Lossy Compression

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The curse of noise for real signals

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8-bits grayscale image.

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What we actually get:

...	148	150	149	151	150	150	153	151	148	147	150	...

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$\rightarrow$ ... but they annihilate lossless compression techniques!

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$\Downarrow$

```
-rw-r--r-- 1 gtochon lrde 177K mars  28 18:00 grenoble.tiff.zip
-rw-r--r-- 1 gtochon lrde 177K mars  28 17:58 grenoble.tiff.gz
-rw-r--r-- 1 gtochon lrde 151K mars  28 17:58 grenoble.tiff.bz2
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```

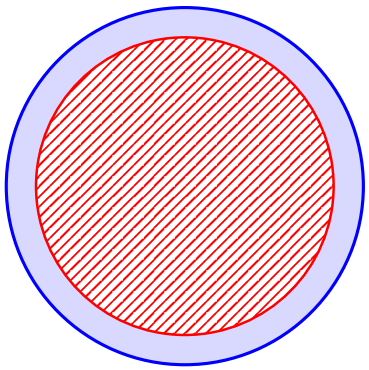

Non-compressible part of the whole image.

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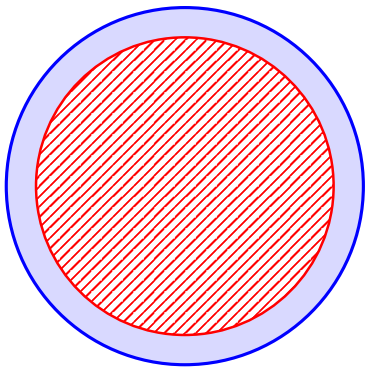

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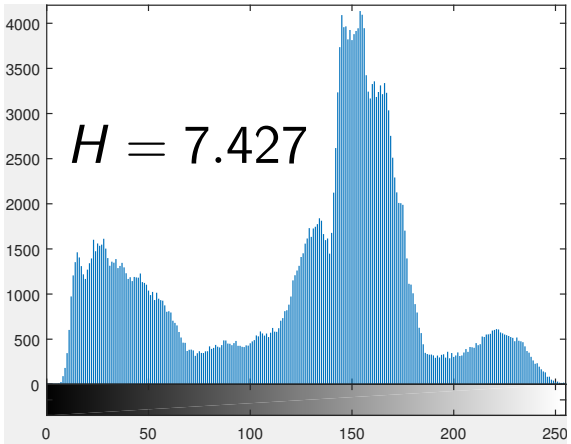

Lossy compression

Data before compression and after decompression are not the same.

Lossy compression algorithms:

- irremediably degrades the data by removing some part of the information.
  → degradation is expected not to be noticeable by the end-user.
- are more efficient than lossless approaches in terms of compression ratio.
  → at the expense of the commited inaccuracies during decompression.
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Lossy compression ≡ trade-off between performance and degradation.

Lossy Compression

Downsampling

A naive compression scheme

Let's take some grayscale image $\mathcal{I}$ with M rows and N columns (that is, some matrix $[\mathcal{I}(m,n)]_{M,N}$) and crudely downsample it by a factor of r ($\mathcal{I}_{\downarrow r} = \mathcal{I}(0{:}r{:}M\text{-}1, 0{:}r{:}N\text{-}1)$).

$\mathcal{I}$

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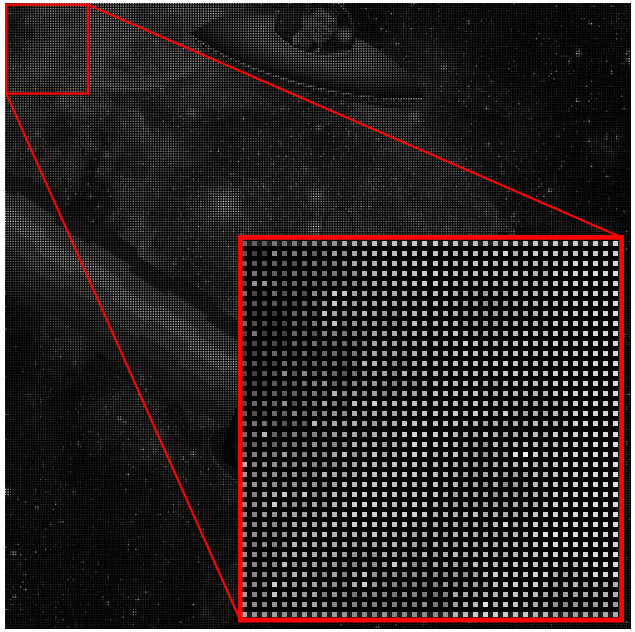

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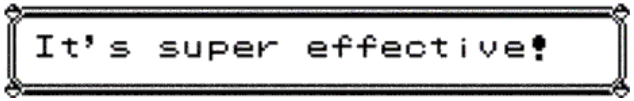
```
-rw-r--r-- 1 gtochon lrde 245K avril 20 13:25 randompic.tif
-rw-r--r-- 1 gtochon lrde  62K avril 20 13:25 randompic_down2.tif
-rw-r--r-- 1 gtochon lrde  16K avril 20 13:25 randompic_down4.tif
```

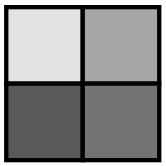


Upsampling

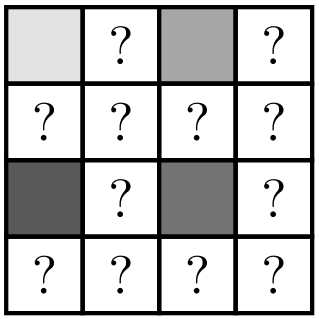
When things are getting tricky

If compression $\equiv$ downsampling, then decompression $\equiv$ upsampling $\equiv$ interpolation.

$\mathcal{I}$



$\tilde{\mathcal{I}} = \mathcal{I}_{\uparrow 2}$



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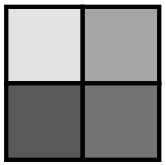
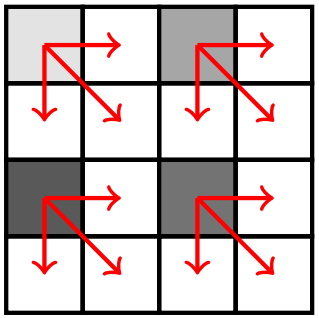


$\tilde{\mathcal{I}}_{NN}$



Nearest-neighbor interpolation:

Copy in an unknown pixel the value of the *closest* known pixel (closest $\equiv$ leftmost and upmost closest pixel).

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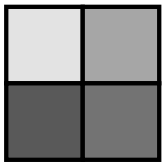

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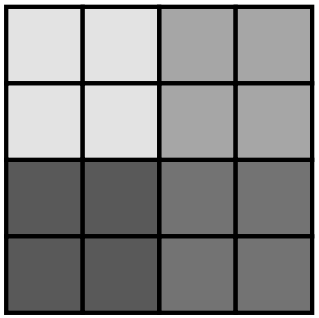

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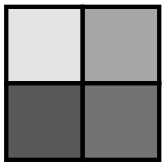

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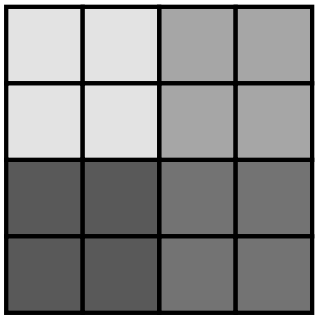

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👎👎 blocking effect 👎👎

$$\mathcal{I} \xrightarrow{\text{downsampling}} \mathcal{I}_{\downarrow 4} \xrightarrow[\text{nearest-neighbor}]{\text{interpolation}} \tilde{\mathcal{I}}_{NN}$$

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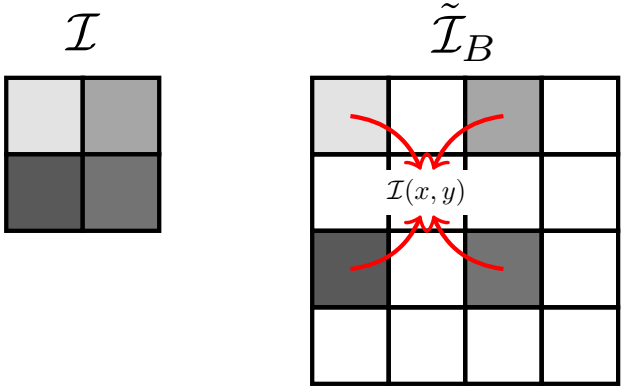

Bilinear interpolation:

Write the pixel value as a bilinear function of its position $\rightarrow \mathcal{I}(x, y) = \alpha x + \beta y + \gamma xy + \delta$.

Use the 4 closest known pixels to determine α, β, γ and δ and plug in previous equation.

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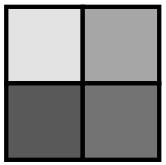
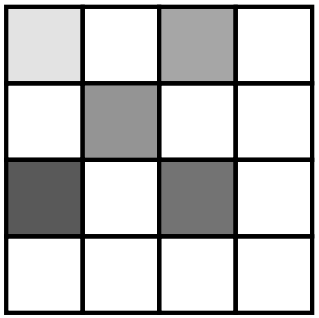


$\tilde{\mathcal{I}}_B$



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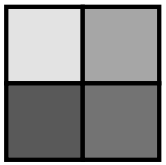

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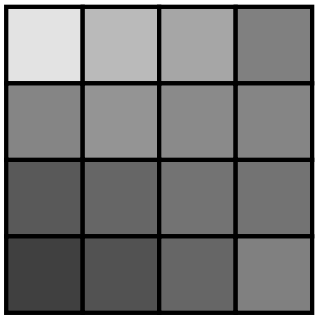

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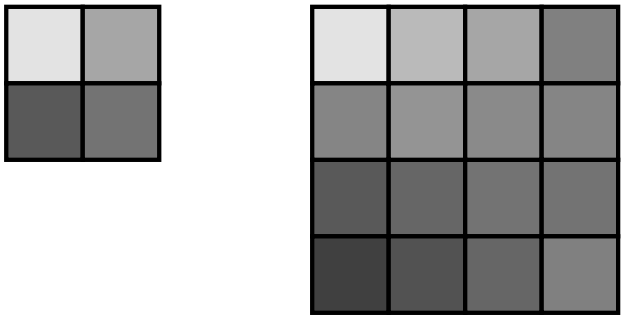

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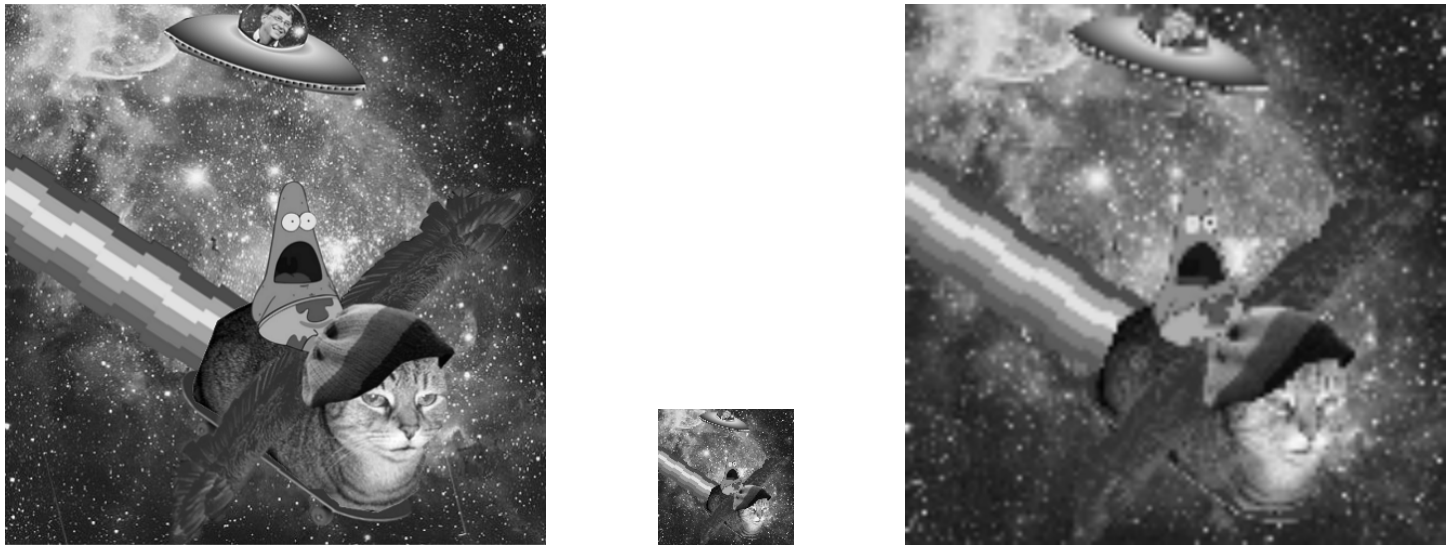
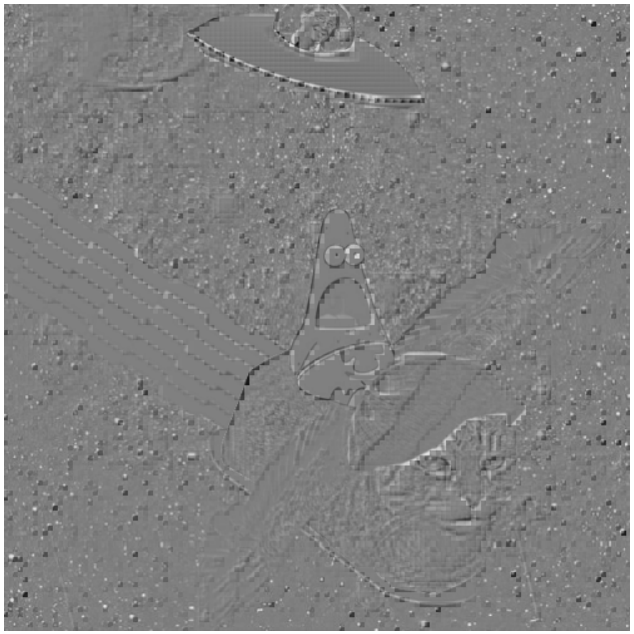
👎👎 blurring effect 👎👎



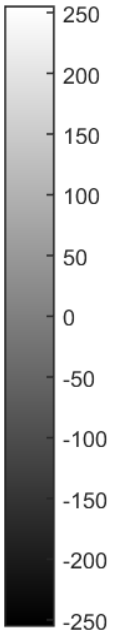
$$\mathcal{I} \xrightarrow{\text{downsampling}} \mathcal{I}_{\downarrow 4} \xrightarrow[\text{bilinear}]{\text{interpolation}} \tilde{\mathcal{I}}_B$$

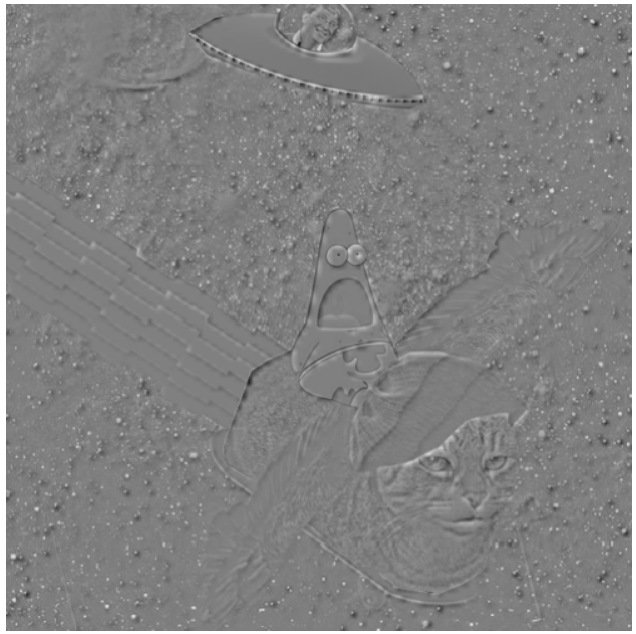
It sure looks bad...

... but how bad?



$\epsilon_{NN} = \mathcal{I} - \tilde{\mathcal{I}}_{NN}$





$\epsilon_{B} = \mathcal{I} - \tilde{\mathcal{I}}_{B}$

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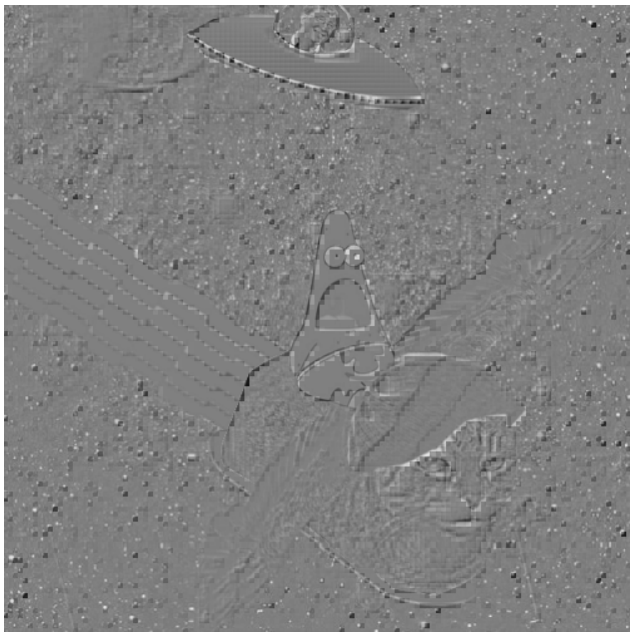
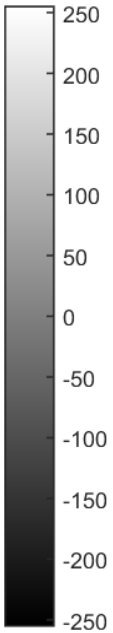
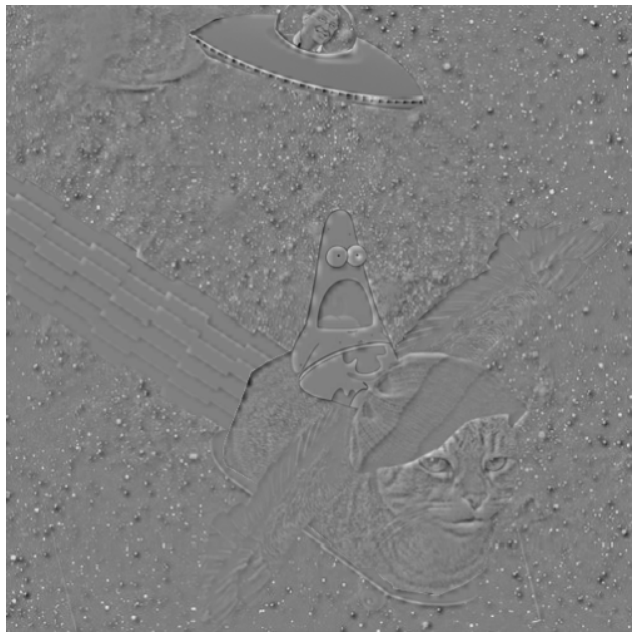
It is possible to quantitatively assess the compression/decompression error $\epsilon = \mathcal{I} - \tilde{\mathcal{I}}$ with numerical measures:

- Root mean square error: $\mathrm{RMSE}(\epsilon) = \sqrt{\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\epsilon(m,n)^2}$
- Signal-to-noise ratio: $\mathrm{SNR}(\epsilon) = 10\log\left(\frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\mathcal{I}(m,n)^2}{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\epsilon(m,n)^2}\right)$







$\epsilon_{NN} = \mathcal{I} - \tilde{\mathcal{I}}_{NN}$ $\epsilon_{B} = \mathcal{I} - \tilde{\mathcal{I}}_{B}$

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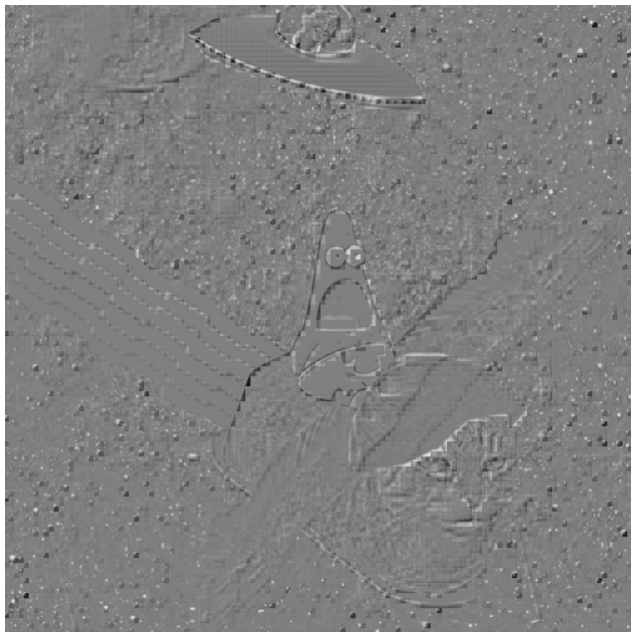

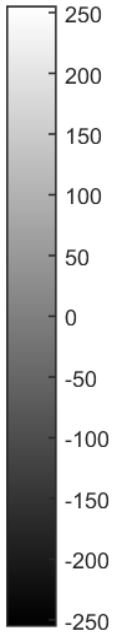

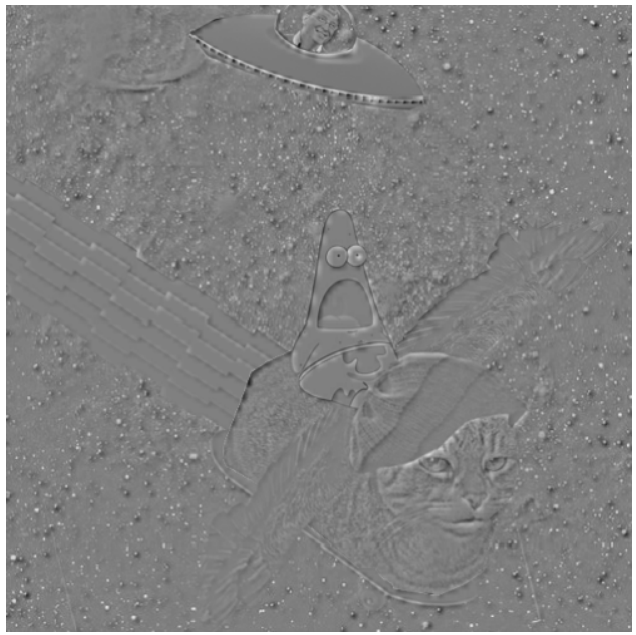

$\epsilon_{NN} = \mathcal{I} - \tilde{\mathcal{I}}_{NN}$
$\mathrm{RMSE}(\epsilon_{NN}) = 24.7$
$\mathrm{SNR}(\epsilon_{NN}) = 12.9$ dB

$\epsilon_{B} = \mathcal{I} - \tilde{\mathcal{I}}_{B}$
$\mathrm{RMSE}(\epsilon_{B}) = 20.7$
$\mathrm{SNR}(\epsilon_{B}) = 14.5$ dB

Frequency decomposition of an image

Any image $\mathcal{I}$ can be expressed as the superposition of a *low-frequency* term $\mathcal{I}_{LF}$ and a *high-frequency* term $\mathcal{I}_{HF}$.

$\mathcal{I}$ $\quad$ $\mathcal{I}_{LF}$ $\quad$ $\mathcal{I}_{HF}$

$\mathcal{I}_{LF} \rightarrow$ Overall structure of the image: uniform areas, color gradients, slowly-varying components.

$\mathcal{I}_{HF} \rightarrow$ Details: sharp edges, rapidly-varying components, **noise**.

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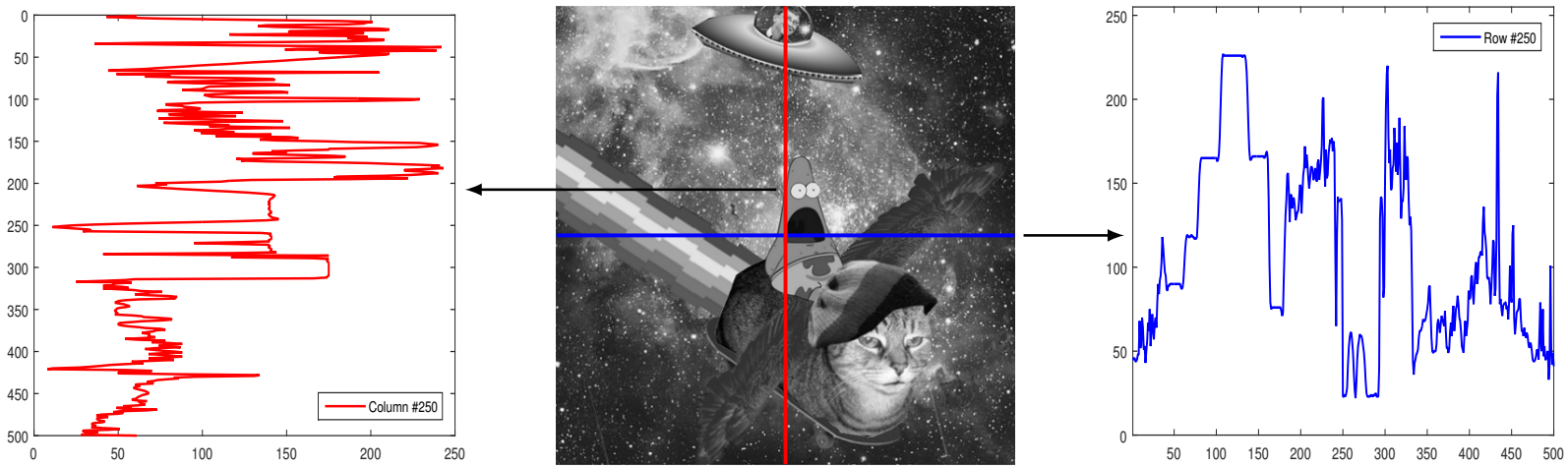
Inaptitude of the spatial representation...

The *spatial* representation is not suited to carry out a frequency analysis of the image:

$\rightarrow$ high frequencies are disseminated eveywhere in the image.

$\rightarrow$ hard to tell whether a quick variation in terms of pixel intensity is due to an edge (important) or the noise (can be discarded).



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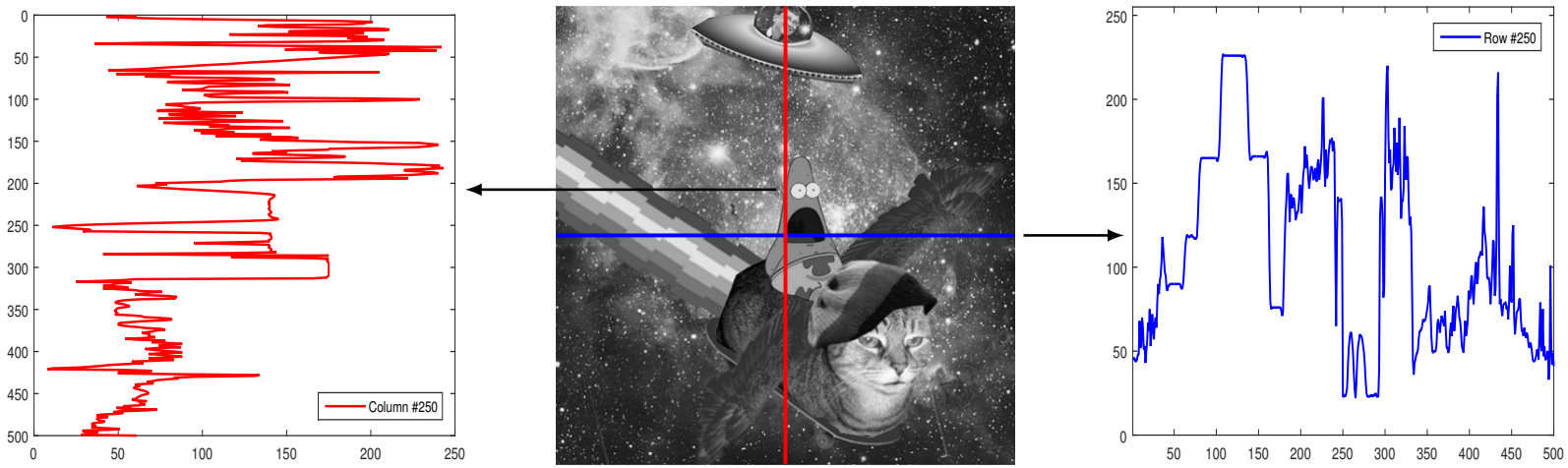
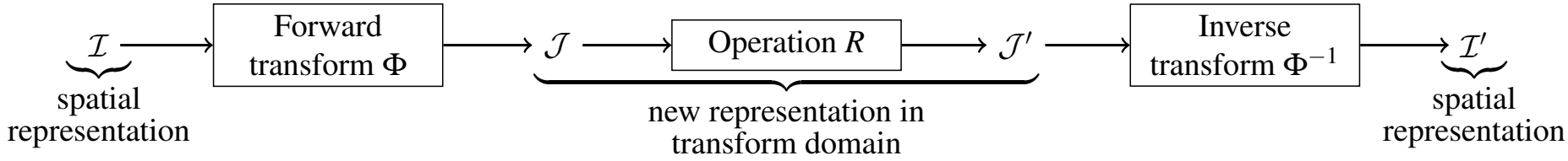
We need a more suited representation to express the image in terms of frequencies



... and the need for a new one

The image $\mathcal{I}$ is transformed into another image $\mathcal{J}$ through some reversible transform Φ such that all further processings to conduct are simpler/more efficient in the transform domain.



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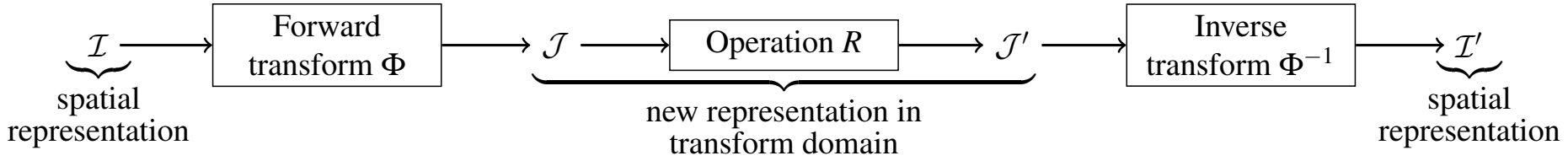
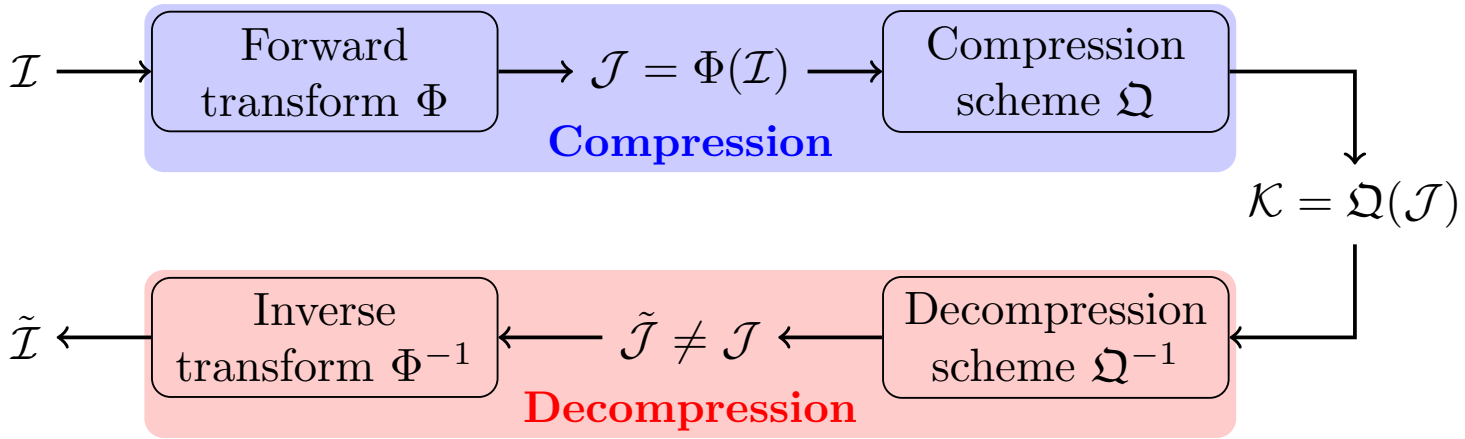


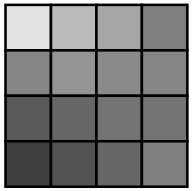
For lossy compression purposes:



The compression efficiency depends both on the used transform Φ/Φ^{-1} and the following compression/decompression scheme $\mathfrak{Q}/\mathfrak{Q}^{-1}$.

But wait...

What did we exactly mean by spatial representation in the first place?



$$\equiv \begin{bmatrix} 227 & 186 & 166 & 127 \\ 133 & 148 & 138 & 133 \\ 89 & 102 & 115 & 115 \\ 64 & 82 & 148 & 127 \end{bmatrix} = \begin{bmatrix} \mathcal{I}(0,0) & \dots & \mathcal{I}(0,3) \\ \vdots & \ddots & \vdots \\ \mathcal{I}(3,0) & \dots & \mathcal{I}(3,3) \end{bmatrix} \equiv [\mathcal{I}(m,n)]_{4,4}$$

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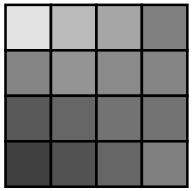

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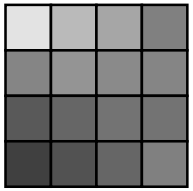

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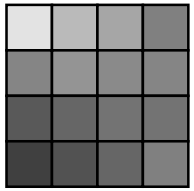

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Spatial representation of an image $\mathcal{I}$

$\Leftrightarrow$

Decomposition of $\mathcal{I}$ over the set of basis images $\left(\mathcal{E}_{00}, \mathcal{E}_{01}, \mathcal{E}_{02}, \dots, \mathcal{E}_{33} \right)$

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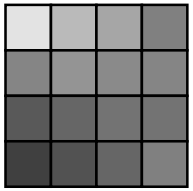

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$\Leftrightarrow$

Pixel value $\mathcal{I}(m,n) \equiv$ coordinate with respect to the corresponding basis image $\mathcal{E}_{mn}$

Changing the representation domain

Changing the representation domain of $\mathcal{I}$

$\Leftrightarrow$

Changing the set of basis images and expressing $\mathcal{I}$ in this new basis

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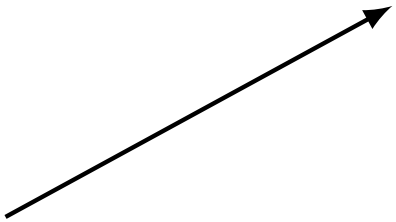

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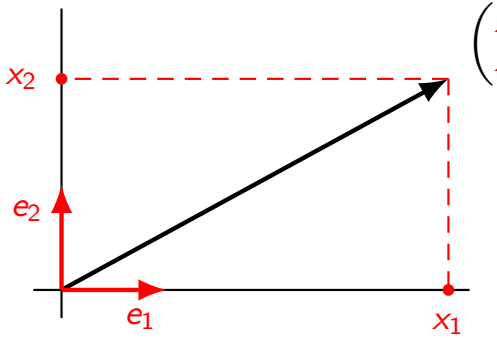

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 e_1 + x_2 e_2$$

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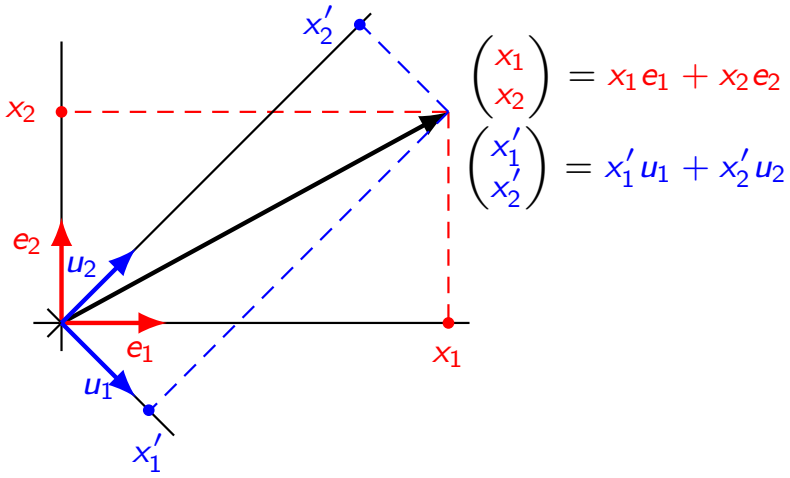

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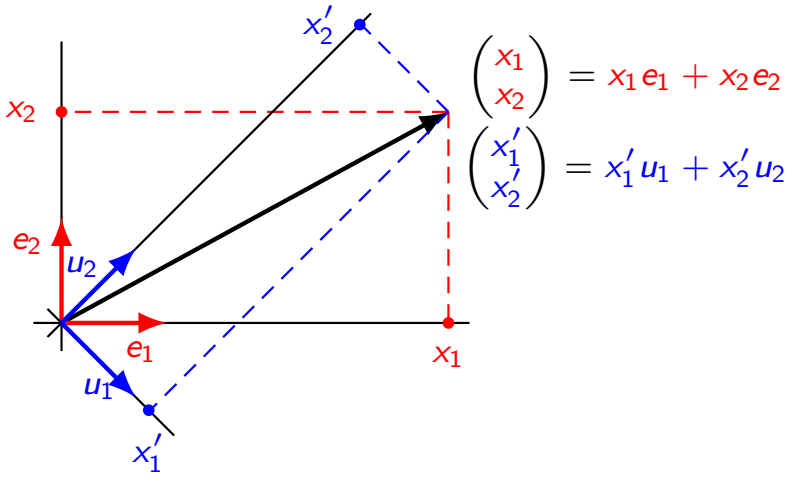

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$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = x_1' u_1 + x_2' u_2$$

$$= \begin{bmatrix} 200 & 150 \\ 50 & 100 \end{bmatrix} = \mathcal{I}$$

$$= 200 \times \mathcal{E}_{00} + 150 \times \mathcal{E}_{01} + 50 \times \mathcal{E}_{10} + 100 \times \mathcal{E}_{11}$$

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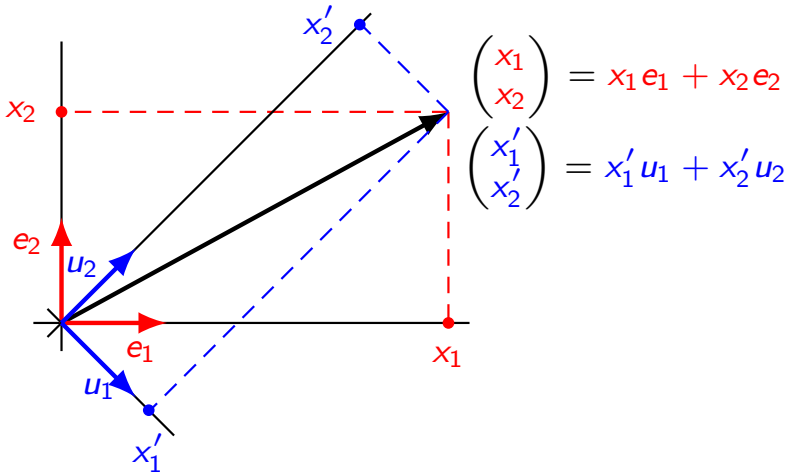

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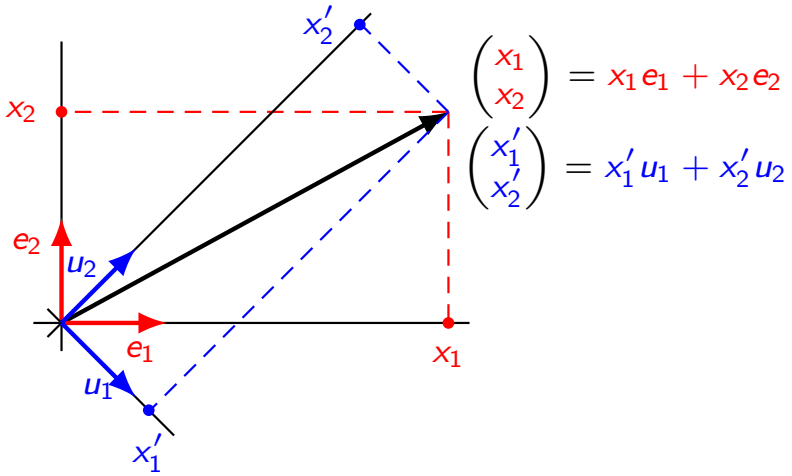

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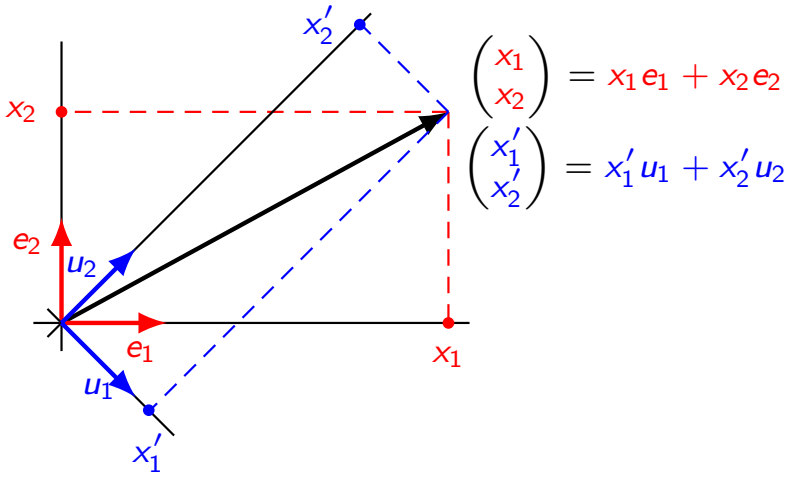

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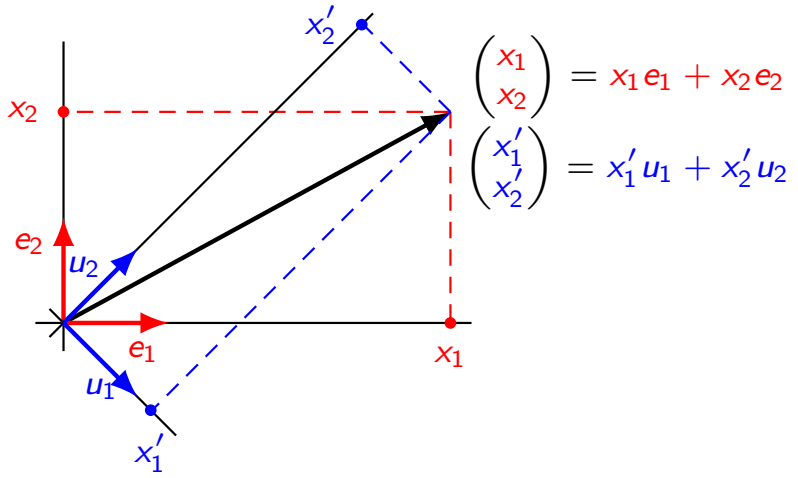

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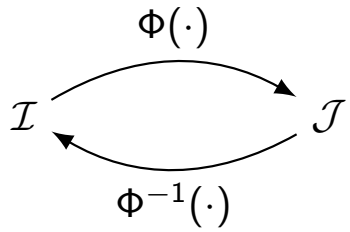
$$= \begin{bmatrix} 200 & 150 \\ 50 & 100 \end{bmatrix} = \mathcal{I} \text{ with respect to basis } \mathcal{E} = \left(\underset{\mathcal{E}_{00}}{\square}, \underset{\mathcal{E}_{01}}{\square}, \underset{\mathcal{E}_{10}}{\square}, \underset{\mathcal{E}_{11}}{\square} \right)$$

$$\equiv \begin{bmatrix} 50 & 0 \\ 100 & 50 \end{bmatrix} = \mathcal{J} \text{ with respect to basis } \mathcal{B} = \left(\underset{\mathcal{B}_{00}}{\square}, \underset{\mathcal{B}_{01}}{\square}, \underset{\mathcal{B}_{10}}{\square}, \underset{\mathcal{B}_{11}}{\square} \right)$$

Two-dimensional discrete linear transform (1/2)

Assume that $\mathcal{I}$ has M rows and N columns and call Φ the *linear* transformation that maps $\mathcal{I} = [\mathcal{I}(m,n)]_{M,N}$ into $\mathcal{J} = [\mathcal{J}(u,v)]_{M,N}$ $\rightarrow \mathcal{J} = \Phi(\mathcal{I})$.



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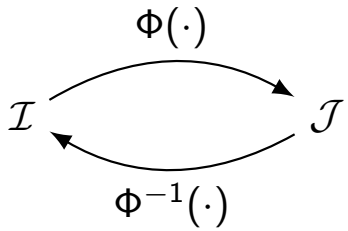

Most classical image transforms Φ are defined such that:

$$\mathcal{J}(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mathcal{I}(m,n)\phi(u,v,m,n)$$

where

- $\phi(u,v,m,n)$ is called the *forward transform kernel*,
- the variables $u = 0, \ldots, M-1$ and $v = 0, \ldots, N-1$ are called the *transform variables*.

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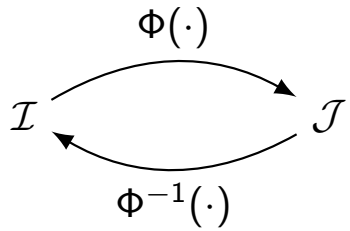

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- $\phi(u,v,m,n)$ is called the *forward transform kernel*,
- the variables $u = 0, \ldots, M-1$ and $v = 0, \ldots, N-1$ are called the *transform variables*.

The inverse transform is given by

$$\mathcal{I}(m,n) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \mathcal{J}(u,v)\psi(u,v,m,n)$$

with $\psi(u,v,m,n)$ being the *inverse transform kernel*.

Two-dimensional discrete linear transform (2/2)

The transform kernels are said to be *separable* if

$$\phi(u, v, m, n) = \phi_M(u, m)\phi_N(v, n)$$
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When the kernels are separable, the transform and inverse tranform can be compactly written in matrix form:

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When $M = N$, the transform kernels are said to be *symmetric* is they are functionnally equivalent (*i.e.* $\phi(u, v, m, n) = \phi(u, m)\phi(v, n)$ and $\psi(u, v, m, n) = \psi(u, m)\psi(v, n)$), and

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Obtaining the new basis images from the old ones

Bases $\mathcal{E}$ and $\mathcal{B}$ are obviously linked through the mapping Φ, but how?

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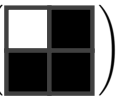
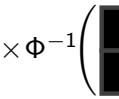
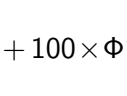
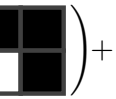
$= 50 \times \Phi^{-1}\Big($  $\Big) + 0 \times \Phi^{-1}\Big($  $\Big) + 100 \times \Phi^{-1}\Big($  $\Big) + 50 \times \Phi^{-1}\Big($  $\Big)$

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$$\Rightarrow \boxed{\mathcal{B}_{uv} = \Phi^{-1}(\mathcal{E}_{uv})}$$


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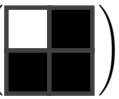
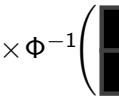
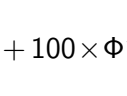
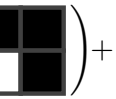
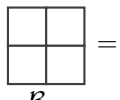
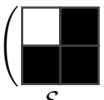
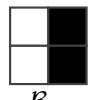
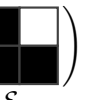
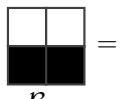
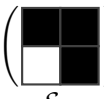
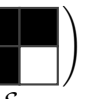
$= 50 \times \Phi^{-1}\left($  $\mathcal{E}_{00}\right) + 0 \times \Phi^{-1}\left($  $\mathcal{E}_{01}\right) + 100 \times \Phi^{-1}\left($  $\mathcal{E}_{10}\right) + 50 \times \Phi^{-1}\left($  $\mathcal{E}_{11}\right)$

 $\mathcal{B}_{00} = \Phi^{-1}\left($  $\mathcal{E}_{00}\right)$  $\mathcal{B}_{01} = \Phi^{-1}\left($  $\mathcal{E}_{01}\right)$

 $\mathcal{B}_{10} = \Phi^{-1}\left($  $\mathcal{E}_{10}\right)$  $\mathcal{B}_{11} = \Phi^{-1}\left($  $\mathcal{E}_{11}\right)$

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→ Named after Joseph Walsh and Jacques Hadamard.

→ Only works for square images of size $2^p \times 2^p$.

→ Can be implemented very efficiently (only requires addition and subtraction).

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Walsh-Hadamard transform

Assuming that $N = 2^p$, a $N \times N$ image $\mathcal{I}$ and its Walsh-Hadamard transform $\mathcal{J}_H$ are linked by

$$\mathcal{J}_H(u,v) = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} (-1)^{\sum_{k=0}^{p-1} m_k u_k + n_k v_k} \mathcal{I}(m,n)$$

$$\mathcal{I}(m,n) = \frac{1}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} (-1)^{\sum_{k=0}^{p-1} u_k m_k + v_k n_k} \mathcal{J}_H(u,v)$$

where

$$\begin{aligned} m &= \langle m_{p-1}, \ldots, m_1, m_0 \rangle_2 &&= 2^{p-1} m_{p-1} + \cdots + 2 m_1 + m_0 \\ n &= \langle n_{p-1}, \ldots, n_1, n_0 \rangle_2 &&= 2^{p-1} n_{p-1} + \cdots + 2 n_1 + n_0 \\ u &= \langle u_{p-1}, \ldots, u_1, u_0 \rangle_2 &&= 2^{p-1} u_{p-1} + \cdots + 2 u_1 + u_0 \\ v &= \langle v_{p-1}, \ldots, v_1, v_0 \rangle_2 &&= 2^{p-1} v_{p-1} + \cdots + 2 v_1 + v_0 \end{aligned}$$

are the base-2 representation of the indices m, n, u, v.

The Walsh-Hadamard kernel matrix

The Walsh-Hadamard transform $\mathcal{J}_{\mathbf{H}}$ (also called *Walsh-Hadamard spectrum*) of a $N \times N$ image $\mathcal{I}$ can be more conveniently defined thanks to the WHT kernel matrix $\mathbf{H}_N$ whose $(j,k)^{\text{th}}$ entry is

$$\mathbf{H}_N(j,k) = \frac{1}{\sqrt{N}}(-1)^{\sum_{n=0}^{p-1} j_n k_n}$$

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$$\mathbf{H}_1 = 1 \qquad \mathbf{H}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{H}_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

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And more generally

$$\forall N = 2^p,\ \mathbf{H}_{2N} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{H}_N & \mathbf{H}_N \\ \mathbf{H}_N & -\mathbf{H}_N \end{bmatrix}$$

The Walsh-Hadamard kernel matrix

The Walsh-Hadamard transform $\mathcal{J}_{\mathbf{H}}$ (also called *Walsh-Hadamard spectrum*) of a $N \times N$ image $\mathcal{I}$ can be more conveniently defined thanks to the WHT kernel matrix $\mathbf{H}_N$ whose $(j,k)^{\text{th}}$ entry is

$$\mathbf{H}_N(j,k) = \frac{1}{\sqrt{N}}(-1)^{\sum_{n=0}^{p-1} j_n k_n}$$

$$\mathbf{H}_1 = 1 \qquad \mathbf{H}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{H}_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

And more generally

$$\forall N = 2^p,\ \mathbf{H}_{2N} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{H}_N & \mathbf{H}_N \\ \mathbf{H}_N & -\mathbf{H}_N \end{bmatrix}$$

Hadamard matrices are symmetric ($\mathbf{H}_N = \mathbf{H}_N^T$) and orthogonal ($\mathbf{H}_N^{-1} = \mathbf{H}_N^T$).

$$\Rightarrow \text{All put together } \begin{cases} \mathcal{J}_{\mathbf{H}} &= \mathbf{H}_N \mathcal{I} \mathbf{H}_N \\ \mathcal{I} &= \mathbf{H}_N \mathcal{J}_{\mathbf{H}} \mathbf{H}_N \end{cases}$$

The Walsh-Hadamard transform for $N = 8$

The Walsh-Hadamard transform can be considered as a kind of Fourier transform $\rightarrow$ suited to perform the frequency analysis of an image.

$$\mathbf{H}_8 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

The Walsh-Hadamard transform for $N = 8$

The Walsh-Hadamard transform can be considered as a kind of Fourier transform $\rightarrow$ suited to perform the frequency analysis of an image.

For that, we must rearrange the rows of $\mathbf{H}_N$ in increasing number of sign changes.

$$\mathbf{H}_8 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

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$$\mathbf{H}_8 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \begin{matrix} \text{\# sign} \\ \text{changes} \\ 0 \\ 7 \\ 3 \\ 4 \\ 1 \\ 6 \\ 2 \\ 5 \end{matrix}$$

The Walsh-Hadamard transform for $N = 8$

The Walsh-Hadamard transform can be considered as a kind of Fourier transform $\rightarrow$ suited to perform the frequency analysis of an image.

For that, we must rearrange the rows of $\mathbf{H}_N$ in increasing number of sign changes.

$$\mathbf{H}_8 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{matrix} \text{\# sign changes} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix}$$

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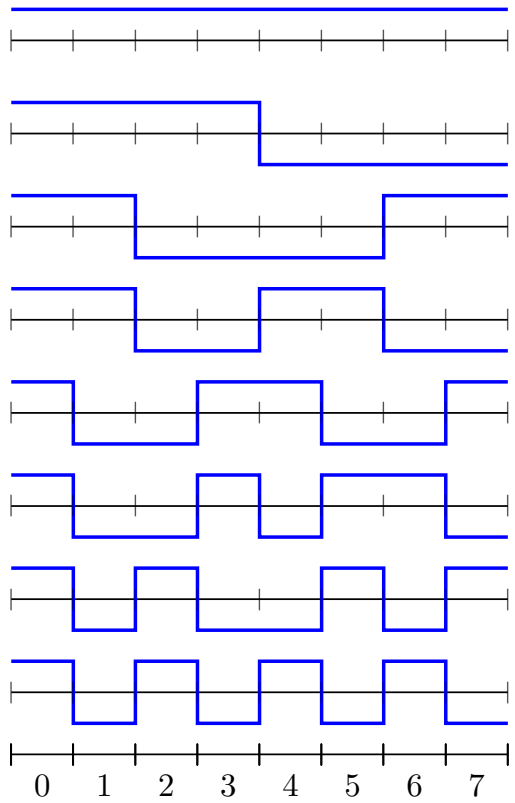
$$\mathbf{H}_8 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$



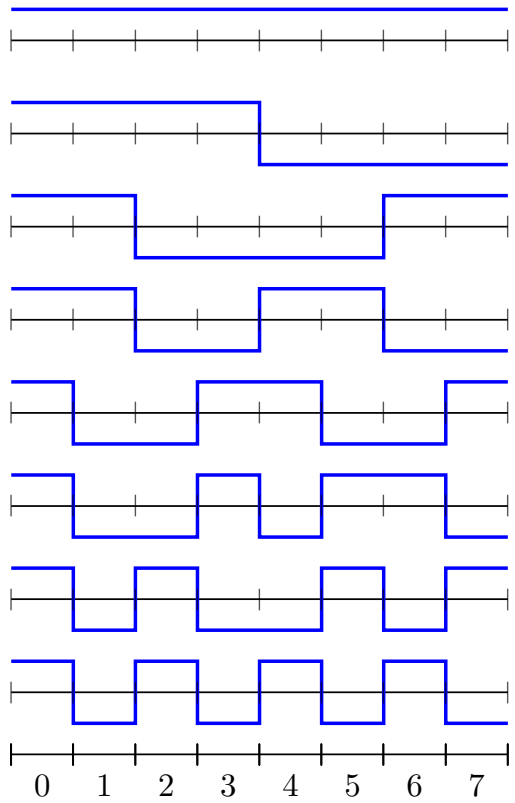
Walsh functions

The Walsh-Hadamard transform for $N = 8$

The Walsh-Hadamard transform can be considered as a kind of Fourier transform $\rightarrow$ suited to perform the frequency analysis of an image.

For that, we must rearrange the rows of $\mathbf{H}_N$ in increasing number of sign changes.

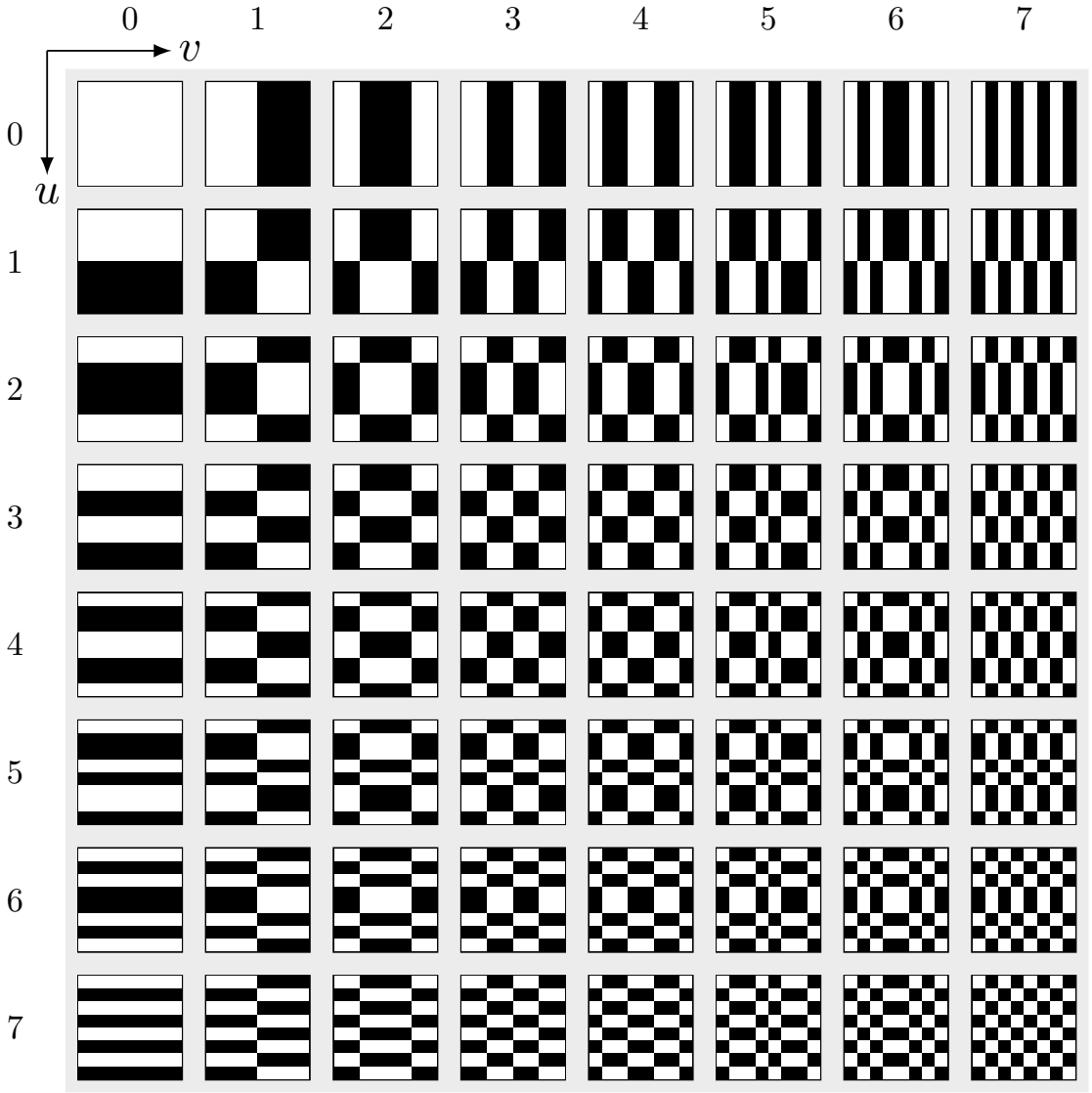
$$\mathbf{H}_8 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$



Walsh functions

Walsh-Hadamard transform $\equiv$ decomposition over the set of Walsh functions.

Walsh-Hadamard basis images for $N = 8$



Walsh-Hadamard basis images $\mathcal{B}_{uv}$ for $N = 8$. $\square = 1$, and $\blacksquare = -1$. The origin of each basis image is at its top-left corner.

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\Rightarrow \mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

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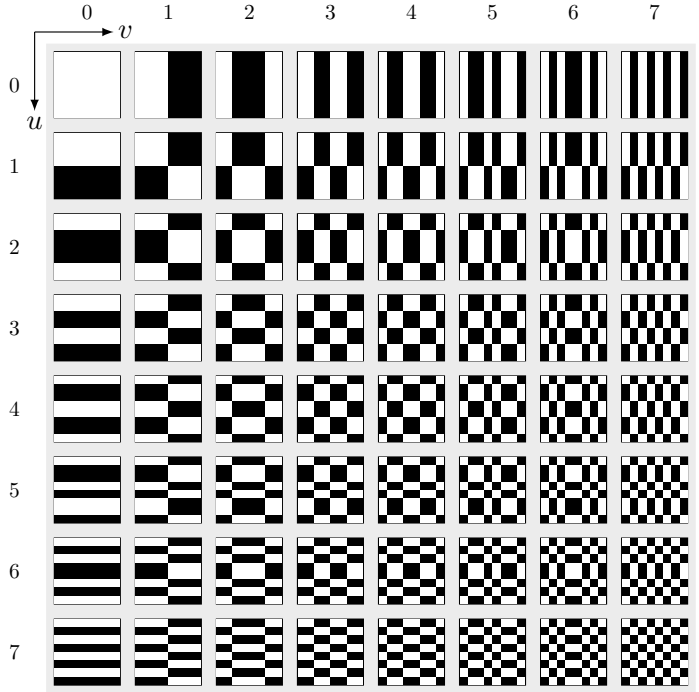
$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

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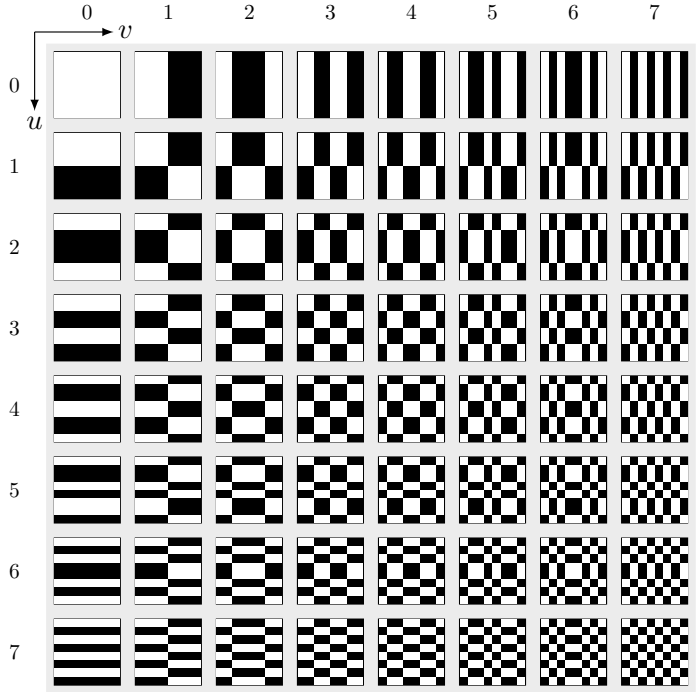

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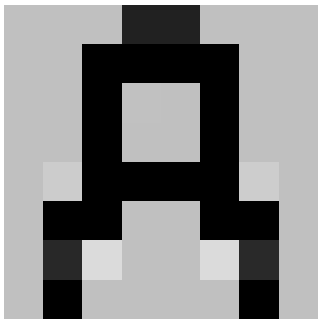
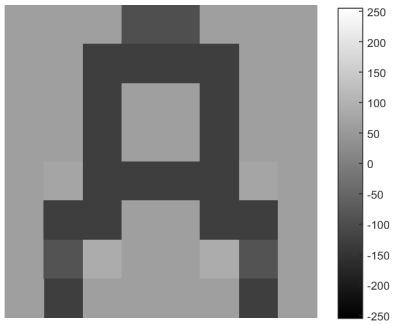


$\mathcal{I}$

$\tilde{\mathcal{I}}_1$



$\mathcal{I} - \tilde{\mathcal{I}}_1$

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

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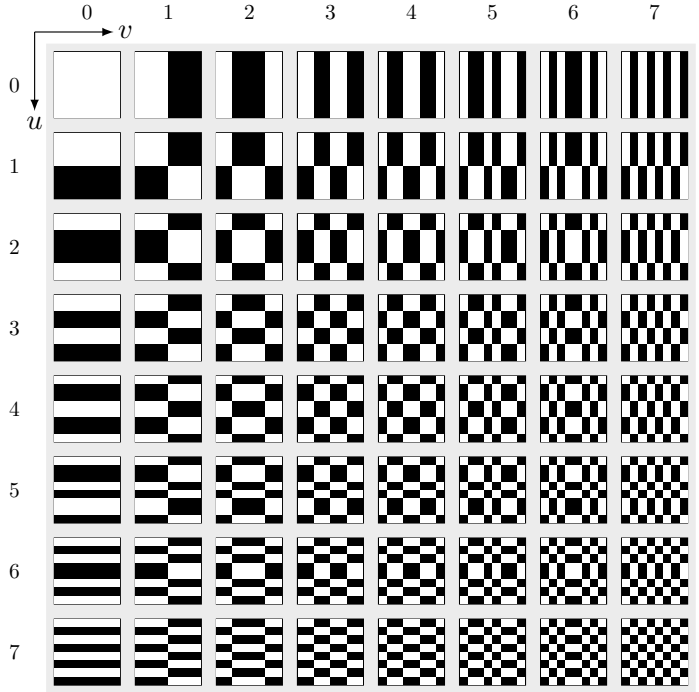

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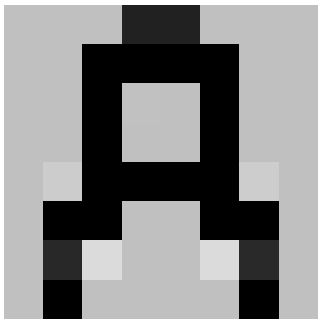
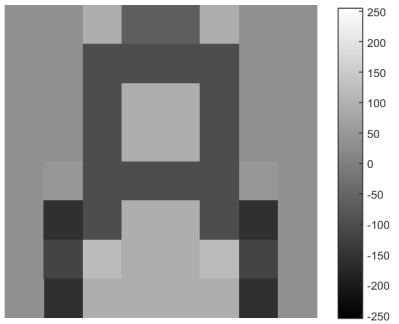


$\mathcal{I}$

$\tilde{\mathcal{I}}_2$



$\mathcal{I} - \tilde{\mathcal{I}}_2$

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

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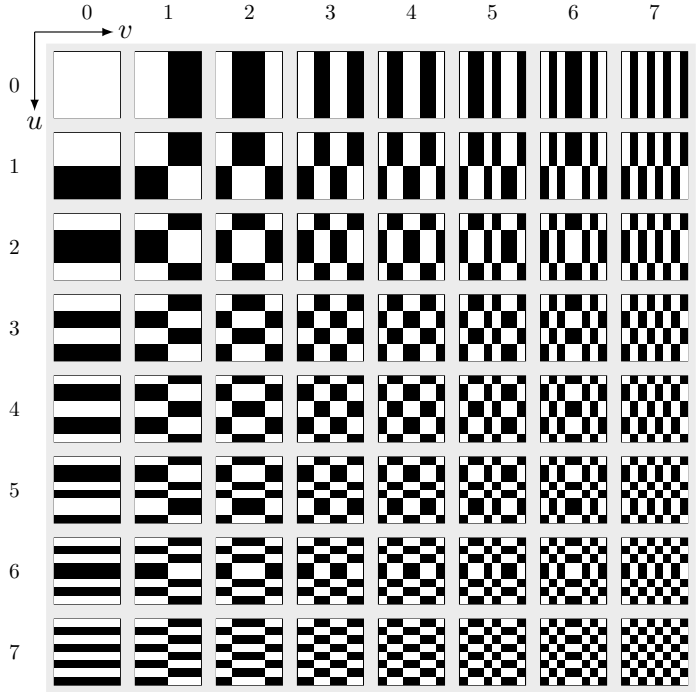

Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

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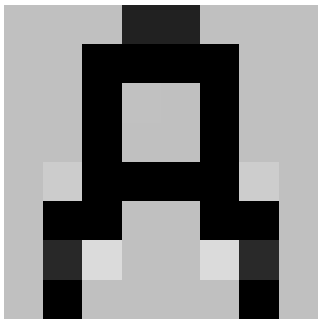
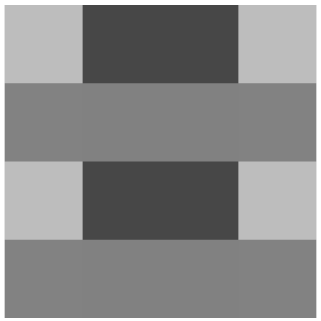
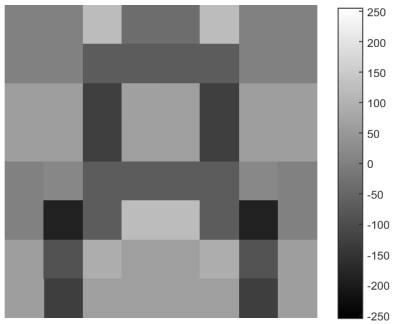


$\mathcal{I}$



$\tilde{\mathcal{I}}_3$



$\mathcal{I} - \tilde{\mathcal{I}}_3$

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

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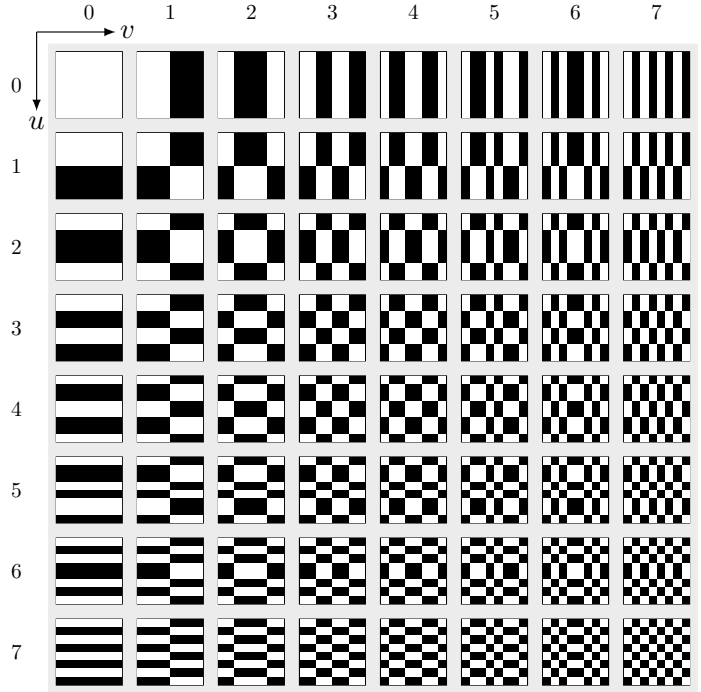
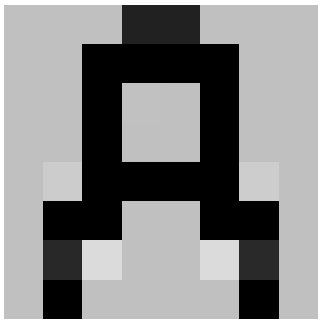
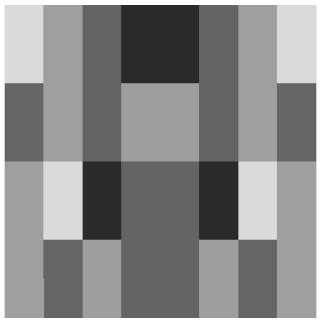


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

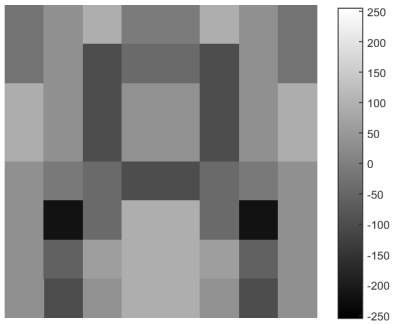
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_4$



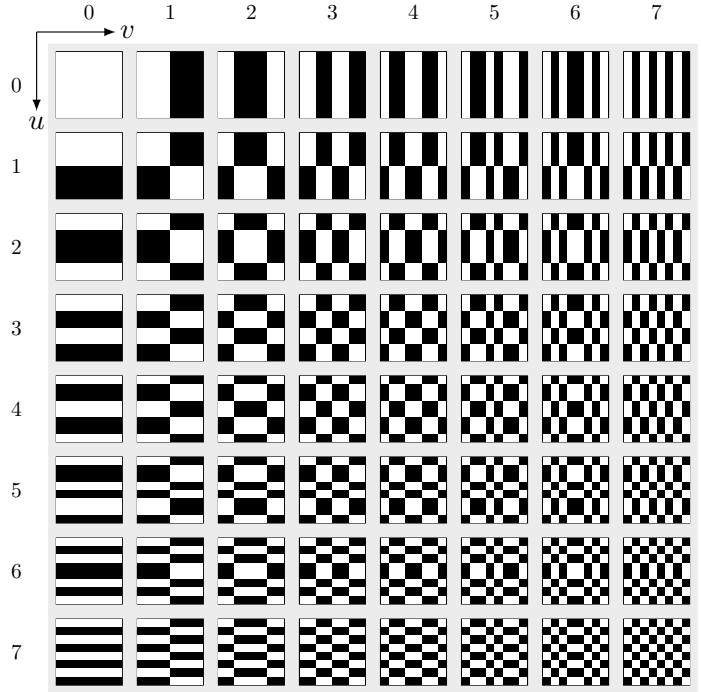
$\mathcal{I} - \tilde{\mathcal{I}}_4$

Example

... finally!

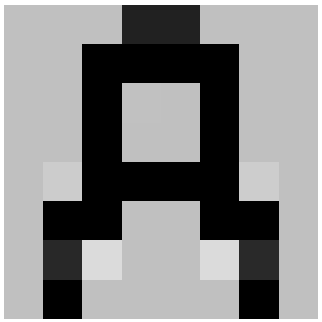
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

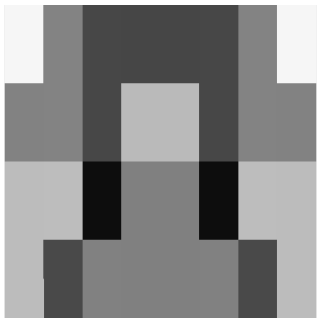


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

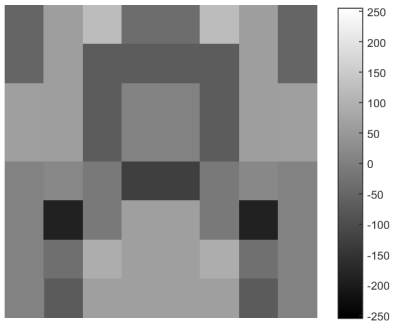
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_5$



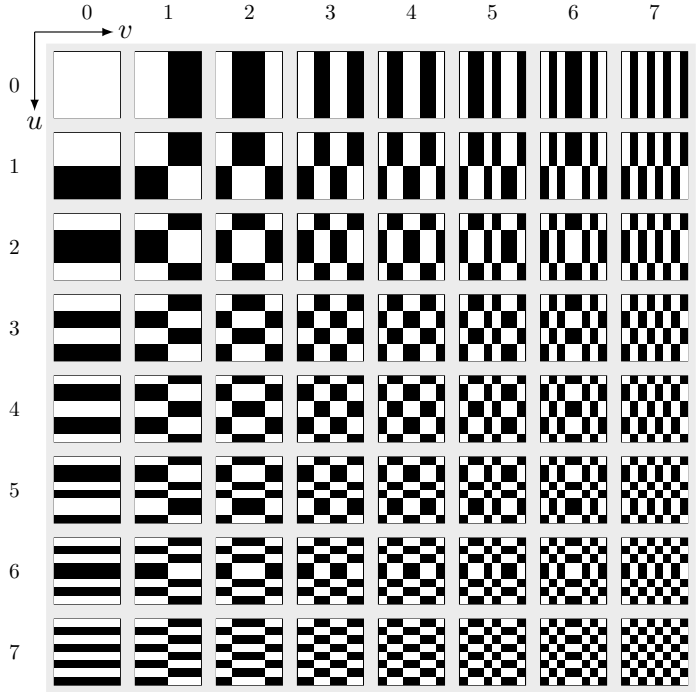
$\mathcal{I} - \tilde{\mathcal{I}}_5$

Example

... finally!

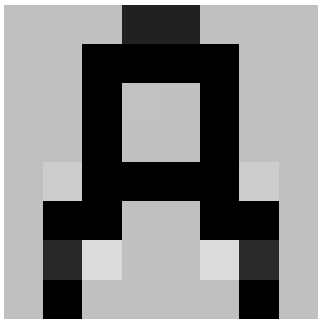
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$



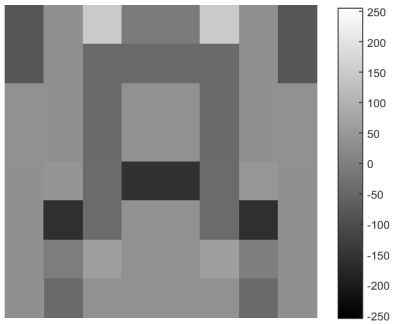
Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$

$\tilde{\mathcal{I}}_6$



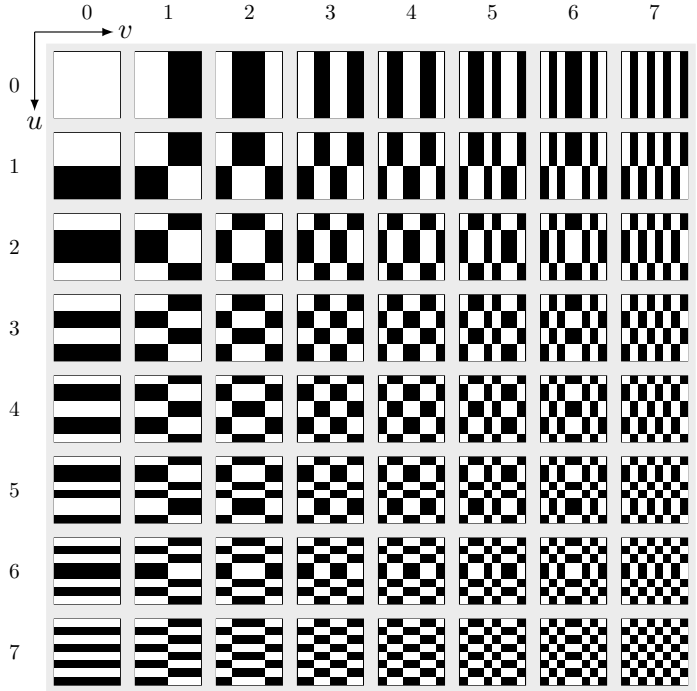
$\mathcal{I} - \tilde{\mathcal{I}}_6$

Example

... finally!

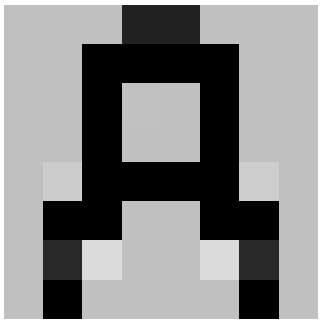
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

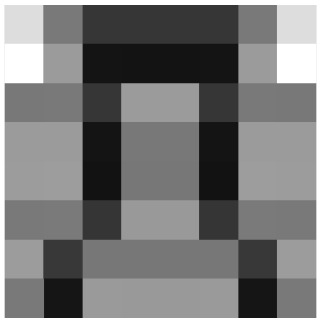


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

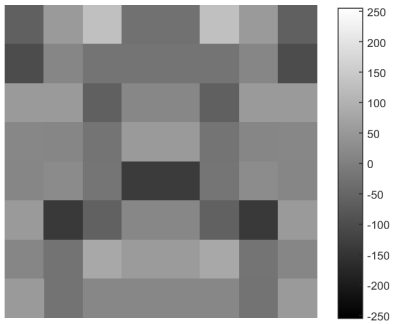
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_7$



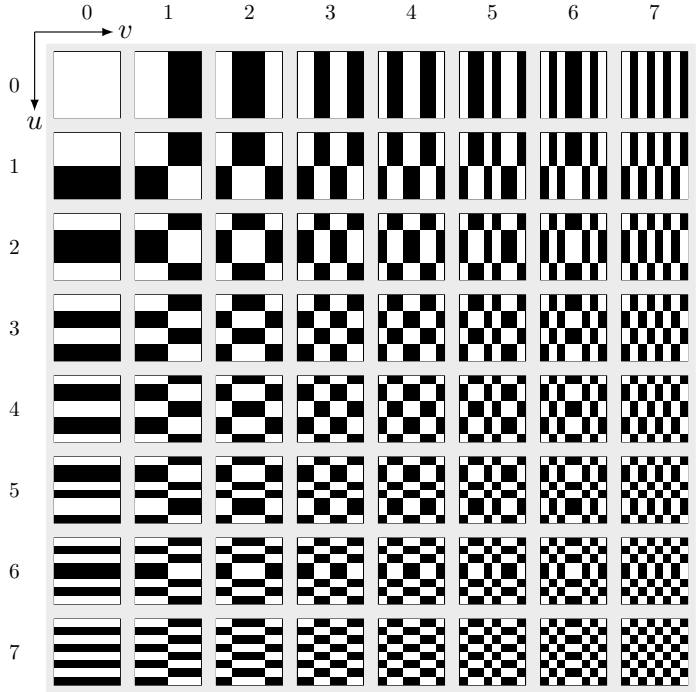
$\mathcal{I} - \tilde{\mathcal{I}}_7$

Example

... finally!

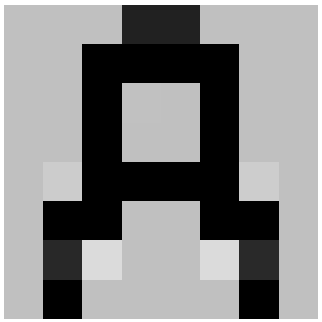
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

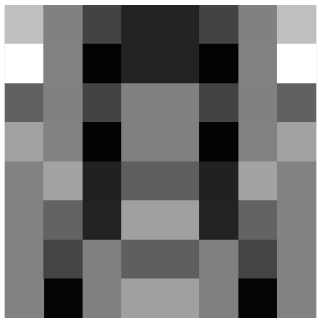


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

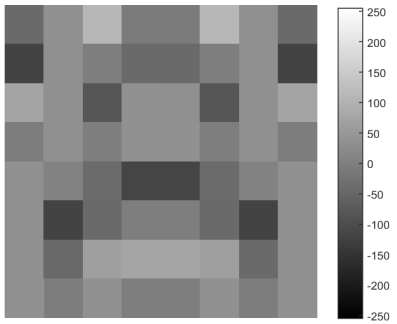
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_8$



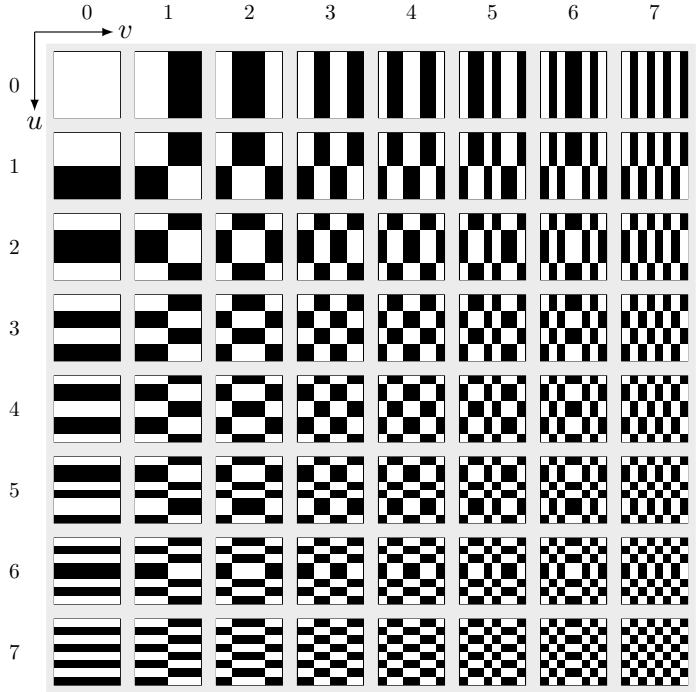
$\mathcal{I} - \tilde{\mathcal{I}}_8$

Example

... finally!

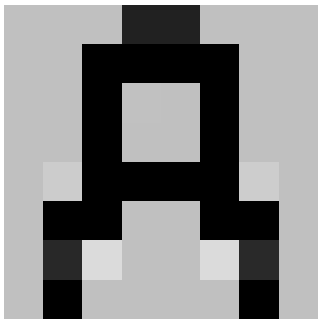
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

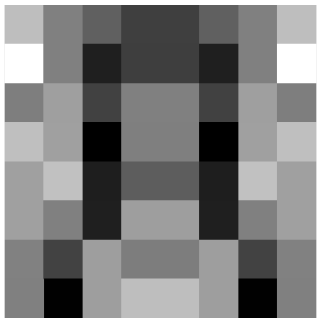


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

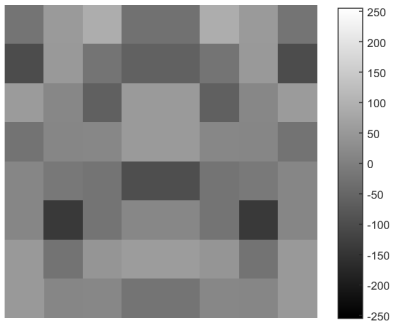
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_9$



$\mathcal{I} - \tilde{\mathcal{I}}_9$

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

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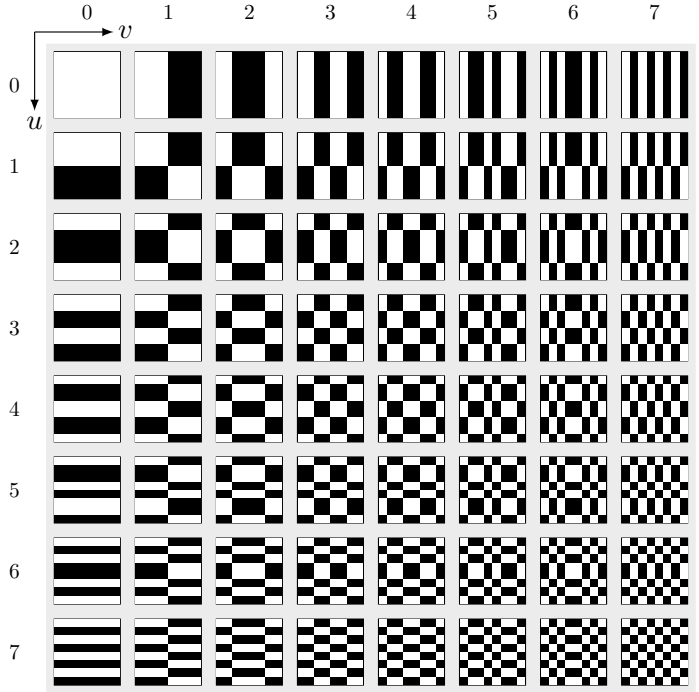
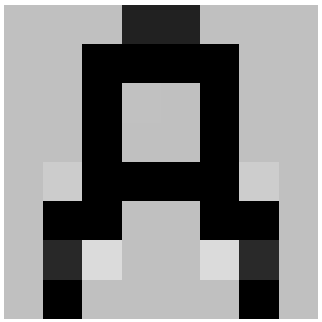
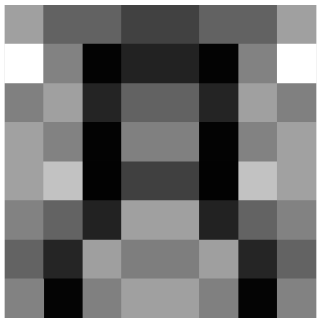


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

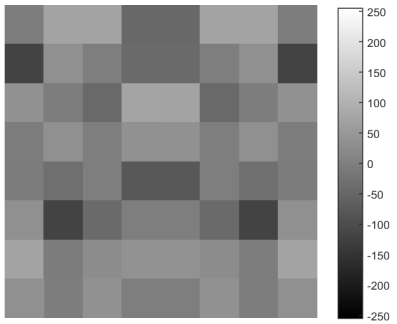
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{10}$



$\mathcal{I} - \tilde{\mathcal{I}}_{10}$

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

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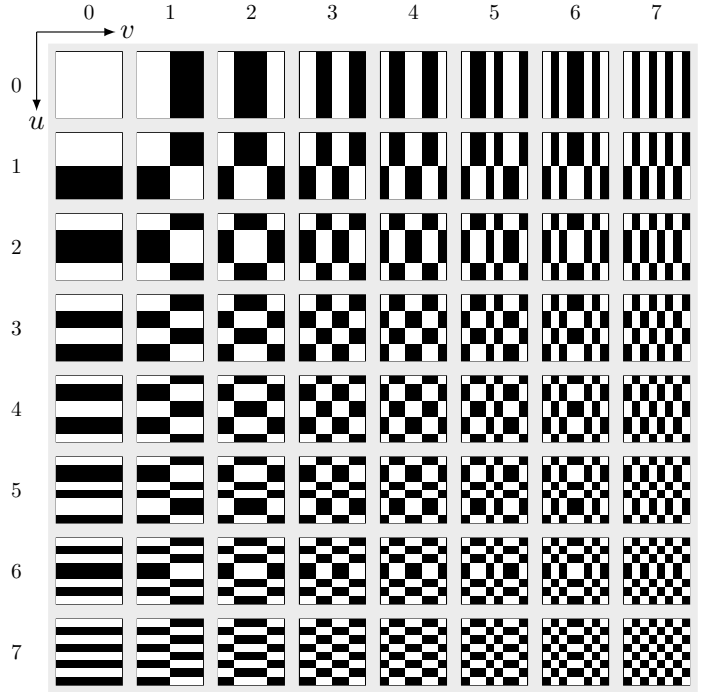
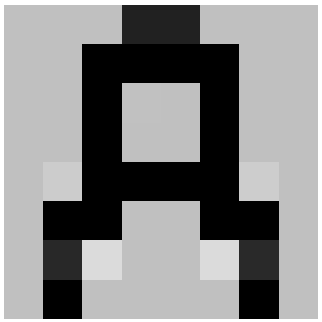
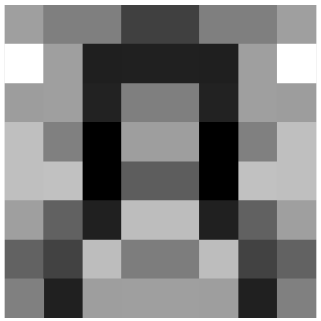


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

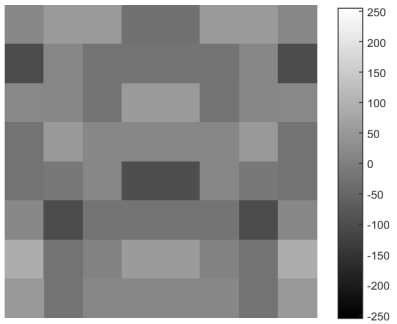
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{11}$



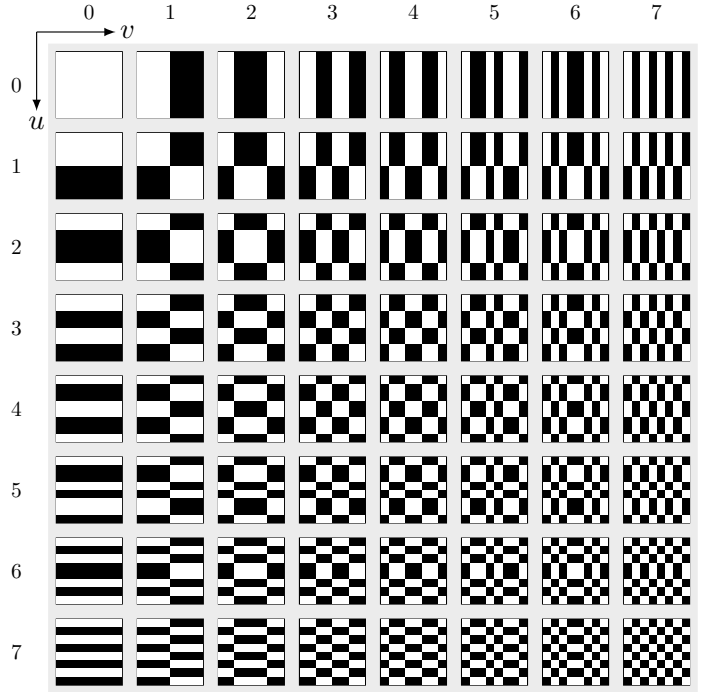
$\mathcal{I} - \tilde{\mathcal{I}}_{11}$

Example

... finally!

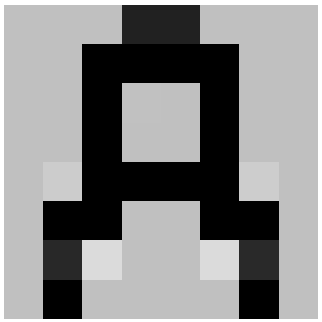
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

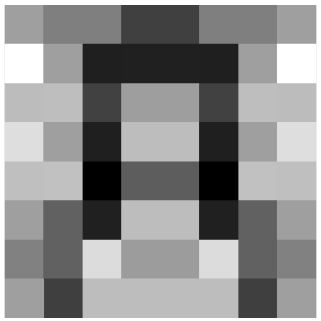


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

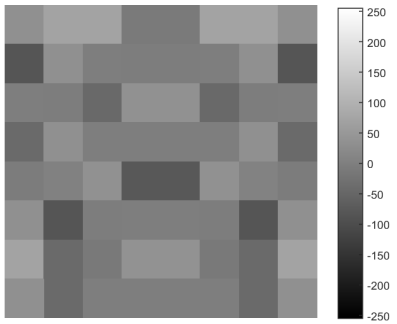
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{12}$



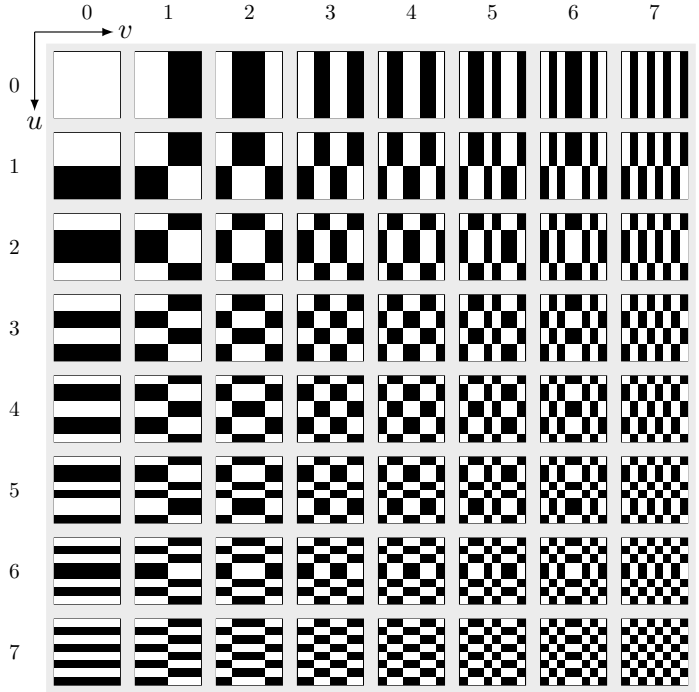
$\mathcal{I} - \tilde{\mathcal{I}}_{12}$

Example

... finally!

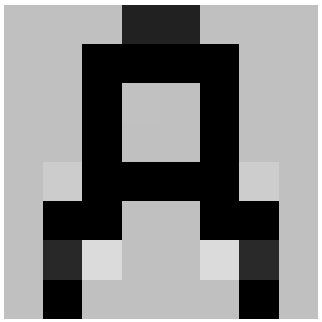
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

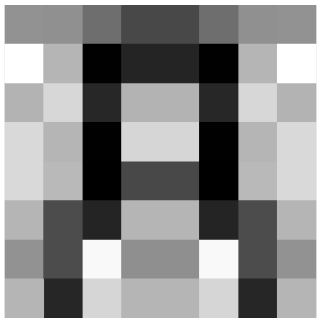


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

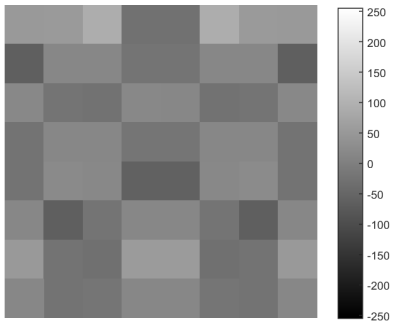
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{13}$



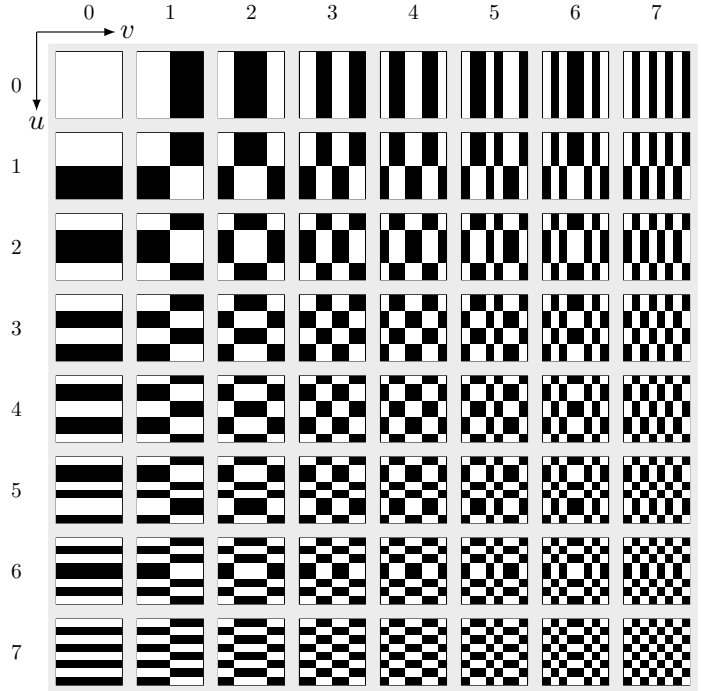
$\mathcal{I} - \tilde{\mathcal{I}}_{13}$

Example

... finally!

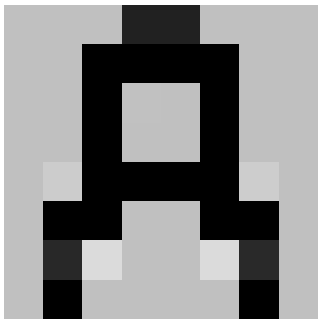
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

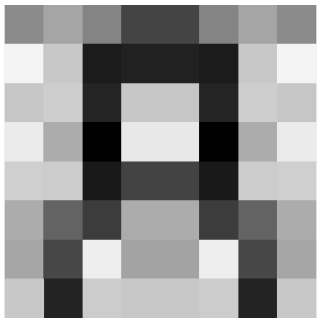


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

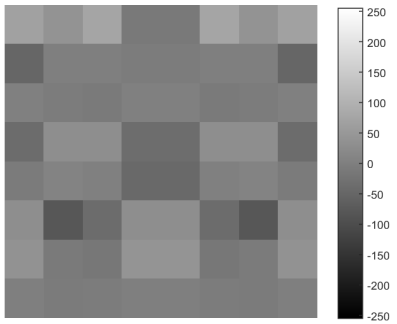
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{14}$



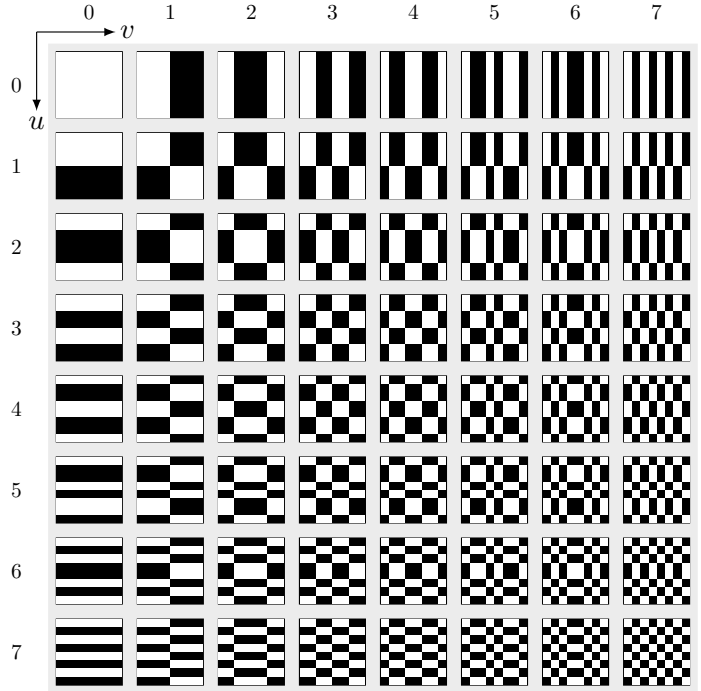
$\mathcal{I} - \tilde{\mathcal{I}}_{14}$

Example

... finally!

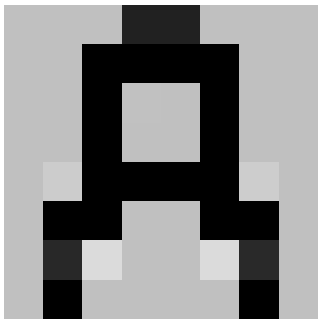
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$



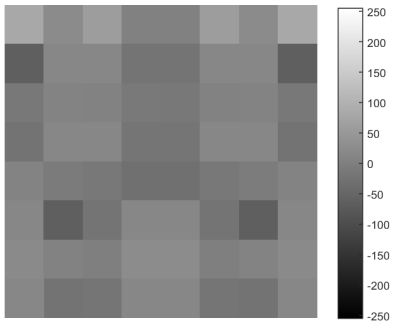
Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$

$\tilde{\mathcal{I}}_{15}$



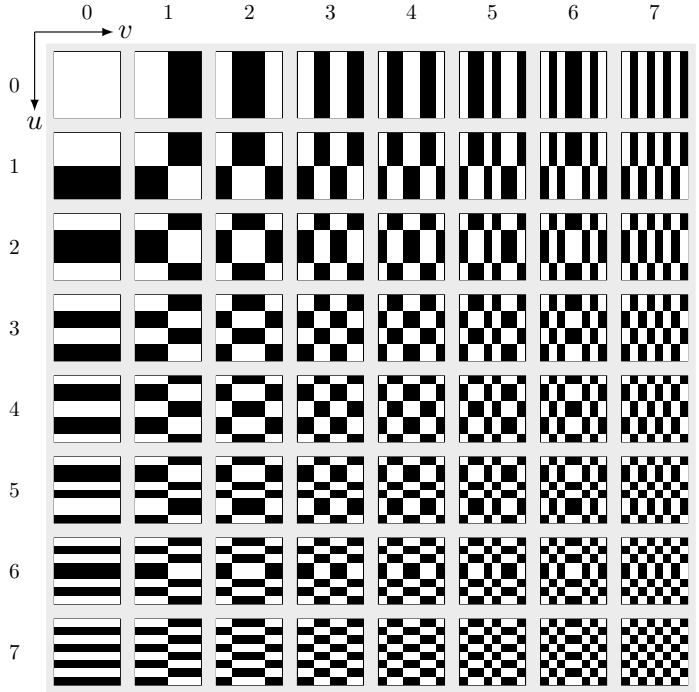
$\mathcal{I} - \tilde{\mathcal{I}}_{15}$

Example

... finally!

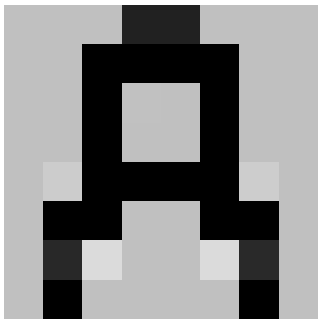
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

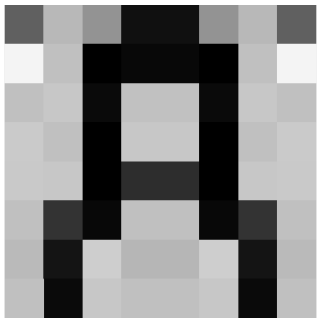


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

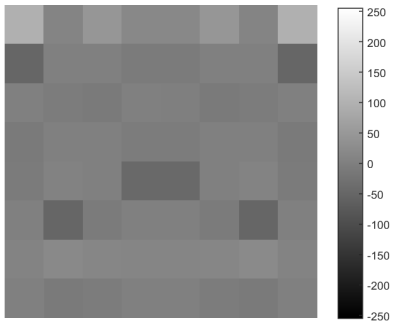
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{16}$



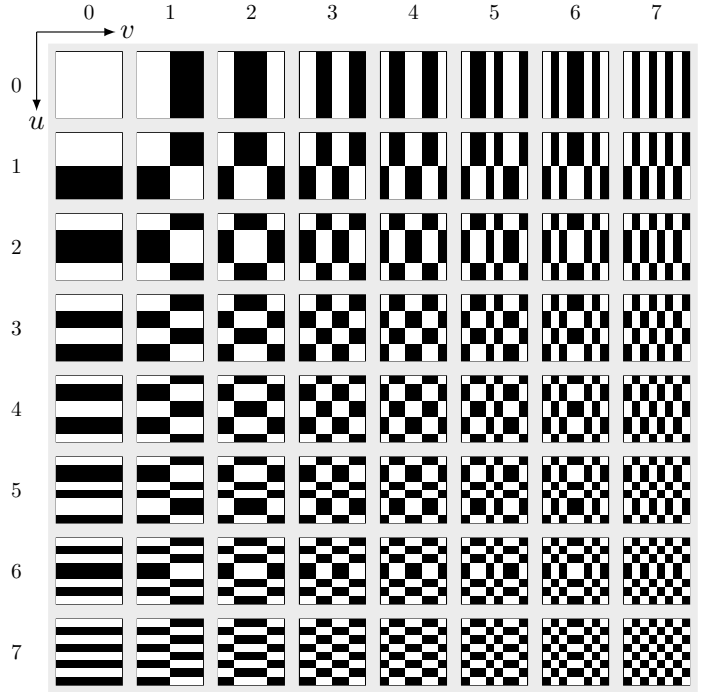
$\mathcal{I} - \tilde{\mathcal{I}}_{16}$

Example

... finally!

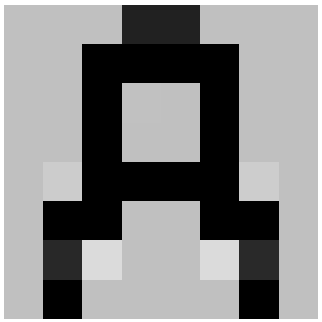
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

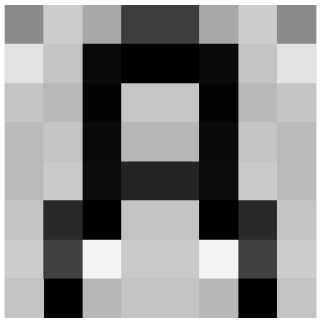


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

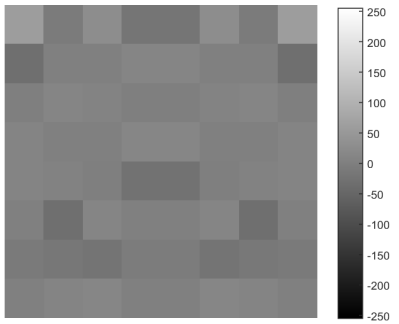
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{20}$



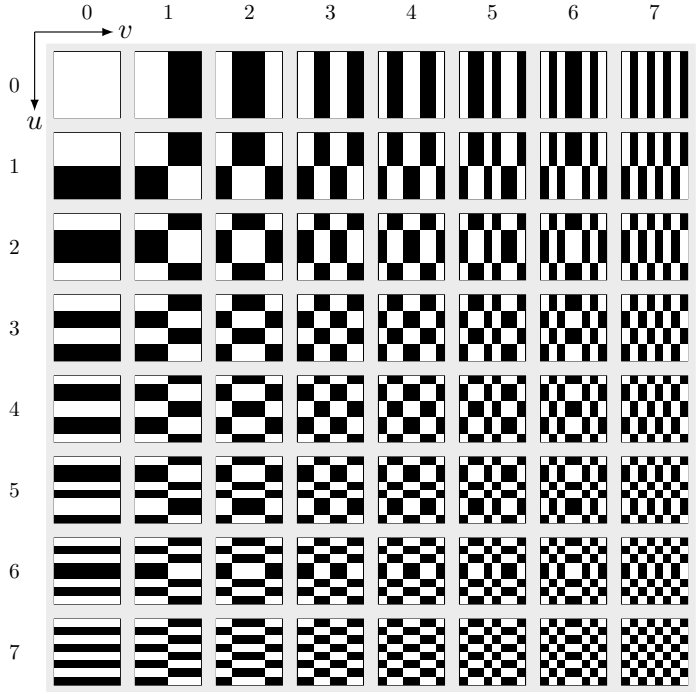
$\mathcal{I} - \tilde{\mathcal{I}}_{20}$

Example

... finally!

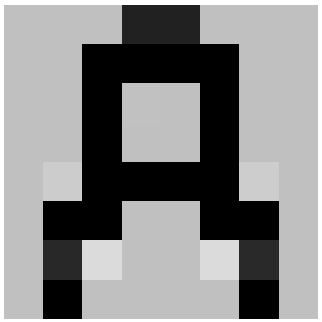
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

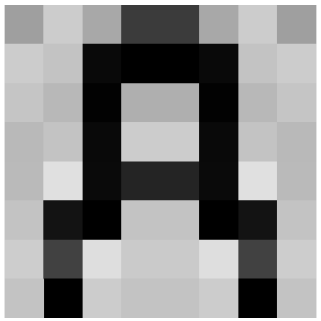


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{24}$

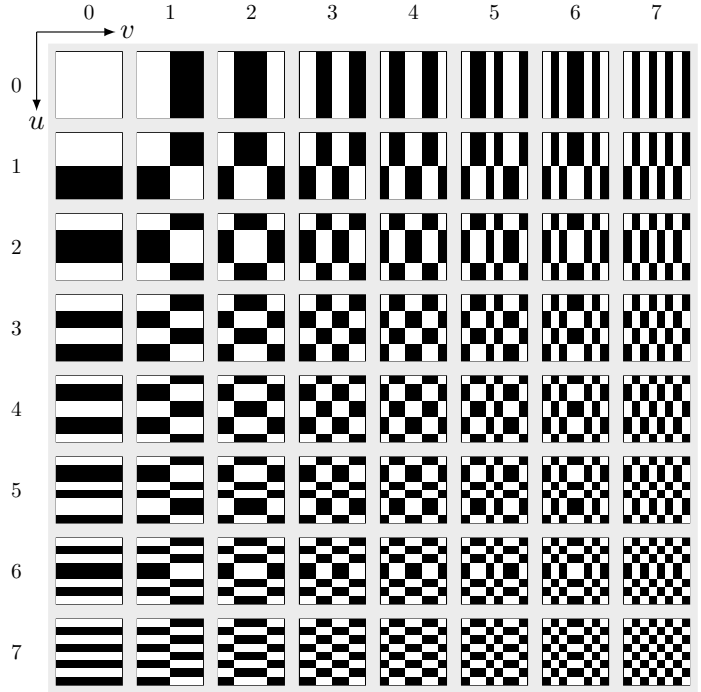
$\mathcal{I} - \tilde{\mathcal{I}}_{24}$

Example

... finally!

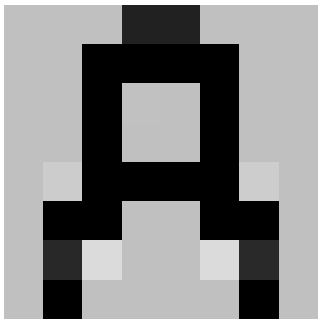
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 265 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 265 & 1 \end{bmatrix}$$

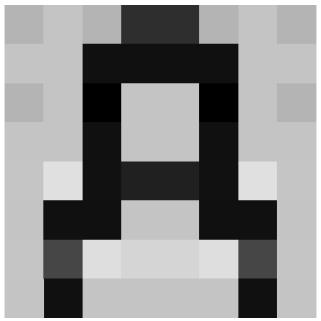


Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

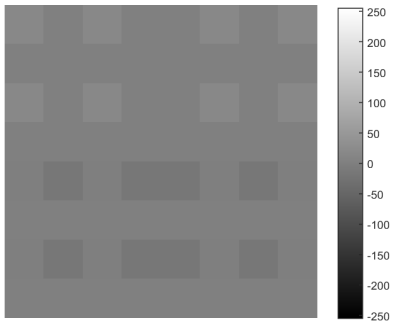
$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$



$\tilde{\mathcal{I}}_{28}$



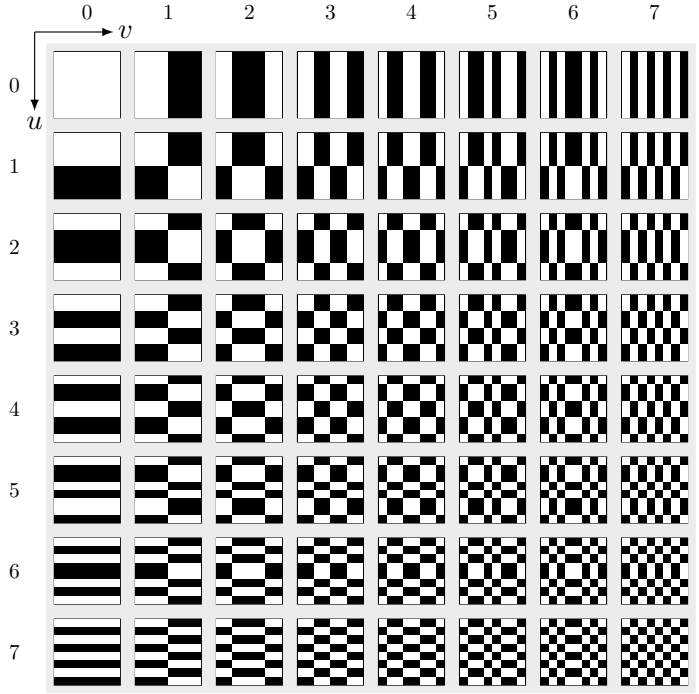
$\mathcal{I} - \tilde{\mathcal{I}}_{28}$

Example

... finally!

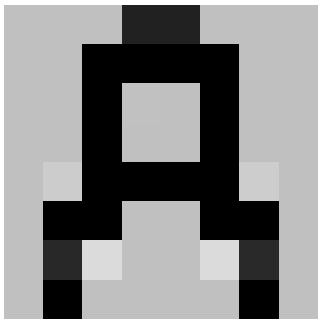
Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 291 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 291 & 1 \end{bmatrix}$$



Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

$\rightarrow$ approximate $\tilde{\mathcal{I}}$ using the leading coefficients of $\mathcal{J}_{\mathbf{H}}$.



$\mathcal{I}$

$\tilde{\mathcal{I}}_{32}$

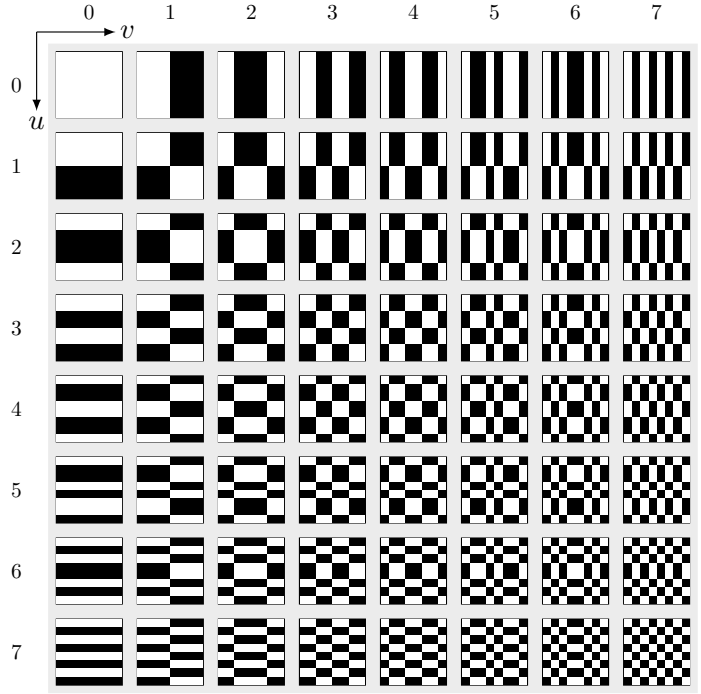
$\mathcal{I} - \tilde{\mathcal{I}}_{32}$

Example

... finally!

Let's take this 8×8 image $\mathcal{I} =$  and compute its Walsh-Hadamard spectrum by $\mathcal{J}_{\mathbf{H}} = \mathbf{H}_8 \mathcal{I} \mathbf{H}_8$.

$$\mathcal{J}_{\mathbf{H}} = \frac{1}{8} \begin{bmatrix} 8291 & -1 & 1905 & -3 & 1827 & 3 & 291 & 1 \\ 291 & 3 & 1801 & 1 & -925 & -1 & -1167 & -3 \\ 559 & -1 & -1215 & 1 & -1197 & -1 & 1853 & 1 \\ -1197 & -1 & 1853 & 1 & -977 & -1 & 321 & 1 \\ 339 & -1 & 317 & 1 & -977 & -1 & 321 & 1 \\ 559 & -1 & -1215 & 1 & 339 & -1 & 317 & 1 \\ 291 & 3 & -1271 & 1 & 611 & -1 & 369 & -3 \\ 611 & -1 & 369 & -3 & -1245 & 3 & 291 & 1 \end{bmatrix}$$



Theory gives $\mathcal{I} = \sum_{u=0}^{7} \sum_{v=0}^{7} \mathcal{J}_{\mathbf{H}}(u, v) \mathcal{B}_{uv}$.

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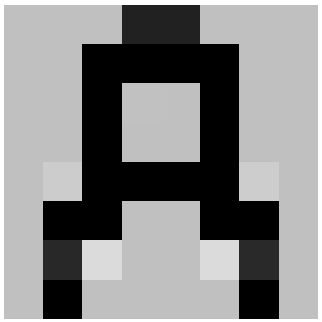


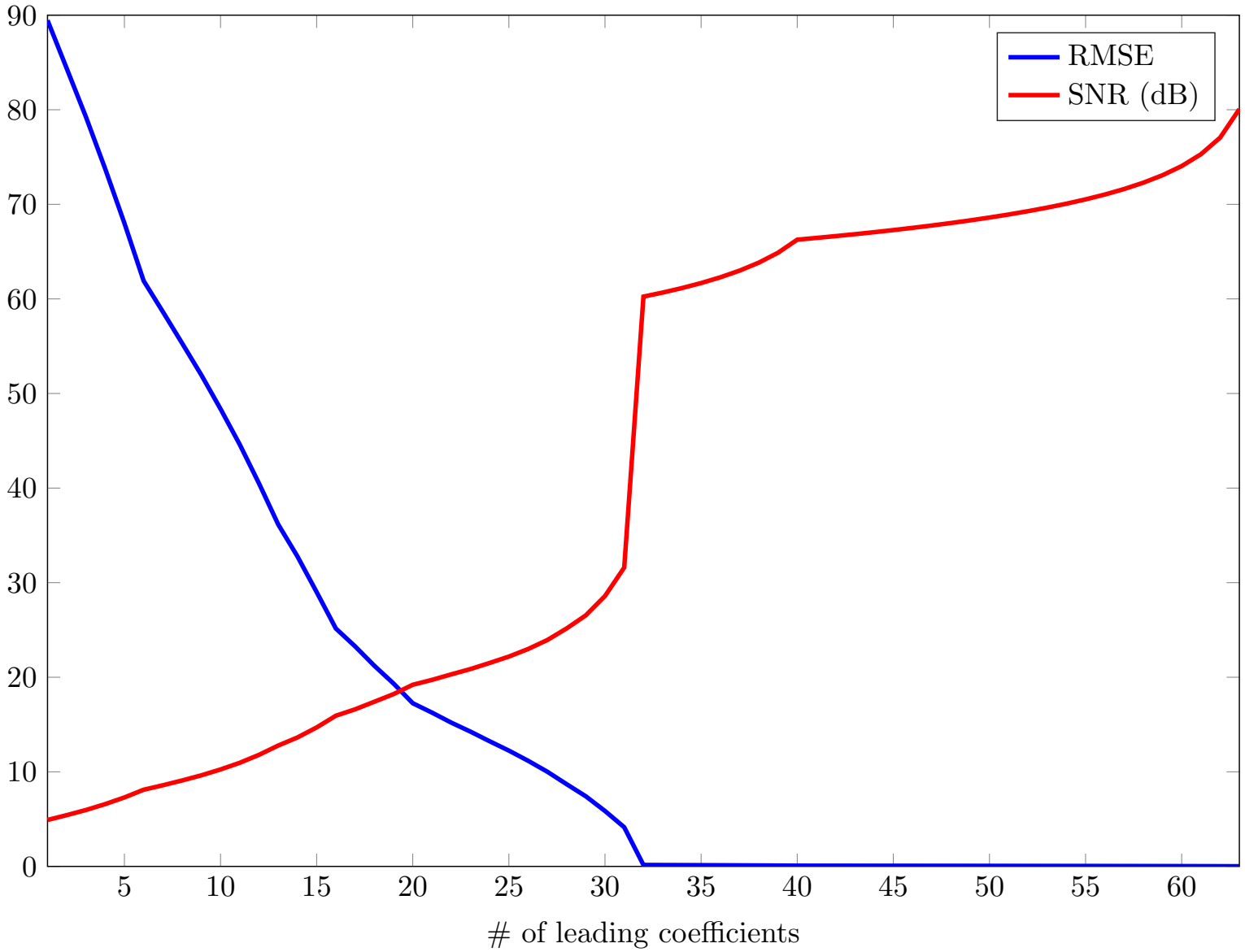
$\mathcal{I}$

$\tilde{\mathcal{I}}_{64}$

$\mathcal{I} - \tilde{\mathcal{I}}_{64}$

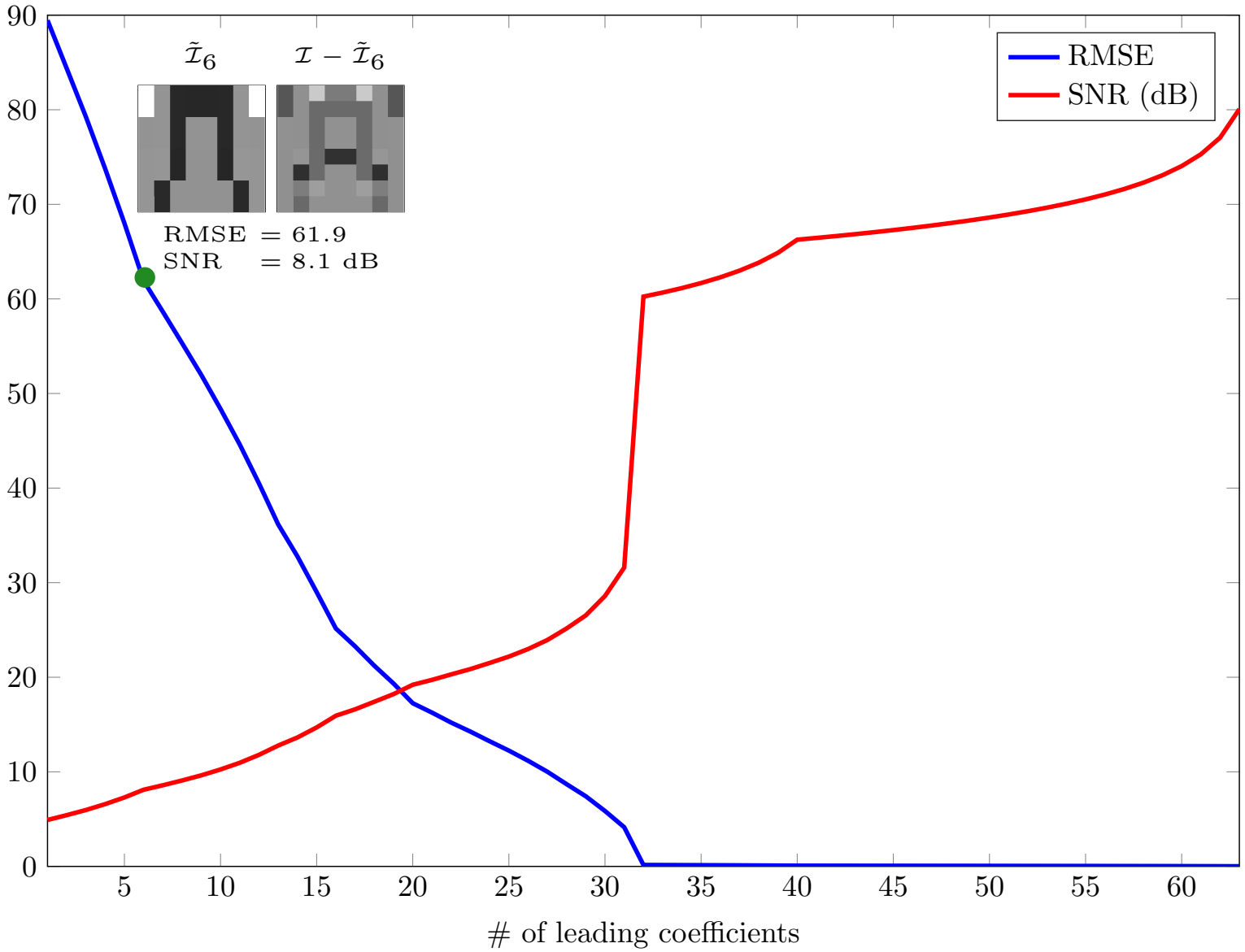
Walsh-Hadamard reconstruction error

What is really happening



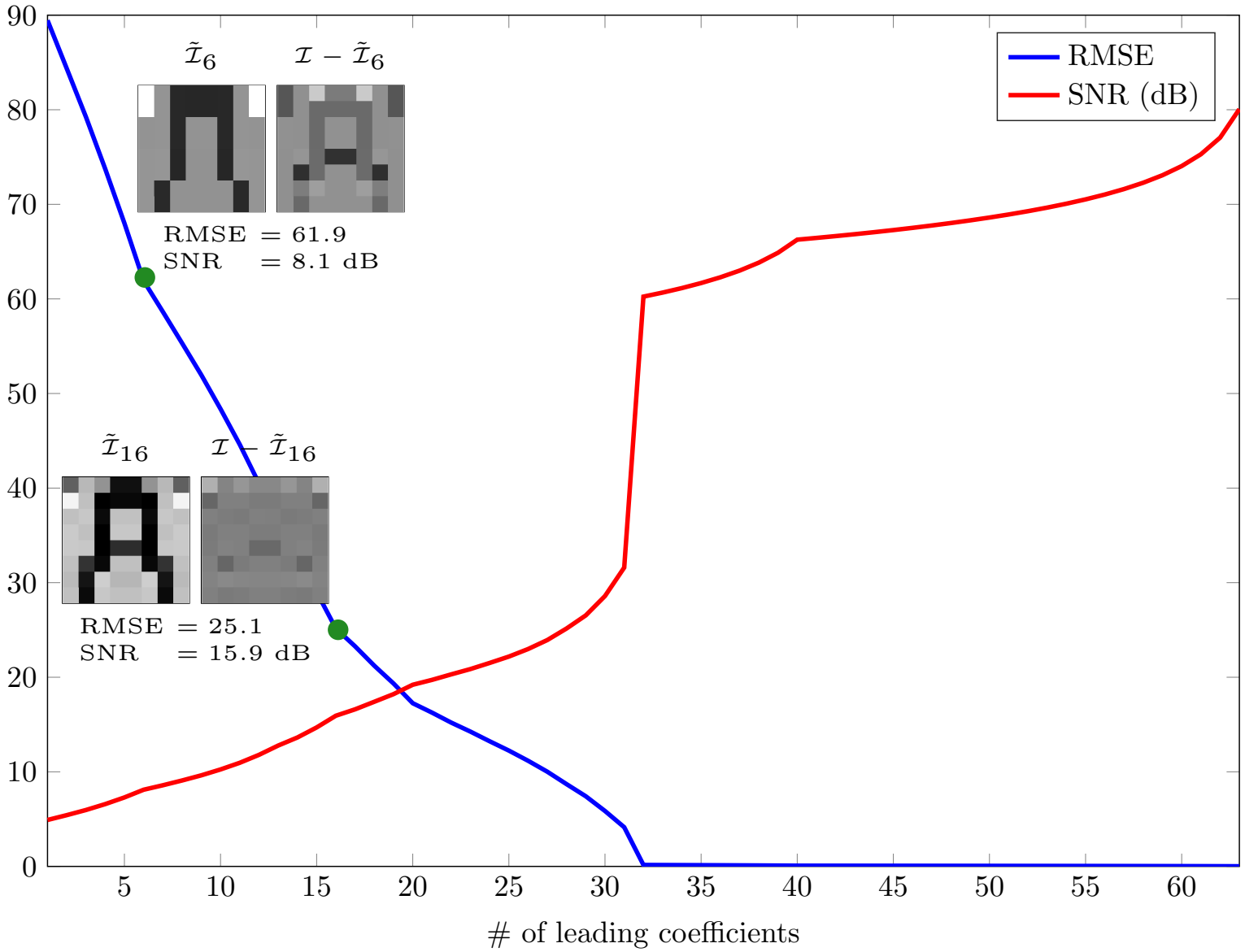
Walsh-Hadamard reconstruction error

What is really happening



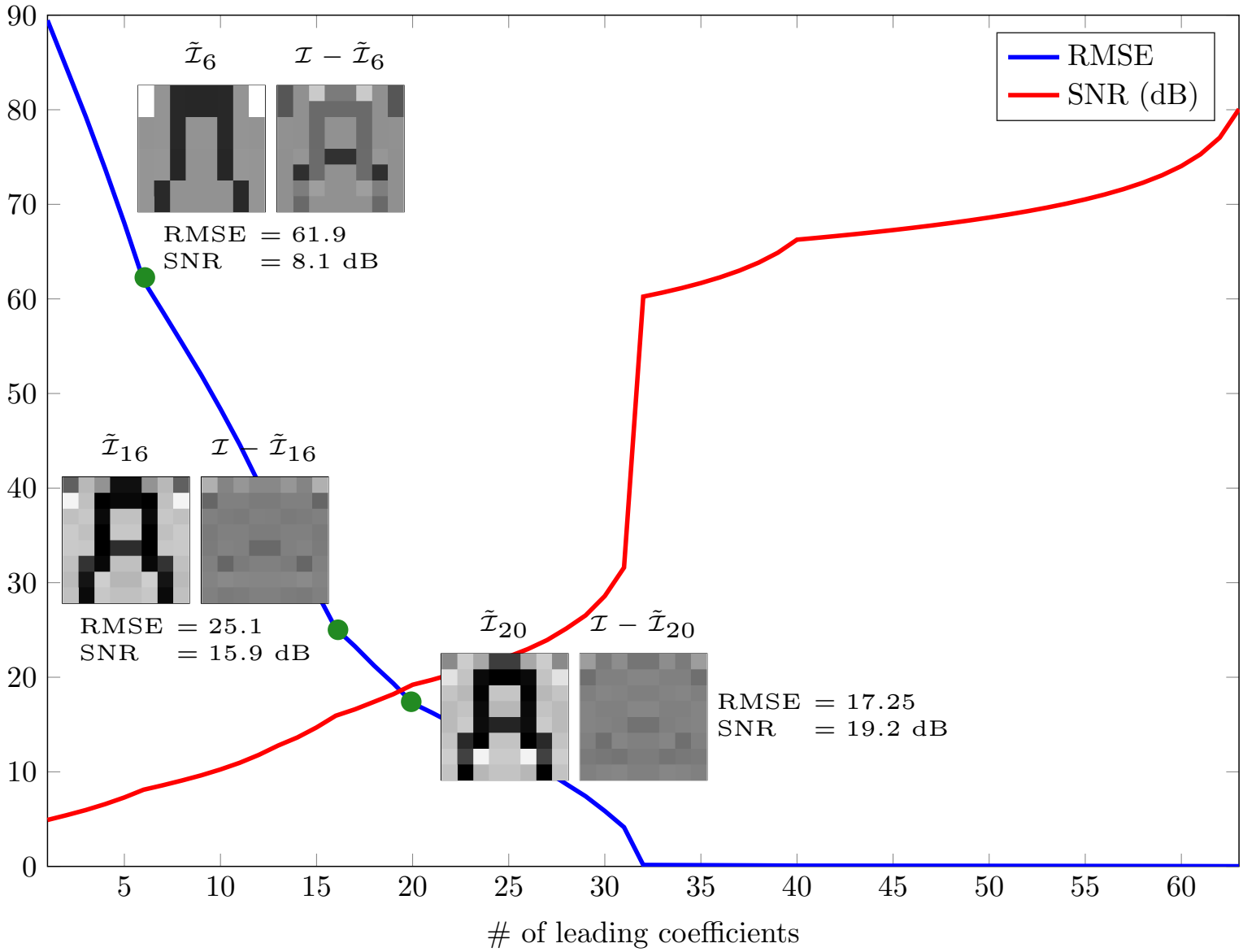
Walsh-Hadamard reconstruction error

What is really happening



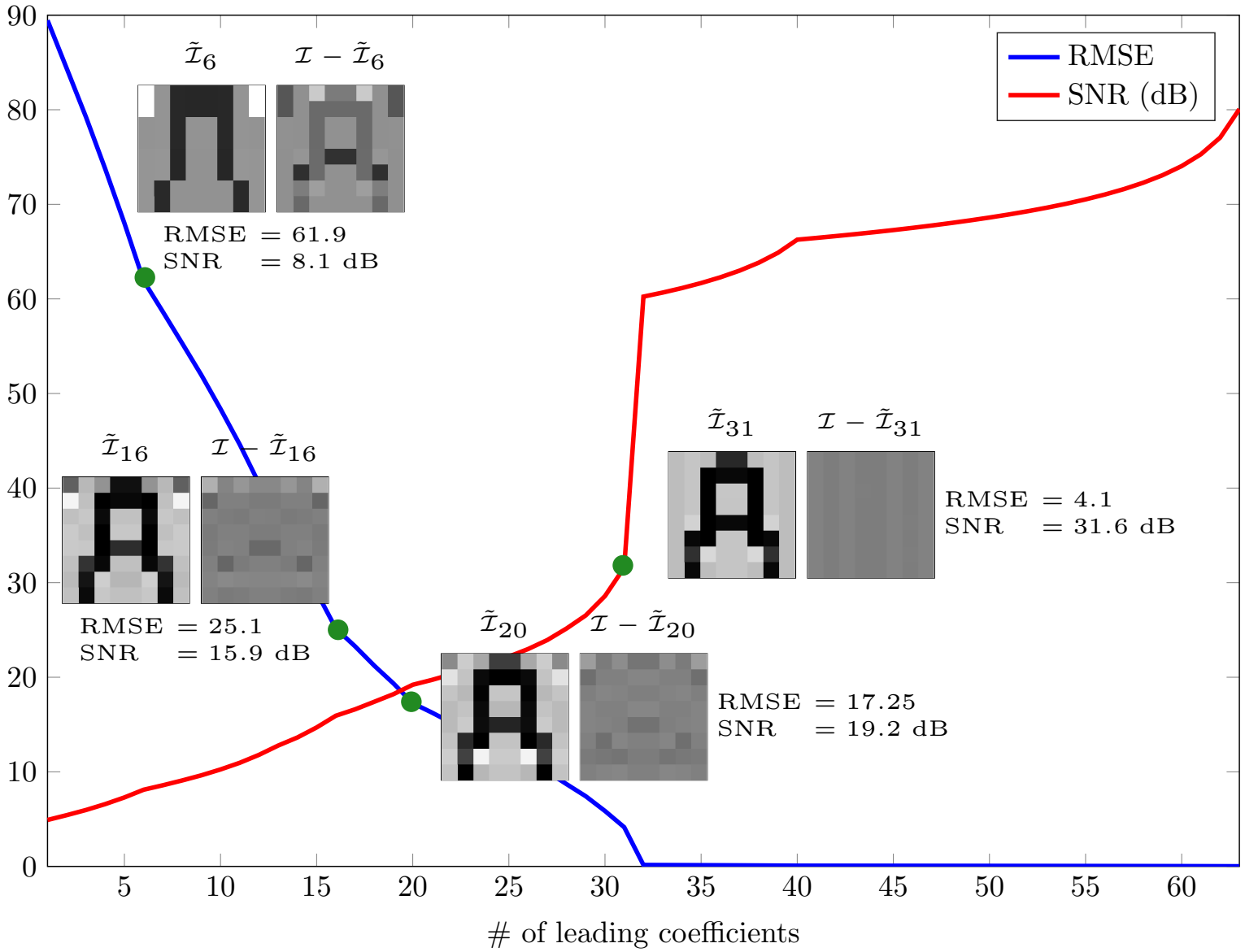
Walsh-Hadamard reconstruction error

What is really happening



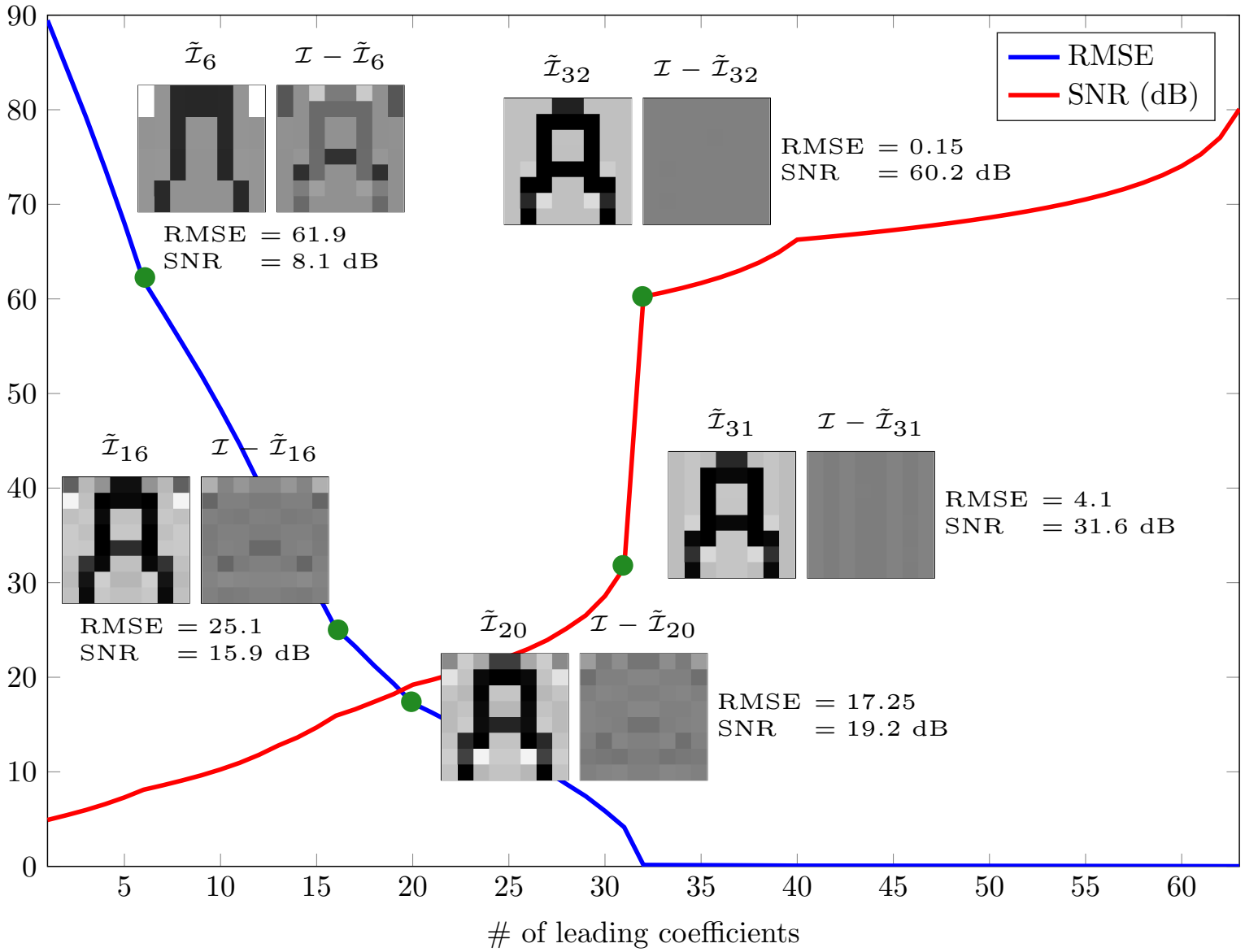
Walsh-Hadamard reconstruction error

What is really happening



Walsh-Hadamard reconstruction error

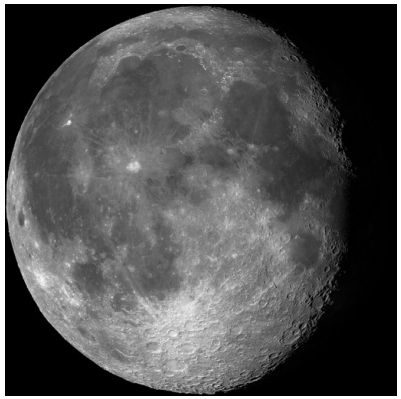
What is really happening



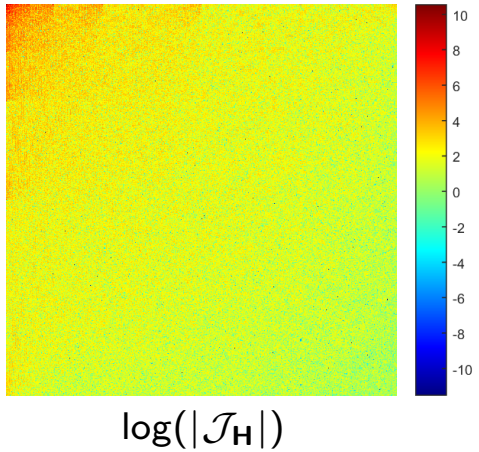
Summary on the Walsh-Hadamard transform

The Walsh-Hadamard transform was used in practice in the 60's in several space missions to compress images of:

- $\rightarrow$ the far side of the moon (*Luna 3* Soviet probe).
- $\rightarrow$ Jupiter, Saturn, Uranus, Neptune and their moons (*Mariner* and *Voyager* space probes).



512×512 image $\mathcal{I}$



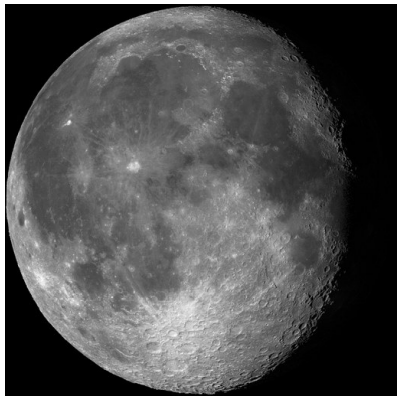
$\log(|\mathcal{J}_{\mathbf{H}}|)$

Summary on the Walsh-Hadamard transform

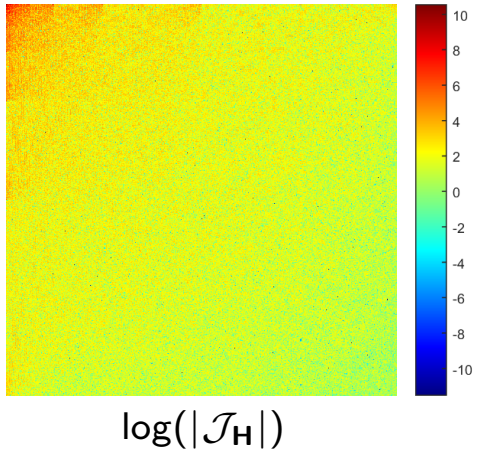
The Walsh-Hadamard transform was used in practice in the 60's in several space missions to compress images of:

→ the far side of the moon (*Luna 3* Soviet probe).

→ Jupiter, Saturn, Uranus, Neptune and their moons (*Mariner* and *Voyager* space probes).



512 × 512 image $\mathcal{I}$



$\log(|\mathcal{J}_{\mathbf{H}}|)$

✓ Suited to perform the frequency analysis of an image.

✓ Very fast, only requires addition and subtraction.

✗ Image dimensions must be a power of 2.

✓ Very efficient FFT-like implementation possible.

✗ Not so good to compact the image energy in a very few coefficients.

In the continuous setting.

If $f : \mathbb{R} \to \mathbb{R}, t \mapsto f(t)$ is a continuous and integrable function, and $\hat{f} : \mathbb{R} \to \mathbb{C}$, $\nu \mapsto \hat{f}(\nu)$ is its Fourier transform, then f and $\hat{f}$ are linked by:

$$\hat{f}(\nu) = \int_{-\infty}^{+\infty} f(t) e^{-i2\pi\nu t} dt \quad \text{and} \quad f(t) = \int_{-\infty}^{+\infty} \hat{f}(\nu) e^{i2\pi\nu t} d\nu$$

The discrete Fourier transform

In 1D

In the continuous setting.

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In the discrete setting.

Let $f = \{f(0), f(1), \ldots, f(M-1)\}$ be a discrete function of length M. Its *discrete* Fourier transform is the complex-valued function $\hat{f} = \{\hat{f}(0), \hat{f}(1), \ldots, \hat{f}(M-1)\}$ of length M defined as:

$$\hat{f}(u) = \sum_{m=0}^{M-1} f(m) e^{-i2\pi \frac{mu}{M}} \quad \text{and} \quad f(m) = \frac{1}{M} \sum_{u=0}^{M-1} \hat{f}(u) e^{i2\pi \frac{mu}{M}}$$

An alternative definition uses a $\frac{1}{\sqrt{M}}$ normalizing coefficient in front of the forward and inverse definitions.

The discrete Fourier transform

In 2D

Straightforward extension from 1D to 2D:

If $\mathcal{I} = \mathcal{I}(m, n)$ is a $M \times N$ image, then its 2D discrete Fourier transform $\mathcal{J}_\mathbf{F}$ is the $M \times N$ complex matrix defined as:

$$\mathcal{J}_\mathbf{F}(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mathcal{I}(m, n) e^{-i2\pi(\frac{mu}{M} + \frac{nv}{N})}$$

$\rightarrow$ The forward DFT kernel is $\phi(u, v, m, n) = e^{-i2\pi(\frac{mu}{M} + \frac{nv}{N})}$

The discrete Fourier transform

In 2D

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The image $\mathcal{I}$ can be retrieved from $\mathcal{J}_\mathbf{F}$ by the 2D inverse discrete Fourier transform:

$$\mathcal{I}(m, n) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \mathcal{J}_\mathbf{F}(u, v) e^{i2\pi(\frac{mu}{M} + \frac{nv}{N})}$$

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The discrete Fourier transform

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$\rightarrow$ The inverse DFT kernel is $\psi(u,v,m,n) = \frac{1}{MN} e^{i2\pi(\frac{mu}{M} + \frac{nv}{N})}$

$\Rightarrow$ Forward and inverse DFT kernels are separable.
$\Rightarrow \mathcal{J}_\mathbf{F} = \mathbf{F}_M \mathcal{I} \mathbf{F}_N^T$ and $\mathcal{I} = \mathbf{F}_M^{-1} \mathcal{J}_\mathbf{F} \mathbf{F}_N^{-T}$.

The DFT kernel matrix

For a $N \times N$ image, the (j, k)-th entry of the Fourier kernel matrix $\mathbf{F}_N$ is

$\mathbf{F}_N(j, k) = e^{-i2\pi \frac{jk}{N}}$

The DFT kernel matrix

For a $N \times N$ image, the (j, k)-th entry of the Fourier kernel matrix $\mathbf{F}_N$ is

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The DFT kernel matrix

For a $N \times N$ image, the (j, k)-th entry of the Fourier kernel matrix $\mathbf{F}_N$ is $\mathbf{F}_N(j, k) = \omega_N^{jk}$ with $\omega_N = e^{-i\frac{2\pi}{N}}$ is a primitive N^{th} root of unity.

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$$\Rightarrow \mathbf{F}_N = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & \omega_N & \omega_N^2 & \omega_N^3 & \dots & \omega_N^{N-1} \\ 1 & \omega_N^2 & \omega_N^4 & \omega_N^6 & \dots & \omega_N^{2(N-1)} \\ 1 & \omega_N^3 & \omega_N^6 & \omega_N^9 & \dots & \omega_N^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_N^{N-1} & \omega_N^{2(N-1)} & \omega_N^{3(N-1)} & \dots & \omega_N^{(N-1)(N-1)} \end{bmatrix}$$

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Clearly, $\mathbf{F}_N = \mathbf{F}_N^T \Rightarrow$ Fourier kernel matrices are symmetric.
But it can also be shown that $\mathbf{F}_N^\dagger \mathbf{F}_N = \mathbf{F}_N \mathbf{F}_N^\dagger = N\mathbf{I}_N \Rightarrow \mathbf{F}_N^{-1} = \frac{1}{N}\mathbf{F}_N^\dagger$ with $\mathbf{F}_N^\dagger = \left(\overline{\mathbf{F}_N}\right)^T = \overline{\mathbf{F}_N^T}$ being the conjugate transpose of $\mathbf{F}_N$.

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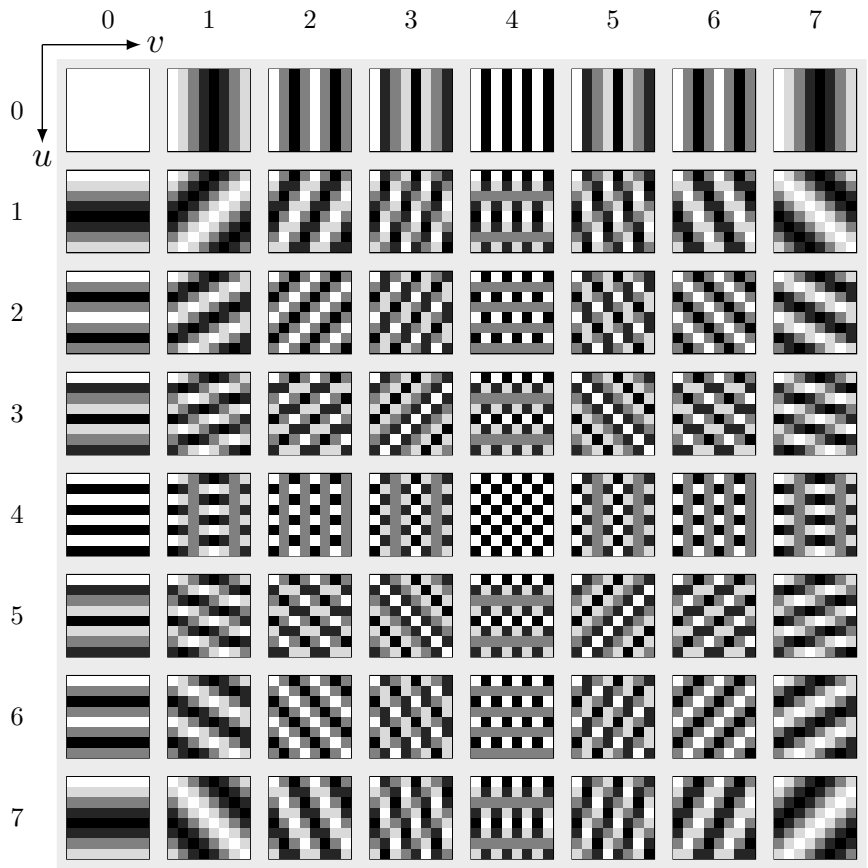
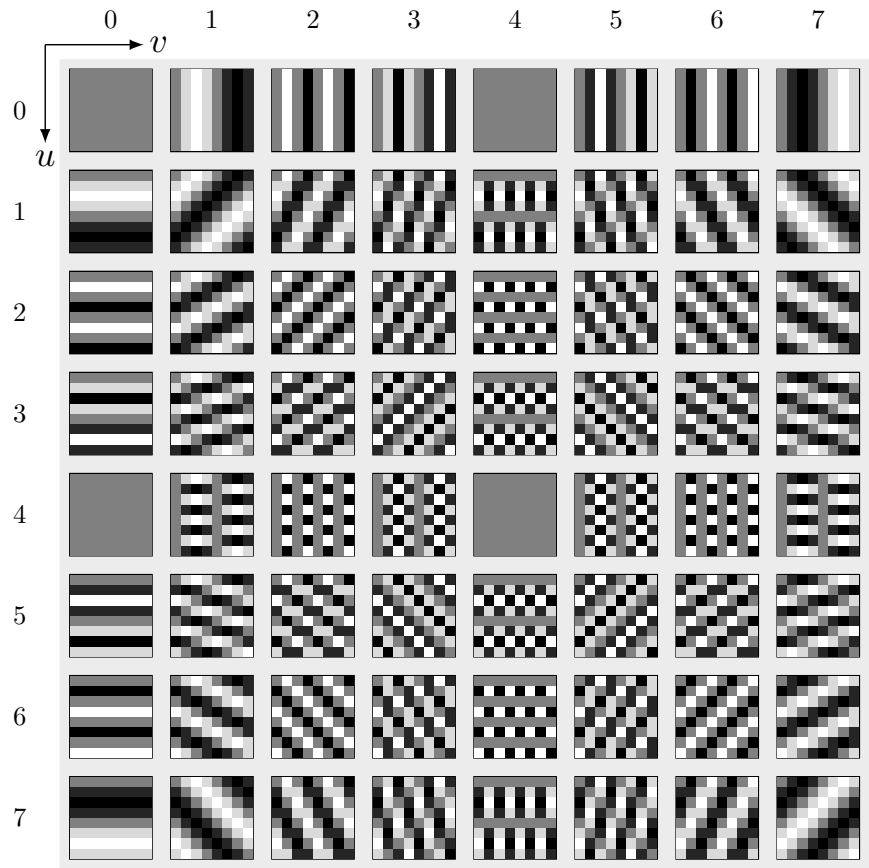
At the end of the day, the DFT matrix $\mathcal{J}_\mathbf{F}$ of a $N \times N$ image $\mathcal{I}$ is given by $\mathcal{J}_\mathbf{F} = \mathbf{F}_N \mathcal{I} \mathbf{F}_N$ and the inverse DFT is $\mathcal{I} = \frac{1}{N^2}\mathbf{F}_N^\dagger \mathcal{J}_\mathbf{F} \mathbf{F}_N^\dagger$

DFT basis images for $N = 8$



$\Re(\mathcal{B}_{uv})$



$\Im(\mathcal{B}_{uv})$

Real and imaginary parts of DFT basis images $\mathcal{B}_{uv}$ for $N = 8$. $\square = \frac{1}{64} = -\blacksquare$. The origin of each basis image is at its top-left corner.

Limits of the DFT for image compression

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DFT spectrum by $\mathcal{J}_{\mathbf{F}} = \mathbf{F}_8 \mathcal{I} \mathbf{F}_8^T$.

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$$\mathcal{J}_{\mathbf{F}} = \left[\right. \quad \left. \right] + i \left[\right. \quad \left. \right]$$

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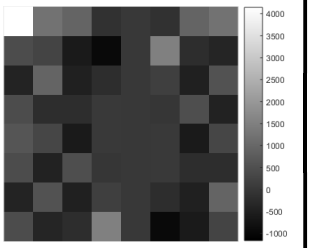
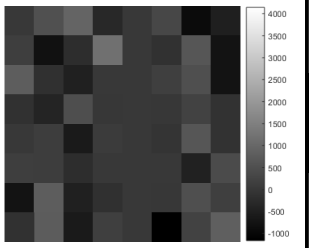
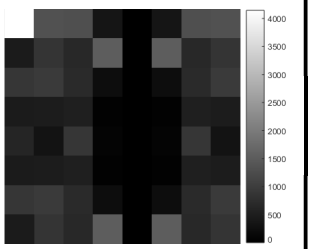
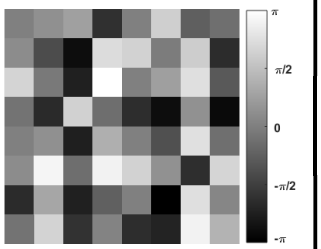
$$\mathcal{J}_{\mathbf{F}} = \left[\begin{array}{c}\text{  }\end{array}\right] + i \left[\begin{array}{c}\text{  }\end{array}\right]$$

$$= \left[\begin{array}{c}\text{  }\end{array}\right] \times \exp\left(i \left[\begin{array}{c}\text{  }\end{array}\right] \right)$$

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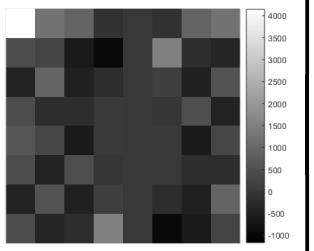
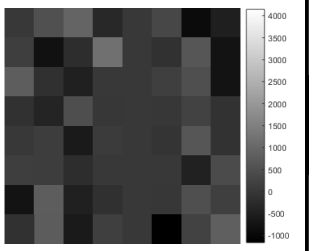
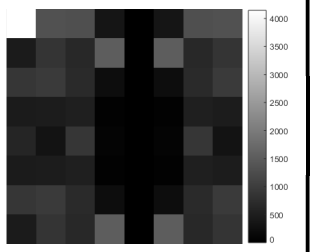
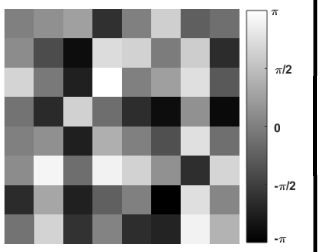
$$\mathcal{J}_{\mathbf{F}} = \left[\begin{array}{c}\text{  }\end{array}\right] + i \left[\begin{array}{c}\text{  }\end{array}\right]$$

$$= \left[\begin{array}{c}\text{  }\end{array}\right] \times \exp\left(i \left[\begin{array}{c}\text{  }\end{array}\right] \right)$$

The DFT is widely used for general spectral analysis applications. But it suffers from two drawbacks for image compression purposes:

- Complex-valued transformed $\rightarrow$ requires double memory for storage.
- Energy compaction is not optimal (spread in both the real and imaginary coefficients of the resulting spectrum).

The discrete cosine transform

→ Published in 1974 by N. Ahmed, T. Natarajan, and K. R. Rao (Discrete cosine transform. *IEEE transactions on Computers*, vol. 100, no 1, pp. 90–93).

→ Real-valued transform, but strongly related to the DFT.
(DCT(*signal*)≡ DFT(*symmetrized signal*)).

→ Several definitions (*types*) exist, depending on the chosen boundary conditions.

Type-II DCT and its inverse transform (Type-III DCT)

A $M \times N$ image $\mathcal{I}$ and its $M \times N$ DCT spectrum $\mathcal{J}_{\mathbf{D}}$ are linked by:

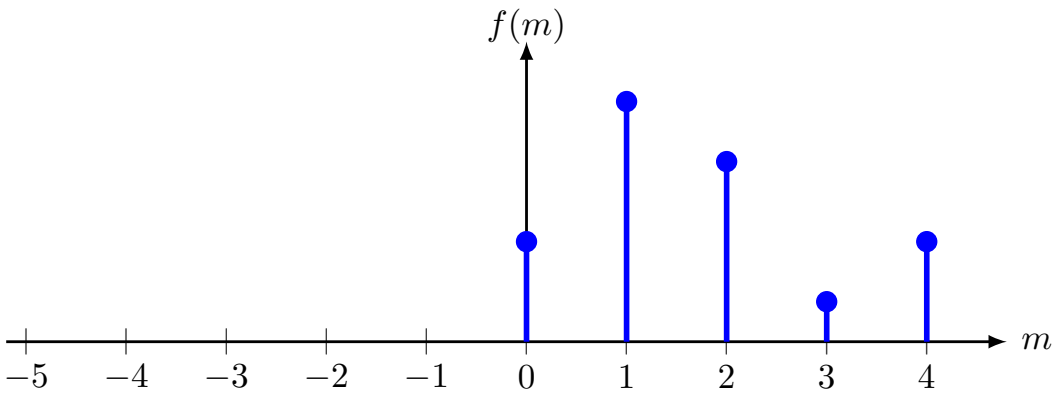
$$\mathcal{J}_{\mathbf{D}}(u,v) = \alpha(u)\alpha(v) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mathcal{I}(m,n) \cos\left(\frac{\pi(2m+1)u}{2M}\right) \cos\left(\frac{\pi(2n+1)v}{2N}\right)$$

$$\mathcal{I}(m,n) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \alpha(u)\alpha(v)\mathcal{J}_{\mathbf{D}}(u,v) \cos\left(\frac{\pi(2m+1)u}{2M}\right) \cos\left(\frac{\pi(2n+1)v}{2N}\right)$$

with $\alpha(u) = \begin{cases} \sqrt{\frac{1}{M}} & \text{if } u = 0 \\ \sqrt{\frac{2}{M}} & \text{if } u = 1, \ldots, M-1 \end{cases}$, $\alpha(v) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } v = 0 \\ \sqrt{\frac{2}{N}} & \text{if } v = 1, \ldots, N-1 \end{cases}$

Going from the DFT to the DCT

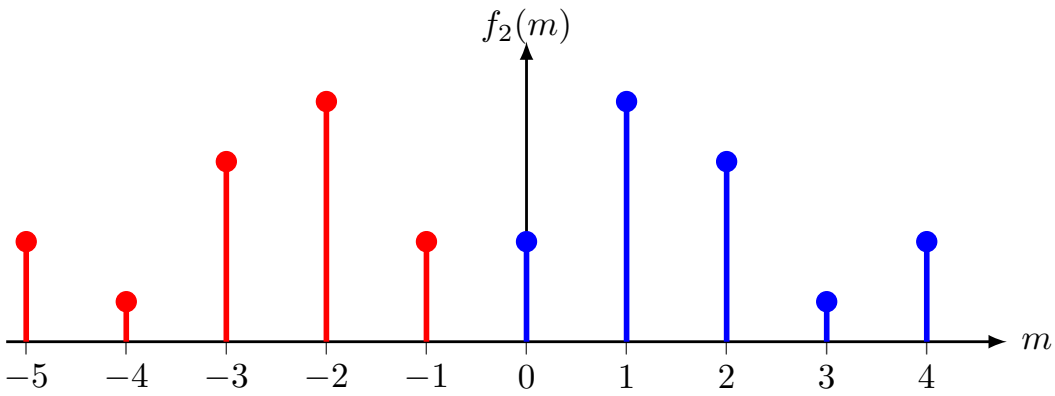
Take some M-points real signal sequence $\{f(0), \ldots, f(M-1)\}$



Going from the DFT to the DCT

Take some M-points real signal sequence $\{f(0), \ldots, f(M-1)\}$

1. Create a symmetric $2M$-point sequence $f_2(m) = \begin{cases} f(m) & (0 \leq m \leq M-1) \\ f(-m-1) & (-M \leq m \leq -1) \end{cases}$.



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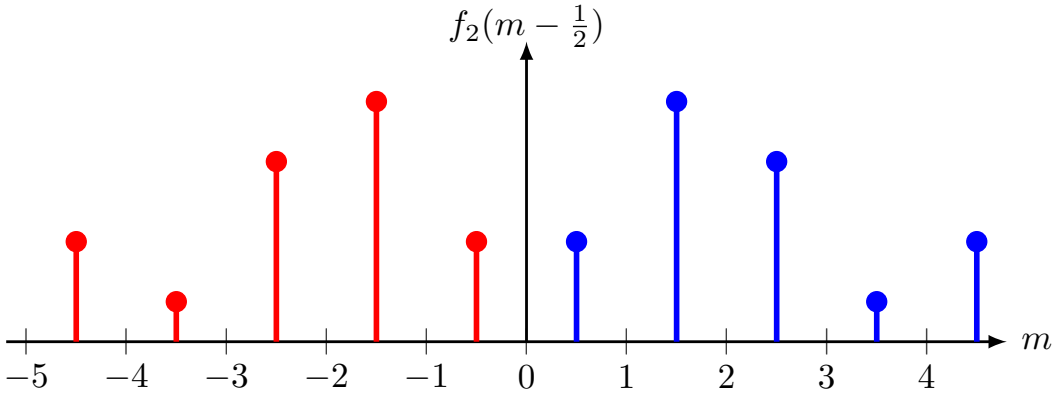
2. Right-shift the resulting sequence by $\frac{1}{2}$.



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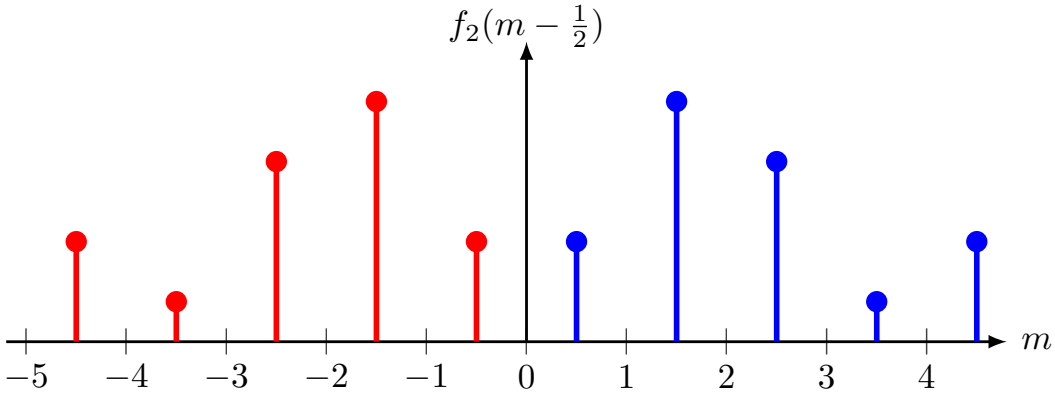
Then $\mathrm{DCT}(f(m)) = \mathrm{DFT}\left(f_2(m - \frac{1}{2})\right)$.



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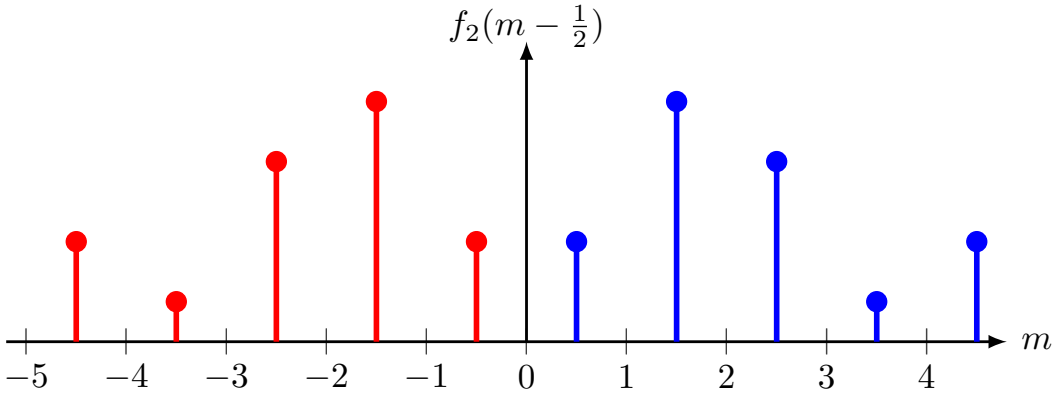
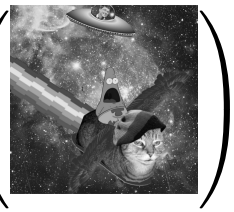


Same goes for images:

$$\text{DCT}\left(\quad\right) \equiv \text{DFT}\left(\quad\right)$$



The DCT kernel matrix

The (j,k)-th entry of the $N \times N$ DCT matrix is $\mathbf{D}_N(j,k) = \alpha(j)\cos\left(\frac{\pi(2k+1)j}{2N}\right)$

with $\alpha(j) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } j = 0, \\ \sqrt{\frac{2}{N}} & \text{otherwise.} \end{cases}$

$$\mathbf{D}_8 = \frac{1}{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos(\frac{\pi}{16}) & \cos(\frac{3\pi}{16}) & \cos(\frac{5\pi}{16}) & \cos(\frac{7\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{11\pi}{16}) & \cos(\frac{13\pi}{16}) & \cos(\frac{15\pi}{16}) \\ \cos(\frac{2\pi}{16}) & \cos(\frac{6\pi}{16}) & \cos(\frac{10\pi}{16}) & \cos(\frac{14\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{22\pi}{16}) & \cos(\frac{26\pi}{16}) & \cos(\frac{30\pi}{16}) \\ \cos(\frac{3\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{27\pi}{16}) & \cos(\frac{33\pi}{16}) & \cos(\frac{39\pi}{16}) & \cos(\frac{45\pi}{16}) \\ \cos(\frac{4\pi}{16}) & \cos(\frac{12\pi}{16}) & \cos(\frac{20\pi}{16}) & \cos(\frac{28\pi}{16}) & \cos(\frac{36\pi}{16}) & \cos(\frac{44\pi}{16}) & \cos(\frac{52\pi}{16}) & \cos(\frac{60\pi}{16}) \\ \cos(\frac{5\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{25\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{45\pi}{16}) & \cos(\frac{55\pi}{16}) & \cos(\frac{65\pi}{16}) & \cos(\frac{75\pi}{16}) \\ \cos(\frac{6\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{30\pi}{16}) & \cos(\frac{42\pi}{16}) & \cos(\frac{54\pi}{16}) & \cos(\frac{66\pi}{16}) & \cos(\frac{78\pi}{16}) & \cos(\frac{90\pi}{16}) \\ \cos(\frac{7\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{49\pi}{16}) & \cos(\frac{63\pi}{16}) & \cos(\frac{77\pi}{16}) & \cos(\frac{91\pi}{16}) & \cos(\frac{105\pi}{16}) \end{bmatrix}$$

The DCT kernel matrix

The (j,k)-th entry of the $N \times N$ DCT matrix is $\mathbf{D}_N(j,k) = \alpha(j)\cos\left(\frac{\pi(2k+1)j}{2N}\right)$

with $\alpha(j) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } j = 0, \\ \sqrt{\frac{2}{N}} & \text{otherwise.} \end{cases}$

- $\mathbf{D}_N$ is not symmetric $\rightarrow \mathcal{J}_\mathbf{D} = \mathbf{D}_N \mathcal{I} \mathbf{D}_N^T$
- But $\mathbf{D}_N$ is orthogonal ($\mathbf{D}_N\mathbf{D}_N^T = \mathbf{D}_N^T\mathbf{D}_N = \mathbf{I}_N$)
  $\rightarrow \mathcal{I} = \mathbf{D}_N^T \mathcal{J}_\mathbf{D} \mathbf{D}_N$

$$\mathbf{D}_8 = \frac{1}{2}\begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\cos(\frac{\pi}{16}) & \cos(\frac{3\pi}{16}) & \cos(\frac{5\pi}{16}) & \cos(\frac{7\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{11\pi}{16}) & \cos(\frac{13\pi}{16}) & \cos(\frac{15\pi}{16}) \\
\cos(\frac{2\pi}{16}) & \cos(\frac{6\pi}{16}) & \cos(\frac{10\pi}{16}) & \cos(\frac{14\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{22\pi}{16}) & \cos(\frac{26\pi}{16}) & \cos(\frac{30\pi}{16}) \\
\cos(\frac{3\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{27\pi}{16}) & \cos(\frac{33\pi}{16}) & \cos(\frac{39\pi}{16}) & \cos(\frac{45\pi}{16}) \\
\cos(\frac{4\pi}{16}) & \cos(\frac{12\pi}{16}) & \cos(\frac{20\pi}{16}) & \cos(\frac{28\pi}{16}) & \cos(\frac{36\pi}{16}) & \cos(\frac{44\pi}{16}) & \cos(\frac{52\pi}{16}) & \cos(\frac{60\pi}{16}) \\
\cos(\frac{5\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{25\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{45\pi}{16}) & \cos(\frac{55\pi}{16}) & \cos(\frac{65\pi}{16}) & \cos(\frac{75\pi}{16}) \\
\cos(\frac{6\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{30\pi}{16}) & \cos(\frac{42\pi}{16}) & \cos(\frac{54\pi}{16}) & \cos(\frac{66\pi}{16}) & \cos(\frac{78\pi}{16}) & \cos(\frac{90\pi}{16}) \\
\cos(\frac{7\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{49\pi}{16}) & \cos(\frac{63\pi}{16}) & \cos(\frac{77\pi}{16}) & \cos(\frac{91\pi}{16}) & \cos(\frac{105\pi}{16})
\end{bmatrix}$$

The DCT kernel matrix

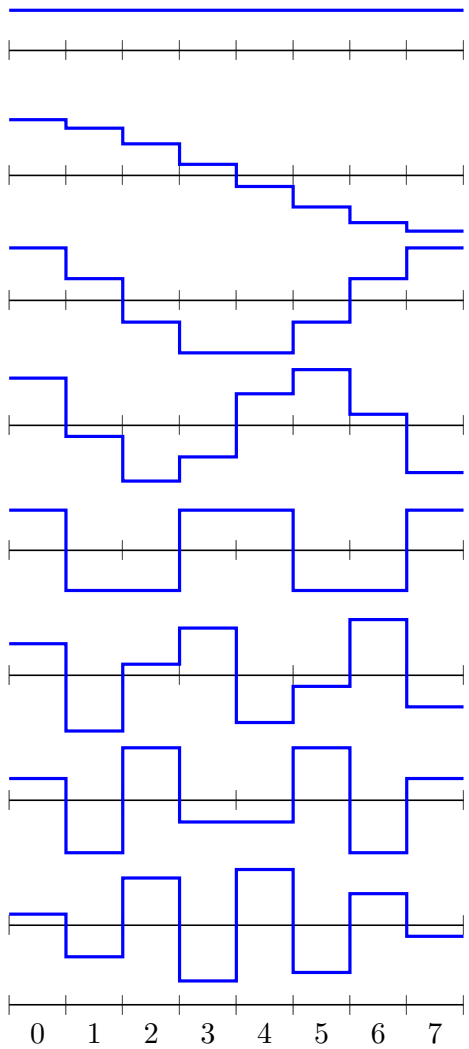
The (j,k)-th entry of the $N \times N$ DCT matrix is $\mathbf{D}_N(j,k) = \alpha(j)\cos\left(\frac{\pi(2k+1)j}{2N}\right)$

with $\alpha(j) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } j = 0, \\ \sqrt{\frac{2}{N}} & \text{otherwise.} \end{cases}$

- $\mathbf{D}_N$ is not symmetric $\rightarrow \mathcal{J}_\mathbf{D} = \mathbf{D}_N \mathcal{I} \mathbf{D}_N^T$
- But $\mathbf{D}_N$ is orthogonal ($\mathbf{D}_N\mathbf{D}_N^T = \mathbf{D}_N^T\mathbf{D}_N = \mathbf{I}_N$)
  $\rightarrow \mathcal{I} = \mathbf{D}_N^T \mathcal{J}_\mathbf{D} \mathbf{D}_N$

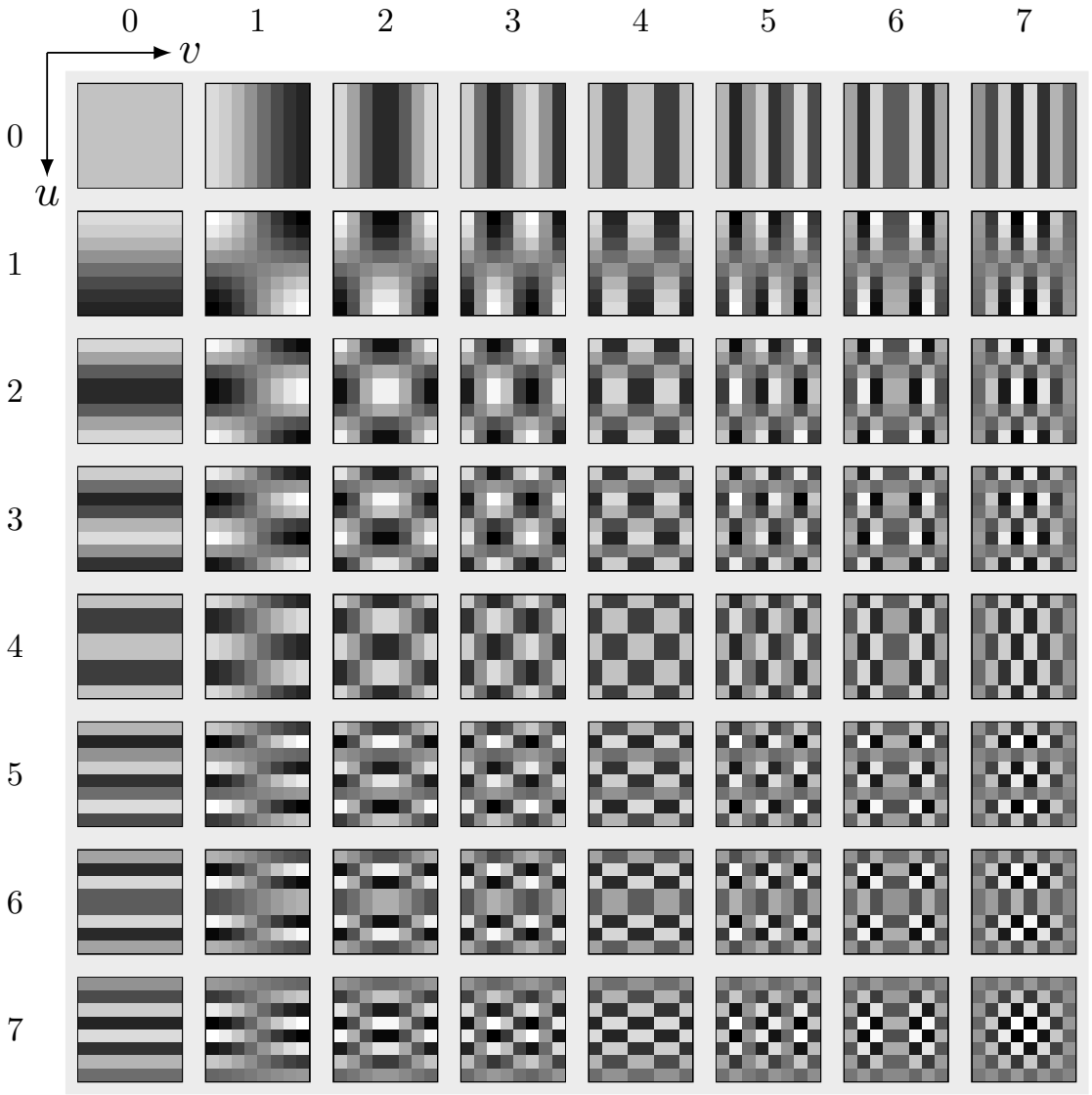
$$\mathbf{D}_8 = \frac{1}{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos(\frac{\pi}{16}) & \cos(\frac{3\pi}{16}) & \cos(\frac{5\pi}{16}) & \cos(\frac{7\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{11\pi}{16}) & \cos(\frac{13\pi}{16}) & \cos(\frac{15\pi}{16}) \\ \cos(\frac{2\pi}{16}) & \cos(\frac{6\pi}{16}) & \cos(\frac{10\pi}{16}) & \cos(\frac{14\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{22\pi}{16}) & \cos(\frac{26\pi}{16}) & \cos(\frac{30\pi}{16}) \\ \cos(\frac{3\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{27\pi}{16}) & \cos(\frac{33\pi}{16}) & \cos(\frac{39\pi}{16}) & \cos(\frac{45\pi}{16}) \\ \cos(\frac{4\pi}{16}) & \cos(\frac{12\pi}{16}) & \cos(\frac{20\pi}{16}) & \cos(\frac{28\pi}{16}) & \cos(\frac{36\pi}{16}) & \cos(\frac{44\pi}{16}) & \cos(\frac{52\pi}{16}) & \cos(\frac{60\pi}{16}) \\ \cos(\frac{5\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{25\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{45\pi}{16}) & \cos(\frac{55\pi}{16}) & \cos(\frac{65\pi}{16}) & \cos(\frac{75\pi}{16}) \\ \cos(\frac{6\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{30\pi}{16}) & \cos(\frac{42\pi}{16}) & \cos(\frac{54\pi}{16}) & \cos(\frac{66\pi}{16}) & \cos(\frac{78\pi}{16}) & \cos(\frac{90\pi}{16}) \\ \cos(\frac{7\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{49\pi}{16}) & \cos(\frac{63\pi}{16}) & \cos(\frac{77\pi}{16}) & \cos(\frac{91\pi}{16}) & \cos(\frac{105\pi}{16}) \end{bmatrix}$$

DCT $\equiv$ decomposition over the set of DCT basis functions.



DCT basis functions

DCT basis images for $M = N = 8$



DCT basis images $\mathcal{B}_{uv}$ for $M = N = 8$. $\square = \frac{1}{4}\cos(\frac{\pi}{16})^2$, and $\blacksquare = -\square$. The origin of each basis image is at its top-left corner.

Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

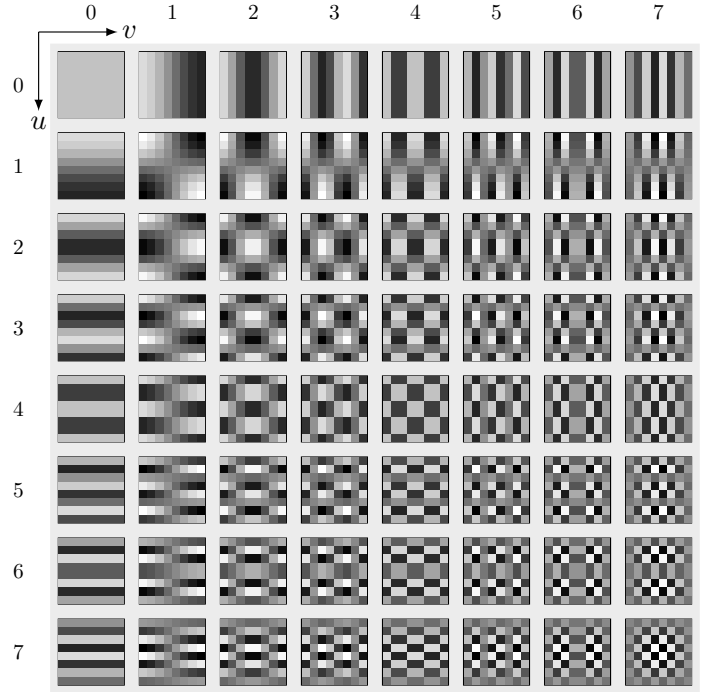
$$\Rightarrow \mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

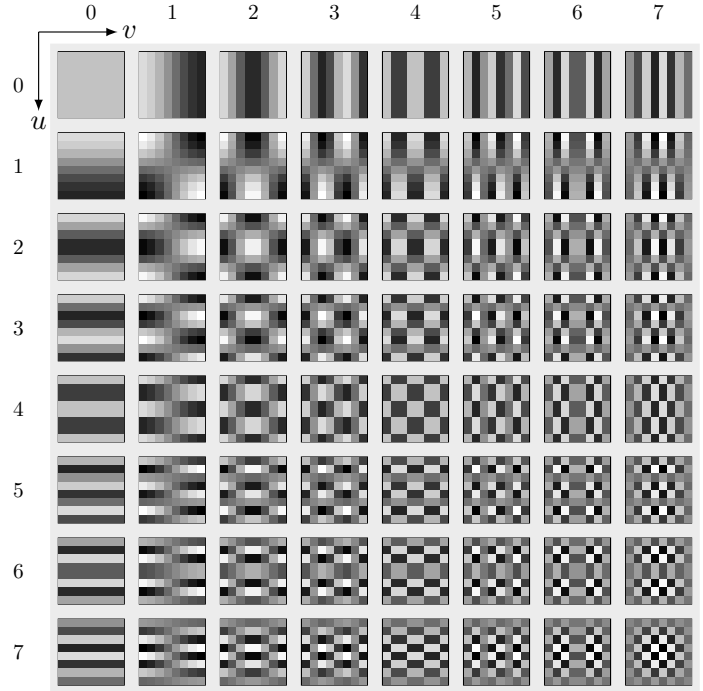


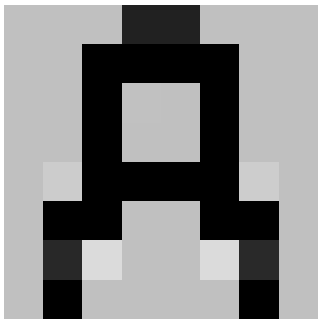
Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

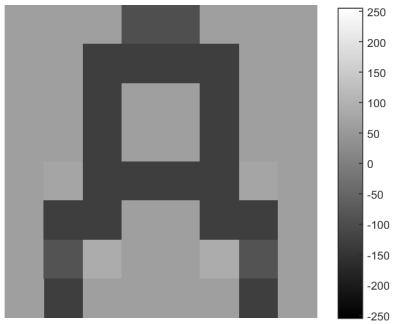
$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$





$\mathcal{I}$

$\tilde{\mathcal{I}}_1$



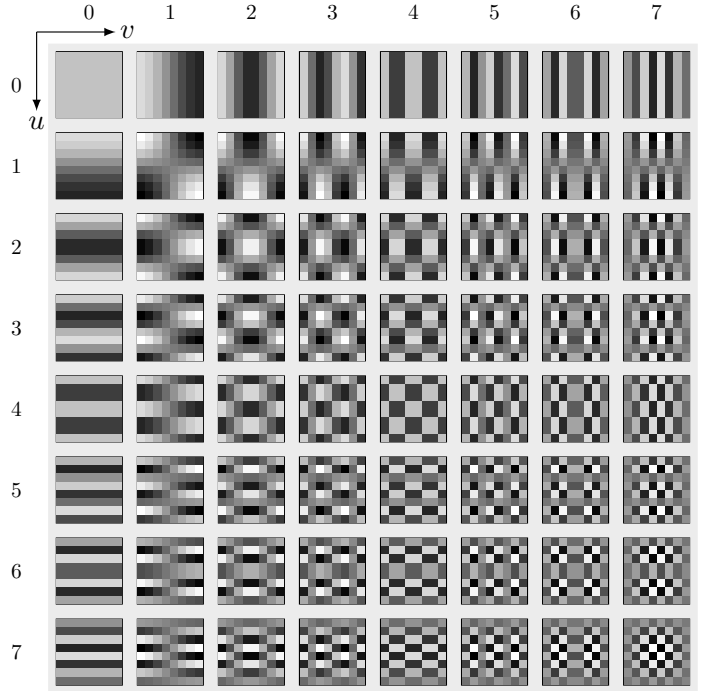
$\mathcal{I} - \tilde{\mathcal{I}}_1$

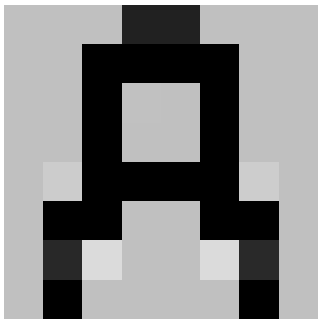
Example

New bottle, same old wine

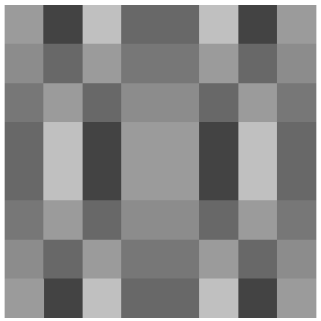
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

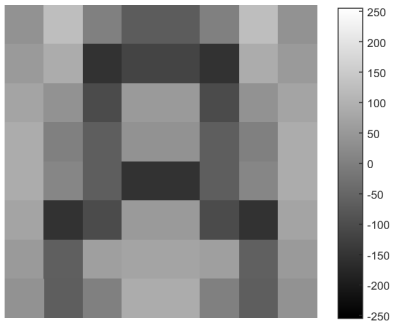




$\mathcal{I}$



$\tilde{\mathcal{I}}_2$



$\mathcal{I} - \tilde{\mathcal{I}}_2$

Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

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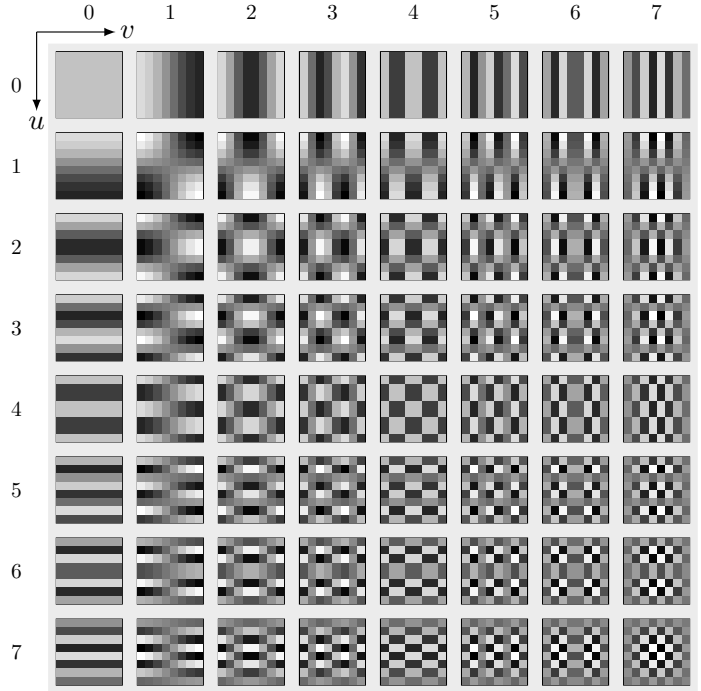
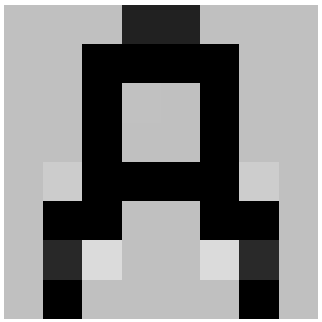
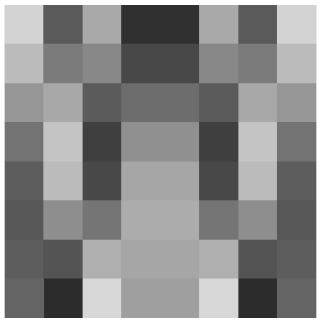
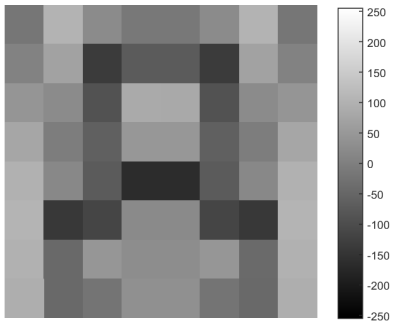




$\mathcal{I}$



$\tilde{\mathcal{I}}_3$



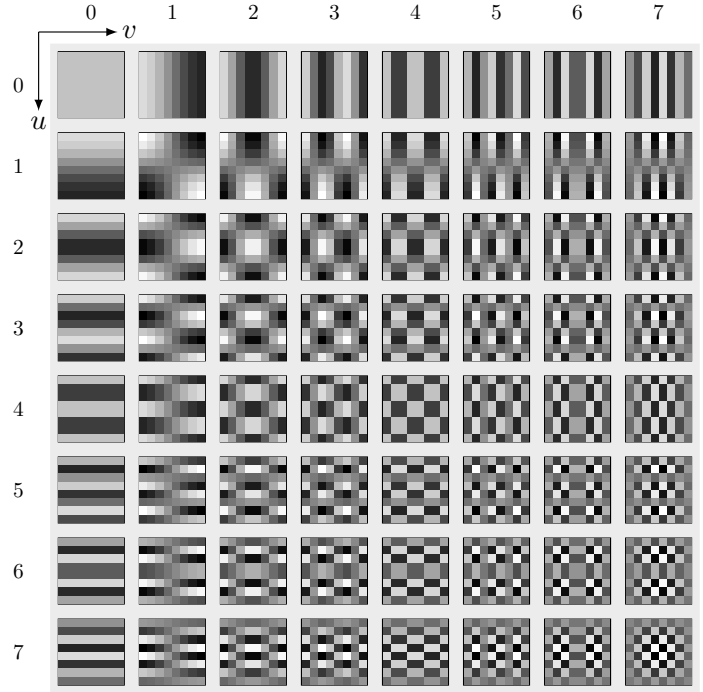
$\mathcal{I} - \tilde{\mathcal{I}}_3$

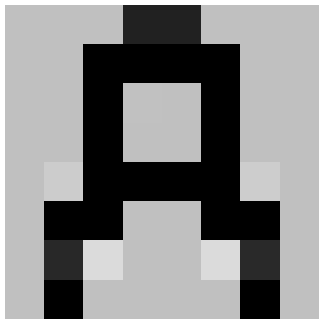
Example

New bottle, same old wine

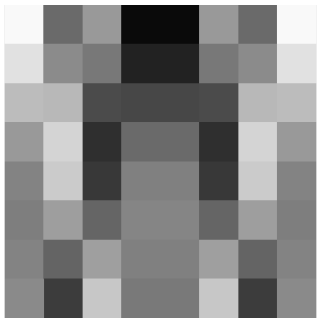
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

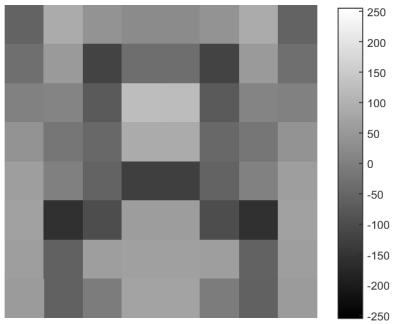




$\mathcal{I}$



$\tilde{\mathcal{I}}_4$



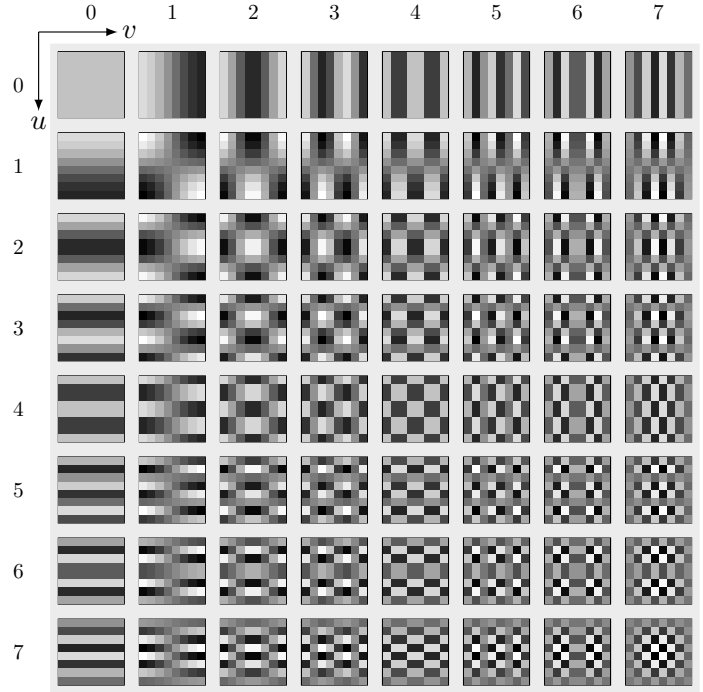
$\mathcal{I} - \tilde{\mathcal{I}}_4$

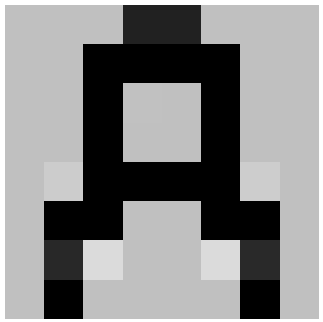
Example

New bottle, same old wine

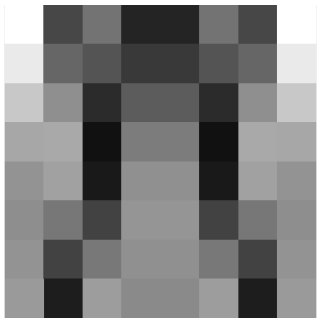
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

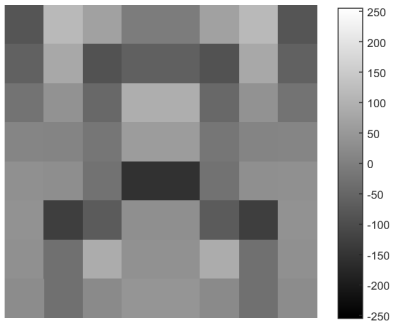




$\mathcal{I}$



$\tilde{\mathcal{I}}_5$



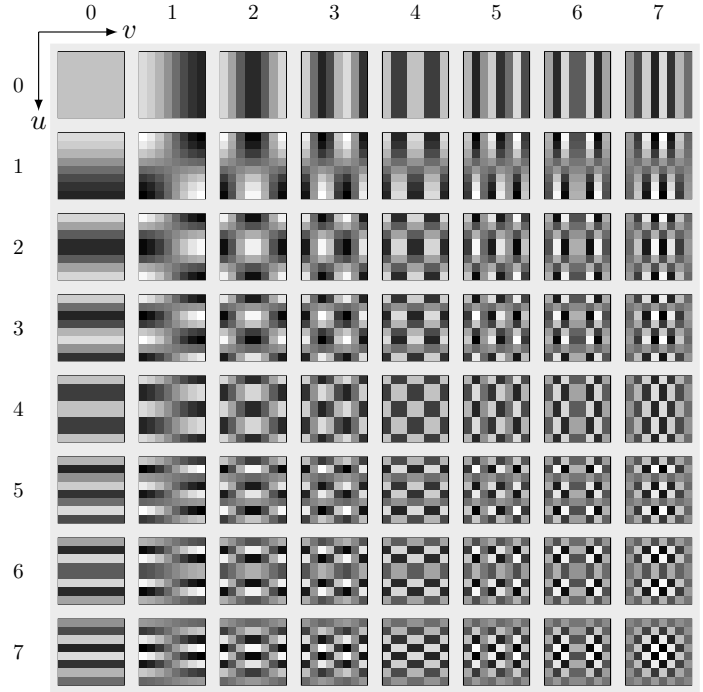
$\mathcal{I} - \tilde{\mathcal{I}}_5$

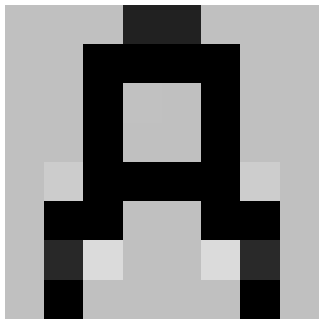
Example

New bottle, same old wine

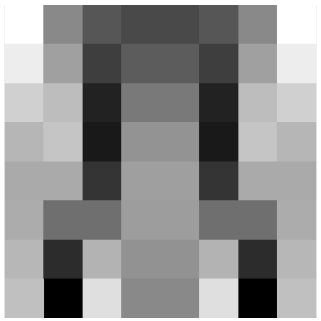
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

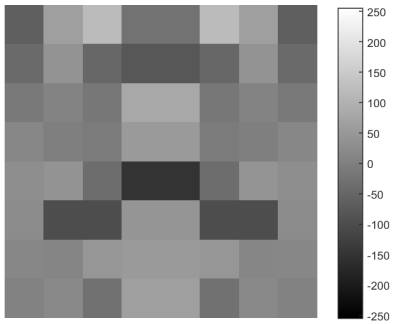




$\mathcal{I}$



$\tilde{\mathcal{I}}_6$



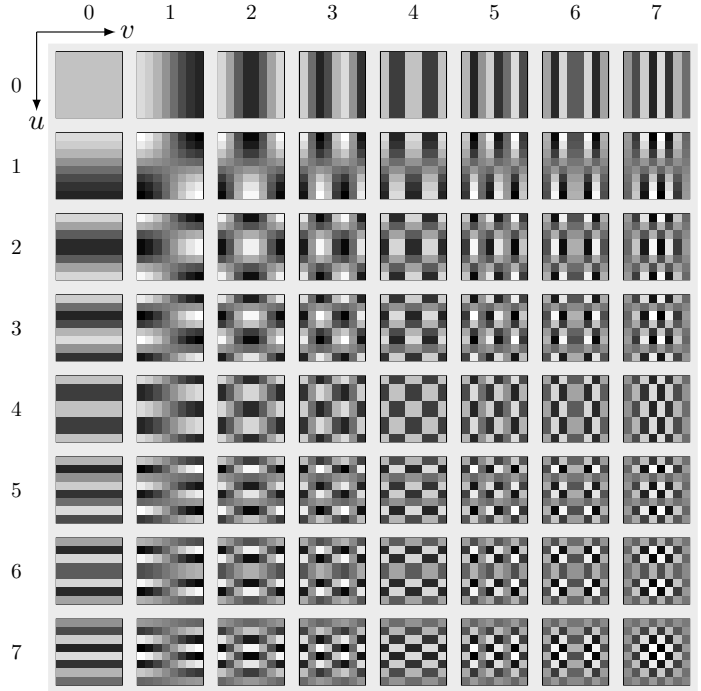
$\mathcal{I} - \tilde{\mathcal{I}}_6$

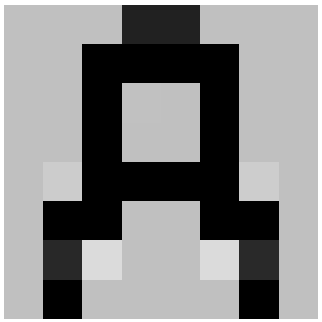
Example

New bottle, same old wine

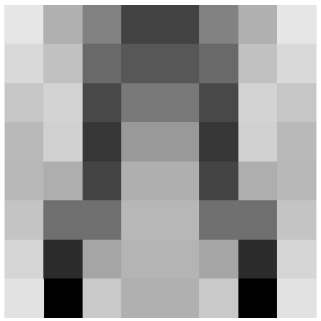
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

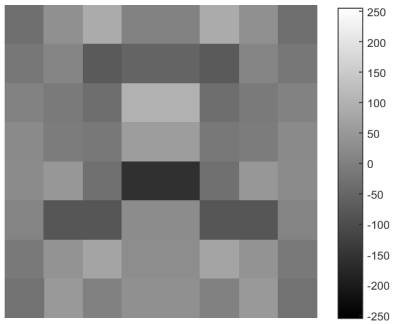




$\mathcal{I}$



$\tilde{\mathcal{I}}_7$



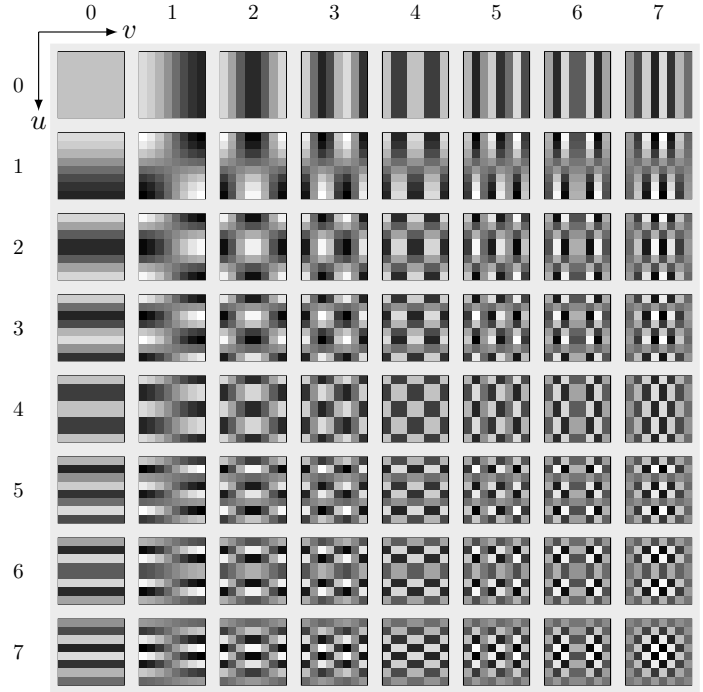
$\mathcal{I} - \tilde{\mathcal{I}}_7$

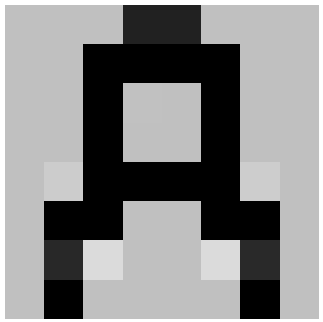
Example

New bottle, same old wine

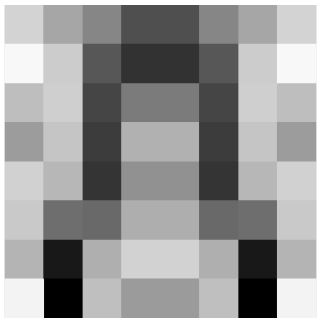
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

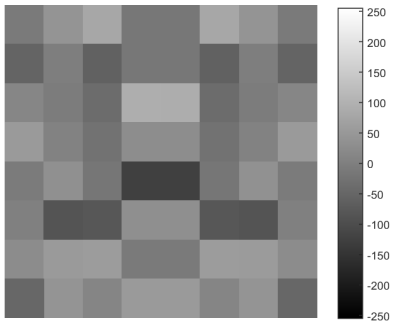




$\mathcal{I}$



$\tilde{\mathcal{I}}_8$



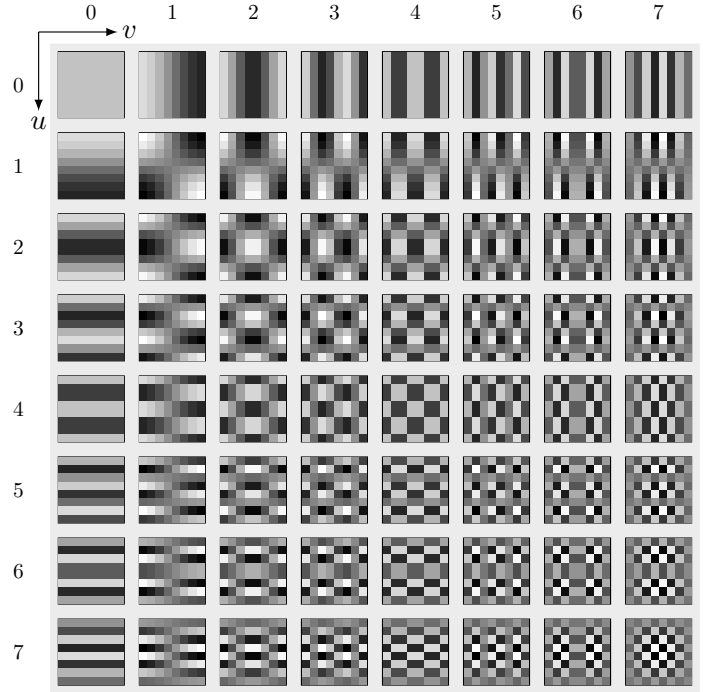
$\mathcal{I} - \tilde{\mathcal{I}}_8$

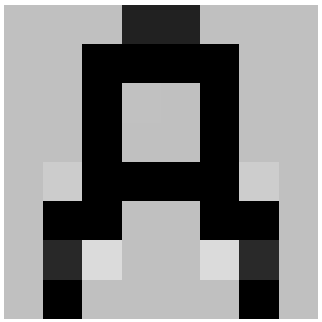
Example

New bottle, same old wine

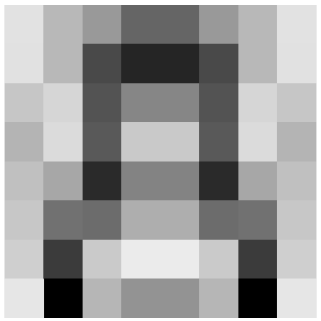
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

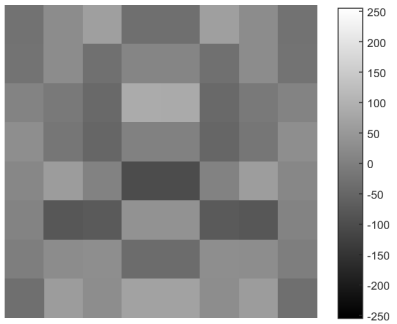




$\mathcal{I}$



$\tilde{\mathcal{I}}_9$



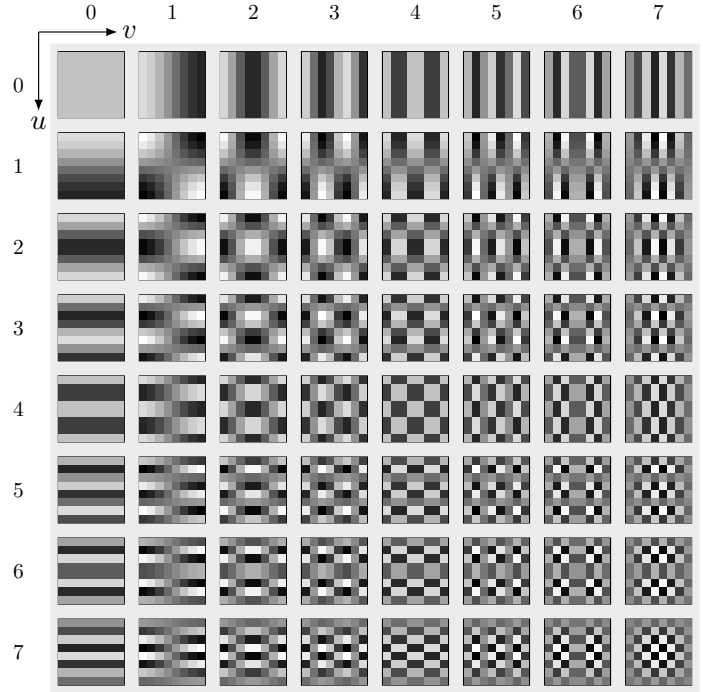
$\mathcal{I} - \tilde{\mathcal{I}}_9$

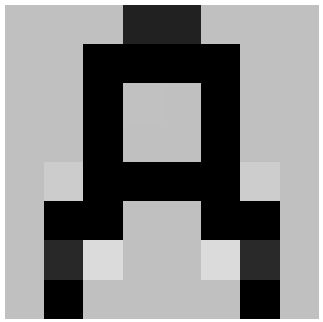
Example

New bottle, same old wine

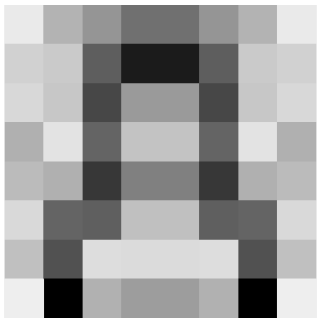
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

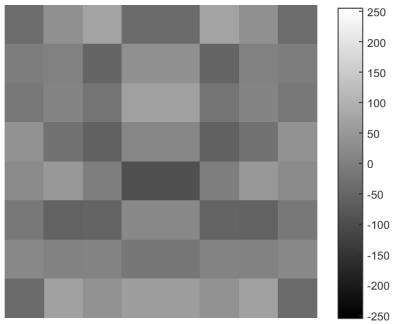




$\mathcal{I}$



$\tilde{\mathcal{I}}_{10}$



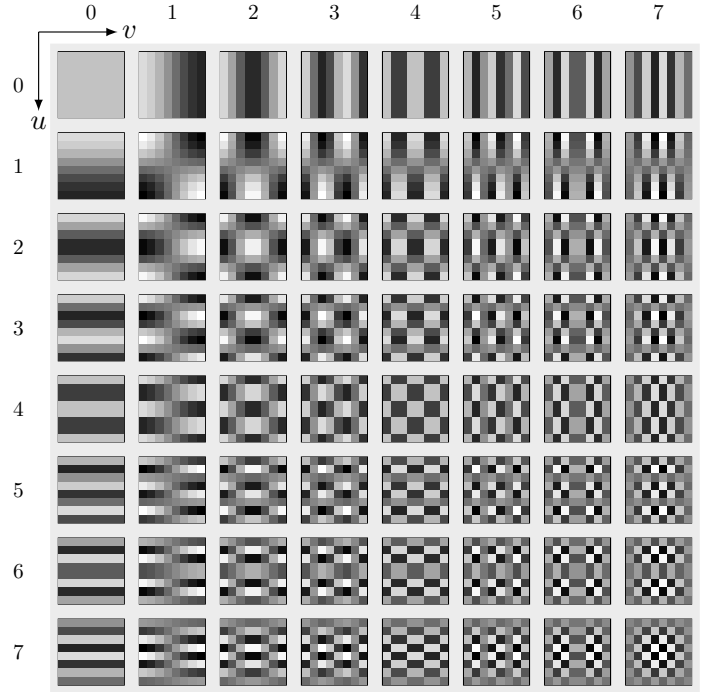
$\mathcal{I} - \tilde{\mathcal{I}}_{10}$

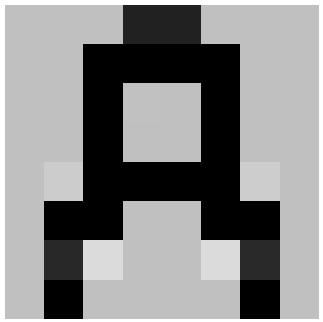
Example

New bottle, same old wine

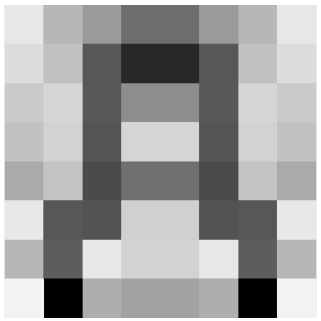
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

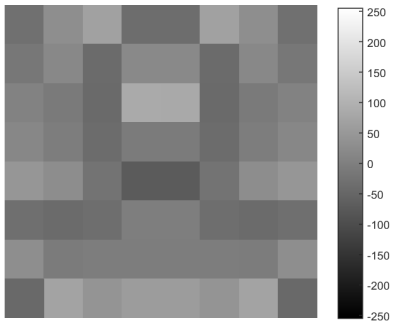




$\mathcal{I}$



$\tilde{\mathcal{I}}_{11}$



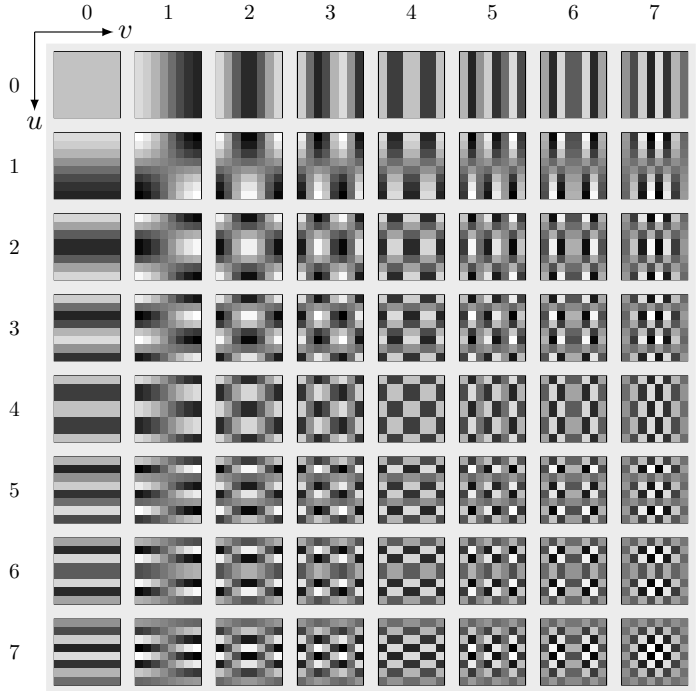
$\mathcal{I} - \tilde{\mathcal{I}}_{11}$

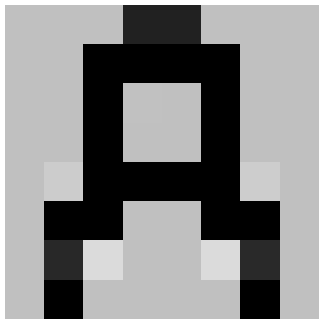
Example

New bottle, same old wine

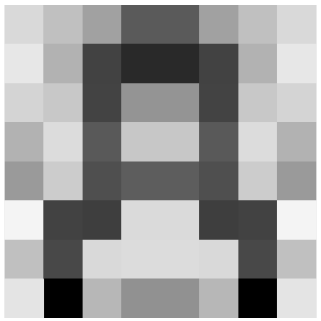
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

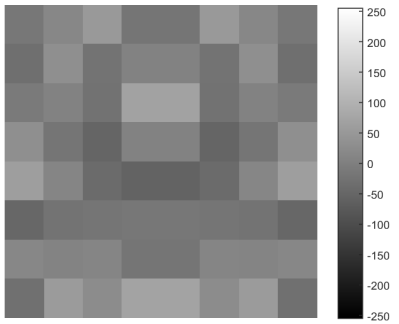




$\mathcal{I}$



$\tilde{\mathcal{I}}_{12}$



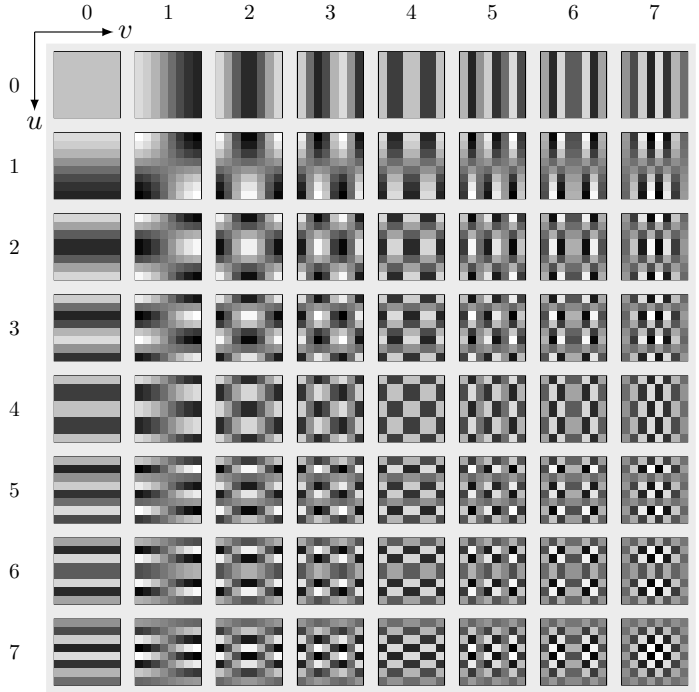
$\mathcal{I} - \tilde{\mathcal{I}}_{12}$

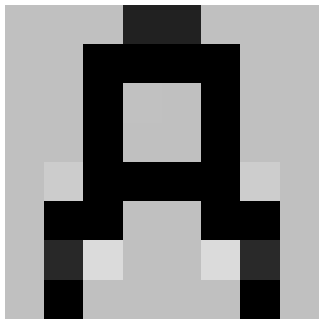
Example

New bottle, same old wine

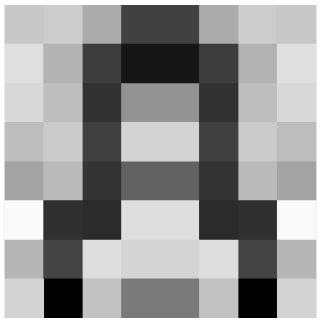
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

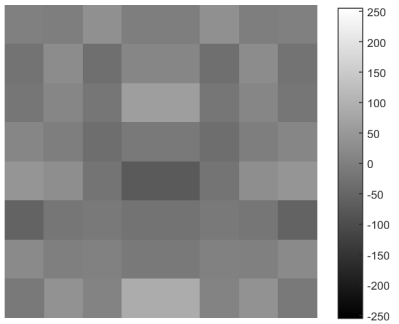




$\mathcal{I}$



$\tilde{\mathcal{I}}_{13}$



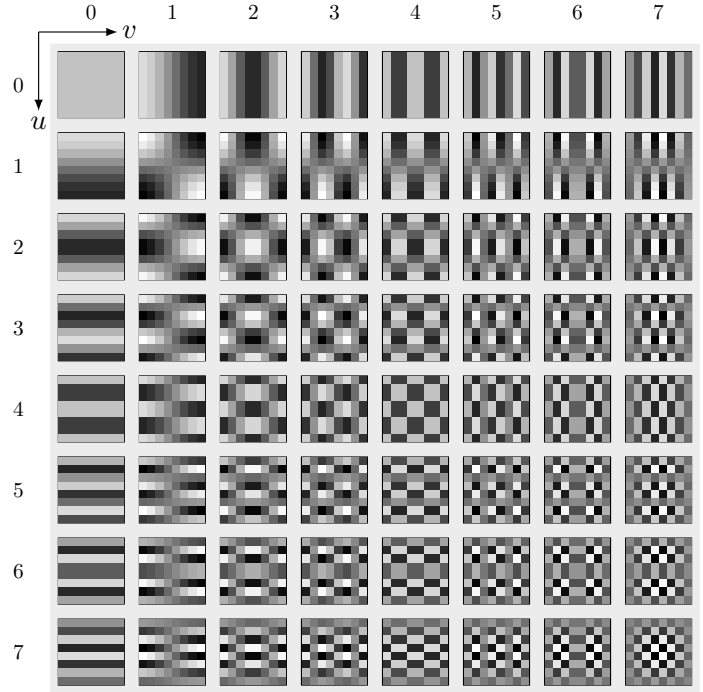
$\mathcal{I} - \tilde{\mathcal{I}}_{13}$

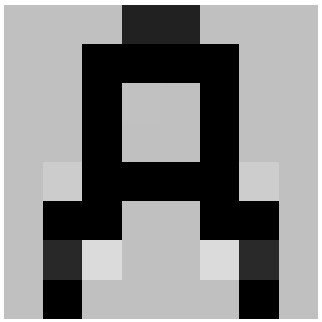
Example

New bottle, same old wine

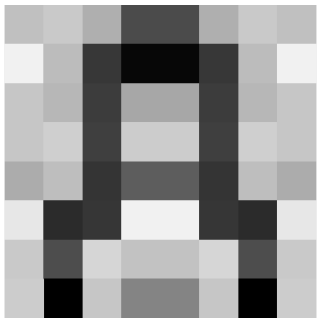
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

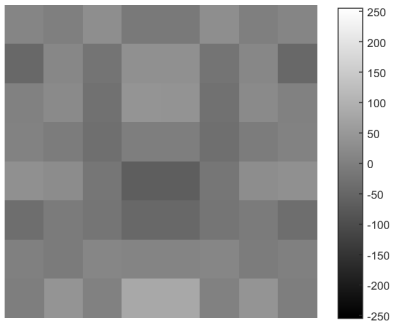




$\mathcal{I}$



$\tilde{\mathcal{I}}_{14}$



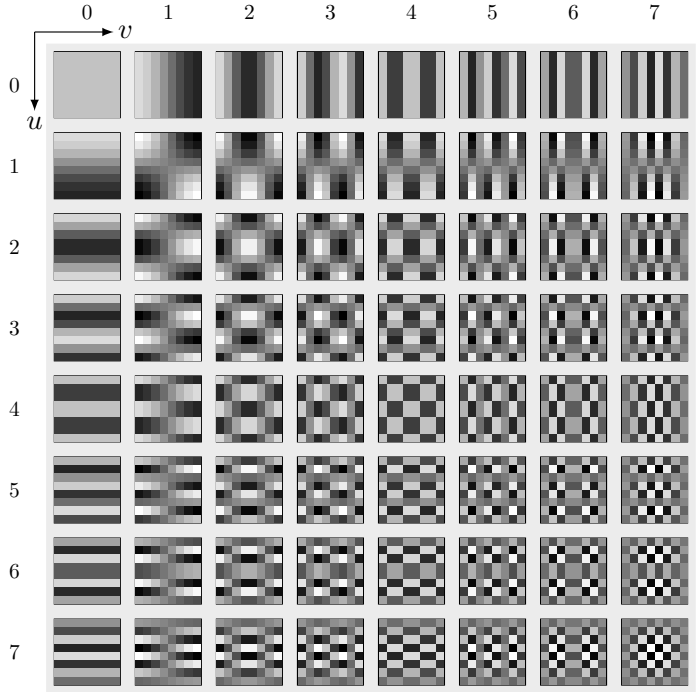
$\mathcal{I} - \tilde{\mathcal{I}}_{14}$

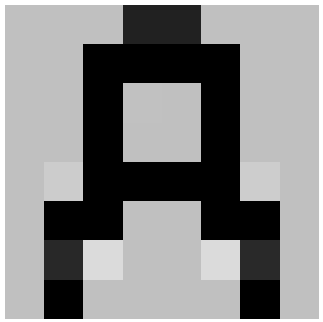
Example

New bottle, same old wine

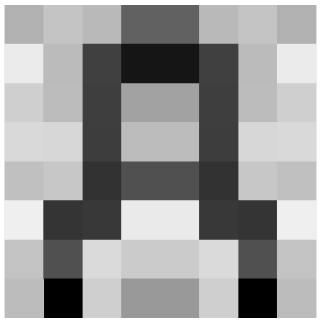
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

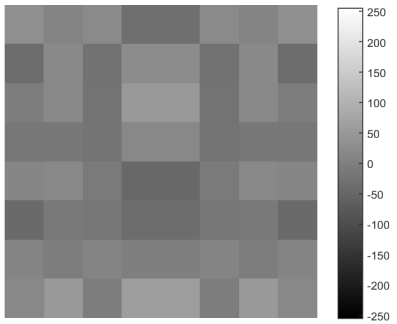




$\mathcal{I}$



$\tilde{\mathcal{I}}_{15}$



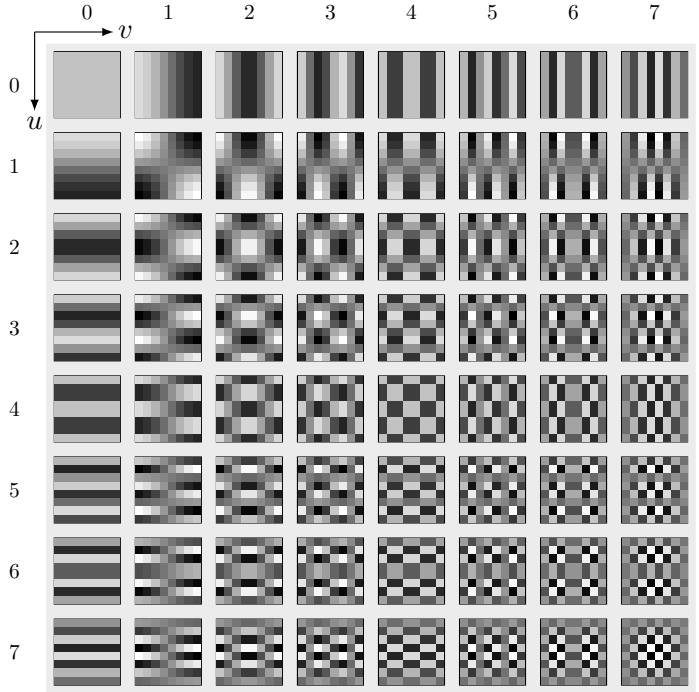
$\mathcal{I} - \tilde{\mathcal{I}}_{15}$

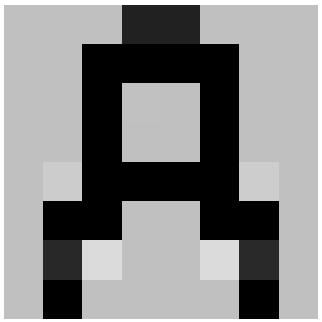
Example

New bottle, same old wine

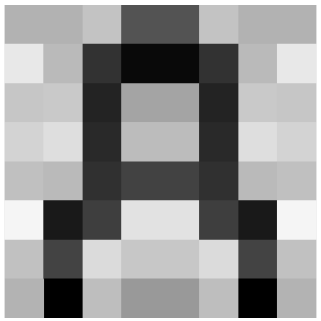
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

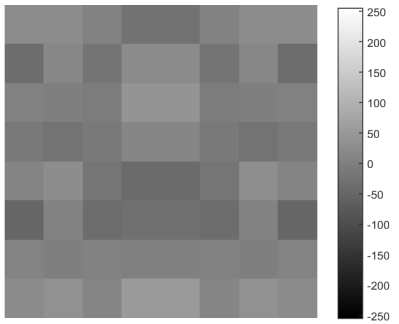




$\mathcal{I}$



$\tilde{\mathcal{I}}_{16}$



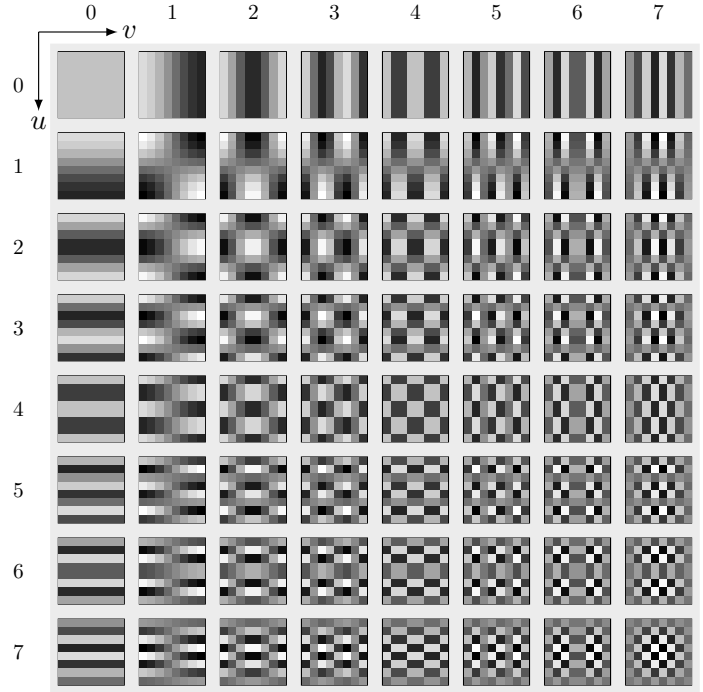
$\mathcal{I} - \tilde{\mathcal{I}}_{16}$

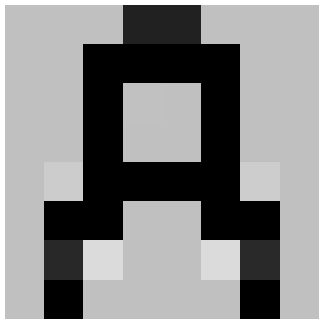
Example

New bottle, same old wine

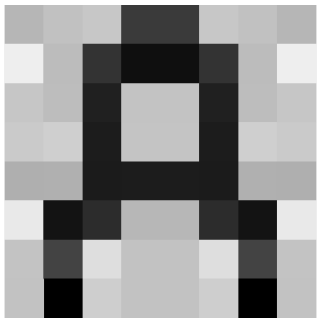
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

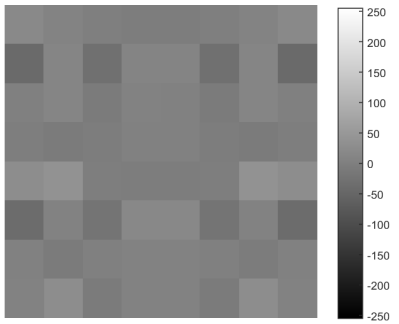




$\mathcal{I}$



$\tilde{\mathcal{I}}_{20}$



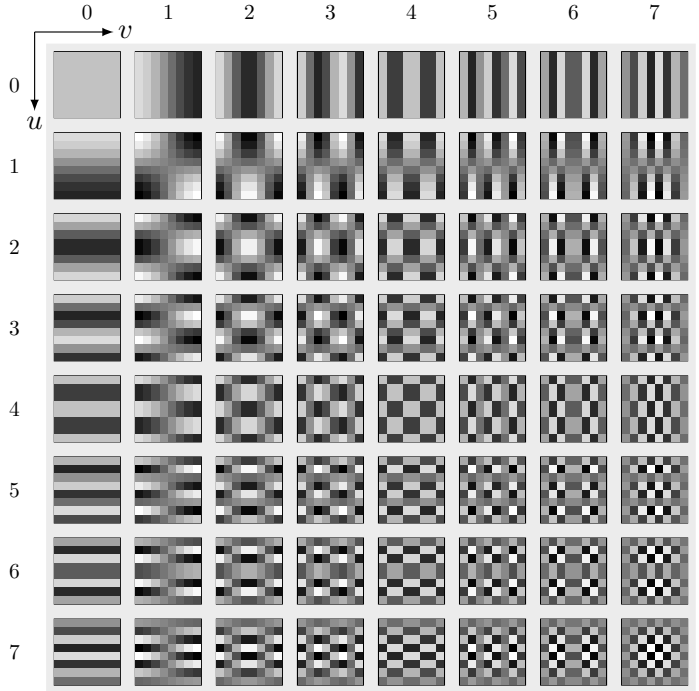
$\mathcal{I} - \tilde{\mathcal{I}}_{20}$

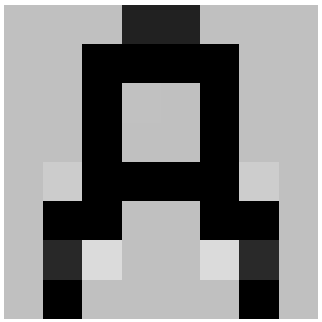
Example

New bottle, same old wine

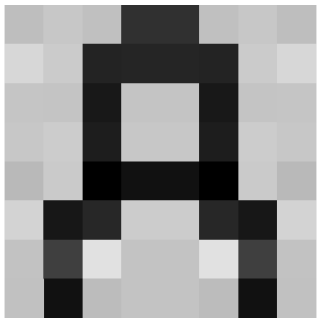
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

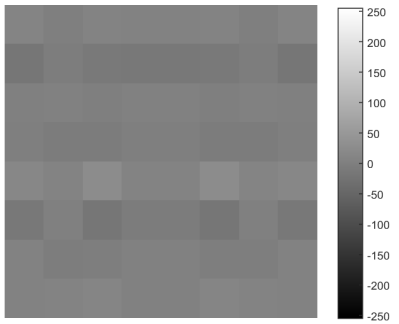




$\mathcal{I}$



$\tilde{\mathcal{I}}_{24}$



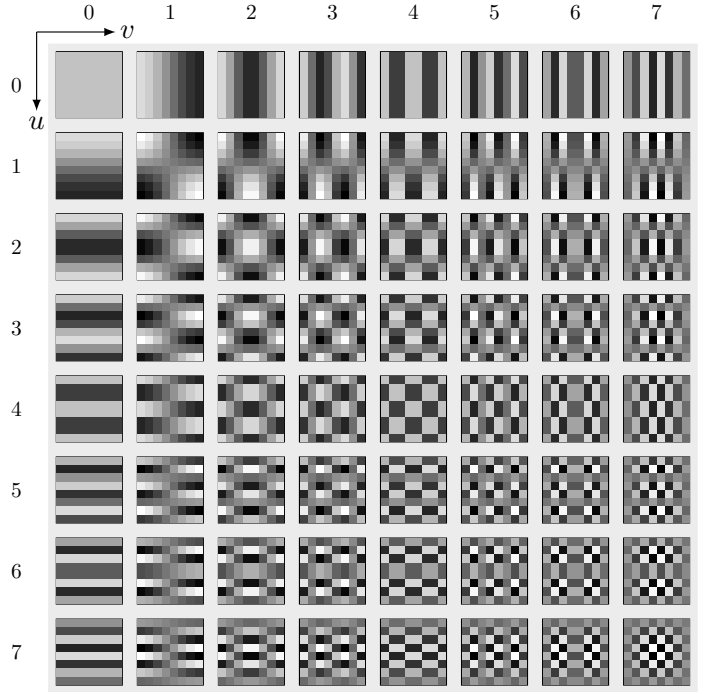
$\mathcal{I} - \tilde{\mathcal{I}}_{24}$

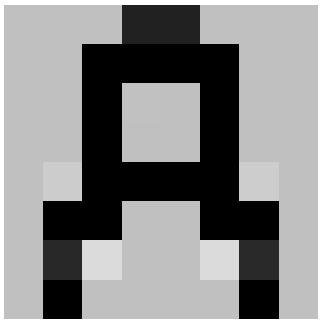
Example

New bottle, same old wine

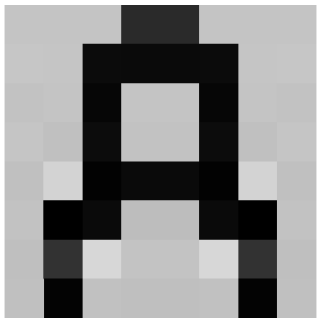
Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$





$\mathcal{I}$



$\tilde{\mathcal{I}}_{28}$

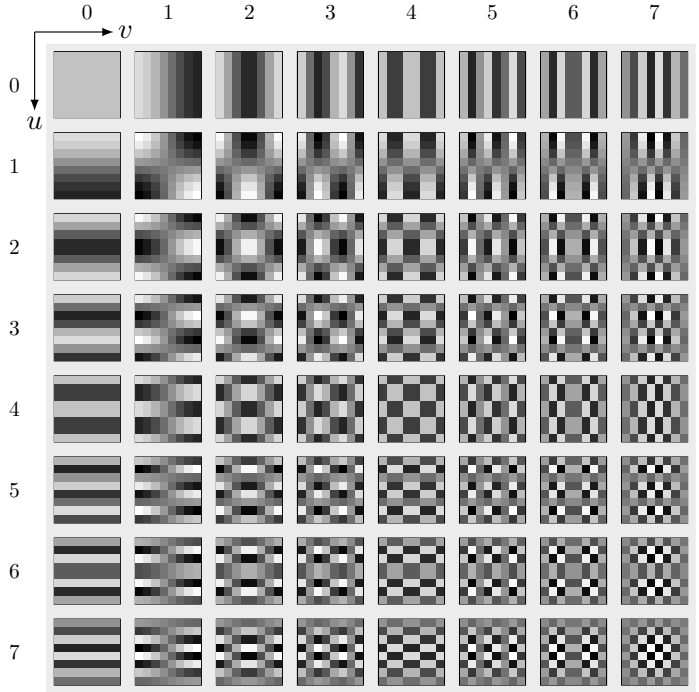
$\mathcal{I} - \tilde{\mathcal{I}}_{28}$

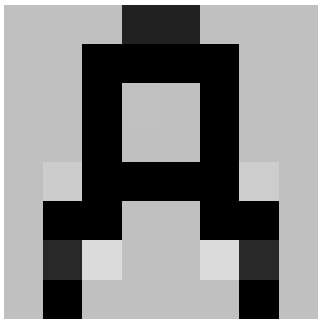
Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$





$\mathcal{I}$

$\tilde{\mathcal{I}}_{32}$

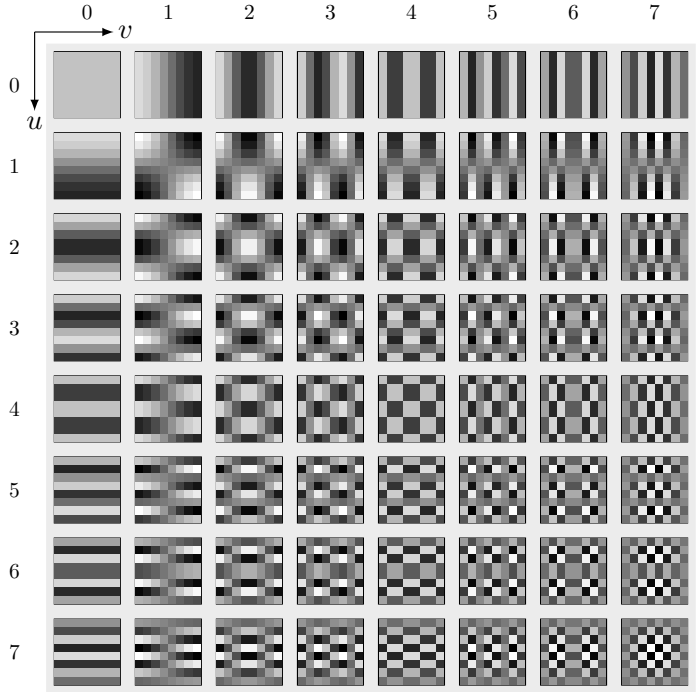
$\mathcal{I} - \tilde{\mathcal{I}}_{32}$

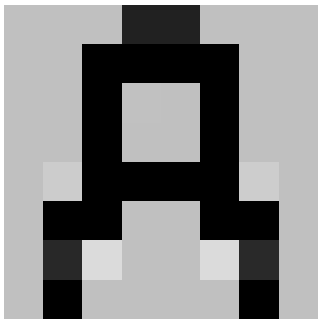
Example

New bottle, same old wine

Let's take again our 8×8 image $\mathcal{I} =$  and compute its DCT spectrum by $\mathcal{J}_{\mathbf{D}} = \mathbf{D}_8 \mathcal{I} \mathbf{D}_8^T$.

$$\mathcal{J}_{\mathbf{D}} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$





$\mathcal{I}$

$\tilde{\mathcal{I}}_{64}$

$\mathcal{I} - \tilde{\mathcal{I}}_{64}$

Rounding the DCT spectrum

A first step toward lossy compression

The (j,k)-th entry of the $N \times N$ DCT matrix is $\mathbf{D}_N(j,k) = \alpha(j)\cos\left(\frac{\pi(2k+1)j}{2N}\right)$ with

$$\alpha(j) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } j = 0, \\ \sqrt{\frac{2}{N}} & \text{otherwise.} \end{cases}$$

$$\mathbf{D}_8 = \frac{1}{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos(\frac{\pi}{16}) & \cos(\frac{3\pi}{16}) & \cos(\frac{5\pi}{16}) & \cos(\frac{7\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{11\pi}{16}) & \cos(\frac{13\pi}{16}) & \cos(\frac{15\pi}{16}) \\ \cos(\frac{2\pi}{16}) & \cos(\frac{6\pi}{16}) & \cos(\frac{10\pi}{16}) & \cos(\frac{14\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{22\pi}{16}) & \cos(\frac{26\pi}{16}) & \cos(\frac{30\pi}{16}) \\ \cos(\frac{3\pi}{16}) & \cos(\frac{9\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{27\pi}{16}) & \cos(\frac{33\pi}{16}) & \cos(\frac{39\pi}{16}) & \cos(\frac{45\pi}{16}) \\ \cos(\frac{4\pi}{16}) & \cos(\frac{12\pi}{16}) & \cos(\frac{20\pi}{16}) & \cos(\frac{28\pi}{16}) & \cos(\frac{36\pi}{16}) & \cos(\frac{44\pi}{16}) & \cos(\frac{52\pi}{16}) & \cos(\frac{60\pi}{16}) \\ \cos(\frac{5\pi}{16}) & \cos(\frac{15\pi}{16}) & \cos(\frac{25\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{45\pi}{16}) & \cos(\frac{55\pi}{16}) & \cos(\frac{65\pi}{16}) & \cos(\frac{75\pi}{16}) \\ \cos(\frac{6\pi}{16}) & \cos(\frac{18\pi}{16}) & \cos(\frac{30\pi}{16}) & \cos(\frac{42\pi}{16}) & \cos(\frac{54\pi}{16}) & \cos(\frac{66\pi}{16}) & \cos(\frac{78\pi}{16}) & \cos(\frac{90\pi}{16}) \\ \cos(\frac{7\pi}{16}) & \cos(\frac{21\pi}{16}) & \cos(\frac{35\pi}{16}) & \cos(\frac{49\pi}{16}) & \cos(\frac{63\pi}{16}) & \cos(\frac{77\pi}{16}) & \cos(\frac{91\pi}{16}) & \cos(\frac{105\pi}{16}) \end{bmatrix}$$

Rounding the DCT spectrum

A first step toward lossy compression

The (j,k)-th entry of the $N \times N$ DCT matrix is $\mathbf{D}_N(j,k) = \alpha(j)\cos\left(\frac{\pi(2k+1)j}{2N}\right)$ with

$$\alpha(j) = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } j = 0, \\ \sqrt{\frac{2}{N}} & \text{otherwise.} \end{cases}$$

$\rightarrow$ Entries of $\mathbf{D}_N$ are floating point numbers.

$$\mathbf{D}_8 = \frac{1}{2}\begin{bmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ 0.4904 & 0.4157 & 0.2778 & 0.0975 & -0.0975 & -0.2778 & -0.4157 & -0.4904 \\ 0.4619 & 0.1913 & -0.1913 & -0.4619 & -0.4619 & -0.1913 & 0.1913 & 0.4619 \\ 0.4157 & -0.0975 & -0.4904 & -0.2778 & 0.2778 & 0.4904 & 0.0975 & -0.4157 \\ 0.3536 & -0.3536 & -0.3536 & 0.3536 & 0.3536 & -0.3536 & -0.3536 & 0.3536 \\ 0.2778 & -0.4904 & 0.0975 & 0.4157 & -0.4157 & -0.0975 & 0.4904 & -0.2778 \\ 0.1913 & -0.4619 & 0.4619 & -0.1913 & -0.1913 & 0.4619 & -0.4619 & 0.1913 \\ 0.0975 & -0.2778 & 0.4157 & -0.4904 & 0.4904 & -0.4157 & 0.2778 & -0.0975 \end{bmatrix}$$

Rounding the DCT spectrum

A first step toward lossy compression

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$$\mathcal{J}_\mathbf{D} = \begin{bmatrix} 1036.38 & -0.26 & 232.68 & -0.03 & 228.38 & 0.49 & -60.52 & 0.02 \\ -14.65 & 0.24 & 243.27 & -0.13 & -181.82 & -0.14 & -224.28 & -0.28 \\ 78.48 & 0.09 & -97.16 & 0.03 & -109.01 & -0.21 & 290.97 & 0.02 \\ -74.41 & -0.20 & 75.28 & 0.17 & -68.36 & -0.02 & 82.06 & 0.34 \\ 42.38 & -0.03 & 51.96 & 0.10 & -122.13 & -0.15 & 21.91 & 0.17 \\ 162.51 & -0.02 & -163.91 & -0.02 & 21.13 & 0.08 & 56.39 & -0.03 \\ 6.87 & 0.32 & -99.53 & -0.19 & 127.82 & -0.13 & -8.59 & -0.41 \\ 47.59 & -0.28 & 52.52 & -0.04 & -126.96 & 0.55 & 19.85 & 0.01 \end{bmatrix}$$

Rounding the DCT spectrum

A first step toward lossy compression

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$\rightarrow$ Entries of $\mathbf{D}_N$ are floating point numbers.

$\rightarrow$ So are those of $\mathcal{J}_{\mathbf{D}}$.

$\rightarrow$ Round the spectrum values $\tilde{\mathcal{J}}_{\mathbf{D}} = \lfloor \mathcal{J}_{\mathbf{D}} \rceil$

$$\tilde{\mathcal{J}}_{\mathbf{D}} = \begin{bmatrix} 1036 & 0 & 233 & 0 & 228 & 0 & -61 & 0 \\ -15 & 0 & 243 & 0 & -182 & 0 & -224 & 0 \\ 78 & 0 & -97 & 0 & -109 & 0 & 291 & 0 \\ -74 & 0 & 75 & 0 & -68 & 0 & 82 & 0 \\ 42 & 0 & 52 & 0 & -122 & 0 & 22 & 0 \\ 163 & 0 & -164 & 0 & 21 & 0 & 56 & 0 \\ 7 & 0 & -100 & 0 & 128 & 0 & -9 & 0 \\ 48 & 0 & 53 & 0 & -127 & 1 & 20 & 0 \end{bmatrix}$$

Rounding the DCT spectrum

A first step toward lossy compression

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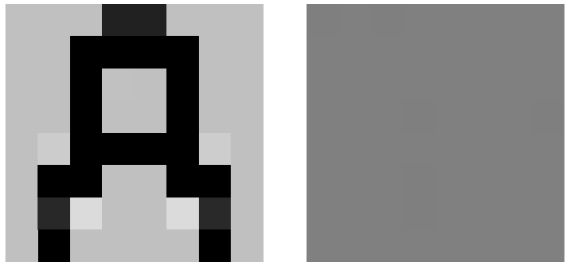
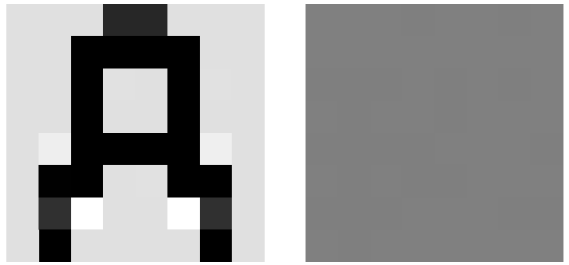
$$\tilde{\mathcal{J}}_{\mathbf{D}} = \begin{bmatrix} 1036 & 0 & 233 & 0 & 228 & 0 & -61 & 0 \\ -15 & 0 & 243 & 0 & -182 & 0 & -224 & 0 \\ 78 & 0 & -97 & 0 & -109 & 0 & 291 & 0 \\ -74 & 0 & 75 & 0 & -68 & 0 & 82 & 0 \\ 42 & 0 & 52 & 0 & -122 & 0 & 22 & 0 \\ 163 & 0 & -164 & 0 & 21 & 0 & 56 & 0 \\ 7 & 0 & -100 & 0 & 128 & 0 & -9 & 0 \\ 48 & 0 & 53 & 0 & -127 & 1 & 20 & 0 \end{bmatrix}$$

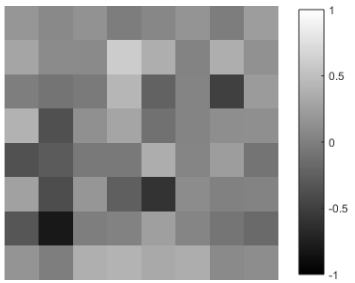
Reconstruction and error:



Without rounding

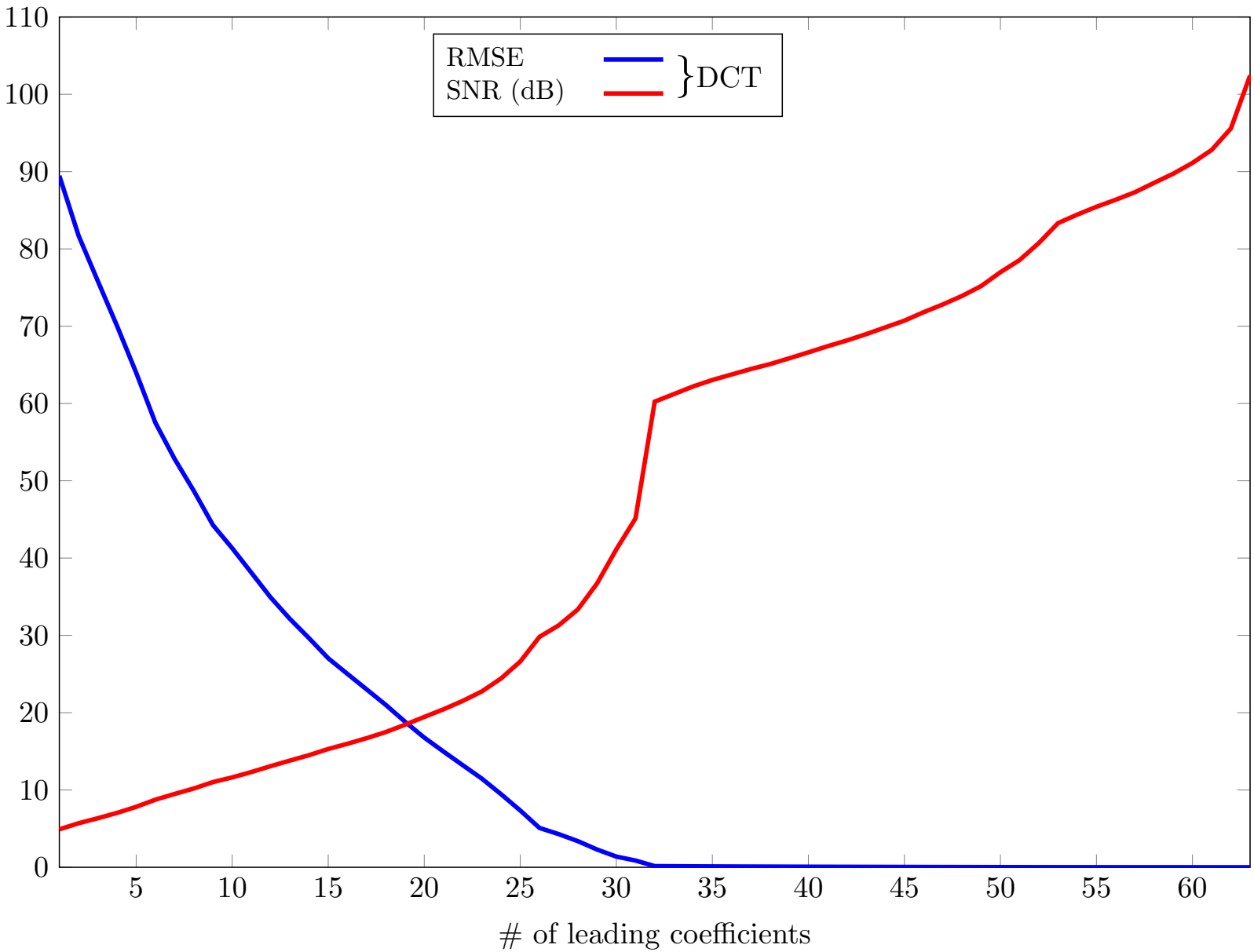


With rounding

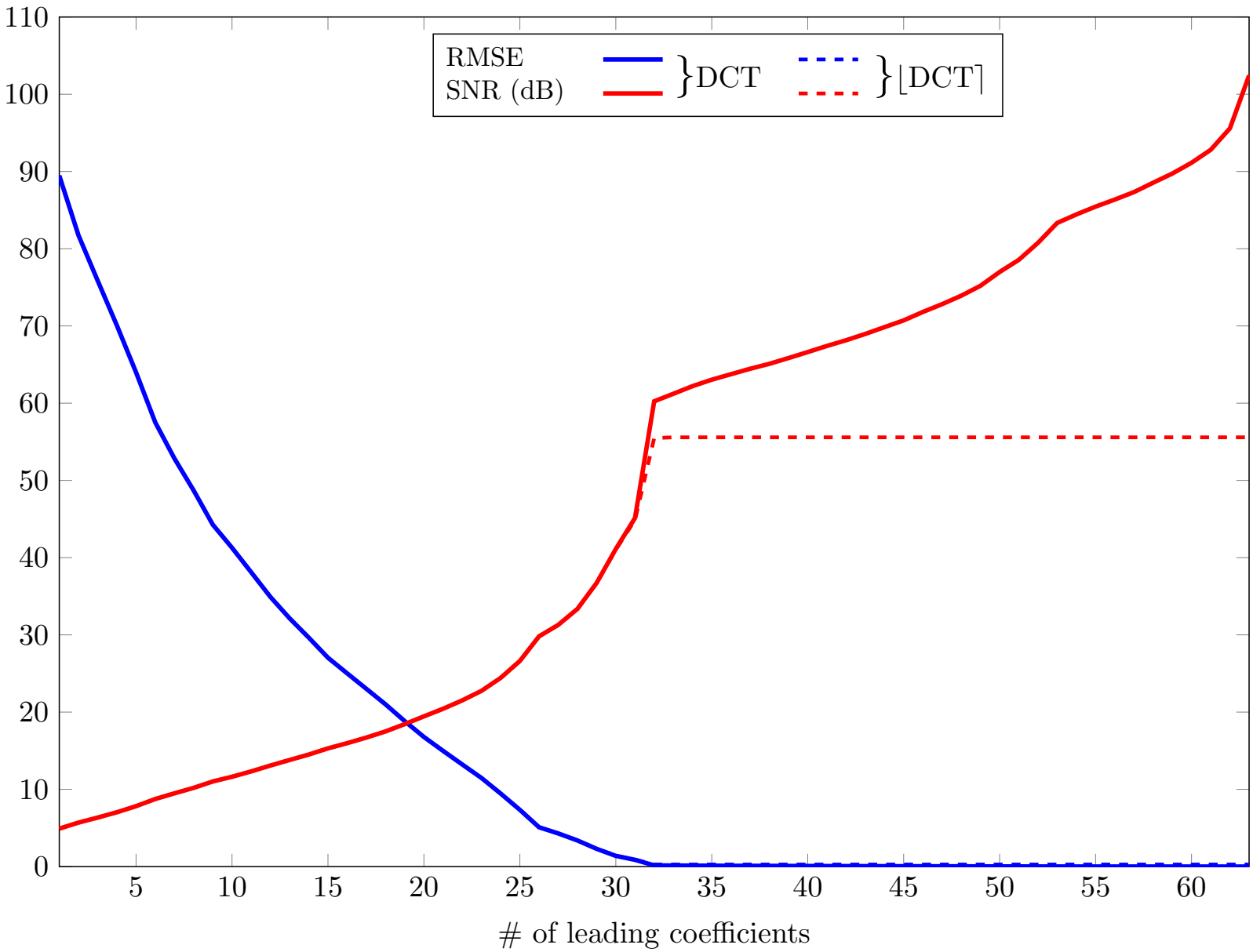


Difference

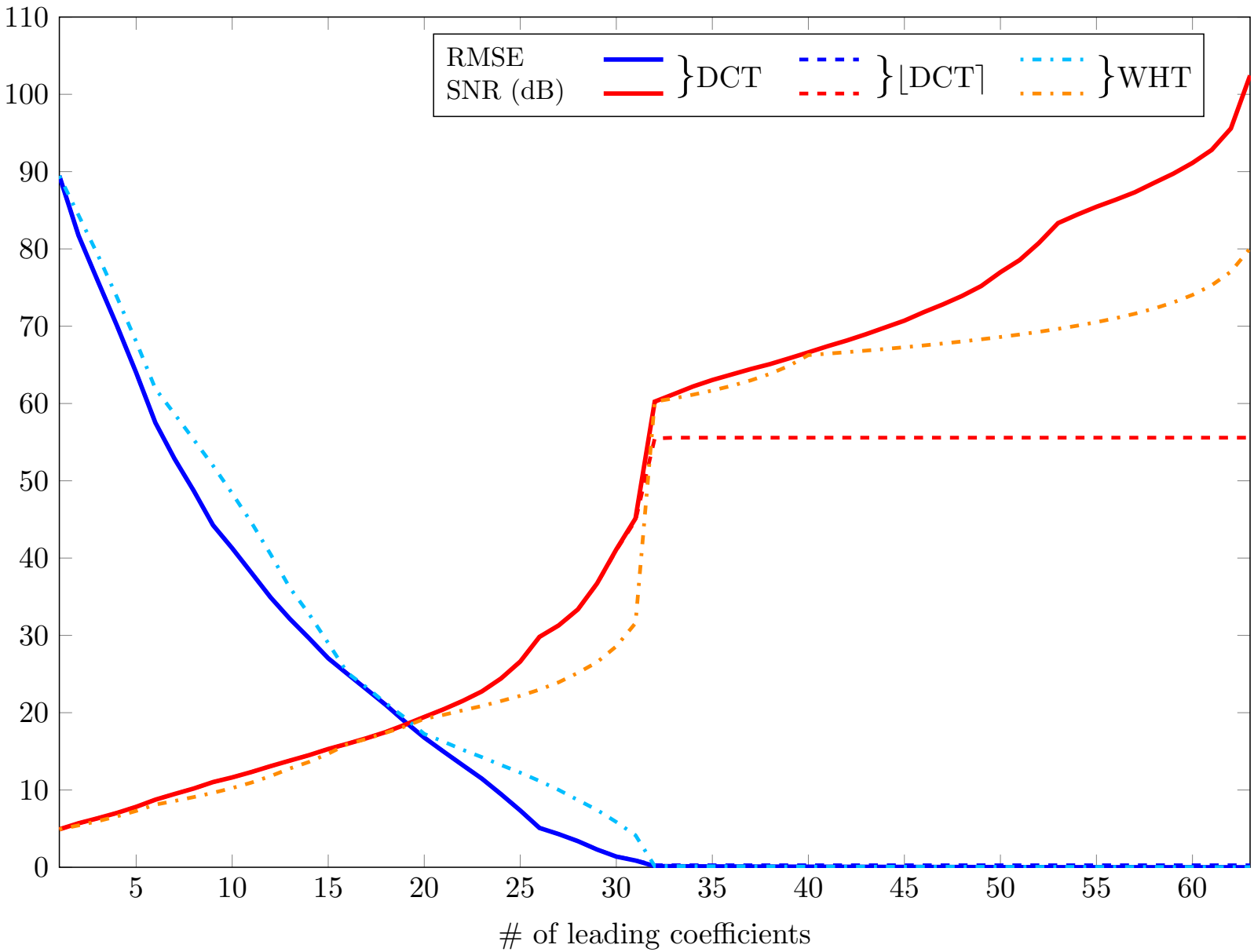
DCT Reconstruction error



DCT Reconstruction error

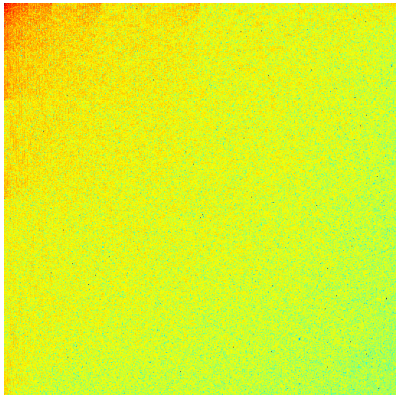


DCT Reconstruction error

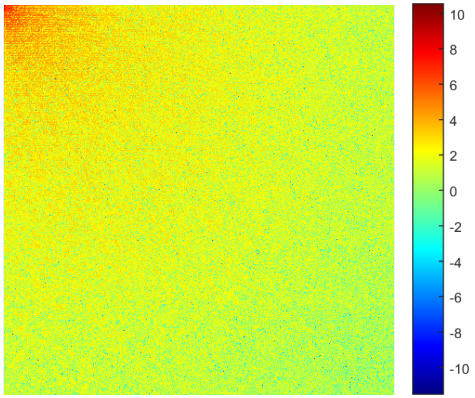


DCT vs. Walsh-Hadamard transform

512×512 image $\mathcal{I}$



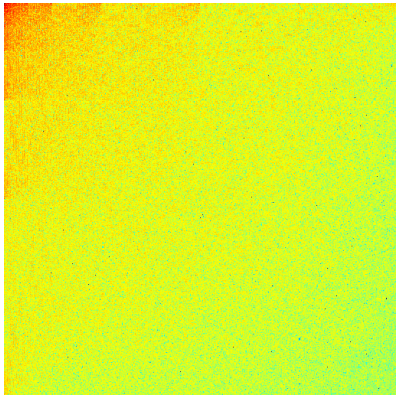
$\log(|\mathcal{J}_{\mathbf{H}}|)$



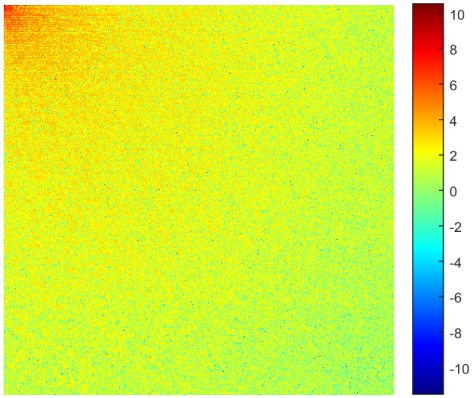
$\log(|\mathcal{J}_{\mathbf{D}}|)$

DCT vs. Walsh-Hadamard transform

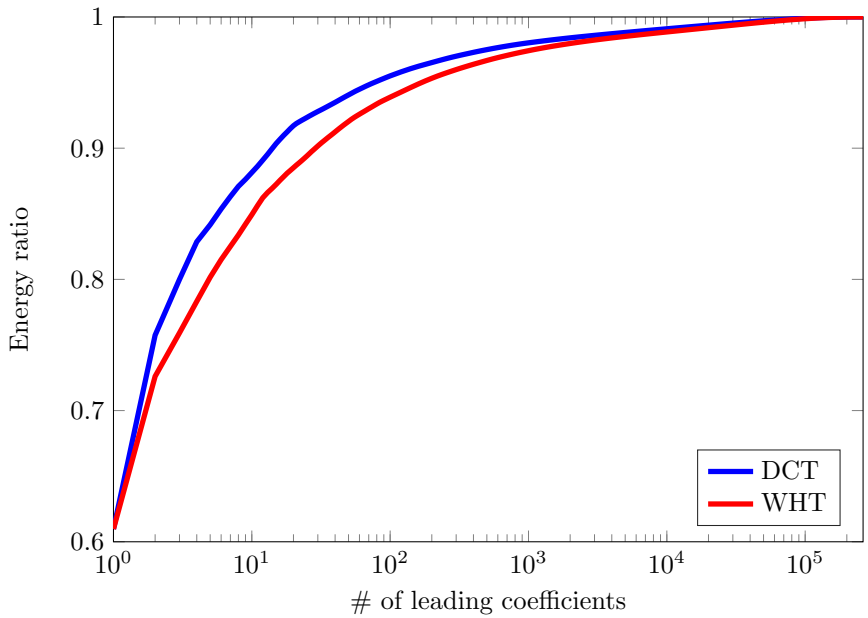
512 × 512 image $\mathcal{I}$



$\log(|\mathcal{J}_{\mathbf{H}}|)$



$\log(|\mathcal{J}_{\mathbf{D}}|)$



Parseval theorem gives $\|\mathcal{I}\|_2^2 = \|\mathcal{J}_{\mathbf{H}}\|_2^2 = \|\mathcal{J}_{\mathbf{D}}\|_2^2$

$\rightarrow$ A efficient transform should compact most of the image energy into as few coefficients as possible.

	80%	90%	95%	99%
DCT	4	15	77	7784
WHT	5	29	169	13501

Number of leading coefficients necessary to reach a given fraction of the total energy.

Frequency analysis

In summary

	WHT	DFT	DCT
Suited for frequency analysis	✓	✓	✓
Real result	✓	✗	✓
Computationally cheap	✓	✗	✗
Adapated to any image dimensions	✗	✓	✓
Efficient implementation available	✓	✓	✓
Energy compaction	✗	✗	✓

Frequency analysis

In summary

	WHT	DFT	DCT
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1. A first naive approach
2. Frequency analysis
3. JPEG compression algorithm
   - Compression scheme
   - Decompression scheme
   - Compression error analysis

JPEG compression algorithm

A brief overview

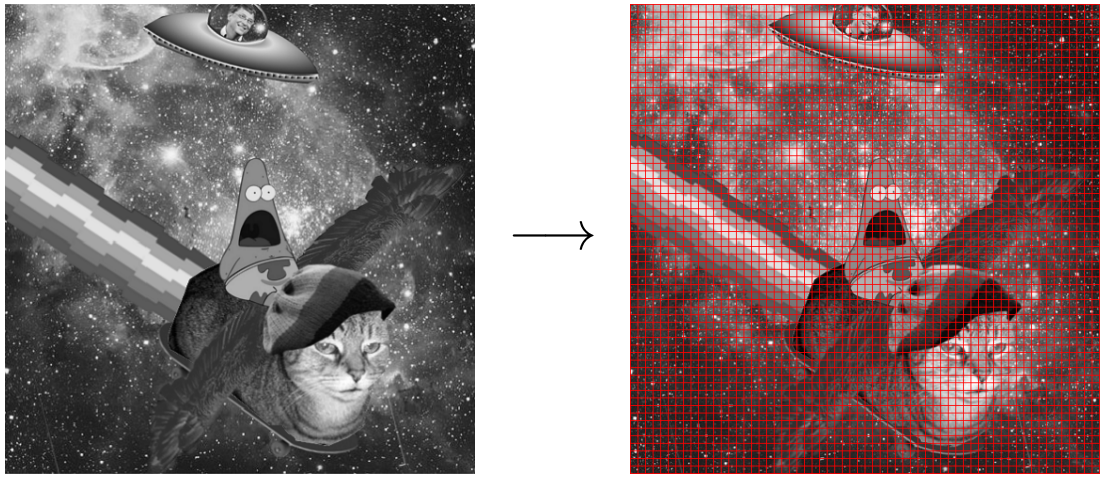
JPEG ≡ Joint Photographic Expert Group.

- → Started in 1986.
- → First standard (Part 1) released in 1992.
  - ↪ Its pet name is ISO/CEI 10918-1 = UIT-T Recommendation T.81.
- → Latest one (Part 6) released in 2013.
- → Still active today (2 or 3 meetings per year).
- → Has spawn many compression standards (JPEG2000, JPEG XR, incorporated in MPEG . . .).

JPEG compression algorithm

Step 1: Block splitting

1st step: the input image is divided into non-overlapping 8×8 macro-blocks.



JPEG compression algorithm

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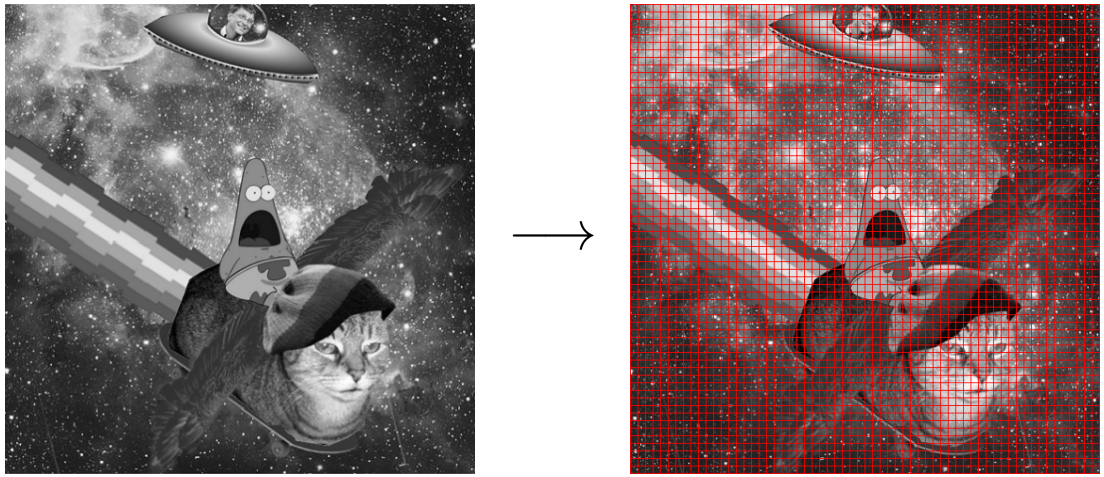
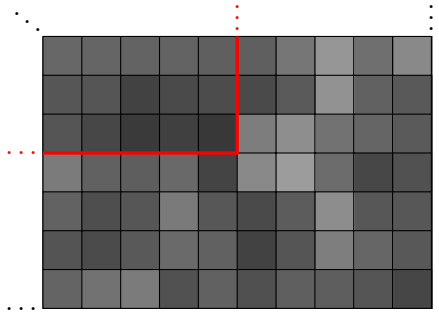


If the dimensions are not divisible in integer numbers of blocks, the image can be padded



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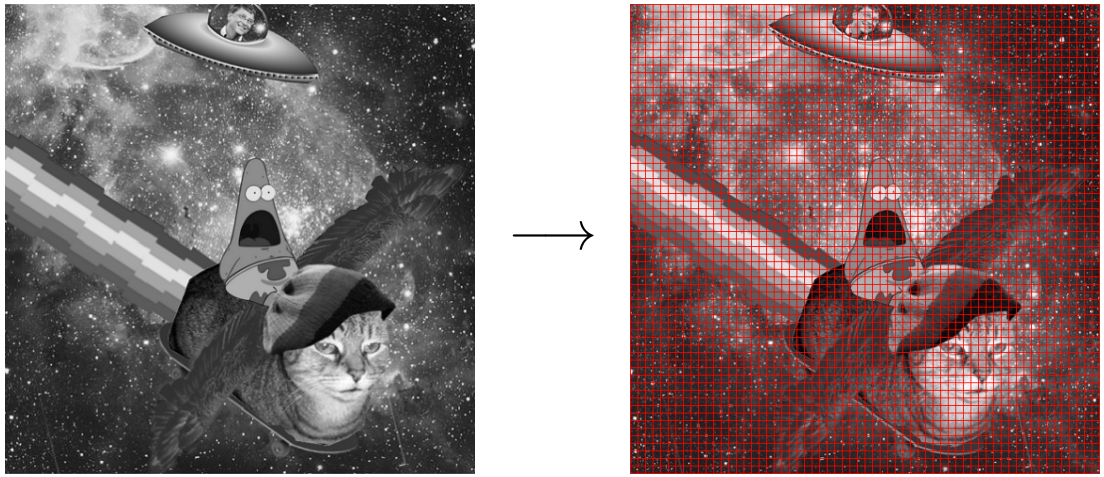
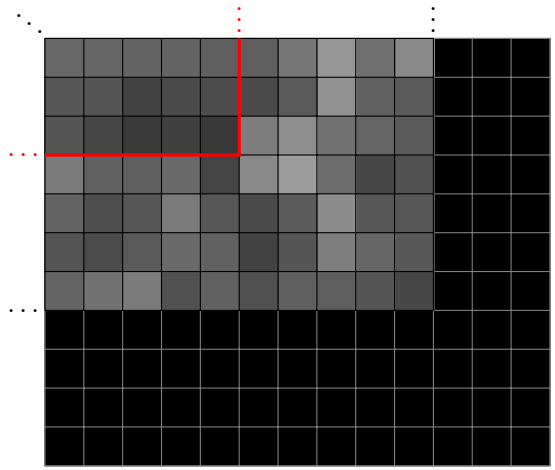


If the dimensions are not divisible in integer numbers of blocks, the image can be padded

→ with black pixels ■ (crude, may create noticeable artifacts).



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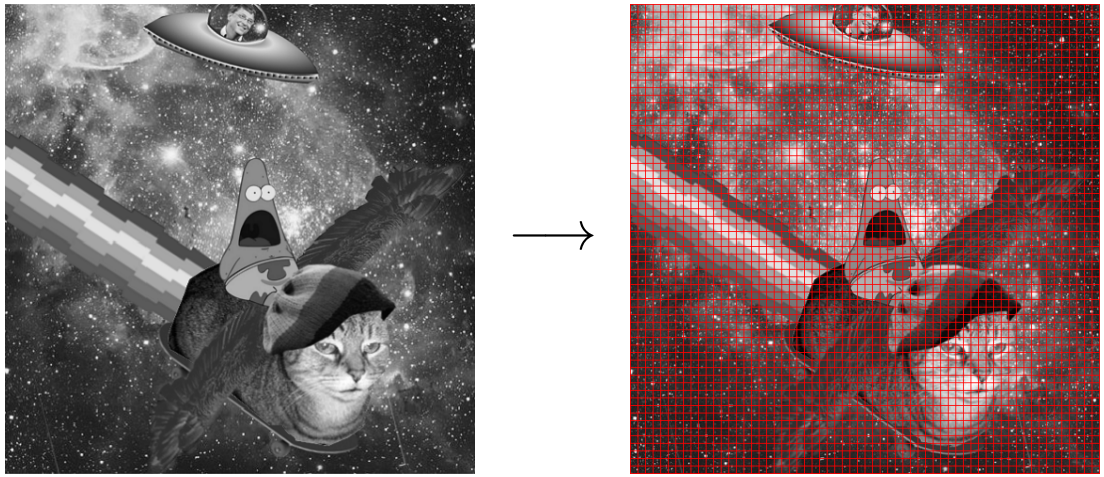

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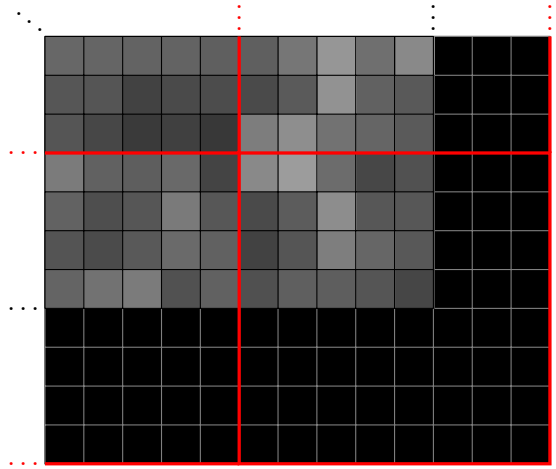

JPEG compression algorithm

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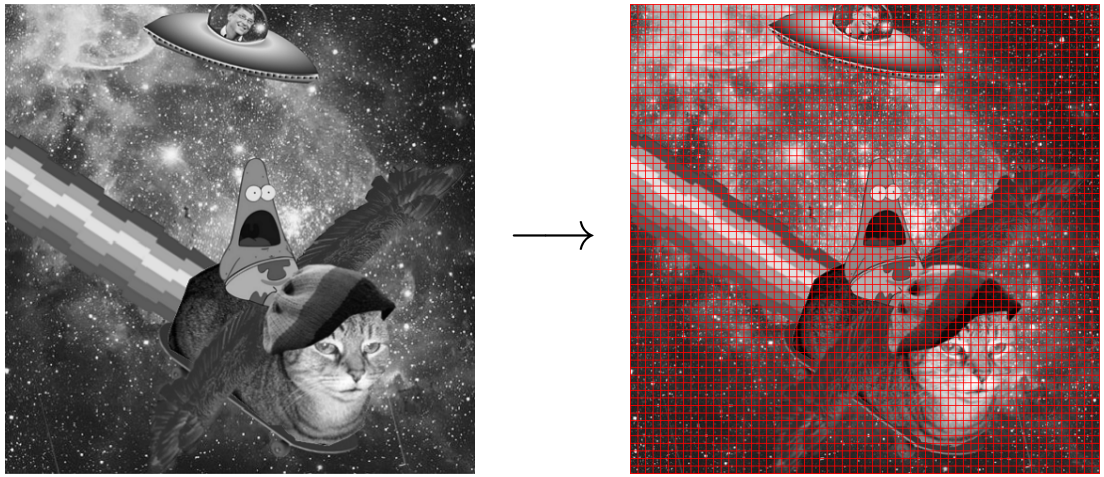
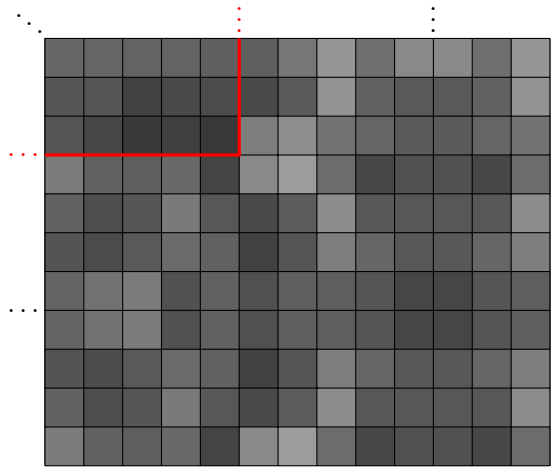


If the dimensions are not divisible in integer numbers of blocks, the image can be padded

- → with black pixels ■ (crude, may create noticeable artifacts).
- → by replicating the border pixels (reduces the artifacts, but not necessarily all of them).



JPEG compression algorithm

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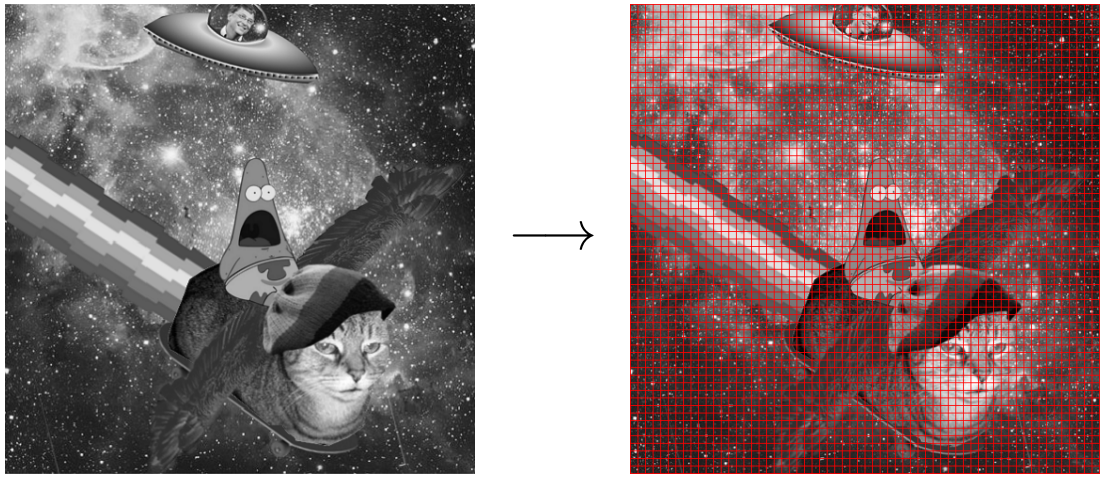

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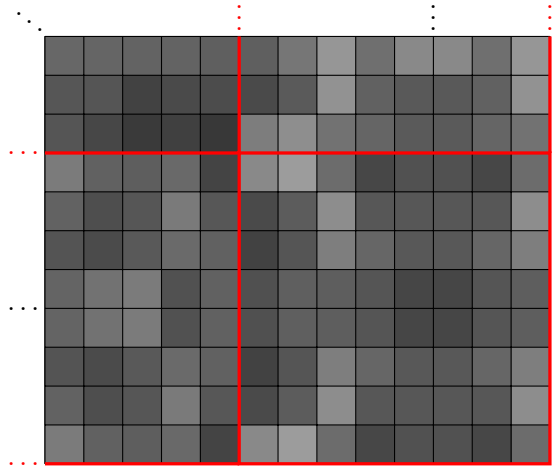
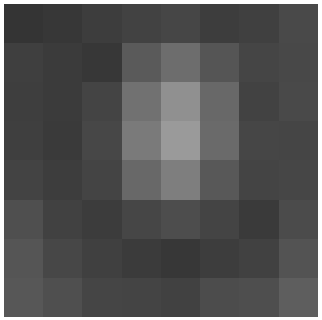


JPEG compression algorithm

Step 2: DCT

2nd step: the DCT of each 8×8 block is computed.

$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

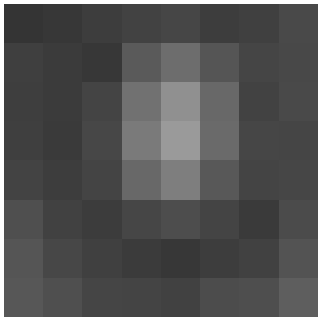


JPEG compression algorithm

Step 2: DCT

2nd step: the DCT of each 8×8 block is computed.

$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$



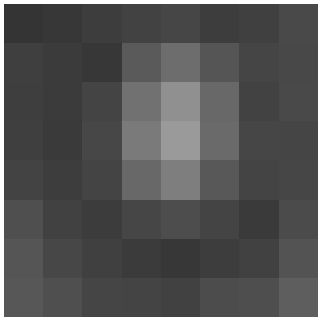
$$\mathcal{J}_{\mathbf{B}_i} = \mathrm{DCT}(\mathbf{B}_i) = \begin{bmatrix} 609 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$

JPEG compression algorithm

Step 2: DCT

2nd step: the DCT of each 8×8 block is computed.

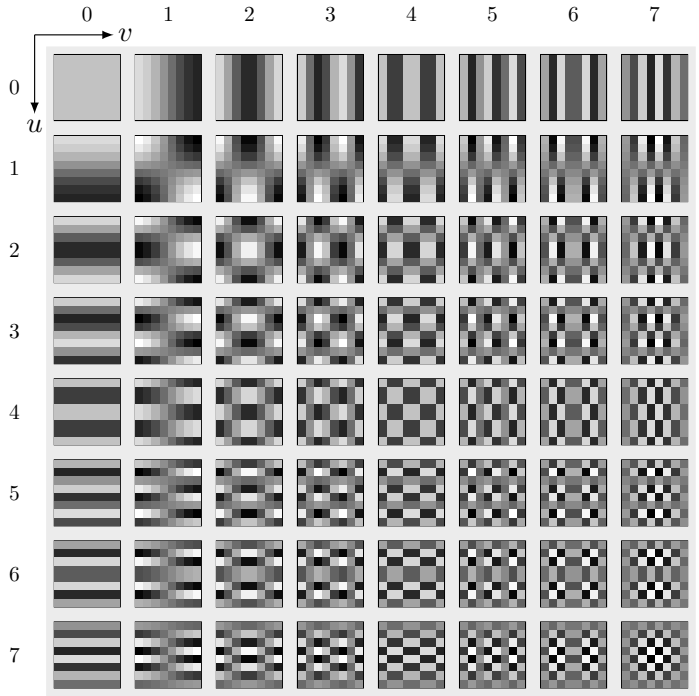
$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$



DC coefficient

$$\mathcal{J}_{\mathbf{B}_i} = \mathrm{DCT}(\mathbf{B}_i) = \begin{bmatrix} 609 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$

AC coefficients

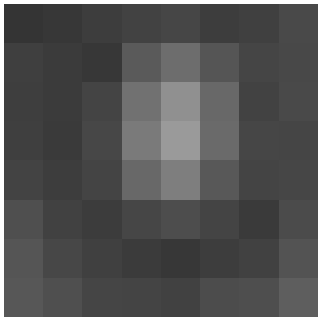


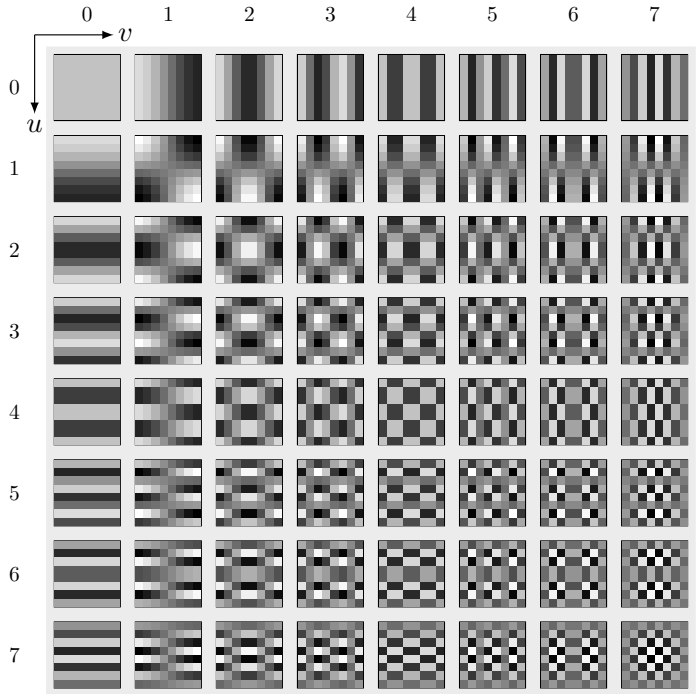
JPEG compression algorithm

Step 2: DCT

2nd step: the DCT of each 8×8 block is computed.

$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$





DC coefficient

AC coefficients

$$\mathcal{J}_{\mathbf{B}_i} = \mathrm{DCT}(\mathbf{B}_i) = \begin{bmatrix} 609 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$

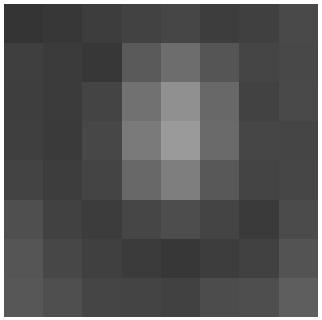
In average, $\mathbf{B}_i$ has a mean value close to 128 $\Rightarrow$ DC coefficient of $\mathrm{DCT}(\mathbf{B}_i - 128)$ should be close to 0.

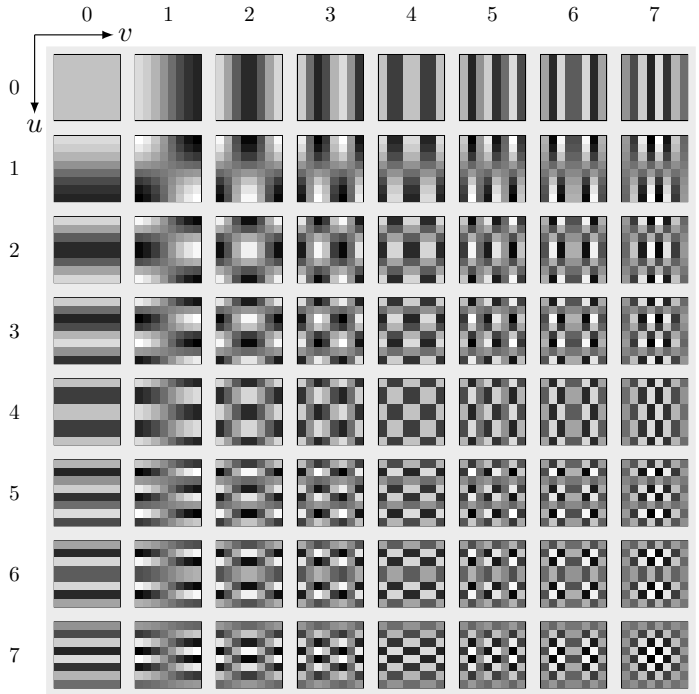
JPEG compression algorithm

Step 2: DCT

2nd step: the DCT of each (8×8 block) -128 is computed.

$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$





$$\mathcal{J}_{\mathbf{B}_i} = \mathrm{DCT}(\mathbf{B}_i - 128) = \begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$

In average, $\mathbf{B}_i$ has a mean value close to 128 $\Rightarrow$ DC coefficient of $\mathrm{DCT}(\mathbf{B}_i - 128)$ should be close to 0.

JPEG compression algorithm

Step 3: quantization

3rd step: the DCT is quantized by some quantification matrix $\mathcal{Q}$.

$$\tilde{\mathcal{J}}_{\mathbf{B}_i} = \lfloor \mathcal{J}_{\mathbf{B}_i} \oslash \mathcal{Q} \rceil$$

element-wise division

JPEG compression algorithm

Step 3: quantization

3rd step: the DCT is quantized by some quantification matrix $\mathcal{Q}$.

$$\tilde{\mathcal{J}}_{\mathbf{B}_i} = \lfloor \mathcal{J}_{\mathbf{B}_i} \oslash \mathcal{Q} \rceil$$

element-wise division

$$= \left\lfloor \begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix} \oslash \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} \right\rceil$$

If you wonder how the hell did they come up with those quantization values, go read *JPEG: Still image data compression standard* by W.B. Pennebaker & J.L. Mitchell, Springer Science & Business Media (1992).

JPEG compression algorithm

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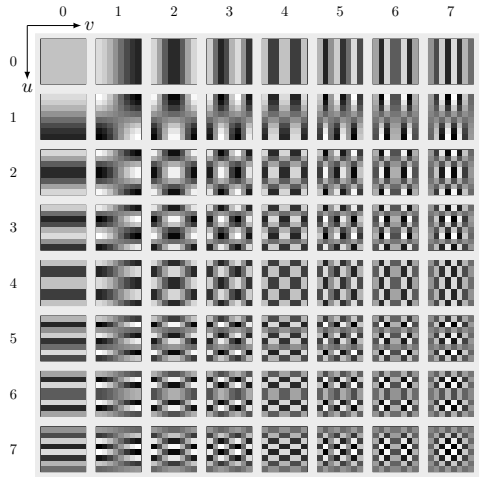
$$\tilde{\mathcal{J}}_{\mathbf{B}_i} = \lfloor \mathcal{J}_{\mathbf{B}_i} \oslash \mathcal{Q} \rceil$$

element-wise division

$$= \left\lfloor \begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix} \oslash \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} \right\rceil$$



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JPEG compression algorithm

Step 3: quantization

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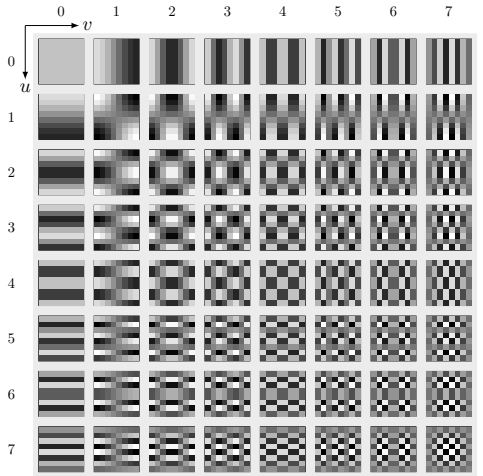
$$\tilde{\mathcal{J}}_{\mathbf{B}_i} = \lfloor \mathcal{J}_{\mathbf{B}_i} \oslash \mathcal{Q} \rceil$$

element-wise division

$$= \left\lfloor \begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix} \oslash \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} \right\rceil$$

$$= \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



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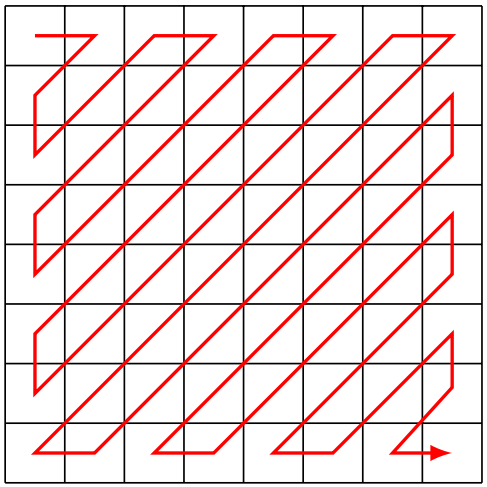
JPEG compression algorithm

Step 4: Zigzag ordering + entropy coding

4^{th} step: Arrange the quantized values in sequence following the zigzag order and use Huffman encoding with pre-determined conversion tables.

$$\tilde{\mathcal{J}}_{\mathbf{B}_i} = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



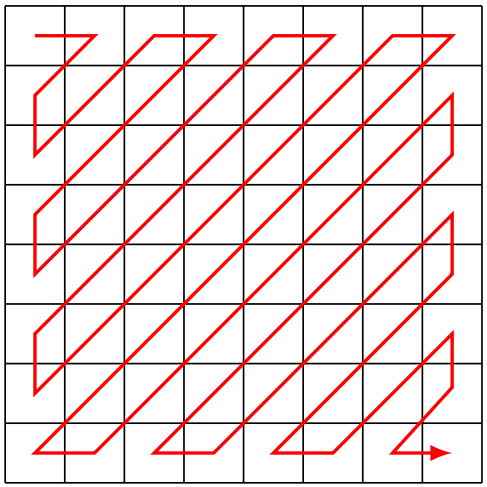
zigzag ordering

JPEG compression algorithm

Step 4: Zigzag ordering + entropy coding

4^{th} step: Arrange the quantized values in sequence following the zigzag order and use Huffman encoding with pre-determined conversion tables.

$$\text{zigzag}(\tilde{\mathcal{J}}_{\mathbf{B}_i}) = \begin{bmatrix} -26 & 3 & -6 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



zigzag ordering

JPEG compression algorithm

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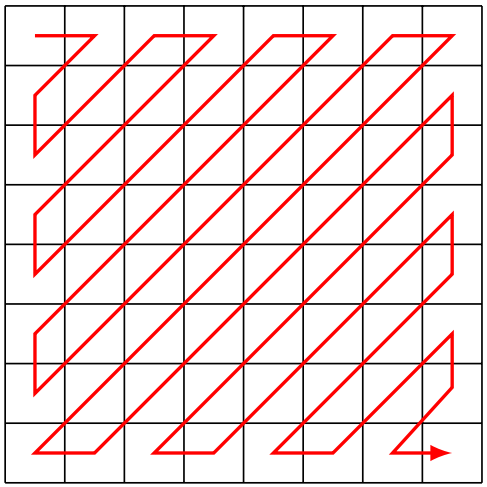
$$\text{zigzag}(\tilde{\mathcal{J}}_{\mathbf{B}_i}) = \begin{bmatrix} -26 & 3 & -6 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



zigzag ordering

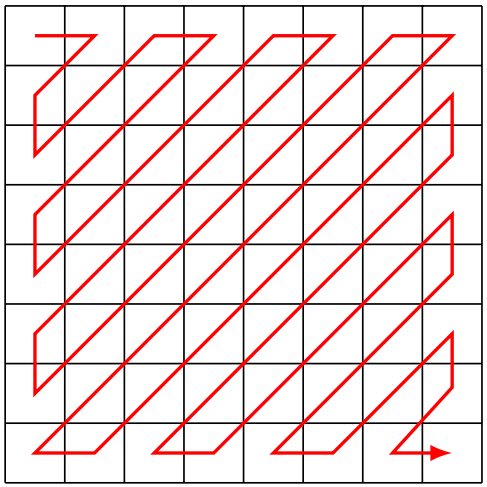
$=$ -26 -3 0 -3 -2 -6 2 -4 1 -3 1 1 5 1 2 -1 1 -1 2 0 0 0 0 0 -1 -1 EOB

JPEG compression algorithm

Step 4: Zigzag ordering + entropy coding

4^{th} step: Arrange the quantized values in sequence following the zigzag order and use Huffman encoding with pre-determined conversion tables.

$$\text{zigzag}(\tilde{\mathcal{J}}_{\mathbf{B}_i}) = \begin{bmatrix} -26 & 3 & -6 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



zigzag ordering

$$= \text{-26 -3 0 -3 -2 -6 2 -4 1 -3 1 1 5 1 2 -1 1 -1 2 0 0 0 0 0 -1 -1 EOB}$$

After applying Huffman with standard JPEG tables, the final encoding is:

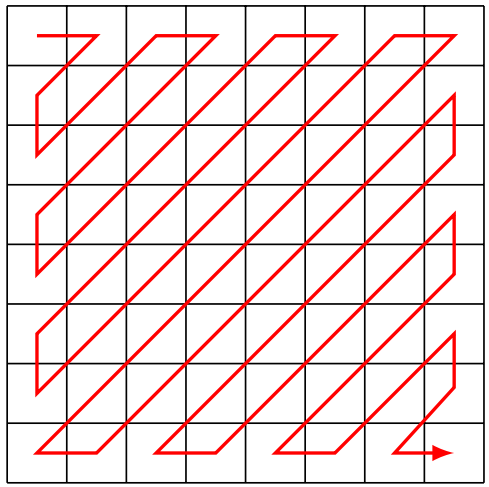
1010110 0100 11100100 0101 100001 0110 100011 001 0100 001 001 100101 001 0110 000 001 000 0110 11110100 000 1010

JPEG compression algorithm

Step 4: Zigzag ordering + entropy coding

4th step: Arrange the quantized values in sequence following the zigzag order and use Huffman encoding with pre-determined conversion tables.

$$\text{zigzag}(\tilde{\mathcal{J}}_{\mathbf{B}_i}) = \begin{bmatrix} -26 & 3 & -6 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



zigzag ordering

$=$ -26 -3 0 -3 -2 -6 2 -4 1 -3 1 1 5 1 2 -1 1 -1 2 0 0 0 0 0 -1 -1 EOB

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JPEG compression algorithm

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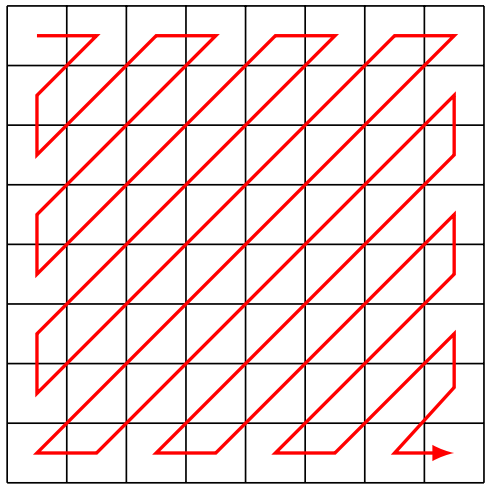
$$\text{zigzag}(\tilde{\mathcal{J}}_{\mathbf{B}_i}) = \begin{bmatrix} -26 & 3 & -6 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



zigzag ordering

$$= \text{-26 -3 0 -3 -2 -6 2 -4 1 -3 1 1 5 1 2 -1 1 -1 2 0 0 0 0 0 -1 -1 EOB}$$

After applying Huffman with standard JPEG tables, the final encoding is:

1010110 0100 11100100 0101 100001 0110 100011 001 0100 001 001 100101 001 0110 000 001 000 0110 11110100 000 1010

Before compression: $8 \times 8 \times 8 = 512$ bits.

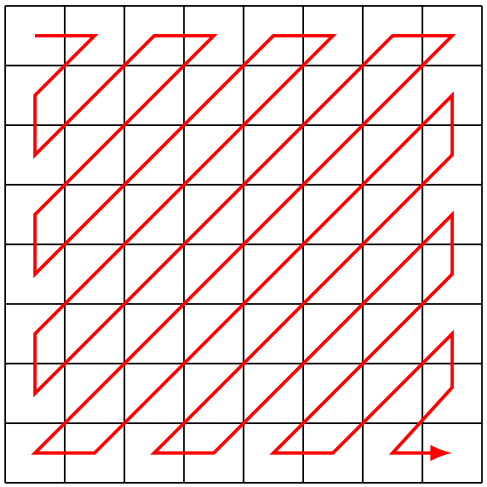
After compression: 94 bits.

JPEG compression algorithm

Step 4: Zigzag ordering + entropy coding

4th step: Arrange the quantized values in sequence following the zigzag order and use Huffman encoding with pre-determined conversion tables.

$$\text{zigzag}(\tilde{\mathcal{J}}_{\mathbf{B}_i}) = \begin{bmatrix} -26 & 3 & -6 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



zigzag ordering

$$= \text{-26 -3 0 -3 -2 -6 2 -4 1 -3 1 1 5 1 2 -1 1 -1 2 0 0 0 0 0 -1 -1 EOB}$$

After applying Huffman with standard JPEG tables, the final encoding is:

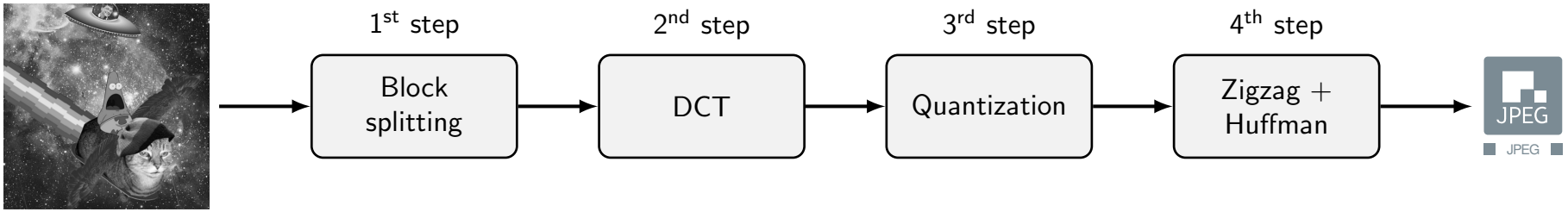
1010110 0100 11100100 0101 100001 0110 100011 001 0100 001 001 100101 001 0110 000 001 000 0110 11110100 000 1010

Before compression: $8 \times 8 \times 8 = 512$ bits.
After compression: 94 bits.

$\Rightarrow$ compression ratio $\cong 5.5$

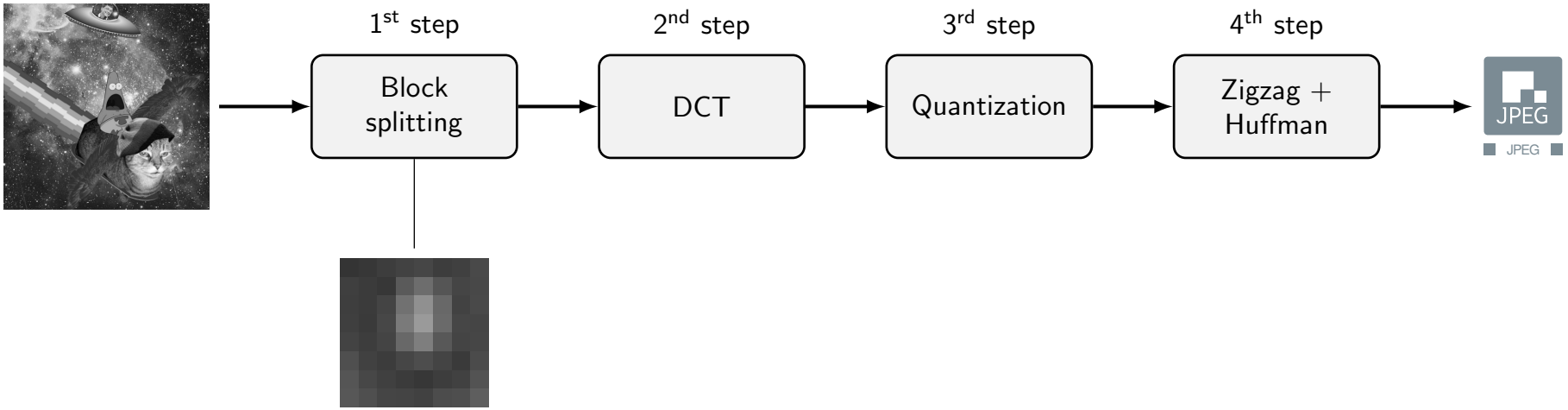
JPEG compression algorithm

Summary of the compression scheme



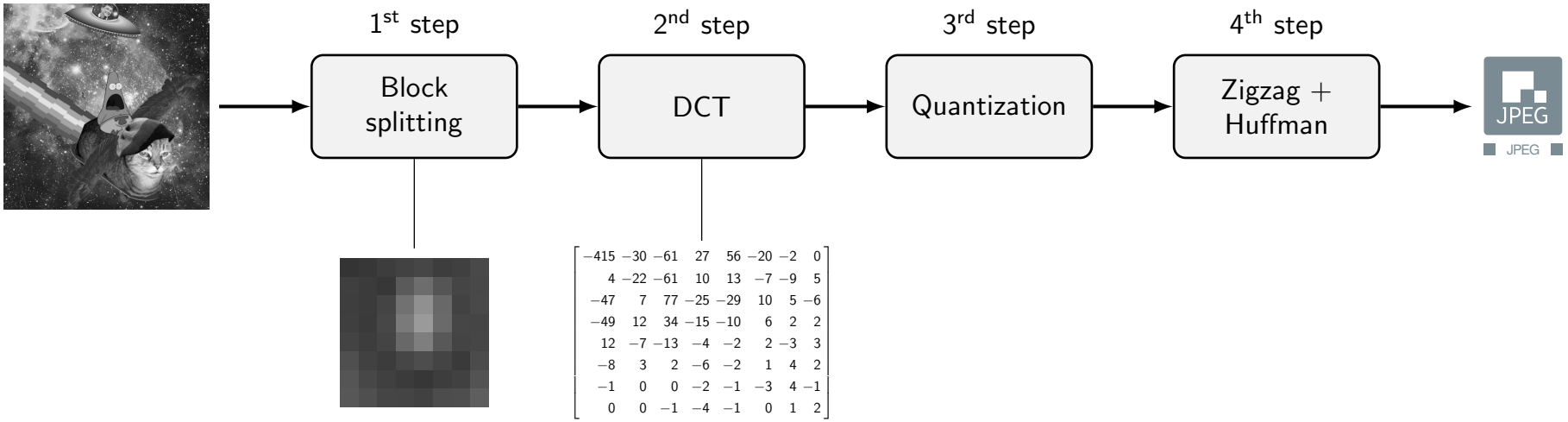
JPEG compression algorithm

Summary of the compression scheme



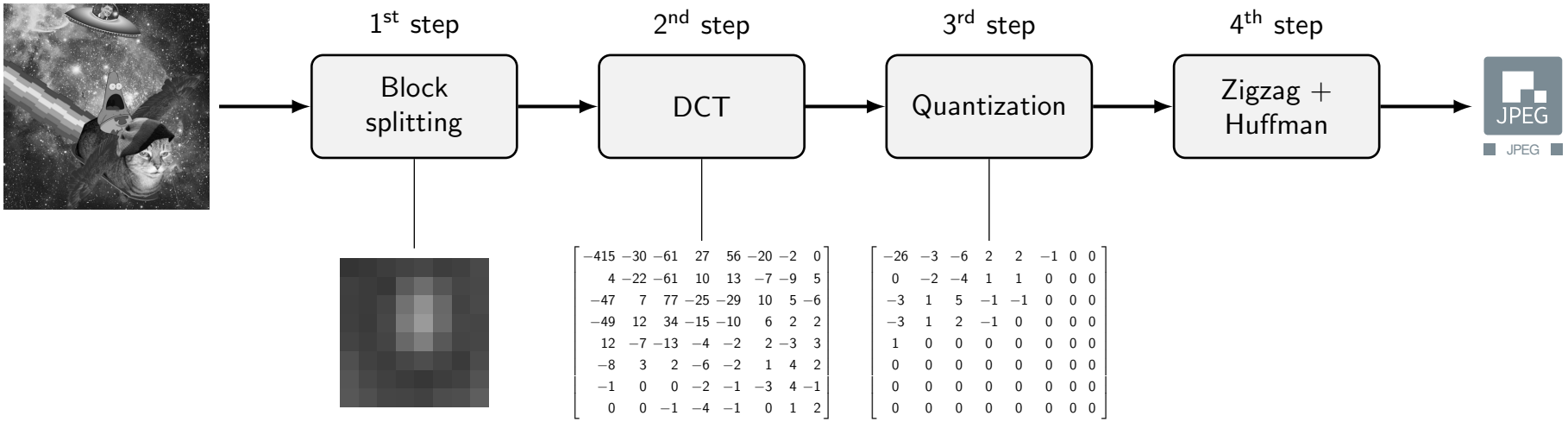
JPEG compression algorithm

Summary of the compression scheme



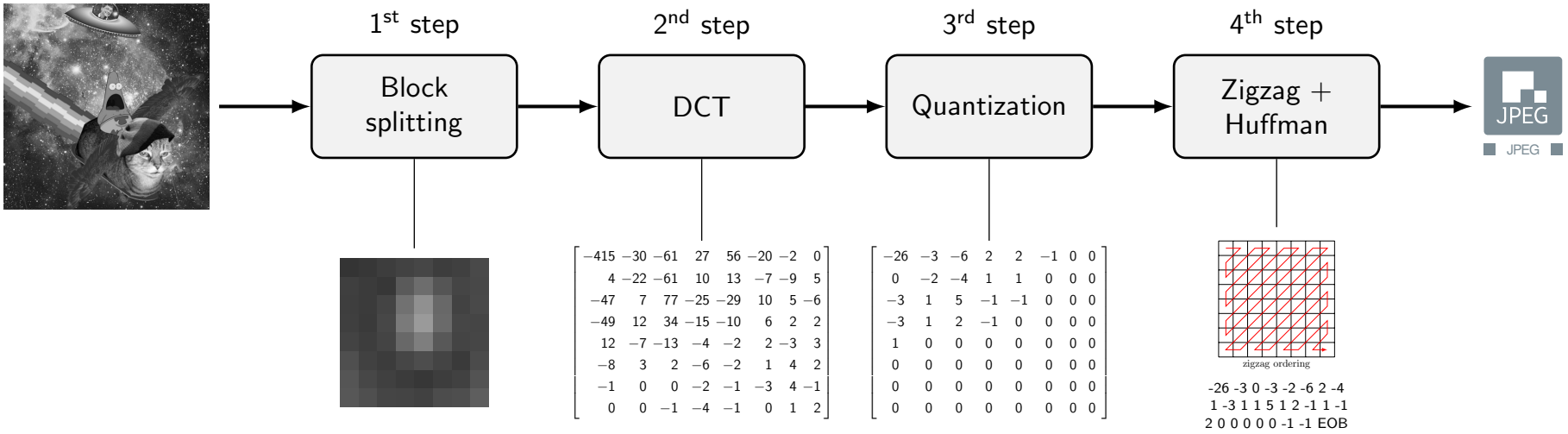
JPEG compression algorithm

Summary of the compression scheme



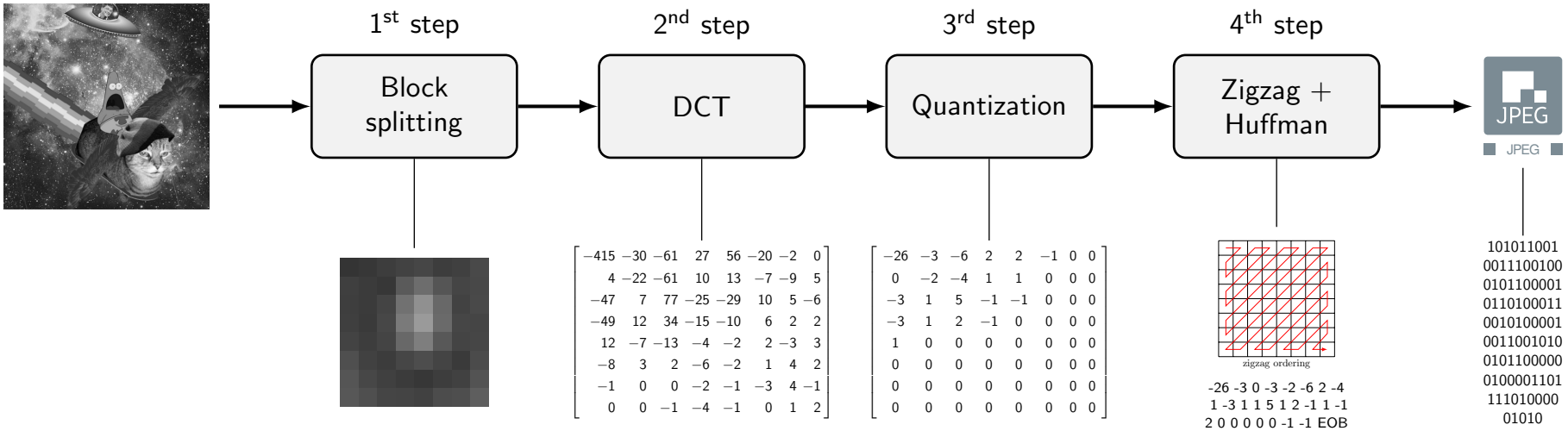
JPEG compression algorithm

Summary of the compression scheme



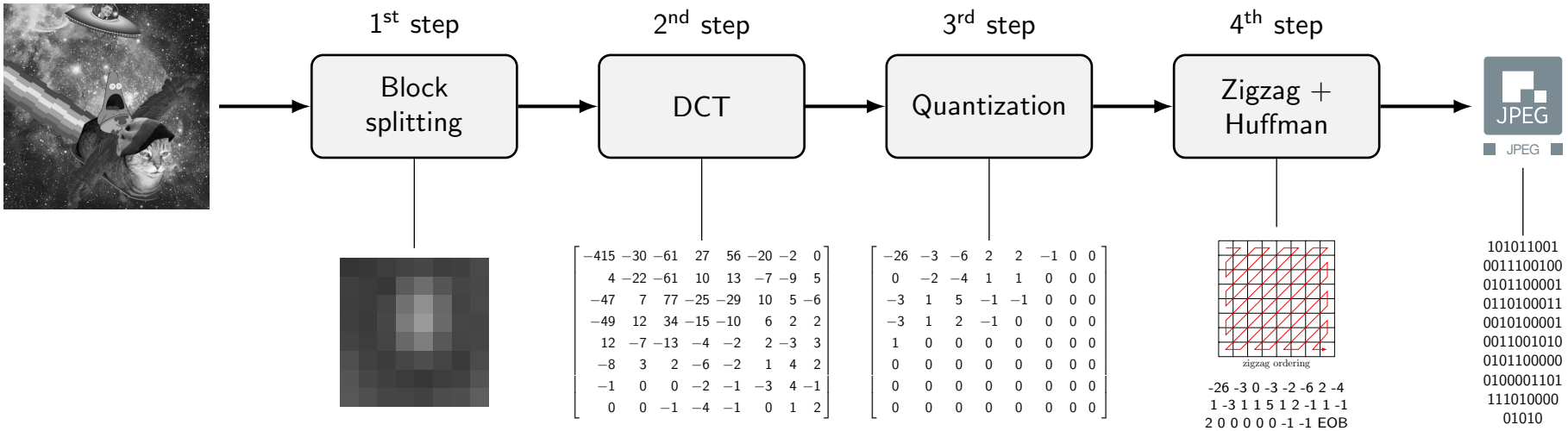
JPEG compression algorithm

Summary of the compression scheme



JPEG compression algorithm

Summary of the compression scheme

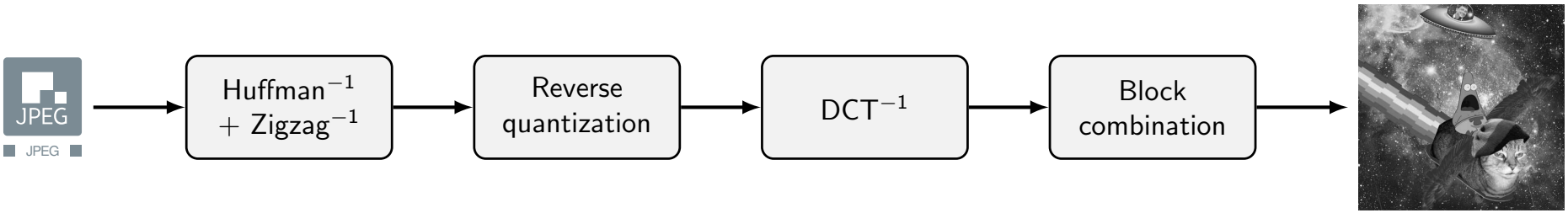



```
-rw-r--r-- 1 gtochon lrde 245K mai   19 13:22 randompic.tif
-rw-r--r-- 1 gtochon lrde  35K mai   19 14:18 randompic.jpg
```

JPEG compression algorithm

Decompression

JPEG decompression process is the exact inverse of the compression scheme.



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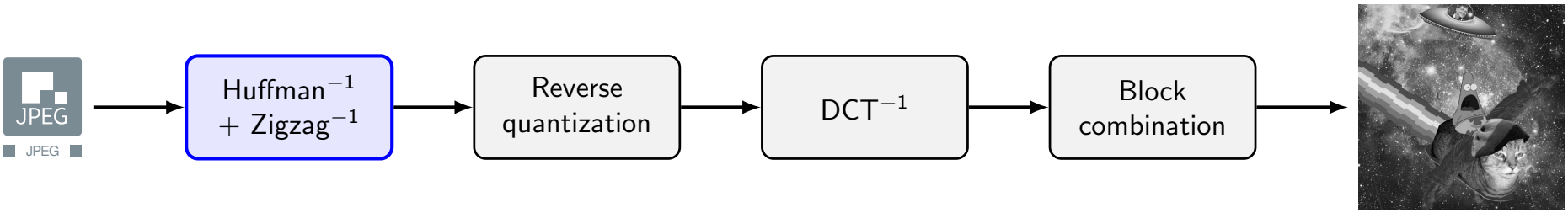

101011001001
110010001011
000010110100
011001010000
100110010100
101100000010
000110111101
000001010

JPEG compression algorithm

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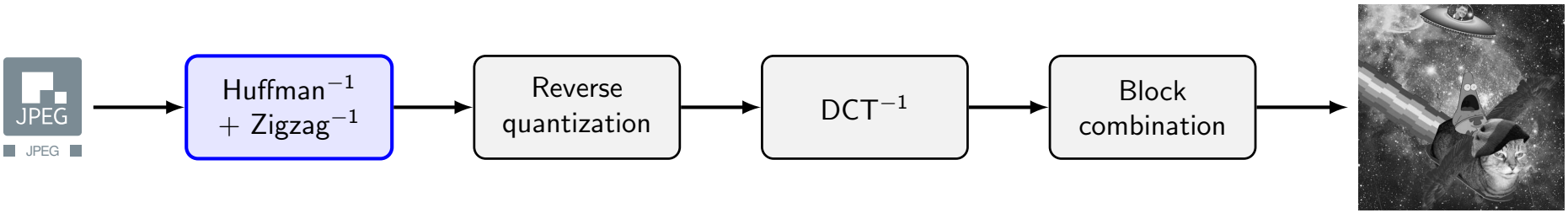


```
101011001001
110010001011
000010110100
011001010000 ⟶ -26 -3 0 -3 -2 -6 2 -4
100110010100    1 -3 1 1 5 1 2 -1 1 -1
101100000010    2 0 0 0 0 0 -1 -1 EOB
000110111101
 000001010
```

JPEG compression algorithm

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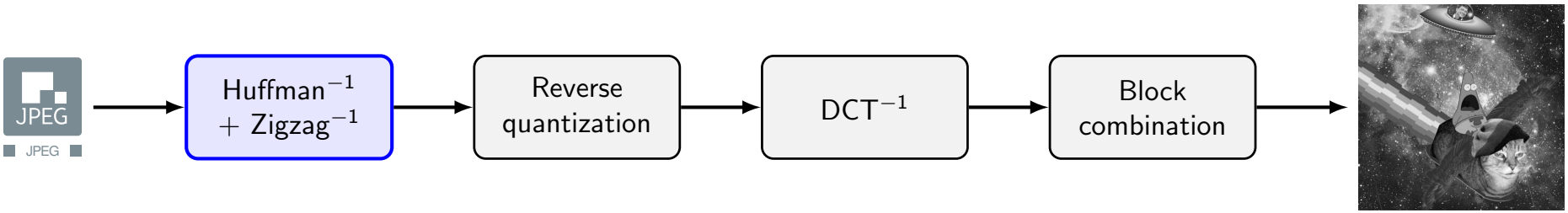

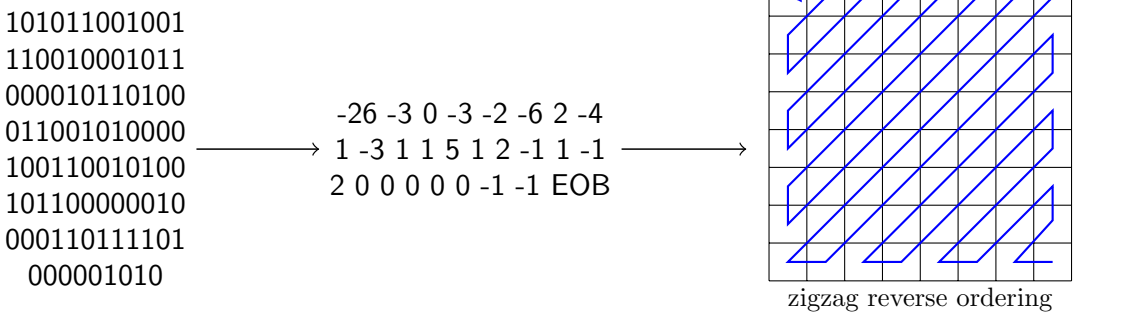

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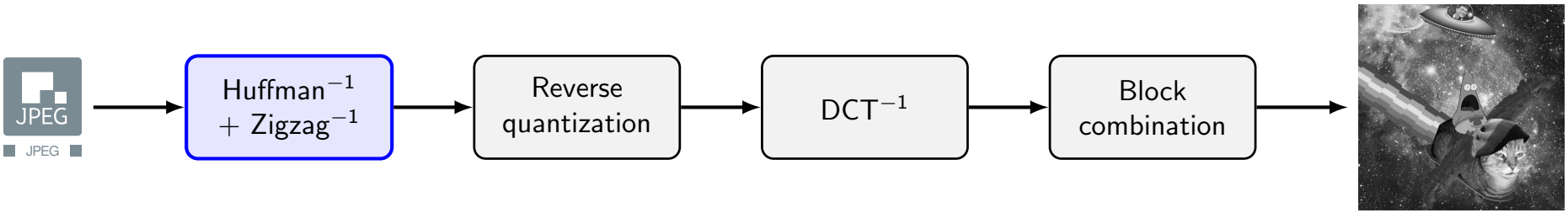

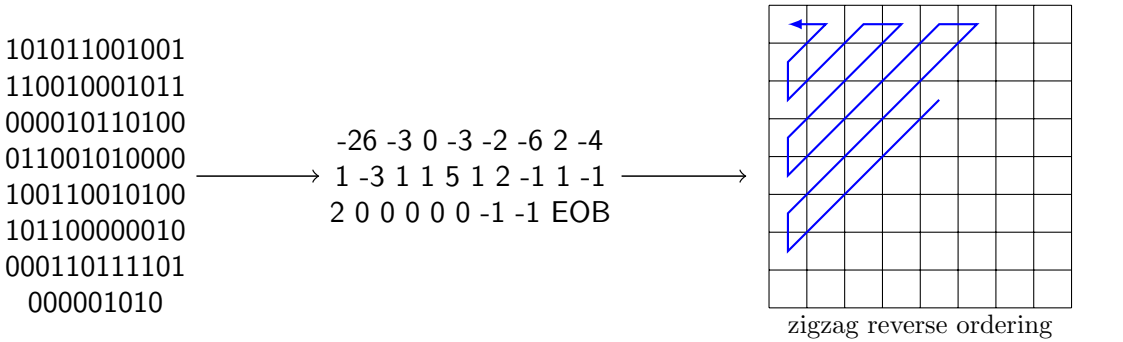

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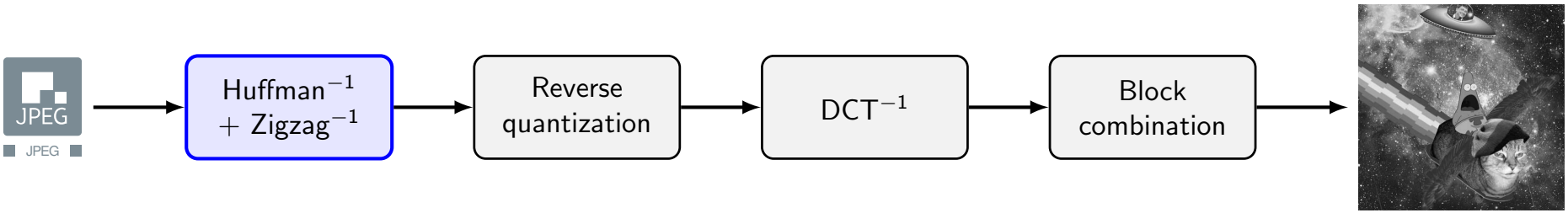

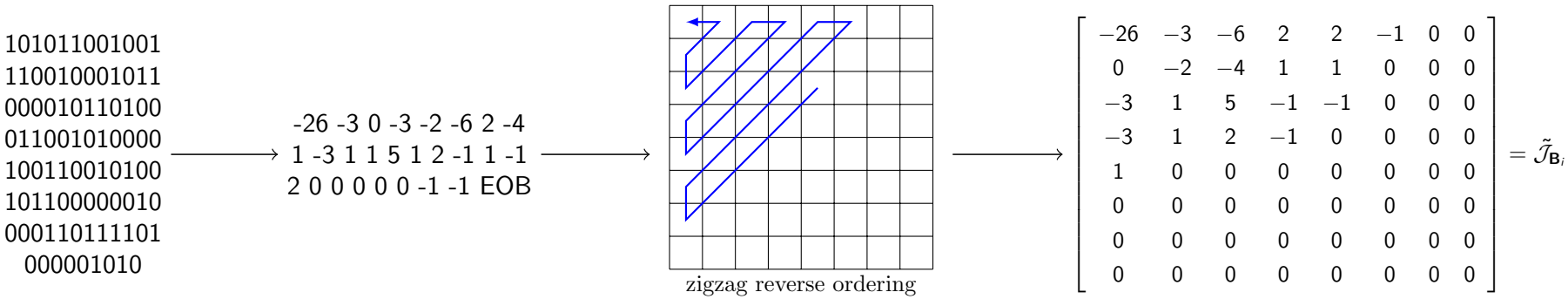

JPEG compression algorithm

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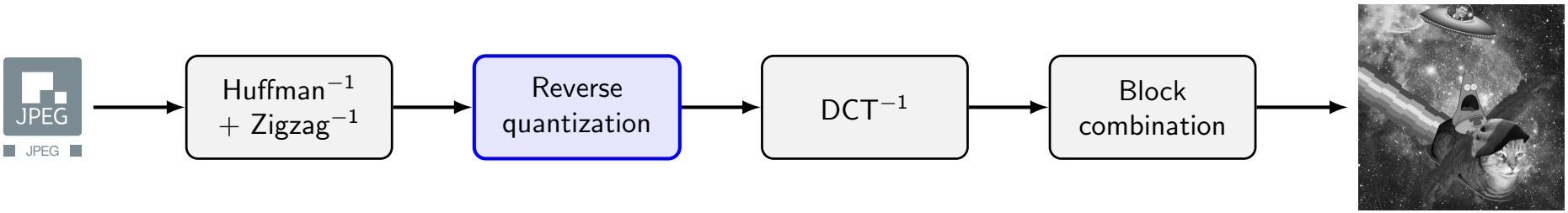

$\tilde{\mathcal{J}}_{\mathbf{B}_i^\star} = \tilde{\mathcal{J}}_{\mathbf{B}_i} \circ \mathcal{Q}$ Hadamard (component-wise) product

JPEG compression algorithm

Decompression

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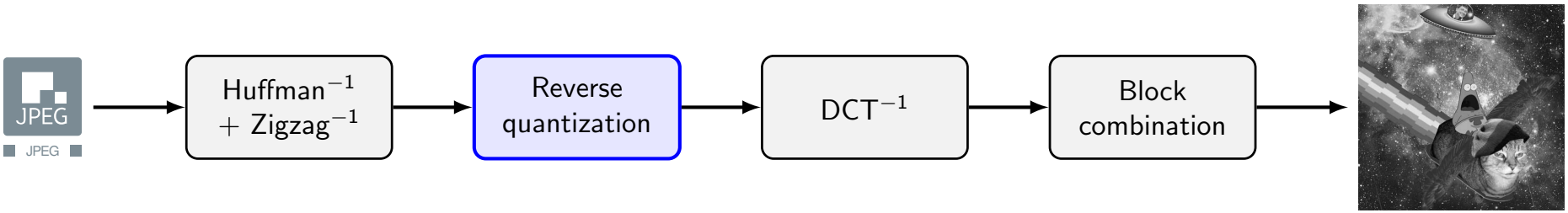


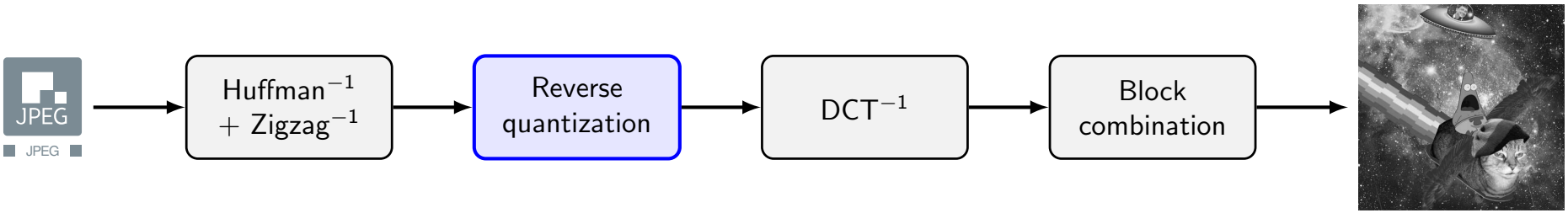
$\tilde{\mathcal{J}}_{\mathbf{B}_i^\star} = \tilde{\mathcal{J}}_{\mathbf{B}_i} \circ \mathcal{Q}$ Hadamard (component-wise) product

$$= \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

JPEG compression algorithm

Decompression

JPEG decompression process is the exact inverse of the compression scheme.



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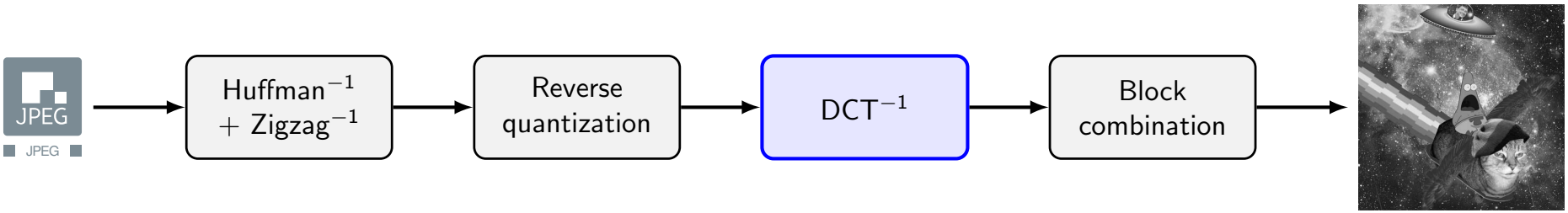
$$= \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} = \begin{bmatrix} -416 & -33 & -60 & 32 & 48 & -40 & 0 & 0 \\ 0 & -24 & -56 & 19 & 26 & 0 & 0 & 0 \\ -42 & 13 & 80 & -24 & -40 & 0 & 0 & 0 \\ -42 & 17 & 44 & -29 & 0 & 0 & 0 & 0 \\ 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

JPEG compression algorithm

Decompression

JPEG decompression process is the exact inverse of the compression scheme.

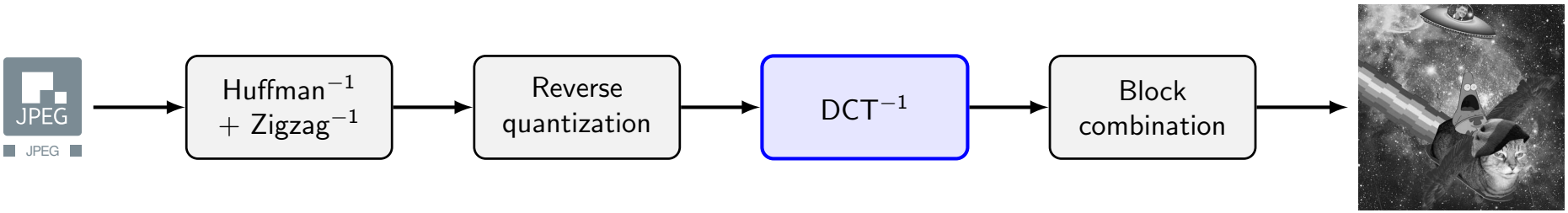


$$\mathrm{DCT}^{-1}\left(\tilde{\mathcal{J}}_{\mathbf{B}_i^\star}\right) = \mathbf{B}_i^\star - 128 = \begin{bmatrix} -66 & -63 & -71 & -68 & -56 & -65 & -68 & -46 \\ -71 & -73 & -72 & -46 & -20 & -41 & -66 & -57 \\ -70 & -78 & -68 & -17 & 20 & -14 & -61 & -63 \\ -63 & -73 & -62 & -8 & 27 & -14 & -60 & -58 \\ -58 & -65 & -61 & -27 & -6 & -40 & -68 & -50 \\ -57 & -57 & -64 & -58 & -48 & -66 & -72 & -47 \\ -53 & -46 & -61 & -74 & -65 & -63 & -62 & -45 \\ -47 & -34 & -53 & -74 & -60 & -47 & -47 & -41 \end{bmatrix}$$

JPEG compression algorithm

Decompression

JPEG decompression process is the exact inverse of the compression scheme.

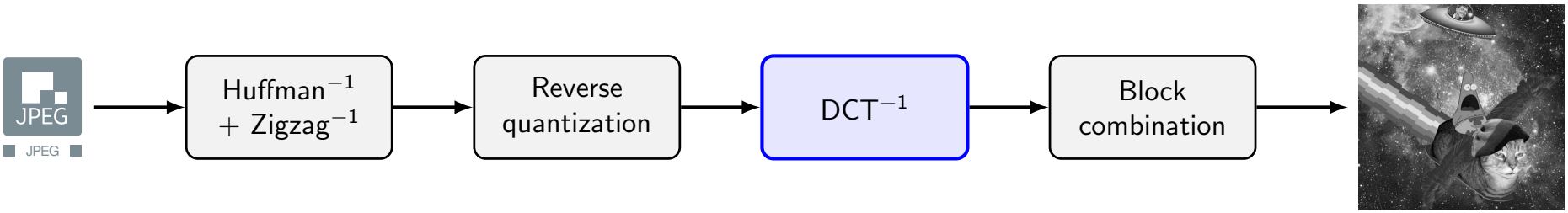


$$\mathbf{B}_i^\star = \mathrm{DCT}^{-1}\left(\tilde{\mathcal{J}}_{\mathbf{B}_i^\star}\right) + 128 = \begin{bmatrix} 62 & 65 & 57 & 60 & 72 & 63 & 60 & 82 \\ 57 & 55 & 56 & 82 & 108 & 87 & 62 & 71 \\ 58 & 50 & 60 & 111 & 148 & 114 & 67 & 65 \\ 65 & 55 & 66 & 120 & 155 & 114 & 68 & 70 \\ 70 & 63 & 67 & 101 & 122 & 88 & 60 & 78 \\ 71 & 71 & 64 & 70 & 80 & 62 & 56 & 81 \\ 75 & 82 & 67 & 54 & 63 & 65 & 66 & 83 \\ 81 & 94 & 75 & 54 & 68 & 81 & 81 & 87 \end{bmatrix}$$

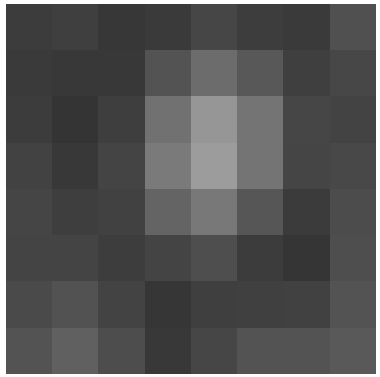
JPEG compression algorithm

Decompression

JPEG decompression process is the exact inverse of the compression scheme.



$$\mathbf{B}_i^\star = \mathrm{DCT}^{-1}\left(\tilde{\mathcal{J}}_{\mathbf{B}_i^\star}\right) + 128 = \begin{bmatrix} 62 & 65 & 57 & 60 & 72 & 63 & 60 & 82 \\ 57 & 55 & 56 & 82 & 108 & 87 & 62 & 71 \\ 58 & 50 & 60 & 111 & 148 & 114 & 67 & 65 \\ 65 & 55 & 66 & 120 & 155 & 114 & 68 & 70 \\ 70 & 63 & 67 & 101 & 122 & 88 & 60 & 78 \\ 71 & 71 & 64 & 70 & 80 & 62 & 56 & 81 \\ 75 & 82 & 67 & 54 & 63 & 65 & 66 & 83 \\ 81 & 94 & 75 & 54 & 68 & 81 & 81 & 87 \end{bmatrix}$$



JPEG compression algorithm

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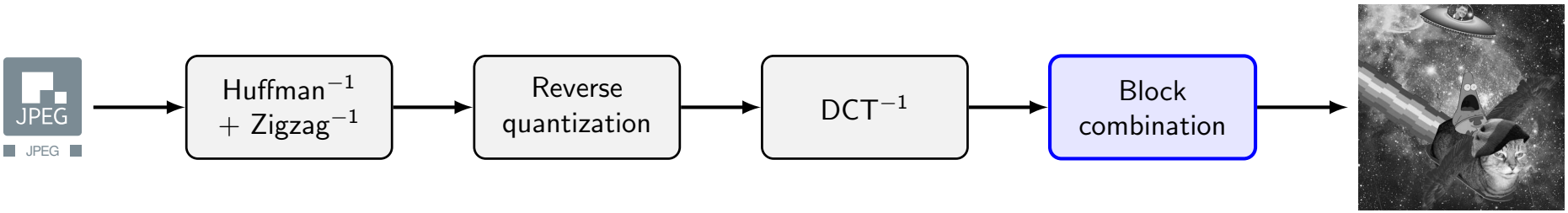

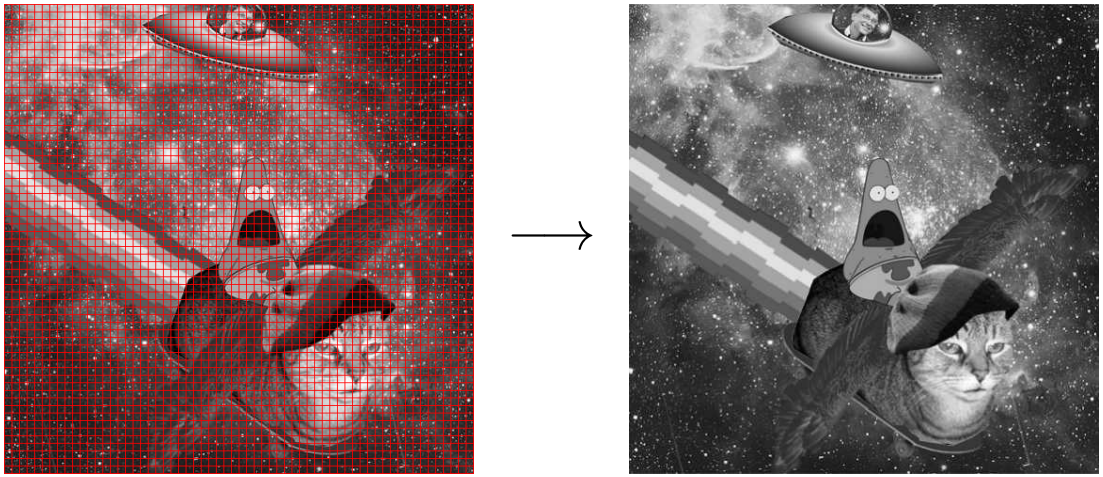

Lossy compression indeed...

... but what is lost in the process?

The only lossy operation in the JPEG process is the rounding $\lfloor \cdot \rceil$ during quantization.

$$\tilde{\mathcal{J}}_{\mathbf{B}_i^\star} = \lfloor \mathcal{J}_{\mathbf{B}_i} \oslash \mathcal{Q} \rceil \circ \mathcal{Q} = \begin{bmatrix} -416 & -33 & -60 & 32 & 48 & -40 & 0 & 0 \\ 0 & -24 & -56 & 19 & 26 & 0 & 0 & 0 \\ -42 & 13 & 80 & -24 & -40 & 0 & 0 & 0 \\ -42 & 17 & 44 & -29 & 0 & 0 & 0 & 0 \\ 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \neq \begin{bmatrix} -415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\ 4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\ -47 & 7 & 77 & -25 & -29 & 10 & 5 & -6 \\ -49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\ 12 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix} = \mathcal{J}_{\mathbf{B}_i}$$

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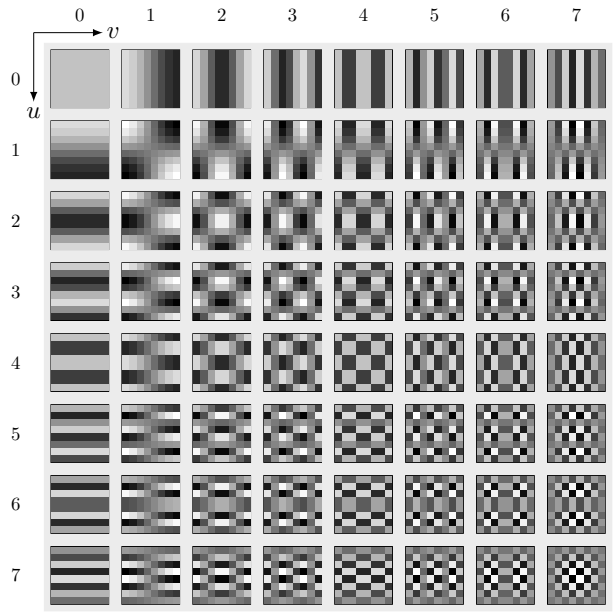
$$\Rightarrow \mathcal{J}_{\mathbf{B}_i} - \tilde{\mathcal{J}}_{\mathbf{B}_i^\star} = \begin{bmatrix} 1 & 3 & -1 & -5 & 8 & 20 & -2 & 0 \\ 4 & 2 & -5 & -9 & -13 & -7 & -9 & 5 \\ -5 & -6 & -3 & -1 & 11 & 10 & 5 & -6 \\ 7 & -5 & -10 & 14 & -10 & 6 & 2 & 2 \\ -6 & -7 & -13 & -4 & -2 & 2 & -3 & 3 \\ -8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\ -1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\ 0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \end{bmatrix}$$



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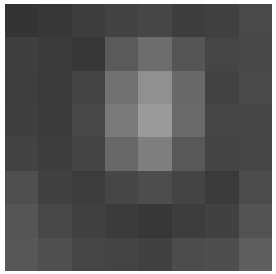
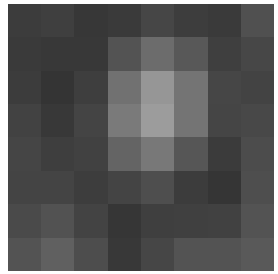
The only lossy operation in the JPEG process is the rounding $\lfloor \cdot \rceil$ during quantization.

$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$



$$\mathbf{B}_i^\star = \begin{bmatrix} 60 & 63 & 55 & 58 & 70 & 61 & 58 & 80 \\ 58 & 56 & 56 & 83 & 108 & 88 & 63 & 71 \\ 60 & 52 & 62 & 113 & 150 & 116 & 70 & 67 \\ 66 & 56 & 68 & 122 & 156 & 116 & 69 & 72 \\ 69 & 62 & 65 & 100 & 120 & 86 & 59 & 76 \\ 68 & 68 & 61 & 68 & 78 & 60 & 53 & 78 \\ 74 & 82 & 67 & 54 & 63 & 64 & 65 & 83 \\ 83 & 96 & 77 & 56 & 70 & 83 & 83 & 89 \end{bmatrix}$$



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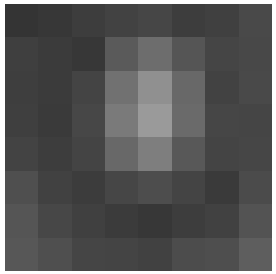
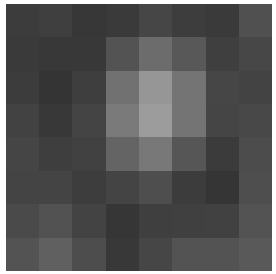
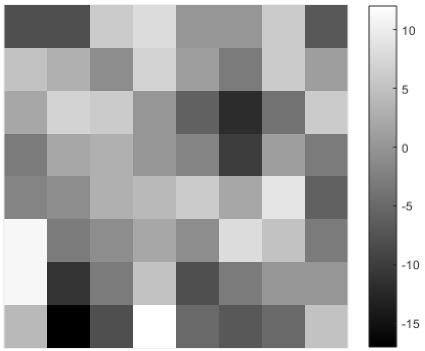


$$\mathbf{B}_i^\star = \begin{bmatrix} 60 & 63 & 55 & 58 & 70 & 61 & 58 & 80 \\ 58 & 56 & 56 & 83 & 108 & 88 & 63 & 71 \\ 60 & 52 & 62 & 113 & 150 & 116 & 70 & 67 \\ 66 & 56 & 68 & 122 & 156 & 116 & 69 & 72 \\ 69 & 62 & 65 & 100 & 120 & 86 & 59 & 76 \\ 68 & 68 & 61 & 68 & 78 & 60 & 53 & 78 \\ 74 & 82 & 67 & 54 & 63 & 64 & 65 & 83 \\ 83 & 96 & 77 & 56 & 70 & 83 & 83 & 89 \end{bmatrix}$$



$$\Rightarrow \epsilon_i = \mathbf{B}_i - \mathbf{B}_i^\star = \begin{bmatrix} -8 & -8 & 6 & 8 & 0 & 0 & 6 & -7 \\ 5 & 3 & -1 & 7 & 1 & -3 & 6 & 1 \\ 2 & 7 & 6 & 0 & -6 & -12 & -4 & 6 \\ -3 & 2 & 3 & 0 & -2 & -10 & 1 & -3 \\ -2 & -1 & 3 & 4 & 6 & 2 & 9 & -6 \\ 11 & -3 & -1 & 2 & -1 & 8 & 5 & -3 \\ 11 & -11 & -3 & 5 & -8 & -3 & 0 & 0 \\ 4 & -17 & -8 & 12 & -5 & -7 & -5 & 5 \end{bmatrix}$$

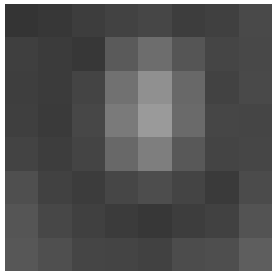


Lossy compression indeed...

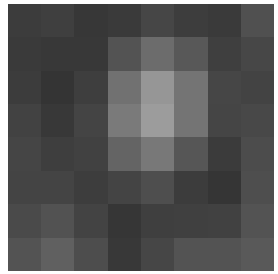
... but what is lost in the process?

The only lossy operation in the JPEG process is the rounding $\lfloor \cdot \rceil$ during quantization.

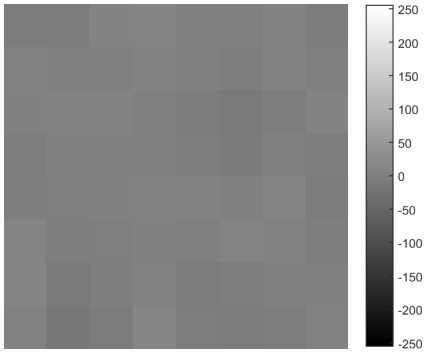
$$\mathbf{B}_i = \begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$



$$\mathbf{B}_i^\star = \begin{bmatrix} 60 & 63 & 55 & 58 & 70 & 61 & 58 & 80 \\ 58 & 56 & 56 & 83 & 108 & 88 & 63 & 71 \\ 60 & 52 & 62 & 113 & 150 & 116 & 70 & 67 \\ 66 & 56 & 68 & 122 & 156 & 116 & 69 & 72 \\ 69 & 62 & 65 & 100 & 120 & 86 & 59 & 76 \\ 68 & 68 & 61 & 68 & 78 & 60 & 53 & 78 \\ 74 & 82 & 67 & 54 & 63 & 64 & 65 & 83 \\ 83 & 96 & 77 & 56 & 70 & 83 & 83 & 89 \end{bmatrix}$$



$$\Rightarrow \epsilon_i = \mathbf{B}_i - \mathbf{B}_i^\star = \begin{bmatrix} -8 & -8 & 6 & 8 & 0 & 0 & 6 & -7 \\ 5 & 3 & -1 & 7 & 1 & -3 & 6 & 1 \\ 2 & 7 & 6 & 0 & -6 & -12 & -4 & 6 \\ -3 & 2 & 3 & 0 & -2 & -10 & 1 & -3 \\ -2 & -1 & 3 & 4 & 6 & 2 & 9 & -6 \\ 11 & -3 & -1 & 2 & -1 & 8 & 5 & -3 \\ 11 & -11 & -3 & 5 & -8 & -3 & 0 & 0 \\ 4 & -17 & -8 & 12 & -5 & -7 & -5 & 5 \end{bmatrix}$$



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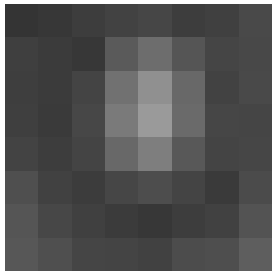
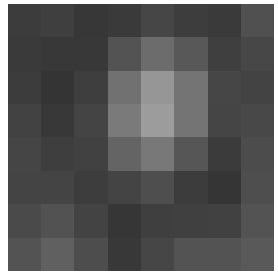


$$\mathbf{B}_i^\star = \begin{bmatrix} 60 & 63 & 55 & 58 & 70 & 61 & 58 & 80 \\ 58 & 56 & 56 & 83 & 108 & 88 & 63 & 71 \\ 60 & 52 & 62 & 113 & 150 & 116 & 70 & 67 \\ 66 & 56 & 68 & 122 & 156 & 116 & 69 & 72 \\ 69 & 62 & 65 & 100 & 120 & 86 & 59 & 76 \\ 68 & 68 & 61 & 68 & 78 & 60 & 53 & 78 \\ 74 & 82 & 67 & 54 & 63 & 64 & 65 & 83 \\ 83 & 96 & 77 & 56 & 70 & 83 & 83 & 89 \end{bmatrix}$$



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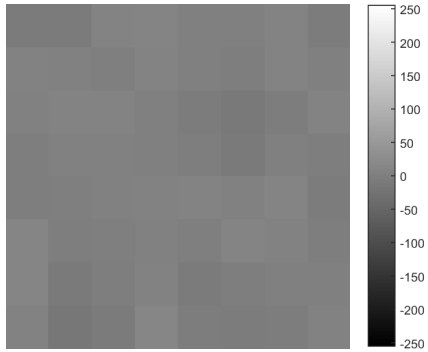


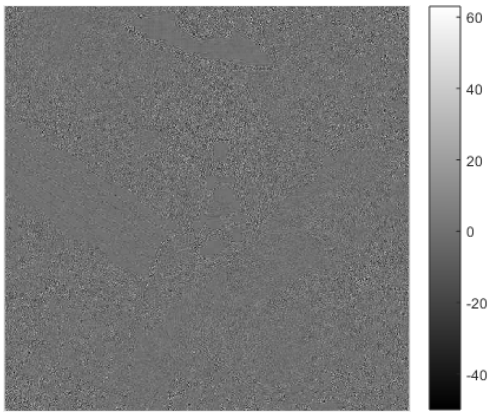
$$\text{RMSE} = 6.01$$
$$\text{SNR} = 22.37 \text{ dB}$$

Reconstruction error at the image scale

The notorious compression artifacts of JPEG

$\mathcal{I}_{\text{TIF}}$

$\mathcal{I}_{\text{JPEG}}$

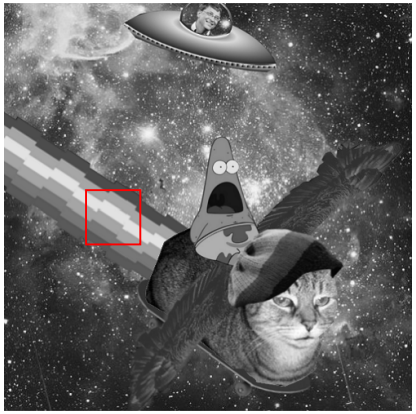


$\epsilon_{\text{JPEG}} = \mathcal{I}_{\text{TIF}} - \mathcal{I}_{\text{JPEG}}$

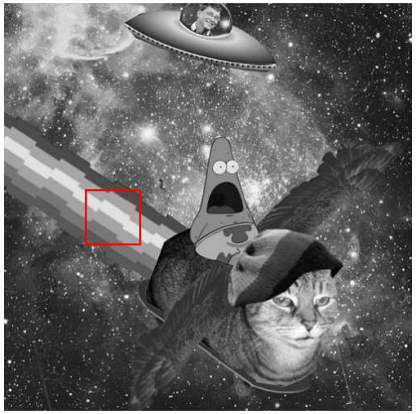
RMSE $= 6.71$
SNR $= 24.29$ dB

Reconstruction error at the image scale

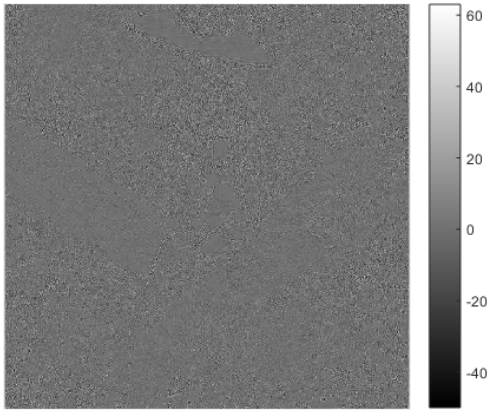
The notorious compression artifacts of JPEG



$\mathcal{I}_{\text{TIF}}$

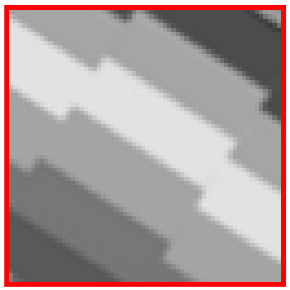


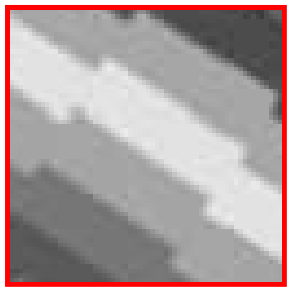
$\mathcal{I}_{\text{JPEG}}$



$\epsilon_{\text{JPEG}} = \mathcal{I}_{\text{TIF}} - \mathcal{I}_{\text{JPEG}}$

RMSE $= 6.71$
SNR $= 24.29$ dB

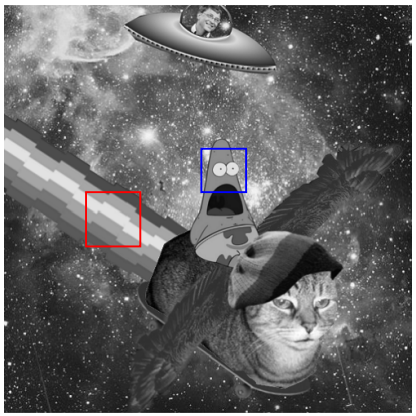




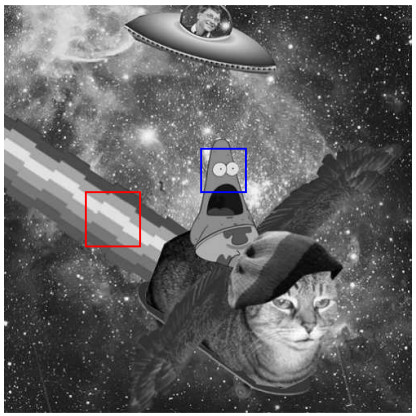
"mosaic effect" in homogeneous areas.

Reconstruction error at the image scale

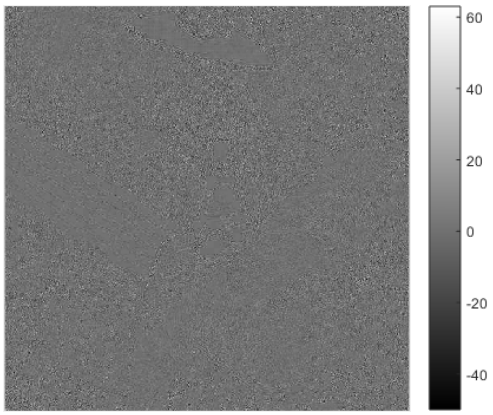
The notorious compression artifacts of JPEG



$\mathcal{I}_{\text{TIF}}$

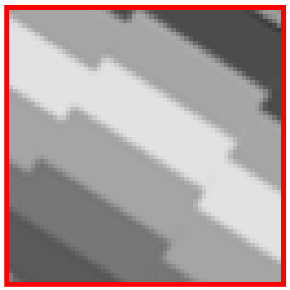


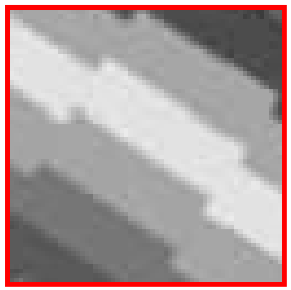
$\mathcal{I}_{\text{JPEG}}$



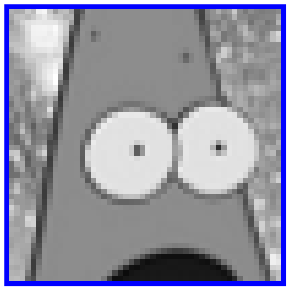
$\epsilon_{\text{JPEG}} = \mathcal{I}_{\text{TIF}} - \mathcal{I}_{\text{JPEG}}$

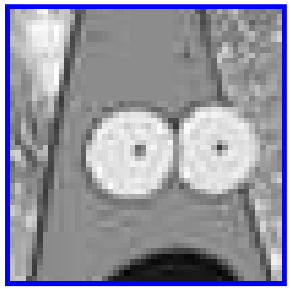
RMSE $= 6.71$
SNR $= 24.29$ dB





"mosaic effect" in homogeneous areas.





"ringing effect" on sharp edges.

Changing the compression quality

It is possible to adjust the compression quality of JPEG by modyfing the quantization matrix

Input: $\mathcal{Q}$, $q \in [1 : 100]$
Output: $\mathcal{Q}_q$
if $q < 50$ **then**
$\quad \alpha = \frac{5000}{q}$;
else
$\quad \alpha = 200 - 2q$;
end
$\mathcal{Q}_q = \left\lfloor \frac{\alpha \mathcal{Q} + 50}{100} \right\rfloor$;

$$\mathcal{Q} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

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$$\mathcal{Q}_{25} = \begin{bmatrix} 32 & 22 & 20 & 32 & 48 & 80 & 102 & 122 \\ 24 & 24 & 28 & 38 & 52 & 116 & 120 & 110 \\ 28 & 26 & 32 & 48 & 80 & 114 & 138 & 112 \\ 28 & 34 & 44 & 58 & 102 & 174 & 160 & 124 \\ 36 & 44 & 74 & 112 & 136 & 218 & 206 & 154 \\ 48 & 70 & 110 & 128 & 162 & 208 & 226 & 184 \\ 98 & 128 & 156 & 174 & 206 & 242 & 240 & 202 \\ 144 & 184 & 190 & 196 & 224 & 200 & 206 & 198 \end{bmatrix}$$

$$\mathcal{Q}_{50} = \mathcal{Q}$$

$$\mathcal{Q}_{75} = \begin{bmatrix} 8 & 6 & 5 & 8 & 12 & 20 & 26 & 31 \\ 6 & 6 & 7 & 10 & 13 & 29 & 30 & 28 \\ 7 & 7 & 8 & 12 & 20 & 29 & 35 & 28 \\ 7 & 9 & 11 & 15 & 26 & 44 & 40 & 31 \\ 9 & 11 & 19 & 28 & 34 & 55 & 52 & 39 \\ 12 & 18 & 28 & 32 & 41 & 52 & 57 & 46 \\ 25 & 32 & 39 & 44 & 52 & 61 & 60 & 51 \\ 36 & 46 & 48 & 49 & 56 & 50 & 52 & 50 \end{bmatrix}$$

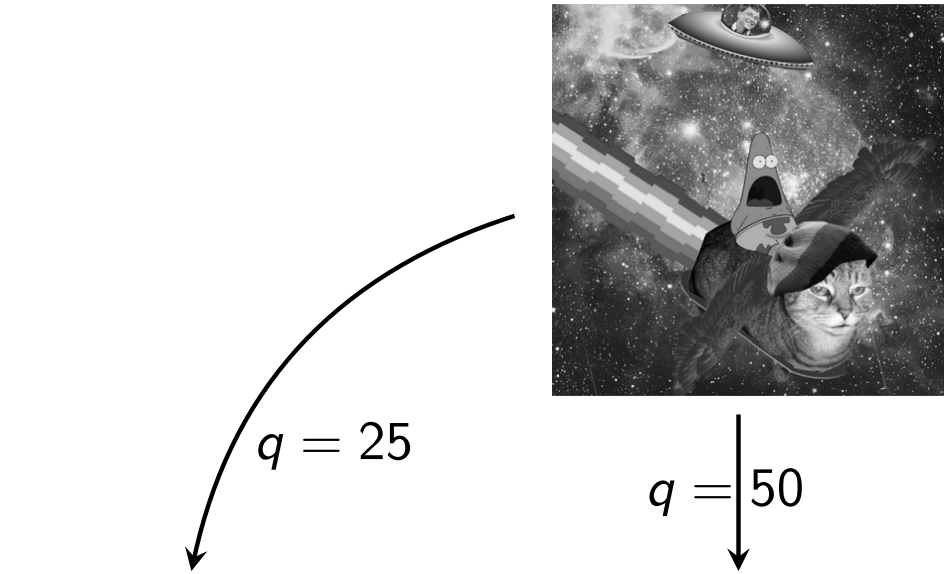
Influence of the quality factor (1/2)

$q = 50$

RMSE $= 6.71$
SNR $= 24.29$ dB

```
-rw-r--r--  35K randompic_Q50.jpg
```

Influence of the quality factor (1/2)



RMSE $= 8.83$
SNR $= 21.90$ dB

```
-rw-r--r--  21K randompic_Q25.jpg
```

RMSE $= 6.71$
SNR $= 24.29$ dB

```
-rw-r--r--  35K randompic_Q50.jpg
```

Influence of the quality factor (1/2)

$q = 25$

$q = 50$

$q = 75$

RMSE = 8.83
SNR = 21.90 dB

```
-rw-r--r--  21K randompic_Q25.jpg
```

RMSE = 6.71
SNR = 24.29 dB

```
-rw-r--r--  35K randompic_Q50.jpg
```

RMSE = 4.82
SNR = 27.16 dB

```
-rw-r--r--  52K randompic_Q75.jpg
```

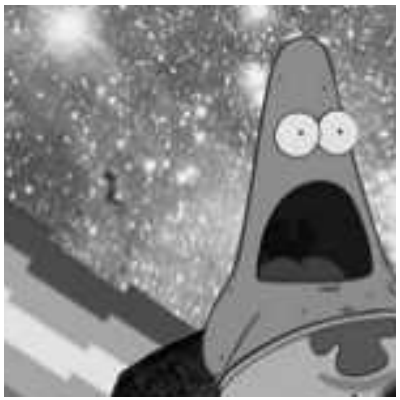
Influence of the quality factor (1/2)

$q = 25$

RMSE = 8.83
SNR = 21.90 dB

-rw-r--r-- 21K randompic_Q25.jpg

$q = 50$



RMSE = 6.71
SNR = 24.29 dB

-rw-r--r-- 35K randompic_Q50.jpg

$q = 75$

RMSE = 4.82
SNR = 27.16 dB

-rw-r--r-- 52K randompic_Q75.jpg

Influence of the quality factor (2/2)