System Identification Under Bounded Noise: Optimal Rates Beyond Least Squares

Xiong Zeng, Jing Yu, and Necmiye Ozay
University of Michigan, Ann Arbor

Introduction: For linear system identification, the ordinary least squares (**OLS**) estimator is **minimax optimal** under **Gaussian** noise. In safety-critical scenarios, **bounded** noise is very common.

Questions:

1. Is **OLS** estimator still **minimax optimal** for bounded noise?
2. **Which** estimator is **minimax optimal** for bounded noise?

Problem Setup: Given a discrete-time linear time-invariant (LTI) system:

$$x_{t+1} = Ax_t + w_t,$$

where $x_t, w_t \in \mathbb{R}^n$. Assume

1). (**bounded noise**) $||w_t||_\infty < \bar{w}$ and w_t are i.i.d. for all t;

2). (**stable**) $\rho(A) < 1$;

3). (**boundary probability**) $\forall \epsilon \in [0, \bar{w}]$, $\exists C > 0$, s. t. $\forall j \in [n]$,

$$\max\left(P\left(w_t^{(j)} < -\bar{w} + \epsilon\right), P\left(w_t^{(j)} > \bar{w} - \epsilon\right)\right) < C\epsilon.$$

Theorem 1 (**our work**). Given a single trajectory $\{x_t\}_{t\in[T]}$. $\mathcal{F}_T$ denotes the σ-algebra generated by $\{x_t\}_{t\in[T]}$ and $\hat{A}_T$ denotes a $\mathcal{F}_T$ -measurable estimator. Then, $\forall \delta \in (0,1)$ and small $\epsilon > 0$,

$$\sup_{\hat{A}_T} \inf_{A \in R^{n\times n}} P(||\hat{A}_T - A||_2 < \epsilon) \geq 1 - \delta$$

only if

$$T > \frac{1}{4\bar{w}C\epsilon}\left(1 - \frac{2\delta}{n}\right).$$

(The best estimator can achieve $\Omega(\frac{1}{\epsilon})$)

		Minimax LB	LB for OLS
Regression	Gaussian	$\Omega(1/\sqrt{T})$ (Wainwright '19)	$\Omega(1/\sqrt{T})$ (Mourtada '22)
	Bounded	$\Omega(1/T)$ (Yi & Neykov '24)	$\Omega(1/\sqrt{T})$ (Rudelson & Vershynin '08)
LTI Sys Id	Gaussian	$\Omega(1/\sqrt{T})$ (Jedra & Proutiere '19)	$\Omega(1/\sqrt{T})$ (Tu et al. '24)
	Bounded	$\Omega(1/T)$ (Thm. 1)	$\Omega(1/\sqrt{T})$ (Thm. 2)

The **ordinary least squares** (OLS) for scalar case:

$$\hat{a}_T^{OLS} = \text{argmin}_a \sum_{t=1}^{T-1} ||x_{t+1} - ax_t||^2$$

Theorem 2 (**our work**). Assume $|a| < 1$. Then, $\forall \delta \in (0,1)$ and small $\epsilon > 0$,

$$P(||\hat{a}_T^{OLS} - a||_2 < \epsilon) \geq 1 - \delta \quad \text{only if} \quad T > \Omega(\tfrac{1}{\epsilon^2}).$$

(OLS only achieves $\Omega(\frac{1}{\epsilon^2})$)

The set membership estimator (**SME**) based on $\{x_t\}_{t\in[T]}$

$$\mathcal{S}_T = \left\{A \in \mathbb{R}^{n\times n}: \left||x_{t+1} - Ax_t|\right|_\infty \leq \bar{w}, \forall t \in [T-1]\right\}.$$

Theorem 3 ([2]). $\forall \delta \in (0,1)$ and small $\epsilon > 0$, with or without knowing $\bar{w}$,

$$\forall \hat{A}_T \in \mathcal{S}_T,\ \ P(||\hat{A}_T - A||_2 < \epsilon) \geq 1 - \delta \quad \text{if} \quad T > \Omega(\tfrac{1}{\epsilon}).$$

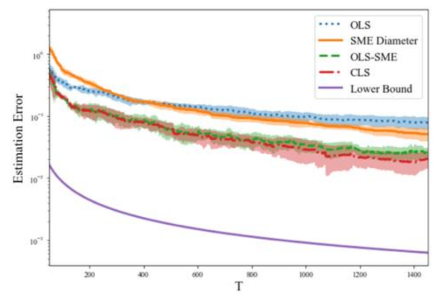
(**SME** achieves the optimal $\Omega(\frac{1}{\epsilon})$)

Algorithm Comparison:

OLS: $\min_A \sum_{t=1}^{T-1} ||x_{t+1} - Ax_t||^2$

OLS-SME: $\min_{A\in\mathcal{S}_T} ||A - \hat{A}_T^{OLS}||$

CLS: $\min_{A\in\mathcal{S}_T} \sum_{t=1}^{T-1} ||x_{t+1} - Ax_t||^2$



OLS achieves $O(\frac{1}{\sqrt{T}})$ ☹

SME, OLS-SME, and CLS achieves $O(\frac{1}{T})$ ☺

References:

[1] Zeng, X., Yu, J., & Ozay, N. (2025). System Identification Under Bounded Noise: Optimal Rates Beyond Least Squares. L-CSS and CDC 2025.

[2]. Y. Li, J. Yu, L. Conger, T. Kargin, and A. Wierman, "Learning the uncertainty sets of linear control systems via set membership: A non-asymptotic analysis," ICML 2024.