Causal.jl:
A Modeling and Simulation Framework for Causal Models

Answers to the Questions/Comments

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We would like to thank Chad Scherrer for contributing to our study with his valuable comments. The text was reviewed accordingly. Below are the specific answers to the questions/comments.

1. (On page 3): *This section is very wordy. and seems to be at a very low level of detail given the length of the article. Possibly more helpful would be*

   - *Comparison to eg. channels or Dagger.jl*
   - *Is this pull based of push based.*

   The paragraph explaining the details of how the data can flow through the connections is removed from the text. Instead, a comparison between the approaches adopted in Causal.jl and Dagger.jl is included.

2. *(On page 3) "outputs are directly dependent on their inputs" is this correct? Aren't outputs always directly dependent on input?*

   The outputs may not *directly* be dependent on the inputs. For example, consider the following simple dynamical system with right-hand-side and readout functions

   $$\dot{x} = f(x, u, t) = -x + u \quad (1)$$

   $$y = g(x, u, t) = x \quad (2)$$

   where x, u, y are the state, input and output at time t, respectively. Note that the output y depends on just the state x, but not the input u.

3. *(On page 4) Typo*

   The typo is corrected.

4. *(On page 4) These quotes are very unusual, especially, the second one. Maybe turn it around a monospace font.*

   The quotes are removed and a monospace font is used, instead.

5. *(On page 4) Very nice plots but there is quite a lot of overplottting. Making the curves semi-transparent would probably help here.*

   Because the chaotic attractors of the dynamical systems given in equations (1) and (2) are very dense naturally, the simulation is performed for a time duration of 15 seconds instead of 100 seconds to make the plots semi-transparent. Also, the line widths of plots are decreased.

6. *(On page 4) "topological" Is this a standard user of this term? I could see it being confused for a topology on the observation space. Not a requirement, but to me "dependency structure" would be more clear.*

   The term "topological structure of a model" is replaced by "dependency structure of a model"

7. *(On page 5) Could there be a better way to visualize this network? Some possibilities:*

   - *Graph layout, eg. GraphViz*
   - *Make nodes smaller ,or maybe even remove.*
   - *Another option - maybe instead visualize ϵ_{ij} matrix directly.*

   The network is re-drawn using GraphPlot.jl. The layout of the network is changed from random-layout to circular-layout make the network be visualized better. Also, the node size is decreased.

8. *(On page 5) A dot is missing in the caption of the Figure 9.*

   The dot is placed.

9. *(On page 5) Is this standard notation? $\boldsymbol{P}$ is often a projection matrix.*

   The notation is not standard. So, the notation $\boldsymbol{P} = diag(p_1, p_2, \dots, p_d)$ is replaced by $\boldsymbol{\Gamma} = diag(\gamma_1, \gamma_2, \dots, \gamma_d)$.

10. *How can a diagonal do this? Wouldn't it instead determine the strength of the interaction?*

    The diagonal matrix $P = diag(p_1, p_2, \dots, p_n)$ determines by which state variable the nodes are connected. For example, consider a very simple network consisting of $n = 2$ nodes each of which is of degree $d = 3$. Consider also that these two nodes are connected to each other with a coupling strength of ϵ, which implies $\epsilon_{11} = -\epsilon_{12} = -\epsilon_{21} = \epsilon_{22} = \epsilon$ From (5), the input to the first node is

    $$\boldsymbol{u}_1 = \epsilon \boldsymbol{P} \boldsymbol{x}_1 - \epsilon \boldsymbol{P} \boldsymbol{x}_2 = \epsilon \boldsymbol{P}(\boldsymbol{x}_1 - \boldsymbol{x}_2) \tag{3}$$

    where $\boldsymbol{x}_i = [x_{i,1}, x_{i,2}, x_{i,3}], i = 1, 2$ and $\boldsymbol{u}_1 = [u_{1,1}, u_{1,2}, u_{1,3}]$. If $\boldsymbol{P} = diag(1, 1, 1)$, than we have,

    $$\begin{aligned} u_{1,1} &= \epsilon(x_{1,1} - x_{2,1}) && (4) \\ u_{1,2} &= \epsilon(x_{1,2} - x_{2,3}) && (5) \\ u_{1,3} &= \epsilon(x_{1,3} - x_{2,3}) && (6) \\ & && (7) \end{aligned}$$

    which implies the nodes are coupled through their all three state variables $x_{i,j},\ i = 1, 2,\ j = 1, 2, 3$. If $\boldsymbol{P} = diag(1, 0, 1)$, then the input to the first node is

    $$\begin{aligned} u_{1,1} &= \epsilon(x_{1,1} - x_{2,1}) && (8) \\ u_{1,2} &= 0 && (9) \\ u_{1,3} &= \epsilon(x_{1,3} - x_{2,3}) && (10) \\ & && (11) \end{aligned}$$

which implies that the nodes are coupled through their first and third states $x_{i,j},\ i = 1, 2,\ j = 1, 3.$. There is no interaction through the second state variables $x_{1,2}$ and $x_{2,2}$.

Therefore, the diagonal matrix $\boldsymbol{P} = diag(p_1, p_2, \ldots, p_d)$ determines through which state variables the nodes are connected. There exists a connection through the state variables $x_{i,j},\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, d$ if $p_j \neq 0,\ j = 1, 2, \ldots, d$.