隐函数及参数方程确定的函数的导数

隐函数

$$y=f(x)(x\in D)$$
$$F(x,y)=0(x\in D),若\forall x\in D,\exists唯一的y(由F(x,y)=0确定)$$
$$称F(x,y)=0确定y为x的隐函数,\frac{dy}{dx}?$$
$$F(x,y)=0=F(x,\Phi(x))$$

$$设e^{2x^2+y}=x^2+y^2确定y是x的函数,求\frac{dy}{dx}.$$

$$e^{2x^2+y}=x^2+y^2\Rightarrow y=\Phi(x)$$
$$\Rightarrow e^{2x^2+y}(4x+\frac{dy}{dx})=2x+2y\frac{dy}{dx}$$
$$\Rightarrow (x^2+y^2)(4x+\frac{dy}{dx})=2x+2y\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}=\frac{4x(x^2+y^2)-2x}{2y-x^2-y^2}$$

$$\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x},求\frac{dy}{dx}.$$

$$\frac{1}{\sqrt{x^2+y^2}}*\frac{2x+2y\frac{dy}{dx}}{2\sqrt{x^2+y^2}}=\frac{1}{1+\frac{y^2}{x^2}}\frac{x\frac{dy}{dx}-y*1}{x^2}$$
$$x+y\frac{dy}{dx}=x\frac{dy}{dx}-y\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y}$$

$$2^{xy}=3x+y,求y'(0).$$

$$x=0\Rightarrow y=1$$
$$2^{xy}\ln 2(y+x\frac{dy}{dx})=3+\frac{dy}{dx}$$
$$x=0,y=1代入,y'(0)=\ln 2-3$$

参数方程确定的函数的导数

一阶

$$\begin{cases}x=\Phi(t)\\y=\phi(t)\end{cases},\Phi(t),\phi(t)可导且,\Phi(t)\neq 0$$

$$y=y(x)$$

$$\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\Phi'(t)\neq 0\Rightarrow\Delta x=O(\Delta t)$$

$$\lim_{\Delta t\to 0}\frac{\Delta y}{\Delta t}=\phi'(t)$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}=\lim_{\Delta t\to 0}\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}=\frac{\phi'(t)}{\Phi'(t)}=\frac{dy}{dt}/\frac{dx}{dt}$$

二阶

$$\Phi(t),\phi(t)\text{二阶可导，且}\Phi(t)\neq 0$$

$$\frac{dy}{dx}=\frac{\phi'(t)}{\Phi'(t)}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{d(\frac{dy}{dx})}{dx}=\frac{d[\frac{\phi'(t)}{\Phi'(t)}]/dt}{dx/dt}$$

$$\text{设函数}y=y(x)\text{由}\begin{cases}x=\arctan t\\y=\ln(1+t^2)\end{cases}\text{确定，求}\frac{d^2y}{dx^2}.$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\frac{2t}{1+t^2}}{\frac{1}{1+t^2}}=2t$$

$$\frac{d^2y}{dx^2}=\frac{d\frac{dy}{dx}}{dx}=\frac{d(2t)/dt}{dx/dt}=\frac{2}{\frac{1}{1+t^2}}=2(1+t^2)$$

$$\text{设函数}y=y(x)\text{由}\begin{cases}x=t-\sin t\\y=1-\cos t\end{cases}\text{确定，求}\frac{d^2y}{dx^2}.$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\sin t}{1-\cos t}$$

$$\frac{d^2y}{dx^2}=\frac{d(\frac{\sin t}{1-\cos t})/dt}{dx/dt}=\frac{(\frac{\sin t}{1-\cos t})'}{1-\cos t}$$