Managing Large Scale, Ultra-Dense Beacon Deployments in Smart Campuses

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***Abstract*—The demand for proximity-based services for mobile device users inside smart buildings has driven a surge in the installation of large-scale, ultradense Bluetooth Low Energy beacon deployments. But managing a large-scale beacon infrastructure presents new systems challenges, including managing battery life, replacing misplaced or stolen beacons, detecting and locating foreign or malicious beacons, and engineering beacon locations to optimize wireless resources. We study two problems whose solutions facilitate large-scale beacon network management. First, we develop a prototype system to demonstrate how conventionally static beacon advertisements can be made time-varying, and argue that time-varying advertisements can convey important device health and network state information. We then consider a fundamental question in site engineering; calculating upper limits on the spatial density of deployed beacons to optimize either the number of advertisements received, or mobile users reached. Using analysis and simulation, we show that with current wireless standards beacon separation should be at least 0.5 meters per advertisement channel. Knowledge of this bound facilitates the construction of topology design rules, and quantifies previously unknown limits on beacon density in ultra-dense deployments.**

I. INTRODUCTION

Beacons are coin-size wireless devices that transmit a short-range Bluetooth Low Energy (BLE) signal to mobile computing devices (e.g., smartphones). Device users are notified of beacon proximity when they move within the wireless device range (e.g., 10 m.). Device applications receive beacon transmissions and present contextual information, typically describing an indoor physical space and its contents. Since attaching beacons to many objects is simple and relatively inexpensive, beacons promise to serve as an enabling technology for the Internet of Things (IoT). Beacons are already enabling new location-based applications, with hundreds of coordinated devices spanning smart buildings and campuses including 'big-box' retailers and airport terminals [1].

Beacons support proximity-based applications and services by periodically broadcasting a short, fixed message with a globally unique identifier using Bluetooth 4.x transmitters [2]. A receiving application typically performs actions such as playing sounds or displaying images, or accessing specific URLs based on a user's proximity to a beacon. The beacon's communication is entirely one-way, protecting mobile user presence and data privacy. As one illustrative application, an airline can issue a gate change notice to travelers near a gate, and later issue a confirmation to those near the new gate, without needing to locally re-configure any beacon hardware, software or physical location.

Beacons are quickly becoming very inexpensive (e.g., US $5-10 [3]), and we anticipate that widespread proliferation is on the horizon. But there is currently a lack of tools and experience to help engineer and manage large-scale deployments (e.g., perhaps thousands installed in seats in a sports stadium [4]), and ad hoc management is quickly becoming untenable. Our goal is to facilitate network management through both device level changes and improved understanding of dense beacon network operation.

- We have developed a proof-of-concept of a *Dynamic* Beacon that implements time-variable announcements. Existing beacons simply repeat the same broadcast at pre-programmed time intervals. Time-variable announcements are crucial for maintaining the integrity of large-scale networks, such as detecting and reporting devices whose batteries are failing. Furthermore, we envision that future beacons will support additional computing and sensing capability, and should be able to report internal states or measurements (e.g. temperature, motion, and ambient noise), and be able to assist network operations through self-management and self-reporting.
- We have developed topology design rules to efficiently organize large-scale beacon deployments. Using analysis and simulation, we have determined bounds on the maximum useful density of beacons sharing a physical space. If beacons are deployed too sparsely then a mobile device user will observe fewer beacons than the device is capable of decoding, however, if the deployment is too dense, then transmissions collide and prevent reliable detection of each transmission.
- We have developed an analytical model of idealized networks with perfect transmission scheduling to find the upper-bound on beacon 'crowding' such that each user can 'hear' each beacon in range. For regular 2D beacon topologies (e.g., a rectangular grid) using a single advertisement channel, we have determined that the minimum beacon separation must be at least 0.5 meters.

The rest of the paper is organized as follows. Section II reviews the basics of beacon broadcasts. In Section III we motivate and describe our prototype implementation of a Dynamic Beacon. We explore the limits of beacon density in Section IV through analysis and simulation. We discuss related work in Section V, and discuss future research directions in Section VI.

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II. Fundamentals of Beacon Broadcasts

Each beacon periodically transmits up to 10 advertisements/sec on each of BLE's three 2 MHz wide advertisement channels in the ISM band. Fig. 1 shows the beacon packet structure; the Protocol Data Unit (PDU) is delineated into 5 distinct fields. All fields –except the transmission power– are statically assigned for a given device.

iBeacon Prefix (fixed) (9B)	Proximity UUID (< 16B)	Major (2B)	Minor (2B)	TxPwr (2B)

Fig. 1. The Apple iBeacon PDU format.

Apple's beacon – known as *iBeacon* – further specifies the structure of each PDU (or packet) as follows:

- **iBeacon Prefix**: The iBeacon prefix is a fixed set of 9 bytes, reserved by Apple to signify that the BLE advertisement packet is an iBeacon packet.
- **Proximity UUID**: The Proximity Universally Unique Identifier (UUID) consists of up to 16 bytes, recommended to serve as an organizational identifier. For example, a specific UUID (or set of UUIDs) may correspond to a retail chain.
- **Major**: The 2 byte Major field is used to group a set of beacons, perhaps by region or location (e.g., a specific store location).
- **Minor**: The 2 byte Minor field is used to uniquely identify a specific beacon (e.g., a store aisle).
- **TxPwr**: The beacon's manufacturer uses the 2 byte Transmit Power (TxPwr) field to broadcast the average Received Signal Strength Indication (RSSI) at 1 meter distance from the device. This value can be used by a receiver to calculate its approximate distance from the Beacon.

III. Dynamic Broadcasts

Nearly all existing beacons simply repeat the same broadcast periodically; earlier generations of beacons could not be reprogrammed. This operation is adequate for many applications and settings, since application updates or backend content can be modified to alter contextual information presented to mobile devices. Reconfiguring a beacon's advertisement to a different fixed advertisement typically requires an administrator to move within range of the device and use a specialized Over-The-Air (OTA) management application [3]. As a result, changing device advertisements is a rare administrative action.

We are aware of no beacons currently on the market that broadcast messages that vary with time. However, we envision that future Beacons will likely support both additional on-board intelligence and sensing capabilities (e.g., measure temperature, pollution, microphone, battery life, etc), and communicating such local measurements will facilitate large-scale beacon network management. Although these features may be energy-intensive and reduce battery life, beacon devices are designed to be deployed using small coin-cell batteries that can easily be replaced: even with dynamic updates, beacon advertisements still follow the low-energy and infrequent transmission schedules of BLE designed to prolong device lifetime. To demonstrate the value of time-varying beacon advertisements, we developed a proof-of-concept of a dynamic beacon using a Raspberry Pi single-board computer.

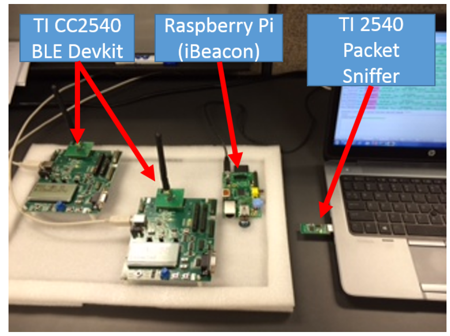


Fig. 2. Dynamic Beacon proof-of-concept testbed.

Fig. 2 shows our experimental testbed comprising a Raspberry Pi with BLE USB dongle programmed to act as an iBeacon, and a TI 2540 Bluetooth Packet Sniffer dongle to receive dynamic advertisements. We sought to verify that the advertisement fields could be updated at the fastest update rate with the following simple counting algorithm:

Algorithm 1 Dynamic Updates to Beacon Packets

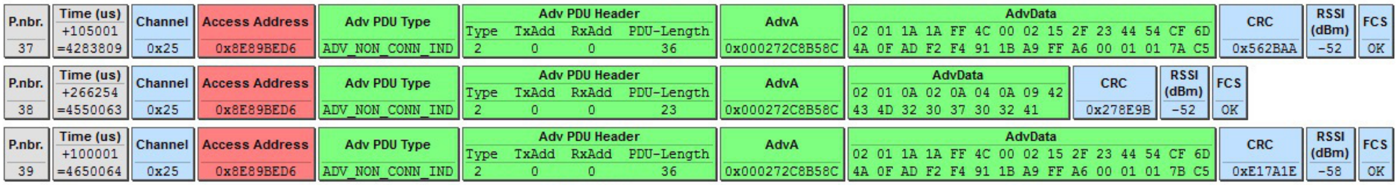
while running **do**
  $minor \leftarrow minor + 1$
  if $minor = 0$ **then**
    $major \leftarrow major + 1$
  end if
  sleep for $[100, 1000]$ ms.
end while

Note that our algorithm permits time-varying control of both advertisement payload and update period.

Fig. 3 shows the captured advertisement transmitted by our Dynamic beacon. While the fields of the static beacon remain unchanged with each transmission, Fig. 3 shows the (Major,Minor) fields incrementing from (00 01 01 7A) to (00 01 01 7B). In a practical setting, of course, a beacon developer might choose to reserve PDU bits to communicate sensor readings or available battery power. When used in this fashion, dynamic advertisements can facilitate large-scale network management by communicating device and network health and integrity, or perhaps even expand beacon applications.

IV. Maximum Beacon Density

We next seek to determine the maximum spatial density of beacons such that the system operates at an optimal communications capacity. Note that, in general, both beacons and receivers might be mobile; examples of applications with mobile beacons include social media 'identity badging' applications [5], or vehicle-attached beacons. For simplicity we consider only fixed location beacons and receivers in a 2-dimensional space. Conceptually, we wish to determine how closely we can pack beacons together such that 1) beacons sharing wireless spectrum don't interfere with each other's transmissions, and 2) the largest number of beacon advertisements are heard by receivers. To begin we make the following initial assumptions:



P.nbr.	Time (us)	Channel	Access Address	Adv PDU Type	Adv PDU Header (Type, TxAdd, RxAdd, PDU-Length)	AdvA	AdvData	CRC	RSSI (dBm)	FCS
37	+105001 =4283809	0x25	0x8E89BED6	ADV_NON_CONN_IND	2 0 0 36	0x000272C8B58C	02 01 1A 1A FF 4C 00 02 15 2F 23 44 54 CF 6D 4A 0F AD F2 F4 91 1B A9 FF A6 00 01 01 7A C5	0x562BAA	-52	OK
38	+266254 =4550063	0x25	0x8E89BED6	ADV_NON_CONN_IND	2 0 0 23	0x000272C8B58C	02 01 0A 02 0A 04 0A 09 42 43 4D 32 30 37 30 32 41	0x278E9B	-52	OK
39	+100001 =4650064	0x25	0x8E89BED6	ADV_NON_CONN_IND	2 0 0 36	0x000272C8B58C	02 01 1A 1A FF 4C 00 02 15 2F 23 44 54 CF 6D 4A 0F AD F2 F4 91 1B A9 FF A6 00 01 01 7B C5	0xE17A1E	-58	OK

Fig. 3. Dynamic Beacon broadcasts captured by a TI CC 2540 packet sniffer with incremented (Major,Minor) values in the advertisement data (AdvData).

- *Regular Topology*: Beacons are deployed in a regular lattice structure on a perfectly uniform plane.
- *Single Channel*: Though the BLE standard identifies 3 advertisement channels, we consider the behavior of only a single advertisement channel in isolation.
- *Static Advertisements*: We assume that each beacon broadcasts only a fixed advertisement.
- *Homogeneous Duty Cycle*: Beacons broadcast with the same duration of transmission and with the same period (same duty-cycle).
- *Lossy Collisions*: When multiple packets are heard by a user simultaneously, all packets are lost.
- *Receiver Capacity*: There exists some maximum number of beacons that a user can observe correctly at any one time.

For the purposes of finding an analytical upper bound on beacon density we initially assume an idealized system constructed of time-synchronized beacons that permit us to control transmissions using Time-Division Multiplexing (TDM) channel access. That is, we can schedule the transmissions of all beacons such that collisions are avoided because all beacons broadcast in a different timeslot. We will relax this assumption in the next section.

Parameters for calculating Maximum Beacon Density	
d	Inter-Beacon lattice distance
t_{dur}	Duration of Beacon Transmission
T	Period of of Beacon Transmission
N_{max}	Maximum number of observable Beacons any one user can decode
R	Maximum radius that a Beacon can broadcast above the noise floor
S_R^i	Set of Beacons within the radius R relative to a given user i
f	Frequency of the transmission in MHz
N	Distance Power Loss Coefficient
$P_f(n)$	Floor Penetration (assumed to be 0)

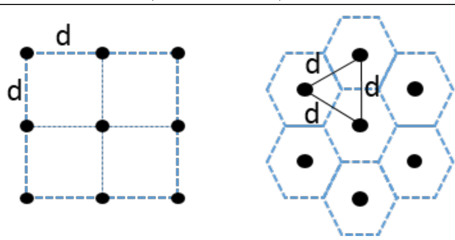


Fig. 4. Two ideal lattices: the (a) Square Lattice, with Beacons located at the intersections of the lattice, and the (b) Hexagonal Lattice, with Beacons located in the center of each cell.

We also assume that beacons are arranged in a regular lattice, with a minimum inter-Beacon lattice distance d between any 2 Beacons (Fig. 4). Next, we assume that each Beacon broadcasts with a fixed period T, with an active broadcast duration t_{dur} at the start of each period (Fig. 5). Figs. 6 and 7 show the difference between the ideal case of Beacon broadcasts, which we address analytically, and the practical case, which we address via simulation. Fig. 6 depicts our idealized system where all beacons are able to broadcast over mutually-exclusive timeslots. The more realistic model – depicted in Fig. 7 and addressed in our simulations at the end of this section – permits broadcast collisions to occur. This initial ideal model is simplified for tractability: it does not include terrain properties, as well as any obstructions between beacons. Expanding the model to cover these topics is left for future work and is not in the scope of this paper.

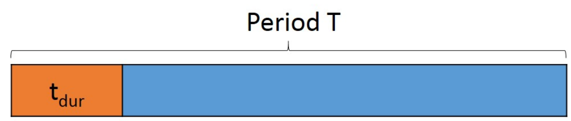


Fig. 5. Relationship between Transmission Duration t_{dur} and Period T. No signal is transmitted during the rest of the period.

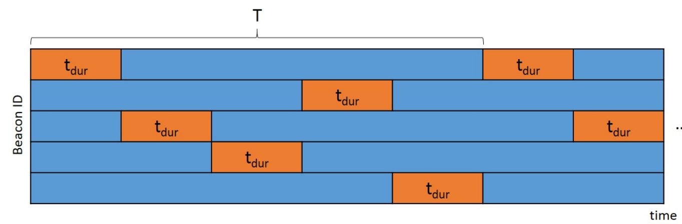


Fig. 6. Ideal Scheduling: TDM Scheduled Packets

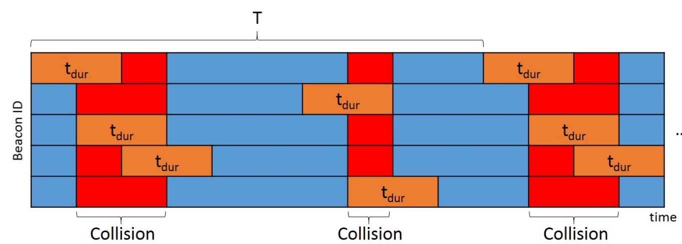


Fig. 7. Practical Scheduling: Uncoordinated with Collisions

A. Theoretical Bounds on Density

Finding the maximum Beacon density in a lattice for the idealized system can be reduced to finding the minimum inter-Beacon spacing d between lattice intersections, and on average, a lattice is simply the average distribution of Beacons over a given geographical area modeled as a uniform distribution. The way we addressed this problem was the following: given that any user in the ideal case is limited to observing N_{max} beacons over any period of time T (due to the nonzero transmission durations t_{dur} of each Beacon), the set of Beacons within range R of a user i, labeled S_R^i, must satisfy $|S_R^i| \geq N_{max}$, where $N_{max} = \frac{T}{t_{dur}}$. For the purposes of this paper, we replace S_R^i with S_R because of the regular nature of the lattice topologies discussed.

Case I: Square Lattice

For the case of a square lattice topology, we seek the minimum distance d such that

$$\arg\min_{R>0} \; (|S_R| \geq N_{max}) \tag{1}$$

subject to

$$S_R = \{(x,y) : (x \cdot d)^2 + (y \cdot d)^2 \leq R^2\} \quad (2)$$
$$\{x,y\} \in \mathbb{Z} \quad (3)$$

This problem can be interpreted as finding the minimum radius R of a circle in an integer lattice such that the circle contains at least N_{max} integer coordinate points. This problem does not have a general closed-form solution. However, for some practical values of t_{dur} and T, setting each Beacon to broadcast for 1ms. with a period of 1 s. (thus giving us $N_{max} = 1000$), we can apply known results for the integer sequence for the number of unit intersections contained within a circle of radius R centered at the origin for a square lattice with spacing d [6]. We obtain for the relationship between R and d the following results: (1) a Square lattice with a user centered directly on an intersection:

$$R = 19d \quad (4)$$

and (2) for a Square lattice with a user placed directly in the center of one of the square cells formed by the lattice, is

$$R = \tfrac{39}{2}d. \quad (5)$$

which is obtained by considering a lattice with spacing $d/2$ and considering only the 4 corner points of a 3x3 intersection grid.

Case II: Hexagonal Lattice

For the case of a hexagonal lattice topology, the equivalent problem can be simplified to seeking the minimum distance d such that

$$\arg\min_{R>0} \left(1 + \sum_{x=1}^{R/d} 6x \geq N_{max}\right) \quad (6)$$

for a user centered on a beacon location. Luckily, this problem has a straightforward solution, and for $N_{max} = 1000$, we get

$$R = 18d \quad (7)$$

Observe that the results Eqs. 4, 5, 7 differ by less than 5% for the different but similar lattices.

Now that we have a relationship between the inter-Beacon distance d and R, the maximum effective physical distance at which the beacon can be observed from Eqs. 5, 7, let's consider a numerical example. Given a known transmit power at the beacon of -59dBm, and a noise floor of -90dBm at which the signal can no longer be decoded, we can use the standard ITU-Path Loss model for indoor propagation [7]

$$Pathloss(dB) = 20\log_{10} f + N\log_{10} R + P_f(n) - 28 \quad (8)$$

to obtain typical values of minimum beacon separation d. Setting $N = 30$, and $P_f(n) = 0$, we find that for the Square lattice with a user on an intersection, $d_{min} = 0.543$ m.; for a Square lattice with a user centered in a square cell, $d_{min} = 0.529$ m.; and $d_{min} = 0.573$ m. for a user on a Hexagonal lattice intersection. Clearly, from a practical perspective a half-meter separation per advertisement channel places little restriction on smart building applications, where we may easily consider dense application scenarios such as as embedding or attaching a beacon to every 'drop ceiling' tile.

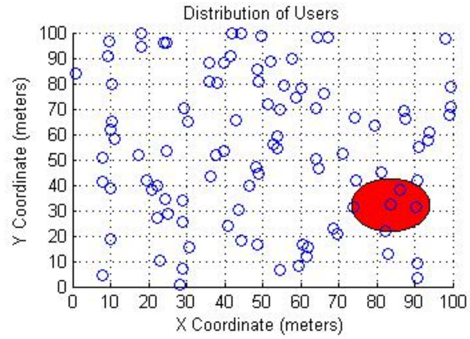


Fig. 8. A realization of the simulation environment with $d = 100$, 121 Beacons and 121 users.

B. Simulation

We next obtain detailed results on beacon crowding in a less idealized system through simulation by relaxing many of the previous assumptions used to calculate a theoretical bound on beacon density. We model an unsynchronized time-slotted system where collisions may occur. We also permit beacons to broadcast with different durations and periods. Finally, we permit a collision resolution scheme where some colliding packets can be successfully decoded.

In our simulation we uniformly distribute receivers across a Square lattice of Beacons. Each Beacon broadcasts in a pre-determined, periodic manner. During each time step, each receiver can hear 0, 1 or multiple beacon broadcasts. Fig. 8 shows an example of such a realization of the simulation setup, with a square lattice of size 100 m^2, and Beacons (placed on the lattice intersections) placed every $d = 10$ m. apart for a total of 11x11=121 beacons. We considered an identical number of randomly placed receivers, each represented by a blue circle. The red region, centered on a chosen user, represents the maximum reception range.

Since it is possible to develop receivers that can decode even colliding advertisements, we have studied various collision resolution techniques. These techniques consider the comparative gain or relative strengths of received signals. In Strongest Signal Decoding (SRD), collisions of signals of sufficiently different strength result in a single successful decoding (the strongest reception); in Iterative Decoding (ID) multiple signal decoding is possible.

In SRD –since any user can see at most one packet successfully decoded per timeslot– our performance metric of choice is the average number of packets decoded. Fig. 9 shows the behavior of this curve with respect to what we call the *Comparative Gain* threshold, or the minimum SNR required to decode the strongest signal. All simulation results in the remainder of this paper average data from 12 runs with parameters $t_{dur} = 1s., T = 5s.$, and a transmit time offset in seconds taken from the uniform integer distribution $U[0,4]$.

Our simulations shows that denser networks with $d = 4, 5$ perform the best at low values of Comparative Gain; during each timeslot, all users can observe a beacon. However, when the Comparative Gain increases, performance drastically falls to being the worst of the cases we examine. We attribute this behavior to the sheer amount of interference observed at such comparable RSSI values creating a situation in which it is very difficult to observe any packets. On the other hand, for very sparse networks such as $d = 20$, one can see that the

Algorithm 2 Strongest Signal Decoding (SRD)

Input: $X = \{x_1, x_2, ..., x_n\}$, a vector of RSSI received from n Beacons, in decreasing RSSI, and Comparative Gain $threshold$
Output: Decoded signal x_1 or absence of decoded signal
if $x_1 - threshold \geq \sum_{i=2}^{n} x_i$ **then**
    x_1 successfully decoded
else
    No signal decoded
end if

Algorithm 3 Iterative Decoding (ID)

Input: $X = \{x_1, x_2, ..., x_n\}$, a vector of RSSI received from n Beacons, in decreasing RSSI, and Comparative Gain $threshold$
Output: Decoded signals X_{out}
$run = true, k = 1$
$X_{out} = \emptyset$
while run **do**
    if $x_k - threshold \geq \sum_{i=k+1}^{n} x_i$ **then**
        x_k successfully decoded
        $X_{out} \leftarrow X_{out} \cup x_k$
        $k = k + 1$
    else
        No signal decoded
        $run = false$
    end if
end while

dependence on Comparative Gain is no longer present; this can be explained by the simple fact that, with $d = 20$, there are only 6x6 beacons in the entire 100 m^2 space. Either a user observes exactly one Beacon, or no Beacon at all, making this a simple constant function that depends on Beacon density.

We studied 3 different metrics for efficient network operation under the Iterative Decoding (ID) scheme; note that we cannot use the same metric as in SRD anymore simply because a user may now potentially decode multiple Beacons in a single timeslot. Thus, we look at three different metrics: 1) the average number of Decoded Beacons per user, 2) the fraction of all transmitted packets observed under ID, and 3) the fraction of packets that were actually decoded compared to the number of packets observed by a user.

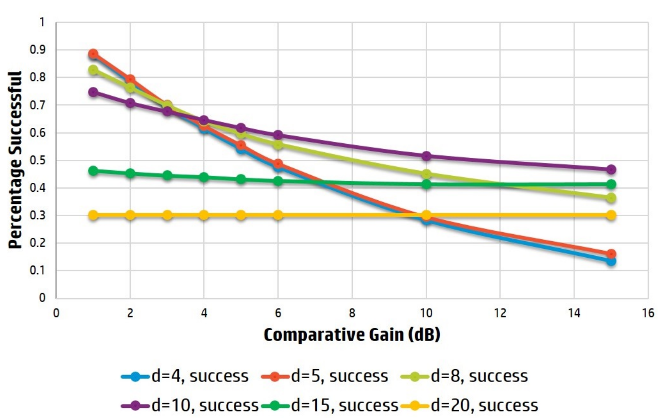


Fig. 9. Average number of timeslots a Packet is successfully decoded under SRD.

Fig. 10 shows the average number of decoded Beacons per user for varying values of d, as a function of Comparative Gain. As in the SRD case, we can see that denser networks function best when the Comparative Gain is small (small differences in RSSI can still allow colliding transmissions to be decoded), but are outperformed by others when the Comparative Gain is large. While performance of dense networks at high Comparative Gain values appears to degrade quickly, modern systems may be able to successfully decode signals with as little as 2-3 dB of Comparative Gain.

Fig. 11 shows the fraction of all transmitted packets observed under ID. First, we immediately notice that for very dense networks, a promising 100% of all packets were observed by at least one user at small Comparative Gain values, which then quickly decreases as we sparsify the network, quickly decaying to a constant function. It is interesting to note how behavior at high Comparative Gain quickly improves with some additional sparsity in the network (e.g. $d = 8$).

Fig. 12 depicts the fraction of packets that were actually decoded vs packets received. The fast decay of dense networks as a function of Comparative Gain is easily noted here.

V. Related Work

BLE, also known as Bluetooth Smart, has gained considerable interest in wearable computing and IoT products since the release of Bluetooth Core Specification 4.0 in 2010. Dedicated single-purpose beacons including Apple iBeacon, Estimote, Qualcomm Gimbal and Stick&Finds are becoming increasingly popular, driving ever larger scale deployments, Virgin Atlantic operates a 300 iBeacon network pilot in Heathrow airport. A 500 iBeacon network to guide the visually-impaired through San Francisco airport is managed by startup Indoo.rs [8]. Few commercial tools to manage large-scale beacon networks have been developed to date, perhaps due to the limited number of large-scale deployments. But this is changing quickly: Apple Computer is aggressively adding system support for beacon reception and handling in iOS, and has deployed iBeacons in every Apple Store.

The study of the performance and management of large-scale beacon networks is closely related to research performed by the distributed sensor network community. These sensors would typically collaboratively gather data or other inputs, and aggregate and process the data to give a more accurate picture regarding the system's state, its environment, or the behavior of other entities in the environment. Beacon self-interference – and bounds on beacon density – were explored as a component of a study of beacon placement [9]. However this work, which predated the emergence of BLE beacons by more than a decade, considered a generic RF beacon technology with a different communications model and interference (noise) model than we have studied here. More recently, Chawathe has considered beacon strategies in dense beacon settings for indoor localization applications [10].

The overwhelming number of dedicated beacon vendors all broadcast a fixed advertisement and have not developed techniques to implement dynamic beaconing. We anticipate this to change as some beacon manufacturers seek to increase

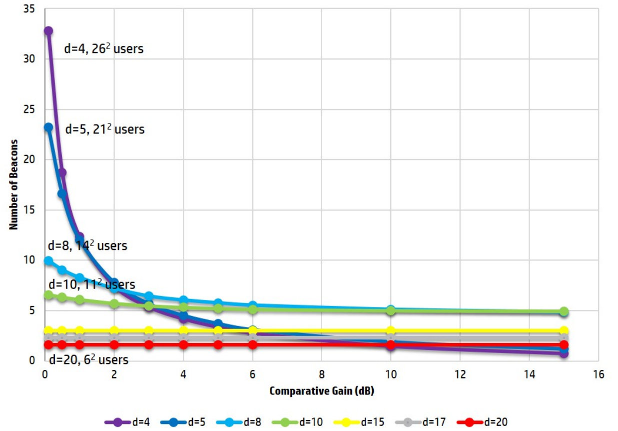


Fig. 10. Average Number of Beacons Decoded per User under ID.

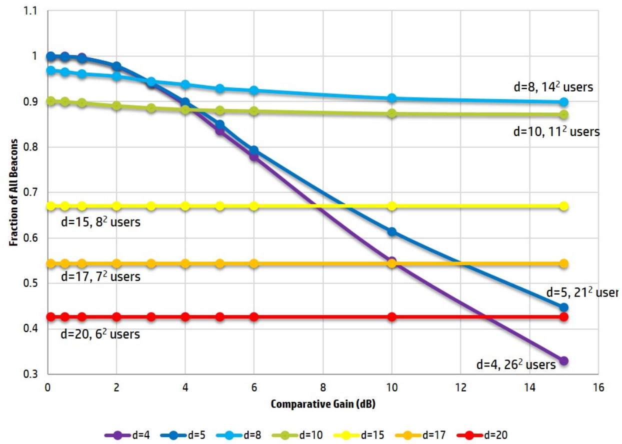


Fig. 11. Fraction of all Transmitted Packets Observed under ID.

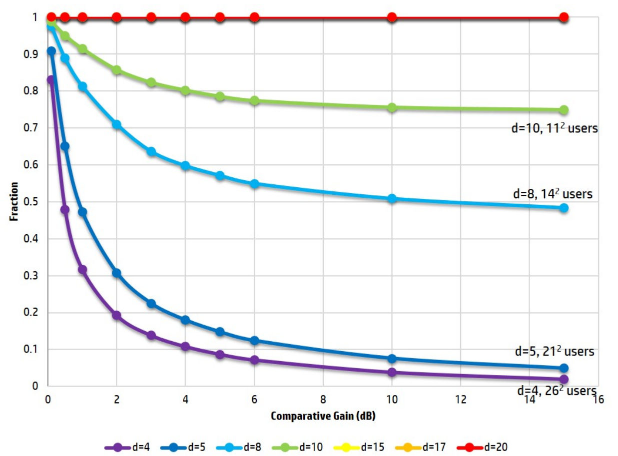


Fig. 12. Fraction of Packets Decoded out of those Received under ID.

the number of on-board sensors they provide. One research group has already called for extensions to the iBeacon PDU format to support new applications [11], and a second has experimented with cyclic advertisements as a 'keep-alive' signal in a building occupancy detection system [12].

VI. Conclusions

The key contributions of this work are the construction of a proof-of-concept of a Dynamic Beacon, and a combined analysis and simulation to quantify the practical limits on placing large numbers of beacons in close proximity. Both contributions facilitate the management of very large, dense beacon networks in smart building and campus settings.

We also established some general guidelines for beacon deployments. The two competing principles dictating beacon system performance are:

- Increased Beacon density may place users in range of additional Beacons, but if the hardware and software on a user's device is insufficient to decode observed packets, then increasing density is unhelpful, and
- Too sparse of a network can lead to a hit-or-miss scenario where users may only observe at most 1 beacon.

For small values of Comparative Gain, our simulation results determine how dense we can make a topology. For large values of Comparative Gain, we may wish to compromise, and select Beacon deployments with sparser distributions in order to optimize the performance per unit cost of the beacons themselves. A network operator may desire that all users see as many Beacons as possible, however, if the network is made too dense for a given Comparative Gain, then the effectiveness of the network with respect to unit cost of the Beacon can result in suboptimal use of resources.

Our future efforts will focus on prototyping a large-scale beacon deployment (e.g., hundreds of devices), and analysis of system performance with mobile users, mobile beacons, or both. Of particular interest are social identification and badging applications (e.g., games), where large numbers of users wearing beacons move through a space while receiving tranmissions from and interacting with each other.

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