Fluxions, Forces, and Fields

Zella Baig

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This document serves as a brief overview of the mathematisation of physics in the modern period, and largely develops the ideas given in the associated presentation.

Pre-Newtonianism

Instruments

To effectively discuss the impact which Newton had on the field of natural philosophy, we must first put into context the state of affairs preceding him. Perhaps unsurprisingly, the early modern period was an age of discovery - the time frame in which we are set immediately precedes the Industrial Revololution, for example, and as such one would expect there to be mechanisms at play as to enable these societal shifts to occur. Nevertheless, I digress.

The first major shift which we must analyse is that of instrumentisation. Natural philosophy before this period had largely followed an Aristoelian (and religious) doctrine: the Heavens were perfect, and we as humans were but passive observers of God's cosmos. Astronomy was a popular study, as indeed it had been for millenia. Royal and holy obervatories were not uncommon, and the study of the behaviour of the celestial bodies was perhaps the most esteemed role a natural philosopher could have, within the mathematical disciplines of course.

Thus, it follows that the key advance in this era which we discuss is the telescope, and indeed the lense itself. And when discussion the telescope, it would be remiss to leave out Galileo. Galileo was vital for societal shifts in natural philosophy for several reasons: casting doubt on scripture with his observations of decidedly imperfect phenomena (sunspots), yes, but also for demonstrating how passive observation led to ignorance. This all ties in, of course, with Copernicanism: Galileo's observations of Venus' & the Moon's phases lended great support to the theory that the Earth was 'just' a planet. Copernicus' bastardised form of natural philosophy - one which employed *mathematics* as well as *measurements* was suddenly something which might have seemed appealing. And indeed, it was this very sort of natural-philosophising which Newton would come to employ with the aid of Flamsteed in order to develop his *Principia*.

Baconism

Somewhat (as you shall soon see) paradoxically, when discussing the mathematics of this era, we must now shift to a look at a natural philosopher who was himself no great mathematician, and in fact could be seen as pushing mathematics away in favour of what we now refer to as *the Baconian Method*; in essence inductive reasoning. Bacon's work came from views in which he saw that Nature should be interrogated, and that knowledge would only come from actively investigating. The way in which he implemented this was by drafting tables of scenarios in which some 'effect' to be investigated occured, scenarios in which it did not occur, and lastly scenarios in which the effect would vary. By comparison one could then eliminate ideas as the true forms of those effects.

This is perhaps best illustrated with an example. One might wish to investigate heat, and for scenarios in which it might occur, let us list 'fire', and 'a pot of boiling water'. Now, the flame exists without any water, and similarly the water may retain heat without any light - and so we may deduce that 'heat' does not arise from either water, or light.

be converted to sidenotes.[1] If you'd like to place ancillary information in the margin without the sidenote mark (the superscript number), you can use the `\marginnote` command.

The specification of the `\sidenote` command is:

`\sidenote[`⟨*number*⟩`][`⟨*offset*⟩`]{`*Sidenote text.*`}`

Both the ⟨*number*⟩ and ⟨*offset*⟩ arguments are optional. If you provide a ⟨*number*⟩ argument, then that number will be used as the sidenote number. It will change of the number of the current sidenote only and will not affect the numbering sequence of subsequent sidenotes.

Sometimes a sidenote may run over the top of other text or graphics in the margin space. If this happens, you can adjust the vertical position of the sidenote by providing a dimension in the ⟨*offset*⟩ argument. Some examples of valid dimensions are:

```
1.0in    2.54cm    254mm    6\baselineskip
```

If the dimension is positive it will push the sidenote down the page; if the dimension is negative, it will move the sidenote up the page.

While both the ⟨*number*⟩ and ⟨*offset*⟩ arguments are optional, they must be provided in order. To adjust the vertical position of the sidenote while leaving the sidenote number alone, use the following syntax:

`\sidenote[][`⟨*offset*⟩`]{`*Sidenote text.*`}`

The empty brackets tell the `\sidenote` command to use the default sidenote number.

If you *only* want to change the sidenote number, however, you may completely omit the ⟨*offset*⟩ argument:

`\sidenote[`⟨*number*⟩`]{`*Sidenote text.*`}`

[1] This is a sidenote that was entered using the `\footnote` command.

This is a margin note. Notice that there isn't a number preceding the note, and there is no number in the main text where this note was written.

The `\marginnote` command has a similar *offset* argument:

`\marginnote[`⟨*offset*⟩`]{`*Margin note text.*`}`

References

References are placed alongside their citations as sidenotes, as well. This can be accomplished using the normal `\cite` command.[2]

The complete list of references may also be printed automatically by using the `\bibliography` command. (See the end of this document for an example.) If you do not want to print a bibliography at the end of your document, use the `\nobibliography` command in its place.

To enter multiple citations at one location,[3] you can provide a list of keys separated by commas and the same optional vertical offset argument: `\cite{Tufte2006,Tufte1990}`.

`\cite[`⟨*offset*⟩`]{`*bibkey1*`,`*bibkey2*`,...}`

[2] The first paragraph of this document includes a citation.

[3] **Tufte2006**; **Tufte1990**.

Figures and Tables

Images and graphics play an integral role in Tufte's work. In addition to the standard figure and tabular environments, this style provides special figure and table environments for full-width floats.

Full page–width figures and tables may be placed in figure* or table* environments. To place figures or tables in the margin, use the marginfigure or margintable environments as follows (see figure 1):

```
\begin{marginfigure}
  \includegraphics{helix}
  \caption{This is a margin figure.}
\end{marginfigure}
```

Figure 1: This is a margin figure. The helix is defined by $x = \cos(2\pi z)$, $y = \sin(2\pi z)$, and $z = [0, 2.7]$. The figure was drawn using Asymptote (`http://asymptote.sf.net/`).

The marginfigure and margintable environments accept an optional parameter ⟨*offset*⟩ that adjusts the vertical position of the figure or table. See the "**??**" section above for examples. The specifications are:

...

...

Figure 2 is an example of the `figure*` environment and figure 3 is an example of the normal `figure` environment.

Figure 3: Hilbert curves of various degrees n. *Notice that this figure only takes up the main textblock width.*

Figure 2: This graph shows $y = \sin x$ from about $x = [-10, 10]$. *Notice that this figure takes up the full page width.*

Table 1 shows table created with the `booktabs` package. Notice the lack of vertical rules—they serve only to clutter the table's data.

Margin	Length
Paper width	8 1/2 inches
Paper height	11 inches
Textblock width	6 1/2 inches
Textblock/sidenote gutter	3/8 inches
Sidenote width	2 inches

Table 1: Here are the dimensions of the various margins used in the Tufte-handout class.

Full-width text blocks

In addition to the new float types, there is a fullwidth environment that stretches across the main text block and the sidenotes area.

```
\begin{fullwidth}
Lorem ipsum dolor sit amet...
\end{fullwidth}
```

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Typography

Typefaces

If the Palatino, Helvetica, and `Bera Mono` typefaces are installed, this style will use them automatically. Otherwise, we'll fall back on the Computer Modern typefaces.

Letterspacing

This document class includes two new commands and some improvements on existing commands for letterspacing.

When setting strings of ALL CAPS or SMALL CAPS, the letterspacing—that is, the spacing between the letters—should be increased slightly.[4] The `\allcaps` command has proper letterspacing for strings of FULL CAPITAL LETTERS, and the `\smallcaps` command has letterspacing for SMALL CAPITAL LETTERS. These commands will also automatically convert the case of the text to upper- or lowercase, respectively.

The `\textsc` command has also been redefined to include letterspacing. The case of the `\textsc` argument is left as is, however. This allows one to use both uppercase and lowercase letters: THE

[4] **Bringhurst2005**.

Initial Letters Of The Words In This Sentence Are Capitalized.

Installation

To install the Tufte-LaTeX classes, simply drop the following files into the same directory as your `.tex` file:

```
tufte-book.cls
tufte-common.def
tufte-handout.cls
tufte.bst
```

More Documentation

For more documentation on the Tufte-LaTeX document classes (including commands not mentioned in this handout), please see the sample book.

Support

The website for the Tufte-LaTeX packages is located at `https://github.com/Tufte-LaTeX/tufte-latex`. On our website, you'll find links to our SVN repository, mailing lists, bug tracker, and documentation.

References

Bacon, Francis (1620). *Novum organum Scientiarum*. Londini.

Boyer, Carl B. (2012). *The History of the Calculus and Its Conceptual Development*. Courier Corporation.

Brown, Harvey R. (2005). *Physical Relativity: Space-Time Structure from a Dynamical Perspective*. Oxford University Press.

Clavius, Christopher (1612). *Opera Mathematica*. Vol. 3, p. 2.

Clifford, W.K. and R. Tucker (1878). *Elements of Dynamic: An Introduction to the Study of Motion and Rest in Solid and Fluid Bodies*. v. 1-3. MacMillan and Company.

Craik, Alex D. D. (1999). "Calculus and Analysis in Early 19th-Century Britain: The Work of William Wallace". In: *Historia Mathematica* 26.3.

Crowe, Michael J. (1994). *A History of Vector Analysis: The Evolution of the Idea of a Vectorial System*. Courier Corporation.

Darrigol, Olivier (1995). "Henri Poincaré's Criticism of Fin De Siècle Electrodynamics". In: *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 26, pp. 1–44.

Fox, Robert (1974). "The Rise and Fall of Laplacian Physics". In: *Historical Studies in the Physical Sciences* 4, pp. 89–136.

Gooding, David (1981). *Final Steps to the Field Theory: Faraday's Study of Magnetic Phenomena, 1845-1850.* Berkeley.

Gooding, David and Frank A. J. L. James (1985). *In Nature's School': Faraday as an Experimentalist*. London: Macmillan Education UK, pp. 105–136.

Greenberg, John L. (1986). "Mathematical Physics in Eighteenth-Century France". In: *Isis* 77.1, pp. 59–78.

Guicciardini, Niccolò (1989). *The Development of Newtonian Calculus in Britain, 1700–1800*. Cambridge: Cambridge University Press.

– (2004). "Dot-Age: Newton's Mathematical Legacy in the Eighteenth Century". In: *Early Science and Medicine* 9.3, pp. 218–256.

Hunt, Bruce J. (1994). *The Maxwellians*. Cornell University Press.

– (2012). "Oliver Heaviside: A First-Rate Oddity". In: *Physics Today* 65.11, pp. 48–54.

Janssen, Michel and John J. Stachel (2004). *The Optics and Electrodynamics of Moving Bodies*. Max-Planck-Institute for the History of Science.

Maxwell, James Clerk (1856). "On Faraday's Lines of Force." In: Cambridge: Cambridge University Press.

– (1861). *On Physical Lines of Force*. London: Royal Society.

– (1865). "A Dynamical Theory of the Electromagnetic Field". In: *Philosophical Transactions of the Royal Society of London* 155, pp. 459–512.

– (1873). *A Treatise on Electricity and Magnetism*. Oxford: Clarendon Press.

– (1877). *Matter and Motion*. Dover, pp. 89–90.

Maxwell, James Clerk and W. D Niven (1965). *The Scientific Papers of James Clerk Maxwell*. New York: Dover Publications.

Merz, J.T. (1896). *A History of European Thought in the Nineteenth Century*. A History of European Thought in the Nineteenth Century. Blackwood & Sons: London, Pp. xiv+, 458.

Newton, Isaac (1670–1710). *MS Add.f. 243r*. Portsmouth Collection.

Newton, Isaac, Bernard Cohen, et al. (1999). *The Principia: Mathematical Principles of Natural Philosophy*. 3rd ed. University of California Press, p. 943.

Newton, Isaac and J. E. McGuire (1970). "Newton's "Principles of Philosophy": An Intended Preface for the 1704 "Opticks" and a

Related Draft Fragment". In: *The British Journal for the History of Science* 5, pp. 178–186.
Newton, Isaac and Derek T. Whiteside (1967–1981). *The Mathematical Papers of Isaac Newton*. Vol. 7. Cambridge.
Poincaré, Henri, Jules Blondin, and Eugene Neculcea (1901). *Electricité et Optique. La lumière et les Théories électrodynamiques (Sorbonne lectures of spring 1888, 1890, and 1899*, pp. 403–420.
Poincaré, Henri and George Bruce Halsted (1929). *The Foundations of Science: Science and Hypothesis, the Value of Science, Science and Method*. Science Press, p. 147.
Rees, Graham (1986). "Mathematics And Francis Bacon's Natural Philosophy". In: *Revue Internationale de Philosophie* 40 (159 (4)), pp. 399–426.
Rindler, Wolfgang (1970). "Einstein's Priority in Recognizing Time Dilation Physically". In: *American Journal of Physics* 38, pp. 1111–1115.
Shapiro, Alan E. (2004). "Newton's "Experimental Philosophy"". In: *Early Science and Medicine* 9.3, pp. 185–217.
Smith, Crosbie (1978). "A New Chart for British Natural Philosophy: The Development of Energy Physics in the Nineteenth Century". In: *History of Science*, pp. 231–279.
Tait, Peter Guthrie (1861). *Letter from Tait to Thomson, 12 December 1861*. Kelvin Papers.
Thomson, William (1851). *On the Dynamical Theory of Heat, with Numerical Results Deduced from Mr Joule's Equivalent of a Thermal Unit, and M. Regnault's Observations on Steam.* Vol. 20. Transactions of the Royal Society of Edinburgh. 261-288.
Thomson, William and Peter Guthrie Tait (1867). *Treatise on Natural Philosophy*. Clarendon Press.
Westfall, Richard S. (1981). *Never at Rest: A Biography of Isaac Newton*. Cambridge: Cambridge University Press.