Fluxions, Forces, and Fields:

An overview of the mathematisation of physics in Europe through the modern period

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Tracing Back

- Let's take a convenient starting point: Einstein, special relativity, & aether theory

- Much of the groundwork had been laid by Lorentz, with his *Theory of Corresponding States*[1]

- Much of the work Lorentz built on was done by George Fitzgerald, who was also influenced greatly by Maxwell

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[1] Janssen and Stachel, 2004.

Beyond Maxwell

- 'Mentored' by William Thomson[2] (later Lord Kelvin)... who was a major figure in shaping physics (or natural philosophy) in C19

- Physics in Thomson's day had been centred around energy (and thus dynamics)...

- ...Which followed from continental development furthered by figures such as Lagrange and Laplace...

- ...Who worked using methods derived from Newton's work on celestial motion

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[2]Smith, 1978.

The Birth of an Era: Isaac Newton

(1643-1727)
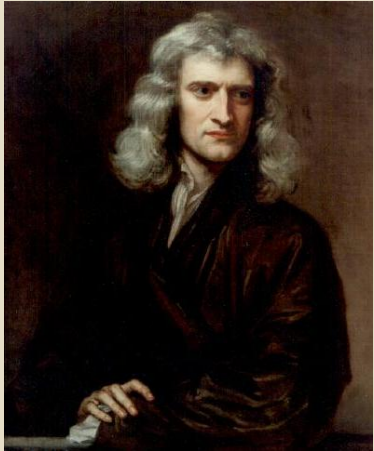
Newtonian Mathematical Ideals

- Saw God as mathematical, with a fondness for geometry
- Fluxions & Fluents[i]
- Clashes with Cartesianism[3]

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[3]Westfall, 1981.

[i]For reading on notational impact, see: Guicciardini, 1989; Guicciardini, 2004; Boyer, 2012

Experimental Philosophy

- The method of resolution & composition:
    1. Observe and "come to the general properties of things"[4]
    2. Assume said properties and describe further phenomena
- Hypotheses non fingo[5] - ignorance is acceptable!
- Nature able to be quantised (e.g. fluxional calculus & Halley's Comet)
    - Concept of forces at a distance

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[4]Newton and McGuire, 1970.
[5]Newton, Cohen, et al., 1999.

The Dynamical Age: Pierre-Simon Laplace

(1749-1827)

Laplacian Physics

- The rise of French scientific dominance
- Physics spearheaded by Laplace (1749-1827) in early C19, who took an "astronomical view of nature"[6]
- Prevalence of central-force based models, for even minute scales[7]
- Competitions set-up with e.g. the Society of Arcueil to promote mathematical collaboration
- Development of light, heat, and electromagnetic theory with various contestants (e.g. Fourier) - via Laplacian methods

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[6]Merz, 1896.
[7]Fox, 1974.

Heating Up:
William Thomson

(1824-1907)
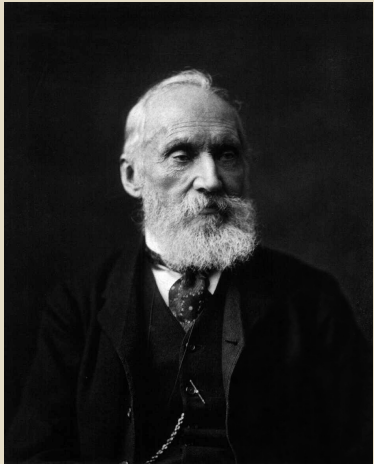
Thomson & Energy

- Ideas of *vis-viva* around since Leibniz, *caloric* furthered by e.g. Carnot in early C19

- Natural philosophy shifted towards a focus on (conservation of) energy - in particular heat[8]

- Thomson follows Newtonian ideals - refusing to assign hypotheses

- There exist "absolute numerical relations" between heat and power[9]

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[8]Smith, 1978.
[9]Thomson, 1851.

**Models and
Mathematics:
James Clerk Maxwell**

(1831-1879)
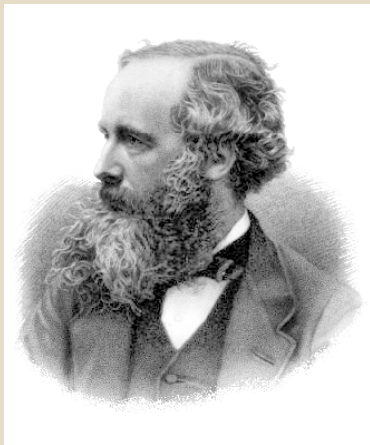
The Birth of Electromagnetism

- Faraday had done groundbreaking work on electromagnetism, but almost entirely qualitative

- Maxwell reticent to attribute any physical cause; *Faraday's Lines*[10] makes no claim of physicality

- *Dynamical Theory*[11] made little note of aether as it was mathematically unnecessary

- *Physical Lines*[12] - first postulates light being electromagnetic, as well as displacement current

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[10]Maxwell, 1856.
[11]Maxwell, 1865.
[12]Maxwell, 1861.

Maxwell & Vectors

- Greatly develops ~~vector~~ calculus out of necessity for his mechanical models - Maxwell didn't have vector notation at his disposal!
- Modern formulation only introduced in 1884, by Heavyside[13]
- Vector notation itself only introduced in 1843, by Hamilton[14]

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[13]Hunt, 2012.
[14]Crowe, 1994.

**Transformations &
Time:
Henri Poincaré**

(1854-1912)

The First Cracks

- Following Maxwell's work, electromagnetism became dominant[15]

- The nature of the aether was hotly debated

- Lorentz develops ideas of 'local time' & 'corresponding states' (co-ordinate transforms - *but only of EM waves*[16])

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[15]Hunt, 1994.
[16]Brown, 2005.

Developing Physicality

- Develops theory to be (functionally) identical to modern Lorentz transformations[17]

- Willing to ignore aether hypotheses, as mathematically unnecessary[18]
    - Unwilling to fully commit to ideas: "Of hypotheses there is never lack"[19]

- Einstein would soon go on to have his *annus mirabilis* and completely shift away from aether theory

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[17]Darrigol, 1995.
[18]Poincaré, Blondin, and Neculcea, 1901.
[19]Poincaré and Halsted, 1929.

Conclusion

To Sum Up...

- Initial shift with Newton's development of *experimental philosophy* and quantisation of nature
- Development of continental force-based physics
- Shift towards disciplinary rigour with Thomson and others
- Maxwellian development of electromagnetic theory
- The final steps away from the aether - after thousands of years

Thank You

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