U-ERRE
Universidad Regiomontana

Métodos Numéricos

Tercer Parcial

Reporte del Método de Romberg

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## Metodo de Romberg:

- Definicion: El metodo de Romberg es un algoritmo de integracion numerica que combina la extrapolacion de Richardson con la regla del trapecio para obtener aproximaciones de integrales definidas con alta precision.

- ~~Definicion~~ Antecedentes

• Surge como mejora a la regla del trapecio compuesta, evitando aumentar el numero de evaluaciones
• Basado en la extrapolacion de Richardson

### Relaciones con otros metodos

• metodo del trapecio
• Cuadratura Gaussiana
• metodo de Simpson

### formula del metodo

$$R(i,j) = R(i,j-1) + \frac{R(i,j-1) - R(i-1,j-1)}{4^j - 1}$$

## Algoritmo

- calcular $R(0,0)$
- Refinar iterativamente (para $i = 1, 2, \ldots N$)
- Detenerse

## Aplicaciones en ITC

- simulaciones de rendimiento
- visión por computadora
- machine learning

### Ejemplo

| | $K_0$ | $K_1$ | $K_2$ |
|---|---|---|---|
| $n=2$ | $T_2^0$ | $T_2^1$ | $T_2^2$ |
| $n=4$ | $T_4^0$ | $T_4^1$ | |
| $n=8$ | $T_8^0$ | | |

Romberg $\dfrac{4\,T_{2n}^{k-1} - T_n^{k-1}}{4^k - 1} = T_n^k$

$$\int_1^3 \frac{x}{x^4+2}\,dx$$

Trapecio

$$\int^3 \frac{x}{x^4+1} = \frac{1}{2}$$

| C1 | $x_n$ | $f(x_n)$ |
|---|---|---|
| $n=2$ | $x_0$ 1 | 0.3 |
| $a=1$ | $x_1$ 2 | 0.11761 |
| $b=3$ | $x_2$ 3 | 0.03654 |
| $n=\frac{3-1}{2}=1$ | | |

$$=\frac{1}{2}\left(0.5+2(0.11264)+0.03\right)$$

$$T_2 = 0.3859$$

$n=4$
$a=1$
$b=3$
$h=\frac{3-1}{4}=0.5$

| $x_n$ | | $f(x_n)$ |
|---|---|---|
| $x_0$ | 1 | 0.5 |
| $x_1$ | 1.5 | 0.24742 |
| $x_2$ | 2 | 0.11764 |
| $x_3$ | 2.5 | 0.0624 |
| $x_4$ | 3 | 0.03658 |
| $x_5$ | 3.5 | 0.02316 |
| $x_6$ | 4 | 0.01336 |

$= \frac{0.5}{2}\left(0.5+2(0.24742)+2(0.11764)\right.$
$+2(0.0624)+2(0.03658)$
$\left.+2(0.02360)+0.01336\right)$

$T_4 = 0.3478$

$n=8$
$a=1$
$b=3$
$h=\frac{3-1}{8}=0.25$

| $x_n$ | $f(x_n)$ |
|---|---|
| 1 | 0.5 |
| 1.25 | 320/881 |
| 1.5 | 24/97 |
| 1.75 | 448/2657 |
| 2 | 2/17 |
| 2.25 | 576/6817 |
| 2.50 | 40/641 |
| 2.75 | 704/14842 |
| 3 | 3/82 |

$$T_8^0 = \frac{0.25}{2}\left(0.5 + 2\left(\frac{320}{331}\right) + 2\left(\frac{24}{97}\right) + 2\left(\frac{448}{2257}\right)\right.$$
$$\left. + 2\left(\frac{2}{17}\right) + 2\left(\frac{576}{6817}\right) + 2\left(\frac{40}{641}\right) + 2\left(\frac{304}{14897}\right) + \frac{3}{32}\right)$$
$$= 0.33983$$

$K_{est} = 0.3372$

| ~~n~~ | $K_0$ | $K_1$ | $K_2$ |
|---|---|---|---|
| $n=2$ | 0.3854 | | |
| $n=4$ | 0.3478 | $T_4^1$ | |
| $n=8$ | 0.3398 | | |

| | $K_0$ | $K_1$ ($T_2^1$) | $K_2$ ($T_2^2$) |
|---|---|---|---|
| $n=2$ | 0.3854 | 0.3351 | 0.3372 |
| $n=4$ | 0.3478 | 0.3371 | |
| $n=8$ | 0.3398 | | |

$$T_2^1 = \frac{4^1 T_4^0 - T_2^0}{4^1 - 1} = \frac{4(0.3478) - 0.3854}{3}$$
$$= 0.3351$$

$$T_4^1 = \frac{4^1 T_8^0 - T_4^0}{4^1 - 1} = \frac{4(0.3398) - 0.3478}{3}$$
$$= 0.3371$$

$$T_2^2 = \frac{4^2 T_4^1 - T_2^1}{4^2 - 1} = \frac{4^2(0.3371) - 0.3351}{15} = 0.3372$$

Tarea

Funcion: $f(x) = \ln(4x^2 + 4)$

intervalo: $a = -3$, $b = 3$

$$\int_{-3}^{3} \ln(4x^2 + 4)\, dx$$

Trapecio: $T_1$

$T_1 \approx 22.1334$

$T_2 \approx 15.2256$

$T_3 \approx 11.4602$

$T_4 \approx 10.5829$

$$I_{j,k} \approx \frac{4^{k-1} \cdot I_{j+1,k-1} - I_{j,k-1}}{4^{k-1} - 1}$$

$$I_{j,2} = \frac{4^1 \cdot I_{j+1,1} - I_{j,1}}{4^1 - 1} = \frac{4 \cdot I_{j+1,1} - I_{j,1}}{3}$$

$T_1$ y $T_2$

$$I_{1,2} = \frac{4 \cdot 15.2256 - 22.1334}{3} = \frac{60.9024 - 22.1334}{3} = \frac{38.76}{3} \approx 12.923$$

$T_2$ y $T_3$

$$I_{2,2} = \frac{4 \cdot 11.4602 - 15.2256}{3} = \frac{45.8408 - 15.2256}{3} = \frac{30.61}{3} \approx 10.205$$

$I_3$ y $I_4$:

$$I_{3,2} = \frac{4 \cdot 10.5829 - 11.4602}{3} = \frac{42.3316 - 11.4602}{3}$$

$$= \frac{30.8714}{3} \approx 10.290$$

Columna 3

$$I_{j,3} = \frac{4^2 \cdot I_{j+1,2} - I_{j,2}}{4^2 - 1} = \frac{16 \cdot I_{j+1,2} - I_{j,2}}{15}$$

$I_{1,2}$ y $I_{2,2}$

$$I_{1,3} = \frac{16 \cdot 10.205 - 12.923}{15} = \frac{163.28 - 12.923}{15}$$

$$= \frac{150.357}{15} \approx 10.024$$

$I_{2,2}$ y $I_{3,2}$

$$I_{2,3} = \frac{16 \cdot 10.290 - 10.205}{15} = \frac{164.64 - 10.205}{15}$$

$$= \frac{154.435}{15} \approx 10.296$$

$$I_{1,4} = \frac{4^3 \cdot I_{2,3} - I_{2,3}}{4^3 - 1} = \frac{64 \cdot 10.296}{65} =$$

$$= \frac{658.944 - 10.024}{63} = \frac{648.92}{63}$$

$$\boxed{\approx 10.302}$$