

M647, Spring 2012, Assignment 6, due Friday March 2

1. [10 pts] For the Gompertz model of U.S. population data analyzed in Problem 5 of Assignment 5, carry out the following analyses.

1a. Compute the mean values μ_t associated with your predictions for the populations at time $t = 150, 220$, and 250 (i.e., 1940, 2010, and 2040), and 95% confidence intervals associated with these means.

1b. Analyze the standardized residuals for your linearized equation at 68% and discuss whether or not the assumption of Gaussian distributed errors is justified. Whether it's justified or not, discuss whether or not the standardized residuals for the Gompertz model suggest that it is a better fit to the data than the logistic model.

2. [10 pts] Fit the data stored in *nlregdata2.mat* (available on the course web site) to the relation

$$y = p_1 x_1^{p_2} x_2^{p_3}.$$

2a. Compute our usual standard deviation estimate for your fit, and also find 95% confidence intervals for your parameter values.

2b. Compute means μ_x with error estimates for $\vec{x} = (10, 10)$ and $\vec{x} = (1, 50)$, at 95% confidence.

2c. Analyze the standardized residuals for your fit at 68%.

3. [10 pts] Fit the data stored in *linearsystemregressiondata.mat* (available on the course web site) to the system

$$y_1 = p_1 + p_2 x_1 + p_3 x_2$$

$$y_2 = p_4 + p_5 x_1 + p_2 x_2.$$

(The last term, $p_2 x_2$ is not a typo; I want at least one parameter to appear in both equations.) In particular, do this by defining an appropriate design matrix F and carrying out the following calculations *without scaling your independent variables*.

3a. Compute our usual standard deviation for your fit, and find 95% confidence intervals for the parameters. I simulated this data by taking a particular choice of parameter values. Can you guess what the choice was?

3b. Compute μ_x (which is now a vector) along with error estimates for $\vec{x} = (0, 0)$ and $\vec{x} = (5, 5)$ at 95% confidence.

3c. Analyze the standardized residuals for your fit at 68%.

4. [10 pts] Repeat Problem 3, except this time scale your independent variables by standard deviation.

5. [10 pts] Fit the data stored in *systemregressiondata2.mat* (available on the course web site) to the nonlinear system

$$y_1 = p_1 x_1^{p_2} + p_3 e^{p_4 x_2}$$

$$y_2 = p_5 e^{p_4 x_1} + p_6 x_2^{p_2}.$$

In particular, carry out the following, scaling your data:

5a. Compute our usual standard deviation estimate for your fit, and find 95% confidence intervals on your parameter estimates.

5b. Compute μ_x (which is now a vector) along with error estimates for $\vec{x} = (2.5, 2.5)$ and $\vec{x} = (10, 10)$ at 95% confidence.

5c. Analyze the standardized residuals for your fit at 68%.