周期为2π的函数$f(x)$：

$f(x)$可否分解为$\frac{a_0}{2}+\sum_{n=1}^{+\infty}(a_n\cos nx+b_n\sin nx)$

$\frac{a_0}{2}$, 直流成份

$a_1\cos x+b_1\sin x$, 一次谐波

$f(x)$与$\frac{a_0}{2}+\sum_{n=1}^{+\infty}(a_n\cos nx+b_n\sin nx)$的关系?

(狄利克雷充分条件)设$f(x)$以2π为周期，在$[-\pi,\pi)$上：

①$f(x)$连续或有有限个第一类间断点

②$f(x)$有有限个极值点

则$f(x)$可以展成$\frac{a_0}{2}+\sum_{n=1}^{+\infty}(a_n\cos nx+b_n\sin nx)$, 且

$$\begin{cases} a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx \\ a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx, n=1,2,\ldots \\ b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx \end{cases}$$

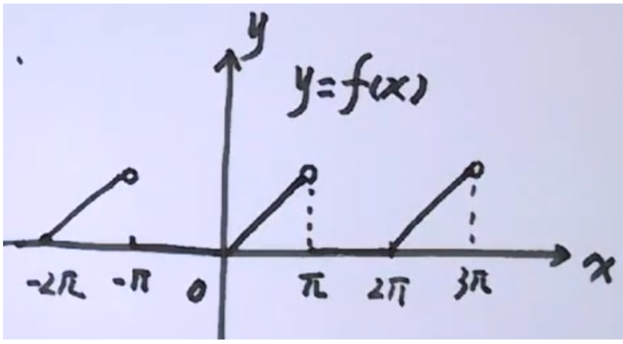
若x为$f(x)$的连续点，则$\frac{a_0}{2}+\sum_{n=1}^{+\infty}(a_n\cos nx+b_n\sin nx)=f(x)$

若x为$f(x)$的间断点，则$\frac{a_0}{2}+\sum_{n=1}^{+\infty}(a_n\cos nx+b_n\sin nx)=\frac{f(x-0)+f(x+0)}{2}$

$f(x)$以2π为周期, 当$x\in[-\pi,\pi)$时,

$$f(x)=\begin{cases} 0, -\pi\le x<0 \\ x, 0\le x<\pi \end{cases}$$

将$f(x)$展成Fourier级数.



1.作$y=f(x)$图

$x=(2k+1)\pi(k\in Z)$为间断点

$$2.a_0=\frac{1}{\pi}\int_0^{\pi}xdx=\frac{\pi}{2}$$

$$a_n=\frac{1}{\pi}\int_0^{\pi}x\cos nxdx=\frac{1}{n\pi}\int_0^{\pi}xd(\sin nx)$$

$$=-\frac{1}{n\pi}\int_0^{\pi}\sin nxdx=\frac{(-1)^n-1}{n^2\pi}$$

$$b_n=\frac{1}{\pi}\int_0^{\pi}x\sin nxdx=-\frac{1}{n\pi}\int_0^{\pi}xd(\cos nx)$$

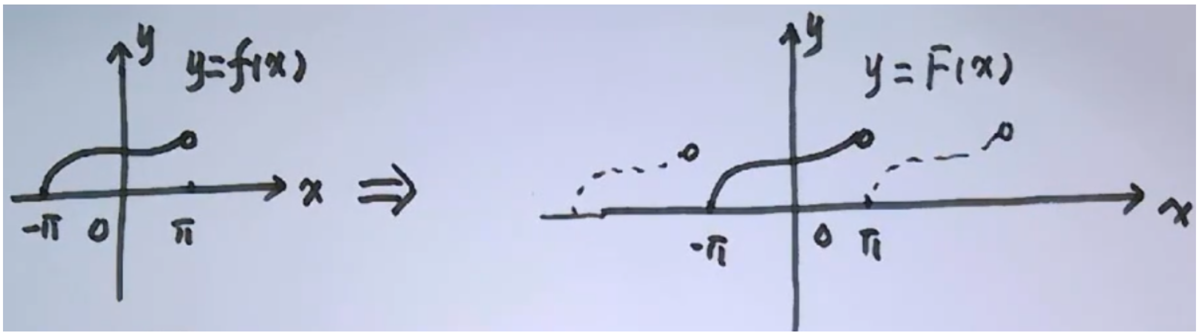
$$3.①f(x)=\frac{\pi}{4}+\sum_{n=1}^{\infty}[\frac{(-1)^n-1}{n^2\pi}\cos nx+\frac{(-1)^{n+1}}{n}\sin nx]$$

$(-\infty<x<+\infty$且$x\neq(2k+1)\pi(k\in Z))$

②当$x=(2k+1)\pi$时$(k\in Z)$

$$\frac{\pi}{4}+\sum_{n=1}^{\infty}[\frac{(-1)^n-1}{n^2\pi}\cos nx+\frac{(-1)^{n+1}}{n}\sin nx]=\frac{\pi}{2}$$

周期延拓



1.$f(x)$定义域$[-\pi,\pi)$, $f(x)$周期延拓：

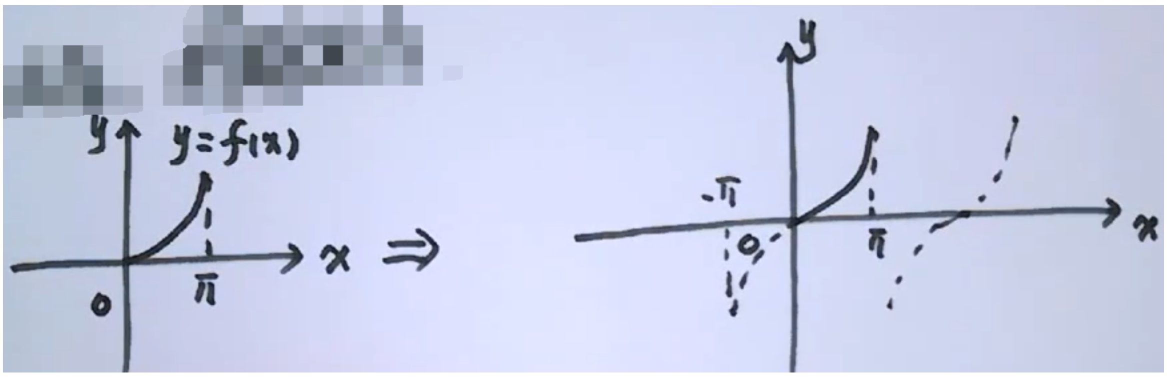
$$2.\begin{cases}a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx\\a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx\\b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx\end{cases}$$

$$3.f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(\dots),(-\pi<x<\pi)$$

奇延拓 展成正弦级数

$f(x)$定义域$[0,\pi)$

$f(x)$奇延拓或展成正弦级数



1.奇延拓、周期延拓

$2.a_0 = 0, a_n = 0, b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx dx$

$3.f(x) = \sum_{n=1}^{\infty} b_n \sin nx, (0 \le x < \pi)$

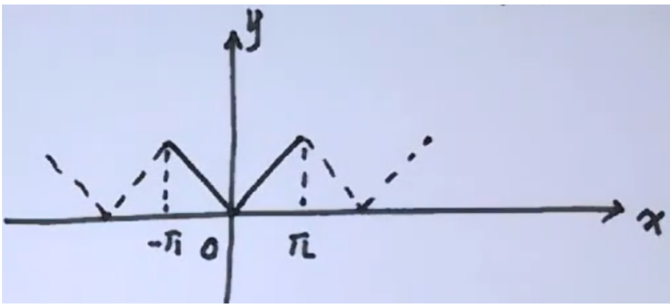
偶延拓 展成余弦级数

1.偶延拓、周期延拓

$2.a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx, a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx, b_n = 0$

$3.f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, (0 \le x \le \pi)$

$$f(x) = |x|(-\pi \le x < \pi)\text{展开}F\text{级数, 求}\sum_{n=1}^{\infty}\frac{1}{n^2} =$$



1.周期延拓

$$2.a_0=\frac{2}{\pi}\int_0^{\pi}xdx=\pi$$

$$a_n=\frac{2}{n\pi}\int_0^{\pi}xd(\sin x)=-\frac{2}{n\pi}\int_0^{2}\sin nxdx$$

$$=\frac{2}{n^2\pi}\cos nx\mid_0^{\pi}=\frac{2[(-1)^n-1]}{n^2\pi}=\begin{cases}-\frac{4}{n^2\pi},n=1,3,5,\dots\\0,n=2,4,6,\dots\end{cases}$$

$$b_n=0(n=1,2,\dots)$$

$$3.|x|=\frac{\pi}{2}-\frac{4}{\pi}(\cos x+\frac{1}{3^2}\cos 3x+\dots)(-\pi\leq x\leq\pi)$$

$$4.x=0\text{时},\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots=\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=\frac{\pi^2}{8}$$

$$\text{令}S=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$=(\frac{1}{1^2}+\frac{1}{1^2}+\dots)+(\frac{1}{1^2}+\frac{1}{1^2}+\frac{1}{1^2}+\dots)$$

$$=\frac{\pi^2}{8}+\frac{1}{4}S$$

$$\Rightarrow\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$

以2l为周期的函数f(x)

1.作图

$$2.\begin{cases}a_0=\frac{1}{l}\int_{-l}^{l}f(x)dx\\a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}dx\\b_n=\frac{1}{l}\int_{-l}^{l}\sin\frac{n\pi x}{l}dx\end{cases}$$

$$3.x\text{为}f(x)\text{的连续点},f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l},(-\infty<x<+\infty,x\neq ?)$$

$$4.x=?,\frac{a_0}{2}+\sum_{n=1}^{\infty}(\dots)=\frac{f(x-0)+f(x+0)}{2}$$

f(x)定义域[-l,l)上

1.周期延拓

$$2.\begin{cases}a_0=\frac{1}{l}\int_{-l}^{l}f(x)dx\\a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi x}{l}dx\\b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi x}{l}dx\end{cases}$$

$$3.f(x)=\frac{a_0}{2}+\dots()$$