APDE: **Charpit:** $F(p,q,u,x,y)=0$ with $u_x=p, u_y=q, \dot{x}=F_p, \dot{y}=F_q$. Then via $F_x, F_y, \&\ p_y=q_x \rightarrow$ $p_\tau=-F_x-pF_u,\ q_\tau=-F_y-qF_u, u_\tau=pF_p+qF_q$. Also, $u0_s=p_0x0_s+q_0y0_s; F_0=0$ - last 2 needed to show u defined on Γ. **Riemann:** $\int_D RLu-uL^*R=\int_D \partial_x(Ru_y+auR)+\partial_y(-uR_x+buR)=\int_{\partial D} dy\,(Ru_y+Rau)+dx\,(uR_x-buR)$. Expand over triangle going B-P-A (B at bottom right) $\rightarrow$ need $R_x=bR@y=\eta, R_y=aR@x=\xi, R(P)=1, L^*R=0$. Also ensure IVP to get R_y, R_x! **Canonical:** For $au_{xx}+2bu_{xy}+cu_{yy}=f$, we need Cauchy-Kowalevski s.t. first derivs defined: $x':=\frac{dx}{ds}$ s.t. on Γ $p_0'=x_0'u_{xx}+y_0'u_{xy}, q_0'=x_0'u_{xy}+y_0'u_{yy}$. Use these 3, solve det A!=0 s.t. $ay_0'^2-2bx_0'y_0'+cx_0'^2\neq 0$. Solve quadratic s.t. $b^2>ac\rightarrow h, b^2<ac\rightarrow e, b^2=ac\rightarrow p$. H: $\lambda_1,\lambda_2\rightarrow\xi,\eta$. E: $\lambda=\lambda_R\pm i\lambda_I; \lambda_R\rightarrow\xi, \lambda_I\rightarrow\eta$. P: $\lambda_1\rightarrow\xi$, choose η independent e.g. xy, x^2. **Green's Fn:** For $u_{xx}+u_{yy}+au_x+bu_y+cu=f$ we have $\int_D GLu-uL^*G=\int_D (u_xG)_x+(u_yG)_y-(uG_x)_x-(uG_y)_y+(auG)_x+(buG)_y=\int_D \boldsymbol{\nabla}\cdot(u_nG-uG_n)+\boldsymbol{\nabla}\cdot\left((a\,b)^T\hat{n}Gu\right)=\int_{\partial D} u_nG-uG_n+(a\,b)^T\hat{n}G$. NB $\hat{n}=(dy,-dx)$. Also note for quarter plane if we have $G_x(0,y)=0, G(x,0)=0$ then we have same sign at $\xi_1=(-x,y)$, opposite sign at $\xi_2=(x,-y)$, and for the third we reflect ξ_2 across y axis so we have an opposite sign to ξ at $\xi_3=(-x,-y)$. **Types:** Quasi: Coeffs don't depend on highest order derivs Semi: Coeffs depend on x,y. **Causality:** For a n-dim prob, we have n characteristics. Shock intersects $2n$. $\exists\, k$ outgoing, $2n-k$ ingoing. Also have n R-H relations, so $3n-k$ pieces of info. Unknowns are n components of $\vec{\boldsymbol{u}}$ on both sides of shock & slope $\Rightarrow 2n+1$ unknowns. We demand $3n-k=2n+1$ so $k=n-1$ outgoing characterisitcs.

SAM: **Dists:** Need linearity and continuity: $\exists N,C$ s.t. $|(u,\phi)|\leq C\sum_{m\leq N}\max_{\in[-X,X]}|\phi^{(m)}|$

NLA: **Cholesky** For matrix $[a_{11},w^*;w,K]=R_1^T\left[I,0;0,K-\frac{ww^*}{a_{11}}\right][\alpha,w^*/\alpha;0,I]$ we have a decomp: for $k=[1,m-1]$: for $j=[k+1,m]$ $R_{j,j:m}=R_{j,j:m}-\frac{R_{kj}}{R_{kk}}R_{k,j:m}$ endfor $R_{k,k:m}=\frac{R_{k,k:m}}{\sqrt{R_{kk}}}$ endfor. $\frac{m^3}{3}$. **Householder** for $k=[1,n]$: $x=A_{k:m,k}; v_k=sgn(x)\|x\|e_k+x; v_k=\frac{v_k}{\|v_k\|}$ for $j=[k,n]$ $A_{k:m,j}=A_{k:m,j}-2v_k\left[v_k^*A_{k:m,j}\right]$ endfor endfor. $\frac{2mn^2}{3}$. **LU** $U=A, L=I$ for $k=[1,m-1]$: for $j=[k+1,m]$ $U_{j,k:m}=U_{j,k:m}-\frac{U_{jk}}{U_{kk}}U_{k,k:m}$ endfor endfor. $\frac{2m^3}{3}$. **MG-S** $V=A$;for $i=[1,n]$: $r_{ii}=\|v_i\|; q_i=\frac{v_i}{r_{ii}}$;for $j=[i+1,n]$ $v_j=v_j-(q_i^Tv_j)q_i; r_{ij}=q_i^Tv_j$ endfor endfor. $2mn^2$. **Givens** $3mn^2$ **SVD:** $=\sum_i^{r:=\min m,n} u_i\sigma_i v_i^T$.

NPDE: **Hyperbolic:** Implicit: $(A-B,A)=\frac{1}{2}(\|A\|^2-\|B\|^2)+\frac{1}{2}\|A-B\|^2$ (time),$(-D_x^+D_x^-U^{m+1}$, $U^{m+1}-U^m)=(D_x^-U^{m+1}-D_x^-U^m, D_x^-U^{m+1})$ (space). Explicit: 1st rewrite in terms of $D_t^{+-}(\Delta t)^{-2}U_j^m+\frac{c^2(\Delta t)^2}{4}D_x^{+-}((\Delta t)^{-2}U_j^m)-c^2D_x^{+-}\left(U_j^{m+1}+2U_j^m+U_j^{m-1}\right)$. Then use $(D(A-B),A+B)=(DA,A)-(DB,B);(D(A+B),A-B)=(DA,A)-(DB,B)$ by multiplying by $U^{m+1}-U^{m-1}$. Finally WTS $\|V_m\|^2-\frac{c^2(\Delta t)^2}{4}\|D_x^-V^m\|^2\geq 0$. Done by noticing: $\|D_x^-V^m\|^2=\sum_i^J\Delta x|D_x^-V_j^m|^2=1/\Delta x\sum_i^J$ $\left(V_j^m-V_{j-1}^m\right)^2\leq 2/\Delta x\sum_i^J(V_j^m)^2+(V_{j-1}^m)^2=4/\Delta x^2\sum_i^{J-1}\Delta x\left(V_j^m\right)^2$ **Max Principle:** For $-\Delta u=f\leq 0\rightarrow \max u\in\partial D$ **P-F Ineq:** $\|V\|_h^2\leq c_\star||D_x^-V]|^2$ **Weak Deriv:** w is a weak derivative of u if $\int dx\,wv=(-1)^{|\alpha|}\int dxu(D^\alpha v)$ **Parseval:** $\int dk\,\hat{u}(k)v(k)=\int dkv(k)\left(\int dx\,u(x)e^{-ixk}\right)=\int dxu(x)\left(\int dk\,v(k)e^{-ixk}\right)=\int dx\,u(x)\hat{v}(x)$. Now $v(k):=\overline{\hat{u}(k)}=\overline{F\left[u(k)\right]}=\overline{\int dk\,u(k)e^{-ixk}}=\int dk\,\overline{u(k)}e^{ixk}=2\pi F^{-1}\left[\overline{u(k)}\right]\Rightarrow \hat{v}(x)=2\pi\overline{u(x)}$