

1 bigass equation

$$\begin{aligned}
\frac{1}{N} \log p(\mathbf{X}) &= \frac{1}{N} \log \int p(\mathbf{X}, \mathbf{Z}) d\mathbf{Z} && \text{taking marginal} \\
&= \frac{1}{N} \log \int \frac{p(\mathbf{X}, \mathbf{Z})}{q(\mathbf{Z})} q(\mathbf{Z}) d\mathbf{Z} && \text{multiplying by 1 inside} \\
&= \frac{1}{N} \log \int \frac{p(\mathbf{X}, \mathbf{Z})}{q(\mathbf{Z})} dq(\mathbf{Z}) && \text{definition of } dq(\mathbf{Z}) \\
&\geq \frac{1}{N} \int \log \frac{p(\mathbf{X}, \mathbf{Z})}{q(\mathbf{Z})} dq(\mathbf{Z}) && \text{Jensen inequality} \\
&= \frac{1}{N} \int \sum_1^N \log \frac{p(\mathbf{x}_i, \mathbf{z}_i)}{q(\mathbf{z}_i)} dq(\mathbf{z}_i) && \text{using the iid property} \\
&= \frac{1}{N} \sum_1^N -\mathcal{L}(q, p, \mathbf{x}_i) && \text{definition of } \mathcal{L}(q, p, \mathbf{x}_i) \\
&= -\mathcal{L}(q, p, X) \triangleq -\mathcal{L}(q, p) && \text{again definition of } \mathcal{L}(p, q) \quad \square
\end{aligned} \tag{1}$$

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