## 矩阵(表格)

## 矩阵

$$1$$
.矩阵 $-$  形如 $A=egin{pmatrix} a_{11} & \dots & a_{1n} \ \dots & & \ a_{m1} & \dots & a_{mn} \end{pmatrix} riangleq (a_{ij})_{m imes n}$ 

- ①若m=n,A-方阵
- ②若 $\forall a_{ij}=0, A=0$
- 2.同型矩阵  $-A_{m\times n}, B_{m\times n}$ 称为同型矩阵

设
$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & \dots & b_{1n} \\ \dots & & & \\ b_{m1} & \dots & b_{mn} \end{pmatrix}$$

若 $orall a_{ij}=b_{ij}, A=B$ 

3.三则运算:

$$egin{array}{cccc} egin{pmatrix} a_{11} & \dots & a_{1n} \ \dots & & & \ a_{m1} & \dots & a_{mn} \end{pmatrix} \pm egin{pmatrix} b_{11} & \dots & b_{1n} \ \dots & & & \ b_{m1} & \dots & b_{mn} \end{pmatrix} = egin{pmatrix} a_{11} \pm b_{11} & \dots & a_{1n} \pm b_{1n} \ \dots & & \ a_{m1} \pm b_{m1} & \dots & a_{mn} \pm b_{mn} \end{pmatrix}$$

$$(3A_{m imes n}=egin{pmatrix} a_{11} & \dots & a_{1n} \ \dots & & & \ a_{m1} & \dots & a_{mn} \end{pmatrix}, B_{n imes s}=egin{pmatrix} b_{11} & \dots & b_{1s} \ \dots & & \ b_{n1} & \dots & b_{ns} \end{pmatrix}$$

 $acksim a_{m1} \ldots a_{mn} / A_{m imes n} B_{n imes s}$ 内标同可乘,外标确定型

$$AB = C_{m imes s} = egin{pmatrix} c_{11} & \dots & c_{1s} \ \dots & & & \ c_{m1} & \dots & c_{ms} \end{pmatrix}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$$

endbackhend

ndpaotheno

engbaocheno

zendbaochend

englaacheng

engbaochens

Notes:  $\bigcirc A \neq 0, B \neq 0 \Rightarrow AB \neq 0$  $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq 0, B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \neq 0$   $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$  $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} 
eq 0, A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$ ③ AB 与 BA 不一定等  $4f(x) = a_n x^n + \ldots + a_1 x + a_0$ 给定 $A_{n\times n}$ :  $f(A) riangleq a_n A^n + \ldots + a_1 A + a_0 E - A$ 的矩阵多项式 如:  $f(x) = x^2 - x - 6 = (x+2)(x-3)$  $f(A) = A^2 - A - 6E = (A + 2E)(A - 3E)$ 又如:  $f(x) = x^2 + x + 2 = (x - 1)(x + 2) + 4$  $f(A) = A^2 + A + 2E = (A - E)(A + 2E) + 4E$ ⑤线性方程组一般形式:  $a_{11}x_1 + \ldots + a_{1n}x_n = 0$  $\langle a_{m1}x_1+\ldots+a_{mn}x_n=0 \rangle$  $\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases}$   $A = \begin{pmatrix} a_{11} & \ldots & a_{1n} \\ \ldots \\ a_{m1} & \ldots & a_{mn} \end{pmatrix}_{\substack{m \times n}} , X = \begin{pmatrix} x_1 \\ \ldots \\ x_n \end{pmatrix}_{\substack{n \times 1}}, b = \begin{pmatrix} b_1 \\ \ldots \\ b_m \end{pmatrix}_{\substack{m \times 1}}$   $AX = \begin{pmatrix} a_{11}x_1 + \ldots + a_{1n}x_n \\ a_{21}x_1 + \ldots + a_{2n}x_n \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n \end{pmatrix}_{\substack{m \times 1}}$   $AX = 0 \Leftrightarrow \begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases}$   $\therefore 3 \Leftrightarrow \exists b \Leftrightarrow \exists a_{11}x_1 + \ldots + a_{mn}x_n = b_m$   $\therefore 3 \Leftrightarrow \exists a_{11}x_1 + \ldots + a_{mn}x_n = b_m$   $\therefore 3 \Leftrightarrow \exists a_{11}x_1 + \ldots + a_{mn}x_n = b_m$   $\therefore 3 \Leftrightarrow \exists a_{11}x_1 + \ldots + a_{mn}x_n = b_m$   $\therefore 3 \Leftrightarrow \exists a_{11}x_1 + \ldots + a_{mn}x_n = b_m$ ::线性方程组的矩阵形式: AX = 0(\*)AX = b(\*\*) $\textcircled{6}A_{m \times n}, B_{n \times s} = (\beta_1, \beta_2, \dots, \beta_s)$  $AB = A(eta_1,eta_2,\ldots,eta_s) = (Aeta_1,Aeta_2,\ldots,Aeta_s)$ 更一般地,A(B,C)=(AB,AC)⑦左列化,右具体化 如: $A=(lpha_1,lpha_2),B=egin{pmatrix} 2 & 1 \ 1 & -5 \end{pmatrix}$ 

如:
$$A=(lpha_1,lpha_2),B=egin{pmatrix}2&1\1&-5\end{pmatrix}$$
 $AB=(2lpha_1+lpha_2,lpha_1-5lpha_2)$ 设:设 $lpha_1,lpha_2,lpha_3$ 为列向量 $(lpha_1,lpha_2,lpha_3)=A$ 

$$label{eq:B} eta B = (2lpha_1 - lpha_2, lpha_1 + 3lpha_2 - lpha_3, 2lpha_2 + lpha_3) = A egin{pmatrix} 2 & 1 & 0 \ -1 & 3 & 2 \ 0 & -1 & 1 \end{pmatrix}$$

$$A_{n imes n} = egin{pmatrix} a_{11} & \dots & a_{1n} \ \dots & & & \ a_{n1} & \dots & a_{nn} \end{pmatrix}$$

$$egin{aligned} 1.|A| = egin{aligned} a_{11} & \dots & a_{1n} \ \dots & & & \ a_{n1} & \dots & a_{nn} \end{aligned} \ 2.orall a_{ij} \Rightarrow M_{ij}(n-1) \Rightarrow A_{ij} \ & A_{ij} \end{aligned}$$

$$2. \forall a_{ij} \Rightarrow M_{ij}(n-1) \Rightarrow A_{ij}$$

$$AA^* = egin{pmatrix} A_{1j} & \Rightarrow M_{ij}(n-1) & \Rightarrow A_{ij} \ A_{11} & A_{21} & \dots & A_{n1} \ A_{12} & A_{22} & \dots & A_{n2} \ \dots & & & & \ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix} - A$$
的伴随矩阵 $AA^* = egin{pmatrix} |A| & 0 & \dots & 0 \ 0 & |A| & \dots & 0 \ \dots & & & \ 0 & 0 & \dots & |A| \end{pmatrix} = |A|E$ 

$$AA^* = egin{pmatrix} |A| & 0 & \dots & 0 \ 0 & |A| & \dots & 0 \ \dots & & & & \ 0 & 0 & \dots & |A| \end{pmatrix} = |A|E$$

Notes: $AA^* = A^*A = |A|E$ 

Q. 矩阵的研究对象?

1.背景:

初一:
$$ax = b$$

$$\bigcirc a \neq 0 : \frac{1}{a} \times a = 1$$

$$2a = 0: \begin{cases} b \neq 0 - \mathbb{E}\mathbf{M} \\ b = 0 - \mathbb{E}\mathbf{M} \end{pmatrix}$$

2.Similar problem:

$$AX = b, X = ?$$

①
$$A_{n \times n}$$
:  $\exists B_{n \times n}$ , 使 $BA = E -$  逆阵理论

$$AX = b \Rightarrow BAX = Bb \Rightarrow X = Bb$$

② 
$$\left\{egin{aligned} A_{n imes n} & op \ A_{m imes n} & op \end{aligned} 
ight.$$
 我理论