Golub for $k = 1 : m, n$: $u_k = (sgn(b_{k,k})\|b_{k:m,k}\|e_1 + b_{k:m,k})$; $u_k := \hat{u}_k$; $U_k := I - 2u_k u_k^T$; $B_{k:m,k:n} := U_k B_{k:m,k:n}$; $U = [I_{k-1,k-1}, 0; 0, U_k]$;for $j = 1 : m, n - 1$: $v_k^T := sgn(b_{k,k+1})\|b_{k,k+1:n}\|e_1 + b_{k:m,k}$; $V_k := I - 2v_k v_k^T$; $B_{1:m,k+1:n} = B_{1:m,k+1:n} V_k$; $V = [I_{k,k}, 0; 0, V_k]$endfor endfor; $2 \cdot (2mn^2 - 2n^3/3)$ **Householder** for $k = [1, n]$: $x = A_{k:m,k}$; $v_k = sgn(x)\|x\|e_k + x$; $v_k = \frac{v_k}{\|v_k\|}$ for $j = [k, n]$ $A_{k:m,j} = A_{k:m,j} - 2v_k [v_k^* A_{k:m,j}]$ endfor endfor. $2mn^2 - \frac{2n^3}{3}$. **MG-S** $V = A$;for $i = [1, n]$: $r_{ii} = \|v_i\|$; $q_i = \frac{v_i}{r_{ii}}$;for $j = [i + 1, n]$ $v_j = v_j - (q_i^T v_j)q_i$; $r_{ij} = q_i^T v_j$ endfor endfor. $2mn^2$. **Arnoldi:** $q_1 := \hat{b}$; $q_{k+1}h_{k+1,k} = Aq_k - \sum_{i=1}^k q_i h_{ik}$; $h_{ik} = q_i^T(Aq_k)$; $h_{k+1,k} := \|v\| \to AQ_k := Q_k H_k + q_{k+1}[0 \ldots h_{k+1,k}]$. **Givens** $3mn^2$ **SVD:** $= \sum_i^{r:=\min m,n} u_i \sigma_i v_i^T$. **QR Algo:** $A_{k+1} = Q_k^T A_k Q_k \to A_{k+1} = (Q^{(k)})^T A Q^{(k)} \& A^k = (Q_1 \ldots Q_k)(R_k \ldots R_1) := Q^{(k)} R^{(k)}$, via induction **GMRES:** $\min \|AQ_k y - b\| \to \min \|H_k y - \|b\|e_1\|$ **CG Convergence:** $\|e_k\| = \min_{p(0)=1} \|p_k(A)e_0\| = \min_{p_k(A)} \max |p_k(\lambda)|\|e_0\| \to \le 2\left((\sqrt{k_2} - 1)/(\sqrt{k_2} + 1)\right)^k$; need $\alpha := 2(\lambda_1 + \lambda_2)$ **Cheb:** $T_k(x) = \frac{1}{2}(z^k + z^{-k})$; $2xT_k = T_{k+1} + T_{k-1}$ **MP:** $\sigma(G) \in [\sqrt{m} - \sqrt{n}, \sqrt{m} + \sqrt{n}] \to k_2 = O(1)$ **Sketch:** with $GA\hat{x} = Gb$, and via $C - F$ $\|G[A,b][v, -1]^T\| \le (s + \sqrt{n+1})\|R[v, -1]^T\|$, similar for lower bound via MP $\to \|A\hat{x} - b\| \le (\sqrt{s} + \sqrt{n+1})/(\sqrt{s} - \sqrt{n+1})\|Ax - b\|$ **Blend:** solve $\left\|(A\hat{R}^{-1})y - b\right\| = 0$ via CG; $k_2(A\hat{R}^{-1}) = O(1)$ with $GA = \hat{Q}\hat{R}$ PROOF: $A = QR$; $GA = GQR = \hat{G}R$. Let $\hat{G} = \hat{Q}\hat{R}$ so $GA = \hat{Q}\hat{R}R \to \tilde{R}^{-1} = R^{-1}\hat{R}^{-1} \to k_2(A\tilde{R}^{-1}) = k_2(\hat{R}^{-1}) = O(1)$ by MP. $O(mn)$ to solve via normal **Bounds:** $\|ABB^{-1}\| \ge \|AB\|\|B^{-1}\| \to \|A\|/\|B^{-1}\| \ge \|AB\|$. **Weyls:** $\sigma_i(A + B) = \sigma_i(A) + [-\|B\|, \|B\|]$ **Rev Δ Ineq:** $\|A - B\| \ge |\|A\| - \|B\||$ **Courant Application:** $\sigma_i([A_1; A_2]) \ge \max(\sigma_i(A_1), \sigma_i(A_2))$ **Schur:** Take $Av_1 = \lambda_1 v_1$; construct $U_1 = [v_1, V_\perp] \to AU_1 = U_1[e_1, X]$. Repeat. **Conditioning** $\kappa_2(A) = \sigma_1/\sigma_n = \|A\|\|A^{-1}\|$ **Similarity:** $A \to P^{-1}AP$, same λ.

CO: **SD:** $\|x_{k+1} - x_*\| \le (k_2(H) - 1)/(k_2(H) + 1)\|x_k - x_*\|$ with H hessian