隐函数及参数方程确定的函数的导数

隐函数

$$y=f(x)(x\in D)$$
 $F(x,y)=0(x\in D)$,若 $orall x\in D$,当唯一的 $y(oxdot F(x,y)=0$ 确定)称 $F(x,y)=0$ 确定 y 为 x 的隐函数, $\dfrac{dy}{dx}$? $F(x,y)=0=F(x,\Phi(x))$

设
$$e^{2x^2+y}=x^2+y^2$$
确定 y 是 x 的函数,求 $\frac{dy}{dx}$.
$$e^{2x^2+y}=x^2+y^2\Rightarrow y=\Phi(x)$$

$$\Rightarrow e^{2x^2+y}(4x+\frac{dy}{dx})=2x+2y\frac{dy}{dx}$$

$$\Rightarrow (x^2+y^2)(4x+\frac{dy}{dx})=2x+2y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x(x^2+y^2)-2x}{2y-x^2-y^2}$$

$$\ln \sqrt{x^2+y^2} = \arctan rac{y}{x}, rac{dy}{dx}. \ rac{1}{\sqrt{x^2+y^2}} * rac{2x+2yrac{dy}{dx}}{2\sqrt{x^2+y^2}} = rac{1}{1+rac{y^2}{x^2}} rac{xrac{dy}{dx}-y*1}{x^2} \ x+yrac{dy}{dx} = xrac{dy}{dx}-y \Rightarrow rac{dy}{dx} = rac{x+y}{x-y}$$

$$2^{xy}=3x+y,$$
 求 $y'(0).$ $x=0\Rightarrow y=1$ $2^{xy}\ln 2(y+xrac{dy}{dx})=3+rac{dy}{dx}$ $x=0,y=1$ 代入 $,y'(0)=\ln 2-3$

参数方程确定的函数的导数

一阶

$$egin{aligned} \left\{ egin{aligned} x = \Phi(t) \ y = \phi(t) \end{aligned}, \Phi(t), \phi(t)$$
可导且, $\Phi(t)
eq 0 \end{aligned}
ight. \ y = y(x) \ \lim_{\Delta t o 0} rac{\Delta x}{\Delta t} = \Phi'(t)
eq 0 \Rightarrow \Delta x = O(\Delta t) \ \lim_{\Delta t o 0} rac{\Delta y}{\Delta t} = \phi'(t) \ \lim_{\Delta t o 0} rac{\Delta y}{\Delta t} = \phi'(t) \ \lim_{\Delta t o 0} rac{\Delta y}{\Delta t} = \lim_{\Delta t o 0} rac{\Delta y}{\Delta t} = rac{\phi'(t)}{\Phi'(t)} = rac{dy}{dt} / rac{dx}{dt} \end{aligned}$

$$rac{dy}{dx} = \lim_{\Delta x o 0} rac{\Delta y}{\Delta x} = \lim_{\Delta x o 0} rac{rac{\Delta y}{\Delta t}}{rac{\Delta x}{\Delta t}} = \lim_{\Delta t o 0} rac{rac{\Delta y}{\Delta t}}{rac{\Delta x}{\Delta t}} = rac{\phi'(t)}{\Phi'(t)} = rac{dy}{dt} / rac{dx}{dt}$$

$$egin{aligned} \Phi(t),\phi(t)$$
二阶可导,且 $\Phi(t)
eq 0 \end{aligned} \ rac{dy}{dx} &= rac{\phi'(t)}{\Phi'(t)} \ \Rightarrow rac{d^2y}{dx^2} &= rac{d(rac{dy}{dx})}{dx} = rac{d[rac{\phi'(t)}{\Phi'(t)}]/dt}{dx/dt} \end{aligned}$

设函数
$$y=y(x)$$
由 $\begin{cases} x=\arctan t \\ y=\ln(1+t^2) \end{cases}$ 确定,求 $\frac{d^2y}{dx^2}$.
$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\frac{2t}{1+t^2}}{\frac{1}{1+t^2}}=2t$$

$$\frac{d^2y}{dx^2}=\frac{d\frac{dy}{dx}}{dx}=\frac{d(2t)/dt}{dx/dt}=\frac{2}{\frac{1}{1+t^2}}=2(1+t^2)$$

设函数
$$y = y(x)$$
由
$$\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$$
确定,求
$$\frac{d^2y}{dx^2}.$$
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{1 - \cos t}$$
$$\frac{d^2y}{dx^2} = \frac{d(\frac{\sin t}{1 - \cos t})/dt}{dx/dt} = \frac{(\frac{\sin t}{1 - \cos t})'}{1 - \cos t}$$