性质

 $1.A_{n imes n}, \lambda_1
eq \lambda_2 \ \lambda_1 E - A
ightarrow \cdots : lpha_1, \ldots, lpha_s$ $\alpha_1 \dots \alpha_s$ 线性无关,且 $A\alpha_1 = \lambda_1 \alpha_1, \dots, A\alpha_s = \lambda_1 \alpha_s$ $\lambda_2 E - A \rightarrow \cdots : \beta_1, \ldots, \beta_t$ $\beta_1 \dots \beta_t$ 线性无关,且 $A\beta_1 = \lambda_2 \beta_1, \dots, A\beta_t = \lambda_2 \beta_t$ $\Rightarrow \alpha_1, \ldots, \alpha_s, \beta_1, \ldots, \beta_t$ 线性无关

$$\begin{split} & \text{iff} : \diamondsuit(k_1\alpha_1 + \ldots + k_s\alpha_s) + (l_1\beta_1 + \ldots + l_t\beta_t) = 0(*) \\ & \because A\alpha_1 = \lambda_1\alpha_1, \ldots, A\alpha_s = \lambda_1\alpha_s \\ & A\beta_1 = \lambda_2\beta_1, \ldots, A\beta_t = \lambda_2\beta_t \end{split}$$

$$egin{aligned} \therefore A imes (*): \ \lambda_1(k_1lpha_1+\ldots+k_slpha_s) + \lambda_2(l_1eta_1+\ldots+l_teta_t) &= 0(**) \ (*) imes \lambda_2 - (**) \end{aligned}$$

$$(\lambda_2-\lambda_1)(k_1lpha_1+\ldots+k_slpha_s)=0 \ orall \lambda_1
eq \lambda_2, \therefore egin{cases} k_1lpha_1+\ldots+k_slpha_s=0 \ l_1eta_1+\ldots+l_teta_t=0 \end{cases}$$

$$\therefore \alpha_1 \dots \alpha_s \beta_1 \dots \beta_t$$
线性无关
 $\therefore k_1 = \dots = k_s = 0, l_1 = \dots = l_t = 0$

$$\therefore \alpha_1 \dots \alpha_s, \beta_1 \dots \beta_t$$
线性无关

 $A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$ $|\lambda E - A| = 0 \Rightarrow \lambda_1 = -1, \lambda_2 = \lambda_3 = 2$ $\lambda_1 = -1: E + A
ightarrow : lpha_1 = egin{pmatrix} 1 \ 1 \ 1 \end{pmatrix}$

$$\lambda_2=\lambda_3=2:2E-A o:lpha_2=egin{pmatrix} -1\ 1\ 0\ \end{pmatrix},lpha_3=egin{pmatrix} -1\ 0\ 1\ \end{pmatrix}$$
是 $\lambda_2=\lambda_3=2:2E-A o:lpha_2=egin{pmatrix} -1\ 0\ 1\ \end{pmatrix}$

 $1.\alpha_1,\alpha_2,\alpha_3$ 线性无关

$$2.Alpha_1=-lpha_1,Alpha_2=2lpha_2,Alpha_3=2lpha_3$$

$$3.(A\alpha_1,A\alpha_2,A\alpha_3)=(-\alpha_1,2\alpha_2,2\alpha_3)$$

$$A(lpha_1,lpha_2,lpha_3) = (lpha_1,lpha_2,lpha_3) egin{pmatrix} -1 & 0 & 0 \ 0 & 2 & 0 \ 0 & 0 & 2 \end{pmatrix}$$

$$\diamondsuit P=(lpha_1,lpha_2,lpha_3)=egin{pmatrix} 1 & -1 & -1 \ 1 & 1 & 0 \ 1 & 0 & 1 \end{pmatrix}$$
可逆

$$AP = P \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

 $(2.A_{n \times n}, \lambda_1 \neq \lambda_2, A\alpha = \overline{\lambda_1}\alpha, A\beta = \lambda_2\beta(\alpha \neq 0, \beta \neq 0))$ \Rightarrow α + β ─定不是特征向量 证:(反)设 $A(\alpha + \beta) = \lambda_3(\alpha + \beta)$ $\Rightarrow \lambda_1 \alpha + \lambda_2 \beta = \lambda_3 \alpha + \lambda_3 \beta$ $\Rightarrow (\lambda_1 - \lambda_3)\alpha + (\lambda_2 - \lambda_3)\beta = 0$ $\therefore \alpha, \beta$ 线性无关, $\lambda_1 = \lambda_2 = \lambda_3, \beta$ 盾 $\therefore \alpha + \beta$ 一定不是特征向量 $3.A\alpha = \lambda_0 \alpha (\alpha \neq 0)$ ②若A可逆,则 $A^{-1}\alpha = \frac{1}{\lambda_0}\alpha, A^*\alpha = \frac{|A|}{\lambda_0}\alpha$ if : ① $A\alpha = \lambda_0 \alpha \Rightarrow A^2 \alpha = A \cdot A\alpha = \lambda_0 A\alpha = \lambda_0^2 \alpha$ $A^3\alpha = \lambda_0^3\alpha, \dots$ $f(A)\alpha = a_n A^n \alpha + \ldots + a_1 A \alpha + a_0 \alpha$ $=(a_0\lambda_0^n+\ldots+a_1\lambda_0+a_0)\alpha=f(\lambda_0)\alpha$ ② :: A可逆, $:: \lambda_0 \neq 0$ $A\alpha = \lambda_0 \alpha \Rightarrow \alpha = \lambda_0 A^{-1} \alpha \Rightarrow A^{-1} \alpha = \frac{1}{\lambda_0} \alpha$

 $Alpha = \lambda_0 lpha \Rightarrow lpha = \lambda_0 A \quad lpha \Rightarrow A \quad lpha = rac{\lambda_0}{\lambda_0} lpha$ $A^*lpha = |A|A^{-1}lpha = rac{|A|}{\lambda_0}lpha$

Notes: A可逆时, A, A^{-1} , A^* 特征向量相同

A可逆, $Alpha=\lambda_0lpha \ ((A^*)^2+E)=[(rac{|A|}{\lambda_0})^2+1]lpha$ 解: $Alpha=\lambda_0lpha\Rightarrow A^*lpha=rac{|A|}{\lambda_0}lpha \ ((A^*)^2+E)lpha=[(rac{|A|}{\lambda_0})^2+1]lpha$

 $A \sim B$ 为4阶阵,A特征值 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ 求 $|B^{-1} - E|$?

解: $A \sim B \Rightarrow A^{-1} \sim B^{-1}$ A^{-1} 特征值 $2, 3, 4, 5 \Rightarrow B^{-1}$ 特征值 $2, 3, 4, 5 \Rightarrow B^{-1} - E$ 特征值1, 2, 3, 4 $\therefore |B^{-1} - E| = 24$

 $4.A_{n \times n}, A$ 可相似对角化 $\Leftrightarrow A$ 有n个线性无关特征向量

pao ter

02

实对称阵

$$A^T = A:$$
 $1.$ 若 $A^T = A \Rightarrow \lambda_1 \in R, \dots, \lambda_n \in R$
 $2.$ 若 $A^T = A, \lambda_1 \neq \lambda_2, A\alpha = \lambda_1\alpha, A\beta = \lambda_2\beta, 则$
 $\alpha \perp \beta$
 $\mathrm{iff}: \alpha^T A^T = \lambda_1\alpha^T \Rightarrow \alpha^T A = \lambda_1\alpha^T$
 $\Rightarrow \alpha^T A\beta = \lambda_1\alpha^T\beta \Rightarrow (\lambda_2 - \lambda_1)\alpha^T\beta = 0$
 $\therefore \lambda_1 \neq \lambda_2, \therefore \alpha^T\beta = 0,$ 即 $\alpha \perp \beta$
 $3.$ 若 $A^T = A, 则 A 一定可相似对角化$

endbaother