求导类型

显函数求导

$$y=f(x)$$

$$y=x\ln(x+\sqrt{x^2+1}),\text{求}y'.$$

$$\begin{aligned}y'=&\ln(x+\sqrt{x^2+1})+x\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot(1+\frac{2x}{2\sqrt{x^2+1}})\\=&\ln(x+\sqrt{x^2+1})+\frac{x}{\sqrt{x^2+1}}\end{aligned}$$

$$y=(1+x)^{\sin x},\text{求}y'$$

$$\begin{aligned}y'=&e^{\sin x\cdot\ln(1+x)}[\cos x\cdot\ln(1+x)+\frac{\sin x}{1+x}]\\=&(1+x)^{\sin x}[\cos x\cdot\ln(1+x)+\frac{\sin x}{1+x}]\end{aligned}$$

隐函数求导

$$F(x,y)=F(x,\Phi(x))=0:y=y(x),\frac{dy}{dx}?$$

$$\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x},\text{确定}y\text{为}x\text{的函数},\text{求}\frac{dy}{dx}$$

$$\begin{aligned}&\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x}\text{两边对}x\text{求导}\\&\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{2x+2y\cdot y'}{2\sqrt{x^2+y^2}}=\frac{1}{1+(\frac{y}{x})^2}\cdot\frac{xy'-y}{x^2}\\&\frac{x+yy'}{x^2+y^2}=\frac{xy'-y}{x^2+y^2}\Rightarrow x+yy'=xy'-y\\&\Rightarrow y'=\frac{x+y}{x-y}\end{aligned}$$

$$e^{xy}=x^2+y,y'(0),y''(0)$$

$$\begin{aligned}&1.\ x=0\text{代入},y=1\\&2.\ e^{xy}(y+xy')=2x+y',\text{代入},y'(0)=1\\&3.\ e^{xy}(y+xy')^2+e^{xy}(2y'+xy'')=2+y'',\text{代入}\\&\quad 1+2=2+y''(0)\Rightarrow y''(0)=1\end{aligned}$$

参数方程求导

$$\begin{cases}x=\Phi(t)\\y=\phi(t)\end{cases},\Phi(t),\phi(t)\text{可导且},\Phi(t)\neq 0$$

$$\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\Phi'(t)\neq 0\Rightarrow \Delta x=O(\Delta t)$$

$$\lim_{\Delta t\to 0}\frac{\Delta y}{\Delta t}=\phi'(t)$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}=\lim_{\Delta t\to 0}\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}=\frac{\phi'(t)}{\Phi'(t)}=\frac{dy}{dt}\Big/\frac{dx}{dt}$$

$$\frac{dy}{dx}=\frac{\phi'(t)}{\Phi'(t)}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{d(\frac{dy}{dx})}{dx}=\frac{d[\frac{\phi'(t)}{\Phi'(t)}]/dt}{dx/dt}$$

$$\text{设函数}y=y(x)\text{由}\begin{cases}x=\arctan t\\ y=\ln(1+t^2)\end{cases}\text{确定, 求}\frac{d^2y}{dx^2}.$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\frac{2t}{1+t^2}}{\frac{1}{1+t^2}}=2t$$

$$\frac{d^2y}{dx^2}=\frac{d\frac{dy}{dx}}{dx}=\frac{d(2t)/dt}{dx/dt}=\frac{2}{\frac{1}{1+t^2}}=2(1+t^2)$$

$$\text{设函数}y=y(x)\text{由}\begin{cases}x=t-\sin t\\ y=1-\cos t\end{cases}\text{确定, 求}\frac{d^2y}{dx^2}.$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\sin t}{1-\cos t}$$

$$\frac{d^2y}{dx^2}=\frac{d(\frac{\sin t}{1-\cos t})/dt}{dx/dt}=\frac{(\frac{\sin t}{1-\cos t})'}{1-\cos t}$$

$$\begin{cases}x=2t^2+t+1\\ e^{yt}=y+3t\end{cases}\text{求}\frac{dy}{dx}\Big|_{x=1}.$$

1. $x=1\text{时}, t=0, y=1$
2. $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$
3. $\dfrac{dx}{dt}=4t+1, \dfrac{dx}{dt}\Big|_{t=0}=1$

   $e^{yt}(y+t\cdot\dfrac{dy}{dt})=\dfrac{dy}{dt}+3$

   $t=0, y=1\text{代入}\dfrac{dy}{dt}\Big|_{t=0}=-2$
4. $\dfrac{dy}{dx}\Big|_{x=1}=\dfrac{dy/dt}{dx/dt}\Big|_{t=0}=-2$

分段函数求导

$$f(x)=\begin{cases}e^{ax}, x<0\\ \ln(1+2x)+b, x\geq 0\end{cases}, f'(0)\exists, \text{求}a, b.$$

$$
\begin{aligned}
&1.\ f(0-0)=1, f(0)=f(0+0)=b \\
&\quad \because f(0-0)=f(0+0)=f(0), \therefore b=1 \\
&2.\ f'_{-}(0)=\lim_{x\to0^-}\frac{f(x)-f(0)}{x}=a \\
&\quad f'_{+}(0)=\lim_{x\to0^+}\frac{f(x)-f(0)}{x}=\lim_{x\to0^+}\frac{\ln(1+2x)}{x}=2 \\
&\quad \because f'_{-}(0)=f'_{+}(0), \therefore a=2
\end{aligned}
$$

$$f(x)=\begin{cases}1, x=0\\ \frac{\sin x}{x}, x\neq 0\end{cases}, \text{求} f'(x)$$

$$x\neq 0\text{时}, f'(x)=\frac{x\cos x-\sin x}{x^2}$$

$$x=0\text{时}, \lim_{x\to0}\frac{f(x)-f(0)}{x}=\lim_{x\to0}\frac{\sin x-x}{x^2}=0\Rightarrow f'(0)=0$$

$$\therefore f'(x)=\begin{cases}0, x=0\\ \frac{x\cos x-\sin x}{x^2}, x\neq 0\end{cases}$$

$$f(x)=\begin{cases}\ln(1+2x), x<0\\ ax^2+bx+c, x\geq 0\end{cases}, f''(0)\exists, \text{求} a, b, c$$

$$
\begin{aligned}
&1.\ f(0-0)=0, f(0)=f(0+0)=c \\
&\quad \because f(x)\text{在}x=0\text{处连续}, \therefore c=0 \\
&2.\ f'(x)=\begin{cases}\frac{2}{1+2x}, x<0\\ 2ax+b, x\geq 0\end{cases} \\
&\quad \because f'(x)\text{在}x=0\text{处连续}, \therefore b=2 \\
&\quad f'(x)\begin{cases}\frac{2}{1+2x}, x<0\\ 2ax+q, x\geq 0\end{cases} \\
&3.\ f''_{-}(0)=\lim_{x\to0^-}\frac{f'(x)-f'(0)}{x}=\lim_{x\to0^-}\frac{\frac{2}{1+x}-2}{x}=-4 \\
&\quad f''_{+}(0)=\lim_{x\to0^+}\frac{f'(x)-f'(0)}{x}=2a \\
&\quad \because f''_{-}(0)=f''_{+}(0), \therefore a=-2
\end{aligned}
$$

高阶导数求导

$$=$$

$$=$$

$$y=f(x)=e^x\sin x, f^{(n)}(x)$$

$$f'(x)=e^x\sin x+e^x\cos x=\sqrt{2}e^x\sin(x+\frac{\pi}{4})$$

$$
\begin{aligned}
f''(x)&=\sqrt{2}[e^x\sin(x+\frac{\pi}{4})+e^x\cos(x+\frac{\pi}{4})] \\
&=(\sqrt{2})^2e^x\cdot\sin(x+\frac{2\pi}{4})
\end{aligned}
$$

$$\therefore f^{(n)}(x)=(\sqrt{2})^n e^x\cdot\sin(x+\frac{n\pi}{4})$$

$$f(x)=\frac{1}{2x+1}, f^{(n)}(0)$$

$$
\begin{aligned}
f(x) &= (2x+1)^{-1} \\
f'(x) &= (-1)(2x+1)^{-2} \cdot 2 \\
f''(x) &= (-1)(-2)(2x+1)^{-3} \cdot 2^2 \\
&\dots \\
f^{(n)}(x) &= (-1)(-2)\dots(-n)(2x+1)^{-(n+1)} \cdot 2^n \\
&= \frac{(-1)^n n! 2^n}{(2x+1)^{n+1}}
\end{aligned}
$$

$$
记:(\frac{1}{ax+b})^{(n)} = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}
$$

$$
f(x) = \ln(2x^2+x-1), 求 f^{(n)}(x)
$$

$$
\begin{aligned}
f(x) &= \ln(x+1)(2x-1) = \ln(x+1) + \ln(2x-1) \\
f'(x) &= \frac{1}{x+1} + 2 \cdot \frac{1}{2x-1} \\
f^{(n)}(x) &= (\frac{1}{x+1})^{n-1} + 2 \cdot (\frac{1}{2x-1})^{n-1} \\
&= \frac{(-1)^{n-1}(n-1)!}{(x+1)^n} + 2 \cdot \frac{(-1)^{n-1} \cdot (n-1)! \cdot 2^{n-1}}{(2x-1)^n}
\end{aligned}
$$