Optimization of The Shortest-Path Routing with Equal-Cost Multi-Path Load Balancing*

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ABSTRACT
In this paper we address the problem of routing optimization in IP networks. We assume that traffic is routed along the shortest paths computed with respect to administrative link metrics. Metrics are distributed in a network by open shortest path first (OSPF) or a similar routing protocol. If it happens that the shortest path is not unique then equal-cost multi-path (ECMP) load balancing principle is applied. It means that the demand traffic destined to specific node is split among all the shortest paths to that node. The problem considered here is to determine the shortest-path routing pattern satisfying traffic demands, and to find appropriate link metrics while link capacities are not exceeded. Besides that many traffic engineering criterias can be used as objective function of the problem, we assume that the residual capacity volume is maximized. In this paper we formulate the problem as a mixed integer programme (MIP) and propose some combinatorial separation cuts for the problem and give an effective method for deriving such cuts.
Keywords: MIP programming, Branch-and-Cut, Cutting Plane, OSPF, ECMP

1. MOTIVATION

In this paper, we discuss an optimization problem related to the shortest-path routing protocols which have gained major popularity due to deployment of the Open Shortest-Path First (OSPF) [7]) and the Intermediate System to Intermediate System (IS-IS) [5])—the most common Internet intra-domain routing protocols. Both, OSPF and IS-IS protocols, use some versions of Dijkstra's shortest path routing algorithm for the local computations of the shortest path. The nodes compute the paths using the known metric for each link in the network. A common practice is to use the fixed value or the inverse of the link rate as the link metrics. Link-state advertisements are flooded periodically, and also immediately after a link failure occurs (failed links receive temporarily infinite weights). Non-zero traffic between end nodes of a demand is routed on the shortest paths with respect to the given link metric system. If there are multiple shortest paths to a particular destination, the routing table contains the next hop for each of the shortest paths to this destination, and the corresponding traffic is equally split over the outgoing links belonging to these shortest paths; this is called equal-cost multi-path (ECMP) split rule and is used in OSPF networks whenever there are multiple shortest paths for a pair of origin-destination nodes. It is important to note that neither OSPF nor IS-IS says what actual link metric values are to be used in the link metric system to fulfill the network performance objectives–this is where the design problems come into picture in the shortest path routing networks. This problem has received considerable attention in the recent years (for example, see [1], [2], [3], [4], [6], [8], [10]).

2. SHORTEST-PATH ROUTING OPTIMIZATION PROBLEM

The OSPF routing optimization problem considered here is as follows. Given link capacities and a set of traffic demands (OD-pairs) with demand volumes, find a system of link metrics $\boldsymbol{w}^*$ which induces a feasible shortest-path flows in accordance with the ECMP splitting principle while the link capacities are not exceeded. As shown in Section 7.10 of [8] this problem is NP-complete. Hence, following [9], we formulate the problem as a MIP programme. To formally state the problem we use the aggregated node-link formulation of the multi-commodity problem notation. In the below formulation, the demand volume to be allocated from node v to node t is given by constant h_{vt}. The starting and the terminating nodes of link e are denoted by $i(e)$ and $j(e)$, respectively. The link capacity is given by constant c_e. We introduce binary routing variables u_{et} which value 1 means that link e is

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chosen to route traffic destined to node t. Variable x_{et} is a continuous counterpart of u_{et} measuring the total flow on link e to t. Positive value of x_{et} induces that u_{et} must be equal to 1. Link metrics are represented by variables w_e. The OSPF protocol requires the integral values of link metrics (in range from 1 to 65536), but since any continuous values can be scaled to integral ones, in this formulation we neglect the integrality condition of $\mathbf{w}$. A distance from node v to node t is represented by variable r_{vt}. An objective function of this problem is to minimize residual capacity given be value of variable z. To model the ECMP split done in node v of traffic destined to node t we introduce variable y_{vt} which is equal to a portion of traffic leaving node v on each active outgoing link.

ECMP Shortest Path Flow Allocation Problem
objective function

$$\text{maximize } z \tag{1a}$$

constraints

$$\textstyle\sum_{\{e:j(e)=t\}} x_{et} = \sum_{s\neq t} h_{st} \qquad t = 1,2,\ldots,V \tag{1b}$$
$$\textstyle\sum_{\{e:i(e)=v\}} x_{et} - \sum_{\{e:j(e)=v\}} x_{et} = h_{vt} \qquad t = 1,2,\ldots,V \quad v = 1,2,\ldots,V \quad v \neq t \tag{1c}$$
$$\textstyle\sum_t x_{et} + z \leq c_e \qquad e = 1,2,\ldots,E \tag{1d}$$
$$0 \leq y_{i(e)t} - x_{et} \leq (1-u_{et})\textstyle\sum_v h_{vt} \qquad t = 1,2,\ldots,V \quad e = 1,2,\ldots,E \tag{1e}$$
$$x_{et} \leq u_{et}\textstyle\sum_v h_{vt} \qquad t = 1,2,\ldots,V \quad e = 1,2,\ldots,E \tag{1f}$$
$$r_{j(e)t} + w_e - r_{i(e)t} \leq (1-u_{et})M \qquad t = 1,2,\ldots,V \quad e = 1,2,\ldots,E \tag{1g}$$
$$1 - u_{et} \leq r_{j(e)t} + w_e - r_{i(e)t} \qquad t = 1,2,\ldots,V \quad e = 1,2,\ldots,E \tag{1h}$$
$$w_e \geq 1 \qquad e = 1,2,\ldots,E \tag{1i}$$
$$\mathbf{u} \in (\mathcal{B})^{E\times V} \tag{1j}$$
$$\mathbf{x} \in (\mathcal{C}_+)^{E\times V} \tag{1k}$$
$$\mathbf{y} \in (\mathcal{C}_+)^{V^2} \tag{1l}$$
$$\mathbf{r} \in (\mathcal{C}_+)^{V^2} \tag{1m}$$

Constraint (1b) assures that the total flow incoming and, at the same time, destined to node t is equal to the demand destined to node t generated in all remaining nodes $s \neq t$. Constraint (1c) forces that demand volume h_{vt} from node v to node t is actually generated in node v. Constraint (1d) is the link capacity constraint. Constraint (1e) assures that if link e belongs to one of the shortest paths from node $i(e)$ to node t then its flow to node t is equal to $y_{i(e)t}$ — a value common to all links outgoing from node $i(e)$ and belonging to the shortest paths to destination t. Constraint (1f) forces the zero flow to t ($x_{et} = 0$) in the case when link e is not on the shortest path to t. Finally, constraint (1g) assures that if $u_{et} = 1$, then link e is on the shortest path to t; constraint (1h) assumes that if $u_{et} = 0$, then link e is not on the shortest path to t. In this case, the link weights need to be ≥ 1, constraint (1i). Note that the unbounded variable z makes the problem always feasible. However, a true feasible solution to problem (1) is obtained when the resulting minimum t is less or equal to 1.

Problem of the shortest-path routing optimization can be decomposed in two subproblems solved alternately. A phase-one subproblem solves an allocation problem consistent with ECMP splitting principle (constraints (1a)-(1f) and (1j)-(1l)). When the optimal solution of the allocation problem is obtained, a weight setting problem is solved. In this problem we want to find link metrics that induce an allocation scheme obtained after solving the phase-one subproblem. If it happens that the phase-two subproblem is infeasible, then we must return to phase-one and exclude the entire phase-one optimal solution previously found by adding new constraint(s) to the problem formulation. When routing variables causing infeasibility can be identified, e.g., two contradictory shortest paths for different root nodes, we can use small subset of appropriate routing variables instead of whole set to formulate an excluding inequality. It helps in excluding at once many phase-one solutions that are infeasible for the same reason. After adding the excluding condition to the problem formulation, new allocation scheme is searched. The procedure ends when a feasible solution of phase-two is obtained. This decomposition is in the field of our current and future interests.

When a load balancing capability is disabled, a single-path counterpart of problem (1) comes on stage. In this case the ECMP split constraints are neglected in the problem formulation. Because of presence of these constraints, to the best of our knowledge, a compact link-path formulation of problem (1) has not been yet proposed. However, its single-path variant can be formulated this way. The link-path formulation, thanks to a path generation technique, makes large-size network optimization problem more scalable. Referring to the two-phase method, the first subproblem becomes a single-path allocation problem. The single-path allocation problem is commonly known and has been studied by many researchers. So, you can expect that this problem can be solved very efficiently.

Our computational investigates revealed that the branch-and-bound method applied to solve problem (1) is inefficient. This is way we want to embed a generation of cutting inequalities in the branch-and-bound method. This approach is known as branch-and-cut. It assumes that valid inequalities are introduced to the problem formulation in order to make it tighter. Thanks to potentially more often cut-offs, this method increases the efficiency of pure branch-and-bound. In this scope issue of efficient cutting inequalities generation is very important. In the following, we present so-called combinatorial separation cuts. Nevertheless, in our research work we study also other types of separation problems. As far as now, they are not so efficient as combinatorial ones. Another issue that is important in case of branch-and-bound method is the procedure of building the searching tree. It appears, that the branching procedure used in standard MIP solvers may not be the best one and needs to be replaced by branching rule making use of the problem structure.

3. COMBINATORIAL SEPARATION CUTS

3.1 Transit Cuts

Let us assume the existence of two paths: $\mathcal{P}$ from v to s and $\mathcal{Q}$ from v to t. $\mathcal{Q}$ contains the terminating node s of path $\mathcal{P}$. Now, if path $\mathcal{Q}$ is a shortest path then of course $\mathcal{P}$ should also be a shortest path. This situation is illustrated with the following figure. On the basis of this observation we can state so-called *transit cut* given by

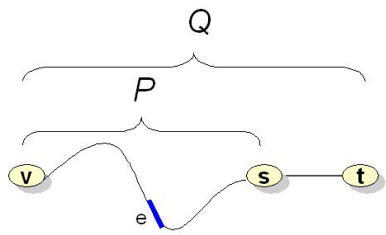

Figure 1: Transit cut

inequality (2). This condition holds for each u_{fs} corresponding to each link f in path $\mathcal{P}$.

$$|\mathcal{Q}| - \sum_{\{e:e\in\mathcal{Q}\}} u_{et} + u_{fs} \geq 1 \qquad f \in \mathcal{P} \tag{2}$$

Note that the above inequality is also valid if we use $\mathcal{P}$ instead of $\mathcal{Q}$.

3.2 Cycle Cuts

Since we consider the shortest-path routing any two oppositely directed paths not containing a root node and forming a cycle are obviously forbidden. Such routing scheme is presented in figure 2. The presented routing

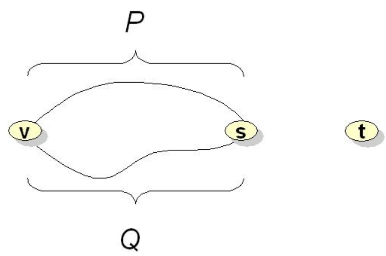

Figure 2: Cycle cut

pattern is excluded from the feasible region of the optimization problem by constraints (1g-h). However, since the integral solution is not found (in relaxed problem) this case is likely to appear. This is why we propose another type of cutting plane called *cycle cut*. Suppose, we have two paths $\mathcal{P}$ and $\mathcal{Q}$ forming a cycle, then inequality (3) helps avoiding situation when traffic destined to node t is routed on both paths simultaneously.

$$\sum_{\{e:e\in\mathcal{P}\}} u_{et} + \sum_{\{e:e\in\mathcal{Q}\}} u_{et} \leq |\mathcal{P}| + |\mathcal{Q}| - 1 \tag{3}$$

3.3 Separation Problem

Let us rewrite inequalities (2) and (3) in the following form:

$$\sum_{\{e:e\in\mathcal{P}\}}(1-u_{et}) \geq (1-u_{fs}) \qquad f\in\mathcal{P} \tag{4}$$

$$\sum_{\{e:e\in\mathcal{P}\}}(1-u_{et}) + \sum_{\{e:e\in\mathcal{Q}\}}(1-u_{et}) \geq 1 \tag{5}$$

The problem that arises here is how to efficiently for the relaxed problem solution derive violated inequalities of both types. Suppose that the solution of the relaxed problem is given by routing variables vector $\mathbf{u}^*$. To solve the separation problem for specific root node t we assign weights $(1-u^*_{et})$ to each edge in the network graph. Next, we compute the tree of shortest paths from every other node to t. Then, for shortest path $\mathcal{P}$ we check its edges in order to find violated inequality of the first type. The same procedure can be applied to every sub-path of $\mathcal{P}$.

To find violated cycle inequalities we run a shortest path algorithm, i.e., Floyd algorithm, with weights $(1-u^*_{et})$, where t is given, to obtain shortest path in every node relation. Now, for each node pair $\{st\}$ we check whether sum of the shortest paths length is smaller than 1. If it is so, then these paths can be used to formulate a cycle cut.

4. FURTHER WORK

In our future studies we want to investigate the issue of link-path formulation of the OSPF/ECMP routing optimization problem. Formulation based on multiple sub-paths can be considered. It assumes that an end-to-end routing path consists of a few sub-paths that are not subject of splitting. Another aspect we want to deal with is searching of new types of cutting inequalities valid for problem (1). Also a problem of routing optimization in failure robust IP networks still remains an open issue.

REFERENCES

[1] W. Ben Ameur and E. Gourdin. Internet routing and releated topology issues. *SIAM J. Discrete Math.*, 17(1):18–49, 2002.

[2] W. Ben-Ameur, E. Gourdin, and B. Liau. Internet routing and topology problems. In *Proc. DRCN 2000*, 2000.

[3] A. Bley. A lagrangean approach for integreted network design and routing in ip networks. In *Proceedings INOC'2003, Evry-Paris*, pages 107–113, 2003.

[4] N. Bourquia, W. Ben Ameur, E. Gourdin, and P. Tolla. Optimal shortest path routing for internet networks. In *Proceedings INOC'2003, Evry-Paris*, pages 119–125, 2003.

[5] R. Callon. Use of OSI IS-IS for routing in TCP/IP and dual environments. *Internt RFC1195*, December 1990. http://www.ietf.org/rfc/rfc1195.txt.

[6] K. Holmberg and D. Yuan. Optimization of Internet protocol network design and routing. *Networks*, 43(1):39–53, 2004.

[7] J. Moy. OSPF version 2. *Internt RFC2328*, 1998. http://www.ietf.org/rfc/rfc2328.txt.

[8] M. Pióro and D. Medhi. *Routing, Flow, and Capacity Design in Communication and Computer Networks*. Morgan Kaufman, 2004.

[9] A. Tomaszewski, M. Pióro, M. Dzida, and M. Zagożdżon. Optimization of administrative weights in ip networks using the branch-and-cut approach. In *The Second International Network Optimization Conference INOC 2005, Lisbon*, March 2005.

[10] M. Zagożdżon, M. Dzida, M. Pióro, and A. Tomaszewski. Optimization of administrative weights in single path networks with ospf routing. In *12th Polish Teletraffic Symposium PSRT 2005, Poznań*, September 2005.