exploration_of_propagation_in_small_graphs

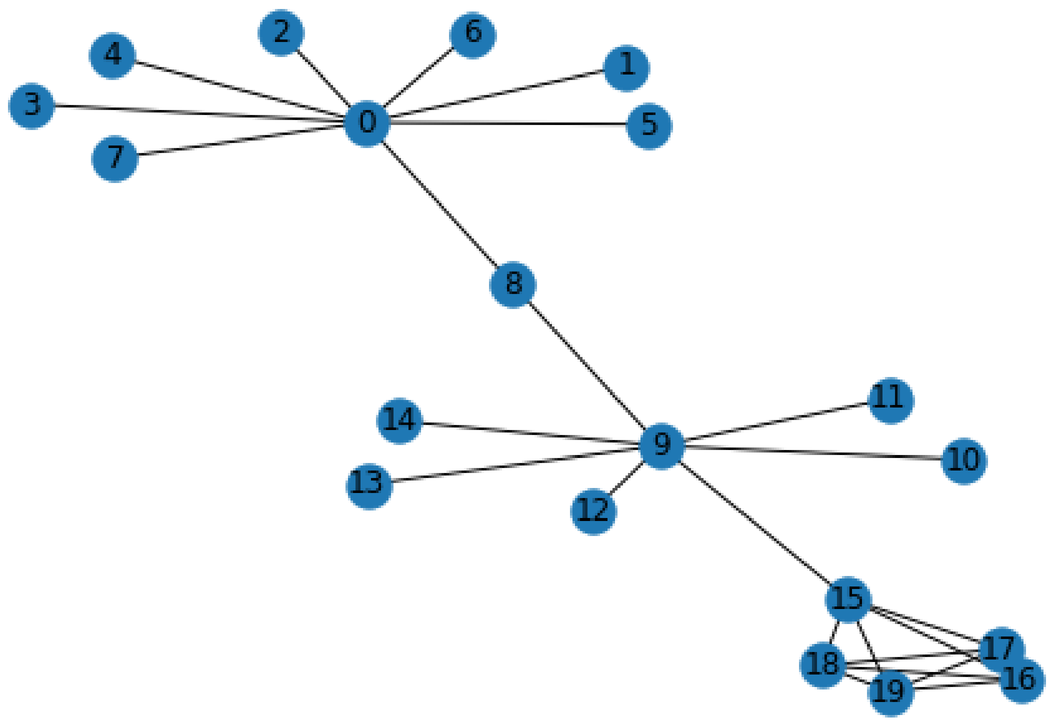
July 13, 2020

1 Propagation Principles in Simple Graphs

We examine graphs that have small vertex-set (up to 20) and have a hub and spoke structure, meaning they have few hubs and the rest of the vertices tend to be leaves. We first consider graphs with no cycles at all.

[1]:
```
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt
import time
import pandas as pd
from scipy.stats import poisson
from tqdm import tqdm

import skimage as ski
from skimage import io

from skimage.transform import rescale, resize, downscale_local_mean


# plt.ioff()
plt.ion()

################ Functions ########################################
from graphsfunctions import *
```

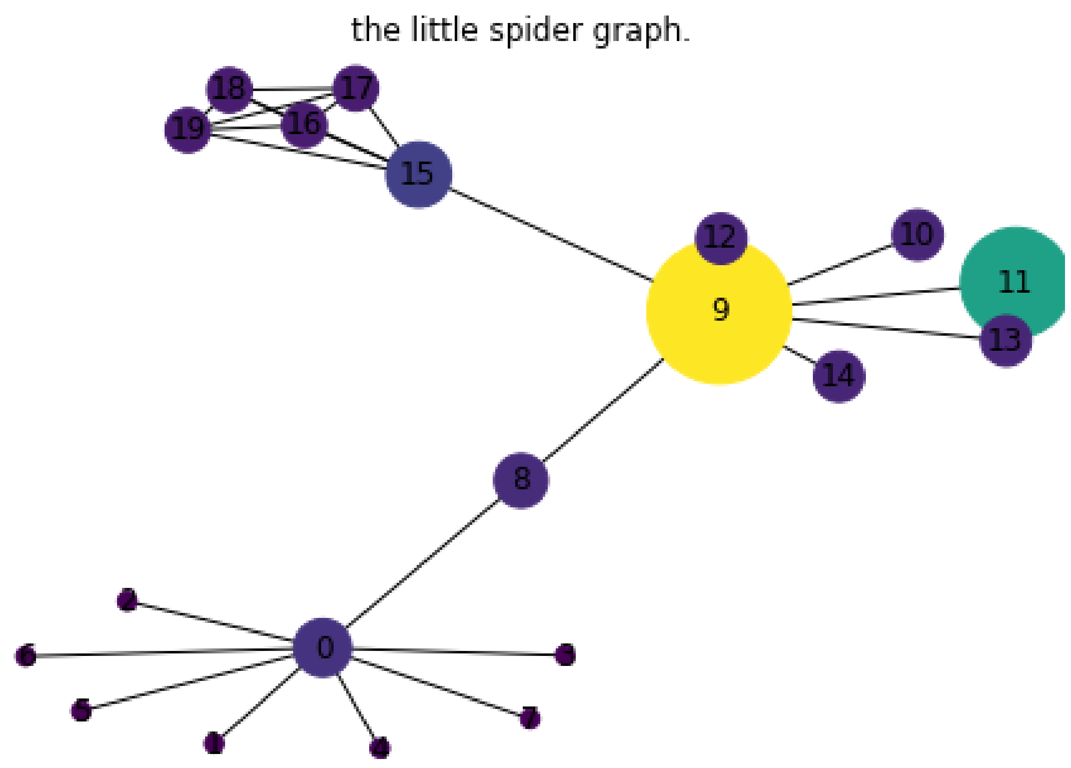
[2]:
```
# Testing a little spider graph
G = nx.Graph()
G.add_nodes_from(range(20))
G.add_edges_from([(0, i) for i in range(1, 8)])
G.add_edge(0, 8)
G.add_edge(8, 9)
G.add_edges_from([(9, i) for i in range(10, 15)])
G.add_edge(15, 9)
G.add_edges_from([(i, j) for i in range(15, 19) for j in range(i + 1, 20)])
nx.draw(G, with_labels=True)
plt.show()
```



The "little Spider" contains a bigger star on 0, connected via 8 to a smaller star on 9, and a K5. We want to explore its propagation properties.

[3]:
```
H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
nx.draw(H, with_labels=True, node_color=W85[11], node_size=W85[11]*10000)
plt.title("the little spider graph.")
plt.show()
```

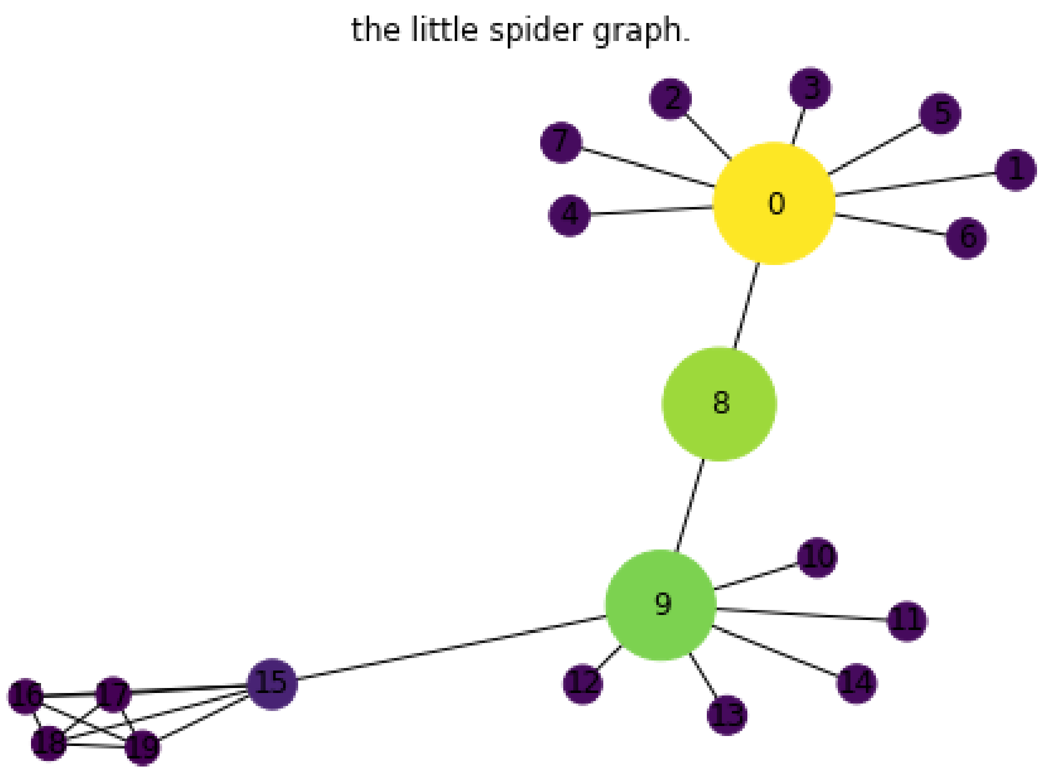
```
100%|          | 20/20 [00:00<00:00, 470.01it/s]
```



The plot shows, both by node size as well as by color code, the stationary propagation with biased concentrated on noe 11, with restart probability of 0.15.

```
[4]: H, W = pageRanksConcentratedBiasG(G, alpha=0.85)
     nx.draw(H, with_labels=True, node_color=W[8], node_size=W[8]*10000)
     plt.title("the little spider graph.")
     plt.show()
```

```
100%|      | 20/20 [00:00<00:00, 395.19it/s]
```

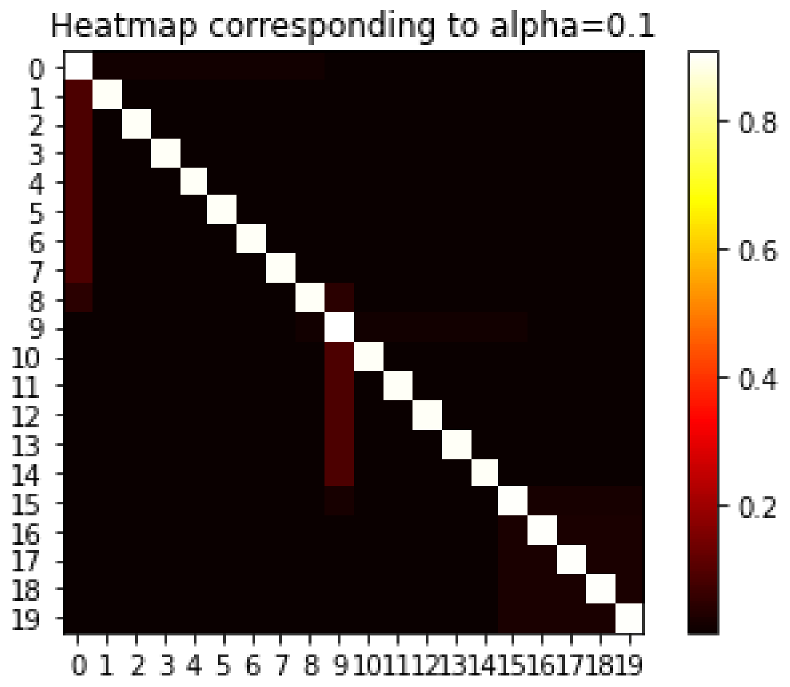


This is the same type of plot, but propagation from 8.

Now we are going top demonstrate how the propgation changes from each vertex, and with various alpha values, by way of heatmaps

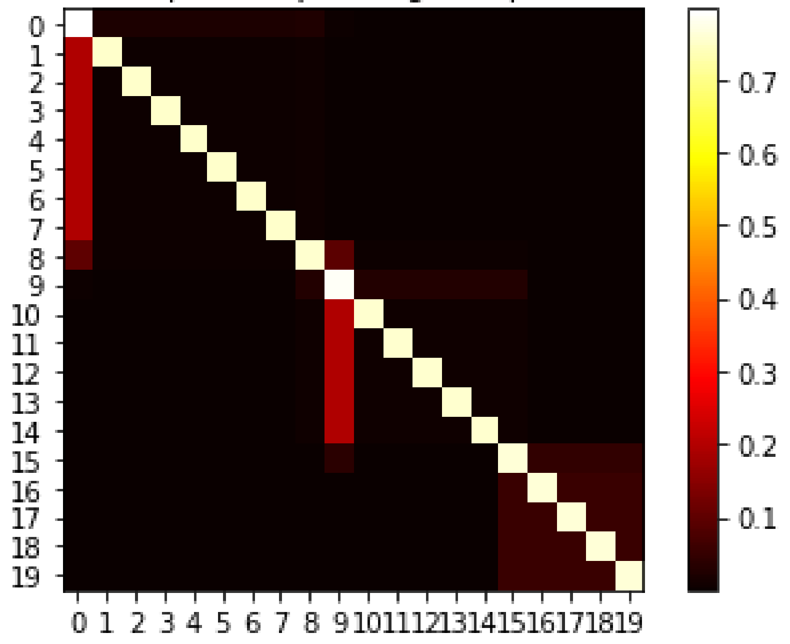
[5]:
```
H, W10 = pageRanksConcentratedBiasG(G, alpha=0.1)
heatmap(W10, "Heatmap corresponding to alpha=0.1")
plt.show()
plt.close()

H, W25 = pageRanksConcentratedBiasG(G, alpha=0.25)
heatmap(W25, "Heatmap corresponding to alpha=0.25")
plt.show()
plt.close()

H, W50 = pageRanksConcentratedBiasG(G, alpha=0.5)
heatmap(W50, "Heatmap corresponding to alpha=0.5")
plt.show()
plt.close()

H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
heatmap(W85, "Heatmap corresponding to alpha=0.85")
plt.show()
plt.close()

H, W90 = pageRanksConcentratedBiasG(G, alpha=0.9)
heatmap(W90, "Heatmap corresponding to alpha=0.9")
plt.savefig("Heatmap Little Spider alpha=0.9.png")
plt.show()
plt.close()
```

```
100%|          | 20/20 [00:00<00:00, 881.66it/s]
```



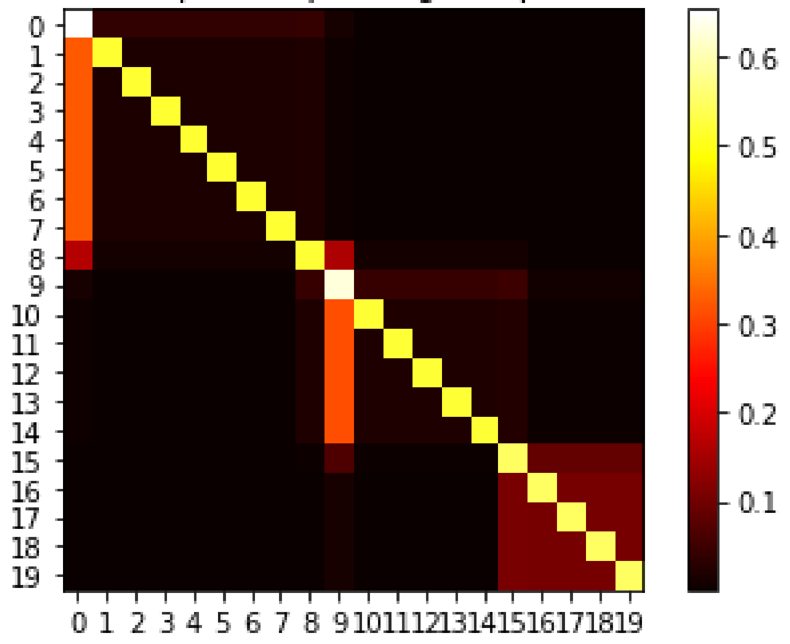
```
100%|          | 20/20 [00:00<00:00, 1297.06it/s]
```

Heatmap corresponding to alpha=0.25

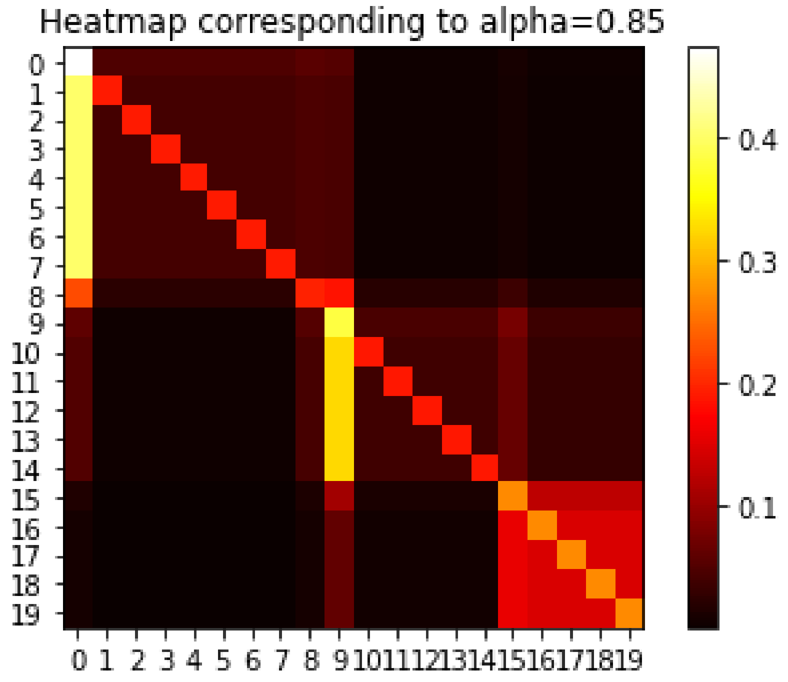


```
100%|          | 20/20 [00:00<00:00, 979.10it/s]
```

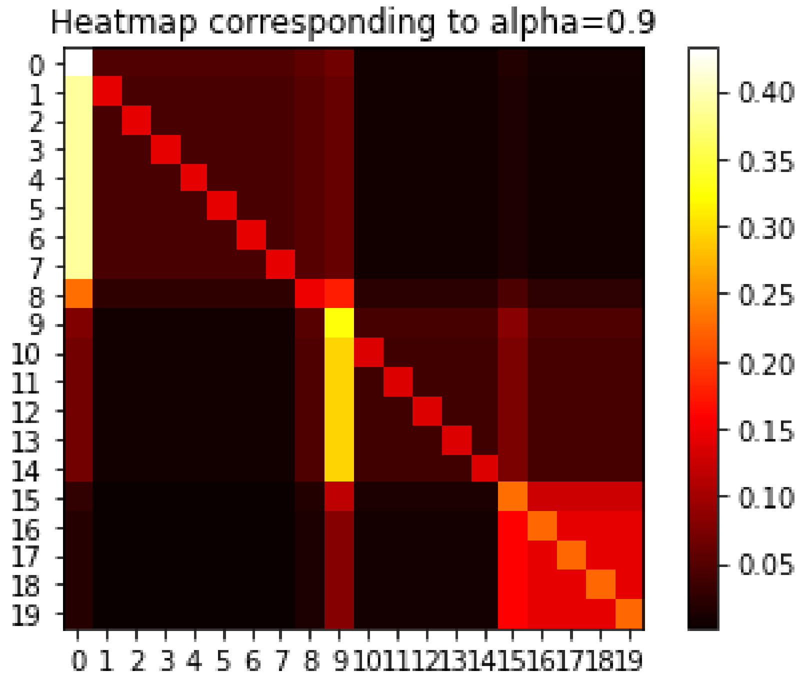
Heatmap corresponding to alpha=0.5



```
100%|          | 20/20 [00:00<00:00, 579.01it/s]
```



```
100%|          | 20/20 [00:00<00:00, 292.37it/s]
```

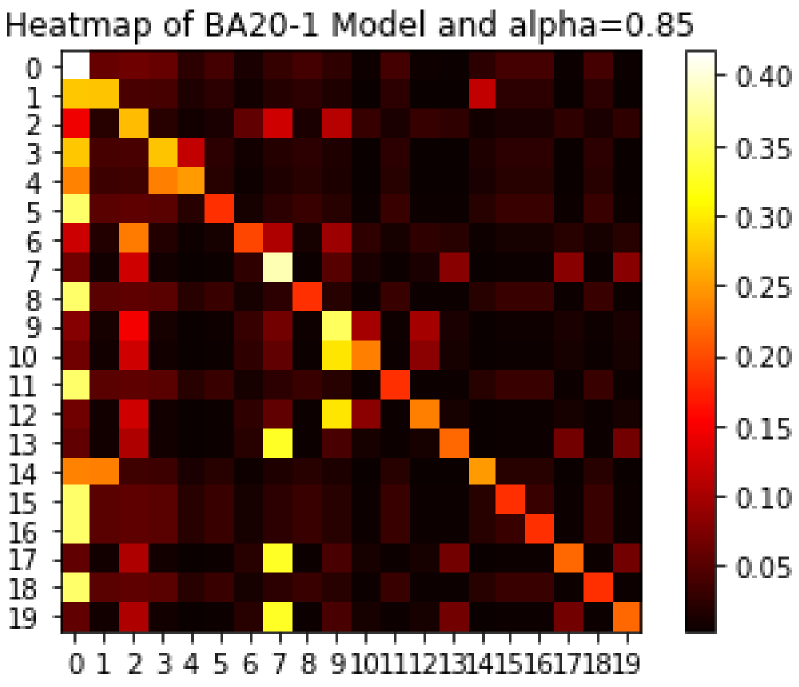


Interestingly the alpha value doesn't seem to change the structure, but just the instensities. But this graph is very simple. We also note some trivial facts such that hubs retain more heat, and the leaves are cold. Because highly connected nodes receive heat from many sources. If we think in terms of random walk, they are more likely to be visited because they are connected to more nodes. Also notice that node 8 which connects 2 hot hubs is also interesting.

Now lets try the smae trick with a different dog, I mean graph…

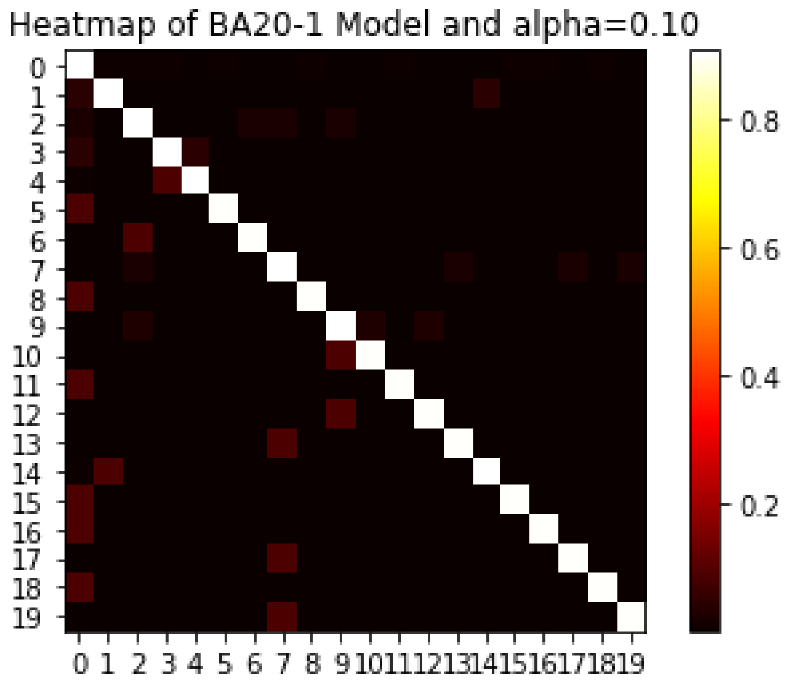
[6]:
```
seed = 42
G = nx.barabasi_albert_graph(n=20, m=1, seed=seed)
H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
heatmap(W85, "Heatmap of BA20-1 Model and alpha=0.85")
```

```
100%|          | 20/20 [00:00<00:00, 358.84it/s]
```



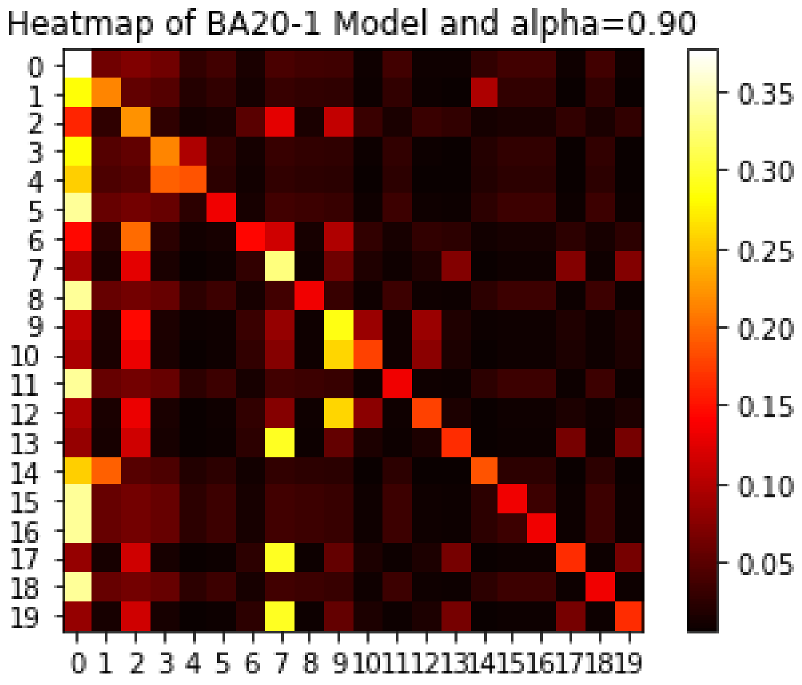
[7]:
```
H, W10 = pageRanksConcentratedBiasG(G, alpha=0.10)
heatmap(W10, "Heatmap of BA20-1 Model and alpha=0.10")
```

```
100%|          | 20/20 [00:00<00:00, 599.88it/s]
```

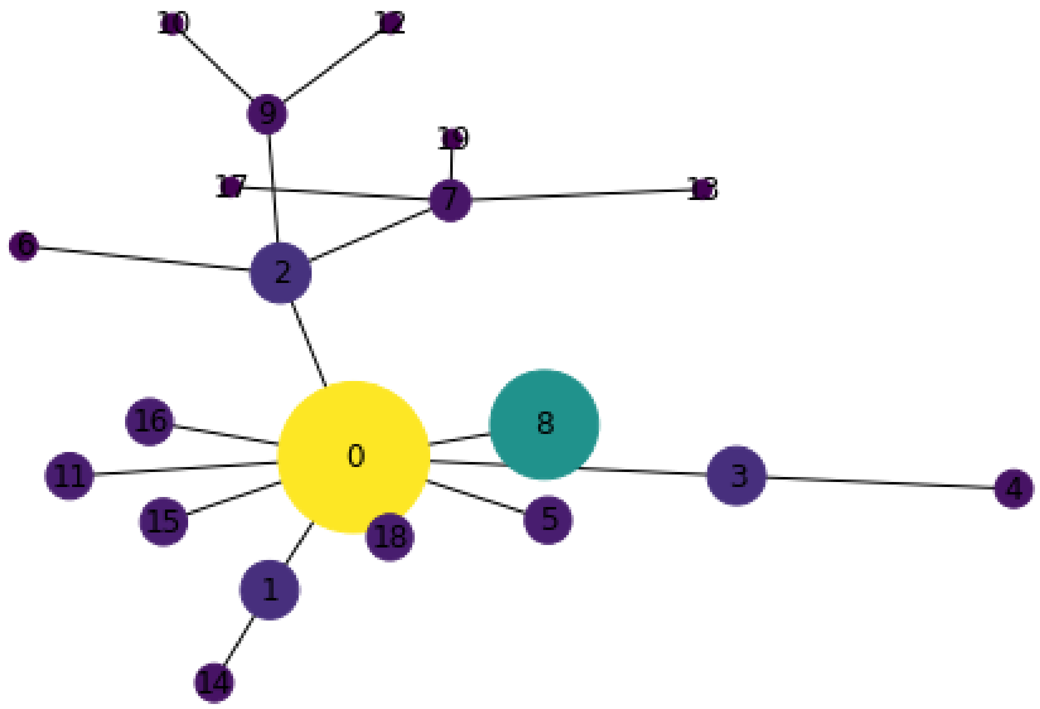


[8]:
```python
H, W90 = pageRanksConcentratedBiasG(G, alpha=0.90)
heatmap(W90, "Heatmap of BA20-1 Model and alpha=0.90")
```

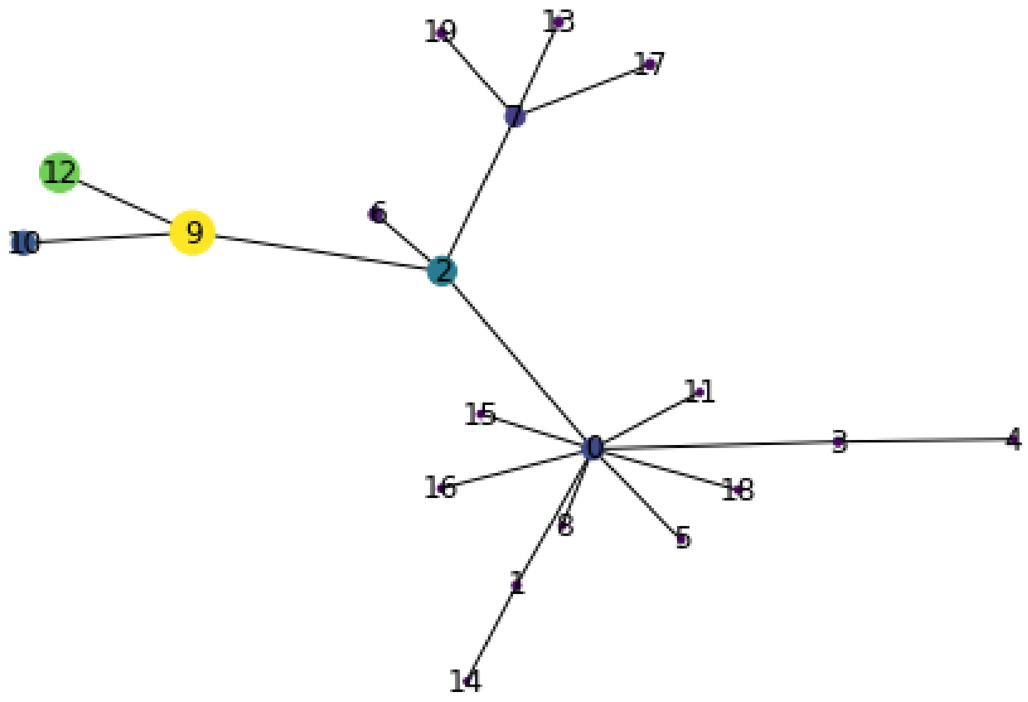
```
100%|          | 20/20 [00:00<00:00, 214.42it/s]
```



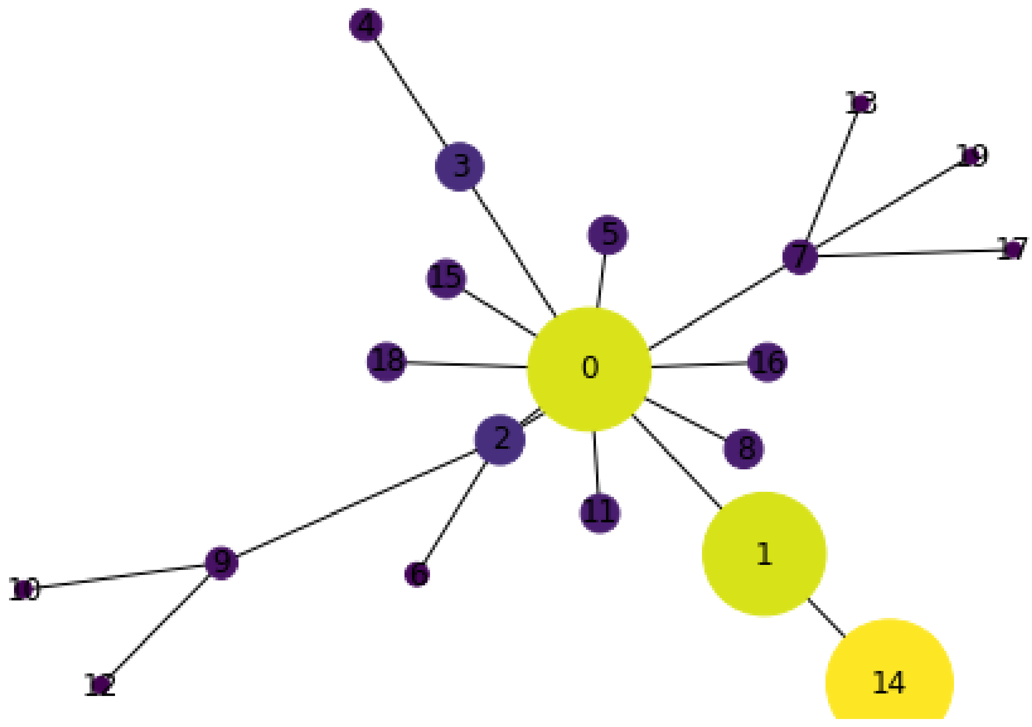
[9]:
```
nx.draw(H, with_labels=True, node_color=W85[8], node_size=W85[8]*10000)
```



[10]:
```
nx.draw(H, with_labels=True, node_color=W85[12], node_size=W85[12]*1000)
```



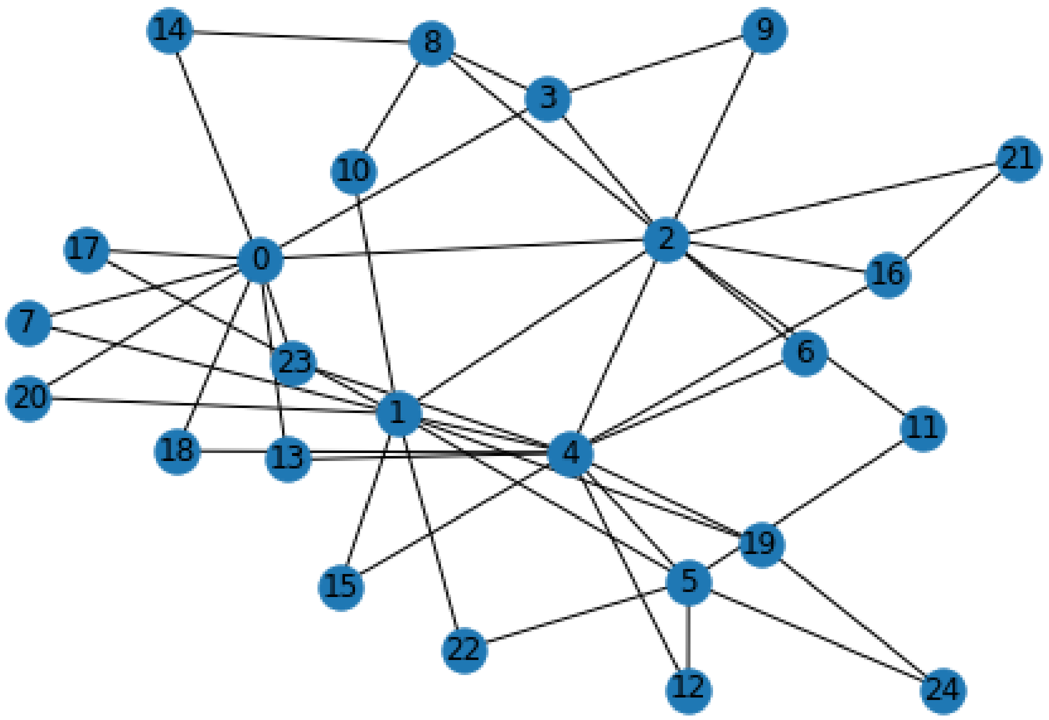
[11]:
```
nx.draw(H, with_labels=True, node_color=W85[14], node_size=W85[14]*10000)
```

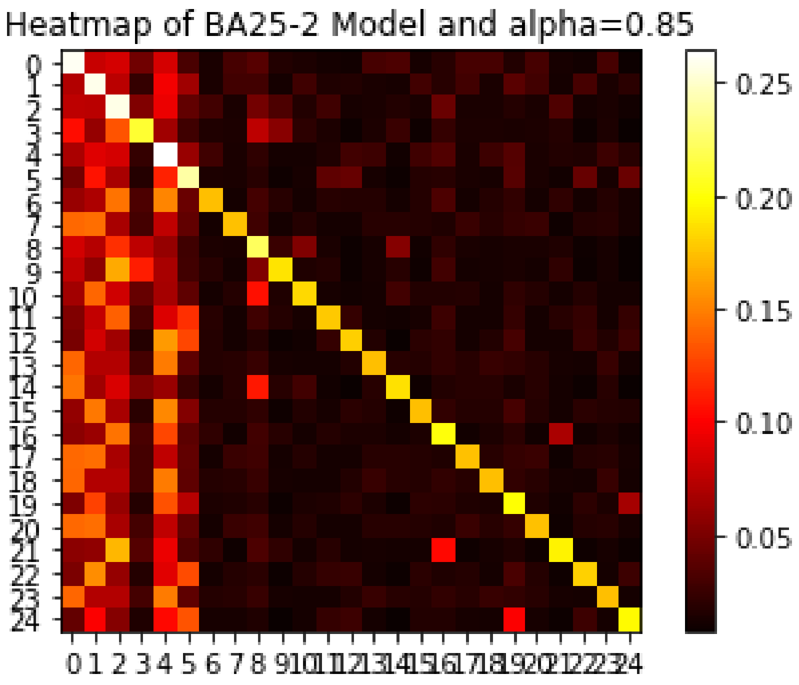


Now we do they same experiment with a more complex graph…

[12]:
```
seed = 42
G = nx.barabasi_albert_graph(n=25, m=2, seed=seed)

nx.draw(G, with_labels=True)

H, W85 = pageRanksConcentratedBiasG(G, alpha=0.85)
heatmap(W85, "Heatmap of BA25-2 Model and alpha=0.85")
```

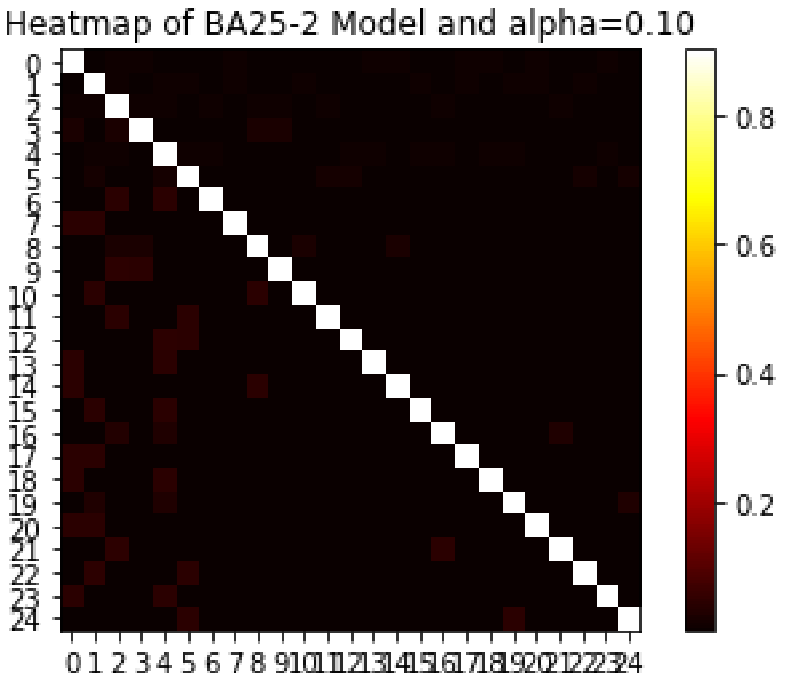
```
100%|          | 25/25 [00:00<00:00, 467.27it/s]
```



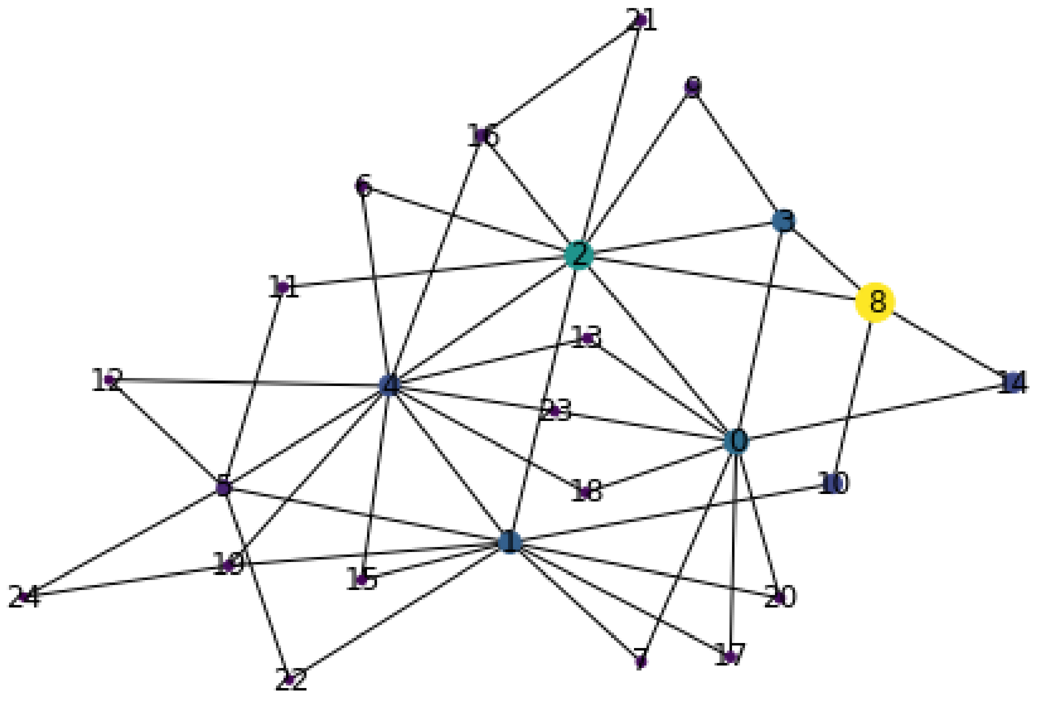


```
[13]: H, W10 = pageRanksConcentratedBiasG(G, alpha=0.10)
      heatmap(W10, "Heatmap of BA25-2 Model and alpha=0.10")
```

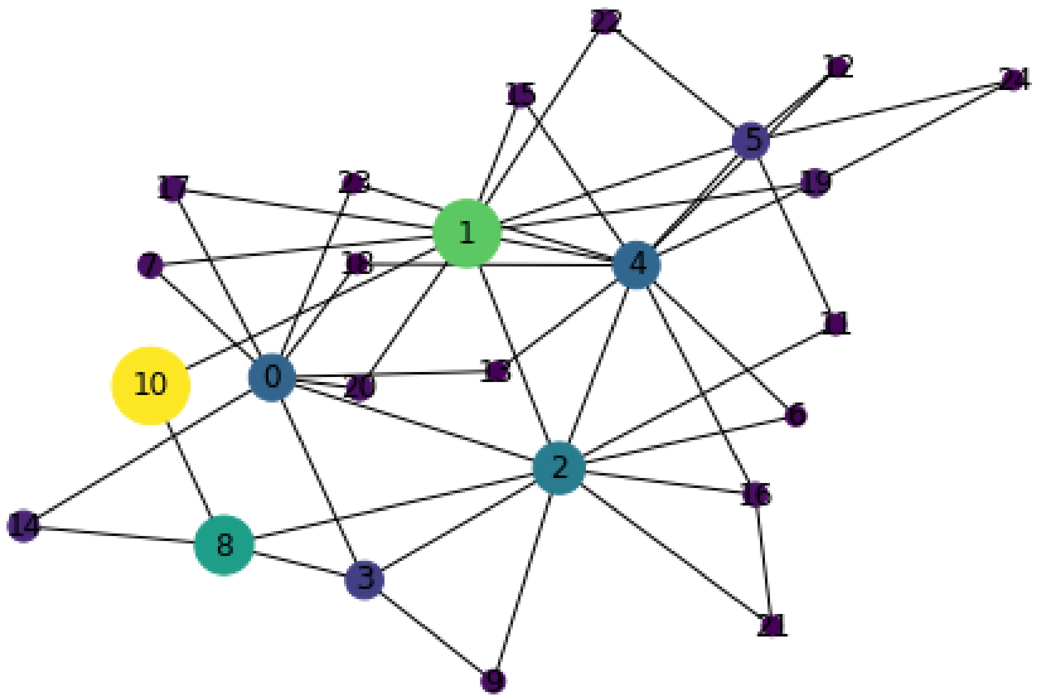
```
100%|      | 25/25 [00:00<00:00, 1185.62it/s]
```



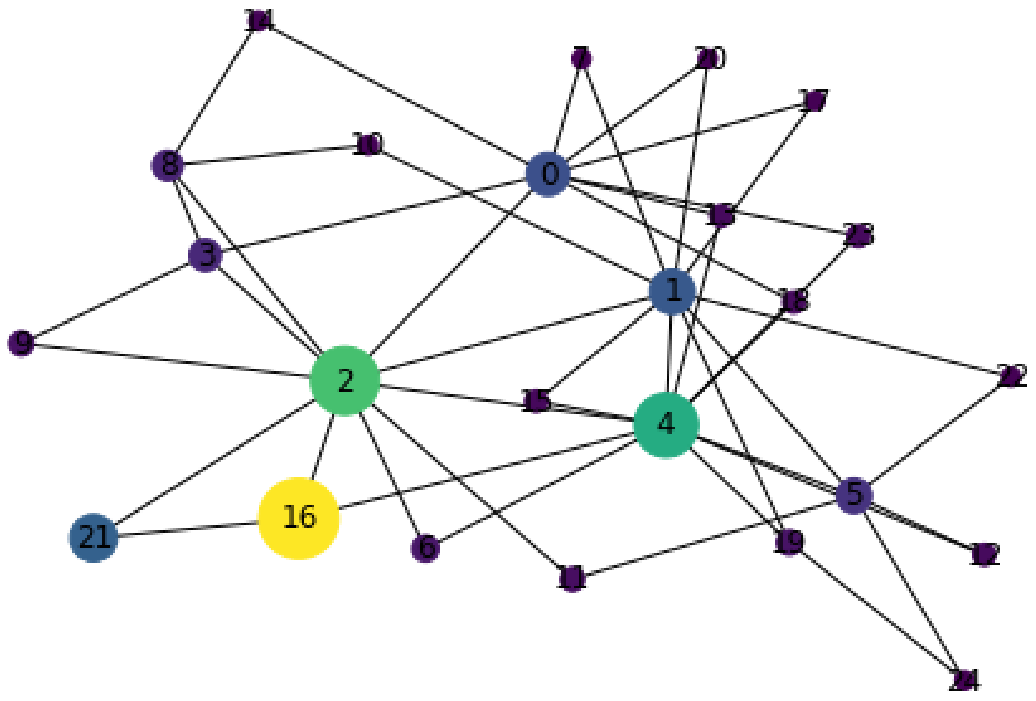
[14]:
```
nx.draw(H, with_labels=True, node_color=W85[8], node_size=W85[8]*1000)
```



[15]:
```
nx.draw(H, with_labels=True, node_color=W85[10], node_size=W85[10]*5000)
```



```
[16]: nx.draw(H, with_labels=True, node_color=W85[16], node_size=W85[16]*5000)
```



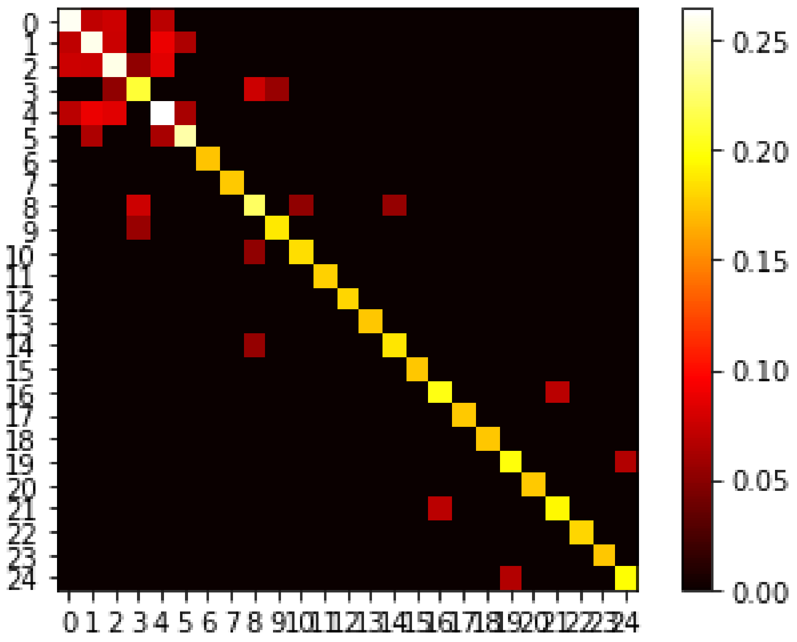
I think it is clear now that propagation strongly depends on the distance from the propagating source and on the degree...

Next we are going to see what the influence graph looks like. The influence of node i on j is pi[j] where pi is the stationary distribution propagated from i (so we look at the j component of it).

The influence graph is a weighted graph on n nodes. where the edge weight of (i,j) is min(pi[j], pj[i]). We also set a threshold δ so we only edges that weigh over delta make the cut.

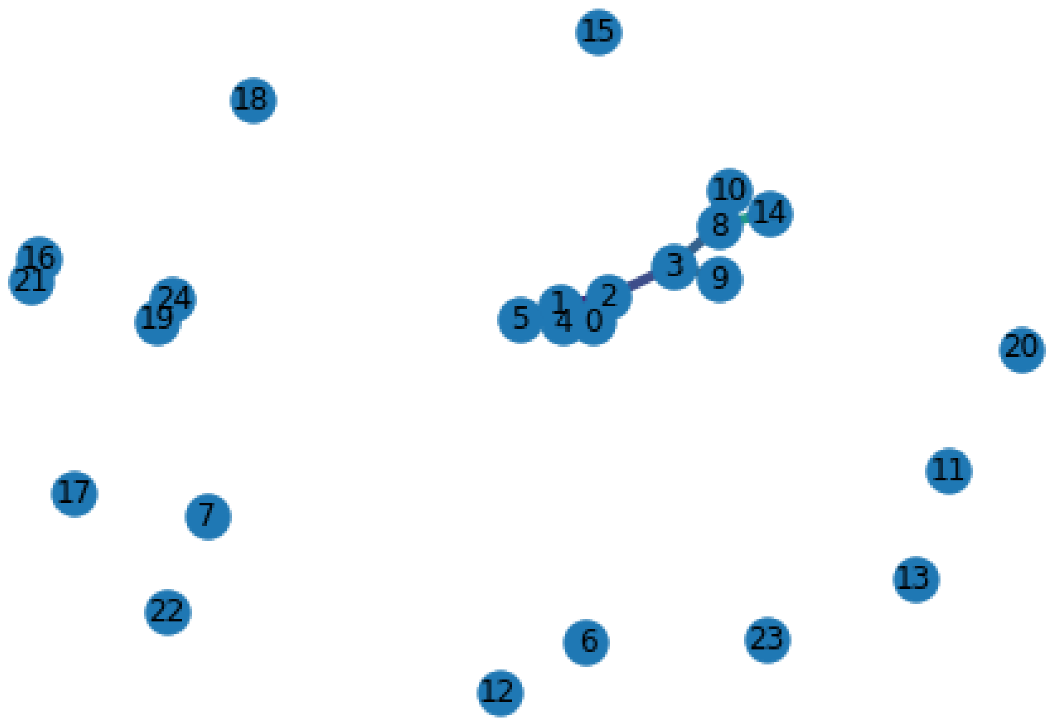
[17]:
```
# reducedInfluenceMatrixG
W = reducedInfluenceMatrixG(G, delta=0.05)

heatmap(W, "")
```

```
100%|          | 25/25 [00:00<00:00, 554.50it/s]
```



[18]:
```
Gd = nx.Graph()
Gd.add_nodes_from(range(25))
edges = [(i,j) for i in range(25) for j in range(i,25) if W[i,j]>0]
Gd.add_edges_from(edges)
l = [W[e] for e in Gd.edges()]
l


nx.draw(Gd, with_labels=True, width=4, edge_color = range(40))
```

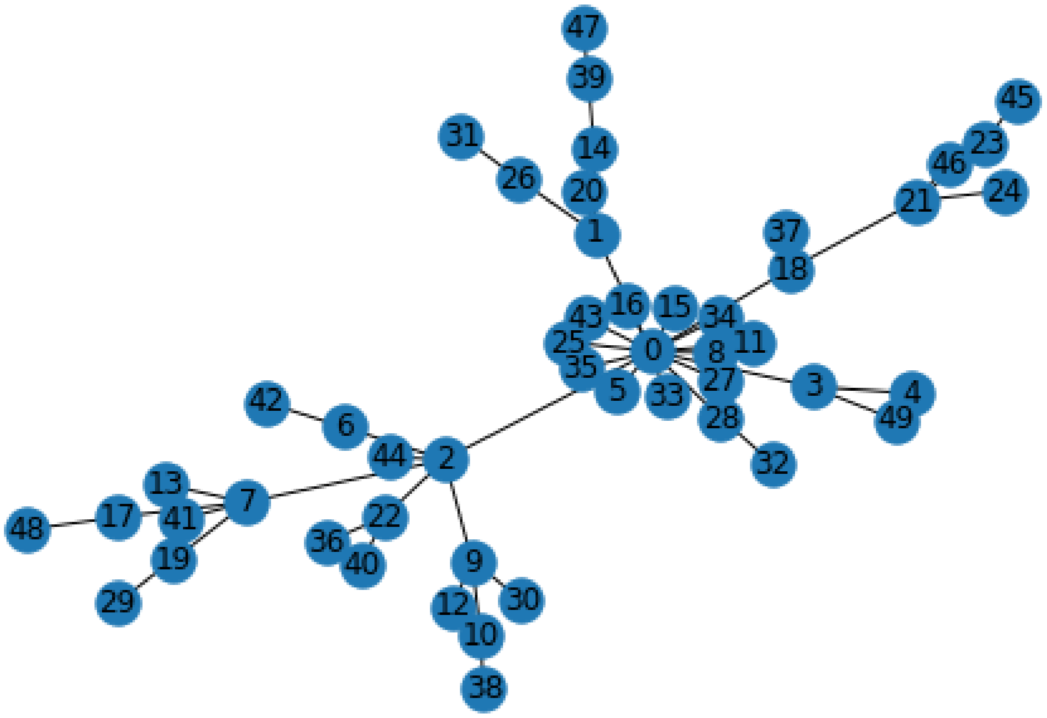
```
plt.show()
```



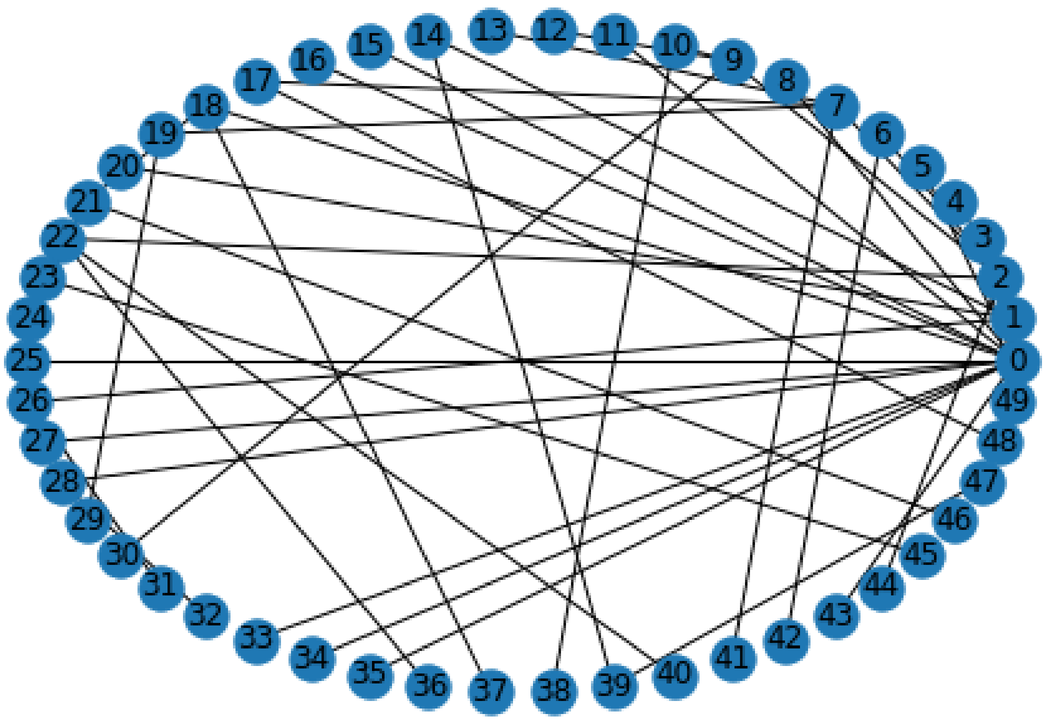
2 Exploring graph clustering methods.

Since we are dealing with propagation and interested in connected scale free graphs such as Barabsi-Albert, I am trying to apply propagation to perform this task. Idea 1: The hottest node is going to form the first cluster. We then extend it by all nodes that are connected to it in a sufficiently hot path. Remove them from the graph and repeat on the smaller graph. Thought should then be dedicated to the matter of setting the parameters: what is 'hot' (delta), how fast we propagate (alpha), how do we test statistical significance and robustness. Define null model? Use connected edge swaps for robustness tests?

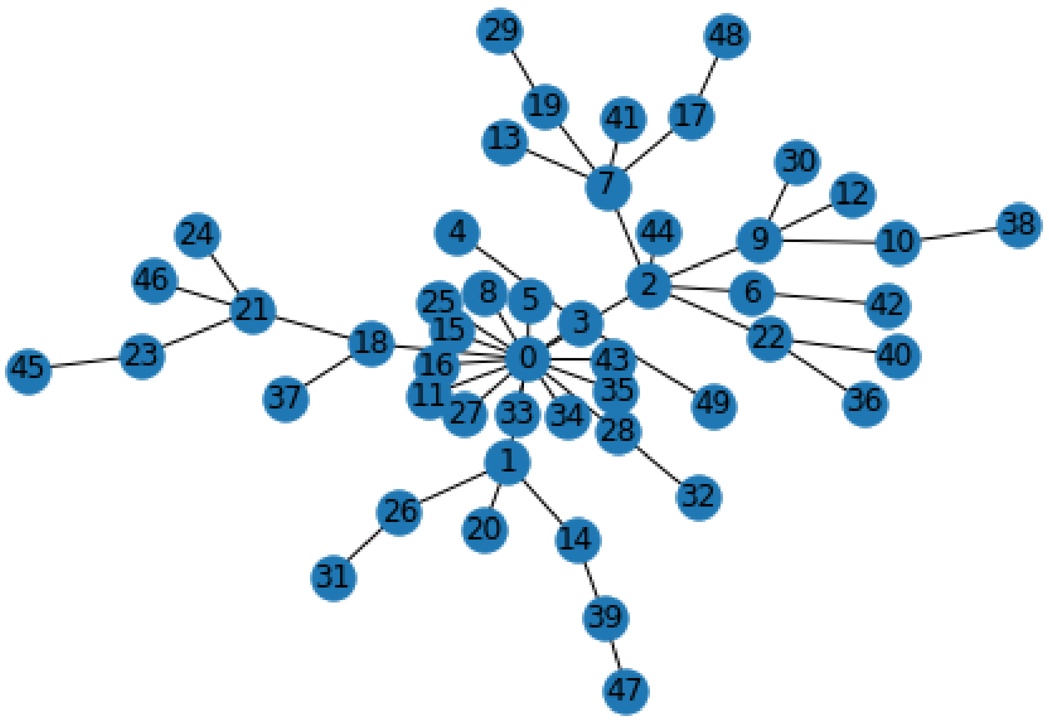
[19]:
```
G = nx.barabasi_albert_graph(n=50, m=1, seed=seed)

nx.draw(G, with_labels=True)
```



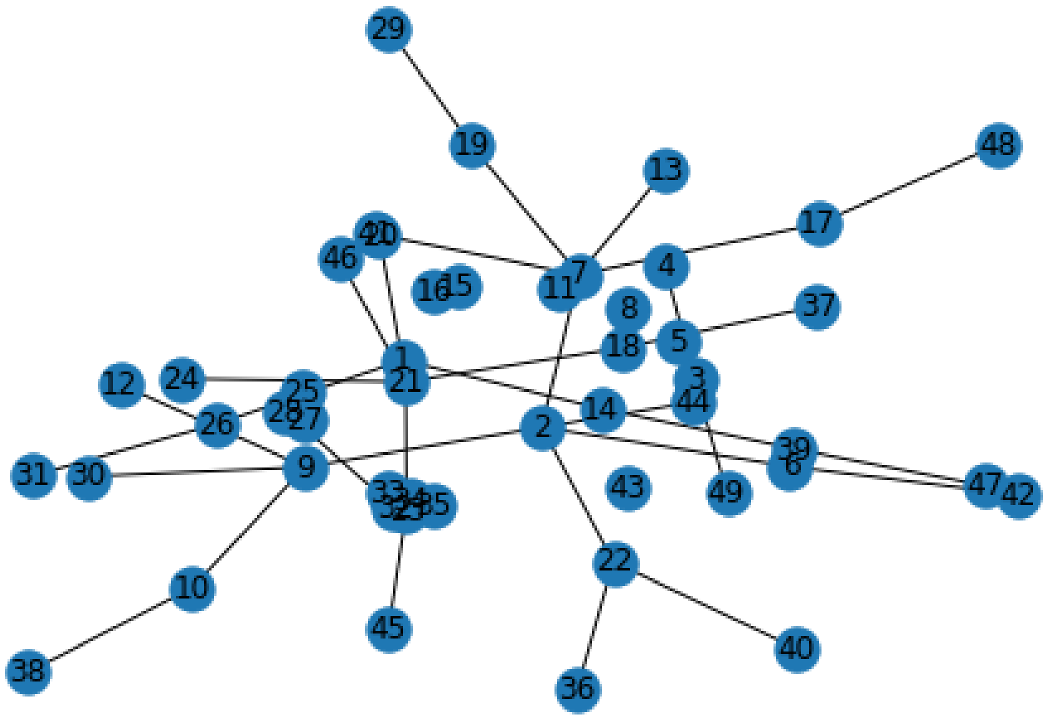
```
[20]: nx.draw_circular(G, with_labels=True)
```



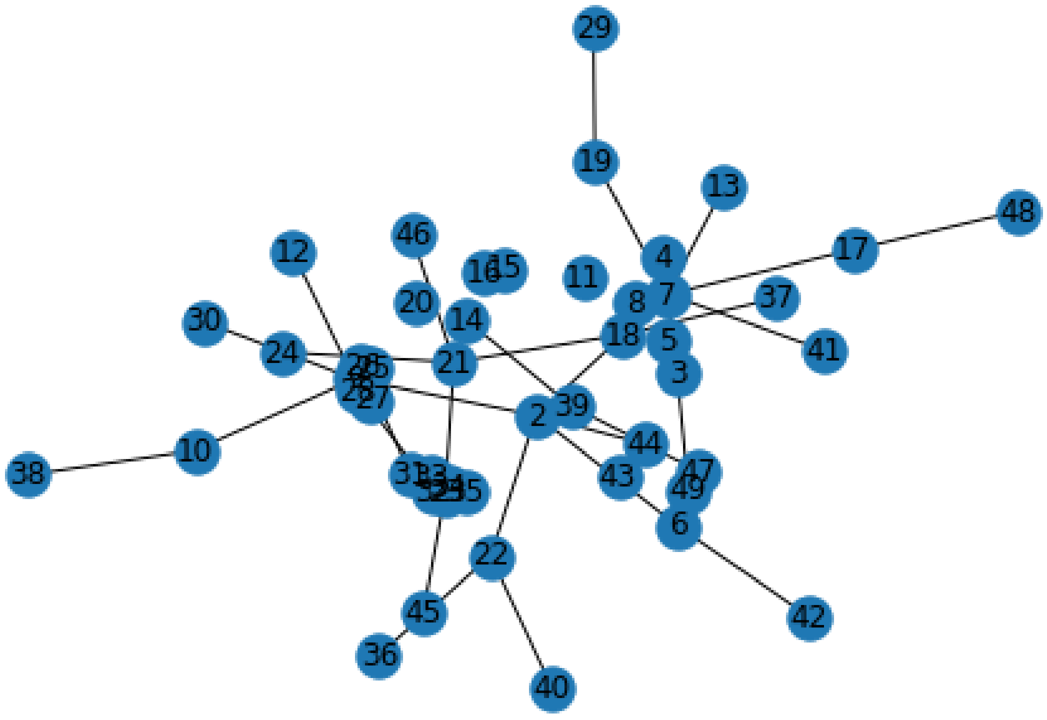
[21]:
```
nx.draw_kamada_kawai(G, with_labels=True)
```



[22]:
```
G.remove_node(0)

nx.draw_kamada_kawai(G, with_labels=True)
```

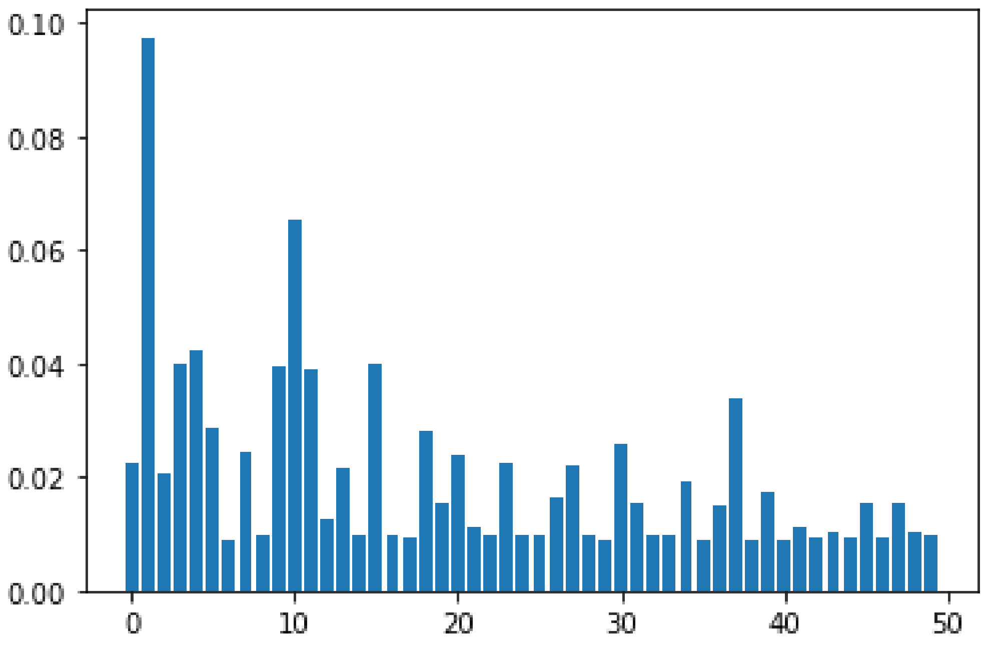


```
[23]: G.remove_node(1)

      nx.draw_kamada_kawai(G, with_labels=True)
```



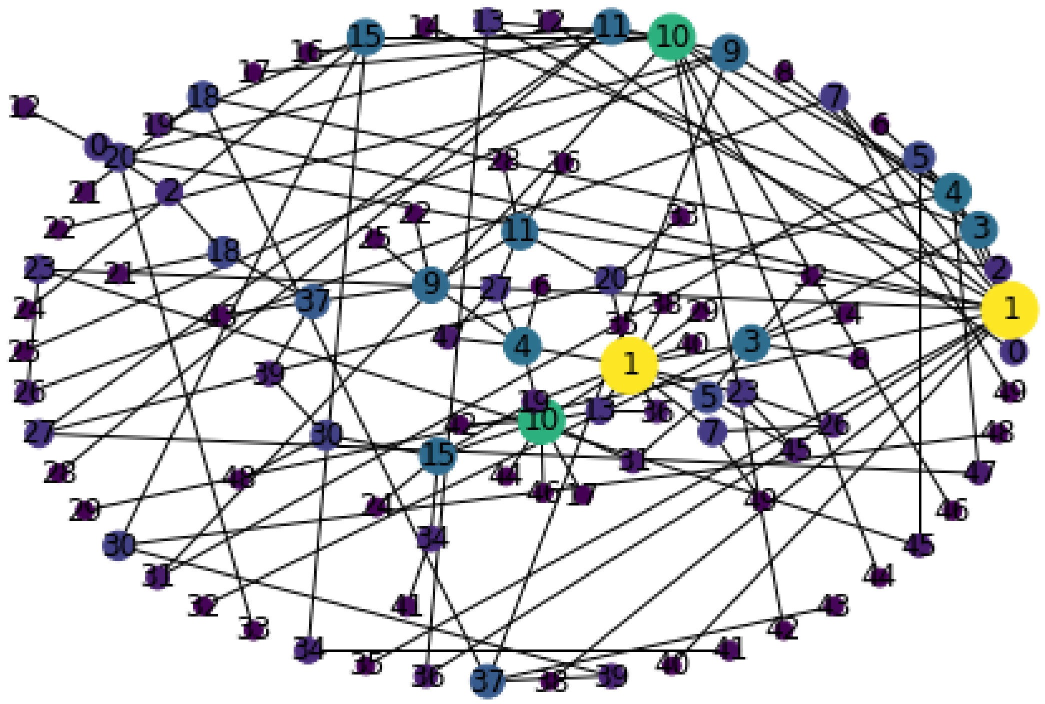
```
[24]: # trying to cluster a graph ...
      G = nx.dual_barabasi_albert_graph(n=50, m1=1, m2=2, p=0.7, seed=seed)
      H = G.copy()
      p, _ = powerIterateG(G, alpha=0.85)
      plt.bar(range(50), p)
```

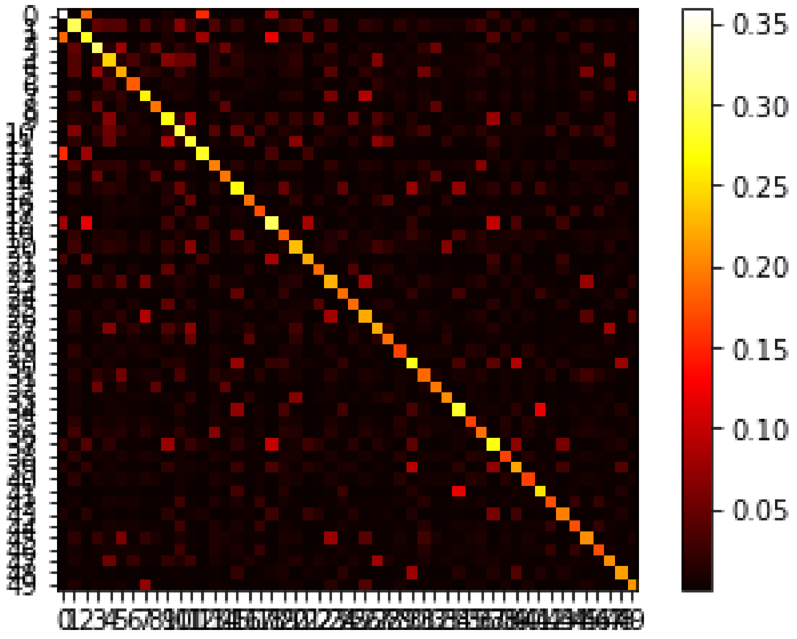
```
[24]: <BarContainer object of 50 artists>
```



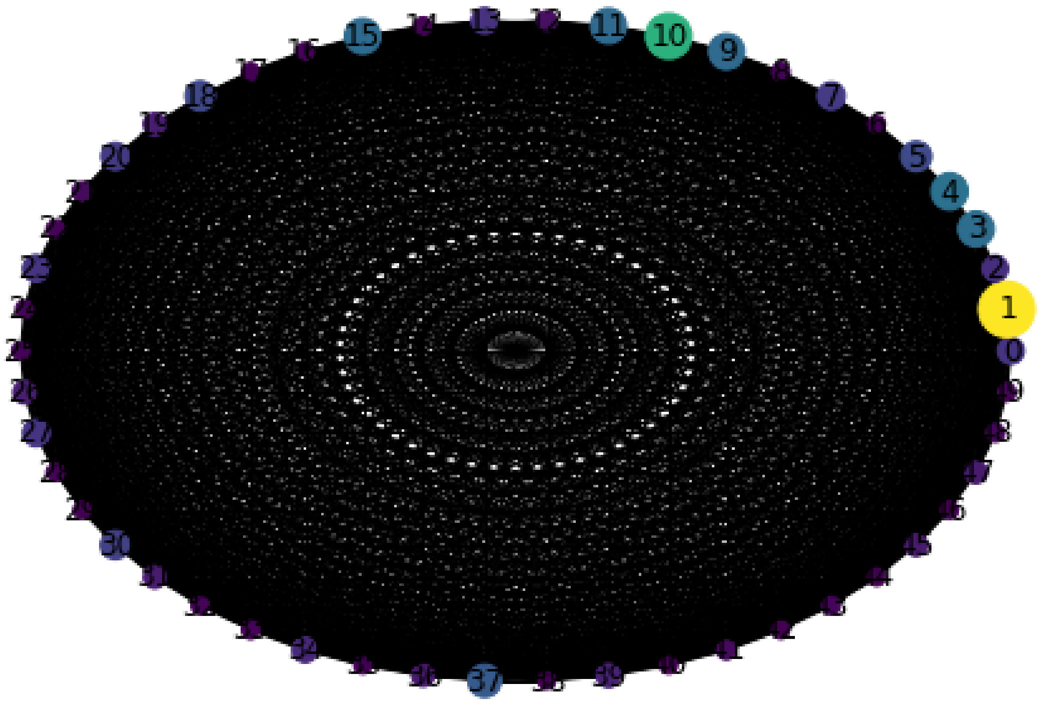
```
[25]: nx.draw_kamada_kawai(G, with_labels=True, node_color=p, node_size=5000*p)

      nx.draw_circular(G, with_labels=True, node_color=p, node_size=5000*p)

      s = np.argmax(p)
      s

      W = reducedInfluenceMatrixG(G, delta=0)
      heatmap(W, "")
```

```
100%|          | 50/50 [00:00<00:00, 529.32it/s]
```




0.35
0.30
0.25
0.20
0.15
0.10
0.05

[26]:
```
H = nx.Graph()
H.add_nodes_from(G.nodes())

edges = [(i,j) for i in range(49) for j in range(i+1,50) if W[i,j]>0]

H.add_edges_from(edges)

nx.draw_circular(H, with_labels=True, node_color=p, node_size=5000*p)
```



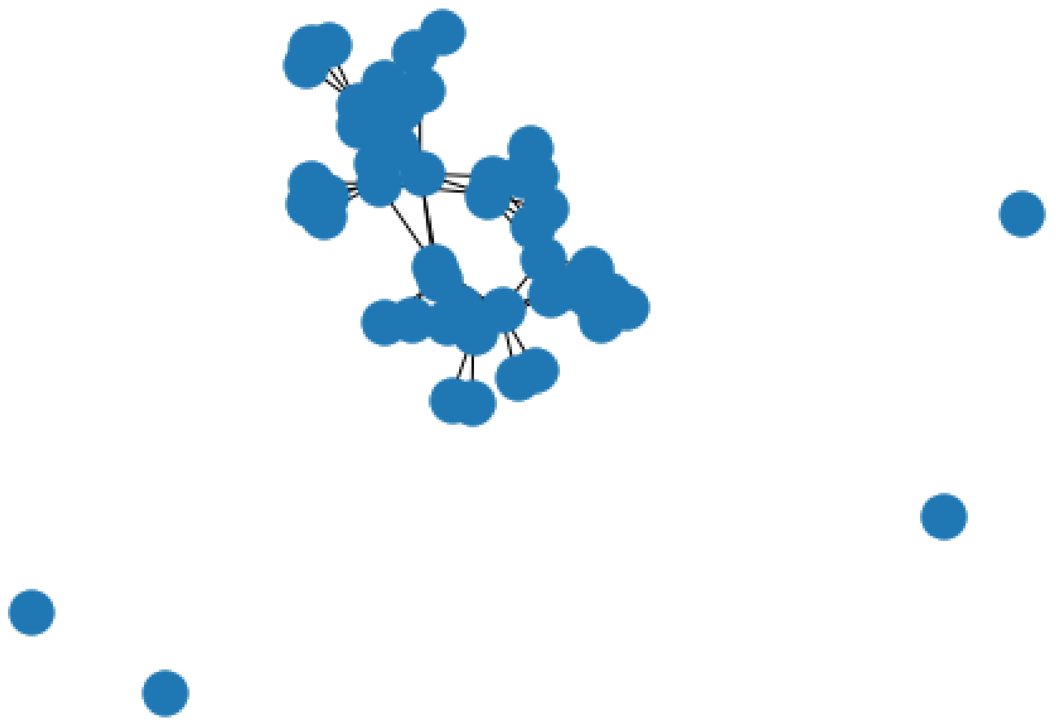
[27]:
```
plt.show()
```

[28]:
```
for delta in np.arange(0.01, 1, 0.01):
    remlist = [e for e in list(H.edges()) if W[e] <= delta]
    H.remove_edges_from(remlist)
    if nx.number_connected_components(H) > 1:
        print("break", len(H.edges()))
        break

len(H.edges())
```

```
break 120
```

[28]:
```
120
```

[29]:
```
nx.draw(H)
plt.show()
```



[30]:
```
# so that method is not very promicing because it tends to prune leaves and␣
↪conCerve one giant connected component
```

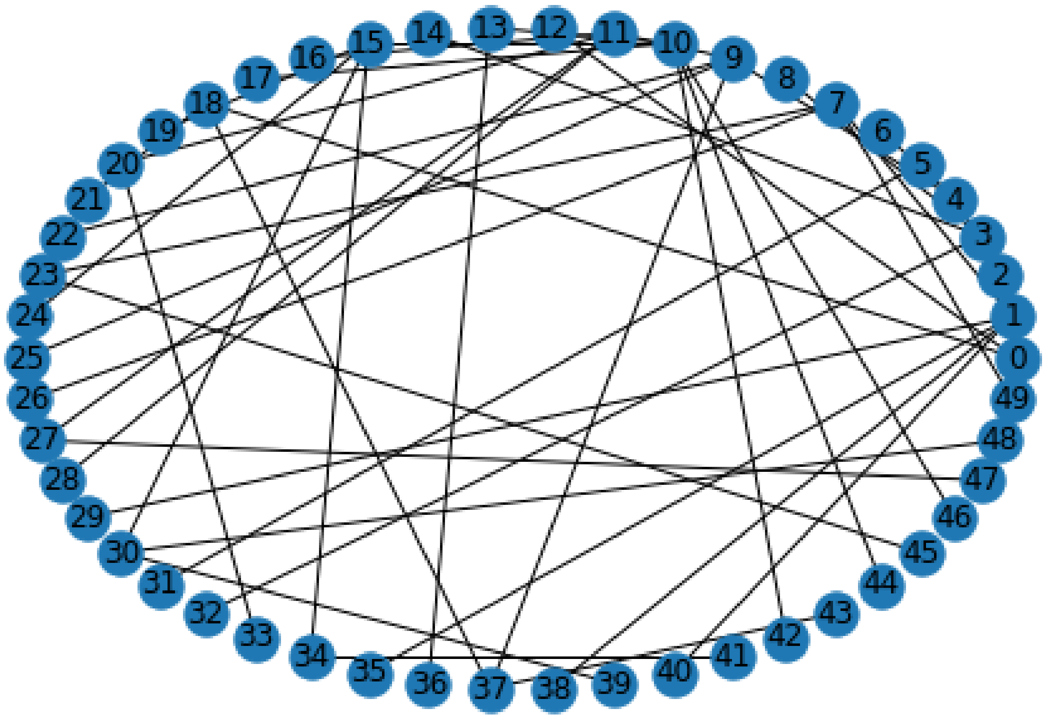
[31]:
```
# We shall now try bottom up method. start from a totally disconnected graph.␣
↪Pick the remaining coldest node and connect it to its nearest
# neighbor in the symmetric influence graph
```

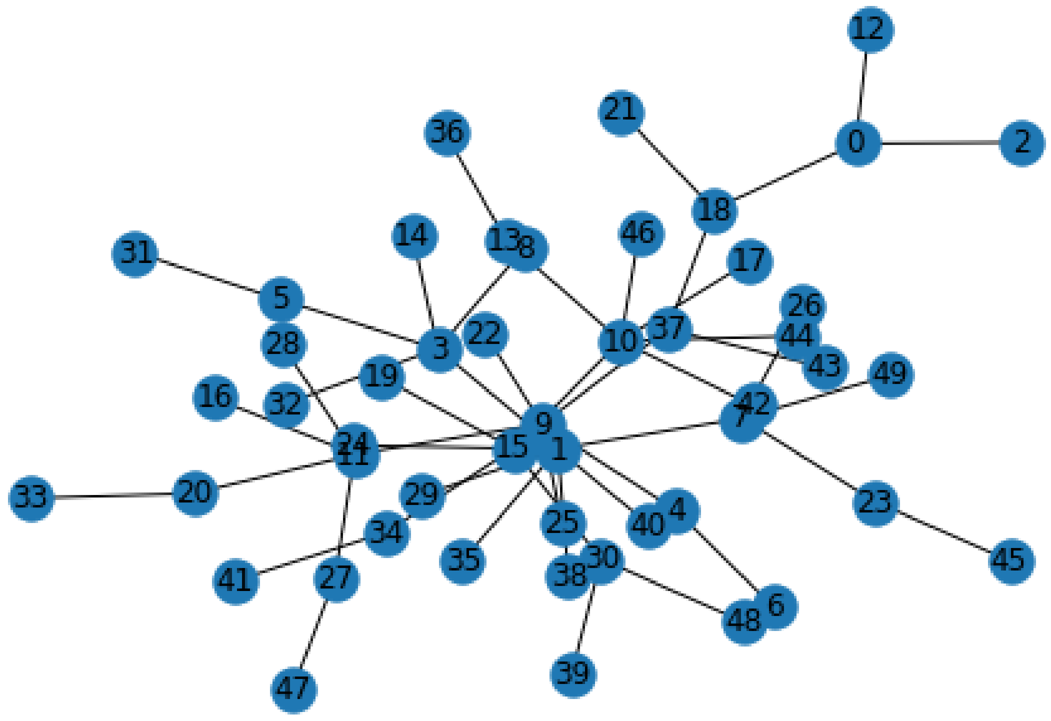
[32]:
```
H = nx.Graph()
H.add_nodes_from(G.nodes())
nlist = list(H.nodes())
while nx.number_connected_components(H) > 3:
    s = np.argmin(p[nlist])
    x = nlist[s]
    print(s,x)
    nlist.pop(s)
    t = np.argmax([W[x,i] for i in nlist])
    H.add_edge(x,nlist[t])

nx.draw_circular(H, with_labels=True )
plt.show()
```

```
nx.number_connected_components(H)

nx.draw_kamada_kawai(H, with_labels=True)
plt.show()
```

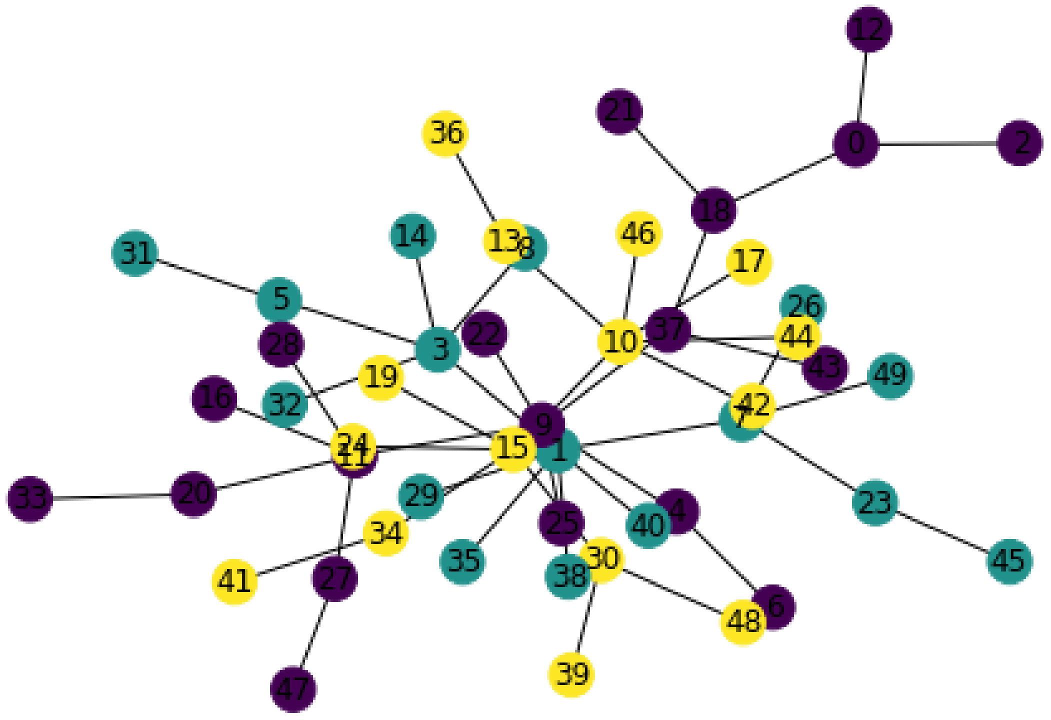
```
29 29
34 35
36 38
37 40
6 6
16 17
36 42
37 44
38 46
15 16
25 28
19 22
21 25
20 24
25 33
7 8
12 14
22 32
31 49
27 43
29 48
16 21
25 41
10 12
21 36
24 47
19 31
22 45
13 19
15 26
19 39
17 34
2 2
9 13
13 27
0 0
11 23
10 20
4 7
9 30
8 18
```

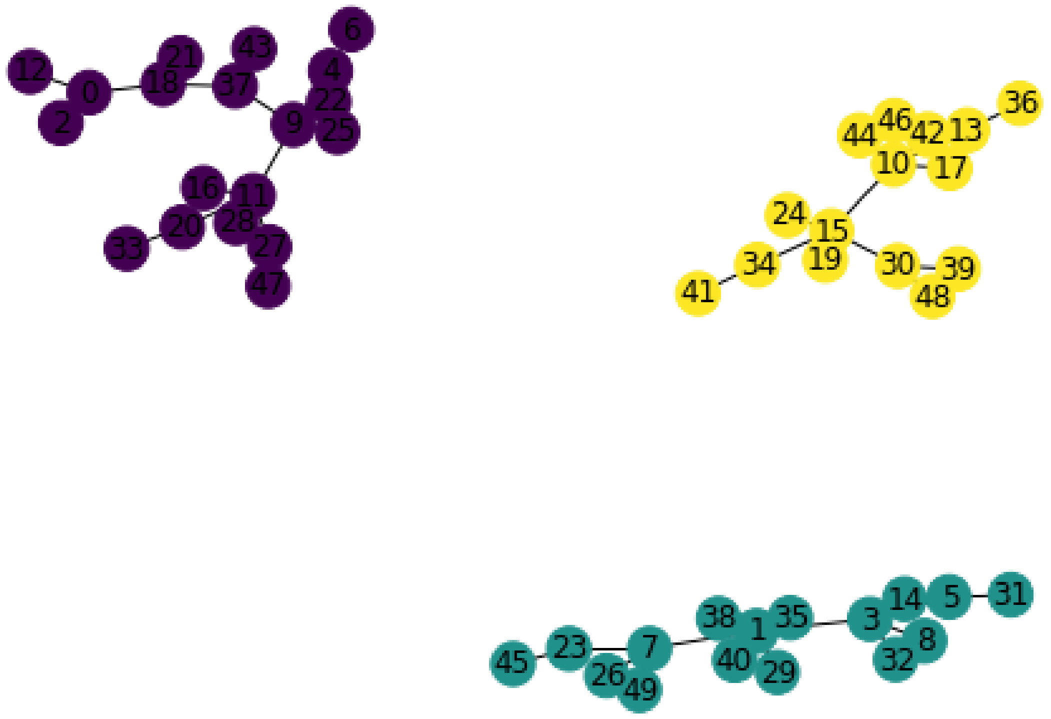
```
3 5
7 37
5 11
3 9
4 15
1 3
```





[33]:
```
CCs = [list(c) for c in nx.connected_components(H)]

CCs

colors = np.zeros(50)
colors[CCs[1]]=1
colors[CCs[2]]=2


nx.draw_kamada_kawai(H, with_labels=True, node_color=colors)
plt.show()

nx.draw_spring(H, with_labels=True, node_color=colors)
plt.show()

nx.draw_spring(G, with_labels=True, node_color=colors)
plt.show()
```


12
21
0
2
36
18
14
13
8
46
31
17
5
26
37
44
28
22
10
3
43
49
19
16
32
9
42
7
24
11
15
1
33
20
29
4
34
25
40
23
45
30
41
27
35
38
6
48
39
47


6
43
21
4
12
0
18
37
22
2
9
25
16
11
28
20
33
27
47
36
44
46
42
13
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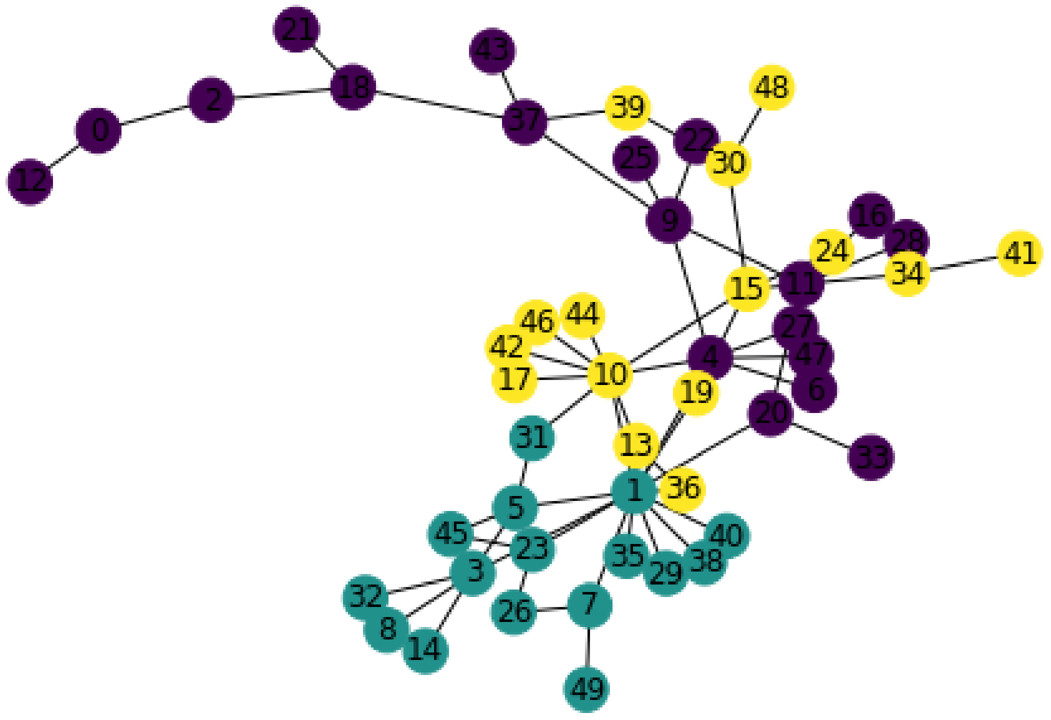
17
24
15
34
19
30
39
41
48
38
35
14
5
31
1
3
8
23
7
40
29
32
45
26
49



[34]:
```
# The 2 upper plots show the clustering on the influence matrix/graph the␣
↪bottom one show the same clustering on the original graph
```

3 Testing the clustering method on images

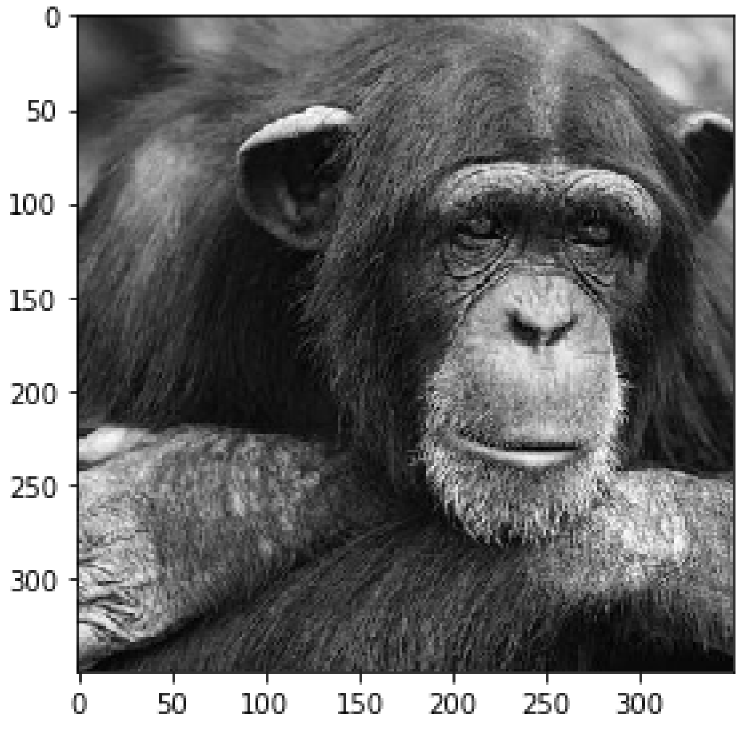
We are going to load a grayscale image, resize it to a compact size. Out of it we are going to create a graph as follows: if the image is an array Y of size n^2, then we create an array of T size n^4. Each row of the bigger array represents a pixel of the image. Implicitly pixels are connected by an edge only if they are neighbors (on the horizontal or vertical, we exclude the diagonal for simplicity). So let $(i,j),(i+1,j)$ be neighboring pixels in T, then we set: $T[i*n+j, i*(n+1)+j] = 1/(1+|y[i,j]-y[i+1,j]|)$ (and then make it symmetric etc…) So the matrix T represents the lattice graph with weighted esges.

We are then going to calculate the influence matrix of that implicit weighted graph, and cluster to k clusters the pixels just as we did above. Then we recreate the image as follows: We create a new n^2 array. The pixels that belong to a cluster, are all going to have the same value, which is the mean of those pixels on Y.

[35]:
```
# now with a real image again
n=50
im = io.imread('chimp-665439.jpg', as_gray=True)
im.shape
x = ski.util.crop(im, ((0,0), (120,120)))
```

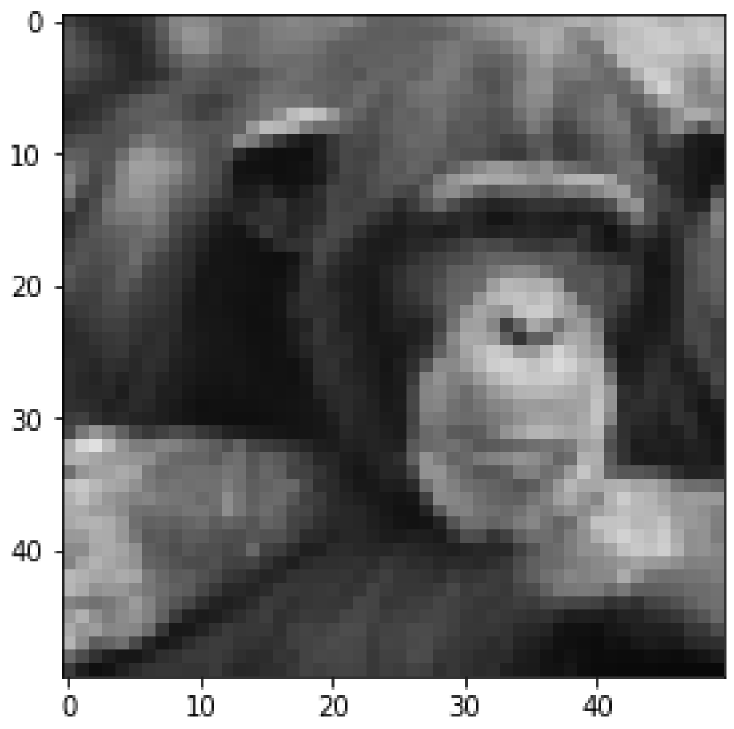
```
x.shape

io.imshow(x)
```

```
[35]: <matplotlib.image.AxesImage at 0x7f335d054550>
```



```
[38]: y = resize(x, (n,n))
      z = y.flatten()
      io.imshow(y)
```

```
[38]: <matplotlib.image.AxesImage at 0x7f335d2cccd0>
```



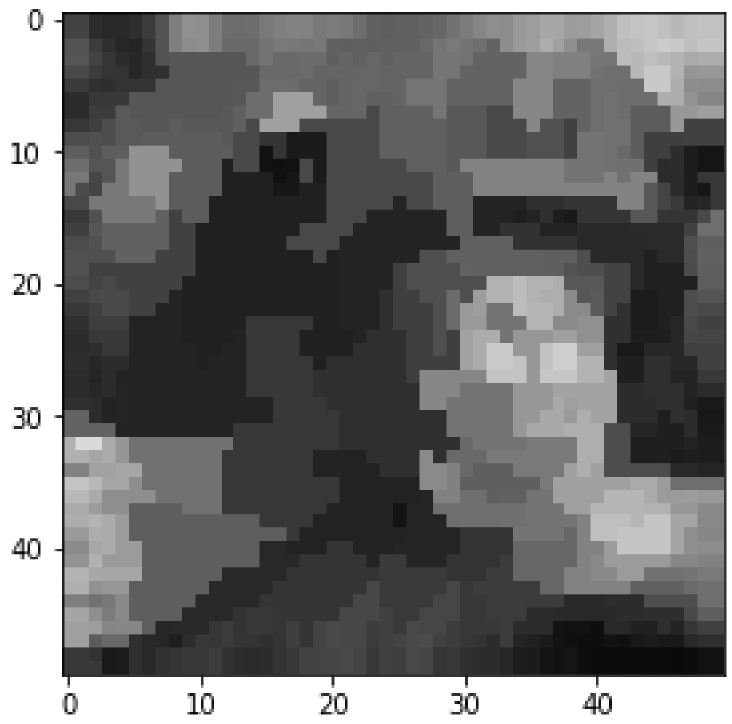
```
[39]: T = np.zeros((n**2,n**2))

      for i in range(n**2 - 1):
          if (i+1) % n > 0:
              r = i // n
              c = i % n
              T[i,i+1] = 1 / (1 + abs(
                  y[r,c] - y[r,c+1]))
          if (i+n) < n**2:
              T[i,i+n] = 1 / (1 + abs(
                  y[r,c] - y[r+1,c]))
```

```
[40]: T = T + np.transpose(T)
      cc = bottomUpCluster(T, 560)
```

```
100%|      | 2500/2500 [13:48<00:00,  3.02it/s]
```

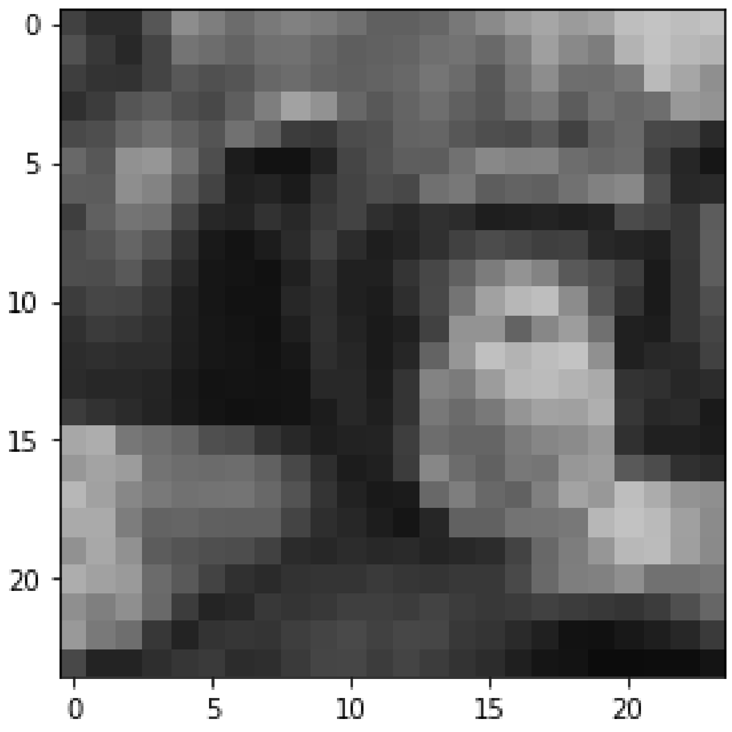
```
[41]: X = np.zeros(n*n)
      for i in range(560):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

[41]: <matplotlib.image.AxesImage at 0x7f335d28af10>



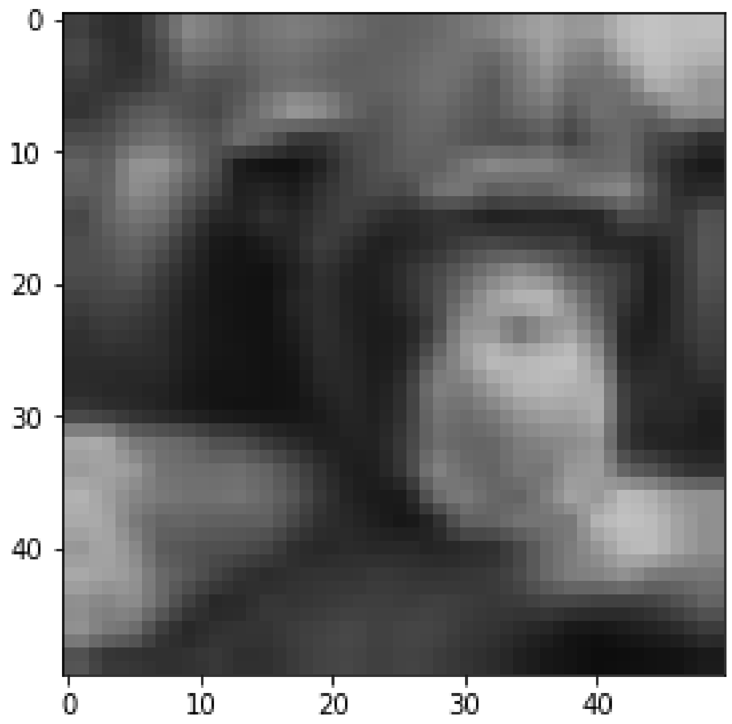
```
[42]: # for comparison, if we resize the image to 24x24 which is equivalent to␣
       ↪partitioning it into
      # 24x24 squares and taking the mean of each, it looks like this:
      io.imshow(resize(y, (24,24)))
```

[42]: <matplotlib.image.AxesImage at 0x7f335de78610>



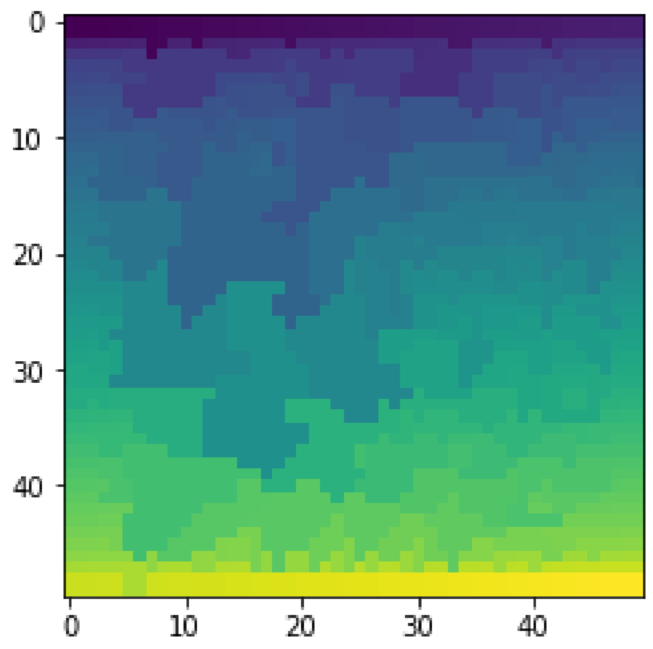
```
[43]: io.imshow(resize(resize(y, (24,24)), (50,50)))
```

```
[43]: <matplotlib.image.AxesImage at 0x7f3356412950>
```



[44]:
```
X = np.zeros(n*n)
for i in range(560):
    #X[cc[i]] = i
    X[cc[i]] = i

plt.imshow(X.reshape(n,n))
```

[44]:
```
<matplotlib.image.AxesImage at 0x7f33563d0b90>
```

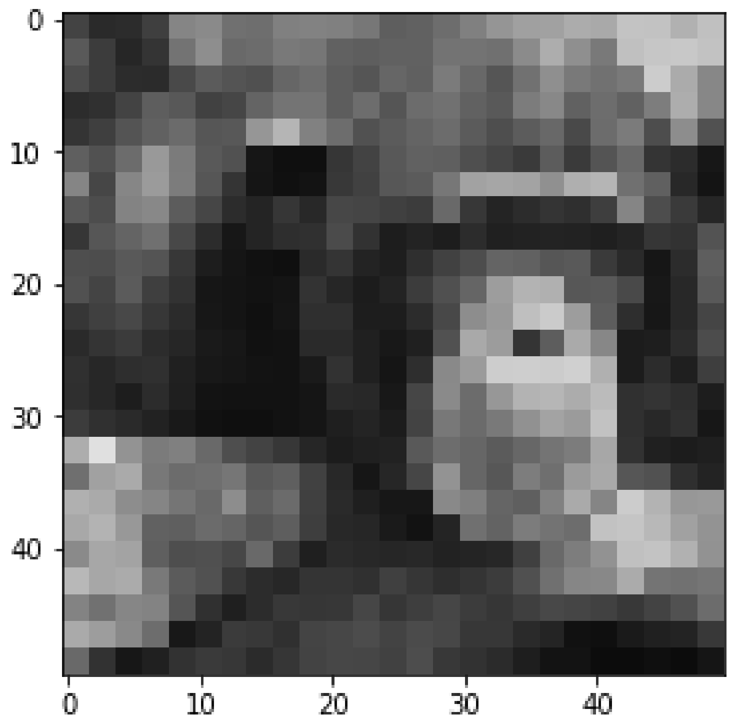


```
[45]: yy = np.zeros((n,n))
      yy.shape
```

```
[45]: (50, 50)
```

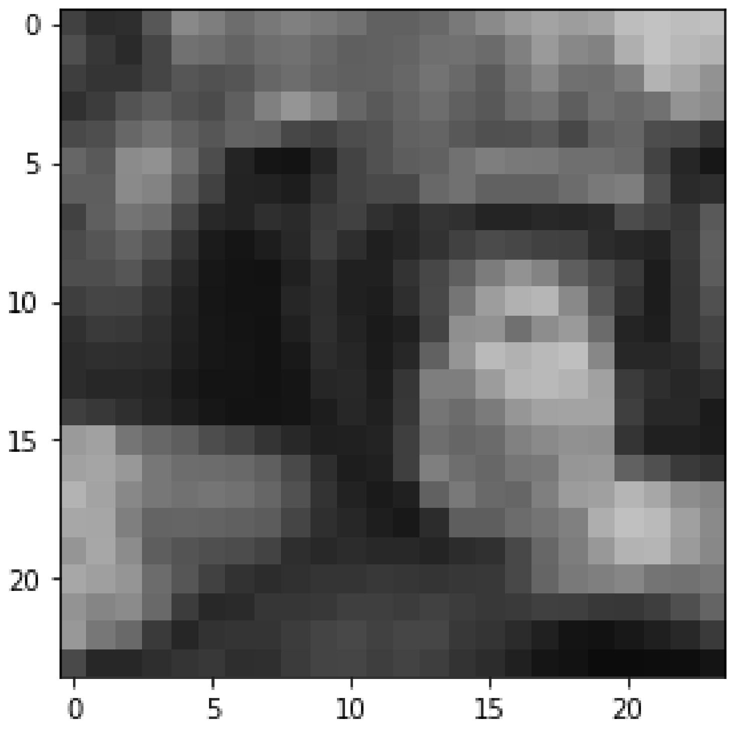
```
[48]: yy = np.zeros((n,n))

      for i in range(0, n, 2):
          yy[i] = y[i]
          yy[i+1] = y[i]
      for i in range(0, n, 2):
          yy[:,i+1] = yy[:,i]

      io.imshow(yy)

      #again creating a 50x50 where each square
```

```
[48]: <matplotlib.image.AxesImage at 0x7f335632c510>
```



```
[50]: # more tests
      n=24
      y = resize(x, (n,n))
      z = y.flatten()
      io.imshow(y)
```

```
[50]: <matplotlib.image.AxesImage at 0x7f3355eff690>
```



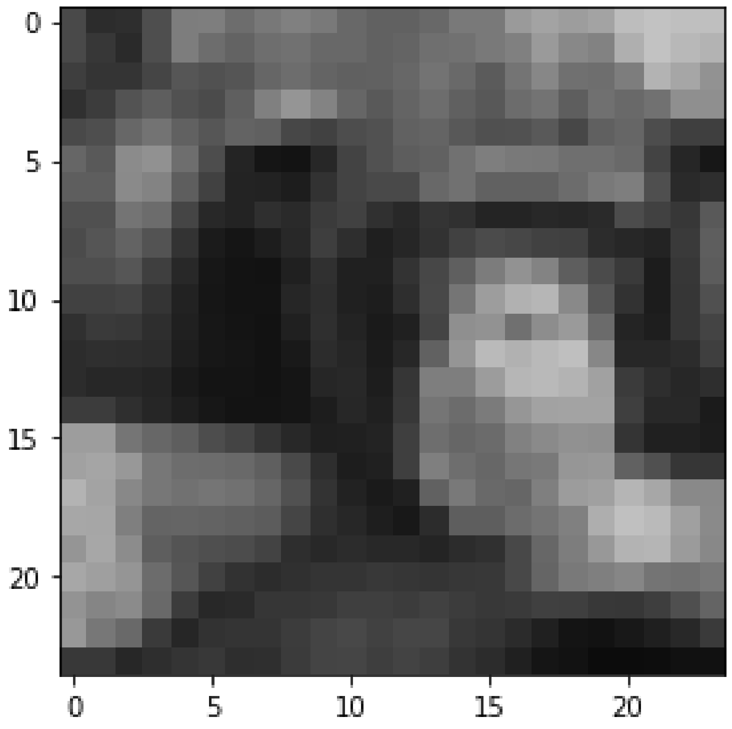
```
[51]: T = np.zeros((n**2,n**2))

      for i in range(n**2 - 1):
          if (i+1) % n > 0:
              r = i // n
              c = i % n
              T[i,i+1] = 1 / (1 + abs(
                  y[r,c] - y[r,c+1]))
          if (i+n) < n**2:
              T[i,i+n] = 1 / (1 + abs(
                  y[r,c] - y[r+1,c]))
```

```
[52]: T = T + np.transpose(T)
      cc = bottomUpCluster(T, 560)
```

```
100%|      | 576/576 [00:08<00:00, 71.01it/s]
```

```
[53]: X = np.zeros(n*n)
      for i in range(560):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

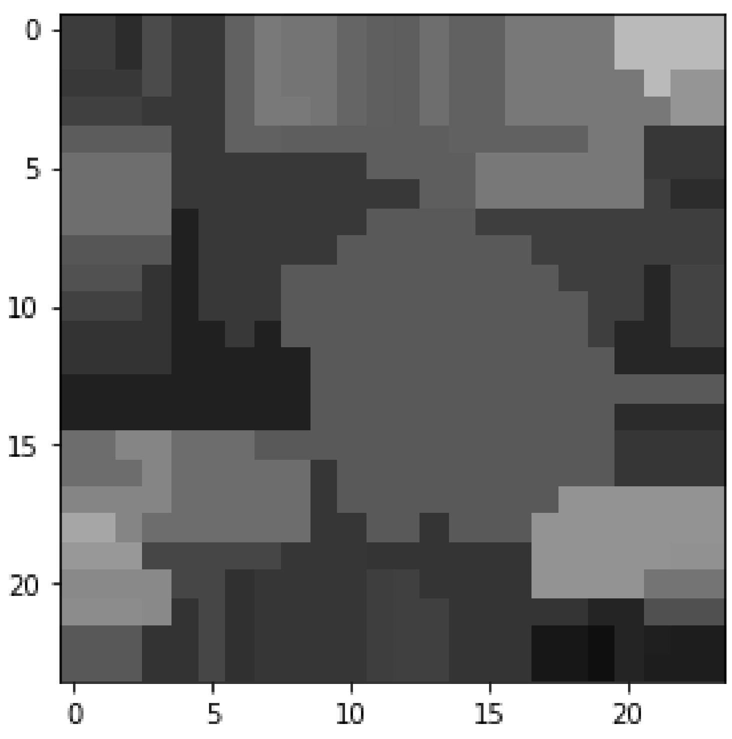
```
[53]: <matplotlib.image.AxesImage at 0x7f3356794b10>
```



```
[56]: # trying to scale down the cluster number ever further
      cc = bottomUpCluster(T, 56)

      X = np.zeros(n*n)
      for i in range(56):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

```
100%|      | 576/576 [00:07<00:00, 73.81it/s]
```

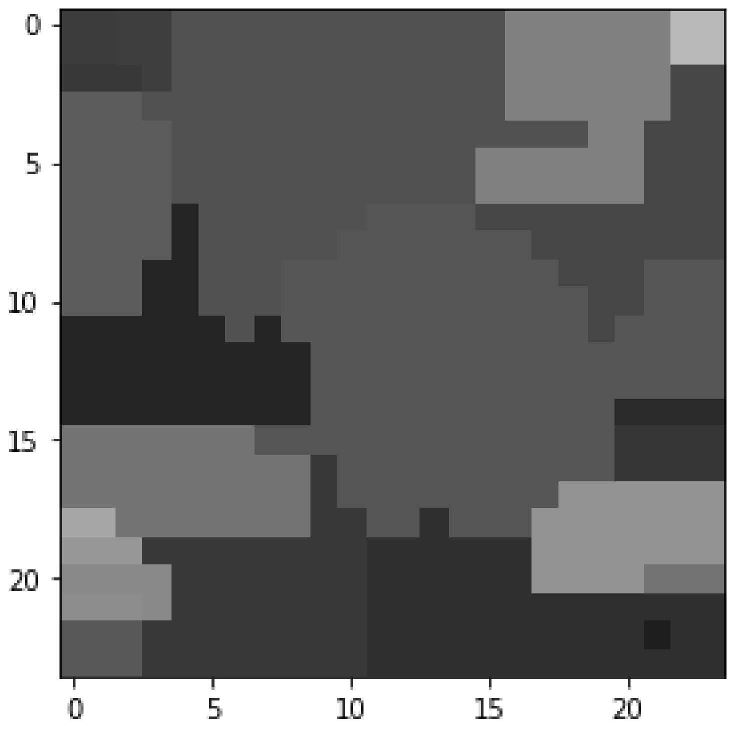
```
[56]: <matplotlib.image.AxesImage at 0x7f3355ed7e10>
```



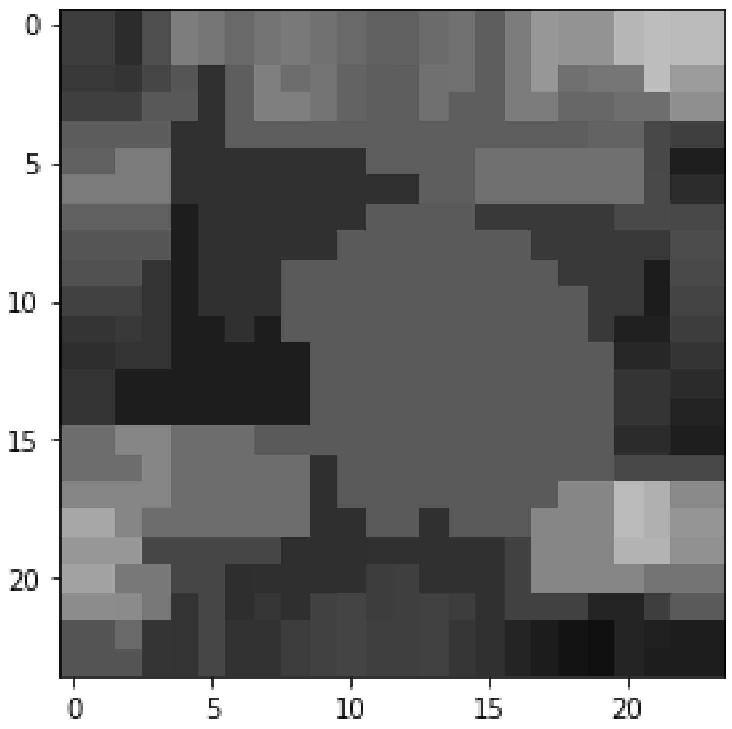
```
[58]: # trying to scale down the cluster number ever further
      cc = bottomUpCluster(T, 24)

      X = np.zeros(n*n)
      for i in range(24):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))
```

```
100%|      | 576/576 [00:07<00:00, 76.35it/s]
```

```
[58]: <matplotlib.image.AxesImage at 0x7f33563eef50>
```



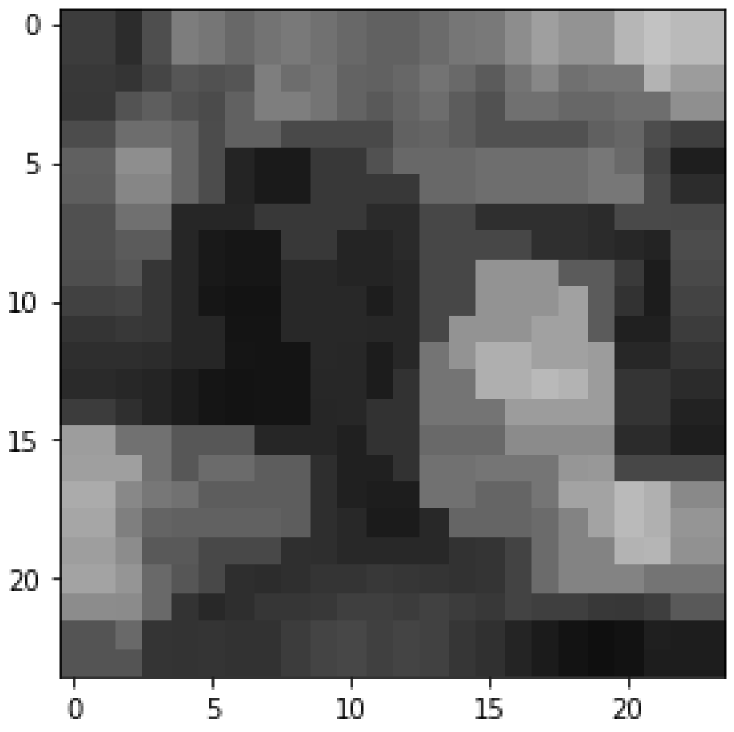
```
[59]: cc = bottomUpCluster(T, 128)

      X = np.zeros(n*n)
      for i in range(128):
          #X[cc[i]] = i
          X[cc[i]] = z[cc[i]].mean()

      io.imshow(X.reshape(n,n))

      100%|      | 576/576 [00:07<00:00, 72.42it/s]

[59]: <matplotlib.image.AxesImage at 0x7f335d08d950>
```



[60]:
```
cc = bottomUpCluster(T, 256)

X = np.zeros(n*n)
for i in range(256):
    #X[cc[i]] = i
    X[cc[i]] = z[cc[i]].mean()

io.imshow(X.reshape(n,n))
```

```
100%|          | 576/576 [00:08<00:00, 71.18it/s]
```

[60]:
```
<matplotlib.image.AxesImage at 0x7f3355c8ce50>
```


0
5
10
15
20
0
5
10
15
20

```
[ ]:
```