

APDE: Charpit: $F(p, q, u, x, y) = 0$ with $u_x = p, u_y = q, \dot{x} = F_p, \dot{y} = F_q$. Then via F_x, F_y , & $p_y = q_x \rightarrow p_\tau = -F_x - pF_u, q_\tau = -F_y - qF_u, u_\tau = pF_p + qF_q$. Also, $u0_s = p_0x0_s + q_0y0_s; F_0 = 0$ - last 2 needed to show u defined on Γ . **Max Principle:** For $-\Delta u = f \leq 0 \rightarrow \max u \in \partial D$. First show contradiction assuming $LU = f < 0$, then try some auxillary function $\psi = U + \alpha(T_{\max})g(x_i, y_i)$ s.t. $L\psi < 0$ so $\max \psi = \max_{\partial D} \psi$. Gets $\max_{e_{i,j}}$; change to $-\alpha$ for $\min_{e_{i,j}}$. **Laplacian:** In $2D : r^{-1}(rf_r)_r + r^{-2}f_{\theta\theta}$. In $3D : r^{-2}(r^2f_r)_r + r^{-2}\sin^{-2}(\theta)f_{\phi\phi} + r^{-2}\sin^{-1}(\theta)(\sin(\theta)f_\theta)_\theta$ **Green's f'n Circle:** For $G = 0|_{\partial D}$ we have $G = \frac{-1}{4\pi} \left(\frac{1}{|x-\xi|} - \frac{1}{|\xi||x-\xi'|} \right)$ **Riemann:** For $u_{xy} + au_x + bu_y + cu = f$ we have $\int_D RLu - uL^*R = \int_D \partial_x (Ru_y + auR) + \partial_y (-uR_x + buR) = \int_{\partial D} dy (Ru_y + Rau) + dx (uR_x - buR)$. Expand over triangle going B-P-A (B at bottom right) \rightarrow need $R_x = bR@y = \eta, R_y = aR@x = \xi, R(P) = 1, L^*R = 0$. Also ensure IVP on $\int_B^P dy Ru_y \rightarrow Ru|_B^P - \int_B^P dy uR_y$. **R-H:** Derived via $P_x\psi + Q_y\psi = R\psi \rightarrow \int_D (P\psi)_x + (Q\psi)_y (= \int_\Gamma \psi Pdy - \psi Qdx)$
 $= \int_D P\psi_x + Q\psi_y + R\psi = \int_{D_1+D_2} P\psi_x + Q\psi_y + R\psi$, where $\int_{D_i} = \int_{D_i} (P\psi)_x + (Q\psi)_y + \psi(R - P_x - Q_y)$. So $\int_\Gamma \psi Pdy - \psi Qdx = \int_{\Gamma+C_1-C_2} \psi Pdy - \psi Qdx$ and so $\int_{C_1+C_2} \psi Pdy - \psi Qdx = 0 \rightarrow dy/dx = [Q]_-^+ / [P]_-^+$
Canonical: For $au_{xx} + 2bu_{xy} + cu_{yy} = f$, we need **Cauchy-Kowalevski** s.t. first derivs defined: $x' := \frac{dx}{ds}$ s.t. on Γ $p'_0 = x'_0u_{xx} + y'_0u_{xy}, q'_0 = x'_0u_{xy} + y'_0u_{yy}$. Use these 3, solve $\det A \neq 0$ s.t. $ay_0'^2 - 2bx_0'y'_0 + cx_0'^2 \neq 0$. Solve quadratic s.t. $b^2 > ac \rightarrow h, b^2 < ac \rightarrow e, b^2 = ac \rightarrow p$. **H:** $\lambda_1, \lambda_2 \rightarrow \xi, \eta$. **E:** $\lambda = \lambda_R \pm i\lambda_I; \lambda_R \rightarrow \xi, \lambda_I \rightarrow \eta$. **P:** $\lambda_1 \rightarrow \xi$, choose η independent e.g. xy, x^2 . **Canonical Differentials:** $u_x = u_\xi \xi_x + u_\eta \eta_x, u_{xx} = u_{\xi\xi} \xi_x^2 + u_{\eta\eta} \eta_x^2 + 2u_{\xi\eta} \xi_x \eta_x + u_\xi \xi_{xx} + u_\eta \eta_{xx}$. Repeat for $\partial_y, \partial_{yy}$ **Green's Fn: DON'T USE GREENS THM USE NORMALS** For $u_{xx} + u_{yy} + au_x + bu_y + cu = f$ we have $\int_D GLu - uL^*G = \int_D (u_x G)_x + (u_y G)_y - (uG_x)_x - (uG_y)_y + (auG)_x + (buG)_y = \int_D \nabla \cdot (u_n G - uG_n) + \nabla \cdot ((a \ b)^T \hat{n} Gu) = \int_{\partial D} u_n G - uG_n + (ab)^T \hat{n} G$. NB $\hat{n} = (dy, -dx)$. **Also note for quarter plane** if we have $G_x(0, y) = 0, G(x, 0) = 0$ then we have same sign at $\xi_1 = (-x, y)$, opposite sign at $\xi_2 = (x, -y)$, and for the third we reflect ξ_2 across y axis so we have an opposite sign to ξ at $\xi_3 = (-x, -y)$. **Types: Quasi:** Coeffs don't depend on highest order derivs **Semi:** Coeffs depend on x, y . **Causality:** For a n -dim prob, we have n characteristics. Shock intersects $2n$. $\exists k$ outgoing, $2n - k$ ingoing. Also have n R-H relations, so $3n - k$ pieces of info. Unknowns are n components of \vec{u} on both sides of shock & slope $\Rightarrow 2n + 1$ unknowns. We demand $3n - k = 2n + 1$ so $k = n - 1$ outgoing characteristics. **d'Alembert:** Consider triangle A-P-B with AB hypoteneuse. Via $\xi = x+t, \eta = x-t$ we get with $R_\eta = 0$ on $\xi = p$, and $R_\xi = 0$ on $\eta = q$, then via riemann f'n $\phi(P) = -\int_D \frac{f}{4}$. $|J| = 2$ so $\phi(r, s) = -\int_D \frac{f}{2} dx dt$. Then have triangle ABP with $AP : \eta = q := r - s \rightarrow x - t = r - s$, $PB : \xi = p := r + s \rightarrow x + t = r + s$, and $AB : y = 0$ so finally $\phi(r, s) = -\frac{1}{2} \int_0^s dt \int_{r-s+t}^{r+s-t} dx f(x, t)$
Integral Derivs $\frac{d}{dt} \int_{b(t)}^{a(t)} dx f(x, t) = a'(t)f(a, t) - b'(t)f(b, t) + \int_{b(t)}^{a(t)} dt f_t(x, t)$
SAM: Dists: Need linearity and continuity: $\exists N, C$ s.t. $|(u, \phi)| \leq C \sum_{m \leq N} \max_{\epsilon \in [-X, X]} |\phi^{(m)}|$. OR $\lim_{n \rightarrow \infty} (u, \phi_n) = (u, \lim_{n \rightarrow \infty} \phi_n)$ for a sequence $\phi_n \rightarrow 0$ as $n \rightarrow \infty$. **Orthog:** $\int_0^\pi \sin(kx) \sin(jx) = \frac{\pi}{2} \delta_{kj}$, same for cos. **S-L Operator** For $T := \alpha y'' + \beta y' + \gamma$, multiply by $\exp(\int dx \beta)$ to get T_{SL}