EEE5069 Adaptive Filter Theory

Adaptive Channel Equalization

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Outline

Intersymbol Interference

Intersymbol interference (ISI)

- is spreading of the transmitted pulses
- is caused by the dispersive nature of the channel
- affects the data transmission rate.

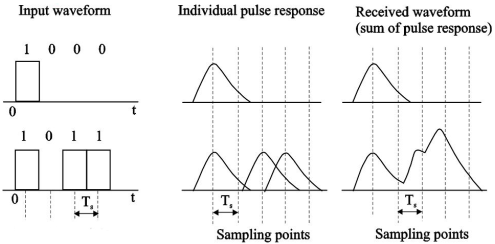


Figure: Intersymbol interference.

Adaptive Channel Equalization

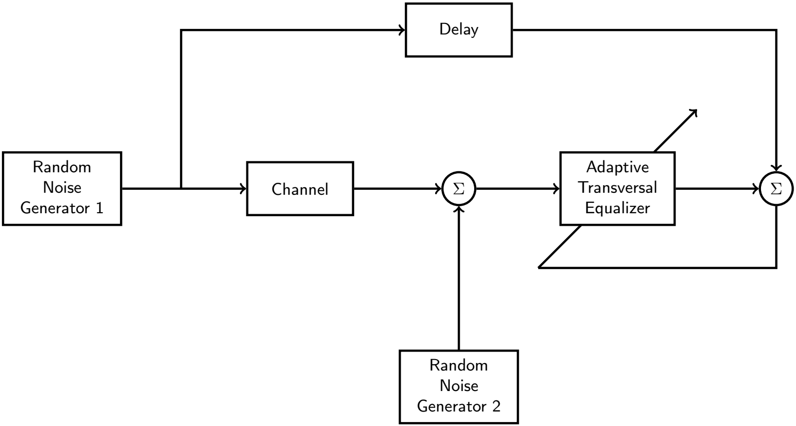


Figure: Block diagram of the adaptive channel equalization.

Tapped-Delay-Line Equalizer

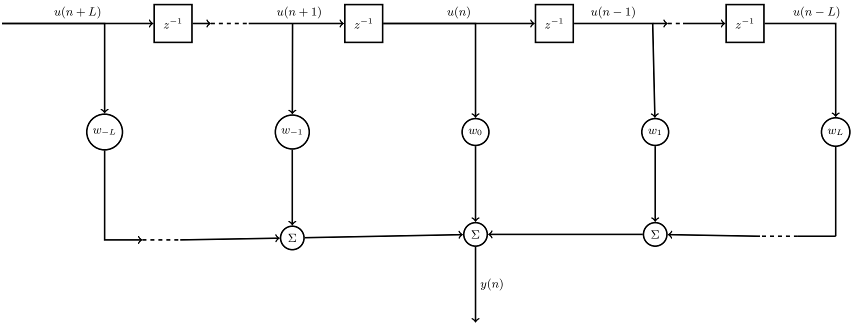


Figure: Block diagram of the tapped-delay-line equalizer.

$$y(n) = \sum_{k=-L}^{L} w_k u(n-k) \tag{1}$$

Problem

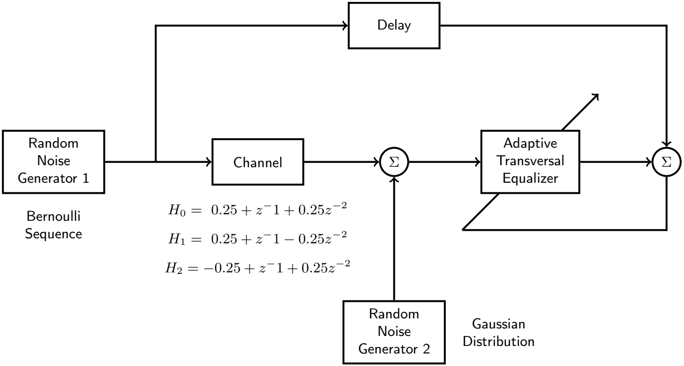


Figure: Block diagram of the channel equalization for the problem.

For three different channel models H_0, H_1, H_2,

- Determine the optimum delay.
- Train the 21-tap equalizer using LMS and RLS.
- Compare the performances.

Least Mean Squares Algorithm

Least Mean Squares (LMS) algorithm,

- is a stochastic gradient algorithm.
- requires no correlation function evaluation or matrix inversion
- favors itself for its simplicity.

Summary of LMS Algortihm

Parameters:	$M =$ the number of taps $\mu =$ step size parameter $0 < \mu < \dfrac{2}{\text{tap input power}}$ tap input power $= \sum_{k=0}^{M-1} E\left[\lvert u(n-k)\rvert^2\right]$
Initialization:	If prior knowledge on the tap-weight vector $\hat{\boldsymbol{w}}(n)$ is available, use it to select an appropriate value for $\hat{\boldsymbol{w}}(0)$. Otherwise, set $\hat{\boldsymbol{w}}(0) = \mathbf{0}$.
Given:	$\boldsymbol{u}(n) = M$-by-1 tap input vector at time n. $d(n) =$ desire response at time n.
To be computed:	$\hat{\boldsymbol{w}}(n+1) =$ estimate of the tap-weight vector at time $n+1$
Computation:	for $n = 0, 1, \ldots$ compute $e(n) = d(n) - \hat{\boldsymbol{w}}^H(n)\boldsymbol{u}(n)$ $\hat{\boldsymbol{w}}(n+1) = \hat{\boldsymbol{w}}(n) + \mu \boldsymbol{u}(n) e^*(n)$

Recursive Least Squares Algorithm

Recursive Least Squares (RLS) algorithm,

- is a an extension of method of least squares
- exhibits fast rate of convergence
- requires weighty computational complexity.

Summary of RLS Algortihm

Parameters:	M = the number of taps δ = step size parameter(small positive constant)
Initialization:	If prior knowledge on the tap-weight vector $\hat{\boldsymbol{w}}(n)$ is available, use it to select an appropriate value for $\hat{\boldsymbol{w}}(0)$. Otherwise, set $\hat{\boldsymbol{w}}(0) = \mathbf{0}$. $\boldsymbol{P}(0) = \delta^{-1}\boldsymbol{I}$
Given:	$\boldsymbol{u}(n)$ = M-by-1 tap input vector at time n. $d(n)$ = desire response at time n.
To be computed:	$\hat{\boldsymbol{w}}(n+1)$ = estimate of the tap-weight vector at time $n+1$
Computation:	for $n = 0, 1, \ldots$ compute

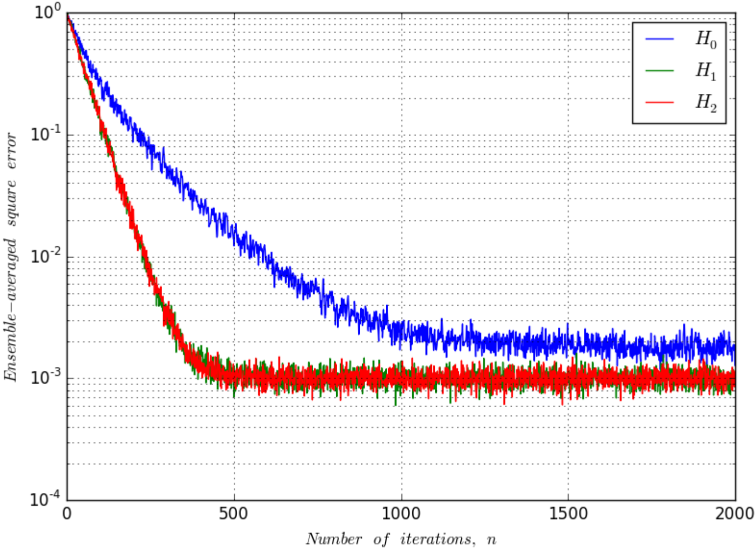
$$\boldsymbol{k}(n) = \frac{\lambda^{-1}\boldsymbol{P}(n-1)\boldsymbol{u}(n)}{1+\lambda^{-1}\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)}$$

$$\xi(n) = d(n) - \hat{\boldsymbol{w}}(n-1)\boldsymbol{u}(n)$$

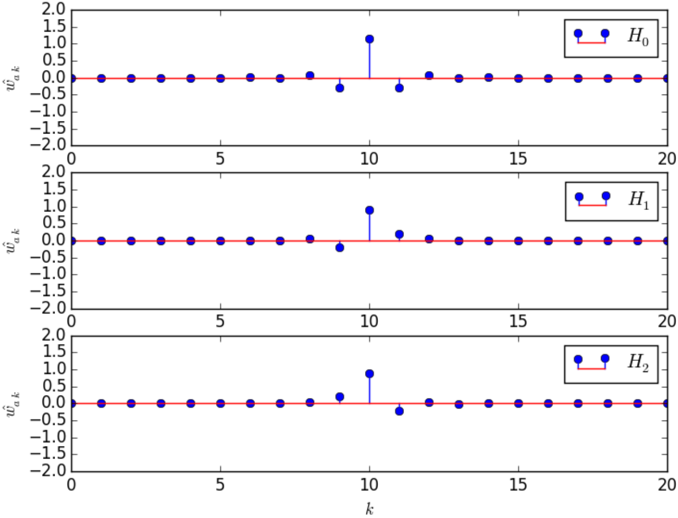
$$\hat{\boldsymbol{w}}(n) = \hat{\boldsymbol{w}}(n-1) + \boldsymbol{k}(n)\xi^*(n)$$

$$\boldsymbol{P}(n) = \lambda^{-1}\boldsymbol{P}(n-1) - \lambda^{-1}\boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)$$

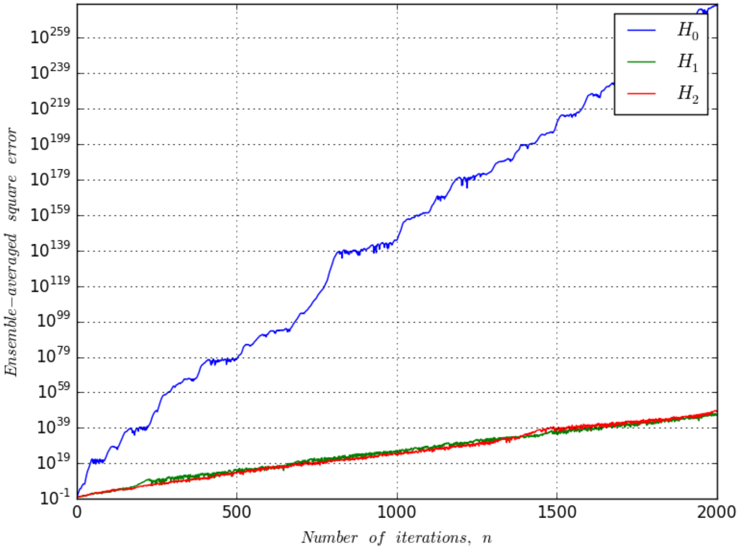
LMS: SNR=30dB, $\mu = 0.01$



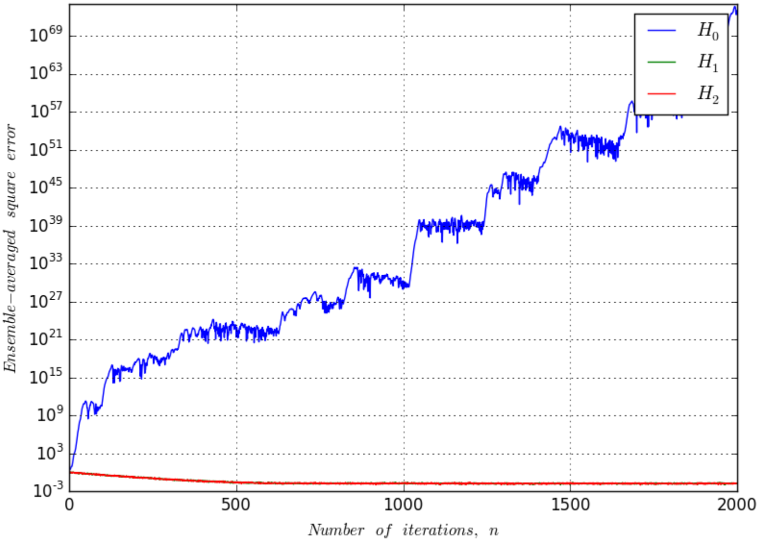
LMS: SNR=30dB, $\mu = 0.01$



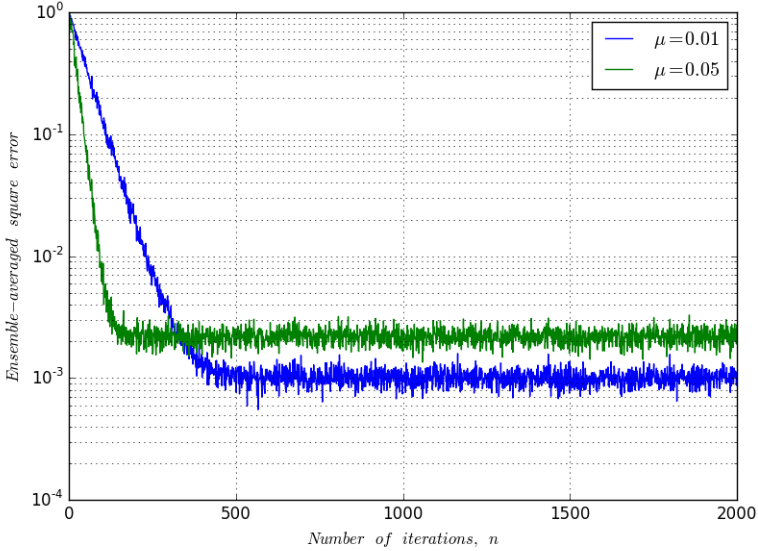
LMS: SNR=30dB, $\mu = 0.1$



LMS: SNR=30dB, $\mu = 0.08$

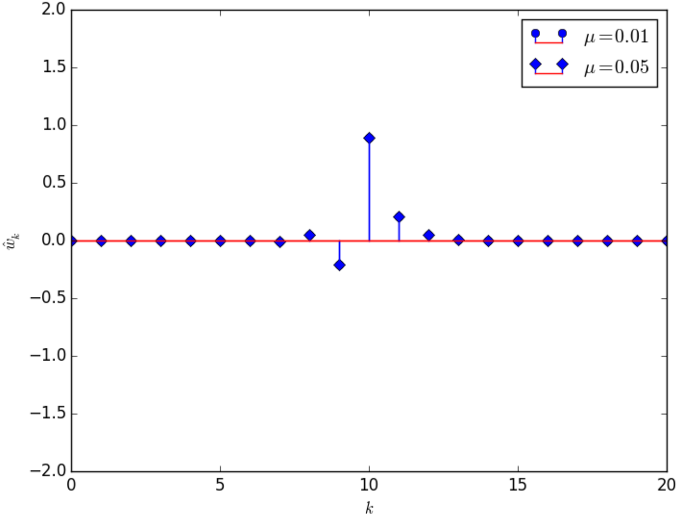


LMS: SNR=30dB

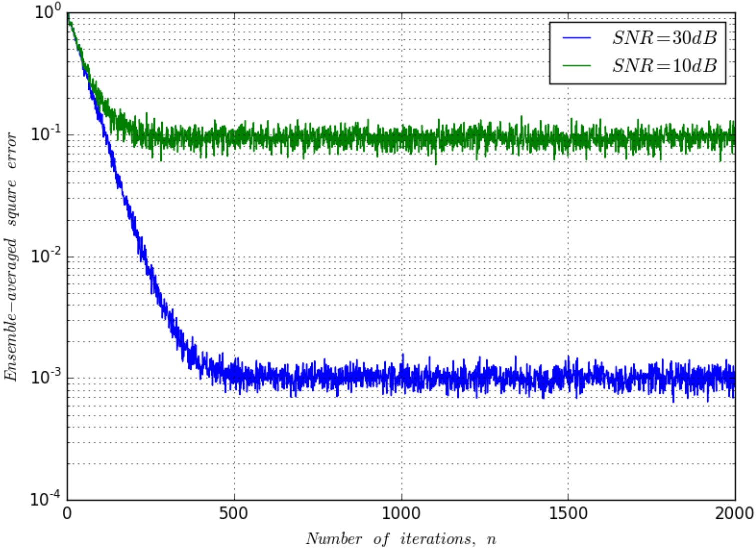


$\mu=0.01$
$\mu=0.05$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
500
1000
1500
2000
Ensemble−averaged square error
Number of iterations, n

LMS: SNR=30dB

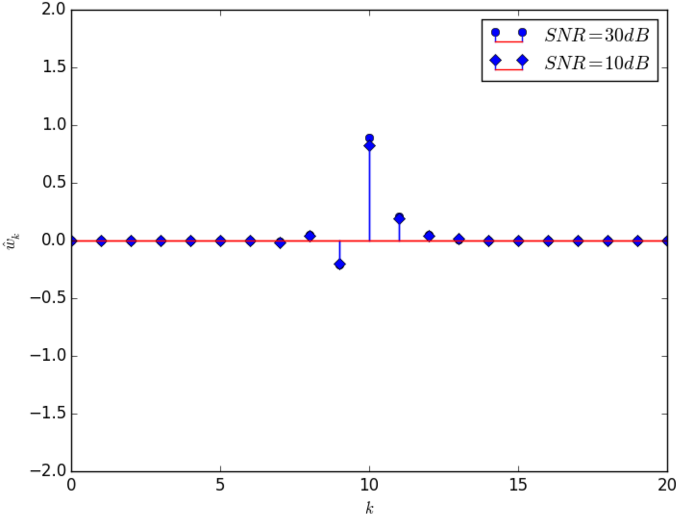


LMS: $\mu = 0.01$

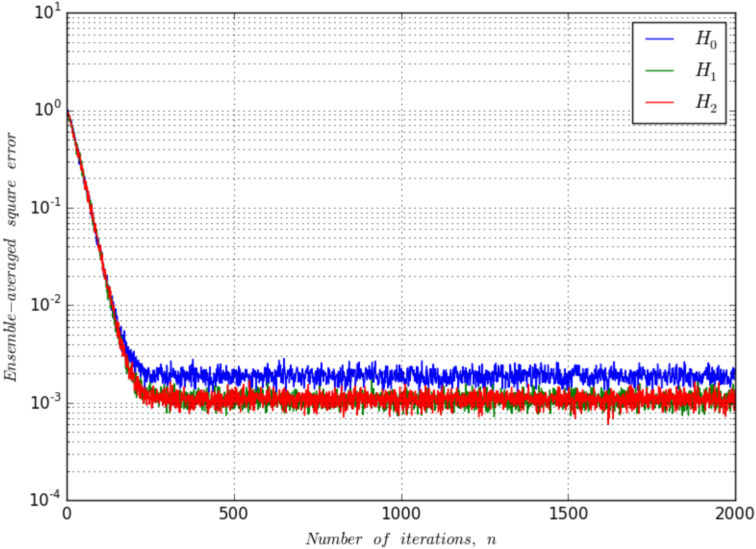


$SNR = 30dB$
$SNR = 10dB$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
500
1000
1500
2000
Ensemble−averaged square error
Number of iterations, n

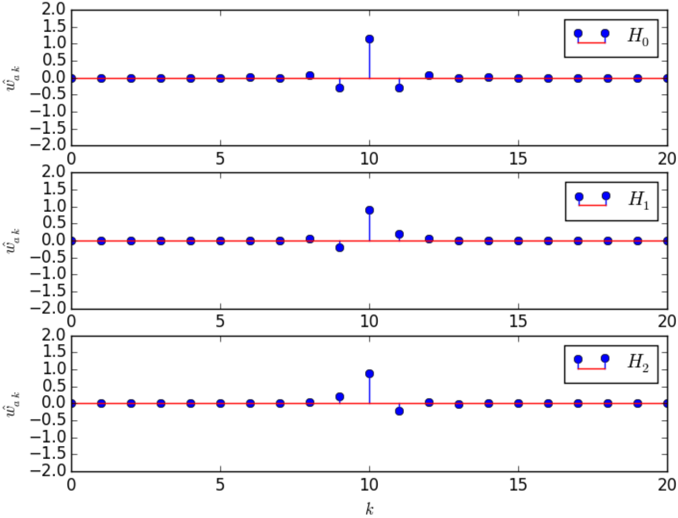
LMS: $\mu = 0.01$



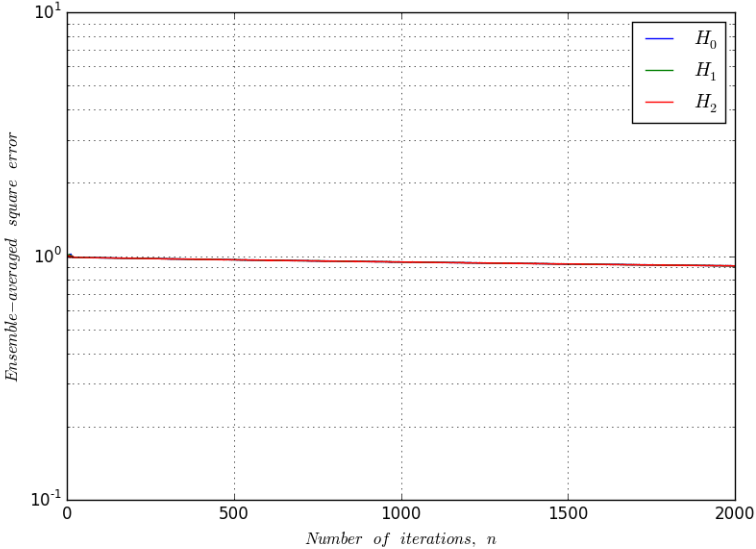
RLS: SNR = 30 dB, $\lambda = 0.98$, $\delta = 0.005$



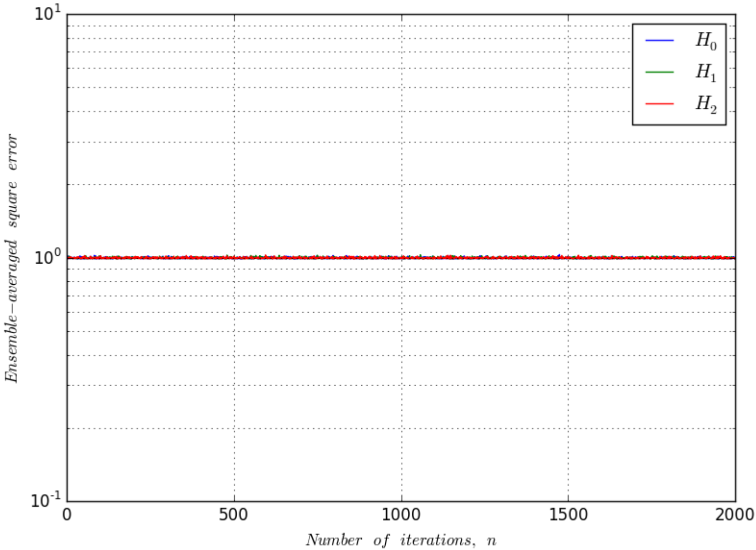
RLS: SNR = 30 dB, $\lambda = 0.98$, $\delta = 0.005$



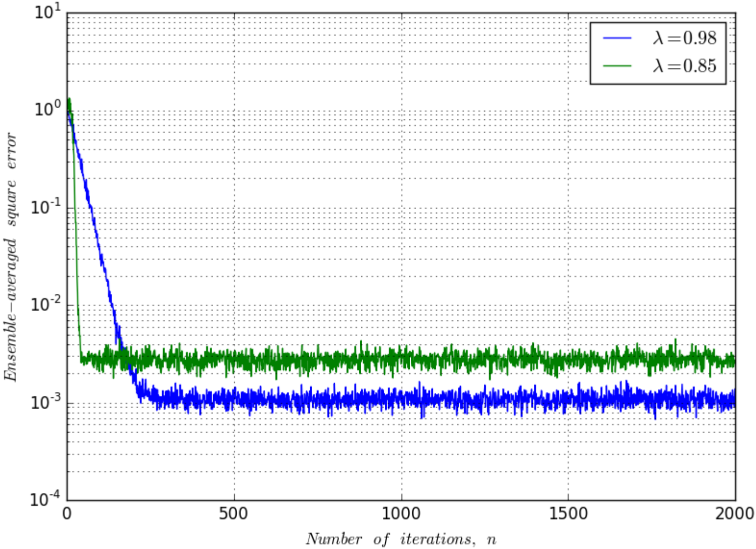
RLS: SNR = 30 dB, $\lambda = 1.0$, $\delta = 0.005$



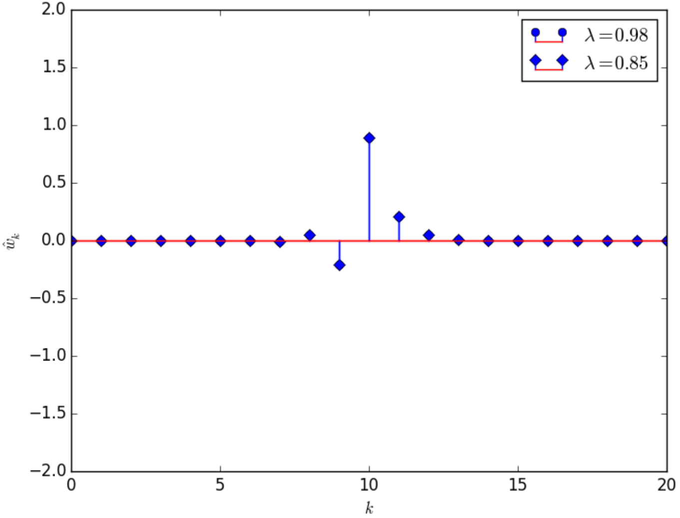
RLS: SNR = 30 dB, $\lambda = 5.0$, $\delta = 0.005$



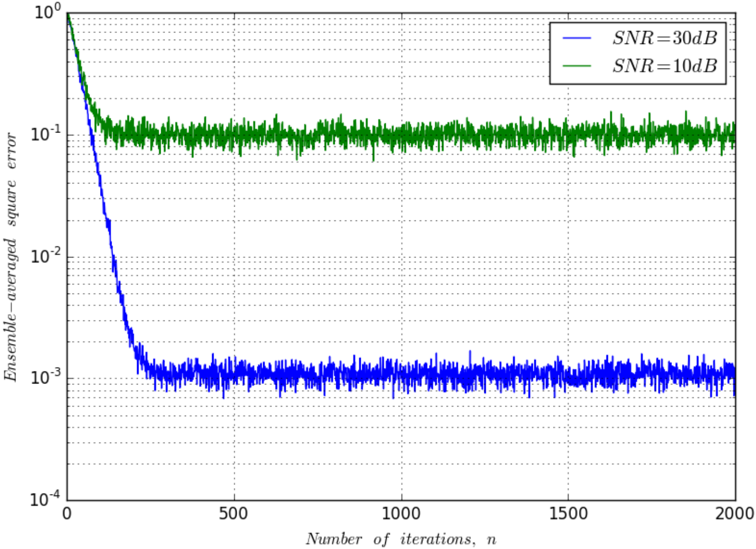
RLS: SNR = 30 dB, $\delta = 0.005$



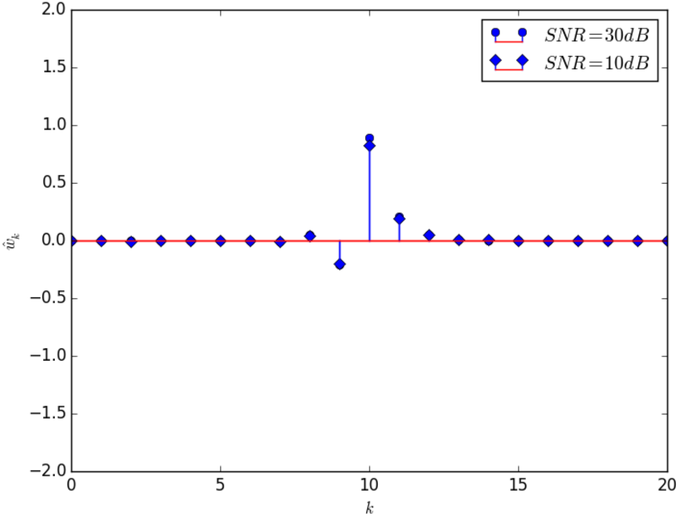
RLS: SNR = 30 dB, $\delta = 0.005$



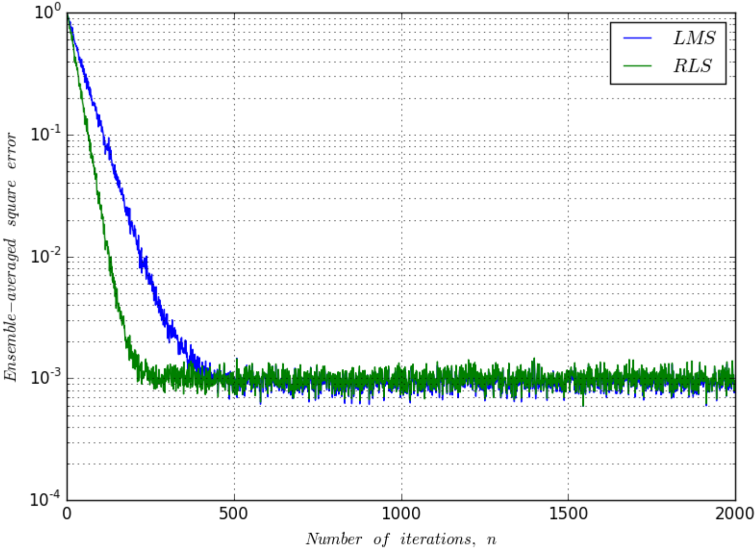
RLS: $\lambda = 0.98$, $\delta = 0.005$



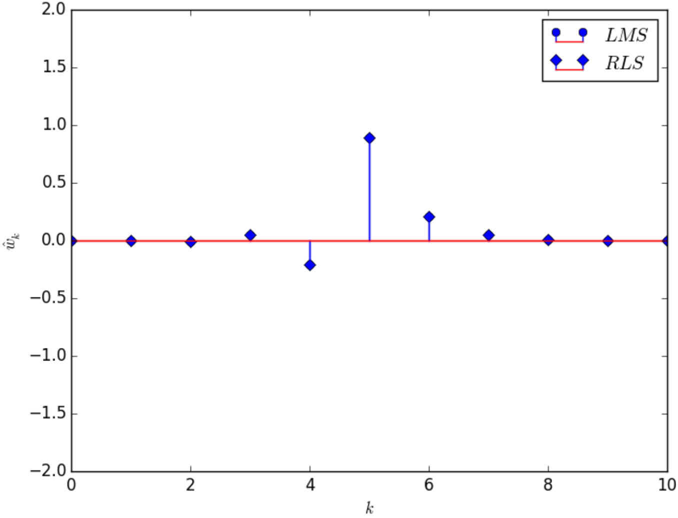
RLS: $\lambda = 0.98$, $\delta = 0.005$



RLS v.s. LMS: SNR $= 30$ dB, $\lambda = 0.98$, $\delta = 0.005$, $\mu = 0.01$



RLS v.s. LMS: SNR $= 30$ dB, $\lambda = 0.98$, $\delta = 0.005$, $\mu = 0.01$



Summary

- Equalizer impulse response is symmetric.
- Channel impulse response symmetry determines equalizer impulse response symmetry.
- For LMS, channel impulse response symmetry matters, but for RLS it does not.
- Ambient noise affects affects the equalizer convergence error.
- LMS is the simpler and RLS is the faster one.