Electric Dipoles

Analysing force and potential

ZellaBaig

February 17, 2021

Important Equations

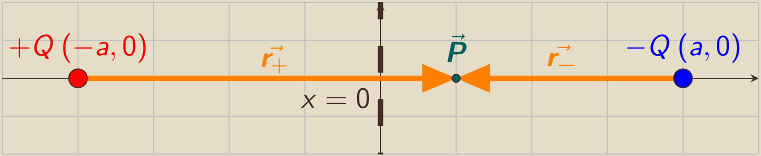
$$W = F \cdot s \cdot \cos\theta \tag{1}$$

$$\vec{\boldsymbol{F}} = \frac{1}{4\pi\epsilon_0}\frac{Q_{source}q_{test}}{r^2}\hat{\boldsymbol{r}} = \vec{\boldsymbol{E}}q \tag{2}$$

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}}\,(V/m) \tag{3}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}\,(V) = \frac{W}{q} \tag{4}$$

The Task



What is the:

a) Force on a particle of charge q located at point $\vec{P}$ on the x axis, as $\vec{P}$ varies?

b) Potential at point $\vec{P}$ (if V at $\infty = 0$) as $\vec{P}$ varies?

a) The Force at $\vec{P}$

$$\vec{\boldsymbol{E}}_{+} = \frac{1}{4\pi\epsilon_0}\frac{(+Q)}{r_{+}^{2}}\hat{\boldsymbol{r}}_{+}$$

but we are only considering the x axis so:

$$\vec{\boldsymbol{E}}_{+} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_{+}^{2}}\hat{\boldsymbol{x}}$$

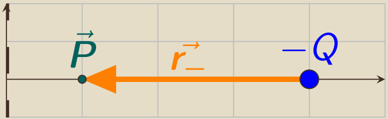
$$\rightarrow \vec{\boldsymbol{F}}_{+} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r_{+}^{2}}\hat{\boldsymbol{x}}$$

a) The Force at $\vec{P}$

now for $\vec{E}_-$:

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{r_-^2}\hat{r}_-$$

$\rightarrow$ where does $\hat{r}_-$ point?



$$\rightarrow \hat{r}_- = -\hat{x}$$

a) The Force at $\vec{P}$

so:

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{r_-^2}(-\hat{x})$$

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_-^2}\hat{x}$$

$$\rightarrow \vec{F}_- = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r_-^2}\hat{x}$$

The Force at $\vec{P}$

$$
\begin{aligned}
\vec{\boldsymbol{F_{total}}} &= \vec{\boldsymbol{F}}_{+} + \vec{\boldsymbol{F}}_{-} \\
&= \frac{Qq}{4\pi\epsilon_0}\left(\frac{1}{r_{+}^{2}} + \frac{1}{r_{-}^{2}}\right)\hat{\boldsymbol{x}}\ (N) \\
&\neq 0 \text{ at origin!}
\end{aligned}
$$

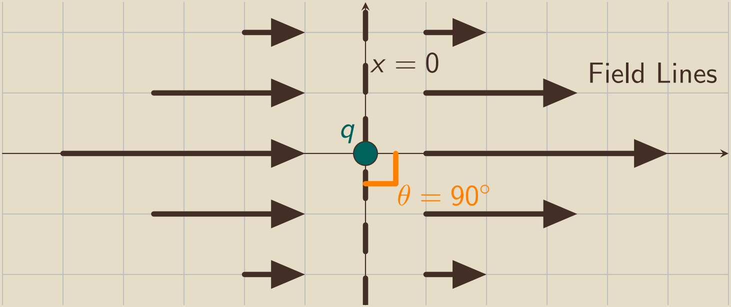
The Potential at $\vec{P}$

$$V_+ = \frac{1}{4\pi\epsilon_0}\frac{(+Q)}{r_+} \qquad V_- = \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{r_-}$$

$$V_{total} = V_+ + V_- = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)(V)$$

$\rightarrow$ **is** 0 at origin!

Why Potential is 0 at the Origin



Recall that $W = F \cdot s \cdot \cos\theta$; but here $\cos\theta = \cos 90 = 0$, so:

$$V = \frac{W}{q} = \frac{F \cdot s \cdot 0}{q} = 0 \text{ along } x = 0!$$