Digital Signal Processing

Module 1: Introduction

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What is a signal?

Description of the evolution of a physical phenomenon

Examples:

- temperature (weather)
- pressure (sound)
- magnetic deviation (recorded sound)
- gray level on paper (photograph)
- ...

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100
90
80
70
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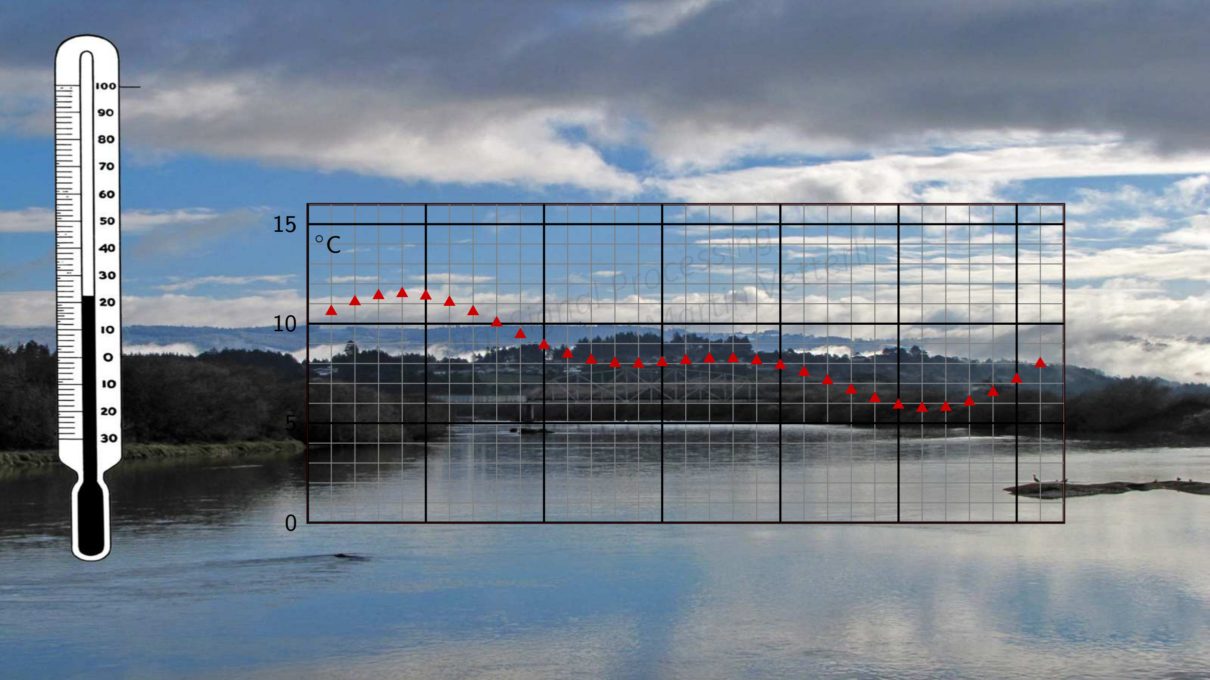
30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Digital Signals

Key ingredients:

- discrete time
- discrete amplitude

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The circle point out (from left to right): Pythagoras, Parmenides, Plato, Euclid. Green for philosophers, red for mathematicians

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A
B


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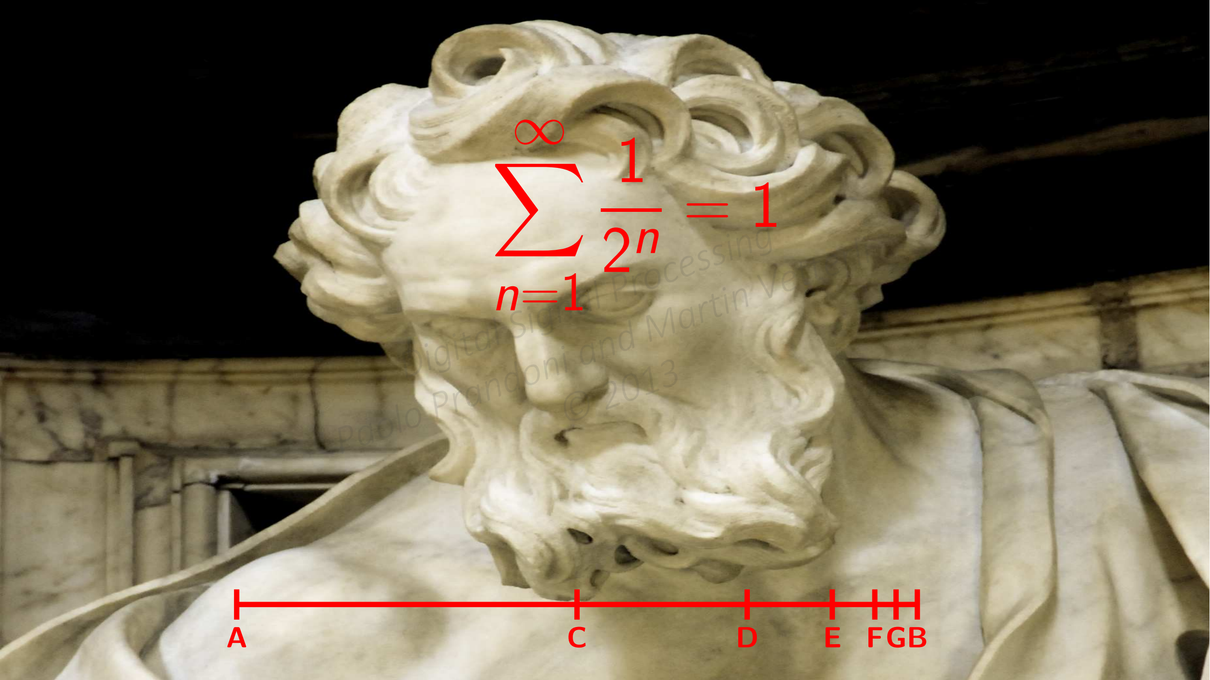
A
C
B


A
C
D
E
F
G
B



$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

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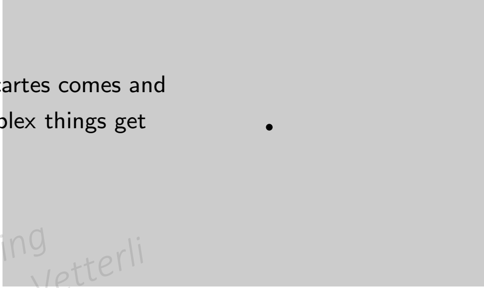
Idea here is: you got a point, nameless geometrical entity. Descartes comes and puts a reference around it, point gets a name. Then more complex things get names (such as lines) and can be described in terms of algebra



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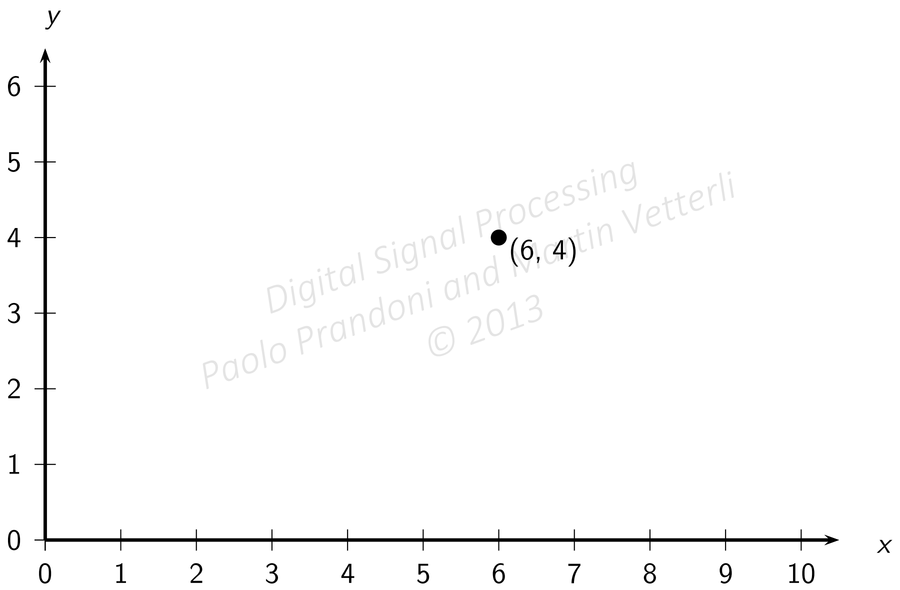

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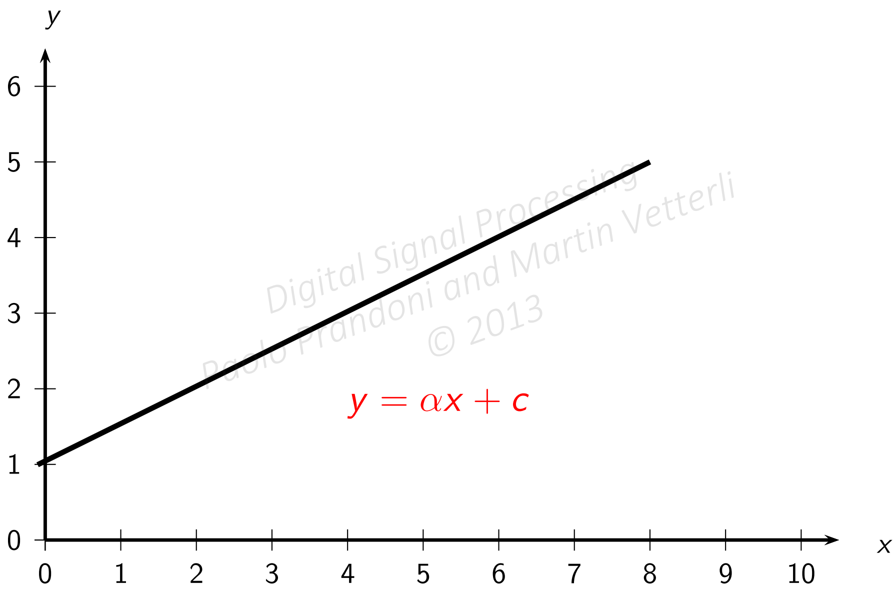

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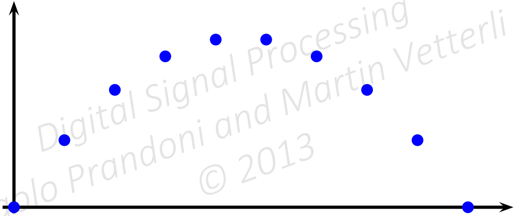

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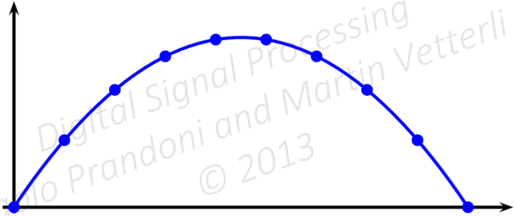
$$\vec{x}(t) = \vec{v}_0 t + (1/2)\vec{g}\, t^2$$

Galileo, 1638

Calculus: ideas at the service of war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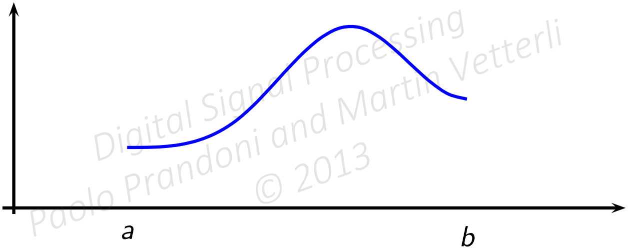
$$\vec{x}(t) = \vec{v}_0 t + (1/2)\vec{g}\, t^2$$

Galileo, 1638

Continuous-time signal processing: the average



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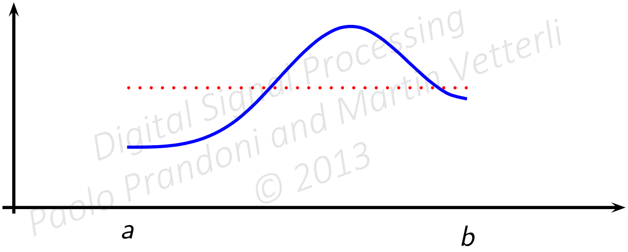
$$\bar{x} = \frac{1}{b-a}\int_a^b f(t)dt$$



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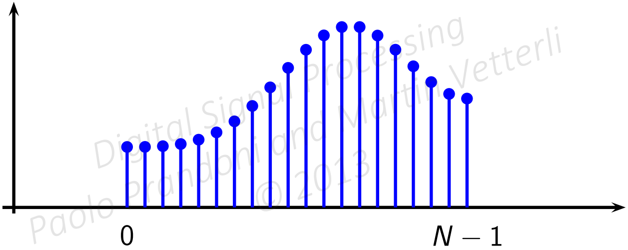
$$\bar{x} = \frac{1}{b-a} \int_a^b f(t)dt$$

Discrete-time signal processing: the average



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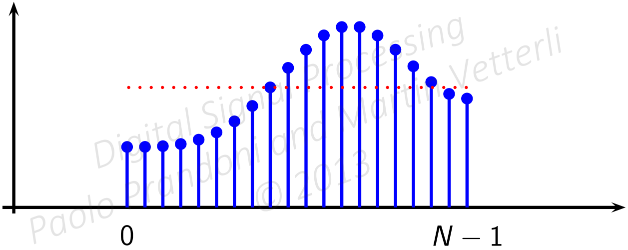
$$\bar{x} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$$

Discrete-time signal processing: the average



$$\bar{x} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]$$

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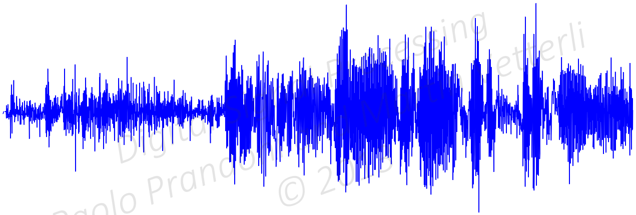
Are we missing pieces?

What if the signal is "too fast"?



$$f(t) = ?$$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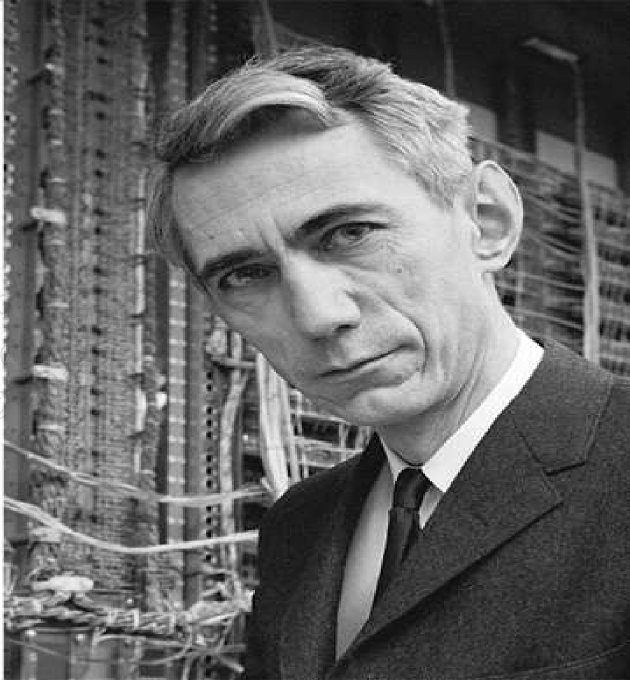

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The Sampling Theorem (1920)

Under appropriate "slowness" conditions for $x(t)$ we have:

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\pi(t - nT_s)/T_s)}{\pi(t - nT_s)/T_s}$$

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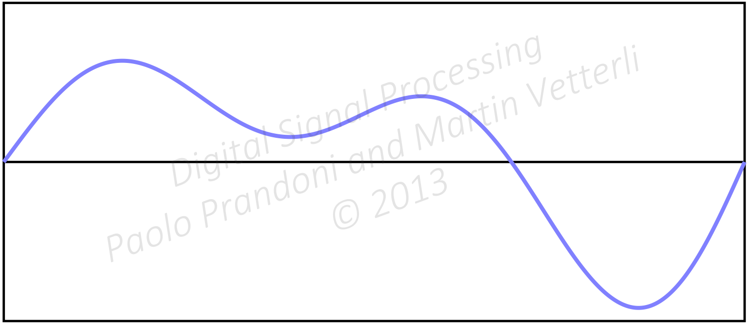

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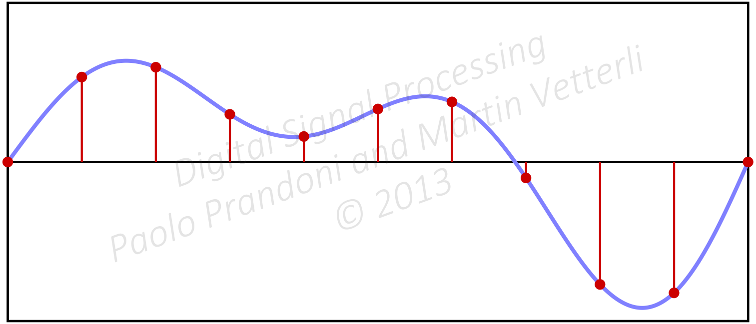

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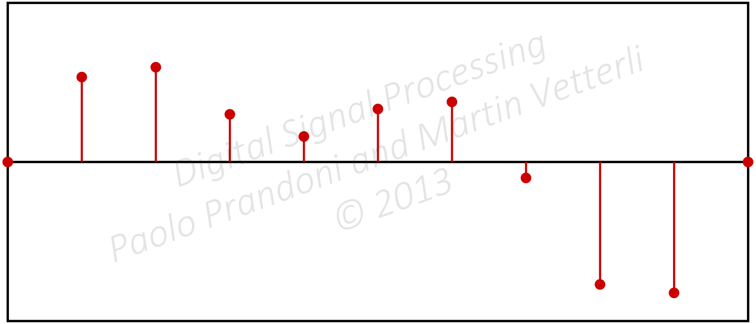

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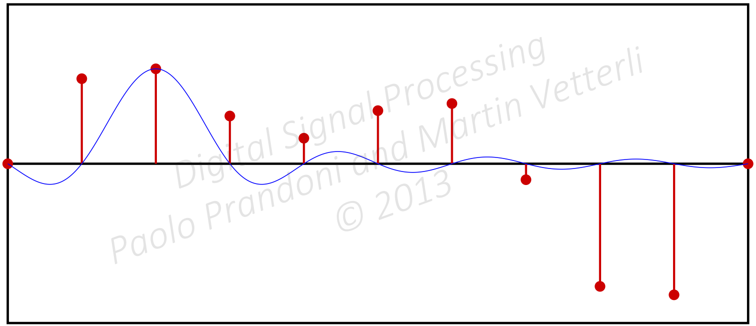

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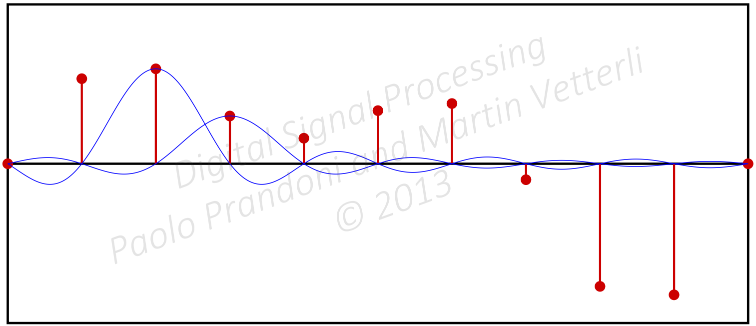
Digital Signals



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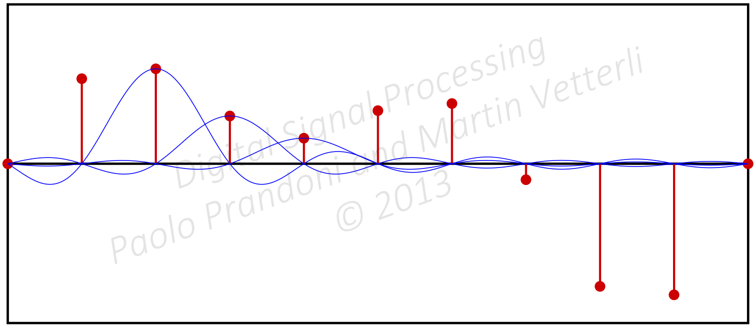

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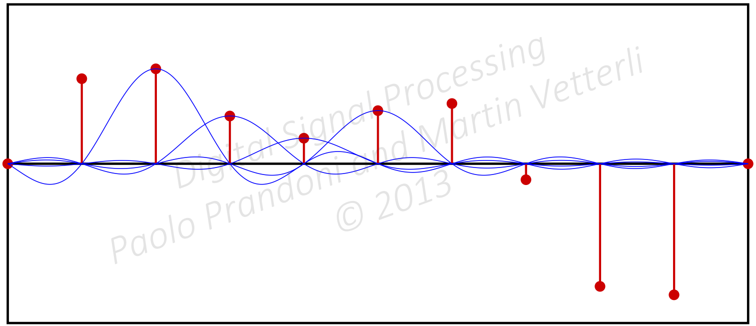

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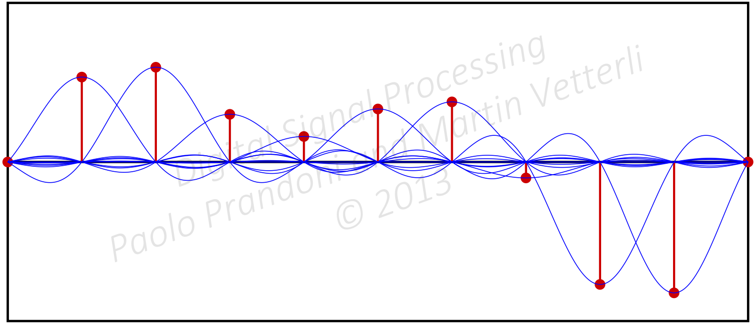

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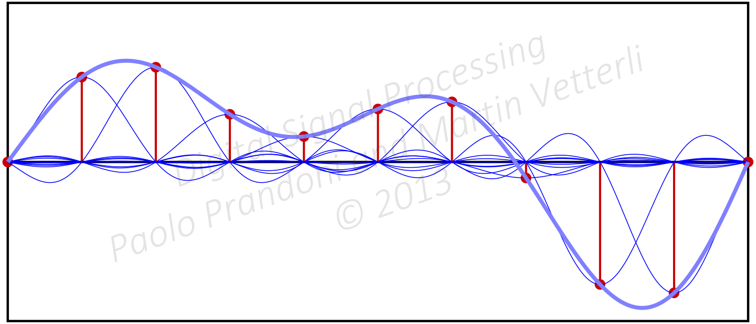

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The world is analog, the computer is digital

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So, what is resolution, really?

Key ingredients:

- discrete time
- **discrete amplitude**

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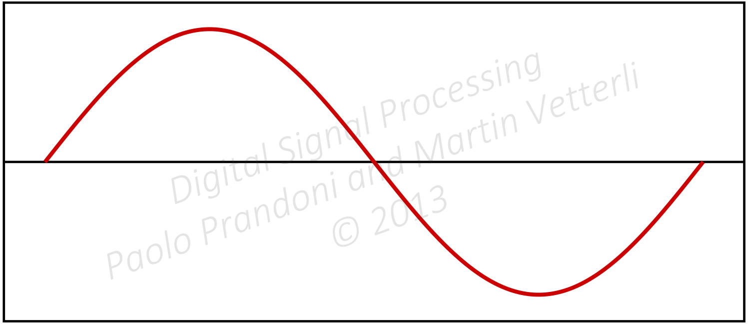
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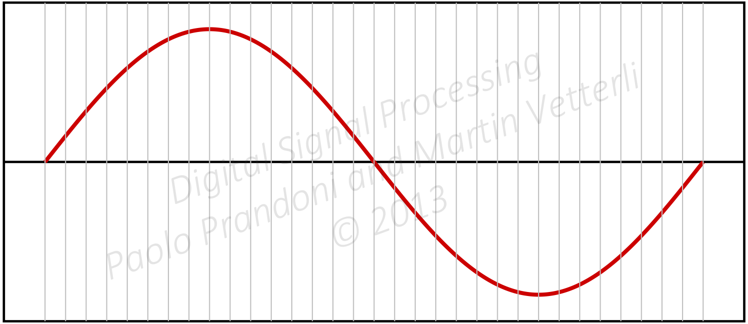
$x(t)$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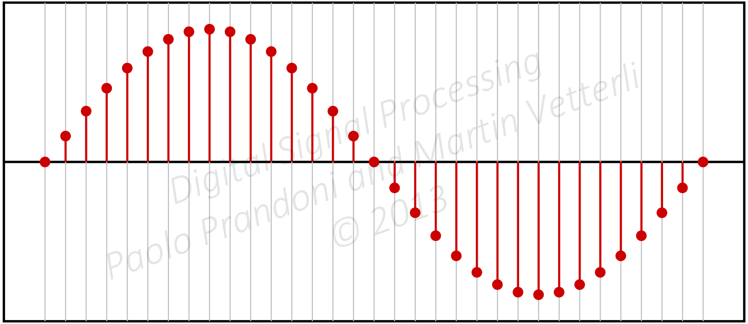
$x(t)$



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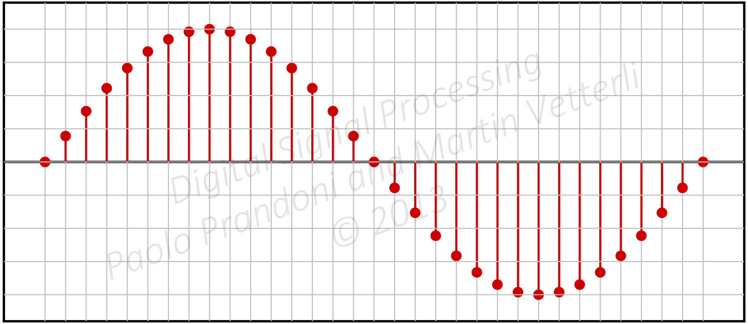
$x[n]$

Digital Signals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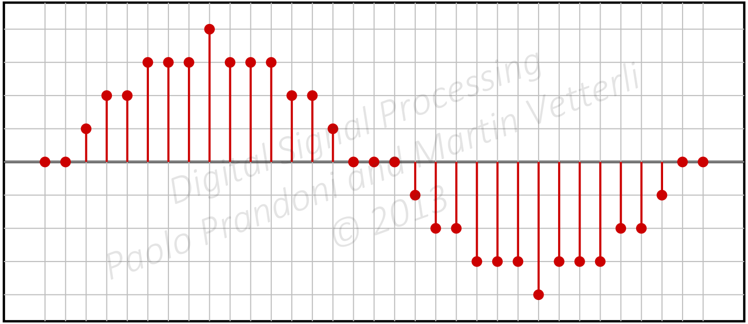
$x[n]$

Digital Signals



$\hat{x}[n]$

Digital amplitude

Why it is important:

- storage
- processing
- transmission

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Data storage

Analog storage:

paper, wax cylinders, reel-to-reel, vinyl, compact cassette, VHS, Betamax, silver plates, Kodachrome, Super8, 8-Track, microfilm, ...

Digital storage:

$\{0, 1\}$

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Digital storage:

$$\{0, 1\}$$


Just one
MicroSD card
stores more than
the rest combined...
25 years
of storage
BeOS
Re formed
4D Compiler
IBM
PRODIGY
Verbatim
DataLifePlus
QNX
radius
redhat
Compact
Auto Switch
Netscape
Navigator Gold
Microsoft Word
MICROSOFT
TURBOLINUX
WORKSTATION
Quarterdeck
CompuServe
HEADLINE HARRY
SPRYNET
Quicken
iMac
COMPAQ
COMPUTER SHOPPER

```
extern double a[N];     // The a's coefficients
extern double b[M];     // The b's coefficients

static double x[M];     // Delay line for x
static double y[N];     // Delay line for y

double GetOutput(double input)
{
  int k;

  // Shift delay line for x:
  for (k = N-1; k > 0; k--)
    x[k] = x[k-1];

  // new input value x[n]:
  x[0] = input;

  // Shift delay line for y:
  for (k = M-1; k > 0; k--)
    y[k] = y[k-1];

  double y = 0;
  for (k = 0; k < M; k++)
    y += b[k] * x[k];
  for (k = 1; k < M; k++)
    y -= a[k] * y[k];

  // New value for y[n]; store in delay line
  return (y[0] = y);
}
```

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Data transmission

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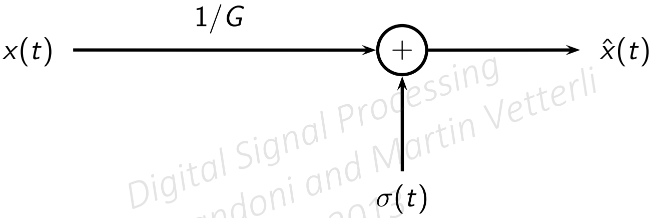
What happens to analog signals



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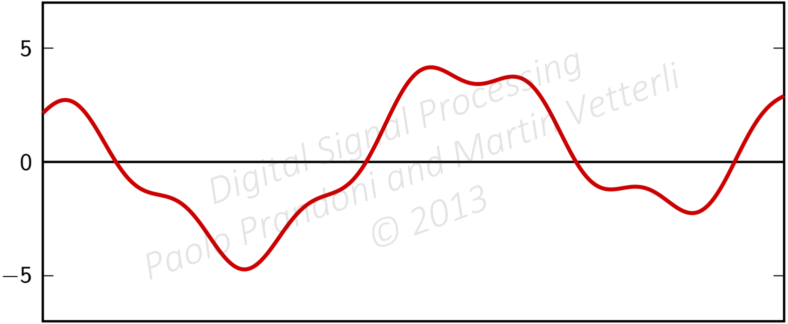
$$\hat{x}(t) = x(t)/G + \sigma(t)$$

What happens to analog signals



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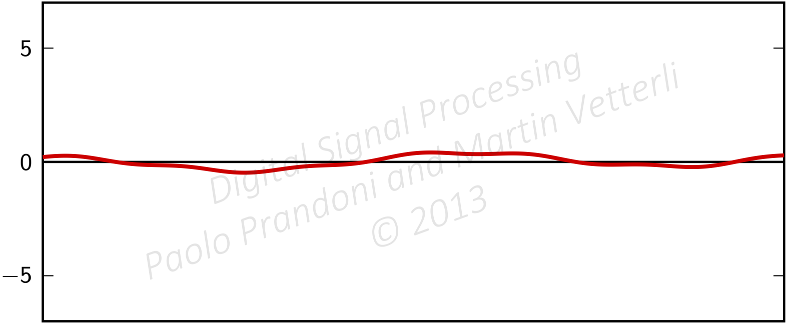
$x(t)$

What happens to analog signals



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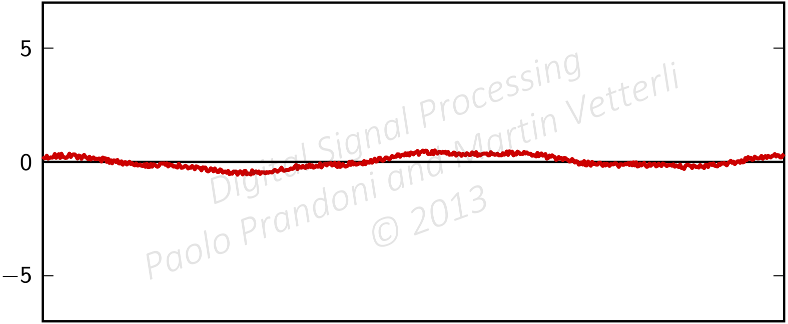
$x(t)/G$

What happens to analog signals



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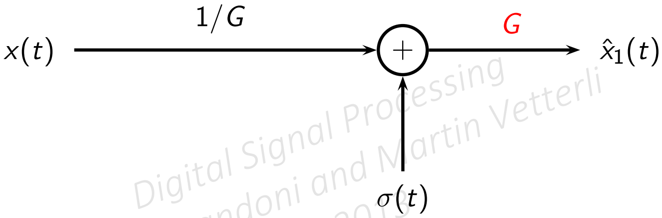
$$x(t)/G + \sigma(t)$$

We can amplify to compensate attenuation



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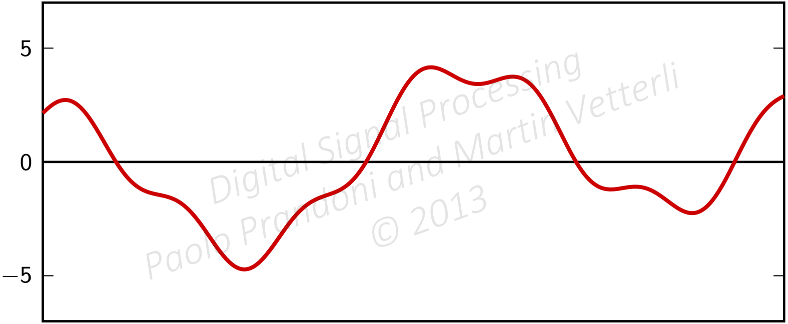
but: $\hat{x}_1(t) = x(t) + G\sigma(t)$

Transmission of analog signals



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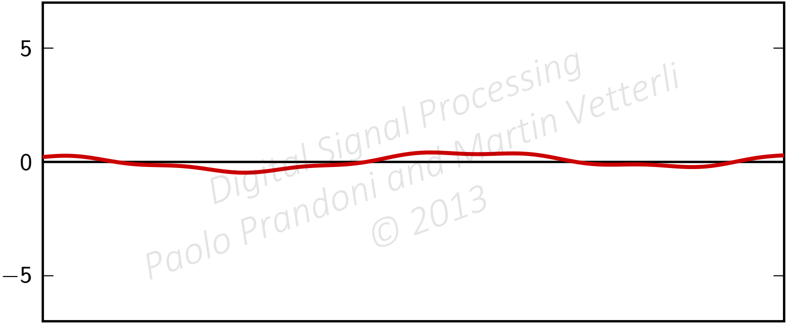
$x(t)$

Transmission of analog signals



$x(t)/G$

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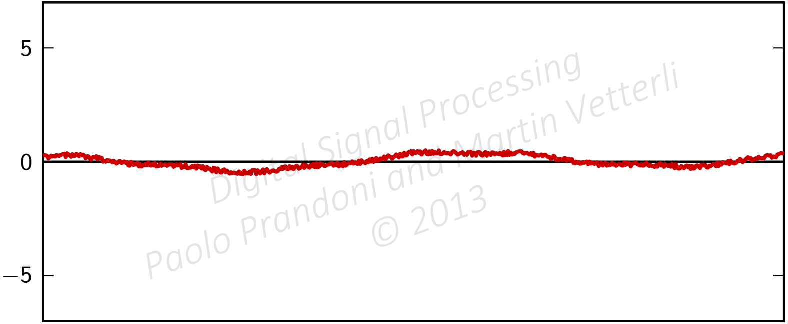
Transmission of analog signals



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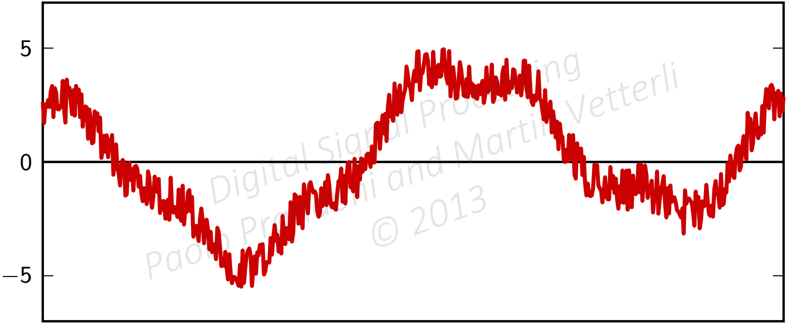
$$x(t)/G + \sigma(t)$$

Transmission of analog signals



$$\hat{x}_1(t) = G[x(t)/G + \sigma(t)] = x(t) + G\sigma(t)$$

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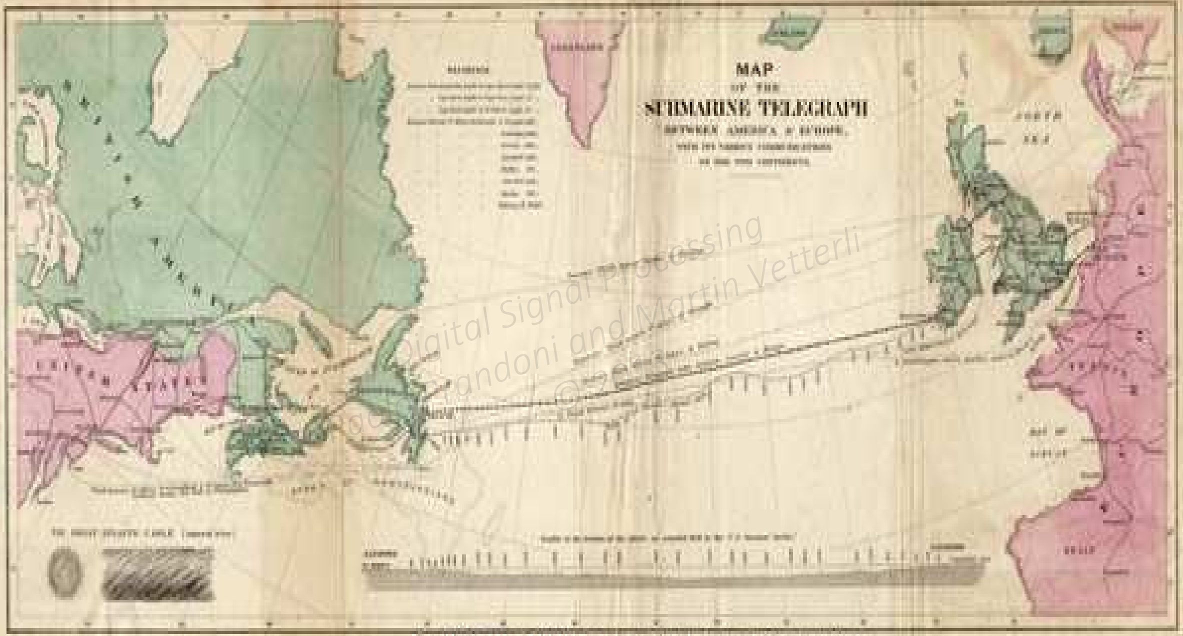

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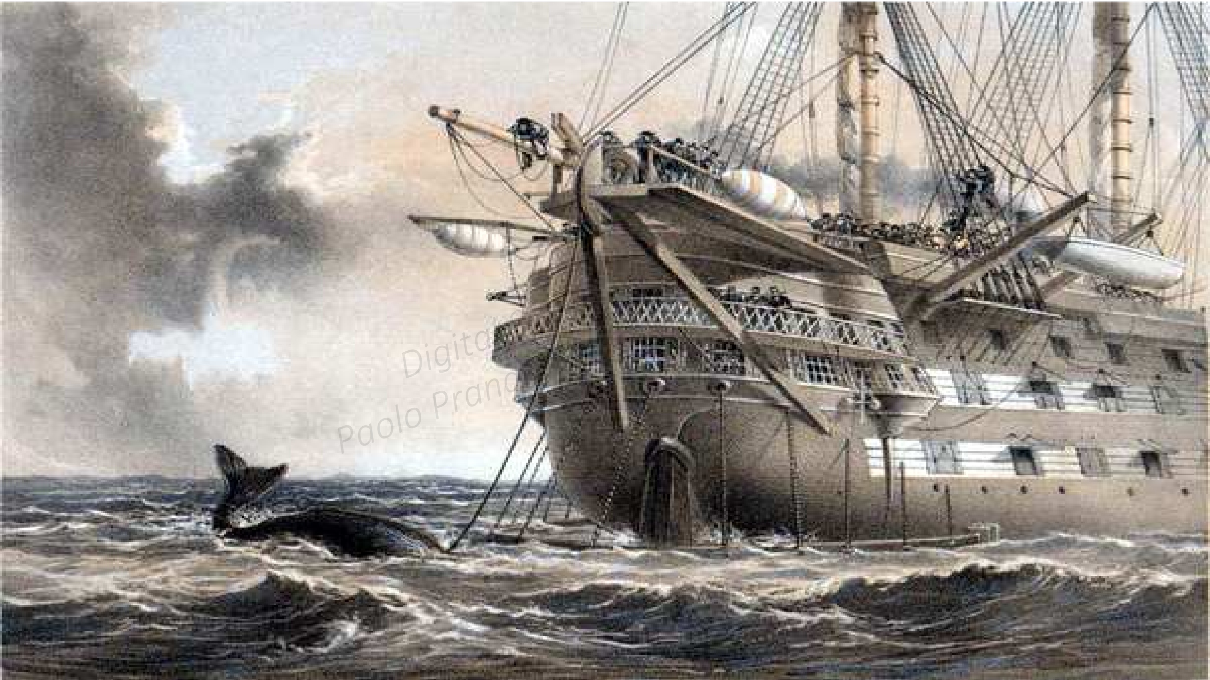
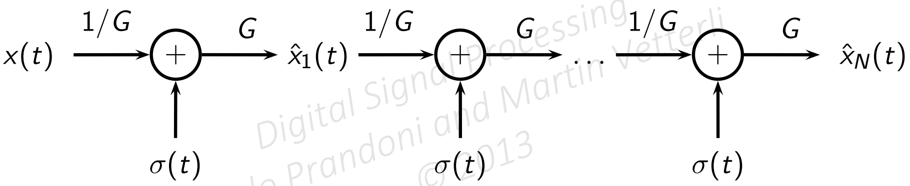


Transmitting a signal overseas

For a long, long channel we need repeaters



$$\hat{x}_N(t) = x(t) + NG\sigma(t)$$

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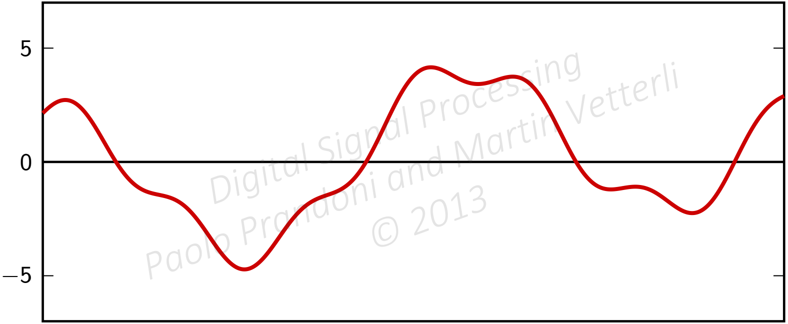
Transmission of analog signals



$x(t)$

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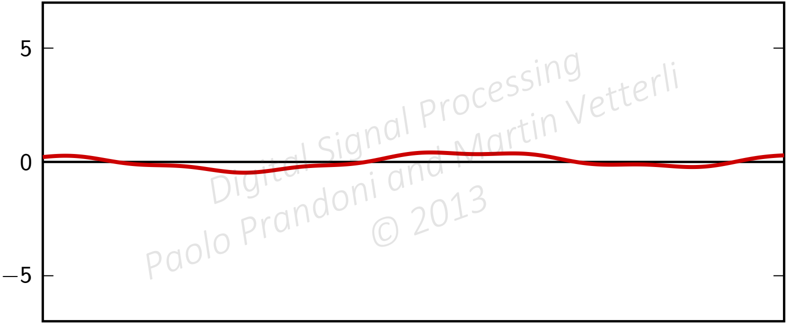

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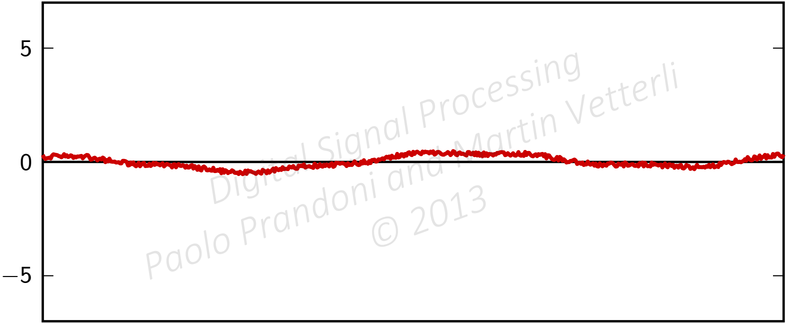
$x(t)/G$



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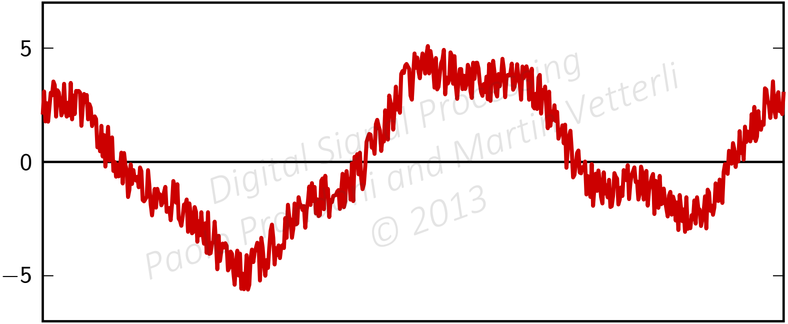
$$x(t)/G + \sigma(t)$$

Transmission of analog signals



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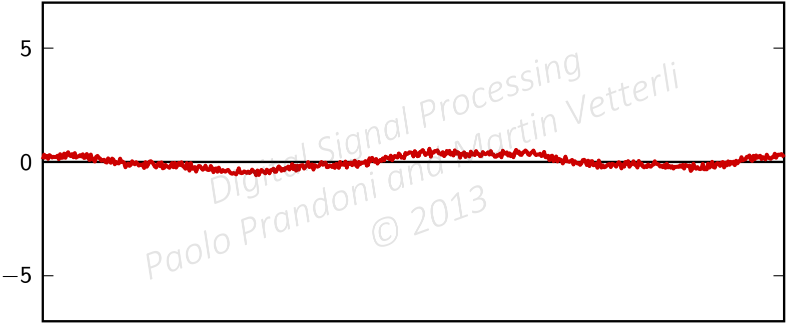
$$\hat{x}_1(t) = G[x(t)/G + \sigma(t)] = x(t) + G\sigma(t)$$

Transmission of analog signals



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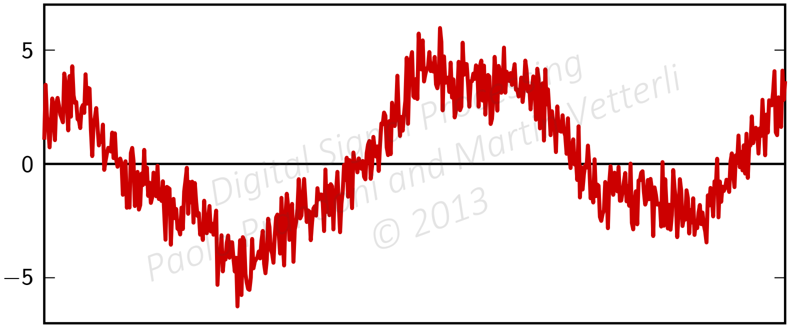
$$\hat{x}_1(t)/G + \sigma(t)$$

Transmission of analog signals



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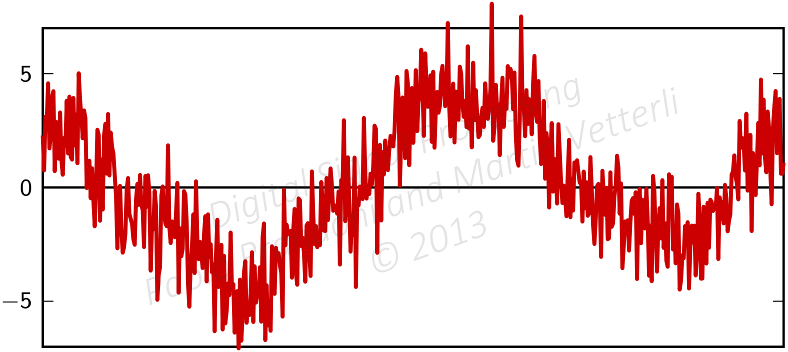
$$\hat{x}_2(t) = G[\hat{x}_1(t)/G + \sigma(t)] = x(t) + 2G\sigma(t)$$



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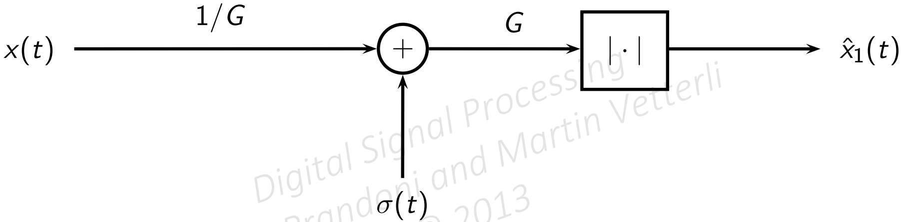
$$\hat{x}_N(t) = x(t) + NG\sigma(t)$$

In digital signals we can threshold



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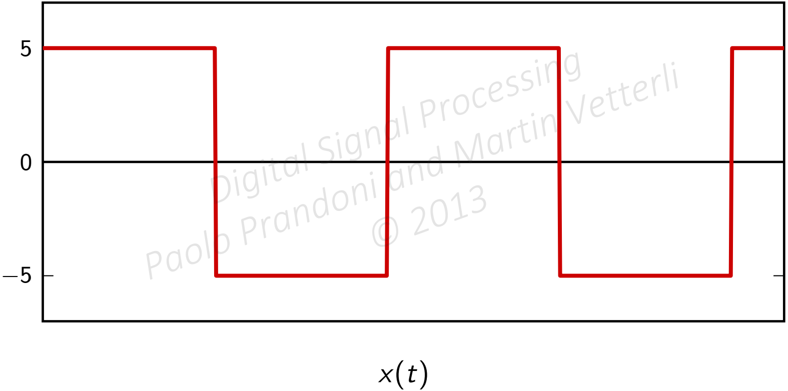
$$\hat{x}_1(t) = \mathrm{sgn}[x(t) + G\sigma(t)]$$


5
0
−5
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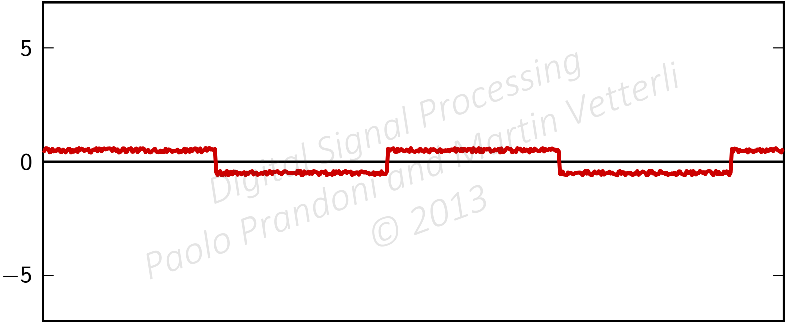
x(t)

Transmission of quantized signals



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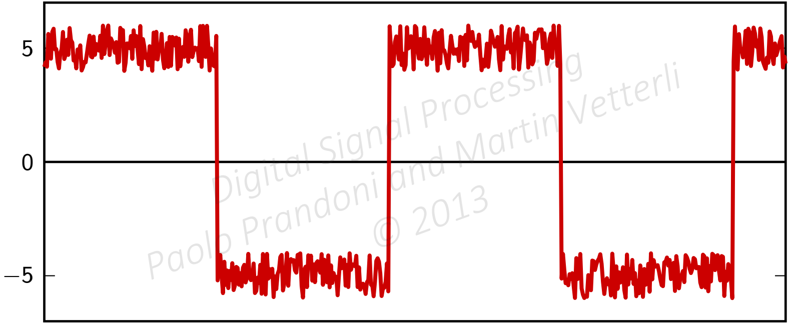
$$x(t)/G + \sigma(t)$$

Transmission of quantized signals



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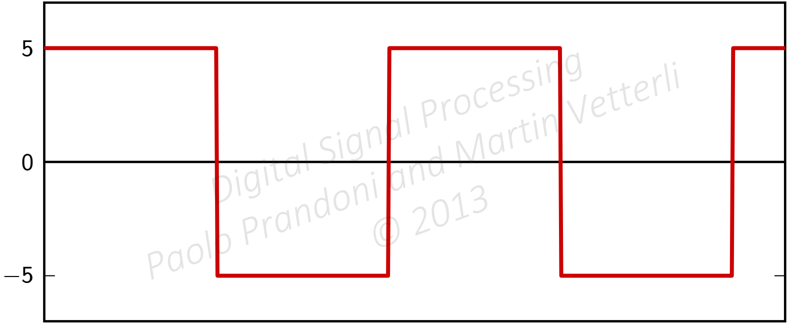
$$G[x(t)/G + \sigma(t)] = x(t) + G\sigma(t)$$

Transmission of quantized signals



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$$\hat{x}_1(t) = G\,\mathrm{sgn}[x(t) + G\sigma(t)]$$

Digital data throughputs

- Transatlantic cable:
  - 1866: 8 words per minute ($\approx$5 bps)
  - 1956: AT&T, coax, 48 voice channels ($\approx$3Mbps)
  - 2005: Alcatel Tera10, fiber, 8.4 Tbps (8.4×10^{12} bps)
  - 2012: fiber, 60 Tbps
- Voiceband modems
  - 1950s: Bell 202, 1200 bps
  - 1990s: V90, 56Kbps
  - 2008: ADSL2+, 24Mbps

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Final words

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END OF MODULE 1

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