Electric Dipoles

Analysing force and potential

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Important Equations

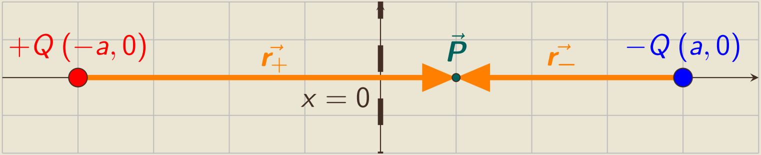
$$W = F \cdot s \cdot \cos\theta \tag{1}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_{source}q_{test}}{r^2}\hat{r} = \vec{E}q \tag{2}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}\,(V/m) \tag{3}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}\,(V) = \frac{W}{q} \tag{4}$$

The Task



What is the:

a) Force on a particle of charge q located at point $\vec{P}$ on the x axis, as $\vec{P}$ varies?

b) Potential at point $\vec{P}$ (if V at $\infty = 0$) as $\vec{P}$ varies?

a) The Force at $\vec{P}$

$$\vec{\boldsymbol{E}}_+ = \frac{1}{4\pi\epsilon_0}\frac{(+Q)}{r_+^2}\hat{\boldsymbol{r}}_+$$

but we are only considering the x axis so:

$$\vec{\boldsymbol{E}}_+ = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_+^2}\hat{\boldsymbol{x}}$$

$$\rightarrow \vec{\boldsymbol{F}}_+ = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r_+^2}\hat{\boldsymbol{x}}$$

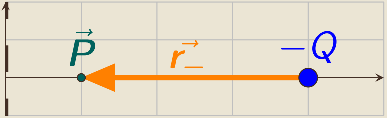
a) The Force at $\vec{P}$

now for $\vec{E}_-$:

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{r_-^2}\hat{r}_-$$

$\rightarrow$ where does $\hat{r}_-$ point?

a) The Force at $\vec{P}$



$$\rightarrow \hat{r_-} = -\hat{x}$$

a) The Force at $\vec{P}$

so:

$$\vec{E_-} = \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{r_-^2}(-\hat{x})$$

$$\vec{E_-} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_-^2}\hat{x}$$

$$\rightarrow \vec{F_-} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r_-^2}\hat{x}$$

The Force at $\vec{P}$

$$\begin{aligned}\vec{\boldsymbol{F}_{\boldsymbol{total}}} &= \vec{\boldsymbol{F}}_+ + \vec{\boldsymbol{F}}_- \\ &= \frac{Qq}{4\pi\epsilon_0}\left(\frac{1}{r_+^2} + \frac{1}{r_-^2}\right)\hat{\boldsymbol{x}}\ (N) \\ &\neq 0 \text{ at origin!}\end{aligned}$$

The Potential at $\vec{P}$

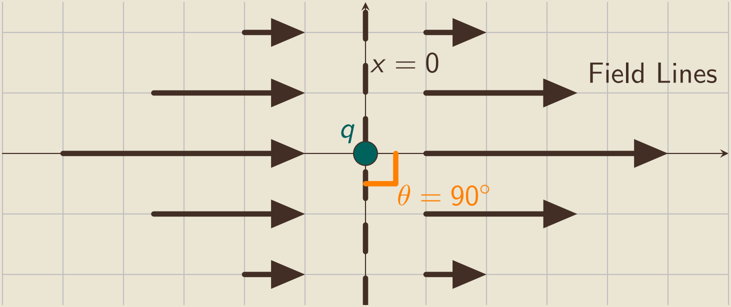
$$V_+ = \frac{1}{4\pi\epsilon_0}\frac{(+Q)}{r_+} \qquad V_- = \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{r_-}$$

The Potential at $\vec{P}$

$$V_{total} = V_{+} + V_{-} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_{+}} - \frac{1}{r_{-}}\right)\ (V)$$

$\rightarrow$ **is** 0 at origin!

Why Potential is 0 at the Origin



Recall that $W = F \cdot s \cdot \cos\theta$; but here $\cos\theta = \cos 90 = 0$, so:

$$V = \frac{W}{q} = \frac{F \cdot s \cdot 0}{q} = 0 \text{ along } x = 0!$$