental distances and $\hat{D}_j$ denotes your model distances. (Notice that in this case you didn't give up any data points for parameter estimation, since the parameter values were obtained from different data.)

4. [10 pts] Continuing with the dart from Problems 2 and 3, we observe that if we have the same amount of confidence in each measurement, we expect to obtain better values for b and $|\vec{v}_0|$ by using all the data in Table 1.

4a. Using the values of b and $|\vec{v}_0|$ from Problem 2 as initial guesses, carry out a nonlinear least squares regression fit for the function $D(\theta; |\vec{v}_0|, b)$, taken to return the horizontal distance traveled by a dart launched with angle of inclination θ, initial velocity $|\vec{v}_0|$, and coefficient of air resistance b.

4b. Compute our standard estimate s for the standard deviation of your fit, and find 95% confidence intervals on your values of $|\vec{v}_0|$ and b from Part (a).

5. [10 pts] The March 4, 1978 issue of a journal called the *British Medical Journal* reported on an influenza epidemic at a boys boarding school. The school had 763 students, and the following data was (more or less[1]) observed:

Day	0	3	4	5	6	7	8	9	10	11	12	13	14
Susceptible	762	740	650	400	250	120	80	50	20	18	15	13	10
Infected	1	20	80	220	300	260	240	190	120	80	20	5	2

In this problem we will model this data with the SIR epidemic model.

5a. Use the central difference derivative approximation to determine values for the parameters a and b. (Notice that the first day you will be able to use is Day 4.) Create a stacked plot of your solutions $y_1(t)$ (susceptible population) and $y_2(t)$ (infected population) to the SIR model, each plotted along with the corresponding data.

5b. Use nonlinear least squares regression to refine your values of a and b from Part (a). Again, create a stacked plot for your model and data.

5c. Compute our standard estimate s for the standard deviation of your fit, and find 95% confidence interval for your parameter values.

[1] I had to take this data from a graph, so it's approximate.