求导类型

显函数求导

$$y = f(x)$$

$$y = x\ln(x+\sqrt{x^2+1}), 求y'.$$

$$\begin{aligned} y' =& \ln(x+\sqrt{x^2+1}) + x\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot(1+\frac{2x}{2\sqrt{x^2+1}}) \\ =& \ln(x+\sqrt{x^2+1}) + \frac{x}{\sqrt{x^2+1}} \end{aligned}$$

$$y = (1+x)^{\sin x}, 求y'$$

$$\begin{aligned} y' =& e^{\sin x\cdot\ln(1+x)}[\cos x\cdot\ln(1+x)+\frac{\sin x}{1+x}] \\ =& (1+x)^{\sin x}[\cos x\cdot\ln(1+x)+\frac{\sin x}{1+x}] \end{aligned}$$

隐函数求导

$$F(x,y) = F(x,\Phi(x)) = 0 : y = y(x), \frac{dy}{dx}?$$

$$\ln\sqrt{x^2+y^2} = \arctan\frac{y}{x}, 确定y为x的函数, 求\frac{dy}{dx}$$

$$\begin{aligned} &\ln\sqrt{x^2+y^2} = \arctan\frac{y}{x}两边对x求导 \\ &\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{2x+2y\cdot y'}{2\sqrt{x^2+y^2}} = \frac{1}{1+(\frac{y}{x})^2}\cdot\frac{xy'-y}{x^2} \\ &\frac{x+yy'}{x^2+y^2} = \frac{xy'-y}{x^2+y^2} \Rightarrow x+yy' = xy'-y \\ &\Rightarrow y' = \frac{x+y}{x-y} \end{aligned}$$

$$e^{xy} = x^2+y, y'(0), y''(0)$$

1. $x=0$代入, $y=1$
2. $e^{xy}(y+xy') = 2x+y'$, 代入, $y'(0)=1$
3. $e^{xy}(y+xy')^2 + e^{xy}(2y'+xy'') = 2+y''$, 代入
   $1+2 = 2+y''(0) \Rightarrow y''(0) = 1$

参数方程求导

$$\begin{cases} x = \Phi(t) \\ y = \phi(t) \end{cases}, \Phi(t), \phi(t)可导且, \Phi(t) \neq 0$$

$$\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\Phi'(t)\neq 0\Rightarrow \Delta x=O(\Delta t)$$

$$\lim_{\Delta t\to 0}\frac{\Delta y}{\Delta t}=\phi'(t)$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}=\lim_{\Delta t\to 0}\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}=\frac{\phi'(t)}{\Phi'(t)}=\frac{dy}{dt}/\frac{dx}{dt}$$

$$\frac{dy}{dx}=\frac{\phi'(t)}{\Phi'(t)}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{d(\frac{dy}{dx})}{dx}=\frac{d[\frac{\phi'(t)}{\Phi'(t)}]/dt}{dx/dt}$$

$$设函数y=y(x)由\begin{cases}x=\arctan t\\ y=\ln(1+t^2)\end{cases}确定,求\frac{d^2y}{dx^2}.$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\frac{2t}{1+t^2}}{\frac{1}{1+t^2}}=2t$$

$$\frac{d^2y}{dx^2}=\frac{d\frac{dy}{dx}}{dx}=\frac{d(2t)/dt}{dx/dt}=\frac{2}{\frac{1}{1+t^2}}=2(1+t^2)$$

$$设函数y=y(x)由\begin{cases}x=t-\sin t\\ y=1-\cos t\end{cases}确定,求\frac{d^2y}{dx^2}.$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\sin t}{1-\cos t}$$

$$\frac{d^2y}{dx^2}=\frac{d(\frac{\sin t}{1-\cos t})/dt}{dx/dt}=\frac{(\frac{\sin t}{1-\cos t})'}{1-\cos t}$$

$$\begin{cases}x=2t^2+t+1\\ e^{yt}=y+3t\end{cases}求\frac{dy}{dx}\mid_{x=1}.$$

1. $x=1时,t=0,y=1$
2. $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$
3. $\frac{dx}{dt}=4t+1,\frac{dx}{dt}\mid_{t=0}=1$

   $e^{yt}(y+t\cdot\frac{dy}{dt})=\frac{dy}{dt}+3$

   $t=0,y=1代入\frac{dy}{dt}\mid_{t=0}=-2$
4. $\frac{dy}{dx}\mid_{x=1}=\frac{dy/dt}{dx/dt}\mid_{t=0}=-2$

分段函数求导

$$f(x)=\begin{cases}e^{ax},x<0\\ \ln(1+2x)+b,x\geq 0\end{cases},f'(0)\exists,求a,b.$$

1. $f(0-0)=1, f(0)=f(0+0)=b$

$\because f(0-0)=f(0+0)=f(0), \therefore b=1$

2. $f'_{-}(0)=\lim\limits_{x\to 0^-}\dfrac{f(x)-f(0)}{x}=a$

$f'_{+}(0)=\lim\limits_{x\to 0^+}\dfrac{f(x)-f(0)}{x}=\lim\limits_{x\to 0^+}\dfrac{\ln(1+2x)}{x}=2$

$\because f'_{-}(0)=f'_{+}(0), \therefore a=2$

$$f(x)=\begin{cases}1, x=0\\ \frac{\sin x}{x}, x\neq 0\end{cases}, 求f'(x)$$

$$x\neq 0时, f'(x)=\frac{x\cos x-\sin x}{x^2}$$

$$x=0时, \lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{\sin x-x}{x^2}=0\Rightarrow f'(0)=0$$

$$\therefore f'(x)=\begin{cases}0, x=0\\ \frac{x\cos x-\sin x}{x^2}, x\neq 0\end{cases}$$

$$f(x)=\begin{cases}\ln(1+2x), x<0\\ ax^2+bx+c, x\geq 0\end{cases}, f''(0)\exists, 求a,b,c$$

1. $f(0-0)=0, f(0)=f(0+0)=c$

$\because f(x)在x=0处连续, \therefore c=0$

2. $f'(x)=\begin{cases}\frac{2}{1+2x}, x<0\\ 2ax+b, x\geq 0\end{cases}$

$\because f'(x)在x=0处连续, \therefore b=2$

$f'(x)\begin{cases}\frac{2}{1+2x}, x<0\\ 2ax+q, x\geq 0\end{cases}$

3. $f''_{-}(0)=\lim\limits_{x\to 0^-}\dfrac{f'(x)-f'(0)}{x}=\lim\limits_{x\to 0^-}\dfrac{\frac{2}{1+x}-2}{x}=-4$

$f''_{+}(0)=\lim\limits_{x\to 0^+}\dfrac{f'(x)-f'(0)}{x}=2a$

$\because f''_{-}(0)=f''_{+}(0), \therefore a=-2$

高阶导数求导

$$=$$

$$=$$

$$y=f(x)=e^x\sin x, f^{(n)}(x)$$

$$f'(x)=e^x\sin x+e^x\cos x=\sqrt{2}e^x\sin(x+\frac{\pi}{4})$$

$$f''(x)=\sqrt{2}[e^x\sin(x+\frac{\pi}{4})+e^x\cos(x+\frac{\pi}{4})]$$

$$=(\sqrt{2})^2e^x\cdot\sin(x+\frac{2\pi}{4})$$

$$\therefore f^{(n)}(x)=(\sqrt{2})^n e^x\cdot\sin(x+\frac{n\pi}{4})$$

$$f(x)=\frac{1}{2x+1}, f^{(n)}(0)$$

$$
\begin{gathered}
f(x)=(2x+1)^{-1} \\
f'(x)=(-1)(2x+1)^{-2}\cdot 2 \\
f''(x)=(-1)(-2)(2x+1)^{-3}\cdot 2^2 \\
\cdots \\
f^{(n)}(x)=(-1)(-2)\ldots(-n)(2x+1)^{-(n+1)}\cdot 2^n \\
=\frac{(-1)^n n!2^n}{(2x+1)^{n+1}}
\end{gathered}
$$

$$
记:(\frac{1}{ax+b})^{(n)}=\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}
$$

$$
f(x)=\ln(2x^2+x-1),求 f^{(n)}(x)
$$

$$
\begin{gathered}
f(x)=\ln(x+1)(2x-1)=\ln(x+1)+\ln(2x-1) \\
f'(x)=\frac{1}{x+1}+2\cdot\frac{1}{2x-1} \\
f^{(n)}(x)=(\frac{1}{x+1})^{n-1}+2\cdot(\frac{1}{2x-1})^{n-1} \\
=\frac{(-1)^{n-1}(n-1)!}{(x+1)^n}+2\cdot\frac{(-1)^{n-1}\cdot(n-1)!\cdot 2^{n-1}}{(2x-1)^n}
\end{gathered}
$$