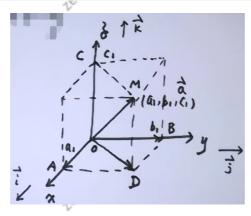
空间解析几何

向量

向量的坐标



$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{OB}$$

$$\overrightarrow{a} = \overrightarrow{OD} + \overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

$$= a_1 \overrightarrow{i} + b_1 \overrightarrow{j} + c_1 \overrightarrow{k}$$

$$\triangleq \{a_1, b_1, c_1\}$$

$$egin{aligned} \ddot{\Xi}A(x_1,y_1,z_1), B(x_2,y_2,z_2) \ \overrightarrow{AB} = \{x_2-x_1,y_2-y_1,z_2-z_1\} \end{aligned}$$

若
$$\vec{a} = \{a_1, b_1, c_1\}, 则$$
① $|\vec{a}| = \sqrt{a_1^2 + b_1^2 + c_1^2}$
② $\vec{a} \circ = \frac{1}{|\vec{a}|} \vec{a} = \{\frac{a_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}, \frac{b_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}, \frac{c_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}\}$

方向角与方向余弦



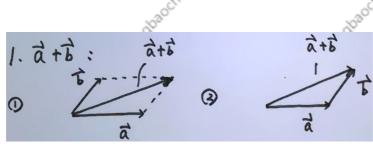
设
$$\vec{a} = \{a_1, b_1, c_1\}$$

$$ec{a}$$
与 x,y,z 轴正方向夹角称为 $ec{a}$ 的方向角, $lpha,eta,\gamma.$ $\coslpha=rac{a_1}{\sqrt{a_1^2+b_1^2+c_1^2}},\coseta=rac{b_1}{\sqrt{a_1^2+b_1^2+c_1^2}},\cos\gamma=rac{c_1}{\sqrt{a_1^2+b_1^2+c_1^2}}$

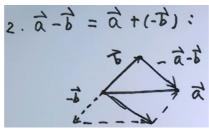
$$2\{\cos\alpha,\cos\beta,\cos\gamma\} = \vec{a}\circ$$

向量运算的刻划

几何刻划



zenajbaciheno



$$3.kec{a}egin{cases} @k>0:kec{a}$$
方向与 $ec{a}$ 同,长为 $ec{a}$ 的 k 倍 $@k=0:kec{a}=ec{0}\ @k<0:kec{a}$ 方向与 $ec{a}$ 反,长为 $ec{a}$ 的 $|ec{k}|$ 倍

endbackerio

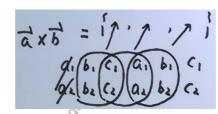
$$egin{aligned} 4.ec{a}\cdotec{b} &= |ec{a}|\cdot|ec{b}|\cdot\cos(ec{a},ec{b}) \ 5.ec{a} imesec{b} &: egin{cases} \dot{eta} &= ec{a}ert \cdot ert ec{b}ert \cdot \sin(ec{a},ec{b}) \ egin{cases} \dot{eta} &= ert ec{a}ert \cdot ert ec{b}ert \cdot \sin(ec{a},ec{b}) \end{aligned}$$



代数刻划

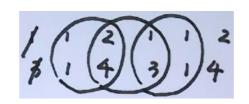
Lengbackens

设
$$\vec{a} = \{a_1, b_1, c_1\}, \vec{b} = \{a_2, b_2, c_2\}$$
 $1.\vec{a} + \vec{b} = \{a_1 + a_2, b_1 + b_2, c_1 + c_2\}$
 $2.\vec{a} - \vec{b} = \{a_1 - a_2, b_1 - b_2, c_1 - c_2\}$
 $3.k\vec{a} = \{ka_1, kb_1, kc_1\}$
 $4.\vec{a} \cdot \vec{b} = \{a_1a_2 + b_1b_2 + c_1c_2\}$
 $5.\vec{a} \times \vec{b} = \{b_1c_2 - b_2c_1, c_1a_2 - c_2a_1, a_1b_2 - a_2b_1\}$



endbacheno

$$ec{a} = \{1,1,2\}, ec{b} = \{3,1,4\} \ ec{a} imes ec{b} = \{2,2,-2\}$$



Lengbaoch.

 $1.ec{a}\cdotec{b}$:

$$@\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$$

$$@\vec{a} \cdot \vec{a} = |\vec{a}|^2$$

$$@\vec{a}\cdot \vec{b} = 0 \Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0 \Leftrightarrow \vec{a}ot \vec{b}$$

 $2.ec{a} imesec{b}$:

$$\bigcirc \vec{a} imes \vec{b} = -\vec{b} imes \vec{a}$$

②
$$\vec{a} imes \vec{b} \perp \vec{a}, \vec{a} imes \vec{b} \perp \vec{b}$$
(法)

$$@ec{a} imesec{b}=ec{0}\Leftrightarrowec{a}\parallelec{b}\Leftrightarrowrac{a_1}{a_2}=rac{b_1}{b_2}=rac{c_1}{c_2}$$

$$|\vec{a}| ec{a} imes ec{b}| = 2 S_{ riangle}$$

andbaochenio





nelbaochenes

 $A(1,-1,0), B(2,0,1), C(1,1,-1), RS_{\triangle ABC}.$

$$\overrightarrow{AB} = \{1,1,1\}, \overrightarrow{AC} = \{0,2,-1\}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \{-3,1,2\}$$

$$|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{14}$$

$$\therefore S_{ riangle ABC} = rac{\sqrt{14}}{2}$$

向量的应用

空间曲面

设 Σ 为空间曲面,F(x,y,z)=0为方程.

若Σ上任一点坐标为F(x,y,z)=0的解

F(x,y,z)=0任一解对应点在 Σ 上

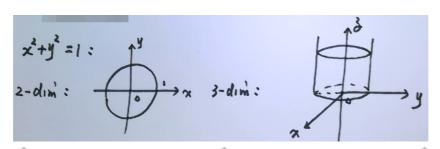
称F(x,y,z)=0为曲面 Σ 的方程, Σ 为F(x,y,z)=0的图

$$x^2 + y^2 = 1$$

enghacher

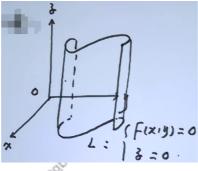
rendbaocheno

engbachens



特殊曲面

柱面,目的功能



 $\Sigma: F(x,y)=0$

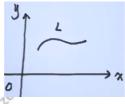
旋转曲面

2-dim

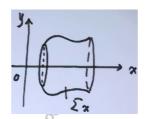
Lengthauthenig

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$$L:egin{cases} f(x,y)=0\ z=0 \end{cases}$$



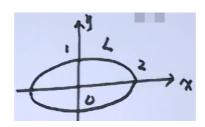
$$egin{aligned} & \mathbb{O}\Sigma_x: f(x,\pm\sqrt{y^2+z^2}) = 0 \ & \mathbb{O}\Sigma_y: f(\pm\sqrt{x^2+z^2},y) = 0 \end{aligned}$$

$$@\Sigma_y: f(\pm\sqrt{x^2+z^2},y)=0$$

Length autherics

Lengbaothens)

 $L: egin{cases} rac{x^2}{4}+y^2=1\ z=0 \end{cases}$



tengbaocheng

zenybaochenis

$$\Sigma_x:rac{x^2}{4}+y^2+z^2=1 \ \Sigma_y:rac{x^2}{4}+y^2+rac{z^2}{4}=1$$

3-dim

平面(退化)

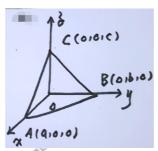
点法式

activens tendpactive



$$M_0(x_0,y_0,z_0)\in\pi, ec{n}=\{A,B,C\}ot\pi \ \pi:A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

截距式



$$\overrightarrow{AB} = \{-a, b, 0\}, \overrightarrow{AC} = \{-a, 0, c\}$$

$$\overrightarrow{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \{bc, ac, ab\}$$

$$\pi : bc(x - a) + ac(y - 0) + ab(z - 0) = 0$$

$$\pi : \frac{x - a}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

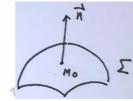
$$\therefore \pi : \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

一般式

$$\pi: Ax + By + Cz + D = 0$$

切平面和法线





$$egin{aligned} \Sigma: F(x,y,z) &= 0, M_0 \in \Sigma \ ec{n} &= \{F_x, F_y, F_z\}_{M_0} \end{aligned}$$

空间曲线

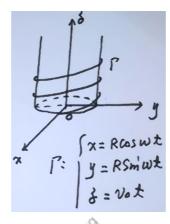
空间曲线的表达形式

1.一般式: 1engbi

$$\Gamma:egin{cases} F(x,y,z)=0\ G(x,y,z)=0 \end{cases}$$

2.参数式:

$$\Gamma: egin{cases} x = \Phi(t) \ y = \phi(t) \ z = \omega(t) \end{cases}$$

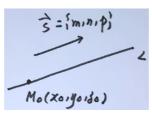


Lengthaochenes

直线(退化)

点向式





$$M_0(x_0,y_0,z_0) \in L, ec{S} = \{m,n,p\} \parallel L \ L: rac{x-x_0}{m} = rac{y-y_0}{n} = rac{z-z_0}{p}$$

参数式

$$L: egin{cases} x=x_0+mt\ y=y_0+nt\ z=z_0+pt \end{cases}$$

-般式

Ø_{De}



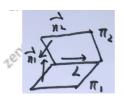
$$L: \begin{cases} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases}$$

tendpaothens)

Q_{De}

$$L: egin{cases} x+y-2z=0 \ 2x-y+z-2=0 \end{cases}$$
 化为点向式.

Q_{De}

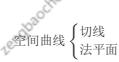


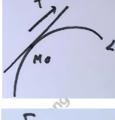
$$1.M_0(1,1,1)\in L$$

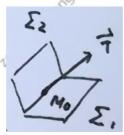
$$2.ec{S} = ec{n_1} imes ec{n_2} = \{1,1,-2\} imes \{2,-1,1\} = \{-1,-5,-3\} \ 3.L: rac{x-1}{1} = rac{y-1}{5} = rac{z-1}{3}$$

$$3.L: \frac{x-1}{1} = \frac{y-1}{5} = \frac{z-1}{3}$$

切线和法平面







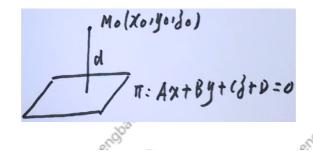
$$egin{aligned} 1.\Gamma : egin{cases} x &= \Phi(t) \ y &= \phi(t) \ , t = t_0
ightarrow M_0(x_0, y_0, z_0) \in \Gamma \ z &= \omega(t) \end{cases} \ ec{T} &= \{\Phi'(t_0), \phi'(t_0), \omega'(t_0)\} \ 2.\Gamma : egin{cases} F(x, y, z) &= 0 \ G(x, y, z) &= 0, M_0(x_0, y_0, z_0) \in \Gamma \end{cases} \ ec{n_1} &= \{F_x, F_y, F_z\}_{M_0}, ec{n_2} &= \{G_x, G_y, G_z\}_{M_0} \ ec{T} &= ec{n_1} imes ec{n_2} \end{cases} \end{aligned}$$

距离

两点之距

$$egin{aligned} A(x_1,y_1,z_1), B(x_2,y_2,z_2) \ d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} \end{aligned}$$

点到平面之距



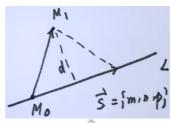
$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

平行平面之距

$$\pi_1: Ax + By + Cz + D_1 = 0$$

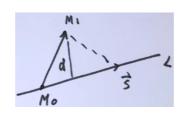
$$\pi_2: Ax + By + Cz + D_2 = 0$$

$$d = \frac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}}$$



$$|\overrightarrow{M_0M_1} imes\overrightarrow{S}|=|\overrightarrow{S}|\cdot d$$
 $\Rightarrow d=rac{|\overrightarrow{M_0M_1} imes\overrightarrow{S}|}{|\overrightarrow{S}|}$

$$M_1(1,2,1), L: egin{cases} x-y-z-2=0 \ 2x+y-z-3=0 \end{cases}$$
,求 M_1 到 L 之距.



$$M_0(1,0,-1)\in L$$

$$ec{S} = \{1, -1, -1\} imes \{2, 1, -1\} = \{2, -1, 3\}$$

$$\overrightarrow{M_0M_1}=\{0,2,2\}$$

$$\overrightarrow{M_0M_1} imes \vec{S} = \{8,4,-4\}, |\overrightarrow{M_0M_1} imes \vec{S}| = 4\sqrt{6} \ |\vec{S}| = \sqrt{14}$$

$$|\vec{S}| = \sqrt{14}$$

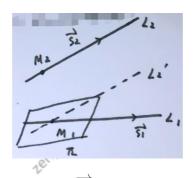
$$\pm |ec{S}| \cdot d = 4\sqrt{6} \Rightarrow d = rac{4\sqrt{6}}{\sqrt{14}}$$

异面直线之距

$$1.L_1, L_2$$
共面 $\Leftrightarrow \overrightarrow{S_1} imes \overrightarrow{S_2} oxdot \overrightarrow{M_1 M_2} \Leftrightarrow (\overrightarrow{S_1} imes \overrightarrow{S_2}) \cdot \overrightarrow{M_1 M_2} = 0$

$$2.L_1, L_2$$
异面 $\Leftrightarrow (\overrightarrow{S_1} imes \overrightarrow{S_2}) \cdot \overrightarrow{M_1 M_2}
eq 0$

$$L_1: \frac{x}{1} = \frac{y-1}{-1} = \frac{z}{0}, L_2: \frac{x-1}{2} = \frac{y}{1} = \frac{z-1}{-1}$$



teriolpaocher.

 $1.M_1(0,1,0) \in L_1, \overrightarrow{S_1} = \{1,-1,0\}$ $M_2(1,0,1) \in L_2, \overrightarrow{S_2} = \{2,1,-1\}$ $\overrightarrow{S_1} \times \overrightarrow{S_2} = \{1,1,3\}, \overrightarrow{M_1M_2} = \{1,-1,1\}$ $\therefore (\overrightarrow{S_1} \times \overrightarrow{S_2}) \cdot \overrightarrow{M_1M_2} = 3 \neq 0, \therefore L_1, L_2$ 异面 $2. ext{过} M_1 ext{作} L_2' \parallel L_2$ L_1, L_2' 成平面 π $\pi: 1 imes (x-0) + 1 imes (y-1) + 3 imes (z-0)$ 即 $\pi: x + y + 3z - 1 = 0$ $3.d = \frac{|1+0+3-1|}{\sqrt{11}} = \frac{3}{\sqrt{11}}$

zengbaachens