German University in Cairo
Faculty of Media Engineering and Technology
Spring 2019

DMET 1002 – Advanced Media Lab

Lab 2 Preparation

K-means Segmentation of Colored Images

1. Objective

This experiment aims at demonstrating the use of K-means clustering in segmenting colored images.

2. Pre-requisites

- Segmentation basics
- MATLAB programming knowledge

3. References

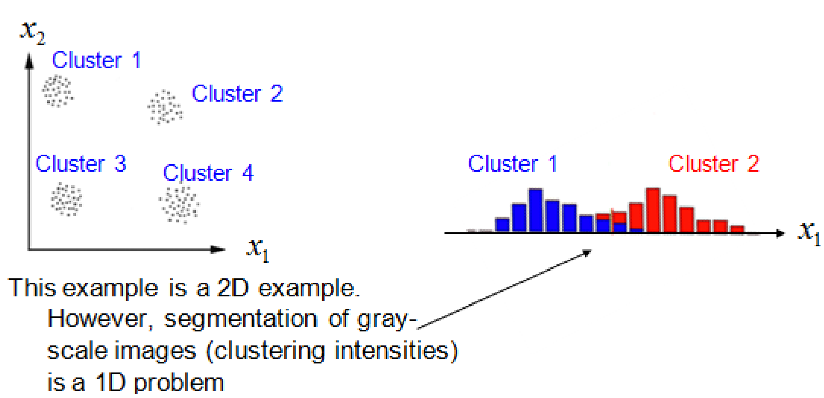
- Section 9.1 of "Pattern Recognition and Machine Learning", Christopher M. Bishop, Springer, 2006

4. Theoretical Background

In this experiment, you will be examining the performance of K-means clustering in segmenting colored images. In the following section, we introduce the K-means clustering algorithm followed by a method that can be used to determine the number of clusters (regions) K.

4.1 K-means Clustering

- Clustering: Grouping together points (colors, intensities) that are similar
- Cluster: A group of data points whose inter-point distances are small compared with distances to points outside the cluster



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- Objective Function: Minimize J

$$J = \sum_{n=1}^{N} \sum_{k=1}^{K} r_{nk} \|\mathbf{x}_n - \boldsymbol{\mu}_k\|^2$$

$\mathbf{x}_n$: Input Data

r_{nk} : Cluster Membership = 1 if $\mathbf{x}_n$ is member of cluster k and 0 otherwise

$\boldsymbol{\mu}_k$: Center of cluster k

N : Number of data points (image pixels)

K : Number of clusters

- K-means Algorithm Steps:

Step 1: Randomly choose clusters center

Step 2: Compute r_{nk}

$$r_{nk} = \begin{cases} 1 & \text{if } k = \arg\min_j \|\mathbf{x}_n - \boldsymbol{\mu}_j\|^2 \\ 0 & \text{otherwise.} \end{cases}$$

(Assign $\mathbf{x}_n$ to the cluster with closest center)

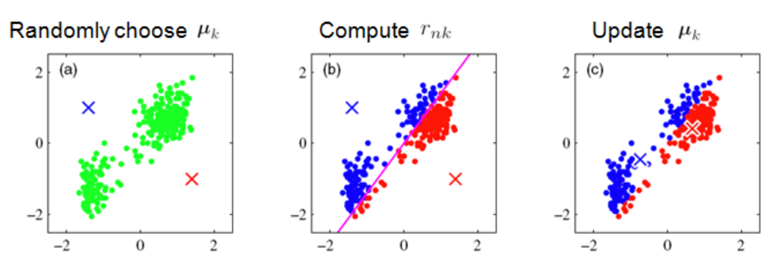
Step 3: Update $\boldsymbol{\mu}_k$

Take derivative of J with respect to $\boldsymbol{\mu}_k$ and equate with zero

$$2 \sum_{n=1}^{N} r_{nk}(\mathbf{x}_n - \boldsymbol{\mu}_k) = 0 \quad \rightarrow \quad \boldsymbol{\mu}_k = \frac{\sum_n r_{nk}\mathbf{x}_n}{\sum_n r_{nk}}$$

Back to Step 2 until convergence

- Example: Consider clustering the following 2-dimensional data. Each point corresponds to a data point (color of one pixel). This is similar to, for example, clustering colored pixels using the Red and Green components (2-dimensional).



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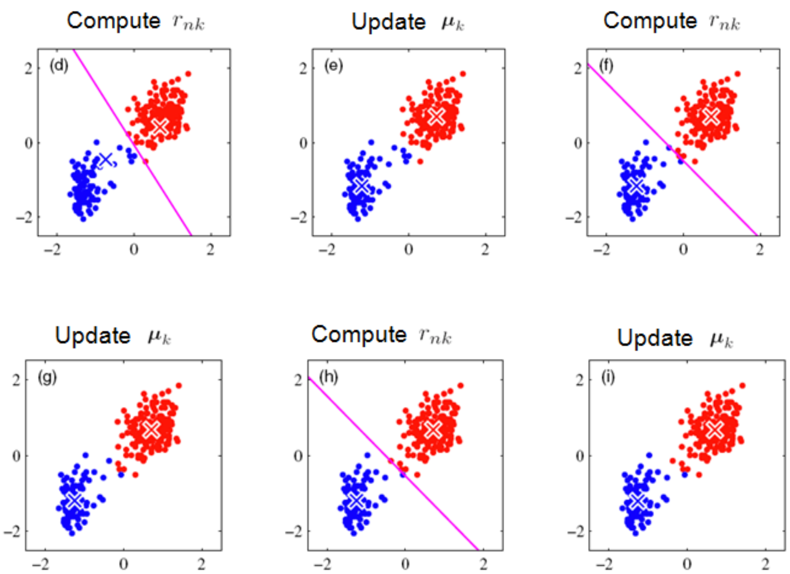
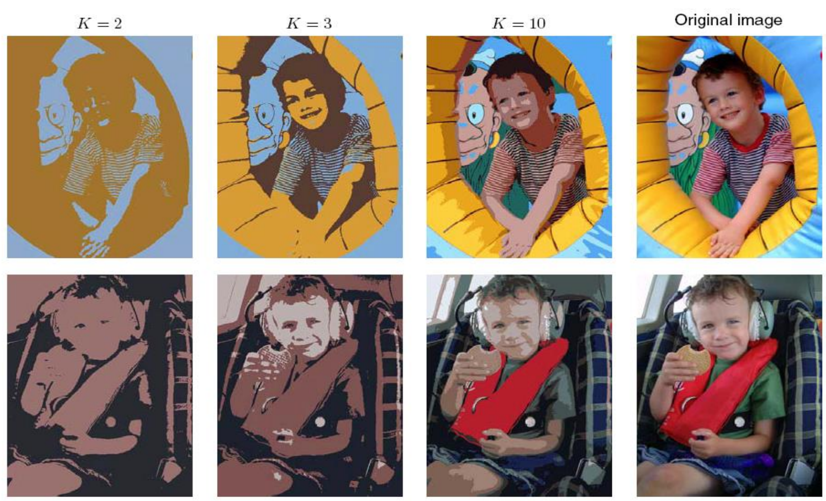


- Example: Segmenting colored image using different values of *K* (the number of regions to find). In this case, each color has three dimensions (Red, Green and Blue). Thus, it is similar to the example above with the difference that the data is now 3 dimensional instead of 2 dimensional.



K-means Segmentation of Colored Images

4.2 Determining the Best Value of K

- As can be seen in the last example, the value of K has a significant impact on the output of the algorithm
- One way to choose the best value of K for a given data (image) is to quantify the quality of clustering
- A good clustering output should satisfy the following two conditions:
    - Small difference between points (colors of pixels) in the same cluster (Compact clusters)
    - Large difference between points (colors of pixels) across different clusters (Distant clusters)
- These conditions can be represented mathematically as minimizing the following quantity:

$$Q = \frac{\frac{1}{K}\sum_{i=1}^{K}\frac{1}{N_i}\left(\sum_{\mathbf{x}_n \in C_i}\left\|\mathbf{x}_n - \boldsymbol{\mu}_i\right\|^2\right)}{\frac{2}{K(K-1)}\sum_{i=1}^{K-1}\sum_{j=i+1}^{K}\left\|\boldsymbol{\mu}_i - \boldsymbol{\mu}_j\right\|^2}$$

  The numerator corresponds to the average squared distance (difference) between the center of each cluster and its corresponding points averaged across clusters. The denominator corresponds to the average squared distance (difference) between the centers of difference clusters. Therefore, minimizing Q corresponds to minimizing the numerator and maximizing the denominator. In this equation, N_i represents the number of points (pixels) in cluster i.
- To find the best K for a given image, run the K-means algorithm with different values of K and compute the corresponding Q for each K. The best segmentation output is the output that gives the minimum value of Q.