行列式(数或式子)

逆序

$$\forall i,j\in N\text{且}i\neq j:\begin{cases}①i<j-(i,j)\text{为顺序}\\②i>j-(i,j)\text{为逆序}\end{cases}$$

$$\begin{aligned}&\tau(312)=2+0=2\\&\tau(2341)=1+1+1=3\\&\tau(35142)=2+3+1=6\end{aligned}$$

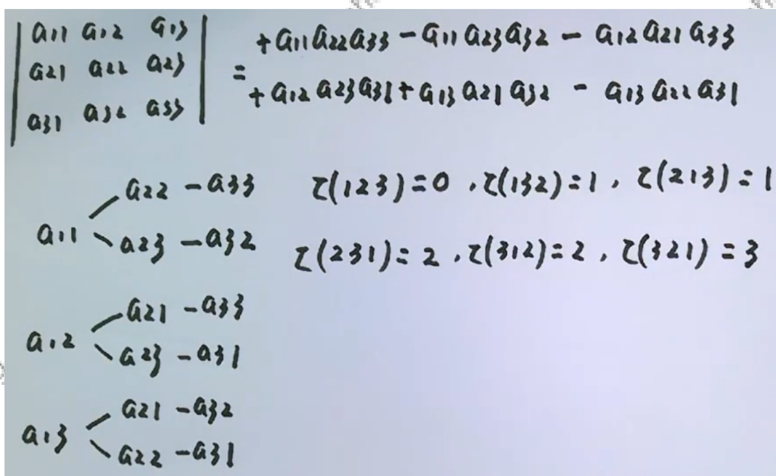
逆序数

$$\begin{aligned}&i_1\ldots i_n\text{为}1,2,\ldots,n\text{的一个排列}\\&i_1\ldots i_n\text{中所含逆序总数称为逆序数, 记}\tau(i_1\ldots i_n)\end{aligned}$$

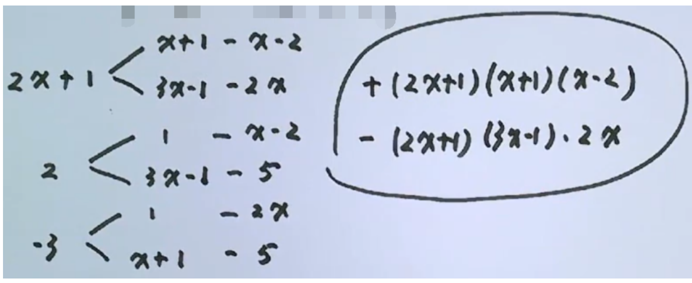
$$\tau(361542)=2+4+2+1=9$$

行列式

$$D\triangleq\begin{vmatrix}a_{11}&a_{12}&\ldots&a_{1n}\\a_{21}&a_{22}&\ldots&a_{2n}\\\ldots\\a_{n1}&a_{n2}&\ldots&a_{nn}\end{vmatrix}$$



$$f(x)=\begin{vmatrix}2x+1&2&-3\\1&x+1&3x-1\\5&2x&x-2\end{vmatrix},x^2\text{系数}$$



余子式与代数余子式

$$D \triangleq \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \\ a_{n1} & \dots & a_{nn} \end{vmatrix}$$

取a_{ij}, D中划去i行j列而成的$n-1$阶行列式.
记$M_{ij}-a_{ij}$的余子式. $A_{ij} \triangleq (-1)^{i+j}M_{ij}-a_{ij}$的代数余子式

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$$\begin{cases} a_{11}=1, M_{11}=11, A_{11}=11 \\ a_{12}=2, M_{12}=5, A_{12}=-5 \\ a_{13}=-1, M_{13}=-1, A_{13}=-7 \end{cases}$$
$$\begin{cases} a_{21}=2, M_{21}=9, A_{21}=-9 \\ a_{22}=3, M_{22}=7, A_{22}=7 \\ a_{23}=1, M_{23}=-5, A_{23}=5 \end{cases}$$
$$\begin{cases} a_{31}=3, M_{31}=5, A_{31}=5 \\ a_{32}=1, M_{32}=3, A_{32}=-3 \\ a_{33}=4, M_{33}=-1, A_{33}=-1 \end{cases}$$

$$1. \begin{cases} 11-10+7=8 \\ -18+21+5=8 \\ 15-3-4=8 \end{cases}$$
$$2. \begin{cases} -9+14-5=0 \\ 5-6+1=0 \\ 22-15-7=0 \\ 10-9-1=0 \end{cases}$$

特殊

对角、上(下)三角行列式

$$\begin{vmatrix} a_{11} & & 0 \\ & \dots & \\ 0 & & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & & 0 \\ & \dots & \\ * & & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & & * \\ & \dots & \\ 0 & & a_{nn} \end{vmatrix} = a_{11}a_{22}\dots a_{nn}$$

范德蒙行列式

$$V_n \triangleq \begin{vmatrix} 1 & 1 & \dots & 1 \\ a_1 & a_2 & \dots & a_n \\ \dots & & & \\ a_1^{n-1} & a_2^{n-1} & \dots & a_n^{n-1} \end{vmatrix} = \Pi_{1\le j<i\le n}(a_i-a_j)$$
$$V_2 = a_2 - a_1$$
$$V_3 = (a_3-a_1)(a_3-a_2)(a_2-a_1)$$
$$V_4 = (a_4-a_1)(a_4-a_2)(a_4-a_3)\cdot(a_3-a_1)(a_3-a_2)\cdot(a_2-a_1)$$
$V_n \neq 0 \Leftrightarrow a_1\dots a_n$两两不等

分块

$$\begin{vmatrix} A & 0 \\ 0 & B \end{vmatrix} = \begin{vmatrix} A & * \\ 0 & B \end{vmatrix} = \begin{vmatrix} A & 0 \\ * & B \end{vmatrix} = |A|\cdot|B|$$

行列式的计算性质

$$\text{如何计算行列式}\begin{cases}\text{①上三角或下三角}\\ \text{②降阶}\end{cases}$$

行列式三角化

$$\begin{aligned}
&1.D^T=D\\
&2.\text{对调两行(或两列)行列式变为相反数}\\
&3.\text{一行(或一列)有公因子可提取}\\
&4.\text{拆：}\\
&\quad\begin{vmatrix}a_1+b_1 & c_1\\ a_2+b_2 & c_2\end{vmatrix}=\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}+\begin{vmatrix}b_1 & c_1\\ b_2 & c_2\end{vmatrix}\\
&5.\text{一行(一列)}k\text{倍加到另一行(另一列),行列式不变}
\end{aligned}$$

$$D=\begin{vmatrix}1 & 2 & -1\\ 2 & 3 & 1\\ 3 & 1 & 4\end{vmatrix}$$

$$D=\begin{vmatrix}1 & 2 & -1\\ 0 & -1 & 3\\ 0 & -5 & 7\end{vmatrix}=\begin{vmatrix}1 & 2 & -1\\ 0 & -1 & 3\\ 0 & 0 & -8\end{vmatrix}=8$$

$$D=\begin{vmatrix}3 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 3\end{vmatrix}$$

$$D=5\begin{vmatrix}1 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 3\end{vmatrix}=5\begin{vmatrix}1 & 1 & 1\\ 0 & 2 & 0\\ 0 & 0 & 2\end{vmatrix}=20$$

$$\text{令}\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=M,\text{求}D=\begin{vmatrix}a_1+b_1 & b_1+c_1 & c_1+a_1\\ a_2+b_2 & b_2+c_2 & c_2+a_2\\ a_3+b_3 & b_3+c_3 & c_3+a_3\end{vmatrix}$$

$$\begin{aligned}
D=&\begin{vmatrix}a_1 & b_1+c_1 & c_1+a_1\\ a_2 & b_2+c_2 & c_2+a_2\\ a_3 & b_3+c_3 & c_3+a_3\end{vmatrix}+\begin{vmatrix}b_1 & b_1+c_1 & c_1+a_1\\ b_2 & b_2+c_2 & c_2+a_2\\ b_3 & b_3+c_3 & c_3+a_3\end{vmatrix}\\
=&\begin{vmatrix}a_1 & b_1+c_1 & c_1\\ a_2 & b_2+c_2 & c_2\\ a_3 & b_3+c_3 & c_3\end{vmatrix}+\begin{vmatrix}b_1 & c_1 & c_1+a_1\\ b_2 & c_2 & c_2+a_2\\ b_3 & c_3 & c_3+a_3\end{vmatrix}\\
=&M+\begin{vmatrix}b_1 & c_1 & a_1\\ b_2 & c_2 & a_2\\ b_3 & c_3 & a_3\end{vmatrix}=2M
\end{aligned}$$

降阶性质

$$\begin{aligned}
&1.a_{i1}A_{i1}+a_{i2}A_{i2}+\ldots+a_{in}A_{in}=|A|(i=1,2,\ldots,n)\\
&\quad a_{1j}A_{1j}+a_{2j}A_{2j}+\ldots+a_{nj}A_{nj}=|A|(j=1,2,\ldots,n)\\
&2.a_{i1}A_{j1}+a_{i2}A_{j2}+\ldots+a_{in}A_{jn}=0(i\neq j)
\end{aligned}$$

$|A| = a_i 1A_{i1} + \ldots + a_{in}A_{in}$
$n-1\text{阶} \Rightarrow n\text{阶}$

Notes:

①. 行列式的一行(一列)中0元素多, 按此行(列)展开

②. $A = \begin{pmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & & \\ a_{n1} & \ldots & a_{nn} \end{pmatrix}$

1.$|A| = \begin{vmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & & \\ a_{n1} & \ldots & a_{nn} \end{vmatrix}$

2.$\forall a_{ij} \Rightarrow M_{ij} \Rightarrow A_{ij}$

3.$A^* = \begin{pmatrix} A_{11} & A_{21} & \ldots & A_{n1} \\ A_{12} & A_{22} & \ldots & A_{n2} \\ \ldots & \ldots & \ldots & \ldots \\ A_{1n} & A_{2n} & \ldots & A_{nn} \end{pmatrix} - A$的伴随矩阵

行列式见A_{ij}或A^*用

$\begin{cases} |A^*| = |A|^{n-1} \\ |A| = a_{i1}A_{i1} + \ldots + a_{in}A_{in} \end{cases}$

$$D = \begin{vmatrix} 1 & a & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & a \\ a & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{aligned} D =& |1 \times A_{11} + a \times A_{12}| = M_{11} - aM_{12} \\ =& 1 - a \begin{vmatrix} 0 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 - a \times a \times A_{12} \\ =& 1 + a^2 M_{12} = 1 - a^4 \end{aligned}$$

$$f(x) = \begin{vmatrix} 2a & -1 & 0 & 0 \\ a^2 & 2a & -1 & 0 \\ 0 & a^2 & 2a & -1 \\ 0 & 0 & a^2 & 2a \end{vmatrix}$$

$$\begin{aligned} D =& 2aA_{11} - A_{12} = 2aM_{11} + M_{12} \\ =& 2a \begin{vmatrix} 2a & -1 & 0 \\ a^2 & 2a & -1 \\ 0 & a^2 & 2a \end{vmatrix} + \begin{vmatrix} a^2 & -1 & 0 \\ 0 & 2a & -1 \\ 0 & a^2 & 2a \end{vmatrix} \\ =& 2a(2aA_{11} - A_{12}) + 5a^4 \\ =& 4a^2 M_{11} + 2aM_{12} + 5a^4 \\ =& 20a^4 + 4a^4 + 5a^4 = 29a^4 \end{aligned}$$