洛必达法则

$$\frac{0}{0}, \frac{\infty}{\infty}$$

$$\frac{\infty}{\infty}$$

$$①\ln^a x(a>0)\to+\infty(x\to+\infty)$$
$$x^b(b>0)\to+\infty(x\to+\infty)$$
$$c^x(c>1)\to+\infty(x\to+\infty)$$

$$\lim_{x\to+\infty}\frac{\ln^{80}x}{\sqrt{x}}=0$$

$$\lim_{x\to+\infty}\frac{x^{60}}{2^x}=0$$

$$② \lim_{x\to+\infty}\frac{b_m x^m+\dots}{a_n x^n+\dots}\begin{cases}=0,m<n\\=\infty,m>n\\=\frac{b_m}{a_n},m=n\end{cases}$$

$$\lim_{x\to+\infty}(\frac{x^2-4x+1}{x-1}-x)$$

$$原式=\lim_{x\to+\infty}\frac{5x+1}{x-1}=5$$

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1. 若① $f(x),g(x)$在$x=a$的去心邻域可导且$f'(x)\neq 0$

$$② \lim_{x\to a}f(x)=\lim_{x\to a}g(x)=0$$

$$③ \lim_{x\to a}\frac{g'(x)}{f'(x)}=A\Rightarrow\lim_{x\to a}\frac{g(x)}{f(x)}=A$$

①若 $\lim\frac{g'(x)}{f'(x)}$不存在，不代表 $\lim\frac{g(x)}{f(x)}$不存在

仅仅代表洛氏法则无效

$$g(x)=x^2\sin\frac{1}{x},f(x)=x,x=0,f'(x)=1\neq0,$$

$$\lim_{x\to0}\frac{g'(x)}{f'(x)}=\lim_{x\to0}(2x\sin\frac{1}{x}-\cos\frac{1}{x})不存在$$

$$而\lim_{x\to0}\frac{g(x)}{f(x)}=\lim_{x\to0}x\sin\frac{1}{x}=0$$

求极限 $\lim_{x\to0}\frac{\arcsin x-x}{x^2\ln(1+2x)}$.

$$(1+\Delta)^a-1\sim a\Delta,\Delta\to0$$

$$\begin{aligned}
\text{原式} &= \frac{1}{2}\lim_{x\to 0}\frac{\arcsin x - x}{x^3} \\
&= \frac{1}{2}\lim_{x\to 0}\frac{(1-x^2)^{-\frac{1}{2}}-1}{3x^2} \\
&\because (1-x^2)^{-\frac{1}{2}}-1 \sim (-\frac{1}{2})(-x^2) = \frac{1}{2}x^2 \\
&\therefore \text{原式} = \frac{1}{6} * \frac{1}{2} = \frac{1}{12}
\end{aligned}$$

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2. 若①$f(x), g(x)$在$x=a$的去心邻域可导且$f'(x)\neq 0$

②$f(x)\to\infty, g(x)\to\infty(x\to a)$

③$\lim\limits_{x\to a}\frac{g'(x)}{f'(x)} = A \Rightarrow \lim\limits_{x\to a}\frac{g(x)}{f(x)} = A$

$$f(x)=x, g(x)=2x+\sin x, \lim_{x\to\infty}\frac{g(x)}{f(x)} : \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{g'(x)}{f'(x)} = \lim_{x\to\infty}(2+\cos x)\text{不存在}$$

$$\lim_{x\to\infty}\frac{g(x)}{f(x)} = \lim_{x\to\infty}(2+\frac{1}{x}\sin x) = 2$$

$$\text{求极限} \lim_{x\to\infty}\frac{\ln^2 x}{2x^2+x+3}.$$

$$\text{原式} = \lim_{x\to\infty}\frac{x^2}{2x^2+x+3} * \frac{\ln^2 x}{x^2} = 0$$