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Final Exam – Economics 713

1. (15 points)
Let $\mathcal{P} = \{\mathcal{A}, \{\Theta_i\}_{i\in\mathcal{A}}, \mu, \mathcal{X}, \{v_i\}_{i\in\mathcal{A}}\}$ with $\mu \in \Delta\Theta$ and $v_i \colon \mathcal{X} \times \Theta \to \mathbb{R}$ be a mechanism design problem. Let g be a social choice function for $\mathcal{P}$.
   (i) What are the domain and range of g? (In other words, what does g take as inputs, and what does it return as outputs?)
   (ii) Explain in words what it means for g to be incentive compatible. Be precise.

2. (15 points)
Consider the following allocation problem: A government would like to efficiently allocate K identical units of an indivisible good to the agents in $\mathcal{A} = \{1, \ldots, n\}$. Agent i has quasilinear preferences, and her type is $\theta_i = (\theta_i^1, \ldots, \theta_i^K)$, $\theta_i^1 > \theta_i^2 > \cdots > \theta_i^K > 0$, where θ_i^k is her marginal benefit from obtaining her kth unit. Thus if agent i obtains $\ell \in \{0, \ldots, K\}$ units and pays a transfer of t, her utility is

$$\sum_{k=1}^{\ell} \theta_i^k - t.$$

The government plans to sell the goods using a *Vickrey auction*, which is the Vickrey-Clarke-Groves mechanism applied in this setting.

Describe this mechanism's allocation and transfer function. In particular, for any profile of type announcements, specify how many units agent i will receive, and his total payment for these units. For convenience, you may focus on profiles $(\hat{\theta}_1, \ldots, \hat{\theta}_n)$ of type announcements in which all of the $\hat{\theta}_i^k$ are distinct. (Hints: Answering this question should require very little calculation. It is easiest to describe the mechanism in words. When thinking about transfer payments, interpret each $\hat{\theta}_i^k$ as a bid.)

3. (20 points)
A manager would like to hire a worker to perform a task. (There is no choice of effort level: the worker either performs the task, or does not.) Performance of the task generates a random amount of money as output. The set of possible outputs is $\{x_1, \ldots, x_m\}$, where $0 < x_1 < \cdots < x_m$, and output x_j occurs with probability p_j. The manager chooses a wage schedule $w_1, \ldots, w_m$, where w_j is the wage paid to the agent when output x_j is realized. (Both x_j and w_j are denominated in dollars.)

The manager's von Neumann-Morgenstern utility for earning y dollars is $v(y)$. The worker's von Neumann-Morgenstern utility for performing the task and earning wage w is $u(w) - c$ for some constant c, and her utility for not performing the task is 0. Both v and u are differentiable, increasing, and strictly concave.

Show that under the manager's optimal wage schedule, wages are increasing in output. (You may assume that u and v are such that the optimal choice of w_1 is positive.)

4. (25 points)

Consider the following two-type adverse selection problem. A principal specifies a menu of quality-price pairs $(q, p) \in \mathbb{R}_+ \times \mathbb{R}$. The agent is privately informed about her marginal return to quality; her type set is $\Theta = \{\theta_l, \theta_h\} = \{20, 50\}$, and the probability that she is of type θ_h is $\pi_h \in (0, 1)$. The utility of an agent of type θ from buying a good of quality q at price p is $u(q, p, \theta) = \theta q - p$, and her utility from not buying a good is 0. The principal's utility for selling a good of quality q at price p is $U(p, q) = p - q^2$.

(i) What it the principal's optimal menu of contracts when $\pi_h = \frac{1}{3}$?

(ii) What is the principal's optimal menu of contracts when $\pi_h = \frac{1}{2}$?

(iii) Provide intuition for any differences between your answers to parts (i) and (ii).

5. (25 points)

Consider the insurance market screening model with a consumer and two competing firms from lecture:

The consumer's initial wealth is w. She may incur a loss of cost ℓ. The consumer's type $\theta \in \Theta \in \{\theta_L, \theta_H\}$, $0 < \theta_L < \theta_H$, is her probability of a loss. She is type θ_L with probability $\alpha_L \in (0, 1)$. An insurance policy is a pair (b, p), representing the benefit in the event of a loss and the price of the policy.

The game proceeds as follows: [0] The consumer learns his type. [1] Each firm i offers a pair of policies, $((b_i^L, p_i^L), (b_i^H, p_i^H))$. [2] The consumer chooses among the available policies, or chooses not to be insured.

Firms maximize expected profit. The utility of a type θ_P consumer from choosing policy (b, p) is $u_P(b, p) = (1 - \theta_P)\, v(w - p) + \theta_P\, v(w - p - \ell + b)$, where v is differentiable, increasing, and strictly concave.

(i) State and prove the single crossing property that holds in this game, and explain what it tells us about the two types' indifference curves.

Suppose we have already argued that in any separating equilibrium, the contract intended for each type is actuarially fair for that type, and that type θ_H's contract provides full insurance.

(ii) Draw a diagram in the space of contracts that represents these facts, and explain how each fact is represented.

(iii) Continuing the argument, reach whatever conclusions are possible about the form a separating equilibrium must take.