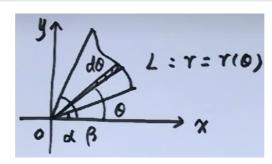
# 定积分的几何应用



L: y=f(x)

$$egin{aligned} 1. orall [x,x+dx] \subset [a,b] \ 2. dA = f(x) dx \end{aligned}$$

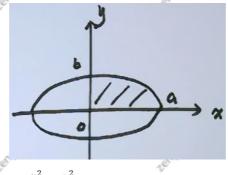
$$3.A = \int_a^b f(x) dx$$



$$1. \forall [\theta, \theta + d\theta] \subset [\alpha, \beta]$$

$$2.dA = \frac{1}{2}r^2(\theta)d\theta$$

$$3.A=rac{1}{2}\int_{lpha}^{eta}r^{2}( heta)d heta$$



求椭圆
$$L:rac{x^2}{a^2}+rac{y^2}{b^2}=1 (a>0,b>0)$$
所围成的区域面积.

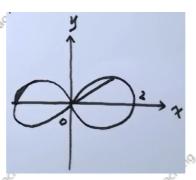
解:法-

$$L_1: y = rac{b}{a} \sqrt{a^2 - x^2} (0 \le x \le a)$$

$$A = 4A_1 = \pi a b$$

$$\Rightarrow \begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$\begin{aligned} & U: r^2 = \frac{a^2b^2}{b^2\cos^2\theta + a^2\sin^2\theta} \\ & A_1 = \frac{a^2b^2}{2} \int_0^{\frac{\pi}{2}} \frac{d\theta}{b^2\cos^2\theta + a^2\sin^2\theta} = \frac{ab^2}{2} \int_0^{\frac{\pi}{2}} \frac{d(a\tan\theta)}{b^2 + (a\tan\theta)^2} = \frac{ab^2}{2} * \frac{1}{b}\arctan\frac{a\tan\theta}{b} \mid_0^{\frac{\pi}{2}} = \frac{ab}{2} * \frac{\pi}{2} = \frac{\pi ab}{4} \\ & A = 4A_1 = \pi ab \end{aligned}$$

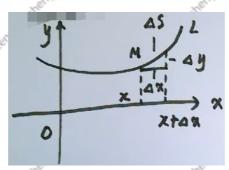


求 $L:(x^2+y^2)^2=4(x^2-y^2)$ 围成面积.

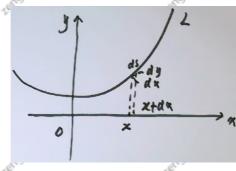
$$\mathbb{R}: \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$L: r^2 = 4\cos 2\theta$$

$$A_1 = \frac{1}{2} \int_0^{\frac{\pi}{4}} 4\cos 2\theta d\theta = \int_0^{\frac{\pi}{4}} \cos 2\theta d(2\theta) = \int_0^{\frac{\pi}{4}} \cos 2\theta d(2\theta) = \int_0^{\frac{\pi}{2}} \cos \theta d\theta$$
$$\Rightarrow A = 4A_1 = 4$$



 $(\Delta s)^2pprox (\Delta x)^2+(\Delta y)^2$ 



 $(ds)^2 = (dx)^2 + (dy)^2$ 

$$1.L: y = f(x)$$

$$egin{aligned} ds &= \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + (rac{dy}{dx})^2} dx \ &= \sqrt{1 + f'^2(x)} dx \end{aligned}$$

$$2.L: \begin{cases} x = \Phi(t) \\ y = \phi(t) \end{cases}$$

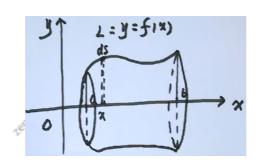
$$y = \phi(t)$$
  $ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\Phi'^2(t) + \phi'^2(t)} dt$  3.

①. 
$$\mathbb{R}[x, x + dx] \subset [a, b]$$

$$=2\pi |y|\cdot \sqrt{1+y'^2}dx$$

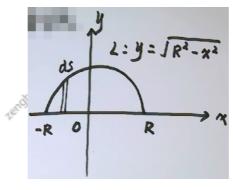
$$\exists A_x = 2\pi \int_a^b |y| \cdot \sqrt{1 + y'^2} dx$$

Lengthacthens





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求半径为*R*的球的表面积

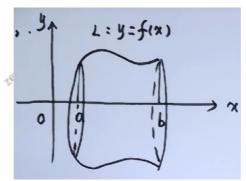
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解: 
$$1.$$
取 $[x, x + dx] \subset [-R, R]$   
 $2.dA = 2\pi y \cdot ds$   
 $= 2\pi \cdot \sqrt{R^2 - x^2} \cdot \sqrt{1 + \frac{x^2}{R^2 - x^2}} dx = 2\pi R dx$   
 $3.A = \int_{-R}^{R} 2\pi R dx = 4\pi R^2$ 

endpactners.

体积

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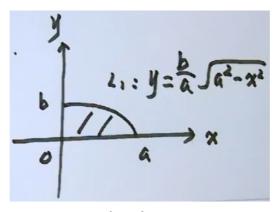
and tracethenes

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$$egin{aligned} 1.$$
與 $[x,x+dx] \subset [a,b] \ 2.dV_x &= \pi f^2(x) dx \ 3.V_x &= \pi \int_a^b f^2(x) dx \end{aligned}$ 

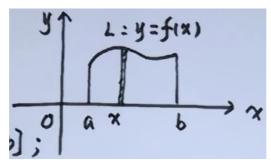
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Lendy action is



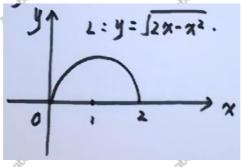
THE TO

$$L:rac{x^2}{a^2}+rac{y^2}{b^2}=1, rac{x}{V_x}$$



$$\begin{split} 1. \mathbb{R}[x, x + dx] \subset [a, b] \\ 2. dV_y &= 2\pi |x| \cdot |y| dx \end{split}$$

$$3.V_y=2\pi\int_a^b|x|\cdot|y|dx$$

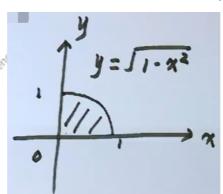


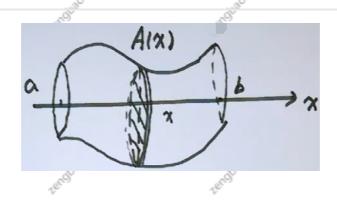
$$1.V_x = \pi \int_0^2 f^2(x) dx = \pi \int_0^2 y^2 dx = \pi \int_0^2 (2x - x^2) dx = \pi (4 - \frac{8}{3}) = \frac{4}{3}\pi$$

$$2.$$
取 $[x,x+dx]\subset [0,2]$ 

$$dV_y = 2\pi x \sqrt{2x - x^2} dx$$

$$V_y = 2\pi\int_0^2 x\sqrt{2x-x^2}dx = 2\pi\int_0^2 [1+(x-1)]\sqrt{1-(x-1)^2}d(x-1) = 2\pi\int_{-1}^1 (1+x)\sqrt{1-x^2}dx = 4\pi\int_0^1 \sqrt{1-x^2}dx = \pi^2$$





$$1.$$
取 $[x,x+dx]\subset [a,b]$   $2.dV=A(x)dx$   $3.V=\int_{a}^{b}A(x)dx$ 

$$1.$$
取 $[x,x+dx]\subset [-2,2]$ 

$$2.dV = 2\pi(3-x) \cdot y \cdot dx = 2\pi(3-x)(4-x^2)dx$$

$$egin{aligned} 1. \mathbb{R}[x,x+dx] &\subset [-2,2] \ 2. dV &= 2\pi(3-x) \cdot y \cdot dx = 2\pi(3-x)(4-x^2) dx \ 3. V &= 2\pi \int_{-2}^2 (3-x)(4-x^2) dx = 12\pi \int_0^2 (4-x^2) dx \ &= 12\pi(8-rac{8}{3}) = 12\pi \cdot rac{16}{3} = 64\pi \end{aligned}$$

弧长

 $egin{aligned} 1.L: y &= f(x) (a \leq x \leq b) \ @\forall [x, x + dx] \subset [a, b] \ @ds &= \sqrt{1 + f'^2(x)} dx \end{aligned}$ 

$$\textcircled{1} \forall [x,x+dx] \subset [a,b]$$

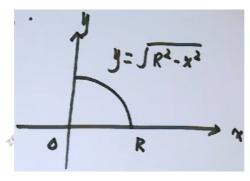
$$@ds = \sqrt{1 + f'^2(x)}dx$$

$$2.L: egin{cases} x = eta_a & \sqrt{1+f^{'}} & (x)dx \ 2.L: egin{cases} x = \Phi(t) \ y = \phi(t) \end{cases} & (lpha \le t \le eta) \ \oplus \mathbb{R}[t,t+dt] \subset [lpha,eta] \ \oplus ds = \sqrt{\Phi'^2(t) + \phi'^2(t)}dt \end{cases}$$

①取
$$[t,t+dt]\subset [lpha,eta]$$

$$@ds = \sqrt{\Phi'^2(t) + \phi'^2(t)} dt$$

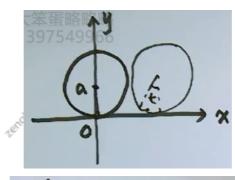
$$\Im l = \int_{lpha}^{eta} \sqrt{\Phi'^2(t) + \phi'^2(t)} dt$$

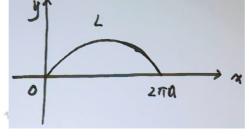


求半径为R的圆周长

解:
$$l = 4 \int_0^R \sqrt{1 + y'^2} dx$$
  
=  $4 \int_0^R \sqrt{1 + \frac{R}{R^2 - x^2}} dx$   
=  $4R \int_0^R \frac{dx}{\sqrt{R^2 - x^2}} = 4R \arcsin \frac{x}{R} \mid_0^R = 2\pi R$ 

#### 摆线

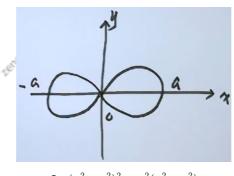




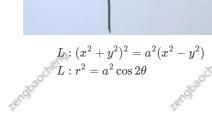
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$$L:egin{cases} x=a(t-\sin t)\ y=a(1-\cos t) \end{cases} (0\leq t\leq 2\pi)$$

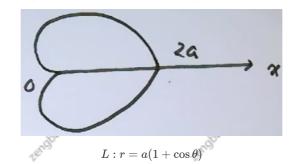
### 双纽线



心脏线







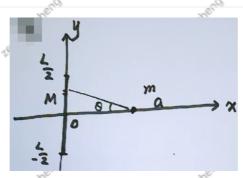
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## 定积分的物理应用

### 引力

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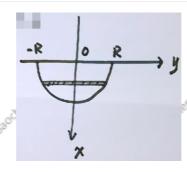


erojtišocheno

$$egin{align*} & egin{align*} \widehat{H}: 1. \mathbb{R}[y,y+dy] \subset [-rac{L}{2},rac{L}{2}] \ & 2. dF = K \cdot rac{m \cdot rac{M}{L} dy}{a^2 + y^2}, dF_x = dF \cdot rac{a}{\sqrt{a^2 + y^2}} \ & dF_x = rac{KamM}{L} \cdot rac{dy}{(a^2 + y^2)^{rac{3}{2}}} \ & 3. F_x = \int_{-rac{L}{2}}^{rac{L}{2}} dF_x = rac{2KamM}{L} \int_{0}^{rac{L}{2}} rac{dy}{(y^2 + a^2)^{rac{3}{2}}} \ & = rac{2KamM}{L} \cdot rac{y}{\sqrt{y^2 + a^2}} \mid_{0}^{rac{L}{2}} \end{aligned}$$

压力

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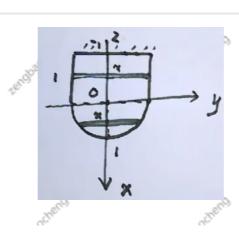
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圆柱形水桶盛一半的水,底面半径为R,将圆柱水平放置,求水对地面的压力.

$$\begin{split} & \Re: 1. \mathbb{R}[x, x + dx] \subset [0, R] \\ & 2. dF = \rho gx \cdot dA \\ & = \rho gx \cdot (y_2 - y_1) dx \\ & \because x^2 + y^2 = R^2, \therefore y_1 = -\sqrt{R^2 - x^2}, y_2 = \sqrt{R^2 - x^2} \\ & \therefore dF = 2\rho gx \sqrt{R^2 - x^2} dx \\ & 3. F = 2\rho g \int_0^R x \sqrt{R^2 - x^2} dx \\ & = -\rho g \int_0^R (R^2 - x^2)^{\frac{1}{2}} d(R^2 - x^2) \\ & = -\frac{2}{3} \rho g(R^2 - x^2)^{\frac{3}{2}} \left|_0^R = \frac{2}{3} \rho g R^3 \right. \end{split}$$

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Hend

解: ①1.取[x, x + dx]  $\subset [0, -1]$   $2.dF_1 = \rho g \cdot (x+1) \cdot 2dx$   $= \rho gx \cdot (y_2 - y_1)dx$   $3.F_1 = 2\rho g \int_{-1}^0 (x+1)dx = 2\rho g(-\frac{1}{2}+1) = \rho g$ ②1.取[x, x + dx]  $\subset [0, 1]$   $2.dF_2 = \rho g \cdot (x+1) \cdot (y_2 - y_1)dx$   $= 2\rho g(x+1)\sqrt{1-x^2}dx$   $3.F_2 = 2\rho g \int_0^1 (x+1)\sqrt{1-x^2}dx$   $= -\rho g \int_0^1 (1-x^2)^{\frac{1}{2}}d(1-x^2) + 2\rho g \frac{\pi}{4}$  $= \frac{2}{3}\rho g + \frac{\pi}{2}\rho g$ 

 $\therefore F = (\frac{5}{3} + \frac{\pi}{2})\rho g$ 

*Lengthauthent* 

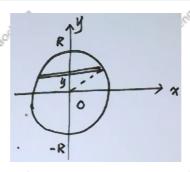
功



有一电荷量为 $q_1$ ,且带正电的固定质点位于原点,在距离原点a处有一电荷量为 $q_2$ 且带正电的活动质点,若固定质点,将活动质点从距离a处排斥到距离b处,求排斥力所做的功.

$$egin{aligned} egin{aligned} \mathbb{R}: 1. \mathbb{X}[x,x+dx] &\subset [a,b] \ 2. dW &= k \cdot rac{q_1 q_2}{x^2} dx \ 3. W &= \int_a^b dW &= -rac{kq_1 q_2}{x} \mid_a^b \ &= kq_1 q_2 (rac{1}{a} - rac{1}{b}) \end{aligned}$$

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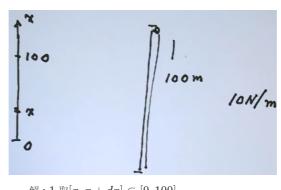


半径为R的球体充满水,将水从顶部抽出,求W.

解: 
$$1.$$
取 $[y, y + dy] \subset [-R, R]$   
 $2.dW = \rho g dV \cdot (R - y)$   
 $dV = \pi x^2 \cdot dy = \pi (R^2 - y^2) dy$   
 $dW = \pi \rho g (R - y) (R^2 - y^2) dy$   
 $3.W = \pi \rho g \int_{-R}^{R} (R - y) (R^2 - y^2) dy$   
 $= 2\pi \rho g R \int_{0}^{R} (R^2 - y^2) dy$   
 $= \frac{4\pi}{3} \rho g R^4$ 

a ocheno

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$$egin{align} \mathscr{M}: 1. \mathbb{X}[x,x+dx] \subset [0,100] \ 2. dW &= 10(100-x) \cdot dx \ 3. W &= \int_0^{100} dW = 10 \int_0^{100} (100-x) dx \ &= 10 * rac{1}{2} * 100 * 100 = 50000(J) \ \end{cases}$$

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