

APDE: Charpit: $F(p, q, u, x, y) = 0$ with $u_x = p, u_y = q, \dot{x} = F_p, \dot{y} = F_q$. Then via $F_x, F_y, \& p_y = q_x \rightarrow p_\tau = -F_x - pF_u, q_\tau = -F_y - qF_u, u_\tau = pF_p + qF_q$. Also, $u0_s = p_0x0_s + q_0y0_s; F_0 = 0$ - last 2 needed to show u defined on Γ . **Riemann:** $\int_D RLu - uL^*R = \int_D \partial_x (Ru_y + auR) + \partial_y (-uR_x + buR) = \int_{\partial D} dy (Ru_y + Rau) + dx (uR_x - buR)$. Expand over triangle going B-P-A (B at bottom right) \rightarrow need $R_x = bR@y = \eta, R_y = aR@x = \xi, R(P) = 1, L^*R = 0$. Also ensure IVP to get $R_y, R_x!$ **Canonical:** For $au_{xx} + 2bu_{xy} + cu_{yy} = f$, we need **Cauchy-Kowalevski** s.t. first derivs defined: $x' := \frac{dx}{dy}$ s.t. on Γ $p'_0 = x'_0 u_{xx} + y'_0 u_{xy}, q'_0 = x'_0 u_{xy} + y'_0 u_{yy}$. Use these 3, solve $\det A! = 0$ s.t. $ay_0'^2 - 2bx_0'y_0' + cx_0'^2 \neq 0$. Solve quadratic s.t. $b^2 > ac \rightarrow h, b^2 < ac \rightarrow e, b^2 = ac \rightarrow p$. **H:** $\lambda_1, \lambda_2 \rightarrow \xi, \eta$. **E:** $\lambda = \lambda_R \pm i\lambda_I; \lambda_R \rightarrow \xi, \lambda_I \rightarrow \eta$. **P:** $\lambda_1 \rightarrow \xi$, choose η independent e.g. xy, x^2 . **Green's Fn:** For $u_{xx} + u_{yy} + au_x + bu_y + cu = f$ we have $\int_D GLu - uL^*G = \int_D (u_x G)_x + (u_y G)_y - (uG_x)_x - (uG_y)_y + (auG)_x + (buG)_y = \int_D \nabla \cdot (u_n G - uG_n) + \nabla \cdot ((ab)^T \hat{n} G u) = \int_{\partial D} u_n G - uG_n + (ab)^T \hat{n} G$. NB $\hat{n} = (dy, -dx)$. **Also note for quarter plane** if we have $G_x(0, y) = 0, G(x, 0) = 0$ then we have same sign at $\xi_1 = (-x, y)$, opposite sign at $\xi_2 = (x, -y)$, and for the third we reflect ξ_2 across y axis so we have an opposite sign to ξ at $\xi_3 = (-x, -y)$. **Types: Quasi:** Coeffs don't depend on highest order derivs **Semi:** Coeffs depend on x, y . **Causality:** For a n -dim prob, we have n characteristics. Shock intersects $2n$. $\exists k$ outgoing, $2n - k$ ingoing. Also have n R-H relations, so $3n - k$ pieces of info. Unknowns are n components of \vec{u} on both sides of shock & slope $\Rightarrow 2n + 1$ unknowns. We demand $3n - k = 2n + 1$ so $k = n - 1$ outgoing characteristics.

SAM: Dists: Need linearity and continuity: $\exists N, C$ s.t. $|(u, \phi)| \leq C \sum_{m \leq N} \max_{\phi \in [-X, X]} |\phi^{(m)}|$

NLA: Cholesky For matrix $[a_{11}, w^*; w, K] = R_1^T \begin{bmatrix} I, 0; 0, K - \frac{ww^*}{a_{11}} \end{bmatrix} [\alpha, w^*/\alpha; 0, I]$ we have a decomp: for $k = [1, m-1] : \text{for } j = [k+1, m] R_{j,j:m} = R_{j,j:m} - \frac{R_{kj}}{R_{kk}} R_{k,j:m} \text{ endfor } R_{k,k:m} = \frac{R_{k,k:m}}{\sqrt{R_{kk}}} \text{ endfor. } \frac{m^3}{3}$. **Householder** for $k = [1, n] : x = A_{k:m,k}; v_k = \text{sgn}(x) \|x\| e_k + x; v_k = \frac{v_k}{\|v_k\|} \text{ for } j = [k, n] A_{k:m,j} = A_{k:m,j} - 2v_k [v_k^* A_{k:m,j}] \text{ endfor endfor. } \frac{2mn^2}{3}$. **LU** $U = A, L = I$ for $k = [1, m-1] : \text{for } j = [k+1, m] U_{j,k:m} = U_{j,k:m} - \frac{U_{jk}}{U_{kk}} U_{k,k:m} \text{ endfor endfor. } \frac{2m^3}{3}$. **MG-S** $V = A; \text{for } i = [1, n] : r_{ii} = \|v_i\|; q_i = \frac{v_i}{r_{ii}}; \text{for } j = [i+1, n] v_j = v_j - (q_i^T v_j) q_i; r_{ij} = q_i^T v_j \text{ endfor endfor. } 2mn^2$. **Givens** $3mn^2$ **SVD:** $= \sum_{i=\min m, n}^r u_i \sigma_i v_i^T$.

NPDE: Hyperbolic: Implicit: $(A - B, A) = \frac{1}{2}(\|A\|^2 - \|B\|^2) + \frac{1}{2}\|A - B\|^2$ (time), $(-D_x^+ D_x^- U^{m+1}, U^{m+1} - U^m) = (D_x^- U^{m+1} - D_x^- U^m, D_x^- U^{m+1})$ (space). **Explicit:** 1st rewrite in terms of $D_t^{+-} (\Delta t)^{-2} U_j^m + \frac{c^2 (\Delta t)^2}{4} D_x^{+-} ((\Delta t)^{-2} U_j^m) - c^2 D_x^{+-} (U_j^{m+1} + 2U_j^m + U_j^{m-1})$. Then use $(D(A - B), A + B) = (DA, A) - (DB, B); (D(A + B), A - B) = (DA, A) - (DB, B)$ by multiplying by $U^{m+1} - U^{m-1}$. Finally WTS $\|V_m\|^2 - \frac{c^2 (\Delta t)^2}{4} \|D_x^- V_m\|^2 \geq 0$. Done by noticing: $\|D_x^- V_m\|^2 = \sum_i^J \Delta x |D_x^- V_j^m|^2 = 1/\Delta x \sum_i^J (V_j^m - V_{j-1}^m)^2 \leq 2/\Delta x \sum_i^J (V_j^m)^2 + (V_{j-1}^m)^2 = 4/\Delta x^2 \sum_i^{J-1} \Delta x (V_j^m)^2$ **Max Principle:** For $-\Delta u = f \leq 0 \rightarrow \max u \in \partial D$ **P-F Ineq:** $\|V\|_h^2 \leq c_* \|D_x^- V\|^2$ **Weak Deriv:** w is a weak derivative of u if $\int dx wv = (-1)^{|\alpha|} \int dx u (D^\alpha v)$ **Parseval:** $\int dk \hat{u}(k) v(k) = \int dk v(k) (\int dx u(x) e^{-ixk}) = \int dx u(x) (\int dk v(k) e^{-ixk}) = \int dx u(x) \hat{v}(x)$. Now $v(k) := \hat{u}(k) = \overline{F[u(k)]} = \overline{\int dk u(k) e^{-ixk}} = \int dk \overline{u(k)} e^{ixk} = 2\pi F^{-1} [\overline{u(k)}] \Rightarrow \hat{v}(x) = 2\pi \overline{u(x)}$ **Iterative:** If $U^{j+1} = U^j - \tau (AU^j - F) \rightarrow U - U^j = (I - \tau A)^j (U - U^0)$ so $\|U - U^j\| \leq \|I - \tau A\|^j \|U - U^0\|$. $\|I - \tau A\| = \sigma_1 = |\lambda_1|$ as symmetric. If $\lambda \in [\alpha, \beta]$ then $\lambda_1 \leq \max \{|1 - \tau\alpha|, |1 - \tau\beta|\}$. Attained when $\tau = 2/(\alpha + \beta) \rightarrow \lambda_1 = \frac{\beta - \alpha}{\beta + \alpha}$. For $-u'' + cu = f$ we have $\lambda_k = c + \frac{4}{h^2} \sin^2(\frac{k\pi h}{2})$. Lower bound via noting $\sin(y) \geq \frac{2\sqrt{2}}{\pi} y$ at $y = \frac{\pi}{4} \rightarrow \lambda_k \geq c + 8$