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Journal Article Reviewed:

Wasu Timpitak, and Nopparat Pochai. (2022). "A Risk Assessment Model for Airborne Infection in a Ventilated Room using the Adaptive Runge-Kutta Method with Cubic Spline Interpolation." AENG International Journal of Applied Mathematics, Volume 52, Issue 4. Retrieved from https://www.iaeng.org/IJAM/issues_v52/issue_4/IJAM_52_4_04.pdf

The research paper entitled "A Risk Assessment Model for Airborne Infection in a Ventilated Room using the Adaptive Runge-Kutta Method with Cubic Spline Interpolation" by Wasu Timpitak, and Nopparat Pochai discusses the modeling and assessment of the risk associated with the concentration of exhaled air in ventilated rooms. The researchers develop a mathematical model to estimate the concentration of exhaled air in spaces equipped with outlet ventilation systems and also assess the corresponding risk of infection.

The abstract provides us with an overview of the research being presented in the paper. They first started with defining the problem about the spread of airborne infections and the associated risk factors, particularly in relation to ventilation rates and the concentration of exhaled air in enclosed spaces. They also presented their objectives well, which is to develop a mathematical model to estimate the concentration of exhaled air and assess the risk of infection in spaces with outlet ventilation systems. In modeling this, they discuss different key factors influencing exhaled air concentration and infection risk, including the actual concentration level, the number of users, and the rate of ventilation. Methodologies employed are also discussed which is the use of adaptive Runge-Kutta and classical fourth-order Runge-Kutta techniques for estimating the model solution. Additionally, Lagrange interpolating polynomials and cubic splines interpolation are mentioned for representing the variability in the number of individuals in the enclosed space over time. From the methodologies discussed, they also mentioned the result that the adaptive Runge-Kutta method with cubic spline interpolation as the good agreement solution which means that the predictions or estimations made by the model are consistent or close to what is observed or anticipated in practical situations.

In the introduction section, they discussed various studies and models related to the transmission of airborne infections. They started by defining the airborne infections caused by bacteria or viruses. From there, they establish a connection between the increased infection rates or clusters and the lack of ventilation or low ventilation rates. Throughout in the discussion, the paper introduced and examined various models, specifically focusing on modeling indoor airborne infection dynamics. Furthermore, they also discussed the importance of ventilation in mitigating infection risks. The paper's significant contribution is utilizing a mathematical model for the estimating the concentration of exhaled air in a space with an outlet ventilation system, as well as the risk of infection.

The paper also discussed the governing equations used for the study. In this section, the research introduces a comprehensive mathematical model aimed at estimating the concentration of exhaled air in a ventilated space and evaluating the associated risk of infection. The model operates under the assumption that an enclosed space like a room with a volume represented as V, has an initial

environmental CO_2 concentration (C_E) of approximately 400 ppm, and is occupied by a certain number of individuals denoted as n.

When there are people in the room who might be disease carriers, there will probably be more exhaled air in the room, which could contain airborne infectious particles. This increase is influenced by both the rate of ventilation (Q) and the number of individuals (n). The paper makes a straightforward assumption that individuals in the room significantly contribute to the production of CO_2, acting as an indicator of exhaled air. The general equation describing the accumulation rate of exhaled air concentration in a room with an initial concentration of C_E is expressed as the sum of the exhaled air rate generated by inhabitants, the rate of C_E, and the subtraction of the rate of ventilation (Q), which removes exhaled air. The equation is given below:

$$V\frac{dC}{dt} = npC_a + QC_E - QC \qquad (1)$$

Where V represents the volume of the indoor space, $\frac{dC}{dt}$ is the rate of change of the concentration of exhaled air (C) over time. It denotes how the concentration of exhaled air is changing over time. The variable n stands for the number of individuals present in the indoor space. The variable p represents the rate of respiration for each person in the room. It measures how fast each person breathes out air, while $C_a$represents the fraction of CO_2 contained in inhaled air. It represents the proportion of carbon dioxide in the air that individuals breathe in during respiration. The product npC_a represents how much exhaled air is being added to the room because of people breathing.

Now, the variable Q is the rate of ventilation and C_E is the initial concentration of carbon dioxide in the room. The product QC_E represents how much exhaled air is being added to the room due to the initial air conditions and ventilation. Lastly, QC presents how much exhaled air is being removed or ventilated out of the room.

The research focuses on modeling airborne infections originating from individuals within an enclosed space. The parameter Q, representing the rate of ventilation, is assumed by Q_{out}, referred to as the outlet ventilation rate. In a simple scenario, the number of people is varied and depends on the time are assumed by $n(t)$. The general equation of the accumulation rate exhaled air concentration in a room with C_E in Eq. (1) can be written as:

$$V\frac{dC}{dt} = n(t)pC_a + Q_{out}C_E - Q_{out}C \qquad (2)$$

for $0 \leq t \leq T$. We believe there may be errors in the paper, as the equation provided in the document is given as:

$$V\frac{dC}{dt} = n(t)pC_a - Q_{out}C \qquad (3)$$

We find this discrepancy puzzling, considering that Eq. (2) is the equation utilized in the simulations and processes, not Eq. (3). And Eq. (3) means that the environmental carbon dioxide in the room is not part of the computation.

The solution to the differential equation in Eq. (1) would provide the concentration of exhaled air as a function of time. the researchers used the classical fourth-order Runge-Kutta method and the

Adaptive Runge-Kutta method to solve the equation and compare this to the analytical solution. We first discuss the analytical solution presented in the paper and this is given by:

$$C(t) = C_E + \frac{npC_a}{Q}\left[1 - e^{-\frac{Qt}{V}}\right] \tag{4}$$

We attempted to find a solution for Eq. (1), and our result matched the initial answer. We find ourselves wondering why there is a necessity to employ numerical solutions when we have already obtained analytical solutions. One reason that we might think is numerical methods can be adapted to fit real-world data, allowing for a more realistic representation of the system's behavior.

Now, we solve the Eq. (1) numerically. For the classical Runge-Kutta Method and the Adaptive Runge-Kutta method, we used the the algorithm stated in the paper and tried to code it using python.

For the classical Runge-Kutta Method, we used the computations stated in the paper below.

$$C \cong C_i \tag{5}$$

$$C_{i+1} = C_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{6}$$

$$k_1 = f(t_i, C_i) \tag{7}$$

$$k_2 = f\left(t_i + \frac{1}{2}h, C_i + \frac{1}{2}k_1 h\right) \tag{8}$$

$$k_3 = f\left(t_i + \frac{1}{2}h, C_i + \frac{1}{2}k_2 h\right) \tag{9}$$

$$k_4 = f\left(t_i + \frac{1}{2}h, C_i + k_3 h\right) \tag{10}$$

For the Adaptive Runge-Kutta Method. We used the computations below.

$$C \cong C_i \tag{11}$$

$$C_{i+1} = C_i + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2917}{4140}k_4 - \frac{1}{5}k_5 \tag{12}$$

$$k_1 = h\, f(t_i, C_i) \tag{13}$$

$$k_2 = hf\left(t_i + \frac{1}{4}h, C_i + \frac{1}{4}k_1\right) \tag{14}$$

$$k_3 = hf\left(t_i + \frac{3h}{8}, C_i + \frac{3}{32}k_1 + \frac{9}{32}k_2\right) \tag{15}$$

$$k_4 = hf\left(t_i + \frac{12h}{13}, C_i + \frac{1932}{2197}k_1 - \frac{7200}{2917}k_2 + \frac{7296}{2197}k_3\right) \tag{16}$$

$$k_5 = hf\left(t_i + \frac{h}{2}, C_i - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{50}k_5\right) \quad (17)$$

where,

$$\frac{dC}{dt} = f(t_i, C_i) \quad (18)$$

$$f(t_i, C_i) = \frac{1}{V}(n(t)pC_a + Q_{out}C_E - Q_{out}C) \quad (19)$$

In the paper, $f(t_i, C_i) = \frac{1}{V}(n(t)pC_a - Q_{out}C)$ is deemed incorrect based on the provided analytic solution that involved C_E.

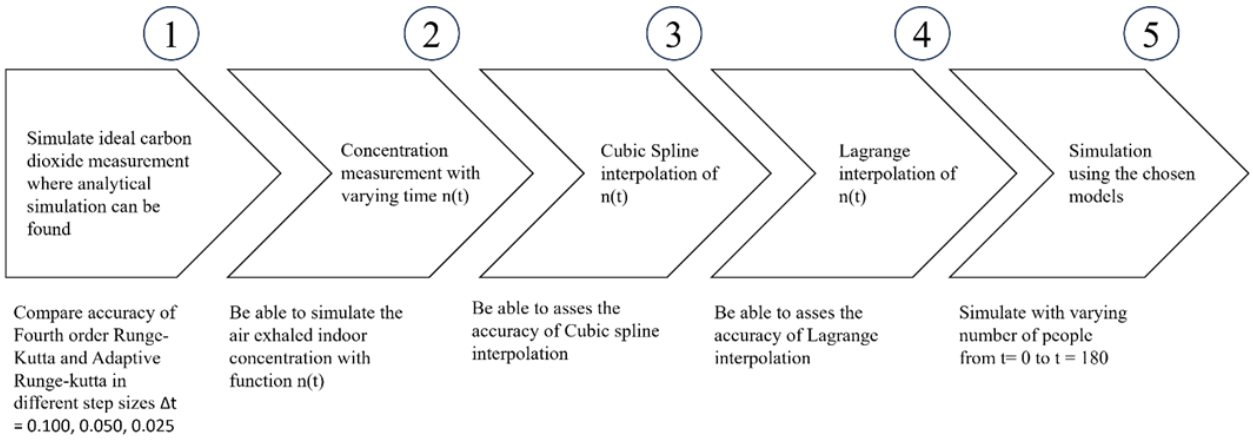

Figure 1: Conceptual framework of their simulations

The methodology for the experiments were separated into five different simulations. The first simulation involves an ideal carbon concentration. The second covers the measuring the concentration of exhaled air, with the variable $n(t)$, The third and fourth simulations involve the use of Cubic spline interpolation and Lagrange interpolation on n(t). And lastly, the fifth simulation covers simulating the risk of normal people who are staying in a room with infectors.

For the entirety of the simulations, they assumed that the respiration rate $p = 0.12\ (L/s)$ and a fraction of the Covid-19 concentration contained inbreathed air $C_a = 0.04$. Now, we discuss the result of the Simulation 1 in the paper. Simulation 1 involves the measurement of an ideal carbon dioxide concentration. The set $C_0 = 0.01$ as the ambient carbon dioxide concentration (ppm) and based on the Table 1 in the paper, the parameters are as follows:

Table 1: List of parameters

$n(t)$	C_E	V	Q
50	0.004	75	8

We believe there are certain concerns with the outcomes of the simulation. We start with their graph,

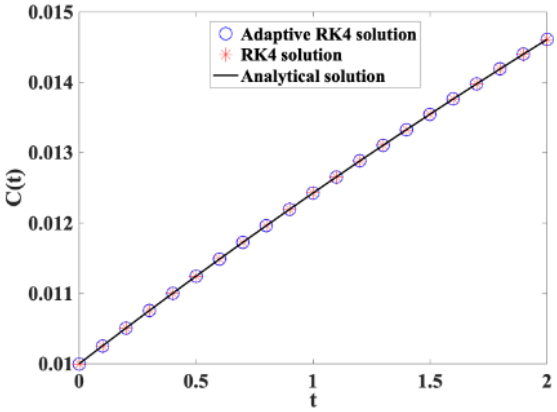

Fig. 2. The approximated air exhaled indoor concentration in a room $T = 180$.

Figure 2. Graph of Simulation 1 in the paper

Figure 2 illustrates the estimated indoor air concentration for a room with $T = 180$. Two notable issues arise from this representation. Firstly, the time range ($T = 180$) depicted on the graph spans only [0,2]. Secondly, concerning the initial condition $C(0)$, analytical solution from Eq. (4) yields $(0) = C_E + \frac{npC_a}{Q}\left[1 - e^{-\frac{Q(0)}{V}}\right] = C_E = 0.004$. However, the graph indicates an initial value of C at $t = 0$ as 0.01. This suggests that there is an inconsistency in the evaluation of C in the graph of the analytical solution. Adjusting the parameter C_E to 0.01 results to the desired outcome.

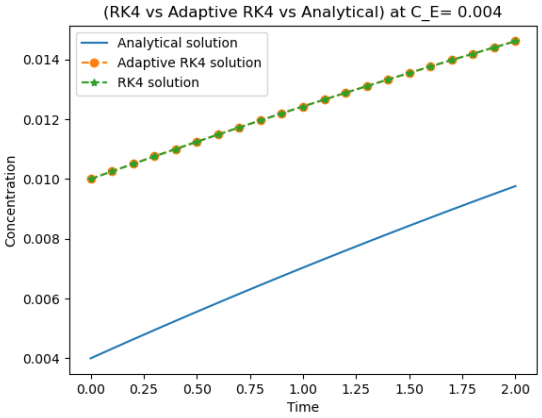

Figure 3: Solutions using RK4, Adaptive RK4, and analytical solution at $C_E = 0.004$

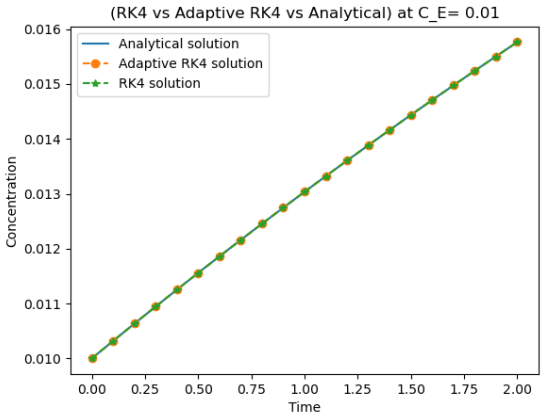

Figure 4: Solutions using RK4, Adaptive RK4, and analytical solution at $C_E = 0.01$

Figures 3 and 4 shows the solutions of the adaptive Runge-Kutta and classical fourth-order Runge-Kutta in comparison to the analytical solution for different values of C_E with step size 0.1. These visual representations showcase the results obtained through our Python scripting, implementing the algorithm detailed in the paper, while varying the values of C_E. We manage to replicate the figure for the solution adaptive Runge-Kutta and classical fourth-order Runge-Kutta stated in the paper but we think that this is the correct graph of the analytical solution. If we move T to 180 the result is given the Figures 5 and 6. We believe that the assumption for the parameter C_E should be changed since this is an initial value problem and the assumption in their paper for the initial value is not equal to the initial value of the analytical solution.

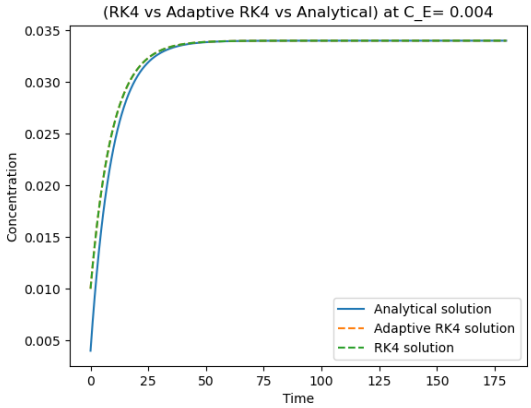

Figure 5: Solutions using RK4, Adaptive RK4, and analytical solution at T=180

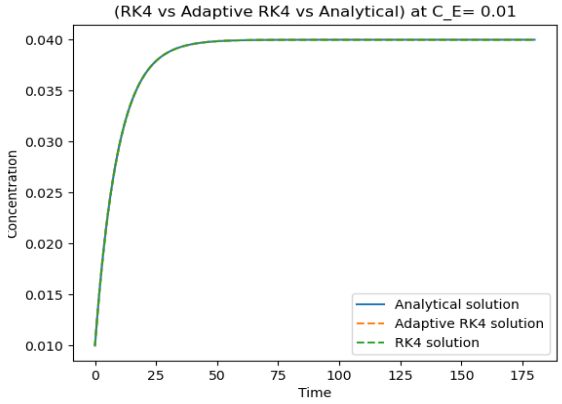

Figure 6: Solutions using RK4, Adaptive RK4, and analytical solution at T=180

Regarding the comparison between the adaptive Runge-Kutta and the classical fourth-order Runge-Kutta methods, the maximum errors consistently show lower values for the adaptive Runge-Kutta across all step sizes. This indicates its superior performance compared to the classical method. This observation aligns with the findings presented in the paper.

In Simulation 2, the investigation focuses on measuring the concentration of exhaled air, with the variable $n(t)$, representing the number of people. Specifically, the assumption is made that $n(t) = 45 + 5\,sin\,(\pi t)$.

For Simulation 2, the researchers tried to measure the concentration measurement of exhaled air with n(t) is function. This means that a number of people are assumed by n(t) = 45 + 5 sin(πt). This is a deviation from Simulation 1, where there was a constant number of people, as Simulation 2 introduces variability in the number of people.

In setting up the parameters, they set the initial value to $C_0 = 0.01$ while C_E is initialized to 0.004. However, based on the analytical solution $C(0) = C_E$. However, according to the analytical solution, this indicates a discrepancy in the initial values. In their paper for simulation 2, they utilized $C_E = 0.004$ to generate the graph. Our attempt to replicate the graph resulted in the following outcomes.

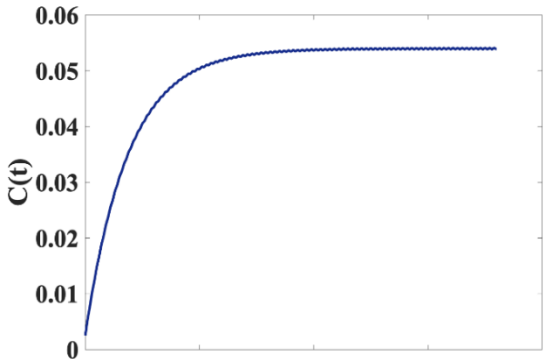

Figure 7: The approximated air exhaled indoor concentration based on the paper.

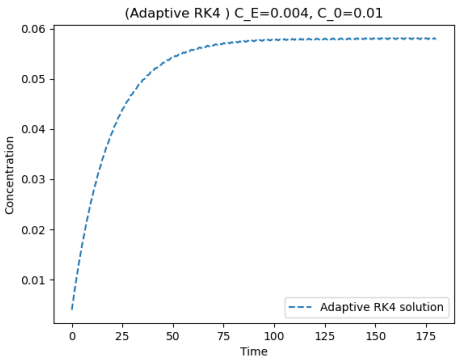

Figure 8: The approximated air exhaled indoor concentration with $C_E = 0.004$ and $C_0 = 0.01$

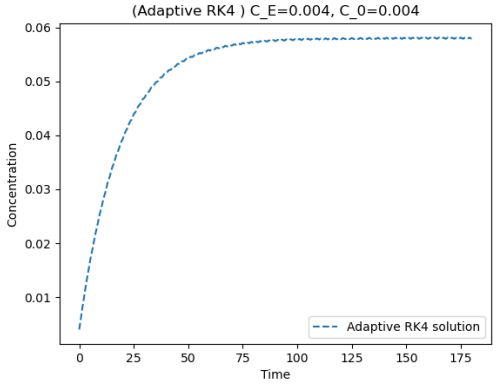

Figure 9: The approximated air exhaled indoor concentration with $C_E = 0.004$ and $C_0 = 0.004$

Figure 7 displays the graph as presented in the research paper. In Figure 8, we depict the graph resulting from the application of parameters defined by the authors in Simulation 2. Moving on to Figure 9, it illustrates the graph of the adaptive Runge-Kutta (RK4) method at $C_0 = 0.004$. Notably, discrepancies persist within the graph in the paper. Through analysis, Figures 6 and 8 appear more similar. This implies the possibility of discrepancies or errors in defining parameters for each simulation. It would be good to thoroughly reevaluate the paper to ensure accuracy and consistency in the parameter definitions across simulations. Nevertheless, we successfully approximated the indoor concentration of exhaled air in a room, considering $n(t)$ as a variable.

In simulation 3 and 4, the authors tried to model the concentration measurement of exhaled air with cubic splines interpolation and Lagrange interpolation of function n(t), respectively. We managed to get the same graph for the cubic spline interpolation when compared to $n(t) = 45 + 5\ sin\ (\pi t)$. Moreover, we tried to graph the simulation and get the result and the same problem of parameter initialization that we encountered in simulations 1 and 2. Results also showed that Cubic Spline interpolation had lower RMSE compared to the Lagrange interpolation. This is also aligned with the results of the paper.

In simulation 5, the authors modeled the risk of normal people being infected by staying in a room with infected people. Compared to previous simulations where $n(t)$ is given by an exact solution,

it is now given as data points. We use cubic-spline to be able to approximate the behavior of the system in terms of number of people and time as it is shown to have better accuracy than the Lagrange method shown in the previous simulation. After approximating n(t), we can now approximate air exhaled indoor concentration in a room and the percentage of exhaled air.

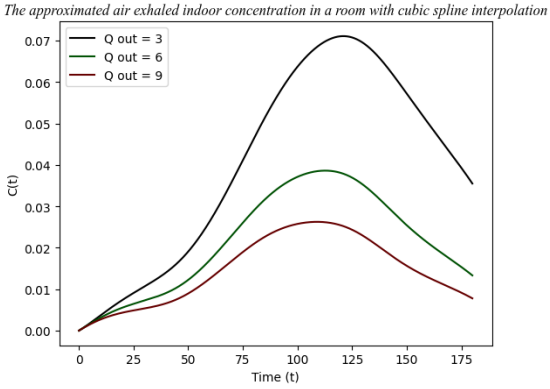

Figure 10: The approximated air exhaled indoor concentration with cubic spline interpolation

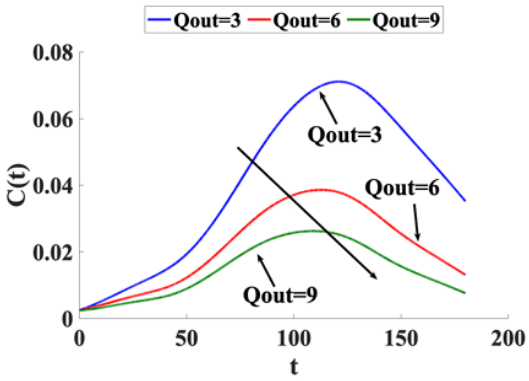

Figure 11: The approximated air exhaled indoor concentration with cubic spline interpolation based on paper

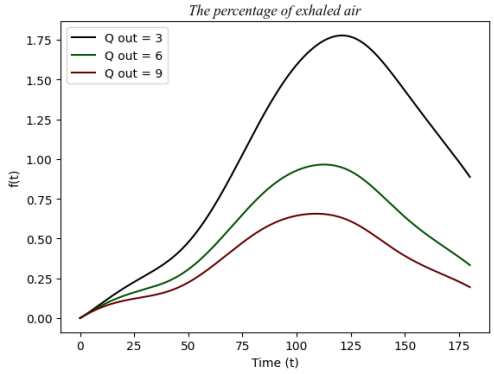

Figure 12: The percentage of exhaled air

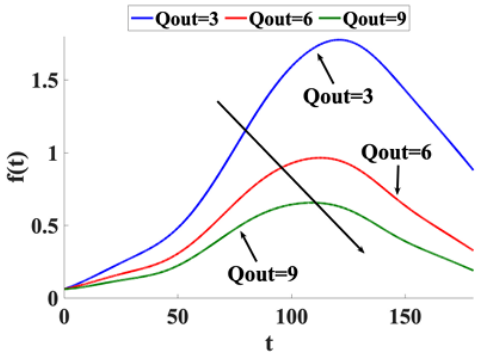

Figure 13: The percentage of exhaled air based on paper.

Figure 11 shows the graph presented in the paper while figure 10 shows our depicted graph given the parameters shown on the paper. We were able to emulate the results as shown above. This led us to believe that we'll be able to emulate all the results from this simulation. However, one barrier stopped us in our tracks and that is not all data points and parameters used for the simulation were presented in the paper.

$$N(t) = \frac{IC(t)(\beta-\mu)}{npC_a} \quad (20)$$

$$\lambda = pt\theta N \quad (21)$$

$$P(t) = 1 - e^{-\lambda(t)} \quad (22)$$

Data points like number of infected people, denoted as I, are not shown as well as parameters like β and μ that could help find the rate of survival of airborne infection particles generated by the infector that reaches its target infected area of the person who is vulnerable to infection at a threshold value and θ which is the respiratory deposition fraction of airborne infectious particles

that successfully reach and deposit at the target infection site of the host. Ultimately, we want to solve for $P(t)$ which is the probability of susceptible individuals with airborne infectors risk. Since these data points and parameters were not presented in the paper, we used random values instead.

Table 2. List of parameters

β	μ	θ	V
1.5	0.5	0.5	75

Table 3. Infected people data points

t	0	20	40	60	80	100	120	140	160	180
I	1	3	6	8	10	15	12	9	7	6

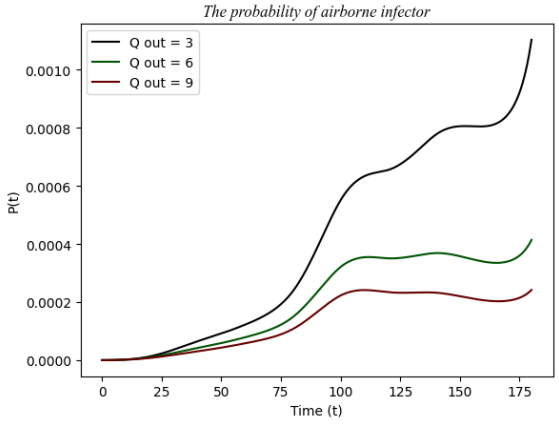

Figure 12: The probability of airborne infector

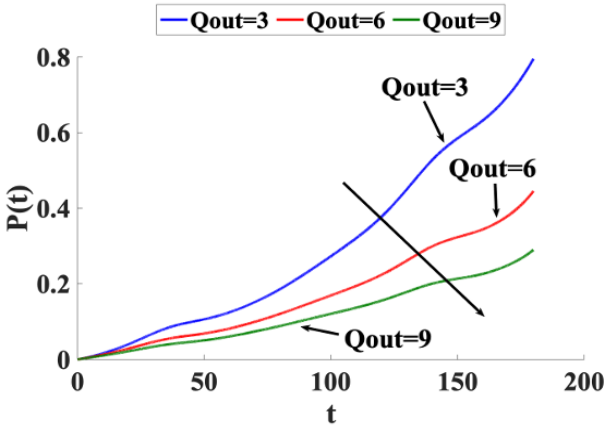

Figure 13: he probability of airborne infector based on paper

Figure 13 shows the probability of the airborne infector chart presented in the paper while figure 12 is our depicted graph. Even with different parameter values and data points used, we can still see that, even with different conditions, having a higher ventilation rate can help lower the chances of having airborne infectors. With given data and conditions, we can model the risk of airborne infections on different airborne diseases for different indoor environments.

Based on the computations we did; we can say that our results aligned with those in the paper. The Adaptive Runge-Kutta solution with cubic spline interpolation had the highest overall accuracy. However, we should reassess the parameters used in the simulations since there are some discrepancies encountered.

For centuries, our humanity has witnessed one of the deadliest airborne diseases that nature can bring to us. Diseases like tuberculosis, influenza, measles, severe acute respiratory syndrome (SARS), and recently SARS-CoV-2 also known as COVID-19 have destroyed millions of lives and changed the way our society behaves. Creating mathematical models that can help us analyze how these infectious diseases are spread paves the way for creating preventive measures and risk mitigating strategies that can prevent such diseases from spreading and potentially save lives. With

this model, we can influence how hospital rooms are operated and designed. Because of the results showing that having a decent ventilation system can reduce the risk of infectious particles from spreading in the room, we can expect more rooms to be ventilated in the future.

Moving forward, we can create more comprehensive studies using real world data against different scenarios like historical outbreaks such as the COVID-19 pandemic. We can also explore different environments that can potentially cause more spreading of infectious diseases like public transportation, schools, malls, concert events among others and through this we can create more complex models by adding elements of human behavior and how they move throughout the area. An example would be in a train station, where most commuters have the tendency of lining up. A model that includes human behavior can create a more comprehensive understanding on how these diseases are spread in public places. We could also take into consideration various measures that have been observed in the COVID-19 pandemic such as the use of face masks, social distancing, and even the case in the Philippines where people were required to use face shields and how these preventive measures were effective in reducing the risk of infections in an indoor setting. One question that we could investigate is if we can optimize the disease control strategy by properly integrating these preventive measures as well as proper ventilation design. And lastly we can apply this model to a user interface to create a software that could be used in places with vulnerable populations like hospitals and healthcare facilities. All in all, this paper has shown what mathematical modeling can do to solve problems like this one that are life threatening.

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