APDE: **Charpit:** $F(p,q,u,x,y)=0$ with $u_x=p, u_y=q, \dot{x}=F_p, \dot{y}=F_q$. Then via $F_x, F_y, \&\, p_y=q_x \rightarrow p_\tau=-F_x-pF_u, q_\tau=-F_y-qF_u, u_\tau=pF_p+qF_q$. Also, $\frac{du_0}{ds}=p_0\frac{dx_0}{ds}+q_0\frac{dy_0}{ds}; F_0=0$ - last 2 needed to show u defined on Γ. **Max Principle:** For $-\Delta u=f\leq 0 \rightarrow \max u\in\partial D$. First show contradiction assuming $LU=f<0$, then try some auxillary function $\psi=U+\alpha\left(T_{\max}\right)g\left(x_i,y_i\right)$ s.t. $L\psi<0$ so $\max\psi=\max_{\in\partial D}\psi$. Gets $\max e_{i,j}$; change to $-\alpha$ for $\min e_{i,j}$. **Laplacian:** In $2D: r^{-1}\left(rf_r\right)_r+r^{-2}f_{\theta\theta}$. In $3D: r^{-2}\left(r^2f_r\right)_r+r^{-2}\sin^{-2}(\theta)f_{\phi\phi}+r^{-2}\sin^{-1}(\theta)\left(\sin(\theta)f_\theta\right)_\theta$ **Green's f'n Circle:** For $G=0|_{\partial D}$ we have $G=\frac{-1}{4\pi}\left(\frac{1}{|x-\xi|}-\frac{1}{|\xi||x-\xi'|}\right)$ **Riemann:** For $u_{xy}+au_x+bu_y+cu=f$ we have $\int_D RLu-uL^*R=\int_D\partial_x\left(Ru_y+auR\right)+\partial_y\left(-uR_x+buR\right)=\int_{\partial D}dy\left(Ru_y+Rau\right)+dx\left(uR_x-buR\right)$. Expand over triangle going B-P-A (B at bottom right) $\rightarrow$ need $R_x=bR@y=\eta, R_y=aR@x=\xi, R(P)=1, L^*R=0$. Also ensure IVP on $\int_B^P dyRu_y\rightarrow Ru|_B^P-\int_B^P dy\, uR_y$. **Riemann Invariants:** If we have $\frac{d}{dx}[u-v]=-f$ on $y=x+c_1$, and $\frac{d}{dx}[u+v]=f$ on $y=-x+c_2$, then we have: $u-v+\int_{-c_1}^x ds\, f(s,s+c_1)=k_1$, and $u+v-\int_{c_2}^x ds\, f(s,-s+c_2)=k_2$ for constants k_1,k_2. **R-H:** Derived via $P_x\psi+Q_y\psi=R\psi\rightarrow\int_D(P\psi)_x+(Q\psi)_y\,(=\int_\Gamma\psi Pdy-\psi Qdx)=\int_D P\psi_x+Q\psi_y+R\psi=\int_{D_1+D_2}P\psi_x+Q\psi_y+R\psi$, where $\int_{D_i}=\int_{D_i}(P\psi)_x+(Q\psi)_y+\psi\left(R-P_x-Q_y\right)$. So $\int_\Gamma\psi Pdy-\psi Qdx=\int_{\Gamma+C_1-C_2}\psi Pdy-\psi Qdx$ and so $\int_{C_1+C_2}\psi Pdy-\psi Qdx=0\rightarrow dy/dx=[Q]_-^+/[P]_-^+$ **Canonical:** For $au_{xx}+2bu_{xy}+cu_{yy}=f$, we need Cauchy-Kowalevski s.t. first derivs defined: $x':=\frac{dx}{ds}$ s.t. on Γ $p_0'=x_0'u_{xx}+y_0'u_{xy}, q_0'=x_0'u_{xy}+y_0'u_{yy}$. Use these 3, solve det A!=0 s.t. $ay_0'^2-2bx_0'y_0'+cx_0'^2\neq 0$. Solve quadratic s.t. $b^2>ac\rightarrow h, b^2<ac\rightarrow e, b^2=ac\rightarrow p$. H: $\lambda_1,\lambda_2\rightarrow\xi,\eta$. E: $\lambda=\lambda_R\pm i\lambda_I;\lambda_R\rightarrow\xi,\lambda_I\rightarrow\eta$. P: $\lambda_1\rightarrow\xi$, choose η independent e.g. xy, x^2. **Canonical Differentials:** $u_x=u_\xi\xi_x+u_\eta\eta_x, u_{xx}=u_{\xi\xi}\xi_x^2+u_{\eta\eta}\eta_x^2+2u_{\xi\eta}\xi_x\eta_x+u_\xi\xi_{xx}+u_\eta\eta_{xx}$. Repeat for ∂_y,∂_{yy} **Green's Fn:** DON'T USE GREENS THM USE NORMALS For $u_{xx}+u_{yy}+au_x+bu_y+cu=f$ we have $\int_D GLu-uL^*G=\int_D\left(u_xG\right)_x+\left(u_yG\right)_y-\left(uG_x\right)_x-\left(uG_y\right)_y+(auG)_x+(buG)_y=\int_D\boldsymbol{\nabla}\cdot\left(u_nG-uG_n\right)+\boldsymbol{\nabla}\cdot\left((a\,b)^T\hat{n}Gu\right)=\int_{\partial D}u_nG-uG_n+(a\,b)^T\hat{n}G$. NB $\hat{n}=(dy,-dx)$. Also note for quarter plane if we have $G_x(0,y)=0, G(x,0)=0$ then we have same sign at $\xi_1=(-x,y)$, opposite sign at $\xi_2=(x,-y)$, and for the third we reflect ξ_2 across y axis so we have an opposite sign to ξ at $\xi_3=(-x,-y)$. **Types:** Quasi: Coeffs don't depend on highest order derivs Semi: Coeffs depend on x,y. **Causality:** For a n-dim prob, we have n characteristics. Shock intersects $2n$. $\exists k$ outgoing, $2n-k$ ingoing. Also have n R-H relations, so $3n-k$ pieces of info. Unknowns are n components of $\vec{\boldsymbol{u}}$ on both sides of shock & slope $\Rightarrow 2n+1$ unknowns. We demand $3n-k=2n+1$ so $k=n-1$ outgoing characterisitcs. **d'Alembert:** Consider triangle A-P-B with AB hypoteneuse. Via $\xi=x+t,\eta=x-t$ we get with $R_\eta=0$ on $\xi=p$, and $R_\xi=0$ on $\eta=q$, then via riemann f'n $\phi(P)=-\int_D\frac{\hat{f}}{4}$. $|J|=2$ so $\phi(r,s)=-\int_D\frac{f}{2}dxdt$. Then have triangle ABP with $AP:\eta=q:=r-s\rightarrow x-t=r-s$, $PB:\xi=p:=r+s\rightarrow x+t=r+s$, and $AB: y=0$ so finally $\phi(r,s)=-\frac{1}{2}\int_0^s dt\int_{r-s+t}^{r+s-t}dx\, f(x,t)$ **Integral Derivs** $\frac{d}{dt}\int_{b(t)}^{a(t)}dx\, f(x,t)=a'(t)f(a,t)-b'(t)f(b,t)+\int_{b(t)}^{a(t)}dt\, f_t(x,t)$ SAM: **Dists:** Need linearity and continuity: $\exists N,C$ s.t. $|(u,\phi)|\leq C\sum_{m\leq N}\max_{\in[-X,X]}|\phi^{(m)}|$. OR $\lim_{n\rightarrow\infty}(u,\phi_n)=(u,\lim_{n\rightarrow\infty}\phi_n)$ for a sequence $\phi_n\rightarrow 0$ as $n\rightarrow\infty$. **Orthog:** $\int_0^\pi\sin(kx)\sin(jx)=\frac{\pi}{2}\delta_{kj}$, same for cos. **S-L Operator** For $T:=\alpha y''+\beta y'+\gamma$, multiply by $\exp\left(\int dx\,\beta\right)$ to get T_{SL}