APDE: **Charpit:** $F(p,q,u,x,y) = 0$ with $u_x = p, u_y = q, \dot{x} = F_p, \dot{y} = F_q$. Then via $F_x, F_y, \&\ p_y = q_x \to p_\tau = -F_x - pF_u,\ q_\tau = -F_y - qF_u, u_\tau = pF_p + qF_q$. Also, $u0_s = p_0 x0_s + q_0 y0_s; F_0 = 0$ - last 2 needed to show u defined on Γ. **Laplacian:** In $2D : r^{-1}(rf_r)_r + r^{-2}f_{\theta\theta}$. In $3D : r^{-2}\left(r^2 f_r\right)_r + r^{-2}\sin^{-2}(\theta) f_{\phi\phi} + r^{-2}\sin^{-1}(\theta)\left(\sin(\theta) f_\theta\right)_\theta$ **Riemann:** For $u_{xy} + au_x + bu_y + cu = f$ we have $\int_D RLu - uL^*R = \int_D \partial_x (Ru_y + auR) + \partial_y(-uR_x + buR) = \int_{\partial D} dy\,(Ru_y + Rau) + dx\,(uR_x - buR)$. Expand over triangle going B-P-A (B at bottom right) $\to$ need $R_x = bR@y = \eta, R_y = aR@x = \xi, R(P) = 1, L^*R = 0$. Also ensure IVP to get R_y, R_x! **R-H:** Derived via $P_x\psi + Q_y\psi = R\psi \to \int_D (P\psi)_x + (Q\psi)_y \left(= \int_\Gamma \psi P dy - \psi Q dx\right) = \int_D P\psi_x + Q\psi_y + R\psi = \int_{D_1+D_2} P\psi_x + Q\psi_y + R\psi$, where $\int_{D_i} = \int_{D_i} (P\psi)_x + (Q\psi)_y + \psi(R - P_x - Q_y)$. So $\int_\Gamma \psi P dy - \psi Q dx = \int_{\Gamma + C_1 - C_2} \psi P dy - \psi Q dx$ and so $\int_{C_1+C_2} \psi P dy - \psi Q dx = 0 \to dy/dx = [Q]_-^+ / [P]_-^+$
Canonical: For $au_{xx} + 2bu_{xy} + cu_{yy} = f$, we need Cauchy-Kowalevski s.t. first derivs defined: $x' := \frac{dx}{ds}$ s.t. on Γ $p_0' = x_0' u_{xx} + y_0' u_{xy}, q_0' = x_0' u_{xy} + y_0' u_{yy}$. Use these 3, solve det A!=0 s.t. $ay_0'^2 - 2bx_0'y_0' + cx_0'^2 \neq 0$. Solve quadratic s.t. $b^2 > ac \to h, b^2 < ac \to e, b^2 = ac \to p$. H: $\lambda_1, \lambda_2 \to \xi, \eta$. E: $\lambda = \lambda_R \pm i\lambda_I; \lambda_R \to \xi, \lambda_I \to \eta$. P: $\lambda_1 \to \xi$, choose η independent e.g. xy, x^2. **Green's Fn:** For $u_{xx} + u_{yy} + au_x + bu_y + cu = f$ we have $\int_D GLu - uL^*G = \int_D (u_xG)_x + (u_yG)_y - (uG_x)_x - (uG_y)_y + (auG)_x + (buG)_y = \int_D \boldsymbol{\nabla}\cdot(u_nG - uG_n) + \boldsymbol{\nabla}\cdot\left((a\,b)^T \hat{n} G u\right) = \int_{\partial D} u_nG - uG_n + (a\,b)^T\hat{n}G$. NB $\hat{n} = (dy, -dx)$. Also note for quarter plane if we have $G_x(0,y) = 0, G(x,0) = 0$ then we have same sign at $\xi_1 = (-x, y)$, opposite sign at $\xi_2 = (x, -y)$, and for the third we reflect ξ_2 across y axis so we have an opposite sign to ξ at $\xi_3 = (-x, -y)$. **Types:** Quasi: Coeffs don't depend on highest order derivs Semi: Coeffs depend on x, y. **Causality:** For a n-dim prob, we have n characteristics. Shock intersects $2n$. $\exists\, k$ outgoing, $2n - k$ ingoing. Also have n R-H relations, so $3n - k$ pieces of info. Unknowns are n components of $\vec{\boldsymbol{u}}$ on both sides of shock & slope $\Rightarrow 2n + 1$ unknowns. We demand $3n - k = 2n + 1$ so $k = n - 1$ outgoing characterisitcs.
SAM: **Dists:** Need linearity and continuity: $\exists N, C$ s.t. $|(u,\phi)| \leq C\sum_{m\leq N} \max_{\in[-X,X]} |\phi^{(m)}|$. OR $\lim_{n\to\infty}(u, \phi_n) = (u, \lim_{n\to\infty}\phi_n)$ for a sequence $\phi_n \to 0$ as $n \to \infty$. **Orthog:** $\int_0^\pi \sin(kx)\sin(jx) = \frac{\pi}{2}\delta_{kj}$, same for cos.