
NS: Invariant Set: $\phi_t(S) \subseteq S$ **Lim Pts:**
FPDE: Types: 1^{st} : \exists scale s.t. solution found, not so for 2^{nd} . **Heat:** $\hat{T} = u(\hat{T}_\infty - \hat{T}_{-\infty}) + \hat{T}_{-\infty}$ **Oil**
Spread: Dims: $x = x_f + \varepsilon\xi, t = \tau$ **Ground Spread:** $(1-s)\phi h_t + Q_x = 0; Q \sim -hh_x, 0 < x_s < x_f$. Have
 $h(x_f) = 0, h_t(x_s) = 0$, and $hh_x|_{x=0, x_f} = 0$ (i.e. no flux at centre and front), and h, hh_x cont. at joint.
Expansions: Let $\xi = z + \varepsilon\eta$ for perturbations **Scale:** Try $x = x_f + \varepsilon\xi$ for groundwater **Stefan:** $S_0 =$
 $C(T_1 - T_m)/L$, condition $= \rho L \dot{s} = kT_x|_{s_-}^{s_+}$ **1ph Stefan:** Bar $= T_h|_{liq|sol} INS$. Use $T = T_m + (T_1 - T_m)u$
s.t. $S_0 u_t = u_{xx}, u = 1 @ x = 0, \{\dot{s} = -u_x, u = 0\} @ x = s, s(0) = 0$. Sim. sol is $s = \beta\sqrt{t}, f = f(x/\sqrt{t})$
2ph Stefan: Use $T = T_m + (T_1 - T_m)u$ s.t. $S_0 u_t = u_{xx} @ 0 < x < s, (S_0/\kappa)u_t = u_{xx} @ s < x <$
 $1, u = 1 @ x = 0, u_x = 0 @ x = 1, \{\dot{s} = Ku_x|_{s_+} - u_x|_{s_-}, u = 0\} @ x = s, \{s = 0, u = -\theta\} @ x = 0$. Here
 $\theta := (T_m - T_0)/(T_1 - T_m), \kappa := c_1 k_1/(c_2 k_2), K := k_2/k_1$ Sim. sol is $s = \beta\sqrt{t}, f = f(x/\sqrt{t})$ **2-Dim:**
 $U_n = \hat{n} \cdot u = K(u_2)_n - (u_1)_n$. If $x = f(y, t)$ then $\hat{n} := \nabla(x - f) = [1, -f_y]^T / \sqrt{1 + f_y^2}$ **Welding:**
Have $0 < s_2 < s_1$. Have cold $x = a$, no flux $x = 0$. $\theta = 1$ in liquid. In mush $\rho L \theta_t = J^2/\sigma$, CoE
 $\rightarrow \theta \rho L \dot{s} + kT_x|_{s_-}^{s_+} = 0$. Have θ cont. ($= 0$) at s_1 . I.e. we have $S_0 u_t = u_{xx} + q, u_x = 0 @ x = 0, u =$
 $-1 @ x = 1, \theta = 0 @ x = s_1$. Also $\theta_t = q$ in mush.
FMM: