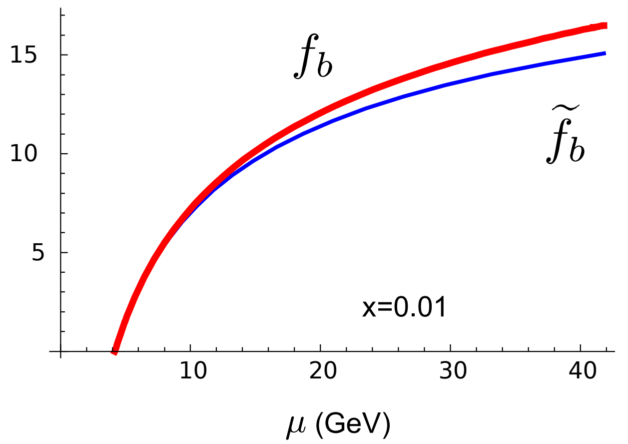

FIG. 1. We may switch to charm; I took from our heavy flavor paper. The LO contributions correspond to the heavy quark (Q) initiated f_Q, and the SUB to $\widetilde{f}_Q$. The cancellation (LO-SUB) is quite precise. If we were to remove LO or SUB, our TOT result would have anomalous contributions (and correspondingly anomalous μ-dependence) in the region $\mu \sim m_Q$.

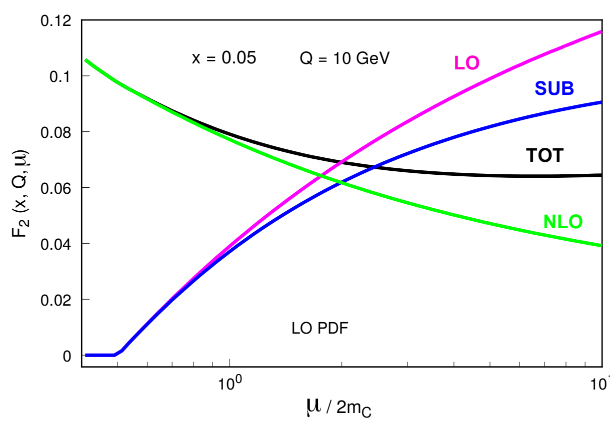

FIG. 2. Calculation of F_2^c in the VFNS illustrating the cancellation of the LO ($\bar{c}W^+ \to \bar{s}$) and the SUB ($g \to \bar{c} \otimes \bar{c}W^+ \to \bar{s}$) contributions in the region $\mu \sim m_c$. *[Placeholder figure]*

Appendix A: Defining F_2^{charm} Beyond Lead Order

The charged current DIS charm production process involves some interesting issues that we will explore here in detail. Because it is a flavor-changing process, two quark masses are involved $\{m_s, m_c\}$; thus, we can examine the mass singularities of the t-channel and u-channel separately. This separation is particularly useful to understand how the individual mass singularities are addressed, and how FFNS and VFNS organize the contributions to the total structure function.

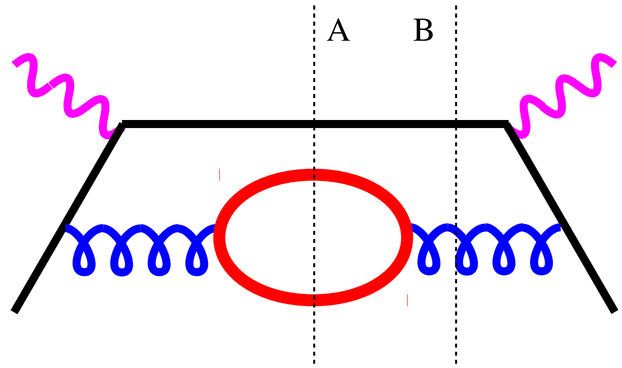

FIG. 3. draft in progress: A higher order Feynman graph illustrating the difficulty in defining an "inclusive" F_2^{charm}. If we have a light quark (q) scattering from a vector boson (V), at higher orders we could have a charm–anti-charm loop. If we cut the amplitude with cut "A" we have charm in the final state and this must be included in F_2^{charm}. If we cut the amplitude with cut "B" there is no charm in the final state, but this process is required to satisfy IR divergences as governed by the Kinoshita-Lee-Nauenberg (KLN) theorem. Also note, since this diagram contributes to the beta function, this highlights the difficulty of using an α_S and hard scattering $\hat{\sigma}$ with differing N_{eff}.

[FRED: REVISED AFTER DISCUSSION]

To be specific, we will consider charged-current DIS production of a charm quark. We first compute this in a 3-flavor (FFNS) scheme where {u,d,s} are light "active" partons in the proton, and charm is considered an external "heavy" particle. This can be implemented in the ACOT scheme for example by using a CWZ renormalization where the light "active" partons are renormalized with normal $\overline{MS}$, and the "heavy" quarks use a zero-momentum subtraction.

[FFNS 3-FLAVORS]

In this scheme, the LO process is $sW^+ \to c$ as illustrated in Fig.***.

At NLO, we then include $gW^+ \to c\bar{s}$ which has both t-channel and u-channel contributions.[1]

[t-channel]

The t-channel process has an intermediate s-quark exchanged, and if we use the strange quark mass m_s to regulate the singularities, this will yield a contribution

[1] Note, there are also corresponding quark-initiated processes; we will focus on the gluon-initiated processes as this is sufficient to illustrate our points.

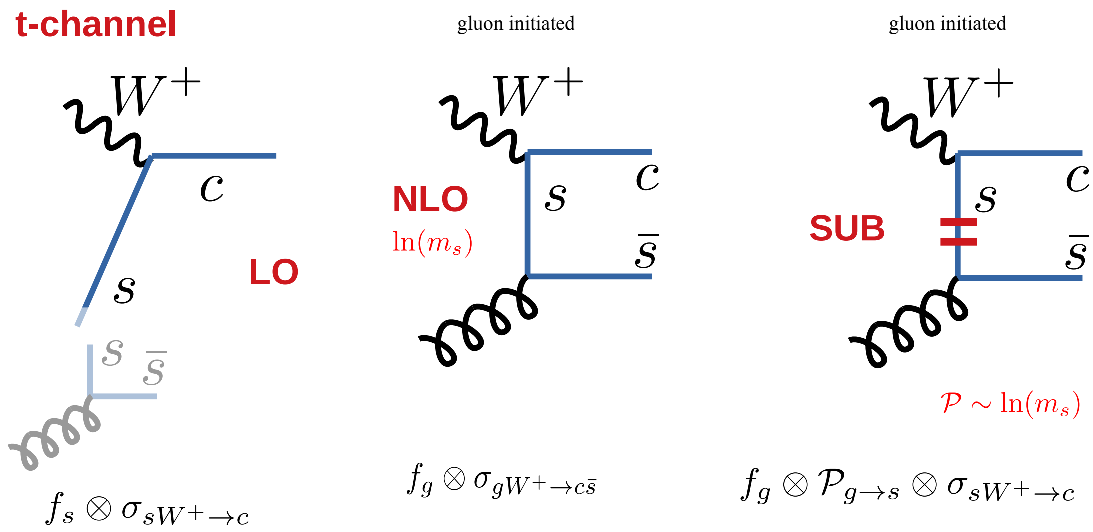

FIG. 4. Gluon NLO t-channel processes

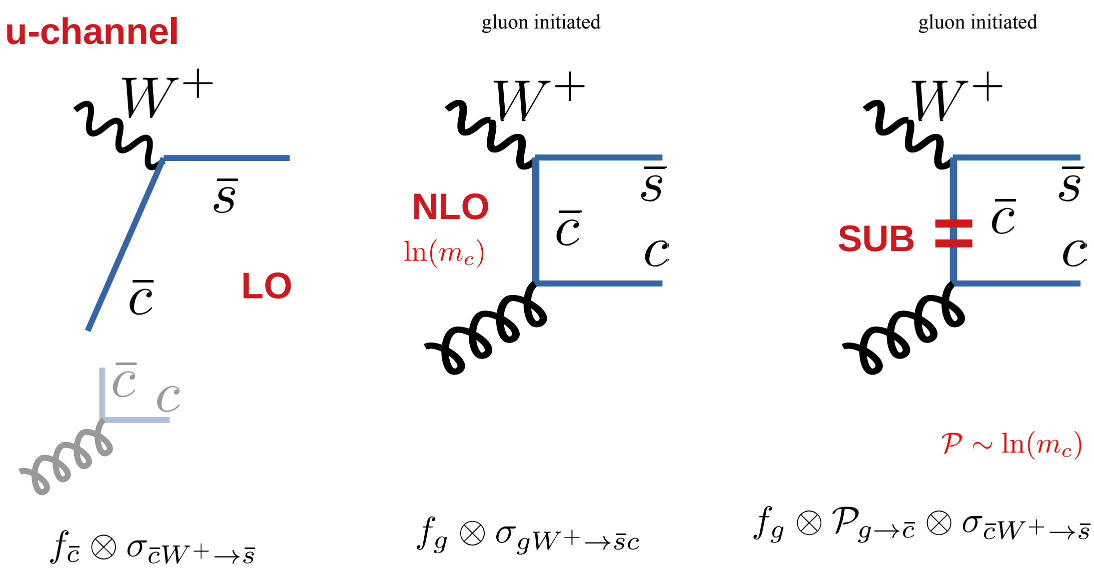

FIG. 5. Gluon NLO u-channel processes

proportional to $\ln(\mu/m_s)$. This mass singularity arises from the region of phase space where the exchanged s-quark becomes collinear and close to the mass-shell; that is, when the phase space of the $gW^+ \to c\bar{s}$ process begins to overlap with that of the $sW^+ \to c$ process. This "double counting" is resolved by a subtraction (SUB) counter-term (which is formally part of the NLO contribution) given by $f_g \otimes \mathcal{P}_{g\to s} \otimes \sigma_{sW^+\to c}$, and this ensures that the NLO calculation is well-defined for $(\mu/m_s) \to \infty$. Here, $\mathcal{P}_{g\to s}$ is the perturbative splitting of the gluon into an $s\bar{s}$ pair; the leading term is proportional to $\frac{\alpha_S}{2\pi} P^{(1)}_{g\to q} \ln(\mu^2/m_c^2)$ where $P^{(1)}_{g\to q}$ is the α_S DGLAP splitting kernel for $g \to q$.

[u-channel]

We next examine the u-channel contribution to the $gW^+ \to c\bar{s}$ process. This has an intermediate c-quark exchanged, and is proportional to $\ln(\mu/m_c)$. In the FFNS where the charm is a "heavy" non-parton, there is no counter-term for this graph, and the resulting observables will retain the $\ln(\mu/m_c)$ dependence. In principle, this means when we go to large μ-scales, these terms will begin to degrade the convergence of our perturbation series. In practice, while this degradation only grows logarithmically, at large scales (such as at the LHC) we do find it convenient to treat the charm on an equal-footing as the {u,d,s} partons.

[VFNS 4-FLAVORS]

We now turn to the 4-flavor (VFNS) scheme where we include the charm quark as an "active" parton.

In this case there is a u-channel counter-term given by $f_g \otimes \mathcal{P}_{g\to\bar{c}} \otimes \sigma_{\bar{c}W^+\to\bar{s}}$ which will cancel the $\ln(\mu/m_c)$ dependence and ensures that the NLO calculation is well-defined for $(\mu/m_c) \to \infty$.

What is less obvious is that we also must include the LO process $\bar{c}W^+ \to \bar{s}$. There are two ways we can understand why this is necessary.

[EXPLANATION #1: LO-SUB MATCHING AT THRESHOLD]

Recall that in the t-channel case, the subtraction term removed the double counting between the LO $sW^+ \to c$ and NLO $gW^+ \to c\bar{s}$ processes.

The u-channel case is analogous in that this subtraction term removes the double counting between the LO $\bar{c}W^+ \to \bar{s}$ and NLO $gW^+ \to c\bar{s}$ processes; both contributions are required to ensure the resulting cross section is insensitive to the μ-scale.

This is apparent in Fig.*** where we plot the individual terms versus the μ scale. In the region of $\mu \sim m_c$, the charm PDF f_c (and hence, the LO contribution) rises very quickly as it is driven by the very large gluon, and coupled with a large $\alpha_S(\mu)$. The SUB subtraction also rises quickly as this is driven by the logarithmic term $\ln(\mu^2/m_c^2)$. The difference $(LO - SUB)$ is the physical contribution to the total $(TOT = LO + NLO - SUB)$, and it is this combination which is smooth across the "turn on" of the charm PDF. We now see that if we neglect the LO $(\bar{c}W^+ \to \bar{s})$ we loose the cancellation between LO and SUB in the region $\mu \sim m_c$, and our structure function (or cross section) would have an anomalous shift at the arbitrarily location where we turn on the charm PDF.

As we vary the unphysical renormalization scale μ, we are simply shifting contribution between the various terms. While the individual $\{LO, NLO, SUB\}$ terms exhibit a large μ-dependence, the combination (TOT) which represents the physical observable is relatively insensitive to μ (up to higher orders).

[EXPLANATION #2: THERE REALLY IS CHARM IN THE FINAL STATE]

A second way to understand why the LO process $\bar{c}W^+ \to \bar{s}$ is required is to consider the regions of phase space covered by each of the sub-processes. The singularity of the u-channel NLO $gW^+ \to c\bar{s}$ processes arises the phase space region when the intermediate $\bar{c}$ quark becomes collinear and close to the mass-shell. This is precisely the phase space region of the LO process $\bar{c}W^+ \to \bar{s}$ where the partonic $\bar{c}$-quark is collinear to the hadron. The SUB term then removes the "double counting" between the LO and NLO contributions; hence, all three contributions $\{LO, NLO, SUB\}$ are necessary to cover the full phase space.

x

Note the combination of the LO and SUB terms ensure a minimal μ-variation at low μ scales, and the combination of SUB and NLO ensure the mass singularities are canceled at high μ scales.

x

Furthermore, note that for both the LO and SUB contributions, the charm quark is collinear with the incoming hadron, and thus exits in the hadron remnants. While this may be experimentally unobservable, because we are asking for an "inclusive" F_2^c, these contributions cannot be simply ignored. We'll now discuss in further detail.

[PROBLEMS DEFINING AN "INCLUSIVE" F_2^c]

The u-channel LO $\bar{c}W^+ \to \bar{s}$ process foreshadows difficulties we encounter if we try and extend the concept of a truly "inclusive" F_2^c to higher orders. We note that in Ref.***, Collins extended the proof of factorization to include heavy quarks such as charm and bottom[2] for an inclusive structure function F_2; there is no proof for an "inclusive" F_2^c as this is ill-defined. What is measured experimentally is a differential cross section for producing a charm quark in the final state in a fiducial region; as we go to higher orders, we must be careful how we define this quantity so that it is singularity-free and μ-independent.

x

To characterize the problems in constructing an F_2^c, we can imagine starting from the (well defined) inclusive F_2, and then dividing the contributions into two sets: one for F_2^c for the "heavy" charm quark, and the rest into $F_2^{u,d,s}$ for the "light" quarks. We will show this division cannot be performed unambiguously.

x

The u-channel LO $\bar{c}W^+ \to \bar{s}$ process does not have any "apparent" charm quark in the final state, but this contribution is essential to balance with the SUB process $f_g \otimes \mathcal{P}_{g\to\bar{c}} \otimes \sigma_{\bar{c}W^+\to\bar{s}}$. Note that for the SUB process, the charm quark arises from a gluon splitting into a collinear $c\bar{c}$ pair which is then part of the hadron remnants. For the LO process, presumably our $\bar{c}$ quark also came from a gluon splitting into a collinear $c\bar{c}$ pair. Thus, if we want to define an "inclusive" F_2^c, it seems we must include those cases where the charm in contained in the hadron remnants.

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[2] Additionally, he specifically addressed (in Sec.***) the case of multiple heavy quarks which can allow for both charm and bottom in a unified framework; in contrast to some of the literature, there is no difficulty including multiple heavy quarks.

[THE BUBBLE DIAGRAM AND AN "INCLUSIVE" F_2^c]

Such difficulties are not unique to the VFNS, and also are encountered in the FFNS (cite Smith & van Neerven). This is succinctly illustrated in Fig. 3 which shows a higher-order DIS process with a quark–anti-quark loop. Let us compute this diagram in a 3-flavor FFNS where the internal loop is a $c\bar{c}$-pair, and the external quark is a light $\{u, d, s\}$. If the final-state is represented by Cut-A, then we have charm quarks in the final state, and this should be included in F_2^c.

However, if we instead use Cut-B as the final state, there is no charm in the final state, so this should not be included in F_2^c. [More precisely, when we renormalize the charm loop with zero-momentum subtraction, this contribution effectively decouples.] Thus, the contribution from Cut-A will be included in F_2^c, but the contribution from Cut-B will not.

If we had included both Cut-A and Cut-B, the Kinoshita-Lee-Nauenberg (KLN) theorem would ensure that the combination was free of IR divergences; but, since these are not uniformly included, KLN does not apply and we will have uncanceled singularities.

[THE PROBLEM WITH FFNS AND $N_{eff} = 4$ α_S running]

The bubble diagram of Fig. 3 also highlights the difficult of using a $N_{eff} = 3$ FFNS with an $N_{eff} = 4$ flavor running α_S. In a $N_{eff} = 3$ FFNS, internal $c\bar{c}$ loops decouple from the theory and are not included in the calculation; however, the $N_{eff} = 4$ β-function requires precisely these $c\bar{c}$ loop contributions. [Note; this deficiency can be patched order-by-order by expanding the β-function and inserting the required terms at each order. Cite: [ask Valerio*** for ref. does this generate other problems???]]

1. The bottom line:

A truly "inclusive" F_2^{charm} is ill-defined; instead, what is measured experimentally is a differential cross section for producing a charm quark in the final state in a fiducial region; as we go to higher orders, we must be careful how we define this quantity so that it is singularity-free and μ-independent. This is an issue for both the FFNS and VFNS.

We can compute in the FFNS, but in the large energy limit, we encounter $\ln(\mu^2/m_c^2)$ divergences and this, in part, gives rise to the observed differences at large Q scales.

The VFNS includes the charm quark as an active parton for μ scales above a threshold value, and this is calculation is free of $\ln(\mu^2/m_c^2)$ divergences. In the region $\mu \sim m_c$, there is a cancellation between the LO and SUB contributions. As this can be delicate to implement numerically, we have the option of displacing the threshold to a larger μ scale where the cancellation is more stable as was demonstrated in Ref.***.[3]

x

Comment: the same holds for bottom.

[3] Impact of the heavy quark matching scales in PDF fits The xFitter Developers Team (V. Bertone (Vrije U., Amsterdam & NIKHEF, Amsterdam) et al.). Jul 17, 2017. 18 pp. Published in Eur.Phys.J. C77 (2017) no.12, 837