exploration_of_propagation_in_small_graphs

July 26, 2020

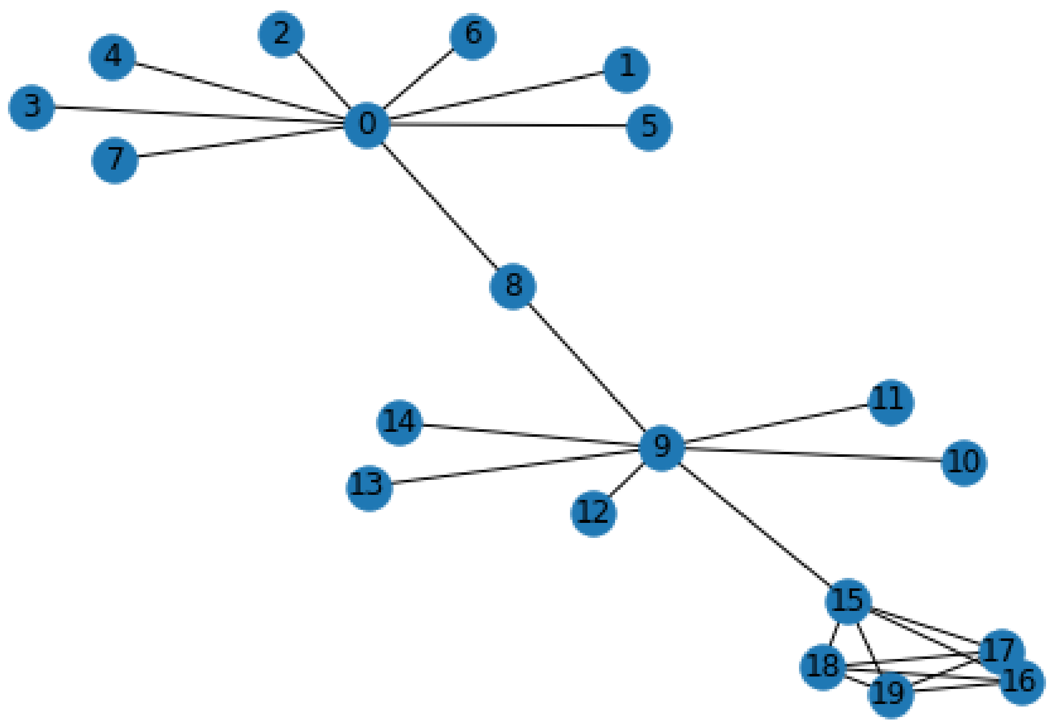
1 Propagation Principles in Simple Graphs

We examine graphs that have small vertex-set (up to 20) and have a hub and spoke structure, meaning they have few hubs and the rest of the vertices tend to be leaves. We first consider graphs with no cycles at all.

[1]:
```
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt
import time
import pandas as pd
from scipy.stats import poisson
from tqdm import tqdm

import skimage as ski
from skimage import io

from skimage.transform import rescale, resize, downscale_local_mean

from scipy.special import rel_entr


# plt.ioff()
plt.ion()

################ Functions ########################################
from graphsfunctions import *
```

[2]:
```
# Testing a little spider graph
G = nx.Graph()
G.add_nodes_from(range(20))
G.add_edges_from([(0, i) for i in range(1, 8)])
G.add_edge(0, 8)
G.add_edge(8, 9)
G.add_edges_from([(9, i) for i in range(10, 15)])
G.add_edge(15, 9)
G.add_edges_from([(i, j) for i in range(15, 19) for j in range(i + 1, 20)])
nx.draw(G, with_labels=True)
```

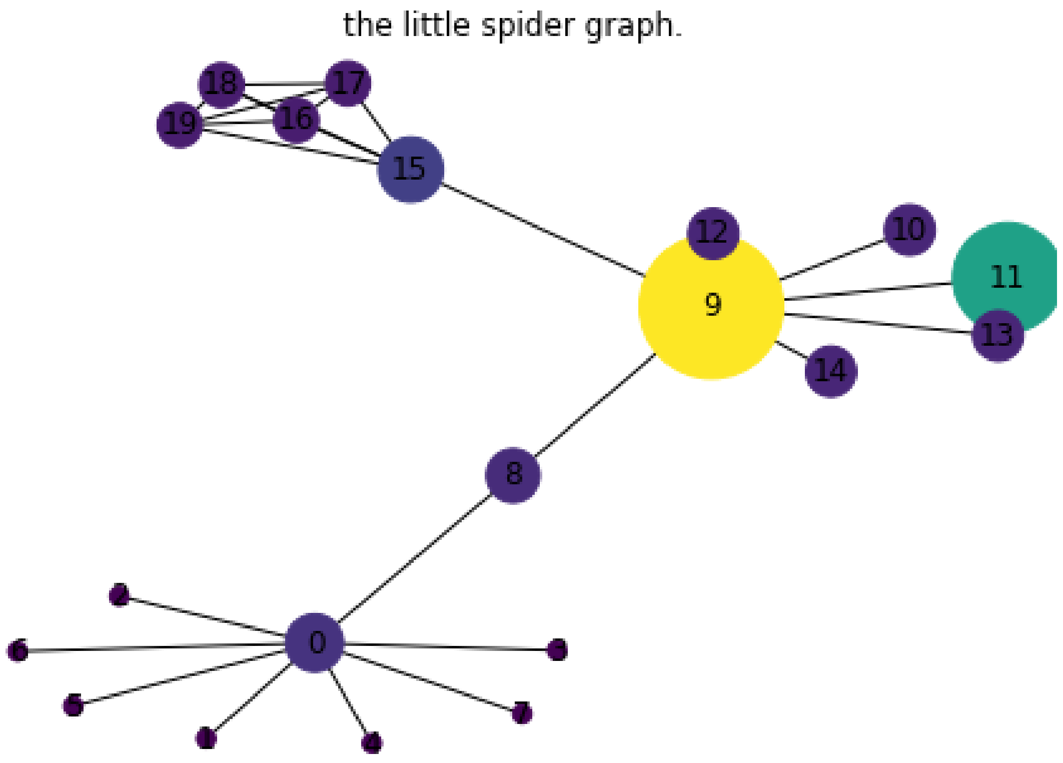
```
plt.show()
```



The "little Spider" contains a bigger star on 0, connected via 8 to a smaller star on 9, and a K5. We want to explore its propagation properties.

[3]:
```
H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
nx.draw(H, with_labels=True, node_color=W85[11], node_size=W85[11]*10000)
plt.title("the little spider graph.")
plt.show()
```

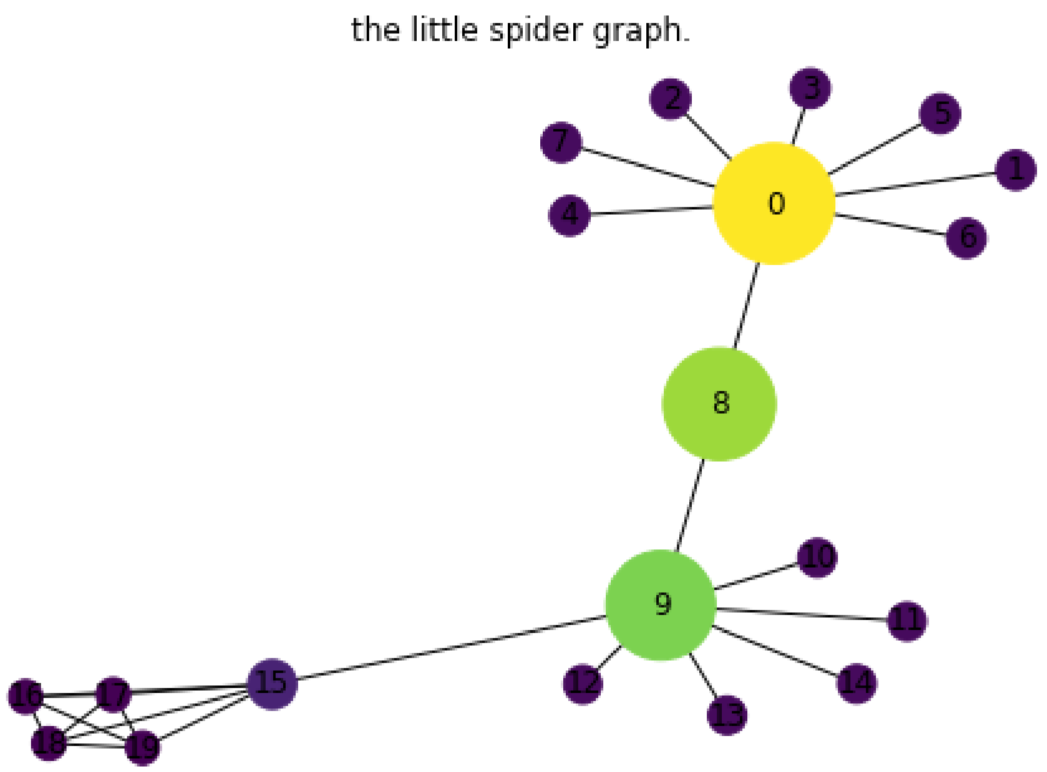
```
100%|      | 20/20 [00:00<00:00, 470.01it/s]
```



The plot shows, both by node size as well as by color code, the stationary propagation with biased concentrated on noe 11, with restart probability of 0.15.

```
[4]: H, W = pageRanksConcentratedBiasG(G, alpha=0.85)
     nx.draw(H, with_labels=True, node_color=W[8], node_size=W[8]*10000)
     plt.title("the little spider graph.")
     plt.show()
```

```
100%|      | 20/20 [00:00<00:00, 395.19it/s]
```

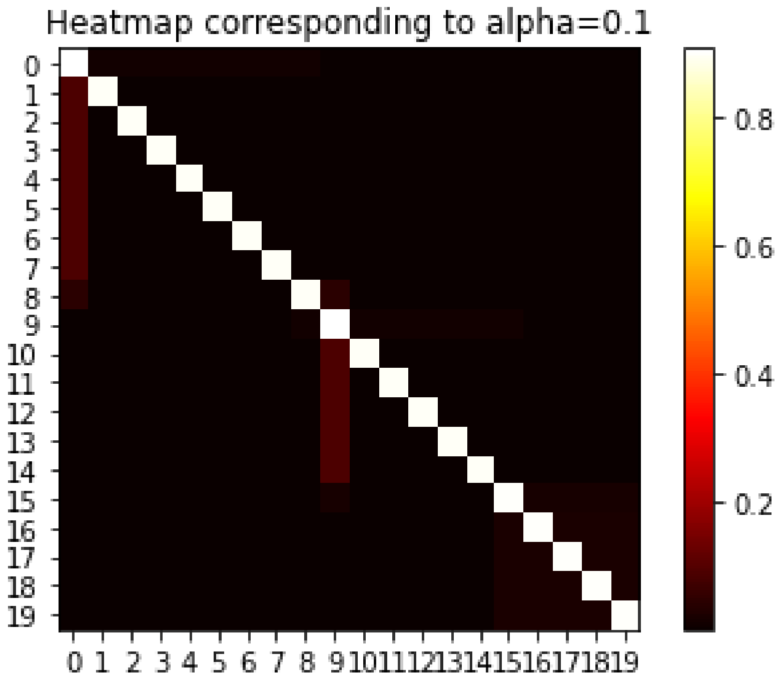


This is the same type of plot, but propagation from 8.

Now we are going top demonstrate how the propgation changes from each vertex, and with various alpha values, by way of heatmaps

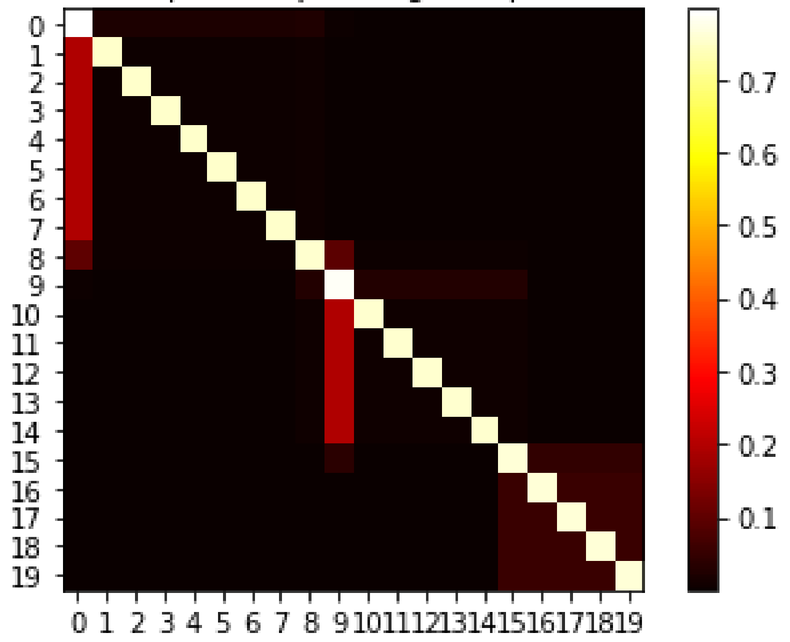
[5]:
```
H, W10 = pageRanksConcentratedBiasG(G, alpha=0.1)
heatmap(W10, "Heatmap corresponding to alpha=0.1")
plt.show()
plt.close()

H, W25 = pageRanksConcentratedBiasG(G, alpha=0.25)
heatmap(W25, "Heatmap corresponding to alpha=0.25")
plt.show()
plt.close()

H, W50 = pageRanksConcentratedBiasG(G, alpha=0.5)
heatmap(W50, "Heatmap corresponding to alpha=0.5")
plt.show()
plt.close()

H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
heatmap(W85, "Heatmap corresponding to alpha=0.85")
plt.show()
plt.close()

H, W90 = pageRanksConcentratedBiasG(G, alpha=0.9)
heatmap(W90, "Heatmap corresponding to alpha=0.9")
plt.savefig("Heatmap Little Spider alpha=0.9.png")
plt.show()
plt.close()
```

```
100%|          | 20/20 [00:00<00:00, 881.66it/s]
```



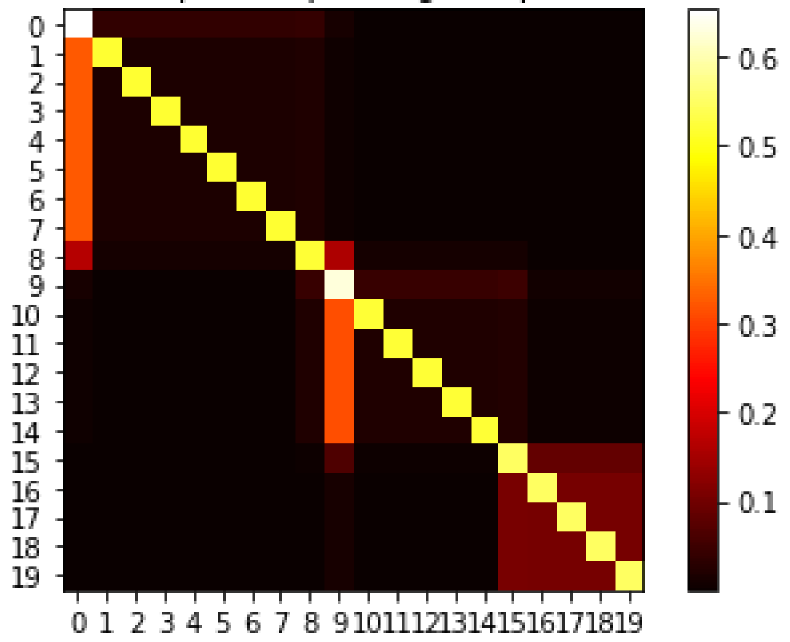
```
100%|          | 20/20 [00:00<00:00, 1297.06it/s]
```

Heatmap corresponding to alpha=0.25

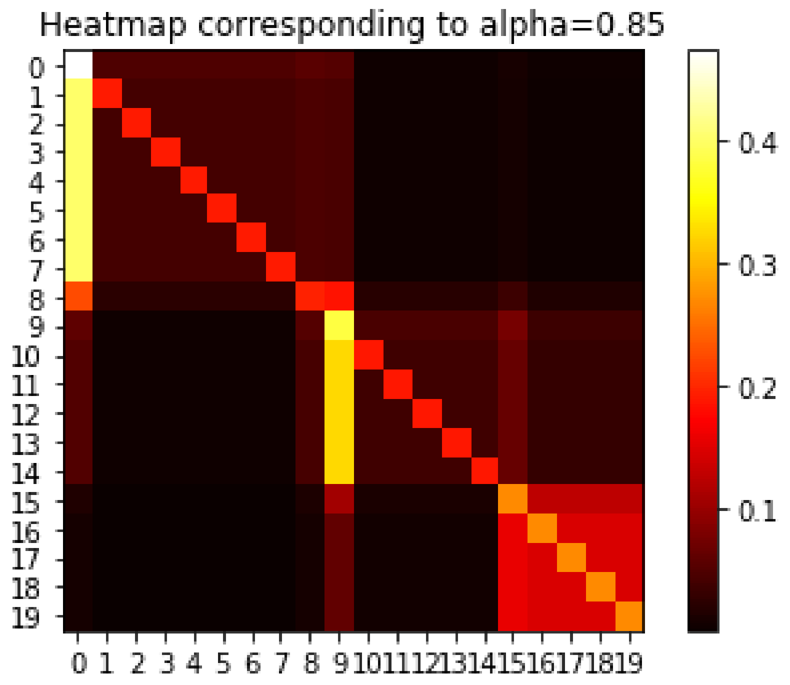


```
100%|          | 20/20 [00:00<00:00, 979.10it/s]
```

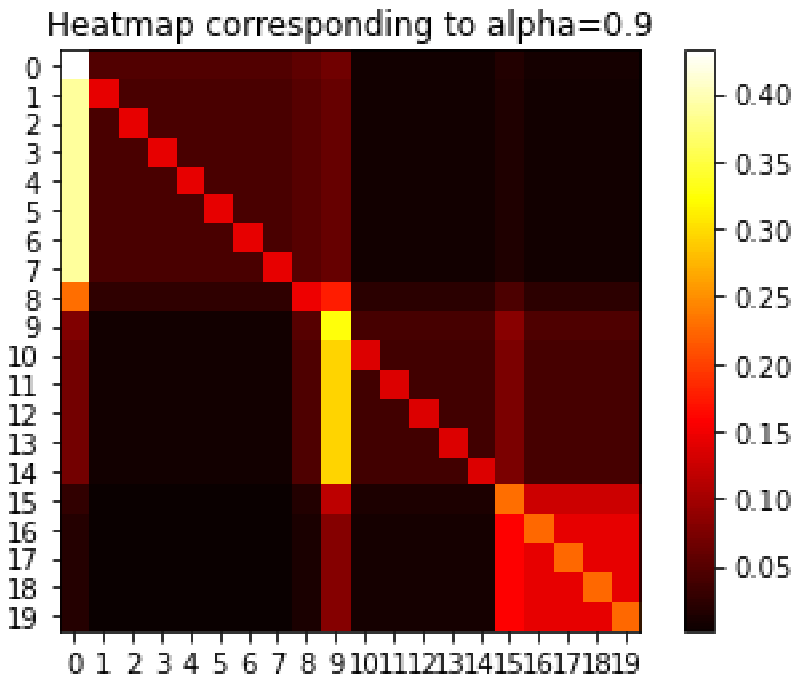
Heatmap corresponding to alpha=0.5



```
100%|          | 20/20 [00:00<00:00, 579.01it/s]
```



```
100%|          | 20/20 [00:00<00:00, 292.37it/s]
```

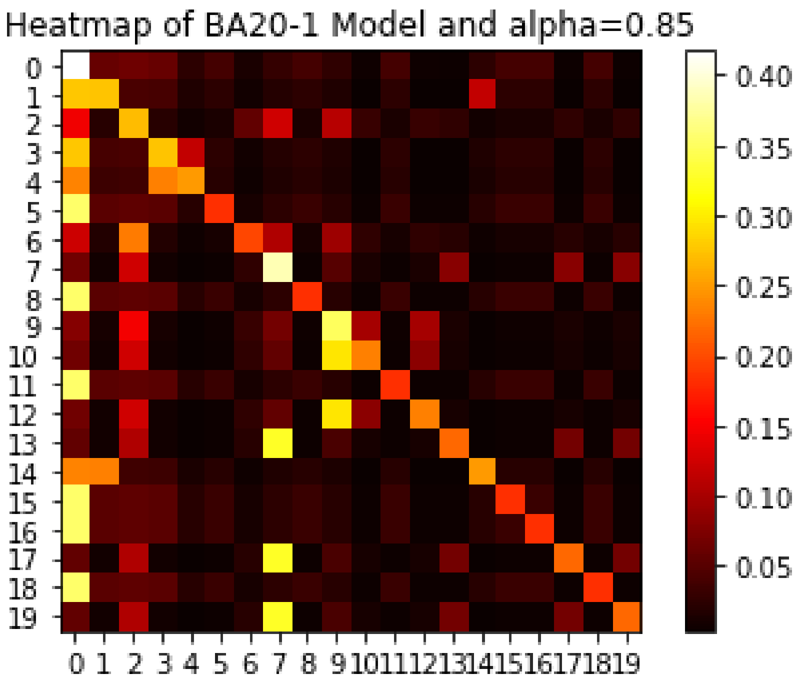


Interestingly the alpha value doesn't seem to change the structure, but just the instensities. But this graph is very simple. We also note some trivial facts such that hubs retain more heat, and the leaves are cold. Because highly connected nodes receive heat from many sources. If we think in terms of random walk, they are more likely to be visited because they are connected to more nodes. Also notice that node 8 which connects 2 hot hubs is also interesting.

Now lets try the smae trick with a different dog, I mean graph...

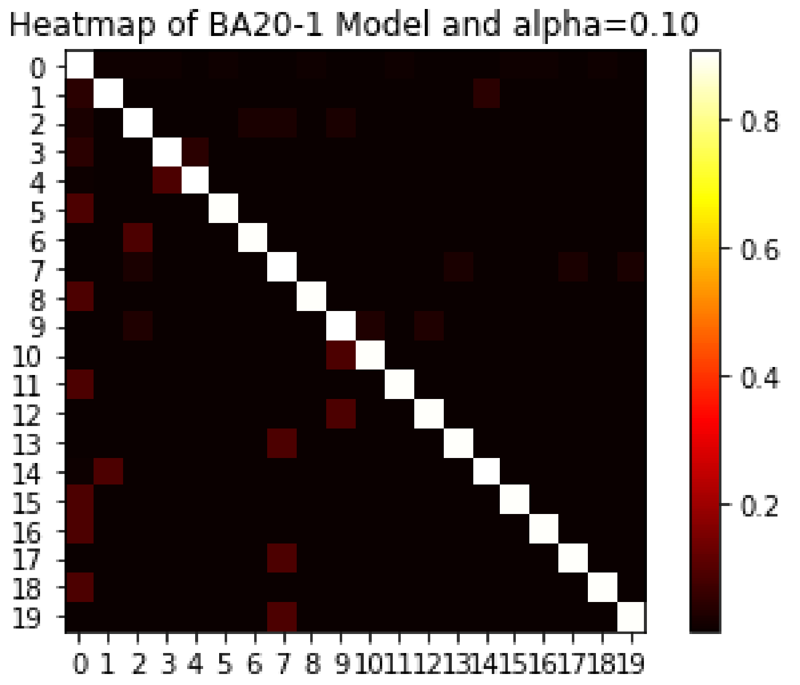
[6]:
```
seed = 42
G = nx.barabasi_albert_graph(n=20, m=1, seed=seed)
H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
heatmap(W85, "Heatmap of BA20-1 Model and alpha=0.85")
```

```
100%|          | 20/20 [00:00<00:00, 358.84it/s]
```



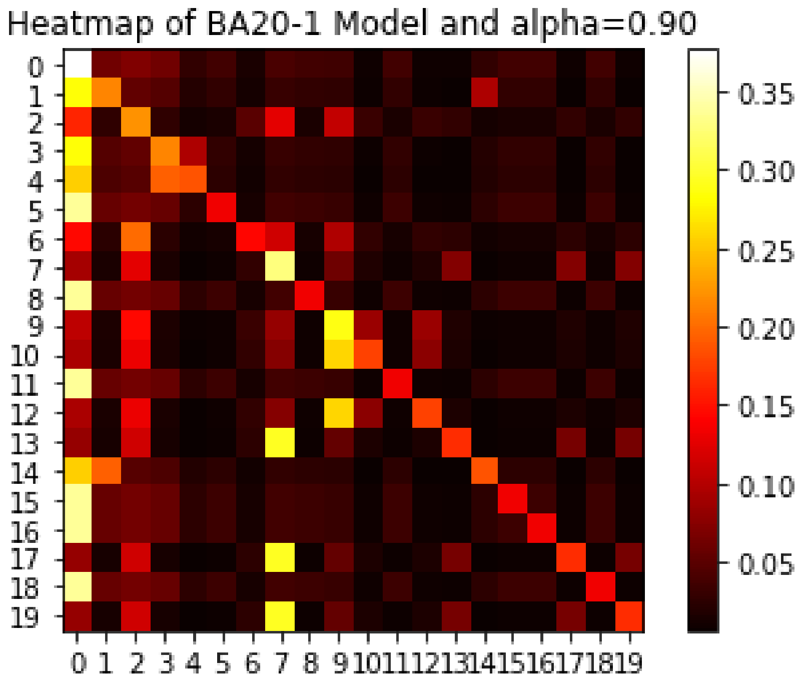
[7]:
```
H, W10 = pageRanksConcentratedBiasG(G, alpha=0.10)
heatmap(W10, "Heatmap of BA20-1 Model and alpha=0.10")
```

```
100%|          | 20/20 [00:00<00:00, 599.88it/s]
```

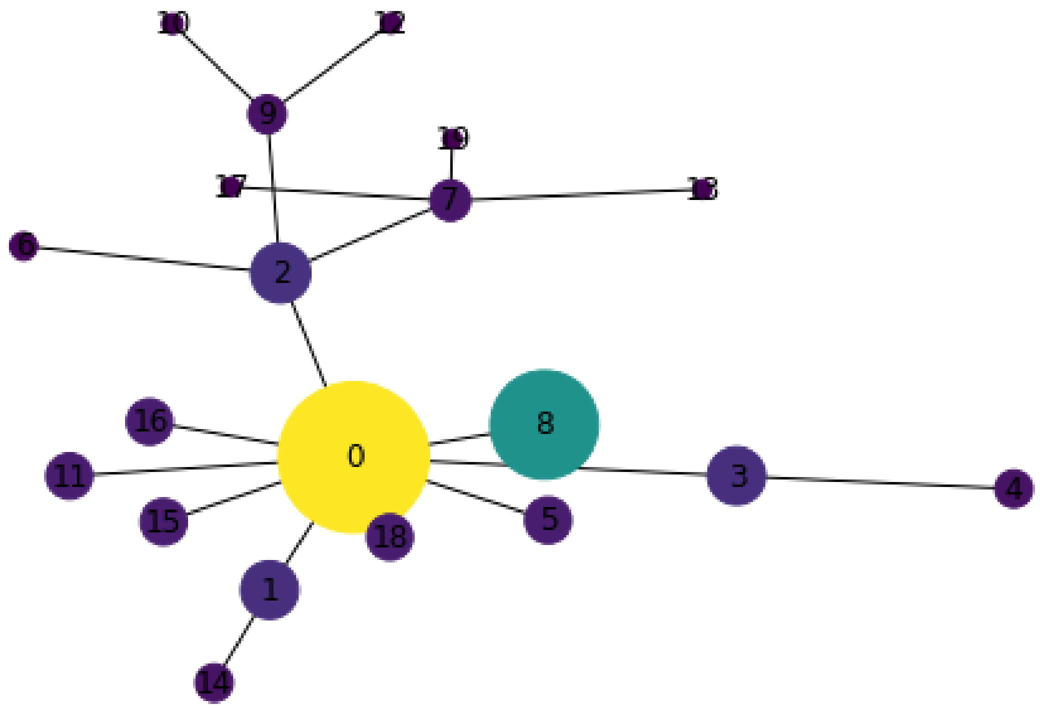


[8]:
```
H, W90 = pageRanksConcentratedBiasG(G, alpha=0.90)
heatmap(W90, "Heatmap of BA20-1 Model and alpha=0.90")
```

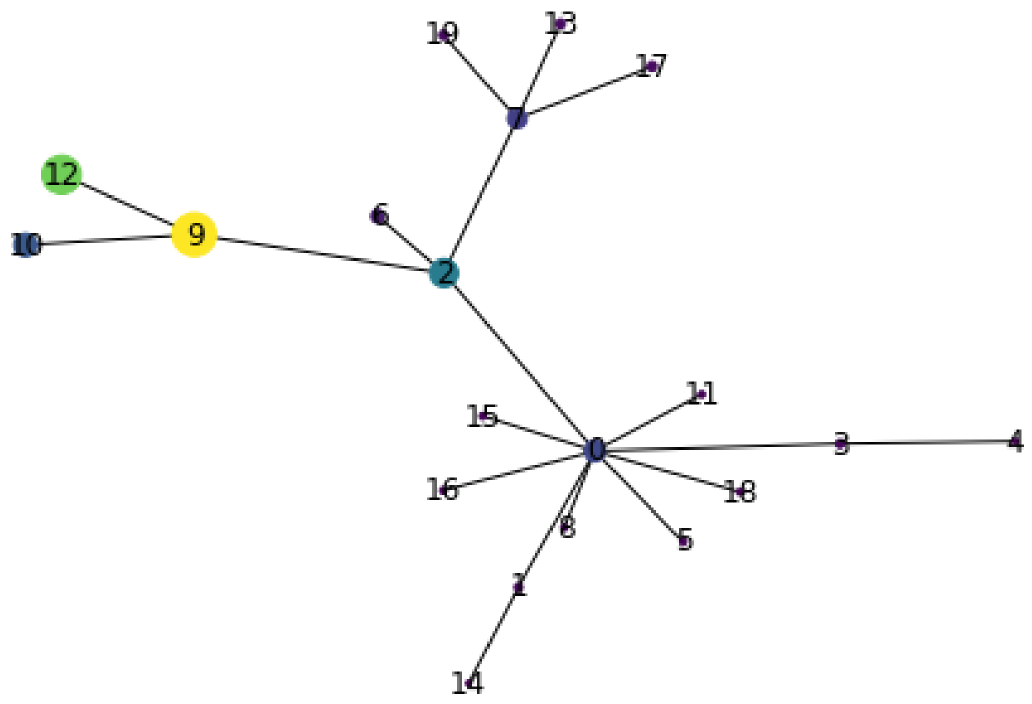
```
100%|          | 20/20 [00:00<00:00, 214.42it/s]
```



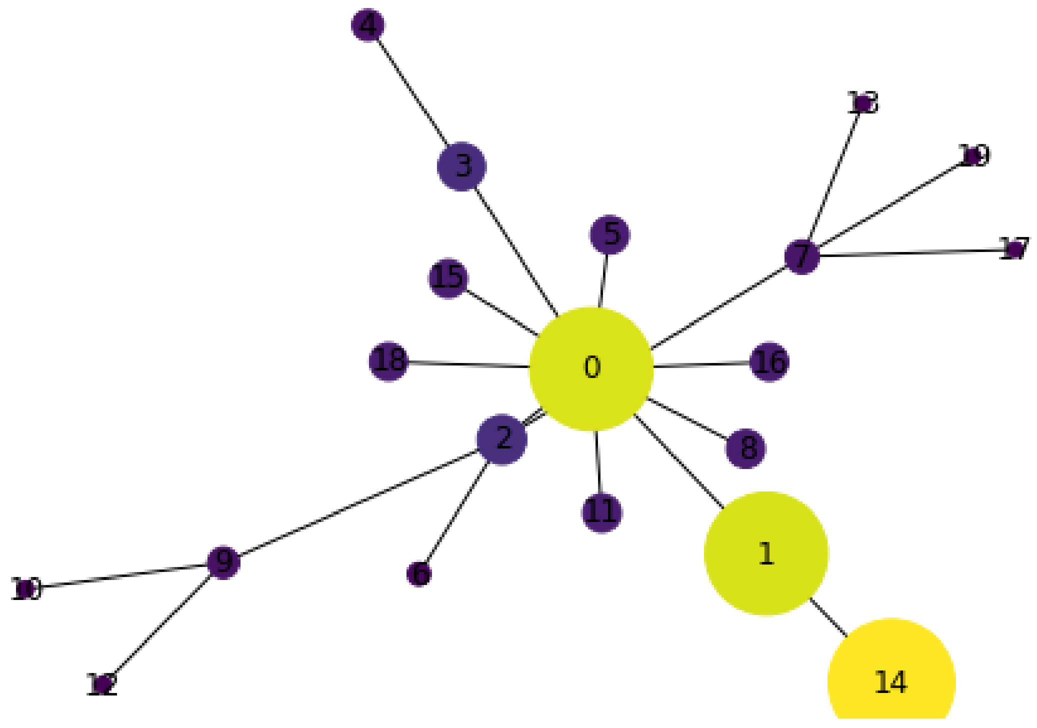
[9]:
```
nx.draw(H, with_labels=True, node_color=W85[8], node_size=W85[8]*10000)
```



[10]:
```
nx.draw(H, with_labels=True, node_color=W85[12], node_size=W85[12]*1000)
```



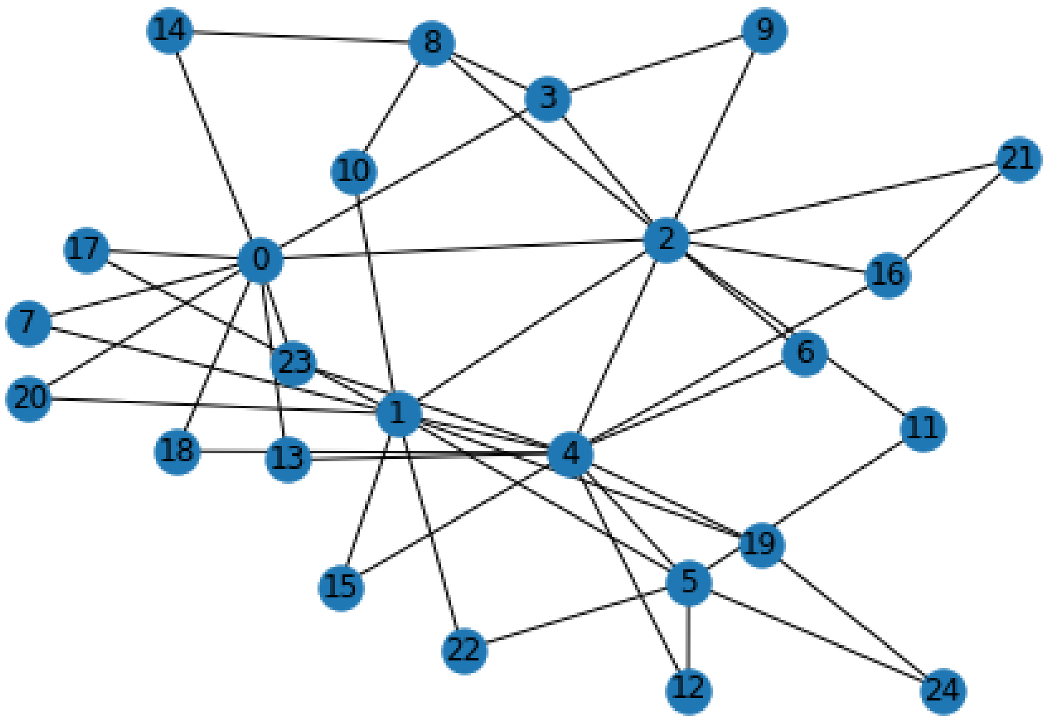
```
[11]: nx.draw(H, with_labels=True, node_color=W85[14], node_size=W85[14]*10000)
```

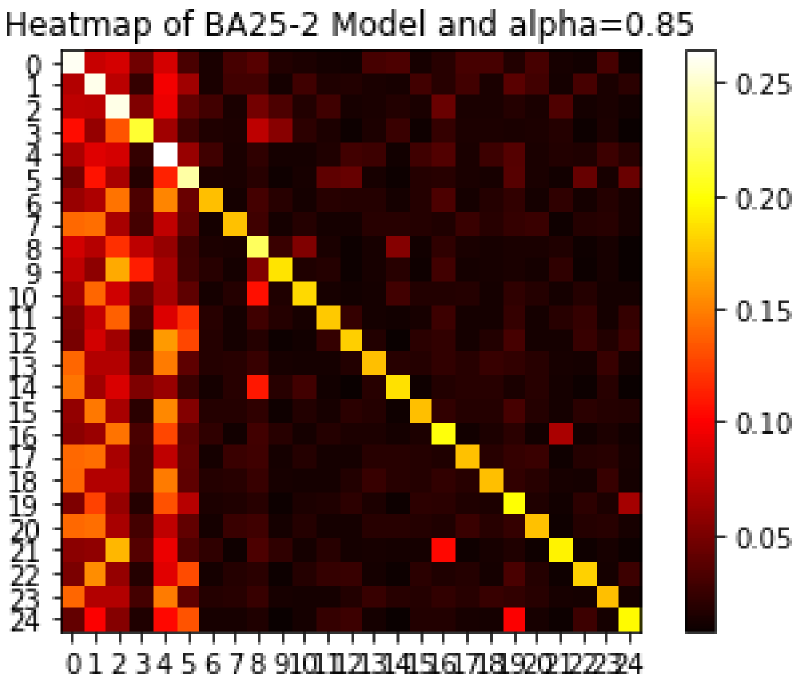


Now we do they same experiment with a more complex graph…

[12]:
```
seed = 42
G = nx.barabasi_albert_graph(n=25, m=2, seed=seed)

nx.draw(G, with_labels=True)

H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
heatmap(W85, "Heatmap of BA25-2 Model and alpha=0.85")
```

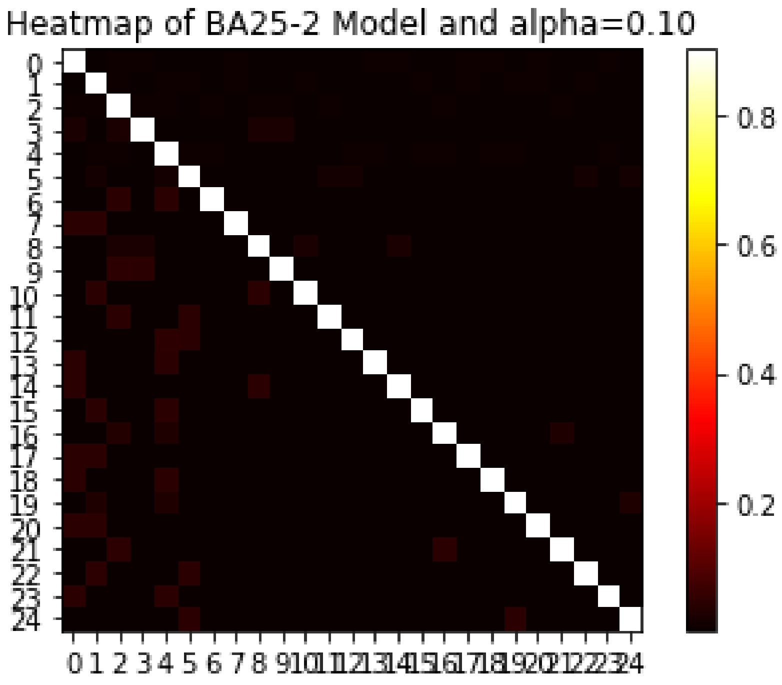
```
100%|      | 25/25 [00:00<00:00, 467.27it/s]
```



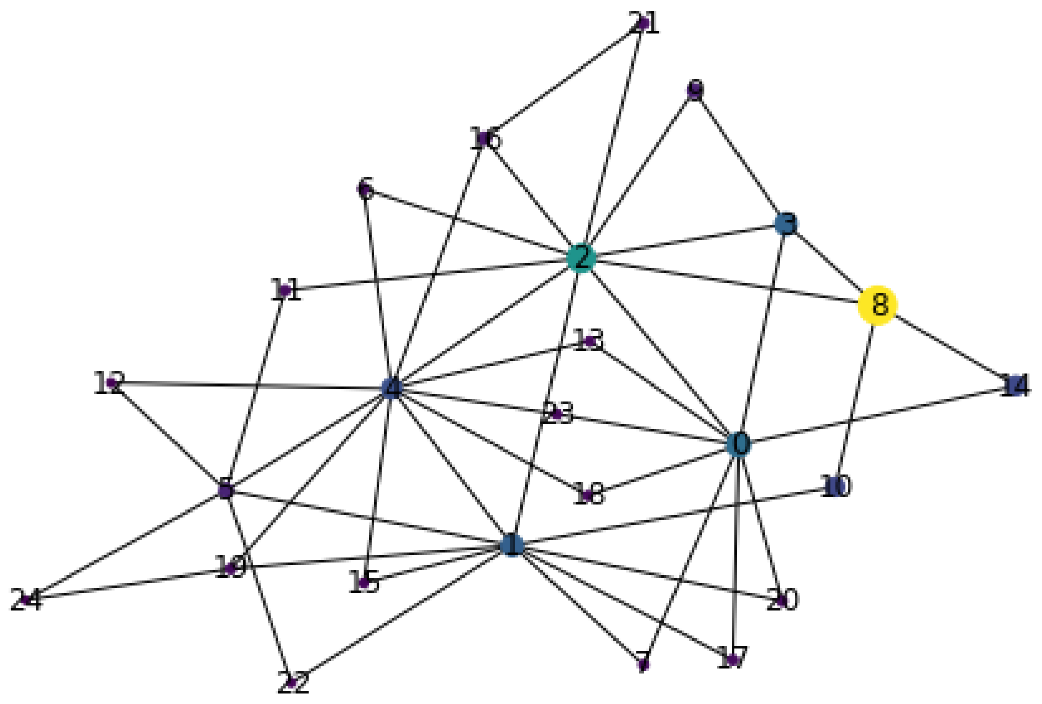


```
[13]: H, W10 = pageRanksConcentratedBiasG(G, alpha=0.10)
      heatmap(W10, "Heatmap of BA25-2 Model and alpha=0.10")
```

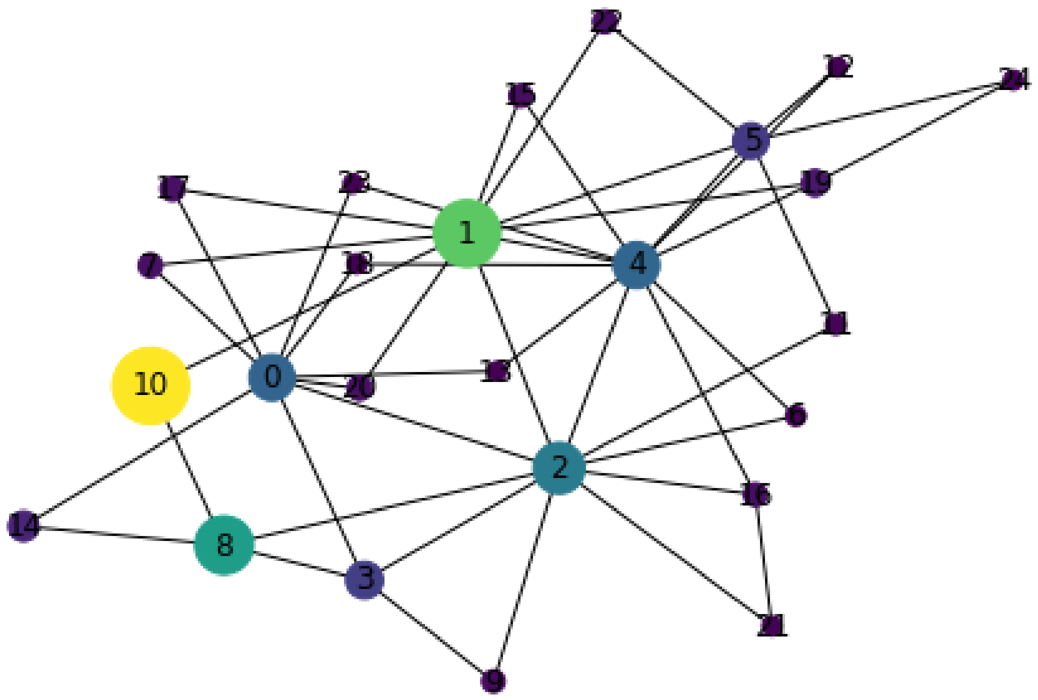
```
100%|      | 25/25 [00:00<00:00, 1185.62it/s]
```



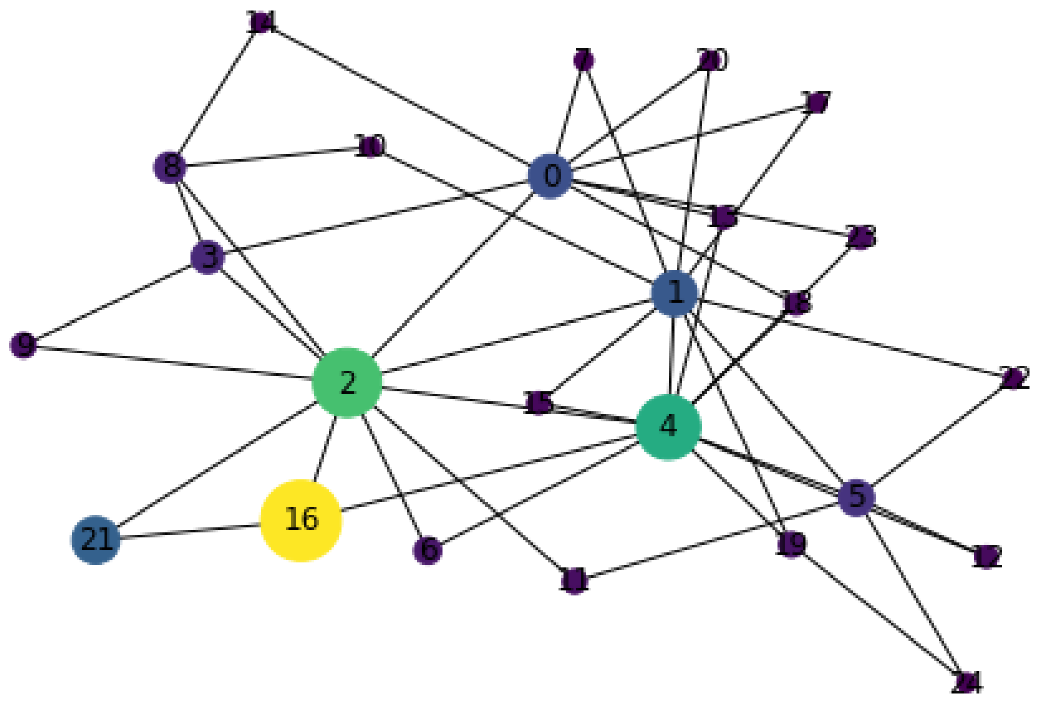
[14]:
```
nx.draw(H, with_labels=True, node_color=W85[8], node_size=W85[8]*1000)
```



[15]:
```
nx.draw(H, with_labels=True, node_color=W85[10], node_size=W85[10]*5000)
```



```
[16]: nx.draw(H, with_labels=True, node_color=W85[16], node_size=W85[16]*5000)
```



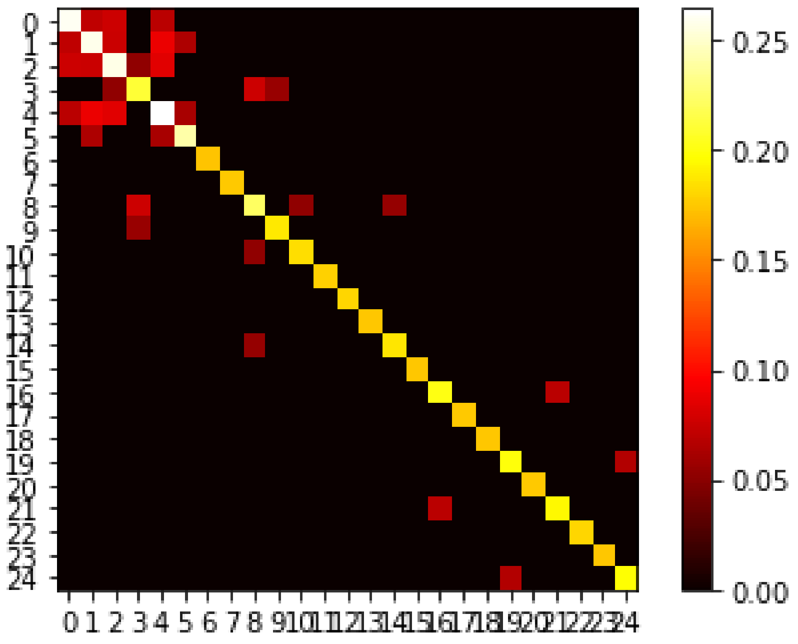
I think it is clear now that propagation strongly depends on the distance from the propagating source and on the degree...

Next we are going to see what the influence graph looks like. The influence of node i on j is pi[j] where pi is the stationary distribution propagated from i (so we look at the j component of it).

The influence graph is a weighted graph on n nodes. where the edge weight of (i,j) is min(pi[j], pj[i]). We also set a threshold δ so we only edges that weigh over delta make the cut.

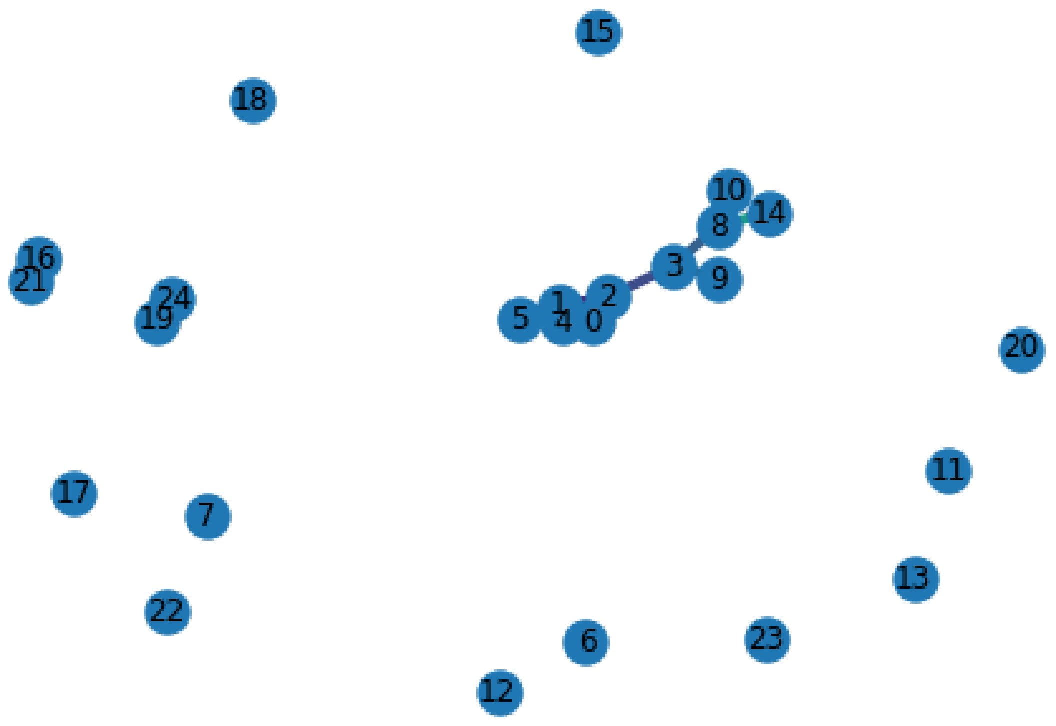
[17]:
```
# reducedInfluenceMatrixG
W = reducedInfluenceMatrixG(G, delta=0.05)

heatmap(W, "")
```

```
100%|      | 25/25 [00:00<00:00, 554.50it/s]
```



[18]:
```
Gd = nx.Graph()
Gd.add_nodes_from(range(25))
edges = [(i,j) for i in range(25) for j in range(i,25) if W[i,j]>0]
Gd.add_edges_from(edges)
l = [W[e] for e in Gd.edges()]
l



nx.draw(Gd, with_labels=True, width=4, edge_color = range(40))
```

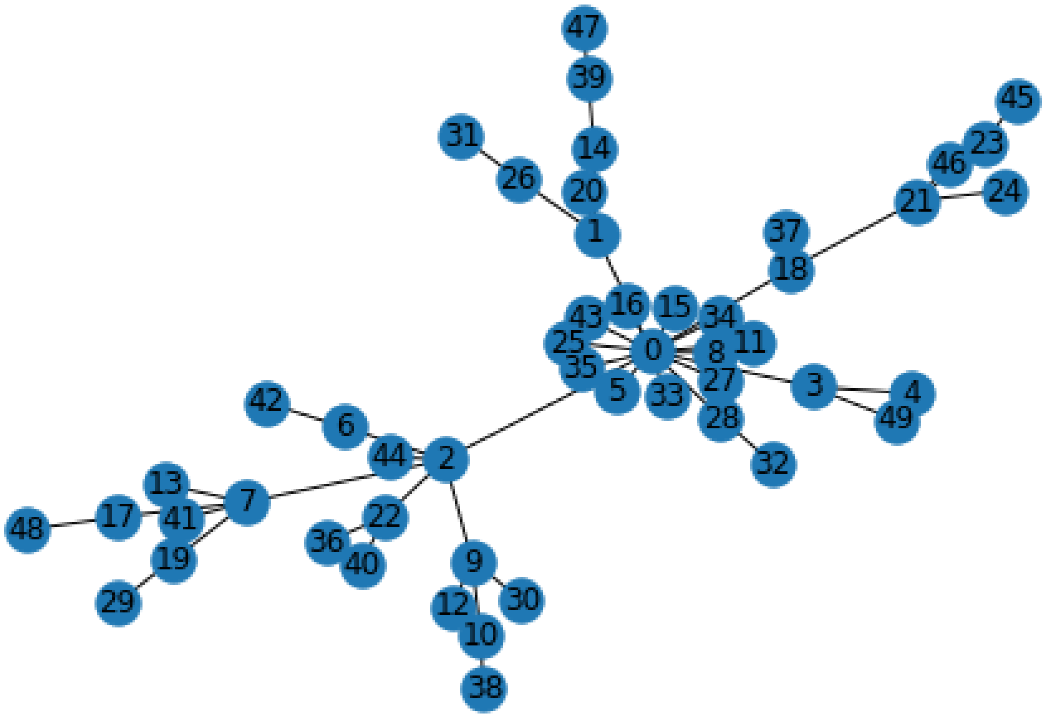
```
plt.show()
```



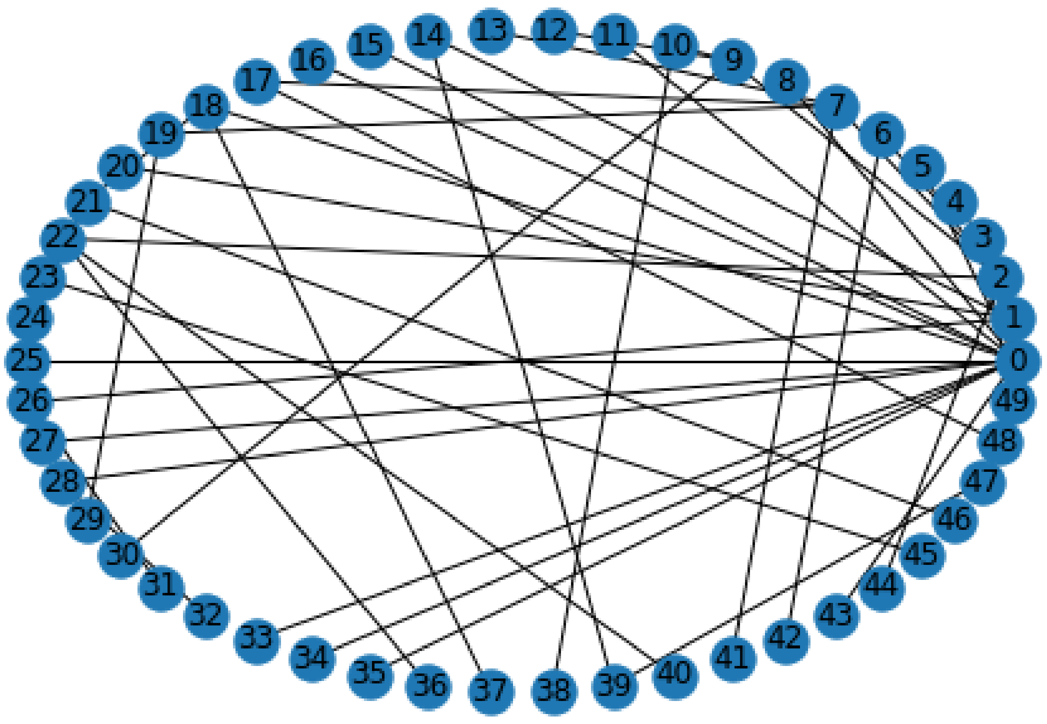
2 Exploring graph clustering methods.

Since we are dealing with propagation and interested in connected scale free graphs such as Barabsi-Albert, I am trying to apply propagation to perform this task. Idea 1: The hottest node is going to form the first cluster. We then extend it by all nodes that are connected to it in a sufficiently hot path. Remove them from the graph and repeat on the smaller graph. Thought should then be dedicated to the matter of setting the parameters: what is 'hot' (delta), how fast we propagate (alpha), how do we test statistical significance and robustness. Define null model? Use connected edge swaps for robustness tests?

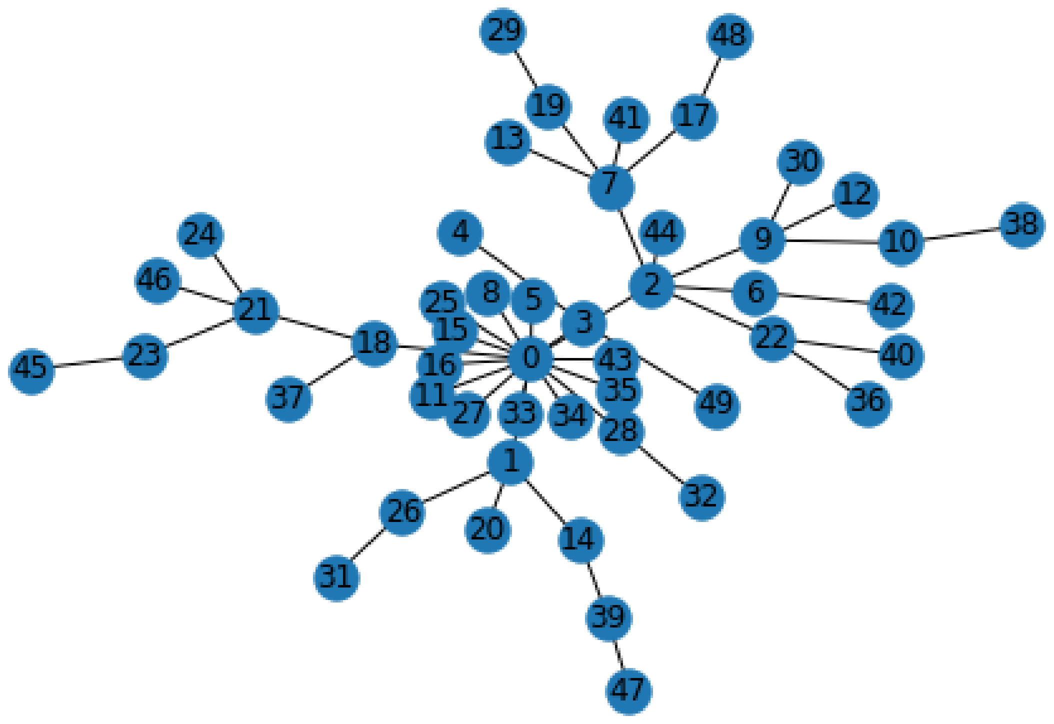
[19]:
```
G = nx.barabasi_albert_graph(n=50, m=1, seed=seed)

nx.draw(G, with_labels=True)
```



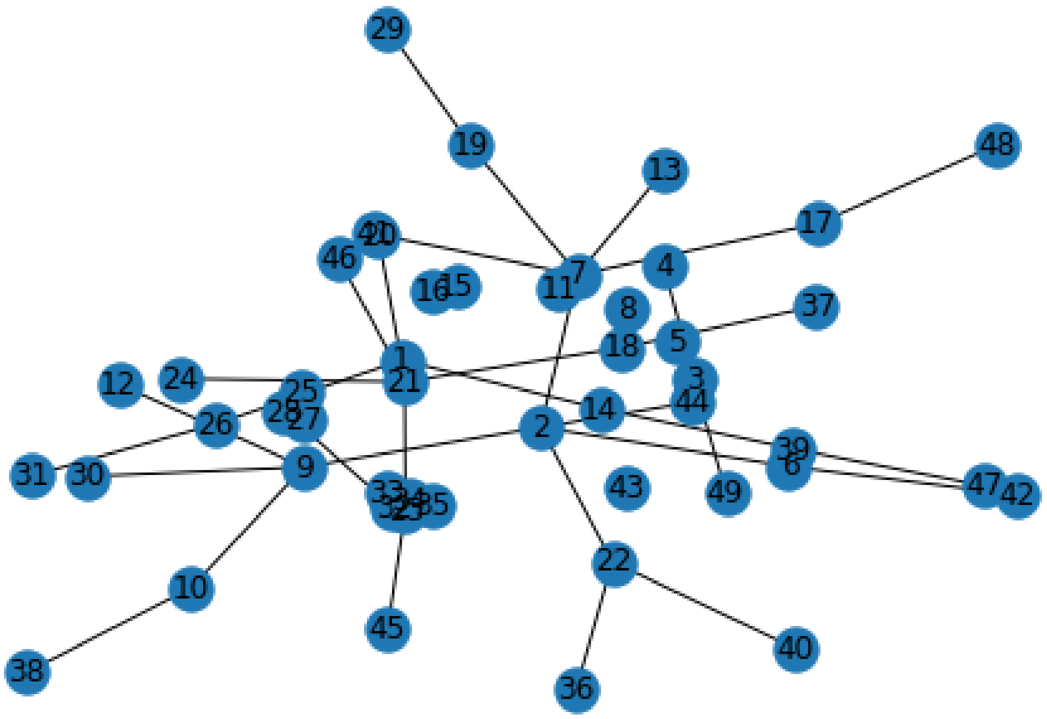
[20]:
```
nx.draw_circular(G, with_labels=True)
```



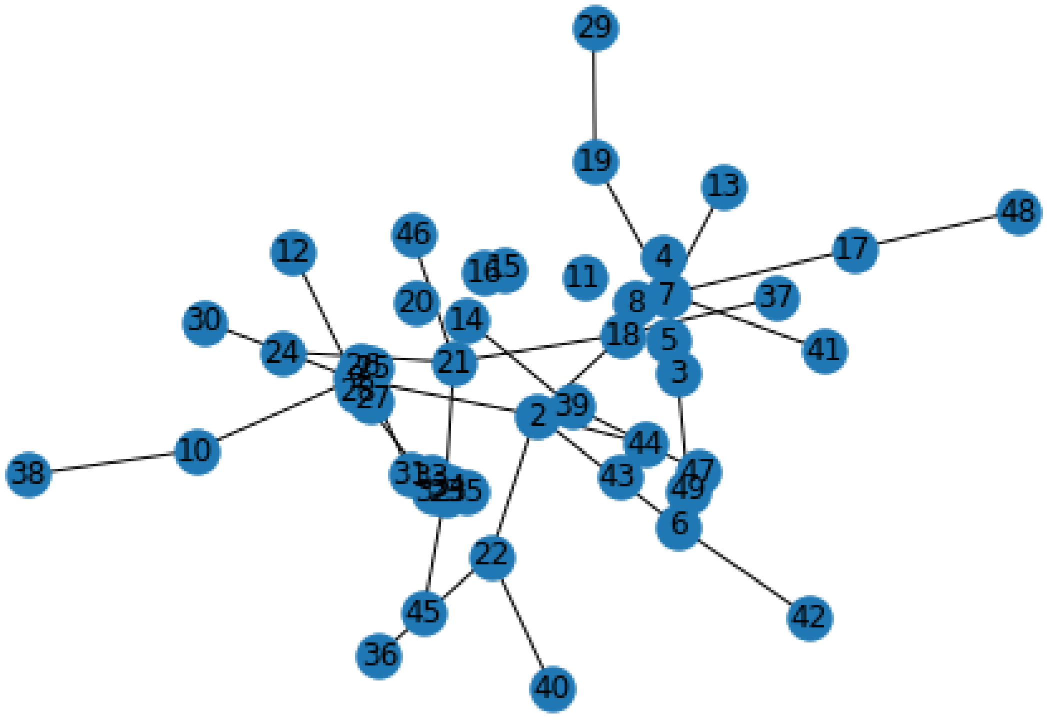
[21]:
```
nx.draw_kamada_kawai(G, with_labels=True)
```



[22]:
```
G.remove_node(0)

nx.draw_kamada_kawai(G, with_labels=True)
```

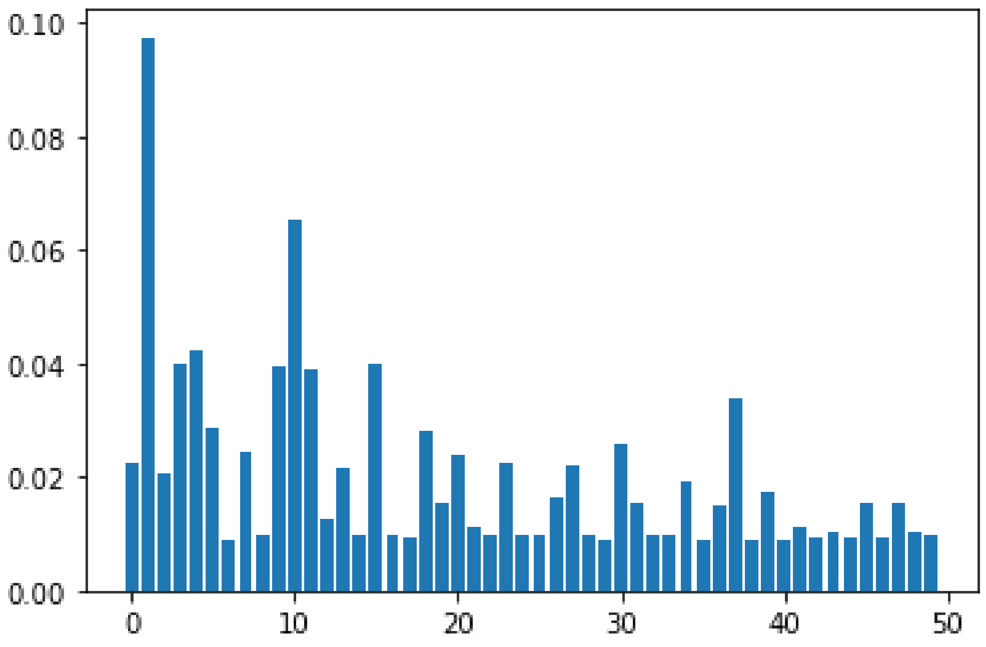


[23]:
```
G.remove_node(1)

nx.draw_kamada_kawai(G, with_labels=True)
```



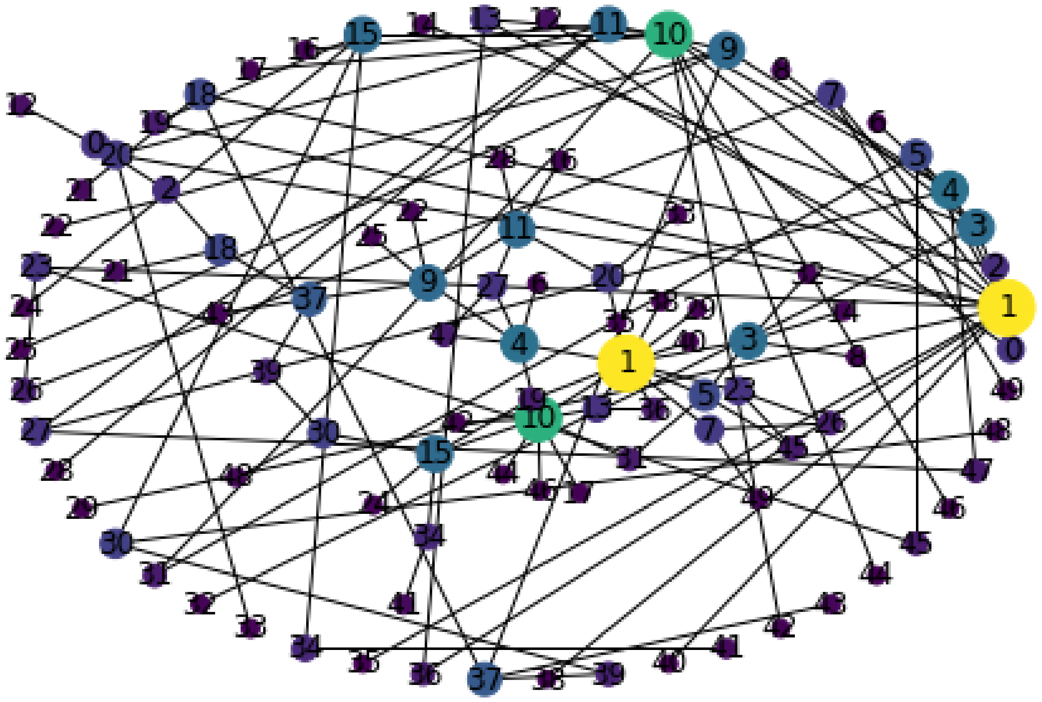
[24]:
```
# trying to cluster a graph ...
G = nx.dual_barabasi_albert_graph(n=50, m1=1, m2=2, p=0.7, seed=seed)
H = G.copy()
p, _ = powerIterateG(G, alpha=0.85)
plt.bar(range(50), p)
```

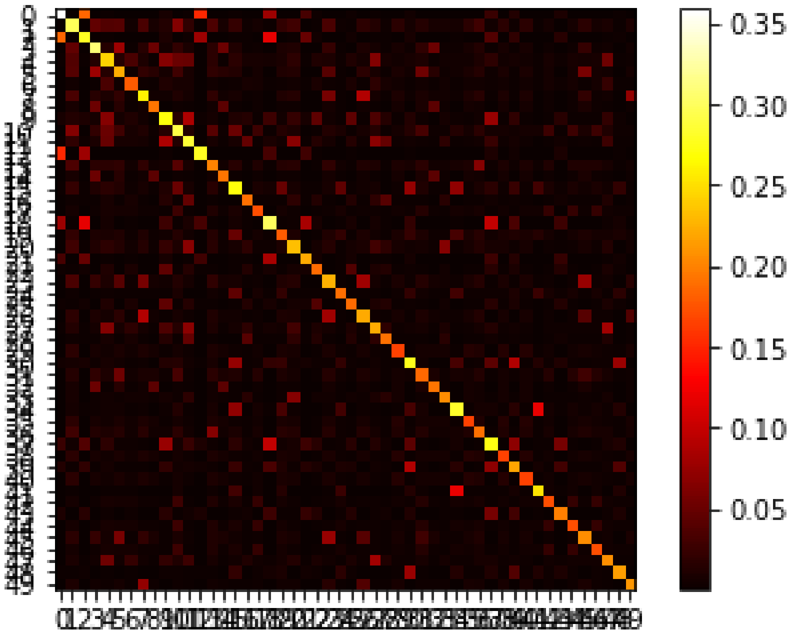
[24]:
```
<BarContainer object of 50 artists>
```



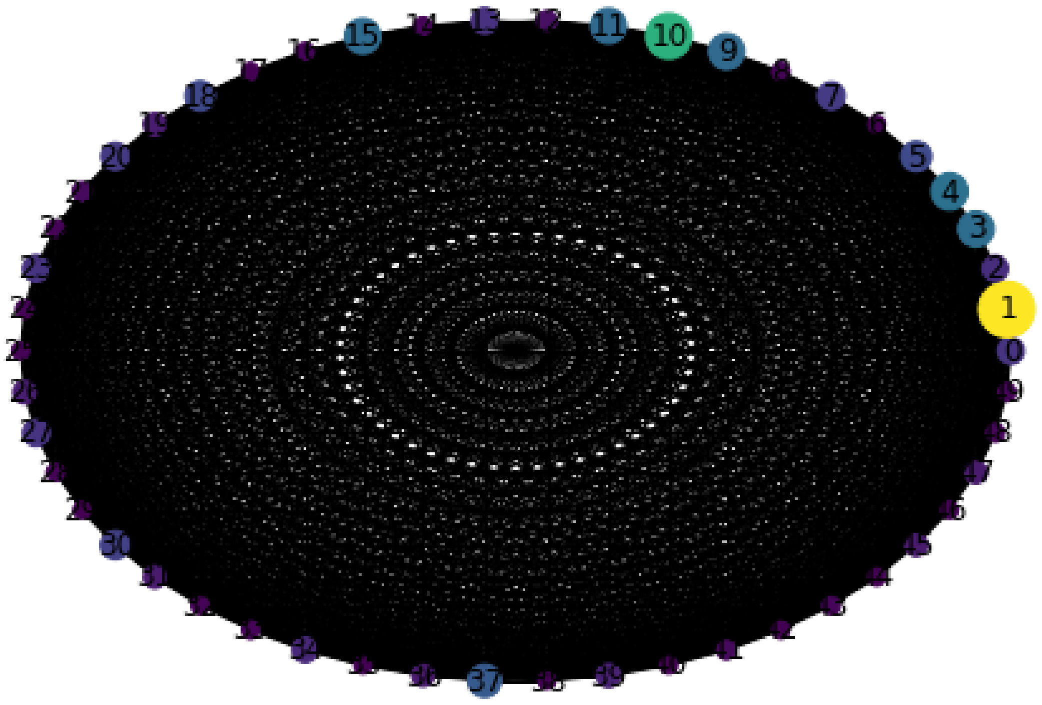
```
[25]: nx.draw_kamada_kawai(G, with_labels=True, node_color=p, node_size=5000*p)

      nx.draw_circular(G, with_labels=True, node_color=p, node_size=5000*p)

      s = np.argmax(p)
      s

      W = reducedInfluenceMatrixG(G, delta=0)
      heatmap(W, "")
```

```
100%|      | 50/50 [00:00<00:00, 529.32it/s]
```




0.35
0.30
0.25
0.20
0.15
0.10
0.05

```
[26]: H = nx.Graph()
      H.add_nodes_from(G.nodes())

      edges = [(i,j) for i in range(49) for j in range(i+1,50) if W[i,j]>0]

      H.add_edges_from(edges)

      nx.draw_circular(H, with_labels=True, node_color=p, node_size=5000*p)
```



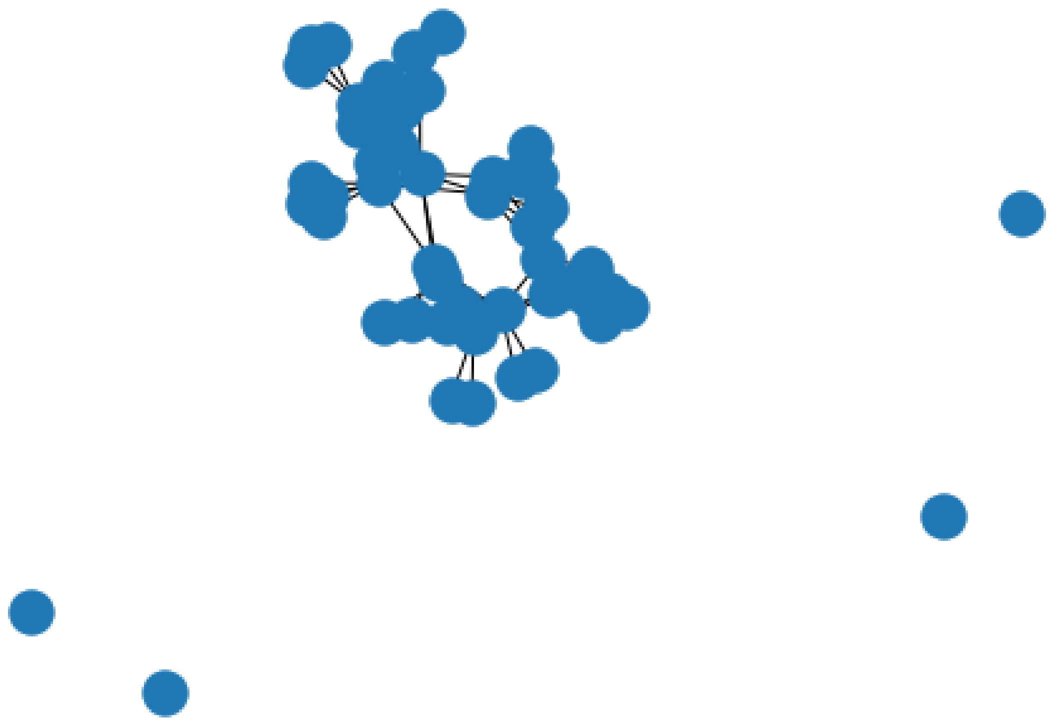
```
[27]: plt.show()
```

```
[28]: for delta in np.arange(0.01, 1, 0.01):
          remlist = [e for e in list(H.edges()) if W[e] <= delta]
          H.remove_edges_from(remlist)
          if nx.number_connected_components(H) > 1:
              print("break", len(H.edges()))
              break

      len(H.edges())
```

```
break 120
```

```
[28]: 120
```

[29]:
```
nx.draw(H)
plt.show()
```



[30]:
```
# so that method is not very promicing because it tends to prune leaves and␣
↪conserve one giant connected component
```

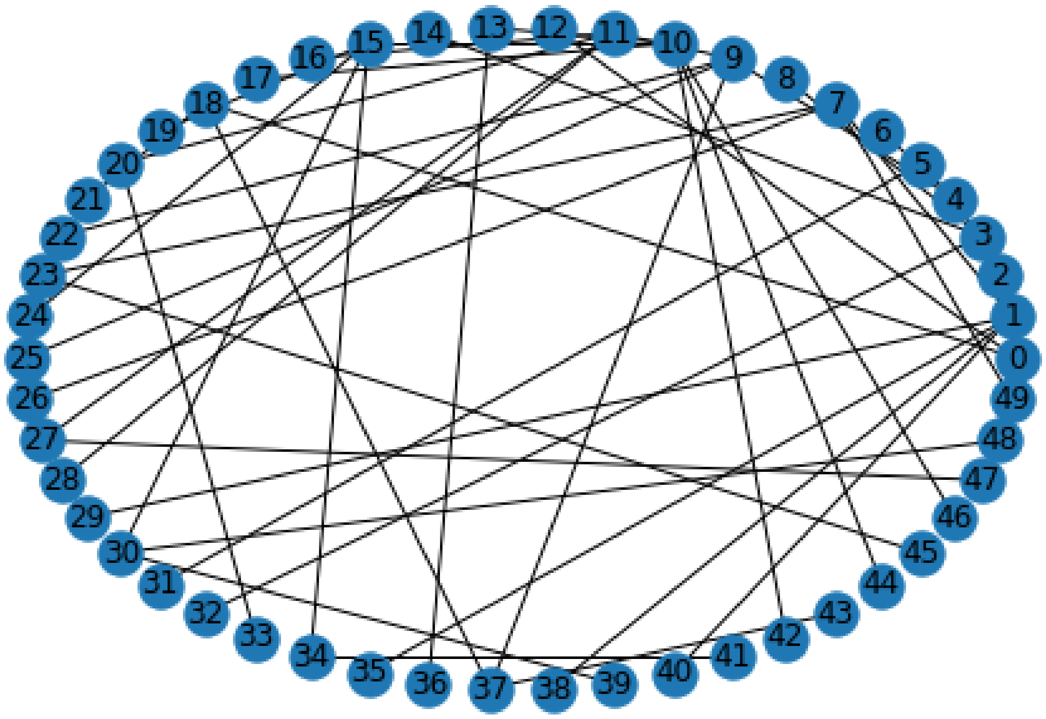
[31]:
```
# We shall now try bottom up method. start from a totally disconnected graph.␣
↪Pick the remaining coldest node and connect it to its nearest
# neighbor in the symmetric influence graph
```

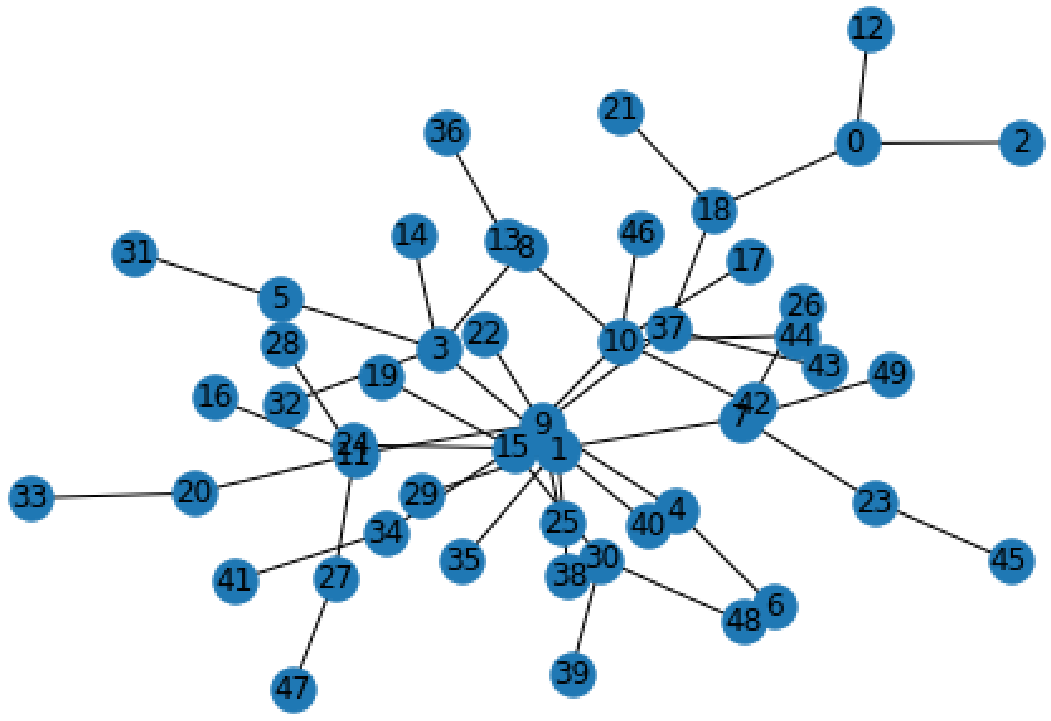
[32]:
```
H = nx.Graph()
H.add_nodes_from(G.nodes())
nlist = list(H.nodes())
while nx.number_connected_components(H) > 3:
    s = np.argmin(p[nlist])
    x = nlist[s]
    print(s,x)
    nlist.pop(s)
    t = np.argmax([W[x,i] for i in nlist])
    H.add_edge(x,nlist[t])

nx.draw_circular(H, with_labels=True )
plt.show()
```

```python
nx.number_connected_components(H)

nx.draw_kamada_kawai(H, with_labels=True)
plt.show()
```

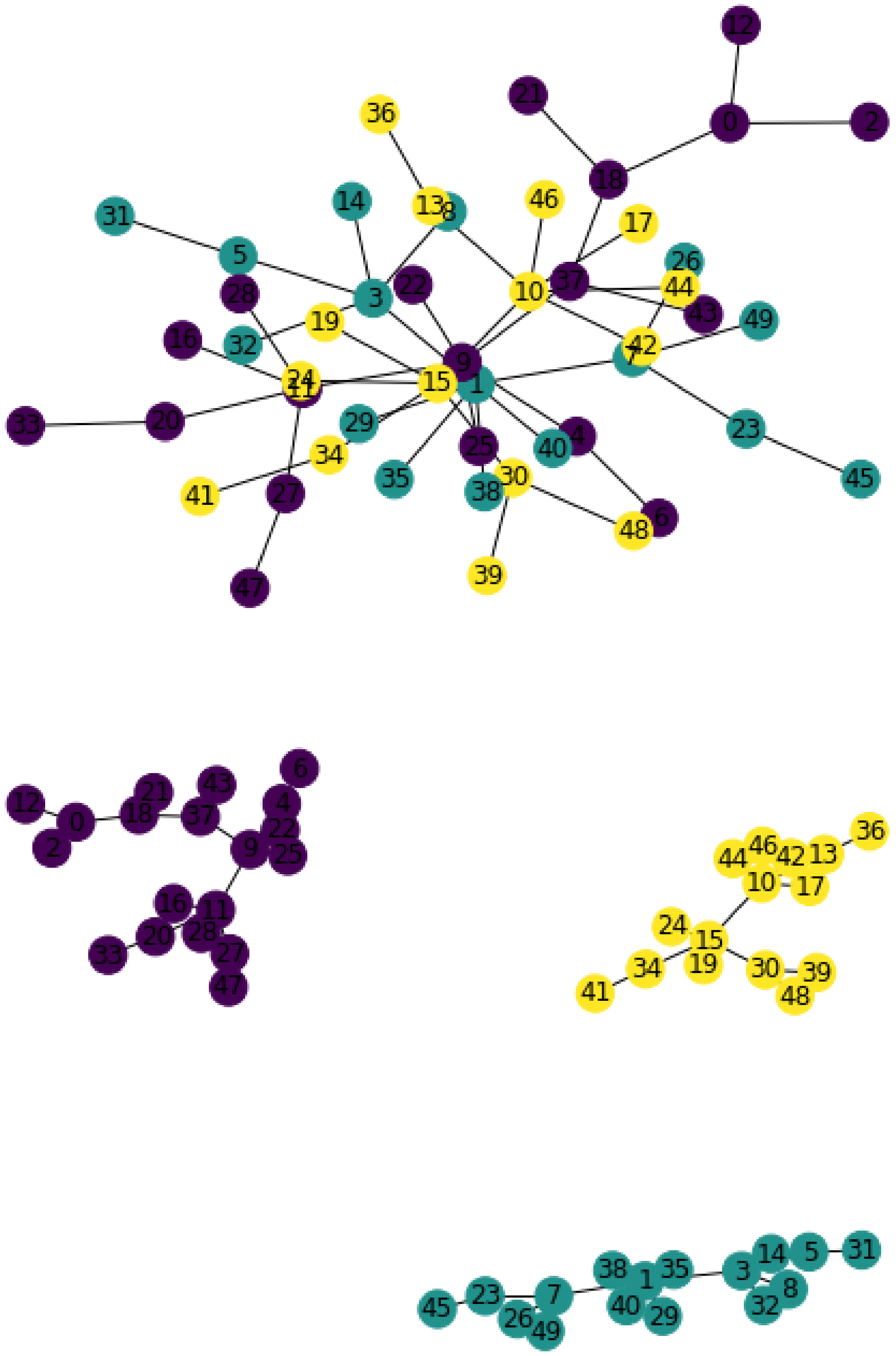
```
29 29
34 35
36 38
37 40
6 6
16 17
36 42
37 44
38 46
15 16
25 28
19 22
21 25
20 24
25 33
7 8
12 14
22 32
31 49
27 43
29 48
16 21
25 41
10 12
21 36
24 47
19 31
22 45
13 19
15 26
19 39
17 34
2 2
9 13
13 27
0 0
11 23
10 20
4 7
9 30
8 18
```

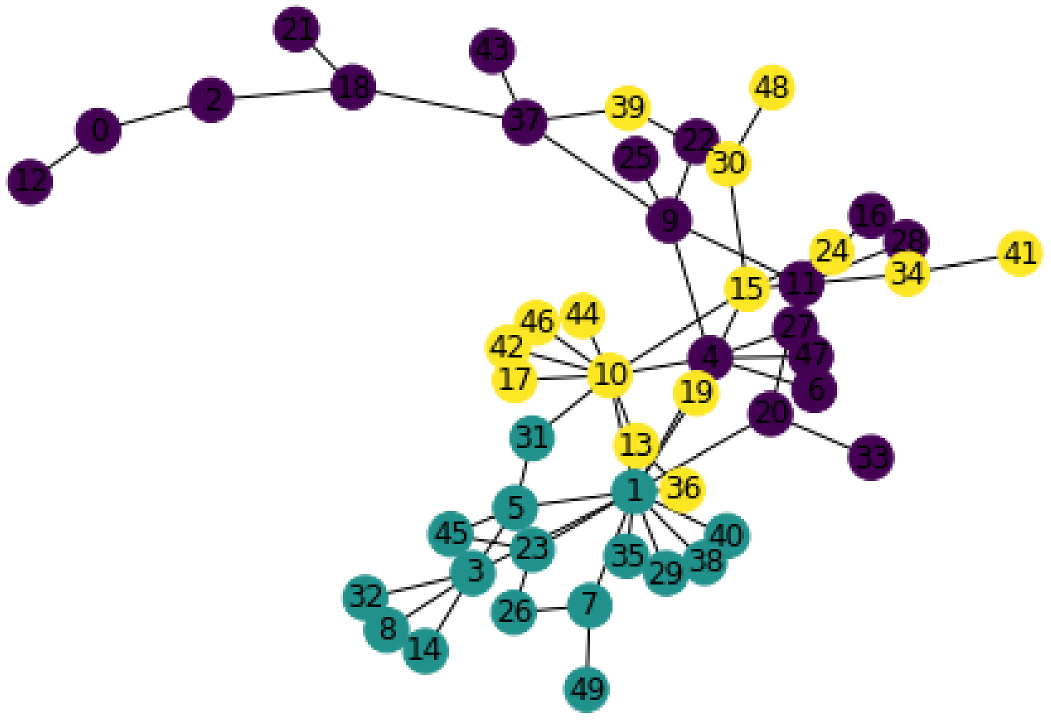
```
3 5
7 37
5 11
3 9
4 15
1 3
```





```
[33]: CCs = [list(c) for c in nx.connected_components(H)]

      CCs

      colors = np.zeros(50)
      colors[CCs[1]]=1
      colors[CCs[2]]=2



      nx.draw_kamada_kawai(H, with_labels=True, node_color=colors)
      plt.show()

      nx.draw_spring(H, with_labels=True, node_color=colors)
      plt.show()

      nx.draw_spring(G, with_labels=True, node_color=colors)
      plt.show()
```





[34]:
```
# The 2 upper plots show the clustering on the influence matrix/graph the␣
↪bottom one show the same clustering on the original graph
```

3 Testing the clustering method on images

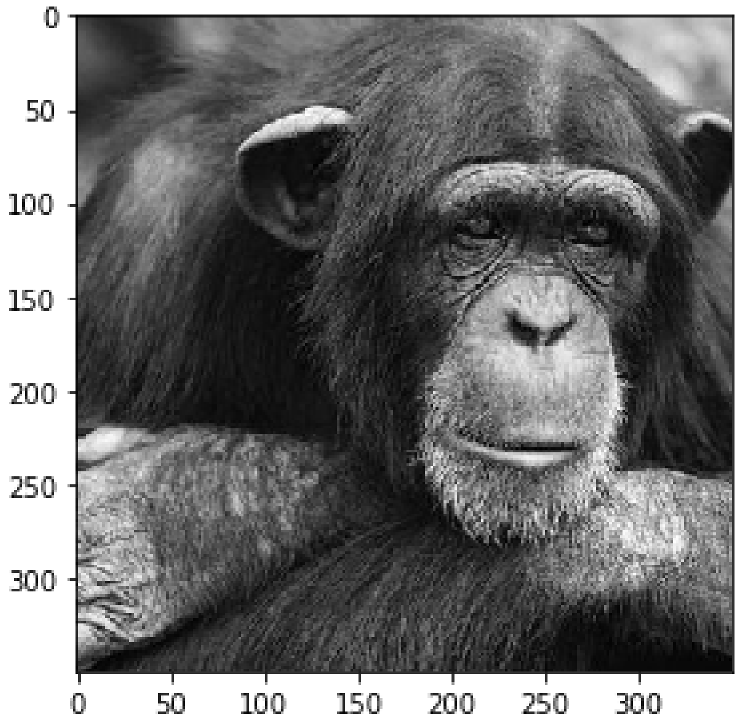
We are going to load a grayscale image, resize it to a compact size. Out of it we are going to create a graph as follows: if the image is an array Y of size n^2, then we create an array of T size n^4. Each row of the bigger array represents a pixel of the image. Implicitly pixels are connected by an edge only if they are neighbors (on the horizontal or vertical, we exclude the diagonal for simplicity). So let $(i, j), (i+1, j)$ be neighboring pixels in T, then we set: $T[i*n+j, i*(n+1)+j] = 1/(1+|y[i, j]-y[i+1, j]|)$ (and then make it symmetric etc...) So the matrix T represents the lattice graph with weighted esges.

We are then going to calculate the influence matrix of that implicit weighted graph, and cluster to k clusters the pixels just as we did above. Then we recreate the image as follows: We create a new n^2 array. The pixels that belong to a cluster, are all going to have the same value, which is the mean of those pixels on Y.

[35]:
```
# now with a real image again
n=50
im = io.imread('chimp-665439.jpg', as_gray=True)
im.shape
x = ski.util.crop(im, ((0,0), (120,120)))
```

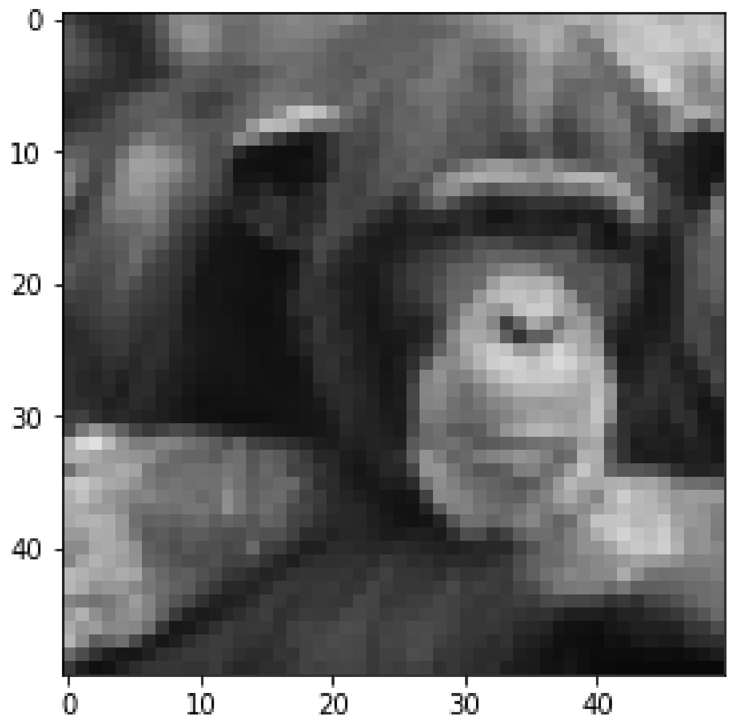
```
x.shape

io.imshow(x)
```

[35]:
```
<matplotlib.image.AxesImage at 0x7f335d054550>
```



[38]:
```
y = resize(x, (n,n))
z = y.flatten()
io.imshow(y)
```

[38]:
```
<matplotlib.image.AxesImage at 0x7f335d2cccd0>
```



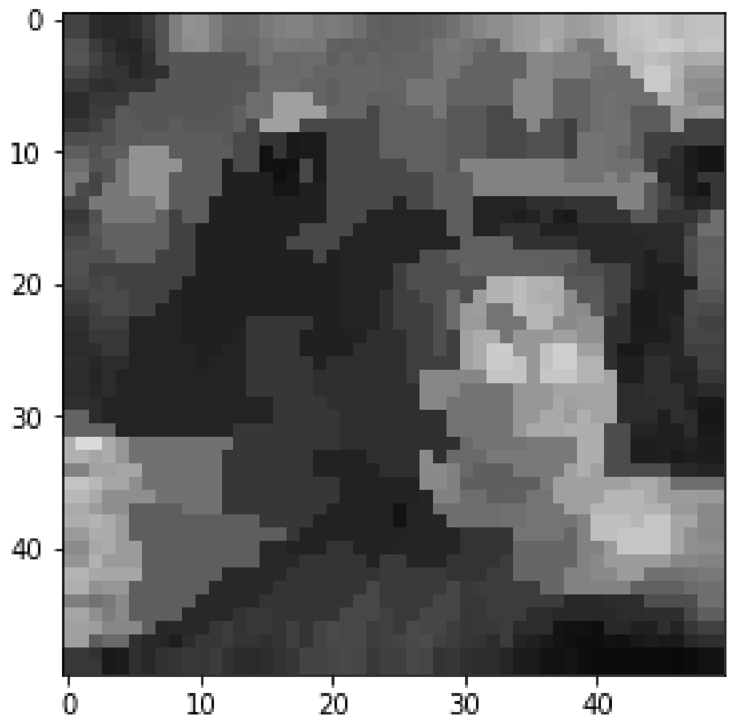
[39]:
```
T = np.zeros((n**2,n**2))

for i in range(n**2 - 1):
    if (i+1) % n > 0:
        r = i // n
        c = i % n
        T[i,i+1] = 1 / (1 + abs(
            y[r,c] - y[r,c+1]))
    if (i+n) < n**2:
        T[i,i+n] = 1 / (1 + abs(
            y[r,c] - y[r+1,c]))
```

[40]:
```
T = T + np.transpose(T)
cc = bottomUpCluster(T, 560)
```

```
100%|      | 2500/2500 [13:48<00:00,  3.02it/s]
```

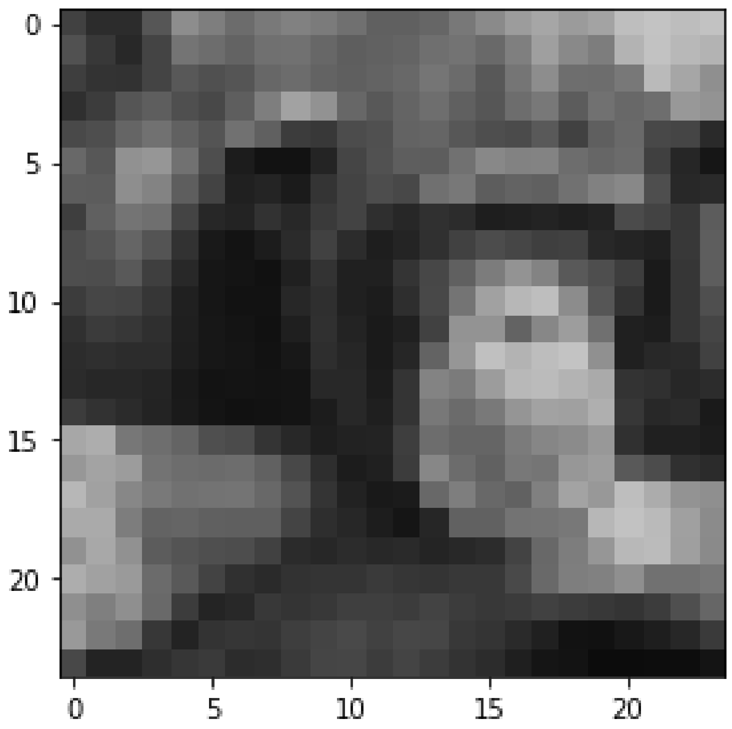
[41]:
```
X = np.zeros(n*n)
for i in range(560):
    #X[cc[i]] = i
    X[cc[i]] = z[cc[i]].mean()

io.imshow(X.reshape(n,n))
```

[41]: <matplotlib.image.AxesImage at 0x7f335d28af10>



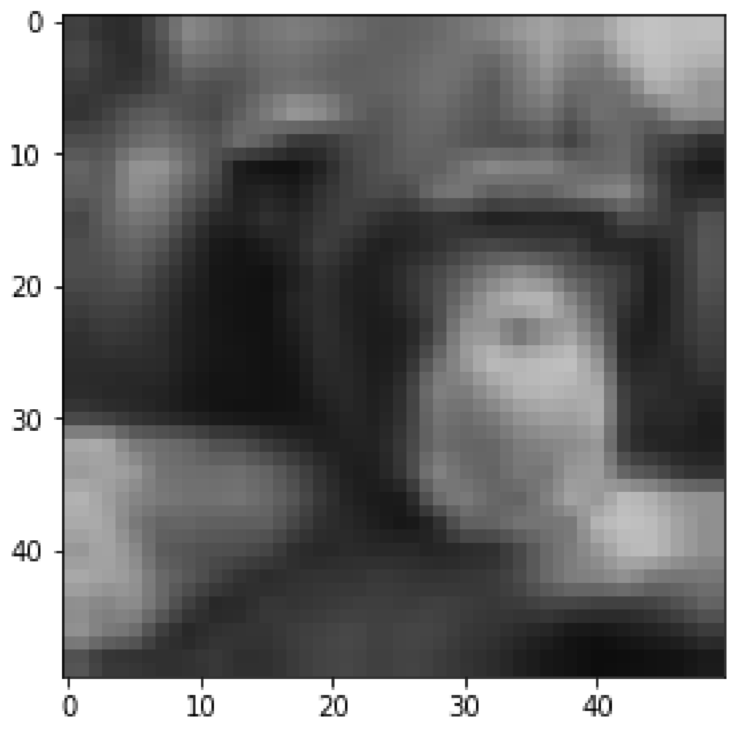
[42]:
```
# for comparison, if we resize the image to 24x24 which is equivalent to␣
↪partitioning it into
# 24x24 squares and taking the mean of each, it looks like this:
io.imshow(resize(y, (24,24)))
```

[42]: <matplotlib.image.AxesImage at 0x7f335de78610>



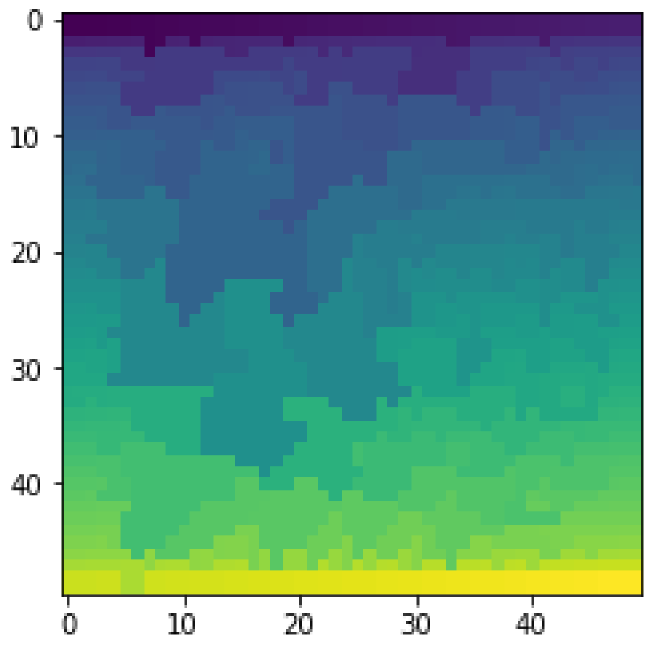
```
[43]: io.imshow(resize(resize(y, (24,24)), (50,50)))
```

```
[43]: <matplotlib.image.AxesImage at 0x7f3356412950>
```



```
[44]: X = np.zeros(n*n)
      for i in range(560):
          #X[cc[i]] = i
          X[cc[i]] = i

      plt.imshow(X.reshape(n,n))
```

```
[44]: <matplotlib.image.AxesImage at 0x7f33563d0b90>
```

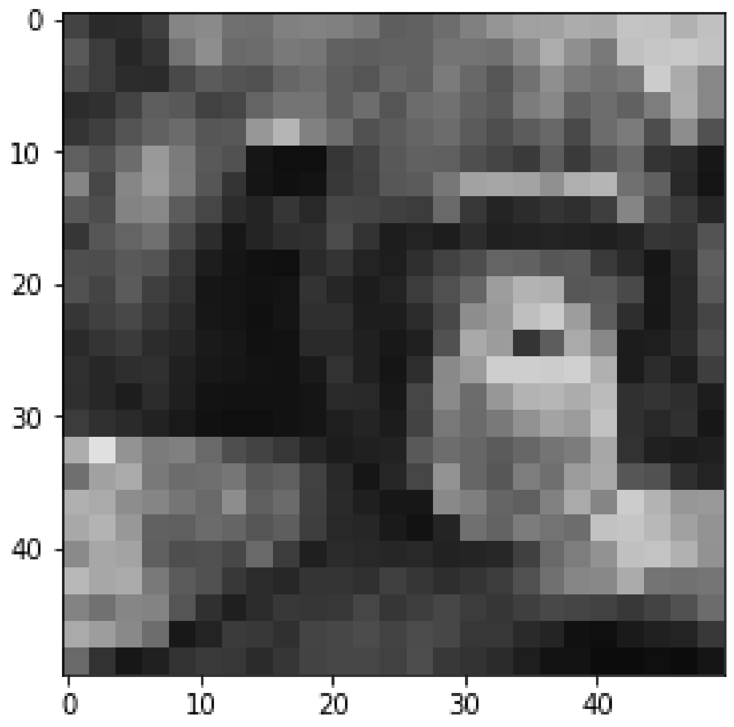


[45]:
```
yy = np.zeros((n,n))
yy.shape
```

[45]:
```
(50, 50)
```

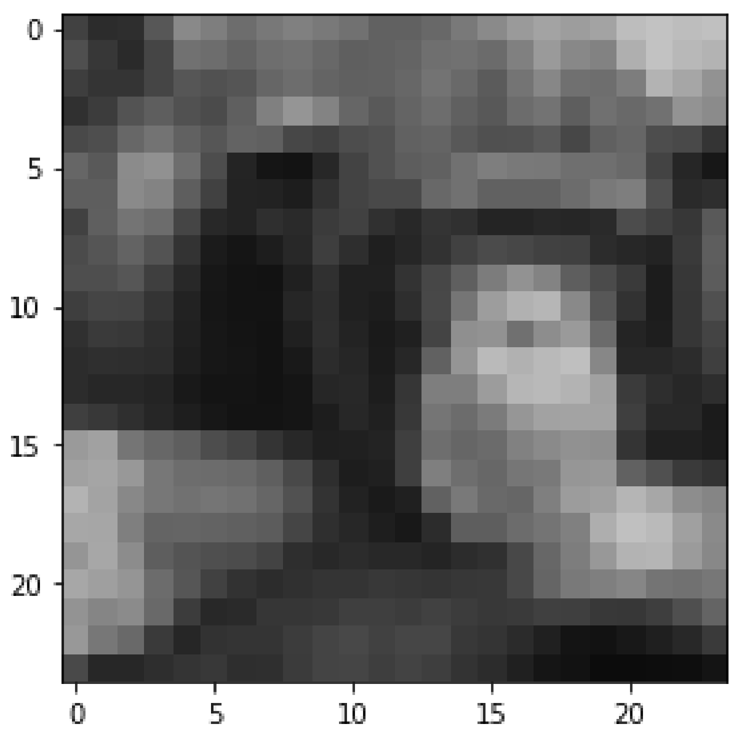
[48]:
```
yy = np.zeros((n,n))

for i in range(0, n, 2):
    yy[i] = y[i]
    yy[i+1] = y[i]
for i in range(0, n, 2):
    yy[:,i+1] = yy[:,i]

io.imshow(yy)

#again creating a 50x50 where each square
```

[48]:
```
<matplotlib.image.AxesImage at 0x7f335632c510>
```



```
[50]: # more tests
      n=24
      y = resize(x, (n,n))
      z = y.flatten()
      io.imshow(y)
```

```
[50]: <matplotlib.image.AxesImage at 0x7f3355eff690>
```



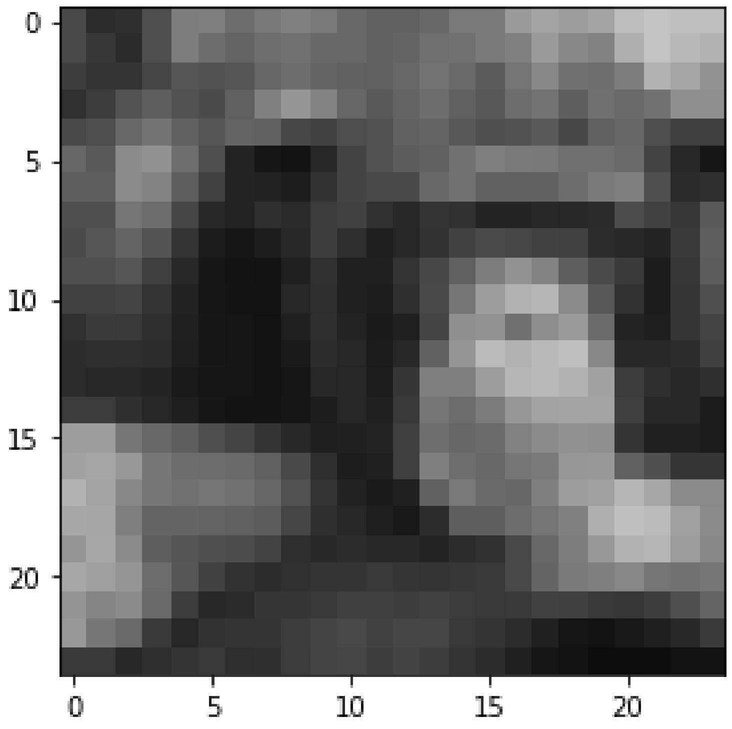
```
[51]: T = np.zeros((n**2,n**2))

      for i in range(n**2 - 1):
          if (i+1) % n > 0:
              r = i // n
              c = i % n
              T[i,i+1] = 1 / (1 + abs(
                  y[r,c] - y[r,c+1]))
          if (i+n) < n**2:
              T[i,i+n] = 1 / (1 + abs(
                  y[r,c] - y[r+1,c]))
```

```
[52]: T = T + np.transpose(T)
      cc = bottomUpCluster(T, 560)
```

```
100%|      | 576/576 [00:08<00:00, 71.01it/s]
```

```
[53]: X = np.zeros(n*n)
      for i in range(560):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

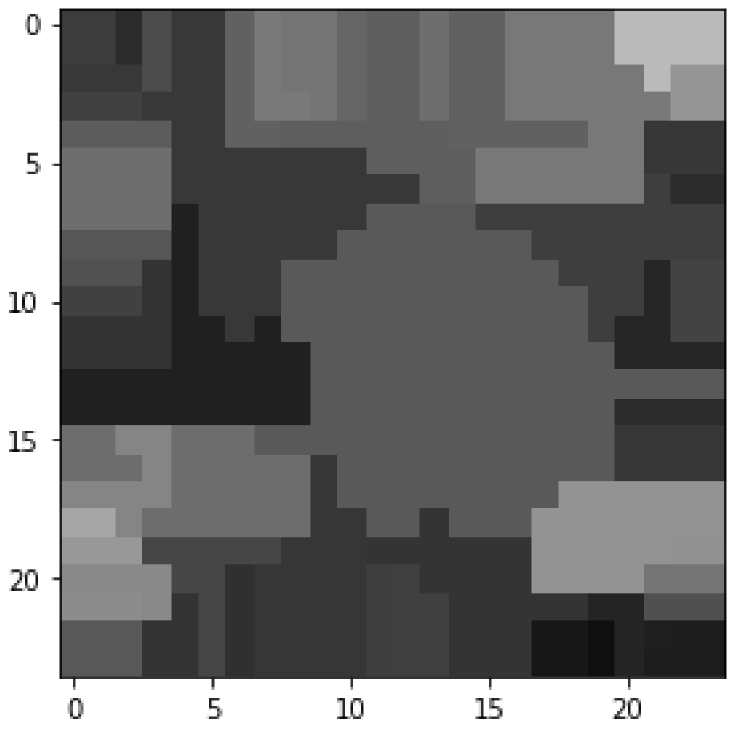
```
[53]: <matplotlib.image.AxesImage at 0x7f3356794b10>
```



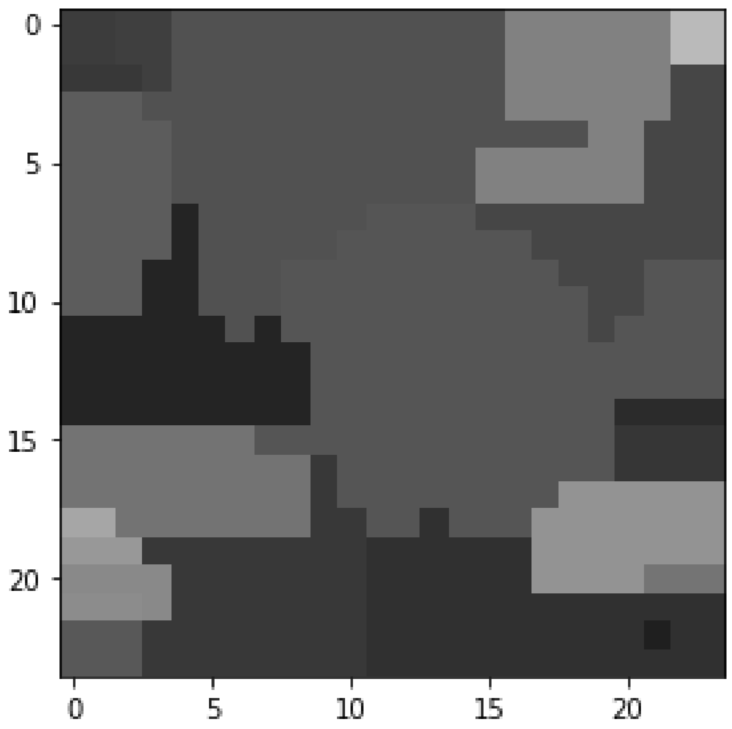
```
[56]: # trying to scale down the cluster number ever further
      cc = bottomUpCluster(T, 56)

      X = np.zeros(n*n)
      for i in range(56):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

```
100%|      | 576/576 [00:07<00:00, 73.81it/s]
```

```
[56]: <matplotlib.image.AxesImage at 0x7f3355ed7e10>
```



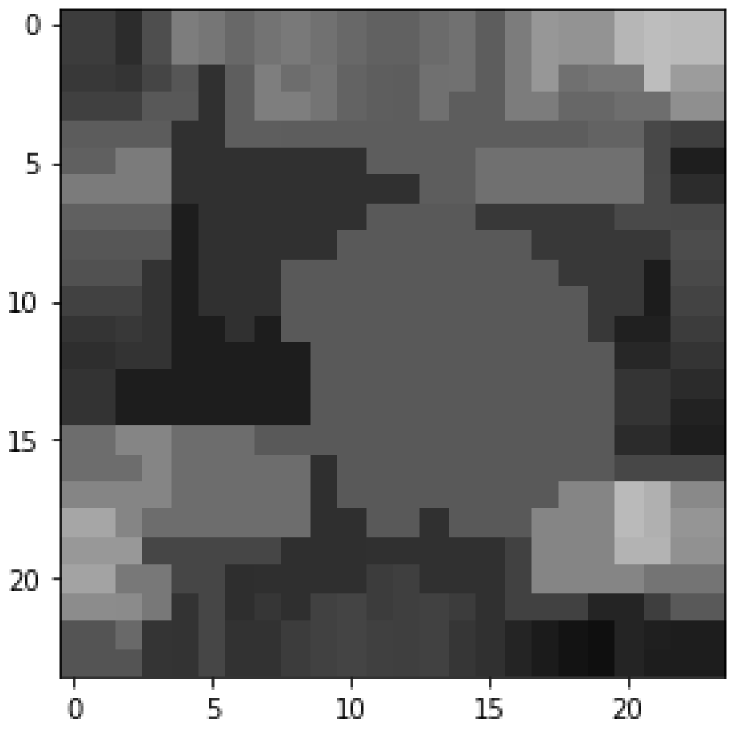
```
[58]: # trying to scale down the cluster number ever further
      cc = bottomUpCluster(T, 24)

      X = np.zeros(n*n)
      for i in range(24):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))

      100%|      | 576/576 [00:07<00:00, 76.35it/s]

[58]: <matplotlib.image.AxesImage at 0x7f33563eef50>
```



```
[59]: cc = bottomUpCluster(T, 128)

      X = np.zeros(n*n)
      for i in range(128):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

```
100%|          | 576/576 [00:07<00:00, 72.42it/s]
```

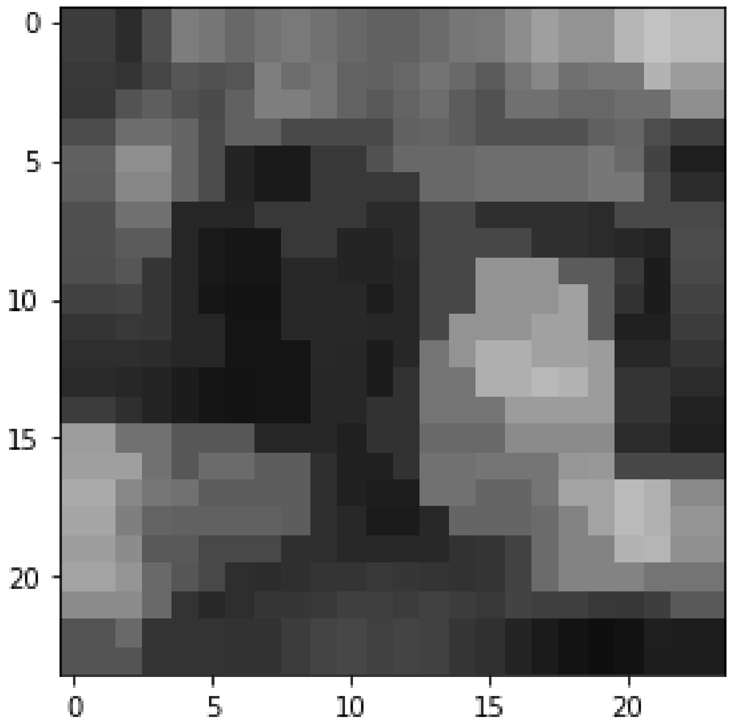
```
[59]: <matplotlib.image.AxesImage at 0x7f335d08d950>
```



```
[60]: cc = bottomUpCluster(T, 256)

      X = np.zeros(n*n)
      for i in range(256):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

```
100%|          | 576/576 [00:08<00:00, 71.18it/s]
```

```
[60]: <matplotlib.image.AxesImage at 0x7f3355c8ce50>
```

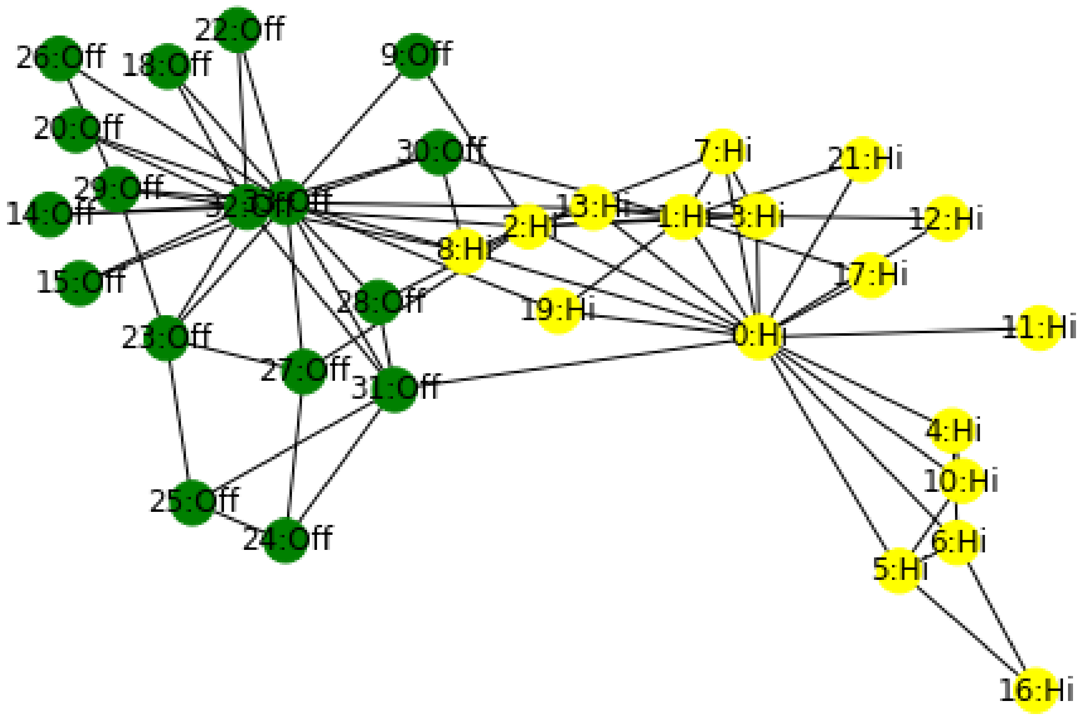


4 Karate Club

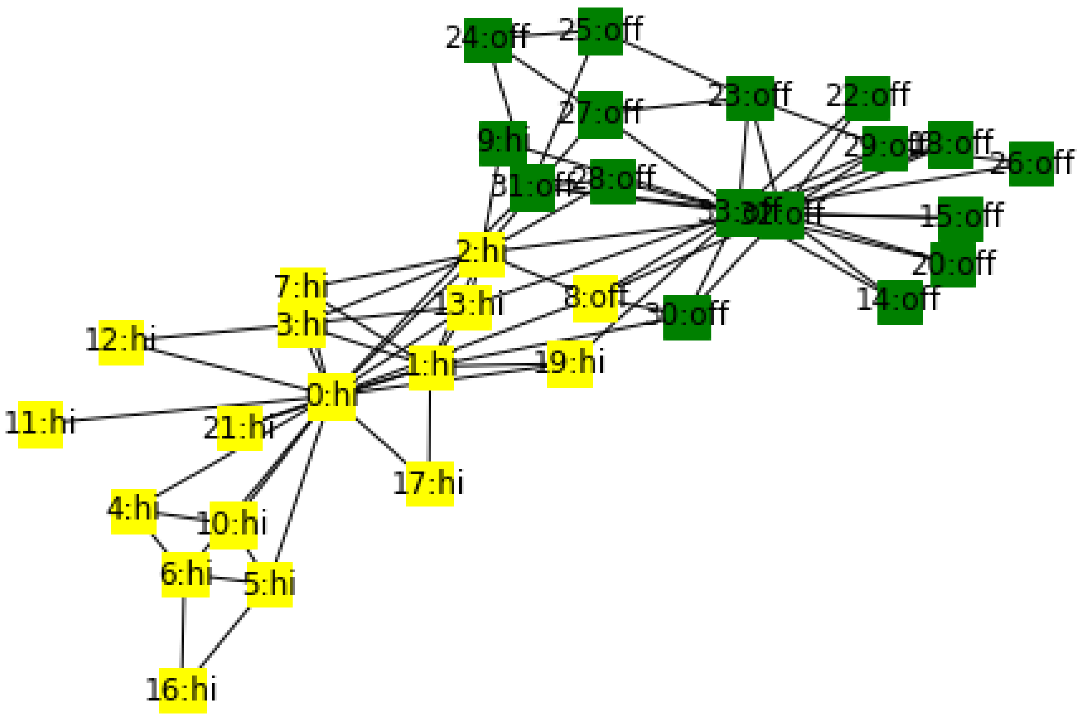
We are now going to try this clustering algorithm on the famous "Karate Club". We shall see how the close the clustering of the algorithm comes to the gound truth.

[25]:
```
# Karate Club
G = nx.karate_club_graph()

colors = [0 if G.nodes[v]['club'] == 'Mr. Hi' else 1 for
        v in G.nodes()]

colors = np.array(colors)

colors

colours = ['yellow' if x==0 else 'green' for x in colors]

clublabel = ['Hi' if i==0 else 'Off' for i in colors]

clublabel = [str(i) + ":" + clublabel[i] for i in G.nodes()]

clubdict = dict(zip(G.nodes(), clublabel))
clubdict
```

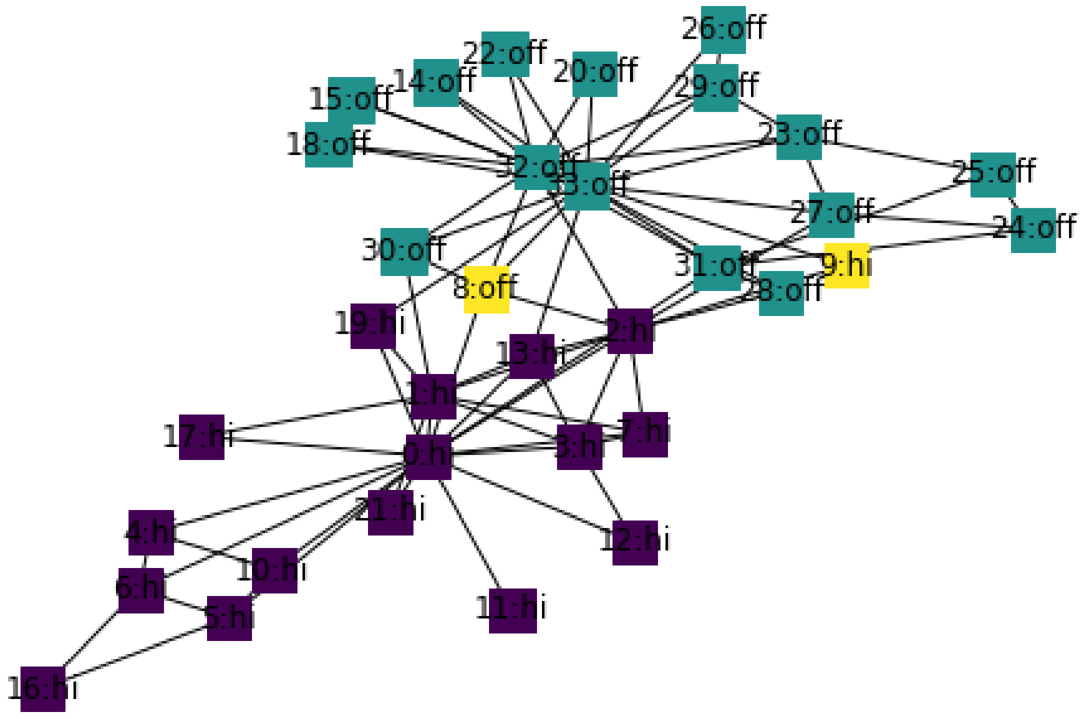
```
nx.draw_spring(G, with_labels=True, node_color=colours, labels=clubdict)
```



[26]:
```
W = reducedInfluenceMatrixG(G, delta=0)

cc = bottomUpClusterG(G, W, 2)

clusters = np.zeros(len(W))

clusters[cc[1]]=1
clusters

clusters == colors
# so this algorithm got it mostly correct, with 2 errors.

W

#for i,j in G.edges:
#    G.edges[i,j]['weight'] = W[i,j]
#
#nx.draw_spring(G, with_labels=True, node_color=colors, k=30, labels=clubdict)

clusterlabel = ['hi' if i==0 else 'off' for i in clusters]
clusterlabel = [str(i) + ":" + clusterlabel[i] for i in G.nodes()]

clusterdict = dict(zip(G.nodes(), clusterlabel))

nx.draw_spring(G, with_labels=True, node_color=colours,
        node_shape='s', labels=clusterdict)
```



[5]:
```
adjcolors = colors.copy()
adjcolors[colors != clusters] = 2

adjcolors
# labels show: node name and cluster according to the algorithm. Colors show␣
 ↪the two groups in reality
# In case the cluster agrees with the ground truth. A third color indicates the␣
 ↪exceptions
nx.draw_spring(G, with_labels=True, node_color=adjcolors,
        node_shape='s', labels=clusterdict)
```



5 New Idea: USe Propagation on the Edge-Graph for clusterin

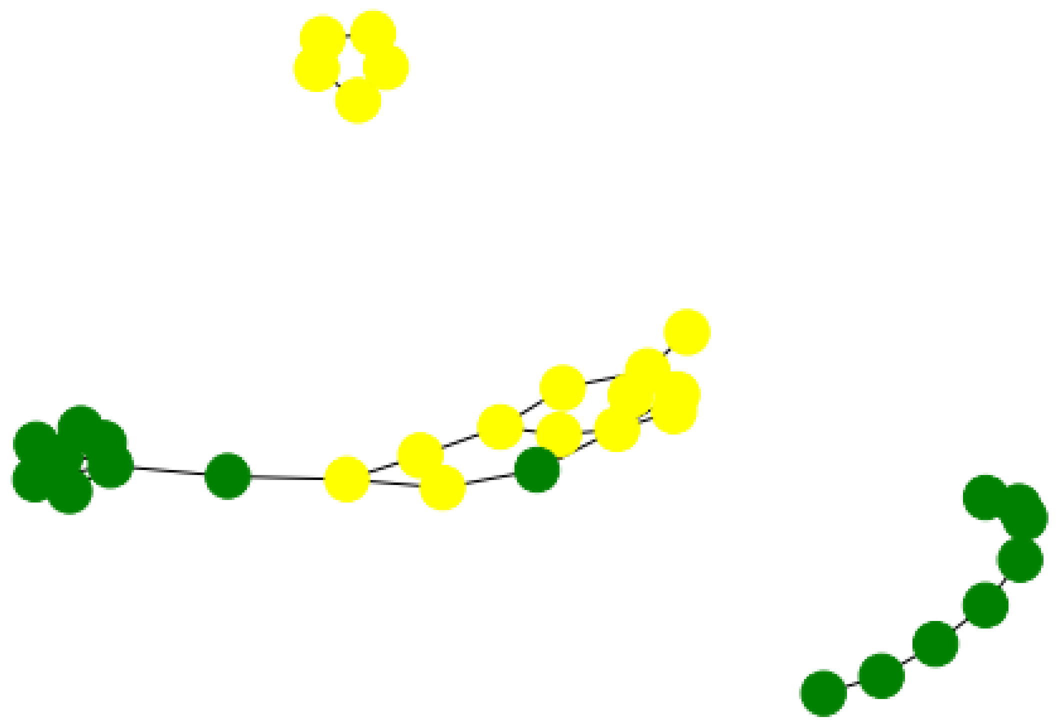
New Clustering Algorithm: Starting from a Graph G, we construct its ‘edge graph’ G’ whose vertices are the edges of G and vertices are connected by an edge in G’ iff their corresponding edges share a vertex. Edges that connect multiple groups probably have high degree of betweeness. In the edge graph G’ I predict that these nodes are going to be highly ranked. Edges that connect members of the same group in G will be nodes of average pagerank because they are sort of all the same as the other inter-group edges. The idea: Create the edge graph and calculate its pagerank. Then from G the edges that correspond to the hottest nodes until G is split into 2 components. Then repeat the procedure on each component. Stopping points can also be defined. In case the pagerank is close to uniform we predict that there is no informative subdivision and stop the process.

Further motivation: There are 2 types of edges: edges that connect 2 nodes within the same group, or edges that connect 2 nodes from 2 different groups, so in short and ecge is either ‘within’ or ‘between’. We are interested in identifying the between edges. Remove them, and the graph is partitioned to connected component. Propagation is computed for nodes on the graph rather than its edges. If we move to the edge graph, the propagation of the nodes of the edge graph is an assessment of the edges of the original graph. a between edge is not necesarily hotter than a within edge. But perhaps the variance of its influence is smaller than a within edge, because it has some influence on multiple groups whereas a ‘within’ edge has most of its influence concetrated within its own group. Hypothesis: If we take a ‘between’ edge, and look at the induced subgraph of that edge and its neighbouring edges in the edge graph, in a sprung layout it will look like an hourglass. If we take a ‘within’ edge, the induced subgraph will resemble a clique. Extra motivation for PPIN: An edge in the PPI represent an interaction, which can either be internal ‘within a function/complex’ or transitional ‘between function/complex’.

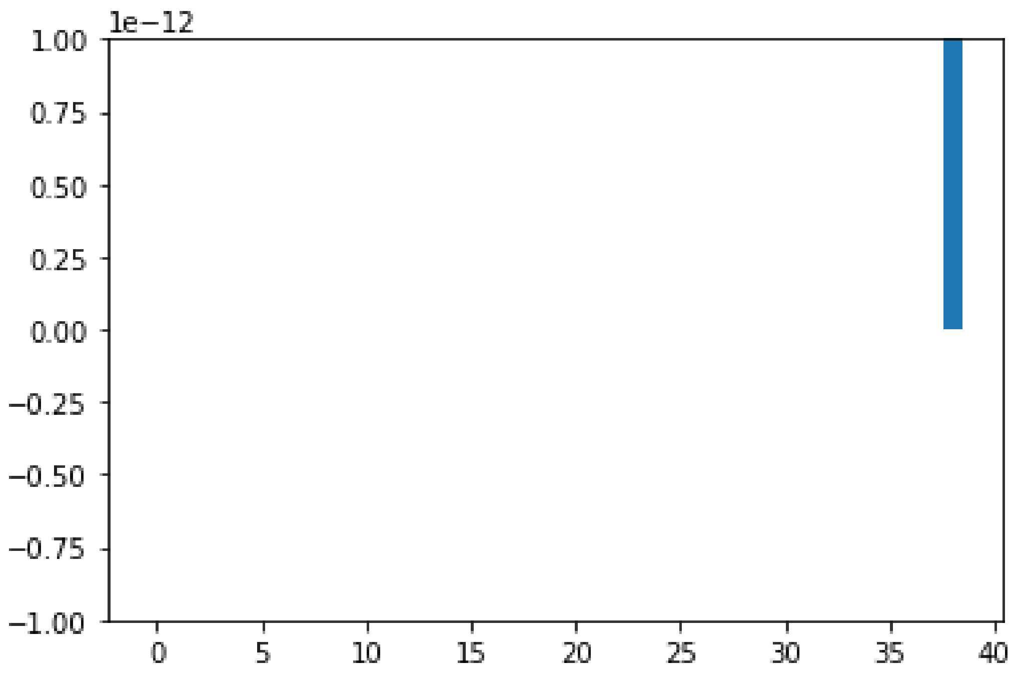
[7]:
```
testG = G.copy()
eG = edgeGraphG(testG)
p,_ = powerIterateG(eG)

x = np.argmax(p)

eG.nodes[x]['edg']

y = tuple( eG.nodes[x]['edg'])

testG.remove_edge(*y)

nx.number_connected_components(testG)
```

[7]:
```
1
```

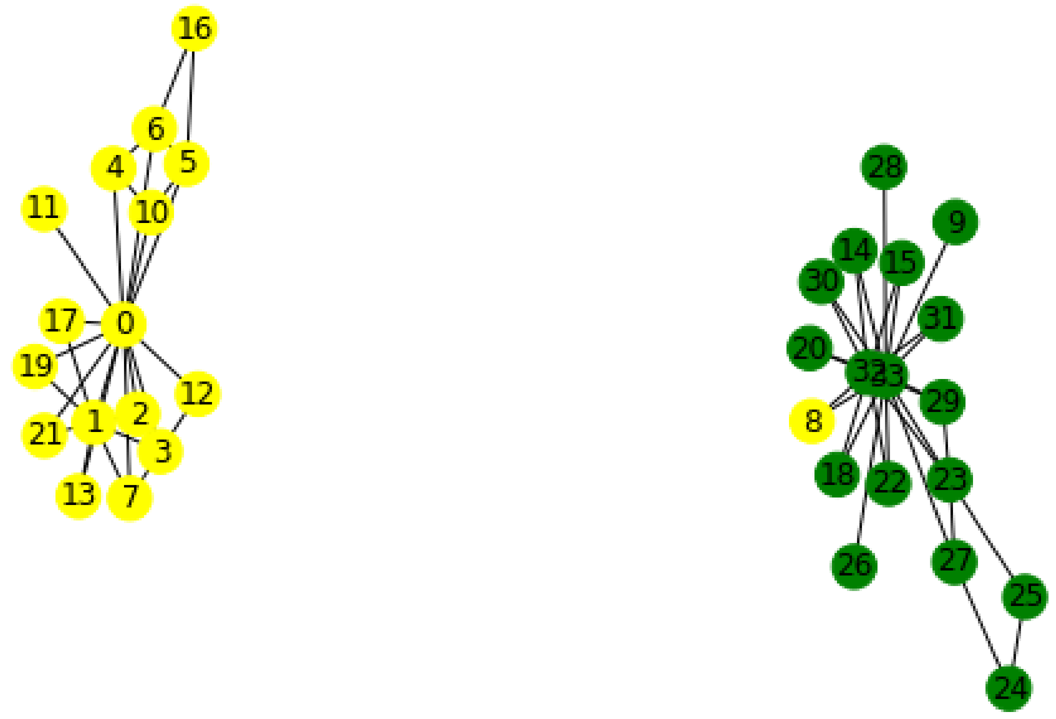
[27]:
```
testG = G.copy()
eG = edgeGraphG(testG)
while nx.number_connected_components(testG) <= 3:
    p,_ = powerIterateG(eG)
    x = np.argmax(p)
    y = tuple( eG.nodes[x]['edg'])
    testG.remove_edge(*y)
    eG = edgeGraphG(testG)

nx.draw_spring(testG, node_color=colours)
# fail :(
```



```
[9]: _ , w = pageRanksConcentratedBiasG(eG)

     w[9].sum()

     w.sum(axis=1)

     w.min(axis = 1)

     plt.bar(range(len(w)), w.min(axis=1))

     x = np.argmax(w.min(axis=1))
     eG.nodes[x]['edg']

     x = np.argmax(w.var(axis=1))
     eG.nodes[x]['edg']
     # nope fail again :(

[9]: array([23, 29])
```



[28]:
```
testG = G.copy()
eG = edgeGraphG(testG)
_ , w = pageRanksConcentratedBiasG(eG)
w.sum(axis=1)
w.sum(axis=0)
w[0].sum()
w[:,0].sum() #row sum to 1, col not

while nx.number_connected_components(testG) <= 1:
    _ , w = pageRanksConcentratedBiasG(eG)
    x = np.argmin(w.var(axis=1))
    eG.nodes[x]['edg']
    y = tuple( eG.nodes[x]['edg'])
    testG.remove_edge(*y)
    eG = edgeGraphG(testG)



nx.draw_spring(testG, node_color=colours, with_labels=True)
```



5.1 What we did in above

We caomputed the influence matrix (non-symmetric, not reduced) of the edge-graph on G. For each row (axis=1) we calculate its variance. We predict that an edge in G that connects two clusters- its corresponding node in the edge-graph will have low variance because its influence extends to both groups. For a ‘within’ edge, its influence exends mostly within its own group. Well maybe its a coincidence but it worked for the karate club... The missed node, 8 is a person with interactions to both clubs so its not immediately clear why the person chose one club over the other. In fact node 8 has 8 interactions with ‘Officer’ vs 2 with ‘Mr Hi” yet it belongs to ’Mr Hi’

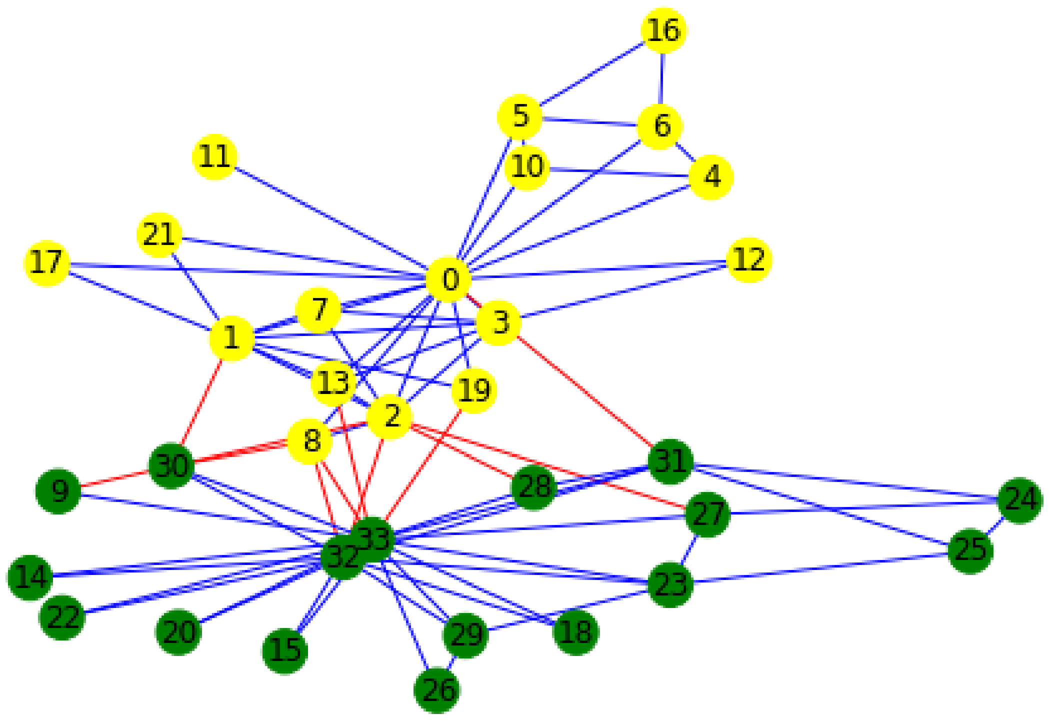
[22]:
```
print(G.nodes[8]['club'], list(G.neighbors(8)))
[G.nodes[x]['club'] for x in G.neighbors(8)]
```

```
Mr. Hi [0, 2, 30, 32, 33]
```

[22]:
```
['Mr. Hi', 'Mr. Hi', 'Officer', 'Officer', 'Officer']
```

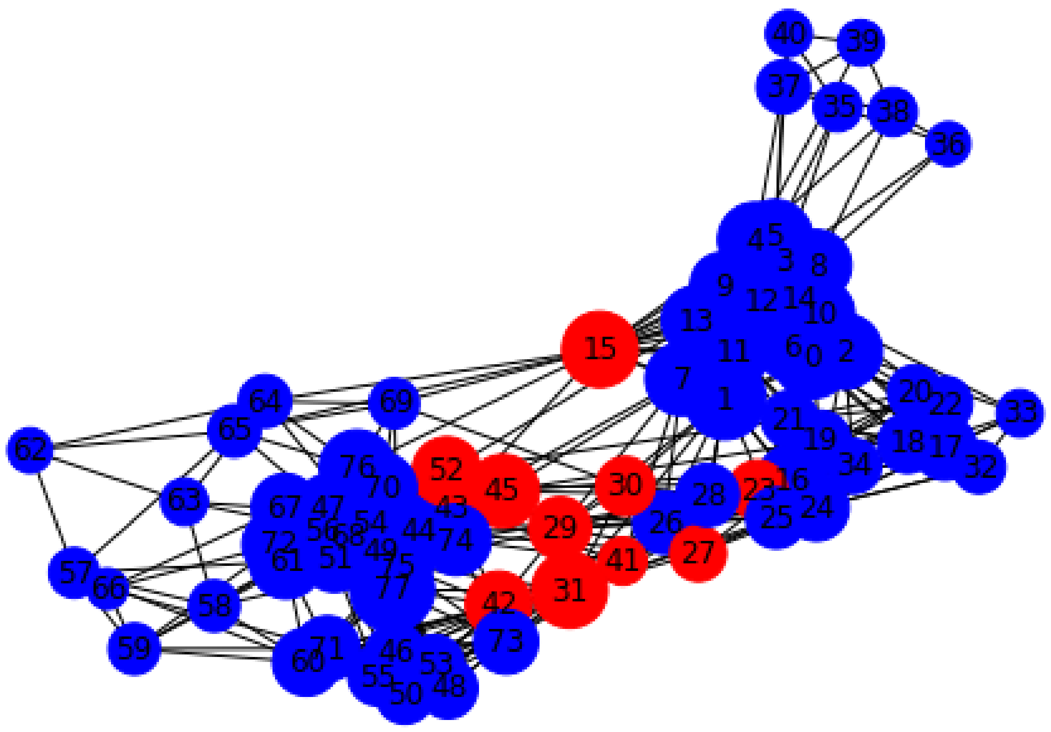
[24]:
```
betweenEdges = [(u,v) for u,v in G.edges()
        if G.nodes[v]['club'] != G.nodes[u]['club']]

betweenEdges

EdgeColors = ['red' if G.nodes[v]['club'] != G.nodes[u]['club'] else 'blue' for
```

```
        u,v in G.edges()]

colours = ['yellow' if x==0 else 'green' for x in colors]

nx.draw_spring(G, node_color=colours, edge_color=EdgeColors, with_labels=True)
```



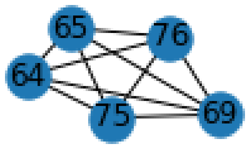
The Karate Club again. Edges in Red are the 'between' edges

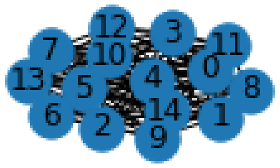
[29]:
```
eG = edgeGraphG(G)
p,_ = powerIterateG(eG)
nx.draw_spring(eG, node_color=EdgeColors, with_labels=True, node_size=p*50000)
```



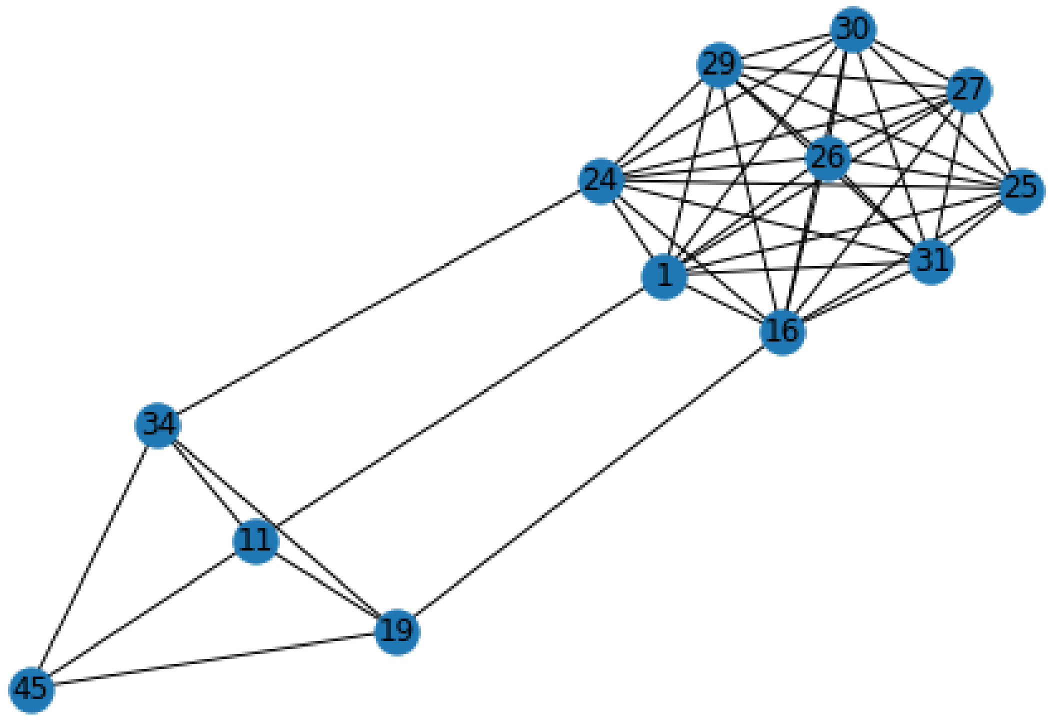
The edge graph of the Karate Club. The Between edges of G are the red nodes of the edge graph.

```
[32]: g = nx.subgraph(eG, eG.neighbors(15))
      nx.draw_spring(g, with_labels=True)
```

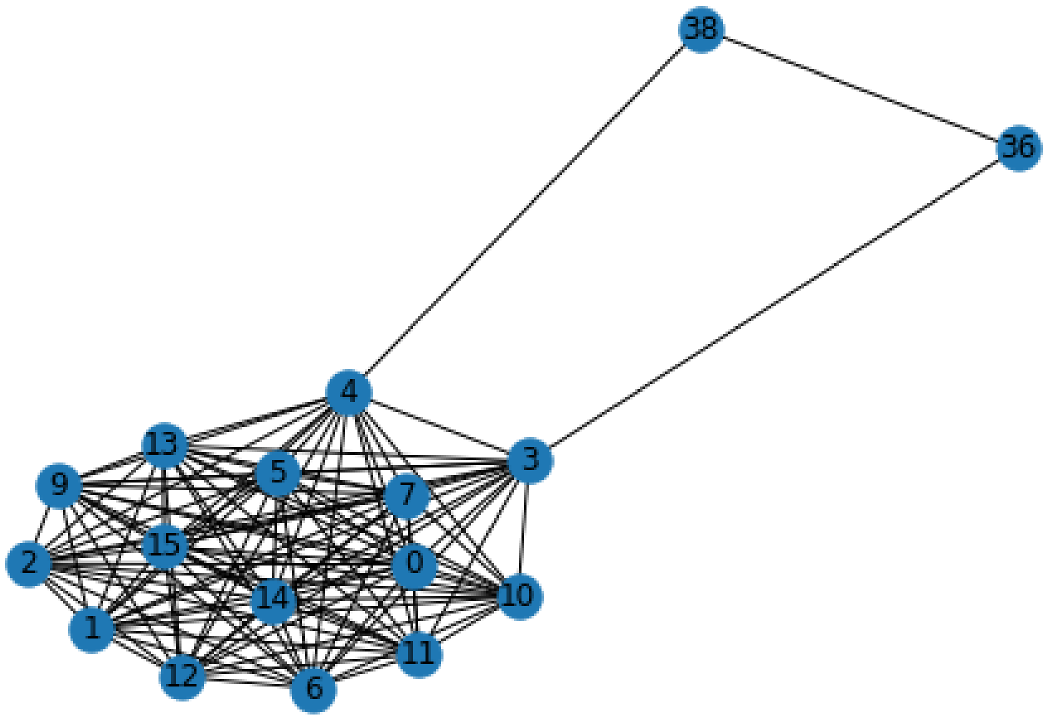




[31]:
```
g = nx.subgraph(eG, eG.neighbors(28))
nx.draw_spring(g, with_labels=True)
```



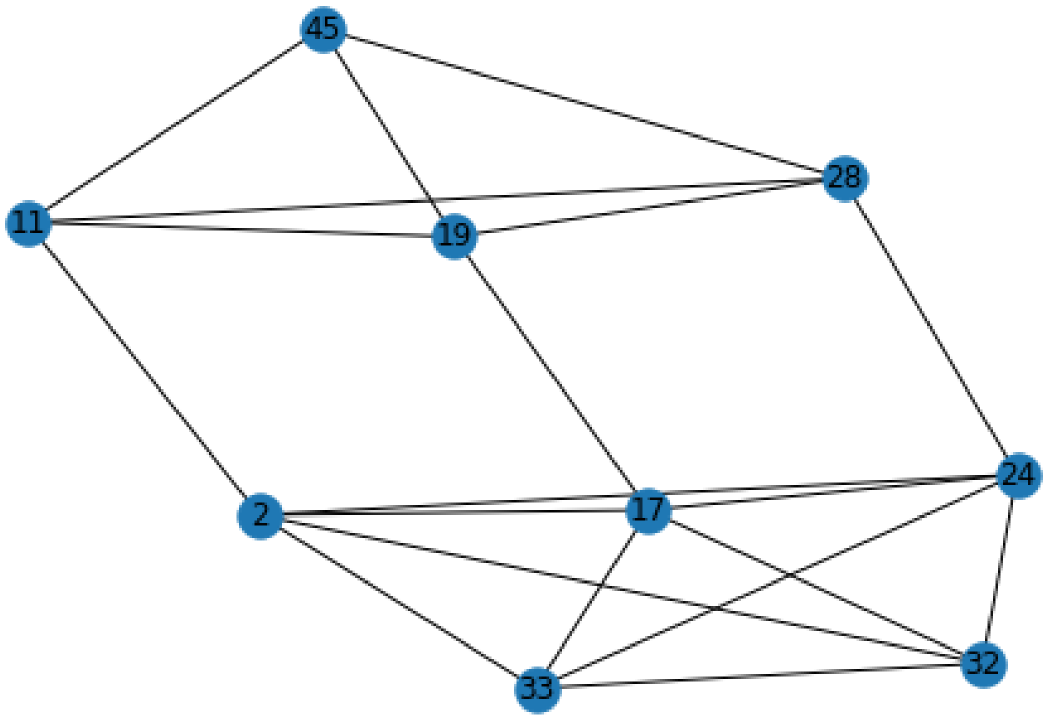
[33]:
```
g = nx.subgraph(eG, eG.neighbors(8))
nx.draw_spring(g, with_labels=True)
```



```
[34]: g = nx.subgraph(eG, eG.neighbors(44))
      nx.draw_spring(g, with_labels=True)
```

27
54
74
51
43
52
76
47
56
67
68
77
70
61
49
45
72

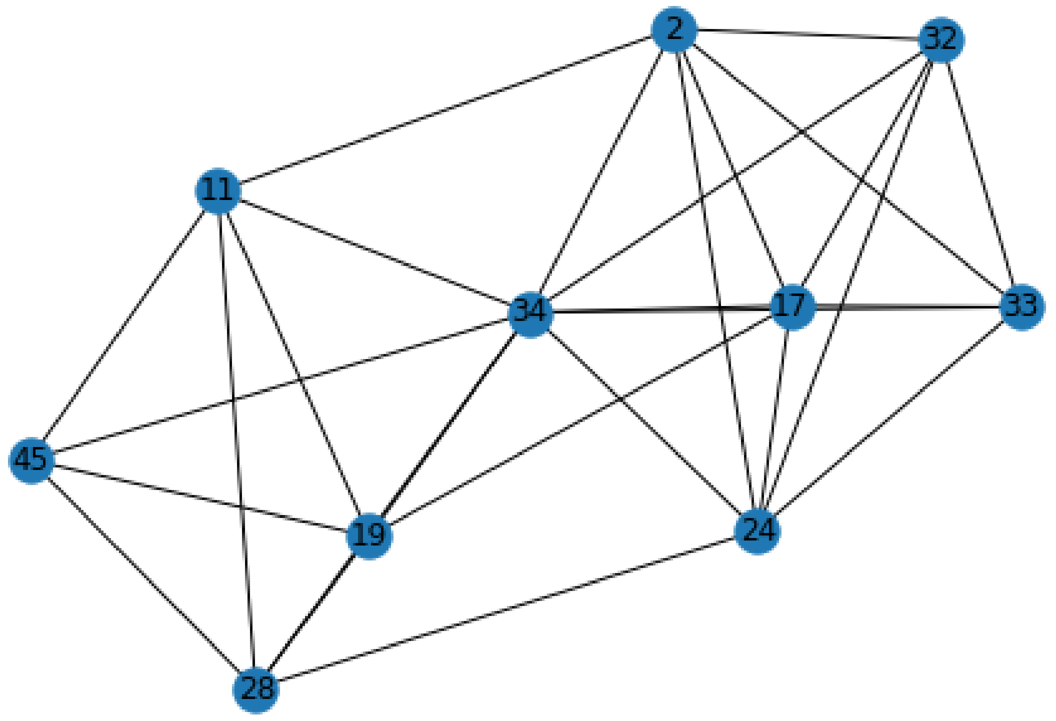
[35]:
```
g = nx.subgraph(eG, eG.neighbors(34))
nx.draw_spring(g, with_labels=True)
```



```
[36]: y = list(eG.neighbors(34))
      y.append(34)
      y
```

```
[36]: [2, 11, 17, 19, 24, 28, 32, 33, 45, 34]
```

```
[37]: g = nx.subgraph(eG,y )
      nx.draw_spring(g, with_labels=True)
```

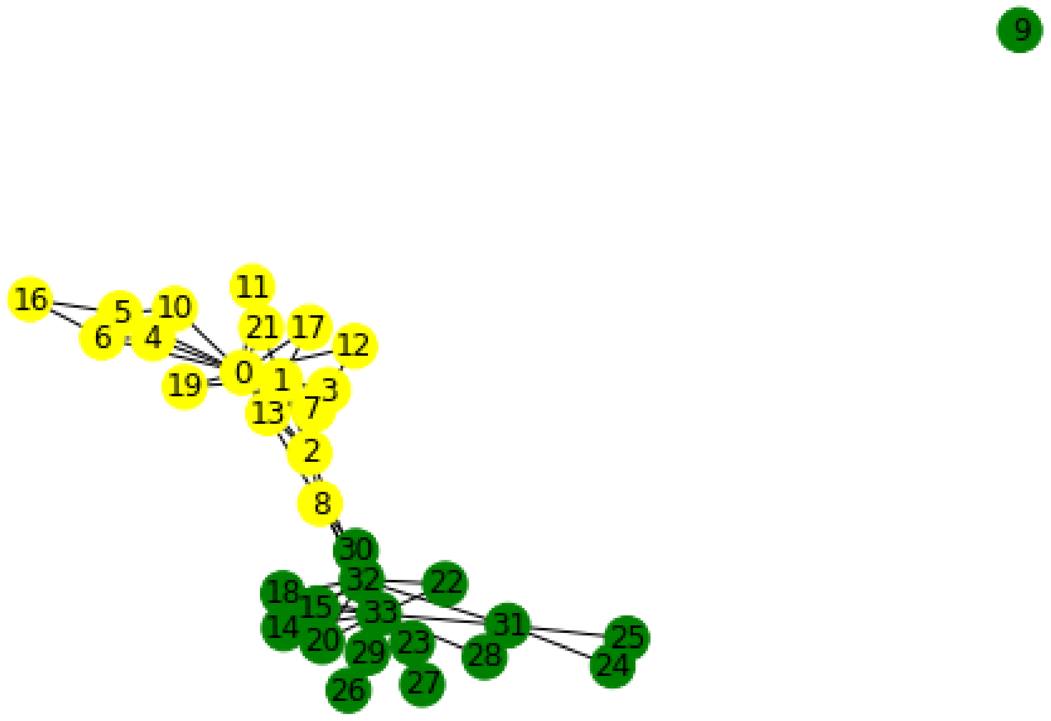


when we consider between edges, in the edge graph, we expect (?) that their neighbors are divided into 2 or more distinct groups. The neigbors of a within edge remain a single group even when the with edge (as a node) is removed! A graph cannot be further partition if in its edgegraph there are no nodes whose neighbors form 2 or mored identifiable groups.

[38]:
```
testG = G.copy()
eG = edgeGraphG(testG)
myList = eG.nodes()
for e in myList:
    g = nx.subgraph(eG, eG.neighbors(e))
    if nx.number_connected_components(g) > 1:
        print(e, eG.nodes[e]['edg'])
        testG.remove_edge(*eG.nodes[e]['edg'])

betweenEdges

nx.draw_spring(testG, node_color=colours, with_labels=True)
```

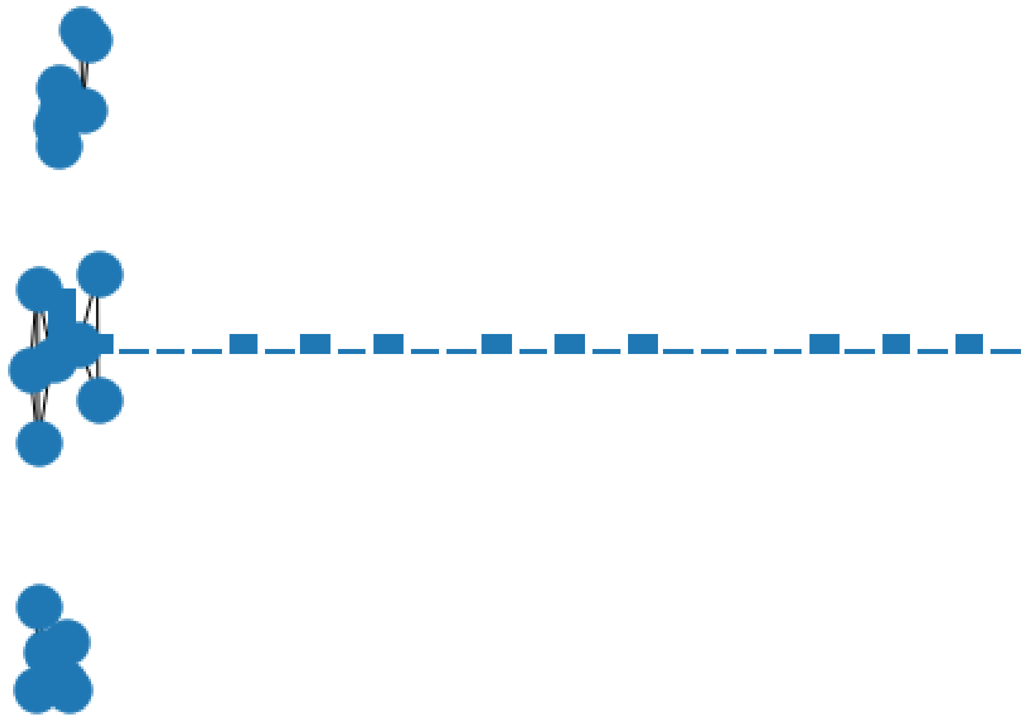
```
15 [ 0 31]
23 [ 1 30]
27 [2 9]
29 [ 2 27]
30 [ 2 28]
44 [ 9 33]
```

```
45 [13 33]
52 [19 33]
57 [23 25]
63 [24 27]
```



[39]:
```python
# We need a looser criterion to qualify a 'between' edge. Instead of require
# complete disconnection we need a measure of 'loosely connected' components.


foo = nx.Graph()
foo.add_nodes_from(range(5))

bar, _ = powerIterateG(foo)

foo.add_edges_from([(0,1),(0,2),(1,2),(3,4)])

bar, _ = powerIterateG(foo)

foo.add_node(5)
foo.add_edges_from([(3,5),(4,5)])
nx.draw_spring(foo)

foo.add_node(6)
foo.add_edges_from([(3,6),(4,6),(5,6)])
nx.draw_spring(foo)

foo.add_edge(2,3)
nx.draw_spring(foo)

bar, _ = powerIterateG(foo)
voo = 1/len(bar) * np.ones_like(bar)

rel_entr(bar,voo).sum()
rel_entr(voo,bar).sum()

_, w = pageRanksConcentratedBiasG(foo)


ag = np.array(nx.adj_matrix(g).todense())

w = pageRanksConcentratedBias(ag)

q = 1/len(w)
q

np.max(1/w * q)

(1/w * q).max(axis=1)

plt.bar(range(len(w)), w[0])
```

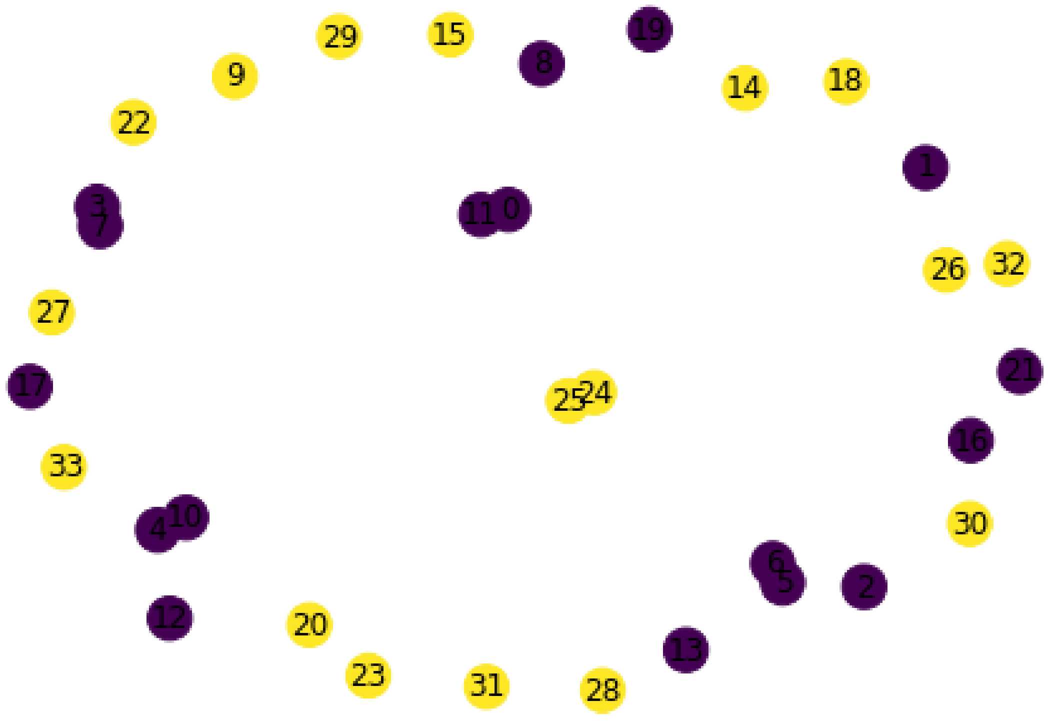
```
[39]: <BarContainer object of 27 artists>
```



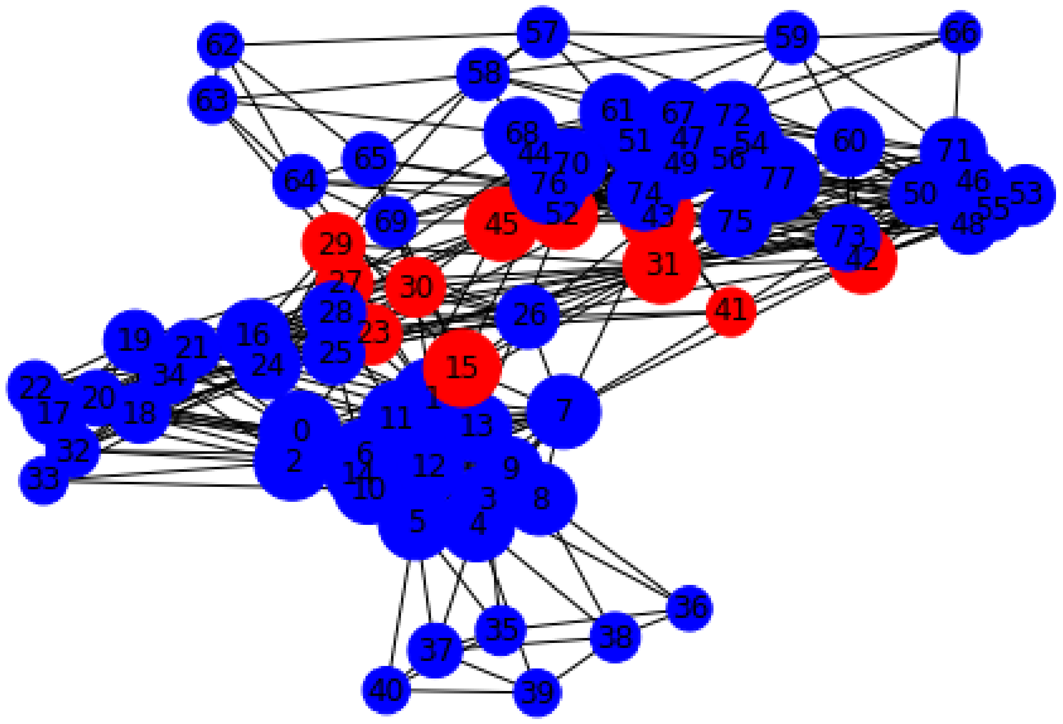
```
[40]: testG = G.copy()
      eG = edgeGraphG(testG)
      myList = eG.nodes()
      for e in myList:
          g = nx.subgraph(eG, eG.neighbors(e))
          if nx.number_connected_components(g) > 1:
              print('discovered ', e, eG.nodes[e]['edg'])
              testG.remove_edge(*eG.nodes[e]['edg'])
          else:
              ag = np.array(nx.adj_matrix(g).todense())
              w = pageRanksConcentratedBias(ag)
              q = 1/len(w)
              if np.max(1/w * q) > 2.5:
                  print('maybe discovered ', e, eG.nodes[e]['edg'])
                  testG.remove_edge(*eG.nodes[e]['edg'])




      betweenEdges

      nx.draw_spring(testG, node_color=colors, with_labels=True)
```

```
maybe discovered  0 [0 1]
maybe discovered  1 [0 2]
maybe discovered  2 [0 3]
maybe discovered  3 [0 4]
maybe discovered  4 [0 5]
maybe discovered  5 [0 6]
maybe discovered  6 [0 7]
maybe discovered  7 [0 8]
maybe discovered  8 [ 0 10]
maybe discovered  10 [ 0 12]
maybe discovered  11 [ 0 13]
maybe discovered  12 [ 0 17]
maybe discovered  13 [ 0 19]
maybe discovered  14 [ 0 21]
discovered  15 [ 0 31]
maybe discovered  16 [1 2]
maybe discovered  17 [1 3]
maybe discovered  18 [1 7]
maybe discovered  19 [ 1 13]
maybe discovered  20 [ 1 17]
maybe discovered  21 [ 1 19]
maybe discovered  22 [ 1 21]
discovered  23 [ 1 30]
maybe discovered  24 [2 3]
maybe discovered  25 [2 7]
maybe discovered  26 [2 8]
discovered  27 [2 9]
maybe discovered  28 [ 2 13]
discovered  29 [ 2 27]
discovered  30 [ 2 28]
maybe discovered  31 [ 2 32]
maybe discovered  33 [ 3 12]
maybe discovered  34 [ 3 13]
maybe discovered  35 [4 6]
maybe discovered  38 [ 5 10]
maybe discovered  39 [ 5 16]
maybe discovered  40 [ 6 16]
maybe discovered  41 [ 8 30]
maybe discovered  42 [ 8 32]
maybe discovered  43 [ 8 33]
discovered  44 [ 9 33]
discovered  45 [13 33]
maybe discovered  46 [14 32]
maybe discovered  47 [14 33]
maybe discovered  48 [15 32]
maybe discovered  49 [15 33]
maybe discovered  50 [18 32]
maybe discovered  51 [18 33]
```

```
discovered  52 [19 33]
maybe discovered  53 [20 32]
maybe discovered  54 [20 33]
maybe discovered  55 [22 32]
maybe discovered  56 [22 33]
discovered  57 [23 25]
maybe discovered  58 [23 27]
maybe discovered  59 [23 29]
maybe discovered  60 [23 32]
maybe discovered  61 [23 33]
discovered  63 [24 27]
maybe discovered  64 [24 31]
maybe discovered  65 [25 31]
maybe discovered  66 [26 29]
maybe discovered  67 [26 33]
maybe discovered  68 [27 33]
maybe discovered  69 [28 31]
maybe discovered  70 [28 33]
maybe discovered  71 [29 32]
maybe discovered  72 [29 33]
maybe discovered  73 [30 32]
maybe discovered  74 [30 33]
maybe discovered  75 [31 32]
maybe discovered  76 [31 33]
maybe discovered  77 [32 33]
```



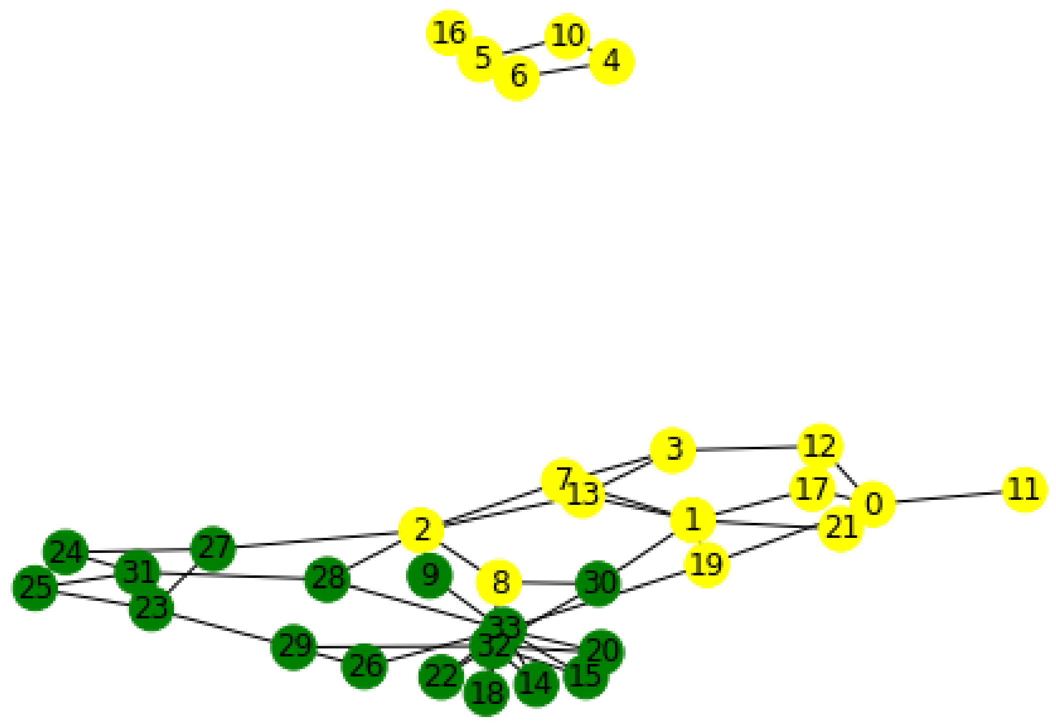
[41]:
```
pos = nx.spring_layout(g)


eG = edgeGraphG(G)
p,_ = powerIterateG(eG)
nx.draw_spring(eG, node_color=EdgeColors, with_labels=True, node_size=p*50000)
```



[44]:
```
testG = G.copy()
eG = edgeGraphG(testG)
myList = eG.nodes()
while nx.number_connected_components(testG) == 1:
    p,_ = powerIterateG(eG)
    e = np.argmax(p)
    print(e, eG.nodes[e]['edg'])
    testG.remove_edge(*eG.nodes[e]['edg'])
    eG = edgeGraphG(testG)

nx.draw_spring(testG, node_color=colours, with_labels=True)
```

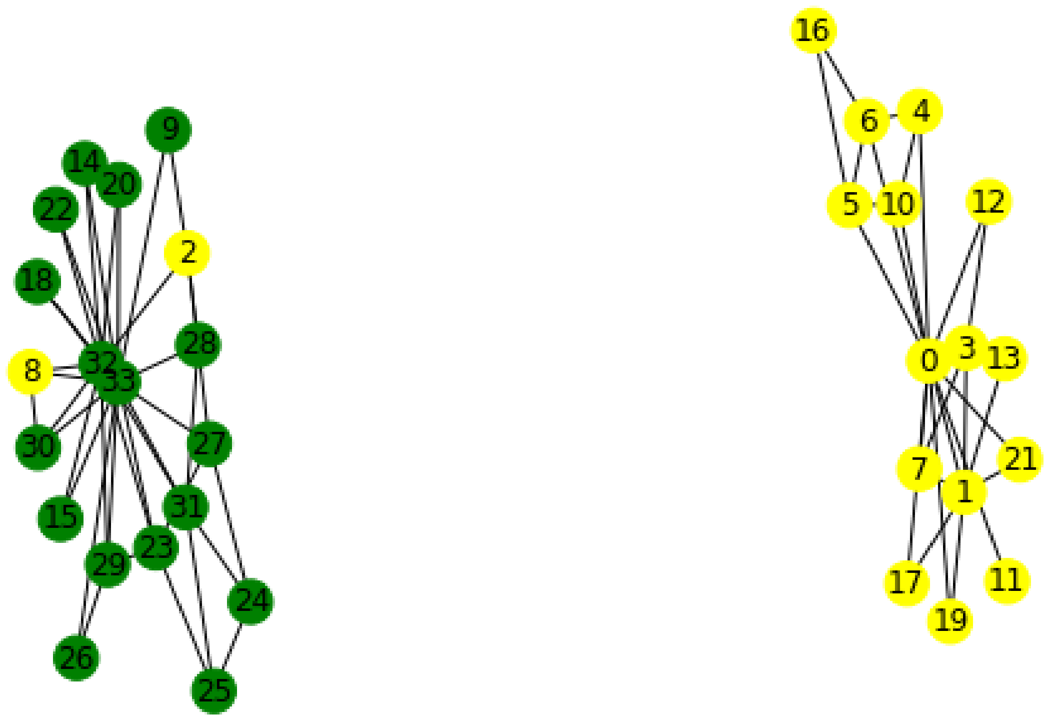
```
77 [32 33]
1 [0 2]
```

```
0 [0 1]
74 [31 33]
29 [ 2 32]
0 [0 3]
12 [ 0 31]
56 [23 33]
40 [13 33]
1 [0 5]
1 [0 6]
10 [1 2]
35 [ 8 33]
57 [27 33]
63 [31 32]
60 [29 33]
50 [23 32]
6 [ 0 13]
59 [30 33]
2 [0 8]
15 [2 3]
1 [0 7]
7 [1 3]
0 [0 4]
0 [ 0 10]
```

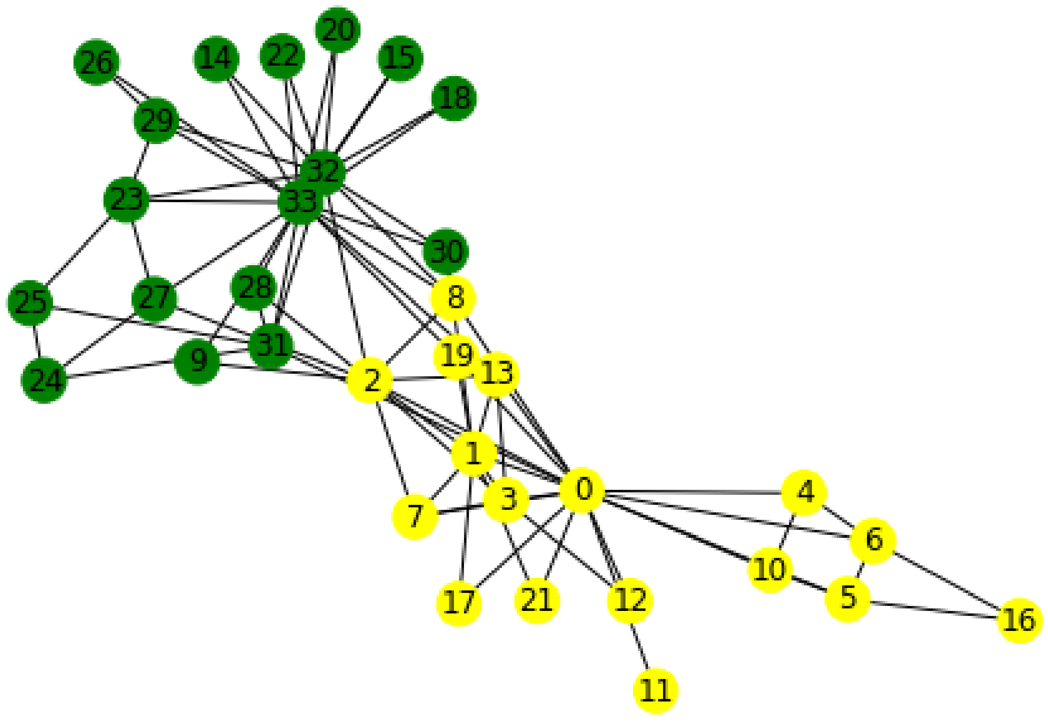


```
[46]: testG = G.copy()
      eG = edgeGraphG(testG)
      myList = eG.nodes()
      while nx.number_connected_components(testG) == 1:
          _, w = pageRanksConcentratedBiasG(eG)
          #w =reducedInfluenceMatrixG(eG)
          e = w.min(axis=0).argmax()
          print(e, eG.nodes[e]['edg'])
          testG.remove_edge(*eG.nodes[e]['edg'])
          eG = edgeGraphG(testG)

      nx.draw_spring(testG, node_color=colours, with_labels=True)
```

```
15 [ 0 31]
25 [2 8]
1 [0 2]
14 [1 2]
41 [13 33]
21 [2 3]
21 [2 7]
22 [ 2 13]
44 [19 33]
6 [0 8]
19 [ 1 30]
```



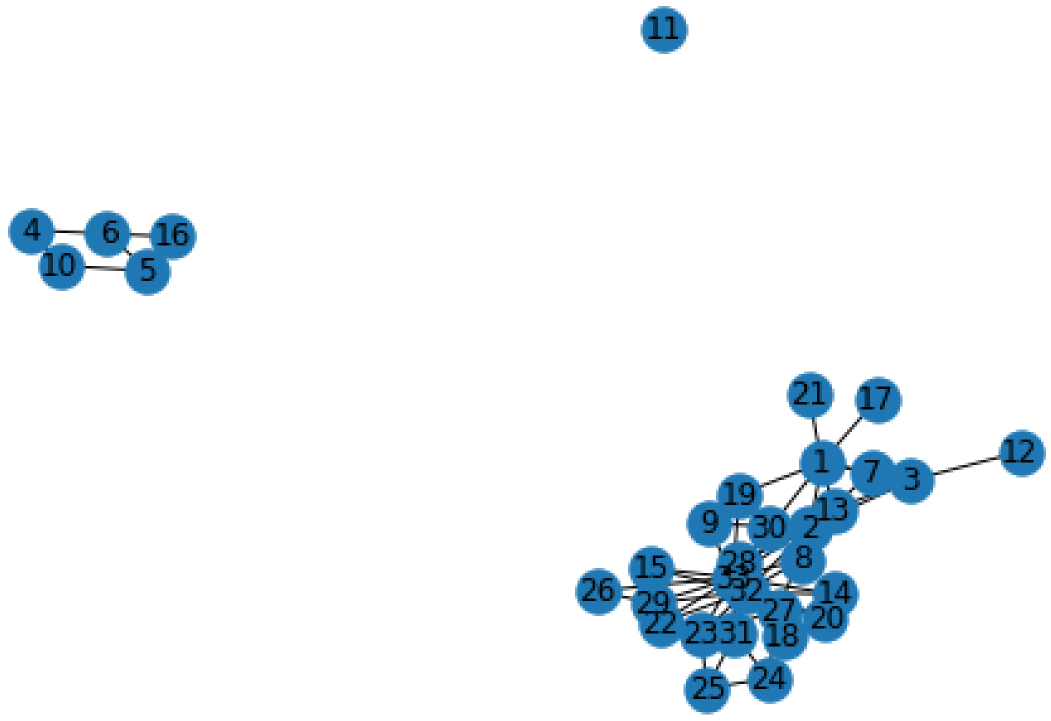
```
[47]: nx.draw_spring(G, node_color=colours, with_labels=True)
```



Above, we again created. This time we identified the edge whose minimal influence is maximal and removed it. Repeat until the Graph is disconnected. The resulting subdivision fails on nodes 8 and 2 which are indeed border nodes and hard to call where they belong.

```
[49]: w =reducedInfluenceMatrixG(eG)
      _, w = pageRanksConcentratedBiasG(eG)

      testG = G.copy()
      eG = edgeGraphG(testG)
      myList = eG.nodes()
      while nx.number_connected_components(testG) == 1:
          _, w = pageRanksConcentratedBiasG(testG)
          #w =reducedInfluenceMatrixG(eG)
          x = w.min(axis=0).argmax()
          print(x)
          testG.remove_node(x)

      nx.draw_spring(testG, with_labels=True)
```

```
0
```



When a try a similar method but on the graph G rather than the