Fluxions, Forces, and Fields

An overview of the mathematisation of physics in Europe through the modern period

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Tracing Back

- Let's take a convenient starting point: Einstein, special relativity, & aether theory
- Much of the groundwork had been laid by Lorentz, with his *Theory of Corresponding States*[1]
- Much of the work Lorentz built on was done by George Fitzgerald, who was also influenced greatly by Maxwell

[1] Janssen and Stachel, 2004.

Beyond Maxwell

- 'Mentored' by William Thomson[2] (later Lord Kelvin)... who was a major figure in shaping physics (or natural philosophy) in C19
- Physics in Thomson's day had been centred around energy (and thus dynamics)...
- ...Which followed from continental development furthered by figures such as Lagrange and Laplace...
- ...Who worked using methods derived from Newton's work on celestial motion

[2] Smith, 1978.

Isaac Newton (1642-1727)

Newtonian Mathematical Ideals

- Saw God as mathematical, with a fondness for geometry[ii]
- Would have grown up with Cartesianism (deductive reasoning), but clashed with it later in life[3]

[3] Westfall, 1981.

[ii] Newton's piety would greatly influence his work

Experimental Philosophy

- The method of resolution & composition:
  1. Observe and "come to the general properties of things"[4]
  2. Assume said properties and describe further phenomena
- Hypotheses non fingo[5] - ignorance is acceptable!
- Nature able to be quantised (e.g. fluxional calculus & Halley's Comet)
  - Concept of forces at a distance

[4] Newton and McGuire, 1970.

[5] Newton, Cohen, et al., 1999.

The Dynamical Age: Continental Physics

Laplacian Physics

- The rise of French scientific dominance
- Physics spearheaded by Laplace in early C19, who took an "astronomical view of nature"[6]
- Prevalence of central-force based models, for even minute scales[7]
- Competitions set-up with e.g. the Society of Arcueil to promote mathematical collaboration
- Development of light, heat, and electromagnetic theory with various contestants (e.g. Fourier) - via Laplacian methods

[6] Merz, 1896.
[7] Fox, 1974.

Energy Physics

William Thomson (1824-1907)

- Ideas of *vis-viva* around since Leibniz, *caloric* furthered by e.g. Carnot in early C19
- Natural philosophy shifted towards a focus on (conservation of) energy - in particular heat[8]
- Thomson follows Newtonian ideals - refusing to assign hypotheses
- There exist "absolute numerical relations" between heat and power[9]

[8] Smith, 1978.
[9] Thomson, 1851.

James Maxwell (1831-1879)

Electromagnetism

- Faraday had done groundbreaking work on electromagnetism, but almost entirely qualitative
- Maxwell reticent to attribute any physical cause; *Faraday's Lines*[10] makes no claim of physicality
- *Dynamical Theory*[11] made little note of aether as it was mathematically unnecessary
- *Physical Lines*[12] - first postulates light being electromagnetic, as well as displacement current

[10] Maxwell, 1856.
[11] Maxwell, 1865.
[12] Maxwell, 1861.

Maxwell & Vectors

- Greatly develops ~~vector~~ calculus out of necessity for his mechanical models - Maxwell didn't have vector notation at his disposal!
- Modern formulation only introduced in 1884, by Heavyside[13]
- Vector notation itself only introduced in 1843, by Hamilton[14]

[13] Hunt, 2012.

[14] Crowe, 1994.

Aether Theory & Special Relativity

The First Cracks

- Following Maxwell's work, electromagnetism became dominant[15]
- The nature of the aether [iv] was hotly debated
- Lorentz develops ideas of 'local time' & 'corresponding states' (co-ordinate transforms - *but only of EM waves*[16])

[15] Hunt, 1994.

[16] Brown, 2005.

[iv] In fact, Newton's initial work necessitated a vacuum in space

Henri Poincaré (1854-1912)

- Develops theory to be (functionally) identical to modern Lorentz transformations[17]
- Willing to ignore aether hypotheses, as mathematically unnecessary[18]
  - Unwilling to fully commit to ideas: "Of hypotheses there is never lack"[19]
- Einstein would soon go on to have his *annus mirabilis* and completely shift away from aether theory

[17] Darrigol, 1995.
[18] Poincaré, Blondin, and Neculcea, 1901.
[19] Poincaré and Halsted, 1929.

Conclusion

To Sum Up...

- Initial shift with Newton's development of *experimental philosophy* and quantisation of nature
- Development of continental force-based physics
- Shift towards disciplinary rigour with Thomson and others
- Maxwellian development of electromagnetic theory
- The final steps away from the aether - after thousands of years

Thank You

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