Orbital Mechanics for the non-mathematically inclined

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Contents

Chapter 1

Introduction

So for some reason you need to understand orbital mechanics, but you don't consider yourself a math wiz. I've been there. This book, despite the name, does include a great deal of math, but don't worry! I will teach you everything you need to know in order to make sense of it.

This book will focus on what is called a two-body problem. This will allow for a close approximation of the orbit of an object around a central body. Technically since all objects with mass exert gravity on all other objects with mass, the orbit of an object will be a little bit different, but for most cases this will work, and the math is significantly easier.

I assume in this book that you have a basic high-school level understanding of math. You should be able to use a graphing calculator and understand algebra. In many cases I have already solved equations for the variables that we are looking for, but if you want to know where these came from, you should be able to do the algebra to figure that out.

I want this book to be useful for anyone who needs it, so it is being made available under the Creative Commons Attribution Share-Alike license. You can see the back of the book for the legalese of what that means, but in a nutshell it means that you can make copies of this book to give to your friends for free, and you are also allowed to make changes to this book if you'd like!

If you have a little bit of money, I would greatly appreciate it if you'd be willing to purchase this book, but if you don't then you shouldn't be deprived of useful knowledge. If you'd like to make modifications to the book, please email me at *zellfaze@zellfaze.org*. I can send you the LaTeX source files for the book. If you have any suggestions or comments, you can also reach me at that email address.

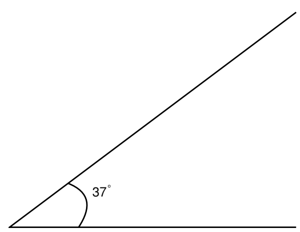
Part I

Required Background

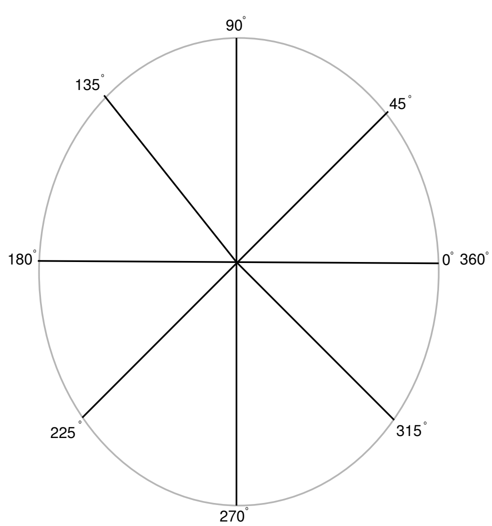
Chapter 2

Angles and Radians

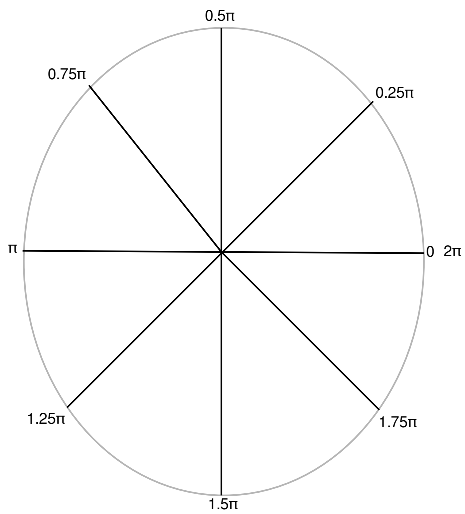
When most people think of angles they think of something that is measured in degrees.



Most people also would know that there are 360° in a circle



While this is a useful way to look at angles for people, for various reasons it is not a very good way to measure angles when working in a mathematical setting. Instead a unit known as Radians is used to measure angles. There are a total of 2π radians in a circle.



Thankfully, it is easy to convert between radians and degrees. The formula for that is:

$$degrees = \frac{radians}{\pi/180}$$

$$radians = degrees * \pi/180$$

For the rest of this document we will be using radians, not degrees, to measure angles. You can always refer back to the above formulas for converting between the two if needed.

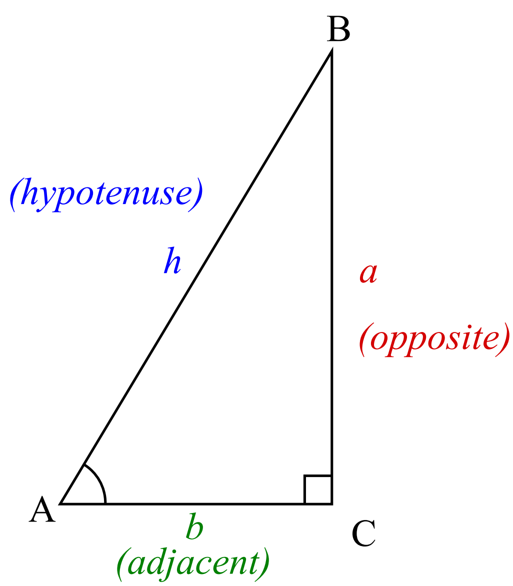
Chapter 3

Basic Trigonometry

A lot of orbital mechanics requires the use of some basic trigonometry to calculate. In this chapter we are going to cover the concepts required.

There are six different trigonometric functions that are you will need to know. These are sine, cosine, and tangent, along with arcsine, arccosine, and arctangent. These are usually abbreviated as sin, cos, tan, arcsin, arccos, and arctan respectively. You will also need to know the Pythagorean Theorem.

To explain these lets start by drawing a right triangle (a triangle where one of the angles is 0.5π radians).



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3.1 Calculating Angles

All of the angles in a triangle will always add up to π radians. Knowing this, if you know two of the angles of a triangle, you can always find the third.

$$\pi = A + B + C$$

3.2 Calculating Sides

The Pythagorean Theorem provides a similar formula for finding the length of any of the sides if you know the length of other two.

$$h^2 = a^2 + b^2$$

We can rearrange this equation for if we need to find a or b given h and the remaining side.

$$a = \sqrt{h^2 - b^2}$$

$$b = \sqrt{h^2 - a^2}$$

3.3 Trigonometric Functions

Alright, so we now know how to find the last angle in a triangle if we have the other angles, and how to find the last side length if we have the other side lengths. We also though need to be able to find angles given the length of a side and how long the sides are given some of the angles in a triangle. These are both found using the six trigonometric functions.

The first three of the trigonometric functions allow you to find the ratio between two of the sides of the triangle if you know one of the angles. In this case, we will assume that we know angle A, but you could change this to knowing angle B with a little reworking.

$$\sin A = \frac{a}{h}$$

$$\cos A = \frac{b}{h}$$

$$\tan A = \frac{a}{b}$$

Your calculator should be able to find the sin, cos, or tan for any given number. It is worth knowing that sin or cos values will always be between -1 and 1. A tan value will never be equal to $\pm\frac{x\pi}{2}$, where x is an odd number. Thus $\pm\frac{\pi}{2}$ and $\pm\frac{3\pi}{2}$ are invalid for tangents. Otherwise all numbers are valid for tangents. The reason for this is not really needed to continue so we won't be getting into it.

Somewhat more obviously useful is to take the side lengths and go the other direction. It is easy to find the ratio between two sides of a triangle and we can use that information to find an angle. To do this we use arcsin, arccos, and arctan which cancel out sin, cos, and tan respectively.

$A = \arcsin \frac{a}{h}$

$A = \arccos \frac{b}{h}$

$A = \arctan \frac{a}{b}$

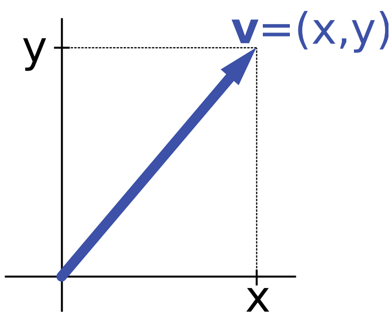
You now know enough trigonometry in order to understand orbital mechanics.

Chapter 4

Vectors

A vector is simply a line segment with a magnitude (aka a length) and a direction (which is just an angle). You can define a vector with its magnitude and direction, but in many cases it is easier to define a vector in terms of coordinates instead. These coordinates are often called the x and y components of the vector.

For example, a vector that is defined as {10,15} is a line segment from {0,0} to the coordinate {10,15}. Since vectors are technically just a magnitude and a direction, all vectors defined by coordinates originate at {0,0}.



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The direction of the vector is the angle made by the vector line and the x axis. The magnitude is the length of the vector line.

4.1 Comparing Vectors

Two vectors are considered to be equal if their direction and magnitude are the same. Vectors that are opposite of each other have the same magnitude, but are pointed in the exact opposite directions. Parallel vectors have the same direction, but different magnitudes.

4.2 Finding Magnitude and Direction

You can find the magnitude of a vector from its coordinates with this formula:

$magnitude = \sqrt{x^2 + y^2}$

You can find the direction of a vector from its coordinates with this formula:

$direction = \arctan \frac{y}{x}$

4.3 Vector Arithmetic

You can perform a number of operations on vectors. We will be covering the most common types.

4.3.1 Basic Operations

Vectors can be added together by adding their coordinates together. For example:

$\{5, 5\} = \{1, 3\} + \{4, 2\}$

Subtraction works in the exact same way.

$\{4, 2\} = \{5, 5\} - \{1, 3\}$

You can multiple a vector by a number (also sometimes called a scalar) by multiplying each of the vectors components with the number.

$\{4, 8\} = \{2, 4\} * 2$

Division works in the exact same way.

$\{2, 4\} = \{4, 8\}/2$

4.3.2 Dot Product

A dot product is sort of like multiplication for two vectors, except that it leaves you with a single number when you are done instead of a new vector. It is relatively easily done.

Let's say that you have two vectors we'll call them V_1 and V_2. We will define them as follows.

$V_1 = \{2, 4\}$

$V_2 = \{3, 5\}$

You can calculate their dot product by multiplying their components together and then adding the resulting components. See the following example:

$V_1 \bullet V_2 = (2 * 3) + (4 * 5) = 26$

More generally you could define it as:

$V_1 \bullet V_2 = (X_1 * X_2) + (Y_1 * Y_2)$

4.3.3 Cross Product

Like dot products, cross products could also be considered something like multiplication. Unlike dot products, the cross product of two vectors is a third vector. Cross products can't be done on 2D vectors, so for the first time, we will be using a 3D vector. Things work just the same for 3D vectors, just we have an additional coordinate. (They also have an additional direction angle, but we won't need that here.)

You can calculate their cross product using the following formula:

$V_1 \times V_2 =$
$X_3 = Y_1 * Z_2 - Z_1 * Y_2$
$Y_3 = Z_1 * X_2 - X_1 * Z_2$
$X_3 = X_1 * Y_2 - Y_1 * X_2$

How about an example? Let's define two vectors again. Again we'll call them V_1 and V_2.

$V_1 = \{2, 4, 6\}$
$V_2 = \{3, 5, 7\}$

$V_1 \times V_2 =$
$X_3 = 4 * 7 - 6 * 5 = -2$
$Y_3 = 6 * 3 - 2 * 7 = 4$
$X_3 = 2 * 5 - 4 * 3 = -2$

$V_1 \times V_2 = \{-2, 4, -2\}$

One of the cool things about cross products is their relationship to each other. They are sort of like the angles or sides of a triangle in that if you know two of them, you can always find the third. They have the following relationship:

$V_3 = V_1 \times V_2$
$V_2 = V_1 \times V_3$
$V_1 = V_2 \times V_3$

Part II

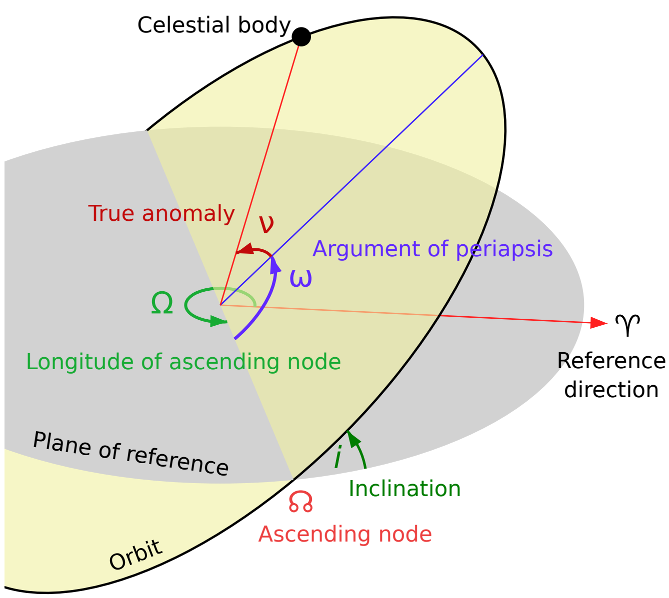
Orbital Mechanics

Chapter 5

Keplerian Orbital Elements

There are many ways to define the orbit of an object, but the one that allows one to most easily see the shape of the orbit itself are the Keplerian Orbital Elements. The other methods of defining an orbit's shape can all be calculated from these, so it is with these that we will start.

Below I have included a diagram showing some of the orbital elements. This is provided so that you may get a general summary of what is to follow.



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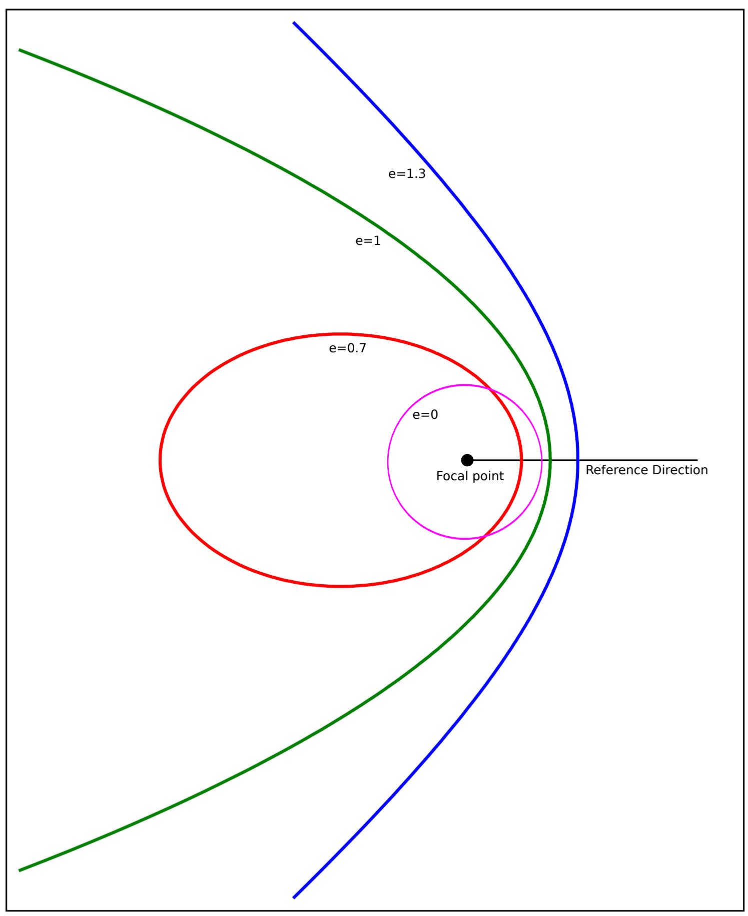
5.1 Eccentricity

Eccentricity is the orbital element that describes the shape of the curve that the orbit takes. It describes the shape of the orbit, but not the orbit's position, the position of the central body, or the position of the orbiting object.

The eccentricity of an orbit can never be negative. An eccentricity of 0 describes an orbit that is perfectly circular. The higher the eccentricity the less like a circle and the more like a line the orbit is.

An eccentricity value greater than 0 but less than 1 is an ellipse. If eccentricity is 1 then it is a parabola, and if eccentricity is greater than 1 then it is a hyperbola. Parabolic and hyperbolic orbits do not form a closed loop.



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5.2 Semi-Major Axis

Along with the eccentricity, the semi-major axis also describes the shape of an orbit. In a circular orbit the semi-major axis is the radius of the circle. In an elliptical orbit, it is half the distance between the two farthest points. In a parabolic or hyperbolic orbit, it is the distance between the central body and the closest approach of the orbiting body.

5.3 Inclination

5.4 Longitude of the Ascending Node

5.5 Argument of Periapsis

5.6 The Anomalies

5.6.1 Mean Anomaly

5.6.2 True Anomaly

5.6.3 Eccentric Anomaly