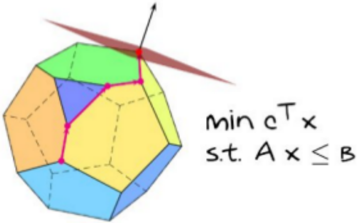
The simplex method

- Finding an initial vertex
- Wrapping it up:
  Solving a linear program with the simplex algorithm



Finding an initial vertex: Phase 1

LP1 feasible $\Leftrightarrow$ OPT VALUE LP2 = $\boxed{0}$

LP1

$$\max \; c^T \cdot x$$
$$Ax \le b$$
$$x \ge 0$$

(x^*, y^*) opt. vertex of LP2, then x^* feasible basic sol. of LP1.

$$Ax \le b \;\rightarrow\; A_1 x \le b_1 \qquad b_1 \ge 0,\; b_1 \in \mathbb{R}^{m_1}$$
$$\rightarrow\; A_2 x \le b_2 \qquad b_2 < 0,\; b_2 \in \mathbb{R}^{m_2}$$

LP2.

$$\min \sum_{i=1}^{m_2} y_i$$
$$A_1 x \le b_1$$
$$A_2 x \le b_2 + y$$
$$x, y \ge 0$$
$$y \le |b_2|$$

Quiz:

which (x^*, y^*) is feas. vertex of auxiliary LP2?

~~O~~ $x^* = 0$, $y^* = |b_2|$

O $x^* = 0$, $y^* = b_2$

(x^*, y^*) feasible

$x \ge 0$, $y \le |b_2|$ are active

$\Rightarrow$ feasible initial vertex.

Solving a linear program with simplex

- Given $\max\{c^T x \colon x \in \mathbb{R}^n,\, Ax \leqslant b\}$
- Re-write $\max\{c^T(y-z) \colon y, z \in \mathbb{R}^n, A(y-z) \leqslant b,\ y, z \geqslant 0\}$
- Find initial vertex or assert that LP is infeasible (Phase 1)
- Solve LP with simplex method using initial vertex (Phase 2)
  Outcome: Optimal solution or assertion that LP is unbounded