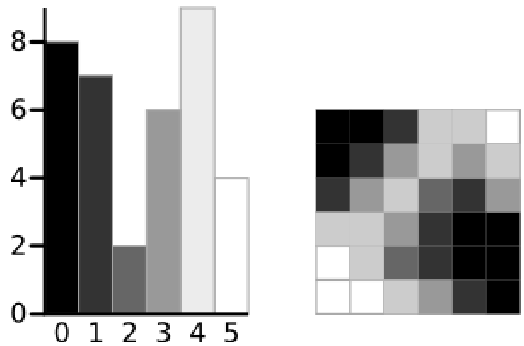
Otsu Thresholding

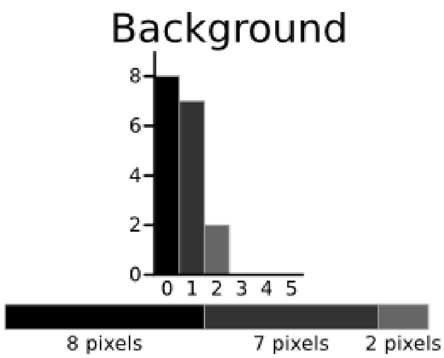
Otsu's thresholding method involves iterating through all the possible threshold values and calculating a measure of spread for the pixel levels each side of the threshold, i.e. the pixels that either fall in foreground or background. The aim is to find the threshold value where the sum of foreground and background spreads is at its minimum.

The algorithm will be demonstrated using the simple 6x6 image shown below. The histogram for the image is shown next to it. To simplify the explanation, only 6 greyscale levels are used.

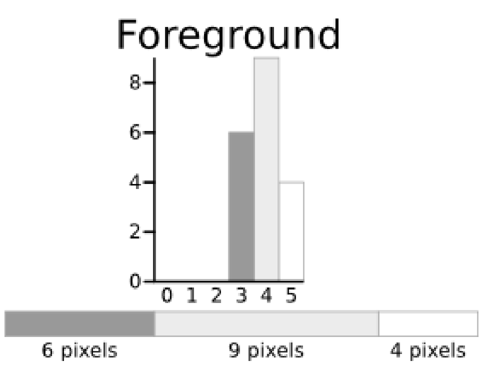


A 6-level greyscale image and its histogram

The calculations for finding the foreground and background variances (the measure of spread) for a single threshold are now shown. In this case the threshold value is 3.



$$
\begin{aligned}
\text{Weight} \quad W_b &= \frac{8+7+2}{36} = 0.4722 \\
\text{Mean} \quad \mu_b &= \frac{(0\times 8)+(1\times 7)+(2\times 2)}{17} = 0.6471 \\
\text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2\times 8)+((1-0.6471)^2\times 7)+((2-0.6471)^2\times 2)}{17} \\
&= \frac{(0.4187\times 8)+(0.1246\times 7)+(1.8304\times 2)}{17} \\
&= 0.4637
\end{aligned}
$$



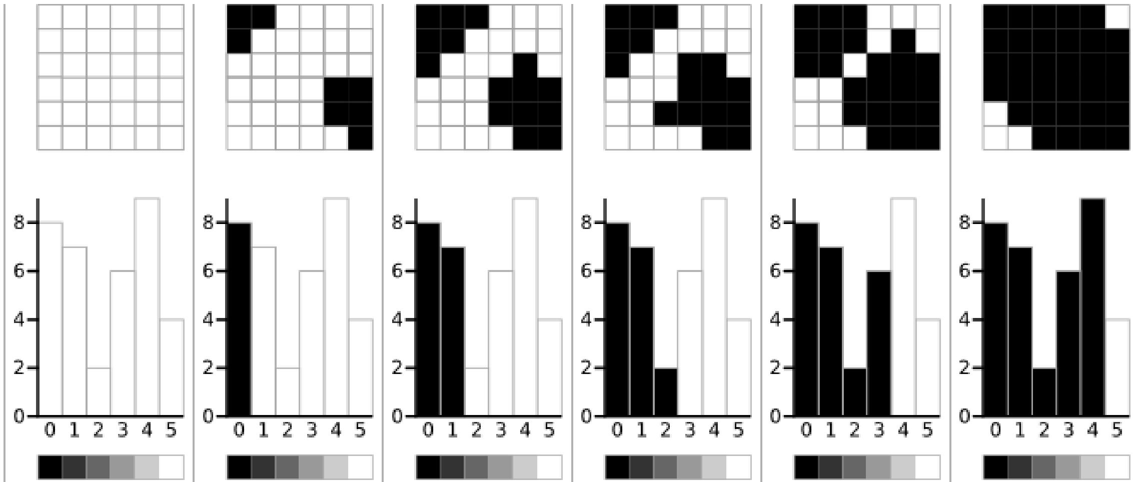
$$
\begin{aligned}
\text{Weight} \quad W_f &= \frac{6+9+4}{36} = 0.5278 \\
\text{Mean} \quad \mu_f &= \frac{(3\times 6)+(4\times 9)+(5\times 4)}{19} = 3.8947 \\
\text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2\times 6)+((4-3.8947)^2\times 9)+((5-3.8947)^2\times 4)}{19} \\
&= \frac{(4.8033\times 6)+(0.0997\times 9)+(4.8864\times 4)}{19} \\
&= 0.5152
\end{aligned}
$$

The next step is to calculate the 'Within-Class Variance'. This is simply the sum of the two variances multiplied by their associated weights.

$$
\begin{aligned}
\text{Within Class Variance} \quad \sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2 &= 0.4722 * 0.4637 + 0.5278 * 0.5152 \\
&= 0.4909
\end{aligned}
$$

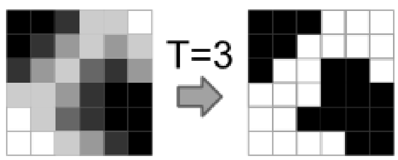
This final value is the 'sum of weighted variances' for the threshold value 3. This same calculation needs to be performed for all the possible threshold values 0 to 5. The table below shows the results for these calculations. The highlighted column shows the values for the threshold calculated above.

Threshold	T=0	T=1	T=2	T=3	T=4	T=5



Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2_b = 0$	$\sigma^2_b = 0$	$\sigma^2_b = 0.2489$	$\sigma^2_b = 0.4637$	$\sigma^2_b = 1.4102$	$\sigma^2_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2_f = 3.1196$	$\sigma^2_f = 1.9639$	$\sigma^2_f = 0.7755$	$\sigma^2_f = 0.5152$	$\sigma^2_f = 0.2130$	$\sigma^2_f = 0$
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\boldsymbol{\sigma^2_W = 0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$

It can be seen that for the threshold equal to 3, as well as being used for the example, also has the lowest sum of weighted variances. Therefore, this is the final selected threshold. All pixels with a level less than 3 are background, all those with a level equal to or greater than 3 are foreground. As the images in the table show, this threshold works well.



Result of Otsu's Method

This approach for calculating Otsu's threshold is useful for explaining the theory, but it is computationally intensive, especially if you have a full 8-bit greyscale. The next section shows a faster method of performing the calculations which is much more appropriate for implementations.

A Faster Approach

By a bit of manipulation, you can calculate what is called the *between class* variance, which is far quicker to calculate. Luckily, the threshold with the maximum *between class* variance also has the minimum *within class* variance. So it can also be used for finding the best threshold and therefore due to being simpler is a much better approach to use.

$$
\begin{aligned}
\text{Within Class Variance} \quad \sigma_W^2 &= W_b\,\sigma_b^2 + W_f\,\sigma_f^2 \quad \text{(as seen above)} \\
\text{Between Class Variance} \quad \sigma_B^2 &= \sigma^2 - \sigma_W^2 \\
&= W_b(\mu_b - \mu)^2 + W_f(\mu_f - \mu)^2 \quad (\text{where } \mu = W_b\,\mu_b + W_f\,\mu_f) \\
&= W_b\,W_f\,(\mu_b - \mu_f)^2
\end{aligned}
$$

The table below shows the different variances for each threshold value.

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\boldsymbol{\sigma^2_W = 0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$
Between Class Variance	$\sigma^2_B = 0$	$\sigma^2_B = 1.5928$	$\sigma^2_B = 2.5635$	$\boldsymbol{\sigma^2_B = 2.6287}$	$\sigma^2_B = 2.1417$	$\sigma^2_B = 0.8705$