**The Hilbert Matrix**

c)

i. Calculating an inverse for a matrix that is larger than 2X2 involves row reducing the matrix along with the identity matrix, which would be highly inefficient. For example, given a 100X100 matrix, it would be necessary to row reduce a 200X200 matrix to find the inverse. Therefore, it is much faster to find the inverse via LU or QR decomposition.

ii. According to our data, LU decomposition is the most effective method in decomposing and solving for x, as it’s error was zero for all decomposition tests, and closest to zero for all |Hx – b| tests. On the other hand, the Householder reflections method performed the worst in the decomposition tests, while the Givens Rotation method performed worst in the |Hx – b| tests.

Additionally, given a matrix A and multiple vectors b (in Ax = 0), it is much more efficient to solve the system using LU or QR decomposition as opposed to Gaussian elimination because it is only necessary to decompose A once, and then solve the different systems for the varying b vectors, while Gaussian elimination would require the entire system of A|b be recomputed with every new b.

**Convergence of the iterative methods**

b)

Jacobi Error: **1.877x10^-6**

Gauss-Seidel: **4.453x10^-6**

c) N\_Jacobi / N\_Gauss-Seidel: 1.97

e) The error obtained by the Jacobi iterations is slightly higher than the error obtained via Gauss-Seidel iterations, while the number of iterations Jacobi required to complete was nearly twice as much as Gauss-Seidel. According to this, it can be concluded that Gauss-Seidel is faster than Jacobi, but less efficient, and vice versa.

Based on the graphs produced, as the initial error increases, the number of iterations Jacobi requires has a tendency to increase, while the number of iterations required by Gauss-Seidel remains the same. These results show that choosing an initial value of x\_0 that is close to the solution helps to decrease the number of iterations that Jacobi requires, but has no effect of Gauss-Seidel.

h) Given a matrix that does not work for