矩阵(表格)

矩阵

1.矩阵 $-$ 形如$A=\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \triangleq (a_{ij})_{m\times n}$

①若$m=n, A-$方阵

②若$\forall a_{ij}=0, A=0$

2.同型矩阵 $-$ $A_{m\times n}, B_{m\times n}$称为同型矩阵

设$A=\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix}, B=\begin{pmatrix} b_{11} & \dots & b_{1n} \\ \dots & & \\ b_{m1} & \dots & b_{mn} \end{pmatrix}$

若$\forall a_{ij}=b_{ij}, A=B$

3.三则运算：

① $\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \pm \begin{pmatrix} b_{11} & \dots & b_{1n} \\ \dots & & \\ b_{m1} & \dots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11}\pm b_{11} & \dots & a_{1n}\pm b_{1n} \\ \dots & & \\ a_{m1}\pm b_{m1} & \dots & a_{mn}\pm b_{mn} \end{pmatrix}$

②$k\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix} = \begin{pmatrix} ka_{11} & \dots & ka_{1n} \\ \dots & & \\ ka_{m1} & \dots & ka_{mn} \end{pmatrix}$

③$A_{m\times n}=\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \\ a_{m1} & \dots & a_{mn} \end{pmatrix}, B_{n\times s}=\begin{pmatrix} b_{11} & \dots & b_{1s} \\ \dots & & \\ b_{n1} & \dots & b_{ns} \end{pmatrix}$

$A_{m\times n}B_{n\times s}$内标同可乘，外标确定型

$AB=C_{m\times s}=\begin{pmatrix} c_{11} & \dots & c_{1s} \\ \dots & & \\ c_{m1} & \dots & c_{ms} \end{pmatrix}$

$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+\dots+a_{in}b_{nj}$

Notes:

①$A \neq 0, B \neq 0 \nRightarrow AB \neq 0$

$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq 0, B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \neq 0$

$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

②$A \neq 0 \nRightarrow A^k \neq 0$

$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0, A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

③AB与BA不一定等

④$f(x) = a_n x^n + \ldots + a_1 x + a_0$

给定$A_{n\times n}$：

$f(A) \triangleq a_n A^n + \ldots + a_1 A + a_0 E - A$的矩阵多项式

如：$f(x) = x^2 - x - 6 = (x+2)(x-3)$

$f(A) = A^2 - A - 6E = (A+2E)(A-3E)$

又如：$f(x) = x^2 + x + 2 = (x-1)(x+2) + 4$

$f(A) = A^2 + A + 2E = (A-E)(A+2E) + 4E$

⑤线性方程组一般形式：

$$\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = 0 \end{cases} \quad (*)$$

$$\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases} \quad (**)$$

$$A = \begin{pmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & & \\ a_{m1} & \ldots & a_{mn} \end{pmatrix}_{m\times n}, X = \begin{pmatrix} x_1 \\ \ldots \\ x_n \end{pmatrix}_{n\times 1}, b = \begin{pmatrix} b_1 \\ \ldots \\ b_m \end{pmatrix}_{m\times 1}$$

$$AX = \begin{pmatrix} a_{11}x_1 + \ldots + a_{1n}x_n \\ a_{21}x_1 + \ldots + a_{2n}x_n \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n \end{pmatrix}_{m\times 1}$$

$$AX = 0 \Leftrightarrow \begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 0 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = 0 \end{cases} \quad (*)$$

$$AX = b \Leftrightarrow \begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{cases} \quad (**)$$

$\therefore$ 线性方程组的矩阵形式：

$AX = 0(*)$

$AX = b(**)$

⑥$A_{m\times n}, B_{n\times s} = (\beta_1, \beta_2, \ldots, \beta_s)$

$AB = A(\beta_1, \beta_2, \ldots, \beta_s) = (A\beta_1, A\beta_2, \ldots, A\beta_s)$

更一般地, $A(B, C) = (AB, AC)$

⑦左列化, 右具体化

如：$A = (\alpha_1, \alpha_2), B = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$

$AB = (2\alpha_1 + \alpha_2, \alpha_1 - 5\alpha_2)$

记：设$\alpha_1, \alpha_2, \alpha_3$为列向量$(\alpha_1, \alpha_2, \alpha_3) = A$

若$B = (2\alpha_1 - \alpha_2, \alpha_1 + 3\alpha_2 - \alpha_3, 2\alpha_2 + \alpha_3) = A\begin{pmatrix} 2 & 1 & 0 \\ -1 & 3 & 2 \\ 0 & -1 & 1 \end{pmatrix}$

伴随矩阵

$$A_{n\times n}=\begin{pmatrix}a_{11} & \dots & a_{1n}\\ \dots & & \\ a_{n1} & \dots & a_{nn}\end{pmatrix}$$

$$1.|A|=\begin{vmatrix}a_{11} & \dots & a_{1n}\\ \dots & & \\ a_{n1} & \dots & a_{nn}\end{vmatrix}$$

$$2.\forall a_{ij}\Rightarrow M_{ij}(n-1\text{阶})\Rightarrow A_{ij}$$

$$3.A^*=\begin{pmatrix}A_{11} & A_{21} & \dots & A_{n1}\\ A_{12} & A_{22} & \dots & A_{n2}\\ \dots & & & \\ A_{1n} & A_{2n} & \dots & A_{nn}\end{pmatrix}-A\text{的伴随矩阵}$$

$$AA^*=\begin{pmatrix}|A| & 0 & \dots & 0\\ 0 & |A| & \dots & 0\\ \dots & & & \\ 0 & 0 & \dots & |A|\end{pmatrix}=|A|E$$

$$\text{Notes:}AA^*=A^*A=|A|E$$

$Q.$ 矩阵的研究对象?

1.背景：

初一：$ax=b$

①$a\neq 0:\dfrac{1}{a}\times a=1$

$ax=b\Rightarrow \dfrac{1}{a}\times ax=\dfrac{1}{a}\times b\Rightarrow x=\dfrac{b}{a}$

②$a=0:\begin{cases}b\neq 0-\text{无解}\\ b=0-\text{无数个解}\end{cases}$

2.Similar problem:

$AX=b,X=?$

①$A_{n\times n}:\exists B_{n\times n},$ 使 $BA=E-$ 逆阵理论

$AX=b\Rightarrow BAX=Bb\Rightarrow X=Bb$

②$\begin{cases}A_{n\times n}\text{不可逆}\\ A_{m\times n}\text{且}m\neq n\end{cases}-$ 秩理论