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Monetary Policy, Overlapping Generations, and Patterns of Growth

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This paper investigates the role of monetary policy in economic growth. Using an infinitely lived overlapping-generations model with a simple convex technology that can yield endogenous growth, we show that money supply behaviour of the government may have significant effects on the long-run economic growth. In addition to the effect on the long-term growth rate of the economy, the policy may determine whether the economy stays in the exogenous growth process restricted by the growth rate of labour supply, or realizes the endogenous growth that sustains continuous growth of *per capita* income and consumption.

Introduction

Beginning with the seminal work of Tobin (1965) and Sidrauski (1967), the effect of inflation on capital accumulation has been one of the central topics in macroeconomics. Using an *ad hoc* model, Tobin shows that a rise in the rate of inflation deepens capital formation, whereas Sidrauski presents an optimizing model in which money is superneutral; that is, inflation has no effect on capital formation in the long run. These studies have been extended by a number of studies such as Dornbusch and Frenkel (1973) and Wang and Yip (1992). However, the concern of these studies is restricted to the analysis of the *level* effect of inflation on capital accumulation. Recent developments in endogenous growth theory present a useful analytical framework for re-examining the effect of inflation on capital accumulation and economic growth.

In contrast to the strong emphasis on the importance of fiscal policy for long-run economic growth seen in Mino (1989), Barro (1990) and King and Rebelo (1990), the role of monetary policy has been mostly ignored in the endogenous growth literature. Recalling that the effect of money growth on capital formation has been the central issue in money and growth literature, it is rather curious that recent studies on endogenous economic growth focus exclusively on the real side of the economy. There is, however, a small number of authors investigating the role of money in endogenously growing economies. Marquis and Reffet (1991) and Mino (1991) introduce money into two-sector models involving human capital accumulation via a cash-in-advance constraint. They conclude that an increase in the rate of nominal money supply (generally) depresses the long-term economic growth, as long as the cash-in-advance constraint applies to investment demand for either physical or human capital.[1] Wang and Yip (1991) extend the Uzawa–Lucas model of endogenous growth by assuming that households allocate their available time between production, transaction and human capital formation. The transaction time is assumed to be a decreasing function of real-money balances, and therefore a reduction in real balances arising from an increase in the monetary expansion rate increases the time needed for transactions, which lowers the long-term growth rate by discouraging human capital accumulation. In a similar way, De Gregorio (1992) constructs a model with endogenous labour supply and transaction costs that depend on real-money balances, and finds a negative effect of inflation on the long-run growth rate. Gomme (1993) studies a cash-in-advance model of endogenous growth with flexible labour supply and evaluates welfare loss caused by inflation taxes by calibrating the model. Jones and Manuelli (1993) also construct two monetary models of endogenous growth, and calculate the steady-state growth and welfare effects of changes in the rate of monetary expansion.

There is, however, one important route, neglected by these studies, through which monetary policy would affect capital accumulation and economic growth. As Weil (1987, 1991) emphasizes, if there are heterogenous agents in an economy, changes in government monetary policy have redistributional effects, and will affect capital accumulation. The studies mentioned above employ representative agent models, and thereby the redistributional effects of government policy are excluded *a priori*. In this paper we focus on these redistributional effects of inflation and monetary policy. Specifically, we extend Weil's infinitely lived overlapping-generations model, in which there are many heterogeneous generations, to a model with endogenous growth, and examine the effect of monetary policy on economic growth through the intergenerational redistributions.[2] Money is introduced based on the standard money-in-the-utility-function (MIUF) approach.[3] In order to construct a tractable framework for discussing the effects of an increase in the rate of money supply in an endogenously growing economy, we employ the simple production technology analysed by Jones and Manuelli (1990). This production function is a mixture of the standard, neoclassical concave function and the so-called 'AK' technology.

By using an infinitely lived representative agents model, Jones and Manuelli (1990) prove that endogenous growth is generally possible in this setting. In contrast, Jones and Manuelli (1992) show that, if we replace the representative agent model with an overlapping-generations model, the asymptotic growth rate must then be zero even under this convex technology. They also show, however, that some intergenerational redistribution policy allows this economy to grow for ever. In our model, as we stress above, monetary policy has intergenerational redistributional effects, and thus it makes the sustainable growth of the economy possible. If monetary policy fails to sustain enough capital accumulation, the economy may converge to a steady state where the rate of economic growth is determined by an exogenously given rate of population growth. By contrast, if the policy successfully stimulates capital accumulation, the endogenous growth that is realized under the 'AK' technology can be attained. In this regime, as the rate of money supply increases, the growth rate of the economy rises.

These results demonstrate the central message of this paper: not only fiscal policy but also monetary policy of the government may play a prominent role in the process of long-run economic growth. Although the endogenous growth model developed below is quite simple, we will show that it can serve satisfactorily to convey our message.

The organization of this paper is as follows. The analytical framework is presented in Section I. Section II derives the basic equations that characterize the dynamic behaviour of the economy. In Section III the existence and stability of steady-state equilibria are examined. The patterns of growth under a convex technology and the effect of monetary policy on these patterns are also analysed. In Section IV the implications of financial development for growth are briefly discussed. Section V concludes with some remarks on the implications of our results for empirical research.

I. The Model

Production

We consider a competitive economy in which there are many identical firms. Each firm has the same production function, such that

$$(1)\qquad Y(t)=\psi K(t)^{\beta}N(t)^{1-\beta}+\mathrm{A}K(t),\qquad 0<\beta<1,\qquad \psi\geq 0,\qquad A\geq 0,$$

where $Y(t)$ is output of final goods, $K(t)$ is the amount of capital employed and $N(t)$ is the amount of labour employed. For simplicity, the number of firms is normalized to unity, and hence (1) represents the aggregate production function as well.

The production technology adopted here is a mixture of the traditional neoclassical environment and the newly developed endogenous-growth approach: if $A=0$, the production function takes the usual Cobb–Douglas form with constant returns to scale; and if $\psi=0$, the production function reduces to the 'AK' technology that has been employed by a number of endogenous-growth models (e.g. Romer 1986; Rebelo 1991; Barro 1990). If the total labour force is fully employed and if there is no exogenous technological change, then, in the case where $A=0$ and $\psi>0$, the long-term growth rate is determined by the growth rate of the labour supply. In contrast, if $\psi=0$ and $A>0$, the steady-growth rate may be determined endogenously and the amount of available labour supply will not restrict the long-run growth process. In the following analysis, we assume $\psi=1$ for simplicity.

Let us denote the competitive rate of return on capital and the real wage rate by $r(t)$ and $w(t)$, respectively. Perfect competition among firms yields

$$(2)\qquad r(t)=\partial Y(t)/\partial K(t)=\beta Y(t)/K(t)+(1-\beta)A,$$

$$(3)\qquad w(t)=\partial Y(t)/\partial N(t)=(1-\beta)[Y(t)/K(t)-A]^{-\beta/(1-\beta)}.$$

Household behaviour

As for the formulation of household behaviour, we use the infinitely lived overlapping-generations model developed by Weil (1987, 1989, 1991).[4] In this model each generation lives for ever but is disconnected from others, because it is assumed that a newly born generation, which is born at the exogenously given rate n, does not receive any share of existing wealth.

The dynamic budget equation at time t of a cohort born at time s is

$$(4)\qquad \mathrm{d}v(s,t)/\mathrm{d}t=r(t)v(s,t)-[r(t)+\pi(t)]m(s,t)+w(t)-c(s,t)+\tau(t),$$

where v, r, c, m, π, w and τ are total non-human wealth, the rate of interest, consumption, real-money balances, the rate of inflation, age-independent wage income and age-independent real per capita transfers, respectively.

The representative household in a cohort born at period s maximizes[5]

(5) $$\int_s^{\infty} \{a \ln [c(s, t)] + (1-\alpha) \ln [m(s, t)]\} \mathrm{e}^{-\rho(t-s)} \mathrm{d}t,$$

subject to (4) and

$$\lim_{t\to\infty} v(s, t) \exp\left(-\int_s^t r(\xi)\, \mathrm{d}\xi\right) = 0.$$

In (5), ρ is the subjective discount rate, which is assumed to be smaller than A.[6] The resulting consumption and money holdings functions are

(6) $$c(s, t) = \alpha\rho[v(s, t) + h(t)],$$

(7) $$m(s, t) = (1-\alpha)c(s, t)/\alpha[r(t) + \pi(t)],$$

where, $h(t)$ is *per capita* human wealth:

(8) $$h(t) = \int_t^{\infty} [w(u) + \tau(u)] \exp\left(-\int_t^u r(\xi)\, \mathrm{d}\xi\right) \mathrm{d}u.$$

Note that $w(t)$, $\tau(t)$ and $r(t)$ are assumed to be common for every cohort, and hence *per capita* human wealth is age-independent.

Defining the aggregate variable for $x(s, t)$ as[7]

$$X(t) = x(0, t) + n\int_0^t x(s, t)\, \mathrm{e}^{ns}\, \mathrm{d}s,$$

we obtain the aggregate consumption function, real-money holdings, the budget equation and the human wealth acquisition function:[8]

(9) $$C(t) = \alpha\rho[V(t) + H(t)],$$

(10) $$M(t) = (1/\alpha - 1)C(t)/[r(t) + \pi(t)],$$

(11) $$\dot{V}(t) = r(t)V(t) + [w(t) + \tau(t)]N(t) - C(t)/\alpha,$$

(12) $$\dot{H}(t) = [r(t) + n]H(t) - [w(t) + \tau(t)]N(t).$$

Differentiating (9), and using (11) and (12), gives the aggregate Euler equation:

(13) $$\dot{C}(t) = [r(t) + n - \rho]C(t) - \alpha n\rho V(t).$$

Market equilibrium

We assume that the government keeps the money growth rate at a constant rate μ, and distributes seigniorage equally among all cohorts at each point in time; that is, newly created money is distributed to each household as a transfer payment in a lump-sum manner. Note that this policy redistributes wealth among different generations since inflation taxes, πm, are proportional to m, which differs among generations, whereas lump-sum transfers are generation-independent. The flow budget constraint of the government is given by

(14) $$\tau N = \mu M.$$

The financial market equilibrium condition is

$$V(t)=M(t)+K(t). \tag{15}$$

Finally, combining (10), (11), (14), (15) and $Y(t)=r(t)K(t)+w(t)N(t)$, we find the equilibrium conditions for final goods:

$$Y(t)=C(t)+\dot{K}(t). \tag{16}$$

II. The Aggregated Dynamic System

In this section we derive a set of dynamic equations that describe the behaviour of state variables. For this purpose, it is convenient to define the following variables:

$$Y(t)/K(t)\equiv y(t), \qquad C(t)/K(t)\equiv c(t),$$
$$M(t)/K(t)\equiv m(t), \qquad H(t)/K(t)\equiv h(t),$$

since on the balanced growth path Y, C, M, H and K grow at the same rate. In the following, we drop t from each variable for simplicity. It should be noted that in what follows c and m are different from $c(s, t)$ and $m(s, t)$ in the previous section.

Using the variables given above, we can rewrite the production function (1) as

$$N/K=(y-A)^{1/(1-\beta)}. \tag{17}$$

Therefore, taking time derivatives of both sides of (17), we obtain

$$\dot{y}=(1-\beta)(y-A)(n-\dot{K}/K). \tag{18}$$

From (16), the rate of capital accumulation is given by

$$\dot{K}/K=y-c, \tag{19}$$

and therefore y follows

$$\dot{y}=(1-\beta)(y-A)(n-y+c). \tag{20}$$

From (3), (12), (14) and (19), human wealth acquisition dynamics are represented as

$$\dot{h}=(r+n+c-y)h-(1-\beta)(y-A)-\mu m. \tag{21}$$

From (2), the real rate of return on capital can be expressed as $r=\beta y+(1-\beta)A$. Thus, (21) can be written as

$$\dot{h}=[n+c-(1-\beta)(y-A)]h-(1-\beta)(y-A)-\mu m. \tag{22}$$

In view of (13) and (19), the motion of c $(=C/K)$ is described by the following equation:

$$\dot{c}=[n-(1-\beta)(y-A)-\rho]c+c^2-\alpha n\rho(1+m). \tag{23}$$

On the other hand, real-money balances change according to $\dot{M}/M=\mu-\pi$, by definition. From (10), $\pi=(1/\alpha-1)(c/m)-r$; hence m $(=M/K)$ evolves according to

(24) $\dot{m}=[\mu+c-(1-\beta)(y-A)]m-(1/\alpha-1)c.$

Equations (20), (23) and (24) give a set of dynamic equations with respect to y, m and c. Human wealth, h, is not involved in (20), (23) and (24); therefore these equations constitute a complete dynamic system.

III. Patterns of Growth in a Monetary Economy and the Role of Monetary Policy

In this section we examine the existence and stability of steady-state equilibria. By utilizing these results, we clarify the patterns of growth of the monetary overlapping-generations economy.

The existence of the steady-state equilibria

Let us first characterize the steady-state equilibria of the economy. As (20) indicates, there are two steady-state conditions for y: $y=A$ and $y=c+n$. Figure 1 depicts the combinations of y and c that satisfy $\dot{y}=0$. The intersection of two loci is given by $(A-n, A)$. In this figure we assume that $A>n$; that is, the economy is dynamically efficient. We maintain this assumption throughout this paper. Later we will discuss the implication of this assumption. Note that y cannot be smaller than A under our production technology (1). Thus, only the solid lines in Figure 1 constitute the set of possible steady-state equilibria.

First, we show the condition under which the exogenously growing steady state exists.

Proposition 1. Suppose that $(A-n)(A-\rho)>\alpha n\rho$ and $n+\rho>A$. Then, for any values of the money growth rate within the range $[\underline{\mu}, \hat{\mu}]$, where $\underline{\mu}=-(1-\beta)(A-n)$ and $\hat{\mu}=(A-n)[\rho n-(A-n)(A-\rho)]/[(A-n)(A-\rho)-\alpha n\rho]$ (>0), there exists a unique exogenously growing steady state if the absolute value of negative monetary transfers is not too large.

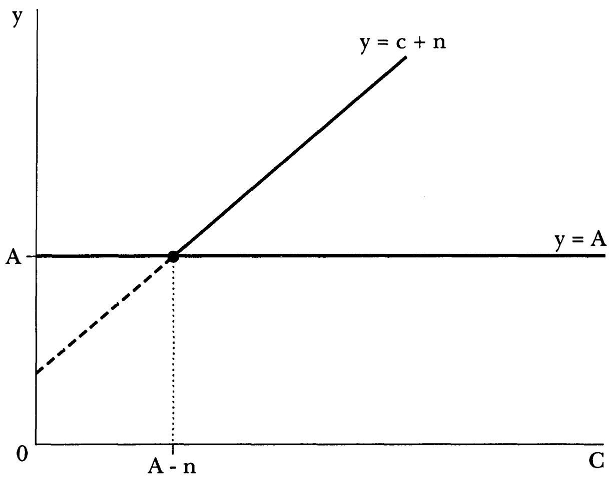

FIGURE 1

Proof. In the steady state in which $y=n+c$, the following conditions are satisfied:

$$(25) \qquad y=n+c,$$

$$(26) \qquad c^2+[n-\rho-(1-\beta)(y-A)]c-\alpha\rho n(1+m)=0,$$

$$(27) \qquad [\mu+c-(1-\beta)(y-A)]m-(1/\alpha-1)c=0.$$

Let us denote the steady-state values of y, c, m and h by (y^*, c^*, m^*, h^*). In this steady state, Y, C, K, H and M grow at the exogenously given population growth rate n, as can easily be seen from (19) and (25). Eliminating m and y from the above equations, we obtain

$$(28) \qquad \beta(n+c)+(1-\beta)A=\rho+\frac{\alpha\rho n}{c}+\frac{\alpha\rho n(1/\alpha-1)}{\mu+\beta c+(1-\beta)(A-n)}.$$

Assuming that $\mu>-(1-\beta)(A-n)\equiv\underline{\mu}$, we can show that this equation has a unique, positive solution for c, and that the value of c increases as μ decreases (see Property 1 in Appendix, Section (a)). Remember now that y cannot be smaller than A under our production technology and therefore the following constraint must be satisfied: $c^*\geq A-n$ (see Figure 1). As Section (a) of the Appendix demonstrates, under the condition $(A-n)(A-\rho)-\alpha\rho n>0$, there exists a value of c^* that satisfies this constraint if $\mu\in[\underline{\mu}, \hat{\mu}]$. Given this c^*, from (25) and (27) y^* and m^* with positive values are also uniquely determined. That is, if $\mu\in[\underline{\mu}, \hat{\mu}]$, then $y^*=c^*+n>A$. Since real wage is given by (3), in the exogenously growing steady state wage is strictly positive, and therefore human wealth, h^*, is also positive if the absolute value of the negative transfers is not too large. Hence if $\mu\in[\underline{\mu}, \hat{\mu}]$, then there exists a unique steady-growth equilibrium in which the long-run growth rate is exogenously specified by the population growth rate. □

Next, consider the other steady state in which $y=A$. When $y=A$, the production technology reduces essentially to the 'AK' technology, and therefore the endogenous growth is realized. We prove the following proposition.

Proposition 2. There exists a unique endogenously growing steady state if and only if the money-growth rate is positive.

Proof. First suppose that μ is non-positive and that an endogenously growing steady state exists. Denote the steady-state values by $(y^{**}, c^{**}, m^{**}, h^{**})$. In the endogenously growing steady state, since

$$\lim_{(N/K)\to 0}(wN/K)=\lim_{(N/K)\to 0}[(1-\beta)\phi(N/K)^{1-\beta}]=0$$

wage in terms of capital reduces to zero. From this and (21), we get

$$h^{**}=\mu m^{**}/(c^{**}+n).$$

Hence μm^{**} must be positive. If μm^{**} is negative, then human wealth takes a negative value. Since a newly born generation is endowed with no financial wealth, its consumption at the birth date, t, is: $c(t, t)=\alpha\rho h(t, t)$. Hence there exists no meaningful equilibrium if μm^{**} is negative. Since the sign of m^{**} is definitely positive, this implies that in an endogenously growing steady state the money growth rate must be positive.

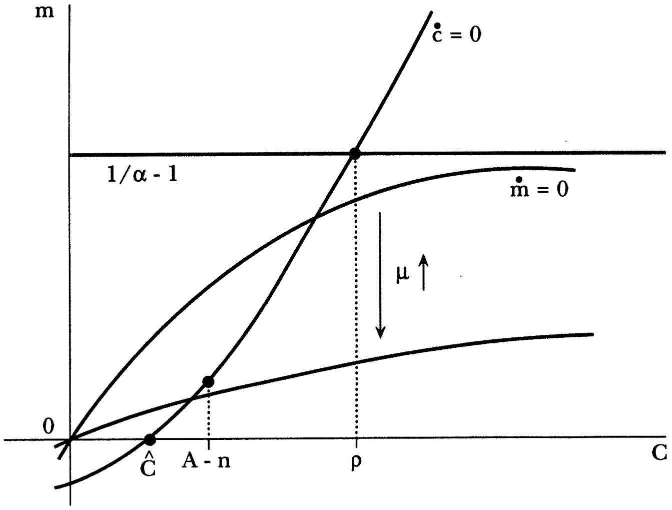

FIGURE 2

Next, suppose that μ is positive. In order to prove the existence of the endogenously growing steady state, it is sufficient to show that there exist positive values of m^{**} and c^{**} since h^{**} is proportional to μm^{**} and since monetary transfers are equally distributed among different generations. Substituting $y=A$ into (23) and (24) and setting $\dot{c}=\dot{m}=0$, we obtain the conditions of the endogenously growing steady state:

$$c^2+(n-\rho)c-\alpha\rho n(1+m)=0, \tag{29}$$

$$(\mu+c)m=(1/\alpha-1)c. \tag{30}$$

Figure 2 depicts (29) and (30). From this figure it is easily verified that there exists a unique set of steady-growth levels of y^{**} $(=A)$, c^{**} and m^{**} (thus positive h^{**}). Thus, if μ is positive, there exists a unique endogenously growing steady state. □

If $\mu<0$ the endogenous growth regime cannot exist, since the wage rate falls to zero as y approaches A and the value of transfers is negative. On the other hand, when $\mu>0$ the monetary transfers assure purchasing power of newly born agents even if wage declines towards zero; therefore an endogenously growing state can exist.

The stability of the steady-state equilibria

Let us next investigate the stability of these two steady states. Section (d) of the Appendix reveals that the exogenously growing steady state, (y^*, c^*, m^*), satisfies the saddle-point stability; i.e., there is a unique, stable perfect-foresight path at least around (y^*, c^*, m^*). On the other hand, Section (e) of the Appendix demonstrates that the dynamic system that linearizes (20), (23) and (24) around (y^{**}, c^{**}, m^{**}) has a unique, stable perfect-foresight equilibrium path that converges to the endogenously growing steady state if and only if

$c^{**} < A - n$; otherwise, (y^{**}, c^{**}, m^{**}) is totally unstable and hence the endogenous steady-growth equilibrium is not generally realized. These arguments can be summarized as follows.

Proposition 3. Suppose that an exogenously growing steady state exists. Then the exogenously growing steady state is saddle-point stable.

Proposition 1. Suppose that an endogenously growing steady state exists. Then the endogenously steady state is saddle-point stable if and only if $c^{**} < A - n$.

Patterns of growth and the effects of monetary policy

Since we have verified that both endogenous and exogenous steady-growth equilibria satisfy saddle-point stability, the next question is: Which steady state will be realized under a given level of money growth? This is the central question of this paper. The answer is given by the following proposition.

Proposition 5. Suppose that the money-growth rate is larger than $\underline{\mu}$. If the money-growth rate is smaller than $\hat{\mu}$, then exogenous-growth equilibrium is realized. If the money-growth rate is greater than $\hat{\mu}$, then endogenous-growth equilibrium is realized.

Proof. From Propositions 1 and 2, it follows that when the money-growth rate is within the range $[\underline{\mu}, 0]$ there exists only an exogenously growing steady state; and when the money-growth rate is within the range $[0, \hat{\mu}]$ two steady states exist, one an endogenous growth equilibrium and the other an exogenous growth equilibrium; and that if $\mu > \hat{\mu}$ there is only an endogenously growing steady state. As properties 1, 2 and 3 show, the steady-state value of c in each regime is a monotonically decreasing continuous function of μ and takes the same value, $A - n$, when $\mu = \hat{\mu}$. Furthermore, when $\mu > \hat{\mu}$, c^{**} becomes smaller than $A - n$. From these observations and Proposition 3, when $\mu \in [\underline{\mu}, \hat{\mu}]$ the exogenously growing steady state is realized. If the money growth rate is within the range $[0, \hat{\mu}]$, the endogenously growing steady state is unstable from Proposition 4, and hence only the exogenously growing steady state is realized. Finally, if μ is greater than $\hat{\mu}$, then, from Property 3, Proposition 2 and Proposition 3, it follows that the endogenously growing steady state is realized. □

In sum, other things being equal, whether an endogenously or exogenously growing steady state is realized depends upon the magnitude of the money growth rate. If the money growth rate exceeds $\hat{\mu}$, then the endogenously growing steady state is the long-run equilibrium. Conversely, if μ is smaller than $\hat{\mu}$, the endogenous steady-growth path is unstable and exogenous steady growth may be realized. As a result, a strategy to get out of the 'low-growth trap', where *per capita* income does not grow, is to set the monetary expansion rate at a level that is higher than $\hat{\mu}$.

Let us here investigate the effect of monetary expansion on the rate of growth in the endogenously growing steady state. From (19), the growth rate in the endogenously growing steady state, g^{**}, is given by

$$g^{**} = A - c^{**}.$$

Since a rise in μ makes the steady-state value of c^{**} lower, as Property 2 in the Appendix (Section (b)) shows, the rise produces a higher growth rate of the economy. Intuitively speaking, a permanent rise in the growth rate of the money supply depresses real-money balances, which would decrease consumption demand through a negative wealth effect. In addition, a higher rate of inflation caused by the rise in the money-growth rate reduces the rate of return on money holdings, whereas the rate of return on capital stays at a constant level, A. Hence asset demand shifts from money to real capital owing to the higher rate of inflation, and thereby the growth rate of the economy is raised through this stimulated capital accumulation. Thus we make the following proposition.

Proposition 6. In the endogenously growing steady state, a rise in the money growth rate increases the growth rate of the economy.

This positive effect of money growth on the long-run economic growth rate is essentially the same as the Tobin effect obtained by Weil (1991). Since Weil does not consider endogenous growth, the Tobin effect in his model is the *level* effect, while in our model it is the *rate* effect.

Finally, we return to the assumption of dynamic efficiency of the economy. We have assumed $A-n>0$ throughout this paper. Conversely, if we assume that $A-n$ is negative, it can easily be seen from Figure 1 that there exists no endogenously growing steady state. Thus, the validity of the assumption is crucial in our analysis. However, in many industrialized countries the population growth rates are sufficiently small, and hence it seems plausible to assume the dynamic efficiency of the economy.[9]

IV. Implications for Financial Development

The money-in-the-utility-function formulation adopted in this paper is simply a device for avoiding complex moulding of a monetary economy in a dynamic setting. This formulation is usually defended on the basis of the functional equivalence between transaction-cost models for money demand and the money-in-the-utility-function approach (e.g. Feenstra 1986). According to this view, financial development, which lowers the necessity of holding cash to conduct transactions, can be represented by a reduction of the weight of utility derived from money holdings. For example, let us redefine the instantaneous utility function of the household as $u(c, m) = \alpha \ln c + \gamma \ln m$ $(\alpha, \gamma > 0)$. Then, a smaller γ indicates a high degree of financial development. It is easy to anticipate that the steady-state effect of a change in γ on c^{**}, i.e. $\mathrm{d}c^{**}/\mathrm{d}\gamma$, is positive. Thus, a reduction in γ decreases c^{**} and puts the economy into the endogenous growth region or raises the long-run growth rate of the economy, g. That is, financial development reduces money holdings relative to physical capital holdings, and hence it accelerates capital accumulation in the long run. Accordingly, as suggested by Roubini and Sala-i-Martin (1992), if the government tries to raise γ by financial repression in order to promote inflation-tax financing, it would depress long-run growth and raise the steady-state rate of inflation.

V. Concluding Remarks

Using an infintely lived overlapping-generations model, we have investigated the effects of monetary policy on long-run economic growth. We have shown that changing the money supply rate may produce significant effects on the patterns of growth as well as on the rate of long-term growth of the economy.

The analysis of this paper hinges on several restrictive assumptions. Besides the obvious restrictions on the forms of utility and production functions, the presumptions that are relevant for our argument are the modelling of endogenous growth and the money-in-the-utility-function formulation. As for the endogenous growth modelling, using a more sophisticated (and more complex) model would provide similar conclusions. The simple convex model of endogenous growth in this paper has satisfactorily served our purpose.

Putting money into the utility function is more critical for our argument. As mentioned in the Introduction, if we use alternative formulations of money and growth (for example, the cash-in-advance constraint model), the positive relation between the money growth rate and the economic growth rate obtained in this paper would not hold. The relation between the long-run growth rate and the rate of inflation has been a controversial empirical issue. For example, based on cross-country regressions and some case studies, Fischer (1991) claims that inflation and government deficits produce a negative impact on growth. He suggests that short-run macroeconomic performances represented by the rate of inflation and budget deficits are relevant for the long-run growth and development. Fischer's study may seem to imply that the result of our analysis contradicts the empirical findings, and hence that our modelling strategy is inappropriate to capture the long-run relation between growth and inflation. However, even in our model, the negative relation between inflation and growth can be attributed to, for example, differences in production technology and in the development of financial institutions. Two countries with the same magnitude of policy variables may attain different rates of inflation and growth if these countries have different levels of technology (i.e. different magnitudes of A) and/or financial institutions at different stages of development (i.e. different values of γ, defined above). A country that has a higher A and/or a lower γ can attain a higher growth rate with a lower rate of inflation. Thus, the negative relation between growth and inflation found in cross-country regressions may simply reveal technological and institutional differences among the countries.

At this stage of development in theoretical and empirical studies, it is fair to say that the question as to which endogenous monetary growth model can successfully explain reality has not yet been settled.

APPENDIX

(a) Property 1

Suppose that $(A-n)(A-\rho)-\alpha\rho n>0$. Then, in an exogenously growing steady state, c^* is a monotonically decreasing function of μ and the value of c^* can be either greater than or equal to $A-n$ depending on the value of μ.

Proof. In an exogenously growing steady state, (28) must hold:

$$\text{(A1)} \qquad \beta(n+c)+(1-\beta)A=\rho+\frac{\alpha\rho n}{c}+\frac{\alpha\rho n(1/\alpha-1)}{\mu+\beta c+(1-\beta)(A-n)}.$$

Figure 3 depicts both sides of the above equation. The left-hand side is a straight line which has a positive slope, whereas the right-hand side is negatively sloped. As μ increases, the right-hand side of (A1) shifts downward, implying a decline in c.

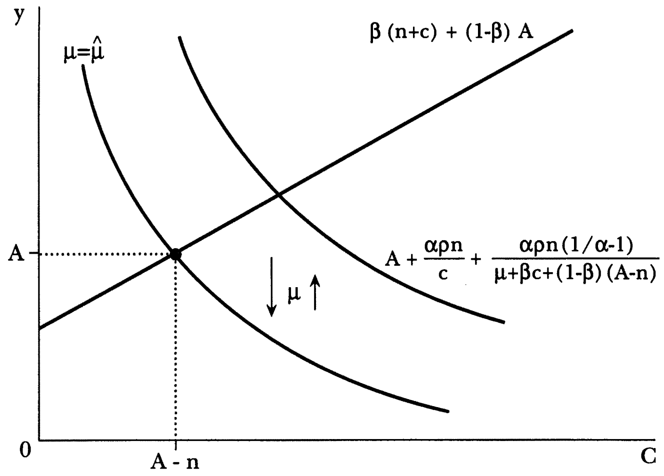

FIGURE 3

Evaluating the left-hand side at $c=A-n$ yields A (>0). At the same time, the right-hand side gives

$$\text{(A2)} \qquad \rho+\frac{\alpha\rho n}{A-n}+\frac{\alpha n\rho(1/\alpha-1)}{\mu+A-n}.$$

Under the assumption

$$[(A-n)(A-\rho)-\alpha\rho n]>0,$$

there exists a value of μ which equates (A2) and A. We call that value $\hat{\mu}$; that is, $\hat{\mu}$ satisfies[10]

$$\text{(A3)} \qquad \hat{\mu}+A-n=\frac{\alpha n\rho(A-n)}{(A-\rho)(A-n)-\alpha n\rho}(1/\alpha-1)\ (>0).$$

Thus, the range in which the level of consumption in the exogenously growing steady state can vary contains the point $A-n$. Since c is a monotonically decreasing function of μ, for all $\mu\leq\hat{\mu}$, $c+n$ is greater than A and there exists an exogenously growing steady state, while for all $\mu<\hat{\mu}$ there is no meaningful exogenously growing steady state since c^* cannot be smaller than $A-n$. □

(b) Property 2

In an endogenously growing steady state, c^{**} is a monotonically decreasing function of μ, and c^{**} can be smaller than $A-n$, depending on the value of μ, if $[(A-n)(A-\rho)-\alpha\rho n]/\alpha\rho n>0$ and $n+\rho>A$.

Proof. The $\dot{c}=0$ locus is given by

$$\text{(A4)} \qquad m=[c^2+(n-\rho)c]/\alpha\rho n-1.$$

The $\dot{m}=0$ locus is positively sloped. These loci are depicted in Figure 2. From Figure 2, we know that $m<1/\alpha-1$. The $\dot{m}=0$ locus shifts upward (downward) as μ decreases (increases). Thus, c^{**} increases as μ decreases. Evaluating the right-hand side of (A4) at $c=A-n$, we get

$$\text{(A5)} \qquad [(A-n)(A-\rho)-\alpha\rho n]/\alpha\rho n.$$

If (A5) takes a positive value, the lower bound of c^{**}, $\hat{c}$, is smaller than $A-n$. Setting $\mu=0$ in (30) gives $m^{**}=1/\alpha-1$. Substituting this into (A4), we obtain $c^{**}=\rho$. Thus, if $A-n<\rho$, c^{**} can be either greater or smaller than $A-n$ depending on μ. □

(c) Property 3

If the money growth rate is equal to $\hat{\mu}$, the steady-state values of consumption in both steady states take the same value.

Proof. At $\mu=\hat{\mu}$, A is equal to $c+n$. Thus from (20), (23) and (24) it follows directly that the values of c and m are the same in the two steady states. □

(d) Proof of Proposition 3

Let us linearize the system consisting of (20), (23) and (24) around the exogenously growing steady state. The coefficient matrix of the linearized system is as follows

$$J_2=\begin{bmatrix} -(1-\beta)(y^*-A) & (1-\beta)(y^*-A) & 0 \\ -(1-\beta)c^* & 2c^*+(n-\rho)-(1-\beta)(y^*-A) & -\alpha n\rho \\ -(1-\beta)m^* & m^*-(1/\alpha-1) & \mu+c^*-(1-\beta)(y^*-A) \end{bmatrix}$$

The necessary and sufficient condition under which the exogenously growing steady state satisfies local stability is that J_2 has one negative and two positive (possibly complex) characteristic roots. Making use of steady-state conditions, and keeping in mind that $y^*>A$, $y^*=c^*+n$ and $(1-\beta)(y^*-A)=y-r^*$, where $r^*=\beta y^*+(1-\beta)A$, we find, from (26), that

(A6) $\quad r^*-\rho=\alpha n\rho(1+m^*)/c^*>0.$

In addition, (27) yields

(A7) $\quad \mu+r^*-n=(1/\alpha-1)c^*/m^*>0.$

Therefore, we can show that

(A8)
$$\begin{aligned}\text{trace } J_2&=-3(1-\beta)(y^*-A)+3c^*+n-\rho+\mu\\ &=(\mu+r^*-n)+(r^*-n)+(r^*-\rho)>0,\end{aligned}$$

because we have assumed that $r>n$ in the steady state.

On the other hand, the determinant of J_2 is given by

(A9)
$$\begin{aligned}\det J_2=(1-\beta)(y^*-A)\{&[\mu+c^*-(1-\beta)(y^*-A)][-2c^*-n+\rho\\ &+(1-\beta)(y^*-A)+(1-\beta)c^*]+\alpha n\rho(1/\alpha-1-\beta m^*)\}.\end{aligned}$$

Since $-2c^*-n+\rho+(1-\beta)(y^*-A)+(1-\beta)c^*=-\beta c^*-(r^*-\rho)$ and $1/\alpha-1-\beta m^*=(1/\alpha-1)(\mu+r^*-n-\beta c^*)/(\mu-n+r^*)$, det J_2 may be rewritten as

$$\det J_2=(1-\beta)(y^*-A)\left[-(\mu-n+r^*)(\beta c^*+r^*-\rho)+\rho n(1-\alpha)-\frac{\rho n(1-\alpha)\beta c^*}{\mu-n+r^*}\right].$$

In view of (A7),

$$-(\beta c^*+r^*-\rho)(\mu-n+r^*)+\rho n(1-\alpha)=[-c^*(\beta c^*+r^*-\rho)+\alpha\rho nm^*]\frac{1-\alpha}{\alpha m^*}.$$

Furthermore, (A6) yields $-c^*(\beta c^*+r^*-\rho)+\alpha n\rho m^*=-\beta c^{*2}-\alpha\rho n<0$. Accordingly, as long as the steady-state values of c, m and y are strictly positive, det J_2 has a negative value. The fact that det $J_2<0$ and trace $J_2>0$ implies that J_2 has one negative and two positive characteristic roots, and hence that there exists a locally stable, unique path converging to the exogenously growing steady state. □

(e) Proof of Proposition 4

When an endogenously growing steady state is realized, the linearized system around the steady state has a coefficient matrix such that

$$J_1=\begin{bmatrix}(1-\beta)(n+c^{**}-A) & 0 & 0\\ -(1-\beta)c^{**} & 2c^{**}+n-\rho & -\alpha n\rho\\ -(1-\beta)m^{**} & m^{**}-(1/\alpha-1) & \mu+c^{**}\end{bmatrix}.$$

One of the characteristic roots of matrix J_1 is $(1-\beta)(n+c^{**}-A)$. The other two roots are the solutions of the following equation:

$$\text{(A10)}\quad \lambda^2-(3c^{**}+\mu+n-\rho)\lambda+(2c^{**}+n-\rho)(\mu+c^{**})+\alpha n\rho(m^{**}+1-1/\alpha)=0.$$

From Figure 2, we know that there is a unique set of steady-growth levels of c^{**} and m^{**} at the point where the $\dot{c}=0$ locus cuts the $\dot{m}=0$ locus from below. This implies the following inequality:

$$\left.\frac{\mathrm{d}m}{\mathrm{d}c^{**}}\right|_{\dot{c}=0}=\frac{2c^{**}+n-\rho}{\alpha\rho n}>\left.\frac{\mathrm{d}m}{\mathrm{d}c^{**}}\right|_{\dot{m}=0}=-\frac{m^{**}+1-1/\alpha}{\mu+c^{**}}.$$

Thus, the last term of (A10) is positive. Furthermore,

$$3c^{**}+\mu+n-\rho=(c^{**}+\mu)+(c^{**}+n-\rho)+c^{**}>0,^{11}$$

and hence we can conclude that (A10) has two unstable roots. Since the system (20), (23) and (24) includes one backward-looking variable, y, and two forward-looking variables, m and c, there should be a one-dimensional stable manifold to establish uniqueness and stability of the perfect-foresight equilibrium. Consequently, the endogenously growing steady state is stable if $c^{**}<A-n$, and it is totally unstable if $c^{**}>A-n$. □

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NOTES

1. If the elasticity of intertemporal substitution is sufficiently small, we can obtain the opposite result (see Mino 1991).
2. Using similar models, van der Ploeg and Alogoskoufis (1993) and Mino and Shibata (1993) analyse the effects of fiscal and monetary policies on the growth rate of the economy. However, the stress of these papers is rather different from ours.
3. Futagami and Shibata (1993) analyse the effect of monetary policy on the growth rate by employing the Weil model. However, money is introduced into the model in a different way; instead of the MIUF approach, they regard money as 'bubbles', which yield no utility.
4. The Weil model is particularly useful for the long-run analysis of monetary and fiscal policies, because in this framework monetary and fiscal disturbances are usually non-neutral even at the steady-state equilibrium, unless the growth rate of population is zero. Hence a number of studies employ the Weil model. For example, Alogoskoufis and van der Ploeg (1991) analyse the international spill-over effects of fiscal policies in an endogenously growing world economy, and Obstfeld (1989) investigates the effects of fiscal policies in open economies. Alhanasios and Solow (1990) develop a monetary growth model by using Blanchard's (1985) finite-horizons model which has properties similar to those in the Weil model. It is now well recognized that the main source of policy non-neutrality obtained in the overlapping-generations models (without bequest motives) is the heterogeneity of agents resulting from uneven distribution of

wealth among generations. Finiteness of time horizon or 'probability death' of agents plays a negligible role (see e.g. Buiter 1988).
5. We can easily extend this utility function to the class of CES utility.
6. This is the standard assumption in 'AK'-type models; see e.g. Barro (1990); Rebelo (1991).
7. Letting $dN(s)$ be the number of members within the cohort born at period s, the total population at t is given by $N(t)=\int_0^t dN(s)$. Assuming $N(0)=1$, we obtain $dN(t)=n\,e^{nt}\,dt$, where n is the rate of population growth.
8. Equation (9) is from (6), (10) from (7), (11) from (4), (10) and $a(t, t)=0$, and (12) from (8).
9. In a stochastic model, Abel *et al.* (1989) develop a criterion for evaluating the dynamic efficiency. Applying this criterion to OECD countries, they conclude that these economies are dynamically efficient.
10. From (2), (27) and $(1-\beta)(y^{**}-A)=y-r^{**}$, $\mu+r^{**}-n=(1/\alpha-1)(c^{**}/m^{**})>0$. Since $r^{**}=A$ at $\mu=\hat{\mu}$, $\hat{\mu}+A-n$ must be positive.
11. Note that, from (29) and (30), $c^*+n-\rho=\alpha n\rho(1+m^*)/c^*>0$, $c^*+\mu=(1/\alpha-1)(c^*/m^*)>0$.

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